THE SENSITIVITY OF HIERARCHICAL LINEAR MODELS TO OUTLIERS

by

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(Under the Direction of Zhenqiu Lu)

ABSTRACT

The hierarchical linear model (HLM) has become popular in behavioral research, and has been widely used in various educational studies in recent years. Violations of model assumptions can have a non-ignorable impact on the model. One issue in this regard is the sensitivity of HLM to outliers. The purpose of this study is to evaluate the sensitivity of two-level HLM to the outliers by exploring the influence of outliers on parameter estimates of HLM under normality assumptions at both levels. A simulation study is performed to examine the biases of parameter estimates with different numbers and types of outliers (3 SD and 5 SD) given different sample sizes. Results indicated that the biases of parameter estimates increased with the growing of standard deviation and the number of outliers. The estimates have very small biases with a few outliers. A robust method Huber sandwich estimator corrects the standard errors efficiently when there is a large proportion of outliers.

INDEX WORDS: Hierarchical linear model, outliers, sensitivity

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CHAPTER 1

INTRODUCTION

Investigating whether the estimation of a model can be easily influenced by extreme observations or not is of great importance. Based on previous studies, statistical models with an assumption of normality can be highly sensitive to outlying cases (Andrew, 1974; Hogg, 1979; Mosteller, 1977). Hierarchical linear models (HLM) can estimate variance components with unbalanced or nested data and divide the variability based on different levels. Residual terms at all levels of the HLM are assumed to be normally distributed. The outliers for a two-level HLM can be the outlying level-1 units that are far from the normal expectation given the regression equation for each level-2 unit and also can be the outlying level-2 units with an atypical regression coefficient. Rachman-Moore and Wolfe (1984) indicated that even one outlier at level-1 can “sour” these estimates of the level-2 unit aggregates and other level-1 unit contributions, impacting the estimation of fixed effects. Several studies have been conducted indicating that point estimates and intervals for fixed effects may be sensitive to outliers at all levels (Seltzer, 1993; Seltzer, Novak, Choi & Lim, 2002). Seltzer and Choi (2003) also asserted that results were not excessively influenced by one or two extreme outlying values at level 1 by reanalyzing the real data with level-1 outliers and finding little change in the fixed effects. Different estimation methods, computational algorithms, assumptions, sample sizes, and severity of outliers may impact the results of parameter estimation. The leading sensitivity analyses for the HLM adopted real data analyses and then compared results across different assumptions (Seltzer, 1993; Seltzer & Choi, 2003; Seltzer, Novak, Choi, & Lim, 2002) or employed robust methods to fit the real data (Rachman-Moore & Wolfe, 1984). Few studies detected the influence of outliers in details for a specific subtype of the HLM by using a simulation study. A well-performed simulation study can explore the bias of parameter estimates in various conditions from the true parameters. Practical instructions can be provided for educational researchers in using the HLM when outliers exist.

Hierarchical linear models (HLM) in educational psychology have earned a good reputation. The applications of the HLM have been explored in various studies in recent years. Pahlke (2013) applied the HLM to test single-sex schooling and mathematics and science achievements, and he concluded that students' performances were not statistically significantly different as a function of whether the students attended a single-sex school or not. This claim did not support the single-sex classrooms or schools perspective proposed by other researchers. (Gurian, Henley, & Trueman, 2001; James, 2009; Sax, 2005). Skibbe (2012) employed HLM to investigate peer effects on classmates' self-regulation skills and children's early literacy growth. The classroom mean of self-regulation represented the peer effects. This study explored the relationship between hierarchical levels to determine if classmates' self-regulation (classroom-mean self-regulation after controlling for the specific individual's self-regulation) can predict students' literacy achievement (individual level) after controlling the individual self-regulation. McCoach (2006) applied a piecewise growth model to evaluate the growth of children's reading abilities during the first 2-year of schooling. A 3-level (time – student – school) HLM can locate the factors (i.e., school-level variables: "percentage of minority students" and "percentage of free-lunch students") that investigate students' performances (student level) across time (time level). The HLM has advantages over ordinary least squares (OLS) regression with multilevel data and accurate estimation of the relative strength of the variables' relationship at level 2 or higher level (Pollack, 1998).

Over the past decades, the failure of many quantitative studies that cannot accommodate and analyze hierarchical or multilevel data has been a prominent methodological criticism in the educational research field (Burstein, 1980; Cronbach, 1976; Haney, 1980; Rogosa, 1978). Compared to the general linear models, the HLM is favored by a number of researchers (Field, 2009; Morris, 1995; Mundfrom & Schults, 2002; Raudenbush, 1988; Raudenbush & Bryk, 2002). Morris (1995) claimed that "hierarchical models are extremely promising tools for data analysis" (p. 85) almost twenty years ago. One of the distributional assumptions of general linear models requires the error terms to be independent and identically distributed (Frank, 1998). For example, subjects need to be randomly assigned to the groups and treatments need to be randomly assigned to the subjects. In reality, educational research studies usually choose several classes within a couple of schools out of interest or convenience. Students are actually nested within their classes, and classes nested within schools, which creates 3-level hierarchical data. Without using multilevel models attending to the hierarchical data, aggregation bias (Cronbach & Webb, 1975; Robinson, 1950) would unavoidably appear. Aggregation bias assumes that what we found about the group can also be true for each individual. The problem of misestimated precision can be caused by ignoring the hierarchy in the data as well (Aitkin, Anderson, & Hinde, 1981; Knapp, 1977; Walsh, 1947). The OLS estimation fails to include covariance components in the standard error estimates when applied to the nested data (Bijleveld et al., 1998). Field (2009) summarized three crucial benefits of HLM that “cast aside the assumption of homogeneity of regression slopes”, “say ‘bye-bye’ to the assumption of independence”, and “laugh in the face of missing data” (p. 729).

The major applications of HLM focus on the estimation of fixed effects within each level and the interrelations among them (McCoach, 2006; Pahlke, 2013; Skibbe, 2012). Different estimation methods have been developed for the HLM. In recent studies, the full maximum likelihood estimation (FMLE) method, restricted maximum likelihood estimation (RMLE) method, and Bayesian method are three popular methods for HLM. The maximum likelihood estimation (MLE) procedure is a breakthrough for HLM since the covariance components estimation is easily accessible even for large datasets (Raudenbush, 1988). FMLE estimates both regression coefficients including the fixed effects (intercepts and slopes) and the random effects (variance components) estimates for the HLM; the RMLE is mainly used for estimating the covariance components. The estimation theory leads to the statistical inferences based on the sample data and also the computational algorithm (Raudenbush & Bryk, 2002). The prevalent choices of computational algorithms for MLE include the Newton-Raphson algorithm, expectation-maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977; Dempster, Rubin, & Tsutakawa, 1981), and the Fisher scoring algorithm (Longford, 1987). These algorithms have been implemented in various software programs. The PROC MIXED procedure in SAS software obtaining the estimates of both the fixed and random effects uses a ridged-stabilized Newton-Raphson algorithm for FMLE or REML. The SPSS MIXED method employs a combination of Fisher's scoring and Newton-Raphson algorithms to obtain the maximum likelihood estimates. The HLM software implements the EM algorithm and Fisher's scoring. The Newton-Raphson algorithm is the default method in the xtmixed command in Stata. Additionally, the R packages lme() and gls() use a combination of EM and the Newton-Raphson algorithm (West, Welch, & Galecki, 2007). Snijders and Bosker (1999) claimed that different computational algorithms would produce the same estimates, but results may vary regarding the convergence and computational speed. Lindstorm and Bates (1988) stated that "a well-implemented Newton-Raphson algorithm is preferable to the EM algorithm or EM algorithm with Aitken's acceleration". The FMLE method with the Newton-Raphson algorithm, as a popular method of estimating the parameters of the HLM, is our interest in this study.

Nonnormality in one of the factors can affect the standard errors for the fixed-effect estimates, and in turn affects the test statistics in HLM. Applying a robust method to correct the standard errors is a practical recommendation. An asymptotically consistent robust method called the "Huber sandwich estimator" is implemented in the SAS software and the Mplus software, which is popular for correcting standard errors. In the SAS software, the Huber sandwich estimator can be used in the PROC MIXED and PROC GLIMMIX procedures to compute the estimated variance-covariance matrix of the fixed-effects parameters. In the Mplus software, the "MLR" estimator provides the parameter estimates and robust standard errors which are sandwich or Huber-White standard errors. Freedman (2006) indicated that the "Huber sandwich estimator" can be useful when the model is misspecified. He addressed that when the model is nearly correct, there is no evident benefits from the robustification of the Huber sandwich estimator for correcting the usual standard errors. Additionally, the cost of increasing robustness requires more computational complexities. If the Huber sandwich estimator does not perform significantly better than the FMLE method, the cost for the robustness of the robust method is compromised in dealing with outliers for the HLM.

The purposes of this study are to investigate the biases in the parameters estimates of the HLM due to outliers, and to check the robustness of the Huber sandwich estimator. Normality assumptions are assumed at both levels. The FMLE method and Newton-Raphson computational algorithms are employed for the estimation procedure. The random-coefficients regression model of the HLM is examined. A simulation study is performed using the SAS software (Version 9.3) (1) to explore the biases of parameter estimates with different sample sizes and different numbers of outliers (3 SD and 5 SD), and furthermore, (2) to examine the sensitivity test for the Huber sandwich estimator in order to evaluate the performance of this robust method in correcting standard errors and test statistics of fixed-effect estimates in the presence of outliers.

CHAPTER 2

THEORETICAL BACKGROUND

Hierarchical linear model

In educational research, hierarchical data are common. Suppose students' mathematical abilities over consecutive years, from 6^{th} grade to 8^{th} grade, within three classes of several schools are measured; this data has four levels as mathematic achievements across years are nested within each student, students are nested within classes, and classes are nested within schools. Teaching effects or school environments may create dependency among the data, given that part of the students are in the same class and part of the classes are in the same school. By using the OLS estimation method to obtain the parameters of a linear regression model, the hierarchical data violates one key assumption of OLS estimation that errors are independent from each other. An HLM is a generalization of traditional linear models. For a 2-level HLM, the regression functions are computed for each unit at the second level given the first level unit characteristics and are also regressed across the units at the second level given the second level unit characteristics. An HLM not only incorporates the hierarchical structure of data, but also partitions the covariance components and test cross-level effects. Apart from that, an HLM can be adjusted for non-independence of error terms and also accommodated to an unbalanced design and missing data.

The HLM share the linearity, normality, and homoscedasticity assumptions with an OLS regression. However, for an OLS regression, the function should be linear, the total residuals should be normally distributed, and residual variances for all should be constant. For HLM, linearity should be met at each level, residuals at each level should be in the univariate or multivariate normal distributions, and the error term at level 1 should be constant. The independence among all the observations that hold by OLS is not required for HLM. In turn, an HLM has a unique assumption regarding independence that residuals at different levels need to be uncorrelated and the observations at the highest level should be independent of each other (Raudenbush & Bryk, 2002).

Raudenbush and Bryk (2002) introduced a general model and six simpler subtypes of a 2-level HLM, among which the simplest subtype for random intercepts and slopes is a random-coefficients regression model. We evaluate this model in this study. The general form of the random coefficients regression model with one random intercept and one random slope is as follows:

Hierarchical form:

Level-1 $Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij}$ with $r_{ij} \sim N\left(0, \sigma^2\right)$

Level-2 $\beta_{0j} = \gamma_{00} + u_{0j}$ with $\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right)$

$\beta_{1j} = \gamma_{10} + u_{1j}$

Model in a combined form:

$$Y_{ij} = \left(\gamma_{00} + \gamma_{10} X_{ij}\right) + \left(u_{0j} + u_{1j} X_{ij} + r_{ij}\right), \quad (1)$$

where i and j represent the level-1 and -2 units, respectively. The ranges for each is based on the sample size in order to keep the same ratio of numbers of units at two levels. β_{0j} and β_{1j} are the random intercept and the random slope of the regression equation correspondingly. r_{ij} represents the error term at level-1 which follows a normal distribution. γ_{00} and γ_{10} represent the fixed-effect estimates of the random intercept β_{0j} and the random slope β_{1j} respectively. In each second level unit, γ_{00} represents the mean of intercept β_{0j} and γ_{10} represents the mean of slope β_{1j} . The random-effect estimates of the regression coefficients - u_{0j} and u_{1j} - form a bi-normal distribution. The variance of u_{0j} denotes as τ_{00} , the variance of u_{1j} denotes as τ_{11} , the covariance of u_{0j} and u_{1j} denotes as τ_{01} / τ_{10}, and the variance of r_{ij} denotes as σ^2. As τ_{01} is equal to τ_{10} , we will only use τ_{01} to represent the covariance of u_{0j} and u_{1j} in the following descriptions. For a 2-level regression-coefficients regression model of an HLM, it has two fixed effects estimates, three random effects estimates, and one error term. The mean structure part of this regression equation is $\left(\gamma_{00} + \gamma_{10} X_{ij}\right)$, and the residual part is $\left(u_{0j} + u_{1j} X_{ij} + r_{ij}\right)$.

Maximum likelihood estimation

The MLE produces consistent and efficient estimates. The MLE is also scaled free and scale invariant. The value of fit functions is the same for the correlation matrix, covariance matrix, or any other change of scale. The scale invariant refers that the parameter estimates are not affected by the transformation of variables. Furthermore, the MLE is the default method of estimation for HLM in most statistical programs (i.e., SAS, HLM, Mplus, and LISREL).

MLE produces parameter estimates of a statistical model that maximize a likelihood function. The FMLE method is employed in the estimation procedure of this study. To maximize the likelihood of the data given the hierarchical models, the probability function is:

$$L\left(\omega|Y\right) = f\left(Y|\omega\right) = \int f\left(Y|u,\omega\right) p\left(u|\omega\right) du \tag{2}$$

where Y is a vector of all level-1 outcomes, u is the vector of all random effects at level 2 or higher levels, ω is a vector with all the parameters needed to be estimated including all regression coefficients and covariance components, $f\left(Y|u,\omega\right)$ is the probability distribution of the outcomes at level 1 given the random effects and parameters, and $p\left(u|\omega\right)$ is the probability distribution of random effects at higher levels given parameters (Raudenbush & Bryk, 2002).

The fixed-effect estimates in random-coefficients regression model based on FMLE are as follows:

$$\hat{\gamma} = \left(\sum_{j}^{m} X_j^{'} V_j^{-1} X_j \right)^{-1} \sum_{j}^{m} X_j^{'} V_j^{-1} y_j \,, \tag{3}$$

and the variance of $\hat{\gamma}$ is estimated by

$$\mathrm{var}(\hat{\gamma}) = (X^{'} \widehat{V}^{-1} X)^{-1} \,, \tag{4}$$

where m represents the number of units at level 2. X is the design matrix stacked by level-2 units, $\hat{V}$ is the variance-covariance components of the model stacked by level-2 units including the estimates of τ_{00}, τ_{01}, τ_{11}, and σ^2 (Sullivan, Dukes, & Losina, 1999). The Newton-Raphson algorithm is one of the numerical analysis methods based on linear approximation finding the roots of an equation through the iterative process (Ypma, 1995). SAS Proc Mixed uses a ridged-stabilized Newton-Raphson algorithm in the log-likelihood maximization (Sullivan, Dukes, & Losina, 1999).

And the log-likelihood function for the variance-covariance component is as below (Littell, Milliken, Stroup, & Wolfinger, 1996; Searle, Casella, & McCulloch, & Schabenberger, O., 2006):

$$L(G,R) = -\frac{1}{2}\log|V| - \frac{N}{2}\log \mathrm{E}'V^{-1}\mathrm{E} - \frac{N}{2}\left[1+\log\frac{2\pi}{N}\right], \tag{6}$$

where E represents the residuals of the model, which has the equation of

$$\mathrm{E} = Y - X\left(X'V^{-1}X\right)' X'V^{-1}Y. \tag{7}$$

The estimates of random effects are generated as follows:

$$\hat{\upsilon} = \hat{G}X'\hat{V}^{-1}\left(Y - X\hat{\gamma}\right), \tag{8}$$

where $\hat{G}$ is the variance-covariance component matrix, including τ_{00}, τ_{01}, and τ_{11} (Sullivan, Dukes, & Losina, 1999).

The Outliers and Huber Sandwich Estimator

However, the disadvantages of FMLE cannot be ignored. One concern about the FMLE is the assumption of multivariate normality. The nonnormality affects the significant tests through the influence of standard errors, even though it does not affect the parameter estimates (Bollen, 1989). Grubbs (1969) stated that an outlier is one that seems to deviate markedly from other data of the sample, and it may be merely an extreme indication of the random variability inherent in the data and it may also be the result of gross deviation from the experiment process or an error in processing the numerical data. Barnett and Lewis (Barnett & Lewis, 1994) defined outliers to be the ones are inconsistent with the rest of the data. In educational settings, the extreme values can be the coding errors or the data entry errors. If it is certain that an outliers is the result from these errors, the value should be corrected or deleted. However, the outlying observations may also arise from the random variability of the data which are in the target population. Researchers would not like to simply delete them. However, the presence of outliers has serious effects on the modeling, monitoring, and diagnosis of data (Zou, Tseng, & Wang, 2014). Before applying the model to fit the data with outliers, it is necessary to do a sensitivity test of the model to the outliers.

As the data become nonnormal, a robust method providing robust standard errors can be applied. Huber (1967) and White (1982) introduced a robust covariance matrix estimator, which is commonly used in the generalized estimating equations (GEE, Diggle, Liang, & Zeger, 1994; Huber, 1967; Liang & Zeger, 1986; White, 1980). The sandwich estimator is also known as robust covariance matrix estimator (Kauermann & Carroll, 1999). The Huber sandwich estimator does not change parameter estimates. It provides the robust standard errors of fixed-effect estimates which in turn corrects the test of significance (King & Roberts, 2012). Huber sandwich estimator is an implemented robust method for correcting standard errors in the SAS software and Mplus software. By using the generalized inverse, it provides a consistent covariance estimation in the presence of heteroskedasticity (White, 1980).

Different from the usual covariance matrix estimator $\mathrm{var}(\hat{\gamma}) = (X'\widehat{V}^{-1}X)^{-1}$ (4), the Huber sandwich estimator based on the quasi-likelihood GEE computes the covariance matrix for the fixed-effect estimates as below:

$$(X'\widehat{V}^{-1}X)^{-}\left(\sum_{j=1}^{m} X_j'\widehat{V}_j^{-1}\hat{\varepsilon}_j\hat{\varepsilon}_j'\widehat{V}_j^{-1}X_j\right)(X'\widehat{V}^{-1}X)^{-}, \tag{9}$$

where j refers to the level-2 units and m is the number of units at level 2. $\hat{\varepsilon}_j = y_j - X_j\hat{\gamma}$, is the estimated residual part of the model. X_j And $\widehat{V}_j$ are the design matrix and the covariance matrix for unit j, respectively. The general inverse in the equation is appropriate as the matrix is singular (Liang & Zeger, 1986).

CHAPTER 3

A SIMULATION STUDY

Simulation Study Design

A simulation study was performed to investigate the sensitivity of HLM to outliers. Data sets with three different sample sizes were simulated based on the random-coefficients regression model. In order to maintain the same ratio of the number of level-1 units to the number of level-2 units, the three sample sizes set as 200 (20 level-1 units and 10 level-2 units), 1250 (50 level-1 units and 25 level-2 units), and 5000 (100 level-1 units and 50 level-2 units).

Step 1, normally distributed data were generated based on the random-coefficients regression model, $Y_{ij} = \left(\gamma_{00} + \gamma_{10} X_{ij}\right) + \left(u_{0j} + u_{1j} X_{ij} + r_{ij}\right)$. The true values of parameters set as follows: γ_{00} = 5, γ_{01} = 1, τ_{00} = 1, τ_{11} = 4, τ_{01} = 1, and σ^2 = 4. The correlation between τ_{00} and τ_{11} was .50. Step 2, two types of outliers (3 SD and 5 SD) were defined based on the sample standard deviations from the sample means. The mathematical equation for creating outliers was $\bar{Y}_{outlier} = \bar{Y} + n * \hat{\sigma}$ where n =3 or 5. The sample mean $\bar{Y}$ and the sample standard deviation $\hat{\sigma}$ were estimated by using the simulated datasets without outliers. The 3 SD outliers have three standard deviations from the sample mean. Similarly, the 5 SD outliers have five standard deviations from the sample mean. All the outliers were created in the positive direction, in order to avoid the trade-off effects of outliers. Several specific numbers of outliers with 3 SD and 5 SD are created for the dependent variable and replaced the same quantity of the simulated data separately.

Therefore, the data sets with different sample sizes have different numbers of outliers. For the datasets with a sample size of 200, 1, 2, 5, 8, 10, and 20 outliers have been created. The percentage of the outliers in the datasets of sample size 200 are .50%, 1.00%, 2.50%, 4.00%, 5.00%, 10.00%. For the data sets with a sample size of 1250, 2, 5, 10, 25, 50, 75, and 125 outliers have been created. The corresponding percentages are .16%, .40%, .80%, 2.00%, 4.00%, 6.00%, and 10.00%. For the data sets with a sample size of 5000, 2, 5, 10, 20, 30, 40, 50, 100, 150, 250, and 500 outliers have been created. The percentages of those outliers are .04%, .10%, .20%, .40%, .60%, .80%, 1.00%, 2.00%, 3.00%, 5.00%, and 10.00%. With the increases of sample sizes, more options are available for the number of outliers. For each condition, 100 replications were conducted to carry out the simulation study. Step 3, the FMLE method with Newton-Raphson algorithm was adopted to estimate parameters with the correctly specified model that the random-coefficients regression model. Step 4, the fixed-effect and random-effect estimates were compared with the true values of the parameters. The indices for the comparison are absolute bias and relative bias. The bias of a statistic $\widehat{\theta}$ is defined as $B(\widehat{\theta}) = E(\widehat{\theta}) - \hat{\theta}$, which is the distance of the estimates' expectation and the estimate. The absolute bias is the absolute value of the difference between estimates and true parameters. The relative bias represents the ratio of the absolute bias to the true parameters, which is the percentage of the relative difference between the expectance of the estimate and the single estimate. Both the absolute bias and relative bias indicate the sensitivity of the model with a specific estimation method to the outliers.

The Q-Q plots of the scaled residuals for the dependent variable were used to display the distribution of residuals. When data are correlated, the vector of residuals instead of each separate residual can be scaled, accounting for the covariances among the observations. The Cholesky residuals were used in the this study. As described in the *SAS/STAT(R) 9.22 User's Guide*, if $\mathrm{var}(Y) = C'C$, then $C'^{-1}Y$ is a vector of uncorrelated variables with unit variance which has uniform dispersion. Those residuals are expressed as $\hat{\varepsilon}_C = C'^{-1}\left(Y - X\hat{\beta}\right)$.

Step 5, the robust method Huber sandwich estimator was also examined for the performance of correcting the standard errors and test statistics on the fixed-effect estimates. The estimated covariance matrix of fixed-effect estimates by the Huber sandwich estimator was compared with those obtained by the FMLE method.

Simulation Study Results

In order to recover the parameters, the random-coefficients regression model was employed to fit the simulated data without outliers. The absolute biases and the relative biases of the estimates were acceptable with all absolute biases less than .20 and relative biases range from .01% to 8.06% (Table 1). The Pearson correlation between true values and the estimates was strongly positive, r = .999, p <.01 which provides evidence the parameters were successfully recovered. The Q-Q plots and histograms for the scaled residual of the dependent variable indicated that the simulated data were normally distributed (Figure 1). With sample sizes increasing, the distribution of the data became more and more normal.

The Q-Q plots of the scaled residuals for the dependent variable showed how the outliers affect the normal distribution of the data. In the presence of a few outliers, the distributions of the scaled residuals appeared normal. With larger numbers of outliers, however, the scaled residuals had more variability around the line (Figure 2-5). For the same numbers of outliers, the model estimations with 5 SD outliers tended to produce more nonnormal residuals than those with 3 SD outliers. In the presence of extreme outliers, the scaled residual did not appear to be normally distributed.

The bias of sample means increased with more outliers. For a sample size of 200, sample mean for data with 10% 3 SD outliers was 1.09 points higher than the sample mean of the data without outliers. The sample mean of the data with 10% 3 SD outliers was 1.07 higher than the normal distributed data for a sample size of 1250, and 1.06 for a sample size of 5000. The sample mean of the data with 10% 5 SD outliers was 1.80 higher than the data without outliers for a sample size of 200, 1.74 for a sample size of 1250, and 1.76 for a sample size of 5000. These results indicated that the sample means were biased when the data sets had outliers. The sample means of the data with 5 SD outliers were higher than the data with 3 SD outliers given the same numbers of outliers and same sample sizes (Figure 6–8); however, the independent t-test indicated no significant mean difference between different types of outliers, $t(46) = -1.45, p = .16$. There is little variation in the sample means across different sample sizes. In addition, the one-way ANOVA test showed that there were no significant differences in sample means for different sample sizes, $F(2,48) = 1.02, p = .37$.

The intercept estimate γ_{00} appeared to be affected evidently by outliers. The absolute biases of the estimates increased as the number of outliers increased. For a sample size of 200, the absolute bias displayed increases (Figure 9). The range of relative biases of γ_{00} for sample size 200 was from .53% to 35.63% (Table 2). From 5% outliers to 10% outliers, the relative bias of γ_{00} estimate increased by 11.23% with 3 SD outliers and by 18.86% with 5 SD outliers. The largest absolute bias of the estimate γ_{00} was 1.78 and its relative bias was 35.63% when there were 10% 5SD outliers. For a sample size of 1250, the absolute bias increased slowly when the numbers of outliers were less than .80%; when the numbers of outliers were larger than .80%, the absolute biases increased rapidly (Figure 10). The range of relative biases of γ_{00} for a sample size of 1250 was from .81% to 13.75% (Table 3). From 6% outliers to the 10% outliers of a sample size of 200, the relative bias of γ_{00} estimate increased by 7.96% with 3 SD outliers and 13.93% with 5 SD outliers. For a sample size of 5000, the absolute bias increased slowly when there were less than 1.00% outliers, but with more than 1.00% outliers, the absolute biases grew fast (Figure 11). The range of relative biases of γ_{00} given a sample size of 5000 was from .15% to 35.11%. From 5.00% outliers to the 10.00% outliers given a sample size of 5000, the relative bias in the estimate of γ_{00} increased 10.45% with 3 SD outliers and 17.97% with 5 SD outliers (Table 4). The one-way ANOVA test for the differences of absolute/relative biases on γ_{00} across sample sizes was not significant, $F(2,45)=1.19, p=.31$. The independent sample t -test for the differences of absolute/relative biases on γ_{00} between different types of outliers was not significant either, $t(46)=-1.4, p=.17$. These results indicate that the relative bias of the intercept estimate γ_{00} did exist but not significantly varying across sample sizes and types of outliers.

The slope estimate of γ_{01} with the range of absolute biases less than .15 was much less susceptible to outliers, compared with the estimate of γ_{00}. The range of relative biases of γ_{01} was from 1.53% to 12.55% for a sample size of 200. The largest absolute bias of γ_{01} estimate was .14 and its relative bias was 13.75% with 10% of 5 SD outliers given a sample size of 200 (Table 2). The variation of the absolute biases of γ_{01} for a sample size of 200 did not have a specific variation pattern (Figure 12). For sample sizes of 1250 and 5000, the absolute biases with more than 6.00% outliers tended to be high (Figure 13-14). The other cases still appeared to have random variation with small absolute biases. The range of relative biases of γ_{01} was from .85% to 13.75% given a sample size of 1250; and the range of relative biases of γ_{01} was from .84% to 12.56% given a sample size of 5000. For sample sizes of 1250 and 5000, the largest absolute biases were .12 and .13 and their relative biases were 11.90% and 12.65% respectively, given 10.00% of 3 SD outliers (Table 4-5). The absolute biases possessed different variation patterns with different sample sizes. The one-way ANOVA test for the differences of absolute/relative biases on γ_{01} across sample sizes was significant, $F(2,45) = 4.34, p < .05$. Additionally, the independent sample t-test indicated that there was no significant difference in the absolute bias in the γ_{01} estimation between 3 SD and 5 SD outliers, $t(77.46) = -1.23, p = .22$. Therefore, the bigger sample sizes would be helpful for the estimation of γ_{01}.

The outliers have the most influential effects on the estimation of σ^2. For a sample size of 200, with the increase of the numbers of outliers, the absolute biases increased rapidly (Figure 15). The range of relative biases was from 16.97% to 733.97%, which are almost 10 to 20 times wider than other estimates. In the presence of 10.00% the outliers in the data, it had the largest absolute bias as 10.92 and relative bias as 272.89%

given 3 SD outliers; and it had the largest absolute bias being 29.36 and relative bias as 733.97% given 5 SD outliers (Table 6). For a sample size of 1250, the absolute biases increased slowly with the less than .80% outliers; but after this point, the absolute biases increased evidently (Figure 16). The range of relative biases was narrow down slightly given a sample size of 1250. The largest absolute biases and relative biases still appeared with 10.00% outliers. For a sample size of 5000, the absolute biases increased steadily with less than 1.00% outliers, but increased sharply with more than 1.00% (Figure 17). The range of relative biases was less wide than it of sample size 1250. The largest absolute biases and relative biases for 3 SD outliers were 10.22 and 255.52%, and those for 5 SD outliers were 28.21 and 702.61% respectively, which are obtained with 10.00% outliers in the data (Table 7). By comparing the results from different sample sizes, it is concluded that the relative biases can be relatively small when we have larger sample sizes. The one-way ANOVA test indicated that there was no significant difference between absolute/relative biases among three sample sizes, $F(2,45) = 1.17, p = .32$. The means of the absolute biases for all the estimates with 3 SD and 5 SD outliers were .98 and 2.36. The independent t-test demonstrates that the absolute biases for all the estimates with 5 SD outliers were significantly higher than those with 3 SD outliers, $t(119.17) = -2.17, p < .05$. The means of the relative biases for the estimates with 3 SD and 5 SD outliers were .30 and .66. The relative biases for the estimates with 5 SD outliers were significantly higher than those with 3 SD outliers as well, $t(118.72) = -2.29, p < .05$. However, there was no significant difference of absolute/relative biases on σ^2 across sample sizes, $F(2,45)=1.17, p=.32$. Thus, the distance of outliers has a significant effect on the σ^2 estimates.

The variance component of the intercept τ_{00} was affected in a similar pattern with γ_{00}. The absolute biases of τ_{00} increased with larger number of outliers (Figure 18-20). For a sample size of 200, the range of relative biases was from 10.47% to 97.99. The highest relative bias of τ_{00} with 3 SD outliers was 70.05% when there are 10.00% outliers. When 10.00% of 5 SD outliers existed in the data sets, the estimate of τ_{00} was .02 with a highest absolute bias .98 led to a 97.99% relative bias (Table 8). There was an interaction between numbers and types of outliers in the plot, however, it was not statistically significant. For a sample size of 1250, the evident increasing linear trend started from .80% outliers (Figure 19). The range of relative biases was from .78% to 76.97%. When there were 10.00% outliers, the estimates of τ_{00} has the largest absolute and the relative biases. The absolute biases of τ_{00} with 5 SD outliers appeared to be higher than those with 3 SD outliers. However, based on the independent sample t-test, there was no significant difference of absolute biases between 3 SD and 5 SD outliers, $t(12)=-.98, p=.35$. For a sample size of 5000, the absolute biases seemed to vary randomly within 1.00%, but increased from the 1.00% point (Figure 20). The range of relative biases of τ_{00}, given a sample size of 5000, was from .79% to 49.29% (Table 9). With less than 1.00% outliers, the relative biases were all less than 10.00%. The largest absolute bias of τ_{00} was .49 and its relative bias was 49.29% when there were 10.00% of 5 SD outliers. The absolute biases of τ_{00} with 5 SD were close to those with 3 SD, and there were no significant differences between them, $t(20) = -.47, p = .64$. With larger sample sizes, the minimum and maximum values of relative biases both decreased. Furthermore, there was an extremely significant difference of absolute/relative biases across three sample sizes, $F(2,45) = 10.56, p < .001$.

The effects of variance component of the slope τ_{11} varied with different sample sizes. For sample size 200, the absolute biases of τ_{11} was high with only .50% outliers; and then increased slowly before 5.00%; however, the 10.00% outliers can totally distorted the estimates of τ_{11} (Figure 21). The highest relative bias of τ_{11} with 3 SD outliers was 30.07% when the percent of outliers is 10.00% (Table 10). When there were 10.00% 5 SD outliers existing in the data sets, the τ_{11} estimate was 3.86 with a standard error being 2.73. It had the lowest absolute bias .14 leading to a 3.51% relative bias. The absolute biases of τ_{11} with 5 SD outliers were not significantly different from those with 3 SD outliers based on the independent sample t-test, $t(10) = .43, p = .68$. For a sample size of 1250, there were more variations of absolute biases with less than.80% outliers. The absolute biases increased from .80% outliers (Figure 22). The range of relative biases was from .71% to 25.86%. The largest relative bias of τ_{11} was 25.86% with 10.00% 3 SD outliers. When there were .40% of 5 SD outliers, the estimate τ_{11} had the lowest absolute bias .03 and relative bias .71% (Table 6). The absolute biases of τ_{11} with 5 SD outliers were close to those with 3 SD outliers. Based on the independent sample t-test, there was no significant difference of absolute biases between 3 SD and 5 SD outliers, $t(12) = .10, p = .93$. For a sample size of 5000, the absolute biases varied randomly within 1.00%, but increased evidently with more than 1.00% (Figure 23). The range of relative biases of τ_{11} given a sample size of 5000 was from .97% to 21.95% (Table 11). With less than 1.00% outliers, the relative biases were all less than 6.00%. The largest absolute bias of τ_{11} was .88 and its relative bias was 21.95% given 10.00% of 3 SD outliers. The absolute biases of τ_{11} with 5 SD were close to those with 3 SD as well; and there was no statistically significant difference between them, $t\left(20\right)=.56, p=.58$. Besides, there was an extremely significant difference of absolute biases for all the estimates of τ_{11} across three sample sizes, $F\left(2,45\right)=11.92, p<.001$.

The covariance of the intercept γ_{00} and slope γ_{10} was τ_{01}. For a sample size of 200, the distribution of absolute bias did not display a clear pattern (Figure 24). The absolute biases appeared to be similar across numbers and types of outliers. The range of relative biases was from 1.92% to 26.14%. For a sample size of 1250, the range of relative biases was from 0.18% to 27.05% (Table 12). The absolute biases had evidently changed when there were more than 4.00% outliers (Figure 25). For a sample size of 5000, the absolute biases increased fast with more than 1.00% outliers (Figure 26). The range of relative biases was from 0.35% to 23.56% (Table 13). For the estimates of τ_{01}, there was little difference between different types of outliers. The independent sample t- test for the differences of absolute biases on the τ_{01} was not significantly varied between 3 SD and 5 SD outliers, $t\left(46\right)=.57, p=.57$. In addition, the one-way ANOVA for testing the differences of absolute biases across sample sizes was significant, $F\left(2,45\right)=7.77, p<.001$.

Comparing FMLE and Huber-sandwich estimator for estimating standard errors of fixed-effect estimates, there was no significant difference between the FMLE standard errors and Huber standard errors, according to the independent sample t- tests as follows. For a sample size of 200, the standard errors of FMLE and of Huber-sandwich estimator were closer to each other with less than 4.00% of outliers (Figure 27). With more than 4.00% of outliers, the Huber-sandwich estimator provided smaller standard errors than the FMLE. With 10.00% of outliers, the difference between standard errors of γ_{00} with 5 SD outliers was larger than those with 3 SD outliers. A similar situation happened to γ_{01} as well, but the mean differences between two estimation methods were not significant given a sample size of 200, $t(46) = .34, p = .74$. For a sample size of 1250, the Huber standard errors of both γ_{00} and γ_{01} with 3 SD outliers were pretty close to the FMLE standard errors. With 10.00% of 5 SD outliers, the Huber standard error was slightly smaller than the FMLE standard error of both γ_{00} and γ_{01} estimates given a sample size of 1250. The mean difference between two estimation methods was also not significant, $t(54) = .01, p = .99$. For a sample size of 5000, it is difficult to distinguish the distinctions between Huber standard errors and FMLE standard errors for both γ_{00} and γ_{01} estimates with either type of outliers. The mean difference between two estimation methods was not significant as well, $t(86) = -.001, p = .999$, which indicated that Huber-sandwich estimator tended to provide robust standard errors when the outliers were extreme.

CHAPTER 4

DISCUSSION

The outliers had influences on the estimates of random-coefficients regression model under the FMLE. The estimate of the σ^2 has been influenced mostly. The outliers contributed the most to the estimate of σ^2. With 10.00% of 5 SD outliers for a sample size of 200, the relative biases were 733.97%. The effects on the γ_{00} was increasing with larger numbers of outliers. The biases of γ_{00} with 5 SD outliers were clearly higher than those with 3 SD outliers. The estimate of τ_{00}, which is the variance of intercept γ_{00}, was affected in a similar pattern to the estimate of γ_{00}. The estimate of γ_{00} is the mean of intercept estimate. The estimate of τ_{00} accounts for the variation of the intercept estimate. With σ^2, γ_{00}, and τ_{00} accounting for a large proportion of variation from outliers, the estimate of γ_{01} has been less influenced when encountered the outliers, as the slope term of the full model. Besides, the estimate γ_{01} had no specific influence pattern of outliers. The estimate τ_{11}, which is the variance of γ_{01}, had more random variations before the numbers of outliers reaching a limit. For example, with a sample size of 200, the biases of all the numbers, except the case with 10.00% of outliers, seemed to have much variation. With a sample size of 1250, the approximate linear growth pattern started from .80% outliers. With a sample size of 5000, the approximate linear trend started from 1.00%. The covariance of the intercept and the slope is τ_{01}, which also displayed much variation. For a sample size of 200, there was no specific pattern for all the estimates of τ_{01}. With a sample size of 1250, the biases of small percents of outliers (less than 4.00%) decreased but still randomly varied. With a sample size of 5000, the biases within less than 5.00% outliers were small and had more random variation.

For the estimate of τ_{11} with a sample size of 200, it had the lowest absolute bias, .14, leading to a 3.51% relative bias given 10.00% of 5 SD outliers, however, the estimation was not convincing. The standard errors which were estimated with 10.00% of 5SD outliers given a sample size of 200 was very high as well. It, in turn, distorted the estimate of τ_{11}. The large proportions of the 5 SD outliers on the positive side made the estimate of τ_{00} being high. The estimates of τ_{01} with sample size 200 had much variation as well. The 10.00% outliers given a sample size of 200 is a large proportion, which can no longer be treated as outliers. We would like to call it noise with more than 10.00% outliers. With 10.00% of 3 SD outliers given a sample size of 200, 1 out of 100 replications of the estimation procedures did not converge with 10^6 iterations. With 5.00% and 10.00% of 5 SD outliers given a sample size of 200, there are 5 out of 100 replications of the estimation procedures diverged with 10^6 iterations separately. Therefore, the stability of parameter estimation of HLM will be compromised with a large proportion of outliers.

No standard acceptable criterion can be established for the absolute biases and relative biases. Based on the tables, the researchers can look up the absolute and relative biases with the corresponding types and numbers of outliers given a specific sample size. Larger sample size is always good for the estimation. Except the biases of the estimates γ_{00} and σ^2, the rest biases of estimates $\gamma_{01}, \tau_{00}, \tau_{11}, \tau_{01}$ have been significantly different across sample sizes. For sample size 5000, the model estimation produced less biased parameter estimates with the same number of outliers given a sample size of 1250.

The Huber-sandwich robust estimator corrected the standard errors efficiently only when there are a large proportion of outliers in the data. Compared to the standard error estimates with 3 SD outliers, the Huber-sandwich estimator was more efficient in correcting the standard errors with 5 SD outliers. Therefore, the Huber-sandwich estimator did not work efficiently in the conditions of this study.

The future studies will investigate the correction for the parameter estimates and standard errors with robust methods compared with the FMLE. A *t* -distribution assumption will be employed in the parameter estimation, compared with the normal distribution assumption. The influence of level-2 outliers to the HLM will be further explored. More subtype models of HLM will be included for further examination.

CHAPTER 5

CONCLUSION

The simulation study investigates the biases of estimates from true values of the parameters due to the outliers with three sample sizes, in order to evaluate the sensitivity of two-level HLM to the outliers under normality assumptions at both levels with FMLE method and Newton-Raphson computational algorithm. The 2 types of outliers (3SD and 5SD) with specific numbers of outliers vary across different sample sizes have been created and replaced the same quantity of simulated data. By adding in different types and numbers of outliers, the model assumption of normality has been violated in various degrees. Violations of model assumptions have a non-ignorable impact on the model. The biases of parameter estimates for σ^2, γ_{00}, and τ_{00} increased with the larger number of outliers. For other estimates, the biases have different extents of random variation in various conditions. The 5 SD outliers have significantly more severe influence than 3 SD outliers on the estimates of σ^2. But for the rest estimates, there is no significant difference of biases between 3 SD outliers and 5 SD outliers. With a limited number of outliers, the estimates have very small biases, but the specific limit number varies across sample sizes. The robust method Huber sandwich estimator corrects the standard errors efficiently only with a large proportion of outliers.

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Appendix A – The SAS Codes

Sample size	SAS codes
200	(see code below)

```
%macro HLM(n);
%do rep= 1 %to 100;
data N&n.R&rep;
  gamma00=5;
  gamma10=1;
  sig1=1;
  sig2=2;
  rho=0.5;

   %do i=1 %to 10;
       macro=&i;
       r1 = rannor(-1);
       r2 = rannor(-1);
       U0j= sig1*r1;
         U1j= rho*sig2*r1+sqrt(1-rho**2)*sig2*r2;

     %do j=1 %to &n;
         micro=&j;
       xij=rannor(-1);
         Rij=rannor(-1)*2;
       yij = gamma00+ gamma10*xij + U0j + U1j*xij + Rij;
      output;
     %end;

 %end;
run;

    data N&n.R&rep;
        set N&n.R&rep;
          if macro<11 & micro<3 then
              yij=5+(3+abs(ranuni(1015)))*3;
/*Define outliers and replace part of simulated data */

      run;

proc means data=N&n.R&rep;
   output out=MN&n.R&rep;
run;

proc mixed data=N&n.R&rep method=ML covtest
MAXITER=1000000;
   class macro;
   model yij = xij / solution chisq;
   random intercept xij/ subject=macro solution type=un g;
   ods output SolutionF=FN&n.R&rep COVPARMS=RN&n.R&rep;

run;
proc mixed data=N&n.R&rep method=ML covtest empirical
MAXITER=1000000;
   class macro;
   model yij =  xij / solution chisq;
   random intercept xij/ subject=macro solution type=un g;
```

```
   ods output SolutionF=EFN&n.R&rep COVPARMS=ERN&n.R&rep;
run;
%end;
%mend HLM;
%HLM(n=20);

data Out5_fixed20;
set Fn20r1 Fn20r2 Fn20r3 Fn20r4 Fn20r5 Fn20r6 Fn20r7 Fn20r8
Fn20r9 Fn20r10 Fn20r11 Fn20r12 Fn20r13 Fn20r14 Fn20r15
Fn20r16 Fn20r17 Fn20r18 Fn20r19 Fn20r20 Fn20r21 Fn20r22
Fn20r23 Fn20r24 Fn20r25 Fn20r26 Fn20r27 Fn20r28 Fn20r29
Fn20r30 Fn20r31 Fn20r32 Fn20r33 Fn20r34 Fn20r35 Fn20r36
Fn20r37 Fn20r38 Fn20r39 Fn20r40 Fn20r41 Fn20r42 Fn20r43
Fn20r44 Fn20r45 Fn20r46 Fn20r47 Fn20r48 Fn20r49 Fn20r50
Fn20r51 Fn20r52 Fn20r53 Fn20r54 Fn20r55 Fn20r56 Fn20r57
Fn20r58 Fn20r59 Fn20r60 Fn20r61 Fn20r62 Fn20r63 Fn20r64
Fn20r65 Fn20r66 Fn20r67 Fn20r68 Fn20r69 Fn20r70 Fn20r71
Fn20r72 Fn20r73 Fn20r74 Fn20r75 Fn20r76 Fn20r77 Fn20r78
Fn20r79 Fn20r80 Fn20r81 Fn20r82 Fn20r83 Fn20r84 Fn20r85
Fn20r86 Fn20r87 Fn20r88 Fn20r89 Fn20r90 Fn20r91 Fn20r92
Fn20r93 Fn20r94 Fn20r95 Fn20r96 Fn20r97 Fn20r98 Fn20r99
Fn20r100;
run;

proc means data=Out5_fixed20 mean;
  var Estimate StdErr;
  class Effect;
run;

data RobustOut5_fixed20;
set EFn20r1 EFn20r2 EFn20r3 EFn20r4 EFn20r5 EFn20r6 EFn20r7
EFn20r8 EFn20r9 EFn20r10 EFn20r11 EFn20r12 EFn20r13
EFn20r14 EFn20r15 EFn20r16 EFn20r17 EFn20r18 EFn20r19
EFn20r20 EFn20r21 EFn20r22 EFn20r23 EFn20r24 EFn20r25
EFn20r26 EFn20r27 EFn20r28 EFn20r29 EFn20r30 EFn20r31
EFn20r32 EFn20r33 EFn20r34 EFn20r35 EFn20r36 EFn20r37
EFn20r38 EFn20r39 EFn20r40 EFn20r41 EFn20r42 EFn20r43
EFn20r44 EFn20r45 EFn20r46 EFn20r47 EFn20r48 EFn20r49
EFn20r50 EFn20r51 EFn20r52 EFn20r53 EFn20r54 EFn20r55
EFn20r56 EFn20r57 EFn20r58 EFn20r59 EFn20r60 EFn20r61
EFn20r62 EFn20r63 EFn20r64 EFn20r65 EFn20r66 EFn20r67
EFn20r68 EFn20r69 EFn20r70 EFn20r71 EFn20r72 EFn20r73
EFn20r74 EFn20r75 EFn20r76 EFn20r77 EFn20r78 EFn20r79
EFn20r80 EFn20r81 EFn20r82 EFn20r83 EFn20r84 EFn20r85
EFn20r86 EFn20r87 EFn20r88 EFn20r89 EFn20r90 EFn20r91
EFn20r92 EFn20r93 EFn20r94 EFn20r95 EFn20r96 EFn20r97
EFn20r98 EFn20r99 EFn20r100;
run;

proc means data=RobustOut5_fixed20 mean;
  var Estimate StdErr;
  class Effect;
run;

data Out5_random20;
set Rn20r1 Rn20r2 Rn20r3 Rn20r4 Rn20r5 Rn20r6 Rn20r7 Rn20r8
Rn20r9 Rn20r10 Rn20r11 Rn20r12 Rn20r13 Rn20r14 Rn20r15
```

```
Rn20r16 Rn20r17 Rn20r18 Rn20r19 Rn20r20 Rn20r21 Rn20r22
Rn20r23 Rn20r24 Rn20r25 Rn20r26 Rn20r27 Rn20r28 Rn20r29
Rn20r30 Rn20r31 Rn20r32 Rn20r33 Rn20r34 Rn20r35 Rn20r36
Rn20r37 Rn20r38 Rn20r39 Rn20r40 Rn20r41 Rn20r42 Rn20r43
Rn20r44 Rn20r45 Rn20r46 Rn20r47 Rn20r48 Rn20r49 Rn20r50
Rn20r51 Rn20r52 Rn20r53 Rn20r54 Rn20r55 Rn20r56 Rn20r57
Rn20r58 Rn20r59 Rn20r60 Rn20r61 Rn20r62 Rn20r63 Rn20r64
Rn20r65 Rn20r66 Rn20r67 Rn20r68 Rn20r69 Rn20r70 Rn20r71
Rn20r72 Rn20r73 Rn20r74 Rn20r75 Rn20r76 Rn20r77 Rn20r78
Rn20r79 Rn20r80 Rn20r81 Rn20r82 Rn20r83 Rn20r84 Rn20r85
Rn20r86 Rn20r87 Rn20r88 Rn20r89 Rn20r90 Rn20r91 Rn20r92
Rn20r93 Rn20r94 Rn20r95 Rn20r96 Rn20r97 Rn20r98 Rn20r99
Rn20r100;
run;

proc means data=Out5_random20 mean;
  var Estimate StdErr;
  class CovParm;
run;

data RobustOut5_random20;
set ERn20r1 ERn20r2 ERn20r3 ERn20r4 ERn20r5 ERn20r6 ERn20r7
ERn20r8 ERn20r9 ERn20r10 ERn20r11 ERn20r12 ERn20r13
ERn20r14 ERn20r15 ERn20r16 ERn20r17 ERn20r18 ERn20r19
ERn20r20 ERn20r21 ERn20r22 ERn20r23 ERn20r24 ERn20r25
ERn20r26 ERn20r27 ERn20r28 ERn20r29 ERn20r30 ERn20r31
ERn20r32 ERn20r33 ERn20r34 ERn20r35 ERn20r36 ERn20r37
ERn20r38 ERn20r39 ERn20r40 ERn20r41 ERn20r42 ERn20r43
ERn20r44 ERn20r45 ERn20r46 ERn20r47 ERn20r48 ERn20r49
ERn20r50 ERn20r51 ERn20r52 ERn20r53 ERn20r54 ERn20r55
ERn20r56 ERn20r57 ERn20r58 ERn20r59 ERn20r60 ERn20r61
ERn20r62 ERn20r63 ERn20r64 ERn20r65 ERn20r66 ERn20r67
ERn20r68 ERn20r69 ERn20r70 ERn20r71 ERn20r72 ERn20r73
ERn20r74 ERn20r75 ERn20r76 ERn20r77 ERn20r78 ERn20r79
ERn20r80 ERn20r81 ERn20r82 ERn20r83 ERn20r84 ERn20r85
ERn20r86 ERn20r87 ERn20r88 ERn20r89 ERn20r90 ERn20r91
ERn20r92 ERn20r93 ERn20r94 ERn20r95 ERn20r96 ERn20r97
ERn20r98 ERn20r99 ERn20r100;
run;

proc means data=RobustOut5_random20 mean;
  var Estimate StdErr;
  class CovParm;
run;

data Out5_means20;
set Mn20r1 Mn20r2 Mn20r3 Mn20r4 Mn20r5 Mn20r6 Mn20r7 Mn20r8
Mn20r9 Mn20r10 Mn20r11 Mn20r12 Mn20r13 Mn20r14 Mn20r15
Mn20r16 Mn20r17 Mn20r18 Mn20r19 Mn20r20 Mn20r21 Mn20r22
Mn20r23 Mn20r24 Mn20r25 Mn20r26 Mn20r27 Mn20r28 Mn20r29
Mn20r30 Mn20r31 Mn20r32 Mn20r33 Mn20r34 Mn20r35 Mn20r36
Mn20r37 Mn20r38 Mn20r39 Mn20r40 Mn20r41 Mn20r42 Mn20r43
Mn20r44 Mn20r45 Mn20r46 Mn20r47 Mn20r48 Mn20r49 Mn20r50
Mn20r51 Mn20r52 Mn20r53 Mn20r54 Mn20r55 Mn20r56 Mn20r57
Mn20r58 Mn20r59 Mn20r60 Mn20r61 Mn20r62 Mn20r63 Mn20r64
Mn20r65 Mn20r66 Mn20r67 Mn20r68 Mn20r69 Mn20r70 Mn20r71
Mn20r72 Mn20r73 Mn20r74 Mn20r75 Mn20r76 Mn20r77 Mn20r78
```

```
Mn20r79 Mn20r80 Mn20r81 Mn20r82 Mn20r83 Mn20r84 Mn20r85
Mn20r86 Mn20r87 Mn20r88 Mn20r89 Mn20r90 Mn20r91 Mn20r92
Mn20r93 Mn20r94 Mn20r95 Mn20r96 Mn20r97 Mn20r98 Mn20r99
Mn20r100;
run;

proc means data=Out5_means20 mean;
  var yij;
  class _STAT_;
run;
```

1250

```
%macro HLM(n);
%do rep= 1 %to 100;
data N&n.R&rep;
  gamma00=5;
  gamma10=1;
  sig1=1;
  sig2=2;
  rho=0.5;

   %do i=1 %to 10;
       macro=&i;
       r1 = rannor(-1);
       r2 = rannor(-1);
       U0j= sig1*r1;
         U1j= rho*sig2*r1+sqrt(1-rho**2)*sig2*r2;

     %do j=1 %to &n;
         micro=&j;
       xij=rannor(-1);
         Rij=rannor(-1)*2;
       yij = gamma00+ gamma10*xij + U0j + U1j*xij + Rij;
      output;
     %end;

 %end;
run;

    data N&n.R&rep;
        set N&n.R&rep;
          if macro<11 & micro<3 then
              yij=5+(3+abs(ranuni(1015)))*3;
/*Define outliers and replace part of simulated data */

      run;

proc means data=N&n.R&rep;
   output out=MN&n.R&rep;
run;

proc mixed data=N&n.R&rep method=ML covtest
MAXITER=1000000;
   class macro;
   model yij = xij / solution chisq;
   random intercept xij/ subject=macro solution type=un g;
   ods output SolutionF=FN&n.R&rep COVPARMS=RN&n.R&rep;

run;
```

	`` proc mixed data=N&n.R&rep method=ML covtest empirical MAXITER=1000000; class macro; model yij = xij / solution chisq; random intercept xij/ subject=macro solution type=un g; ods output SolutionF=EFN&n.R&rep COVPARMS=ERN&n.R&rep; run; %end; %mend HLM;%HLM(n=50); data Out5_fixed50; set Fn50r1 Fn50r2 Fn50r3 Fn50r4 Fn50r5 Fn50r6 Fn50r7 Fn50r8 Fn50r9 Fn50r10 Fn50r11 Fn50r12 Fn50r13 Fn50r14 Fn50r15 Fn50r16 Fn50r17 Fn50r18 Fn50r19 Fn50r20 Fn50r21 Fn50r22 Fn50r23 Fn50r24 Fn50r25 Fn50r26 Fn50r27 Fn50r28 Fn50r29 Fn50r30 Fn50r31 Fn50r32 Fn50r33 Fn50r34 Fn50r35 Fn50r36 Fn50r37 Fn50r38 Fn50r39 Fn50r40 Fn50r41 Fn50r42 Fn50r43 Fn50r44 Fn50r45 Fn50r46 Fn50r47 Fn50r48 Fn50r49 Fn50r50 Fn50r51 Fn50r52 Fn50r53 Fn50r54 Fn50r55 Fn50r56 Fn50r57 Fn50r58 Fn50r59 Fn50r60 Fn50r61 Fn50r62 Fn50r63 Fn50r64 Fn50r65 Fn50r66 Fn50r67 Fn50r68 Fn50r69 Fn50r70 Fn50r71 Fn50r72 Fn50r73 Fn50r74 Fn50r75 Fn50r76 Fn50r77 Fn50r78 Fn50r79 Fn50r80 Fn50r81 Fn50r82 Fn50r83 Fn50r84 Fn50r85 Fn50r86 Fn50r87 Fn50r88 Fn50r89 Fn50r90 Fn50r91 Fn50r92 Fn50r93 Fn50r94 Fn50r95 Fn50r96 Fn50r97 Fn50r98 Fn50r99 Fn50r100; run; proc means data=Out5_fixed50 mean; var Estimate StdErr; class Effect; run; data RobustOut5_fixed50; set EFn50r1 EFn50r2 EFn50r3 EFn50r4 EFn50r5 EFn50r6 EFn50r7 EFn50r8 EFn50r9 EFn50r10 EFn50r11 EFn50r12 EFn50r13 EFn50r14 EFn50r15 EFn50r16 EFn50r17 EFn50r18 EFn50r19 EFn50r20 EFn50r21 EFn50r22 EFn50r23 EFn50r24 EFn50r25 EFn50r26 EFn50r27 EFn50r28 EFn50r29 EFn50r30 EFn50r31 EFn50r32 EFn50r33 EFn50r34 EFn50r35 EFn50r36 EFn50r37 EFn50r38 EFn50r39 EFn50r40 EFn50r41 EFn50r42 EFn50r43 EFn50r44 EFn50r45 EFn50r46 EFn50r47 EFn50r48 EFn50r49 EFn50r50 EFn50r51 EFn50r52 EFn50r53 EFn50r54 EFn50r55 EFn50r56 EFn50r57 EFn50r58 EFn50r59 EFn50r60 EFn50r61 EFn50r62 EFn50r63 EFn50r64 EFn50r65 EFn50r66 EFn50r67 EFn50r68 EFn50r69 EFn50r70 EFn50r71 EFn50r72 EFn50r73 EFn50r74 EFn50r75 EFn50r76 EFn50r77 EFn50r78 EFn50r79 EFn50r80 EFn50r81 EFn50r82 EFn50r83 EFn50r84 EFn50r85 EFn50r86 EFn50r87 EFn50r88 EFn50r89 EFn50r90 EFn50r91 EFn50r92 EFn50r93 EFn50r94 EFn50r95 EFn50r96 EFn50r97 EFn50r98 EFn50r99 EFn50r100; run; proc means data=RobustOut5_fixed50 mean; var Estimate StdErr; class Effect; run; ```

```
data Out5_random50;
  set Rn50r1 Rn50r2 Rn50r3 Rn50r4 Rn50r5 Rn50r6 Rn50r7
Rn50r8 Rn50r9 Rn50r10 Rn50r11 Rn50r12 Rn50r13 Rn50r14
Rn50r15 Rn50r16 Rn50r17 Rn50r18 Rn50r19 Rn50r20 Rn50r21
Rn50r22 Rn50r23 Rn50r24 Rn50r25 Rn50r26 Rn50r27 Rn50r28
Rn50r29 Rn50r30 Rn50r31 Rn50r32 Rn50r33 Rn50r34 Rn50r35
Rn50r36 Rn50r37 Rn50r38 Rn50r39 Rn50r40 Rn50r41 Rn50r42
Rn50r43 Rn50r44 Rn50r45 Rn50r46 Rn50r47 Rn50r48 Rn50r49
Rn50r50 Rn50r51 Rn50r52 Rn50r53 Rn50r54 Rn50r55 Rn50r56
Rn50r57 Rn50r58 Rn50r59 Rn50r60 Rn50r61 Rn50r62 Rn50r63
Rn50r64 Rn50r65 Rn50r66 Rn50r67 Rn50r68 Rn50r69 Rn50r70
Rn50r71 Rn50r72 Rn50r73 Rn50r74 Rn50r75 Rn50r76 Rn50r77
Rn50r78 Rn50r79 Rn50r80 Rn50r81 Rn50r82 Rn50r83 Rn50r84
Rn50r85 Rn50r86 Rn50r87 Rn50r88 Rn50r89 Rn50r90 Rn50r91
Rn50r92 Rn50r93 Rn50r94 Rn50r95 Rn50r96 Rn50r97 Rn50r98
Rn50r99 Rn50r100;
run;

proc means data=Out5_random50 mean;
  var Estimate StdErr;
  class CovParm;
run;

data RobustOut5_random50;
  set ERn50r1 ERn50r2 ERn50r3 ERn50r4 ERn50r5 ERn50r6
ERn50r7 ERn50r8 ERn50r9 ERn50r10 ERn50r11 ERn50r12 ERn50r13
ERn50r14 ERn50r15 ERn50r16 ERn50r17 ERn50r18 ERn50r19
ERn50r20 ERn50r21 ERn50r22 ERn50r23 ERn50r24 ERn50r25
ERn50r26 ERn50r27 ERn50r28 ERn50r29 ERn50r30 ERn50r31
ERn50r32 ERn50r33 ERn50r34 ERn50r35 ERn50r36 ERn50r37
ERn50r38 ERn50r39 ERn50r40 ERn50r41 ERn50r42 ERn50r43
ERn50r44 ERn50r45 ERn50r46 ERn50r47 ERn50r48 ERn50r49
ERn50r50 ERn50r51 ERn50r52 ERn50r53 ERn50r54 ERn50r55
ERn50r56 ERn50r57 ERn50r58 ERn50r59 ERn50r60 ERn50r61
ERn50r62 ERn50r63 ERn50r64 ERn50r65 ERn50r66 ERn50r67
ERn50r68 ERn50r69 ERn50r70 ERn50r71 ERn50r72 ERn50r73
ERn50r74 ERn50r75 ERn50r76 ERn50r77 ERn50r78 ERn50r79
ERn50r80 ERn50r81 ERn50r82 ERn50r83 ERn50r84 ERn50r85
ERn50r86 ERn50r87 ERn50r88 ERn50r89 ERn50r90 ERn50r91
ERn50r92 ERn50r93 ERn50r94 ERn50r95 ERn50r96 ERn50r97
ERn50r98 ERn50r99 ERn50r100;
run;

proc means data=RobustOut5_random50 mean;
  var Estimate StdErr;
  class CovParm;
run;

data Out5_means50;
  set Mn50r1 Mn50r2 Mn50r3 Mn50r4 Mn50r5 Mn50r6 Mn50r7
Mn50r8 Mn50r9 Mn50r10 Mn50r11 Mn50r12 Mn50r13 Mn50r14
Mn50r15 Mn50r16 Mn50r17 Mn50r18 Mn50r19 Mn50r20 Mn50r21
Mn50r22 Mn50r23 Mn50r24 Mn50r25 Mn50r26 Mn50r27 Mn50r28
Mn50r29 Mn50r30 Mn50r31 Mn50r32 Mn50r33 Mn50r34 Mn50r35
Mn50r36 Mn50r37 Mn50r38 Mn50r39 Mn50r40 Mn50r41 Mn50r42
Mn50r43 Mn50r44 Mn50r45 Mn50r46 Mn50r47 Mn50r48 Mn50r49
```

	`` Mn50r50 Mn50r51 Mn50r52 Mn50r53 Mn50r54 Mn50r55 Mn50r56 Mn50r57 Mn50r58 Mn50r59 Mn50r60 Mn50r61 Mn50r62 Mn50r63 Mn50r64 Mn50r65 Mn50r66 Mn50r67 Mn50r68 Mn50r69 Mn50r70 Mn50r71 Mn50r72 Mn50r73 Mn50r74 Mn50r75 Mn50r76 Mn50r77 Mn50r78 Mn50r79 Mn50r80 Mn50r81 Mn50r82 Mn50r83 Mn50r84 Mn50r85 Mn50r86 Mn50r87 Mn50r88 Mn50r89 Mn50r90 Mn50r91 Mn50r92 Mn50r93 Mn50r94 Mn50r95 Mn50r96 Mn50r97 Mn50r98 Mn50r99 Mn50r100; run; proc means data=Out5_means50 mean; var yij; class _STAT_; run; ```
5000	``` %macro HLM(n); %do rep= 1 %to 100; data N&n.R&rep; gamma00=5; gamma10=1; sig1=1; sig2=2; rho=0.5; %do i=1 %to 10; macro=&i; r1 = rannor(-1); r2 = rannor(-1); U0j= sig1*r1; U1j= rho*sig2*r1+sqrt(1-rho**2)*sig2*r2; %do j=1 %to &n; micro=&j; xij=rannor(-1); Rij=rannor(-1)*2; yij = gamma00+ gamma10*xij + U0j + U1j*xij + Rij; output; %end; %end; run; data N&n.R&rep; set N&n.R&rep; if macro<11 & micro<3 then yij=5+(3+abs(ranuni(1015)))*3; /*Define outliers and replace part of simulated data */ run; proc means data=N&n.R&rep; output out=MN&n.R&rep; run; proc mixed data=N&n.R&rep method=ML covtest MAXITER=1000000; class macro; model yij = xij / solution chisq; ```

	`` random intercept xij/ subject=macro solution type=un g; ods output SolutionF=FN&n.R&rep COVPARMS=RN&n.R&rep; run; proc mixed data=N&n.R&rep method=ML covtest empirical MAXITER=1000000; class macro; model yij = xij / solution chisq; random intercept xij/ subject=macro solution type=un g; ods output SolutionF=EFN&n.R&rep COVPARMS=ERN&n.R&rep; run; %end; %mend HLM;%HLM(n=100); /*n=100;*/ data Out5_fixed100; set Fn100r1 Fn100r2 Fn100r3 Fn100r4 Fn100r5 Fn100r6 Fn100r7 Fn100r8 Fn100r9 Fn100r10 Fn100r11 Fn100r12 Fn100r13 Fn100r14 Fn100r15 Fn100r16 Fn100r17 Fn100r18 Fn100r19 Fn100r20 Fn100r21 Fn100r22 Fn100r23 Fn100r24 Fn100r25 Fn100r26 Fn100r27 Fn100r28 Fn100r29 Fn100r30 Fn100r31 Fn100r32 Fn100r33 Fn100r34 Fn100r35 Fn100r36 Fn100r37 Fn100r38 Fn100r39 Fn100r40 Fn100r41 Fn100r42 Fn100r43 Fn100r44 Fn100r45 Fn100r46 Fn100r47 Fn100r48 Fn100r49 Fn100r50 Fn100r51 Fn100r52 Fn100r53 Fn100r54 Fn100r55 Fn100r56 Fn100r57 Fn100r58 Fn100r59 Fn100r60 Fn100r61 Fn100r62 Fn100r63 Fn100r64 Fn100r65 Fn100r66 Fn100r67 Fn100r68 Fn100r69 Fn100r70 Fn100r71 Fn100r72 Fn100r73 Fn100r74 Fn100r75 Fn100r76 Fn100r77 Fn100r78 Fn100r79 Fn100r80 Fn100r81 Fn100r82 Fn100r83 Fn100r84 Fn100r85 Fn100r86 Fn100r87 Fn100r88 Fn100r89 Fn100r90 Fn100r91 Fn100r92 Fn100r93 Fn100r94 Fn100r95 Fn100r96 Fn100r97 Fn100r98 Fn100r99 Fn100r100; run; proc means data=Out5_fixed100 mean; var Estimate StdErr; class Effect; run; data RobustOut5_fixed100; set EFn100r1 EFn100r2 EFn100r3 EFn100r4 EFn100r5 EFn100r6 EFn100r7 EFn100r8 EFn100r9 EFn100r10 EFn100r11 EFn100r12 EFn100r13 EFn100r14 EFn100r15 EFn100r16 EFn100r17 EFn100r18 EFn100r19 EFn100r20 EFn100r21 EFn100r22 EFn100r23 EFn100r24 EFn100r25 EFn100r26 EFn100r27 EFn100r28 EFn100r29 EFn100r30 EFn100r31 EFn100r32 EFn100r33 EFn100r34 EFn100r35 EFn100r36 EFn100r37 EFn100r38 EFn100r39 EFn100r40 EFn100r41 EFn100r42 EFn100r43 EFn100r44 EFn100r45 EFn100r46 EFn100r47 EFn100r48 EFn100r49 EFn100r50 EFn100r51 EFn100r52 EFn100r53 EFn100r54 EFn100r55 EFn100r56 EFn100r57 EFn100r58 EFn100r59 EFn100r60 EFn100r61 EFn100r62 EFn100r63 EFn100r64 EFn100r65 EFn100r66 EFn100r67 EFn100r68 EFn100r69 EFn100r70 EFn100r71 EFn100r72 EFn100r73 EFn100r74 EFn100r75 EFn100r76 EFn100r77 EFn100r78 EFn100r79 EFn100r80 EFn100r81 EFn100r82 EFn100r83 EFn100r84 EFn100r85 EFn100r86 EFn100r87 EFn100r88 EFn100r89 EFn100r90 EFn100r91 EFn100r92 EFn100r93 EFn100r94 EFn100r95 ```

	`` EFn100r96 EFn100r97 EFn100r98 EFn100r99 EFn100r100; run; proc means data=RobustOut5_fixed100 mean; var Estimate StdErr; class Effect; run; data Out5_random100; set Rn100r1 Rn100r2 Rn100r3 Rn100r4 Rn100r5 Rn100r6 Rn100r7 Rn100r8 Rn100r9 Rn100r10 Rn100r11 Rn100r12 Rn100r13 Rn100r14 Rn100r15 Rn100r16 Rn100r17 Rn100r18 Rn100r19 Rn100r20 Rn100r21 Rn100r22 Rn100r23 Rn100r24 Rn100r25 Rn100r26 Rn100r27 Rn100r28 Rn100r29 Rn100r30 Rn100r31 Rn100r32 Rn100r33 Rn100r34 Rn100r35 Rn100r36 Rn100r37 Rn100r38 Rn100r39 Rn100r40 Rn100r41 Rn100r42 Rn100r43 Rn100r44 Rn100r45 Rn100r46 Rn100r47 Rn100r48 Rn100r49 Rn100r50 Rn100r51 Rn100r52 Rn100r53 Rn100r54 Rn100r55 Rn100r56 Rn100r57 Rn100r58 Rn100r59 Rn100r60 Rn100r61 Rn100r62 Rn100r63 Rn100r64 Rn100r65 Rn100r66 Rn100r67 Rn100r68 Rn100r69 Rn100r70 Rn100r71 Rn100r72 Rn100r73 Rn100r74 Rn100r75 Rn100r76 Rn100r77 Rn100r78 Rn100r79 Rn100r80 Rn100r81 Rn100r82 Rn100r83 Rn100r84 Rn100r85 Rn100r86 Rn100r87 Rn100r88 Rn100r89 Rn100r90 Rn100r91 Rn100r92 Rn100r93 Rn100r94 Rn100r95 Rn100r96 Rn100r97 Rn100r98 Rn100r99 Rn100r100; run; proc means data=Out5_random100 mean; var Estimate StdErr; class CovParm; run; data RobustOut5_random100; set ERn100r1 ERn100r2 ERn100r3 ERn100r4 ERn100r5 ERn100r6 ERn100r7 ERn100r8 ERn100r9 ERn100r10 ERn100r11 ERn100r12 ERn100r13 ERn100r14 ERn100r15 ERn100r16 ERn100r17 ERn100r18 ERn100r19 ERn100r20 ERn100r21 ERn100r22 ERn100r23 ERn100r24 ERn100r25 ERn100r26 ERn100r27 ERn100r28 ERn100r29 ERn100r30 ERn100r31 ERn100r32 ERn100r33 ERn100r34 ERn100r35 ERn100r36 ERn100r37 ERn100r38 ERn100r39 ERn100r40 ERn100r41 ERn100r42 ERn100r43 ERn100r44 ERn100r45 ERn100r46 ERn100r47 ERn100r48 ERn100r49 ERn100r50 ERn100r51 ERn100r52 ERn100r53 ERn100r54 ERn100r55 ERn100r56 ERn100r57 ERn100r58 ERn100r59 ERn100r60 ERn100r61 ERn100r62 ERn100r63 ERn100r64 ERn100r65 ERn100r66 ERn100r67 ERn100r68 ERn100r69 ERn100r70 ERn100r71 ERn100r72 ERn100r73 ERn100r74 ERn100r75 ERn100r76 ERn100r77 ERn100r78 ERn100r79 ERn100r80 ERn100r81 ERn100r82 ERn100r83 ERn100r84 ERn100r85 ERn100r86 ERn100r87 ERn100r88 ERn100r89 ERn100r90 ERn100r91 ERn100r92 ERn100r93 ERn100r94 ERn100r95 ERn100r96 ERn100r97 ERn100r98 ERn100r99 ERn100r100; run; proc means data=RobustOut5_random100 mean; var Estimate StdErr; class CovParm; run; ```

	<pre>data Out5_means100; set Mn100r1 Mn100r2 Mn100r3 Mn100r4 Mn100r5 Mn100r6 Mn100r7 Mn100r8 Mn100r9 Mn100r10 Mn100r11 Mn100r12 Mn100r13 Mn100r14 Mn100r15 Mn100r16 Mn100r17 Mn100r18 Mn100r19 Mn100r20 Mn100r21 Mn100r22 Mn100r23 Mn100r24 Mn100r25 Mn100r26 Mn100r27 Mn100r28 Mn100r29 Mn100r30 Mn100r31 Mn100r32 Mn100r33 Mn100r34 Mn100r35 Mn100r36 Mn100r37 Mn100r38 Mn100r39 Mn100r40 Mn100r41 Mn100r42 Mn100r43 Mn100r44 Mn100r45 Mn100r46 Mn100r47 Mn100r48 Mn100r49 Mn100r50 Mn100r51 Mn100r52 Mn100r53 Mn100r54 Mn100r55 Mn100r56 Mn100r57 Mn100r58 Mn100r59 Mn100r60 Mn100r61 Mn100r62 Mn100r63 Mn100r64 Mn100r65 Mn100r66 Mn100r67 Mn100r68 Mn100r69 Mn100r70 Mn100r71 Mn100r72 Mn100r73 Mn100r74 Mn100r75 Mn100r76 Mn100r77 Mn100r78 Mn100r79 Mn100r80 Mn100r81 Mn100r82 Mn100r83 Mn100r84 Mn100r85 Mn100r86 Mn100r87 Mn100r88 Mn100r89 Mn100r90 Mn100r91 Mn100r92 Mn100r93 Mn100r94 Mn100r95 Mn100r96 Mn100r97 Mn100r98 Mn100r99 Mn100r100; run; proc means data=Out5_means100 mean; var yij; class _STAT_; run;</pre>

Table 1

Parameter Recovery with Normally Distributed Data.

Sample sizes	Parameters	True values	Estimates	Standard errors	Absolute biases	Relative biases
200	γ_{00}	5	5.00	.32	.00	.01%
	γ_{01}	1	1.01	.61	.01	1.11%
	σ^2	4	3.96	.42	.04	1.06%
	τ_{00}	1	.93	.51	.07	6.63%
	τ_{01} / τ_{10}	1	.97	.73	.03	3.36%
	τ_{11}	4	3.80	1.80	.20	4.89%
1250	γ_{00}	5	5.00	.20	.00	.08%
	γ_{01}	1	1.00	.40	.00	.28%
	σ^2	4	4.04	.16	.04	.88%
	τ_{00}	1	.98	.30	.02	2.22%
	τ_{01} / τ_{10}	1	.92	.45	.08	8.06%
	τ_{11}	4	3.90	1.13	.10	2.38%
5000	γ_{00}	5	5.00	.14	.00	.04%
	γ_{01}	1	.97	.28	.03	3.30%
	σ^2	4	3.99	.08	.01	.18%
	τ_{00}	1	1.00	.21	.00	.08%
	τ_{01} / τ_{10}	1	.96	.32	.04	3.89%
	τ_{11}	4	3.86	.78	.14	3.40%
Mean					.04	2.33%
SD					.05	2.40%

Note: The correlation between true values and estimates is $r = 0.999$ ($p < .01$).

Table 2

Biases of the Fixed-Effect estimates with Sample Size 200.

Type of outliers	Percents	Parameters	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.50%	γ_{00}	5	5.03	.32	.03	0.53%
		γ_{01}	1	.93	.60	.07	7.19%
3 SD	1.00%	γ_{00}	5	5.13	.33	.13	2.56%
		γ_{01}	1	.98	.60	.02	1.99%
3 SD	2.50%	γ_{00}	5	5.24	.32	.24	4.79%
		γ_{01}	1	.97	.60	.03	3.12%
3 SD	4.00%	γ_{00}	5	5.44	.32	.44	8.88%
		γ_{01}	1	1.07	.59	.07	7.35%
3 SD	5.00%	γ_{00}	5	5.53	.32	.53	10.64%
		γ_{01}	1	.95	.60	.05	4.65%
3 SD	10.00%	γ_{00}	5	6.09	.32	1.09	21.87%
		γ_{01}	1	.93	.58	.07	7.12%
5 SD	.50%	γ_{00}	5	5.10	.33	.10	2.10%
		γ_{01}	1	1.04	.59	.04	4.42%
5 SD	1.00%	γ_{00}	5	5.19	.33	.19	3.78%
		γ_{01}	1	.98	.58	.02	1.53%
5 SD	2.50%	γ_{00}	5	5.40	.36	.40	7.99%
		γ_{01}	1	.92	.62	.08	8.01%
5 SD	4.00%	γ_{00}	5	5.71	.34	.71	14.11%
		γ_{01}	1	1.12	.62	.12	12.49%
5 SD	5.00%	γ_{00}	5	5.84	.34	.84	16.77%
		γ_{01}	1	.91	.64	.09	9.12%
5 SD	10.00%	γ_{00}	5	6.78	.41	1.78	35.63%
		γ_{01}	1	.87	.73	.13	12.55%

Table 3

Biases of the Fixed-Effect estimates with Sample Size 1250.

Type of outliers	Percents	Parameters	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.16%	γ_{00}	5	4.96	.20	.04	.81%
		γ_{01}	1	.98	.38	.02	2.32%
3 SD	.40%	γ_{00}	5	5.02	.21	.02	.39%
		γ_{01}	1	.99	.39	.01	1.21%
3 SD	.80%	γ_{00}	5	5.08	.20	.08	1.65%
		γ_{01}	1	1.08	.39	.08	7.74%
3 SD	2.00%	γ_{00}	5	5.23	.20	.23	4.66%
		γ_{01}	1	.94	.39	.06	5.86%
3 SD	4.00%	γ_{00}	5	5.43	.20	.43	8.64%
		γ_{01}	1	.91	.37	.09	8.75%
3 SD	6.00%	γ_{00}	5	5.66	.20	.66	13.20%
		γ_{01}	1	.94	.38	.06	6.23%
3 SD	10.00%	γ_{00}	5	6.06	.18	1.06	21.16%
		γ_{01}	1	.86	.36	.14	13.75%
5 SD	.16%	γ_{00}	5	5.02	.20	.02	.46%
		γ_{01}	1	1.01	.38	.01	.95%
5 SD	.40%	γ_{00}	5	5.06	.21	.06	1.26%
		γ_{01}	1	.99	.40	.01	1.14%
5 SD	.80%	γ_{00}	5	5.13	.21	.13	2.67%
		γ_{01}	1	1.01	.38	.01	.85%
5 SD	2.00%	γ_{00}	5	5.37	.20	.37	7.42%
		γ_{01}	1	1.03	.39	.03	2.75%
5 SD	4.00%	γ_{00}	5	5.71	.20	.71	14.10%
		γ_{01}	1	1.01	.39	.01	1.02%
5 SD	6.00%	γ_{00}	5	6.03	.20	1.03	20.61%
		γ_{01}	1	.92	.39	.08	7.69%
5 SD	10.00%	γ_{00}	5	6.73	.19	1.73	34.54%
		γ_{01}	1	.88	.38	.12	11.90%

Table 4

Biases of the Fixed-Effect estimates with 3 SD Outliers for Sample Size 5000.

Percents	Parameters	True values	Estimates	Standard errors	Absolute biases	Relative biases
.04%	γ_{00}	5	4.99	.14	.01	.15%
	γ_{01}	1	.99	.28	.01	1.27%
.10%	γ_{00}	5	5.02	.14	.02	.33%
	γ_{01}	1	.98	.28	.02	2.17%
.20%	γ_{00}	5	5.03	.14	.03	.56%
	γ_{01}	1	1.01	.28	.01	.84%
.40%	γ_{00}	5	5.05	.14	.05	1.01%
	γ_{01}	1	.99	.28	.01	1.08%
.60%	γ_{00}	5	5.07	.14	.07	1.42%
	γ_{01}	1	1.01	.28	.01	1.13%
.80%	γ_{00}	5	5.08	.14	.08	1.67%
	γ_{01}	1	.98	.28	.02	1.62%
1.00%	γ_{00}	5	5.11	.14	.11	2.22%
	γ_{01}	1	.98	.28	.02	2.28%
2.00%	γ_{00}	5	5.22	.14	.22	4.36%
	γ_{01}	1	.98	.27	.02	2.23%
3.00%	γ_{00}	5	5.34	.14	.34	6.88%
	γ_{01}	1	.99	.27	.01	.89%
5.00%	γ_{00}	5	5.53	.14	.53	10.57%
	γ_{01}	1	.95	.27	.05	4.74%
10.00%	γ_{00}	5	6.05	.13	1.05	21.02%
	γ_{01}	1	.87	.25	.13	12.65%

Table 5

Biases of the Fixed-Effect estimates with 5 SD Outliers for Sample Size 5000.

Percents	Parameters	True values	Estimates	Standard errors	Absolute biases	Relative biases
.04%	γ_{00}	5	4.99	.14	.01	.25%
	γ_{01}	1	.95	.28	.05	5.25%
.10%	γ_{00}	5	5.02	.14	.02	.48%
	γ_{01}	1	.99	.28	.01	.88%
.20%	γ_{00}	5	5.04	.14	.04	.88%
	γ_{01}	1	.98	.28	.02	1.62%
.40%	γ_{00}	5	5.06	.14	.06	1.15%
	γ_{01}	1	.99	.28	.01	1.05%
.60%	γ_{00}	5	5.11	.14	.11	2.12%
	γ_{01}	1	1.02	.28	.02	2.32%
.80%	γ_{00}	5	5.14	.14	.14	2.71%
	γ_{01}	1	.97	.28	.03	2.50%
1.00%	γ_{00}	5	5.18	.14	.18	3.66%
	γ_{01}	1	.99	.28	.01	1.28%
2.00%	γ_{00}	5	5.34	.14	.34	6.81%
	γ_{01}	1	.98	.28	.02	2.20%
3.00%	γ_{00}	5	5.50	.14	.50	10.03%
	γ_{01}	1	.96	.28	.04	4.42%
5.00%	γ_{00}	5	5.86	.14	.86	17.14%
	γ_{01}	1	.97	.27	.03	3.40%
10.00%	γ_{00}	5	6.76	.13	1.76	35.11%
	γ_{01}	1	.90	.27	.10	9.66%

Table 6

Biases of σ^2 with Sample Sizes 200 and 1250.

Sample size	Types of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
200	3 SD	.50%	4	4.68	.49	.68	16.97%
		1.00%	4	5.20	.55	1.20	30.11%
		2.50%	4	6.83	.72	2.83	70.79%
		4.00%	4	8.51	.89	4.51	112.82%
		5.00%	4	9.87	1.04	5.87	146.63%
		10.00%	4	14.92	1.56	10.92	272.89%
	5 SD	.50%	4	5.64	.59	1.64	40.99%
		1.00%	4	7.43	.78	3.43	85.68%
		2.50%	4	12.12	1.27	8.12	202.97%
		4.00%	4	16.77	1.75	12.77	319.13%
		5.00%	4	19.97	2.07	15.97	399.16%
		10.00%	4	33.36	3.42	29.36	733.97%
1250	3 SD	.16%	4	4.21	.17	.21	5.31%
		.40%	4	4.46	.18	.46	11.47%
		.80%	4	4.94	.20	.94	23.58%
		2.00%	4	6.31	.26	2.31	57.64%
		4.00%	4	8.49	.35	4.49	112.36%
		6.00%	4	10.46	.43	6.46	161.61%
		10.00%	4	14.21	.58	10.21	255.32%
	5 SD	.16%	4	4.57	.19	.57	14.35%
		.40%	4	5.28	.22	1.28	32.07%
		.80%	4	6.58	.27	2.58	64.52%
		2.00%	4	10.46	.43	6.46	161.44%
		4.00%	4	16.30	.67	12.30	307.53%
		6.00%	4	21.92	.89	17.92	447.96%
		10.00%	4	32.12	1.31	28.12	703.02%

Table 7

Biases of the σ^2 with Sample Size 5000.

Type of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.04%	4	4.07	.08	.07	1.64%
	.10%	4	4.11	.08	.11	2.79%
	.20%	4	4.22	.09	.22	5.54%
	.40%	4	4.44	.09	.44	10.99%
	.60%	4	4.69	.09	.69	17.37%
	.80%	4	4.92	.10	.92	23.08%
	1.00%	4	5.15	.10	1.15	28.64%
	2.00%	4	6.23	.13	2.23	55.63%
	3.00%	4	7.29	.15	3.29	82.31%
	5.00%	4	9.39	.19	5.39	134.78%
	10.00%	4	14.22	.29	10.22	255.52%
5 SD	.04%	4	4.13	.08	.13	3.25%
	.10%	4	4.31	.09	.31	7.86%
	.20%	4	4.63	.09	.63	15.78%
	.40%	4	5.25	.11	1.25	31.15%
	.60%	4	5.91	.12	1.91	47.83%
	.80%	4	6.52	.13	2.52	63.12%
	1.00%	4	7.15	.14	3.15	78.68%
	2.00%	4	10.11	.20	6.11	152.81%
	3.00%	4	13.12	.26	9.12	227.88%
	5.00%	4	18.83	.38	14.83	370.71%
	10.00%	4	32.10	.65	28.10	702.61%

Table 8

Biases of τ_{00} with Sample Sizes 200 and 1250.

Sample size	Types of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
200	3 SD	.50%	1	.85	.50	.15	14.95%
		1.00%	1	.85	.51	.15	14.69%
		2.50%	1	.70	.49	.30	29.61%
		4.00%	1	.65	.50	.35	35.26%
		5.00%	1	.54	.51	.46	46.50%
		10.00%	1	.30	.58	.70	70.05%
	5 SD	.50%	1	.90	.54	.10	10.47%
		1.00%	1	.74	.53	.26	26.18%
		2.50%	1	.70	.66	.30	29.69%
		4.00%	1	.37	.67	.63	63.02%
		5.00%	1	.14	.65	.86	85.76%
		10.00%	1	.02	.90	.98	97.99%
1250	3 SD	.16%	1	.96	.29	.04	4.42%
		.40%	1	1.02	.32	.02	2.39%
		.80%	1	.95	.30	.05	5.48%
		2.00%	1	.88	.29	.12	11.97%
		4.00%	1	.82	.28	.18	18.43%
		6.00%	1	.76	.28	.24	23.61%
		10.00%	1	.56	.24	.44	43.65%
	5 SD	.16%	1	.92	.29	.08	7.97%
		.40%	1	.99	.31	.01	0.78%
		.80%	1	.95	.31	.05	5.49%
		2.00%	1	.78	.28	.22	22.08%
		4.00%	1	.70	.29	.30	30.20%
		6.00%	1	.54	.28	.46	46.21%
		10.00%	1	.23	.27	.77	76.91%

Table 9

Biases of the τ_{00} with Sample Size 5000.

Type of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.04%	1	.96	.20	.04	4.47%
	.10%	1	.99	.21	.01	.79%
	.20%	1	.95	.20	.05	5.37%
	.40%	1	.93	.19	.07	6.97%
	.60%	1	.96	.20	.04	3.74%
	.80%	1	.97	.20	.03	3.00%
	1.00%	1	.89	.19	.11	10.50%
	2.00%	1	.89	.19	.11	11.27%
	3.00%	1	.90	.19	.10	9.91%
	5.00%	1	.83	.18	.17	17.21%
	10.00%	1	.71	.17	.29	29.06%
5 SD	.04%	1	1.01	.21	.01	.61%
	.10%	1	1.02	.21	.02	1.77%
	.20%	1	1.01	.21	.01	1.11%
	.40%	1	.96	.20	.04	3.70%
	.60%	1	.96	.20	.04	3.53%
	.80%	1	.93	.20	.07	6.64%
	1.00%	1	.93	.20	.07	7.44%
	2.00%	1	.86	.19	.14	14.28%
	3.00%	1	.84	.20	.16	15.68%
	5.00%	1	.76	.19	.24	24.32%
	10.00%	1	.51	.17	.49	49.29%

Table 10

Biases of τ_{11} for Sample Sizes 200 and 1250.

Sample size	Types of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
200	3 SD	.50%	4	3.57	1.72	.43	10.81%
		1.00%	4	3.45	1.68	.55	13.83%
		2.50%	4	3.42	1.71	.58	14.41%
		4.00%	4	3.17	1.64	.83	20.67%
		5.00%	4	3.31	1.73	.69	17.36%
		10.00%	4	2.80	1.70	1.20	30.07%
	5 SD	.50%	4	3.40	1.67	.60	14.95%
		1.00%	4	3.15	1.61	.85	21.24%
		2.50%	4	3.35	1.83	.65	16.27%
		4.00%	4	3.12	1.84	.88	22.00%
		5.00%	4	3.24	2.04	.76	19.04%
		10.00%	4	3.86	2.73	.14	3.51%
1250	3 SD	.16%	4	3.62	1.05	.38	9.57%
		.40%	4	3.74	1.08	.26	6.57%
		.80%	4	3.87	1.12	.13	3.33%
		2.00%	4	3.74	1.10	.26	6.46%
		4.00%	4	3.41	1.01	.59	14.78%
		6.00%	4	3.36	1.01	.64	16.03%
		10.00%	4	2.97	.92	1.03	25.86%
	5 SD	.16%	4	3.62	1.05	.38	9.41%
		.40%	4	4.03	1.17	.03	0.71%
		.80%	4	3.56	1.05	.44	11.10%
		2.00%	4	3.63	1.09	.37	9.27%
		4.00%	4	3.53	1.10	.47	11.66%
		6.00%	4	3.41	1.10	.59	14.65%
		10.00%	4	3.07	1.06	.93	23.21%

Table 11

Biases of the τ_{11} with Sample Size 5000.

Type of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.04%	4	3.80	.77	.20	4.95%
	.10%	4	3.96	.80	.04	.97%
	.20%	4	3.80	.77	.20	5.01%
	.40%	4	3.78	.76	.22	5.60%
	.60%	4	3.88	.79	.12	2.90%
	.80%	4	3.89	.79	.11	2.74%
	1.00%	4	3.82	.77	.18	4.62%
	2.00%	4	3.74	.76	.26	6.62%
	3.00%	4	3.65	.75	.35	8.71%
	5.00%	4	3.51	.72	.49	12.22%
	10.00%	4	3.12	.65	.88	21.95%
5 SD	.04%	4	3.86	.78	.14	3.55%
	.10%	4	3.84	.78	.16	4.11%
	.20%	4	4.04	.82	.04	1.11%
	.40%	4	3.83	.78	.17	4.17%
	.60%	4	3.92	.80	.08	2.12%
	.80%	4	3.93	.80	.07	1.72%
	1.00%	4	3.94	.80	.06	1.58%
	2.00%	4	3.76	.77	.24	5.90%
	3.00%	4	3.71	.77	.29	7.22%
	5.00%	4	3.55	.75	.45	11.25%
	10.00%	4	3.25	.71	.75	18.83%

Table 12

Biases of τ_{01} for Sample Sizes 200 and 1250.

Sample size	Types of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
200	3 SD	.50%	1	.78	.68	.22	21.98%
		1.00%	1	.93	.70	.07	6.69%
		2.50%	1	.80	.69	.20	20.49%
		4.00%	1	.82	.67	.18	17.89%
		5.00%	1	.87	.69	.13	12.78%
		10.00%	1	.77	.66	.23	22.70%
	5 SD	.50%	1	.98	.72	.02	1.92%
		1.00%	1	.74	.66	.26	26.14%
		2.50%	1	.88	.78	.12	12.02%
		4.00%	1	.76	.74	.24	23.87%
		5.00%	1	.91	.78	.09	8.51%
		10.00%	1	1.25	1.16	.25	24.74%
1250	3 SD	.16%	1	.92	.43	.08	7.51%
		.40%	1	.94	.45	.06	5.63%
		.80%	1	.95	.45	.05	4.99%
		2.00%	1	.93	.43	.07	6.77%
		4.00%	1	.90	.42	.10	10.20%
		6.00%	1	.88	.41	.12	11.59%
		10.00%	1	.73	.36	.27	26.64%
	5 SD	.16%	1	.89	.42	.11	10.53%
		.40%	1	1.00	.47	.00	0.18%
		.80%	1	.92	.44	.08	8.48%
		2.00%	1	.91	.43	.09	8.98%
		4.00%	1	.94	.44	.06	6.17%
		6.00%	1	.85	.42	.15	14.60%
		10.00%	1	.73	.38	.27	27.05%

Table 13

Biases of the τ_{01} / τ_{10} with Sample Size 5000.

Type of outliers	Percents	True values	Estimates	Standard errors	Absolute biases	Relative biases
3 SD	.04%	1	.92	.31	.08	7.78%
	.10%	1	1.03	.32	.03	3.46%
	.20%	1	.94	.31	.06	5.53%
	.40%	1	.94	.30	.06	5.99%
	.60%	1	.94	.31	.06	6.14%
	.80%	1	.98	.31	.02	1.98%
	1.00%	1	.93	.30	.07	6.62%
	2.00%	1	.91	.30	.09	8.61%
	3.00%	1	.93	.30	.07	7.31%
	5.00%	1	.87	.29	.13	12.66%
	10.00%	1	.76	.26	.24	23.56%
5 SD	.04%	1	1.00	.32	.00	.35%
	.10%	1	1.01	.32	.01	.82%
	.20%	1	1.05	.33	.05	4.52%
	.40%	1	.97	.31	.03	2.77%
	.60%	1	.94	.31	.06	5.57%
	.80%	1	.95	.31	.05	5.02%
	1.00%	1	1.00	.32	.00	.36%
	2.00%	1	.92	.30	.08	7.82%
	3.00%	1	.92	.30	.08	7.74%
	5.00%	1	.93	.30	.07	7.42%
	10.00%	1	.81	.27	.19	19.17%

Sample sizes	Q-Q plots	Histograms
200		
1250		
5000		

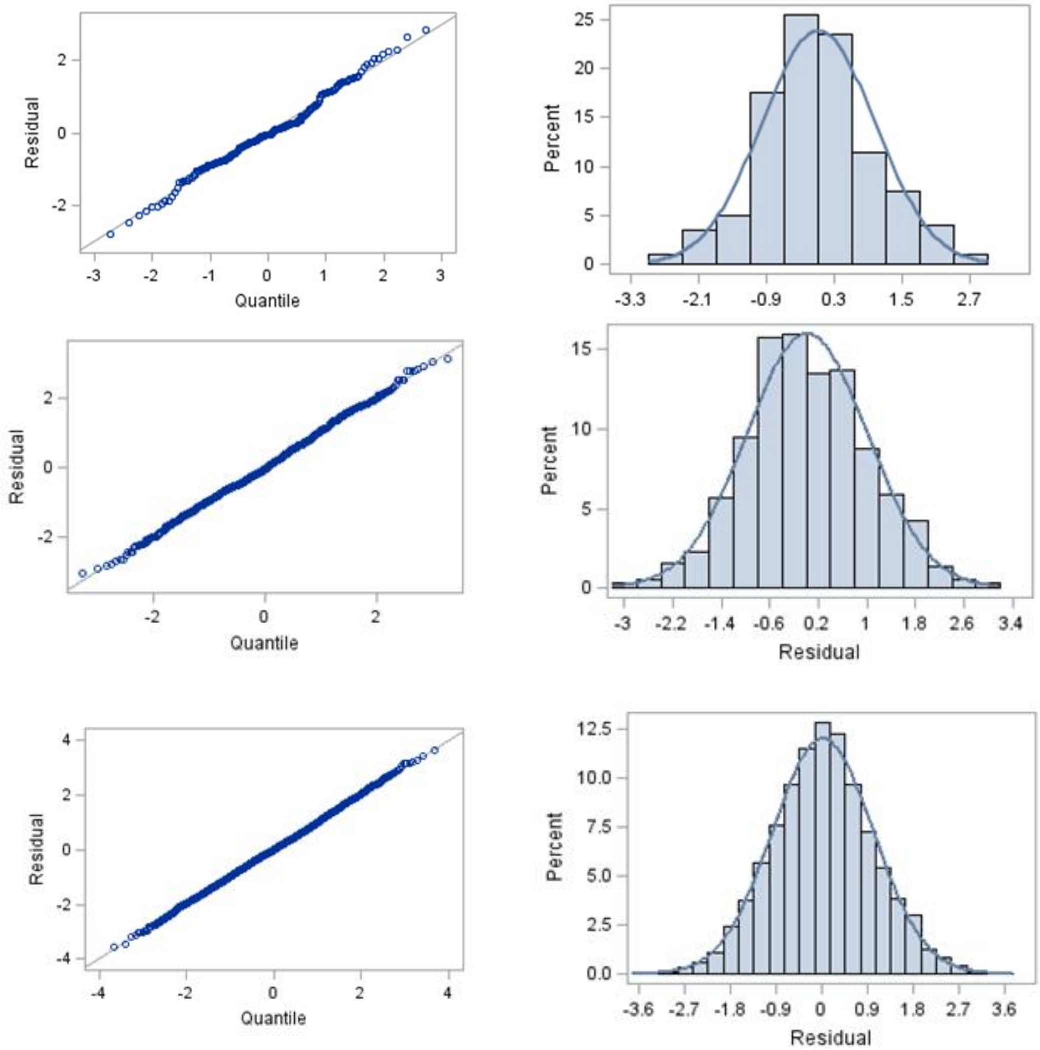
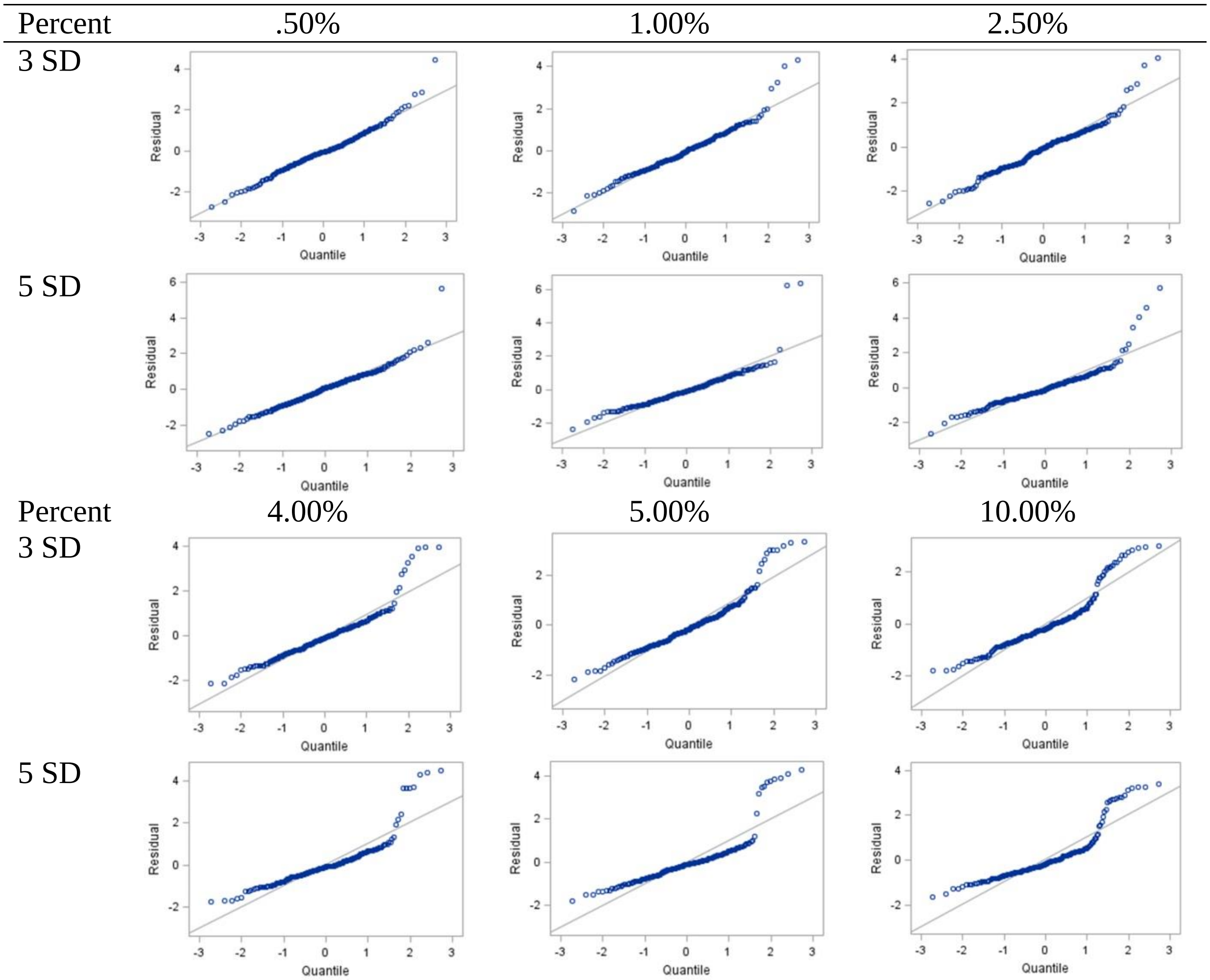
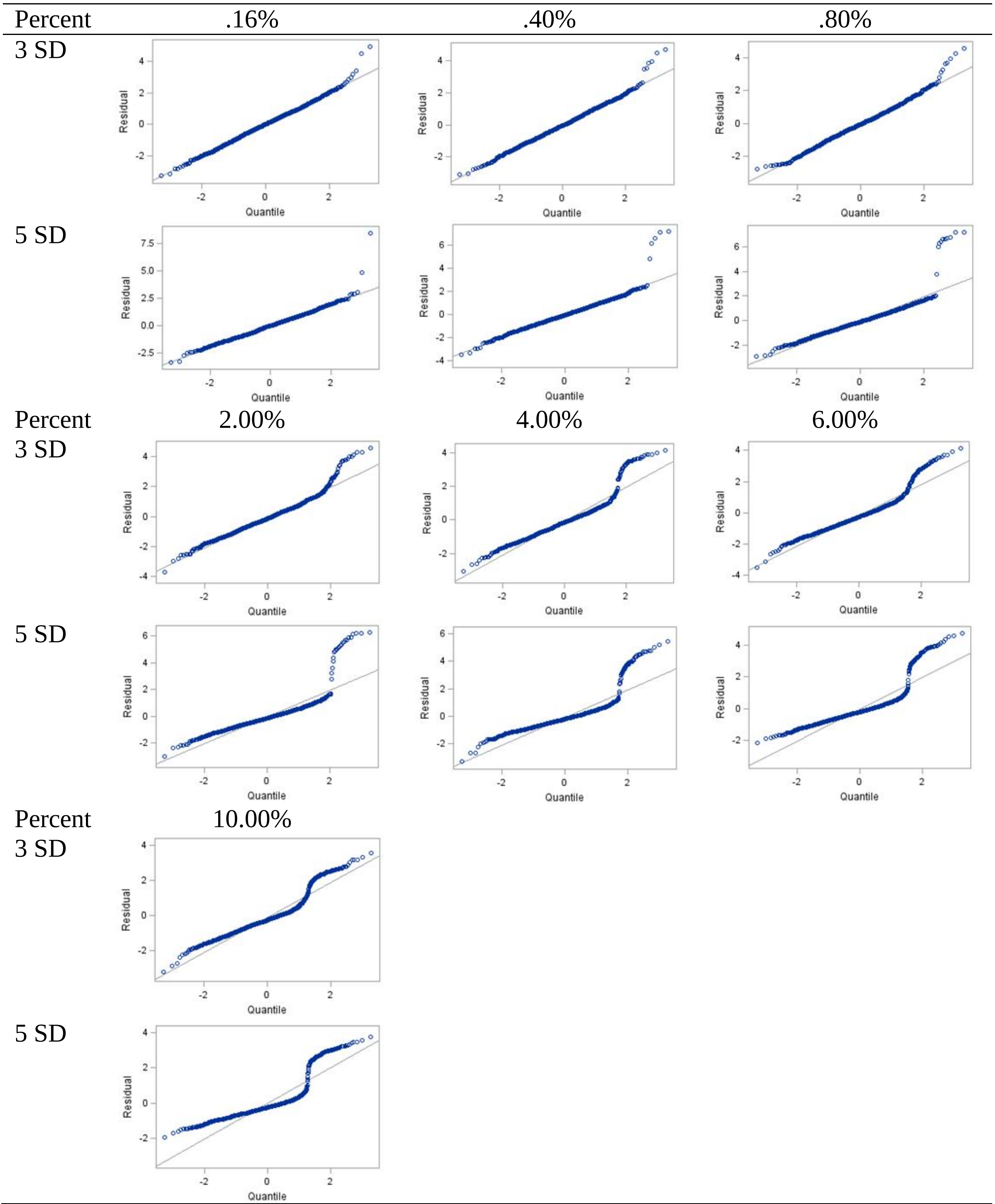


Figure 1. The Q-Q Plots and Histograms for the Scaled Residuals with 1 Replication of Normally Distributed Data.



Note: 3 SD and 5 SD are the types of outliers.

Figure 2. Q-Q Plots for the Scaled Residuals with 1 Replication on Sample Size 200.



Note: 3 SD and 5 SD are the types of outliers.

Figure 3. Q-Q plots for the Scaled Residuals with 1 Replication on Sample Size 1250.

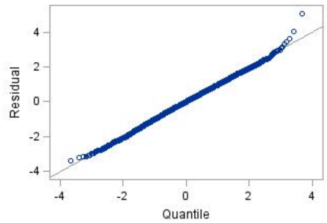
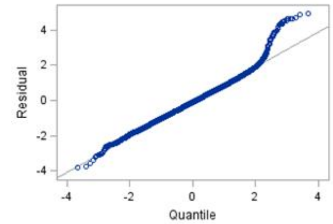
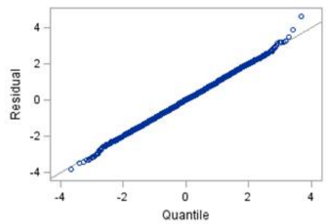
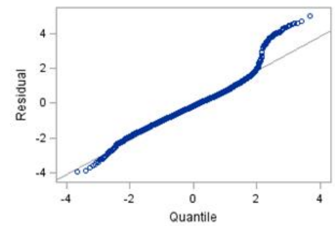
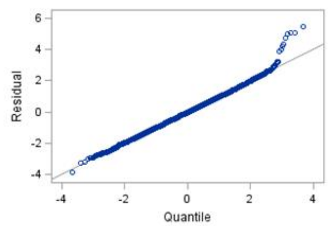
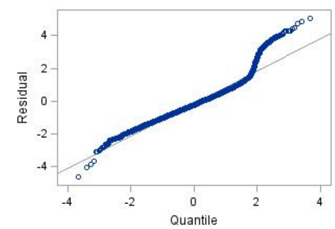
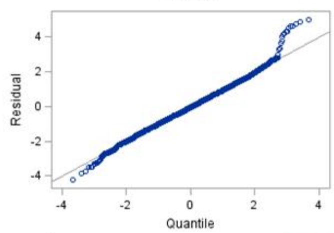
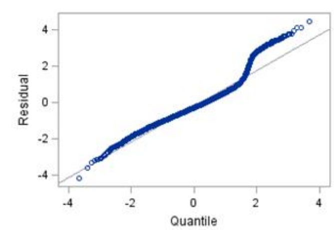
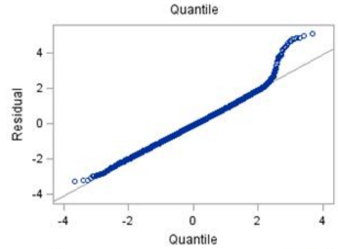
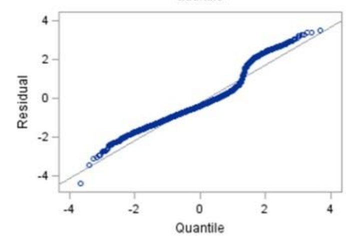
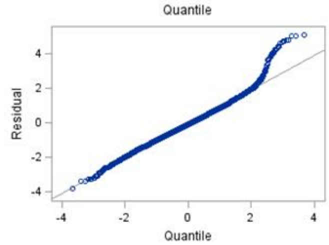
Percent	Q-Q plots	Percent	Q-Q plots
.04%		1.00%	
.10%		2.00%	
.20%		3.00%	
.40%		5.00%	
.60%		10.00%	
.80%			

Figure 4. Q-Q plots for the Scaled Residuals with 1 Replication of 3 SD on Sample Size 5000.

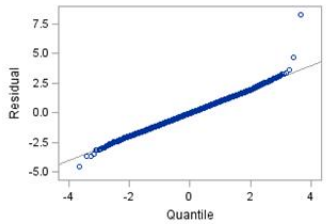
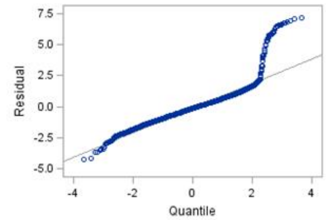
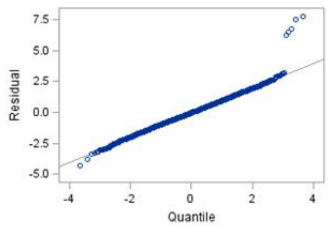
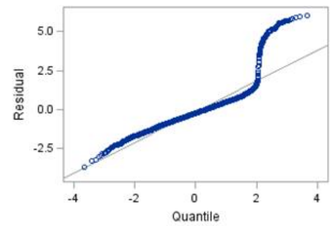
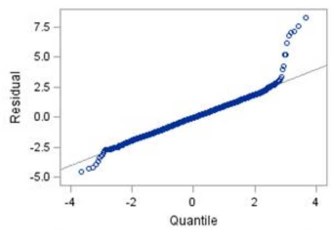
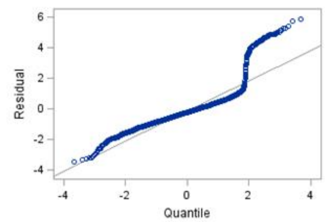
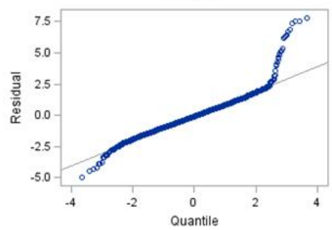
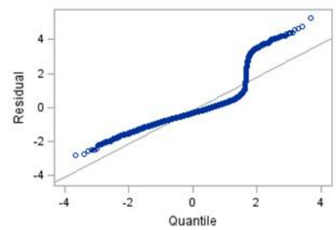
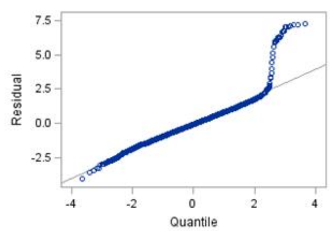
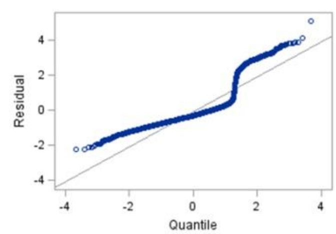
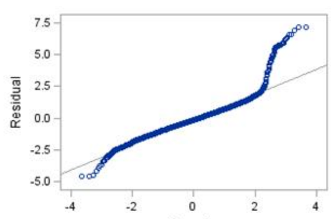
Percent	Q-Q plots	Percent	Q-Q plots
.04%		1.00%	
.10%		2.00%	
.20%		3.00%	
.40%		5.00%	
.60%		10.00%	
.80%			

Figure 5. Q-Q plots for the Scaled Residuals with 1 Replication of 5 SD on Sample Size 5000.

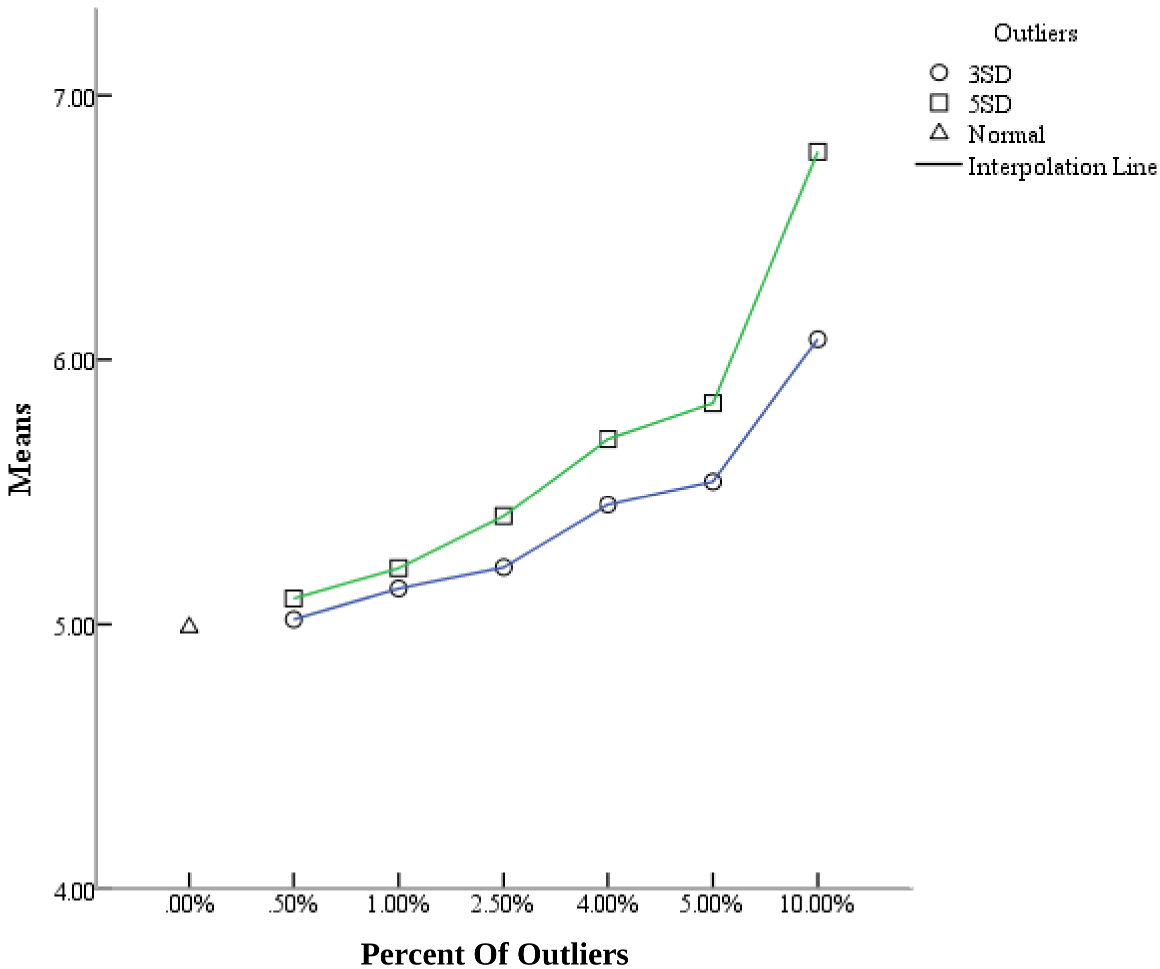


Figure 6. Means of the Dependent Variable of Sample Size 200.

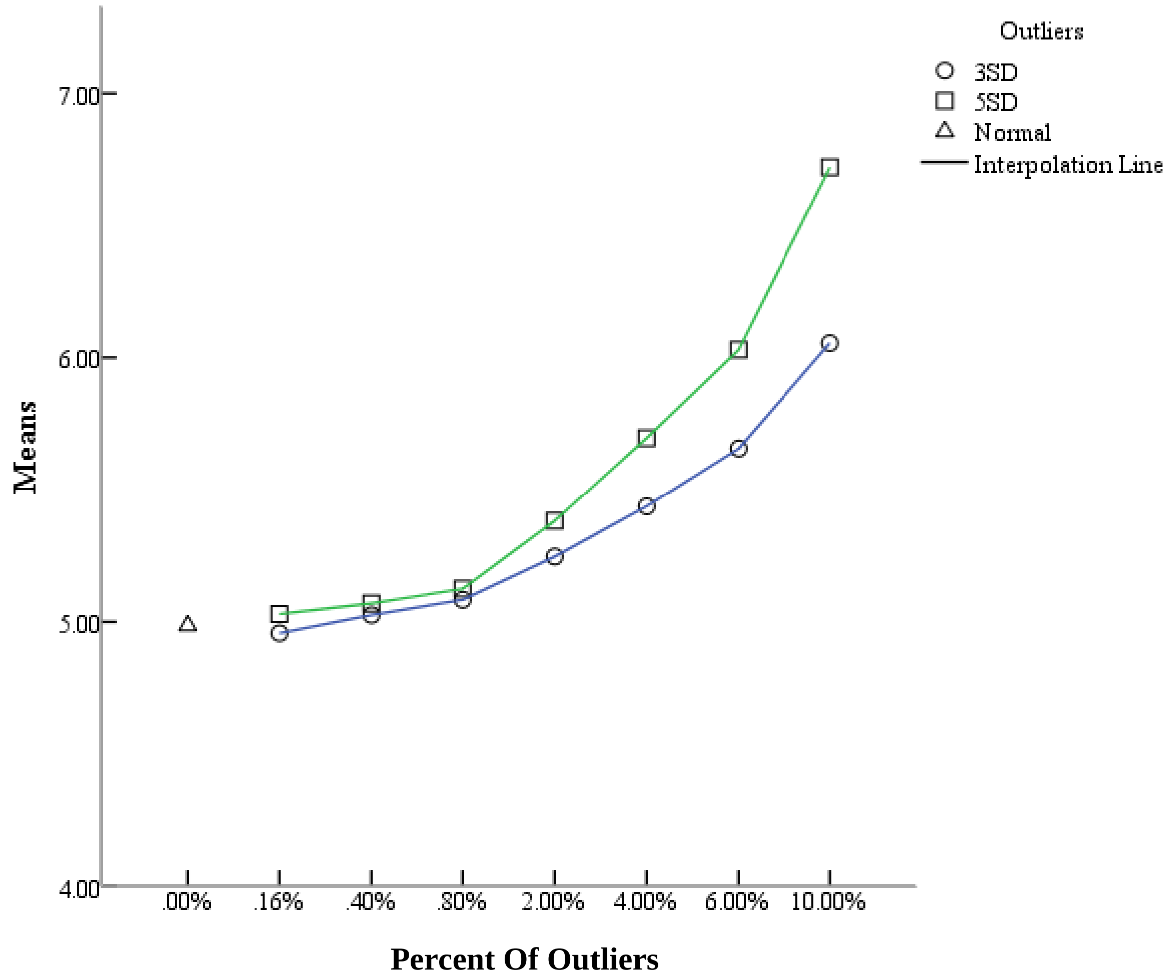


Figure 7. Means of the Dependent Variable of Sample Size 1250.

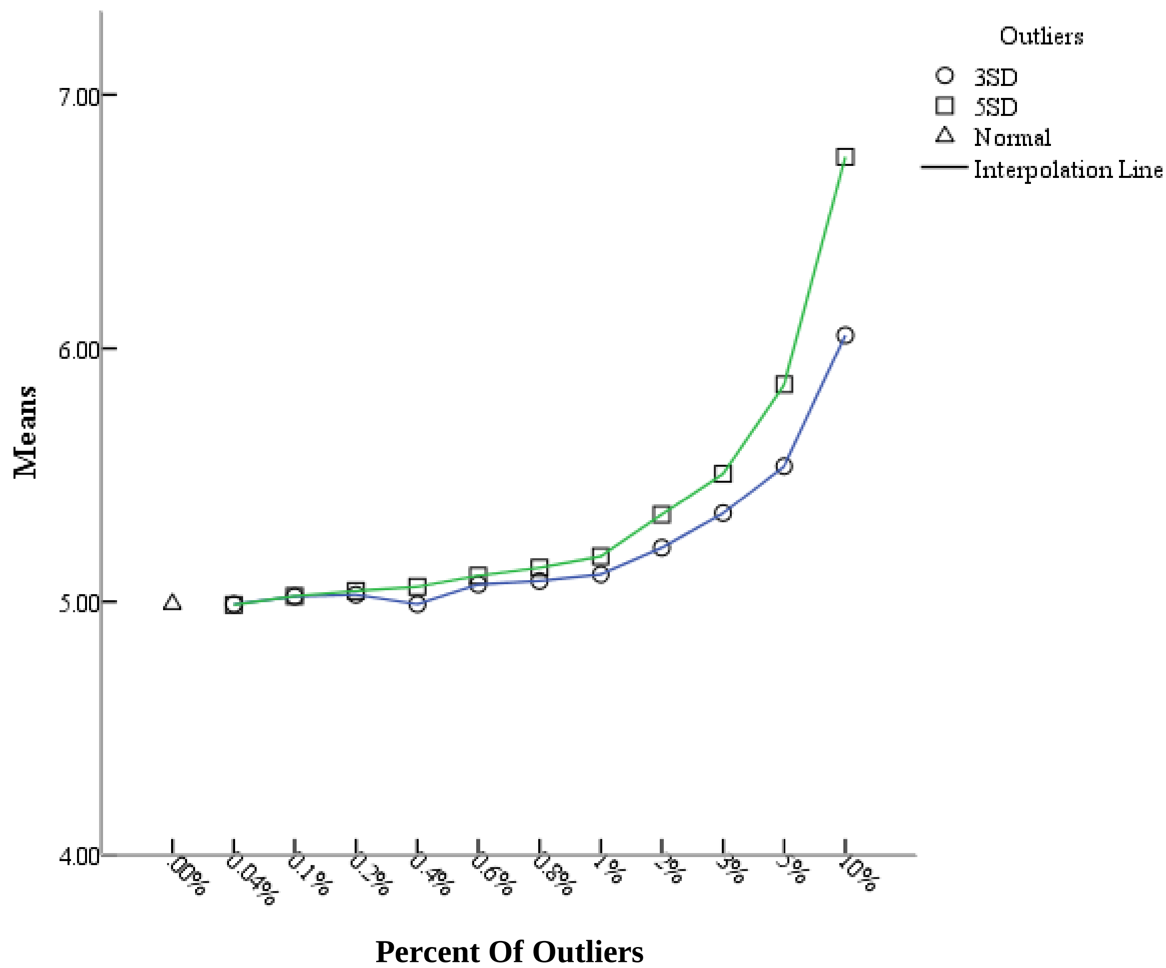


Figure 8. Means of the Dependent Variable of Sample Size 5000.

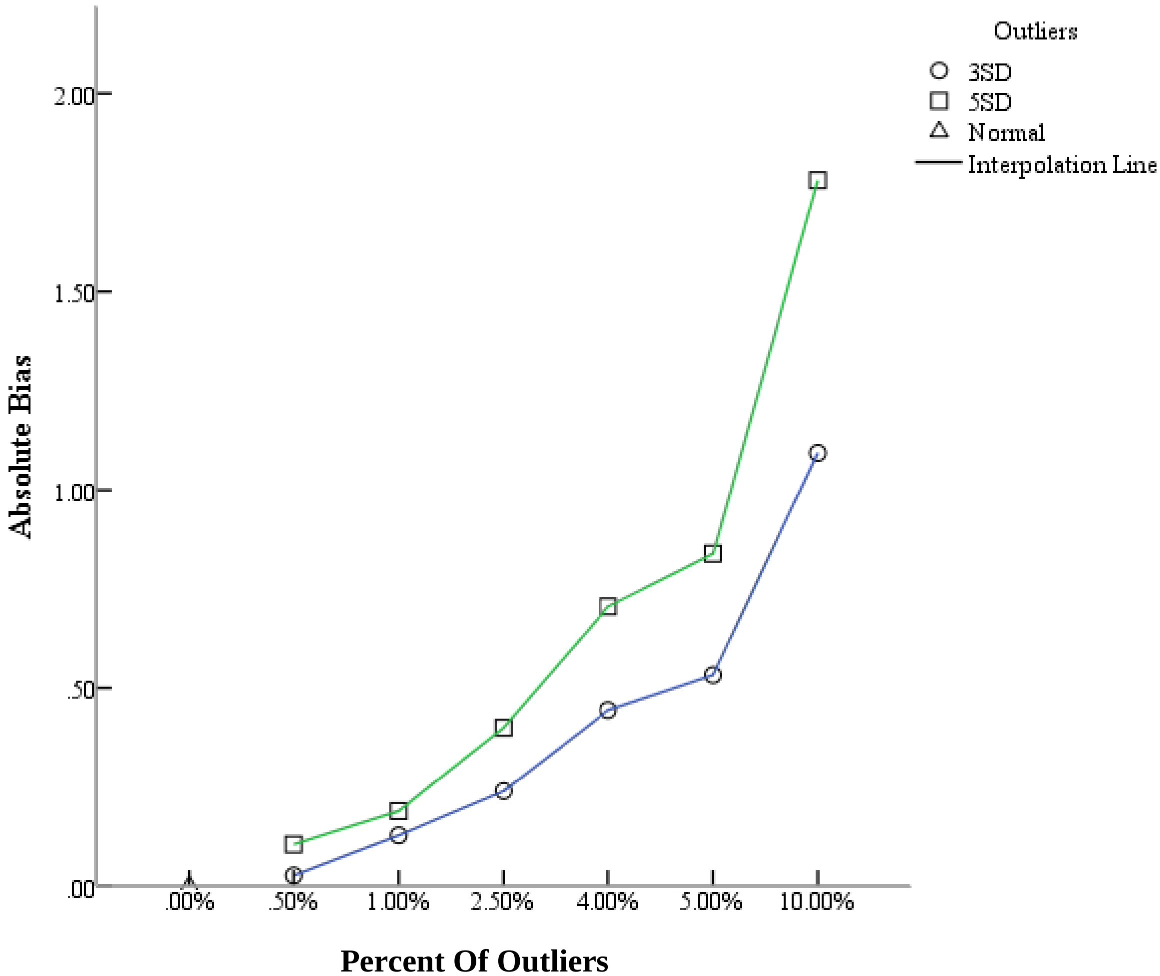


Figure 9. Absolute Bias of the Estimate of γ_{00} of Sample Size 200.

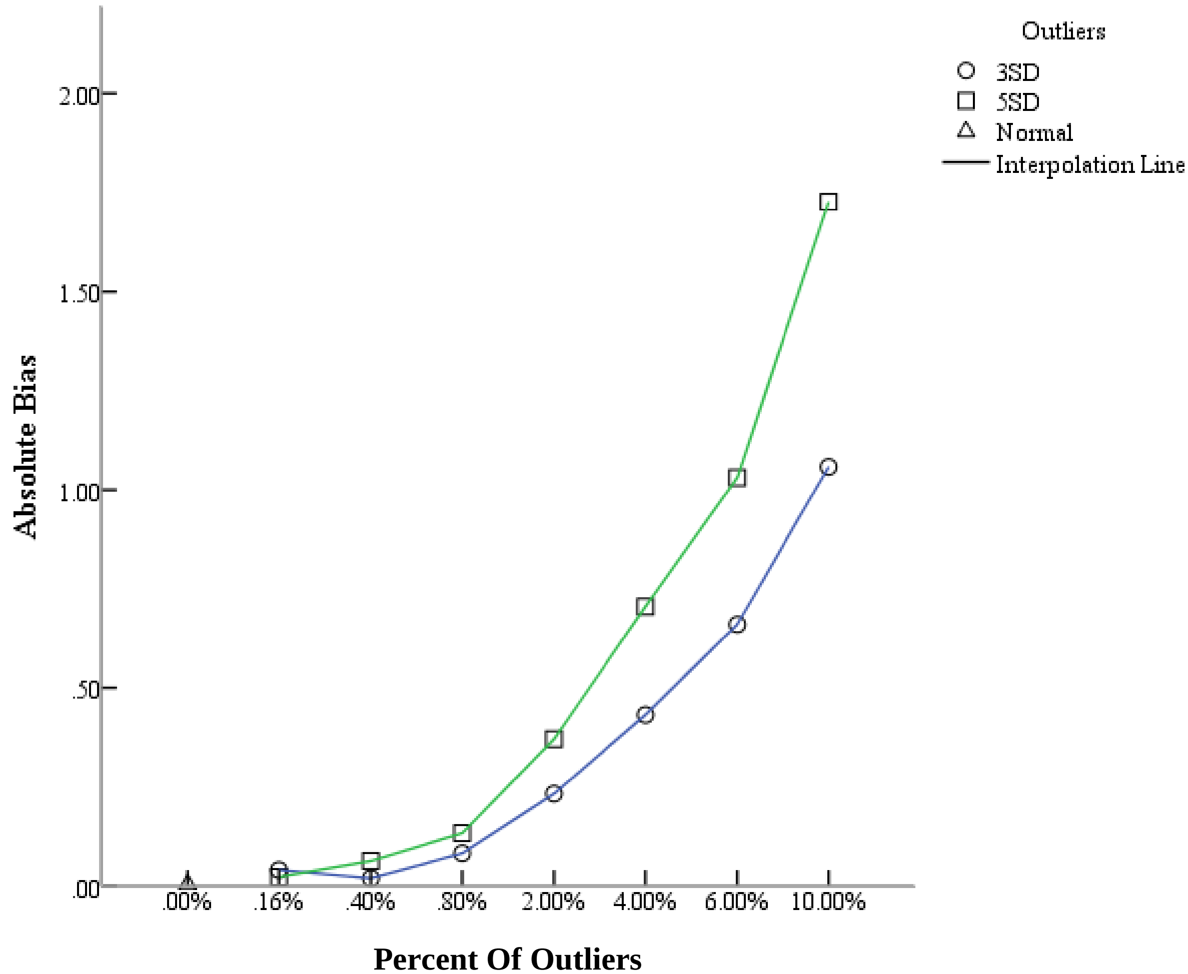


Figure 10. Absolute Bias of the Estimate of γ_{00} of Sample Size 1250.

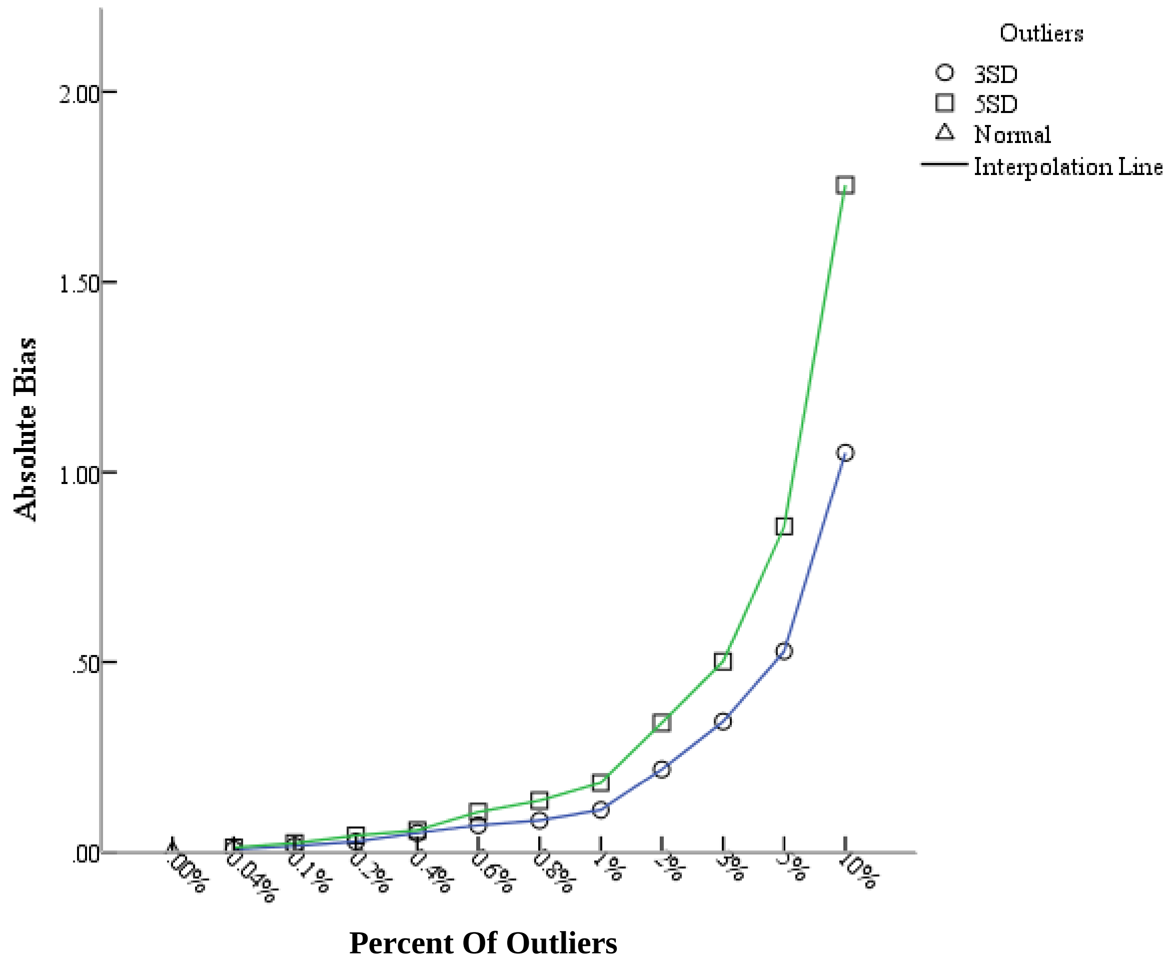


Figure 11. Absolute Bias of the Estimate of γ_{00} of Sample Size 5000.

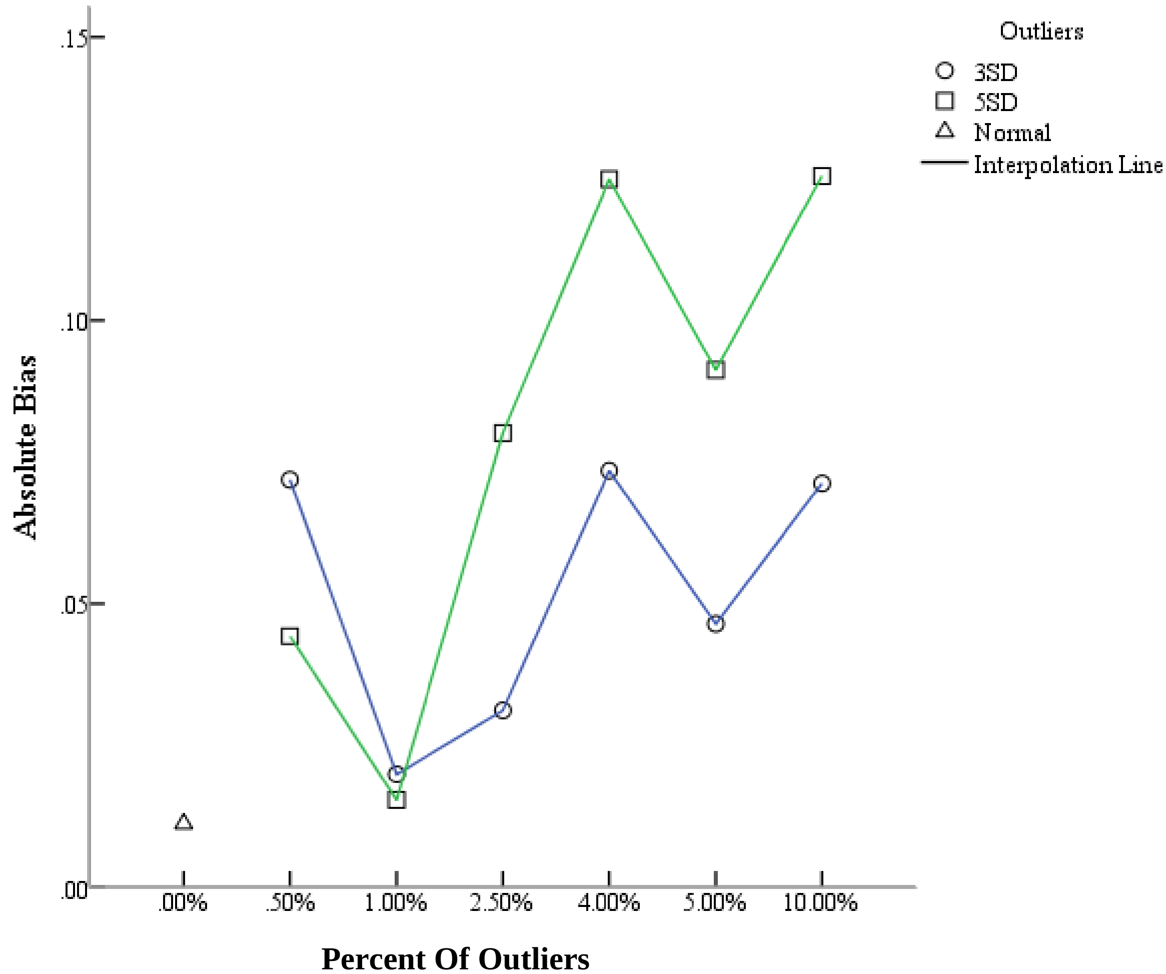


Figure 12. Absolute Bias of the Estimate of γ_{10} of Sample Size 200.

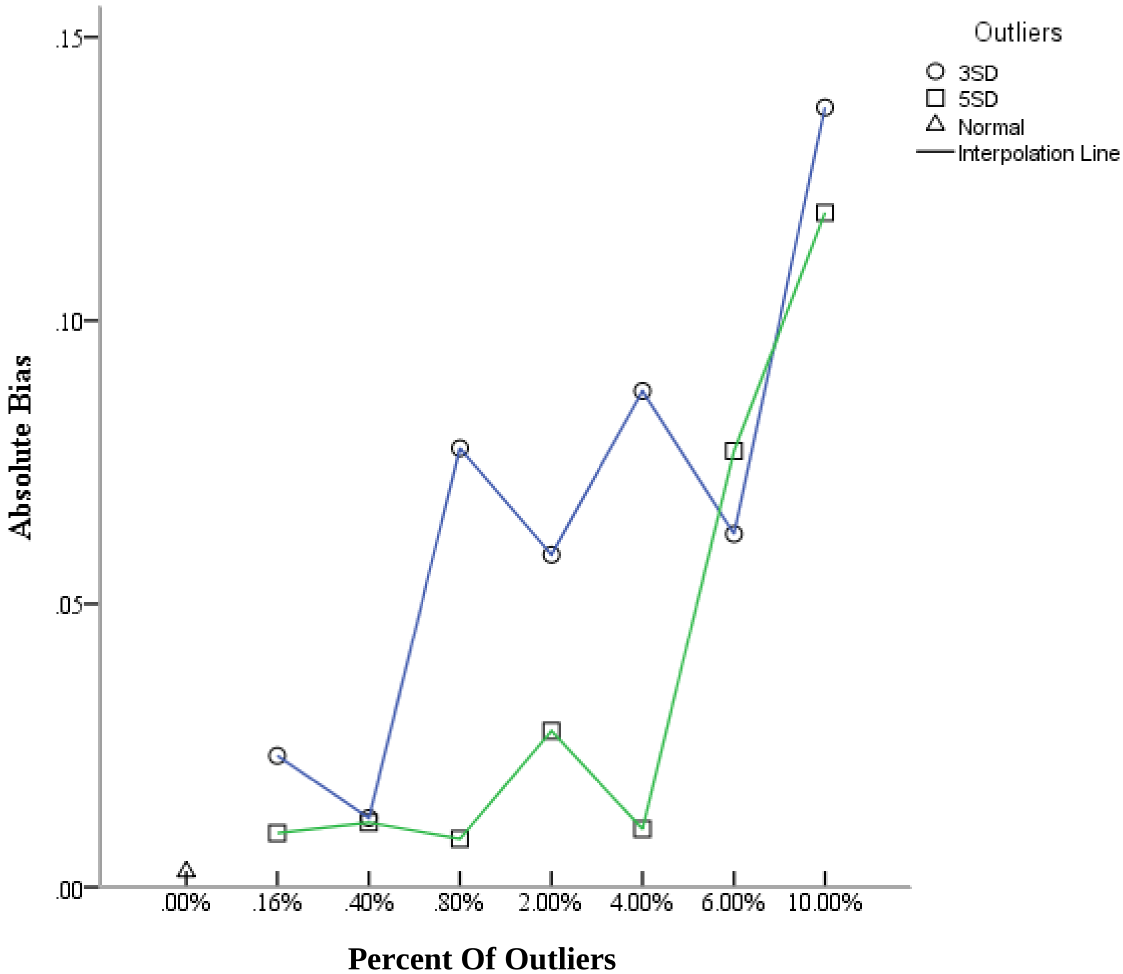


Figure 13. Absolute Bias of the Estimate of γ_{10} of Sample Size 1250.

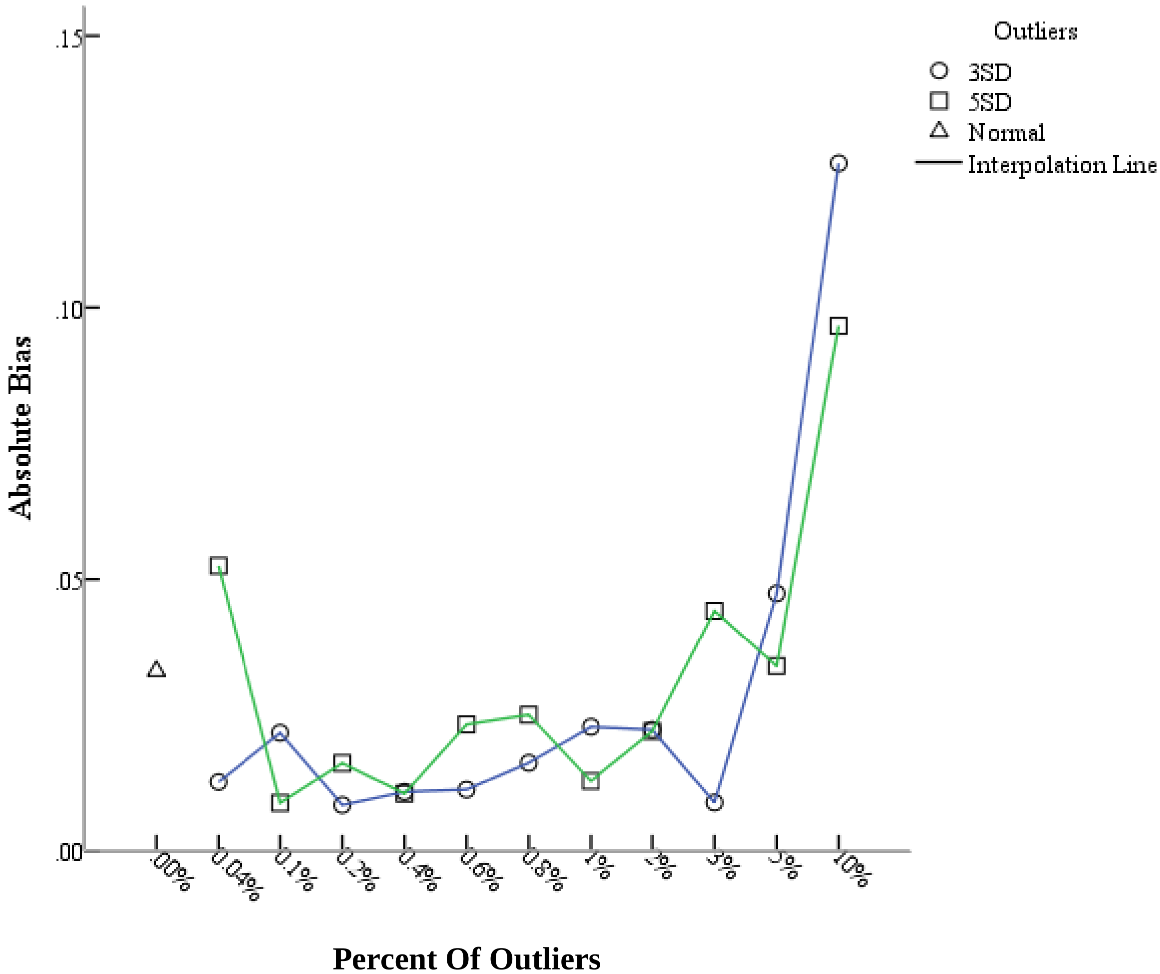


Figure 14. Absolute Bias of the Estimate of γ_{10} of Sample Size 5000.

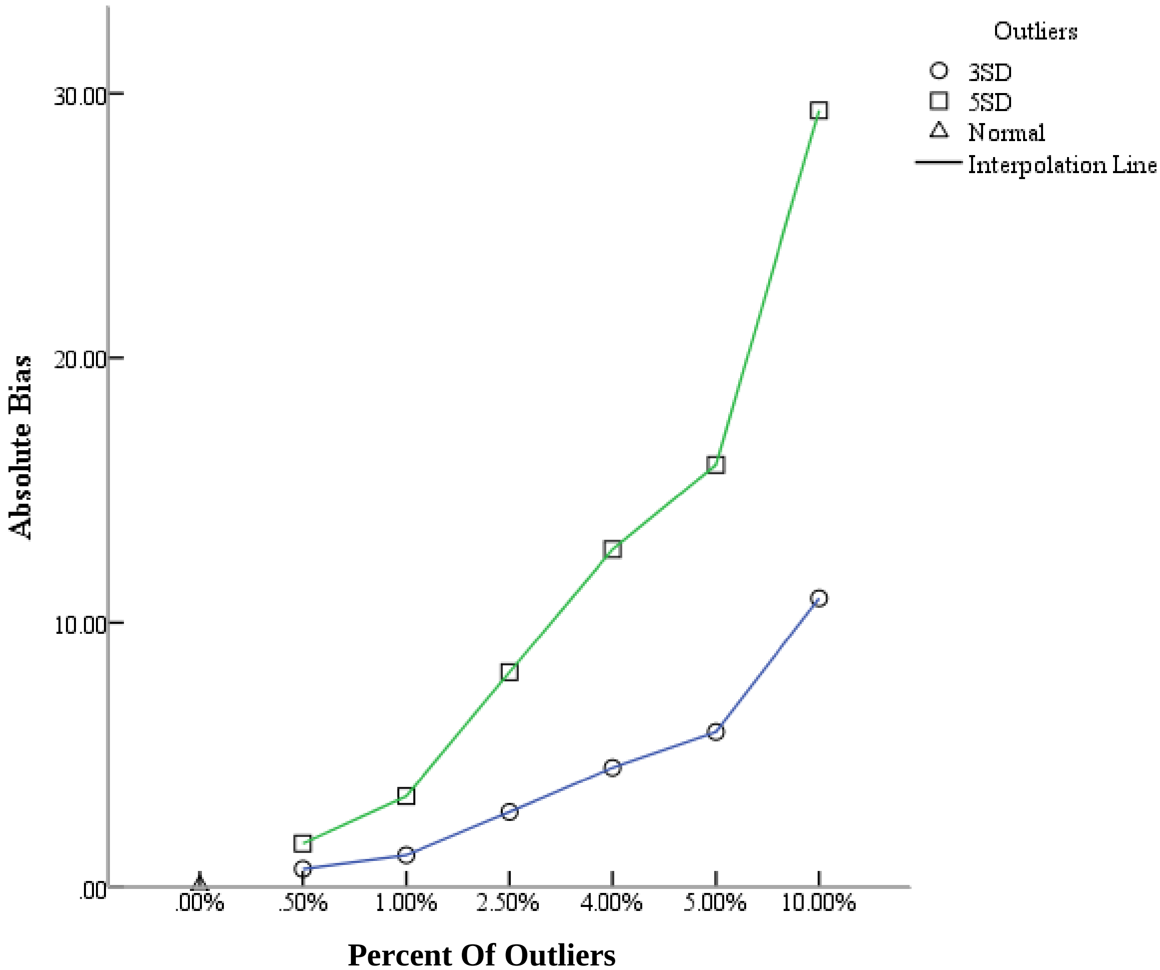


Figure 15. Absolute Bias of the Estimate of σ^2 of Sample Size 200.

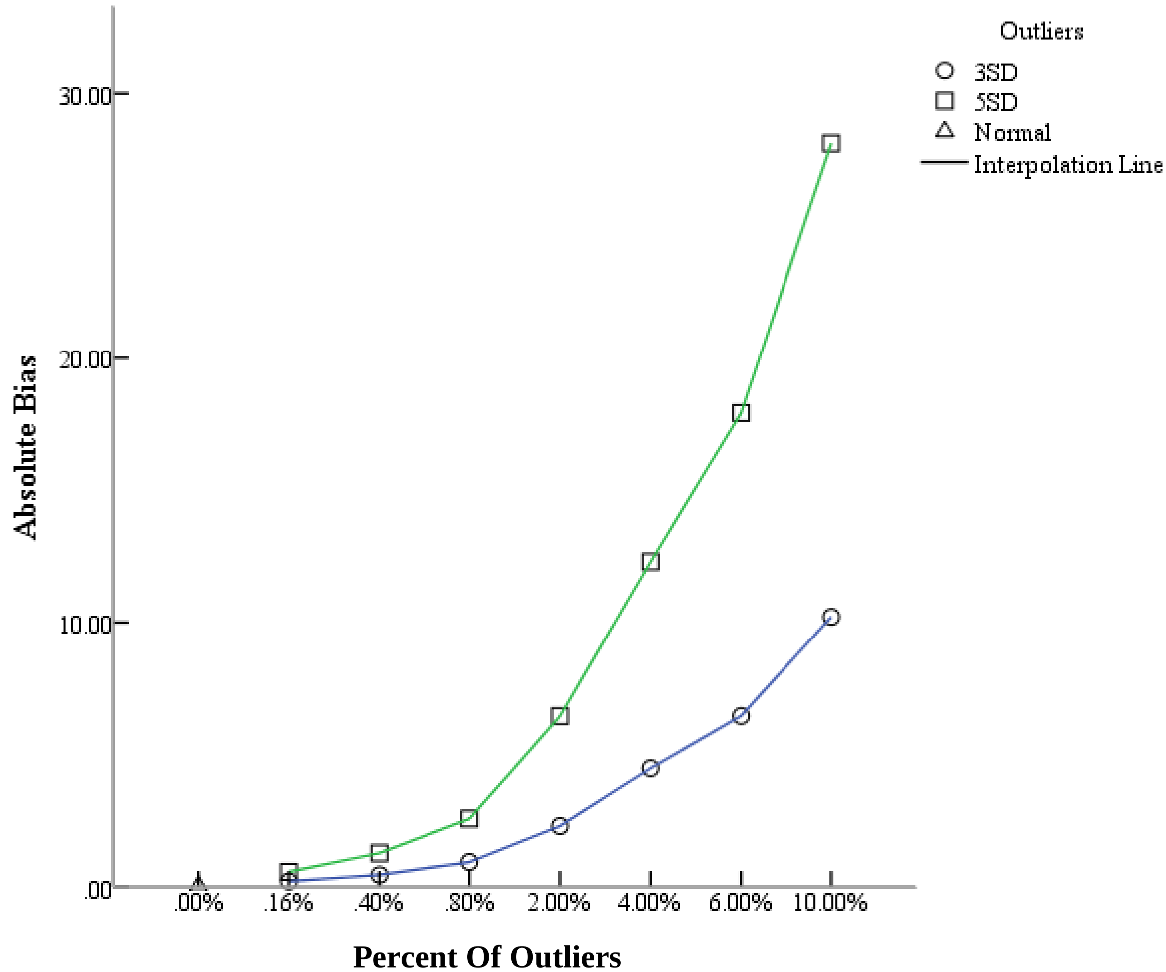


Figure 16. Absolute Bias of the Estimate of σ^2 of Sample Size 1250.

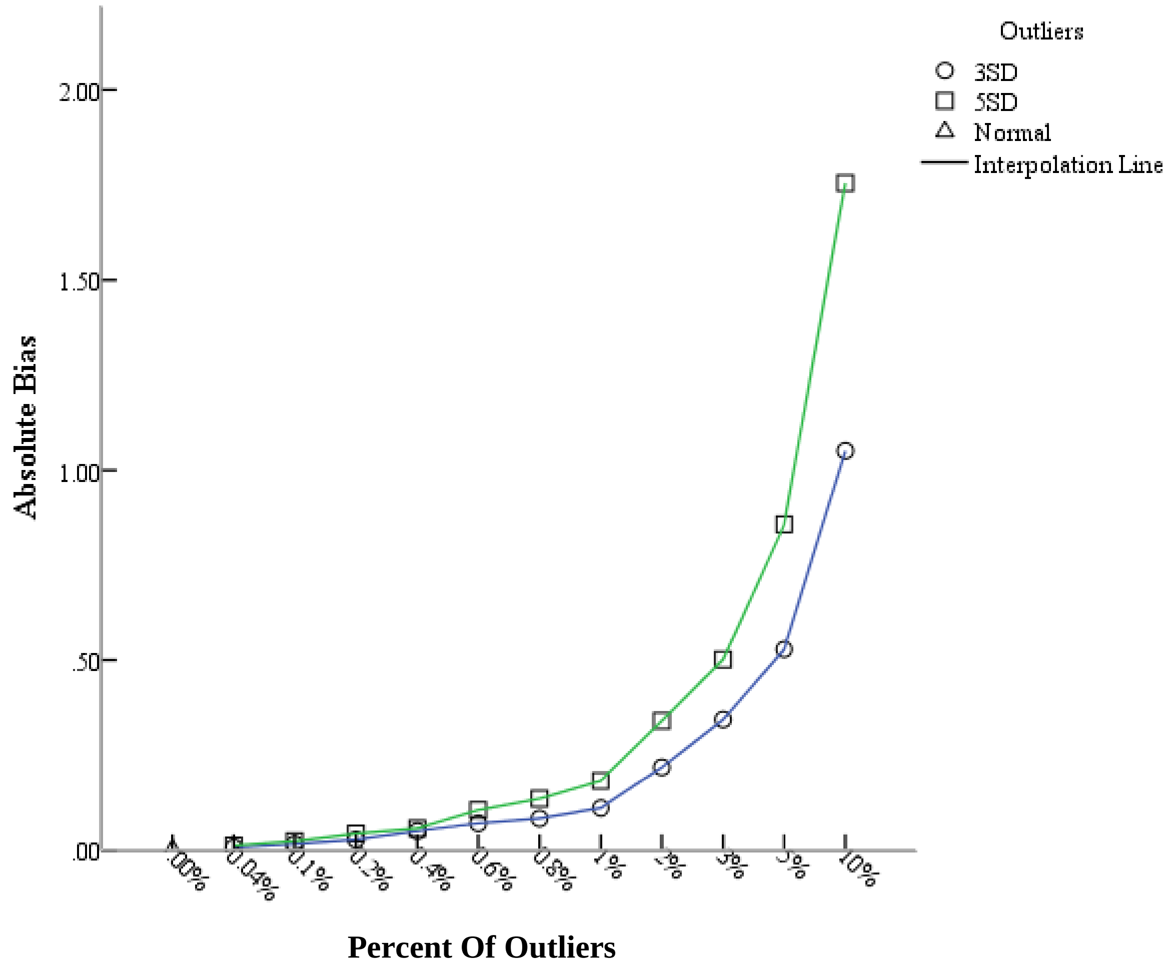


Figure 17. Absolute Bias of the Estimate of σ^2 of Sample Size 5000.

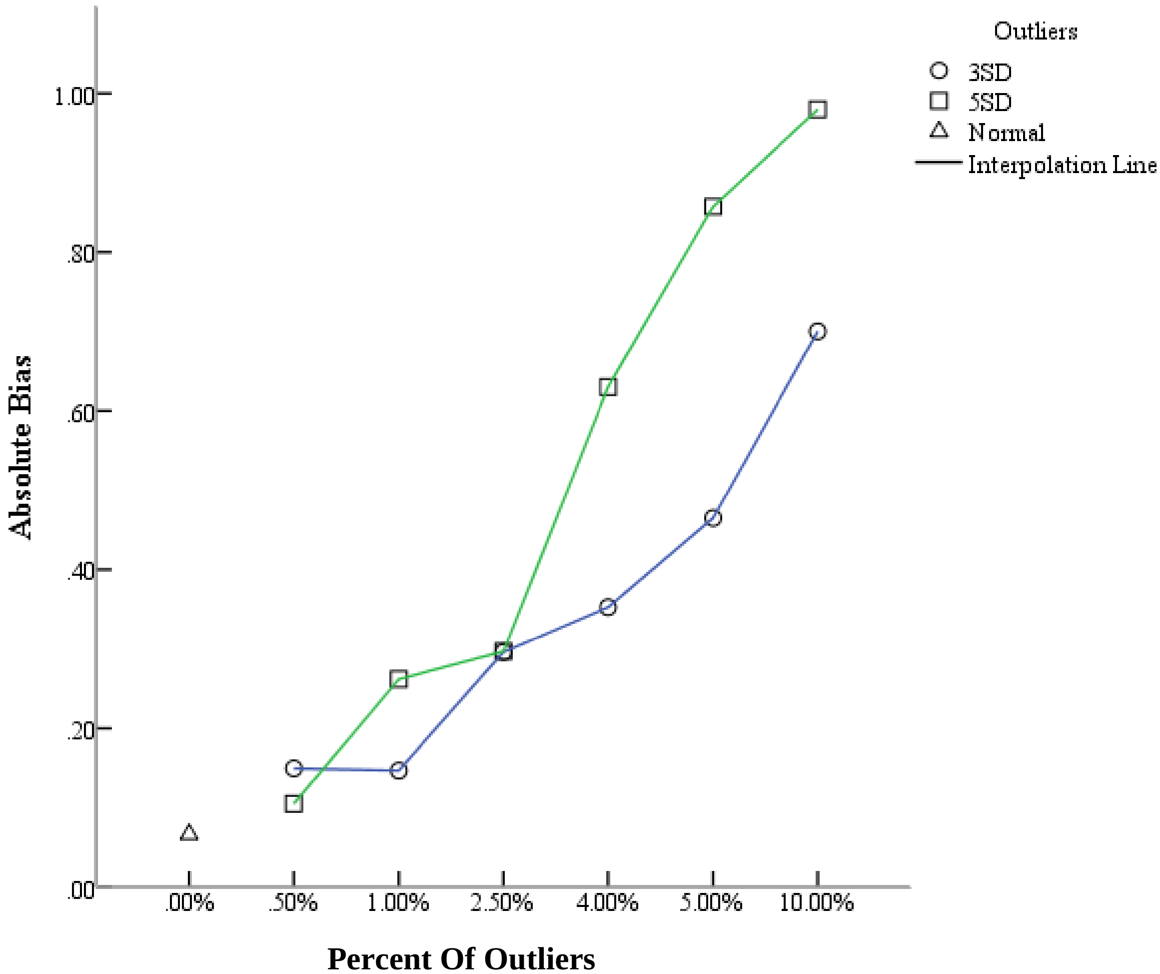


Figure 18. Absolute Bias of the Estimate of τ_{00} of Sample Size 200.

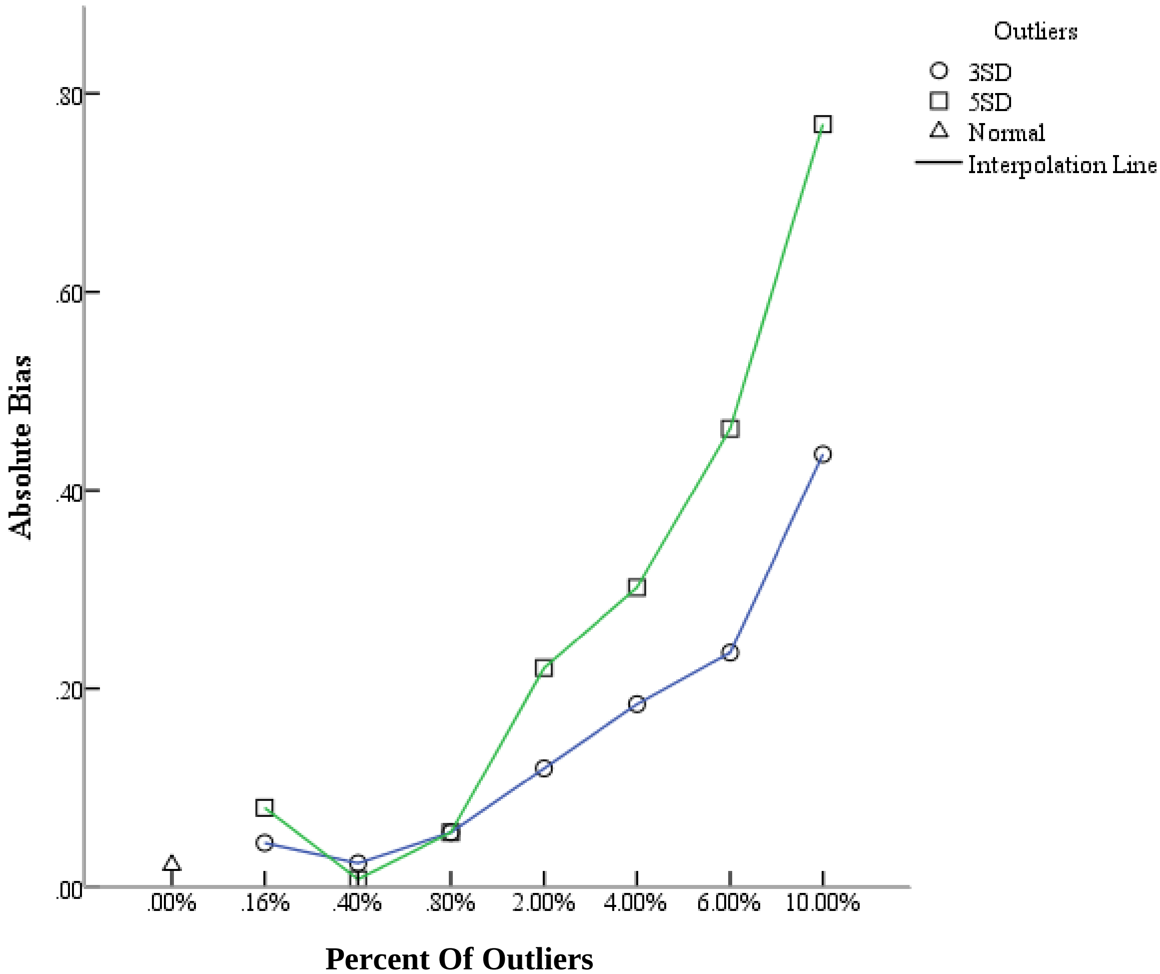


Figure 19. Absolute Bias of the Estimate of τ_{00} of Sample Size 1250.

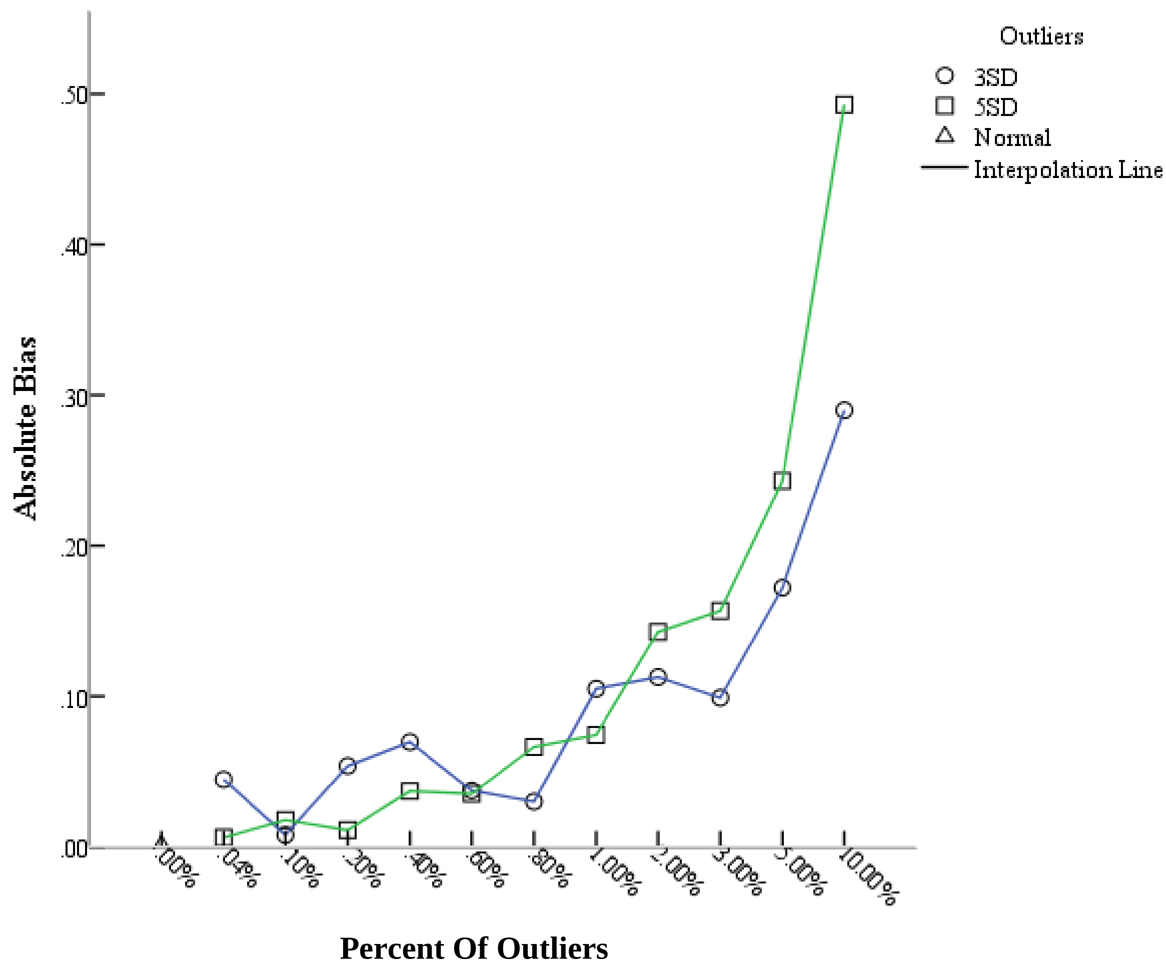


Figure 20. Absolute Bias of the Estimate of τ_{00} of Sample Size 5000.

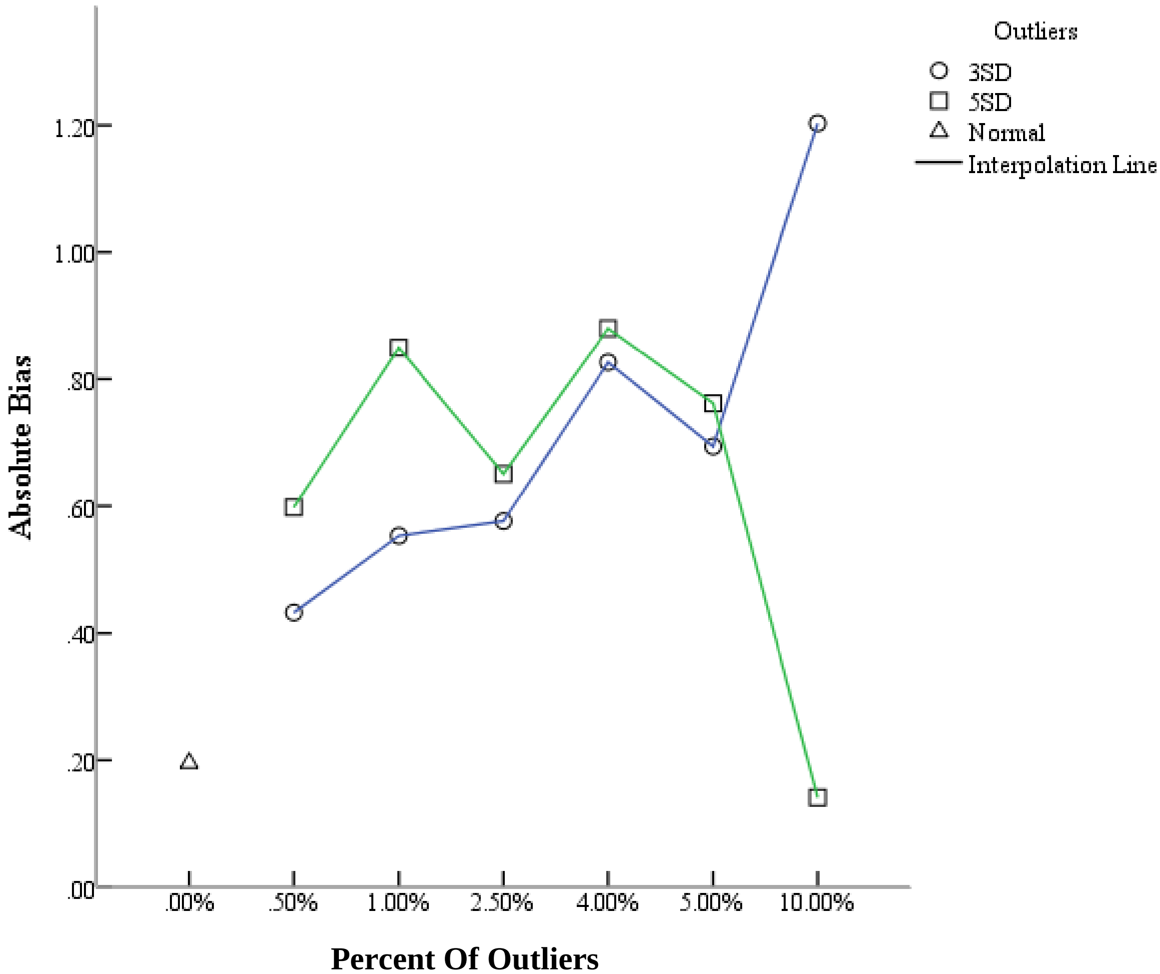


Figure 21. Absolute Bias of the Estimate of τ_{11} of Sample Size 200.

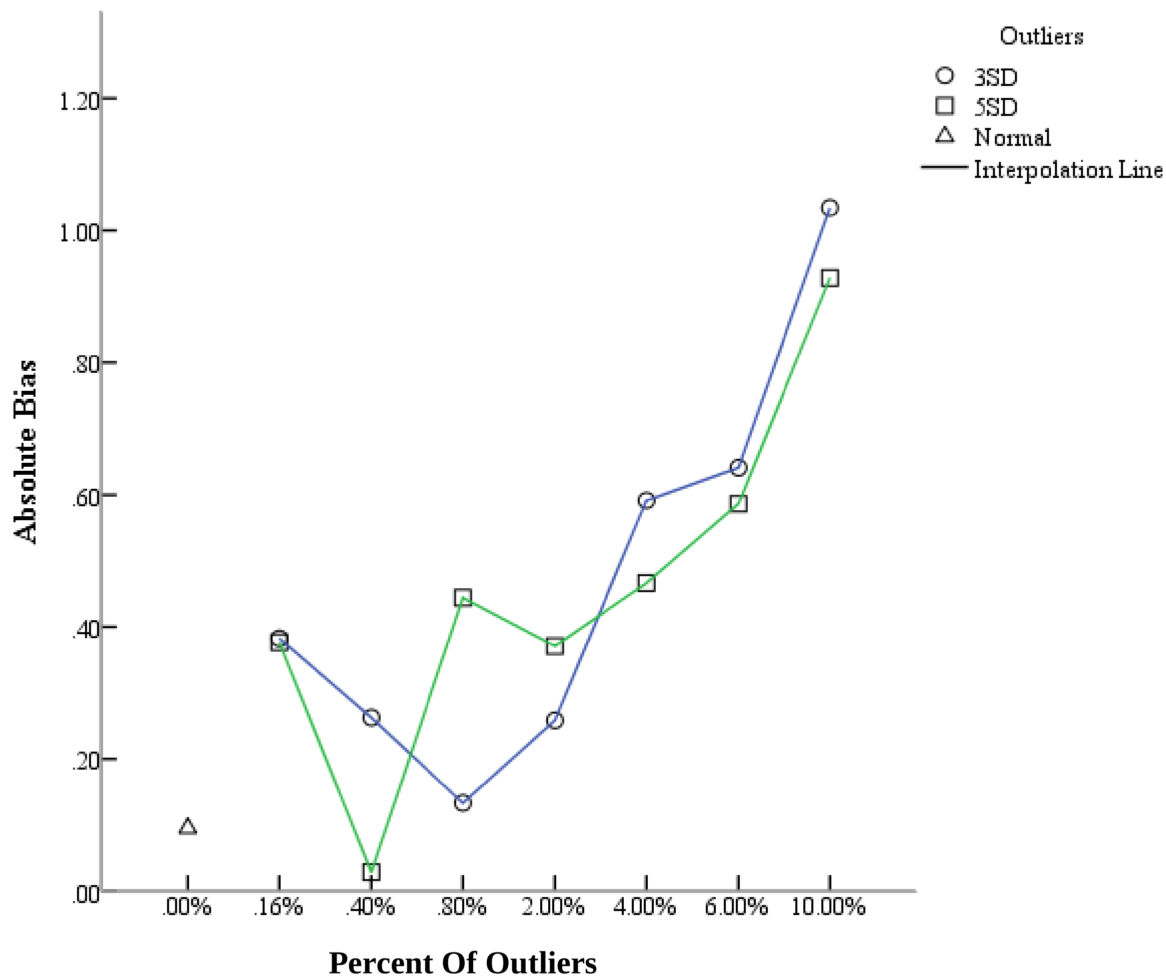


Figure 22. Absolute Bias of the Estimate of τ_{11} of Sample Size 1250.

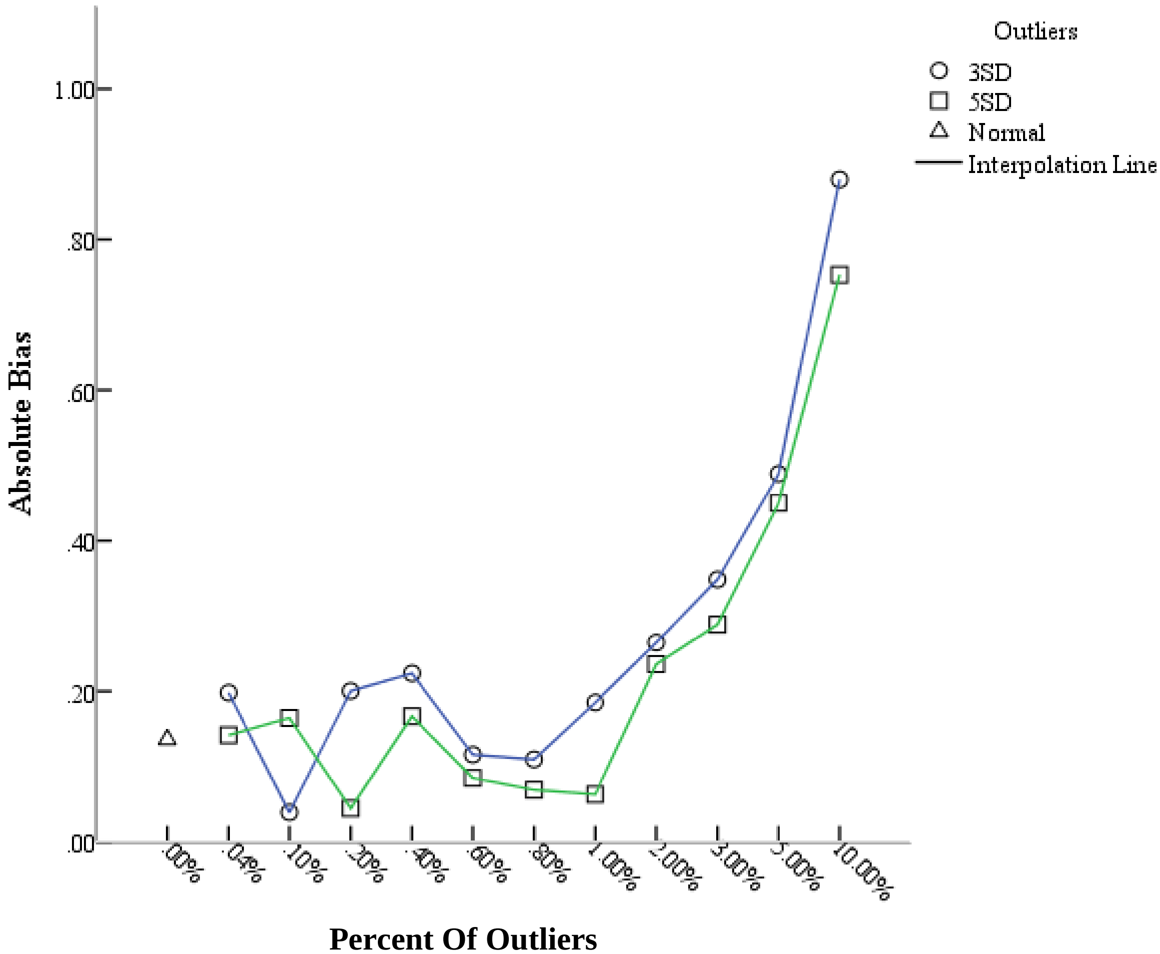


Figure 23. Absolute Bias of the Estimate of τ_{11} of Sample Size 5000.

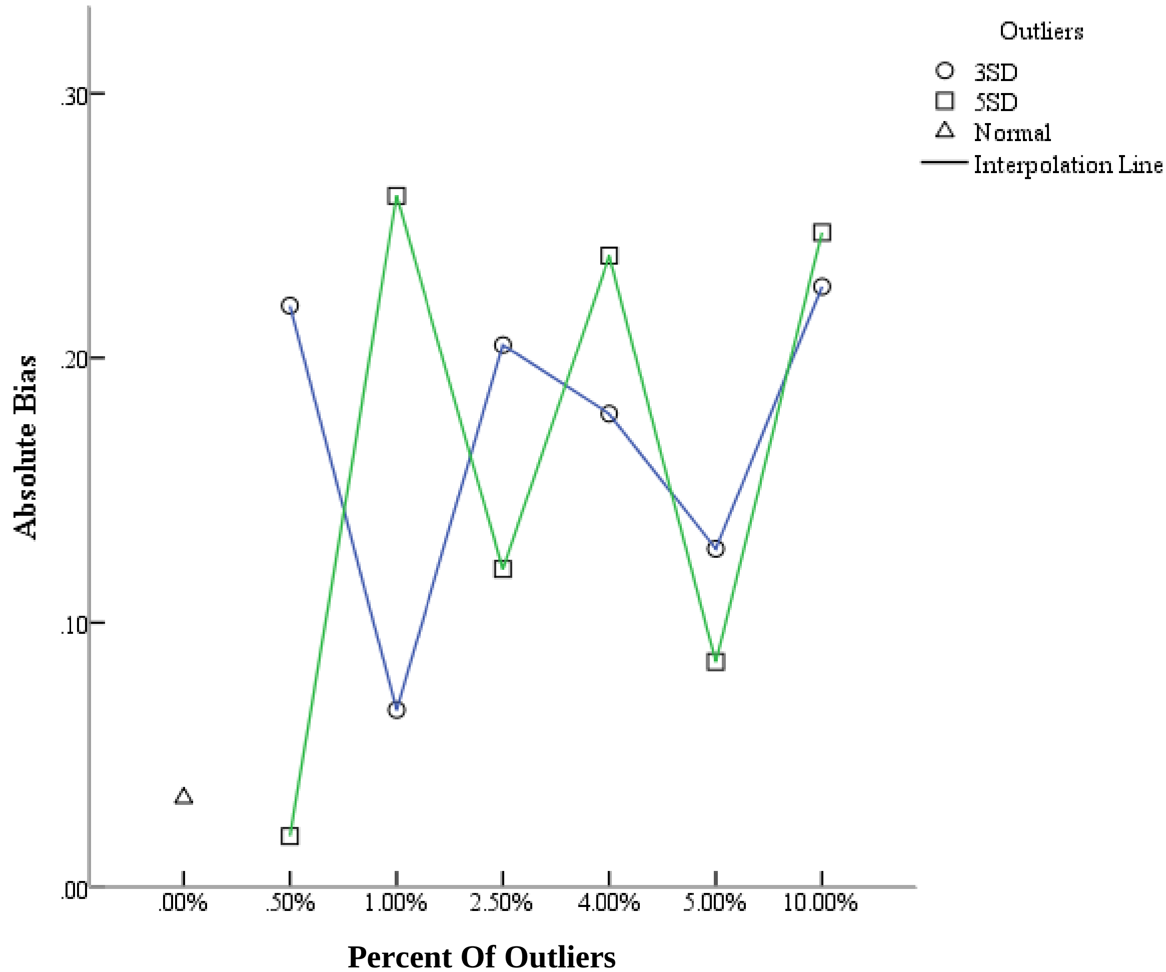


Figure 24. Absolute Bias of the Estimate of τ_{01} of Sample Size 200.

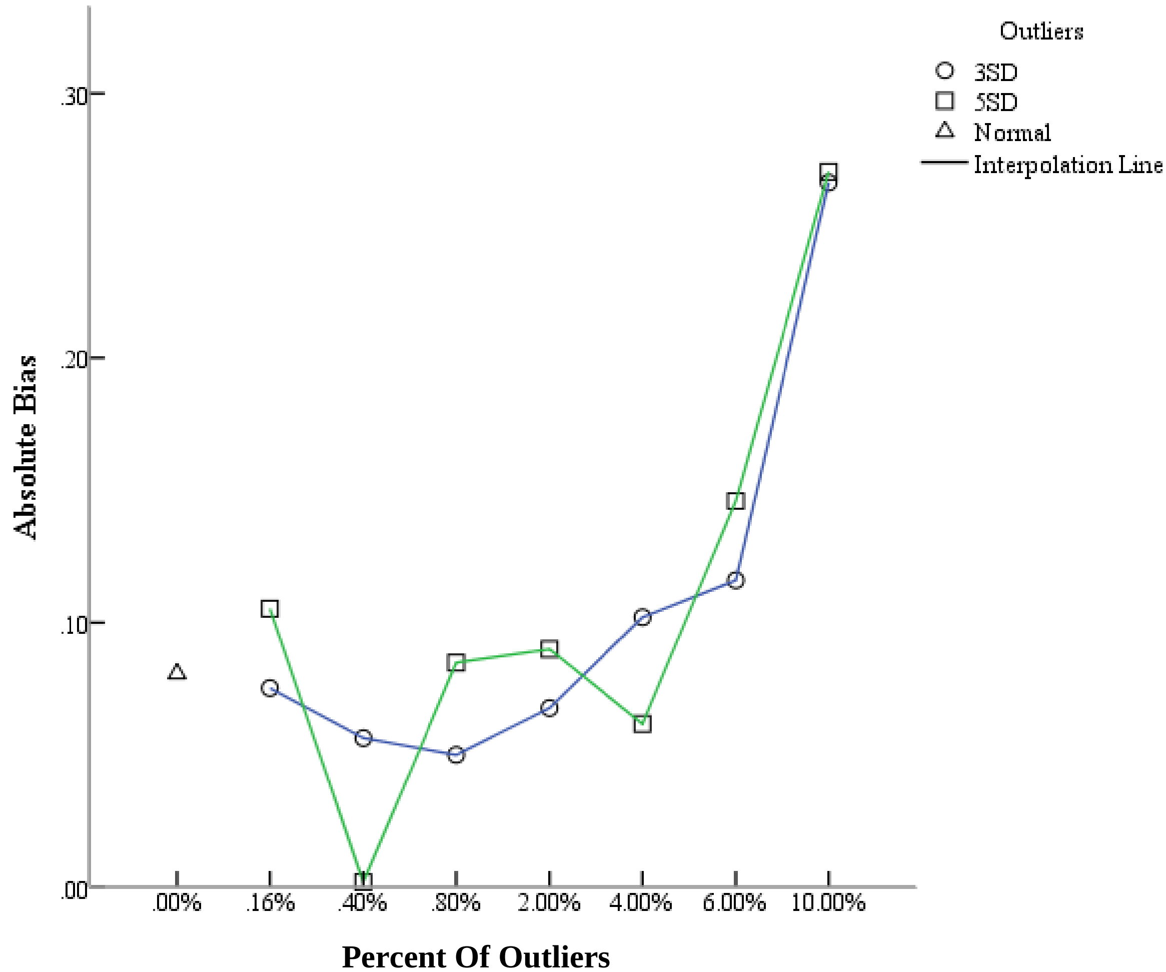


Figure 25. Absolute Bias of the Estimate of τ_{01} of Sample Size 1250.

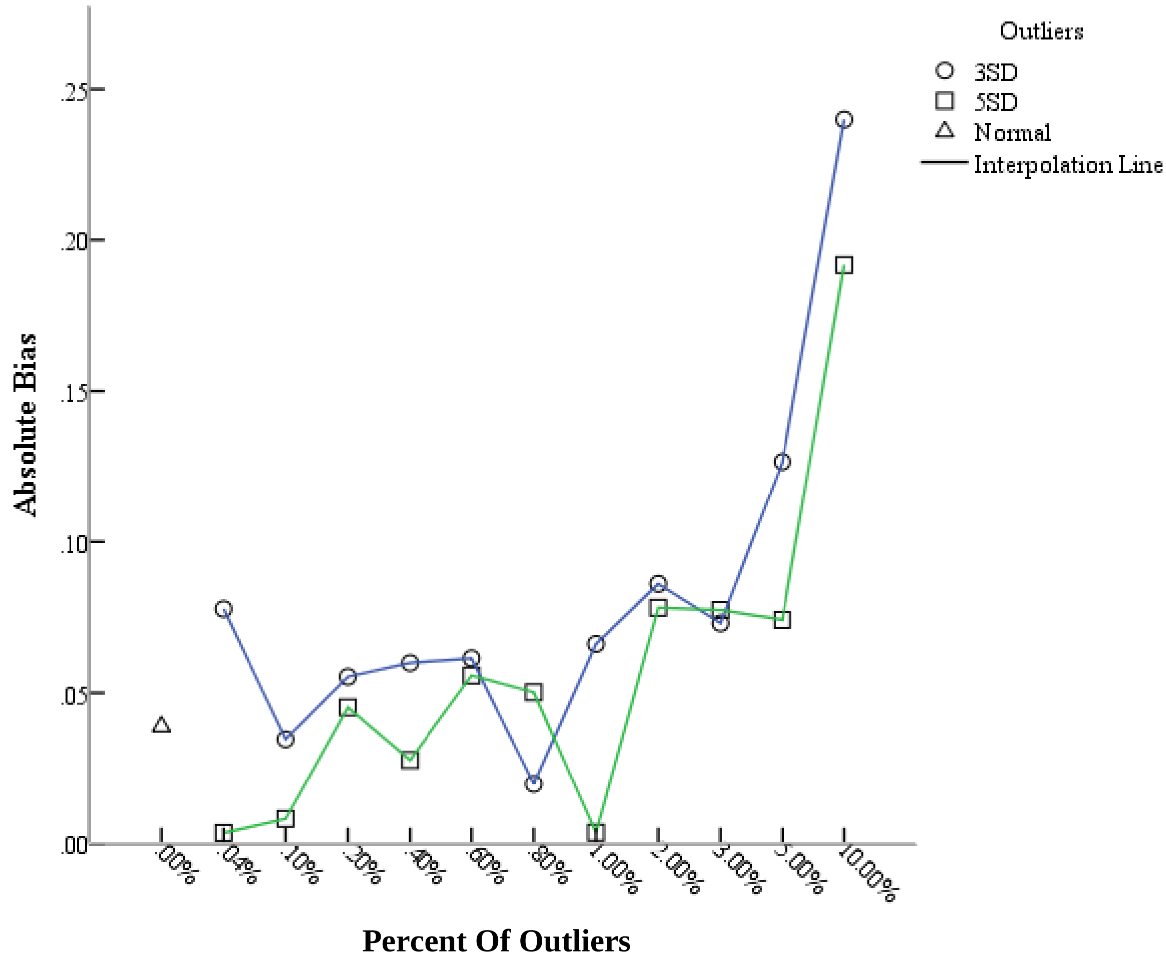
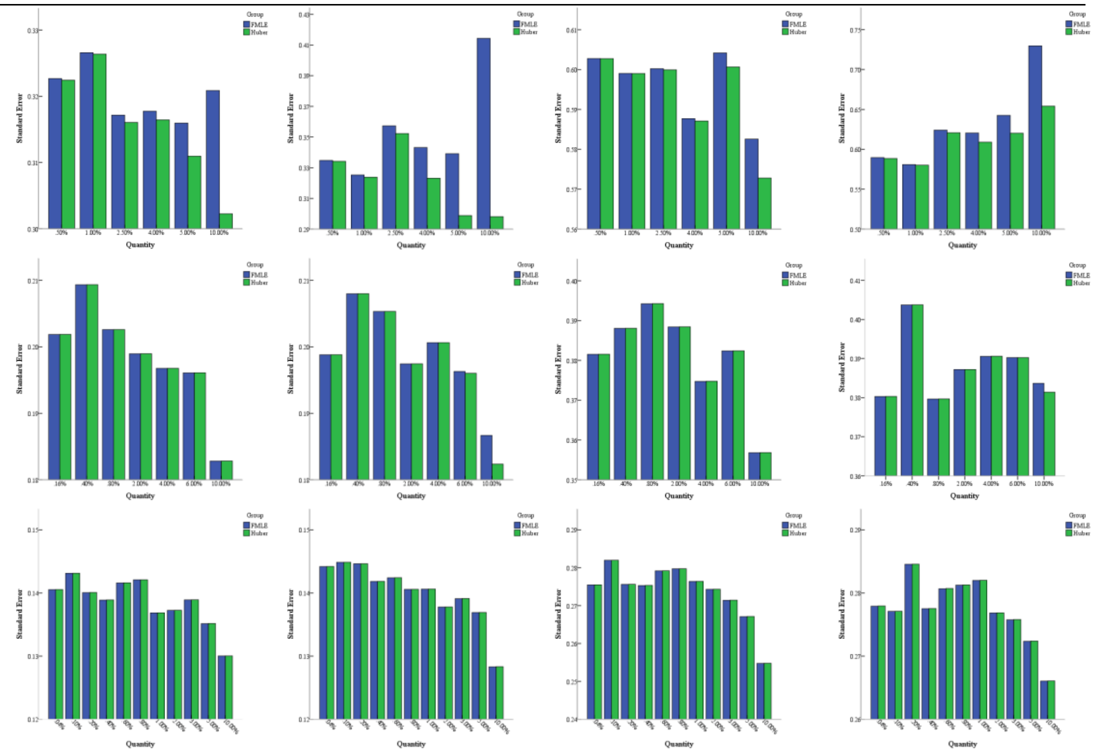


Figure 26. Absolute Bias of the Estimate of τ_{01} of Sample Size 5000.

N	Standard error of γ_{00} 3 SD	Standard error of γ_{00} 5 SD	Standard error of γ_{01} 3 SD	Standard error of γ_{01} 5 SD
200				
1250				
5000				



Notes: N denotes sample size. 3 SD and 5 SD are the types of outliers.

Figure 27. Compare the FMLE and Huber-Sandwich Estimator on Standard Errors Estimation.