

**TIME SERIES ANALYSIS OF VOLATILITY IN FINANCIAL MARKETS  
IN HONG KONG FROM 1991 TO 2004**

by

JIDONG ZHANG

(Under the Direction of Tharuvai N. Sriram)

**ABSTRACT**

Bond market and stock market are the two most important financial markets. Study on the volatility of these two markets has always received considerable great attention because volatility is a major risk factor for investors and portfolio managers who regularly make asset-allocation decisions between the two markets. An appropriate statistical analysis of historical and present volatility relationship between these two markets is essential in order to obtain supportive information to make this decision.

Global investment is one of the most common methods for diversification. Historical data indicates that, on the average, overseas market outperforms the United States financial markets in terms of rate of returns. Last decade, Hong Kong has become an important international financial center in Asia. It is believed that understanding the volatility in stock and bond market in Hong Kong can shed light on returns and risks to make correct investment diversification decision in US markets. This thesis focuses on the volatility of stock and bond market in Hong Kong from 1991 to 2004. We build time series models to analyze the stock returns volatility, bond returns volatility and the ratio of the two in order to understand the volatility in these two markets.

**INDEX WORDS:** Volatility, Hong Kong, stock market, bond market, exchange fund, Hang Seng Index (HSI), GARCH model

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## **DEDICATION**

To my wife, Huina Guo, and thanks a lot for her to give me so much support for this thesis; I will remember this during my whole life.

To my parents, aiyun zhang and jingchuan zhang, thank both of them so much to give me the life, to educate me and cultivate me.

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## **CHAPTER 1**

### **INTRODUCTION**

The purpose of this study is to examine the bond market volatility (variability), stock market volatility (variability), and their relationship in the Hong Kong financial markets. In this study, we want to investigate the following two questions: (a) how do we model the volatility of the Hong Kong stock market, the volatility of the bond market, and the volatility ratio of the two market indices? And (b) does the bond market in Hong Kong exhibit a similar pattern to its stock market, as is the case in some other countries such as the United States (U.S.) or the United Kingdom?

Research on volatility in financial markets originated in the U.S. in 1970 and primarily focused on the U.S. stock market. During 1950-1979, the volatility of the U.S. bond market was significantly smaller than that of the stock market. In fact, bond volatility was, on the average, only about one third of the volatility of the stock market (Reilly, 2000). It was widely accepted that the bond market was the most important diversification vehicle for people who invested in the stock market.

However, during the early 1980s the volatility of the U.S. bond market increased significantly (Coleman et al., 1993), which stimulated more research bond volatility. Many bond derivative instruments and bond portfolio-management techniques sprang up in the late 1980s and early 1990s. Change in the volatility of the U.S. bond market is widely believed to be a major risk factor in bond investment (Longstaff & Schwartz, 1993) and fixed-income securities with embedded options (Dunetz and Mahoney, 1988; Fabozzi et al., 1997). And

Change in the volatility also has a major impact on the bond yield spread, one of the measurements for bond risk. (Dialynas and Edington, 1992).

Reilly (2000) furthered the research by verifying that the bond and stock markets have different volatility patterns in U.S. According to his results, the annual volatility of the U.S. bond market depends on the previous year's volatility and exhibits a regular, systematic pattern over time. Additionally, Reilly (2000) showed that an Autoregressive Conditionally Heteroscedastic (ARCH) could be used to model the bond volatility series. However, he also showed that the U.S. stock volatility does not exhibit a predictable time series behavior and that the ARCH (1) model does not track the actual stock volatility very well.

While many previous studies have analyzed the volatility of stock and bond market rates of return in the U.S., there is very little study on volatility in bond/stock markets of other countries. Schwert (1998) showed that the U.K. stock market also exhibits a similar volatility pattern as the U.S. stock market, including the fact that the volatility in these markets in U.K. returned to normal levels quickly after the 1987 stock market crash in the U.S. He also made another interesting observation that the 1973-1975 OPEC crisis (first oil crisis) had a much larger effect on the volatility of UK stock market than on US stock market. Johnson and Young (2002) showed that during 1970-2000, volatility in the UK bond market was not significantly increasing relative to its stock market volatility. Furthermore, they showed that a lack of trend in the ratio of bond-stock standard deviations and in correlation between stocks and bonds indicates that U.K. bonds continue to provide an effective diversification vehicle for people invest in the U.K. financial market.

We are interested in conducting similar studies for the Hong Kong financial markets because Hong Kong is one of the world's most open and dynamic economies (The U.S. State of Department, 2004, footnote 1), and one of the largest developed markets in the world (the International Finance Corporation (IFC), 1997, footnote 2). The openness of the market, the absence of control on foreign capital flow, the high liquidity in markets, the long history of the international financial center, and being a “gateway” to china, also make the Hong Kong financial markets an ideal candidate for global diversification.

In chapter two, we give a brief introduction about the Generalized Autoregression Conditionally Heteroscedastic (GARCH) models and their applications in financial data analysis. In chapter three, we give a background on the Hong Kong economy and its financial markets; we also describe our datasets in detail. In chapter four, we carry out the data analysis and fit models for 3 series: the monthly standard deviation of the Hong Kong bond market, the monthly standard deviation of Hong Kong stock market, and the ratio of the standard deviations corresponding to the bond market and the stock market. In each of these three analyses, we begin with a theoretical introduction, followed by data analysis and model building, and end by drawing a conclusion.

We find that the Hong Kong bond market has the simplest pattern: very small and almost constant standard deviation. Then we continue to prove that the mean model is sufficient for the bond market, which means that the Hong Kong bond market monthly standard deviation data is a constant variable, without any statistically significant change over the period of 1990 to 2004). We also find that the Hong Kong bond market has very low risk.

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<sup>1</sup> <http://www.state.gov/r/pa/ei/bgn/2747.htm>

<sup>2</sup> IFC, a member of the World Bank Group, is the largest multilateral source of financing for private sector companies in developing countries. [www.ifc.org](http://www.ifc.org).

The Hong Kong stock market, however, seems to exhibit a different pattern: monthly volatility of the Hong Kong stock market depends on the previous month volatility. Here, we find that ARCH models are more suitable to predict the stock monthly standard deviation than standard autoregressive integrated moving average (ARIMA) model.

The analysis of volatility ratio between bond and stock market provides more details to help make portfolio allocation decisions. We find that the ratio series has non-constant variance pattern, which leads us to an analysis using a GARCH models that is more accurate. Note that Reilly (2000) and Jason and Young (2002) only give trend description on the volatility ratio of bond market over stock market, in the U.S. and in the U.K. respectively in their studies.

In the last chapter, we draw conclusions: (a) switching from the stock market in Hong Kong to its bond market can decrease the overall risk of portfolio; and (b) accurate fund allocation between these two markets should be based on the volatility ratio forecasting.

In a summary, because of the significant impact that bond market volatility has on yield-spreads and security-values, it is important for investors in the global market to be informed of the volatility patterns in these markets as well as the relative volatility of the bond market to the stock market. We find that the bond and stock market volatility trends exhibited in the U.S. markets are not found in the Hong Kong markets. The result of this study indicates that it is unwise to assume that the patterns observed in the U.S. markets are also present in other markets. This finding has implications on portfolio asset-allocation decisions for investors who have invested or will invest in Hong Kong bond/stock market.

Thus, volatilities in Hong Kong bond market and Hong Kong stock market influence many areas of investments and are a topic worthy of further study.



## CHAPTER 2

### Literature review

#### 2.1 ARCH/GARCH model

##### 2.1.1 Introduction

###### ENGLE'S original ARCH (p) model

Engle (1982) introduced and studied Autoregressive Conditionally Heteroscedastic time series models, popularly known as ARCH models, for modeling a time-varying volatility clustering phenomenon, frequently exhibited in financial time series data, such as rate of return for financial assets.

Prior to Engle's study, researchers mainly focused on rolling volatility or historical volatility estimators. For example, suppose  $p_t$  is the asset price at time  $t$ . For convenience we assume a continuous time process and model the instantaneous rate of returns at time  $t$  as

$$u_t = \partial p_t / \partial t$$

Then it can be shown that  $E(u_t) = 0$ . The rolling volatility or historical volatility estimator is  $h_t$  defined by

$$h_t = \frac{1}{N} \sum_{q=1}^N u_{t-q}^2$$

Because  $E(u_t) = 0$ ,  $h_t$  is population variance. There are two assumptions built into rolling volatility or historical volatility estimator: (1) weights are equal for  $j < N$ ; (2) weights are zero for  $j > N$ . For example, you can obtain unlimited observations of rate of return if you track the data back to a long time period in the past, but you have to define a time range in which you

want to calculate the variance, say,  $N=30$ . When you calculate the population variance, you assume that (1) weights are equal for  $q < 30$ ; (2) weights are zero for  $q > 30$ , in order to obtain the population variance result. One main problem for rolling volatility or historical volatility method is how to determine appropriate term of  $N$ .

Engle (1982) proposed an ARCH (p) model with the key idea that weights can be estimated. <sup>①</sup>

$$h_t = \sum_{q=1}^N \alpha_q u_{t-q}^2$$

This model has a simple intuitive interpretation as a model for volatility clustering: (1) General speaking, the assumption that all the historical volatilities have the same affect (equal weight) on current volatility does not hold in real world. Recent volatility should have a greater impact on the current volatility than the volatility long-time-ago; (2) large values of past squared returns give rise to a large current volatility values; there is symmetric pattern due to squaring operation.

ARCH model has other characteristics: (1) The distribution of the returns, conditioned on past returns, may be a specified heavy-tailed non-Gaussian distribution, such as a students-t distribution, which can generate large outliers; (2) the model parameters can be estimated by a maximum-likelihood method; (3) the model cannot solve asymmetry problems; (4) In fact, according to generally accepted notation of GARCH (p, q), the ARCH model is a special form of GARCH when  $p=0$ , so, accurately, the ARCH should be called ARCH (q) or GARCH (p=0, q); (5) ARCH model is the simplification of regime shifting model, in fact, ARCH is one-regime model.

Standard ARCH model is following.

$$y_t = x_t \beta + u_t$$

$$u_t = \sqrt{h_t} \cdot v_t$$

$$h_t = \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

$$u_t | \Psi_{t-1} = N(0, h_t)$$

Here,  $\Psi_{t-1}$  is the information upto time t-1.

In our study,  $y_t$  is (1) monthly standard deviation of return rate for Hong Kong bond and Hong Kong stock; (2) ratio of the standard deviations of Hong Kong bond and Hong Kong stock, respectively.

Engle's work inspired the academic community and the financial world to carry out further studies on the use of these simple models. Incidentally, Engle won the Noble prize in 2003 for Economics for developing methods of analyzing economic time series with time-varying volatility. <sup>②</sup>

#### Standard GARCH (p, q) model (symmetric)

A usual practical problem encountered in fitting ARCH (p) models to financial returns data is that in order to obtain a good fitting model, the order p is fairly large, e.g., often in excess of 10 or more. To overcome this, Bollerslev (1986) introduced and studied a Generalized Autoregression Conditionally Heteroscedastic (GARCH) models. <sup>③</sup>

The GARCH (p, q) process models the error of a time series regression in the following way. Suppose  $y_t = x_t \beta + u_t$

Where the error  $u_t$  is modeled as  $u_t = \sqrt{h_t} \cdot v_t$ , with

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

And  $v_t$  is Normal (0, 1).

In many instances, GARCH (p, q) model with relatively small values of p and q can provides a good model for volatility. For the purpose of stationary, the following constraints are placed on the coefficients of the GARCH model:

$$k > 0$$

$$\delta \geq 0$$

$$\alpha \geq 0$$

$$\sum_i^q \alpha_i + \sum_j^p \delta_j < 1$$

If we change the last constraint to a new constraint (given below), then the new model is called as an IGARCH model:

$$\sum_i^q \alpha_i + \sum_j^p \delta_j = 1$$

The GARCH constraints described above are sufficient conditions for stationary but not necessary. One can also modify the GARCH model by placing other constraints. <sup>④</sup>

### GARCH applications in finance

Estimates of asset return volatility are used to assess the risk of many financial products. Accurate measures and reliable forecasts of volatility are crucial for derivative pricing techniques as well as trading and hedging strategies that arise in portfolio allocation problems.

Financial time series of returns frequently exhibit characteristics that render invalid common assumptions. In particular, (1) Financial return volatility data is influenced by time-dependent information flows, which result in pronounced temporal volatility clustering

(time-varying volatilities); (2) individual instruments often have non-Gaussian distributions; (3) collections of instruments, always experience the time-varying correlations between pairs of returns and the non-Gaussian multivariate distributions.

The GARCH process is a popular stochastic process, and is fairly successful in modeling financial time series. <sup>⑤</sup> It is known that GARCH models provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample volatility forecasts. <sup>⑥</sup>

## 2.1.2 UNIVARIATE GARCH MODELS

### Standard (symmetric) model

The standard (symmetric) regression-GARCH (p, q) model with Gaussian shocks takes the following form:

$$y_t = x_t\beta + u_t$$

$$u_t = \sqrt{h_t} \cdot v_t$$

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

$$u_t | \Psi_{t-1} = N(0, h_t)$$

Here,  $\Psi_{t-1}$  is the information set on the time of t-1.

In general, the GARCH (p, q) process has (p+q+1) parameters, which must be estimated by the data. GARCH (1, 1) is the simplest form of this class with 3 parameters.

### A Simple GARCH Model with Normally Distributed Errors

A simple GARCH(p, q) model can be expressed as follows:

$$y_t = x_t\beta + u_t$$

The error  $u_t$  is modeled as  $u_t = \sqrt{h_t} \cdot v_t$ , where  $v_t$  is i.i.d., with zero mean and unit variance, and where  $h_t$  is expressed as

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

In a standard GARCH model  $v_t$  has the unit Normal density:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{v_t^2}{2}}$$

Alternative models can be specified by assuming different distributions for  $v_t$ , for example, the t distribution, Cauchy distribution, etc.

### GARCH-M Model (mean)

Another type of GARCH model is the GARCH-M model, which adds the heteroscedasticity term directly into the mean equation. In this example, consider the following specification:

$$y_t = x_t \beta + \gamma \sqrt{h_t} + u_t$$

The residual  $u_t$  is modeled as

$$u_t = \sqrt{h_t} \cdot v_t$$

where  $v_t$  is i.i.d. with zero mean and unit variance

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

In the SAS command, The AUTOREG procedure enables you to specify the GARCH-M model with the MEAN= sub-option of the GARCH= option. The MEAN= option specifies the functional form of the GARCH-M model.

The values of the MEAN= option are

LINEAR, specifies the linear function	$y_t = x_t\beta + \gamma h_t + u_t$
LOG, specifies the log function	$y_t = x_t\beta + \gamma \ln h_t + u_t$
SQRT, specifies the square-root function	$y_t = x_t\beta + \gamma \sqrt{h_t} + u_t$

### GARCH Model with t-Distributed Residuals

In SAS command, to estimate a GARCH model with t-distributed errors, you can use the AUTOREG procedure. You can specify the GARCH (p,q) process with the GARCH=(p=q) option, and specify the t distributed error structure with the DIST= option.

### GARCH Model with Generalized Error Distribution Residuals (GED)

In SAS command, you can also estimate a GARCH model with GED (generalized error distribution) residuals with the MODEL procedure. <sup>⑦</sup>

The log likelihood function for GARCH with GED residuals is expressed as

$$\mathcal{L} = T \{ \log(\nu/\lambda) - (1 + \nu^{-1}) \log(2) - \log(\Gamma(1/\nu)) \} - (1/2) \sum_{t=1}^T | (y_t - x_t\beta) / (\lambda \cdot \sqrt{h_t}) |^\nu - (1/2) \sum_{t=1}^T \log(h_t)$$

where T is the sample size,  $\Gamma(\cdot)$  is the gamma function,  $\lambda$  is a constant given by

$$\lambda = \left\{ \frac{2^{(-2/\nu)} \Gamma(1/\nu)}{\Gamma(3/\nu)} \right\}^{1/2}$$

and  $\nu$  is a positive parameter governing the thickness of the tails of the distribution. Note that for  $\nu = 2$ , constant  $\lambda = 1$ , and the GED is the standard normal distribution.

### GARCH parameter estimation

$$y_t = b_0 + x_t^T b + \varepsilon_t, \quad \varepsilon_t | \psi_{t-1} = N(0, b_t)$$

$$b_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j b_{t-j}$$

All the GARCH processes above are uniquely described by the parameter vector  $\theta$ , where  $\theta = (b_0, b^T, \omega^T)$ ,  $\omega^T = (\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_q, \beta_1, \beta_2, \dots, \beta_p, \gamma)$  and  $b^T = (b_1, \dots, b_k)$ . One

method of estimating GARCH model parameters is by finding the value which maximizes the conditional log-likelihood (objective) function:

$$lf = \frac{1}{2} \sum_{t=1}^T \left( \log(b_t) + \frac{\varepsilon_t^2}{b_t} \right)$$

Here T is the number of terms in the sequence. This can be achieved by starting with an initial approximation for  $\theta$  and then using numerical optimization to iterate to an acceptable solution. The standard errors for the parameter estimates can then be computed by using the well known result that the maximum likelihood estimate for  $\theta$  is asymptotically normal with mean  $\theta$  and covariance matrix  $\mathfrak{I}^{-1}$  where  $\mathfrak{I}$  (The Fisher Information Matrix) is given by:

$$\mathfrak{I} = E \left[ \sum_{t=1}^T \frac{\partial^2 lf}{\partial \theta \partial \theta^T} \right]$$

The difficulty of modeling a GARCH sequence depends on both p and q and also on how much volatility memory there is in the process. Higher values of the  $\beta_i$  parameters give rise to more volatility memory and are therefore harder to model accurately. Increasing the number of model parameters will also make the model more complicated simply because there are more variables to numerically optimize. This suggests the following order of difficulty ARCH (1), ARCH( 2), ARCH( 3), GARCH( 1,1), GARCH( 1, 2), GARCH( 2, 2), ..., etc.

## 2.2. Research for the U.S. financial markets

### 2.2.1 Motivation for Bond research -- US

In United States, the most obvious example of high risk in stock market is the great depression during the 1930s. This was the most volatile period in stock price volatility in



terms of daily percentage returns to market portfolio before 1980s. <sup>®</sup> But the situation has changed dramatically after the World War Two. In fact, after 1950 the volatility of bond market, on the average, has increased faster than the volatility of stock market. According to Frank [2000], in 1981 the volatility of bond market is almost the same as that of stock market. In Frank's research, we can find that if we plot the ratio of standard deviation of returns for treasury bonds to that of S&P500 stocks, between 1950-2000, there is a significant positive trend in the ratio, as shown by the least squares trend line; see the following graph. The ratio of the standard deviation of returns for bonds versus that of stocks is as high as 0.8419, compared to the ratio of 0.0435 in 1963. (Reilly Frank K 2000). This showed that it is not wise to ignore the volatility of bond market. These observations sparked research on bond return volatility.

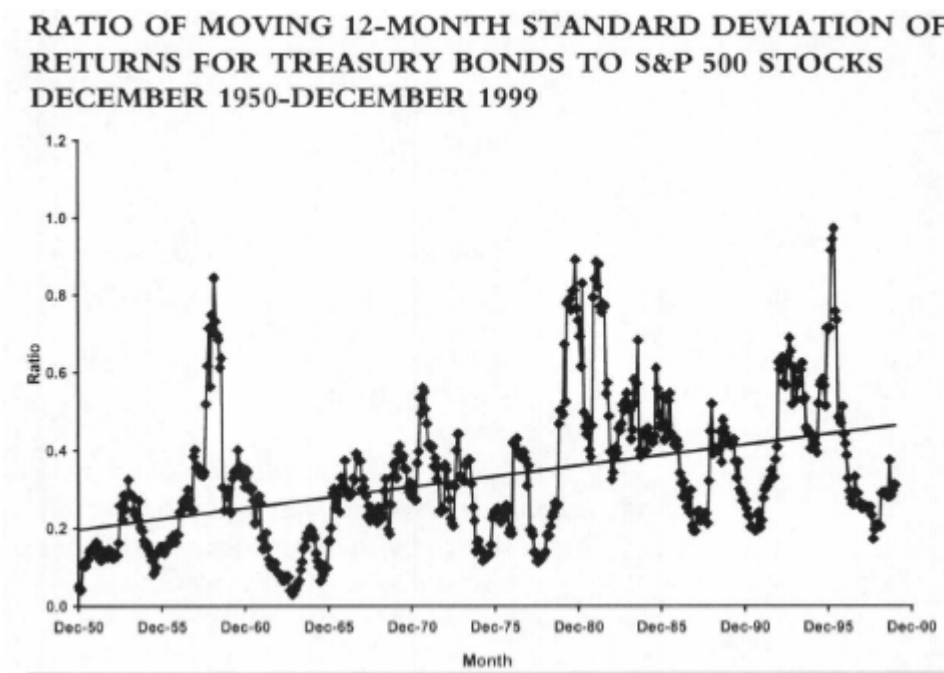


Figure 1: Bond market volatility compared to stock market volatility by Frank K Reilly

### 2.2.2 Research on stock Market Volatility – US

There are many factors that affect the stock return volatility of which the most important one is the interest rate volatility. Other factors include (1) risk premium that is expected as an extra compensation to the risk free rate, (2) the changes in the expected growth of earning and (3) cash flows for corporations. In general, the above three factors are obviously more volatile than interest rate volatility.

In 1970, Fisher and Lorie [1970] considered all the stock listed then on the NYSE and studied the changes in the variance of their returns over time. This was the first study of stock market variance over time. The range of their study is from 1926 to 1965.

Officer [1973] also studied the standard deviation of returns in stock markets, but he used a new method involving 12-month moving standard deviation of returns. Officer's study range (1897-1969) is larger than one considered in Fisher and Lorie and furthermore includes more volatile years. Officer found that during the great depression of 1930-1942 the volatility was significantly higher than at other times. He also found that there was no significant difference between volatility during other two time periods, namely 1897-1930 and 1942-1969. From these he concluded that the stock returns reverted back to the normal level of variability. <sup>⑨</sup>

Schwert [1989, 1990] provided a list of the highest and lowest daily percent returns during the 105-year period from 1885 to 1989, and found that almost all the largest single day decline, the single day increase and the monthly increase were during the 1929-1939 period (but the lowest daily percentage return was on October 19, 1987; a -20.39% decline). Schwert also found that stock return volatility was higher during the period of economic

recessions. Moreover, he concluded that any improvement in the trading system or new technical innovations cannot decrease the volatility. <sup>⑩</sup>

### **2.2.3 Research on bond Market Volatility -- US**

There are many factors that affect the bond return volatility of which the most important one is the interest rate volatility. Others include (1) maturity (2) coupon, and (3) term structure of the market. These three factors can influence the bond duration and the convexity of the bond market. However, compared to the factors influencing stock return volatility, the factors influencing bond return are less volatile.

While stock market volatility has been studied detail, there are not many studies on the bond market volatility. The main reason for this is the lack of well-specified bond index with adequate history. Only in 1973, the first comprehensive bond market index of Lehman Brothers index came into being. Research on bond volatility is limited because the interest rate volatility which is one of the main factors influencing the bond volatility can be traced back only to the year 1926. Based on interest rate volatility, Coleman, Fisher, and Ibbotson [1993] concluded that the bond volatility continued to increase from 1950 to 1987, reaching its highest volatility during 1980-1987. <sup>11</sup>

In his work, Frank [2000] concluded that the changes in stock returns are more like bond returns which is consistent with Bernstein [1992]'s findings. <sup>12</sup> The rising trend in the stock-bond correlation shows that the following 3 factors of stock volatility, namely, (1) the risk premium, (2) the expected growth of earning, and (3) the cash flow, all become more stable. This trend also in turn makes the interest rate factor play a more and more important

role in stock pricing, and stocks become more and more “bond-like”. The phenomenon that Stocks are “bond-like” was also observed in the work of Leibowitz [1987],<sup>13</sup> and Reilly, and Brown [2000].<sup>14</sup>

### **2.3. Research in other countries**

Bollerslev [1992] advocated the need for empirical investigation of stock market volatility in countries other than the United States. Robert and Philip [2002]<sup>15</sup> studied the stock and bond market in the United Kingdom (UK) and concluded that the UK bond is still a effective diversification vehicle to investors who invest in UK stock market, and there is no evidence to support that the UK stock is bond-like.

Taufiq [1997]<sup>16</sup> conducted research on Stock spot market and stock index futures market in Australia, Hong Kong and Japan, but his research does not investigate the returns volatility in the corresponding bond markets. Taufiq showed that as stock spot price and stock futures market prices move further apart in the short run, stock returns volatility increases. His results also indicate significant volatility clustering in the stated stock markets, and strong interaction between the stock spot market and stock index futures markets. But Taufiq did not conduct research on bond markets in these countries/regions.

## **CHAPTER 3**

### **Hong Kong Data description**

#### **3.1 Background on Hong Kong financial system**

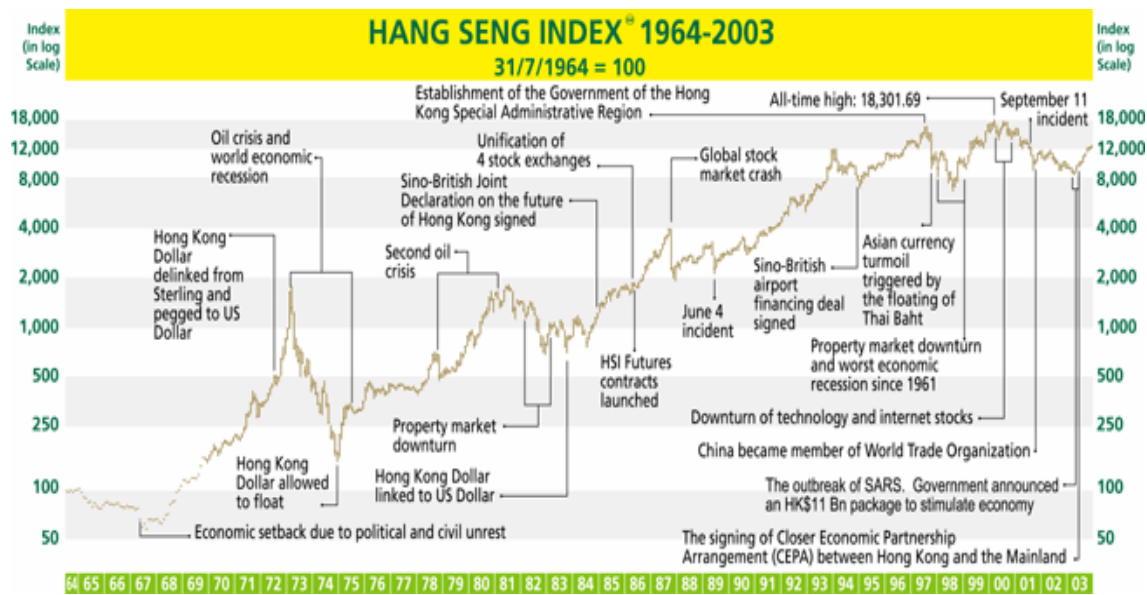
. There are two reasons for why Hong Kong financial markets are attractive to worldwide investors. First, Hong Kong has been a free economic society for at least hundred years under the British rule with a long history of having an International Financial Center in Asia whose stock exchange is ranked sixth in the world. Secondly, economic reforms of 1978 have transformed People's Republic of China (PRC) from a planned economy to the current market economy. As a result of this significant change, PRC is experiencing a high rate of growth and economic interdependency. Since Chinese stock markets are not completely open to overseas investors, Hong Kong stock market provides an avenue for these investors to take advantage of the economic growth in China.

#### **3.2. Hong Kong Stock market**

Stock market index is a tool for measuring the performance of an entire stock market or group of related stocks. These indices are often associated with particular stock exchanges or industries. They exist because changes in a market index can reflect a more general price trend than a change in individual stock prices.

Hang Seng Index (HSI) is the main indicator of the overall market performance in Hong Kong. It is a capitalization-weighted stock market index, and is used to record and monitor daily changes in the 33 largest companies listed in the Hong Kong stock market. This

represents about 70% of capitalization of the Hong Kong Stock Exchange. <sup>3</sup>



Source Hang Seng service company, <http://www.hsi.com.hk>

Figure 2: Hang Seng Index 1964-2003

Hang Seng index daily data provided by Reuters can be found under the code of (^HSI) in the Yahoo financial Website (<http://finance.yahoo.com>)<sup>4</sup>. It ranges from 12/31/1986 to the present date. In our study we use the data from July 1st, 1991 to May 31st, 2004 primarily because we want to match the HK stock data with the available HK bond data. Our dataset has 3345 observations in all.

<sup>3</sup> source: <http://encyclopedia.thefreedictionary.com/Hang%20Seng%20Index>  
<http://www.hsi.com.hk> is Hang seng Index official Website, and you can find the updated index and historical index data under the catalog of statistics, but it is only for monthly data.

<sup>4</sup> <http://finance.yahoo.com/q?s=%5Ehsi>  
According to description in yahoo Website, “ Historical chart data and daily updates provided by Commodity Systems, Inc. (CSI). Quote data provided by Reuters.”



Figure 3: Hong Kong Hang Seng index 1986-2004

After obtaining the Hang Seng index daily data, we convert it into daily rate of returns and annual rate of return by following formula:

$p_t$  = the daily Hang Seng index, which can be taken as the weighted stock price.

The daily rate of return =  $(p_t - p_{t-1}) / p_{t-1}$ , which is capital gain only, does not include the dividend gain. Annual rate of return = Daily rate of return \* 365. Here we assume that all the days have the same rate of return, so the annual rate of return is just the product of daily rate of return multiplied by the numbers of days in one year. We do not use the compounding method to calculate the annual rate of return.

After obtaining the annual rate of return for each day, we continue to calculate the mean and standard deviations for each month (July 1991 to May 2004). For example, we take the data from July 1<sup>st</sup> 1991 to July 31<sup>st</sup> 1991 and calculate July 1991 monthly mean and

standard deviation based on these data. We do the same for the data from Aug 1<sup>st</sup> 1991 to Aug 31st 1991 and obtain the Aug 1991 monthly mean and standard deviation. More accurately, this method was called the mean and standard deviation for a discrete, non-overlapping one-month calendar time period. [Reilly 2000]

Our final dataset ranges from July 1991 to May 2004, with a total of 155 observations. Given below is a plot of the monthly standard deviation of HK stock market.

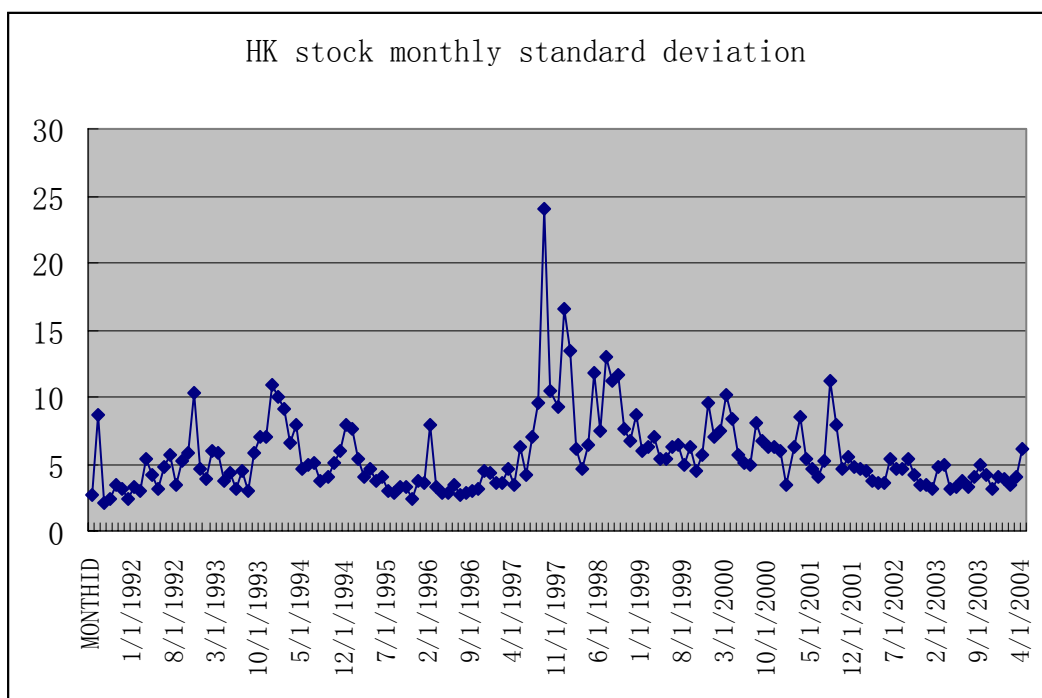


Figure 4: Hong Kong stock monthly standard deviation

For SAS command, refer to the footnote seventeen (17). For the whole monthly dataset, refer to the Appendix: Hong Kong monthly dataset.



### **3.3 Hong Kong Bond market**

Before analyzing the Hong Kong Bond market, we give a brief summary of the Hong Kong financial authority. The Hong Kong Monetary Authority (HKMA) was established in April 1993 by merging the Office of the Exchange Fund with the Office of the Commissioner of Banking. The HKMA is the government authority in Hong Kong responsible for maintaining monetary and banking stability. Its main policy objective is to maintain currency stability, within the framework of the linked exchange rate system, through sound management of the Exchange Fund, monetary policy operations and other means deemed necessary.

The exchange fund in Hong Kong has the main goal of supporting the linked exchange rate system since this system was created in 1983. The exchange fund also has a second goal of maintaining Hong Kong monetary and banking stability since 1992 amendment. Until 2003, Hong Kong exchange fund had never been used for funding the budget deficits by the Hong Kong government. As of September 2004, the fund has almost 1018 billion Hong Kong dollars, which is equal to 130 billion US dollars, of the exchange fund in Hong Kong outstanding.

Meanwhile, the U.S. treasure debt, after being raised through the capital markets, served as the mechanism for funding the large budget deficits incurred by the federal government. As of October 2004, the U.S. treasure has almost 7.383 trillion US dollars of treasure debts.

Compared to the U.S. bond market, the Hong Kong Bond Market value is relatively small (130 billion vs. 7.383 trillion), and even compared to Hong Kong stock market, the

Hong Kong Bond Market is still premature (130 billion vs. 495 billion, as of 2002, the Hong Kong stock market has 3868 billion Hong Kong dollars market value, which is equal to 495 billion U.S. dollars.) Based on above reasons, the Hong Kong financial authority has tried their best to increase their bond market size for a long time.

The yield rate data on exchange fund bills/notes can be obtained from the Hong Kong Monetary Authority (HKMA).<sup>5</sup> For a detailed description of this dataset, please refer to the footnotes.<sup>6</sup>

Table 1: Hong Kong Monetary Authority Yield of Exchange Fund (daily data)

MATURITY	FROM	TO
91 day Bill	Jun-91	current
182 day Bill	Jun-91	current
1 year Bill	Jun-91	current
2 year Note	May-93	current
3 year Note	Oct-93	current
5 year Note	Sep-94	current
7 year Note	Nov-95	current
10 year Note	Oct-96	current

We pick the 1-Year Bill data as our research objective, with range from June 10th, 1991 to May 31st, 2004, and we choose the whole month data from July 1st, 1991 to May 31st, 2004. Since data is only available for business days, there are 3345 observations.

---

<sup>5</sup> Data source: Official HKMA Website, under the following catalog: 5. Exchange Fund Bills & Notes -- 5.3 Yield of Exchange Fund Bills & Notes -- 5.3.1 End of period figures Download-- 5.3.2 Period average figures Download--5.3.3 Daily figures. ([http://www.info.gov.hk/hkma/eng/statistics/msb/new\\_msb\\_tables\\_b.htm#exchange\\_fund\\_bills\\_and\\_notes](http://www.info.gov.hk/hkma/eng/statistics/msb/new_msb_tables_b.htm#exchange_fund_bills_and_notes))

<sup>6</sup> "Table 5.3 : Yield of Exchange Fund Bills & Notes1, 2. give the following information. (1) Before 16 December 2002, the yield figures are calculated as the arithmetic mean of 4 quotes collected from 4 designated banks. Following the introduction of the HKMA EFBN Fixings on 16 December 2002, the yield figures are calculated as the arithmetic mean of the middle 8 quotes, after excluding the 2 highest and 2 lowest quotes, collected from 12 designated banks. (2) Yield figures powered by Reuters.

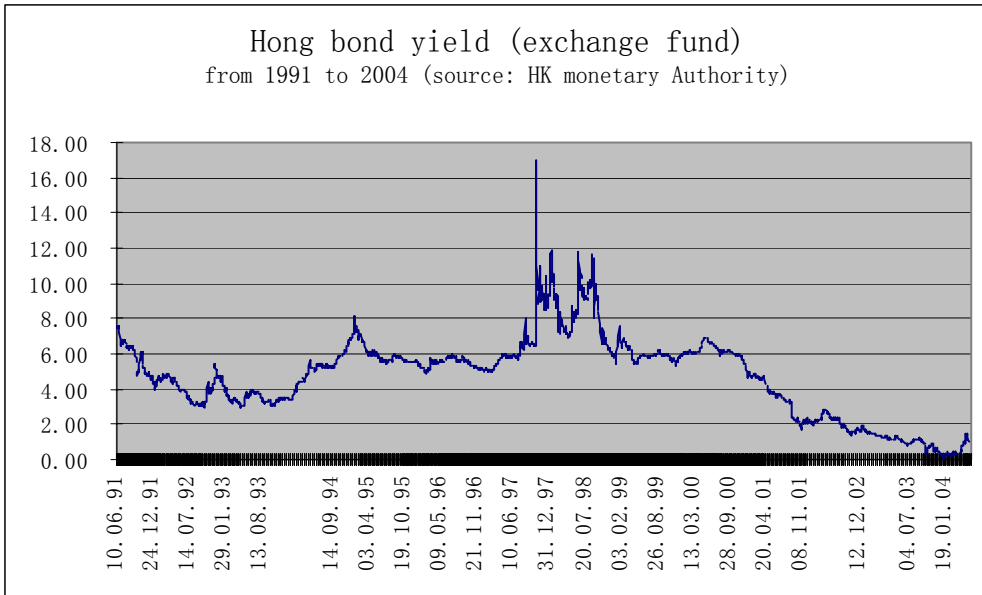


Figure 5: Hong Kong bond Daily data 1991-2004

We used a SAS program to calculate the average/mean and the standard deviation of Hong Kong bond market for each month. This SAS program is similar to HK stock market one. Our final monthly data ranges from July 1991 to May 2004, with 155 observations. For the whole dataset, see the Appendix: Hong Kong monthly dataset.

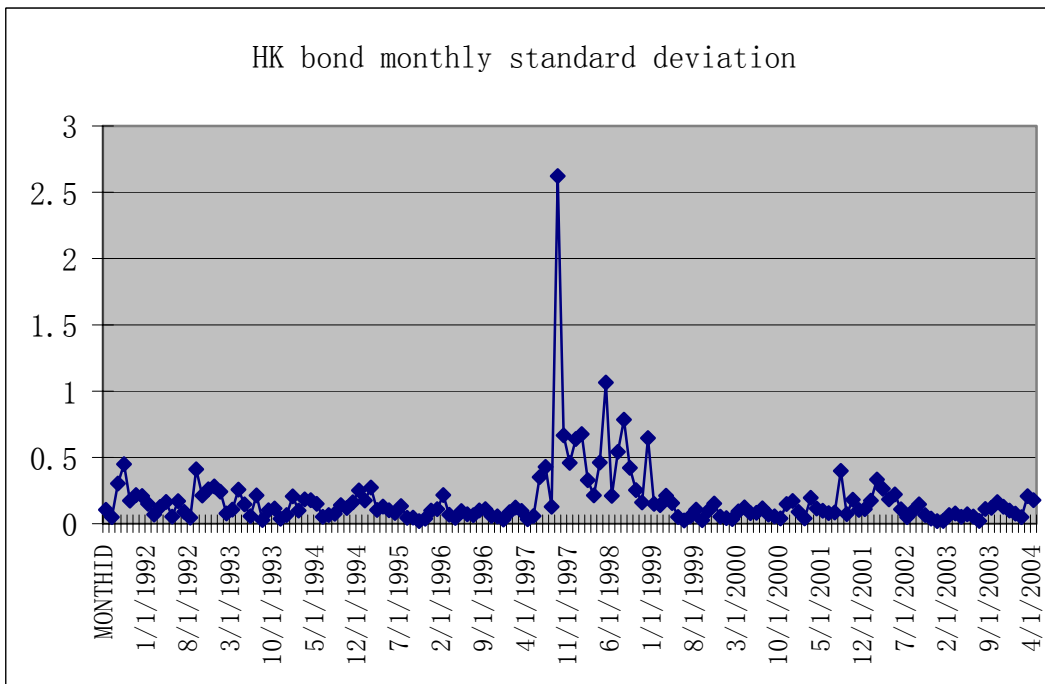


Figure 6: Hong Kong bond monthly standard deviation 1991-2004

## CHAPTER 4

### HONG KONG FINANCIAL MARKET ANALYSIS

#### 4.1 Analysis of squared standard deviation for HK bond returns

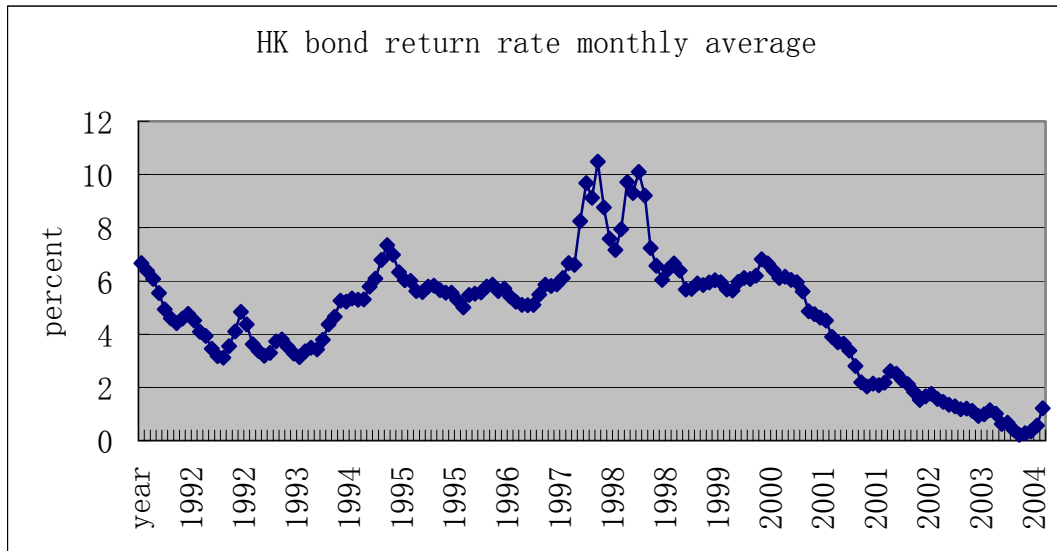


Figure 7: Hong Kong bond monthly average rate of return

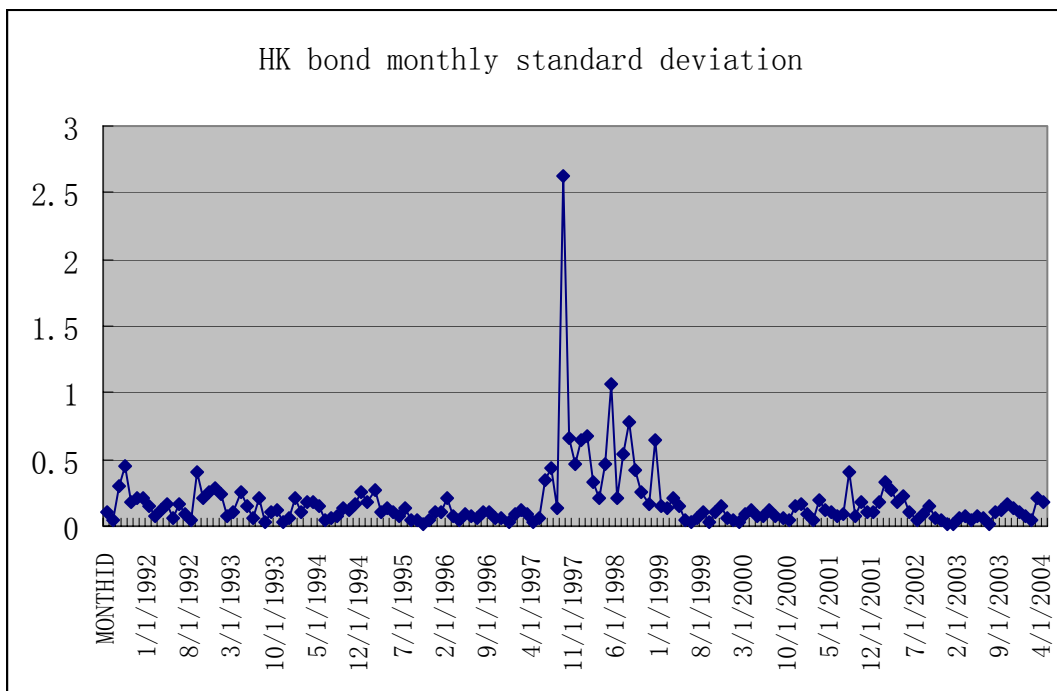


Figure 8: Hong Kong bond monthly standard deviation

## Re-parametrize ARCH model to an AR model

The GARCH model can be expressed as follows:

$$y_t = x_t \beta + u_t$$

$$u_t = \sqrt{h_t} \cdot v_t$$

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

$$u_t | \Psi_{t-1} = N(0, h_t)$$

In our study,  $y_t$  is (1) monthly standard deviation of return rate for the Hong Kong bond. We carry out the GARCH analysis by checking whether variance has autoregressive or moving average pattern. Variance is equal to the squared standard deviation.

A special case of ARCH model is the following:

$$y_t = \sigma_t \varepsilon_t$$

where  $\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2$

and  $\varepsilon_t \sim$  independent  $N(0,1)$

$y_t$  here is the monthly standard deviation of Hong Kong bond market.

$y_t^2 =$  square of standard deviation = variance.

We can re-write the model as follows:

$$y_t^2 = \sigma_t^2 + y_t^2 - \sigma_t^2$$

$$= (\alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2) + (\sigma_t \varepsilon_t)^2 - \sigma_t^2$$

$$= \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2 + \sigma_t^2 (\varepsilon_t^2 - 1)$$

$$= \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2 + v_t$$

where  $v_t = \sigma_t^2 (\varepsilon_{t-1}^2)$

Actually, the ARCH (m) process on  $\sigma_t^2$ , changes to AR(m) process for  $y_t^2$ .  
Moreover, both the processes have the same parameters.

### ARIMA fitting

Following are the plot of square of standard deviation (variance)

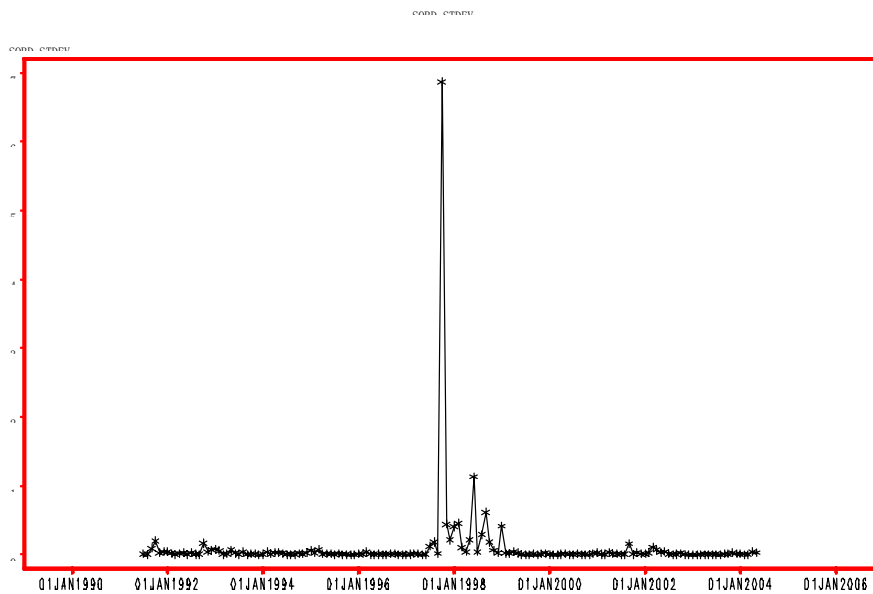


Figure 9: Square of standard deviation for Hong Kong bond

### ACF /PACF and unit root test

#### ACF /PACF graph for hk bd\_stdev sq

### SQBD\_STDEV

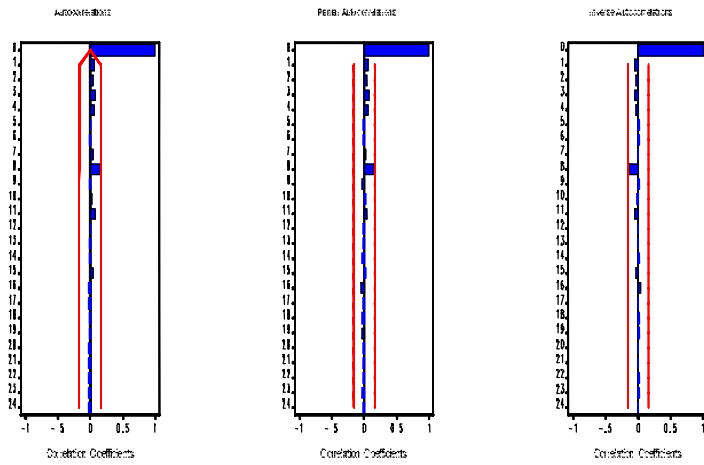


Figure 10: ACF/PACF graph for Hong Kong bond standard deviation square

### Unit root test for hk bd\_stdev sq

### SQBD\_STDEV

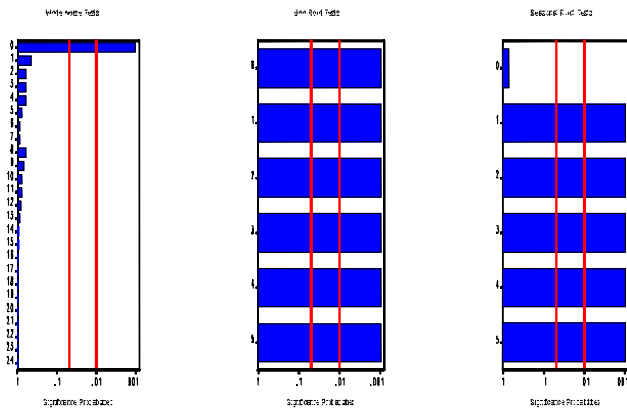


Figure 11: white noise/ Unit root test for Hong Kong bond standard deviation square

Both ACF/PACF shows the  $y_t^2$  do not have significant AR or MA pattern. Even though the PACF has some spike at lag term of 8, but this spike is not significant. White noise check gives the same information, that there is NO autocorrelation of the residuals, and unit root test do not suggest further step of simple difference.

### MINIC method

Error series model: AR(20)

Minimum Table Value: BIC(3, 0) = 0.564131

SAS output, please refer to 18

We ran the MINIC to determine the best model according this criterion, and this result suggests an AR (3) process. We proceed to analyze this model further.

### Model selection From forecasting system

Given below is a table consisting of all the models that we considered for the squared standard deviation series of Hong Kong bond market. Here, we have also used transformations such as *log* and *square root* in some cases. We used the SAS time series forecasting system to fit a model automatically. In the table below, we refer to ARCH(p) as AR(p) using the above re-parametrization. Finally, we also fitted the simplest model of ARCH (1), since the ARCH (1) is good enough in many cases.

Table 2: Model selection for HK bond

Forecasting system	AIC	BIC	Summary	Conclusion
Automatic model fitting: mean	-177	-174	Lowest AIC, BIC	Best model
AR(1)	-173	-167	Coefficient of AR1 is not significant	
sqrt AR(1)	-173	-167		
log AR(1)	-172	-166		
AR(3)	-170	-158	Coefficient of AR1, AR2, AR3 are not significant	
Log AR(3)	-171	-159		
Sqrt AR(3)	-171	-159		
sqrt AR(8)		-138		
AR(8)	-164	-137		
MA(8)	-164	-137		
log AR(8)	-161	-134		

These show that the mean model is the best model for the squared standard



deviation series of Hong Kong bond market. Therefore, it is not necessary to apply an ARCH model.

**Final model: mean (SAS automatic model fitting result)**

	Estimate	std error	T	prob of T
Intercept	0.09492	0.0452	2.1008	0.0373
Model Variance (sigma squared)	0.31644			

**Conclusion for Hong Kong bond analysis**

The Hong Kong bond market has a constant monthly variance of 0.09492, and it is not necessary to apply an ARCH model.

**4.2 Analysis of squared standard deviation for HK stock returns**

In this section, we write a GARCH(r, m) as an ARMA model for  $y_t^2$ . Recall that

$y_t = \sigma_t \varepsilon_t$  where

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \dots + \alpha_m y_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_r \sigma_{t-r}^2$$

- 1)  $\varepsilon_t \sim$  independent  $N(0,1)$
- 2)  $\sigma_{t-1}^2, \dots, \sigma_{t-r}^2$  are all unobservable.
- 3) If  $r=0$ , then  $GARCH(r,m) = ARCH(m)$ .

For illustration, we only consider a GARCH(1,1) process. Write

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$y_t^2 = (\sigma_t^2) + (y_t^2 - \sigma_t^2)$$

$$\begin{aligned}
&= (\alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2) + (y_t^2 - \sigma_t^2) \\
&= (\alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2) + (\beta_1 y_{t-1}^2 - \beta_1 y_{t-1}^2) + (y_t^2 - \sigma_t^2) \\
&= \alpha_0 + (\alpha_1 + \beta_1) y_{t-1}^2 - \beta_1 (y_{t-1}^2 - \sigma_{t-1}^2) + (y_t^2 - \sigma_t^2) \\
&= \alpha_0 + (\alpha_1 + \beta_1) y_{t-1}^2 - \beta_1 v_{t-1} + v_t
\end{aligned}$$

where  $v_t = y_t^2 - \sigma_t^2 = \sigma_t^2 \varepsilon_t - \sigma_t^2 = \sigma_t^2 (\varepsilon_t - 1)$  plays the role of an error series “ $w_t$ ” in a regular ARMA(1,1) model. The GARCH (1, 1) process on  $\sigma_t^2$ , changes to an ARMA (1,1) process on  $y_t^2$ . The table below summarizes the algebra above.

Table 3: Parameters estimation comparison between GARCH and ARMA

	Intercept estimate	Parameter associated with $y_{t-1}^2$	Other parameters
GARCH (1,1) on $\sigma_t^2$	$\alpha_0$	$\alpha_1$	$\beta_1$ : parameter associated with $\sigma_{t-1}^2$
ARMA (1,1) on $y_t^2$	$\alpha_0$	$\alpha_1 + \beta_1$	$-\beta_1$ : parameter associated with $v_{t-1}$

### ARIMA fitting

Firstly, we square monthly standard deviation of Hong Kong stock market, and then obtain the monthly variance of the Hong Kong stock market rate of return.

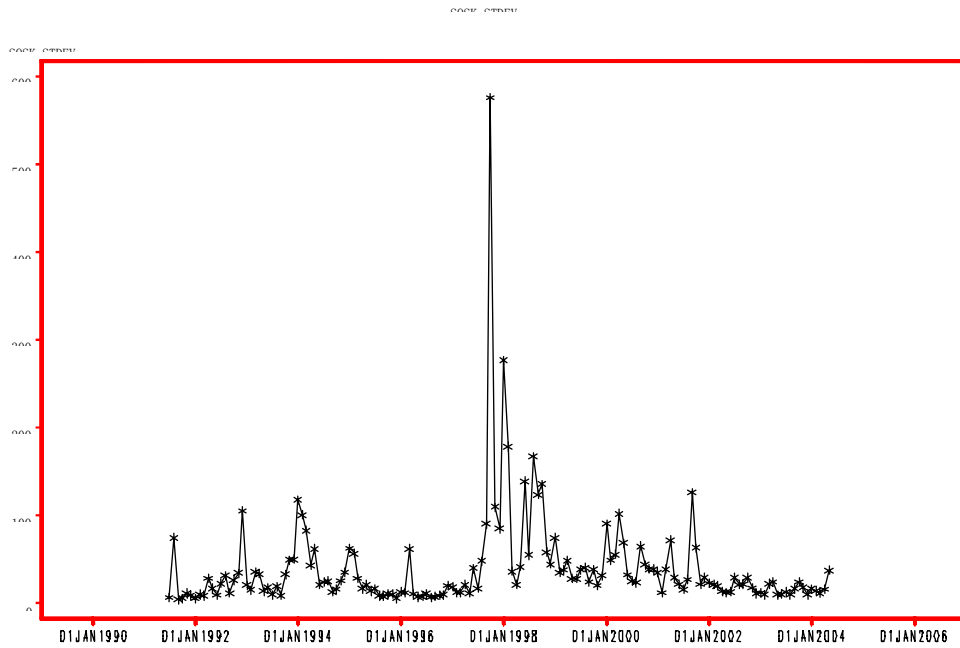


Figure 12: Hong Kong stock monthly variance of rate of return

ACF /PACF and unit root test

**ACF /PACF graph for sk\_stdev\_sq**

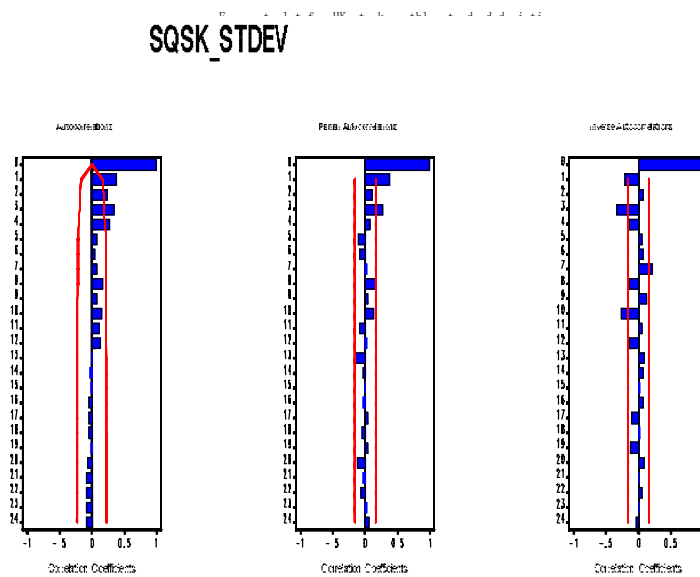


Figure 13: ACF /PACF graph for Hong Kong stock monthly variance

ACF cuts off after lag term 4, and PACF dies off after lag term 3. Both of them suggest MA (4) process and AR(3) process.

## Unit root test

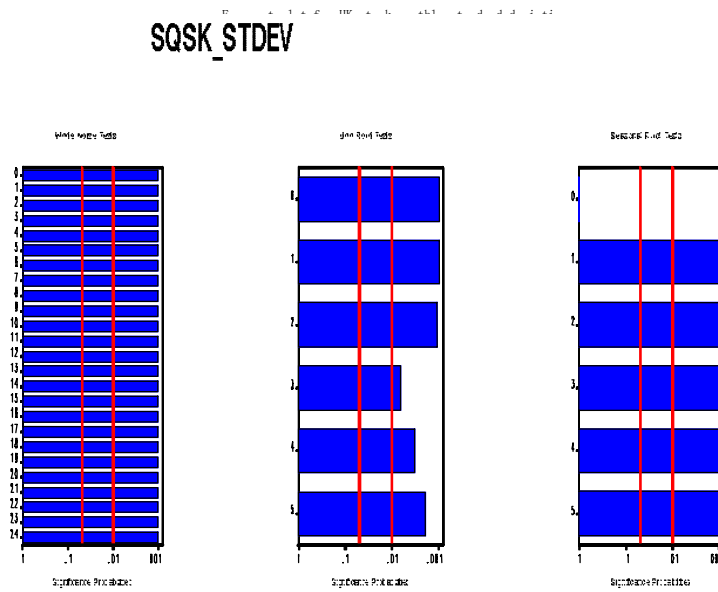


Figure 14: white noise/ unit root test for Hong Kong stock monthly variance

The white noise check shows that there is a significant auto regression pattern.  
The unit root test is ok so we do not need make simple difference for this series.

## From forecasting system

Based on above information, we try the following models.

Firstly, the ACF results suggest MA (4), with log and square root transforming, respectively.

Secondly, the PACF results suggest AR(3), and we also try AR(2) and AR(1).

Finally, we try ARMA (3, 4) and automatic model fitting.

Table 4: first model selection for Hong Kong stock monthly volatility

From forecasting system	AIC	BIC	Summary	Conclusion
Log MA (4)	1215	1230	Lowest BIC, lowest AIC, ACF/PACF spike on lag 3, all other test passed.	Best model
MA (4)	1216	1231	Coefficient of MA2 is not significant	
Sqrt MA (4)	1217	1232		
AR(3)	1220	1232		
AR(4)	1221	1236		
automatic:log simple exponential smoothing	1234	1237		
AR(2)	1229	1238		
MA(1)	1232	1238		
MA(3)	1227	1239		

MA (2)	1233	1243		
ARMA (3, 4)	1219	1243		

Conclusion:

(1) Log transforming is necessary.

(2) MA(4) as candidate for best model for log(variance).

### MINIC method

Then we try MINIC to confirm the above result, but NINIC result suggest the AR(1) model.

Error series model: AR(20)

Minimum Table Value: BIC(1, 0) = -1.14569

MINIC output, please refer to SAS output 19

### Model selection

Combining above results, we check the ACF/ PACF with more lag terms, and apply SAS command to specific lag terms, not to all lag terms.

Dependent variable: log (variance) = log (square of stock standard deviation)

Table 5: second model selection for Hong Kong stock monthly volatility

From SAS command	AIC	BIC	Summary	Conclusion
From MINIC result AR(1) Estimate p=1 q=0	312	319	Autocorrelation Check of Residuals fails till lag of 12, ACF/PACF spike on lag 4	
MA(4) Estimate p=0 q=4	312	327	Autocorrelation Check of Residuals fails till lag of 12, ACF spike on lag 5	
ACF die off, and PACF has spike on lag 8, so we try AR (8) Estimate p=8 q=0	308	336	Only lag 1, 8 has significant coefficients.	
p=(1, 8) q=0	308	317	All tests passed.	Final model

Please refer to AR (8) SAS output in footnote of 20

Please refer to AR (p=1, 8) SAS output in footnote of 21

## Normality check for residual

Residual plot from  $P=(1, 8)$  model

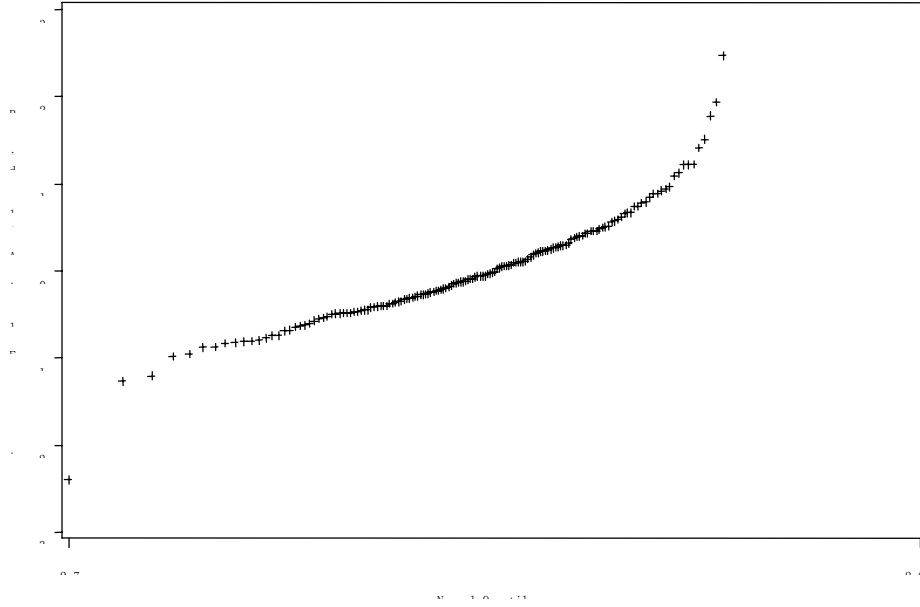


Figure 15: Residual plot from  $P = (1, 8)$  model for Hong Kong stock

## Final model $p=(1, 8)$ $q=0$

Parameter	Estimate	Error	t Value	Pr >  t	Lag
MU	3.18889	0.21725	14.68	<.0001	0
AR1,1	0.60937	0.06142	9.92	<.0001	1
AR1,2	0.16405	0.06222	2.64	0.0084	8

Constant Estimate	0.72256
Variance Estimate	0.418231
Std Error Estimate	0.646708
AIC	308.5911
SBC	317.7214
Number of Residuals	155

To	Chi-Square	DF	Pr > ChiSq	-----Autocorrelations-----					
Lag									
6	6.90	4	0.1410	-0.087	0.019	0.101	0.152	-0.032	0.026
12	11.62	10	0.3110	-0.048	0.034	-0.027	-0.001	-0.004	0.154
18	13.61	16	0.6277	-0.038	0.036	0.032	-0.036	-0.065	-0.045
24	18.19	22	0.6948	0.077	-0.050	-0.107	-0.036	-0.057	0.032

30      29.80      28      0.3730      -0.175      -0.050      0.066      -0.132      0.010      -0.077

Autoregressive Factors

Factor 1: 1 - 0.60937 B\*\*(1) - 0.16405 B\*\*(8)

ARCH model estimation

The process is ARCH model.

Xt: monthly standard deviation for Hong Kong stock rate of returns

$$Y_t = \log(\text{squared } X_t) = \log[(x_t)^2] = 2 * \log(x_t)$$

$$y_t = \sigma_t \varepsilon_t$$

$$\text{where } \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2$$

$$y_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \dots + \alpha_m y_{t-m}^2 + v_t$$

$$\text{where } v_t = \sigma_t^2 (\varepsilon_t^2 - 1)$$

$$y_t^2 \text{ est} = 3.18889 + 0.60937 * y_{t-1}^2 \text{ est} + 0.16405 * y_{t-8}^2 \text{ est}$$

$$\hat{\sigma}_t^2 = 3.18889 + 0.60937 * y_{t-1}^2 \text{ est} + 0.16405 * y_{t-8}^2 \text{ est}$$

Series plot

Forecasting data

Forecasts for variable logsqsk_stdev				
Obs	Forecast	Std Error	95% Confidence Limits	
156	3.4659	0.6467	2.1984	4.7334
157	3.3095	0.7573	1.8252	4.7938
158	3.1216	0.7945	1.5645	4.6788
159	3.0868	0.8078	1.5035	4.6702
160	3.0429	0.8127	1.4500	4.6359
161	2.9851	0.8146	1.3886	4.5816
162	3.0022	0.8152	1.4044	4.6001
163	3.1492	0.8155	1.5508	4.7475
164	3.2101	0.8240	1.5951	4.8252
165	3.2216	0.8353	1.5844	4.8588
166	3.1978	0.8443	1.5430	4.8526
167	3.1776	0.8500	1.5115	4.8436

Series plot for entire data (logsqsk\_stdev)

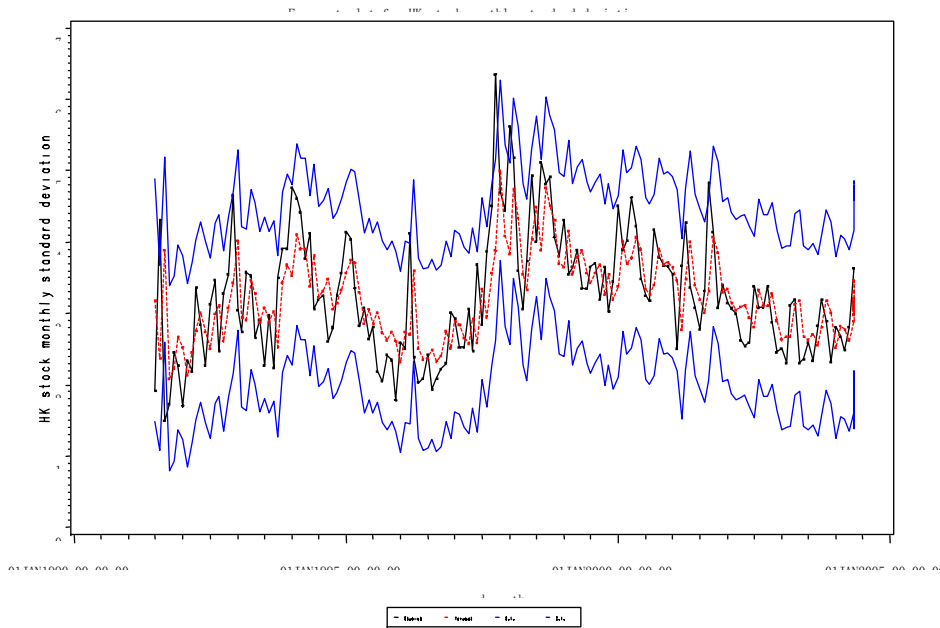


Figure 16: Series plot for entire Hong Kong stock monthly variance

In above graph, for the value of log\_sq\_sk\_stdev, the black line is actual line, the red line is the predicted value, and blue values are 95% upper and lower confidant intervals.

Series plot for forecasting data (logsqsk\_stdev)

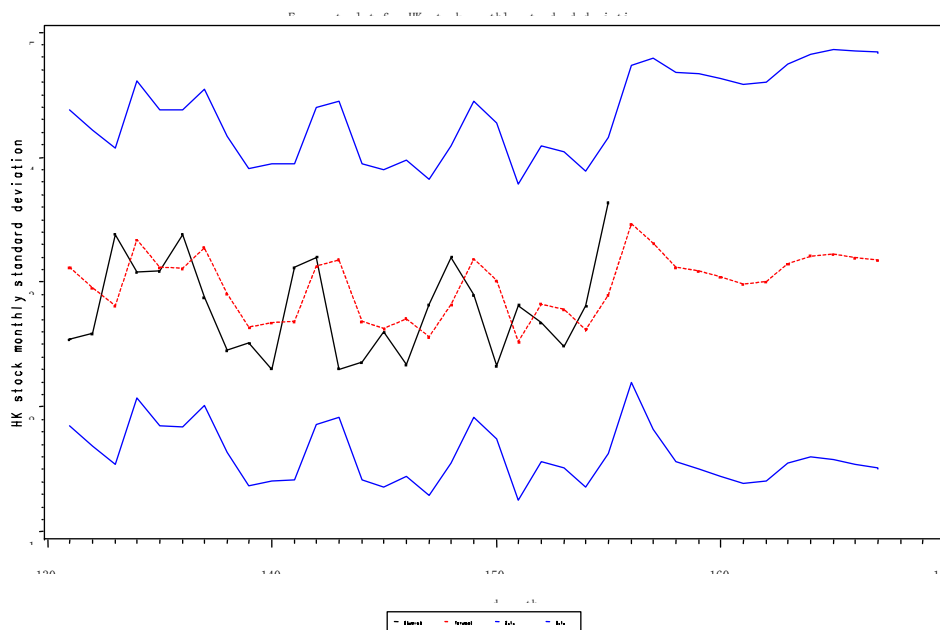


Figure 17: Series plot for forecasting Hong Kong stock monthly variance



Analysis on  $\log_{sq\_sk\_stdev}$  is shown in above graph, and the black line is actual line, the red line is the predicted value, and blue values are 95% upper and lower confident interval.

### **Conclusion for Hong Kong stock analysis**

Firstly, all ARCH parameters are statistically significant, and then the ARCH model is necessary to predict the monthly standard deviation of Hong Kong stock market.

In Hong Kong, the bond and stock markets have different volatility. (1)According to my research, monthly stock return volatility is dependent on the monthly volatilities of prior month and 8 months ago. Then it exhibits a regular, systematical pattern over time and ARCH ( $q=(1\ 8)$ ) model can be used to predict bond volatility series closely. (2) But Hong Kong bond volatility does not exhibit a predictable time series behavior and ARCH model do not track the actual stock volatility very well.

It is very interesting that my finding in Hong Kong is different with the Reilly [2000] finding in the U.S.: US bond market annual standard deviation can be fitted by ARCH model, but the U.S stock market annual standard deviation can not be fitted by ARCH.

### 4.3 Ratio analysis (standard deviation of bond over stock)

In this section, we focus on the relative standard deviation, the monthly standard deviation ratio of Hong Kong bond market over stock market.

$$\text{ratio} = \frac{\text{Hong Kong bond monthly standard deviation}}{\text{Hong Kong stock monthly standard deviation}}$$

### General process to build GARCH models and comments

We can apply ARIMA model to original series of  $X_t$  and get residual of  $Y_t$ .

$$y_t = \sigma_t \varepsilon_t \quad \text{where}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \dots + \alpha_m y_{t-m}^2$$
$$+ \beta_1 \sigma_{t-1}^2 + \dots + \beta_r \sigma_{t-r}^2$$

$\varepsilon_t \sim \text{independent } N(0,1)$

#### Comments on the method

1. Build an ARIMA model for the observed time series  $X_t$  to remove any autocorrelation in the data. Usually, this just means making first differences. Sometimes we need a more complex ARIMA model to remove all patterns of trend, auto recession and moving average, in order to get the residual from the ARIMA. We call the residual as  $y_t$ .

2. Examine the squared residuals  $y_t^2$  for conditional heteroscedasticity. This can be done by checking ACF and PACF plots.

3. If the true process is ARCH (m) model, we need construct an AR model for  $y_t^2$ . And we can expect PACF to cut off after lag term of m, which is the way to determine m. (please refer to the HK stock standard deviation squared analysis part)

4. If the true process is GARCH (r, m) model, how to determine value of r is a

tough problem. Pena, Tiao, and Tsay (2001) stated, “The identification of GARCH models in practice is not simple. Only lower-order GARCH models are used in most applications.”<sup>7</sup>

Chan (2002) also did not suggest methods to determine the value of  $r$  in GARCH model.<sup>8</sup>

5. From the view of volatility memory, the difficulty of modeling a GARCH sequence depends on both  $r$  and  $m$ , and also on how much volatility memory there is in the process.

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \dots + \delta_p h_{t-p} + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

Higher values of the  $\delta$  parameters give rise to more volatility memory and therefore make it to accurately model the process. Increasing the number of model parameters will also make the modeling more difficult simply because there are more variables to numerically optimize. (Please refer to later explanation for numerical optimization)

In a summary, above reasons suggest the following order of difficulty: ARCH (1), ARCH(2), ARCH(3), GARCH(1,1), GARCH(1,2), GARCH(2,2), ..., etc.<sup>9</sup>

6. In real world practice, one solution is trying a few different models and finds a group of models with acceptable/satisfactory residuals. Among these candidate models, choose the one (1) with the smallest number of parameters, and (2) the smallest BIC and AIC.

7. One estimating method for GARCH parameters is maximizing the conditional log-likelihood function. Starting with an initial approximation for all parameters and then using numerical optimization to iterate to an acceptable solution can achieve this.

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<sup>7</sup> Pena, D., Tiao, G. C., and Tsay, R. S. (2001). A Course in Time Series Analysis. Wiley. p.257

<sup>8</sup> Chan, N. H. (2002). Time Series: Applications to Finance. Wiley.

<sup>9</sup> An Introduction to GARCH Models in Finance By George Levy, From the June 2001 issue of Financial Engineering News, [www.fenews.com/fen22](http://www.fenews.com/fen22)

8. Apply ARIMA again to fit  $y_t^2$ , we get the residual, we call that r.
9. SAS perform maximum likelihood estimation to get the parameters. To restore the parameters of ARCH or GARCH, please refer to re parameters in previous part.
10. Check the residual r by plotting ACF/PACF of  $r^2$ , the white noise check, unit root check and chi-square check. Make sure that GARCH/ARCH model is better and necessary.
11. Above separate process is not identical with joint process. But the difference is very small. <sup>10</sup>
12. In order to get the most accurate GARCH model estimate, we need to run joint models. Because when we simultaneously estimated the ARMA (p, q) model and GARCH (r, m) model, we are able to retest the parameters in joint model. In many cases, we can find that some parameters or all parameters in ARMA (p, q) are no longer necessary, then we can modify or drop the ARMA (p,q) model and continue to modify GARCH (r, m) model for best fitting.

### **First ARIMA model fitting**

Firstly, we plot the standard deviation ratio between Hong Kong bond and Hong Kong stock market.

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<sup>10</sup> Christopher R. Bilder, GARCH model, OSU STAT 5053 - Time Series Analysis, page 11, [www.chrisbilder.com](http://www.chrisbilder.com). Results compared with page 259, of PTT, Pena, D., Tiao, G. C., and Tsay, R. S. (2001). A Course in Time Series Analysis. Wiley.

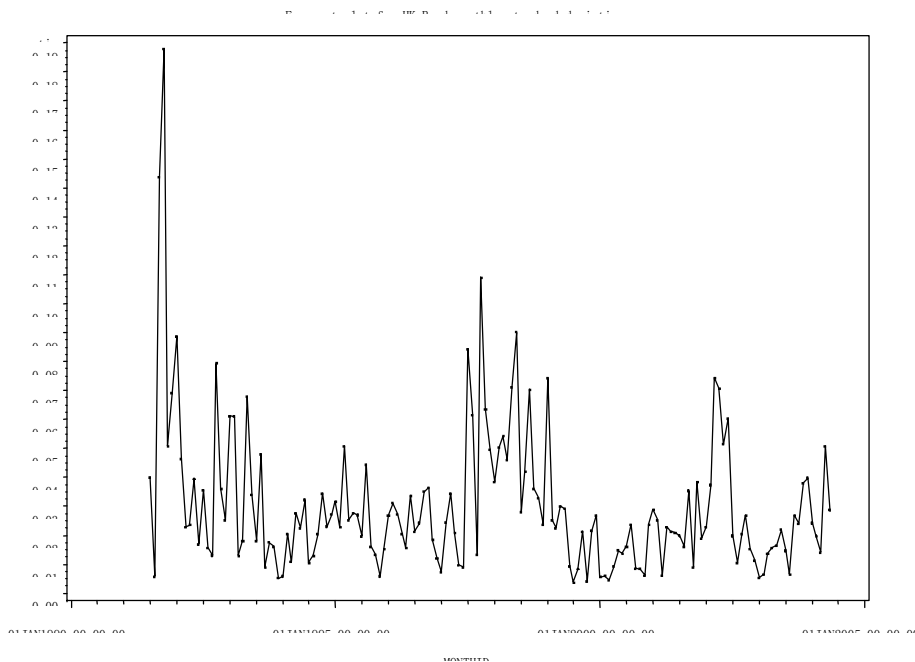


Figure 18: Hong Kong standard deviation ratio  
 Among 155 months, only 2 ratios are above 0.10, they are  
 Oct 1991 0.188  
 Oct 1997 0.109

ACF /PACF and unit root test

**RATIO: ratio**

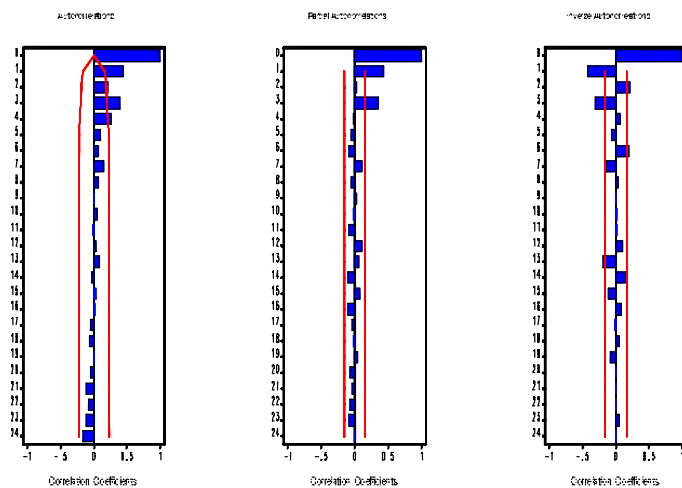


Figure 19: ACF /PACF graph for Hong Kong standard deviation ratio

ACF cut off after lag 4, maybe MA (4)

PACF cut off after lag 3, maybe AR (3), and from PACF, the lag 2 is not significant.

## ratio: ratio

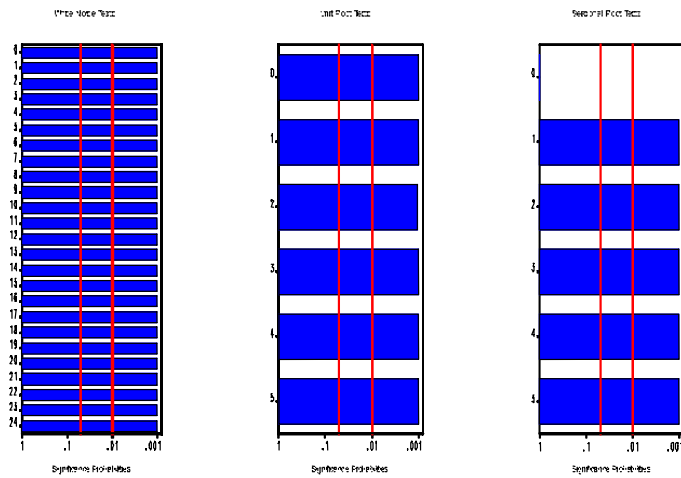


Figure 20: white noise/ unit root test for standard deviation ratio

Unit root test result: HK ratio series is stationary, but strongly auto correlated.

### MINIC method

SAS output 22

Error series model: AR(16)

Minimum Table Value: BIC(3, 0) = -8.22917

So the MINIC also confirmed that AR (3) is the appropriate model.

### Model selection: First ARIMA model

Table 6: Transforming check for standard deviation ratio

#### **Results from Time series forecasting system**

	BIC	AIC	
Automatic fitting: simple exp smoothing	-1162	-1165	
AR(3)	-1161	-1173	Best model
Sqrt AR(3)	-1159	-1171	
MA(4)	-1155	-1170	
Log AR(3)	-1151	-1163	
ARMA (3,4)	-1143	-1167	
Log IAR(3,1) not intercept	-1126	-1135	we try to include the intercept, but no significant

Conclusion: we do not need transforming of log or square root.

### Some other models

We run SAS command to confirm the above results, and still try other possible models.  
 Attention: SAS command result in different AIC/BIC from that in Time series forecasting system.

Table 7: first ARIMA model selection for standard deviation ratio

	AIC	BIC	Summary	Conclusion
AR(3) no int	-739	-730	ACF spike on lag 6, Coefficient of lag 2 has p value of 0.0762	
AR(3) with intercept	-748	-736	All test passed.	Best model
$p=(1\ 3\ 6)\ q=0$	-737	-728	Coefficient of lag 6 is not significant	
$p=(1\ 3)\ q=0$	-739	-733	PACF spike on lag 6	

### First ARIMA model: AR(3) with intercept

#### Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	0.03130	0.0052379	5.98	<.0001	0
AR1,1	0.42548	0.07445	5.71	<.0001	1
AR1,2	-0.18593	0.08117	-2.29	0.0220	2
AR1,3	0.44352	0.07298	6.08	<.0001	3
Constant Estimate			0.009921		
Variance Estimate			0.000453		
Std Error Estimate			0.021287		
AIC			-748.712		
SBC			-736.538		
Number of Residuals			155		

#### Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	-----Autocorrelations-----					
6	5.85	3	0.1191	-0.007	0.083	-0.013	-0.082	0.023	-0.147
12	8.29	9	0.5052	0.057	0.008	-0.075	0.019	-0.073	-0.001
18	14.28	15	0.5047	0.127	-0.086	0.096	-0.033	-0.021	-0.018
24	16.65	21	0.7323	0.030	0.045	-0.057	0.008	-0.063	-0.052
30	19.19	27	0.8631	-0.075	-0.020	-0.051	0.015	0.021	-0.064

## Normality check for residual

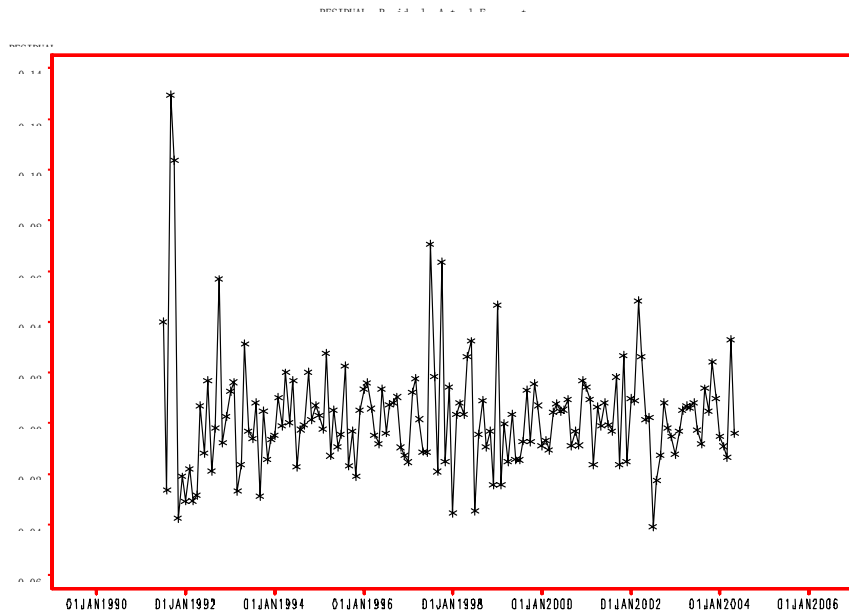


Figure 21: residual plot from first ARIMA model

From the graph, there is an obvious patten of non-constant variance.

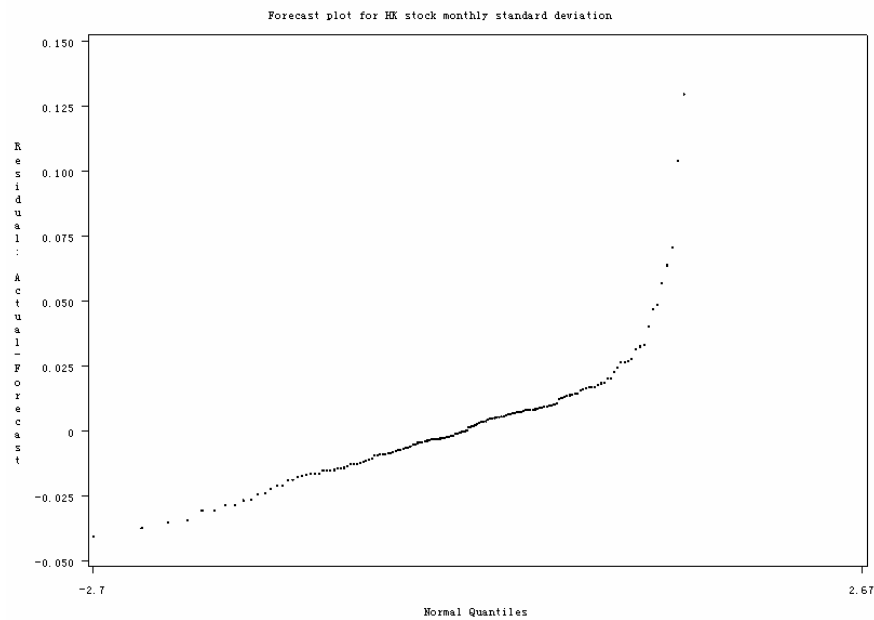


Figure 22: Normality check for residual from first ARIMA model

QQ plot shows the residual is not normal distribution.



## ARIMA--GARCH separately (consequently) fitting

In fact, it is sq\_residual analysis after first ARIMA. The process is: getting the residual from first ARIMA model; then Taking residual squared, and use ACF, PACF to check the Heteroscedasticity, and use ingre-parameter method to estimate GARCH model from second ARIMA model.

### ACF/PACF and unit root test for residual sq

Get residual, then take square, and plot as following.

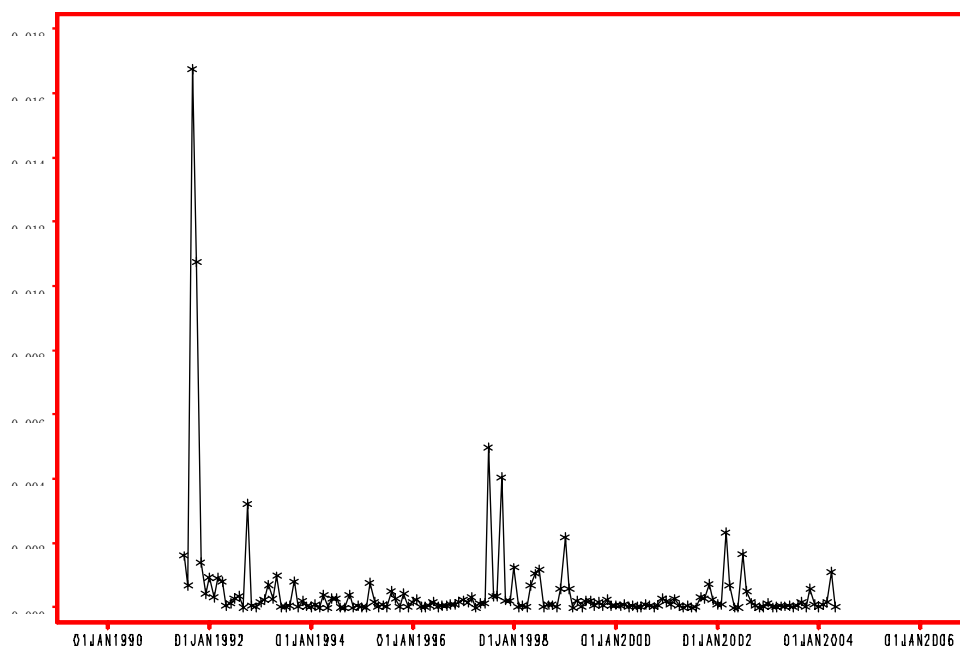


Figure 23: Square of residual from the first ARIMA model

sqres

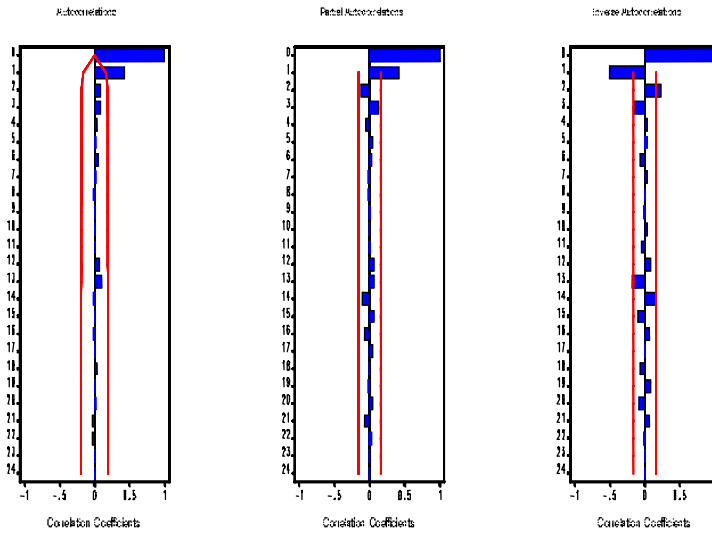


Figure 24: ACF/PACF graph for Residual Square

ACF/PACF suggest the MA(1), and it is still worth to try AR(3).

sqres

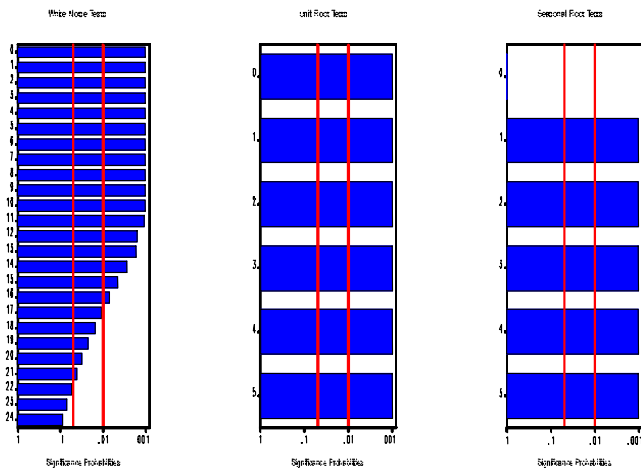


Figure 25: white noise/ unit root test for Residual Square

Normality check

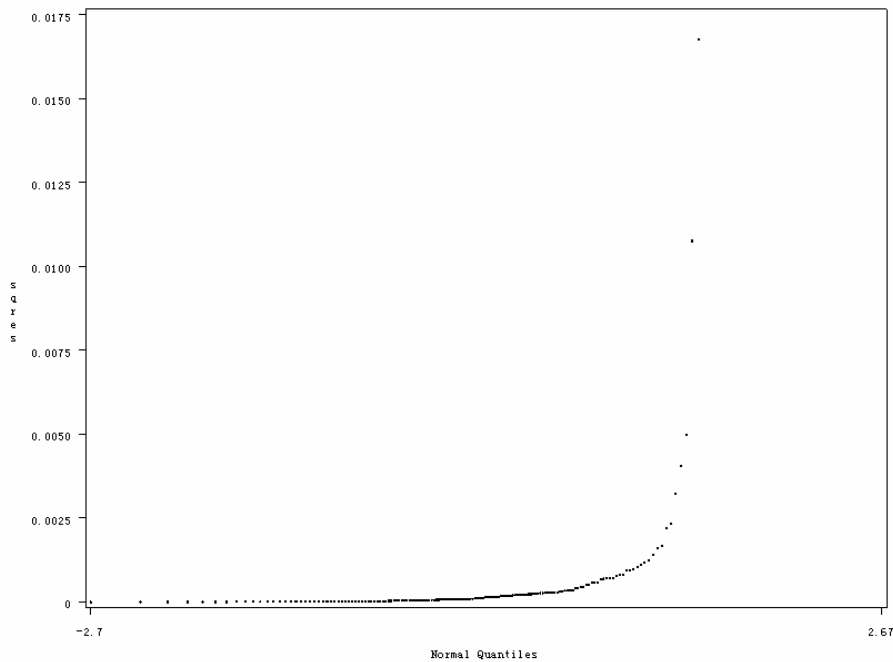


Figure 26: Normality check for Residual Square after first ARIMA

MINIC for squared residual

SAS output <sup>23</sup>

Error series model: AR(17)  
 Minimum Table Value: BIC(3, 0) = -14.9977

MINIC results suggest AR (3)

But ACF/PACF is MA (1) or AR (3)

Table 8: separately fitting model selection for standard deviation ratio

Var=sqres	AIC	BIC	Summary	Conclusion
AR(3) with intercept	-1607	-1595	Autocorrelation Check of Residuals fails. Spike of ACF/PACF of lag=4	Intercept with p-value of 0.0468, we could drop the intercept. Another reason is the first ARIMA model should remove all constant components.
AR(3) no int	-1606	-1597	Autocorrelation Check of Residuals fails totally, both ACF/PACF spike on lag 3,4	
MA(1) no int	-1603	-1600	All test passed.	

MA(1) with int	-1607	-1601	All test passed. Intercept is significant	Best model
AR (1) no int	-1588	-1588	Autocorrelation Check of Residuals fails to lag 6. and spike on ACF/PACF with lag=2	
AR (1) with int	-1591	-1585	Autocorrelation Check of Residuals fails to lag 6. and spike on ACF/PACF with lag=2	
AR(2)	-1597	-1598	Spike of PACF of lag=3	

Result is perfect.

Conclusion: GARCH exist in ratio series

**GARCH model built on MA(1)**

Maximum Likelihood Estimation									
Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag				
MU	0.0004726	0.0001784	2.65	0.0081	0				
MA1,1	-0.66414	0.06542	-10.15	<.0001	1				
Constant Estimate			0.000473						
Variance Estimate			1.799E-6						
Std Error Estimate			0.001341						
AIC			-1607.97						
SBC			-1601.88						
Number of Residuals			155						
Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	-----Autocorrelations-----					
6	1.65	5	0.8956	-0.094	0.021	0.022	0.003	-0.011	0.022
12	2.45	11	0.9962	0.023	-0.027	0.022	-0.028	0.033	-0.034
18	8.55	17	0.9534	0.137	-0.103	0.058	-0.041	-0.002	0.030
24	9.70	23	0.9930	-0.038	0.041	-0.040	-0.007	-0.010	0.039
30	10.61	29	0.9993	-0.041	0.026	-0.038	0.012	-0.030	-0.001

Check the residual sq after the second ARIMA

## sqres

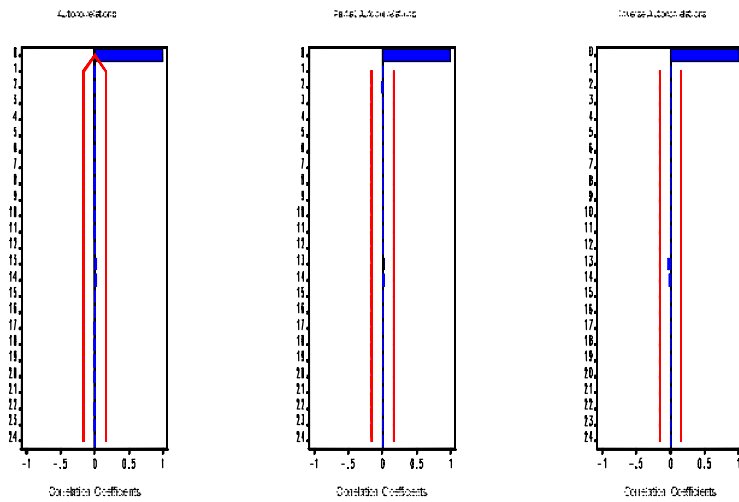


Figure 27: ACF/PACF graph for Residual Square after the second ARIMA

From the above graph, the residual do not have heterchoterchisty after the second ARIMA fitting.

### GARCH model

Firstly: AR (3) estimate

Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	0.03130	0.0052379	5.98	<.0001	0
AR1, 1	0.42548	0.07445	5.71	<.0001	1
AR1, 2	-0.18593	0.08117	-2.29	0.0220	2
AR1, 3	0.44352	0.07298	6.08	<.0001	3

$Z_t$  is the ratio of standard deviation

$$X_t = Z_t - 0.03130$$

$$X_t - 0.42548 * X_{t-1} + 0.18593 * X_{t-2} - 0.44352 * X_{t-3} = W_t = y_t = \sigma_t \varepsilon_t$$

where  $w_t \sim N(0, \hat{\sigma}_t^2)$ .

Secondly: Separate GARCH with MA (1)

Parameter	Estimate	Error	t Value	Pr >  t	Lag
MU	0.0004726	0.0001784	2.65	0.0081	0

MA(1, 1)      -0.66414      0.06542      -10.15      <.0001      1

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$y_t^2 = (\sigma_t^2) + (y_t^2 - \sigma_t^2) = \alpha_0 + (\alpha_1 + \beta_1) y_{t-1}^2 - \beta_1 v_{t-1} + v_t$$

Where  $v_t = y_t^2 - \sigma_t^2 = \sigma_t^2 \varepsilon_t - \sigma_t^2 = \sigma_t^2 (\varepsilon_t - 1)$  plays the role of “wt” in a regular ARMA (1,1) model.

Since there is no AR effect, so  $(\alpha_1 + \beta_1) = 0$ .

-  $\beta_1 = -0.66414$ , then  $\beta_1 = 0.66414$ ,  $\alpha_1 = -0.66414$

Actually, the GARCH (1, 1) process on  $\sigma_t^2$ , change to ARMA (1, 1) process on  $y_t^2$ , and the parameters change too.

Table 9: parameters estimation for separate fitting model

	Intercept estimate	Parameter on $y_{t-1}^2$	Parameter
GARCH (1,1) on $\sigma_t^2$	$\alpha_0 =$ 0.0004726	$\alpha_1 = -0.66414$	$\beta_1 = 0.66414$ , Parameter on $\sigma_{t-1}^2$
ARMA (1,1) on $y_t^2$	$\alpha_0 =$ 0.0004726	$\alpha_1 + \beta_1 = 0$	- $\beta_1 = -0.66414$ , Parameter on $v_{t-1}$

So the parameter will changes, except the intercept will keep the same.

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$\hat{\sigma}_t^2 = 0.0004726 - 0.66414 * y_{t-1}^2 \text{est} + 0.66414 * \sigma_{t-1}^2 \text{est}$$

Attention: actually, it changes to GARCH (1, 1) model.

In a summary, the separately fitting process has the result of

$Z_t$  is the ratio of standard deviation

$$X_t = Z_t - 0.03130$$

$$X_t = 0.42548 * X_{t-1} + 0.18593 * X_{t-2} - 0.44352 * X_{t-3} = W_t = y_t = \sigma_t \varepsilon_t$$

$$\hat{\sigma}_t^2 = 0.0004726 - 0.66414 * y_{t-1}^2_{est} + 0.66414 * \sigma_{t-1}^2_{est}$$

### AR--GARCH jointly (simultaneously) fitting

Table 10: jointly fitting model selection

GARCH RATIO	BIC	AIC	Summary	Conclusion
ARCH(1), not lag	-738	-747	ARCH1 estimate is bigger than 1, not common	
ARCH(2), not lag	-733	-745	Coefficient of ARCH2 is not significant.	
Jointly AR(3) with ARCH(1) garch = (q=1) nlag=3	-775	-793	Coefficient of AR2 is not significant.	
Jointly AR(3) with ARCH(2) garch = (q=2) nlag=3	-775	-793	Coefficient of AR2 and ARCH2 are not significant.	
Jointly AR(3) with ARCH(3) garch = (q=3) nlag=3	-770	-791	Coefficient of AR2 and ARCH2, ARCH3 are not significant.	
Jointly nlag (1 3) with ARCH(1) garch = (q=1) nlag=(1 3)	-777	-792	All Coefficient are significant. ARCH1 estimate is bigger than 1, not common	BEST MODEL. Lowest AIC and BIC
Jointly nlag (1 3) with ARCH(2) garch = (q=2) nlag=(1 3)	-777	-792	Coefficient of ARCH2 is not significant.	
Jointly nlag (1 3) with GARCH(1, 1) garch = (p=1,q=1) nlag=(1 3)	-777	-792	Coefficient of GARCH1 is not significant.	

### Estimators of Jointly nlag (1 3) with ARCH (1) model

Variable	DF	Estimate	Standard Error	t Value	Approx Pr >  t
Intercept	1	0.0292	0.002275	12.84	<.0001
AR1	1	-0.2641	0.0517	-5.11	<.0001
AR3	1	-0.2889	0.0482	-5.99	<.0001
ARCH0	1	0.000143	0.0000282	5.06	<.0001
ARCH1	1	1.1035	0.2364	4.67	<.0001

$Z_t$  is the ratio of standard deviation

$$X_t = Z_t - 0.0292$$

$$X_t + 0.2641 * X_{t-1} + 0.2889 * X_{t-3} = W_t = y_t = \sigma_t \varepsilon_t$$

Where  $w_t \sim N(0, \hat{\sigma}_t^2)$ .

$$y_t^2 \text{ est} = 0.000143 + 1.1035 * y_{t-1}^2 \text{ est}$$

$$\hat{\sigma}_t^2 = 0.000143 + 1.1035 * y_{t-1}^2 \text{ est}$$

Check residual sq from ARCH model with ACF/PACF

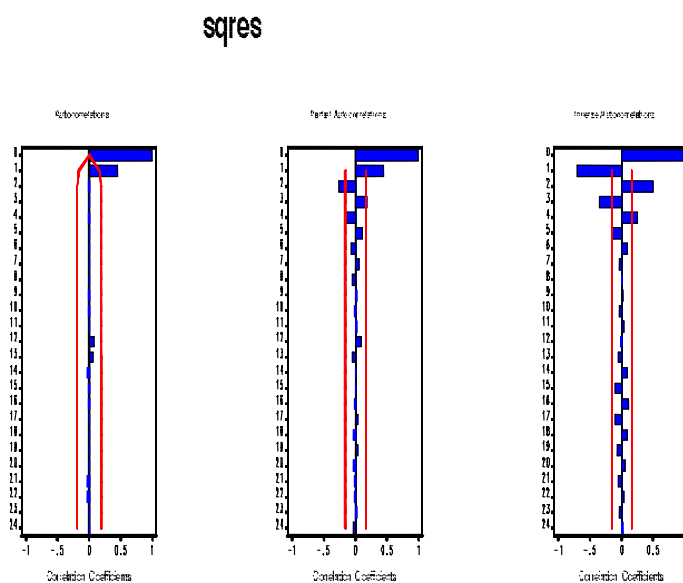


Figure 28: ACF/PACF graph for Residual Square of jointly fitting model

From the first Joint model ( garch = (q=1), nlag=(1 3)), the residual sq ACF/PACF, it looks bad.

We try some other jointly models

Table 11: jointly fitting model selection with SAS command

GARCH RATIO	BIC	AIC	Summary	Conclusion
Jointly nlag (1 3) with ARCH(1) garch = (q=1) nlag=(1 3)	-777	-792	All Coefficients are significant. ARCH1 estimate is bigger than 1, not common	Lowest AIC and BIC, but residual sq has problem



Jointly AR(3) with ARCH(1) garch = (q=1) nlag=3	-775	-793	Coefficient of AR2 is not significant.	residual sq has problem, similar to Jointly nlag (1 3) with ARCH(1) model
Jointly AR(3) with GARCH(1,1) garch = (p=1,q=1) nlag=3	-775	-793	Coefficient of GARCH1 is not significant.	
Jointly nlag (1 3) with GARCH(1, 1) garch = (p=1,q=1) nlag=(1 3)	-777	-792	Coefficient of GARCH1 is not significant.	

### Conclusion:

(1) In the terms of residual squared, we did not find the good enough joint model to fit the ratio series.

(2) Even we skip the residual square checking, the coefficient of ARCH1 is bigger than one, that is still a problem to keep the process stationary.

### Compare jointly and separately models

$Z_t$  is the ratio of standard deviation

Separately fitting process has the result (**reliable**)

$$X_t = Z_t - 0.03130$$

$$X_t - 0.42548 * X_{t-1} + 0.18593 * X_{t-2} - 0.44352 * X_{t-3} = W_t = y_t = \sigma_t \varepsilon_t$$

$$\hat{\sigma}_t^2 = 0.0004726 - 0.66414 * y_{t-1}^2 \text{ est} + 0.66414 * \sigma_{t-1}^2 \text{ est}$$

Jointly fitting process has the result (**not reliable**)

$$X_t = Z_t - 0.0292$$

$$X_t + 0.2641 * X_{t-1} + 0.2889 * X_{t-3} = W_t = y_t = \sigma_t \varepsilon_t$$

$$\hat{\sigma}_t^2 = 0.000143 + 1.1035 * y_{t-1}^2 \text{ est}$$

### **Conclusion for volatility ratio analysis (Hong Kong bond/stock)**

GARCH model is really good in Hong Kong ratio series. More specifically, we use separate model to get GARCH model and it can be used to accurately predict the relative standard deviation between Hong Kong bond market and stock market.

## CHAPTER 5

### CONCLUSIONS ON HONG KONG FINANCIAL MARKETS

According to the portfolio theory, if an asset has a lower standard deviation than that of another, it is always of great benefit to diversify between these two assets. Both Schwert [1989, 1990] and Frank [2000] indicated that the average volatility for the U.S. bond market is about one third of that for the U.S. stock market from 1950s to 1970s. However, because the volatility of the U.S. bond market dramatically increased to around eight-tenth of that of the U.S. stock market during the 1980s and 1990s, the U.S. bond market had been losing more and more diversification function.

Portfolio managers and fund managers alike, who want to make global diversification by investing in the Hong Kong bonds and stocks, should have information about the volatility of the Hong Kong bond market and the stock market, respectively, and their relationships. Research is very critical to make decisions on asset-allocation and balance the two main investment goals, namely, maximum returns and minimum risks.

In this study, the results show that in the period 1991-2004 of interest the average of monthly standard deviation for Hong Kong bond market is significantly smaller than that of the corresponding stock market, and the volatility for Hong Kong bond market is about one-tenth of the volatility for the Hong Kong stock market. These facts strongly suggest the efficiency of using Hong Kong Bond market as a diversification vehicle for people who invest in Hong Kong stock market.

### Hong Kong bond market volatility

Study on the monthly volatility of the Hong Kong bond market shows that none of the parameters in the GARCH model is statistically significant. Hence, the assumption of constant variance in Hong Kong Bond market still holds and it is not necessary to involve a rather sophisticated GARCH model. In other words, the GARCH model is not superior to the commonly employed ARIMA model. In fact, a simple mean model is sufficient to model Hong Kong bond market.

### Hong Kong stock market volatility

As for the Hong Kong stock market volatility, the coefficients of lagged terms of stock volatility series are significantly different from zero, suggesting that the volatility for Hong Kong stock market depends on that of the previous month. In addition, it also shows that Hong Kong stock market has pattern of non-constant variance, which makes the ARIMA model not an appropriate one for Hong Kong stock market. We find that a GARCH model is appropriate and that all of the GARCH parameters are statistically significant. In conclusion, a GARCH model is preferable to predict the volatility for Hong Kong stock market.

### Hong Kong bond/stock volatility ratio characters

Our analysis on volatility of the ratio of Hong Kong bond market and the stock market gives detailed information to make portfolio-allocation decisions. Only when the ratio is forecasted accurately, the fund can be properly allocated. For example, given the maximum

acceptable level of risk (volatility), if the ratio increases, more funds need to be allocated to the bonds, and less funds to stocks. Real data in Hong Kong shows that GARCH model is preferable to predict the volatility ratio.

In summary, we conclude that (1) the Hong Kong Bond market has constant variance and a mean/null model is appropriate, (2) the Hong Kong stock market has the non-constant variance pattern and a GARCH model is appropriate, and (3) the volatility ratio has the non-constant variance pattern and GARCH model is appropriate.

#### Future works

It is a challenge to deal with outliers, as observed in the year 1997. Some reasons might contribute to such pattern. First, Hong Kong was handed over to China in 1997; secondly, the breakout of Asian financial crisis began in the same year.

Beyond the aforementioned problems, some improvement or extension of the model, such as seasonal adjustment model, event study model and multivariate model, can also be our future direction of research.

GARCH model has some built-in weakness, and updated models can be created to mitigate these shortcomings. For example, asymmetric GARCH model and regime shifting model offers more flexibility in predicting the volatility than traditional GARCH model.

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## APPENDICES

### Appendix A: Hong Kong financial market dataset, monthly, July 1991 to May 2004

MONTHID	YEAR	MONTH	B_DAY_SUM	BD_AV	BD_STDEV	SK_AV	SK_STDEV	RATIO
7/1/1991	1991	7	23	6.667391	0.104804	1.422367	2.612895	0.04011
8/1/1991	1991	8	21	6.382857	0.049713	0.052402	8.636481	0.005756
9/1/1991	1991	9	20	6.0725	0.303191	-0.18367	2.109098	0.143754
10/1/1991	1991	10	22	5.544091	0.448581	0.347899	2.385752	0.188025
11/1/1991	1991	11	21	4.937143	0.17413	0.487183	3.414945	0.050991
12/1/1991	1991	12	20	4.6015	0.215511	0.650565	3.107167	0.069359
1/1/1992	1992	1	22	4.420455	0.208749	1.144797	2.351069	0.088789
2/1/1992	1992	2	17	4.586471	0.149998	1.491575	3.224497	0.046518
3/1/1992	1992	3	22	4.764545	0.06885	0.042595	2.989132	0.023033
4/1/1992	1992	4	20	4.5085	0.126503	1.568545	5.370406	0.023555
5/1/1992	1992	5	21	4.098095	0.163848	2.188919	4.152059	0.039462
6/1/1992	1992	6	20	3.9345	0.052663	0.083606	3.115327	0.016905
7/1/1992	1992	7	22	3.455909	0.170424	-0.5863	4.789998	0.035579
8/1/1992	1992	8	20	3.174	0.089643	-0.75855	5.656877	0.015847
9/1/1992	1992	9	22	3.118182	0.045527	-0.3513	3.460429	0.013157
10/1/1992	1992	10	21	3.55619	0.410859	2.079793	5.171494	0.079447
11/1/1992	1992	11	21	4.10381	0.21306	-1.05392	5.891944	0.036161
12/1/1992	1992	12	22	4.839545	0.259587	-0.73272	10.26657	0.025285
1/1/1993	1993	1	18	4.369444	0.281121	0.888813	4.588456	0.061267
2/1/1993	1993	2	20	3.623	0.241336	1.837341	3.95222	0.061063
3/1/1993	1993	3	23	3.365652	0.077625	0.13971	5.986786	0.012966
4/1/1993	1993	4	19	3.193158	0.106304	1.329295	5.83736	0.018211
5/1/1993	1993	5	21	3.303333	0.2573	1.347447	3.779368	0.06808
6/1/1993	1993	6	20	3.731	0.145743	-0.66365	4.293251	0.033947
7/1/1993	1993	7	22	3.807727	0.056563	-0.2469	3.129089	0.018076
8/1/1993	1993	8	21	3.531905	0.213509	1.369372	4.441565	0.048071
9/1/1993	1993	9	21	3.272857	0.027775	0.301111	3.05826	0.009082
10/1/1993	1993	10	20	3.146	0.101224	3.619209	5.765286	0.017558
11/1/1993	1993	11	22	3.362273	0.114681	-0.30175	7.048858	0.016269
12/1/1993	1993	12	22	3.494545	0.036869	4.48006	7.07767	0.005209
1/1/1994	1994	1	21	3.426667	0.064679	-0.4415	10.85801	0.005957
2/1/1994	1994	2	18	3.787222	0.205936	-1.85811	10.04073	0.02051
3/1/1994	1994	3	23	4.37087	0.098899	-2.1417	9.12117	0.010843
4/1/1994	1994	4	18	4.656667	0.183431	-0.08727	6.606825	0.027764



5/1/1994	1994	5	22	5.262727	0.177205	1.135351	7.896213	0.022442
6/1/1994	1994	6	20	5.2255	0.150139	-1.55425	4.649647	0.03229
7/1/1994	1994	7	21	5.34381	0.052103	1.41555	4.956212	0.010513
8/1/1994	1994	8	22	5.295455	0.065591	0.798013	5.089622	0.012887
9/1/1994	1994	9	21	5.30619	0.075065	-0.71109	3.682839	0.020382
10/1/1994	1994	10	20	5.7845	0.13983	0.259709	4.063739	0.034409
11/1/1994	1994	11	22	6.092727	0.118007	-2.1238	5.102164	0.023129
12/1/1994	1994	12	20	6.783	0.16203	-0.55647	5.961034	0.027182
1/1/1995	1995	1	20	7.3425	0.250974	-1.90731	7.935583	0.031626
2/1/1995	1995	2	18	6.986667	0.17476	2.634187	7.574001	0.023074
3/1/1995	1995	3	23	6.33087	0.272845	0.52606	5.363543	0.05087
4/1/1995	1995	4	17	6.026471	0.103498	-0.5521	4.103756	0.02522
5/1/1995	1995	5	23	6.005217	0.129224	1.904362	4.661015	0.027724
6/1/1995	1995	6	20	5.6235	0.101581	-0.37561	3.746852	0.027111
7/1/1995	1995	7	21	5.581429	0.080702	0.481846	4.077621	0.019792
8/1/1995	1995	8	22	5.78381	0.13336	-0.47506	2.991903	0.044573
9/1/1995	1995	9	21	5.820952	0.045156	0.872484	2.784664	0.016216
10/1/1995	1995	10	22	5.652273	0.045349	0.247188	3.354085	0.01352
11/1/1995	1995	11	21	5.56619	0.019359	0.06848	3.237685	0.005979
12/1/1995	1995	12	19	5.558421	0.037898	0.510655	2.450493	0.015465
1/1/1996	1996	1	22	5.289545	0.098922	2.016768	3.669933	0.026955
2/1/1996	1996	2	18	5.006667	0.109491	-0.40576	3.517469	0.031128
3/1/1996	1996	3	21	5.472381	0.215567	-0.18144	7.898029	0.027294
4/1/1996	1996	4	19	5.526842	0.067909	0.027004	3.311831	0.020505
5/1/1996	1996	5	23	5.573478	0.043444	0.438898	2.761656	0.015731
6/1/1996	1996	6	18	5.787222	0.095536	-0.43296	2.839243	0.033648
7/1/1996	1996	7	23	5.866957	0.071442	-0.48127	3.366486	0.021222
8/1/1996	1996	8	21	5.622381	0.064258	0.770189	2.643609	0.024307
9/1/1996	1996	9	21	5.714762	0.09963	1.133145	2.836779	0.035121
10/1/1996	1996	10	22	5.418636	0.110423	0.795844	3.03561	0.036376
11/1/1996	1996	11	21	5.22381	0.058521	1.246777	3.162829	0.018503
12/1/1996	1996	12	20	5.109	0.054955	0.105152	4.531509	0.012127
1/1/1997	1997	1	22	5.095455	0.031582	-0.13636	4.330884	0.007292
2/1/1997	1997	2	18	5.104444	0.086448	0.132933	3.542007	0.024406
3/1/1997	1997	3	19	5.492632	0.121876	-1.26267	3.535008	0.034477
4/1/1997	1997	4	22	5.866364	0.096587	0.509891	4.627191	0.020874
5/1/1997	1997	5	22	5.818636	0.033707	2.250222	3.449385	0.009772
6/1/1997	1997	6	19	5.870526	0.057491	0.615174	6.325085	0.009089
7/1/1997	1997	7	21	6.119048	0.351339	1.312682	4.160443	0.084447
8/1/1997	1997	8	20	6.669	0.428828	-2.60022	6.949882	0.061703
9/1/1997	1997	9	21	6.604762	0.12793	1.208599	9.531497	0.013422
10/1/1997	1997	10	20	8.243	2.621836	-5.57091	24.00093	0.109239

11/1/1997	1997	11	20	9.675	0.666945	-0.02541	10.49213	0.063566
12/1/1997	1997	12	21	9.120952	0.458584	0.432945	9.243971	0.049609
1/1/1998	1998	1	18	10.47778	0.638164	-2.61896	16.63151	0.038371
2/1/1998	1998	2	20	8.7645	0.676216	4.177474	13.36471	0.050597
3/1/1998	1998	3	22	7.583182	0.328654	0.103346	6.058047	0.054251
4/1/1998	1998	4	19	7.16	0.214139	-1.95938	4.641521	0.046136
5/1/1998	1998	5	21	7.940476	0.461514	-2.54722	6.489366	0.071118
6/1/1998	1998	6	22	9.714545	1.06454	-0.56134	11.8018	0.090201
7/1/1998	1998	7	22	9.300455	0.209841	-1.14856	7.424888	0.028262
8/1/1998	1998	8	21	10.094	0.54228	-1.29275	12.92596	0.041953
9/1/1998	1998	9	22	9.209545	0.784429	1.49587	11.15344	0.070331
10/1/1998	1998	10	18	7.235	0.421527	5.341137	11.66267	0.036143
11/1/1998	1998	11	21	6.576667	0.25293	0.494632	7.650373	0.033061
12/1/1998	1998	12	22	6.041364	0.160039	-0.51467	6.707796	0.023859
1/1/1999	1999	1	20	6.442	0.645287	-0.9124	8.657452	0.074535
2/1/1999	1999	2	17	6.660588	0.149268	0.8257	5.927299	0.025183
3/1/1999	1999	3	23	6.389565	0.139691	1.708933	6.19935	0.022533
4/1/1999	1999	4	19	5.675263	0.210696	3.879486	7.009163	0.03006
5/1/1999	1999	5	21	5.708571	0.15599	-1.57802	5.345737	0.02918
6/1/1999	1999	6	21	5.915	0.050315	1.918032	5.329379	0.009441
7/1/1999	1999	7	21	5.851905	0.024417	-0.39887	6.222198	0.003924
8/1/1999	1999	8	22	5.942857	0.053772	0.421659	6.392544	0.008412
9/1/1999	1999	9	21	6.033333	0.106599	-0.96078	4.972064	0.02144
10/1/1999	1999	10	19	5.952632	0.026842	0.824993	6.215326	0.004319
11/1/1999	1999	11	22	5.677273	0.098134	2.496523	4.540471	0.021613
12/1/1999	1999	12	21	5.640476	0.152298	1.750342	5.639023	0.027008
1/1/2000	2000	1	21	5.965714	0.053439	-1.40661	9.524536	0.005611
2/1/2000	2000	2	19	6.108947	0.042413	1.993802	7.028384	0.006034
3/1/2000	2000	3	23	6.073913	0.035386	0.290803	7.466707	0.004739
4/1/2000	2000	4	17	6.196471	0.093337	-2.31992	10.11117	0.009231
5/1/2000	2000	5	21	6.817619	0.122756	-0.83637	8.287857	0.014812
6/1/2000	2000	6	21	6.65619	0.07934	1.671143	5.731646	0.013842
7/1/2000	2000	7	21	6.406667	0.081813	0.756359	5.079374	0.016107
8/1/2000	2000	8	23	6.109565	0.116012	0.271429	4.902518	0.023664
9/1/2000	2000	9	20	6.1605	0.068401	-1.52725	8.089352	0.008456
10/1/2000	2000	10	20	6.0465	0.056033	-0.84239	6.646462	0.008431
11/1/2000	2000	11	22	5.955455	0.039608	-0.99361	6.288254	0.006299
12/1/2000	2000	12	19	5.603684	0.148669	1.522188	6.244847	0.023807
1/1/2001	2001	1	19	4.865789	0.171669	1.287608	5.918881	0.029004
2/1/2001	2001	2	20	4.7545	0.089176	-1.53476	3.505811	0.025437
3/1/2001	2001	3	22	4.625909	0.039359	-2.38602	6.288723	0.006259
4/1/2001	2001	4	17	4.511176	0.19522	1.121326	8.492817	0.022987

5/1/2001	2001	5	22	3.904091	0.115001	-0.22632	5.391667	0.021329
6/1/2001	2001	6	20	3.6965	0.098155	-0.15508	4.681471	0.020967
7/1/2001	2001	7	19	3.65	0.080277	-1.07722	4.025525	0.019942
8/1/2001	2001	8	23	3.377826	0.084743	-1.62426	5.245406	0.016156
9/1/2001	2001	9	20	2.804	0.398872	-1.80541	11.24592	0.035468
10/1/2001	2001	10	21	2.193	0.072627	0.296087	7.952803	0.009132
11/1/2001	2001	11	22	2.035909	0.180598	1.908188	4.677193	0.038613
12/1/2001	2001	12	19	2.141053	0.103703	0.238993	5.470936	0.018955
1/1/2002	2002	1	22	2.085	0.110227	-0.97622	4.828641	0.022828
2/1/2002	2002	2	17	2.179444	0.174372	-0.46329	4.653515	0.037471
3/1/2002	2002	3	20	2.612857	0.332868	0.960884	4.476624	0.074357
4/1/2002	2002	4	20	2.516667	0.263521	0.771576	3.717941	0.070878
5/1/2002	2002	5	21	2.275455	0.183581	-0.28171	3.567988	0.051452
6/1/2002	2002	6	20	2.119048	0.221222	-1.15322	3.653412	0.060552
7/1/2002	2002	7	22	1.825455	0.108309	-0.48764	5.431736	0.01994
8/1/2002	2002	8	22	1.523182	0.049894	-0.33666	4.665164	0.010695
9/1/2002	2002	9	21	1.655	0.096053	-1.73549	4.689378	0.020483
10/1/2002	2002	10	21	1.760909	0.146252	0.731935	5.432553	0.026921
11/1/2002	2002	11	21	1.564545	0.0653	1.145012	4.210642	0.015508
12/1/2002	2002	12	20	1.4605	0.038862	-1.39179	3.417113	0.011373
1/1/2003	2003	1	21	1.350476	0.019099	-0.10052	3.506404	0.005447
2/1/2003	2003	2	19	1.283158	0.020831	-0.27179	3.166615	0.006578
3/1/2003	2003	3	21	1.180476	0.064998	-0.92525	4.755013	0.013669
4/1/2003	2003	4	20	1.199	0.077521	0.205706	4.955992	0.015642
5/1/2003	2003	5	20	1.1225	0.052202	1.561387	3.166904	0.016483
6/1/2003	2003	6	20	0.926	0.071333	0.185509	3.248579	0.021958
7/1/2003	2003	7	22	0.989091	0.054064	0.957705	3.666782	0.014744
8/1/2003	2003	8	21	1.148571	0.021044	1.295221	3.218226	0.006539
9/1/2003	2003	9	21	1.01	0.11077	0.526098	4.101736	0.027006
10/1/2003	2003	10	22	0.621364	0.119335	1.396077	4.956786	0.024075
11/1/2003	2003	11	20	0.6735	0.162457	0.21332	4.252247	0.038205
12/1/2003	2003	12	21	0.42381	0.127925	0.374466	3.2077	0.039881
1/1/2004	2004	1	19	0.22	0.099331	1.083336	4.088811	0.024293
2/1/2004	2004	2	20	0.2725	0.075941	0.848877	3.815584	0.019903
3/1/2004	2004	3	23	0.349565	0.049311	-1.44492	3.470673	0.014208
4/1/2004	2004	4	19	0.562632	0.207441	-1.12945	4.071929	0.050944
5/1/2004	2004	5	20	1.212	0.177456	0.4357	6.172168	0.028751

## Appendix B: SAS Command

### SAS code for HK bond stdev sq analysis

```
/*====hk bond stdev sq analysis====*/
option formdlim='=';

/* import the dataset with name of simon2*/
/* create new series of log and sqroot */
data total;
set simon2;
sqbd_stdev=(bd_stdev)**2;
logsqbd_stdev=log(sqbd_stdev);
Num=_N_;
proc print;
run;

/*====bond stdev sq analysis====*/
/* model selection*/
proc arima data= total;
  identify var=logsqbd_stdev minic p=(0:10) q=(0:10);
run;

/*==== use Forecasting system, not SAS command to get result====*/

proc arima data= total;
  identify var=logsqbd_stdev minic p=(0:10) q=(0:10);
  estimate p=1 q=0 plot method=ml printall;
  estimate p=3 q=0 plot method=ml printall;
  estimate p=0 q=8 plot method=ml printall;
  estimate p=8 q=0 plot method=ml printall;
run;
```

### SAS code for HK sk stdev sq analysis

```
/*====sk stdev sq analysis====*/
option formdlim='=';
```

```

/* import the dataset with name of simon2*/
/* create new series of log and sqroot */
data total;
set simon2;
sqsk_stdev=(sk_stdev)**2;
logsqsk_stdev=log(sqsk_stdev);
Num=_N_;
proc print;
run;

/*====sk stdev sq analysis====*/
/* model selection*/
proc arima data= total;
  identify var=logsqsk_stdev minic p=(0:10) q=(0:10);
  estimate p=1 q=0 plot method=ml printall;
  estimate p=0 q=4 plot method=ml printall;
  estimate p=0 q=5 plot method=ml printall;
  estimate p=8 q=0 plot method=ml printall;
  estimate p=(1,8) q=0 plot method=ml printall;
forecast out=ar18_out id=monthid lead=12 alpha=0.05;
run;

/* find the best model p=(1 3)(12) */

/* forecast from best model*/
proc arima data= total;
  identify var=logsqsk_stdev;
  estimate p=(1,8) q=0 plot method=ml printall;
  forecast out=ar18_out id=monthid lead=12 alpha=0.05;
run;

/* 2 method of QQ plot*/
proc univariate data=ar18_out plot;
  var residual; /* var sqres*/
run;

proc univariate data=ar18_out;
qqplot residual/ normal;
run;

proc print data=ar18_out;
run;

```

```

data ar18_out2;
set ar18_out;
num=_N_;
forecast_stdev=exp(forecast);
run;

/* plot of entire data*/

proc gplot data=ar18_out;
  plot logsqsk_stdev*monthid forecast*monthid L95*monthid U95*monthid / overlay
haxis=axis1
      vaxis=axis2 frame legend=legend1;
  axis1 label = ("year and month");
  axis2 label = (a=90 "HK stock monthly standard deviation");
  title2 "Forecast plot for HK stock monthly standard deviation";
  symbol1 i=join h=.1 v=dot cv=black l=1 ci=black;
  symbol2 i=join h=.1 v=dot cv=red l=2 ci=red;
  symbol3 i=join l=1 r=2 ci=blue;
  symbol5 i=join h=.25 v=dot l=3 ci=green;
  legend1 label = none
        position = (bottom center outside)
        across = 4
        down = 1
        mode = reserve
        frame
        offset =(0.5, 0.5)
        shape = line(0.5)
        value = (j=1 h=0.3 'Observed' 'Forecast' 'C.I.' 'C.I.');
```

```

run;

/* plot of forecasting data*/
proc gplot data=ar18_out2;

where num> 130;      /* where command to limit the display data*/
/* here monthid has some problem, it keep the same during the forecasting, so
i
change to plot based on num*/

plot logsqsk_stdev*num forecast*num L95*num U95*num / overlay haxis=axis1
      vaxis=axis2 frame legend=legend1;
  axis1 label = ("year and month");
  axis2 label = (a=90 "HK stock monthly standard deviation");

```

```

title2 "Forecast plot for HK stock monthly standard deviation";
symbol1 i=join h=.1 v=dot cv=black l=1 ci=black;
symbol2 i=join h=.1 v=dot cv=red l=2 ci=red;
symbol3 i=join l=1 r=2 ci=blue;
symbol5 i=join h=.25 v=dot l=3 ci=green;
legend1 label = none
      position = (bottom center outside)
      across = 4
      down = 1
      mode = reserve
      frame
      offset =(0.5, 0.5)
      shape = line(0.5)
      value = (j=1 h=0.3 'Observed' 'Forecast' 'C.I.' 'C.I.');
```

**run;**

```

/* forecast from best model*/
proc arima data= total;
  identify var=log((sk_stdev)**2);      * this line is wrong;
  estimate p=(1,8) q=0 plot method=ml printall;
  forecast out=ar18_auto_out id=monthid lead=12 alpha=0.05;
run;
```

## SAS code for ratio analysis

```

/*====ratio analysis====*/
option formdlim='';

/* import the dataset with name of simon2*/
/* create new series of log and sqrt */
data total;
set simon2;
logratio=log(ratio);
sqrtratio=sqrt(ratio);
* proc print;
run;
```

```

/*====ratio analysis====*/
/* model selection*/
proc arima data= total;
* identify var=logratio(1) minic p=(0:10) q=(0:10);
  identify var=ratio minic p=(0:10) q=(0:10);
  estimate p=3 q=0 plot method=ml noint printall;
```

```

estimate p=(1 3 6) q=0 plot method=ml noint printall;
estimate p=(1 3) q=0 plot method=ml noint printall;
estimate p=3 q=0 plot method=ml printall;
forecast out=ratio_sepout id=monthid lead=12 alpha=0.05;
run;

/* find the first ARIMA model of AR(3) with intercept */
/* forecast from best model*/
proc arima data= total;
  identify var=ratio;
  estimate p=3 q=0 plot method=ml printall;
  forecast out=ratio_sepout id=monthid lead=12 alpha=0.05;
run;

/* create garch dataset, take square of residual*/
data ratio_sep_garch;
set ratio_sepout;
sqres=residual**2;
num=_N_;
proc print;
run;

/* 2 method of QQ plot*/
proc univariate data=ratio_sep_garch plot;
  var residual; /* var sqres*/
run;

proc univariate data=ratio_sep_garch;
qqplot residual sqres/ normal;
run;

/* minic to check the garch model */
proc arima data=ratio_sep_garch;
identify var=sqres minic p=(0:10) q=(0:10);
run;

/* consequently GARCH fitting */
proc arima data=ratio_sep_garch;
  identify var=sqres minic p=(0:10) q=(0:10);
  estimate p=3 q=0 plot method=ml noint printall;
  estimate p=3 q=0 plot method=ml printall;

```



```

estimate p=0 q=1 plot method=ml noint printall;
estimate p=0 q=1 plot method=ml      printall;

estimate p=1 q=0 plot method=ml noint printall;
estimate p=1 q=0 plot method=ml      printall;

estimate p=2 q=0 plot method=ml      printall;
estimate p=4 q=0 plot method=ml      printall;

run;

/* consequently GARCH fitting final model MA(1) with int */
proc arima data=ratio_sepgarch;
  identify var=sqres minic p=(0:10) q=(0:10);
  estimate p=0 q=1 plot method=ml      printall;
  forecast out=check_sepout id=monthid lead=12 alpha=0.05;
run;

/* check residual sq */
data check_sepgarch;
set check_sepout;
sqres=residual**2;
num=_N_;
proc print;
run;

proc arima data=check_sepgarch;
  identify var=sqres minic p=(0:10) q=(0:10);
run;

/* ===== jointly model ===== */
/* ===== ratio ===== */

proc autoreg data =total;

model ratio = /garch = (p=1, q=1) ;

/*
model ratio = /garch = (p=1, q=2) ;
model ratio = /garch = (p=0, q=1)      nlag=3;
model ratio = /garch = (p=0, q=2)      nlag=3;
model ratio = /garch = (p=1, q=3)      nlag=3;

```

```

model ratio = /garch = (p=0, q=1)          nlag=(1 3);
model ratio = /garch = (p=0, q=2)          nlag=(1 3);
model ratio = /garch = (p=1, q=1)          nlag=(1 3);
*/
output out=comb_out cev=cev p=p r=residual;
run;

/* ===== jointly model final model: garch = (q=1)nlag=(1 3)===== */

proc autoreg data =total;
model ratio = /garch = (q=1)nlag=(1 3) ;
output out=joint_out cev=cev p=p r=residual;
run;

/* check residual sq */
data check_jointgarch;
set joint_out;
sqres=residual**2;
num=_N_;
proc print;
run;

proc arima data=check_jointgarch;
  identify var=sqres minic p=(0:10) q=(0:10);
run;

/* ===== 2nd jointly model final model: garch = (q=1)nlag=(1 3)===== */

proc autoreg data =total;
model ratio = /garch = (q=1)nlag= 3 ;
output out=joint_out cev=cev p=p r=residual;
run;

/* check residual sq */
data check_jointgarch;
set joint_out;
sqres=residual**2;
num=_N_;
proc print;
run;

```

```
proc arima data=check_jointgarch;
  identify var=sqres minic p=(0:10) q=(0:10);
run;
```

## Appendix C: footnotes

---

① Engle, R. F. (1982). "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation", *Econometrica*, 50, pp. 987-1006.

② [The press release from The Royal Swedish Academy of Sciences can be viewed at <http://www.nobel.se/economics/laureates/2003/press.html>.]

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Daniel B. Nelson and Charles Q. Cao, 1992, "Inequality Constraints in the Univariate GARCH Model", *Journal of Business and Economic Statistics*, 10:229-235.

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Tim Bollerslev, Ray Y. Chou, and Kenneth F. Kroner. ARCH modeling in finance. *Journal of Econometrics*, 52:5-59, January 1992.

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⑧

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## Appendix D: SAS output

17

```
proc means data=two;
class year month;
var x;
output out=xout mean=bdmean std=bdstd N=daysnum;
run;

proc print data=xout; run;
```

18

---

Minimum Information Criterion

Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	0.944819	0.881865	0.881196	0.845898	0.821917	0.833188
AR 1	0.597084	0.588907	0.617896	0.623661	0.653635	0.67522
AR 2	0.60428	0.62046	0.639708	0.653372	0.682833	0.701658
AR 3	0.564131	0.592455	0.622347	0.649201	0.681377	0.703893
AR 4	0.594105	0.624516	0.653906	0.680958	0.713336	0.736344
AR 5	0.615747	0.646525	0.676326	0.704216	0.736613	0.766502
AR 6	0.639214	0.669659	0.699817	0.726903	0.759435	0.784534
AR 7	0.603699	0.633534	0.662728	0.691091	0.72129	0.749773
AR 8	0.622339	0.64756	0.675848	0.703185	0.733296	0.751266
AR 9	0.594912	0.625534	0.658071	0.686085	0.716277	0.735321
AR 10	0.601141	0.632256	0.664565	0.693981	0.724929	0.746328

Minimum Information Criterion

Lags	MA 6	MA 7	MA 8	MA 9	MA 10
AR 0	0.845271	0.770232	0.724205	0.75315	0.760407
AR 1	0.694472	0.628062	0.635675	0.611913	0.605449
AR 2	0.722992	0.659889	0.667415	0.63644	0.632829
AR 3	0.72781	0.676619	0.682181	0.662643	0.658997
AR 4	0.760273	0.708031	0.709356	0.688264	0.687995
AR 5	0.789648	0.736592	0.741569	0.716261	0.718046
AR 6	0.816682	0.754124	0.761831	0.746565	0.750542
AR 7	0.775669	0.786598	0.791787	0.773766	0.780579
AR 8	0.779325	0.801477	0.803201	0.801542	0.807346
AR 9	0.762003	0.787085	0.815927	0.834038	0.839744
AR 10	0.772334	0.80068	0.830029	0.838693	0.871002

Error series model: AR(20)

Minimum Table Value: BIC(3, 0) = 0.564131

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Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	-0.49003	-0.63836	-0.67919	-0.7224	-0.7989	-0.80261
AR 1	-1.14569	-1.14156	-1.11316	-1.09443	-1.06978	-1.07127
AR 2	-1.12366	-1.11263	-1.08101	-1.06213	-1.03758	-1.03988
AR 3	-1.11626	-1.0952	-1.06268	-1.03022	-1.00505	-1.00749
AR 4	-1.09014	-1.0647	-1.0324	-1.00147	-0.97251	-0.97648

---

AR 5	-1.0818	-1.06289	-1.0307	-0.99835	-0.96973	-0.94482
AR 6	-1.05474	-1.0411	-1.00863	-0.97623	-0.94469	-0.91419
AR 7	-1.03435	-1.02414	-0.99167	-0.95932	-0.927	-0.89594
AR 8	-1.03824	-1.02389	-0.99135	-0.95928	-0.92748	-0.8951
AR 9	-1.02602	-1.00159	-0.96907	-0.93674	-0.90457	-0.87203
AR 10	-0.99429	-0.96938	-0.93685	-0.90449	-0.87239	-0.83985

Lags	MA 6	MA 7	MA 8	MA 9	MA 10
AR 0	-0.79957	-0.77785	-0.77214	-0.7995	-0.79571
AR 1	-1.04181	-1.02987	-1.01622	-0.99612	-0.96554
AR 2	-1.0097	-0.99738	-0.9843	-0.96361	-0.93313
AR 3	-0.9774	-0.96729	-0.95432	-0.932	-0.90261
AR 4	-0.94609	-0.93525	-0.92227	-0.89971	-0.87007
AR 5	-0.9143	-0.90299	-0.88973	-0.86763	-0.83864
AR 6	-0.88252	-0.8737	-0.85919	-0.83733	-0.80668
AR 7	-0.86573	-0.84345	-0.82823	-0.8063	-0.77554
AR 8	-0.8626	-0.83049	-0.80207	-0.77778	-0.74805
AR 9	-0.83949	-0.80704	-0.7771	-0.74778	-0.7195
AR 10	-0.80733	-0.77495	-0.74557	-0.71643	-0.68772

Error series model: AR(20)

Minimum Table Value: BIC(1,0) = -1.14569

20

Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	3.19567	0.24684	12.95	<.0001	0
AR1, 1	0.51313	0.08103	6.33	<u>&lt;.0001</u>	1
AR1, 2	0.05749	0.08909	0.65	0.5187	2
AR1, 3	0.15314	0.08849	1.73	0.0835	3
AR1, 4	0.13217	0.08746	1.51	0.1307	4
AR1, 5	-0.15305	0.09088	-1.68	0.0922	5
AR1, 6	0.02075	0.09182	0.23	0.8212	6
AR1, 7	-0.12236	0.09116	-1.34	0.1795	7
AR1, 8	0.20379	0.08141	2.50	<u>0.0123</u>	8

---

Constant Estimate	0.622961
Variance Estimate	0.402997
Std Error Estimate	0.634821
AIC	308.87
SBC	336.2608
Number of Residuals	155

21

Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	3.18889	0.21725	14.68	<.0001	0
AR1,1	0.60937	0.06142	9.92	<.0001	1
AR1,2	0.16405	0.06222	2.64	0.0084	8

Constant Estimate	0.72256
Variance Estimate	0.418231
Std Error Estimate	0.646708
AIC	308.5911
SBC	317.7214
Number of Residuals	155

22

Minimum Information Criterion

Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	-7.96746	-8.01546	-8.01279	-8.07958	-8.14101	-8.15672
AR 1	-8.13331	-8.11301	-8.09056	-8.14566	-8.13295	-8.13011
AR 2	-8.12032	-8.089	-8.08627	-8.11351	-8.10123	-8.09788
AR 3	-8.22917	-8.19973	-8.17691	-8.14611	-8.12024	-8.12152
AR 4	-8.21465	-8.18509	-8.17081	-8.14036	-8.12312	-8.1125
AR 5	-8.20191	-8.1746	-8.15563	-8.13222	-8.10494	-8.08032
AR 6	-8.18178	-8.15609	-8.14546	-8.11292	-8.09343	-8.06121
AR 7	-8.19475	-8.16229	-8.13661	-8.1105	-8.07992	-8.04762
AR 8	-8.16528	-8.13302	-8.10909	-8.0802	-8.04826	-8.01579
AR 9	-8.15738	-8.12514	-8.0938	-8.0632	-8.03086	-8.0018
AR 10	-8.12816	-8.09575	-8.06394	-8.03355	-8.0011	-7.97138

Minimum Information Criterion

Lags	MA 6	MA 7	MA 8	MA 9	MA 10
AR 0	-8.12962	-8.1632	-8.17145	-8.14701	-8.11792
AR 1	-8.10078	-8.13776	-8.14012	-8.11457	-8.08546
AR 2	-8.06892	-8.10601	-8.10879	-8.08205	-8.05296
AR 3	-8.11643	-8.10545	-8.09116	-8.06329	-8.04132
AR 4	-8.09178	-8.07382	-8.0595	-8.03253	-8.01155
AR 5	-8.0593	-8.04134	-8.03897	-8.00907	-7.98727
AR 6	-8.02968	-8.01507	-8.00646	-7.97679	-7.95755
AR 7	-8.01536	-7.98296	-7.97432	-7.94484	-7.92531
AR 8	-7.98434	-7.95199	-7.95049	-7.91924	-7.91328
AR 9	-8.00669	-7.99671	-7.96463	-7.93845	-7.92343
AR 10	-7.99081	-7.97694	-7.94557	-7.91387	-7.89089

Error series model: AR(16)

Minimum Table Value: BIC(3, 0) = -8.22917

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Minimum Information Criterion

Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	-14.9001	-14.9179	-14.8909	-14.9609	-14.9762	-14.9468
AR 1	-14.889	-14.8879	-14.8603	-14.9286	-14.944	-14.9149
AR 2	-14.8616	-14.9315	-14.919	-14.8961	-14.9136	-14.8834
AR 3	-14.9977	-14.9743	-14.9422	-14.9109	-14.8817	-14.852
AR 4	-14.9665	-14.9449	-14.9138	-14.8815	-14.8498	-14.8243
AR 5	-14.9392	-14.9136	-14.8817	-14.8496	-14.8191	-14.7917
AR 6	-14.9104	-14.8839	-14.8514	-14.819	-14.7884	-14.7597
AR 7	-14.894	-14.8667	-14.8364	-14.8138	-14.7863	-14.7551
AR 8	-14.8665	-14.8405	-14.8111	-14.7843	-14.754	-14.7236
AR 9	-14.8386	-14.8116	-14.7824	-14.7553	-14.725	-14.6925
AR 10	-14.8298	-14.8004	-14.772	-14.7463	-14.7151	-14.6829

Minimum Information Criterion

Lags	MA 6	MA 7	MA 8	MA 9	MA 10
------	------	------	------	------	-------



---

AR 0	-14.9151	-14.8857	-14.8628	-14.8391	-14.8202
AR 1	-14.884	-14.8545	-14.834	-14.8096	-14.7895
AR 2	-14.8518	-14.8234	-14.802	-14.7771	-14.757
AR 3	-14.821	-14.7959	-14.778	-14.7513	-14.7347
AR 4	-14.7934	-14.7729	-14.7553	-14.7253	-14.7081
AR 5	-14.7734	-14.7515	-14.7295	-14.7032	-14.6786
AR 6	-14.7478	-14.7197	-14.6991	-14.6713	-14.6466
AR 7	-14.727	-14.6981	-14.6672	-14.639	-14.6158
AR 8	-14.6974	-14.6672	-14.6347	-14.6103	-14.5858
AR 9	-14.6705	-14.6388	-14.6064	-14.5818	-14.5561
AR 10	-14.6504	-14.6183	-14.5857	-14.5551	-14.5268

Error series model: AR(17)

Minimum Table Value: BIC(3, 0) = -14.9977