COGNITIVE STRATEGY INTERVENTION FOR SECOND GRADERS EXPERIENCING MATHEMATICS DIFFICULTIES

by

SARA E. WOODRUFF

(Under the Direction of MARTHA CARR)

ABSTRACT

The majority of research in mathematics for primary-aged children has been concentrated on students who are achieving at grade level, neglecting a much larger group, students with mathematics difficulties. The current study is a cognitive strategy intervention for second grade students experiencing mathematics difficulties. Students (n = 51) were randomly assigned to the treatment or control conditions. Students in the treatment group received six weeks of instruction addressing several components of number sense and fluency. Students were pretested and posttested on addition and subtraction accuracy, cognitive strategy use, place value, and spatial ability. ANCOVAs revealed no significant differences between the treatment group and control group following the intervention. While these results were not expected, implementation of the intervention earlier in the school year may have produced different results. Improvement for students experiencing mathematics difficulties will necessitate a shift in instructional emphasis and sustained intervention.

INDEX WORDS: Low-ability, Mathematics, Difficulties, Cognitive, Strategy Use

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SARA E. WOODRUFF

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by

SARA E. WOODRUFF

Major Professor: Martha Carr

Committee: Stacey Neuharth-Pritchett
Jonathan Templin

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
December 2009

DEDICATION

This work is dedicated to my late stepfather, William J. Battle, II and my mother, Sylvia Battle. Your love, encouragement, and persistent questions about my graduation date were integral to the completion of this thesis.

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It takes a village to raise a graduate student and I would like to recognize family, friends, committee members and other faculty and staff whose support and encouragement have been indispensable.

TABLE OF CONTENTS

Page

ACKNOWLEDGEMENTS v

LIST OF TABLES viii

LIST OF FIGURES ix

CHAPTER

1 INTRODUCTION 1

Purpose of the Study 1

How this study is original 1

2 COGNITIVE STRATEGY INTERVENTION FOR SECOND GRADERS WITH MATHEMATICS DIFFICULTIES 2

Number sense 13

3 CURRENT STUDY 14

4 METHOD 16

Participants 16

Procedure and Materials 17

Measures 17

Intervention Sessions 21

5 RESULTS 23

Did the Intervention Improve Performance? 23

Correlations 26

6 DISCUSSION 30

Number Magnitude 31

Cognitive strategy use and accuracy 32

Instructional Implications 34

Conclusion 35

REFERENCES 37

APPENDICES

A *Intervention Sessions* 45

B UNO™ Game Problems 52

C Problems Solving Strategies (Session 4) 54

D Problem Solving Strategies (Session 5) 55

E No Choice/Constructivism Activity (Session 6) 57

LIST OF TABLES

Page

Table 1: *Estimated Marginal Means (Posttest)* 23

Table 2: *Pretest and Posttest Means and Standard Deviations*...25

Table 3: *Pretest Correlations*..27

LIST OF FIGURES

Page

Figure 1: Number Magnitude Estimation Control Group 28

Figure 2: Number Magnitude Estimation Treatment Group 29

CHAPTER 1

INTRODUCTION

Purpose of the Study

The focus of this study was on improving number sense and fluency for arithmetic in second graders who struggle in mathematics. The current study attempts to address several skills and concepts proposed by NCTM (2006) to be essential for mathematics achievement for primary-aged children: developing an understanding of the base-ten number system, place value concepts, quick retrieval of addition and subtraction facts, and fluency with multi-digit addition and subtraction problems (NCTM, 2006). For the purposes of this study, number sense was assessed through cognitive strategies, place value concepts, and the mental number line. Fluency was operationalized as the ability to complete 10 addition and subtraction problems under a timed condition. NCTM (2006) emphasizes that such skills and concepts are not intended to be taught in isolation and are connected to one another. The instruction provided in the intervention, therefore, included instruction on both the base-ten system and fluency, cognitive strategy use, and the mental number line.

How this study is original

One of the key motivations for this study is absence of research on primary-aged students who face challenges in mathematics (Baker, Gersten, & Lee, 2002) who are not formally identified with a specific intellectual or learning disability. Mathematics intervention research that has been conducted with samples of children with disabilities has focused on students who are identified, and receiving services through special education programs (Bryant, 2005). Only six to seven percent of grade school children have neurological or cognitive deficits that qualify them as mathematically disabled (MD) (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974). There is, however, another much larger group of students who exhibit low performance in mathematics but who do not qualify as mathematically disabled (Fuchs et al., 2005). These students are referred to in the literature as having mathematics difficulties (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008). The paucity of research on this group of children is unfortunate because they comprise a much larger percentage of the general population. The focus of this study was on students who have mathematics difficulties, not those who are formally identified with disabilities. Specifically, the focus is on children who are performing below average and who are likely to fail mathematics, but who have not been formally diagnosed with a mathematics disability.

CHAPTER 2

COGNITVE STRATEGY INTERVENTION FOR SECOND GRADERS WITH MATHEMATICS DIFFICULTIES

The premise of this study was to examine whether instruction would advance students' number sense in the form of place value knowledge, understanding and use of the mental number line, and children's cognitive strategy use, and mathematical fluency. Also examined in this study was the impact of spatial ability on the intervention. Spatial ability is usually neglected in research on mathematics difficulties and disabilities but it has been found to predict mathematics achievement (Geary, 1993). Therefore, the study investigated whether spatial ability would impact the effectiveness of instruction and whether instruction affects spatial ability. Below is a review of the literature on number sense, fluency, and spatial ability.

Number sense

Number sense is not a singular characteristic or ability, but rather a set of skills that result in mastery of basic number properties and advanced number skills (Aunio, 2005). Berch (2005) describes number sense as "an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line" (p. 1). Gersten and Chard (1999) depict number sense as a child's ability to fluidly and flexibly work with numbers, make numerical comparisons, and perform mental mathematics. In addition, Aunio (2005) operatioanlizes number sense as the dynamic ability to understand the meaning of numbers and form correct mathematical statements. Number sense has also been defined as capacity to compare and classify numbers, exhibit one-to-one correspondence, and seriate (Smith, 2002). Kalchman, Moss, and Case (2001) define number sense as being comprised of the following: "(a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representation for a given situation, and (e) ability to represent the same number or function in multiple ways, depending on the context and purpose of this representation" (p. 2). This definition of number sense was the one chosen for this study given its explicit description of the skills expected of second grade children.

While the fundamentals of number sense are generally developed during the primary years of elementary school, deficits in this area are also witnessed and prevalent in students experiencing mathematics difficulties at this grade level. Children in second grade who are having difficulties in mathematics typically show deficits in their understanding of place value (Hanick, Jordan, Kaplan, & Dick, 2001), their mental number line and the ability to deal with number magnitude (Geary et al., 2007), poorer and less advanced strategy use (Russell & Ginsburg, 1984), and the inability to decompose and recompose number.

Place value. Place value is a critical part of number sense and is frequently misunderstood by students (Nataraj & Thomas, 2007). Children often have trouble understanding place value of a number and what the quantities represent in terms, specifically, of ones, tens, and hundreds (Varelas & Becker, 2007). For example, a student may view the number 23 as having a value of 2 and separately 3, neglecting the position of the 2 in the tens place as representing a value of 20. Additionally, while children may be able to identify a number with respect to the column in which it is located, they may be unable to discern the relationship between the numbers, particularly when the number changes, altering the value in all the columns. Ward (1979) conducted a study in which he asked 10-year olds to name the number after 06299. Only 41% percent of students were able to complete this task suggesting that even 10-year olds frequently have a poor number sense for place value (as cited in Thompson, 2000).

The literature on children with mathematics difficulties indicates that these children are less likely than typically developing students to understand place value (Hanich, Jordan, Kaplan, & Dick, 2001). This finding has prompted recommendations to introduce place value concepts as soon as students being to work with two-digit numbers through discussing the nature of two-digit numbers during the primary grades as a foundation for strong place value understanding (Baroody, 1990). For students with mathematics difficulties, Baroody (1990) also suggests explicit focus on place value may minimize the reinforcement of numbers as unitary, which may make it more difficult to transition to a multi-digit understanding of number. In an intervention conducted with first and second grade students with mathematics difficulties, Bryant et al. (2008) dedicated additional instructional time to double-digit numbers, and the idea of zero as a place-holder. Both concepts are difficult for children with mathematics difficulties. There was no significant effect of the intervention for first-grade students. There was a positive main effect of the intervention for second-grade students in the at-risk (for difficulties) group resulting from the addition and subtraction subtest. Aside from the studies mentioned above, there is a lack of substantive research on place value knowledge in students with mathematics difficulties. Only four studies have measured place value knowledge in written calculation in students with mathematics difficulties (Andersson, 2008).

Mental number line and number magnitude. The mental number line is defined as the system in which numerical quantities are internally encoded on a continuum (Izard & Dehaene, 2008). The mental number line is thought to be a horizontal line in which smaller numbers are on the left and larger numbers are on the right (van Galen & Reitsma, 2008). Number magnitude is the differentiation of numbers based on their relative value (Girelli, Lucangeli, & Butterworth, 2000). The ability to discriminate the magnitude of larger multi-digit numbers is critical as students advance to first and second grade, confronting double digit addition and subtraction problems (Siegler & Booth, 2004). During kindergarten, first, and second grade, numerical estimation on the number-line task correlated with overall math achievement (Booth & Siegler, 2006). Students that represent number linearly solve arithmetic problems with greater accuracy and are more likely to correctly retrieve the correct answer (Booth & Siegler, 2008).

Students experiencing mathematics difficulties possess an informal (non school-taught) concept of number line in that they have some understanding of the relationship between numbers, such that the child understands that 12 is closer to 10 than it is to 20 (Russell & Ginsburg, 1984). Informal knowledge may stem from everyday experiences with spontaneous counting, using non-mathematical terms. For example, this may occur when children count rocks on the playground or grouping action figures. Students may experience difficulties because of dissonance between this informal knowledge and formal written instruction, which also includes formal mathematical terminology. Even when children begin to develop formal school knowledge, they assimilate, rather than abandon, informal knowledge (Ginsburg & Russell, 1981). Therefore, we would expect that children who fail to successfully assimilate this knowledge and abandon some practices, when appropriate, will experience difficulties.

Children tend to represent number either linearly or logarithmically (Siegler & Booth, 2004). When children use logarithmic representations, small numbers (i.e. 8 and 9) are more widely spaced apart in the number line and large numbers are compressed (Fisher & Campenas, 2009). Children may find the logarithmic representation useful when estimating unfamiliar quantities, particularly because it exaggerates the difference between numbers in the higher ranges and allows the child to discriminate more accurately between numbers in the higher range (Siegler & Booth, 2004). Consistent with this finding, discrimination between quantities that are similar in magnitude will result in more overlap and prove more difficult to discriminate than quantities of greater magnitude (Siegler & Opfer, 2003).

There is a significant relationship between age and experience with respect to number magnitude estimation. Between kindergarten and second grade, most children shift from a logarithmic pattern of estimation of a linear representation of number when estimating to 100 (Booth and Siegler, 2008). Second grade students produced a generally logarithmic pattern of numerical estimation for quantities between 0 and 1000, and an increasingly linear estimation pattern for quantities between 0 and 100 (Booth & Siegler, 2006). Children that represent number linearly are more likely to produce accurate responses to larger addition problems or near misses when responding incorrectly. In addition, students that represent number linearly are more likely to use retrieval for correct responses (Booth & Siegler, 2008).

Cognitive Strategy Use. Cognitive strategies are described as conscious, intentional, self-aware cognitive activities (Bjorklund, Huberts, & Reubens, 2004) utilized to solve mathematics problems. Cognitive strategies are a form of number sense in that they reflect a move away from concrete representations to a more abstract representation and manipulation of number. Children typically make this transition in elementary school. Ilg and Ames (1951) found that children of ages five and six tended to use counting all with a focus on counting concrete objects, but that eight and nine year old children were more likely to use count mentally or retrieval to answer the same problems.

The difficulty in the transition from concrete representations to more abstract representations is articulated in Siegler's (1996) overlapping waves theory. The primary tenant of overlapping waves theory is that children of all ages employ a variety of strategies competing for use in their mathematics problem solving. Bjorklund and Ronsenblum (2001), for example, found that young children applied diverse strategies of varied complication levels. Strategies vary in adaptability depending on the problem being solved with some strategies more appropriate than others. Overlapping waves theory articulates the complexity in this transition because as children learn new strategies, they do not completely abandon old strategies and often create new strategies by adapting old strategies. Transition from concrete representations to cognitive representations involves adapting old strategies. Fuson and Secada (1986) note that counting on requires that a student simply count from the second addend; for the same problem, a student would start counting from 4 to "5, 6, 7." Counting on can involve the use of manipulatives or can be entirely cognitive in the absence of symbols to represent the number. However, problems arise when children do not effectively adapt strategies based on the problem and situation. Children who do not adopt new strategies or adapt older strategies will continue to employ immature strategies resembling those of young children's mathematics problem solving (Opfer & Sielger, 2007).

One cognitive strategy, decomposition, was of particular interest for this study because its use indicates an advanced number sense requiring the comparison of larger numbers and often involving place value. Decomposition strategy use is a columnwise procedure that can be used in addition and subtraction problems (Beishuizen, 1993). One distinguishing feature of the decomposition is that numbers are decomposed first from the left column (tens place) and then right (ones place). The two most prominent non-column strategies are the 10 + 10 (1010) and the

N + 10 (N10). The first strategy involves taking a number and adding the bases of ten first, and then adding the ones (e.g. 53 + 21 is solved by utilizing 50 + 20=70 and then adding 3+1 to 70; 70+3=73). The latter strategy, N10, differs in that the same problem would be solved in the following manner: 53 + 21 becomes 53 + 20= 73 and then adding 1. Because decomposition is a complex cognitive strategy that utilizes children's understanding of place value it was targeted for intervention in this study. Another goal of the intervention study described here was to improve children's use of the min strategy using entirely mental representations of number. The current study focused on the min strategy because it is a more sophisticated and efficient strategy than the sum strategy (Bjorklund & Rosenblum, 2001) and may be considered a backup strategy in the absence of retrieval.

Fluency. Retrieval is the process of recalling math facts from memory to solve problems (Geary & Hoard, 2001) and is characteristic of children who are highly fluent. For any student to quickly and to accurately solve addition and subtraction problems, they must possess substantial knowledge of basic math facts. Students experiencing math disabilities are slow at retrieving mathematics facts and face challenges with place value and estimation (Barnes, et. al, 2006). In studies comparing strategy use of students with mathematics difficulties and typically developing students, typically developing students used retrieval significantly more than children with disabilities (Jordan and Montani, 1997). Over time, students with mathematics disabilities continue to retrieve fewer facts from memory, experience higher error rates, and exhibit unsystematic retrieval speeds (Geary, 1994). Jordan and Montani (1997) found that time constraints are important to students with specific mathematics difficulties. When orally presented with problem stories and math facts, student with specific mathematics difficulties (as opposed to general mathematics difficulties) only performed well on the untimed tasks, relative to average students. There is no research that could be found on fluency in children experiencing difficulties in mathematics. It was assumed that these children would have similar problems to children with learning disabilities.

Furthermore, a neglected factor is how fluency changes over time and how it is accompanied by increased use of retrieval. Typically developing students increased their fluency over time and relied more on retrieval as a means for solving computation problems (Geary, 1993). A useful approach to gaining insight to the ways children select, and more importantly, apply a variety of strategies, specifically when children shift from retrieval to counting strategies, is outlined in the strategy choice model. Under the strategy choice model, a strategy is chosen by assessing the distribution of associations between a problem and all possible solutions (Seigler & Shrager, 1984). This process consists of three phases: retrieval, elaboration of the representation, and counting. The problem solving process begins with the retrieval phase when the child sets the confidence criterion and then the search length time, which represents that maximum number of times that a child will attempt to retrieve the correct answer (Siegler & Shrager, 1983). Retrieval then continues as long as the confidence criterion is greater than the associated strength of each retrieved answer and as long as the maximum search length time has not been reached. If these conditions are violated, the second phase in which the children implements a “backup” counting strategy begins. Children vary in their confidence intervals. Some students tend to have low thresholds and retrieve when they are less confident. The result is typically low achievement. Other children are perfectionists and must be very confident before they will retrieve whereas other students are “good students” in that they set an appropriate confidence threshold and tend to retrieve accurately when they do (Seigler, 1988)

In contrast to average students, students with MD are less consistent and less fluent than children who are typically developing and do not experience the same level of improvement as those students. In a study comparing strategy choice and speed of processing in typically developing, gifted, and those experiencing mathematics difficulties, Geary and Brown (1991) found several cognitive dimensions affecting students in these groups. The long-term memory component is the central dimension that distinguishes students in each group and was suggested as an explanation for mathematics difficulties in first and second grade students (Geary, 1990). Other research indicates that poor working memory may negatively influence mathematics skill development and may explain procedural and retrieval deficits that plague MD students (Butterfield & Ferretti, 1987; Geary, 1990). If fluent computation and retrieval relies on both a well-organized number system in long-term memory and working-memory, then interventions that strengthen the associations between problems and answers in memory (Ashcraft, 1982; Stadler, Geary, & Hogan, 2001) will improve fluency. The likelihood of accurate retrieval is also increased when students represent number linearly, signaling a better calibration of mental number line (Booth & Siegler, 2008). This increased fluency, in turn, may support the emergence of cognitive strategies.

Several intervention studies have tested the effectiveness of fluency instruction on student achievement. Codding et al. (2007) tested the effectiveness of two interventions said to improve computational fluency: cover-copy-compare (CCC) and explicit timing (ET). Cover-copy-compare involved the follow steps: (a) looking at the problem with the correct answer provided, (b) covering the problem with the answer, (c) recording the answer, (d) uncovering the problem with the answer, and (e) comparing the two answers. Explicit time involved marking the progress in one-minute intervals so students can monitor the number of problems completed per interval. There was no significant difference between the performance of the control group and the two treatment conditions before and after the intervention. However, students with a higher number of digits incorrect per minute improved most in the explicit timing group compared to the control and CCC groups. In another study, Rhymer, Skinner, Henington, D'Reaux, and Sims (1998) conducted an explicit timing intervention with African-American third graders, consisting of three daily sessions for three consecutive days. Participants in all three groups showed improvement in the mean number of problems completed correctly following implementation of explicit timing. The lack of control group made it impossible to determine whether these results were due to the instruction.

No study to date has examined whether instruction to improve fluency will impact the development of other forms of number sense and children's accuracy in computation. Increased fluency should be accompanied by improvements in the mental number line if it improves the quality of the associations between problems and their answers in memory. Similarly, it should be correlated with better cognitive strategy use if quick and accurate retrieval is needed for cognitive strategies to emerge.

Spatial Ability

Like number sense, spatial ability is not a singular characteristic or trait, but is comprised of a number of skills. Linn and Peterson (1985) divided spatial skills into three categories: spatial perception, which requires assessing the "spatial relationship with respect to the orientation of their own body (p. 1482),"mental rotation, the ability to mentally rotate two or three-dimensional objects with accuracy and speed, and spatial visualisation, the ability to manipulate data that are presented spatially, requiring several complex rotations. An additional component of spatial ability is subitising, instantaneously recognizing the composites of a spatial structure (Bobis, 2008). Spatial skills are of particular interest here because spatial ability is correlated with mathematics performance (Casey et al., 2008).

More specifically, there is evidence that spatial skills are correlated with both functional skills that are needed in the problem solving process (i.e. the alignment of numbers in a complex addition problems) and conceptual knowledge and understanding of concepts such as place value (Geary, 1993). In addition, Casey (2004) and Casey et al. (2008) hypothesized that spatial ability may support mathematics achievement by allowing for the discovery of mathematical patterns and functions. In a study with sixth grades males, Hegarty and Kozhevnikov (1999) found that spatial ability promotes successful problem solving.

Despite its importance, spatial skills are rarely assessed in studies of mathematics disabilities (Geary, 1993). An analysis of data in a longitudinal study on strategy use was conducted and students were clustered into three levels of cognitive function. The finding was that students with mathematics difficulties performed lower than their typically developing peers on a spatial ability task (Janes, 2007). However, there is a paucity of research on spatial ability with students experiencing mathematics difficulties. In the current study, spatial ability was assessed as a potential correlate for overall mathematics ability.

CHAPTER 3

THE CURRENT STUDY

The research on mathematics interventions for children with mathematics difficulties and disabilities (MD) is quite limited, particularly in comparison to reading difficulties and disabilities (Fuchs & Fuchs, 2005). Fuchs and Fuchs (2005) also point out that the little research that has been done disproportionately focuses on simple math facts rather than more complex problem solving. The focus on early developing skills is warranted because early deficits in number sense are thought to impede the comprehension of more complex mathematics concepts and problem solving (Gersten & Chard, 1999).

Despite the evidence that children with MD are slower to develop number sense and are less fluent, few intervention studies have been done to determine whether teaching number sense and improving fluency will result in better performance. One study done by Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) involved an intervention designed to improve number sense in children with MD. Bryant and colleagues conducted a Tier 2 intervention, which they characterize by flexible groupings, responsive instructional decisions, and evidence-based interventions. Students in first and second grade received instruction in small, grade-level based groups of three to four students. Students completed instructional tasks related to number concepts, place value and base 10, and addition and subtraction combinations. This intervention consisted of 15 minute “booster” sessions over 18 weeks. Sessions were supplementary to state mathematics standards taught in the classroom. There was no significant effect in pre-test and post-test scores for first grade students; there was a significant main effect for second grade students, indicating a positive effect of the intervention.

The primary goal of the current study is to improve the use of cognitive strategy use in second graders experiencing mathematics difficulties. Because the majority of research has focused on mathematics disabilities, this research is essential because it addresses the weakness of a much larger percentage of the student population—students with mathematics difficulties. The increase in cognitive strategy use is predicated on improving students' place value knowledge, number line estimation, and addition and subtraction accuracy and fluency. The instruction in this study was intended to specifically address weaknesses that lead to poor accuracy in addition and subtraction problem solving.

In order to address these weaknesses, instruction was provided to improve addition and subtraction accuracy as well as improve mastery of the *min* strategy without the use of manipulatives or concrete representations. Students received instruction on number line estimation to improve their understanding of the relationship between numbers of varying magnitude and how estimation is connected to addition and subtraction conceptually. Students also received instruction on basic math facts in order to improve the frequency of accurate retrieval. Students then received instruction on the decomposition strategy as another cognitive strategy for solving double-digit addition and subtraction problems, and strengthening place value concepts. Finally, students received an opportunity to apply these strategies in practice sessions, discover and utilize their own cognitive strategies.

CHAPTER 4

METHOD

Participants

The sample included 51 students (24 males and 27 females) from seven second-grade classrooms in a rural elementary school approximately twenty miles outside Athens, GA. A Title I school where 45% of students qualified for free or reduced-price lunch, all students were identified by their teachers as performing in the second or third quartile in mathematics, excluding students that receive special education services.

Students were randomly assigned to the treatment group and control groups. Initially 30 students were in the control group and 30 students were in the treatment group. After students with incomplete data were excluded, the final sample consisted of 29 treatment students and 22 control students. Several students that were in the control group were either absent from testing, or were not tested prior to the start of the intervention, making it necessary to exclude these participants. The mean age of the participants was 8 years, 2 months (one parent/guardian of a participant declined to provide a date of birth).

One of the obstacles to studying students with MD stems from the way students are classified. A key issue of concern in identification of MD students is the numerous cutoffs used by researchers to identify students with mathematics disabilities. Geary (2004) highlights the common practice of identifying students as having a mathematics disability by scoring below the 20^{th} to 25^{th} percentile, with a comparably higher IQ score. Baker, Gersten, and Lee (2002) conducted a review of studies including children with mathematics difficulties They found that selection criterion included a wide range of cutoffs: all students who scored below the 34^{th} percentile on the Iowa Test of Basic Skills (ITBS) to scores below the 50^{th} percentile on a standardized math assessment. Some of these studies included students with mathematics disabilities. Students with these cognitive or neurological deficits are considered to have a mathematics disability in this study, and were excluded in the present study.

Procedure and Materials

The participants were pretested and posttested on strategy use, accuracy in solving addition and subtraction problems, spatial ability, number magnitude estimation, and understanding of place value. Pretest data were collected for each participant over the course of two weeks. Posttest data were collected from individual students within the week following the last intervention session. Following the pretest, participants were randomly assigned to the treatment group. Participants attended six intervention sessions, conducted once per week. These sessions each lasted 45 minutes. The researcher, a graduate student, conducted two sessions per day, over the six weeks, and participants attended 1 of the 2 sessions. Sessions were conducted outside the classroom. At the conclusion of the six intervention sessions, posttest data was collected for all participants.

Measures

Strategy Use and Accuracy (Addition and Subtraction). Participants were assessed on their ability to use cognitive strategies on ten arithmetic problems (five addition, five subtraction). Of the five addition problems, all were double digit, while two required regrouping. Three of the subtraction problems were double-digit and none required borrowing or regrouping. Problems were scored for cognitive strategy use (1 = yes, 0 = no), and accuracy, (correct = 1, incorrect = 0). Scores for each section were derived independently, with a maximum value of 10 points for each section. The score for accuracy had no bearing on cognitive strategy use and the converse was true as well. Cognitive strategy use was scored by observation of problem solving technique. The use of concrete representations (such as fingers or tick marks) was not considered cognitive and was awarded 0 points, whereas simple pen and paper strategies using mental methods and traditional algorithms (such as regrouping on paper) were considered cognitive and awarded one point. If both non-cognitive strategies were utilized along with cognitive strategies, the student received 0 points. The maximum score was 10 for cognitive strategy use and 10 for accuracy.

Spatial Ability. We measured spatial ability using a test consisting of a series of 10 3-D transformations created by Casey (2007). To instruct the participant in the task, the participant was presented with two identical 3-D figures constructed of interlocking cubes. The participant was given two practice rotations that were not scored, which were intended to control for error arising from misunderstanding of activity instructions. During the practice problems, participants were given feedback about their responses and shown how rotations could be made to make the newly oriented object identical to the first. If the child's first attempt was correct, they were told so, and if it was not, the researcher referred to the object that had not been moved as a reminder of the objective. If the child incorrectly rotated the object again, the researcher continued testing with problem 1, making note the second attempt was incorrect as well. No feedback was given from this problem forward, regardless of whether or not the response was correct or incorrect.

For each of the 10 problems, a screen was placed in front of the display blocking the participant's view. The two identical items were placed on a table parallel to one another, with identical orientations. The screen was then removed and the participant verbally verified that the two figures were identical. Following the verification, the screen was placed in front of the figures and the figure to the left was rotated either three dimensionally or two dimensionally. (If the child was left- handed the figure to the right is rotated.) The screen was then removed and the participant attempted to rotate the figure to the left to match the other figure's orientation. This protocol was repeated for the subsequent 9 items.

The researcher timed the number of seconds required to complete the task by measuring the number of seconds between the moment the participant first touched the figure, until he or she is finished by either successfully rotating the figure or by giving up. The researcher verified verbally that the participant was finished if he or she was not sure if the rotation was complete. The researcher also recorded whether the answer was correct.

The participant had 10 transformations to complete, but the activity was terminated if the participant attempted 3 successive problems incorrectly. Children were encouraged to continue to attempt to try the rotations even if he or she expressed an inability to do so. No student refused to complete the activity.

A score for the number of correct items (spatial accuracy) was derived by summing the number of items the participant correctly rotated within the 10-second time limit. Possible scores for accuracy on this measure ranged from 0 to 10. The researcher also recorded whether the participant used trial and error or whether he or she correctly rotated the figure initially. The time of completion for each item was recorded and the researched noted whether the item was rotated correctly or incorrectly regardless of the time for completion. However, only items correctly rotated within the 10-second limit were scored correct, receiving 1 point. Reliability (internal consistency) for this measure is .85. A score for time was also created (spatial time) and that score ranged from 30 to 232 seconds.

Number Magnitude. The participant's ability to determine number magnitude (number line), the approximate location and distance between numbers, was measured using a number line that was incomplete. Participants were given a sheet of paper with a number line with only the numbers 0 and 100 placed at the left and right ends of the line. A number was presented to the right of each blank line, allowing only the current problem to be visible. The children estimated the location of 22 numbers in this way, placing a line estimating the location of the corresponding number on the number line. The numbers 0 to 20 were oversampled (2, 5, 8, 11, 13, 17, 18, 20) given that children between 5 and 10 are apt to represent smaller numbers linearly, but are slow to extend this linear representation to larger numbers (Siegler & Opfer, 2007). The remaining numbers presented were: 35, 38, 40, 41 (presented twice), 50, 53, 59, 64, 70, 77, 83, 89, 95) (presented in Appendix B). The order of presentation of numbers was random.

In order to determine the impact of the intervention on number line estimation, the actual estimates for each number scores were summed across individuals to ascertain a mean score for each number. In order to score number line data, a transparent number line was placed over each number to compare the actual location of the number on the number line to the participant's estimate. The data were scored by three trained scorers and the accuracy of data entry was checked by a second individual who was not one of the three trained scorers. Number line estimations of second graders tend to conform best to a linear function, so evidence of the contrary may predict overall mathematics difficulties for participants in this age group.

Place Value. To assess number sense and place value we gave children 12 numbers, ranging from 0 to 100. Participants were required to list, in a separate column, the number of tens and ones for each number. For example, if the participant was given the number 54, he or she was expected to list 5 in the tens column, and 4 on the ones column. Ten of twelve problems were double-digit numbers. The final two problems required addition of two double-digit numbers and the subsequent placement of the solution in the appropriate column with respect to tens and ones. There were 12 possible points.

Intervention Sessions

Following the pretest, participants were randomly assigned to the treatment and control groups. Children in the treatment group attended six intervention sessions, conducted once per week. These sessions each lasted 45 minutes. The researcher, a graduate student, conducted all of the intervention sessions. Sessions were conducted outside the classroom. Children in the control group received no instruction and participated in the usual classroom activities. Participants attended six intervention sessions. Materials and instructions for the six sessions are presented in detail in Appendix A, B, C, D, and E.

Session 1 was focused on a number magnitude activity. Participants practiced counting up and down using a modified UNO™ game. Students randomly selected a card from the deck and counted up or down depending on the quantity selected. If the selected card was smaller than the prior card, participants counted down; if the selected card was larger than the prior number, the participant counted up. In the second portion of the session, participants used a paper number line as a tool to rehearse addition and subtraction problems. Participants were instructed on addition and subtraction with the number line using the following examples: 5+4, 6+5, 8+10, 12-9, 19-7. Students then received paper number lines and a worksheet to complete addition and subtraction problems independently with the use of the number line.

Session 2 was dedicated to fluency. Students rehearsed common math facts with flash cards. Because students in this sample were performing below average, the flash cards included numbers between 0 and 20. Students received an average of approximately 30-45 cards. Participants utilized the duration of the session for this activity.

Session 3 focused on cognitive strategy use practice solving addition and subtraction problems. Participants played an addition and subtraction game using UNO™ cards drawn from a deck to represent the addends and minuends. Cards ranged from 0 to 9. Participants received a paper to fill in the addends and minuends drawn from the deck. Participants received points based on the problem solving method. Participants received 3, 2, or 1 points based on the use of cognitive strategies, the number line, or fingers, respectively.

Session 4 included instruction on the use of the decomposition strategy for addition problems. After receiving instruction, and guided practice, students were given a worksheet with four addition problems to complete independently. Problems are listed in Appendix A.

Session 5 included instruction on the use of the decomposition strategy for subtraction problems. After receiving instruction, and guided practice, students were given a worksheet with four subtraction problems to complete independently Problems are listed in Appendix A.

Session 6 served as a practice session to apply the strategies learned during the intervention. Students were asked to apply self-discovered strategies (constructivism) to solve five addition and subtraction problems. Students were allotted fifteen minutes to complete these problems. Students were allowed to work in pairs. For the second portion, of the session, participants received a worksheet with 19 addition and subtraction problems; they had 10 minutes to complete the problems.

CHAPTER 5

RESULTS

Did the Intervention Improve Performance?

To examine whether the intervention affected students' accuracy, cognitive strategy use, spatial accuracy, spatial time, and place value knowledge ANCOVAs were performed with posttest assessments as the dependent variable, group membership as the independent variable and pretest scores as the covariates. Adjusted means and standard errors are presented in Table 1; means and standard deviations are presented in Table 2. Number line data for the control and treatment groups are presented in Figure 1 and Figure 2, respectively.

Table 1

Estimated Marginal Means (Posttest)

Measure	Control *M (S.E.)*	Treatment *M (S.E.)*
Addition/Subtraction Accuracy	7.201 (.443)	6.639 (.389)
Cognitive Strategy	7.155 (.690)	6.704 (.690)
Spatial Accuracy	7.673 (.311)	7.414 (.270)
Spatial Time	97.300 (9.528)	78.967 (8.298)
Place Value	6.282 (.928)	5.494 (.788)

There was no significant effect of the intervention on addition and subtraction accuracy after controlling for pretest addition and subtraction scores, $F(1, 48) = .39$, $p = .54$). Analysis of covariance results on cognitive strategy scores indicated no significant effect of the intervention on cognitive strategy score after controlling for pretest cognitive strategy scores, $F(1, 48) = .382$, *p* = .54. There was also no significant effect of the intervention on posttest place value scores after controlling for pretest place value scores, $F(1, 48) = .570$, $p = .454$. Nor was there a significant effect of the intervention on spatial ability scores after controlling for pretest spatial scores, $F(1,48) = .413$, $p = .52$). There was no significant effect of the intervention on spatial ability time after controlling for spatial ability time pretest scores, $F(1, 48) = 2.13$.

Effect sizes for each measure were calculated using Cohen's *d*: addition and subtraction $d = .17$; cognitive strategy use $d = .16$; spatial accuracy $d = .19$; spatial time $d = .42$; place value $d = .26$. All effect sizes are considered small with the exception of spatial time, which is considered a moderate effect size.

Table 2

Pretest and Posttest Means and Standard Deviations

Measure	Control				Treatment			
	M		*SD*		*M*		*SD*	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Addition/Subtraction	7.50	7.09	2.03	1.63	7.44	6.72	1.94	2.48
Cognitive Strategy	5.77	7.36	3.49	3.42	4.66	6.83	4.04	3.21
Spatial Accuracy	6.86	7.68	3.49	1.43	7.10	7.41	2.19	1.43
Spatial Time [a]	86.27	97.36	34.81	51.33	85.34	79.03	39.92	37.87
Place Value	7.86	6.64	3.99	4.48	5.52	5.59	4.14	3.79

[a] Seconds

In order to determine the impact of the intervention on number line estimation, the actual magnitude of each number was graphed against the estimate of the participant. Both linear regression and logarithmic regression were performed to determine which function best fit the data. Figure 1 shows the pretest and posttest estimates for the control group. Figure 2 shows the pretest and posttest estimates for the treatment group. Both the control group and treatment group transitioned from estimates that were best fit by a logarithmic regression, to estimates that were best fit by a linear regression.

Correlations

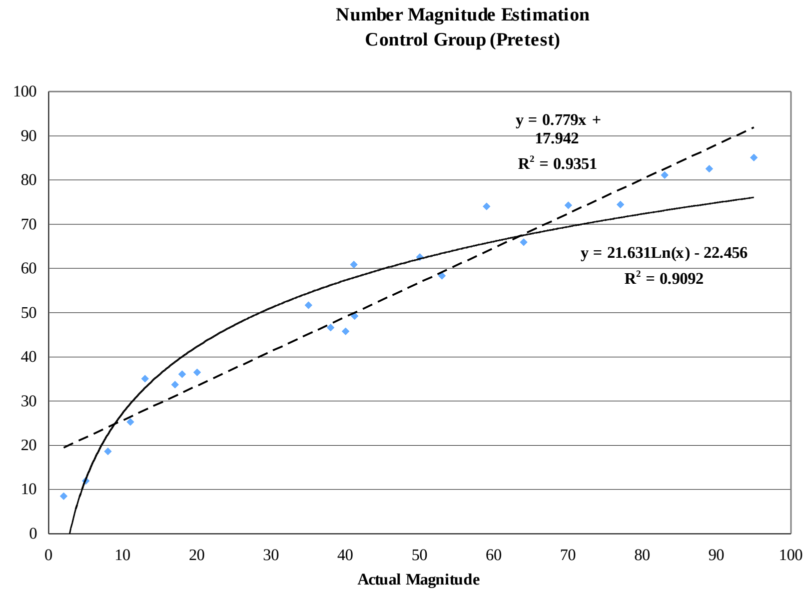
Pretest correlations were run to determine the relationship between variables. Correlations are displayed in Table 3. Variables were only correlated at pretest to avoid confounding by the intervention and classroom instruction that was received between pretest and posttest. Spatial ability and spatial time were expected to predict overall mathematics ability. The remaining variables, addition and subtraction accuracy, cognitive strategy use, and place value are all related to number sense, and were expected to be highly correlated. As can be seen in Table 3, all variables were weakly correlated with the exception of spatial accuracy and spatial time. There was a significant negative correlation between spatial time and spatial accuracy, $r = -.72$, $p = .01$. Therefore, students who were more accurate at the spatial rotation task also completed the task more quickly. This is not surprising given the time constraint imposed on the task. Even if the rotation was completed accurately, no point was awarded because it was not completed within the time limit. The correlations between addition and subtraction accuracy and place value were surprisingly low. This may be explained in several ways. First, the pretest means for the control group and treatment group were 7.47 (out of 10 possible points), resulting in a possible ceiling effect given the small range for improvement. Secondly, students performed poorly on the place value task, with a means of 6.53 (out of 12 possible points) at pretest, suggesting a floor effect for this task. Given the nature of the place value task, the two-column nature of the response form, students may have experienced consistent errors in the placing of the appropriate number in the appropriate column. A third, and more likely explanation, is that students have poor understanding of the meaning and relationship between numbers (Valeras & Becker, 1997), which may not be revealed in a child’s ability to accurately solve double-digit addition and subtraction problems.

Table 3

Pretest Correlations

	Addition/Subtraction Accuracy	Cognitive Strategy Use	Spatial Accuracy	Spatial Time	Place Value
Addition/ Subtraction Accuracy		.218	-.228	.199	.028
Cognitive Strategy Use			.023	.055	.111
Spatial Accuracy				-.719**	.368
Spatial Time					-.024

**Correlation is significant at the .01 level



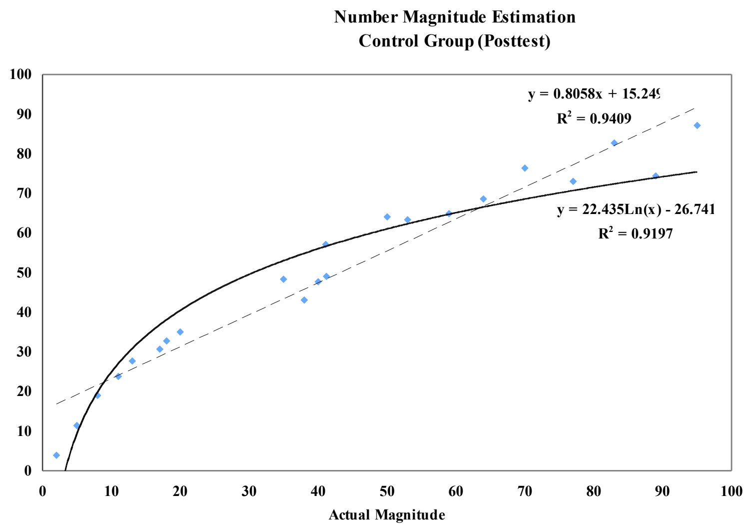
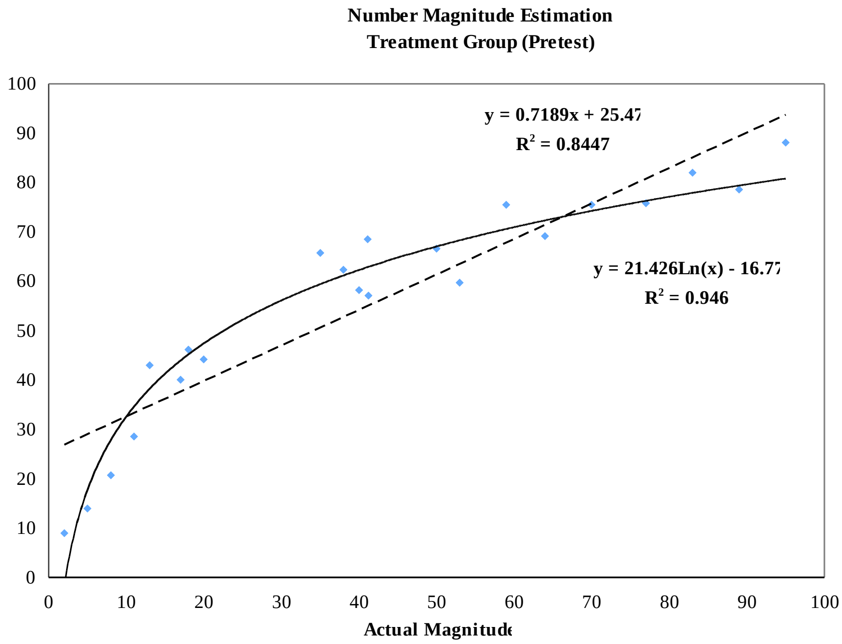


Figure 1. Number estimation means for control group at pretest and posttest. Figure contains regression for linear and logarithmic functions. R^2 allows comparison of the regression function that best fits the data.



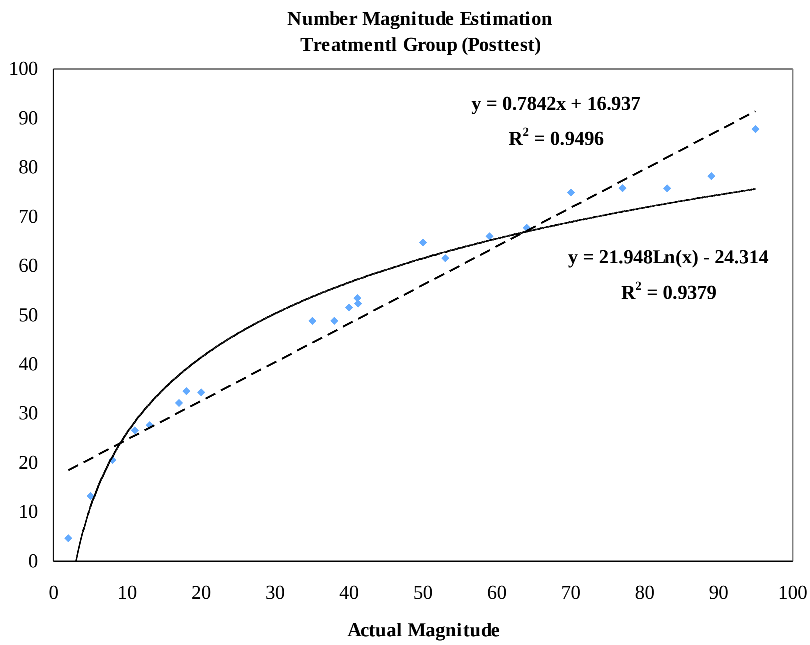


Figure 2. Number estimation means for treatment group at pretest and posttest. Figure contains regression for linear and logarithmic functions. R^2 allows comparison of the regression function that best fits the data

CHAPTER 6

DISCUSSION

This study examined second graders who experience difficulties in mathematics and their use of cognitive strategies to solve mathematics problems. There is very limited research on students with mathematics difficulties, although this population represents a moderate percentage of classroom students. On average teachers volunteered 25% (or more) of students in their classes. This is of significant concern because a vast number of students are below grade level in mathematics performance. Teachers made these decisions based on classroom assessments, and presumably, standardized test results.

The hypothesis of this study was that providing students with instruction on fluency and number line estimation would support the shift to cognitive strategy use in students that received instruction from the intervention. Number line estimation was the only task that produced expected results following the intervention. Several factors may have impacted the ability to detect differences in the intervention and non-intervention groups. With the exception of the number line task, there are reliability concerns with the other assessments. Spatial ability was expected to predict mathematics performance (Casey, et al., 2008). However, for the spatial task, as with the place value task, and the addition and subtraction task, scores were relatively high, limiting their predictive ability. Although reliability had been achieved for the spatial ability task, it may have proven to be a rudimentary task for second graders in the latter part of the school year.

Number Magnitude

One of the most surprising findings is the data resulting from the number line task. This sample, focused primarily on students experiencing math difficulties, exhibited number line estimation skills that initially resembled first grade students. Siegler and Booth (2004) found that second grade number line estimation was best represented by a linear function. In contrast, the data from this sample were most accurately represented by a logarithmic function, which confirms the findings of prior studies that children with poor estimation ability are more likely to score lower on achievement tests (Siegler & Booth, 2004; Booth & Siegler, 2006). This finding suggests that students with mathematics difficulties may fail to transition to more accurate number line estimation skills compared do their peers that do not experience difficulties. Booth and Siegler (2006) found that students in second grade were more likely to follow a logarithmic pattern of numerical representation when estimating numbers between 0 and 1000, although their pattern of representation transition to a linear function with numbers between 0 and 100. This was not supported for the estimation trends for students in this sample. For both the treatment and control groups, their estimation patters were best fit by a logarithmic function on the pretest. However, at posttest, both groups most closely fit a linear function.

Data from this study suggest that prior models fail to capture the numerical estimation patterns of students with mathematics difficulties. Distinct models and time trajectories may be necessary students that perform below grade level expectations. Future research may need to place greater focus on numerical estimation within grade levels for students at various ability levels. Prior data may have failed to capture variability in estimation patters, at different grade levels, because data was not disaggregated by overall level of mathematics achievement. Geary et. al (2008) found that higher error in numerical estimation was associated with lower IQ scores, and estimation that conformed closely to a linear function was also associated with higher IQ scores. Further research in this area is imperative to fully understand the numerical estimation abilities of students with mathematics difficulties. In addition, the improvement in numerical estimation of the treatment and control groups may reveal the fact that students benefit from instruction on number line as well as simply being exposed to the number line task.

Cognitive strategy use and accuracy

Cognitive strategy use improved for the treatment group ($M = 6.83$) and control group ($M = 7.36$) although there was no significant effect of the intervention. In contrast, there was a slight decrease in addition and subtraction accuracy for the treatment group ($M=6.72$) and control group ($M=7.09$). There are several possible explanations for this result. First, completion of the addition and accuracy task was timed, therefore, some participants may have been unable to complete all problems within the allotted 10- minute constraint, thereby receiving no points for incomplete problems. Furthermore, although cognitive strategy use and accuracy were measured independently, they do not function in isolation from the other. First, students in the treatment group may have been more inclined to attempt a cognitive strategy following the intervention—with various degrees of success. Schwenk, Bjorklund, and Schneider (2007) found that utilization deficiencies are most prevalent in individuals in the treatment condition, and less so with those in the control group. Prompting students to attempt certain strategies increased the instance of utilization deficiencies. In the present study, the treatment group, who were encouraged to use cognitive strategies as opposed to non-cognitive strategies. The control group, on the other hand, had little incentive to transition towards cognitive strategy use. For those students, they may have achieved accuracy with other non-cognitive strategies, like finger counting. For other students, ineffective use of cognitive strategies may have negatively impacted accuracy.

Although short-term memory resources are not a focus of the present study, this may be a critical issue facing the students with difficulties in this sample. This may be particularly true because there is an emphasis on the application of new strategies learned and applied within a relatively short time frame. In line with the free resources perspective, if students have learned multiple strategies, the student will expend resources retrieving multiple strategies and then selecting a strategy to attempt (McNeil & Alibali, 2004). In addition, speed of processing issues persist for students with MD, compared those without disabilities or difficulties. Jordan and Montani (1997) found that students with MD performed poorly on timed addition and subtraction tasks, but prevailed in the absence of time constraints. Bull and and Johnston (1997) also found speed of processing to be a critical component of problem solving. Using fixed-order multiple regression analysis, they found that speed of processing was the best predictor of overall mathematics achievement in their sample. Speed of processing and short-term memory resources affect students' ability to efficiently apply cognitive strategies. Surprisingly, students may expend more time counting when applying the min strategy than using the sum strategy (Bjorklund & Rosenblum, 2001), which may have affected addition and subtraction accuracy scores given the time constraint. Students in the treatment group appeared to struggle with dual demands of applying a cognitive strategy and solving the problem correctly.

One of the limitations of this study was the decision to sum across strategies. Siegler (1987) cautions against this procedure because it fails to capture the diversity of strategies used by individuals and the group, as well as the frequency in which each strategy is applied. Because students' strategy use was scored as cognitive and non-cognitive by observation, a very limited range of strategies was observed. This scoring method did not account for both the use of cognitive and non-cognitive strategies simultaneously. Nor does it capture the extent to which certain strategies were used consistently by certain children or whether they applied a variety of strategies. Therefore, it was not possible to fully capture the impact of the intervention, and whether components of instruction such as number line or fluency were equally effective or ineffective at impacting addition and subtraction accuracy as well as cognitive strategy use.

A second limitation of this study is the duration and the timing of the intervention. Because this intervention took place in the second half of the school year, it is plausible that students may have experienced greater benefit if intervened on at the beginning of the school year when many concepts were initially introduced. It is likely that six weeks was inadequate given the deficits experienced by the participants in this study. Furthermore, since effect sizes were small for almost all measures, a larger sample size may have resulted in statistically significant results.

Instructional Implications

Given the number of students that experience difficulties with mathematics, these deficits suggest curricular and instructional dissonance with the skills that are most important to mathematics development and success. This intervention targeted two of the focal points listed for second graders by the NCTM (2006). Although not fully captured by assessment tools in this intervention, students continued to experience difficulties with these key components. Place value and fluency were two components directly addressed in this study that were emphasized by NCTM. While the place value task appeared rudimentary, students still struggled on the task. However, it is critical for students to understand the relationship between numbers, particularly with respect to the extent to which place value knowledge facilitates multiple representations of numbers. For example, 835 is 8 hundreds, 3 tens, and 5 ones (NCTM, 2006). Therefore, it is imperative for teachers to include tasks that require students to demonstrate an understanding of the base-ten system and how these numbers change position when numbers are added and subtracted.

Second, teachers need to provide adequate instruction on fluency and basic math facts. The intervention included instruction and practice with math facts, given its essential relationship to fluency. Although fluency information was not systematically collected during this study, it was evident through the intervention sessions that many of the students continued to struggle with basic math facts that included single digit numbers. Unfortunately, this may require teachers to commit to providing supplemental instruction, beyond that which is mandated by the curriculum and state standards.

In addition, students had limited to no exposure to the number line prior to this intervention. This pattern is disconcerting because the understanding of numerical magnitude may increase performance on numerical tasks (Laski & Siegler, 2007). For the control group, even limited exposure the number line improved numerical estimation, suggesting even just a small allocation of instructional time on number magnitude could positively affect mathematics achievement with negligible time investment.

Conclusion

There is limited research on students that are experiencing mathematics difficulties (Fuchs et. al, 2005), although the number of students that fall within this category is pervasive and troublesome. In addition to the paucity of research focused on students with mathematics difficulties, even fewer interventions have targeted this group of students. Given the proportion of students that are experiencing difficulties, as opposed to those diagnosed with disabilities, there is an urgent need to address the failure of these students. The number of students with difficulties makes it difficult to disconnect these performance deficiencies with instructional practices that facilitate or exacerbate these struggles.

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Appendix A

Intervention Sessions

1. Number sense reinforcement/ number line

Check for and reinforce number sense and number line through a card game similar to UNO™. *The researcher said* : *"the goal of the game is to initially just count up and down as shown by the cards—model 3, count up to 4, count up to 5, or down to 4, etc."* This is game was be played for several minutes until all children had adequate opportunities, and grasped the concept. Children were allowed to correct their own mistakes, and those of each other should the need arise. The researcher intervened to discuss errors as necessary.

The second component of this session was focused on the use of a paper number line to perform addition and subtraction problems. Each student received an individual paper number line for use during the the activity.

The researcher said: *"think of ways that the number line might be useful to perform addition and subtraction operations. Keep in mind the UNO™ game we played earlier and brainstorm how they might be related to one another.*

Participants were given 5-7 minutes for independent thought and exploration on how to solve problems using this method.

The researcher said: "You may have come up with some ideas of how to use the number line to add and subtract. Even if you did, I am going to demonstrate how you can do that and then I will give you time to work independently." The researcher then performed the examples with the number line saying for each problem, "*the first problem* *is 5 plus 4, so we would start at the number 5 and count up four places on the number line: 1, 2, 3, 4. This takes us to the number 9, so 5 plus 4 equals nine."*

The researcher continued in the same manner through one or more problem. *"Please count with me on the next few problems. I will then ask you to do this on your own."*

Examples included: 5+4, 6+5, 8+10, 12-9, 19-7. Participants were given problems to work independently (with number line) and paper and pencil to record answers. The researcher reviewed correct answers, while demonstrating with the number line. The researcher reiterated addition and subtraction as movement along number line. The researcher explained: *"addition and subtraction can also be thought of as movement along a number line. What we are working towards, however, is a mental number line that does not require a number line in front of us."*

2. <u>Fluency</u>

The researcher said: *"Last session we spent time working on counting up and down and using the line for addition and subtraction. Remember, we are working towards working these problems in our heads. Today we will focus on math facts that will make it easier for us to solve problems without a number line or our fingers."*

3. <u>Strategy Use (Addition/Subtraction)</u>

The researcher began the session by reviewing the concepts of decomposition, number line, and counting up and down. She explained that these concepts extend to mental addition and subtraction. On this occasion, the instructor reintroduced the UNO™ cards and placed them in two stacks. Students worked in pairs. Each turn, student turned over one card from each stack. They will be required to add the two numbers that are revealed. A different point value was earned for each correct answer based on the strategy used. Students kept score for their partner on a pre-printed score sheet with five turns for each student. At the end of the game, participants tallied their scores. The game was scored as follows: mental solving (cognitive strategy): 3 points, using number line: 2 points, and finger counting: 1 point.

The researcher said: "Today we will play UNO™ in a different way than before. Each pair or group will receive a deck of cards. Then, you will place the cards in approximately even stacks; it is not important that the stacks are exact. Now, each of you has a score sheet that is numbered from 1 to 10. These numbers go with each turn you will take. What makes this game a little different is that there are three possible scores for each turn that you take; 1, 2, or 3. You will get 1 point for counting on your fingers, 2 points if you use the number line, and 3 points if you solve the problem all in your head. Remember: the goal is to solve these problems in your head. I will now place you in teams and allow you a chance to practice once [each person will take one turn.] I will comment as necessary; please ask questions if you do not understand."

4. <u>Problem Solving Strategies (Decomposition with Addition)</u>

The researcher introduced the session by saying: *"Please raise your hand to share some of the addition and subtraction strategies that we have worked on…"* Students were given 2-3 minutes to generate and discuss responses. *"Today, you are going to learn a new strategy that will probably be completely new to you; it is called decomposition."*

She then explained: "*decomposition is a tool we can use to make larger numbers into smaller numbers so that they are easier to use. Take a minute and think of the number 10: tell me two numbers that we can combine that would equal 10..."*

Students were given a moment to think of a number combination, then allow 2-3 minutes for the sharing of answers; the researcher expected a variety of responses.

"That is the basic idea behind decomposition. Each large number can be made smaller, which will make it easier to add and subtract. Here is an example of how to use decomposition to solve an addition problem: "First, I would think of a way to break 12 into a smaller number…ah, I remember a way, 10+2. I would write those numbers on top of one anther. Then, I will think of two smaller numbers that add up to 17… 10 and 7. I then write those numbers above one another. Look, I now have two separate addition problems:

12 +17 = ?
⇓ ⇓
10 + 10 = 20
⇓ ⇓ ⇒ *20 + 9 = ?*
2 + 7 = 9

"Now we have two addition problems that will be easier for us to solve: 10+9 =19.

"Let's try another one:

19 +1 8
⇓ ⇓
10 + 10 = 20
⇓ ⇓ ⇒ *20 + 17= ?*
9 + 8 = 17

"Here is another problem that we can now solve: 20 +17= 37.

Students then received a set or problems for independent practice. This will consist of five problems and approximately 20-25 minutes will be allotted to work through these problems, and then we will review possible solutions as a group. Once again, multiple correct responses were expected and accepted.

"Next week, we will continue with decomposition, but we will focus on subtraction rather than addition."

5. <u>Problem Solving Strategies (Decomposition with Subtraction)</u>

"Think back to last week…can someone tell me what strategy we worked on?" That is correct, decomposition. If you recall, this week we will also work on decomposition, but our focus will be subtraction instead of addition."

Here is an example:

17 - 9 =?
⇓ ⇓
10 – 6 = 4
⇓ ⇓ ⇒ 4 + 4 = ?
7 - 3 = 4

"First, we must break the number 17 into two smaller numbers. The two easiest numbers that come to mind for me are 10 and 7. Now, I must make 9 into two smaller numbers. I remember one of my math facts, 6+3, yeah that works."

"Now, we have two simple subtraction problems: 10- 6=4 and 7- 3 =4. We almost have the answer, but we are not quite there. We must add the two solutions to our subtraction problems to get the final answer: 4+4 =8. So, we know by breaking the problem into smaller numbers that 17 -9 =8!"

Say: "You may notice that decomposition with subtraction is very similar to decomposition with addition. You still break the first digit into smaller numbers and do the same with the second digit as well. The main difference is that once you get two smaller numbers you subtract instead of add. BUT, it is very important that once you have the answers from your two subtraction problems that you add the two numerals together to get the final answer to the problem.

"Now, you will solve four problems on your own using this worksheet that I will give you."

6. Practice Session

Individually, and in pairs, students received time to perform a series of addition and subtraction problems, recording the answers on a pre-written sheet. Participants were not permitted to use any manipulatives or concrete objects. This session served as their opportunity to apply strategies learned in prior sessions as well as try, and share their constructed strategies with other participants. The researcher noted some of the strategies students utilized, allowing time for sharing at the conclusion of the session. In this final session, students were required to solve a series of problems presented in written form. Students were encouraged to use mental strategies, given the time limit imposed on this activity.

The following problems were utilized for the constructivist portion of this session. Again, students were permitted to work in pairs and discuss potential mental strategies for solving the following problems:

17 - 9 = ?

12 + 7= ?

19 – 9 = ?

13 + 6 = ?

8 + 11= ?

Students were allotted 15 minutes to solve and discuss these problems. They had pencil and paper to show work and record their responses. Following the activity, students completed the “no choice” mental strategy only problems. This was a timed activity with a 10-minute time limit.

Appendix B

UNO™ Game Problems
(Session 3)

1. ______ + _______ = _______ 1 2 3

2. ______ + _______ = _______ 1 2 3

3. ______ + _______ = _______ 1 2 3

4. ______ + _______ = _______ 1 2 3

5. ______ + _______ = _______ 1 2 3

6. ______ - _______ = _______ 1 2 3

7. ______ - _______ = _______ 1 2 3

8. ______ - _______ = _______ 1 2 3

9. ______ - _______ = _______ 1 2 3

10. ______ - _______ = _______ 1 2 3

Appendix C

Problem Solving Strategies (Session 4)

29 + 16 = ?
⇓ ⇓

_____ + _____ = _____

⇓ ⇓

⇒ _____ + _____ = _____

_____ + _____ = _____

22 + 12 = ?
⇓ ⇓

_____ + _____ = _____

⇓ ⇓

⇒ _____ + _____ = _____

_____ + _____ = _____

19 + 11 = ?
⇓ ⇓

_____ + _____ = _____

⇓ ⇓

⇒ _____ + _____ = _____

_____ + _____ = _____

13 + 17 = ?
⇓ ⇓

_____ + _____ = _____

⇓ ⇓

⇒ _____ + _____ = _____

______ + ______ = ______

Appendix D

Problem Solving Strategies (Session 5)

28 - 17 = ?
⇓ ⇓

_____ - _____ = _____

⇓ ⇓

_____ - _____ = _____ ⇒ _____ + _____ = _____

25 - 13 = ?
⇓ ⇓

_____ - _____ = _____

⇓ ⇓

_____ - _____ = _____ ⇒ _____ + _____ = _____

24 - 12 = ?
⇓ ⇓

_____ - _____ = _____

⇓ ⇓

_____ - _____ = _____ ⇒ _____ + _____ = _____

27 - 15 = ?
⇓ ⇓

_____ -_____ = _____

$\Downarrow$ $\Downarrow$

$\Rightarrow$ ______ + ______ = ______

______ - ______ = ______

Appendix E

No Choice/Constructivism Activity
(Session 6)

1. 18 + 9 = _____
2. 22 + 7= ______
3. 16 + 10 = ______
4. 11 + 18= ______
5. 13 + 15= ______
6. 29 - 14 = ______
7. 23 - 12 = _____
8. 19 - 17= ______
9. 38 - 22 = ______

10. 47 - 13 = ________

11. 83-21 =________

12. 39-37= ________

13. 28+41=________

14. 75-49= ________

15. 56+50= ________

16. 89-80= ________

17. 73+19= ________

18. 84-33= _________

19. 52-22= _________