AUTOMATICITY AND THE LEARNING OF MATHEMATICS

by

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(Under the Direction of Denise A. Spangler)

ABSTRACT

The purpose of this study was to examine the relationship between student automaticity of basic mathematics facts and their standardized test scores. Moreover, this study sought to determine the changes in student automaticity assessment ratios and standardized test scores in the presence of an automaticity intervention treatment for an academic year. A correlation design with a pretest-posttest paradigm was used to collect data on an automaticity diagnostic assessment and a nationally normed standardized test for middle school students in three Kentucky school districts at the beginning and end of an academic school year. No control groups were used.

Descriptive statistics analysis showed significant increases in both automaticity quotient ratios and standardized test scores from fall to spring of the academic year. Increases in introduction to algebra readiness and algebra readiness benchmarks, as defined by the test publishers, were also observed. Regression analyses revealed a positive strong relationship between student automaticity quotient ratios and standardized test scores. For each unit increase in automaticity quotient ratios, student standardized test scores increased one point. Slightly more than one third of the change in standardized test scores can be predicted from the automaticity quotient ratios, showing that automaticity is an important component for the learning of mathematics.

INDEX WORDS: Automaticity; Automaticity with basic mathematics facts; Computational fluency and automaticity; Number sense and automaticity

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DEDICATION

This dissertation is dedicated to Marilyn Faye Wyscarver, my sweet sister and my second mother, who never stopped believing in me. I shall forever cherish her wisdom and kindness.

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CHAPTER 1

INTRODUCTION

In the technological environment of the dawn of the 21^{st} century, people have amazing opportunities to learn mathematics through new mediums with interactive capability. But people have also been robbed of their opportunity to attain memorized number sequences or number facts naturally. No longer must they instantly recall strings of numbers to call someone on the telephone; they simply press a key that brings up the stored number. No longer must they instantly recognize the fractional part of an hour; they simply read a digital clock. No longer must they count change from a cash transaction; they simply swipe a card that pays for the exact purchase through the bank. And, no longer must they memorize basic number facts; they simply press buttons on a calculator.

In the past few decades of mathematics education reform, less importance has been placed on the memorization of basic mathematics facts; rather, understanding the underlying mathematics along with fluency in problem solving has been emphasized. But throughout this era of reform, researchers have warned educators that a synthesis of these components is optimal for students to learn meaningful mathematics. Gains in student achievement have not increased as expected with reform changes in mathematics education (Loveless, 2008; Hanushek, Peterson, and Woessmann, 2010; OECD, 2013). One possible explanation for this disappointment is that students may lack automaticity in basic skills to advance to higher mathematics.

Background of Study

In order to understand the importance of automaticity of basic mathematics facts in learning mathematics, a discussion on prevailing ideas associated with automaticity and basic mathematics facts is warranted. In addition, a consensus on definitions for these concepts must be reached for purposes of this study.

Automaticity of Basic Mathematics Facts

According to LeFevre and Bisanz (1987), automaticity "occurs without intention" (p. 3). Singer-Dudek and Greer (2005) cited a multitude of research studies from the 1980s when they defined *automaticity* as "performance of skills without 'conscious attention' [at a] fast rate or fluency" (p. 261). Woodward (2006) discussed automaticity in mathematics facts as the "ability to retrieve facts directly or automatically" (p. 269). Automaticity in mathematics refers to the learning of fundamental mathematics skills to mastery levels.

The terminology used to explain automaticity is unclear and often conflicting. Memorization, quick recall, fluency, unconscious processing, involuntary or obligatory effort, declarative knowledge, and fast retrieval are a few of the ideas invoked in definitions of *automaticity*. Artigue, de Shallit, and Ralston (2006) referred to learning of basic skills as "technical facets of mathematical learning" (p. 1647).

Cumming and Elkins (1999) referred to Ashcraft's (1994) explanation of automaticity in addition facts as "solution by fact recall or fast unconscious processing of the facts" (p. 150), and they connected fluency with rapid and accurate solutions. In Caron's (2007) discussion of multiplication facts, he considered that students "can recite the multiplication tables automatically when [students] know them in such depth that these facts are embedded in memory, allowing [students] to call them up at a moment's notice" (p. 281).

Robert Gagne provided an interesting contrast of two examples on the notion of automaticity given in a critique by Wachsmuth (1983) of an earlier Gagne (1983) article that discussed issues in the psychology of mathematics instruction:

> Wachsmuth's examples for automaticity of skills are $3 + 5 = 8$ and $(a + b)^2 = a^2 + 2ab + b^2$, which he discusses as propositions of declarative knowledge. The first of these is indeed declarative knowledge, which mathematics educators often call "number facts." The second, however, I consider to be the verbal statement of a rule, an intellectual skill. (Gagne, 1983, p. 215)

Gagne (1983) went on to say that computation "is most readily performed by learners as a set of intellectual skills that have been brought to the stage of automaticity" (p. 215). Crawford (2004) offered an interesting mathematical depiction of how students develop automaticity of basic mathematics facts and described student performance once it is achieved.

> The purpose of practice on math facts is to learn them to the level of automaticity. Automaticity is the third stage of learning. First, students learn facts to the level of accuracy – they can do them correctly if they take their time and concentrate. Next, if they continue practicing, they can develop fluency. Then they can go quickly without making mistakes. Finally, after fluency, if students keep practicing they can develop automaticity. Automaticity is when students can go quickly without errors and without much conscious attention, when they can perform other tasks at the same time and still perform quickly and accurately. Automaticity with math facts means students can answer any math fact instantly and without having to stop and think about it. In fact, one good description of automaticity is that it is "obligatory" – one can't help but do it. Students who are automatic in decoding can't help but read a word if you hold it up in front of them. Similarly, students who are automatic with their math facts can't help but think of the answer to a math fact when they say the problem to themselves. (p. 43)

Agreeing with Crawford (2004) as well as Logan, Taylor, and Etherton (1996), Burns (2005) explained the notion of automaticity as being "obtained when it is faster to solve the problem through recall than it is to perform a mental algorithm for completing the current task" (p. 238). Pegg, Graham, and Bellert (2005) further defined automaticity as "pupils' fluency and facility with basic academic facts in mathematics" (p. 49).

Perhaps the confusion in terminology defining automaticity lies in the use of strategy. Some explanations unquestionably deny use of strategies, while others are not clear. Are students quickly recalling and using strategies with rapid processing? Do memory-based processes include use of memorized strategy? When a student answers quickly, is the fast use of strategy discounted? Does fluency imply efficient and accurate use of strategy? Researchers who dismiss the use of strategy when studying automaticity are unable to confirm this condition completely.

Determining student automaticity capability involves the number of correct facts given by the student per minute for a designated quantity of facts. Two or three items per minute are suggested for students to be considered automatic (Frawley, 2012; Van de Walle & Lovin, 2006). For purposes of this study, an automaticity quotient (AQ) ratio of *the number of correct items* to *time used* on an automaticity diagnostic was created as a measure of student automaticity with basic mathematics facts.

Basic Mathematics Facts

According to Baroody and Dowker (2003), automaticity of mathematics facts refers to the memorization of arithmetic operations with numbers 0 through 9. But when Ball et al. (2005) discussed the "automatic recall of basic facts," they defined basic number facts as "addition and multiplication combinations of integers 0 through 10" (p. 1056). Beyond these statements, little explanation is offered as to why these ranges of numbers are preferred. Hence, many arguments for this section are subjective.

Examining studies on the development of counting knowledge in young children from Siegler and Shrager (1984), Gersten, Jordan, and Flojo (2005) concluded, "children use an array of strategies when solving simple counting and computational problems." (p. 295). These strategies, with 3 + 8, for example, range from the unsophisticated counting out of two sets of objects, 3 objects and 8 objects, and then counting them all together, to the inefficient counting that begins with 3 and counts up 8 more; from the more efficient counting that begins at 8 and then counts 3 more, to the sum stored in memory (Gersten et al., 2005).

For addition, finger range allows for solving combinations up to 10 + 10 by the novice learner, beginning the count by saying "10" and counting 10 more on the fingers. Repeated use of these strategies culminates in the ultimate strategy of storing the sums into memory. One argument to support the case for learning up to 12 + 12 is the idea that adding numbers beyond ten puts the sum out of finger range whereby students begin the count with "12" and cannot count "+12" on their fingers without running out of fingers. Such sums invite repeated practice of more advanced strategies of counting for eventual storage into memory. Flashing 10 fingers for a required number of times allows for multiplication combinations up to 10 × 10. While the previously stated research involves addition, one can assume that it can apply to the repeated addition model of multiplication. The case for stopping at 9 × 9 has no advantage in this context.

Jordan (2007) supported these arguments further when she explained, "Kindergartners, who use their fingers on simple number combinations, often stop using them in 1st grade and develop fluency by 2nd grade. By 3rd grade, most students can add or subtract combinations without external supports" (p. 64). However, students with mathematics difficulties "start using their fingers later (in 1st grade) and depend on them for longer periods of time. Their fingers are less reliable with larger combinations, and such students often fail to develop the calculation fluency necessary for higher-level math classes" (p. 64).

Butterworth (2005) stated that "some cultures do not teach the whole set of multiplication facts from 1 × 1 to 9 × 9 in tabular forms." (p. 11). From a personal communication with Yin

Wengang (n.d.), Butterworth learned that “in China, they only teach one half of the set, beginning with 2×2 (the 1× table being considered trivial) to 2×9; but since 2×3 has already been learned, the 3× table begins with 3×3, and so on. In this way, only 36 facts have to be acquired, and the equivalence of the commuted pairs has to be learned” (p. 11). Because the main counting system used in the United States is a decimal system, argument can be made for memorization of basic facts, now deemed basic combinations, up to 10 for both addition and multiplication. Number combinations of addition and multiplication include their relationship operations of subtraction and division.

However, arguments for combinations to 12 in both addition and multiplication examine the difficulty children have “crossing the decades of counting” and the difficulty American children have with the numbers 11 and 12. Combinations to $12 + 12$ or 12×12 reap the benefit of practicing these numbers in sequence while inviting memory storage of the sums and products. Furthermore, just as multiplication by 10 produces easily memorized products such as 10, 20, 30, and so forth, multiplication by 11 up to 11×9 yields easily memorized products, such as 11, 22, 33, … 99. Multiplication of 11×11 invokes the memorization of doubles, a familiar strategy used by children. Multiplication to 12 increases the memorization load by only a few more combinations.

Americans employ the English system of measure that includes units of 12 inches to a foot and 12 items to a dozen. In addition, the base 60-system is evident in time measures of 60 minutes to an hour and 60 seconds to a minute, and each can be broken down into increments of 12. Real situations that students experience may involve buying a dozen items for a particular price or measuring items needed for construction involving feet or inches, for example. Knowing combinations to 12 allows computational facility in such circumstances.

Mathematical situations that students encounter often involve computations beyond the tens combinations. An appropriate example is factoring, where factors of 10 are 1, 2, 5, and 10, whereas factors of 12 are 1, 2, 3, 4, 6, and 12. Memorized combinations to 12 facilitate the learning of factoring 12 and many combinations beyond 12. Considering the previous discussion, the argument for combinations to 12 is strong and seems reasonable. Therefore, basic mathematics facts refer to combinations of integers 0 through 12 with addition and multiplication and their relationship operations of subtraction and division. For purposes of this study, a consensus of these ideas on automaticity and basic mathematics facts was given the following definition: *automaticity of basic mathematics facts is the instant recall of number combinations from 0 to 12 using operations of addition, subtraction, multiplication, and division from memory without use of computational strategies.*

Statement of Problem

Many educators and researchers agree that automaticity is one goal of mathematics learning that must be achieved before some other goals can be reached. Burns (2005) argued "some children may lack prerequisite skills for higher-order tasks and must first master the basic information in order to move to higher levels" (p. 238). He contended this reasoning to be "especially true for mathematics, given its hierarchical nature" (p. 238).

Long-term trend National Assessment of Education Progress (NAEP) results from the 1980s through the 1990s showed little gain in scores of 9 year olds and fourth-grade students. Evidence from a small number of items on computation skills from trend NAEP assessments at the national level suggested students' computation skills have declined. Those skills are basic arithmetic skills students are expected to master at the fourth grade level (Loveless, 2003).

A synthesis of the Trends in International Mathematics and Science Study (TIMSS) mathematics results of U.S performance across international assessments of student achievement between 1995 and 2007 reported an increase in U.S. 4^{th}-grade students' average scores. However, students from the same grade in many countries, including Taiwan, consistently outperformed their U.S. peers in mathematics (Organization for Economic Co-operation and Development, 2013). Upon examination of why American students test scores are lower than Taiwanese students, Wei and Eisenhart (2011) suggested one factor could be Taiwanese mathematical curriculum, which emphasizes computational skills.

Kentucky Core Academic Standards [KCAS] (2013) expect students to know from memory all sums of two one-digit numbers by the end of the second grade and to know from memory all products of two one-digit numbers by the end of the third grade. Mathematics textbooks and curricula are written with the expectation that students beyond elementary grades have acquired these basic skills. However, many students entering the middle grades lack the pre-requisite skills they need to be successful in middle school mathematics courses. Those students might benefit from opportunities to develop automaticity of basic mathematics facts to achieve understanding of higher-level mathematics.

Purpose of Study and Research Questions

The purpose of this study was to examine students' automaticity of basic mathematics facts and their learning of mathematics. The main research question that guided this study is as follows:

> *What is the relationship of students' automaticity of basic mathematics facts to their learning of mathematics?*

In particular, the intention of the study was to address these subquestions:

- What is the relationship between the automaticity quotient ratios and Measure of Academic Progress standardized test scores of middle school students from three school districts in Kentucky?
- How did standardized test scores change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?
- How did automaticity quotient ratios change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?

Middle school students from three districts in an Appalachian region of Kentucky took part in an automaticity intervention treatment for 5 minutes daily during the 2011–2012 academic year. The students were administered a pretest diagnostic assessment of their automaticity skills at the beginning of the school year and a posttest diagnostic assessment at the end of the school year. The administrations of the automaticity diagnostic assessment coincided with district-wide administrations of the Measure of Academic Progress (MAP) standardized test. Data were collected from the students' automaticity diagnostic assessments and the MAP test scores. An automaticity quotient (AQ) ratio to describe students' automaticity of basic skills was developed for each student from the number of correct answers and the time taken to complete the assessment.

Rationale for Study

In order to consider the importance of the study, it is necessary to understand the significance of automaticity in mathematics. In addition, it is important to recognize the relationship that poverty and education, particularly mathematics courses, have to career opportunities. An examination of achievement test scores of students in recent years reveals a connection among these issues.

In the earlier part of this century, several publications addressed the place of computation in learning mathematics. The National Council of Teachers of Mathematics' (NCTM, 2006) publication of the *Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics* defined its position on basic skills with an importance placed on building students' automatization of basic mathematics facts and procedures in order to develop computational fluency as a tool for learning higher level mathematics. The National Research Council's publication *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001) stated proficiency includes both computational fluency and an understanding of the mathematics involved. They recommended a blend of explicit instruction in procedures with opportunities to apply procedures to open-ended problems with contextual relevance.

In the American Mathematics Society's publication *Reaching for Common Ground in K–12 Mathematics Education* (Ball et al., 2005), the editors argued that

> "Basic skills with numbers continue to be vitally important for a variety of everyday uses. They also provide a crucial foundation for higher-level mathematics essential for success in the workplace, which must now also be part of a basic education. … Today's students need proficiency with computational procedures" (p. 1056).

Proficiency with computations was deemed important for learning mathematics.

An examination of educational policies regarding high stakes testing revealed most universities require an entrance examination such as the ACT (2014) that predicts the college readiness of students. These entrance examinations are timed so that every second saved answering one question can be used for answering more complex questions. Although certain calculators are permitted on the mathematics portion of the assessment, time used to enter basic mathematics facts into calculators for computation puts students at a disadvantage with respect to their counterparts who have automatized those facts. Students with higher scores on the entrance examination are given preferential status for choosing the university they wish to attend and the field they wish to pursue. Opportunity for more lucrative jobs is likely.

Beyond knowledge of basic mathematics facts, students must perform mathematical problem-solving tasks with accuracy on high stakes tests. According to Goldman (1989), instant accessibility to basic mathematics facts is important for application of strategies needed to solve problems. These strategies rely on data in the form of basic number facts. Acquired strategies become part of the knowledge base for mathematics (Goldman, 1989). Hence, “lack of fluency in recalling basic math facts interferes with the development of higher-order mathematical thinking and problem-solving” (Hasselbring, Lott, & Zydney, 2005, p. 4).

Cummings and Elkins (1999) suggested that automaticity is important for performing more complex tasks. They also suggested students who have not developed automaticity might be deficient in other areas. Woodward (2006) agreed when he said that automaticity is important for skills such as estimation, mental calculation, and approximation, all of which are connected to the development of number sense.

Cummings and Elkins (1999) indicated educators and researchers erroneously assume that strategies developed naturally or by implicit instruction eventually lead to automaticity. They cited research indicating automaticity is not naturally developed in children with learning disabilities. The success of low achieving students (LA) and students with mathematics disabilities (MD), along with students who have specific memory disorders has been linked to cognitive difficulties associated with working and long-term memory deficiencies.

Accompanying these deficiencies are educational and behavioral difficulties that can impair success on academic measures (Hood & Rankin, 2005). Working memory deficits are linked with reduced retrieval of basic mathematics facts. Long-term memory difficulties involve failures to recall events or facts that were learned minutes to hours or longer previously (Hood & Rankin, 2005).

Jordan (2007) and colleagues, with supporting evidence from Gersten et al. (2005), found early screening of children for number-sense "helps identify students at risk for learning difficulties and disabilities in math" (Jordan, 2007, p. 65). Additional evidence indicated "the strong predictive value of kindergarten number sense holds into 2nd grade on a calculation fluency measure" (p. 65).

According to Gersten and Chard (1999), "number sense is facilitated by environmental circumstances" (p. 20). They further explained environmental conditions promoting "number sense are, to some extent, mediated by informal teaching by parents, siblings, and other adults" (Gersten & Chard, 1999, p. 20). They used results of Griffin, Case, & Siegler (1994) as an example that children beginning kindergarten have differing abilities on questions that ask, when given two one-digit numbers, which number is bigger "even when they controlled for student abilities in counting and working simple addition problems in the context of visual materials" (p. 20). They offered statistics that "high socioeconomic status (SES) children answered the question correctly 96% of the time, compared with low SES children who answered correctly only 18% of the time" (p. 20).

Analysis of the statistics displayed in Table 1 shows the low socio-economic status of the state population in this study. Conclusions that students who are newly entering school are underprepared are plausible.

Table 1

Kentucky School Demographics 2010

Student school membership	644,284
Per-pupil expenditure	~ $10,000
Free and reduced lunch	57%
Average freshmen graduation rate	77%
Number of districts/schools	174/1194

Note: Kentucky District Data Profiles School Year 2010, Research Report No.382, Legislative Research Commission, Frankfort, KY, lrc.ky.gov, 2011. http://www.lrc.ky.gov/lrcpubs/rr382.pdf

In January 2014, the Bureau of Labor Statistics published a display on their website that correlates the 2013 unemployment rates of salaried workers 25 years or older aggregated to their education attained. Figure 1 illustrates the display.

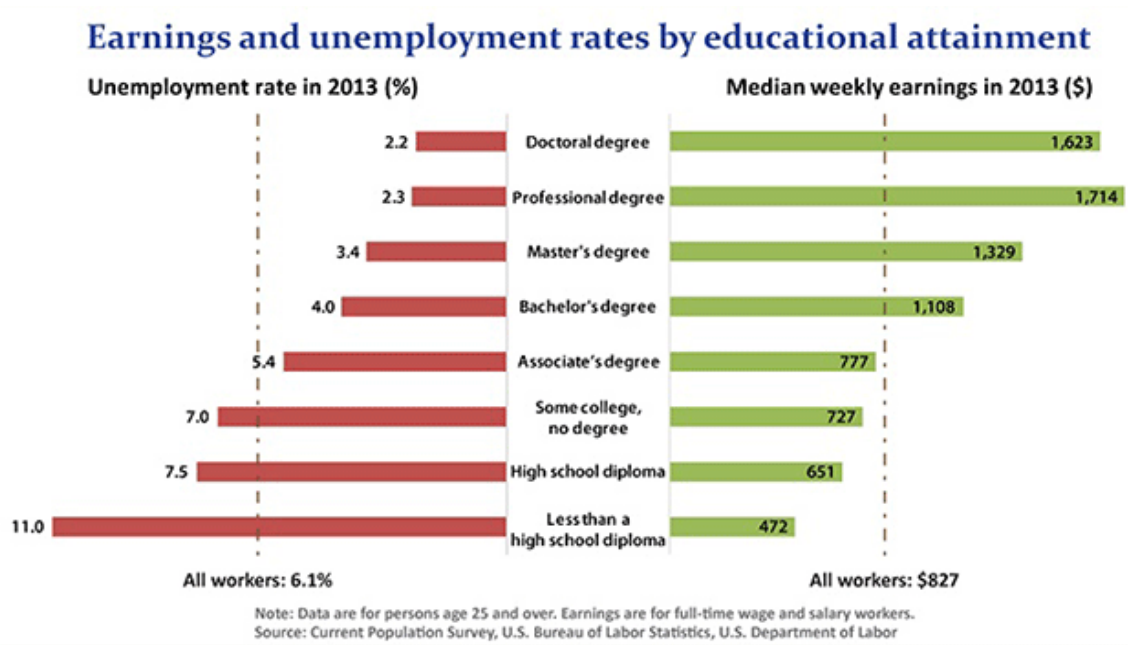

Figure 1. Bar graph showing the 2013 unemployment rates of salaried workers 25 years or older aggregated to their education attained.

The population without a high school diploma garnered the lowest earned wages and highest unemployment rate. The population earning less than an associate's college degree earned less than the national median weekly salary and reported more than the national average unemployment rate.

The statistics emphasize the role education plays in career success with respect to salary and employment. Analysis of data from the U.S. Census Bureau (2010) in Table 2 corroborates these statistics with respect to Kentucky, the state in this study.

Table 2

National and State Demographics

Demographics	Kentucky	National
Population 2010	4,339,367	308,745,538
Number of households (2007–2011)	1,681,085	114,761,359
Median income per household (2007–2011)	$42,245	$52,762
Average income per capita (2007–2011)	$23,033	$27,915
High school graduate or higher (2007–2011) age 25+	81.7%	85.4%

http://quickfacts.census.gov/qfd/states/21000.html

Rose and Betts (2001) conducted a longitudinal study of a representative national sample of students who were in Grade 10 in 1980 and examined information about the mathematics courses they took in high school, their graduation rates, the highest degree earned, and wages earned 10 years after high school. Results of data analysis indicated, "mathematics curriculum has a very large effect on earning" (p. 152), with an approximate 9 percent return for taking a one-unit algebra/geometry course. According to Rose and Betts (2001), "it is not simply the number of math courses that matters; what matters more is the extent to which students take more demanding courses such as algebra and geometry" (p. 3).

The Legislative Research Commission (LRC) (2009) for Kentucky concluded, "Kentucky students' mathematics knowledge and skills have been improving over time but are still at levels below the national average" (p. 46). To support this conclusion, they stated that trends comparing "national and state data for performance on NAEP assessments longitudinally indicate that Kentucky has a slight deficiency in scores for 4th grade but not 8th grade" (p. 36).

Upon inspection of data from college-readiness exams, the LRC (2009) also concluded that "a sizable portion of the state's high school graduates are not ready for the postsecondary education and careers of today, much less for the increasing demands of tomorrow's workplace. Achievement gaps are substantial with respect to income, race, English language proficiency, and disability" (p. 46).

Table 3

Kentucky ACT Benchmark Data

	Juniors		Graduates	
ACT	Mean benchmark	Met benchmark	Mean benchmark	Met benchmark
Mathematics	18.3	21.6%	18.8	25.2%
Composite	18.5		19.0	
Students tested	44,390		40,876	

Note: Legislative Research Commission. (2011). Kentucky District Data Profiles School Year 2010, Research Report No.382. Frankfort, KY, lrc.ky.gov. Retrieved from http://www.lrc.ky.gov/lrcpubs/rr382.pdf.

The state ACT benchmark data in Table 3 support the conclusions that students in the state's schools are not college and career ready, lowering their chances at future successful employment. “For each test, ACT identified the minimum scores, or benchmarks, on each test that predict future success. On the ACT exam, students scoring 22 have a 75% chance of earning a C and a 50% chance of earning a B in a college algebra class” (LRC, 2009, p.31). The state statistics discussed reveal a cycle of poverty and low education levels that can only be broken with better educational preparation of students to become college ready.

According to the ACT (2011a) mathematics test description, the test is “designed to measure the mathematical skills students have typically acquired in courses taken by the end of 11th grade” (Mathematics test description section, para. 1). The questions require the use of reasoning skills to solve practical problems in mathematics and test-takers need “knowledge of basic formulas and computational skills” (ACT, 2011a, Mathematics test description section, para. 3). Some types of calculators are permitted, and the test has a time limit.

The *Washington Post* (Strauss, 2011) reported that 45% of all ACT-tested students in the 2011 high school graduating class met the mathematics benchmark score of 22. This benchmark is the predictor of student success in significant college courses. “Success is defined as a 50% or higher probability of earning a B or higher in the corresponding college course or courses” (ACT, 2014, College readiness benchmarks section, para. 1). ACT recommends that if students are to be ready for college or career when they graduate, their progress must be monitored closely so that deficiencies in foundational skills can be identified early and remediated immediately in upper elementary and middle school (ACT, 2011b).

The Iowa Test of Basic Skills (ITBS) purports to “provide a comprehensive assessment of student progress in major content areas … the math test emphasizes the ability to do quantitative reasoning and to think mathematically in a wide variety of contexts” (ITBS, 2010, Math section, para. 1). The mathematics portion of the test has three components: concepts, problem solving, and computation. The computation portion of the test evaluates “operations with whole numbers, fractions, decimals, and various combinations of these, depending on the test level” (ITBS, 2010, Math section, para. 5).

Calculator use is restricted to the concepts portion of one level of the test and a time limit is imposed (ITBS, 2010). Table 4 gives a summary of ITBS statistics from the Kentucky Department of Education for Grades 3 through 7 in 2010 and 2011 (Kentucky Department of Education, 2011).

Table 4

Kentucky ITBS Scores for Grades 3 to 7

	Grade				
Year	3	4	5	6	7
2010	60	56	52	45	48
2011	61	57	54	45	48

The scores were low and for the most part decreased each year of instruction at these grade levels. A breakdown of the results for demographically similar school districts revealed that computation scores were between 10–20% lower than the scores on the concepts and problem-solving portions of the test (Thomas, Crowe, & Williams, 2011). These results suggested that student weakness in computational skills influenced the composite score totals of the three components on the standardized assessment. When students lack the opportunity to attain automaticity of basic mathematics facts naturally, an artificial automaticity treatment might be beneficial.

This study investigated the relationship of automaticity of basic mathematics facts and students' standardized test scores in the presence of an automaticity intervention treatment. It also examined the changes in students' automaticity of basic mathematics facts as well as their standardized test scores in the presence of an automaticity intervention treatment. Addressing the research questions posed for the study requires an understanding of automaticity and how it is situated within two important components of learning mathematics known as number sense and computational fluency.

Definitions

Throughout this chapter, some of the terminology discussed was contentious and definitions were challenging to develop. When necessary, the developed definitions were truncated here to facilitate reading.

- Automaticity – instant recall of mathematic facts from memory without use of strategy
- Automaticity Quotient – ratio of number correct items to time used on the automaticity diagnostic
- Basic Mathematics Facts – number combinations from 0 to 12 using operations of addition, subtraction, multiplication, and division
- Computational Fluency – having efficient, flexible and accurate methods for computing (NCTM, 2000)
- Correlation – statistical dependence relationship between variables
- MAP – Measure of Academic Progress® nationally normed standardized assessment published by Northwest Evaluation Association

- Number Sense - an intuitive way of thinking about numbers that can be developed into a conceptual structure of understanding mathematics
- Working Memory – part of the executive function that temporarily stores and manages information needed to complete complex cognitive tasks

CHAPTER 2

REVIEW OF THE LITERATURE

Three major arenas of literature inform this review: automaticity and learning mathematics, computational fluency, and number sense. To advance their knowledge of mathematics, students must have a meaningful understanding of mathematics concepts. This understanding requires a strong foundation in number sense as well as computational fluency.

Number sense and computational fluency allow students to apply their understanding of mathematics concepts to confidently solve mathematics problems with flexible use of efficient strategies and accurate calculations. (NCTM, 2000). Therefore, number sense and computational fluency have a symbiotic relationship that fosters mathematics development and understanding. Automaticity of basic mathematics facts is an important element of both these components.

Automaticity and Learning Mathematics

Cumming and Elkins (1999) conducted a study that was "designed to explore the hypotheses that lack of automaticity on the basic addition facts contributes to failure on a more complex task" (p. 150). The results indicated that processing efficiency in the addition facts affected performance on multi-digit addition sums.

Geary, Liu, Chen, Saults, and Hoard (1999) conducted a study that compared 237 U.S. and 218 Chinese college along with 55 U.S. and 80 Chinese high school students' performances on arithmetical computational and reasoning tests as well as IQ and spatial reasoning tests. Except for spatial reasoning, which showed no differences, the Chinese students' performance was only slightly better than the U.S. students' performance after controlling for IQ and computational fluency. However, differences in computational fluency were substantial. These results are "consistent with the position that the East Asian advantage in computational abilities contributes to the advantage in arithmetical reasoning." (p. 716)

More specifically, Crawford (2004) noted, "Students who are automatic with math facts find learning new computation algorithms much easier and are able to use mental math to solve problems as well" (p. 43). According to Woodward (2006), "finding common multiples when adding fractions with unlike denominators or factoring algebraic equations are but two examples from secondary-school mathematics where automaticity in math facts can facilitate successful performance" (p. 269).

In a review of literature of cognitive studies, Geary (2004) argued that the emphasis placed on arithmetic fact retrieval depends on the type of instruction emphasized. Conceptual-based instruction is less likely to stress facts and procedures, whereas a scientific-based approach will likely promote them. He suggested that one strategy to study mathematical competencies without interference from instructional issues is "applying the theories and methods used by cognitive psychologists to study mathematical competencies in typically achieving children to the study of children with mathematics learning disabilities (MLD)" (p. 4).

Geary (2004) surveyed such cognitive research along with studies of dyscalculia and brain imaging of mathematical processing to understand the "cognitive and brain systems that support mathematical competency and any associated learning disabilities" (p. 4). He provided a research-based chronological view of "typical development in the counting and arithmetic domains, along with patterns that have been found with the comparison of children with MLD to their typically achieving peers" (p. 4). He defined children with MLD as having low scores in mathematics relative to IQ.

Geary (2004) cited multiple studies to describe the typical child's counting development according to Gelman and Gallistel's (1978) five implicit principles: one-to-one correspondence, stable order, cardinality, abstraction, and order irrelevance. Children also develop beliefs in unessential features of counting called *standard direction* and *adjacency*. "By 5 years of age, many children know the essential features of counting but also believe that adjacency and standard direction are essential features of counting" (Geary, 2004, p. 6).

Studies of children in first grade showed that the children with MLD had deficiencies in different components of the implicit principles and unessential beliefs. Although many of these children understood cardinality and stable order, they demonstrated difficulty with order irrelevance and adjacency. For example, referring to Hitch and McAuley (1991), Geary (2004) stated that the inability of the child to recognize double-counts at the beginning or end of counting a set suggests "difficulties holding information in working memory while monitoring the act of counting" (p. 6).

Deficiencies in counting contribute to delayed arithmetical progress. In the development of arithmetic, the typical child changes the "distribution of procedures, or strategies" used in solving problems (Geary, 2004, p. 7). For example, in first learning to add, finger counting or verbal counting strategies are generally employed. The two procedures typically used are *counting all* "when children count both addends starting at one" and *counting on* "when children state one addend and count the value of the other addend" (Geary, 2004, p. 7).

Citing work from Geary, Bow-Thomas, & Yao (1992) and Briars and Siegler (1984), Geary (2004) explained that as their conceptual understanding of number improves, typical children shift from counting all to counting on with less finger or verbal support. With examples from Geary et al. (1999) and Geary, Hamson, and Hoard (2000), Geary (2004) noted that "first and second grade children with MLD committed more counting errors and used the developmentally-immature counting all procedure more frequently" and for a "longer time than typically achieving children," relying heavily on finger or verbal strategies (p. 7).

As the typical child uses these counting procedures, he or she develops "memory representations of basic facts" (Geary, 2004, p. 7), which are stored in long-term memory. "Once formed, these long-term memory representations support the use of memory-based problem solving processes," of which the most common are "direct retrieval of arithmetic facts and decomposition" (p. 7). Direct retrieval involves retrieving from long-term memory the answer to a problem. With decomposition, the child "reconstructs the answer based on the retrieval of a partial sum" (p. 7) using, for example, doubles to add (6 + 7) as (6 + 6) + 1.

Using multiple research sources, Geary (2004) determined that as the collection of strategies develops, for example, fact retrieval or decomposition, "children solve problems more quickly because they use more efficient memory-based strategies" (p. 7) and with practice take less time to execute them. He added that the "eventual automatic retrieval of basic facts and the accompanying reduction of the working memory demands" (p. 7) allow for the finding of solutions to more complex problems with less error.

LeFevre and Bisanz (1987) agreed with Geary (2004) in their claim that "one aspect of the development of arithmetic skill is the increasingly efficient and automatic retrieval of stored facts" (p. 5). Children with MLD "do not show a shift from procedure-based problem solving to memory-based problem solving that is commonly found in typically achieving children, suggesting difficulties in storing arithmetic facts in or accessing them from long-term memory" (Geary, 2004, p. 8).

Many mathematics education and cognitive researchers have investigated the correlation between memory and automaticity of basic arithmetic facts. According to Gagne (1983), "The desirability of automatizing intellectual skills continues to be strongly implied by contemporary research on human cognition" and he suggested, "it is a prerequisite for the understanding of mathematics" (p. 216). Binder, Haughton, and Bateman (2002) agreed when they claimed that "another way to understand the effects of fluency or "automaticity" is that it frees up attention for higher order application rather than overloading attention with the mechanics of performance. Fluency in foundation skills frees attention for application, creativity, and problem-solving – the higher-order activities that make education valuable and fun" (p. 5).

Caron (2007) argued, "developing automaticity frees up cognitive capacity for problem solving" (p. 278). Using the example of multiplication facts, he further explained his position from his experience with high school students.

> Without this seemingly simple set of knowledge, students are virtually denied anything but minimal growth in any serious use of mathematics or related subjects for the remainder of their school years and, most likely, the rest of their lives. This includes both single and multiple digits, whether on a computation sheet or in a word problem. (p. 278)

Referring to Gagne (1983), Caron (2007) concluded, "Without automatization of these basic computations their solution uses cognitive capacity needed for successful problem solving" (p. 279).

Cognitive experts have conducted many studies of the working memory as it relates to fact retrieval practices as well as the effects of speed of processing. "Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes" (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007, p. 1345). Ehrenstein, Schweickert, Choi, and Proctor (1997) asserted "humans must often use working memory to execute processes one at a time because of its limited capacity" (p. 766). However, they do allow that "items can be held in storage while other tasks are being performed" (p. 767), and they claimed that "time to complete a second task increases as the number of items to be held in memory increases" (p. 767).

This literature indicates the eventual development of arithmetic skills leads to the subsequent increase in more advanced arithmetic skills. The repeated retrieval of stored facts in working memory enables transfer of knowledge into long-term memory for more efficient processing, which frees short-term memory for more advanced mathematical thinking. Many of the researchers developed a progression of learning skills that when automatized, build the foundation for the successive development of new mathematics knowledge. But what if students do not develop automaticity of these skills naturally?

To include repetition of previously learned mathematics with new lessons being taught is considered optimal for committing new learning to memory, but not all students naturally automatize what they learn. An artificial means of committing basic mathematics facts to memory might help those students who need more repetition than their teacher offers.

Some controversy surrounds artificial strategies to automatize basic number facts. Kamii and Dominick (1997) proposed that automatizing mathematics before understanding the underlying concepts is harmful to students learning mathematics. They suggested that forced automaticity stifles students' mathematics creativity and restricts them from being able to understand the underlying mathematics once algorithms have been memorized. So what components should be integrated into artificial strategies that promote automaticity of basic mathematics facts without restricting conceptual learning?

Components of Effective Automaticity Intervention Treatments

Supporting previous discussion, Smith-Chant (2010) and Korn (2011) contended that children could solve more advanced mathematics problems successfully when less demand is made on the working memory. When arithmetic facts are stored in long-term memory, the working memory is more efficiently used for advanced skills requiring more effort. Motivated by Barrouillet (2005), Korn (2011) advocated, "Repeated practice of simple arithmetic problems reinforces arithmetic facts that overtime become stored in long-term memory" (p. 3). She contended, "Memorizing arithmetic facts is best taught through repeated practice" and noted "practice and quizzes are not only useful for testing knowledge of arithmetic facts, but they are helpful in strengthening memorized arithmetic facts as well" (p. 5).

According to Frawley (2012), timed drills are also an important factor in developing automaticity. "Math fluency is often calculated by determining a student's digits correct per minute for a specific set of facts (e.g., addition, division). Students who possess fluency can recall facts with automaticity, which means they typically think no longer than two seconds before responding with the correct answer" (Frawley, 2012, What is math fact fluency? section, para. 1).

Interventions such as flashcards, drills, timed tests, and so forth, are available for developing automaticity of mathematics facts. Frawley (2012) referred to a meta-analysis by Codding, Burns, and Lukito (2011) to determine which intervention components are effective in helping students recall basic mathematics facts. They examined 17 single-case design studies with 55 elementary students identified as struggling mathematics learners. "Codding et al. (2011) found that math interventions that contain the components of practice with modeling and drill produce the largest treatment effects" (Frawley, 2012, How can teachers assist students develop math fact fluency? section, para. 1). They determined that adding “auditory and/or visual models with practice provide students additional repetitions that can lead to increased retention and recall of math facts” and that “the multiple practice opportunities reinforce the accuracy of the students’ answers and result in increased proficiency” (Frawley, 2012, What is practice with modeling? section, para.1).

The literature shows that students can solve more advanced problems when less demand is made on the working memory. Repetition allows facts to be stored in long-term memory for later retrieval. Repeated practice with modeling that supplies immediate feedback is effective for students to improve automaticity skills. Interventions that incorporate these components described are effective in helping students develop automaticity of basic facts.

Computational Fluency

It seems the lack of clarity concerning fluency in mathematics computation also lies within definitions. Terminology such as computational fluency, algorithmic facilitation, and algorithmic computation are used interchangeably. In a discussion involving the definition of computational fluency Corlu, Capraro, and Corlu (2011) offered this discourse on the confusion:

> Computational fluency has been misunderstood by many as the set of rules of arithmetic; similar to problem solving which was once interpreted as students solving simple word problems so that algorithmic calculations could be avoided. The lack of a common definition caused researchers to use concepts such as algorithmic thinking, algorithms, computation, arithmetic, etc. interchangeably, and even sometimes incorrectly (pp. 72–73)

According to Mabott and Bisanz (2003), “computational skill usually is indexed by the accuracy and speed with which problems are solved as well as by the solution procedure that is used” (p. 1092). From examples given in Fuchs, Fuchs, Compton, Powell, Seethaler, Capizi, Schatschneider, and Fletcher (2006) and Carpenter, Fennema, and Franke (1996), Corlu et al. (2011) determined that (3 + 2) is considered arithmetic because it incorporates automaticity and

(35 + 29) is considered algorithmic computation because a strategy for solving is involved. If a strategy were used for (3 + 2)—counting up, for example—to find the answer, then it too would be algorithmic. They offered that “algorithmic computation involves systematic processes comprised of operation(s) and relative symbols to reach the solution rather than memorized answers to a mathematical problem” (p. 73). Thus, the distinction between arithmetic and an algorithm is not an inherent characteristic of the problem; it is a characteristic of the solver.

It seems that Ball et al. (2005) added an automaticity component to fluency in their discussion of automatic recall of basic facts claiming, “certain procedures and algorithms in mathematics are so basic and have such wide application that they should be practiced to the point of automaticity” (p. 1056). They emphasized computational fluency in whole number arithmetic and asserted that “crucial ingredients of computational fluency are efficiency and accuracy” and that “ultimately, fluency requires automatic recall of basic number facts” (p. 1056).

Binder et al. (2002) advocated that “fluency goes beyond mere accuracy to include the pace, or speed of performance” (pp. 3–4). They claimed that “fluency is true mastery: accuracy + speed” (p. 3). Caron (2007) referred to Gagne’s (1983) emphasis for processes of computation, which he believed to be the basis of all problem solving, to be “not just learned, not just mastered, but automatized” (Caron, 2007, p. 278).

Computational Fluency and Automaticity

A study by Cumming and Elkins (1999) of students in Grades 3 through 6 examined computational facility and the relationship between automaticity, as defined by the efficient processing of addition facts, and success in more complex tasks, in the way of multi-digit sums. Implications from the study showed that “the cognitive demands caused by inefficient solutions of basic facts made the multidigit sums inaccessible" (p. 149). Corlu et al. (2011) agreed with their findings and claimed "efficiency, in the sense of automaticity, should still be an essential target of algorithmic computation teaching as well as flexibility, and accuracy" (p. 73).

Binder et al. (2002) identified examples of informal experience and scientific research which "suggest that fluency contributes directly to three types of critical learning outcomes: Retention and maintenance; endurance; and application" (p. 4). Similarly, Crawford (2004) advocated that automaticity allows students to "focus their mental energies on the problem solving steps rather than the facts" (p. 43).

After allowing for the considerable information indicated with respect to automaticity of mathematics facts and fluency in mathematics computation, it is important to bear in mind the National Mathematics Advisory Panel's (2008) report on mathematics education in the U.S. that recognizes the "mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic recall of facts" (p. xiv). It is also essential to consider the strands of mathematical proficiency promoted in *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001) supporting the view that proficiency includes both computational fluency and understanding of the underlying mathematical ideas and principles. The understanding referred to by both of these sources is known as number sense.

Number Sense

According to Gersten et al. (2005), "no two researchers have defined number sense in precisely the same fashion" (p. 296). Many researchers' attempts to define number sense quoted Case's (1998) position (e.g., Gersten & Chard, 1999; Gersten, Jordan, & Flojo, 2005; Malofeeva, Day, Saco, Young, & Ciancio, 2004).

In his struggle to define *number sense*, Case (1998) stated the following:

> Number sense is difficult to define but easy to recognize. Students with good number sense can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They can invent their own procedures for conducting numerical operations. They can represent the same number in multiple ways depending on the content and purpose of this representation. They can recognize benchmark numbers and number patterns: especially ones that derive from the deep structure of the number system. They have a good sense of numerical magnitude and can recognize gross numerical errors, that is, errors that are off by an order of magnitude. Finally, they can think or talk in a sensible way about the general properties of a numerical problem or expression – without doing any precise computation. (p. 1)

Kalchman, Moss, and Case (2001) attempted to "operationalize" (Gersten et al., 2005, p. 297) the concept of "good number sense" with these characteristics:

- "Fluency in estimating and judging magnitude
- Ability to recognize unreasonable results
- Flexibility when mentally computing
- Ability to move among different representations and to use the most appropriate representation" (p. 2).

Gersten et al. (2005) also contended that magnitude comparison and the ability to use some type of number line are two essential foundational principles of number sense. This assertion has support from Gersten and Chard's (1999) reference to research of the mid 1990s when they noted that a mental number line, which allows comparison of number magnitude, "appears to be the critical 'big idea' necessary for solving addition and subtraction problems common in first grade" (p. 23).

Berch (2005) compiled a list of presumed features of number sense from a collection of literature in the domains of mathematical cognition, cognitive development, and mathematics education. From this list he contended, "number sense reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line" (p. 333). He further asserted that

> Possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information. (pp. 333–334)

Capturing ideas from Case, Harris, and Graham (1992), Gersten et al. (2005) summarized the notion of number sense as "a conceptual structure that relies on many links among mathematical relationships, mathematical principles (e.g., commutativity), and mathematical procedure" (p. 297). These linkages support growth in students' higher order mathematical thinking and insights in problem solving. The development of mathematical proficiency is connected to early development of these linkages, and "number sense can be enhanced by informal or formal instruction prior to entering school" (p. 297). Children who have not created such linkages may need intervention in order to build them.

From a multitude of compiled references, Berch (2005) offered 30 "alleged components of number sense" (p. 334). An inspection of these components "reveals that number sense reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or number line" (p. 333)

Agreeing with some of these components, Jordan (2007) stated that "number sense refers to intuitive knowledge of numbers" (p. 64) and gave examples such as "the ability to grasp and compare quantities (6 versus 8); internalize counting principles (the final number in a count indicates the quantity of a set, numbers are always counted in the same order); and estimate quantities on a number line" (p. 64). Drawing from multiple sources that included Case and Sandieson (1991), Case (1998), Baroody and Wilkins (1999), and Gersten and Chard (1999), Malofeeva et al. (2004) gave a broad definition of number sense as "an understanding of what numbers mean and of numerical relationships" (p. 648). They further explained their definition "reveals that number sense includes a collection of concepts about quantities (e.g., more and less, one-to-one correspondence, cardinality, ordinality, and understanding of the relative size of numbers) and their interrelationships" (p. 648).

The synthesis and analysis of ideas concerning number sense seem to emerge with similar philosophies and principles of what one is able to do if one has good number sense. This emergence likely occurs because most researchers cite Case's stance on number sense. The definition of number sense as a *conceptual structure* (also from Case) is intriguing, as is the *intuitive knowledge of numbers,* as well as *an understanding of what numbers mean,* but little insight is offered in the previous discussions as to their meanings. These definitions merely relate the notion to links on which gaining number sense relies to make mathematical connections or give examples of selected concepts. Defining let alone understanding the notion of number sense seems difficult; examining the developmental versus ability debate may shed further light.

Berch (2005) discussed what he considered the origins of number sense. "Some [theorists] consider number sense to be part of our genetic endowment, whereas others regard it as an acquired skill set that develops with experience" (p. 334). He explained that the major difference between these philosophies concerns the "lower order" interpretation of number sense as a biological "perceptual" nous of number and a "higher order" interpretation as a developed "conceptual sense making" of mathematics (p. 334).

Drawing from research by Dehaene (1997, 2001) and Geary (1995), Berch (2005) asserted that the genetic viewpoint confines components of number sense to basic insights about quantity, “including the rapid and accurate perception of small numerosities and the ability to compare numerical magnitudes, to count, and to comprehend simple arithmetic operations” (p. 334). Drawing from research by Greeno (1991) along with Verschaffel and DeCorte (1996), Berch (2005) contended that while such components are included in the higher order viewpoint, number sense from the naturalist standpoint is deemed to be “much more complex and multifaceted in nature” (p. 334).

Berch (2005) further explained the naturalist position that number sense ability

> Comprises a deep understanding of mathematical principles and relationships, a high degree of fluency and flexibility with operations and procedures, a recognition of and appreciation for the consistency and regularity of mathematics, and a mature facility in working with numerical expressions – all of which develop as a byproduct of learning through a wide array of mathematics education activities. (p. 334)

Within this understanding of number sense, Berch (2005) listed other types of mathematical “senses” elicited from Arcavi (1994), Linchevski and Livneh (1999), Picciotto Wah (1993), and Slavit (1998): “an operation sense, a graphic sense, a spatial sense, a symbol sense, and a structure sense” (p. 337).

Examining theories such as those from Gersten et al. (2005) and Robinson et al. (2002), Berch (2005) claimed, “If number sense is viewed as a skill or a kind of knowledge rather than an “intrinsic” process, it should be teachable” (p. 336). So, if number sense is embedded in “our biological makeup, what are the implications of this perspective for “teaching” it or at least fostering its development?” (p. 336).

One answer lies in Berch’s (2005) contention that most naturalist researchers who believe number sense encompasses “a long evolutionary history and a specialized cerebral substrate” do not believe that number sense is permanent or absolute (p. 336). He asserted that children’s development of basic number sense abilities takes place “spontaneously without much explicit instruction" (quoting Dehaene, 1997, p. 245) within desirable childhood environments (Berch, 2005, p. 336). He also contended "the neurocognitive systems supporting these elementary numerical abilities include what has been referred to as *skeletal principles*, as denoted by Gelman and Meck (1992) and Gelman (1990), because they provide just the foundational structure for the acquisition of these abilities" (Berch, 2005, p. 336).

In their analysis of research by Griffin, Case, and Siegler (1994), Gersten et al. (2005) stated that development of certain number sense concepts is "linked to the amount of informal instruction that students receive at home on number concepts" (p. 297). In addition, Gersten and Chard (1999) reported:

> On average, in well-educated middle-class homes, there is a good deal of informal instruction about numbers and concepts related to numbers such as two more or double and, on average, significantly less of this type of instruction in low socio-economic status (SES) homes. (p. 20)

According to Gersten and Chard (1999), "number sense is facilitated by environmental circumstances" (p. 20). They further explained that environmental conditions promoting "number sense are, to some extent, mediated by informal teaching by parents, siblings, and other adults" (p. 20). As an example, they referred to research from Griffin et al. (1994) to state that "entering kindergartners differed on questions such as 'which number is bigger, 5 or 4?' even when they controlled for student abilities in counting and working simple addition problems in the context of visual materials" (Gersten & Chard, 1999, p. 20). They offered statistics that "high socioeconomic status (SES) children answered the question correctly 96% of the time, compared with low SES children who answered correctly only 18% of the time" (p. 20).

However, Gersten et al. (2005) contended that some students with delays in early mathematical development are able to "catch up with their peers … when provided with appropriate instruction in pre-school, kindergarten, or first grade in the more complex aspects such as quantity discrimination" (p. 297). Straddling both camps of thought, Berch (2005) asked this essential question: "What are the pedagogical implications of viewing number sense as a much more complex and multifaceted construct than 'simply' possessing elementary intuitions about quantity?" (p. 336).

Berch (2005) drew from Reys (1994), Verschaffel and De Corte (1996), and Greeno (1991) to form an answer to his question. Reys (1994) contended that number sense constitutes "a way of thinking that should permeate all aspects of mathematics teaching and learning" (p. 114). Verschaffel and De Corte (1996) argued that components of number sense "cannot be compartmentalized into special textbook chapters or instructional units" (p. 109). Greeno (1991) suggested, "It may be more fruitful to view number sense as a by-product of other learning than as a goal of direct instruction" (p. 173). From these sources Berch (2005) maintained, "[Number sense] development does not result from a selected subset of activities designed specifically for this purpose" (p. 336).

So, added to the previous ambiguous notions of number sense as a *conceptual structure*, *intuitive knowledge of numbers*, and *an understanding of what numbers mean*, it is also described as *a way of thinking* about numbers and *a by-product of other learning beyond direct instruction* along with dichotomies of *genetic intuition of quantity* and *conceptual understanding of mathematics*. Perhaps a useful synthesis of ideas from these discussions is that *number sense is a way of thinking about numbers initially embedded in genetic intuitions of quantity that can later be developed formally or informally into a conceptual structure of understanding mathematics as a by-product of other learning beyond direct instruction.*

Number Sense and Automaticity

From works of Ginsburg (1998) and Hiebert, Carpenter, Fennema, Fuson, Murray, Oliver, Human, & Wearne (1997), Bottge et al. (2007) contended "an important feature of [mathematics education] reform is the merging of basic skills instruction (e.g., procedural knowledge) with problem-solving instruction (e.g., conceptual understanding) so students become literate in both areas." (p. 96). Gersten and Chard (1999) discussed models of learning that incorporate "the number sense concept" (p.19) and that rely on Cobb's (1995) "conceptualization of constructivism as a joint approach" (Gersten & Chard, 1999, p. 19). These models provide what they deemed "a sensible middle ground in the mathematics reform debate" (p. 18). They warned educators to consider that "along with increased competence and fluency with basic addition and subtraction facts, children also develop—or fail to develop—a number sense" (p. 19).

For example, Gersten and Chard (1999) considered "the problems students with learning disabilities have with subtraction that requires regrouping" (p. 23). One interpretation offered for such difficulty was that "this is the first math skill for which the child needs number sense to solve problems and, without such a sense, performance breaks down" (p. 23). They reminded educators that "at some point in time, even basic arithmetic facts are problems to be solved by naïve learners … and therefore, mere drill and practice on basic math facts will be insufficient for developing students who are competent in mathematics" (p. 25). Using research by Siegler and Shrader (1984), Gersten et al. (2005) later supported these notions by reminding us that "a basic arithmetic combination such as 2 + 9 is at some time in a person's life a complex, potentially intriguing problem to be solved" and that "only with repeated use does it become a routine "fact" that can be easily recalled" (p. 295).

Gersten and Chard (1999) argued that even students lacking automaticity with basic facts should still "be engaged in activities that promote the development of number sense and mathematical reasoning" (p. 25). Chard et al. (2008) asserted that "number sense not only leads to automatic use of math information taught in school, it is also is a key ingredient in the ability to solve basic arithmetic problems" (p. 13). But both Gersten and Chard (1999, p. 20) and Chard et al. (2008, p. 13) stressed the importance of students memorizing to automaticity "more than 100 basic addition facts" to support the exploration of number combinations.

An explanation of this contention comes from Gersten et al. (2005), who offered "if a child can easily retrieve some basic combinations (e.g., 6 + 6), then he or she can use this information to help quickly solve another problem (e.g., 6 + 7) by using decomposition (e.g., 6 + 6 + 1 = 13)" (p. 295). In addition, they stressed, "the ability to store this information in memory and easily retrieve it helps students build both procedural and conceptual knowledge of abstract mathematical principles, such as commutativity and the associative law" (p. 295). They further asserted that immature counting strategies such as finger counting or counting objects offers little opportunity for children to learn such principles.

Additional explanation came from Gersten and Chard (1999), who said

> A key strategy that most children learn is the "min" strategy – that it is more efficient to start with the larger number than the smaller one when trying to find the answer to either 3 + 8 or 8 + 3 if using one's fingers, manipulatives, or stick figures. Acquisition of this mini-strategy is an essential predictor of success in early mathematics. Children often learn or deduce this strategy (p. 23).

They referred to Siegler's (1988) findings that "some children do not acquire the strategy readily" and contended "these less successful children seem to represent students with learning disabilities and students who are at risk for school failure" (Gersten & Chard, 1999, p. 23).

According to Gersten and Chard (1999), strategies such as the "min" strategy can be difficult to teach since children need some "factual automaticity" to remember, for example, 8 is greater than 3. But they agree that children also need number sense to help gain access or automaticity in retrieving these facts. Therefore, children must master these three components: "problem-solving strategies, verbal comprehension, and automaticity with relevant facts" (p. 24). They suggested "children differ in their sense of numbers, their representation of problems, and their application of strategies that integrate all the previous components to solve even basic arithmetic problems" (p. 24).

Findings from years of research of Siegler and colleagues suggested to Gersten and Chard (1999) the existence of "an intricate relationship between conceptual understanding and consistent use of efficient strategies for computation and problem solving" (p. 26). Examination of Baroody and Rosu (2006) led Jordan (2007) to contend, "mastery of number combinations is tied to knowledge of fundamental number concepts" (p. 64). She further argued, "Although calculation fluency is not sufficient for succeeding in advanced math, such as algebra, it is a necessary foundation" (p. 65). She explained, "Weak computational fluency, a distinguishing feature of math difficulties, reflects basic deficiencies in number sense. Helping students build number sense right from the start gives the background they need to achieve in later years" (p. 65).

Gersten and Chard (1999) summarized the findings of the Siegler and Stern (1998) studies on efficient strategy development of children for problem solving, which asserted "both computational speed and accuracy and conceptual understanding influenced development of the strategy for efficiently solving the problem" and that "math instruction needs to take all these facets into account" (p. 24). To further this idea of an interdependent relationship, Rittle-

Johnson and Alibali (1999) concluded “conceptual and procedural knowledge may develop interactively, with gains in one leading to gains in the other, which in turn trigger new gains in the first. Thus procedural knowledge could also influence conceptual understanding… [and] under some circumstances, children first learn a correct procedure and later develop an understanding of the concepts underlying it” (p. 6). A primary example of such circumstances is the child who counts by rote at a very young age and then later learns one-to-one correspondence when counting objects. To further understand the relationship of automaticity and number sense, it may help to examine students who lack automaticity and analyze the cause.

Low-Achieving Students

Gersten et al. (2005) suggested from research of Geary and colleagues that there is a “link between mathematics difficulties (MD) and efficient, effective counting strategy use” (pp. 294–295). Gersten and Chard (1999) used studies by Geary (1993) and McCloskey and Macaruso (1995) as examples of increasing empirical support to contend that [lack of] informal home instruction may be “related to underlying deficits in learning disabilities” (p. 20). They also claimed that existing “early interventions focusing on pre-numeracy skills attempt to expose children to experiences lacking in their home or in preschool” (p. 24).

Research from Griffin et al. (1994) supported Gersten and Chard (1999) in their claim that “instruction including number sense activities leads to significant reductions in failure in early mathematics” (p. 20). In addition, Gersten and Chard (1999) asserted that “simultaneously integrating number sense activities with increased number fact automaticity rather than teaching these skills sequentially appears to be important for both reduction of difficulties in math for the general population and for instruction of students with learning disabilities” (p. 20).

Furthermore, Gersten and Chard (1999) summarized findings from Griffin (1998), which demonstrate that “schools could provide guided instruction that builds number sense in kindergartners who enter with deficits in the area of abstract mathematical reasoning “(Gersten & Chard, 1999, p. 23). They then proposed, “the goal of this instruction is, in large part, for students to develop an elaborate and integrated schema that centers on a mental number line, allowing students to solve a variety of addition and subtraction problems” (p. 23).

According to Gersten and Chard (1999), as teachers increasingly offer reform-oriented teaching of mathematics, “students’ comprehension of what the teacher is discussing is likely to be limited because the teacher assumes such automaticity as a basis for explanations” (p. 21). They summarized conclusions of Woodward and Baxter (1997) that “students with disabilities in mathematics tended to make significantly less growth in discussion-oriented classrooms then students with disabilities taught in more traditional methods” (Gersten & Chard, 1999, p. 21). They also contended from studies of Geary (1993) and from Swanson (1987) that “students with learning disabilities… tend to have great difficulties abstracting principles from experiences and support is invariably necessary” (Gersten & Chard, 1999, p. 26).

Referring to several studies including Jordan et al. (2003), Geary (2004), and Hanich et al. (2001), Gersten et al. (2005) concluded that “although students with MD [mathematical difficulties] often make good strides in terms of facility with algorithms and procedures and simple word problems when provided with classroom instruction, deficits in the retrieval of basic combinations remain” (p. 294). They further stated that these deficits likely inhibit students’ “ability to understand mathematical discourse and to grasp the more complex algebraic concepts that are introduced” (p. 294). They also noted this inability to automatically “retrieve basic combinations, such as 8 + 7 often makes discussions of the mathematical concepts involved in algebraic equations more challenging" (p. 294).

However, a study by Bottge et al. (2007) "uncovered ways that students with mathematics learning disabilities (MLD) could participate in the kinds of learning activities consistent with those emphasized in current math reform" (p. 107). They showed that students with low computation skills "learned relatively complex concepts, which in many cases far exceeded teacher expectations" (p. 96). In addition, they cited Goldman, Hasselbring, and the Cognition and Technology Group at Vanderbilt (1998) and Hickey, Moore, and Pellegrino (2001) to support their conclusion that "teaching concepts for understanding does not always have to wait until all related procedural skills (e.g., algorithms) are mastered" (Bottage et al., 2007, p. 96).

Gersten and Chard (1999) indicated that Ginsburg (1997) offered a partial solution that suggested instruction focusing on understanding and emphasizing students' solving number facts. They noted such instruction would benefit "students whose learning disabilities may lie in memory deficits that preclude their moving through the hierarchy of skills" (Gersten & Chard, 1999, p. 25).

From their observations on the unsuccessful efforts of special educators, Pugac and Warger (1993) stated it is necessary "to foster cognitive-mediational or strategic learning strategies," noting the teachers "isolate the strategies... from meaningful instruction and teach them as efficient prescriptions" (p. 134). Gerten and Chard (1999) argued special education intervention focuses on "computation rather than mathematical understanding" (p. 25). Bottge et al. (2007) also questioned "the adequacy of instructional strategies and the richness of the math content used with low-achieving students" (p. 96).

Chard et al. (2008) offered as a reminder “while children may be born with a predisposition for making quantitative distinctions, an inability to develop a refined understanding of number has been implicated as a key predictor of later mathematics difficulties” (p. 12). In discussion on direction for improvement of mathematics instruction for students with learning disabilities, Gersten and Chard (1999) considered “if beginning math instruction were focused in part on building number sense, many students with learning disabilities would benefit” (p. 20).

Discussing some goals of early intervention, Gersten et al. (2005) stated that “one goal is increased fluency and accuracy with basic arithmetic combinations” and they connected this goal to other related goals such as “the development of more mature and efficient counting strategies and the development of some of the foundational principles of number sense – in particular, magnitude comparison and ability to use some type of number line.” They added, “It is quite likely that other aspects of number sense are equally important goals” (p. 300).

Researchers in the late 1980s consistently found that “students who struggled with mathematics in the elementary grades were unable to automatically retrieve what were then called arithmetic facts, such as 4 + 3 = 7 or 9 × 8 = 72” (Gersten et al., 2005, p. 294). Through their research, Gersten and Chard (1999) linked reading intervention based on phonemic awareness and reading skills with math intervention based on number sense ability and math skills. They further compared “how the number sense concept can inform and significantly enhance the quality of mathematics interventions for students with learning disabilities, just as the concept of phonemic awareness has informed the field of reading” (p. 18). They concluded, “Many children who show phonologically based reading difficulties exhibit difficulties in arithmetic retrieval as well” (p. 25).

Number sense and computational fluency are important components in the learning of mathematics. Automaticity of basic mathematics facts is an essential element of both components. However, the literature indicates that while automaticity of basic mathematics facts is necessary for gaining knowledge of meaningful mathematics, it is not sufficient for students to achieve success in mathematics and mathematics learning.

CHAPTER 3

METHODOLOGY

This chapter discusses the history of the preliminary initiative for the study and the quantitative research design of the study. In addition, selection of participants, development of instrumentation, and data collection are examined along with the implementation of treatment and data analyses methods. Limitations of the study are then considered.

History of the Initiative

In the first years of the 21st Century, a master's level cohort of special education teachers from a small university in Kentucky were assigned units of 'lab' time to collaborate with high school students with special education needs in mathematics. The teachers reported to their professor, Dr. Robert Thomas, that the students were deficient in their automatization of multiplication facts. After some research on the issue, Dr. Thomas came across an intervention developed by a high school mathematics teacher that yielded positive results in student performance following implementation in the teacher's algebra class. He sought and gained permission from the teacher to use an adapted version of the intervention.

Upon implementation of the intervention with their students with special needs, the cohort reported to Dr. Thomas that their students needed remediation with addition before they tried to master multiplication. In particular, they lacked automaticity of basic arithmetic facts with the four basic operations. He used the multiplication model of the intervention to develop an addition intervention to help the students develop automaticity. The teachers indicated successful results in their reports on student progress, and thus born was the first version of automaticity intervention for the university's Mathematics Transition Initiative (R. Thomas, personal communication, September 8, 2012).

Preliminary Initiative

Several years after the initial automaticity intervention was implemented, the professor was offered a fellowship grant to consult with a high-achieving school district to improve students' mathematics scores on the Measure of Academic Progress (MAP)®, a nationally normed standardized test, and the Commonwealth Accountability Testing System (CATS), the state normed standardized test given from 1999 to 2009. After 6 months of analyzing district, school, and student data, he noticed a paucity of computational requirements in curriculum and assessments. From this perceived deficit, he determined that student proficiency in automatization of basic mathematics facts would provide a good foundation for intervention.

The professor decided to use the interventions he developed for his special education cohort as an intervention in seventh-grade classrooms district-wide. In order to assess the success of the intervention, he needed a diagnostic instrument of addition and multiplication facts to determine the initial automaticity skills of the students and to track their progress during the semester. Many of his cohort teachers were successfully using the intervention in their own classrooms and they had created a diagnostic instrument to track individual and class success.

Dr. Thomas decided to use their diagnostic instrument as it had already shown to be practical and effective. The diagnostic instrument had 105 addition and multiplication problems involving one- and two-digit numbers in vertical form on the front and back of one sheet of paper. Students were given at most 15 minutes to complete the diagnostic. Teachers graded their own students' assessments, recording time used and errors made by each student. Uncompleted problems were marked as incorrect.

The professor worked with the teachers to develop protocols for implementing the intervention, and all seventh-grade classes in the district received the intervention from February through May 2008. No other mathematics curriculum support was being implemented in the district, according to the district curriculum coordinator (R. Thomas, personal communication, September 8, 2012). The automaticity diagnostic instrument was administered to the students before and after intervention implementation for pretest-posttest tracking of data to determine changes in assessment results. Spring MAP scores and automaticity diagnostic data were recorded by the teachers and submitted for analysis.

The district then administered the MAP to beginning eighth-grade students the following fall to assess changes in students' scores. The analysis of the data collected from the students' spring 2008 and fall 2008 MAP scores in mathematics reported a grade level mean increase of 3% and a median increase of 7% over the summer. These results permitted the district administrators and teachers to be cautiously optimistic that the intervention was successful.

Extension of Preliminary Initiative

For the 2008–2009 academic year, the automaticity intervention was administered to students from the fourth grade through the eighth grade in all mathematics classes in the district. No other mathematics programs supplemental to the curriculum were implemented during the year (R. Thomas, personal communication, September 8, 2012). The pre- and post-automaticity diagnostics were administered to the students within 2 weeks of their taking the MAP in both the fall of 2008 and the spring of 2009. The data showed a median increase of 10 percentile points for the district for the academic year. The data also showed a range of increases in scores from 4 percentile points to 16 percentile points at individual grade levels.

A post-analysis meeting of participating teachers and administrators revealed that teachers who administered the intervention with fidelity and compliance to protocols saw the greatest improvements in students' MAP scores; the fifth grade showed the greatest increase in mathematics test scores at 16 percent. Students of teachers who ignored the protocols or sporadically offered the intervention showed the least improvement; the sixth grade showed the least increase in mathematics test scores at 4 percent. Even these sporadic treatments afforded some student progress. Had an interim diagnostic instrument been administered or had data from scores of the MAP administered in the winter been collected, the data would have alerted the team that protocols were not being followed and measures might have been taken to bring the teachers onboard.

Students in the third grade and ninth grade did not receive the automaticity intervention treatment, and no improvements in their test scores occurred. At the time, Kentucky standards dictated that basic mathematics facts with the four basic operations were not accountable at third grade level. The district opted to supplement instruction with the intervention beginning with fourth grade through eighth grade. The high school did not participate.

As word of the improved results circulated throughout the state, other districts requested assistance from the university to help improve students' scores. Thus, the university's Mathematics Education Team (MET), consisting of Dr. Robert Thomas, Dr. Cheryll Crowe, and the researcher of this study, established the Mathematics Transitions Initiative to work with teachers across the state to assist with improvement of students' automaticity of basic mathematics number facts. The present study used part of the data from the ongoing initiative.

Quantitative Research Design

This descriptive study used a correlational design with a pretest-posttest paradigm with three Kentucky districts at the middle grade level. Analyses were implemented to define changes in data from fall to spring in the presence of an automaticity intervention treatment and to determine the relationship between assessment variables.

Middle school teachers in three Kentucky school districts collected data for the fall, winter and spring of the 2011–2012 academic year. These data included automaticity diagnostic information, which was converted to an automaticity quotient (AQ), and standardized test (ST) scores for each middle grades student. All middle school students in the district received automaticity intervention treatment for the academic year after the initial assessments.

First, the researcher analyzed the data with descriptive statistics including means, medians, and percentages to determine changes from fall to spring. Also included in the analyses were the numbers of students reaching scores at benchmark for introduction to algebra readiness and for algebra readiness and for changes that occurred between the fall and spring after treatment. Next, the researcher performed regression analyses to determine if a relationship existed between students' automaticity quotient ratios and standardized test scores. Using actual student information from the winter report, the researcher addressed missing data through imputation. Because each superintendent required all students in his or her district receive the automaticity treatment, the university could not withhold treatment, and no control groups were used in this study. This ethical mandate limited the type of analyses that could be implemented.

Description of Study Setting

The preliminary district and the three participating districts were located in the Appalachian region of the state of Kentucky. The economics of the region depended on farming, factories, and coal mining. The main diversity of the population was socio-economic; the region of the state in which these districts were located had little cultural or racial diversity.

Selection and Participation of Districts

The demographics of school districts participating in the high school readiness component of the Mathematics Transitions Initiative were examined and compared to those of the preliminary district. The demographic data chosen for comparison were comprised of both community and district data. In particular, these districts were similar to each other in student population, socio-economic status (SES) as measured by free and reduced lunch eligibility, per pupil (PP) property assessment, PP state and local revenue, and standardized test scores as measured by the ITBS, ACT, and MAP.

Table 5

District Data Profiles School Year 2011

Data	District A	District B	District C	District D
Student population	2643	2271	2898	1987
Free/reduced lunch	54%	67%	74%	68%
$PP property assessment	221,546	262,498	378,977	228,198
$PP local revenue	1722	1750	2579	1377
$PP state Revenue	5300	5621	5104	5430

Note: Legislative Research Commission, Frankfort, Kentucky, June 12, 2011, Director Robert Sherman

Table 5 shows demographic data for the preliminary district, designated District A, and the two districts with similar demographics, designated District B and District C. District D had a larger school population and greater expenditures per pupil, but the standardized test scores were similar.

State ACT statistics for 2011-2012 academic year were as follows:

- State approved college readiness ACT math benchmark (18)
- ACT approved college readiness ACT math benchmark (22)
- State mean ACT mathematics students who met benchmark (18) 21.6%
- State mean ACT mathematics score 18.3
- Total 44,390 students tested

Table 6 compares mathematics ACT scores during the academic year 2010–2011 for juniors from the districts in the study.

Table 6

Junior Mean ACT Mathematics Scores

District	District A	District B	District C	District D
Mean ACT mathematics	18.8	18.6	18.6	17.2
Met benchmark	22.4%	22.1%	27.0%	25.5%

Note: Legislative Research Commission, Frankfort, Kentucky, June 12, 2011, Director Robert Sherman

In 2009, the Kentucky state senate adopted a bill that required all students to be college and career ready upon high school graduation. One aspect of implementing the bill was the state adoption in 2010 of the Common Core State Standards (National Governors Association & Council of Chief State School Officers, 2010). Previous to this adoption, students in all school districts were given the Commonwealth Accountability Testing System (CATS) state criterion-referenced test that assessed the state standards. Other standardized assessments were optional.

With the adoption of Common Core State Standards (CCSS) came the obligation of a statewide nationally normed assessment. The state used the Iowa Test of Basic Skills (ITBS) as a temporary assessment for Grades 3 through 7 until a regularly administered nationally normed assessment was adopted. All districts in the state were required to administer the ITBS in the spring of 2010 and again in the spring of 2011.

Table 7

District B: State and District ITBS Score Comparisons (2010-2011)

		Grade				
Level	Year	3	4	5	6	7
State	2010	60	56	53	43	47
District	2010	62	66	60	52	57
State	2011	61	57	54	45	48
District	2011	71	68	64	62	52

Note: Compiled by R. Thomas, 2011

Table 8

District C: State and District ITBS Score Comparisons (2010-2011)

		Grade				
Level	Year	3	4	5	6	7
State	2010	60	56	53	43	47
District	2010	62	59	59	52	51
State	2011	61	57	54	45	48
District	2011	71	62	60	50	52

Note: Compiled by R. Thomas, 2011

Table 9

District D: State and District ITBS Score Comparisons (2010-2011)

		Grade				
Level	Year	3	4	5	6	7
State	2010	60	56	53	43	47
District	2010	62	66	60	52	57
State	2011	61	57	54	45	48
District	2011	80	63	64	62	64

Note: Compiled by R. Thomas, 2011

The statewide administration of the ITBS allowed a comparison of state and district scores for school districts with similar demographics. Tables 7, 8, and 9 show similar scores for the three districts.

Conditions of District and School Participation

Before joining the Mathematics Transitions Initiative, district superintendents and curriculum supervisors, school principals and guidance counselors, and teachers were required to sign a *Memorandum of Understanding* (MOU) with the university. This MOU allowed the Mathematics Education Team to work and collect data within the school districts and committed the participating districts to follow protocols with fidelity and compliance. See Appendix A for a partial sample MOU.

The Mathematics Education Team met with participating district administrators and teachers in school, district, and regional settings throughout the academic year to provide professional development and support. The professional development provided background information, implementation protocols, and analysis of data for the automaticity intervention.

Pedagogical, mathematical, and technical support allowed customized interventions that fit the needs of each school and district but did not violate protocols.

Instrumentation for Study

The two assessment instruments used for this study were a pretest-posttest automaticity diagnostic instrument and the Measure of Academic Progress (MAP), a nationally normed standardized test. The automaticity diagnostic assessments were administered within 2 weeks of students taking the MAP in the fall, winter, and spring of the 2011–2012 academic year.

Automaticity Diagnostic Assessment

The pretest-posttest automaticity diagnostic instrument was composed of 105 addition and multiplication problems using single-digit and two-digit numbers placed in vertical orientation. Space was provided below the vinculum for the students to write their answers. The middle school students were given 15 minutes to complete the assessment. The students recorded the time they used to complete the assessment and the teachers recorded the number of incorrect answers in the spaces provided.

Reliability of Automaticity Diagnostic Instrument

As previously noted, approximately 15 practicing teachers in a masters level special education course at the university developed the diagnostic instrument to use in their classrooms with their students. The automaticity diagnostic instrument was designed for use in their classes, and no validity or reliability measurements were made.

Reliability indicates the consistency of a measurement and can be estimated by determining the extent to which to different measures yield comparable results. One way to estimate the reliability of a measure is to establish the degree to which two different measures of knowledge or skill produce comparable results. Within the extended preliminary initiative protocols, teachers administered two versions of the automaticity diagnostic instrument, Diagnostic B two days after Diagnostic A. No differences in their results were reported and thus the reliability of the automaticity diagnostic instrument was likely high (R. Thomas, personal communication, September 8, 2012).

Validity is the degree to which items on an instrument represent the construct of a study and can be assessed by a panel of experts, who evaluate the content of instrument. Practicing secondary teachers working on their masters' degree developed the instrument and could reasonably be considered experts in the realm of number combinations with basic operations; their development and use of the assessment endorsed the content along with the intended measure. Hence, the validity of the automaticity diagnostic instrument is plausible. See a sample portion of the automaticity diagnostic instrument in Appendix B.

MAP Assessment

The Measure of Academic Progress® (MAP) assessment is published by the Northwest Evaluation Association (NWEA, 2014). Categories of questions asked on the mathematics assessment include Number Sense/Number Systems, Estimation and Computation; Algebra; Geometry; Measurement; Statistics and Probability; and Problem Solving, Reasoning, and Proof. The computerized assessment adapts to students' responses to each question while students take the test. When students answer a question correctly, a more challenging question appears next. When students give an incorrect answer, a less challenging question is offered.

The assessment reports students' results in a Rasch Unit (RIT) developmental scale that estimate students' instructional levels as well as academic progress and measures students' understanding regardless of grade level (NWEA, 2014). According to NWEA (2014), "RIT assigns a value of difficulty to each item, and with an equal internal measurement, so the difference between scores is the same regardless of whether a student is at the top, bottom, or middle of the scale" (http://www.nwea.org/products-services/assessments/map). RIT scale charts are offered on the NWEA website (http://www.nwea.org) with examples of work that students within a certain score can do.

Reliability and Validity of MAP Assessment

The MAP assessments were considered reliable and valid according to the publishers (NWEA, 2004). To measure reliability, NWEA (2004) uses a test-retest approach they consider more rigorous because their method includes a parallel form of reliability for a longer amount of time than other test-retest methods. The publishers report strong correlation coefficients in the mid-80s to the low 90s from NWEA Norm Studies in 1999 and 2002 for middle grades (NWEA, 2004).

NWEA (2004) documents validity in the form of concurrent validity, which measures the correspondence of MAP scores on the RIT scale to scores from an established test with a different scale in the same subject. Using twelve different assessments from 1998 to 2003, the publishers report strong correlation coefficients in the low to high 80s for the middle grades, although not all middle grades were reported each year. An evaluation of the reliability and validity of the MAP assessment by the Institute of Education Sciences corroborates this claim for the most part; the correlation coefficients were still considered strong from the low 70s to the high 80s, but the predictive validity of the assessment was deemed unavailable (Brown & Coughlin, 2007).

Introduction to Algebra Readiness and Algebra Readiness Benchmarks

In the Principles and Standards for School Mathematics (NCTM, 2000), the algebra standard for grades 6-8 includes the following concepts middle school students need to know:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

However, middle school students also need to know the concepts from the number and operations standard for grades 6-8 to be successful learning the algebraic concepts:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates (NCTM, 2000)

According to the Southern Regional Education Board report *Getting students ready for algebra I: What middle grades students need to know and be able to do* (Bottoms, 2003), Algebra Readiness Indicators are “the essential mathematics skills and concepts that students must master by the end of the middle grades to succeed in Algebra 1” (Bottoms, p. 2). These indicators were not only influenced by the *Principles and Standards for School Mathematics* (NCTM, 2000), but were also “developed from the curriculum materials underlying the National Assessment of Education Progress (NAEP) and curriculum guides from the SREB Middle Grades Consortium member states and selected other states” (Bottoms, p. 2).

The five process Readiness Indicators “are problem solving, reading and communicating, estimating and verifying answers and solutions, logical reasoning, and using technology” (Bottoms, p. 5). These skills and concepts “should be incorporated into mathematics and all grade levels and in all courses” (Bottoms, p. 3).

The 12 content-specific Readiness Indicators address "essential content-specific skill and concepts that prepare students for Algebra 1" (p. 2) and follow in a bulleted list:

- "Read, write, compare, order and represent in a variety of forms: integers, fractions, decimals, percents, and numbers written in scientific and exponential form.
- Compute (add, subtract, multiply and divide) fluently with integers, fractions, decimals, percents, and numbers written in scientific notation and exponential form, with and without technology.
- Determine the greatest common factor, least common multiple and prime factorization of numbers.
- Write and use ratios, rates and proportions to describe situations and solve problems.
- Draw with appropriate tools and classify different types of geometric figures using their properties.
- Measure length with appropriate tools and find perimeter, area, surface area and volume using appropriate units, techniques, formulas and levels of accuracy.
- Understand and use the Pythagorean relationship to solve problems.
- Gather, organize, display and interpret data.
- Determine the number of ways events can occur and the associated probabilities.
- Write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality.
- Represent, analyze, extend and generalize a variety of patterns.
- Understand and represent functions algebraically and graphically" (Bottoms, p. 11).

The NWEA (2011) Growth Norm Study developed norms not only from empirical data but also from a model-based post–stratification variable that enabled a more efficient comparison of schools. In addition, a statistical model for estimating status and growth controlled for factors that may affect those estimates, including starting RIT scores, standard errors of measurement, sample weighting based on school characteristics, and instructional weeks.

The data for the study were retrieved from the Growth Research Database (GRD), a repository of test event information, demographics of the test taker, information about test items and their characteristics, and links to datasets from external agencies. NWEA created and maintains the GRD, which, at the time of the 2011 study, held more than 200 million test event records, 99% of which came from MAP tests since 2002. Items considered for the 2011 study included the time frame of spring 2009 through fall 2010, as well as other criteria.

According to the NWEA website, a score of 230 suggests student readiness for *Introduction to Algebra* and a score of 235 suggests student readiness for *Algebra 1*. These benchmark scores were developed from data reported on their 6+ Mathematics Survey with Goals test and analyzed in the 2011 NWEA Growth Norm study.

The algebra components of the test include student understanding and application of algebraic concepts such as extending patterns, simplifying expressions, solving equations and inequalities, using coordinate graphing, and solving functions and matrices. Students who score in the 231-240 difficulty range are likely to answer questions on these algebraic concepts with a 50% success rate (NWEA, 2014).

Administration of Automaticity Diagnostic Instrument

Before administering the automaticity diagnostic assessment, participating teachers took part in professional development sessions that informed them about state mandates, research relating to automaticity, pilot and extended pilot study results, the Mathematics Transitions Initiative website access, and the diagnostic, treatment, and data reporting protocols. The teachers were given a password that allowed access to the automaticity diagnostic instruments, treatments, data reporting template, and grade level protocols on the website. Protocol documents explained for each grade level how to implement the automaticity diagnostic and treatments, what data to collect, and how to report and submit the data. Subsequent visits from the Mathematics Education Team members as well as access to team member email addresses allowed questions and concerns of participating teachers and administrators to be addressed.

Teachers in participating districts administered the automaticity diagnostic instrument in the same pretest-posttest paradigm as the preliminary and extended preliminary initiative district at the beginning and end of the academic year. Because the results of the extended preliminary initiative district raised the issue of compliance and fidelity to protocols, the teachers also administered the automaticity diagnostic at the end of the first semester or the beginning of the second semester. These time frames coincided with the districts' administration of the fall, winter, and spring MAP assessments.

The teachers were directed to administer the automaticity diagnostic assessment to students within 2 weeks of the MAP assessment. In addition, the teachers were directed to give students a second opportunity to take the diagnostic after waiting one extra day before re-administration. This dual administration of the automaticity diagnostic not only allowed previously absent students to participate but also allowed students to be more comfortable with the assessment and the procedures. Teachers recorded the best time with accompanying errors to ensure that the best performances were represented in the data.

The mathematics teachers administered the automaticity diagnostic to their students at the beginning of their mathematics classes. The teachers could display prepared directions for students as well as review them orally before administering the diagnostic instrument. In addition to procedure protocols, a few sample directions included no calculator use, no talking, time is important, and so forth. See Appendix C for the automaticity diagnostic test instructions.

The middle school students were allowed 15 minutes to complete the automaticity diagnostic assessment of 105 simple one- and two-digit addition and multiplication problems.

The teachers projected an electronic stopwatch for their students to see with the time counting down from 15:00 minutes to 0:00 minutes. They signaled students when to begin working and then started the countdown of the clock. When the students completed the automaticity assessment, they recorded at the top of their paper the time displayed on the stopwatch; students raised their hands when finished for the teacher to verify the time recorded and collect the assessments. The students were allowed no more than the allotted time; if they did not finish the assessment in the 15-minute time period, they discontinued taking the diagnostic and recorded the time used as 15 minutes. The teachers collected and scored the automaticity diagnostic assessments and recorded the number of answers incorrect; problems without answers were considered incorrect. The teachers also converted and recorded the time remaining, documented by students on their papers, to the time used by students to complete the assessment.

Data Collection

On an electronic spreadsheet prepared and provided by the Mathematics Education Team, teachers recorded automaticity time and errors for each student as well as the recent student MAP Rasch Unit (RIT) scores that evaluate student achievement. A district representative electronically collected data from the mathematics teachers at each school, removed student names from the documents, and uploaded the document with the accumulated district data onto the Mathematics Transitions Initiative website following the given protocols. The website was password protected, and the district representatives used a password different from that of the teachers to submit the data. Each student was given a district assigned number for district bookkeeping purposes; the university was not privy to protocols for assignation of student numbers in order to preserve the anonymity of the students. Members of the team had password access to the data submitted.

Automaticity Treatment

For this study, an intervention treatment was developed to help students increase their automaticity of basic mathematics facts. The treatment and treatment protocols have research recommended components for effective automaticity practice that include computational fluency and number sense features. Some of these components are the following:

- Students understand the mathematics underlying number combinations before memorizing them.
- Students complete a mixture of previously memorized number combinations and new combinations.
- Students practice number combinations daily.
- Students understand that quick recall with accuracy is the goal; counting strategies are discouraged in the presence of the treatment.
- Students are given time limits to complete treatments to foster speed along with accuracy.
- Students receive immediate feedback on their answers.
- Students individually practice number combinations they have not memorized; students will not likely progress at the same rate nor should they be expected to do so. Competition is discouraged.
- Students track their own progress toward self-improvement thereby developing traits of a self-directed learner.

- Student automaticity status is assessed only two times during the academic year after the initial assessment to avoid the "drill and kill" of historical types of practice. At no time are grades assigned to assessments or treatments.
- Students can be given "mad minute" quizzes weekly only to determine if their progress is genuine or if they need to return to previous number combinations for added practice.

The automaticity intervention treatment was administered to students in addition to the curriculum and instruction offered by their teachers.

Implementation of Treatment

The teachers provided daily automaticity intervention treatment with practice sheets that had addition, multiplication, subtraction, or division problems to solve. The problem components were aligned vertically with space to provide answers under the vinculum; the problems were situated in rows and columns on one side of a sheet of paper. The sheets were designated by number and operation to be learned and the given problems for each set were randomly placed. See Appendix D for a sample automaticity intervention treatment.

Students practiced addition first beginning with a (2+) sheet, and then going on to a (3+) sheet, and so forth until the (12+) sheet was mastered. The design of the rest of the practice with multiplication, subtraction, and division was the same: one operation per practice sheet, one set of numbers to learn (2s, 3s, 4s, … 12s).

Answers to the problems were provided for the students to access if they did not automatically know the solution. These answers were situated horizontally at the top of each sheet. For example, the (4x) practice sheet displayed $4 \times 1 = 4$, $4 \times 2 = 8$, and $4 \times 3 = 12$, and so forth through $4 \times 12 = 48$ for middle school students. Access to these answers allowed students repeated reference to number facts they had not yet memorized so they could build automaticity. The answers were situated horizontally rather than vertically as for the problems in order to provide another cognitive step during treatment and to prevent students from parroting answers without reading the information. Answers to subtraction and division were given as addition and multiplication combinations to facilitate understanding of the corresponding combination relationships. See Appendix D for sample automaticity treatment questions for multiplication by 6 or (×6).

The students were given 5 minutes to complete the practice at the beginning of their mathematics class in place of the bell ringers that the teachers previously used. When they finished the practice or when 5 minutes ended, students graded their own papers, correcting any errors and completing the ones they left blank. Correcting their own papers allowed them another round of repetition with the facts on which they were working. They recorded the time and number of errors on a recording instrument of the teacher's choice. The teachers created protocols in their classrooms on how to disperse and collect material as well as how to record progress. The university team did not collect data on the individual teacher's implementation of treatment protocols.

Although students were allowed 5 minutes for practice, the goal was for students to develop automaticity as measured by decrease in time and errors. Students had to complete each practice sheet in two minutes or less with two or fewer errors before continuing to the next practice sheet. For example, when a student completed the (6+) sheet in 2 minutes or less with two or fewer errors, he or she advanced to the (7+) sheet. When all of the addition sheets were mastered, the student began multiplication practice. The order of the intervention for middle school starting with (2+) was addition, multiplication, subtraction, and division.

After multiple intervention sessions, students were practicing operations with numbers that may be different than their fellow students but attended to their individual needs in a differentiated learning situation. Teachers provided quick interim assessments of their choice to monitor student progress. The interim assessments were usually in the form of “mad minutes” that corroborated student reported progress. The diagnostic instrument was used only for the three data collection times and for no other purpose. When the students finished the four operation sets, they were instructed to begin the intervention process again but could not continue to the next practice sheet unless they finished under 1 minute with two or fewer errors. Some adaptations were made at the teacher’s discretion for those students who became stalled on one particular practice sheet. For example, if a student were unable to advance past the (6–) sheet for a lengthy period of time, the teacher could make the decision to let the student move forward to (7–) for a few days before returning to (6–). These types of adaptations allowed for individualized attention to students’ needs to make progress and to avoid increased student frustration levels. See Appendix E for automaticity treatment protocol sample.

Data Analysis Methods

The study was a correlational design with a pretest-posttest paradigm for measuring automaticity treatment effects on district standardized test scores. Data were analyzed with descriptive statistics in the form of means, medians, standard deviations, and percentages to determine changes in results from fall to spring. Also included in the analyses were the numbers of students reaching scores at benchmark for introduction to algebra readiness and for algebra readiness along with changes that occurred between the fall and spring after treatment. Regression analyses were performed to determine the type of relationship that existed between students’ automaticity quotient results and standardized test scores. Missing data were addressed through imputation using actual student information from the winter report. No control groups were used in this study.

According to Hill (2009), a pretest-posttest design carries issues that must be considered, such as student learning from other sources, for example, curriculum and instruction. Another issue for concern is fairness with implementation. A third issue could stem from group selection. To separate the effect of the treatment from these issues, group self-selection, teacher access to protocols through various mediums, and multiple instances of the treatment with varieties of sources were part of the design as previously described.

Students in more than 40 districts across the state participated in the Mathematics Transitions Initiative. Without solicitation, districts requested to join the Initiative, and three of the participating districts similar in demographics were chosen as participants in the study. This study examined data from those three districts with an approximate combined population of 2500 students.

Hill (2009) stated, “Documenting the effect of a treatment requires some sort of measure of that effect” (p.44). The treatment was designed for students to automatize basic mathematics facts and the automaticity diagnostic instrument measured the differences in students’ performances in the presence of the automaticity treatment. To analyze the automaticity diagnostic assessment data, ratios of *correct answers* given to *time used* for completing the diagnostic were considered. The diagnostic instrument had 105 questions with a time limit of 15 minutes for middle school students to complete the assessment; incomplete answers were considered incorrect.

The *time* component provided a meaningful statistic; it was reasonably assumed that given more time to answer questions with greater opportunity for use of strategies, students would produce a greater number of accurate answers. The automaticity ratio allowed a single statistic that considered student performance—a result for actual student participation. Hence, the ratio was a sensible ranking—one that allowed a quantitative ranking of students that fit well with the teacher's observation of student learning. Descriptive analyses were performed to measure changes in student performance between fall and spring in the presence of the treatment. Regression analyses were performed on spring data to determine the type of relationship that existed between the AQ ratios and the ST scores.

Treatment of Unusable Data

Because this study relied on multiple layers of self-reporting, the occurrence of unusable data, while not desired, was expected. Unusable data occurred in two categories: missing data and uncountable data.

Missing data referred to items in which student information was missing in more than one of the reporting intervals of fall, winter, or spring so that comparisons were not possible. Uncountable data referred to items with miskeyed information (e.g. alpha input as opposed to numerical input; test scores less than or greater than possible ranges; unreasonable automaticity times (e.g., 22 seconds to complete 105 items); items in which excess time was given for automaticity diagnostic (more than 15 minutes), and so forth. In several large blocks of information, automaticity information was duplicated in the fall and spring reporting spreadsheet; those data were also considered uncountable. The following table displays the amount in percent of missing or uncountable data for standardized test scores (ST) and automaticity quotients (AQ) of each district in the fall, winter, and spring.

Table 10

Unusable Data: Missing or Uncountable Data

	Fall		Winter		Spring	
District	ST	AQ	ST	AQ	ST	AQ
District B	7.3%	7.4%	6.4%	8.3%	12.6%	13.4%
District C	2.8%	3.1%	3.5%	3.5%	3.0%	3.0%
District D	4.4%	8.0%	67.0%	10.0%	6.7%	10.0%

Possible explanations for missing individual items might be student absences or student relocation, which would be considered random occurrences. Missing chunks of data may be the unintentional or intentional failure to report data from an entire classroom. Unintentional failure to report data would be considered random and would not result in biased analysis results. Intentional failure to report entire class data might be the result of absence of intervention in the classroom; such an occurrence would not be random, and data reported in these situations would skew any inferences based on the automaticity intervention. Therefore, the reasons for the missing data beyond intentional non-reporting seemed to be arbitrary or random. The uncountable data whether miskeyed, duplicated, or impossible occurrences, were considered arbitrary or random as well. According to Baraldi and Enders (2010), if the missing values are unrelated to other variables in the data set, then the data are considered missing completely at random (MCAR) and “the observed data can be thought of as a random sample of the hypothetically complete data” (p. 7).

Importance of Missing Data

According the Dong and Peng (2013), the impact of missing data on quantitative research may produce biased results. Lost data decreases the ability to discover relationships in a set of data, and measures of central tendency may be affected depending on the distribution of the missing data. Unfortunately, the research community has not agreed upon thresholds for allowable amounts of missing data (Dong & Peng, 2013). The most conservative value recommended is 5% missing data and the widest parameter accepted is 20% missing data (Osborne, 2008).

However, much of the literature on missing data discusses information gathered from surveys using a Likert-type scale. Dong and Peng (2013) offer Schafer's (1999) claim that 5% or less of missing data is inconsequential and Bennett's (2001) assertion that 10% or more of missing data produces biased statistical analysis (p. 2). Beyond the proportions of missing data, researchers should consider the impact of missing data mechanisms on research results.

Missing Data Mechanisms

Missing data theory offers three mechanisms from which missing data occur: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). With MAR, the probability of missing data depends on observed data and not the missing data; the MAR missing data mechanism is considered ignorable as it minimally impacts research results. MCAR is a special case of MAR whereby the likeliness of missing data depends on the neither the observed data nor the missing data; the MCAR missing data mechanism is considered a random sample of the complete data and therefore is considered the least threat to research results. With MNAR, the probability of missing data depends on the missing data; this missing data mechanism is not ignorable and renders a greater impact on research results (Dong & Peng, 2013).

Missing Data Analysis

Several explanations for missing data in this study were considered as the underlying deficiencies of self-reporting data. Large chunks of vacant entries in the reporting spreadsheet indicate either (a) unrecorded data from entire classes or (b) classes that met only one semester during the academic year and had opportunity for one assessment implementation. For the first case (a), teachers may have failed to report data intentionally because they did not offer the treatment to their students or they unintentionally did not remember to record the data. For the second case (b), teachers were unable to report data that was unavailable. In either case, the missing data did not depend on the observed data or the missing data, rendering the missing data mechanism MCAR.

Other indiscriminate missing data may have signified students who were absent during assessment administrations or relocated during the academic year. The lack of information generated by these situations would be unintentional and therefore random. Research reveals that students who are habitually absent or frequently move are less likely to perform well in school. Missing information from student absences or relocations would depend on the observed data, not the missing data, and would be considered MAR. While the missing individual student items in this study may not be completely at random, it was still ignorable and would minimally impact the research results if proportions were not more than 10% (Dong & Peng, 2010).).

All missing data in this study for Districts B and C were cautiously considered MAR, and proportions of missing data no more than 10% were considered ignorable. Missing data for District D were considered both MAR for individual student missing items and MCAR for large chunks of data assumed to be missing from whole classes. Although total proportions of missing data for District D were 12%, the large amounts of MCAR data rendered the missing data ignorable. All uncountable data were considered MCAR and therefore ignorable as well.

Missing Data Techniques

Many techniques for analysis of data involving missing information have been implemented in research; two of these techniques are methods of deletion and methods of imputation. Deletion methods are considered less than desirable as they bias results if remaining data does not represent the population or complete sample. For MCAR data such as uncountable data, missing items are considered comparable to non-missing items for analysis purposes (Wayman, 2003). In this case, complete cases analysis (listwise deletion) should not bias results although a loss of power may occur.

Imputation methods use likely values to complete missing data items. Some imputation methods are considered acceptable depending on the model of imputation. For the analysis of data in this study, additional data points on the same measures administered at different times are used to impute missing data. Examples of this method include interpolating missing scores based on existing scores or using the previous, nearest, or last score before the missing score.

Data in this study were collected in the fall, winter, and spring of the 2011–2012 academic year. Differences from the fall and spring data were analyzed to determine changes in data as well as relationships between data. Information collected from the winter cycle was used to impute missing data for automaticity components and test scores. The ensuing chart displays the unusable data results after the winter data imputation.

Table 11

Unusable Data After Imputation of Winter Data

District	Missing data	Uncountable data
District B	8%	6%
District C	3%	1%
District D	<12%	11%

Examination of the chart reveals proportions of missing data in two of the three districts were below10%, one of these districts was even less than 5%; proportions in the third district were less than 12% after adjusting for deletion of MCAR data. Utilizing the existing data for imputation minimized impact on research results. Because the period of time between diagnostic assessments is shortened for missing data - fall to winter or winter to spring - analysis results could be considered cautious.

Limitations of the Study

Four areas of limitation were identified for this study: lack of control group, self-reporting of data, selection of participants, and insufficient verification of fidelity and compliance to protocols.

Lack of Control Group

Superintendents of each school district required that all students in their districts receive the automaticity intervention treatment. Omitting any student from the treatment would be a breach of the MOU between the university and the districts. Education perspectives render the withholding of treatment from students unethical. Had a control group been available, examination of data could have rendered cause and effect analyses.

Self-reporting of Data

Most teachers are not professional researchers, and formalized data collection can be challenging. The university Mathematics Education Team relied on district representatives appointed by the superintendents to submit complete data according to the protocols. The team worked with more than 45 districts in the state and gathered information for more than 100,000 students. Examination of information collected revealed unusable data that was either missing or uncountable from inaccurate reporting. These unusable data affected the strength of the conclusions of analyses. In addition, deletion of uncountable data accounted for obvious errors. Other data entry errors may have occurred that were not obvious and could skew the results as well.

Selection of Participants

In order to select districts for the study, several factors were considered. First, the demographics and location of school districts were examined for similarity to the pilot district demographics. Next, the districts administering the same assessments were considered and in particular, districts using nationally normed standardized assessments. At the time the study took place, the state did not require all school districts to administer the same assessments beyond the state normed assessment. Finally, proportions of unusable data were examined to determine if reasonable results from data analyses could be rendered. These considerations narrowed the search field of districts that could be selected.

Insufficient Verification of Fidelity and Compliance to Protocols

From the results of the extended pilot and from teacher testimony, some teachers did not follow protocols with fidelity and compliance. The absence of daily implementation possibly weakened the relationship of the AQ ratios with ST scores. Upon this discovery, interim assessments were administered in the winter to determine adherence to protocols. This modification did not produce the expected result, as time required for reporting and data analysis was inadequate for implementation of appropriate changes. However, the interim data were useful for imputation of missing data as well as assessing necessity for subsequent professional development for teachers.

CHAPTER 4

DATA ANALYSIS

This chapter presents analyses of data collected for the study in two major sections: descriptive analysis and inferential analysis at the district and middle grade levels. The researcher analyzed data both before and after deletion of unusable data.

Descriptive Analysis

After imputation of winter information into missing fall and spring items, the researcher analyzed data to investigate answers for these research questions.

- How do standardized test scores change in the presence of the automaticity intervention treatment of middle school students in three Kentucky school districts?
- How do automaticity quotient results change in the presence of the automaticity intervention treatment of middle school students in three Kentucky school districts?

Because the researcher performed parts of the investigation before deletion of unusable data, those analyses are described first.

Analysis Before Deletion of Unusable Data

To examine changes in automaticity quotient (AQ) ratios and MAP scores in the presence of the automaticity intervention treatment, the researcher generated descriptive statistics for standardized test (ST) scores and automaticity quotient results for Fall 2011 and Spring 2012. These measures included the mean and median as well as number of students who improved their AQ ratios and ST scores, which were calculated after imputing winter information into the missing items but before deletion of unusable data. In addition, the researcher documented the percent of students with RIT scores of 230 or greater and 235 or greater. These scores are benchmarks that indicate student readiness for introduction to algebra and algebraic readiness, respectively, according to NWEA.

The researcher then calculated the differences between students' fall and spring AQ ratios and fall and spring MAP scores, as well as the mean, median, and the standard deviation of these differences. The researcher also computed the percent of students who improved their AQ ratios and MAP scores and tabulated the percent of students who improved their RIT scores to 230 and beyond and to 235.

Summary of Descriptive Analyses Before Unusable Data Deletion

All districts exhibited student increases in every category of descriptive statistics. Districts B and C showed the greatest increase in median and mean AQ ratios and ST scores as well as percent of student improvement in AQ ratios. Student improvement in ST scores differed little in percent measurement improvement across the three districts, showing close to a 75% gain for each.

All districts improved substantially in the number of students who scored 230 or more and scored 235 or more, with close to 12% or greater increases in introduction to algebra readiness and algebra readiness benchmarks. But District C showed nearly triple the gain of the other districts at the 230-benchmark introduction to algebra with more than a 35% increase. To facilitate examination of the descriptive statistics for the three districts, comparison data analyses are displayed in Table 18.

Table 18

District Comparison Data Analysis After Imputation From Fall 2011 to Spring 2012

	District B		District C		District D	
Statistics	ST	AQ	ST	AQ	ST	AQ
Median of difference	5.0	4.0	5.0	4.4	4.0	3.1
Mean of difference	5.8	4.6	5.3	4.5	5.0	3.2
% Students improved	73.2%	86.4%	75.5%	89.6%	74.1%	75.9%
% Improved 230 or above	12.0%		35.6%		13.5%	
% Improved 235 or above	11.9%		14.0%		12.6%	

After imputation of the winter data into missing items, unusable data were deleted in order to further investigate the changes in AQ results and ST scores.

After Deletion of Unusable Data

After imputation of winter data into missing items and after deletion of unusable items, the researcher added the standard deviation to the descriptive statistics. In addition, the researcher created histograms to describe the frequency distributions of ST differences data and AQ differences data and developed confidence intervals to investigate strength of population sample data.

District B Descriptive Statistics and Confidence Intervals

After the deletion of the unusable data, District B retained data for a sample population of 414 students. All districts exhibited student increases in every category of descriptive statistics. A comparison of the median and the mean differences in standardized test (ST) scores between fall and spring showed little difference. The median and the mean differences in automaticity quotient (AQ) ratios had slightly more than a one-unit difference. When the mean and median are comparable in value, a normal distribution is expected. The standard deviation (SD) for the difference in standardized test scores was nearly 8 points and the SD for the difference in automaticity quotient ratios was less than 6 units. Table 19 displays these difference statistics.

Table 19

District B: Difference Statistics from Fall 2011 to Spring 2012 (Sample Population 414)

Statistics	ST	AQ
Median of difference	5.0	2.9
Mean of difference	5.6	4.1
SD of difference	7.9	5.5
% Students improved	73.7%	86.0%
% Improved 230 or above	11.8%	
% Improved 235 or above	12.3%	

Figures 2 and 3 display histograms that illustrate frequency distributions of the spring ST and AQ differences for District B. Both the AQ and ST distributions show a bell curve with the AQ distribution skewed slightly to the right.

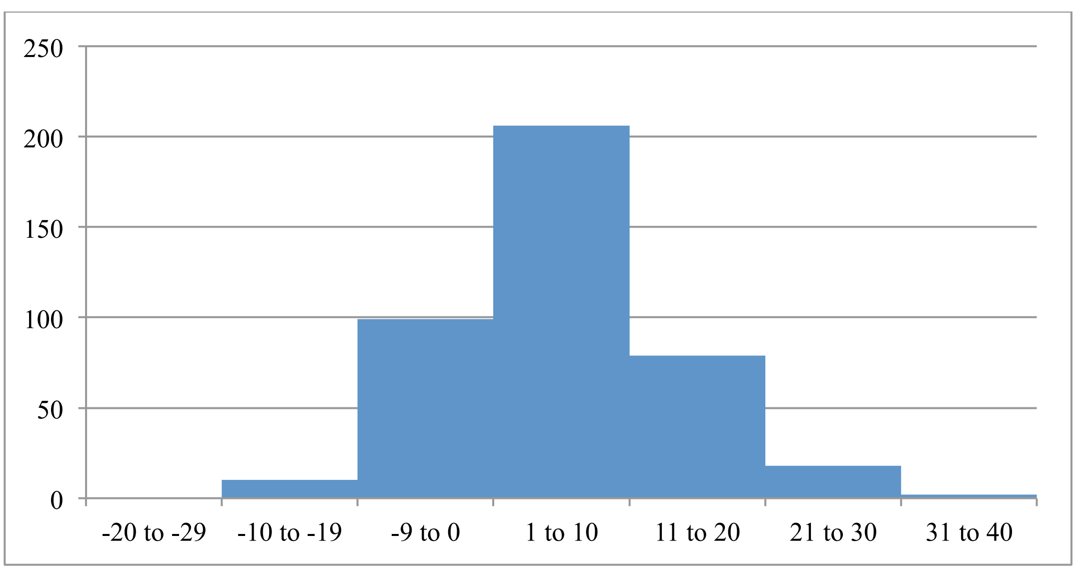

Figure 2. Histogram of standardized test (ST) score differences from District B spring data.

Confidence intervals at a 95% confidence level were computed with margins of error from z-scores as is acceptable for large sample populations. With a 95% confidence level, the true population mean (μ) for the difference in District B standardized test (ST) scores from Fall 2011 to Spring 2012 fell between 4.9 and 6.4 points.

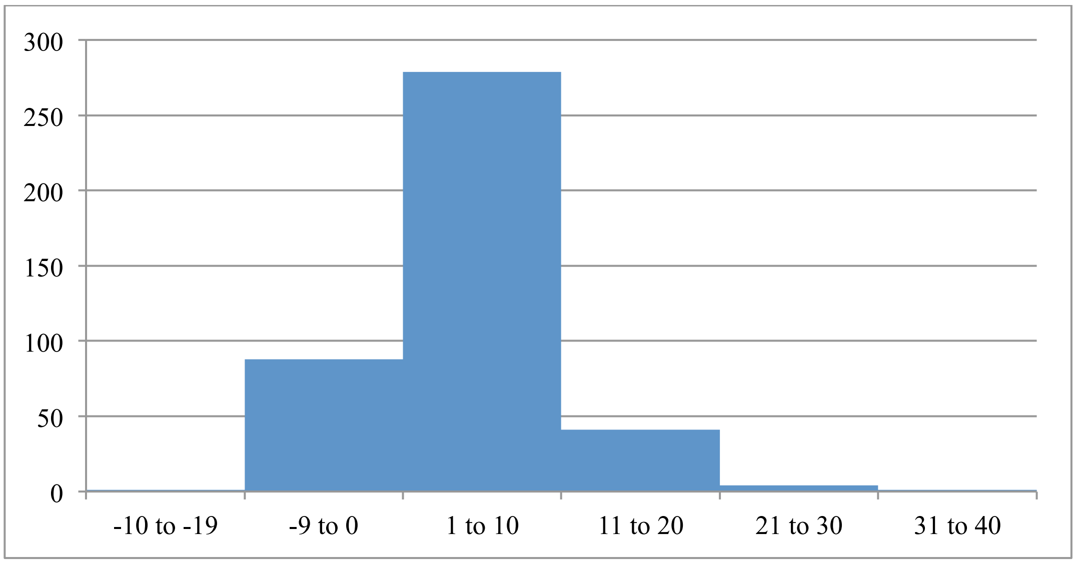

Figure 3. Histogram of automaticity quotient (AQ) ratio differences from District B spring data.

With a 95% confidence level, the true population mean (μ) for the difference in District B automaticity quotient (AQ) ratios from Fall 2011 to Spring 2012 fell between 3.5 and 4.6 points. Table 20 shows the frequency distributions for both ST and AQ differences from District B spring data.

Table 20

District B Spring Data: Frequency Distributions for ST and AQ Differences

Standardized test (ST) scores				Automaticity quotient (AQ) ratios			
Interval	Quantity	Statistic	Measure	Interval	Quantity	Statistic	Measure
-20 to -29	0	Median	5.0	-10 to -19	1	Median	2.9
-10 to -19	10	Mean	5.6	-9 to 0	88	Mean	4.1
-9 to 0	99	SD	7.9	1 to 10	279	SD	5.5
1 to 10	206			11 to 20	41		
11 to 20	79			21 to 30	4		
21 to 30	18			31 to 40	1		
31 to 40	2			-10 to -19	1		

District C Descriptive Statistics and Confidence Intervals

After the deletion of the unusable data, District C retained data for a sample population of 522 students. All districts exhibited student increases in every category of descriptive statistics. A comparison of the median and the mean differences in standardized test (ST) scores and in automaticity (AQ) ratios between fall and spring showed less than one-point differences. When the mean and median are comparable in value, a normal distribution is expected. The standard deviation (SD) for the difference in ST scores was slightly more than 7 points and the SD for the difference in AQ ratios was less than 4 units. These difference statistics are displayed in Table 21.

Table 21

District C: Difference Statistics from Fall 2011 to Spring 2012 (Sample Population 522)

Statistics	ST	AQ
Median of difference	6.0	3.9
Mean of difference	5.4	4.3
SD of difference	7.2	4.8
% Students improved	75.9%	85.5%
% Improved 230 or above	18.2%	
% Improved 235 or above	14.2%	

Figures 4 and 5 display histograms that illustrate the bell curve frequency distributions of the spring ST and AQ differences for District C. Both the AQ and ST distributions show a bell curve with the AQ distribution skewed slightly to the right.

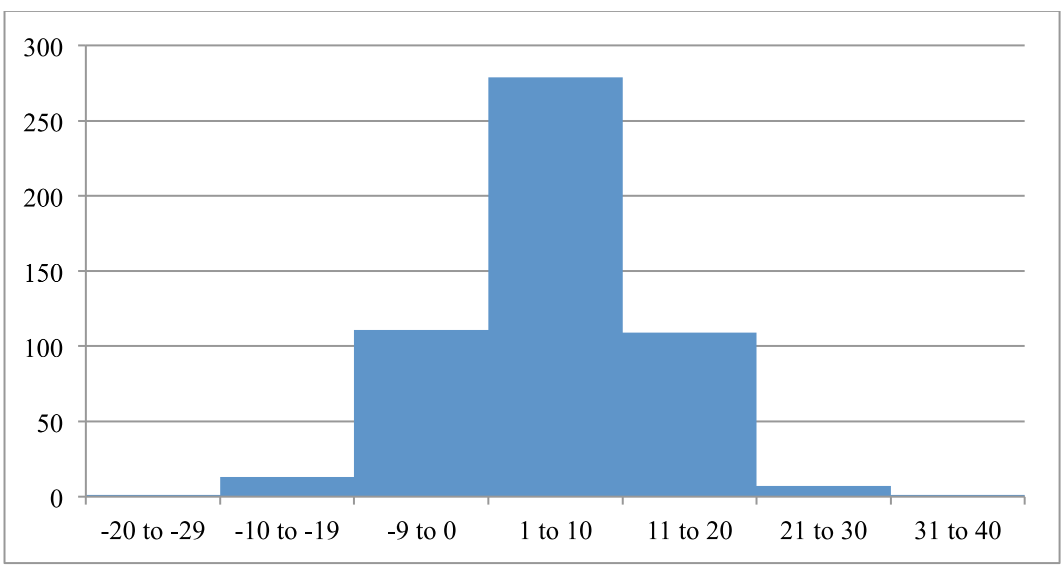

Figure 4. Histogram of standardized test (ST) score differences from District C spring data.

Confidence intervals at a 95% confidence level were computed with margins of error from *z*-scores as is acceptable for large sample populations. With a 95% confidence level, the true population mean (μ) for the difference in District C standardized test (ST) scores from Fall 2011 to Spring 2012 fell between 3.9 and 6.0 points.

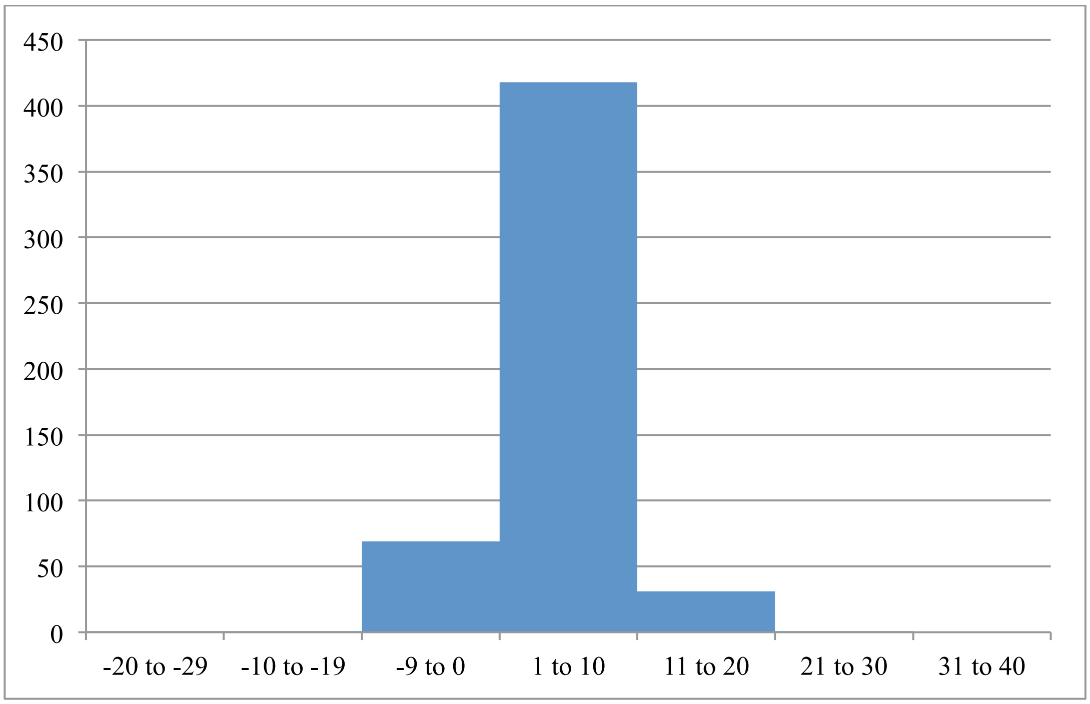

Figure 5. Histogram of automaticity quotient (AQ) ratio differences from District C spring data.

With a 95% confidence level, the true population mean (μ) for the difference in District C automaticity quotient (AQ) ratios from Fall 2011 to Spring 2012 fell between 3.9 and 4.7 points. Table 22 shows the frequency distributions for both ST and AQ differences from District C spring data.

Table 23

District C Spring Data: Frequency Distributions for ST and AQ Differences

Standardized test (ST) scores				Automaticity quotient (AQ) ratios			
Interval	Quantity	Statistic	Measure	Interval	Quantity	Statistic	Measure
-20 to -29	1	Median	6.0	-20 to -29	1	Median	3.9
-10 to -19	13	Mean	5.4	-10 to -19	1	Mean	4.4
-9 to 0	111	SD	7.2	-9 to 0	69	SD	4.8
1 to 10	279			1 to 10	418		
11 to 20	109			11 to 20	31		
21 to 30	7			21 to 30	1		
31 to 40	1			31 to 40	0		

District D Descriptive Statistics and Confidence Intervals

After the deletion of the unusable data, District D retained data for a sample population of 1551 students. All districts exhibited student increases in every category of descriptive statistics. A comparison of the median and the mean differences in standardized test (ST) scores between fall and spring showed little difference. The median and the mean differences in automaticity quotient (AQ) ratios showed less than a one-point difference. When the mean and median are comparable in value, a normal distribution is expected. The standard deviation (SD) for the difference in ST scores was approximately 8½ points and the SD for the difference in AQ ratios was approximately 5½ units. These difference statistics are displayed in Table 24.

Table 24

District D: Difference Statistics from Fall 2011 to Spring 2012 (Sample Population 1551)

Statistics	ST	AQ
Median of difference	5.0	3.1
Mean of difference	5.1	3.7
SD of difference	8.4	5.4
% Students improved	75.5%	80.7%
% Improved 230 or above	14.7%	
% Improved 235 or above	13.1%	

Figures 6 and 7 display histograms that illustrate the bell curve frequency distributions of the spring ST and AQ differences for District D. Both the AQ and ST distributions show a bell curve with the AQ distribution skewed slightly to the right.

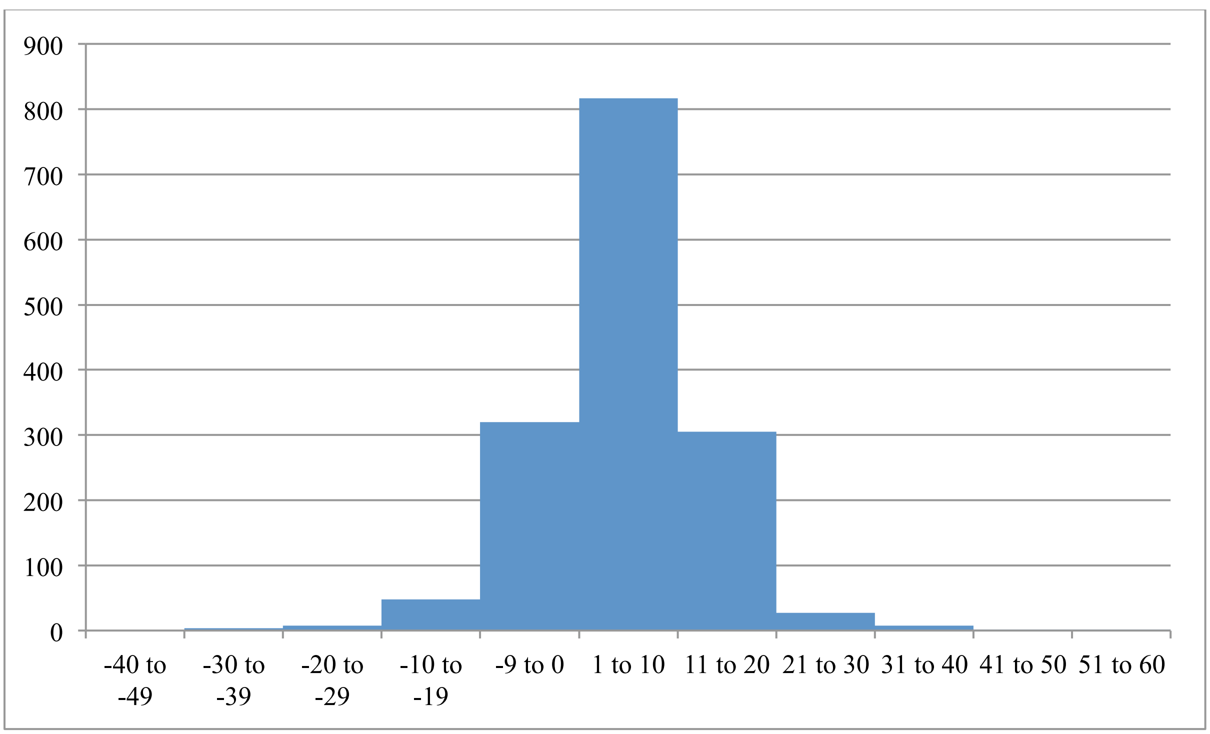

Figure 6. Histogram of standardized test (ST) score differences from District D spring data.

Confidence intervals at a 95% confidence level were computed with margins of error from *z*-scores as is acceptable for large sample populations. With a 95% confidence level, the true population mean (μ) for the difference in District D standardized test (ST) scores from Fall 2011 to Spring 2012 fell between 4.7 and 5.5 points.

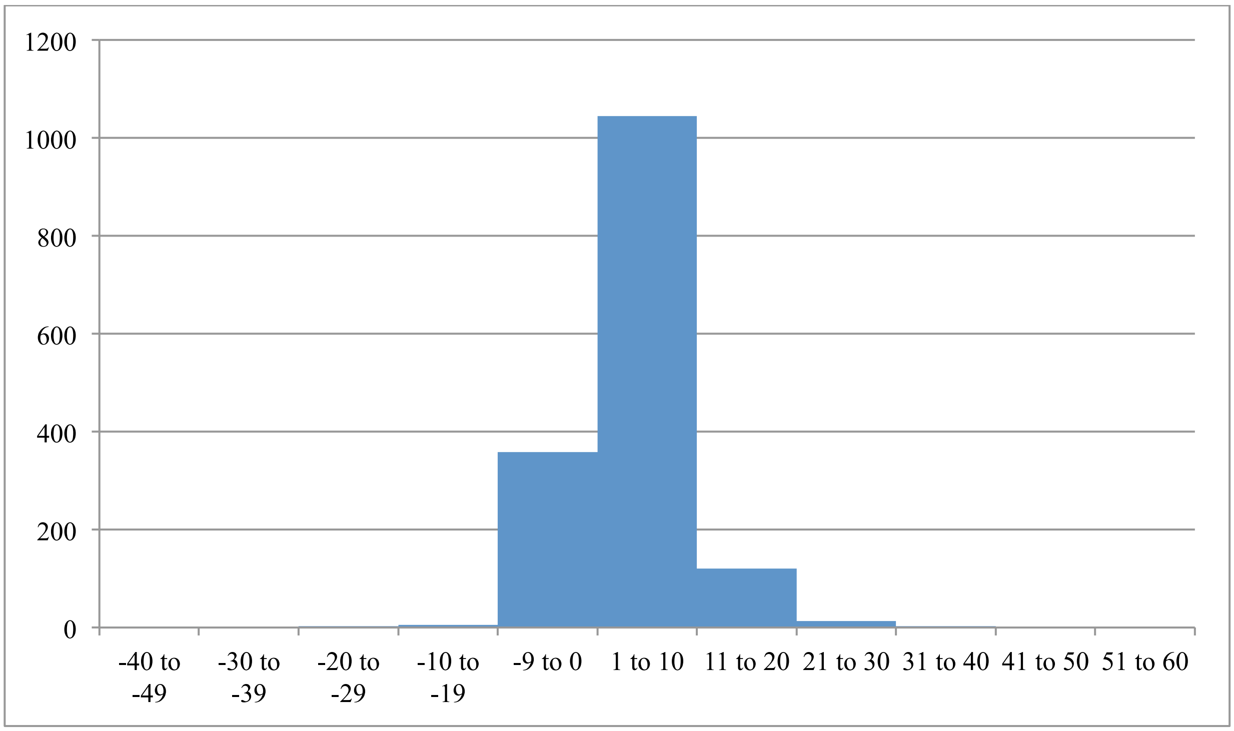

Figure 7. Histogram of automaticity quotient (AQ) ratio differences from District D spring data.

With a 95% confidence level, the true population mean (μ) for the difference in district D automaticity quotient (AQ) ratios from Fall 2011 to Spring 2012 fell between 3.5 and 4.0 points. Table 25 shows the frequency distributions for both ST and AQ differences from District D spring data.

Table 25

District D Spring Data: Frequency Distributions for ST and AQ Differences

Standardized test (ST) scores				Automaticity quotient (AQ) ratios			
Interval	Quantity	Statistics	Measures	Interval	Quantity	Statistics	Measures
-40 to -49	1	Median	5.0	-40 to -49	1	Median	3.1
-30 to -39	4	Mean	5.1	-30 to -39	0	Mean	3.7
-20 to -29	8	SD	8.4	-20 to -29	3	SD	5.4
-10 to -19	48			-10 to -19	6		
-9 to 0	320			-9 to 0	358		
1 to 10	817			1 to 10	1045		
11 to 20	305			11 to 20	121		
21 to 30	27			21 to 30	13		
31 to 40	8			31 to 40	3		
41 to 50	2			41 to 50	0		
51 to 60	2			51 to 60	0		

Note: SD refers to standard deviation

Summary of Descriptive Analyses After Unusable Data Deletion

All districts exhibited student increases in every category of descriptive statistics. Districts C showed the greatest increase in all categories except standard deviation. Student improvement in ST scores varied little in percent measurement improvement across the three districts, showing close to a 75% gain for each. All districts showed student improvement in AQ ratios of more than an 80% gain.

All districts improved substantially in the number of students who scored 230 or more and scored 235 or more; District C showed the highest gain at the 230 benchmark, with more than an 18% increase. To facilitate examination of the descriptive statistics for the three districts, comparison data analyses are displayed in Table 26.

Table 26

District Comparison Data Analysis After Imputation and Deletion Fall 2011 to Spring 2012

	District B		District C		District D	
Statistics	ST	AQ	ST	AQ	ST	AQ
Median of difference	5.0	2.9	6.0	3.9	5.0	3.1
Mean of difference	5.6	4.1	5.4	4.3	5.1	3.7
SD of difference	7.9	5.5	7.2	4.8	8.4	5.4
% Students improved	73.7%	86.0%	75.9%	85.5%	75.5%	80.7%
% Improved 230 or above	11.8%		18.2%		14.7%	
% Improved 235 or above	12.3%		14.2%		13.1%	

Comparison statistics of the undeleted data and the deleted data rendered minor differences with one exception: a difference of 18% for the number of students who improved their scores to 230 or above -benchmark for introduction to algebra readiness. The analysis of undeleted data showed twice the increase in the number of students who improved their scores to 230 or greater. A credible conclusion from the missing data analyses is that, with imputation of the winter data along with deletion of unusable data, the findings were cautious and reasonable.

Inference Analysis

After imputation of winter information into missing fall and spring items and deletion of unusable data, remaining data were analyzed to investigate answers for the final research question.

- What is the relationship between Automaticity Quotient ratios and MAP scores of middle school students from three school districts in Kentucky?

To determine what relationship exists between student automaticity quotient (AQ) ratios and standardized test (ST) scores, least squares linear regressions were performed on the spring data of the three districts after automaticity intervention treatment. These regressions were performed on district middle grades data as well as for each grade level, sixth through eighth. Independent variables were AQ ratios and dependent variables were ST scores.

The regression analyses examined trend lines for predicted values, regression coefficients, Pearson product-moment correlation coefficients, and coefficients of determination to ascertain relationships and strengths of associations between the variables. An examination of the residual plot, the graph of the difference between the observed value of the dependent variable and the predicted variable, for these regressions showed that a linear regression model was appropriate for these data. In addition, confidence intervals and confidence levels along with standard errors were defined to determine the likeliness of predictions. Guidelines for evaluating Pearson's coefficient are show in Table 27.

Table 27

Scale for Evaluating Pearson's Coefficient (r)

Strength of Association	Positive Association	Negative Association
Weak	0.1 to 0.3	-0.1 to -0.3
Moderate	0.3 to 0.5	-0.3 to -0.5
Strong	0.5 to 1.0	-0.5 to -1.0

Note: No values of exactly 0.3 or 0.5 were accrued in the analysis of these data.

District B Middle Grades Regression Analysis

In figure 8, scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for middle grade levels in District B. Spring data that included all middle grades showed a linear relationship between automaticity quotients and standardized test scores. The trend line ($\hat{y} = b_1x + b_0$) for predicted values for district middle grades indicated a positive correlation between the independent (AQ) and dependent (ST) variables.

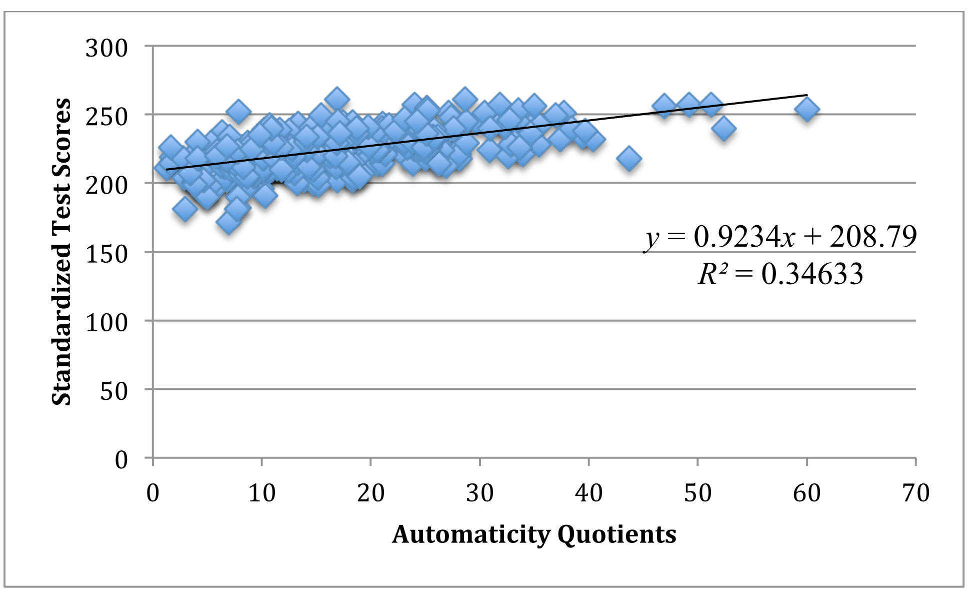

Figure 8. Scatter plot showing regression line of best fit from District B spring data for district grades 6, 7, and 8.

For spring data that includes all middle grades, the regression coefficient (b_1) showed that for every increment of increase in AQ ratios, the average increase in ST scores was nearly one point. The correlation coefficient (*r*) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 35% of the change in ST scores was predictable from AQ ratios.

District B Grade Level Regression Analyses

In figures 9, 10, and 11 scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for each middle grade level in District B. Grades 6, 7, and 8 showed linear relationships between the automaticity quotient ratios and the standardized test scores. The trend lines ($\hat{y} = b_1x + b_0$) for predicted values in each grade indicated positive correlations between the independent (AQ) and dependent (ST) variables.

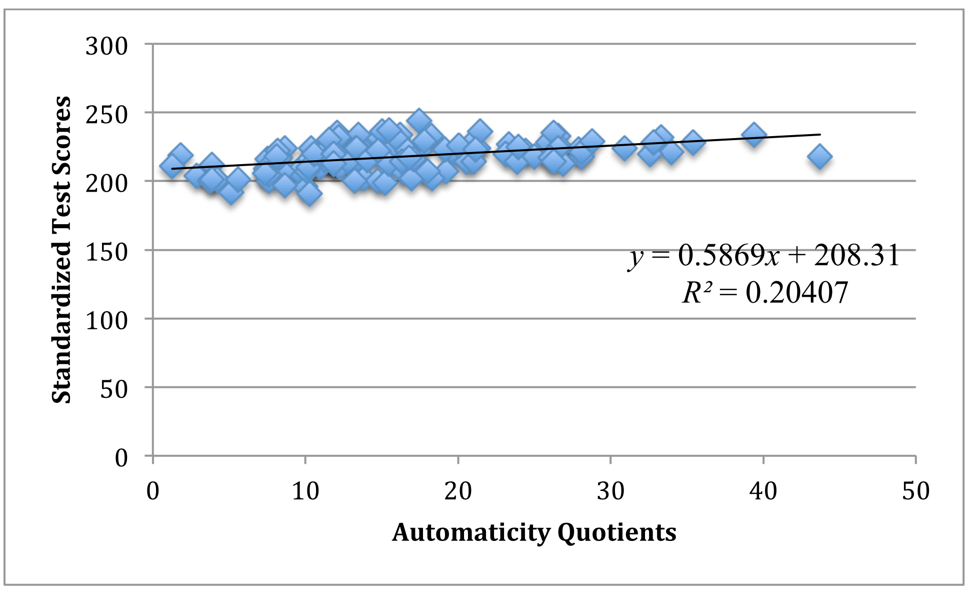

Figure 9. Scatter plot showing regression line of best fit from District B spring data for grade 6.

For Grade 6, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was approximately one-half point average increase in ST scores. The correlation coefficient (*r*) revealed a positive moderate association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 20% of the change in ST scores was predictable from AQ ratios.

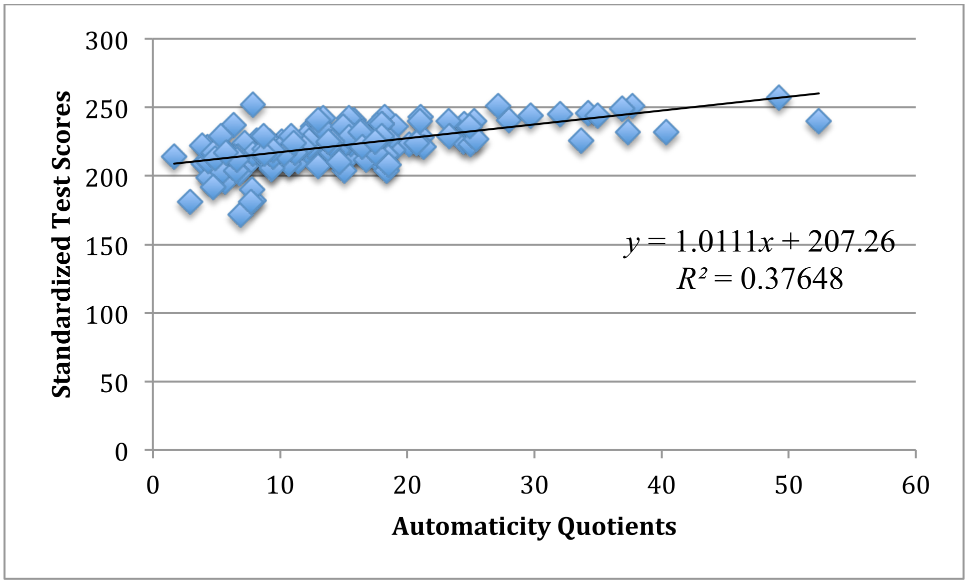

Figure 10. Scatter plot showing regression line of best fit from District B spring data for grade 7.

For Grade 7, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was approximately one point average increase in ST scores. The correlation coefficient (*r*) revealed a positive strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 38% of the change in ST scores was predictable from AQ ratios.

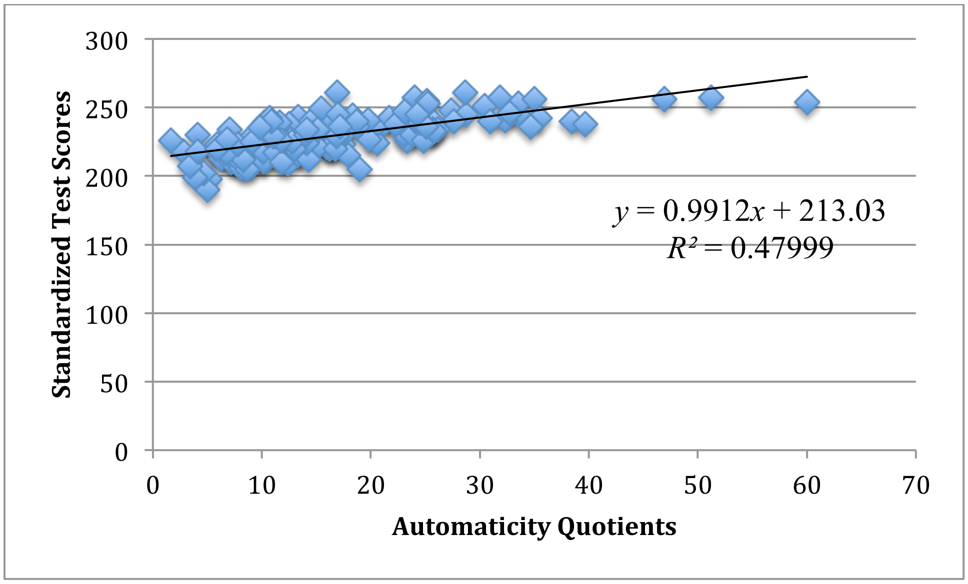

Figure 11. Scatter plot showing regression line of best fit from District B spring data for grade 8.

For Grade 8, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was approximately one point average increase in ST scores. The correlation coefficient (*r*) revealed a positive strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 48% of the change in ST scores was predictable from AQ ratios.

Table 28 organizes the grade level regression analyses for AQ ratios as independent variable and ST scores as dependent variable for District B spring data. Both the correlation coefficient (*r*) and the coefficient of determination (R^2) show a significant increase from 6th grade through 8th grade levels.

Table 28

District B Spring Data: Grade Level Regression Analyses with Independent Variable AQ and Dependent Variable ST

Grade level	Regression line ($\hat{y} = b_1x + b_0$)	Regression coefficient (b_1)	Correlation coefficient (r)	Coefficient of determination (R^2)
All middle grades	$\hat{y} = 0.923x + 208.79$	0.92	0.59	0.35
6th grade	$\hat{y} = 0.59x + 208.31$	0.59	0.45	0.20
7th grade	$\hat{y} = 1.01x + 207.26$	1.01	0.61	0.38
8th grade	$\hat{y} = 0.99x + 213.03$	0.99	0.69	0.48

District C Middle Grades Regression Analysis

In figure 12, scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for middle grade levels in District C. Spring data that included grades 6, 7, and 8 showed a linear relationship between automaticity quotient ratios and standardized test scores. The trend line ($\hat{y} = b_1x + b_0$) for predicted values for district middle grades indicated a positive correlation between the independent (AQ) and dependent (ST) variables.

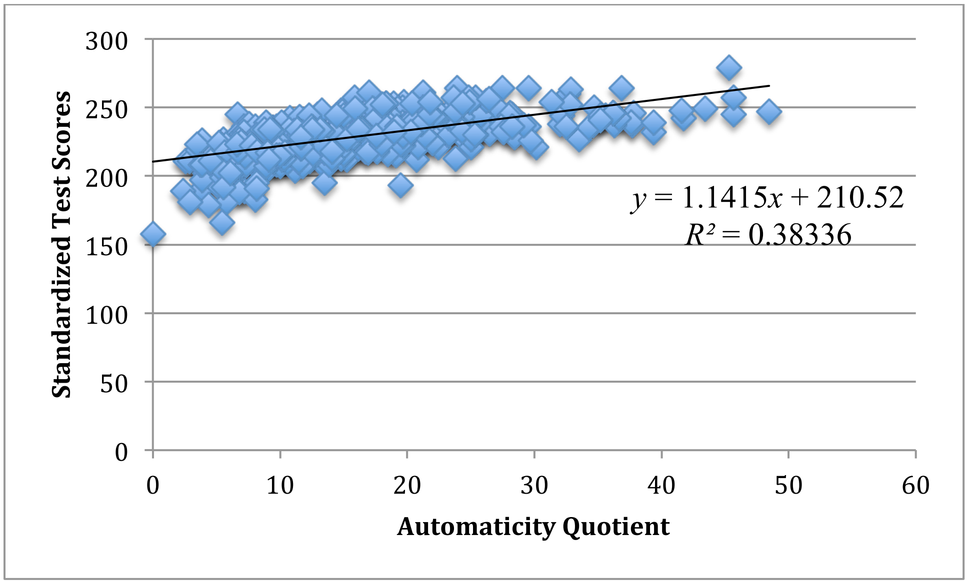

Figure 12. Scatter plot showing regression line of best fit from District C spring data for district grades 6, 7, and 8.

For spring data that includes all middle grades, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was more than one point average increase in ST scores. The correlation coefficient (*r*) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 38% of the change in ST scores was predictable from AQ ratios.

District C Grade Level Regression Analyses

In figures 13, 14, and 15, scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for each middle grade level in District C. Spring data for individual grades 6, 7, and 8 showed linear relationships between the automaticity quotient ratios and the standardized test scores. The trend lines ($\hat{y} = b_1x + b_0$) for predicted values in each grade indicated positive correlations between the independent (AQ) and dependent (ST) variables.

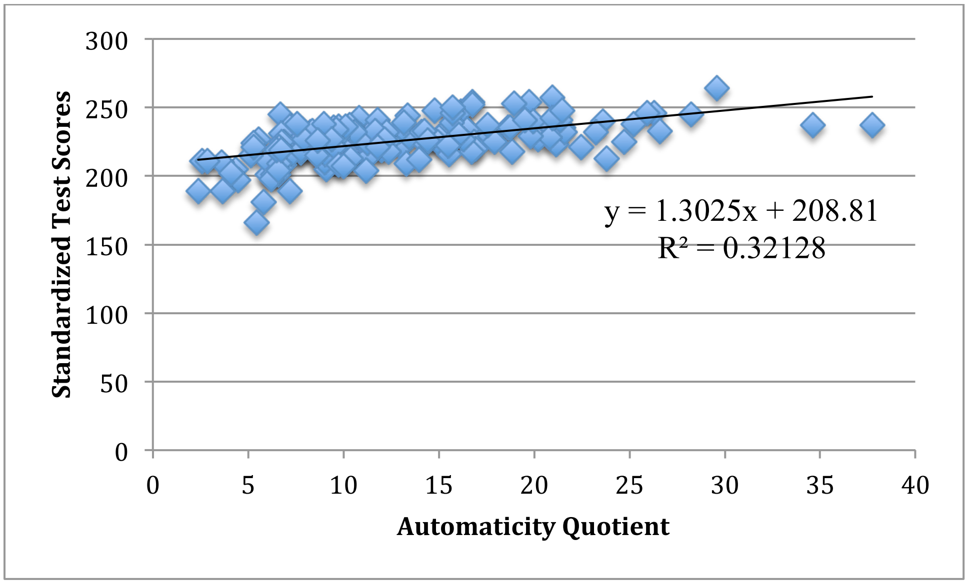

Figure 13. Scatter plot showing regression line of best fit from District C spring data for district grade 6.

For Grade 6, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was nearly one and a third point average increase in ST scores. The correlation coefficient (*r*) revealed a positive strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 32% of the change in ST scores was predictable from AQ ratios.

For Grade 7, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, there was nearly a one point average increase in ST scores. The correlation coefficient (*r*) revealed a positive strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 37% of the change in ST scores was predictable from AQ ratios.

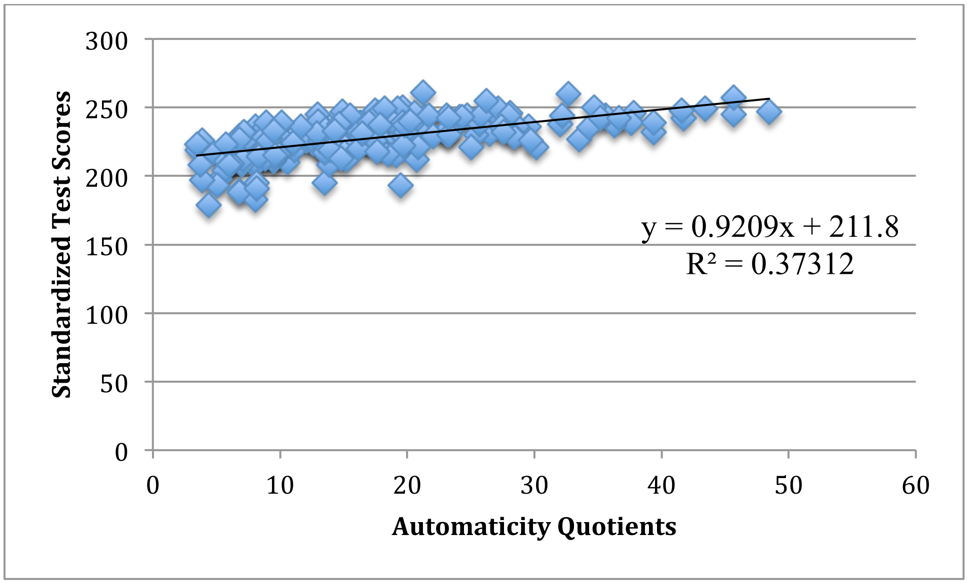

Figure 14. Scatter plot showing regression line of best fit from District C spring data for district grade 7.

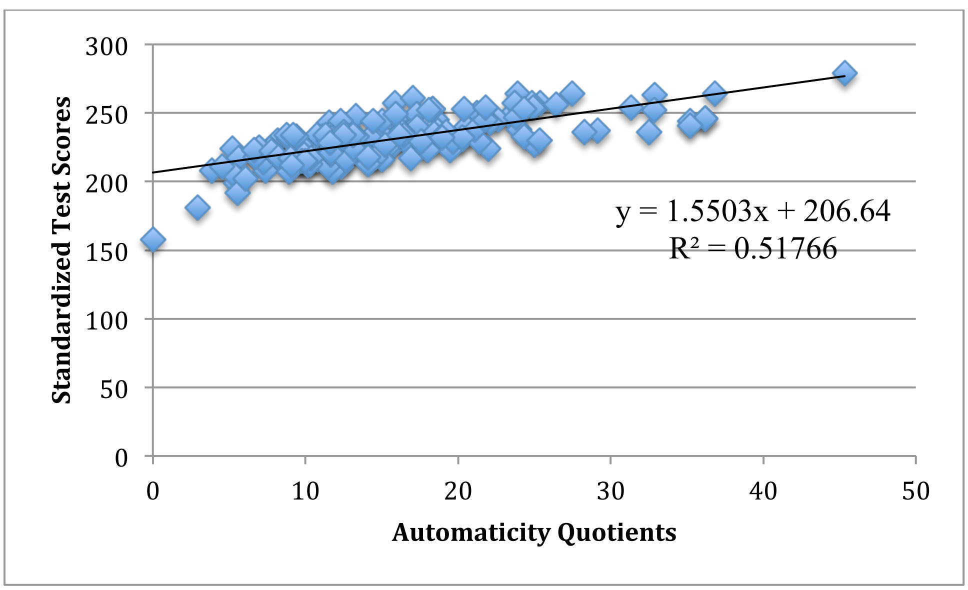

Figure 15. Scatter plot showing regression line of best fit from District C spring data for district grade 8.

For Grade 8, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, the average increase in ST scores was more than one and a half points. The correlation coefficient (r) revealed a positive strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 52% of the change in ST scores was predictable from AQ ratios.

Table 29 organizes the grade level regression analyses for AQ ratios as the independent variable and ST scores as the dependent variable for District B spring data. Both the correlation coefficient (r) and the coefficient of determination (R^2) show a significant increase from 6th grade through 8th grade levels.

Table 29

District C Spring Data: Grade Level Regression Analyses with Independent Variable AQ and Dependent Variable ST

Grade level	Regression line ($\hat{y} = b_1x + b_0$)	Regression coefficient (b_1)	Correlation coefficient (r)	Coefficient of determination (R^2)
All middle grades	$\hat{y} = 1.14x + 210.52$	1.14	0.62	0.38
6th grade	$\hat{y} = 1.30x + 208.81$	1.30	0.56	0.32
7th grade	$\hat{y} = 0.92x + 211.80$	0.92	0.61	0.37
8th grade	$\hat{y} = 1.55x + 206.64$	1.55	0.72	0.52

District D Middle Grades Regression Analysis

In figure 16, scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for middle grade levels in District D. Spring data that included grades 6, 7, and 8 showed a linear relationship between automaticity quotient ratios and standardized test scores. The trend line ($\hat{y} = b_1x + b_0$) for predicted values for district middle grades indicated a positive correlation between the independent (AQ) and dependent (ST) variables.

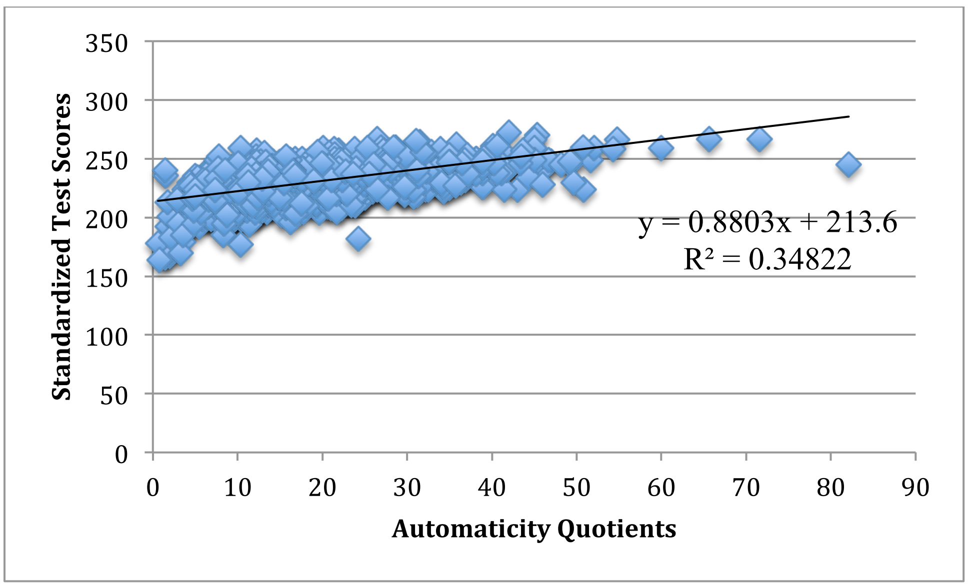

Figure 16. Scatter plot showing regression line of best fit from District D spring data for district grades 6, 7 and 8.

For spring data that included all middle grades, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, the average increase in ST scores was slightly less than one point. The correlation coefficient (r) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 35% of the change in ST scores was predictable from AQ ratios.

District D Grade Level Regression Analyses

In figures 17, 18, and 19, scatter plots were created with AQ ratios as independent variables and ST scores as dependent variables for each middle grade level in District D. Spring data for individual grades 6, 7, and 8 showed linear relationships between the automaticity quotient ratios and the standardized test scores. The trend lines ($\hat{y} = b_1x + b_0$) for predicted values in each grade indicated positive correlations between the independent (AQ) and dependent (ST) variables.

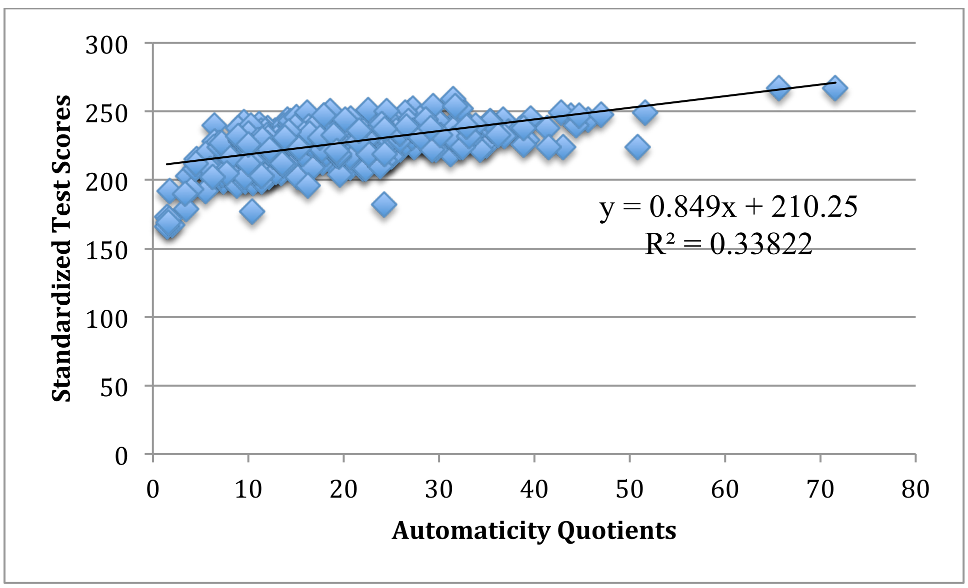

Figure 17. Scatter plot showing regression line of best fit from District D spring data for district grade 6.

For Grade 6, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, the average increase in ST scores was more than three-fourths points. The correlation coefficient (r) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 34% of the change in ST scores was predictable from AQ ratios.

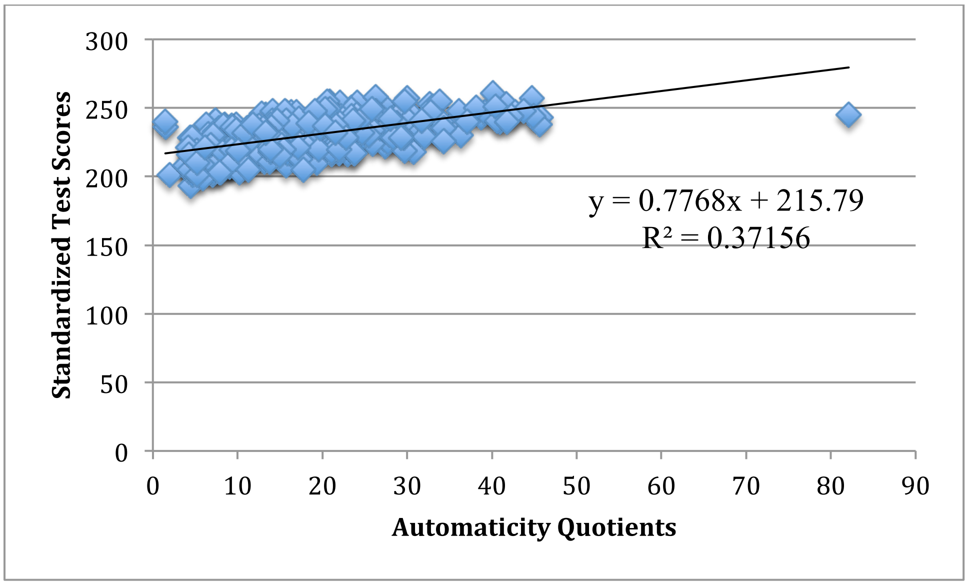

Figure 18. Scatter plot showing regression line of best fit from District D spring data for district grade 7.

For Grade 7, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, the average increase in ST scores was approximately three-fourths points. The correlation coefficient (r) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 37% of the change in ST scores was predictable from AQ ratios.

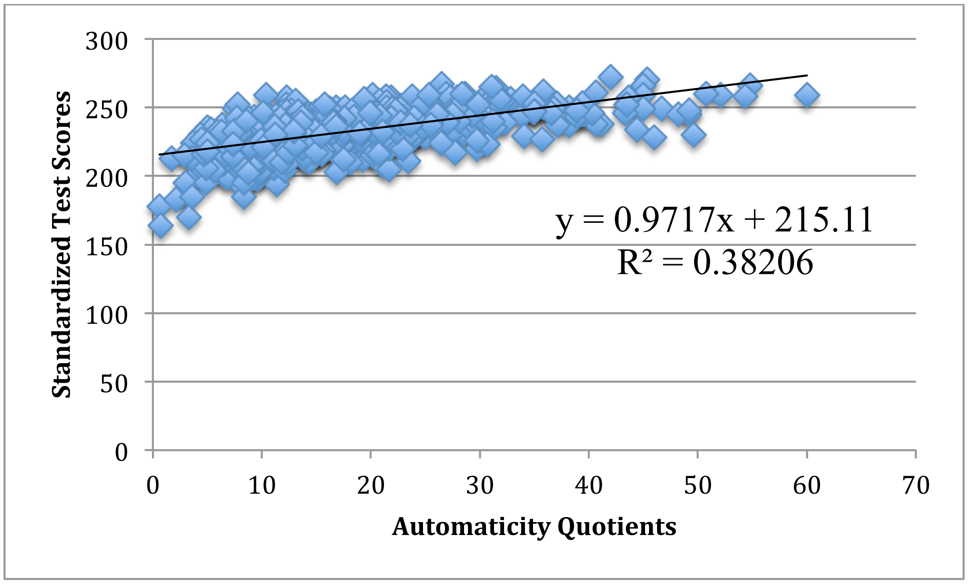

Figure 19. Scatter plot showing regression line of best fit from District D spring data for district grade 8.

For Grade 8, the regression coefficient (b_1) showed that for every increment of change in AQ ratios, the average increase in ST scores was nearly a point. The correlation coefficient (*r*) revealed a positive and strong association between AQ ratios and ST scores. The coefficient of determination (R^2) indicated that approximately 38% of the change in ST scores was predictable from AQ ratios.

Table 30 organizes the grade level regression analyses for AQ ratios as the independent variable and ST scores as the dependent variable for District B spring data. Both the correlation coefficient (*r*) and the coefficient of determination (R^2) show a significant increase from 6th grade through 8th grade levels.

Table 30

District D Spring Data: Grade Level Regression Analyses with Independent Variable AQ and Dependent Variable ST

Grade level	Regression line ($\hat{y} = b_1x + b_0$)	Regression coefficient (b_1)	Correlation coefficient (r)	Coefficient of determination (R^2)
All middle grades	$\hat{y} = 0.88x + 213.60$	0.88	0.59	0.35
6th grade	$\hat{y} = 0.85x + 210.25$	0.85	0.58	0.34
7th grade	$\hat{y} = 0.78x + 215.79$	0.78	0.61	0.37
8th grade	$\hat{y} = 0.97x + 215.11$	0.97	0.62	0.38

Summary of Data Analysis

In order to address the research questions for this study, analyses of descriptive and inferential statistics were implemented. First, descriptive statistics were developed to measure changes in the automaticity quotient ratios and standardized test scores after implementation of the automaticity intervention between fall and spring. Second, inference statistics were developed to examine relationships between automaticity quotient ratios and standardized test scores after implementation of the automaticity intervention.

Unusable data occurred in two forms – missing and uncountable. When possible, winter data collected was used to replace missing fall or spring data. Unusable data were within acceptable proportions when missing data mechanisms were considered.

Data analyses occurred after winter data imputation into missing fall or spring data. Descriptive analyses occurred both before and after deletion of unusable data. Measures of central tendency and percentage of improvements were analyzed before and after deletion of unusable data. Adjusted measures of central tendency along with standard deviations were added as well as frequency distributions and confidence intervals for analysis after deletion of unusable data. With few exceptions, analyses with undeleted and deleted data showed similar statistics. Inferential analysis from least squares regression occurred after deletion of unusable data.

Summary of Descriptive Analysis

A comparison of descriptive analyses of differences in data from fall to spring for the three districts is compiled in Table 31.

Table 31

District Comparison Data Analysis After Imputation and Deletion From Fall 2011 to Spring 2012

	District B		District C		District D	
Statistics	ST	AQ	ST	AQ	ST	AQ
Median of difference	5.0	2.9	6.0	3.9	5.0	3.2
Mean of difference	5.6	4.1	5.4	4.3	5.1	3.7
Standard deviation	7.9	5.5	7.2	4.8	8.2	5.4
% Students improved	73.7%	86.0%	75.9%	85.5%	75.5%	80.7%
% Improved 230 or above	11.9%		18.2%		14.7%	
% Improved 235 or above	12.3%		14.2%		13.1%	

All districts exhibited student increases in every category of descriptive statistics. Median and mean differences varied little across the three districts and standard deviations were within a point across districts for each variable. At least 73% of students improved their ST scores and at least 80% improved their AQ ratios.

All district improved substantially in the percent of students who scored 230 points or more and scored 235 points or more, showing at least 12% increases. But District C had a higher gain of students at the 230 benchmark for introduction to algebra, with more than an 18% increase.

Frequency histograms approximated bell curves for both the AQ and ST difference data, slightly skewing to the right for AQ ratios. Table 32 illustrates the statistics used for confidence intervals for AQ and ST differences from fall to spring in each district.

Table 32

District Confidence Intervals Population Mean (μ) for Difference in ST and AQ

	District B		District C		District D	
Statistics	ST	AQ	ST	AQ	ST	AQ
Confidence level	95%	95%	95%	95%	95%	95%
Standard deviation	7.9	5.5	7.2	4.8	8.4	5.4
Sample size	414	414	522	522	1551	1551
Margin of error	0.8	0.5	0.6	0.4	0.4	0.3
Mean statistic	5.6	4.1	5.4	4.3	5.1	3.7
Confidence interval	$4.9<\mu<6.4$	$3.5<\mu<4.6$	$4.7<\mu<6.0$	$3.9<\mu<4.7$	$4.7<\mu<5.5$	$3.5<\mu<4.0$

Comparison of district confidence intervals indicated similar AQ ranges for the population mean at a 95% confidence level, with the lower range at 3.5 points and the upper range at 4.7 points. The ST ranges for the population mean at a 95% confidence level were from 4.7 to 6.7 points.

Summary of Regression Analyses

Scatter plots from district and grade level data showed positive, linear relationships between student automaticity quotient (AQ) ratios and standardized test (ST) scores. Table 33 summarizes the coefficients developed from the regression analyses of spring data for the three districts.

Table 33

Coefficients From Regression Analyses for Spring District Data

District B	Regression coefficient (b_1)	Correlation coefficient (r)	Coefficient of determination (R^2)
Middle grades	0.92	0.59	0.35
6th grade	0.59	0.46	0.20
7th grade	1.01	0.62	0.38
8th grade	0.99	0.69	0.48
District C	**Regression coefficient (b_1)**	**Correlation coefficient (r)**	**Coefficient of determination (R^2)**
Middle grades	1.14	0.62	0.38
6th grade	1.30	0.56	0.32
7th grade	0.92	0.61	0.37
8th grade	1.55	0.72	0.52
District D	**Regression coefficient (b_1)**	**Coefficient correlation (r)**	**Coefficient of determination (R^2)**
Middle grades	0.88	0.59	0.35
6th grade	0.85	0.58	0.34
7th grade	0.78	0.61	0.37
8th grade	0.97	0.62	0.38

Most of the districts showed a regression coefficient of approximately one point, indicating an approximate 1-point increase in ST scores for each increase in AQ quotient units. District C showed the highest district regression coefficient of 1.14; their 8th grade posted the highest regression grade level coefficient with 1.55. Two grade levels showed lower regression coefficients; the sixth grade for District B had a regression coefficient of 0.59 and the seventh grade for District D had a regression coefficient of 0.78.

The range for most of the correlation coefficients was 0.56 to 0.62 for both district and grade level data, showing a positive, strong relationship between the ST scores and AQ ratios. Outside of this range, the eighth grade for District C posted the highest correlation coefficient with 0.72 followed closely by 8th grade for District B with 0.70. The sixth grade for District B had the lowest correlation coefficient with 0.45.

The range for most of the coefficients of determination was 0.32 to 0.38 for district and grade level data, showing approximately one-third of the variance in ST scores was predictable from AQ ratios. Outside of this range, the eighth grade for District C posted the highest coefficient of determination with 0.52, followed closely by 8th grade for District B with 0.48. The 6th grade for District B posted the lowest coefficient of determination of 0.20.

In each district both the correlation coefficients (r) and the coefficients of determination (R^2) show a significant increase from 6th grade through 8th grade levels. This progression of improvement indicates that the higher the grade level, the more important it is to remediate automaticity of basic mathematics facts. It is a confirmation of the importance of automaticity as students progress though higher levels of school, which coincide with higher levels of mathematics.

Conclusions

The conclusions from this study addressed the research questions through descriptive and inferential analyses. For two of the three questions, descriptive statistics were analyzed to investigate changes that occurred in students AQ ratios and ST scores after administration of the automaticity intervention treatment to middle school students.

- How did automaticity quotient ratios change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?
- How did standardized test scores change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?

Descriptive analyses were performed at the district level for middle school students.

A least squares linear regression analysis was performed for the third research question to investigate the relationship between AQ ratios and ST scores.

- What is the relationship between Automaticity Quotient ratios and MAP scores of middle school students from three school districts in Kentucky?

Regression analyses were performed at district middle grades level and for each individual middle grade – 6^{th}, 7^{th}, and 8^{th} grades.

Conclusions were made from the overall district data for the middle grades, individual district data for middle grades, and individual middle grade level data for each district. Additional trends of student automaticity and standardized test performance were analyzed for a more robust snapshot of student performance relating to automaticity and standardized test scores.

Each of the three districts showed improvements in automaticity quotient ratios and standardized test scores in the presence of an automaticity intervention treatment during the academic year. In addition, the number of students who improved their test scores for introduction to algebra readiness and algebra readiness increased. With a 95% confidence level, the confidence interval for the mean population fell in the range of 4.7 and 6.4 points for ST scores and 3.5 and 4.0 units for AQ ratios.

Median MAP Mathematics Scores

The following charts show the MAP mathematics nationally normed median Rasch Unit (RIT) scores for 6th, 7th, and 8th grade in Fall 2011 and nationally normed median RIT scores adjusted for expected growth in Spring 2012. Individual grade level RIT scores for fall and spring are also shown for the three districts.

The Fall 2011 data revealed that only two grade levels in different districts began the academic year at nationally normed levels as published by Northwest Evaluation Association (NWEA, 2011). In District B the 6th grade began the academic year 4 points behind the norm, the 7th grade began 9 points behind the norm, and the 8th grade began 5.5 points behind the norm. While the 6th grade in District C began at normed levels, the 7th grade began 1 point behind and the 8th grade began 6 points behind the norm. In District D, the 6th grade began the academic year at the normed level but the 7th and 8th grades each began 1 point behind the norm. Table 36 shows the median MAP mathematics and district RIT scores and norms for Fall 2011.

Table 36

Median MAP Mathematics Nationally Normed and District RIT Scores Fall 2011

Grade level	MAP norms	District B	District C	District D
6th	220	216	220	220
7th	226	217	225	225
8th	230	225	224	229

The normed scores are highlighted in yellow

Note: NWEA 2011 Normed RIT Scores at the 50th Percentile

The Spring 2012 data showed that the same two grades beginning the academic year on normed score level also maintained the adjusted normed score level at the end of the academic year. In addition, the 7th grade in District D ended the academic year at the adjusted normed score level. The remaining district grade levels did not reach the adjusted normed scores by the end of the academic year.

No middle grades in District B reached normed score levels for the end of the academic year. Sixth grade remained 6 points behind while 7th and 8th grade ended 7 points and 4.5 points behind, respectively. The 6th grade for District C remained on normed score level but the 7th and 8th grades were both behind 2 points. The 6th grade for District D remained on normed score level, 7th grade gained normed score level, and 8th grade remained 1 point below normed score level at the end of the academic year. Table 37 shows the median MAP mathematics and district RIT scores and norms for Fall 2011.

Table 37

Median MAP Mathematics Nationally Normed and District RIT Scores Spring 2012

Grade level	MAP norms	District B	District C	District D
6th	226	220	226	226
7th	230	223	228	230
8th	234	230	232	233

Normed scores are highlighted in yellow

Note: NWEA 2011 Normed RIT Scores at the 50th Percentile

All but two grade levels in the three districts met annual learning growth expectations; three of them exceeded growth expectations. Although the 7th and 8th grade levels for District B did not meet normed score levels for the end of the year, they exceeded expected growth rates published by NWEA (2011). The 6th grade in District B did not meet expected growth rates, gaining only 4 of the 6 expected growth points.

The 6th grade for District C met expected growth rates of 6 points and the 8th grade doubled the expected growth rate of 4 points, gaining 8 points by the end of the academic year. But the 7th grade did not meet the expected growth rate, gaining only 3 of the 5 expected points.

All of the grade levels in District D met the expected growth rates, 6 points for 6th grade, 5 points for 7th grade, and 4 points for 8th grade. MAP mathematics and annual learning RIT and district growth rates are compiled in Table 38. Expected growth rates are highlighted in yellow; exceeded growth rates are highlighted in blue.

Table 38

MAP Mathematics Annual Learning RIT and District Growth Rates

Grade level	MAP expected	District B	District C	District D
6th	6	4	6	6
7th	5	6	3	5
8th	4	5	8	4

Expected growth rates are highlighted in yellow; exceeded growth rates are highlighted in blue

Note: NWEA 2011 Normed RIT Scores at the 50th Percentile

Seconds per Item

Automaticity of basic mathematics facts demands instant recall of number combinations with basic operations. Van de Walle and Lovin (2006) suggested that 3 seconds was enough time to recall a single fact "without resorting to non-efficient means such as counting" (p. 74). Using this parameter and Table 39, students who completed the automaticity diagnostic assessment in 5.25 minutes without error could be considered automatic.

Table 39

Automaticity Rates

Seconds per item	Minutes per 105 items
1	1.75
2	3.50
3	5.25
4	7.00
5	8.75
6	10.5
7	12.25
8	14.00
8.5	14.88
9	15.75

Students were given 15 minutes to complete the diagnostic. If they used the full 15 minutes, they took an average of more than 8 seconds to complete each item. These times are highlighted in yellow.

Table 40 shows an array of time and errors on the automaticity diagnostic assessment for District B. An examination of the range of time in minutes used with the number of errors made on the automaticity diagnostic shows a pattern of mean student scores. The decrease of mean scores with the increase of time used along with 2 or less errors on the diagnostic was one pattern observed. The decrease of mean scores also occurred with more than 10 errors. This decrease was evident across ranges of 3 to 10 errors, but non-sequential mean scores were sporadically recorded in these ranges.

Mean scores that dictated introduction to algebra readiness (230+) as well as algebra readiness (235+) were recorded within the ranges with less than 5 minutes used for completing the diagnostic. None of the benchmark mean scores were noted within ranges of 5 minutes or greater, and none of them were noted in the range of more than 10 errors. The range of 6-10 errors has one mean algebra readiness score recorded; this score was a non-sequential mean score in these time and error ranges.

Table 40

District B Spring 2012 Time and Error Ranges

Range of time minutes (Algebra)	Range of scores (errors 0-2) Mean	Range of scores (errors 3-5) Mean	Range of scores (errors 6-10) Mean	Range of scores (errors $^{+}$10) Mean
1:00-1:59	254-240 247	X	X	X
2:00-2:59	257-232 243	218 218	X	X
3:00-3:59	260-213 238	245-220 230	240 240	X
4:00-4:59	257- 214 233	242-214 230	241-221 220	227-205 218
5:00-5:59	244-204 228	242-208 225	220-202 210	227-215 221
6:00-6:59	261-202 227	242-199 225	212-240 219	221-213 217
7:00-7:59	236-200 224	242-207 222	243-217 216	224-210 217
8:00-8:59	241-213 226	229-209 219	226-221 224	225-172 213
9:00-9:59	242-209 222	229 229	210-191 201	205-229 214
10:00^{+}	252-206 223	207-227 217	237-197 217	230-181 208

Green: time intervals with scores 230 and above; Yellow: mean of range scores; Grey: non-sequential scores

Table 41 shows an array of time and errors on the automaticity diagnostic assessment for District C. The data shows no mean scores were recorded in the less than 2-minute range. The decrease of mean scores with the increase of time used along with 2 or less errors on the diagnostic was observed with the exception of the mean scores in the 8:00 to 8:59 minute range. A decrease of mean scores also occurred with more than 10 errors with one non-sequential exception. This decrease was evident across ranges of 3 to 10 errors, but more non-sequential mean scores were sporadically recorded in these ranges.

Mean scores that dictated introduction to algebra readiness (230+) as well as algebra readiness (235+) were recorded within the ranges with less than 7 minutes used for completing the diagnostic, not including the non-sequential score in the 8:00-8:59 minute range within the two or less error range previously mentioned. Excluding this errant score, none of the algebraic benchmarks were observed in the ranges of 7 minutes or greater with 6 or more errors.

Table 42 shows an array of time and errors on the automaticity diagnostic assessment for District D. A decrease of mean scores with the increase of time used along with 2 or less errors on the diagnostic was observed. A decrease of mean scores also occurred with more than 10 errors with two non-sequential exceptions after the 6:00 to 6:59 minute time range. This decrease was evident across the 3 to 5 error range with one non-sequential exception in the 9:00 to 9:59 time range. A decrease in test scores occurred in the 6 to 10 error range with 2 non-sequential mean scores in the 2:00 to 2:59 range and the 6:00 to 6:59 range.

Table 41

District C Spring 2012 Time and Error Ranges

Range of time minutes (Algebra)	Range of scores (errors 0-2) Mean	Range of scores (errors 3-5) Mean	Range of scores (errors 6-10) Mean	Range of scores (errors +10) Mean
1:00-1:59	X	X	X	X
2:00-2:59	279-232 246	250-235 240	X	X
3:00-3:59	264-221 241	264-228 245	X	193 193
4:00-4:59	264-221 241	257-213 230	253-223 238	239-212 220
5:00-5:59	253-212 236	254-215 234	240-216 229	249-216 226
6:00-6:59	261-216 235	254-217 232	249-222 235	244-208 221
7:00-7:59	248-208 228	245-195 226	233-216 226	240-207 220
8:00-8:59	241-221 232	244-204 225	324-221 227	225-183 211
9:00-9:59	243-210 224	234-212 224	242-216 225	234-205 218
10:00^{+}	239-205 223	245-188 222	238-201 223	236-158 211

Green: time intervals with scores 230 and above; Yellow: mean of range scores; Grey: non-sequential scores

Mean scores that dictated introduction to algebra readiness (230+) as well as algebra readiness (235+) were recorded within all ranges of less than 9 minutes used for completing the diagnostic. Excluding one errant score, none of the algebraic benchmarks were observed in the ranges of 7 minutes or greater with 6 or more errors. None of these benchmarks occurred in the +10 error range.

Table 42

District D Spring 2012 Time and Error Ranges

Range of time minutes (Algebra)	Range of scores (errors 0-2) Mean	Range of scores (errors 3-5) Mean	Range of scores (errors 6-10) Mean	Range of scores (errors +10) Mean
1:00-1:59	267-258 260	X	X	X
2:00-2:59	272-224 245	244-236 239	246-245 246	218 218
3:00-3:59	267-219 241	252-222 235	246-217 232	235-216 224
4:00-4:59	259-182 236	259-208 234	249-217 227	238-222 221
5:00-5:59	256-208 233	247-204 229	242-205 225	242-196 218
6:00-6:59	252-209 233	247-202 228	247-210 229	232-185 216
7:00-7:59	249-206 231	244-205 226	252-211 225	234-204 221
8:00-8:59	257-202 230	241-203 223	236-199 220	244-192 213
9:00-9:59	242-194 227	259-209 227	234-177 217	233-198 218
10:00+	230-202 224	252-199 223	245-199 220	241-164 211

Green: time intervals with scores 230 and above; Yellow: mean of range scores; Grey: non-sequential scores

With the exception of a few non-sequential scores, all three districts showed a distinct pattern of performance on the diagnostic: students' scores decreased when they used more time and incurred more errors. Students whose scores increased with less time used and incurred fewer errors had a greater probability of meeting algebra ready benchmarks.

Regression Analyses Results From District Spring Difference Data

To investigate relationships that exist between automaticity quotient ratios and standardized test scores, regression analyses were used. Least squares regression analyses were performed on spring data at the district level for middle school students and at each middle grade level after a full year of the automaticity intervention treatment. Independent variables were AQ ratios and dependent variables were ST scores.

Conclusions were made from the district data for middle grades and individual middle grade level for each district – 6^{th}, 7^{th}, and 8^{th} grades. The regression analyses examined trend lines for predicted values, regression coefficients, Pearson product-moment correlation coefficients, and coefficients of determination to ascertain relationships and strengths of associations existing between the variables.

Table 44 summarizes the coefficients developed from the regression analyses of spring data for Districts B, C, and D with independent variable AQ and dependent variable ST.

Examination of coefficients from analyses shows differences among the districts and district grade levels. The regression coefficients (b_1) vary significantly across grade levels in each district, ranging from values of 0.59 to 1.55. District C shows the largest regression coefficients at all grade levels with one exception, and therefore represents the greatest increase in ST scores for each increase in AQ ratios.

Table 44

Comparison of Coefficients Developed from Regression Analyses for District Spring Data

	Regression coefficient (b_1)			Correlation coefficient (r)			Coefficient of determination (R^2)		
District	B	C	D	B	C	D	B	C	D
Middle grades	0.92	1.14	0.88	0.59	0.62	0.59	0.35	0.38	0.35
6th grade	0.59	1.30	0.85	0.45	0.56	0.58	0.20	0.32	0.34
7th grade	1.01	0.92	0.78	0.61	0.61	0.61	0.38	0.37	0.37
8th grade	0.99	1.55	0.97	0.69	0.72	0.62	0.48	0.52	0.38

Examination of coefficients from analyses shows differences among the districts and district grade levels. The regression coefficients (b_1) vary significantly across grade levels in each district, ranging from values of 0.59 to 1.55. District C shows the largest regression coefficients at all grade levels with one exception, and therefore represents the greatest increase in ST scores for each increase in AQ ratios.

Examination of correlation coefficients (r) from analyses shows moderate variation among the districts and grade levels with a range of values from 0.45 to 0.72. All correlation coefficients are positive representing a positive correlation between variables. District C shows the largest correlation coefficient value of 0.72 representing the strongest correlation of ST scores with AQ ratios.

Examination of the coefficients of determination (R^2) reveals moderate variation among the districts and grade levels with values ranging from 0.20 to 0.52. District C shows the largest coefficient of determination of 0.52 indicating the largest change in ST scores that was predictable from AQ ratios.

In each district, both the correlation coefficients (*r*) and the coefficients of determination (R^2) show a significant increase from 6th grade through 8th grade levels. This progression of improvement indicates that the higher the grade level, the more important it is to remediate automaticity of basic mathematics facts. It is a confirmation of the importance of automaticity as students progress though higher levels of school, which coincide with higher levels of mathematics.

Table 45

Mean Coefficients from Regression Analyses for District Middle Grades

District	Regression coefficient (b_1)	Correlation coefficient (r)	Coefficient of determination (R^2)
District B	0.92	0.59	0.35
District C	1.14	0.62	0.38
District D	0.88	0.59	0.35
Mean totals	0.98	0.60	0.36

Without a control group, no cause and effect can be claimed. But the regression analysis showed three important results: 1) A positive, strong relationship between automaticity and standardized test scores existed; 2) For each increment that students' automaticity quotient ratios increased, their standardized test scores improved by one point; 3) Approximately one-third of the change in ST scores can be predicted from AQ ratios; the other two-thirds of change in ST scores can be predicted from other factors.

What does this information mean for mathematics teachers? The results showed that automaticity is important in student mathematics learning as measured with standardized mathematics assessments. As students improved their automaticity of basic mathematics facts, their test scores improved. But the results also showed students need more than basic fact automaticity to succeed in mathematics; they need other mathematics experiences to help them apply the mathematics in various contexts. In other words, automaticity of basic mathematics facts is one important factor in the learning of mathematics, but it is not the only factor.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The purpose of this study was to examine students' automaticity of basic mathematics facts and their learning of mathematics. The main research question that guided this study was:

What is the relationship of students' automaticity of basic mathematics facts to their learning of mathematics?

In particular, the intention of the study was to answer these sub-questions:

- What is the relationship between the automaticity quotient ratios and Measure of Academic Progress standardized test scores of middle school students from three school districts Kentucky?
- How did standardized test scores change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?
- How did automaticity quotient ratios change in the presence of the automaticity intervention treatment of middle school students from three school districts in Kentucky?

Summary of Study

Students who lack automaticity with basic facts can still be engaged in activities that promote number sense and mathematical reasoning. Number sense can lead to memorization of mathematics facts and is as necessary to problem solving as automaticity. The development of mathematical reasoning and problem solving strategies requires both number sense and automaticity of basic mathematics facts. Conceptual and procedural knowledge may develop interactively with gains in one leading to gains in the other, which in turn trigger new gains in the first.

Students with mathematics difficulties often demonstrate inefficient and ineffective counting strategies. Integrating number sense activities with number fact automaticity can lead to reduction in mathematics difficulties for both typical students and students with learning difficulties. But teachers often assume students' automaticity of basic facts and therefore students with learning difficulties are less successful in reform-oriented classrooms. The lack of automaticity renders the discussion of abstract principles such as algebra challenging. Instruction that involves both conceptual understanding and solving number facts would help students move through the hierarchy of mathematics.

Lack of number sense development in students entering kindergarten can later affect students' understanding of mathematics. Early mathematics intervention with younger students gives them the opportunity to catch-up with their peers. Isolating deficiencies and practicing targeted skills allows low achieving students to make gains in understanding mathematics.

For this study, data were analyzed from a larger initiative that provided an intervention treatment to help students increase their automaticity of basic mathematics facts. The treatment and treatment protocols incorporated research recommended components for effective automaticity practice that included computational fluency and number sense features. The purpose of this study was to examine students' automaticity of basic mathematics facts and their learning of mathematics.

The quantitative methodology of the study was a correlational approach with a pretest posttest paradigm. Three districts with similar demographics that used the same nationally normed standardized test to assess middle school students' knowledge of mathematics were chosen as participants in the study.

Students were administered automaticity diagnostic assessments and the Measures of Academic Progress® (MAP) assessment during the fall, winter, and spring of the 2011–2012 academic year. The automaticity intervention treatment began after the initial assessments were given. Teachers followed protocols to administer the treatment daily; students progressed through the treatment at their own pace.

Data reported by teachers through a district representative were the automaticity quotient (AQ) ratios of the number of correct items to the time used on the automaticity diagnostic assessment and the standardized test (ST) scores for middle school students of the three districts. The researcher performed descriptive analyses to measure changes in statistics between fall and spring in the presence of the automaticity intervention treatment. The researcher also performed regression analyses on spring data to determine the type of relationship that existed between the AQ ratios and ST scores. Limitations of the study were as follows: lack of control group, self-reporting of data, selection of participants, and insufficient verification of fidelity and compliance to protocols.

Analysis of Data

Assessment data consisting of the automaticity diagnostic and the Measure of Academic Progress (MAP) were documented in the fall, winter, and spring of the 2011–2012 academic year. Winter statistics were imputed for missing data before unusable data were eliminated. Analyses of data consisted of descriptive statistics, median MAP mathematics Rasch Unit (RIT) scores, expected growth rates, patterns of performance, and regression analysis.

Descriptive Statistics

All districts exhibited student increases in every category of descriptive statistics. All districts improved substantially in the percentage of students who scored at a level deeming them ready for introduction to algebra and scored at a level that deemed them to be ready for algebra. But District C had a higher gain of students at the 230 benchmark for introduction to algebra ready, with more than an 18% increase.

Frequency histograms approximated bell curves for both the AQ and ST difference data for spring data. Comparison of district confidence intervals for AQ and ST differences from fall to spring in each district indicated similar AQ ranges for the population mean at a 95% confidence level.

Median MAP Mathematics RIT Scores

The Fall 2011 data revealed that only two grade levels in different districts began the academic year at nationally normed levels. The Spring 2012 data showed that the same two grades beginning the academic year on normed score level also maintained the adjusted normed RIT score level at the end of the academic year. In addition, the seventh grade in District D ended the academic year at the adjusted normed score level. The remaining district grade levels did not reach the adjusted normed scores by the end of the academic year.

Expected Growth Rates

All but two grade levels in the three districts met annual learning growth expectations; three of them exceeded growth expectations. While the seventh and eighth grade levels for District B did not meet normed RIT score levels for the end of the year, they exceeded expected growth rates published by Northwest Evaluation Association (NWEA, 2011). The sixth grade did not meet expected growth rates, gaining only 4 of the 6 expected growth points.

The sixth grade for District C met expected growth rates and the eighth grade doubled the expected growth rate. But the seventh grade did not meet the expected growth rate. All of the grade levels in District D met the expected growth rates.

Patterns of Performance on the Automaticity Diagnostic

A pattern of performance concerning time used and errors incurred on the automaticity diagnostic occurred for the three districts. The results show that test scores decreased when students used more time and made more errors. Using less time and making fewer errors increased students' probability of meeting algebra readiness benchmarks.

Regression Analyses

Regression analyses were employed to investigate relationships that exist between automaticity quotient results and standardized test scores. Least squares regression analyses were performed on spring data at the district level for middle school students and at each middle grade level after a full year of the automaticity intervention treatment. Independent variables were AQ ratios and independent variables were ST scores. Conclusions were made from the district data for middle grades and individual middle grade level for each district – 6^{th}, 7^{th}, and 8^{th} grades.

The regression analyses examined trend lines for predicted values, regression coefficients, Pearson product-moment correlation coefficients, and coefficients of determination to ascertain relationships and strengths of associations existing between the variables. The mean regression coefficient for the three district middle grades data was 0.98 showing that, for every increment of change in AQ ratios, ST scores increased virtually one point. The mean correlation coefficient (*r*) was 0.60, revealing a positive and strong association between AQ ratios and ST scores. The mean coefficient of determination (R^2) was 0.36, indicating that 36% of the variance in ST scores was predictable from AQ ratios.

Conclusions from Analyses

The conclusions from this study reinforce the literature base concerning the relationship between automaticity of basic facts and learning higher-level mathematics. The results from the descriptive analysis of the study showed that improvements in the mean, median, and standard deviations for AQ ratios and ST scores occurred in the presence of the automaticity intervention treatment. With 95% confidence levels, the interval for the population mean difference of all three grade levels for AQ ratios was between 3.5 and 4.7 and for ST scores was between 4.7 and 6.4, and therefore expected improvements would fall within these intervals for similar populations. Improvements in the number of students who increased readiness for introduction to algebra readiness and algebra readiness also occurred. Increased algebra readiness allows for improved performance in algebra and other higher-level mathematics courses.

The results of the study support the literature base that, in order for students to be successful in higher-level mathematics, automaticity of basic facts in an important component of their mathematics foundation. The descriptive statistics show that the automaticity treatment improved the basic fact automaticity of middle school students.

The results from the inferential analysis of the study show that a strong relationship exists between student AQ ratios and ST scores; that for each increment of improvement for AQ ratios, ST scores increased one point; and that one-third of the change in ST scores could be predicted from the change in AQ ratios. These outcomes support the literature base that indicates automaticity with basic mathematics facts is an important component for the learning of mathematics.

Implications

The outcomes of this study present opportunity for teachers to foster student learning of mathematics through automaticity in three arenas: remediation and intervention, reinforcement, and enrichment. The outcomes also present opportunity for enhanced algebraic instruction offered to students at earlier grades.

Low achieving students or students with mathematics difficulties can be diagnosed to determine if they are deficient in automaticity of basic mathematics facts. Teachers can then offer automaticity remediation to students who fail to meet expected grade level criteria. In addition, students who are identified as low achievers in mathematics or with mathematics learning disabilities can be offered the automaticity treatment as part of their intervention plan.

Teachers can and should help students avoid mathematics difficulties from lack of automaticity in basic mathematics facts by initiating automaticity treatment at the beginning of the school year. According to the information in Tables 40, 41, and 42, for Districts B, C, and D respectively, students within 3 minutes and 0-2 errors earned high RIT scores on the MAP assessment. Offering automaticity treatment to all students gives them the opportunity to decrease time and error resulting in increased opportunity to engage in higher-level mathematical thinking. In addition, the automaticity treatment offers reinforcement in connecting basic operations for combinations of addition and subtraction as well as multiplication and division. The automaticity diagnostic can be used as a tool to identify student fluency.

Students with automaticity of basic mathematics facts provide fertile ground for teachers to bring depth in the learning of mathematics to their students. In particular, students can build readiness for introduction to algebra and readiness for algebra. An increase of students ready for introduction to algebra and ready for algebra is a mixed blessing for school districts. School districts may not be prepared for the type of algebra class essential for middle grades students who are concrete learners and not ready for the abstract thinking needed for learning algebra at the high school level. A dialog with developmental experts, mathematics educators, and district administration along with professional development for middle school mathematics teachers must take place in order to provide appropriate experiences for students with less sophisticated learning capabilities to learn higher-level mathematics.

Recommendations for Future Research

This study supports the literature base asserting that automaticity of basic mathematics facts is an important factor in the learning of mathematics. Because much of the research in mathematics automaticity involves elementary students, this study also enhances the literature base because it involves an underrepresented population—middle school students. Future research could further support and enhance the existing literature through improvement of the existing study, modifications to the existing study, or development of new studies.

Adjustments to the design of the study would improve the data collection and analysis processes. Such adjustments could include collecting surveys of information from teachers at the end of each data report concerning change in student performance in other measures of learning, developing an instrument to measure teachers' fidelity and compliance to protocols, and revision of the automaticity quotient for more predictive outcomes.

Modifications to the study for supporting the existing literature are analysis of outcomes for different populations of students to determine effectiveness of automaticity treatment. These modifications could include more diverse grade levels of students, one level of students across the state, one class level, or individual students for enhanced data collection and analysis. Another study with a population modification could examine students with low-test scores versus students with high-test scores. Such modifications could possibly show if the automaticity treatment has different degrees of effectiveness at different developmental levels, grade levels, or ability levels. Future studies could also remove outliers to examine the change in results. These analysis modifications would necessitate the defining of the parameters of which data would be considered outliers.

Another analysis modification might include using test data scores rather than differences in scores to calculate the coefficients. Another alternative analysis would be a canonical correlation where the two independent variables are the quotient fall and spring and the dependent variables are the test scores in fall and spring. Other modifications might extend automaticity beyond basic facts, including fractions, decimals, and percents; perfect squares and square roots; perfect cubes and cube roots; Pythagorean triples; and trigonometric functions, for example, to enhance student fluency in more advanced mathematics concepts. Additional studies could perform fine grain analysis of student success with categories of questions on the MAP assessment to determine the effects of automaticity on particular mathematics concepts.

Introduction of a control group into a similar study would benefit the existing literature base on the effects of automaticity with basic mathematics facts on student learning. With a control group, the study would then be a quasi-experimental design and could show a cause and effect relationship between automaticity and mathematics learning. Outcomes from such a study could lay to rest many doubts by the mathematics education community concerning the importance of automaticity for student learning of meaningful mathematics.

During the development of this study, care was taken to ensure that the rationale, implementation, analyses, and outcomes were accessible to classroom teachers. Appreciating the connections of automaticity to student learning of mathematics, supporting the application of the automaticity diagnostic and treatment, and understanding the data analyses and outcomes are important for teacher involvement in improving the teaching and learning of mathematics in the classroom. While informing the existing body of literature for the mathematics education community through research is necessary, it is not sufficient if it is not reachable for classroom teachers.

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APPENDIX A

Memorandum of Understanding

High School Readiness Transition Initiative
(Name of University) 2011-2012

In order for a district/school to participate in thin initiative, district administrators, school administrators and counselors must all agree to support the initiative. All teachers in all schools with students participating in the initiative must participate as follows:

Phase 1:

- A teacher representative per grade level from each school participating in the High School Readiness Initiative will attend an informational meeting to discuss the initiative.
- School districts will establish a district main contact to communicate and report data to (the university).
- Participating teachers will attend professional development sessions to discuss automaticity diagnosis and analysis.
- Participating teachers will complete the automaticity diagnostic with every student at their respective schools and then aggregate, analyze, and report the data through the district main contact using the given format.

Phase 2:

- Participating teachers will prepare and discuss student automaticity diagnostic data including teacher analysis.
- (The university) will provide professional development regarding automaticity and discuss implementation of strategies for addressing gaps in automaticity in fall, winter, and spring.
- Schools will share relevant data with (the university) through the district main contact using the given format attending to student anonymity.
- (The university) will collect and analyze data attending to diligent confidentiality practices.
- Disaggregation and distribution of data will be permitted within accepted rules of protocol.

Name of School District: ____

Name of District Main Contact: ____

Signature of participants:

APPENDIX B

Sample Questions from Automaticity Diagnostic Grades 4 to Adult

9 +6	9 +12	5 +4	6 +5	8 +3	7 +4	9 +2
7 +7	7 +8	12 +4	9 +11	9 +6	7 +8	4 +7
3 +5	12 + 9	7 +15	19 + 8	92 +7	76 +12	54 +45
17 − 8	11 − 8	16 −9	19 −11	9 −6	15 − 8	49 − 37
33 − 5	12 − 9	47 −15	104 − 43	19 − 8	92 −7	76 −12
5 ×11	5 ×1	5 ×5	5 ×4	5 ×12	5 ×8	5 ×2
5 ×7	5 ×9	12 × 5	5 ×10	5 ×6	5 ×3	5 ×0

APPENDIX C

Automaticity Diagnostic Test Instructions

Please read the following instructions to your students:

This assessment is a mental math test that measures your automaticity skills that are basic math skills. This test is not for a grade but is very important.

Directions:

- You may not use a calculator.
- You may not ask your teacher for help.
- When you are finished you must record your completion time in the upper left corner of the test and then turn your paper face down.
- No talking is allowed until everyone is finished. You may read a book until time is up.
- Remember there is a time limit posted on the board.
- Time is an important factor in the overall assessment of this test.
- Quickness/speed is most important.
- If you do not know an answer, you can guess but do not stop to count on your fingers, etc.

Maximum Times for Automaticity Diagnostic Test

Grade	Maximum Time (in minutes)
2	10
3	20
4	15
5	15
6	15
7	15
8	15
9-12	12

APPENDIX D

Sample Automaticity Intervention Treatment for (×6)

6 × 1 = 6	**6 × 2 = 12**	**6 × 3 = 18**	**6 × 4 = 24**	**6 × 5 = 30**	**6 × 6 = 36**
6 × 7 = 42	**6 × 8 = 48**	**6 × 9 = 54**	**6 × 10 = 60**	**6 × 11 = 66**	**6 × 12 = 72**

6	**6**	**6**	**8**	**6**	**6**	**6**
×8	**×1**	**×7**	**×6**	**×4**	**×4**	**×2**
6	**6**	**6**	**6**	**6**	**6**	**6**
×5	**×9**	**×6**	**×10**	**×12**	**×11**	**×3**
6	**6**	**12**	**6**	**6**	**8**	**6**
×4	**×8**	**×6**	**×12**	**×5**	**×6**	**×7**
6	**6**	**6**	**6**	**6**	**6**	**11**
×5	**×3**	**×7**	**×4**	**×10**	**×11**	**×6**
6	**6**	**8**	**6**	**6**	**6**	**9**
×1	**×11**	**×6**	**×4**	**×8**	**×5**	**×6**
6	**6**	**6**	**6**	**6**	**6**	**6**
×6	**×12**	**×10**	**×11**	**×7**	**×1**	**×7**
6	**6**	**6**	**6**	**6**	**6**	**6**
×3	**×8**	**×4**	**×5**	**×12**	**×11**	**×5**
6	**6**	**6**	**6**	**6**	**6**	**6**
×7	**×8**	**×1**	**×6**	**×3**	**×11**	**×10**

APPENDIX E

Automaticity Treatment Protocol Guidelines Middle/High School (Grades 6 to 12)

The automaticity treatment for students is completed at the beginning of class during the “warm up” or “bell ringer” time. The total remediation exercise can take 6-7 minutes. Students are allowed 30 seconds to procure the appropriate remediation sheet.

To begin, the teacher projects the timer or stopwatch set at 0 minutes. [Runs up to time limit] Link to online stopwatch: http://www.online-stopwatch.com/full-screen-stopwatch/

- Once started, each student works on a worksheet for up to 5 minutes. Once completed, the student can grade the sheet (answers are at the top) using a colored pen or pencil. The student then records (in an individual folder) the elapsed time and number of problems missed or not completed.
- Students are allowed to progress onto the next worksheet when the amount of time required completing the sheet drops to 2 minutes or below and there are 2 or less problems incorrect. [Teacher discretion advised]
- Students can work through the addition worksheets completely before moving to the multiplication, subtraction and division worksheets.
- Order of completion: Addition; Multiplication; Subtraction; Division
- Once mastered, students should strive toward a maximum time of less than 1 minute per sheet. (Ultimate goal is automaticity)

These exercises are supplementary to regular mathematics instruction.
Teachers are cautioned to avoid “drill and kill” when implementing these regimens.
Daily drill and repetition are encouraged, but not to the exclusion of regular arithmetic and mathematics instruction. A little bit each day or at regular intervals show the best promise for increasing fluency.
Note: The 105 question diagnostic assessment is to be administered only in August, January and May.

Getting started:
Many teachers start all students on the same sheet for the first exercise. Students that complete each sheet in the targeted time with 2 or less errors progress to the next sheet at the next exercise. Within 2 weeks, students will be working at different levels and on different remediation worksheets.
[Differentiated Learning]

Teachers are asked to remind students that they are **not** competing with each other, but with themselves.

Recommendation:
- *Teachers adjust targeted times as appropriate.*
- *Teachers supplement the fluency practice with oral activities and flash cards.*
- *Flash cards should only have 1 problem per card with the answer on the back.*
- *Students should be shown the answer after each query right or wrong (immediate feedback).*

Please keep track of students that are not progressing through the regimen.
This may be an indication of other, more serious mathematical or learning difficulties or deficiencies.