PROBLEMATIZING SPACE AND PERSPECTIVE: A MIDDLE SCHOOL MATHEMATICS EXPERIENCE

by

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(Under the Direction of Theodore J. Kopcha)

ABSTRACT

This dissertation contains three articles accompanied by an introduction and conclusion chapter. The first article, *Chapter Two: Middle School Learners' Ontological 'Trying-on' of Dimensions: A Phenomenological Investigation,* details a post-intentional phenomenological research study with 5th and 6th grade learners and serves as the pilot study connecting the three articles. The second article, *Chapter Three: The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site*, provides the theoretical and design framework guiding the design of a hypermedia site used with learners in a second iteration of the design-based research agenda with *Space and Perspective*. The third article, *Chapter Four: Productive Problematization: Moments When Perspectives Shift in Middle School Mathematics,* presents findings from a post-intentional phenomenological study with 21 eight-grade learners investigating *Space and Perspective* with the hypermedia site. The study aimed to understand what it is like for learners to experience a shift in perspective.

INDEX WORDS: Design-based research, post-intentional phenomenological research, middle school education, mathematics education, space, perspective, shift, problematization

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SPACE AND PERSPECTIVE: THE DESIGN AND EXPERIENCE OF MIDDLE SCHOOL LEARNERS CONFRONTED WITH CASES AS ALTERNATIVE PERSPECTIVE

by

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DEDICATION

I dedicate this dissertation to my parents, Peggy Cochran Daniel and Anthony Michael Duncan, my husband, Patrick Corey Valentine, and my son, Simon “Blue” Alistar Valentine. Collectively, you brought me into this world and told me I could do whatever I set my mind to.

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CHAPTER 1

INTRODUCTION

> Yet I exist in the hope that these memoirs, in some manner, I know not how, may find their way to the minds of humanity in Some Dimension, and may stir up a race of rebels who shall refuse to be confined to limited Dimensionality. (Abbott, 1991, p. 102)

Introduction

Reflecting upon my time as a middle school teacher, the activity of "stirring up" rebellions remains the memorable part, especially in the geometric sense that Edwin A. Abbott expresses in the book, *Flatland: A Romance of Many Dimensions* (1884/1991). When any of us recall pivotal moments from our early schooling, they are usually related to an artifact from a long project, a teacher taking the time to entertain our curiosity, or maybe even contemplating a problem with no known answer. This dissertation project evolved in large part from those moments and seeks to help others see "an iterative exploration of the possibility space of designs" (Jacobson & Kapur, 2012, p. 312). The "possibility space" in this project refers to the variant ways we might engage learners in mathematics. The learners in this project were "a race of rebels" in a sense – remaining open to what space, perspective, and dimension might be rather than adhering to singular notions of Euclidean geometry and space. It is in this disruption of the normal, everyday being, that we (the learners and myself) co-constructed many satiable moments.

Disseminating the Silence

Listening to "The Sound of Silence" (Simon & Garfunkel, 1966) two days ago led me to think about the way I might go about introducing this dissertation project. I could tell you it was

the entire meaning of lyrics, but really, that's not how I hear songs. Similar to "This Man He Weeps Tonight" (The Kinks), I spent many drives to and from campus shaking my head almost like a Metallica song was playing in the car – something about the energy of The Kinks. I remember actually taking the time to listen to the words. Now the song makes me cry. It's an interesting phenomenon though, that the song isn't just the words or the music or their combination. It's more how we let it in. Back to "The Sound of Silence," I feel a little of both – the words and the music. The music makes my eyes swell up and the words make me want to act. It was interesting this morning then to find Paul Simon's (2014) words from his National Public Radio interview with Terry Gross:

> I think about songs that it's not just what the words say but what the melody says and what the sound says. My thinking is that if you don't have the right melody, it really doesn't matter what you have to say, people don't hear it. They only are available to hear when the sound enters and makes people open to the thought. Really the key to 'The Sound Of Silence' is the simplicity of the melody and the words, which are youthful alienation…it resonated with millions of people. Largely because it had a simple and singable melody. (para. 5)

My aim similarly is to create "a simple and singable melody" in this work that has had a profound impact on me as a scholar and educator. Many dreams arriving during the night have found their way in these pages that follow – something similar to this passage:

> Hello darkness, my old friend,
> I've come to talk with you again,
> Because a vision softly creeping,
> Left its seeds while I was sleeping,
> And the vision that was planted in my brain
> Still remains
> Within the sound of silence.

My hope is that this vision doesn't always remain silent. And similar to Edwin A. Abbott (1991), "I exist in the hope that these memoirs…may stir up a race of rebels who shall refuse to be confined to limited Dimensionality" (p. 102). Stirring up rebellion on the part of adolescents is a fun and stimulating activity, but I've been more reticent to stir up my committee. I do, however, ask that you bring an attitude of openness to the fore when you consider the phenomenon of learners' problematizing space and moments when they find their perspective shifting.

What's Your Passion?

The question, "What's your passion?" probably comes up more than most others in academic settings (e.g., conferences, interviews, friends and family). It's taken me a while to articulate my passion in a way that ties together the theory, context, and method chosen for my study. My research is driven by a passion to productively problematize the taken-for-granted aspects of being in the world in an attempt to counter reductive thoughts and support a space where art, problem solving, and innovation spring forth. I value blocks in the road, impasse, struggle, but in a productive way and in a subject (mathematics) that is notorious for heartache and headache. My passion stems from my first teaching experience in Clarke County. My fifth grade class of 28 students came to me with multiple labels (e.g., emotional behavior disorder, English as a second language, free and reduced lunch). Since the pull out model was still active in schools in 2002, there were times of the day where my class of 28 was, shockingly, reduced to 2 students. These practices changed the following year with "mainstreaming," but this quandary introduced me to a new set of challenges – meeting the needs of fifth graders who struggled in various ways. I decided that I wanted to teach those that struggled the most in mathematics and that I was going to teach them as though I was assigned the gifted class. Every day my students engaged together where the norm was to solve problems, to find problems, to investigate,

explore, think critically – the practices usually reserved for advanced learners. They ALL rose to the challenge and this really didn't surprise me. Some were still struggling to memorize their multiplication facts in fifth grade, but that reality did not mean that their instruction was simplified and reduced to solely procedural, rote learning. This experience became the backbone of my passion – to sustain challenge, equity, and access for all learners in the way the National Council of Teachers of Mathematics express in *Principles to Actions: Ensuring Mathematical Success for All* (2014).

This dissertation project gave me a unique opportunity to take this passion to an extreme of sorts. From the theories driving the project (Cognitive Flexibility Theory, Realistic Mathematics Education), to the topic (space, perspective, dimension concepts), to the learners (adolescents), and to the method (post-intentional phenomenology and design-based research), complexity is the overarching theme. Cognitive Flexibility Theory argues against reductive notions, simplification practices, and "textbookization." Realistic Mathematics Education argues for guided reinvention, mathematics as activity, and the messy real world as the source and ending point of mathematics investigations. Space, perspective, and dimension tie together art, story, and mystery in relation to pattern and structure, illustrating the complex, ill-structured nature of embodied space. Even the research methods aim to hold the context as central to answer complex questions.

Finding and Naming

Being asked to explain my research on a number of occasions the past several years led to the moments where I would conjure up the most self-doubt. The doubt centered on not being able to clearly articulate the phenomenon of interest in my work. Somehow that seemed like it should have been the easiest part. After suffering years of this recurring feeling, I started to

realize that a phenomenon isn't a thing that can be stated holistically in a phrase. This is the reason for the investigation – something concerning the phenomenon is unexplored. But my struggle was more than that. I was having difficulty naming it. I've settled on the phenomenon of learners' shift in perspective for the article in Chapter 4, although that doesn't really capture the entire problematization aspect of the investigation. Maybe it will help to take a moment and trace the path that led to this naming.

When I was working as a middle school mathematics teacher, a student brought in the film, *Flatland: The Movie* (Travis & Johnson, 2007) following a month long investigation into properties of polygons (e.g., angles, tiling, regularity). From my perspective, a video in math class was a rare treat. The fifth and sixth graders were excited from the moment the animated film began. I still remember other middle school students coming by my room the rest of the week (seventh and eighth graders I didn't teach) asking if they could watch the film in my room during lunch. My fifth and sixth graders started questioning incessantly – mainly asking, is there a fourth dimension? Honestly, at the time, I'd never considered the notion of a higher dimension. I couldn't answer their questions, but I could certainly provide the space to conjecture. It turns out this space to talk, to throw around ideas, to construct spatial models – this learning environment would turn out to drive this research project. And the phenomenon I couldn't name, were these moments middle school learners found their world problematized – the moment they were in the words of one student, "no longer the highest, or the best, or you know." They came to realize that dimensionality is something they could try on, like a coat. This particular phenomenon is central to the Chapter 2 article. Before discussing the articles in this dissertation, I will introduce design-based research and detail the context for this project.

Design-based Research

Barab and Squire (2004) characterize a central difference between design-based research and psychological experimentation, where "design-based research focuses on understanding the messiness of real-world practice, with context being a core part of the story and not an extraneous variable to be trivialized" (p. 3). Design-based research seeks to inform both theoretical and design concerns within the context of practice, usually in conjunction with classroom teachers. Design-based research is an iterative process of refining these simultaneous concerns, with an understanding that theory and design are strongly tied to context.

Winn (2003), responding to Mayer's concern about a lack of scientific rigor, argues that "just as educational researchers must not reject experimental research, neither must they reject useful research methods that were added to their toolkit in the last two decades" (p. 369). Winn takes issue with a myopic view of the medical model experimental research approach, advocating for an integration of findings from experimental studies with non-experimental research in order to understand the big picture and details of phenomena. This *used inspired* research is important to medicine, technology, engineering, and to education as well (Stokes, 1997). Jacobson and Kapur (2012) convey the value of design research as "rich accounts of an iterative exploration of the possibility space of designs" (p. 312) and echo Winn (2003) that "once promising or effective design features are identified, experiential methods may be used together with descriptive methods to document and explain the emergence of learning in collaborative settings" (p. 312). Winn (2003) specifically advocates for design experiments (see Brown, 1992; Collins, 1992) and claims that design research retains the ecological validity of "messy" places where learning occurs (p. 369).

Space and Perspective, the hypermedia project running throughout all three articles in this dissertation, is an exploratory, design-based research project. Rather than try to manipulate and control the complex fifth and eighth-grade learning environments comprising the context for this project, the focus is to understand the ways this environment affects the lives of learners, specifically their relationship with space. Cognitive Flexibility Theory (CFT) and Realistic Mathematics Education (RME) guide this project. CFT (Spiro, Coulson, Feltovich, & Anderson, 1988) is a meta-theory informing advanced knowledge acquisition in ill-structured domains. RME (Freudenthal, 1973) is a mathematics specific theory informing learners' re-invention of mathematics starting with real-world phenomenon. Gravemeijer and Cobb (2006) argue that "you have to understand the innovative forms of education that you might want to bring about in order to produce them" (p. 17). This project seeks to support learners' complex concept development in geometry, specifically developing multiple perspectives of space and dimension (versus the solely traditional Euclidean form taught in school). The innovative forms of education that *Space and Perspective* seeks "to bring about" relate strongly to supporting learners' "grasping of space," a concept Freudenthal advocated (1973):

> And since it is about the education of children, [geometry] is grasping that space in which the child lives, breathes and moves. The space the child must learn to know, explore, conquer, in order to live, breath and move better in it. Are we so accustomed to this space that we cannot imagine how important it is for us and for those we are educating? (p. 403)

The importance Freudenthal allocates to this idea of "grasping" space is emphasized time and time again from those in geometry education. According to Hansen (1998), as part of the International Commission on Mathematical Instruction's (ICMI) *Perspectives on the Teaching of*

Geometry for the 21st Century, "the necessary and sufficient criteria" for the "selection of new matters to include in a core curriculum" are explicated as follows: (1) "it contributes in a significant way to the grasping of space" and (2) "it is learnable (not the same as teachable)" (p. 241). Similarly, Lehrer and Chazan (1998) suggest:

> [R]easoning about space can and should be successfully integrated with other forms of mathematics, starting at the elementary level and continuing through high school. Reintegrating spatial reasoning into the mathematical mainstream (indeed, placing it at the core of K-12 mathematics environments that promote learning with understanding) will mean increased attention to problems in modeling, structure, and design and reinvigoration of traditional topics like measure, dimension, and form: Geometry education should include contributions to the mathematics of space that were developed after those of the Greeks. (p. ix)

In this project, I sought to "understand the innovative forms" a "grasping" of space might take in a middle school classroom.

Context of the Project

This project started while I was still teaching fifth and sixth-grade mathematics in a K-12 independent school in the Northeast United States. As a former middle school mathematics teacher, I encountered many dilemmas when designing learning experiences for students, but none more palpable than investigating the fourth dimension after watching *Flatland: The Movie*. What started as an informal class discussion soon evolved into a series of investigations and yearlong discussions problematizing three-dimensional space – a taken-for-granted space that

neither the students nor I ever thought to question. Although Euclidean frameworks guided one way to consider space, we soon realized that there are a multiplicity of ways to reason about space and dimension.

Multiple Article Format

The structure of this dissertation is a multiple article format and includes five chapters: an introduction, three articles, and a concluding chapter. The first article, *Middle School Learners' Ontological 'Trying-on' of Dimensions: A Phenomenological Investigation,* presents findings from a pilot study conducted in 2011. The pilot study informs both the framework article (Chapter 3) and the most recent study (Chapter 4). This first article, a post-intentional phenomenological case study, blends the lived quality of learners investigating dimensionality alongside interview data targeting persistent learning outcomes. The second article, *The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site*, presents one aspect of the theoretical/design framework for the design-based research project, the design of cases as alternative perspective. The framework is grounded in the constructivist epistemological perspective of Cognitive Flexibility Theory (CFT) and Realistic Mathematics Education (RME). These perspectives detail instructional design principles to support flexible development of complex concepts in ill-structured, real-world contexts. The strategy of having learners examine multiple perspectives of space (e.g., noticing surprising similarities and surprising differences between variational instances of a concept) as a way to understand the inherent complexity of concepts is central to the framework.

The last article, *Productive Problematization: Moments When Perspectives Shift in Middle School Mathematics*, investigates moments when learners experience a shift in perspective, helping show the way the framework was lived-out by learners. This third article

expands the design-based research project from the pilot study (Chapter 2) and is guided by the theoretical/design framework (Chapter 3). The articles are described in more detail in the following sections.

Chapter Two: Middle School Learners' Ontological 'Trying-on' of Dimensions: A Phenomenological Investigation

The pilot study represents my first attempt at conducting research as a doctoral student. I only had one prior experience with action research as a classroom teacher. This project felt similar to action research, mainly because I was interviewing former students about a learning experience I orchestrated. When I was a fifth and sixth grade mathematics teacher (2008-2010), I showed *Flatland: The Movie* to my students after investigating two-dimensional shape properties and transformations. What came out of this experience surprised me, and my students alike. I also believe it shows that there is an important issue with perspectives or point-of-view that hasn't been explored, namely, can learners conjecture about perspective of another dimension not their own (0D, 1D, 2D, 4D)? If so, what implications does this have for the learner in terms of deepening their understanding of their own three-dimensional world?

After watching the movie, students seemed most interested in understanding the fourth dimension. They wondered what it would be like to live in a four-dimensional world – to eat, to move, to see. I couldn't really answer their questions, so we just talked as a class and eventually created three-dimensional shapes out of straws and pipe cleaners that we submerged in bubble solution. The translucent bubble material allowed us to make physical models of higher dimensional shapes. In addition, we viewed animations of rotating tesseracts and even 5-dimensional hypercubes online.

A year later, I conducted a post-intentional phenomenological study with students focused on their experience. I asked ten students to describe their experience learning about the fourth dimension, both as lived-experience descriptions and through interviews. Although students talk about many aspects of their experience, the paper here presents student experiences that show their "ontological trying-on" of different dimensions – 0D, 1D, 2D, 4D – and even a problematizing of their three-dimensional world.

Almost three years ago to the day (May, 2011), I signed up for JoBeth Allen's Qualitative Writing Retreat. I had all the transcriptions from the seven interviews, the lived-experience descriptions from 10 students, and phenomenology research books ranging from van Manen (1990) to Moustakas (1994). I was trying desperately to apply the method according to Moustakas but continued to run into huge disconnects. I didn't know if the problem involved the young age of participants or if the whole textural/structural method was the problem for me. In the end, my time certainly wasn't wasted and I ultimately met Mark Vagle a week or two later in an Introduction to Phenomenology Workshop.

The paper reflecting this research is titled, *Middle School Learners' Ontological "Trying-on" of Dimensions: A Phenomenological Investigation*. The paper was accepted as a full paper conference proceeding for the International Conference of the Learning Sciences in Boulder, Colorado this June (2014). The overall acceptance rate for submissions was 39% (30% for full papers). The study indicates experiences manifest as an ontological 'trying-on' of geometric dimensions (e.g., through sight, perception, and motion), leading learners to conjecture about dimensional relationships. Page length restrictions did not allow for much detail concerning methods. However, the third article in chapter 4 provides a rich description of post-intentional phenomenological methods.

Chapter Three: The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site

The third article focuses on design relating to one of the five high-level conjectures guiding the *Space and* Perspective project, namely, conveying the complex, ill-structured nature of concepts (e.g., multiple-perspectives through cases as alternative perspective) supports flexible concept development that is more transferable (usable) to a greater number of situations. Rather than try to assemble a complete recount of the entire design of the Space and Perspective project, Sandoval (personal communication, April 3, 2014) recommends, "thinking about what sort of story you have to tell about how the mediating processes you've observed in your work are tied to embodied features of your design and their interaction." When I first started writing the framework paper, I was trying to recount every detail of the 4 foot by 4 foot conjecture map I created, showing over 70 design and theoretical conjectures. It was too massive and wouldn't be productive (or publishable) as a journal article. I decided an important story to tell, and one that would benefit the knowledge base, concerned the embodiment of cases as alternative perspective on the hypermedia site, Space and Perspective (see http://spaceandperspective.com/). The framework paper shows the integration of CFT, RME, and hypermedia design principles (Jacobson, 2008) as well as the way they inform the design of cases as alternative perspective. In particular, images of the cases are shared as embodied on the hypermedia site.

Chapter Four: Productive Problematization: Moments When Perspectives Shift in Middle School Mathematics

Deep explications of the learners' experiences adds a sorely missing voice to the conversation about teaching and learning. When I find myself in a conversation with people, lamenting the ever-increasing prominence and high stakes standardized tests, the conversation

usually points out the flaw with the format of multiple choice. These conversations sometime point to the value added with student writing as a different sort of indicator. Post-intentional phenomenology has become an alternative of sorts, but also different than the essay. It's a story, an experience – good and/or bad – in the learner's words. It brings about their voice and finds meaning in the ways they experience their learning environment and world. In this study, learners articulated their experiences through a variety of sources (e.g., lived experience descriptions, interviews, conceptual maps of their knowledge, etc.). Several manifestations of the phenomena emerged, including moments of realization that their current perspective (concept) was incomplete. Learners talk about feeling provoked by the elusivity of the cases and in turn troubling their own experiences in order to understand complex phenomena more fully. In addition learners experienced impasse and dissonance, feeling discomfort and acceptance at times. Implications concerning learning, designing cases as alternative perspective, the act of problematizing, and mediating technologies are discussed in this article.

Chapter 5: Conclusion

This dissertation ends with a concluding chapter looking across all three articles and summarizes the implications of the larger body of work. A discussion of the articles in the context of mathematics education addresses issues of access and equity and a further discussion about the concept of "grasping" space. A discussion within the unifying themes of learning, design, and technology shows the way these article relate the fields of learning sciences, mathematics education, instructional design, and educational technology. The chapter ends by highlighting future directions for research.

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CHAPTER 2

MIDDLE SCHOOL LEARNERS' ONTOLOGICAL 'TRYING-ON' OF DIMENSIONS: A PHENOMENOLOGICAL INVESTIGATION[1]

[1] Valentine, K. D., & Kopcha, T. J. (2014). Middle school learners' ontological 'trying on' of dimensions: A phenomenological investigation. In J. L. Polman, E. A. Kyza, D. K. O'Neill, I. Tabak, W. R. Penuel, A. S. Jurow, K. O'Connor, K. Lee, L. D'Amico (Eds.), *Learning and becoming in practice: The International Conference of the Learning Sciences (ICLS) 2014, Volume 2,* (pp. 745-752). Boulder, CO: International Society of the Learning Sciences.

Abstract

This paper shares findings from a post-intentional phenomenological study aimed at understanding learners' experience investigating space and dimension concepts in a fifth and sixth grade mathematics class. Findings indicate experiences in this study manifest as an ontological 'trying-on' of geometric dimensions (e.g., through sight, perception, and motion), leading learners to conjecture about dimensional relationships. The paper discusses implications informing the second iteration of the design-based research project.

Introduction

Becoming is an ontological notion explicated in the phenomenological philosophical tradition of Heidegger and Merleau-Ponty. Heidegger questions the epistemological focus (i.e., consciousness of something) of phenomenological investigations, recognizing that experience is already situated in the world of being. Throughout his major work, *Being and Time* (1927/2008), he uses the phrase 'Being-in-the-world' to indicate the ontological phenomenological starting point. According to Heidegger, phenomenology's task is to question what it is to be in the everyday world. Merleau-Ponty (1945/2002) agrees with Heidegger's notion of 'Being-in-the-world' and makes embodiment a central theme in his work – a subject embedded in a certain position and time in space. This is exemplified in his work with the primacy of perception.

This paper shares middle school learners' experiences conjecturing about the relationships between dimensions following the film *Flatland* (Travis & Johnson, 2007). The film offers viewers an opportunity to contemplate analogously within and between dimensions. In addition to the film and subsequent discourse, learners constructed models to demonstrate 3D/4D relationships. For learners, the investigation was taken up ontologically as they 'tried on' the lenses, motion, and perceptions of imagined characters.

Issue Being Addressed

Rarely do learners reflect on the ontological experience of *becoming* in mathematics class. Yet, children form conceptions of space early in life simply by *being* in the world. They engage in everyday activities like "looking, walking, drawing, building, and manipulating objects" (Lehrer, Jenkins, & Osana, 1998, p. 169). They develop intuitions about spatial structure before formalized words are attached to their concepts (Freudenthal, 1983; Gravemeijer, 1998;

Piaget & Inhelder, 1948/1967; van Hiele, 1986). It is possible that their pre-formalized concepts formed by *being* in the world could come in conflict with Euclidean geometric concepts formally taught in school.

These conflicts are shown to relate to the materials and processes used in schools (Lehrer et al., 1998). For example, most textbooks and teachers present images of triangles to students with a horizontal base, although this is not a defining characteristic. This results in students primarily recognizing prototypical examples of triangles to the exclusion of non-prototypical examples. These limited conceptions can inhibit students from reaching the intended goals of instruction and will most likely resurface in later geometry learning. Spiro, Coulson, Feltovich, and Anderson (1988) describe reciprocal misconception compounding as a problem resulting from simplifying complex concepts. Many misconceptions have been identified in geometry and their origins might lie in this simplification process as well as instruction that fails to incorporate learners' prior knowledge (e.g., Carroll, 1998; Monaghan, 2000).

The current educational climate rarely provides opportunities for learners to problematize space in geometry, rather it appears taken-for-granted that everyone shares the same concept – a Euclidean mathematical space represented by x, y, and z coordinates. Even more problematic is that school explorations of space start in an abstract manner, building on idealizations of the point (0 space), the line (1 space), and the plane (2 space) – spaces that are actually impossible to *find* in the world. In this study, we created a learning environment where learners could build on prior 3D experiences and the experiences of characters from the film *Flatland*.

Context

Forty-three 5th and 6th grade mathematics students were shown *Flatland* as a way to follow up on a recent unit investigating properties of polygons. After watching the movie,

students questioned what it would be like to live in a four-dimensional world – to eat, to move, to *see.* There were no simplistic answers to their questions, so we conjectured as a class and eventually created three-dimensional shapes out of straws and pipe cleaners. These were dunked in bubbles, which allowed students to make physical models of hyper-shapes (i.e., representations of 4D shapes). A year later, we conducted a post-intentional phenomenological study with students to explore their lived experiences. In particular, we explored if and how children take on the perspective of another dimension not their own (0D, 1D, 2D, 4D).

Theoretical Perspective

Our perspective acknowledges that humans' experience of phenomena in the world is complex. Phenomena emerge over time, are context dependent, and exhibit complex dynamics such as emergence, ambiguity, and adaptation (e.g., Davis & Sumara, 2006; Jacobson & Wilensky, 2006). Not only is the concept of space and dimension taken up as complex and ill-structured, but designing learning environments is viewed as a complex, iterative process informing and developing theory as well as practice (Barab & Squire, 2004; Gravemeijer & Cobb, 2006). At the early stages in design, our aim is to explicate phenomenological understandings concerning the ways shifts in perspective manifest in complex, emergent environments integrating innovative practices (e.g., modeling, mathematical discourse) and emerging forms of technology (e.g., animations, video cases). In particular, we used conceptual change strategies from science education including a discrepant event and bridging analogies as described by Clement (1993, 2008). These manifestations of shifts in perspective were then used to inform refinements in the design and theoretical conjectures.

There is little insight concerning middle school learners investigating the fourth dimension or even how this may play out using multimedia animations. Banchoff (1990) is one

who passes along geometric insights to teachers of young children. He writes "[t]he invitation to examine coordinates from a dimensional standpoint is available at all times: We only have to make students aware of what they are seeing" (p. 33). We agree that the invitation is available but that it also encompasses more than sight. Freudenthal (1973), attributed with developing Realistic Mathematics Education, describes geometry as embodied:

> since it is about the education of children, [geometry] is grasping that space in which the child lives, breathes and moves. The space the child must learn to know, explore, conquer, in order to live, breath and move better in it. (p. 403)

Clements (1998) describes children's investigation of shape through various actions (e.g., touching, drawing, discussing, moving); yet, one of his descriptions stands out – "development of perspective taking" (p. 3). He describes the "three mountains" task, where a doll is placed in several positions around a mountain scene. Children are asked to describe the scene from the perspective of the doll as it is moved, but always describe their own viewpoint instead. He writes, "it is not just familiarity or experience, but *connecting different* viewpoints, that develops perspective-taking ability" (p. 3). How does one connect different viewpoints? Taking on an external perspective is nuanced because children would also have to make a transition from two to three-space. It may help to think about the context of maps to understand viewpoint. According to Presson (1987 as cited in Clements, 1998) children:

> must grow in their ability to treat the spatial relations as separate from their immediate environment. These secondary meanings require people to take the perspective of an abstract frame of reference ("as if you were there") that conflicts with the primary meaning. (p. 13)

Methodological Approach

Vagle's (2010) post-intentional phenomenological (PIP) research method was used in this study to capture the lived-experiences of middle school students investigating space and dimension. PIP brings post-structural thinking in conversation with phenomenology philosophers, such as Husserl (1901/1970) Heidegger (1927/2008), Merleau-Ponty (1945/2002), Gadamer (1975/1994), and methodologists such as van Manen (1990), and Dahlberg, Dahlberg, and Nyström (2008). Phenomena, and humans experience living with them, are seen as *tentative manifestations* – dynamic and continuously changing. Experiences are not only interpreted, but are lived in tentative, partial, and fleeting ways. Vagle (2010) has designed a five-component process for conducting PIP research: (1) Identify a phenomenon in its multiple, partial, and varied contexts, (2) devise a clear, yet flexible process for collecting data appropriate for the phenomenon under investigation, (3) make a bridling plan, (4) read and write your way through your data in a systematic, responsive manner, and (5) craft a text that captures tentative manifestations of the phenomenon it its multiple, partial, and varied contexts.

Phenomenology is the study of experience, or as Dahlberg et al. (2008) write, "the science of the world and its inhabitants, the "things of experience" understood as the world of experience" (p. 33). One cannot investigate phenomena without due attention to the central tenet of intentionality. Intentionality is best understood as an embodied relationship to things and beings in the world. Although similar to the word "intention," or action (i.e., I intend to go to bed early), intentionality is different. We choose to draw on Vagle's (2009) notion of intentionality as related to PIP research, thus situating intentionality within this study. Vagle writes (2010) "intentionality is shifting and forever partial and thus can be read through post-structural frames" (p. 2). This presents a point of departure from other forms of phenomenology, such as

transcendental phenomenology, which seeks to describe essences. Rather, we ascribe to Vagle's approach that identifies phenomena as *tentative manifestations*.

Phenomenological philosophers and methodologists have discussed in length and with great detail the phenomenological attitude, sometimes referred to as a scientific or open attitude. In choosing Vagle's approach, we chose a phenomenological attitude as one of openness, not of phenomenological reduction described by researchers such as Giorgi (1997), who uses transcendental notions of bracketing and Epoché to capture the essence of a phenomena. An "open attitude" as described by Dahlberg et al. (2008) means, "having the capacity to be surprised and sensitive to the unpredicted and unexpected" (p. 98). Bridling, described below, is one way both Dahlberg and Vagle suggest remaining open to the phenomena under investigation.

Bridling

Bridling is a way to negotiate complexity with data rather than being rigid and lockstep. The practice better encompasses the way towards an increased understanding of the phenomenon, and to us, the validity of the investigation. Bridling includes three main activities on the part of the researcher: (1) questioning pre-understandings (including assumptions), (2) remaining open, and (3) joining in an ongoing dialogue about the phenomenon. Therefore, bridling does not remove, set aside, or render the researcher non-influential but "animates and illuminates the researcher more fully" (Vagle, 2009, p. 592). The bridling entries in this study were made in three different formats: (1) as comments inserted alongside/in the margins of transcripts, (2) as individual, dated Word document files after analysis sessions, and (3) as notes inserted in books and articles.

Statement of Phenomena of Interest and Questions

The phenomena of interest in this study aims to show the reader "the lived quality and significance" of middle school students experience investigating dimensional relationships in a deep and meaningful way. The primary research question asks, "What is the lived-experience investigating dimensional relationships, including the fourth dimension?" Secondary questions ask: (1) What role does the video, *Flatland,* play in the experience for students? (2) Are there indications that persistent, or long-term learning occurred?

Data sources by primary research question

All 43 students (21 fifth grade and 22 sixth grade at the time of instruction) from a K-12 independent school in the Northeastern United States were asked to participate in the study. Three fifth-graders (1 female and 2 males) and seven sixth-graders (6 females and 1 male) agreed to generate lived-experience descriptions one year after instruction. These were used to guide follow-up interviews where all 10 students articulated their experiences more fully and/or clarified their experiences. Second-round interviews were conducted as needed to clarify and expand upon key aspects of the experience. Although the 6^{th} grade interviewees were mostly female, this was representative of the student body (18 females and 4 males).

Because we interviewed former students, we used a "conversational approach" to help alleviate possible power structures (Denzin, 1989). According to Denzin, interviewing "should *not* be a relationship where one party does all the talking and the other only asks questions. When interviews turn into this form, they become asymmetric, authoritarian social relations in which the power of social science determines the information given" (p. 43). To achieve a conversational approach, we focused on having students describe their experience with as much detail and ability to capture multi-faceted aspects of the phenomenon.

During the instruction, student participation and work was captured through video and photographs. Additional data sources include lesson artifacts and bridling entries. These were used to guide interview questions, support/challenge interview data, and generate hunches as described by Glesne (2011). These additional data sources were also used to generate a multifaceted description of the phenomenon and answer secondary research questions.

Analysis: Whole-Part-Whole

We draw on both Vagle (2010) and Dahlberg et al.'s (2008) suggestions for data analysis, using a whole-part-whole approach. Following is a summary, describing the way data was analyzed both within and across sources:

- *Whole.* The first reading focused on the whole data collection event, where all data pieces were brought together and read (not analyzed) as a whole.
- *Part.* This was followed by multiple line-by-line readings with researcher notes and follow-up interview questions for each participant. Subsequent line-by-line readings articulated meanings and considered notes, markings, follow-ups, and bridling entries. After saving documents for each participant, the last line-by-line reading articulated analytic thoughts for each part for each participant.
- *Whole.* Subsequent readings started to identify tentative manifestations across the data.

Findings

For this paper focused on becoming, we will explicate a particular manifestation of the learners' experiences *trying on* various dimensions (0D, 1D, 2D, 3D, and 4D). The phenomenon was evidenced through learners' conjectures about what it might be like to see, perceive, and move in the dimensions. Notable in the descriptions from learners was their attention to relationships within the dimensions. For example, when describing sight, perception, and motion

in 3D space, they conjectured what it might be like to be a point, a line, a square, a cube, and even a hypercube in 3D space.

Learners described *Flatland* as the point when they started considering dimensional relationships. They related to the 2D Flatlanders' struggle to visualize three-dimensional objects. Students had an analogous struggle visualizing the fourth dimension. Students talked most about the last scene of *Flatland*. This is the point where they saw the fourth dimension animated for the first time and where they watched Spherius, the main 3D character in the movie, deny the existence of a fourth dimension. This denial prompted a lengthy discourse among students about close-mindedness. Students expressed being upset with Spherius for denying a higher dimension in a similar way that 2D characters in the movie denied a third dimension.

Taking on Sight and Perception from the Various Dimensions

Almost every student talked about what it might be like to *be* in a dimension. For some, this took the form of articulating what it actually means to be in 3D space and how they are only seeing parts of solids, but not all sides at once. They called this *seeing in 2D*. They talked about perceiving depth and *thinking* that we see in 3D. Somehow it was easy for them to then imagine by analogy what it must be like to see and perceive in the fourth dimension – they called this *seeing in 3D*.

Students engaged in their learning experience by trying on different dimensional lenses (or contemplating being other dimensions) and then visiting other dimensions, even the fourth. Alan is one student who attempted to generalize sight and perception for all dimensions and used Euclidean mathematical notions of the coordinate system in his discussion. He demonstrated an ability to perceive as "creatures" from different dimensions, related to the movie *Flatland*:

> Pointland is a land of the zero dimension, where all that exists is a point. No axis. Lineland is only a line, the first dimension with the x-axis (or y). Flatland is the second dimension, with the x and y-axis. The creatures in the movie could only see a line wherever they looked. Spaceland is the third dimension, where we exist. With the x, y, and z-axis, it is most likely the largest 'land.' The creatures of this land can see a 2D image that can be perceived in 3D.

Interesting here was his distinction between sight and perception in 2D Flatland and 3D Spaceland. His ability to connect dimensions to the x, y, and z-axis demonstrated what Freudenthal termed *mathematization* (1973). He continued the process as he connected the fourth dimension into this same x, y, and z coordinate framework:

> If someone asked me to describe the fourth dimension, I would say something along the lines of...basically a dimension more complex than ours as another axis or existence…starts with none in the zero, x in the first, y in the second, z in the third. We'll have to make a new letter in the alphabet for the fourth dimension. Or 'a'…it would probably work, yeah a different sort of direction. The fourth axis is probably where the idea of how I perceived hypercube came from. Cause it's a cube connected by another axis…like a cube in the center connected by the line coming from the outer corner. And I was thinking of those lines as being a fourth axis.

Alan was the only student who talked about the x, y, z, and "a" axis in relation to the dimensions, attributing *Flatland* as his starting point. He said previously he had only learned about the second dimension, but not the zero, first, third, or fourth dimensions. He talked about his experience as a whole:

> Investigating the fourth dimension is to learn what the world is like mathematically and also to teach us more about the dimension we exist in. The fourth dimension was to explain why things happen, why the world is like it is. Well, before that, I didn't really understand. I had a vague understanding of the second dimension, but not much of the first, zero, third, or fourth. The biggest thing I learned is that humans exist in the third dimension but we can only actually see the second dimension – we just perceive the third. As we evolved to – not evolved – as we exist, only able to see the dimension below it and not the actual dimension it's in.

Dimensions Lower than Three (0-2)

This section incorporates students' discussion of the zero, first, and second dimensions. Students described these in various ways, taking the perspectives of the different types of dimensional creatures (0D – Point; 1D – Linelander; 2D – Flatlander; 3D - Spacelander). For example, taking on a Linelander's perspective of its own dimension, or a Linelander's perspective of a Flatlander. In addition, they described the dimensions either by appearance, motion, perception, and even the sight available to a particular dimension.

Students talked least about the zero dimension, but almost all students recalled the "me" song that the Point character from *Flatland* sang. In the following description, Annie talked about the perspective of a point:

> I had never really thought about it before, like a line can only see side to side of where it is or a point only notices itself because it can't - it's not really anything else, I thought that was really cool because I'd never really taken that in perspective before.

Students' discussion of the first dimension highlights their attention to limitations of sight and motion available to a line. For example, Susie expressed, "their [Linelanders] only way of

traveling was right and left and only it couldn't have any sort of up and down. It was only just in the two directions." Edward, Sabrina, and Daria also described Lineland, but they talked more about the Linelanders' point of view when other dimensional beings came into their line. For example, Edward talked about one Linelander's point of view towards Arthur Square, a Flatlander visiting the line in the movie:

> In Lineland it could only see down it's line so it like it couldn't see up or down so it wouldn't be able to see him…it would only be able to see him if his eye or if his - if he was right on it's line, so I thought that was kind of cool how it had no other power to see anywhere else.

Students talked even more about the second dimension. Mary felt like the movie gave her a better understanding of the second dimension. "If you had a piece of paper, it helped me understand what it would be like if you had a circle that could, or a sphere that could sink into it." Although this description may seem simplified upon first reading, Mary started articulating a foundational calculus concept of iteratively slicing a sphere. Another student, Susie, described the Flatlander's point of view and motion. Here, she started to insert herself into the second dimension showing how she took on the role of dimensional motion:

> It's kind of crazy for me thinking about actually only living in the flat surface. I sort of remember always thinking about the idea of how do they move? Do they slide along on the floor or what? Or crawl? I remember when we started watching, that was basically one of the first things that popped out to me. I was like, how are they moving? Its kinda' cool to think about how – wasn't he saying that they only have northwards and southwards or something? That they don't have up and down? Yeah cause it's flat.

The second dimension is typically taught as a concrete dimensional plane. The movie allowed her to contemplate motion in this dimension, whereas her prior experiences limited her to only seeing two-dimensional drawings in books. It created another way of conceptualizing the second dimension, an unexpected (from the teacher's point of view) experience of taking on the role of seeing and moving in different dimensions.

The Third Dimension

Students' descriptions of the third dimension reflected a problematizing of their own 3D space. These students had little formal instruction concerning the third dimension at the start of the lesson. Edward gave a prototypical example of problematizing sight from his own third dimension, "We don't see circles, we see spheres. I mean we see circles but…and you can't really get anything completely flat except for like if you're looking at a screen. I guess that like what's inside would be totally flat." In his description, there is tension as he tried to reconcile seeing "flat" but at the same time seeing, or perceiving, space. Although he did not use the word perceive, many students did. For example, Alan talked about seeing and perceiving. "The third dimension is something we can only perceive, and that we can't really understand further than that in a visual way."

The Fourth Dimension

Students conjectured most about the fourth dimension. *Flatland* only gave a glimpse of a rotating tesseract at the end of the movie, but this seemed to be the scene that affected them most. During class, student questions about the existence of a fourth dimension were persistent and eventually led to the hands-on investigation with straw and bubble models. This led to several conjectures about the fourth dimension, elaborated below.

Sight in the fourth dimension. Mary talked about sight and being in the fourth dimension. It's as if she not only learned about the fourth dimension but also strengthened her understanding of the 0, 1st, 2nd, and 3rd dimensions and their relationship. She talked about what it would be like to see and perceive from each of these dimensions:

> I want to know more about what it would actually be like to be in a fourth dimension. I have an idea. You'd be able to see all the sides of an object at once, but it would be like in Flatland. The square guy could only see the sphere when he was in his field of vision. So it would be weird to think that there would be things that we can't see…you can see all sides of something. I'm continuously getting the vision of your eyes popping out of your head and like curving around an object so you could see the back of it.

Mary's curiosity led her to think about seeing in a fourth dimension. Other students experienced this as well. For example, Edward said:

> To see every side of a cube, your eyes would have to be…probably have to have one like, like coming out of like something that's like this [makes motion of hand making a hook coming from forehead curving out to front of face] I guess so you could see like this way toward…like you could see yourself…and you could see like anywhere I guess. That would be very difficult to see ourselves, like our eyes.

Alan also talked about sight in the fourth dimension, distinguishing between sight and perception:

> Assuming that any creature can perceive in, can perceive a dimension below it – can SEE the dimension below it, but perceive its own. Well, that would imply the fourth dimensional creature would have to be able to see a third dimension. So basically what I imagine a fourth dimensional creature being like is a sphere with eyes on the inside of it,

> so it could look at something from every angle. And it would have to have some way of getting, getting things in there to look at, but I hadn't gotten past that. Yeah, you'd have to be able to see all sides at once. First my…could imagine seeing all three sides of a cube – all six sides of a cube at once.

Motion in the fourth dimension. In addition to conjecturing about the sight of four-dimensional beings, students also discussed motion. For example, Annie inserted the human form into the fourth dimension. "I think it would be pretty insane to live in the fourth dimension, especially if we were 4D humans, cause that'd mean like our guts would be like rotating inside of us." Edward used motion to think about what it must be like to eat a hamburger. "It will be forever going in and out of itself so you'd never get a bite with both [the bun and the meat] in it cause sometimes you'd get just the meat and sometimes you'd get just the bun." When Annie was asked how she would describe the fourth dimension to someone else, she said, "I would tell them that it's kind of like…I'm still not like 100% sure, but it's kinda' like an object…it…attached kind of inside an object, but it's kind of rotating so it's always on the inside or the outside." It's interesting that she was okay admitting not being 100% sure about describing the fourth dimension, yet continued trying. This seemed to happen for all the students interviewed.

Fourth dimension appearance. In addition to talking about sight and motion in the fourth dimension, students made conjectures about the appearance of fourth dimensional shapes. In these descriptions students used terms like "hyper-human." For example, Susie drew upon her science class to conjecture about "a human inside of a human" as a hyper-shape. Susie's remarks indicated that she was trying to picture humans as they might exist in these conjectured spaces.

An Open Mind

Students did more than describe the fourth dimension. They also talked about being open-minded and close-minded in relation to dimensions other than our own. For example, Mary said:

> Before [learning about the 4th dimension] I always thought the 3rd dimension is as far as you can go in the dimensions or like, I don't know, not really the best, but we're the highest you can go or whatever. But now it's like…there might be something above us…not above us, but something different, or more, or whatever.

Mary talked about thinking that the third dimension was the highest. It's interesting that she chose the phrase, "not really the best, but we're the highest you can go or whatever." Edward did a similar thing:

> He's [Spherius] like but there could never be a fourth dimension. So I think they can believe like numbers lower than them but they don't think there's anything higher than them because they all want to be the best, I guess. Well, I thought that it would be hard to believe there was a fourth dimension.

This idea seems provocative and indicative of most human thinking, similar to people believing they are smarter than animals. Several students discussed Flatland characters' denial of higher dimensions, arguing about whether you could perceive other dimensions if you could never actually experience them. These discussions indicated a tension regarding whether students' perception of the world was closed or open.

Discussion/Conclusions

We chose these particular manifestations of the phenomenon to show how the experience led them to think about the various dimensions in provocative ways (motion, sight, perception, etc.). The findings indicate that *Flatland* served as what Zeck et al. (1998) refer to as a video

anchor, a place to start conjecturing about dimensions. The fact that their conjectures persisted one year after their experience suggests that experiencing dimensional relationships ontologically may be a way to sustain student learning over time. Students were able to articulate specific scenes from the movie, as well as use them to continue conjecturing during the interviews. It may be that the video anchor, which provided a powerful analogy, was able to promote thinking and allow learners to *try on* a being outside their own dimension.

Although we asked non-phenomenological questions, such as conjecturing questions, we feel like their responses give researchers and teachers insight into the capability of middle school students' ability to reason deeply about space and dimension. We admit not attempting to formalize their learning by connecting the investigation to abstract symbols, although we did look at patterns between the dimensions (e.g., number of vertices, edges, faces). The main focus was encouraging them to engage in a conjecturing activity related to their interests following the film. Allowing the space for this to happen gives students an opportunity to engage in mathematical processes, much like a scientist problematizing their world. This seems in line with many of the Realistic Mathematics Education (RME) principles (e.g., Cobb, Zhao, & Visnovska, 2008; Gravemeijer, 1998; van den Heuvel-Panhuizen & Wijers, 2005).

Future Directions

We have already designed and are currently analyzing a second iteration of an instructional program that builds on this study. The revised program incorporates 70 cases of space and perspective in a hypermedia format and collaborative forms of reflective discourse to support the development of robust, flexible concepts among learners (Jacobson, 2008; Kolodner, 2006; Spiro et al., 1988). As part of our research, we have developed a framework grounded in the constructivist perspectives of Cognitive Flexibility Theory (e.g., Spiro et al., 1988) and RME.

At this point, we have developed a media rich set of cases for learners to examine multiple perspectives of space (e.g., noticing surprising similarities and surprising differences between variational instances of a concept) as a way to understand their inherent complexity. One investigation asks students to consider the various ways we capture and represent space. Cases include emergent video techniques that capture sporting events using slow motion or gyro cameras. Concerning invisible spaces, students are asked to consider fractal and hypercube animations representing dimension and spatial relationships we were unable to visualize until recently. At this still emergent stage of design, we are most interested in the ways students experience their engagement with the cases and the learning environment as a whole. Guided by the primary question, what is it for students to find themselves perceiving space as mediated in a variety of ways through technology, we are using post-intentional phenomenological methods to continue our investigation.

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CHAPTER 3

LITERATURE REVIEW

THE EMBODIMENT OF CASES AS ALTERNATIVE PERSPECTIVE IN A MIDDLE SCHOOL GEOMETRY HYPERMEDIA SITE[2]

[2] Valentine, K. D., & Kopcha, T. J. To be submitted to *Educational Technology Research and Development*

Abstract

This design framework paper describes the embodiment of a particular case type, cases as alternative perspectives (Jonassen, 2011). Using the design-based research strategy of conjecture mapping (Sandoval, 2014), the design of cases for a hypermedia site in middle school geometry is described through embodiments of the learning environment (e.g., tools, materials, structures, practices), mediating processes these embodiments are conjectured to support, and outcomes relating to these processes. Theories informing the design are described and include Cognitive Flexibility Theory, Realistic Mathematics Education, and hypermedia design. Design implications from preliminary analysis are detailed regarding cases as alternative perspective and includes curating learners' blog postings to populate the case bank and revisiting video with overlays or segmenting video.

Keywords: cases as alternative perspective, cognitive flexibility theory, realistic mathematics education, hypermedia design, mathematics education, geometry, middle school education, conjecture mapping, design-based research

Instructional approaches tend to simplify and compartmentalize concepts and learners encounter problems making use of these concepts later in applied settings (Spiro, Vispoel, Schmitz, Samarapungavan, & Boerger, 1987). The design described in this paper seeks to address this more general learning problem and those specific to mathematics education stemming from traditional procedural-based teaching and learning (e.g., memorize procedures, formulas, and terminology). This project is contextualized in middle school geometry, specifically space, dimension, and perspective concepts where embodied ways of knowing can support one's understanding. Embodiment was not a construct typically associated with mathematics education until recently (e.g., Alibali & Nathan, 2012; Hall & Nemirovsky, 2012; Núñez, 2012), with the exception of earlier writings by Núñez and colleagues (e.g., Núñez, Edwards, & Matos, 1999). Yet, experiencing space is a constant activity humans engage with informally through the senses. A lack of meaningful learning opportunities to connect to learners' existing spatial concepts may limit their construction and transfer to future problems (see Bransford, Brown, & Cocking, 2000).

This paper is organized by first operationalizing cases as alternative perspective, the need for a design framework, and the aims the framework seeks to achieve. Next a more detailed discussion of particular problems leading to the design effort broadly and as situated in the context of middle school geometry is explicated. Following is a deeper review of the theories and design principles from Cognitive Flexibility Theory, hypermedia design, and Realistic Mathematics Education. Next, elements of the conjecture mapping process are detailed accompanied by high-level conjectures and more specific design and theoretical conjectures used to design the cases. Several figures are included to help visualize the interactions embodying the design. The paper concludes by suggesting limitations, changes, and next steps in the design.

Operationalizing Cases for the Framework

This section operationalizes both cases and cases as alternative perspective to illustrate this relatively new term for the framework.

Cases

Jacobson (2008a), Jacobson and Spiro (1993), and Spiro, Coulson, Feltovich, and Anderson (1988) implement strategies such as using multiple cases, examples, and instances of a particular concept as a way to support the goals of problem solving, prevent misconceptions, and foster flexible use of concepts in new contexts. These cases are contained in a cognitive flexibility hypermedia system, where cases are depicted as "digitally encoded representations of knowledge such as text and symbols, visual images, animations, video, and 2D and even 3D computer models and simulations (Jacobson and Archodidou, 2000)" (Jacobson, 2008a, p. 10). In case-based reasoning, cases are portrayed as scripts or stories, including "a setting, the actors and their goals, a sequence of events, the expected results, what actually happened (if it is known), and explanations linking outcomes to goals and means" (Kolodner, 2006, p. 226). In Realistic Mathematics Education (RME) (Freudenthal, 1973, 1983), the term case isn't used. Yet, the purpose of RME is to start with phenomenon in the everyday world to mathematize. With this perspective, a thing to investigate, a phenomenon, could serve as a case or starting point, such as a cracked sidewalk, the shifting shadows from a building, and even an animation of a hypercube.

Cases as Alternative Perspective

The design framework described in this paper aims to support learning of space and dimension concepts by using cases as alternative perspective (Jonassen, 2011) in middle school geometry. The cases challenge learners' everyday way of being, seeing, and moving by drawing

attention to similarities and differences across multiple contexts (e.g., art, photography, video, gaming). Cases as alternative perspective (CAPs) is a term first used by Jonassen (2011) to aid in solving ill-structured problems. Ill-structuredness refers to phenomena (e.g., problems, concepts, domains) that lack generalizability. Related to teaching and learning, there is no prescription for thinking and acting (Spiro, Collins, & Ramchandran, 2007a). In Jonassen's discussion of CAPs, he draws on Cognitive Flexibility Theory (CFT), a context-independent meta-theory that informs constructing adaptive knowledge in ill-structured domains (Spiro, Collins, Thota, & Feltovich, 2003; Spiro et al., 1988). CFT's "most sweeping recommendation" when designing learning environments is an emphasis on "*multiplicity,*" recognizing that a prepackaged schema is too rigid and limiting (Spiro et al., 1987, p. 6). According to Jonassen, "ill-structured problems requires that the ill-structuredness be conveyed, not eliminated" (2011, p. 210). In order to convey this complexity, "cases need to be presented that convey the multiple perspectives that are implicit in ill-structured problems" (p. 210). For example, investigating a concept such as space can be examined with the multiple perspectives of architecture, photography, and even gaming.

For this project, CAPs are operationalized as anything that allows learners to investigate multiple perspectives inherent in complex concepts. They serve as the context for investigating space and perspective. Employing microcosm-macrocosm relationships, they can take the form of video, text, symbols, photos, animations, and simulations (Jacobson & Spiro, 1993; Spiro et al., 2007a; Spiro, Collins, & Ramchandran, 2007b; Spiro et al., 2003, 1988). The goals of CAPs for this project is the demonstrate variability of concepts in real-world instances, convey complexity to improve flexibility and usability of concepts, and support multiple/alternative perspective.

The Need for a CAPs Framework

In this section, it is argued that designers can benefit from a CAPs framework specifying design decisions and the theoretical outcomes these are conjectured to bring about. This section addresses the promise CAPs hold for addressing learning problems with complex concepts. Unfortunately, there is little practical knowledge concerning the design of CAPS. A discussion follows about the importance of considering context, the aims of this particular project, and the prior work guiding the design.

Promising for Addressing Learning Problems

CAPs are promising for addressing problems caused by rigid concept development because they promote perspective building, which supports learners flexible concept development. According to Spiro et al. (1987), learners need to draw on multiple aspects of a concept to avoid reductive biases and potential misconceptions. CFT, used to develop CAPs, seeks to address four main goals (Spiro et al., 2003): (1) Supporting learners with difficult content matter. Rather than skimming the surface of concepts, value is place on deep and meaningful understandings. (2) Supporting learners to develop adaptable and flexible knowledge for later use in real-world settings. This goal addresses the transfer problem where prior understandings and experiences vary greatly from the initial conditions of learning. Rather than design that focuses on routinized schemas, CFT emphasizes "an adaptively creative response to new situations" (p. 5). (3) Challenging reductive ways of thinking. "The kind of knowledge one constructs, and the way that knowledge is deployed, depends on the "lenses" that filter one's view of the world" (p. 5). Spiro and colleagues have found that learners' worldviews tend to be built on overly simplistic assumptions that need to be interrogated in order to apply knowledge to various real-world contexts. (4) This fourth goal concerns the development of hypermedia

learning environments in order to support flexibility and use on the part of learners. Computer learning environments allow a "nonlinear and multi-perspectival organization of material" (p. 5), which supports flexible concept development. This last goal of CFT requires the development of CAPs, described in this framework.

Educational designers have become increasingly interested in design that conveys complexity to learners. In June 2013, the Educational Technology Research & Development journal released a special issue entitled, *Towards Innovation in Complex Problem Solving Research.* Jonassen writes, "Cognitive-flexibility theory conveys this complexity by presenting multiple representations of the same information and different thematic perspectives on that information" (p. 212). Although CFT and the design of CAPs is promising for conveying complexity in ill-structured domains, there is little practical knowledge in a middle school mathematics context.

Little Practical Knowledge Concerning Hypermedia Design with CAPs

Shapiro and Niederhauser (2004) conducted an extensive review of the literature concerning hypertext (mixed with work in hypermedia). Overall they noted "a marked lack of coherence in the field" (p. 617) concerning the language. Researchers interchange meanings attributed to hypertext, hypermedia, and multimedia. There are many overlaps between hypertext and hypermedia, but multimedia is not considered synonymous. The main difference between hypertext and hypermedia is that hypertext concerns linked text without graphics, audio, video, or other media; hypermedia contains text and media. Shapiro and Niederhauser (2004) draw three main conclusions relevant to this project: (1) well-defined structures (i.e., hierarchies) are best at supporting factual knowledge acquisition, where "ill-structured systems are often beneficial for deep learning, especially for advanced learners" (p. 613); (2) the affordances of a

hypertext system include supporting learners development of relationships, ideas, and multiple aspects of a concept as well as conceptual understanding; (3) "hypertext alone does not necessarily promote active learning" (p. 616). They conclude by suggesting the following:

> In the right circumstances, though, hypertext can enhance learning. It does so by presenting environments that offer greater opportunities for students to engage in the type of cognitive activities recognized by theorists as encouraging learning: active, metacognitive processing aimed at integrating knowledge and boosting understanding. In short, while hypertext does not offer any shortcuts for learners, it offers rich environments in which to explore, ponder, and integrate information. (p. 618)

The issues noted in 2004 by Shapiro and Niederhauser continue to persist today. Most notable is the finding that CFT driven learning environments are better at supporting learners with complex thinking (referenced in the literature as problem solving, considering multiple perspectives, solving ill-structure problems, transfer, etc.) than they are at supporting factual knowledge acquisition (Jacobson & Spiro, 1995; Li & Jonassen, 1996; Lima, Koehler, & Spiro, 2004; Wells & McCrory, 2011). However, other researchers have found that criss-crossing cases, one strategy from CFT, is not effective at improving learner transfer. Conflicting findings concerning hypermedia learning environments have been noted by several researchers (Jacobson & Azevedo, 2008; Shapiro & Niederhauser, 2004; Wells & McCrory, 2011).

Some remaining problems and unresolved issues concern disorientation on the part of the learner (e.g., Heller, 1990) and more recent findings of poor transfer (Strobel, Jonassen, & Ionas, 2008). Strobel at al. (2008) also show learners' difficulty accommodating and creating multiple perspectives and difficulty applying causal reasoning. However, learners' thinking about concepts from multiple perspectives is shown to improve in Zydney's (2005, 2010) two design

iterations. According to Sprio et al. (1988), ill-structured domains are shaped by multiple perspectives and these need to be conveyed to avoid reductionism and possible misconceptions. However, the research is inconclusive concerning the benefits of hypermedia to support multiple perspectives on the part of learners, an important concern for designing CAPs.

Another gap in the hypermedia learning environment literature concerns design with younger learners. Most research has been mainly conducted in medical, teacher, and business education (e.g., Heath, Higgs, & Ambruso, 2008; Lima, Koehler, & Spiro, 2010). A few exceptions exist with high school learners (Jacobson & Archodidou, 2000; Spiro et al., 1987; Swan, 1994; Zydney, 2005, 2010; Zydney & Grincewicz, 2011) and only one study was identified with middle school learners (Azevedo, Moos, Greene, Winters, & Cromley, 2008). Although CFT informs hypermedia design for advanced knowledge acquisition, research with younger learners may benefit the literature base.

One last issue that relates to hypermedia design, including the design of CAPs, concerns scaffolding within the hypermedia system. There are numerous suggestions for scaffolding to support learners with complex concepts and navigating the hypermedia site (Jacobson, 2008; Jacobson & Archodidou, 2000; Shapiro, 2008; Zydney, 2005, 2010). Zydney (2005, 2010) suggests multiple scaffolds are not as effective as a single scaffold. However, Shapiro (2008), Jacobson (2008), and Jacobson and Archodidou (2000) use multiple scaffolds in the hypermedia learning environment. For example, Shapiro (2008) offers design suggestions for embedding scaffolds to support learners with low prior knowledge, address specific learning goals, and even scaffold for metacognition and learners with high prior knowledge. These range from providing

site maps, including metacognitive prompts, and highlighting important links. Jacobson (2008) and Jacobson and Archodidou (2000) use guided conceptual criss-crossing and Story Maker as part of the Scaffolding Connected Knowledge Framework.

Although there are many studies, including design-based research investigating hypermedia systems, most all of the studies are conducted with university students with only a few exceptions for younger learners. Furthermore, none of the studies contextualize cases, cognitive flexibility, or hypermedia systems in mathematics education. Geometry, when viewed as an ill-structured domain, is well positioned for design research concerning CAPs. In addition, hypermedia systems with middle school learners can add to the knowledge base concerning design for younger learners.

Considering Context

While CFT and CAPs are promising, it is unclear how to design in the context of mathematics, in particular with the concept of geometric space. Jacobson (2008a) and Jonassen (2012) recommend iterative, design-based research to better understand the integration of CFT and CAPs in practice. Although CFT is a context-independent theory, it requires the integration of context-dependent theories to inform the design of cases and the learning environment more generally. Geometric space is an area where offering multiple perspectives of space and dimension seems likely to improve the leaners' ability to develop a flexible concept. Rather than start with formalized naming conventions (e.g., vertex, coordinate grid), learners might conjecture about relationships between seeing and perceiving between dimensions.

CFT principles guide the design of CAPs, but the context-dependent nature of applying mathematics requires considering context in design. For this project, geometry, specifically spatial reasoning and visualization, is considered an ill-structured domain, in large part because

of its situated nature in the real world. In an interview with Spiro (personal communication, October 16, 2012), he describes both well and ill-structured aspects of mathematics instruction:

> There are some aspects of math that fall so far towards the well-structured side of things that it would be silly to teach them in the way CFT teaches things. For example – multiplication tables – if you need to know multiplication tables, you might as well just memorize them. But that's not an understanding in mathematics and it's certainly not an application of mathematics. One of the things I have said is even the most well-structured aspects of the most well-structured domains become ill-structured when they touch the messy real world. So the application of mathematics knowledge tends to become much more ill-structured.

Implementing the design in a variety of contexts is important to understanding how the cases manifest, especially in K12 where a large literature gap exists. In many examples using cases, the cases typically include expert perspectives in order to help convey the complexity of a concept or problem. Yet, learning about space, dimension, and perspective does not lend itself to expert perspectives. This framework seeks to show a possible way to interpret CAPs in the context of mathematics, integrating the mathematics specific design framework of Realistic Mathematics Education (RME).

Aims of the Framework

The aims of the design framework are to both inform the design of CAPs for a hypermedia system in middle school geometry and inform the related learning theories. The framework seeks to address problems in learning caused by reductive practices, instead focusing

on supporting the flexibility and usability of concepts in variable situations. Using conjecture mapping (Sandoval, 2014) as part of the design-based research project allows the researcher to articulate both design and theoretical conjectures that can later be revised.

Prior Work

The framework builds on a previous design with 5th and 6th grade learners investigating space and dimension. The first design iteration challenged learners perception of space with the video, *Flatland: The Movie* (Travis & Johnson, 2007). The video helped learners visualize a 2D perspective, consider relationships between dimensions, and spurred discourse concerning the possibility of higher dimensions (Valentine & Kopcha, 2014). Most palpable from the study was the learners' experience of ontologically trying-on dimensions, 0D, 1D, 2D, and 4D. For example, some learners conjectured about sight, motion, and perspective in the fourth dimension based on analogies between the second and third dimensions. In this sense, learners tried on different/alternative perspectives about space and dimension.

The current framework was developed as part of a design-based research project. Design-based research is described as "a series of approaches, with the intent of producing new theories, artifacts, and practices that account for and potentially impact learning and teaching in naturalistic settings" (Barab & Squire, 2004, p. 2). This is in line with other design researchers who characterize design research as interventionist, iterative, process oriented, utility oriented, and theory oriented (Gravemeijer & Cobb, 2006; The Design-Based Research Collective, 2003; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006, p. 5). As design-based research, this story is framed through Sandoval's (2014) activity of conjecture mapping. Conjecture mapping is touted as a promising way design researchers can articulate their conjectures and show how these are embodied within the design. A thorough explanation of the conjecturing process as it

relates to the project will be discussed. First, the problems this framework hopes to rectify are detailed below.

Problem Analysis

This section describes the general and more specific math problem briefly discussed earlier that led to the design framework presented in this paper. Described first are the general problems resulting from oversimplifying complex concepts. Next, problems specific to middle school geometry are articulated, including lagging geometric reasoning portrayed in international assessments and misconceptions arising from traditional "textbookization" practices. It is argued that the application of mathematics, in particular geometric and spatial reasoning, is ill-structured and benefits from attention to embodiment and alternative perspective taking in the learning environment. This section ends by reviewing gaps in the literature specific to designing CAPs.

Oversimplification

In an attempt to make complex concepts easier to learn, instructional approaches tend to simplify and compartmentalize concepts. Spiro et al. (1988) report that oversimplifying complex concepts results in many reductive biases, inhibits flexible concept development, and interferes with later learning. Reductive biases include "overreliance" on singular mental representations, "ridged compartmentalization," "context-independent conceptual representations," among others (p. 3-4). Theorists, designers, and researchers, such as Spiro and colleagues (Jacobson & Spiro, 1993; Spiro et al., 2007a, 2003, 1988, 1987), have sought theories, principles, and practices that convey the complexity of concepts in ill-structured domains in order to increase their flexibility and usability in subsequent situations. They argue that simplifying and compartmentalizing complex concepts not only inhibits flexible concept development but also contributes to "reciprocal misconception compounding" (1988, p. 9). This is described as misconceptions

becoming more robust in subsequent learning. Spiro et al. (1987) find evidence from their two experimental studies with high school history students that success depends on the "*flexibility* with which the relevant prior knowledge is represented in memory, and the mastery or *control* the individual has over those flexible representations" (p. 2-3). In this sense, accessibility to varied experiences early in the learning trajectory inhibits narrow and inflexible concept formation. However, mathematics is traditionally taught as a well ordered, unitary system will little attention to the variant nature of concepts in application. As a result, students in the U.S. are largely unsuccessful in geometry.

Lacking Attention in Geometry

There is a need for focused attention in the domain of geometry in the U.S., especially in the middle grades. Within the field of Mathematics Education in the U.S., geometry has been referred to as *the forgotten strand* (Lappan, 1999) and *a tad off to the side* (Shaughnessy, 2011). The Trends in International Mathematics and Science Study (TIMSS) during the years of 1999, 2003, and 2007 shows the grade 8 geometry and measurement strand as weakest (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005; Gonzales et al., 2009; Mullis et al., 2000). The TIMSS Advanced 1995 Assessment, the only year in which the U.S. participated, shows high school seniors were near the bottom in mathematics and last compared to all participating nations in geometry (Mullis et al., 1998). Although the most recent TIMSS (2011) results shows increases in geometry for 4th graders and no significant change in 8th grade, "nearly all of the ninth grade and benchmarking participants had a relative weakness in geometry" (Provasnik et al., 2012, p. 147) as compared to overall mathematics achievement.

K-8 mathematics curriculum materials in the U.S. pay little attention to geometry concept development (American Association for the Advancement of Science [AAAS], 1999; NCTM, 2000). In addition, the National Mathematics Advisory Panel ([NMAP], 2008), while focused on increasing algebraic readiness, state that geometry and measurement along with number concepts "are the Critical Foundations of Algebra" (p. xvii). Designers and researchers should seriously consider increased attention to geometry in the middle grades in order to change these concerning trends.

However, instead of solely focusing on increasing test scores, this may be an optimal time to re-question the positivist assumptions about what mathematics is and is not. Although it seems mathematics, and geometry in particular, can be taken up as a well-structured domain, in application, mathematics is an ill-structured domain. A concept such as space requires, "the ability to "see," inspect, and reflect on spatial objects, images, relationships, and transformations" (Battista, 2007, p. 843). Besides memorization of multiplication tables and possibly vocabulary, there is little in math education that is taught without the ultimate expectation that it be applied in the "messy" real world. Ironically, most of K-12 education is not applied; rather, it is a subject acting more as a gateway for higher studies – a litmus test for our society (Zhao, 2012).

K12 mathematics is a place where applications of CFT and CAPs can be developed, implemented, and studied. Both the National Council of Teachers of Mathematics (NCTM) and Common Core State Standards for Math (CCSSM) try to accomplish cognitive flexibility in a similar manner to CFT, but there is still a tendency to reduce complexity and adopt lower standardized expectations (Zhao, 2012). There is very little examining design efforts using CFT in mathematics, but there is a strong need for it. In mathematics education, Silver and Herbst

(2007) remark, "Our capacity to see the teaching and learning of mathematics where others just see teaching and learning, and yet others see only social interaction, attests to the different purposes for which individuals and organized fields construct theories" (p. 61). In order to impact the Mathematics Education community, researchers will need to focus on the way theories play out in specific mathematical contexts. In the case of designing CAPs for mathematics education, design-based research can help inform both the design and theory.

Problems Specific to Geometry

There are several problems specific to geometry teaching and learning, with some leading to learner misconceptions (e.g., prototypical images of a triangle with a horizontal base interfere with recognition of less prototypical triangles). This section describes the following: 1) taken-for-granted assumptions that geometry is a well-structured domain, 2) scarcity of attention to embodiment and the learners' perspective when designing learning experiences, and 3) a lack of meaningful learning opportunities to connect to learners' existing spatial concepts.

Taken-for-granted assumptions. Space is complex concept as indicated in the various domains that take it up quite differently (e.g., mathematics, architecture, geography, art, engineering). When an assumption is made that space is a well-structured concept, routinized instructional design practices may follow, such as rote demonstration of categorization and/or use of abstract formulas. Rarely are learners encouraged to problematize space in the context of geometry. We may be too reliant on the Euclidean mathematical notion of space represented by x, y, and z coordinates. Even more problematic is that school explorations of space start in an abstract manner, building on idealizations of the point (0 space), the line (1 space), and the plane (2 space) – spaces that are actually impossible to *find* in the world.

Embodiment and perspective are largely ignored in design. Many attempts to re-conceptualize learning environments in mathematics (especially constructivist/social constructivist) still move towards supporting students' construction of concepts and process standards that are pre-determined outside the learner. For example, school mathematics standards (NCTM, 2000), although a worthy structure to guide teaching and learning, do not always serve as starting points for investigations in geometry. Ideas of embodiment and perspective are largely ignored. Rather, designing geometry learning environments should consider returning to embodied perspectives (see Gibson, 1986; Merleau-Ponty, 1945/2002, 1948/2004), transforming perspectives (e.g., through technology tools) (see Ihde, 1993, 1977/2012), and fostering reflection among learners in the process (NCTM, 2000). Both Husserl's (1936/1970) task of a new inquiry and Freudenthal's (1973, 1983) RME advocate for a *rediscovery* approach, mathematizing phenomenon in the real world as a starting point.

Design focused on problematizing and thus, opening up perspective, has the potential to support learners to develop important visualization skills, reflect on variants in multifold real-world phenomena, and learn to connect to mathematics in powerful ways. Merleau-Ponty's radio talk (1948/2004) offers an embodied perspective for those aiming to embrace the inherent complexity of space:

> In psychology as in geometry, the notion of a single unified space entirely open to a disembodied intellect has been replaced by the idea of a space which consists of different regions and has certain privileged directions; these are closely related to our distinctive bodily features and our situations as beings thrown into the world. Here, for the first time, we come across the idea that rather than a mind *and* a body, man is a mind *with* a body, a

> being who can only get to the truth of things because its body is, as it were, embedded in those things. (p. 43)

Merleau-Ponty describes a geometric space that cannot be transcended by intellect, but one where humans are intimately connected – always moving and relating to their particular bodily position and perspective. Gibson (1986), founder of ecological psychology (which rejects stimulus-response theories), writes that "To perceive is to be aware of the surfaces of the environment and of oneself in it. The interchange between hidden and unhidden surfaces is essential to this awareness" (p. 255). He adds, "Knowing is an *extension* of perceiving" (p. 258). In this sense, to come to know space requires perception and according to Gibson, is not solely the environment or the self, but the embodied self.

Learning about geometric space is an activity wrought with complex relations. The process begins the moment humans are born, relying on perception to make sense of objects in relation to each other and the self. Yet, geometry instruction is rarely approached with the goal of problematizing ideas of space and perspective. Recently, the National Council of Teachers of Mathematics (NCTM) released *Principles to Actions* (2014). One of the eight teaching practices they highlight includes, "Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (p. 48). Problematizing space and perspective is a form of productive struggle the project and CAPs seeks to support.

Sometimes K-12 education assumes that if students can categorize and identify shapes in geometry, an understanding of spatial relationships is evident. However, the spatial concepts students develop and refine result from experiences that challenge their current perspective and are necessarily rooted in how humans perceive their world. These preexisting conceptions built

by naturally perceiving the world should not only be considered as starting points, but challenged for the inherent complexity of spatial concepts to develop. There is much to learn by turning attention to problematizing perception, not just as a way to design, but also as an object of study.

Freudenthal (1973) best captures this space when describing geometry at its most foundational level:

> And since it is about the education of children, [geometry] is grasping that space in which the child lives, breathes and moves. The space the child must learn to know, explore, conquer, in order to live, breath and move better in it. Are we so accustomed to this space that we cannot imagine how important it is for us and for those we are educating? (p. 403)

It is possible that the poor performance in geometry stems from instructional practices that ignore embodiment and perspective in the learning environment design. Instead of starting with abstract reference points, such as nomenclature, it seems promising to support learners to take on their embodied perspective in relation to the world to understand the roots of geometry as that of mathematizing the everyday world? In this sense we can support productive struggle in learning.

Lack of meaningful learning opportunities regarding the space concept. The complex world where geometry originates is rarely drawn on as a starting point for investigating space, rather transcendent views of space are assumed (Husserl, 1936/1970; Ihde, 1993; Merleau-Ponty, 1945/2002). If designers, instructors, and learners remain in an unreflective, everyday attitude, they may not to see all the variables and interactions at play. Additionally, the learners pre-existing conceptions may not be drawn on and/or challenged, leading to the development of misconceptions (Bransford et al., 2000).

When ‘New Math’ became popular in the U.S., instructional approaches operated by replacing the learner’s insight with that of the adult mathematician’s (Greer & Verschaffel, 2007). Although students’ ways of operating in mathematics classrooms are not always in line with their teacher’s adult mathematical models, children are *rational beings* and their mathematical models should be understood and developed, not extinguished (Steffe, 1994). At the same time, students have been shown to hold misconceptions. In geometry, for example, students form conceptions of space early in life simply by *being* in the world. They engage in everyday activities like “looking, walking, drawing, building, and manipulating objects” (Lehrer, Jenkins, & Osana, 1998, p. 169). They develop intuitions and informal knowledge about spatial structure before formalized words are attached to their concepts (Freudenthal, 1983; Gravemeijer, 1998; National Mathematics Advisory Panel, 2008; Piaget & Inhelder, 1967; van Hiele, 1986). It is possible that this ontological, or *being* in the world, means pre-formalized schooling concepts could come in conflict with Euclidean geometric concepts formally taught in school (Lehrer et al., 1998). Additionally, students’ misconceptions can be caused by the materials and processes used in schools (Lehrer et al., 1998). For example, most textbooks and teachers present images of triangles to students with a horizontal base, although this is not a defining characteristic of triangles. This results in students primarily recognizing prototypical examples of triangles to the exclusion of non-prototypical examples.

These misconceptions can inhibit students from reaching the intended goals of instruction and will most likely resurface in later geometry instruction. A recent literature review shows several misconceptions held by students related to geometric reasoning. The four main areas include misconceptions of geometric figures and their properties, geometric transformations,

measurement, and static versus dynamic geometric relationships within and between dimensions. These misconceptions are highlighted below.

Geometric figures and their properties. Many exemplars of triangles, rectangles, and other polygons are not recognized (Carroll, 1998). Even in instances where students memorize verbal definitions of polygons, students do not incorporate shape properties into their concept of shape. Students make comments such as, 'it doesn't *look like* a shape' (Clements, 2003). Unimportant characteristics (e.g., configuration and orientation) of shape can limit students from recognizing instances of a particular shape.

Geometric transformations. According to Goldenberg, Cuoco, and Mark (1998) students tend to reason discretely instead of continuously. This becomes problematic in multiple geometric situations, especially transformations, because students are not able to distinguish what stays the same and what changes in a situation. Students sometimes lack the notion that transformation is the mapping of all two-space or three-space and operate as if only the pictured figure, not the entire plane, is transformed (Edwards, 1991).

Measurement. According to Driscoll (2007), student performance is weak if they, "come to use measurement formulas as procedures with no connections to their meaning" (p. 71). This instruction-induced limited conception causes students to rely on formulas as rote procedures. The effects of this show up as limited conceptions of what length, area, and volume actually mean.

Static versus dynamic geometric relationships within and between dimensions. Students have a predisposition to the static, not dynamic nature of geometry. This has been exemplified across geometry strands. For example, students' concept of shape as image-based conflicts with the geometric understanding of shape properties of varying configurations and orientations

(Carroll, 1998; Clements, 2003; Clements & Battista, 1992; Monaghan, 2000). In Keiser, Klee, and Fitch's (2003) study, most students thought that a scaled up angle (from a map to the real world) would actually increase in size because the distance between the rays would increase. These difficulties are associated with static representations of angle. These misconceptions are similar in nature to the phenomenon described earlier by Spiro et al. (1988) concerning reciprocal misconception compounding, a problem resulting from simplifying complex concepts. Many misconceptions have been identified in geometry and their origins might lie in this simplification process as well as instruction that fails to incorporate learners' prior knowledge.

Theory and Design Principles

The design is presented in the form of a conjecture map. Conjecture mapping is a beneficial way to capture conjectures concerning design and theory and give design researchers a tool to inform future revisions of design and/or theory. Conjecture maps are informed by multiple theories and design principles. In addition, conjectured relationships between embodiments of the learning environment (e.g., tools and materials, activity/task structures, participant structures, discursive practices), processes expected to occur, and outcomes related to these processes are conjectured in advance. The theories and principles most relevant are CFT, RME, and hypermedia design. Together these inform the design, development, and implementation of CAPs, suggesting both the content of the cases and the way they are used in the learning environment.

Cognitive Flexibility Theory

Spiro et al. (1988) define cognitive flexibility as that which "involves the *selective* use of knowledge to *adaptively fit* the needs of understanding and decision making in a particular situation" (p. 5). CFT is a context-independent meta-theory to address the goal of supporting

flexible concept development. However, the application of the theory requires attention to context-dependency in use. CFT is a theory that "builds bridges" (Spiro et al., 2003), not a competitor with other theories. It is a "constructivist theory of learning and instruction" (Spiro, Feltovich, Jacobson, & Coulson, 1995, p. 85). A central claim is that to acquire advanced knowledge, learners must revisit, or "*the landscape must be criss-crossed in many directions*" to acquire advanced knowledge (Spiro et al., 1988, p. 6). In this way, the learner better distinguishes variations in multiple real-world instances. CFT draws on the Wittgenstein metaphor of *criss-crossing a landscape* wherein teaching and learning:

> one proceeds from case to case (example to example) following different routes of organization on successive traversals of the knowledge landscape. Sometimes one returns to the same site (case), but coming from a different direction, bringing a different set of perspectives. Thus different facets of each case are highlighted when juxtaposed to varying other cases (and seeing those multiple facets is essential in producing transferable knowledge). Thus, in CFT, revisiting is not repeating. (Spiro et al., 2003, p. 6)

Initially, CFT utilized flexible hypertext systems, eventually expanding to Cognitive Flexibility Hypermedia systems, which has shown to be successful at enabling transfer (e.g., Jacobson, 2008; Jacobson & Spiro, 1993).

The eight principles below include those that Spiro and colleagues emphasize repeatedly in multiple renditions of CFT and that support problematizing space in order to support learners' development of a robust, flexible, and non-rigid concept. Table 3.1 contains a summary of each principle and also describes the implications for design.

Table 3.1

Summary of Eight Principles of Cognitive Flexibility Theory and Design Implications

Principle	**Summary**	**Design Implications**
1. Multiple Knowledge Representations	In order to *use* knowledge in different ways, it has to be *represented* in different ways (Jacobson & Spiro, 1993; Spiro et al., 2003, 1988). The focus is giving learners opportunities to form multiple perspectives, revealing the true complexity of the situation or concept being learned.	Designers may use multiple themes, analogies, intellectual points of view, cases, etc. to support learners' development of multiple perspectives.
2. Interconnectedness	Teaching and learning should convey the interconnectedness of the multiple knowledge representations to support the flexible organization of concepts (Jacobson & Spiro, 1993; Spiro et al., 2007b, 2003, 1988)8).	Learners need to be given opportunities to revisit and rearrange the multiple representations of a concept. CFT recommends a coding system where a large number of cases are coded with multiple themes producing many "possible retrieval routes in memory" (Spiro et al., 2007a, p. 22).
3. Context-dependency and Conceptual Variability	Learners develop a richer sense of meaning when they consider how concepts are used in various contexts. Interfaces, or other systems in place for learners to come into contact with these multiple and interconnected representations need to allow for "*flexible*, recombinable structures" (Spiro et al., 1988, p. 8).	Illustrate the variable use of concepts across contexts by showing variability in patterns of real world use. Juxtapose cases to draw attention to surprising similarities and differences.
4. Cases and Minicases	Cases serve as context. "Transfer depends on having a rich store of experiences" (Spiro et al., 2007a, p. 6)6).	Learners need experience with many cases, employing microcosm-macrocosm relationships to experience nuances and variability of concepts in real world use (Jacobson & Spiro, 1993; Spiro et al., 2007a, 2007b, 2003, 1988). They should be revisited in the course of learning.

(table continues)

Principle	Summary	Design Implications
5. Early Introduction of Complexity	Simplification practices are replaced with introducing complexity in a "manageable manner" as with the use of minicases. (Jacobson & Spiro, 1993, p. 3).	Spiro et al. (2003) describe Cognitive Flexibility Hypermedia systems that begin with minicases containing complexity in "bite size chunks" (p. 7) in order to incorporate complexity from the onset of instruction.
6. Concentrated Effort to Change Underlying Ways of Thinking	The goal is to change the learners' underlying way of thinking, such as "complacency of perception, thought, and action that we so frequently experience" (R. Spiro, personal communication, October 16, 2012).	Dissonance strategies can be used to shake up learners' thinking. For example, video affordances such as overlays, manipulation of film (speed, magnification), etc. can change the way learners can look at phenomenon.
7. Conceptual Variability Search	Conceptual variability refers to the various ways concepts are used. By creating cases related to this search, learners are supported in developing flexible uses of the concept.	One of the first steps involves conducting a concept variability search. This search focuses on the variety of ways the concept is used, supporting learners to employ the concept with greater flexibility.
8. Focus on Experience Acceleration	Experience acceleration refers to supporting learners to build "cognitive momentum," (Spiro et al., 2007a, p. 6) accelerating the experience acquisition process.	Learners should be given opportunities to revisit prior instances of a concept to notice distinctions. Revisiting part of a case can bring other aspects into being without taking as much time. This can be done with the use of minicases and video overlays.

Hypermedia

Jacobson (2008) presents four components of an educational hypermedia systems design framework that support learners' development of flexible concepts: representational affordances of hypermedia, knowledge in context, learning scaffolds, and learning tasks. Representational affordances include texts & symbols, visual imagery, animations, digital video, and 2D & 3D models and simulations. Cases consist of these affordances and are "authored and selected to have contrasting surface features while sharing important structural conceptual components related to the particular domain being studied (i.e., ''big ideas'') (Jacobson, 2008, p. 11). Knowledge in context (computer mediated modular cases, conceptual minicases, and conceptual

explanations) provides learners with various contextual experiences and explicates important conceptual perspectives in the domain. Learning scaffolds (representational, conceptual, ontological, problem solving, and metacognitive) are integrated into the system in order to support learners' development of cognitive flexibility and include the various representations, mini-lessons, and explanations.

The last framework component concerns learning tasks. Learners' work with cases and mini-lessons with the inclusion of simulations, multimedia, and even the writing/feedback features. The goal is for learners to construct or further develop their conceptual framework along with opportunities for application in multiple contexts. The second focus is on guided conceptual criss-crossing and project-based extensions.

Realistic Mathematics Education

CFT and hypermedia design encompass design principles for developing instructional materials and considering the teaching and learning of complex concepts in ill-structured domains. With CFT viewed as an umbrella theory, additional perspectives, such as the mathematics domain-specific perspective of RME, work together to form the content of the cases. RME offers principles for design, teaching, and learning in the context of mathematics that draws on complex, real-world phenomenon (Freudenthal, 1983). The most central recommendation from RME is selecting real-world phenomenon to mathematize. This section describes the domain-specific theory, RME, and the associated principles related to designing CAPs.

RME was developed in the Netherlands in the early 1970s and the foundations are attributed to Freudenthal (1968, 1973, 1983). The goal of RME is to guide learning and instruction in mathematics as a human activity of grasping everyday reality. Math reasoning is

not separated from real world but rather learners use the world both as the starting point for investigations and also as the context to fold back their reasoning (Freudenthal, 1973, 1983; Pirie & Kieren, 1994). "Folding back" is an activity where a learner revisits former informal noticings or even more developed understandings with a mathematical lens. For example, most learners have experienced sharing a candy bar but may not have considered this phenomenon in terms of equality or fractional relationships. Freudenthal felt students should reinvent mathematics in guided activity, creating the opportunity for learners to *mathematize* everyday life and their own mathematical activity. Cobb, Zhao, and Visnovska (2008) clarify that mathematization supports students' learning through guided reinvention – "a process in which students formalize their informal understandings and intuition" (p. 2).

As the name implies, learners must perceive the starting points of mathematics as experientially real. Van den Heuvel-Panhuizen and Wijers describe the importance of the term *realistic* related to the aims of teaching. "The Dutch translation of "to image" is "zich REALISEren." It is this emphasis on making something real in your mind that gave RME its name" (p. 288). Problems should be connected to the real world, but can also be from a fantasy world or the formal world of mathematics as long as they are real in the student's mind. An example of a problem context that connects to the real, *imaginary* world is *Flatland: The Movie* (Travis & Johnson, 2007). In the film, a three-dimensional sphere visits a two-dimensional polygon (square). They explore various perspectives and relationships between dimensions on their journey, mimicking the human world or work, school, travel, and conversation.

RME as a design theory includes six principles, described by van den Heuvel-Panhuizen and Wijers (2005) among others (Cobb et al., 2008; Figueiredo, van Galen, & Gravemeijer, 2009; Gravemeijer, 1998; Gravemeijer & Terwel, 2000), and is rooted in Freudenthal's work

(1968, 1973, 1983). The principles and brief summary along with implications for designing CAPs for the hypermedia site is shown in Table 3.2 below.

Table 3.2

Summary of Realistic Mathematics Education Principles and Implications for Designing Cases as Alternative Perspective

Principle	**Summary**	**Implications for Designing CAPs**
Activity Principle	• Learning as doing • Students are active participants in the learning process – they develop tools and insights • Students are confronted with problem situations	Cases as alternative perspective (CAPs) are part of more encompassing investigations on the hypermedia site asking learners to engage in activity with the phenomenon.
Reality Principle	• Learning mathematics originates in mathematizing reality • Starts with rich contexts • Enables students to apply mathematics • Recognized as the starting point (source) and end point (application)	CAPs are made up of real-world phenomenon.
Level Principle	• Students pass through levels of understanding • Students move to the next level by reflecting on activity; interacting • Models bridge the gap between informal (model of) and formal (model for) i.e., the number line • Focuses on relations between what has been learned earlier and what will be learned later	Reflecting on activity can be supported with the use of prompts and Web 2.0 tools.
Intertwine-ment Principle	• Mathematics domains are linked within and between • Rich context problems means applying a range of tools and understandings	Rich problems draw on big ideas. In geometry, CAPs are chosen that illustrate the complex nature of space, perspective, and dimension. CAPs are designed that illustrate connections between domains (geometry, algebra, engineering).
Interaction Principle	• Learning mathematics is a social activity • Opportunities to share strategies and inventions • Interaction can evoke reflection, increasing understanding • Students have their own trajectories; differentiation • Means to support students' progressive reorganization of mathematical reasoning	Integration of tools and practices supporting social discursive practices, such as blogs, class discussion, small group discussions, group projects.
Guidance Principle	• Students are guided to re-invent mathematics • Students have room to construct mathematical insights and tools • Teacher provides environment for this construction • Teacher anticipates students understandings and skills • Always includes the perspective of long-term learning process based on goals • The how and what meet each other • Provides students with suitable tasks	Designs that allow for guidance, choice, and time to step away as part of the hypermedia site and teaching design.

Conjecture Mapping

Design is an iterative process and benefits from activities that allow the design/research team to make their conjectures and decisions explicit. Conjecture mapping (Sandoval, 2014) is the activity used to organize conjectures in this project. Sandoval writes that designing learning environments is a theoretical activity and "as researchers (and not just designers) we have an obligation to be as explicit as possible, *in advance"* (p. 20) about how learning happens or can be designed to happen. The process starts with the development of high-level conjectures. This is a "theoretically principled idea of how to support some desired form of learning articulated in general terms and at too high a level to determine design" (Sandoval, 2014, p. 22). High level conjectures are then used to guide the hypothetical learning trajectories made up of design and theoretical conjectures. Sandoval writes, "It is crucial to understand such trajectories are not hypothesized in abstracted notions of learners' capabilities (such as developmental level or reasoning ability), but explicitly in relation to the given means of support" (p. 24-25). In the following sections, components of the *Space and Perspective* project conjecture map are shared as they relate to CAPs embodiment on the *Space and Perspective* hypermedia site (see http://spaceandperspective.com/). First, high-level conjectures for the entire project are shared to show the larger context in which the design of CAPs resides. Following are embodiments of the learning environment (e.g., tasks, tools, participant structures), mediating processes, and conjectured outcomes concerning CAPs.

High-level Conjectures

Five high level conjectures about how to support learning in middle school geometry make up the entire project. These include conjectures related to: 1) leveraging technological affordances, 2) supporting flexible concept development with CAPs, 3) guiding learners to re-

invent mathematics, 4) creating opportunities for learners to connect embodiment to geometry learning, and 5) cultivating socio-mathematical practices in the learning environment. These high-level conjectures guide the formation of conjectured interactions among embodiments of the learning environment, mediating processes these embodiments are thought to bring about, and outcomes related to these processes.

Conjecture Map for Space and Perspective

Figure 3.1 shows specific embodiments, mediating processes, and conjectured outcomes operationalizing the high-level conjectures. The relationships between these components are expressed as design conjectures and theoretical conjectures. Design conjectures connect the embodiments to the mediating processes they are thought to bring about. Theoretical conjectures then describe the outcomes of these mediating processes. For example, a design conjecture associated with CFT may be stated as follows: if learners engage in investigations incorporating a criss-crossing of cases and minicases, opportunities are created to investigate multiple perspectives of space as they relate to real world uses of the concept. The theoretical conjecture associated with this design conjecture, then, is as follows: if learners examine/explore/investigate multiple perspectives of space through a variety of cases, they will construct a flexible, usable concept. Figure 3.1 shows these relationships with arrows between the three columns.

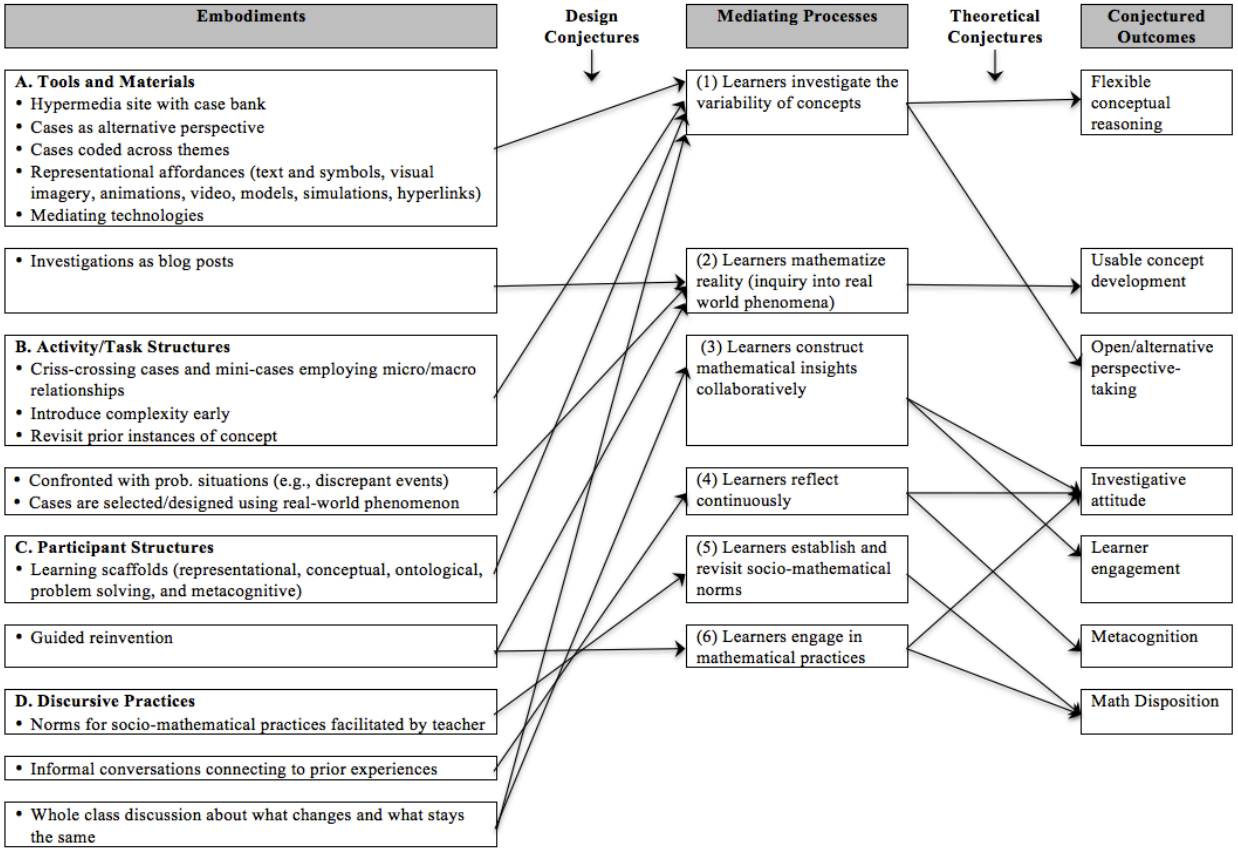

Figure 3.1. Conjecture map for *Space and Perspective* containing the embodiments, mediating processes, and conjectured outcomes associated with a design framework that integrates theoretical perspectives from CFT, hypermedia design, and RME. Arrows indicate the conjectured relationships associated with the design and theories.

The Embodiment of CAPs in a Hypermedia Site

The conjecture map presented in Figure 3.1 maps out five high-level conjectures associated with *Space and Perspective.* In this section, we focus our attention on the high-level conjecture associated with CAPs. As noted earlier in this paper, the design of CAPs is an area of

interest to other designers and is relatively absent from much of the literature. While each of the theoretical perspectives informing the design of *Space and Perspective* are important, the conjecture associated with CAPs provides tremendous insight into the nature of the design of CAPs.

The high-level conjecture associated with CAPs states that conveying the complex, ill-structured nature of concepts (e.g., multiple-perspectives through CAPs) supports flexible concept development that is more transferable (usable) to a greater number of situations. To illustrate this conjecture fully, the embodiments of the learning environment are first depicted as they relate to the tools and materials, activity and task structures, and participant structures. Screen shots are then presented to demonstrate how CAPs were embodied on the *Space and Perspective* hypermedia site. The associated mediating processes and conjectured outcomes are then discussed to illustrate how a high-level conjecture can help inform the conceptualization of research questions associated with the conjectures and potentially inform design knowledge in the future.

Embodiments of the Learning Environment

Any learning environment is embodied with tools, materials, structures, and practices. As a design researcher, it is important to conjecture the design relationship between these components and the subsequent processes. When processes manifest or fail to occur, embodiments may be revised in future iterations. Figure 3.2 contains the embodiments of the high-level conjecture associated with CAPs. These are categorized by tools and materials, activity and task structures, participant structures, and discursive practices. Arrows with solid

black lines indicate the embodiments concerning CAPs. Please note that discursive practices are conjectured as essential components for realizing the desired processes and outcomes, but the focus here is on CAPs as embodied in the hypermedia site.

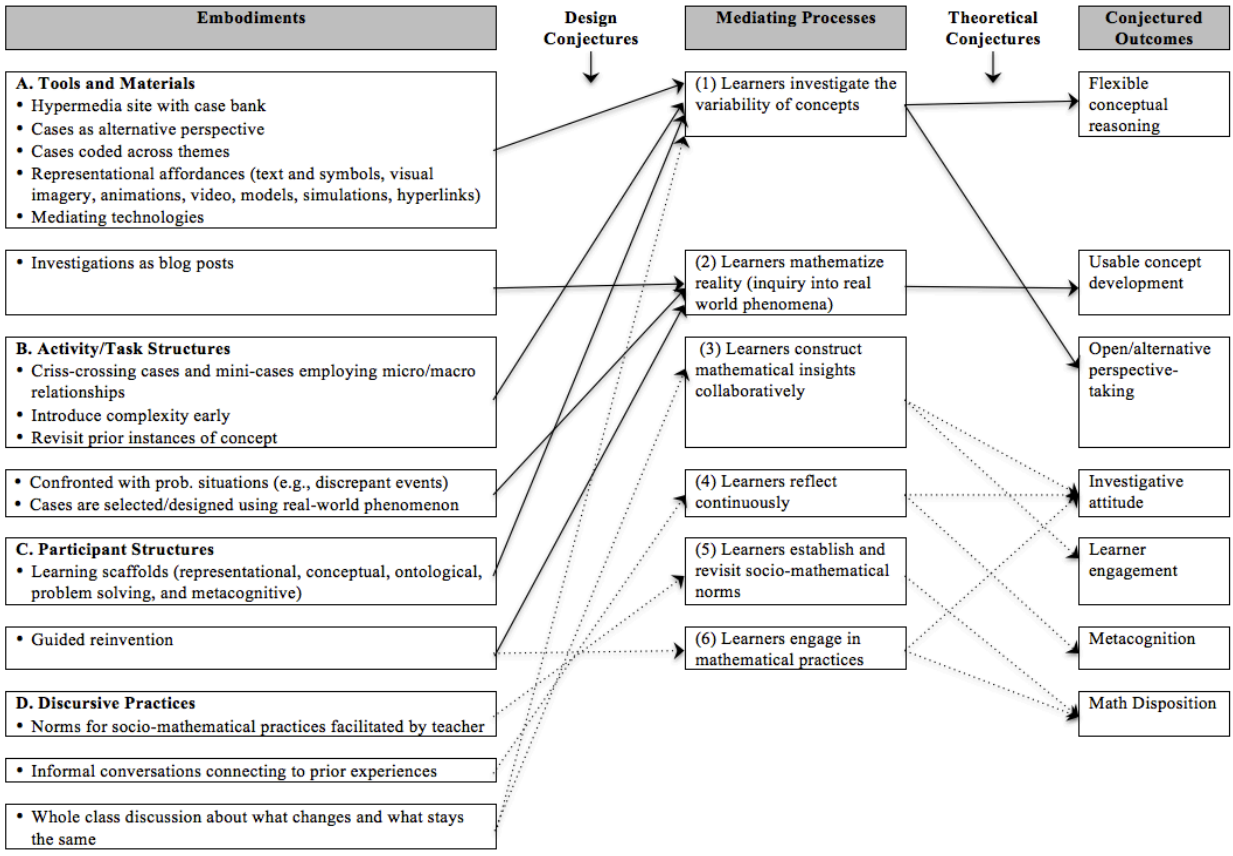

Figure 3.2. Embodiments concerning cases as alternative perspective are shown with solid black arrows.

Tools and materials. Tools and materials are elements of the learning environment that might include resources, handouts, media, and even traditional tools such as calculators. In *Space and Perspective,* the tools and materials include a hypermedia site (created in Wordpress) containing 70 cases in the form of blog posts (see http://spaceandperspective.com/). The site organizes cases by multiple tags (coded across 16 themes). CFT suggests that learners need to be given opportunities to revisit and rearrange the multiple representations of a concept. In

hypermedia systems, coding cases with multiple themes supports many "possible retrieval routes in memory" (Spiro et al., 2007a, p. 22). These are easily accessible on the home page as shown in Figure 3.3.

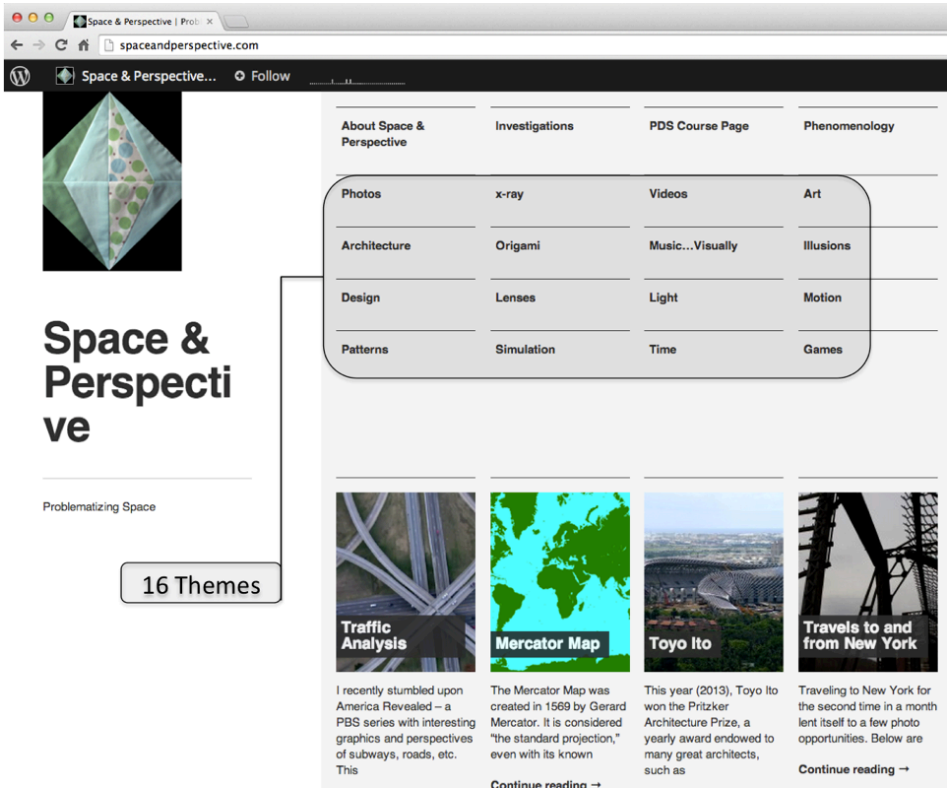

Figure 3.3. View of homepage showing cases coded across 16 themes.

The 16 themes include: photography, x-ray, video, art, architecture, origami/maps, music…visually (sound), illusions, design, lenses, light, motion, patterns, simulation, time, and games. Selecting a theme from the homepage, such as "light" (see Figure 3.4), links the learner to a single web page displaying all of the cases assigned with the light tag.

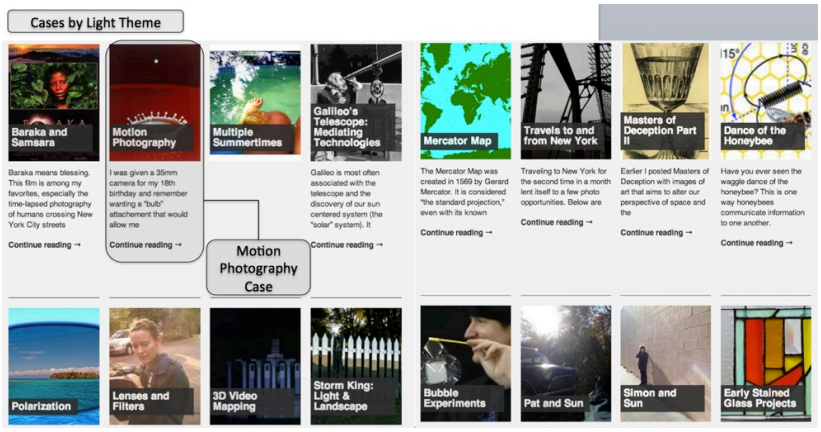

Figure 3.4. After clicking on the light theme, all of the cases assigned with that tag are displayed on a single web page.

When a case, such as "Motion Photography" is selected (see Figure 3.5), the case displays as a web page. Notice the case is also coded in motion, photos, time, and videos. All cases are coded across several themes to support criss-crossing and revisiting of cases, conjectured to be important in developing cognitive flexibility.

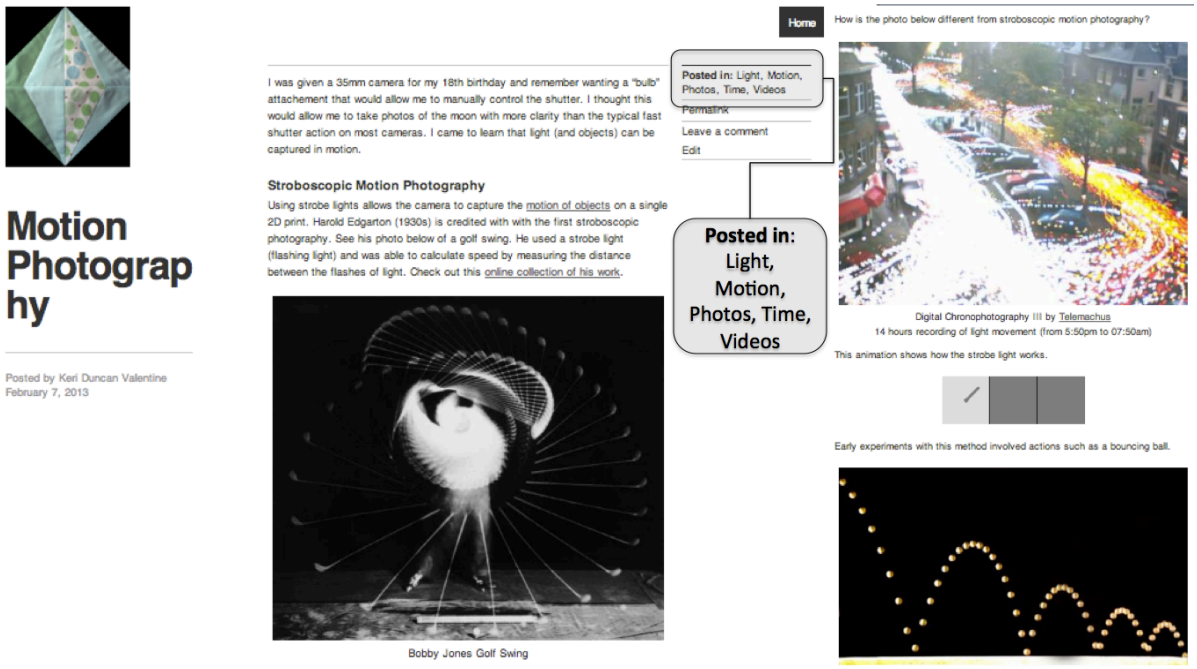

Figure 3.5. Cases, such as "Motion Photography," shown here, are tagged under multiple themes. This case is tagged as light, motion, photos, time, and videos.

One more consideration concerning the design of CAPs is the make-up of a case. A common perspective associated with case-based reasoning (Kolodner, 2006) is that cases present the thinking of an expert in some way. With CAPs, the goal is to present a variety of perspectives concerning a concept, not necessarily a video or text explicating a human viewpoint. For example, an animation of fractal growth is considered a case as alternative perspective. In *Space and Perspective*, cases and minicases are conjectured to include representational affordances such as text and symbols, visual imagery, animations, video, models, simulations, and hyperlinks as advocated by Jacobson (2008a). Figure 3.6 shows the features of a typical case in *Space and Perspective* using "The Fractal Dimension" as an example. The figure shows the title overlaid on an open-source Wikimedia image of a Mandelbrot set. The image is taken from the animation ("Animation of the growth of the Mandelbrot set as you iterate towards infinity"), also part of the case web page. The title plus image created for each case serves as a symbol for the case in order to support learners when revisiting the case in the future. The title plus images allows them to bring the case back to mind quicker when criss-crossing with other cases. The cases also include open-source photos and embedded video. Photos, animations, or videos created for the site are also clearly marked as open-source for others to modify/use for future use. Lastly, all cases include text and hyperlinks to convey relationships, direct the learner, and pose questions for investigation.

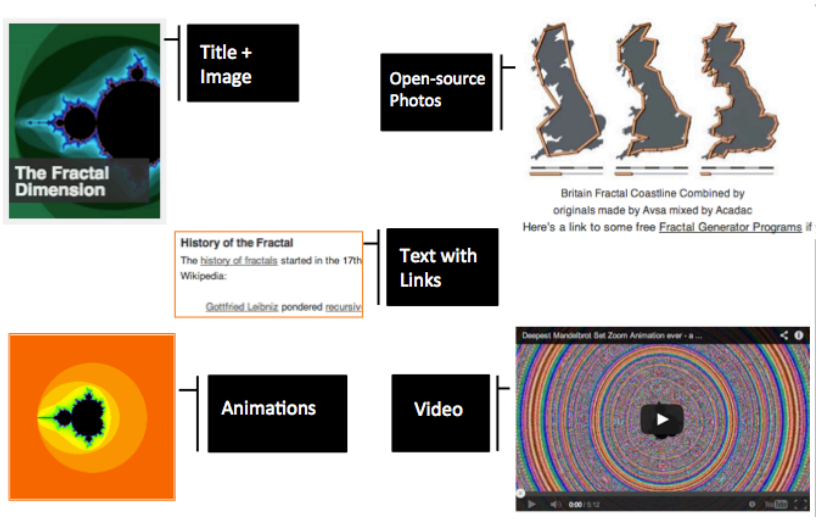

Figure 3.6. Features of a typical case include: a title + image, animations, open-source photos, videos, and text with hyperlinks.

Activity/task structures. Activity/task structures indicate the structure that the activity or task contains, such as the aims and goals. In *Space and Perspective,* the activity/task structures take the form of investigations (also displayed as blog posts). In addition to organizing cases by themes, cases are also curated into 7 investigations as indicated in Figure 3.7. In mathematics, investigations better reflect the tools/materials and learning activity. The 7 investigations include: 1) Norms and setting up blog sites, 2) Flatland: The Movie (Considering the dimensions of space), 3) Considering the dimensions of space: Dimensional mapping, 4) Considering the dimensions of space: Additional cases, 5) Capturing and representing space, 6) Representing impossible and invisible spaces, and 7) Culminating project: Innovative uses of flexible space concepts.

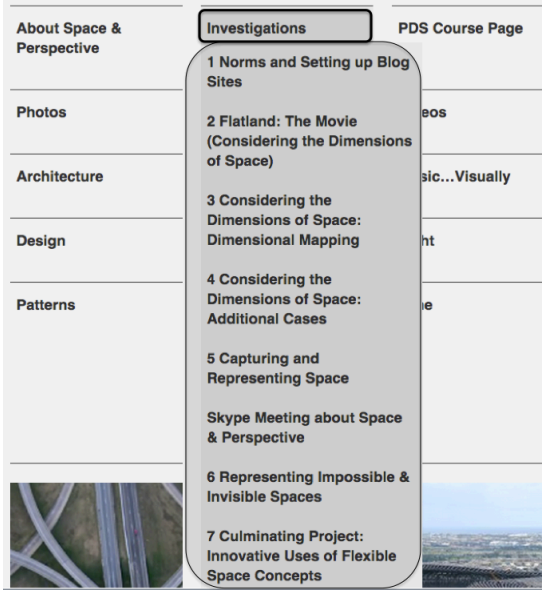

Figure 3.7. Drop down menu displaying 7 investigations.

When an investigation is selected, the learner is directed to a web page that displays multiple sections of the investigation. Within each section, links are provided to the cases. For any single investigation, there may be up to 30 cases, but a section usually includes 4-6 cases for learners to juxtapose and includes a brief introduction. This is similar to the organization of cases, which contain hyperlinks to related cases and media content. Organization of the cases by investigations offers another way to revisit and juxtapose when designing for CAPs. During learning, the offering and linking of cases can present learners with connections among cases, conveying the complexity of the concepts by encouraging learners to criss-cross cases. For example, Figure 3.8 contains sections 5.3 (Mediating Technologies) and 5.4 (Lenses and Filters), which are associated with the investigation, *Capturing and Representing Space*. In Case 5.4, a brief description of the case is presented and hyperlinks to cases are offered for students to investigate. If a learner were to select one of the hyperlinks, they would be directed to the case, which would offer more links to related cases or media content. Section 5.8 (Photographing

Space Activity) is part of the investigation and describes associated projects and activities. In addition, each investigation for *Space and Perspective* includes blog prompts for learners.

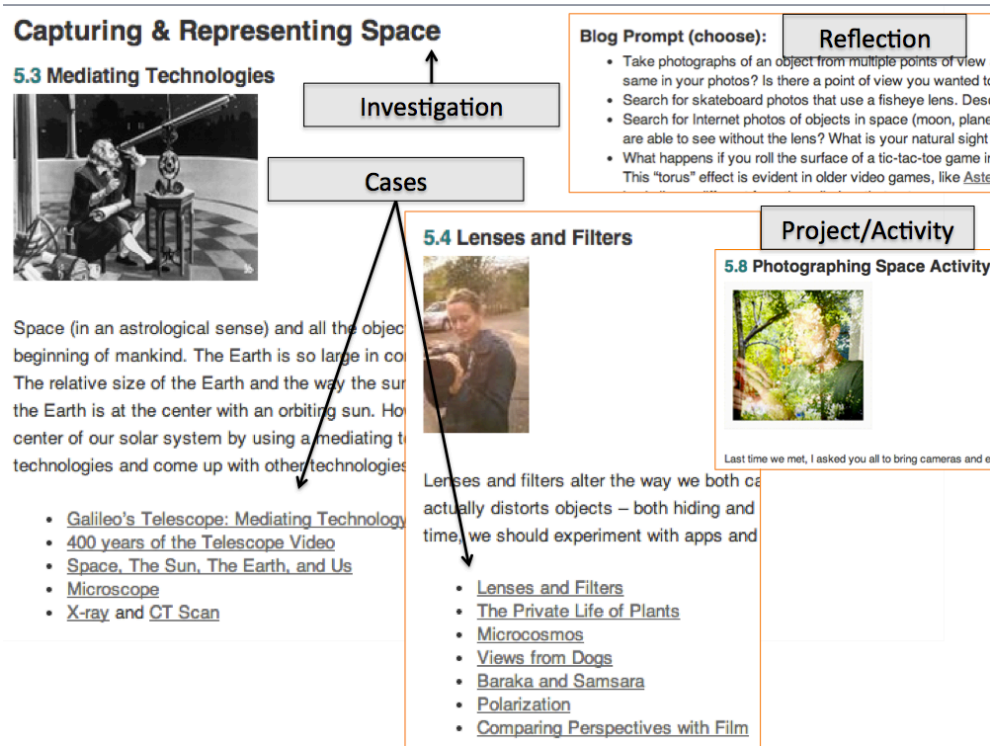

Figure 3.8. Features of "Capturing and Representing Space" investigation include sections, cases, reflections (as blog prompts), and a project/activity.

Jacobson and Spiro (1993) discuss the idea that "there is substantial variability in the application of abstract concepts to specific case situations" (p. 6). In order to illustrate this variability, cases may sometimes contain "smaller units referred to as minicases" (p. 6) Because minicases are contained within and associated with a single case, they provide learners with an opportunity to focus on variability of a particular aspect of a concept. Figure 3.9 presents the cases and minicases associated with Investigation 4, Section 2, "2D/3D…Even 4D Games." As learners investigate the case, "G4 Top 100 Video Games of All Time", they draw on the minicases (i.e., video game play of Asteroids) to consider the questions and prompts offered within the investigation and/or larger case. In Case 4.2, the minicases were comprised of

different games. For each game, learners might watch a video of the actual game play and talk about perspectives inherent in that game. After doing this for each game, learners can then compare games in the context of the larger case. In this sense they are provided with an activity structure that allows them to compare instances of the larger case to better understand the case itself. For example, they might juxtapose the "torus-like" movements in Asteroids with the multiple perspectives offered in a game like Gran Turismo and 4D vantage points offered in a game like Portal.

Figure 3.9. Sub-part of the investigation, "Considering the Dimensions of Space" showing how cases and minicases are linked and include video, prompts, and questions to guide the investigation.

Participant structures. Participant structures indicate the way the teacher and learner activity, participation, and roles. In *Space and Perspective*, participant structures include both learning scaffolds as explicated in Jacobson's (2008) hypermedia design principles and structures to encourage guided reinvention (Freudenthal, 1973). These structures are built into the hypermedia site in several ways. Questions and prompts are posed to learners to guide their

mathematization process as shown in Figure 3.9. Select technologies that support individual and social cognition related to the cases include open-source resources, such as *Flatland: A Romance of Many Dimensions* (Abbott, 1992). Google Drive documents and Edmodo, a course management system, are also linked or embedded in the hypermedia site as mediating technologies to support learners. Figure 3.10 shows these various structures.

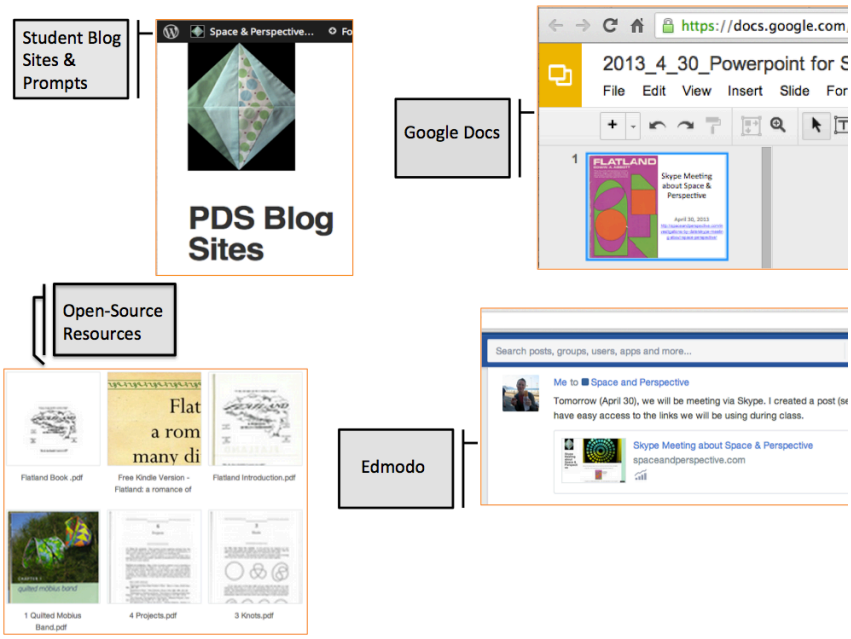

Figure 3.10. Structures to support learners include prompts/question, open-source materials, Google Drive documents, and Edmodo.

Mediating Processes

Mediating processes connect to both the embodiments (as design conjectures) and to the outcomes (as theoretical conjectures). According to Sandoval (2014), "Designs do not lead directly to outcomes" (p. 23). Rather, mediating processes that occur in the learning environment are evident as observable interactions and participant artifacts within a design. The mediating processes associated with S&P are presented in the center column with solid black arrows in Figure 3.2. They include: (1) Learners investigate the variability of concepts, and (2) Learners

mathematize reality (inquiry into real world phenomena). Sandoval clarifies "processes" as a "process-outcome link":

> In learning environments, the use of particular tools for specific tasks enacted in specific ways is intended to produce certain kinds of activity and interaction that are hypothesized to produce intended outcomes. These hypothesized interactions mediate the production of those outcomes. We could refer to these as meditational means from within a Vygotskian (Vygotsky, 1978) perspective, or we could think of them as the functions enabled by the structures of a design from an engineering perspective (cf., Middleton et al., 2008). I label them *processes* to emphasize the process–outcome link of concern to design research. (p. 23)

For *Space and Perspective,* the first process, investigating variability, is conjectured to manifest in several ways. Learners may show multiple ways of thinking about space, dimension, and perspective while engaged in class investigations. Most notable, this comes across when learners start to see the unexpected and engage in class conversations or write on their blogs questioning their own assumed perspective or the perspectives of their classmates. The second process, mathematizing reality, shows itself similarly. Learners apply a range of tools and understandings to rich, contextual problems. Most noticeable is again discourse and writing demonstrating a problematization of their current concept. Where design conjectures connect the embodiments to the mediating processes, theoretical conjectures describe the relationship between mediating processes and conjectured outcomes, which are discussed more below.

Conjectured Outcomes

Conjectured outcomes are articulated as the theorized result of the mediating processes within the learning environment. With regard to the high-level conjecture about CAPs (indicated

with solid black arrows in Figure 3.2), the conjectured outcomes are found in the column to the right. The outcomes associated with CAPs are: flexible conceptual reasoning, usable concept development, and open/alternative perspective-taking. Specifying the conjectured outcomes is central in design-based research projects as Sandoval (2014) stresses:

> [T]he success of any design endeavor requires making some commitment to articulating what desired outcomes will look like and how they might be observed or measured. This is true even if researchers' ideas about what the desired outcomes should be change as a result of study (cf. O'Neill, 2012). Conjecture maps for particular designs should be as specific as possible about what the desired outcomes are. (p. 23-24)

For *Space and Perspective,* these outcomes are conjectured to appear in various sources: whole and small group discussions, writing, interviews, concept maps, projects, and other class artifacts. One of the outcomes, flexible conceptual reasoning is conjectured to show itself as learners apply concepts to new situations.

Exploratory Data/Suggested Changes

This paper illustrates how a high-level conjecture can be mapped by a design team into specific embodiments, processes, and outcomes. This processes not only helps the team specify the relationships between design and theory, but also helps inform the conceptualization of research questions associated with the conjectures. Conjecture mapping is promising for potentially informing design knowledge in the future.

The *Space and Perspective* project was implemented over the course of two and a half months with 21 eighth-grade students as a supplement to the existing mathematics curriculum. The existing curriculum focused solely on algebraic concepts. Currently the project as a whole is being investigated phenomenologically to understand how shifts in perspective manifest in

variant ways for the learners. There are many data sources including: pre/post concept maps, student blog posts, audio from all class discussions (whole and small group), pre/post survey, projects, lesson artifacts, and in depth interviews. It is beyond the scope of this paper to report the phenomenological findings, but design implications from preliminary analysis are detailed below regarding CAPs.

Learners Adding to the Case Bank

Blogging, although not related to the design of CAPs, was used as a way to support learners' reflection during the investigations. It is possible that these posts could become part of the case bank, such as one student's photos from a trip to Washington, D.C., taken from various perspectives. Although the blogging activity was successful in that students posted reflections, ideas, photos, and artifacts from investigations, these remained mostly isolated from the hypermedia site. Although a link to "PDS Blog Sites" is included on the homepage, a page with 21 links is not ideal. Students are not likely to browse fellow classmates' pages. In addition, to protect the privacy of young learners, control settings prevent outsiders from accessing student posts. One possible revision includes curating the posts as cases and minicases through the main site, stripping learner identifiers from the post or allowing learners to pick a nickname. In this way, the case bank will grow, learners can easily access classmates' posts, and it's possible that learners will be more motivated to see their post published as part of the main site.

Revisiting Video

One of the favored cases for investigating space and perspective was *Flatland: The Movie* (Travis & Johnson, 2007). This particular case shows the perspective of Arthur Square, a Flatlander from the 2nd dimension. The 30 minute video is full of scenes where Arthur dreams of visiting Pointland and Lineland. He is even visited by Spherius from the 3rd dimension. There are

many moments in the film that would be advantageous to revisit, such as where he and his granddaughter Hex talk about the "axiom of the day" or his speculations of a 4^{th} dimension accompanied by an animated hypercube. Yet, watching a 30 minute film multiple times is not feasible. Spiro et al. (2007a) suggests overlays or segmenting video to revisit parts of cases. This particular suggestion seems highly likely to make *Flatland: The Movie* a more usable case. However, with copyright laws (not an open-source video), this dilemma is still confounding. This is one reason that the *Space and Perspective* site intentionally demarcates media as open-source for others to modify as needed.

Importance of Case Variety

In interviews with 5 learners, they each talk about several cases that resonated with them. Although they all mention *Flatland*, the movie and book, they talk about addition cases that vary across students. For one student, images of cubist paintings was most memorable and credited with changing the way he now looks at art. For another student, the video game cases influenced his current video game design work. These two learners in particular talk about the relatability of the cases being important for them. For another student, the activity of taking photos and making straw and bubble models was most pivotal for shaping her concept of space and perspective. CFT conjectures the importance of variability in the cases for conveying complexity. It seems that the variability also permits more chance for learners to connect to relatable cases.

Next Steps

There are several more lessons learned from implementing the project with learners. However these other considerations relate to the project as whole and are mainly concerned with attending to socio-cultural norms and allowing time for impasses to arise for robust discussions. After the phenomenological analysis is complete and all data sources from the project are

analyzed holistically, the conjecture map will be revised. This will guide the next iteration of design and plans will be made to implement the project as a supplementary program during or after school. In addition, an attempt to reduce the researcher's role in the delivery of instruction implies creating supports for others using the hypermedia site. The hope is that this resource can supplement geometry, art, or design curriculums or be used in after school programs or camps with learners in grades 5-10. Although design research does not seek to "generalize," extending the design to other contexts and learners may help to inform the design of CAPs and hypermedia design more broadly. Current ideas of interest include science, art, music, and history concepts that lend themselves to alternative perspectives.

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CHAPTER 4

THE EMBODIMENT OF CASES AS ALTERNATIVE PERSPECTIVE IN A MIDDLE SCHOOL GEOMETRY HYPERMEDIA SITE[3]

[3] Valentine, K. D. To be submitted to *Journal of the Learning Sciences*

Abstract

This post-intentional phenomenological research study (Vagle, 2010b) investigates moments of shift in one's perspective. The primary research question asks what it is like for learners (middle school) to find themselves perceiving space that is problematized. A secondary question addresses design and asks what role the learning environment plays in the experience for learners? Twenty-one eighth grade learners participated in a three-month program using cases as alternative perspective to investigate space, dimension, and perspective concepts. Data includes lived-experience descriptions, interviews, observational data (audio, video, photo), artifacts from lessons, student blog postings, and the researcher's post-reflexion journal entries. The data was analyzed using Vagle's five-component method, including a whole-part-whole analysis. An intense manifestation includes learners' moment of realization that their current concept is incomplete. This moment is brought on by other manifestations of experience, such as mathematical and bodily elusiveness, impasse and dissonance, discomfort and acceptance. In addition, these manifestations are mediated by cases as alternative perspective (especially pivotal cases) and various means of discourse and reflection.

Keywords: perspective, shift, phenomenology, post-intentional phenomenology, middle school, mathematics education, cognitive flexibility, cases as alternative perspective, geometry, problematization

Introduction

This post-intentional phenomenological research study (Vagle, 2010b) investigates moments of shift in one's perspective. This is a broad phenomenon, not bound to particular contexts. However, one area seemed promising for investigating the phenomenon and created the border, or boundary for this investigation – a middle school mathematics classroom (a complementary context comprises the residual manifestations the learners carried from their initial experiences) (Hiebert et al., 1996). Twenty-one eighth grade learners investigated space, perspective, and dimension concepts using a hypermedia site (see http://spaceandperspective.com/). The two classes with twelve and nine learners meet 12 times over the course of 3 months, for an average of 100 minutes each meeting. Mostly the classes were separate, but the third day we met together for "D Day," a three hour time slot used for service learning projects and non-traditional learning foci, which the *Space and Perspective* project falls under. *Space and Perspective* is also the name of the hypermedia site housing the tools and materials to guide the investigations. The hypermedia site described in, *The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site*, seeks to problematize space, perspective, and dimension to help learners consider multiple perspectives, develop flexible concepts of space and dimension, and provoke learners to consider their blind spots, or assumptions as part of the learning environment. In this study, learners were asked about this experience investigating *Space and Perspective*, in particular about moments of shift. Before describing the post-intentional phenomenological methods, tentative manifestations (findings), and implications of this research, I first share the impetus spurring this investigation, potential solutions, and then describe the aims of the study.

Problem

Instructional approaches tend to simplify concepts in an attempt to make learning easier. This compartmentalization or "textbookization" can sometimes leads to misconceptions and rigid knowledge assembly that is inflexible and unusable (Spiro, Coulson, Feltovich, & Anderson, 1988; Spiro, Vispoel, Schmitz, Samarapungavan, & Boerger, 1987, p. 9). In the area of mathematics education, geometry has been recognized as the more neglected domain (Lappan, 1999; Shaughnessy, 2011). Therefore it is not surprising that the most recent TIMSS (2011) results show "a relative weakness in geometry" as compared to overall mathematics achievement for "nearly all of the ninth grade and benchmarking participants" (Provasnik et al., 2012, p. 147). Still, these test scores only indicate weakness on a narrow margin of concepts specific to geometric and spatial reasoning. Understanding space concepts, for example, requires "the ability to "see," inspect, and reflect on spatial objects, images, relationships, and transformations" (Battista, 2007, p. 843). This particular application of mathematics is what Spiro et al. (1988) and Jonassen (1997, 2011) would term "ill-structured."

I have identified problems specific to mathematics earlier (see framework paper), including the assumption that space is a well-structured concept where I conjecture, "the current educational climate rarely provides opportunities for learners to problematize space in the context of geometry" (Valentine & Kopcha, 2014, p. 745). In addition, designers rarely attend to embodiment and perspective when designing and developing learning experiences. In the previous chapter I wrote, "Design focused on problematizing and thus, opening up perspective, has the potential to give learners opportunities to develop important visualization skills, reflect on variants in multifold real-world phenomena, and learn to connect to mathematics in powerful ways" (p. 56). Lastly, I identified four main misconceptions concerning geometric reasoning,

likely resulting from a lack of opportunities for learners to connect to their existing spatial concepts and a disconnect between learning and mathematics phenomenon in the real world. These four main misconceptions concern:

- Geometric figures and their properties. For example, learners don't always incorporate the properties of a shape into their concept, excluding non-prototypical examples (see Carroll, 1998; Clements, 2003).
- Geometric transformations. For example, learners have a tendency to reason discretely instead of continuously, thus not being able to tell what changes and what stays the same in situations of transformation (see Edwards, 1991; Goldenberg, Cuoco, & Mark, 1998).
- Measurement. For example, formulas are not connected to their meaning, thus learners may not actually know what area and volume mean (see Driscoll, 2007).
- Static versus dynamic geometric relationships within and between dimensions. For example, shape as image-based conflicts with shape properties with variant configurations and orientations (see Carroll, 1998; Clements, 2003; Clements & Battista, 1992; Keiser, Klee, & Fitch, 2003; Monaghan, 2000).

Possible Solution

One way to address the problem of oversimplification in learning is articulated in Cognitive Flexibility Theory (Spiro et al., 1988). The theory and subsequent applications of the theory suggests ways to design learning environments that support learners' development of flexible, usable concepts (see Jacobson & Spiro, 1993; Spiro, Collins, & Ramchandran, 2007a, 2007b; Spiro, Collins, Thota, & Feltovich, 2003; Spiro et al., 1988; Spiro & Jehng, 1990; Spiro et al., 1987). This is primarily done with the use of cases and mini-cases to support learners to represent knowledge in multiple and interconnected ways. The cases offer learners the space to

consider how concepts are used across various contexts. In addition, applying the theory means introducing complexity early and includes the goal of changing learners underlying ways of thinking. Spiro (personal communication, October 16, 2012) identifies these underlying ways of thinking as complacencies in "thought and action."

In this project, a hypermedia site, called *Space and Perspective* (see http://spaceandperspective.com/) was used with two eighth-grade mathematics classes over a three-month period. The site contains 70 cases and mini-cases designed by integrating theories and principles from Cognitive Flexibility Theory (CFT), Realistic Mathematics Education (Freudenthal, 1973, 1983), and hypermedia design principles (Jacobson, 2008). These cases are called cases as alternative perspective (CAPs), a term used by Jonassen (2011) to support ill-structured problem solving. The CAPs are intended to problematize space, dimension, and perspective in a middle school geometry context and thus change learners underlying ways of thinking that Spiro sees as central to cognitive flexibility and usability. The CAPs problematize being, seeing, and moving by supporting learners to attend to similarities and differences across multiple contexts (e.g., art, photography, video, gaming). Jonassen (2011) emphasizes that "ill-structured problems requires that the ill-structuredness be conveyed, not eliminated" (2011, p. 210). CAPs for the design project (see framework paper) are operationalized as follows:

> [A]nything that allows learners to investigate multiple perspectives inherent in complex concepts. They serve as the context for investigating space and perspective. Employing microcosm-macrocosm relationships, they can take the form of video, text, symbols, photos, animations, and simulations (Jacobson & Spiro, 1993; Spiro et al., 2007a, 2007b, 2003, 1988). The goals of CAPs for this project is to demonstrate variability of concepts

> in real-world instances, convey complexity to improve flexibility and usability of concepts, and support multiple/alternative perspectives. (p. 44)

Although CFT and specifically CAPs are promising, there is little practical knowledge concerning the design, development, and implementation in a middle school mathematics context. What is available occurs in teacher education, medical education, business education, and undergraduate education (e.g., Heath, Higgs, & Ambruso, 2008; Jacobson & Spiro, 1993; Lima, Koehler, & Spiro, 2010; Strobel, Jonassen, & Ionas, 2008). Jacobson (2008) provides a particularly useful framework for developing educational hypermedia systems describing four components to support learners' development of flexible concepts: representational affordances of hypermedia, knowledge in context, learning scaffolds, and learning tasks. Still, the contextual-dependent nature of the CAPs highlights the importance of designing in various contexts, with various age learners, and focusing on exploratory research that can help designers and those interested in learning better understand what it is like for learners to engage in hypermedia cases intended to shift their perspective.

This problem is addressed first in the framework paper, which conjectures the embodiments of the learning environment necessary for supporting the processes and outcomes related to cognitive flexibility and CAPs in the context of middle school mathematics. The purpose of this paper is exploratory and seeks to open up an understanding of the ways learners experienced these moments of shift in perspective.

Aims of the Study

Using post-intentional phenomenological research (Vagle, 2010b), this study seeks to primarily understand what it is like for learners to find themselves perceiving space that is problematized. To connect to the previous framework paper, the study also investigates the secondary question: what role does the learning environment play in the experience for learners? By understanding the ways middle school learners experience shifts in perspective, we may better understand how shifts in perspective and learning are related. In addition, we may learn what (and how) design elements afford these moments for learners. Although this study does not seek to address a purely mathematics education concern, mathematics learning was chosen as the context because of lagging achievement and potential to investigate the ill-structuredness and complexity in geometry. Exploring learners' shifts in this context is hoped to provide practical, theoretical, and design knowledge usable to multiple research disciplines (e.g., learning sciences, mathematics education) and educators as well.

Mediating Technologies

With the ubiquitous nature of mediating technologies (e.g., Google glasses, slow motion video cameras, video game environments), this post-intentional phenomenological investigation seeks to understand shifting perspectives these technologies may bring about as related to concepts of space and dimension. Ubiquitous refers to the phenomenon of appearing and existing everywhere. I assert that in most parts of the U.S., technologies that mediate perspectives are more a normative experience, especially with younger learners. Although older generations may lament the pre-smartphone days, these technologies that travel with us and create access to information, ideas, and media, are only becoming exponentially intermingled with our everyday

being. For learning, this can be leveraged to open up complex concepts, consider multiple perspectives, access simulations and animations, etc.

The exploratory nature of post-intentional phenomenological research methods can support researchers seeking to understand the various manifestations of experience. Post-intentional phenomenology is especially well suited for design work because attention to variants, which this perspective captures, is often critical at early stages and throughout the design process. It creates a more nuanced (or deep) representation of learners' experiences in applied settings. Researchers and designers can then build on these insights to continue revising or supporting theory and design.

Method

This study used Vagle's (2010b, 2014) post-intentional phenomenological research method to investigate the phenomenon of shifts in perspective among middle school learners. Post-intentional phenomenology is a recent form of phenomenological research that draws on and addresses the shortcomings of previous paradigms (descriptive and interpretative). Phenomena, and humans experience living with them, are seen as *tentative manifestations* – dynamic and continuously changing with time, context, mood, or any number of factors. Experiences are not merely described or interpreted, but are lived in tentative, partial, and fleeting ways. Phenomenological inquiry is exploratory in nature and helps us answer questions about living with, in, and through phenomenon. Vagle (2010b, 2014) has designed a five-component process for conducting post-intentional phenomenological research. Table 4.1 indicates the associated research activities for each of the five components.

Table 4.1

Vagle's (2010b) Five Component Process for Conducting Post-Intentional Phenomenological Research and Associated Research Activities

Research Component	**Research Activities**
1. Identify a phenomenon in its multiple, partial, and varied contexts (Vagle, 2010b, p. 9; 2014)	a. State the research problem b. Partial review of the literature c. Philosophical claim d. Statement of the phenomenon (research question(s)), including an intentionality statement e. Contexts f. Participant selection (Vagle, 2010b, p. 10-12; 2014)
2. Devise a clear, yet flexible process for gathering data appropriate for the phenomenon under investigation (Vagle, 2010b, p. 9; 2014)	a. Select data sources b. Align data sources with research questions (Vagle, 2010b, p. 15; 2014)
3. Make a post-reflexion plan (Vagle, 2014)	a. Create a post-reflexion journal b. Write an initial post-reflexion statement c. Post-reflex as you gather and analyze data (Vagle, 2014)
4. Read and write your way through your data in a systematic, responsive manner (Vagle, 2010b, p. 9; 2014)	Whole-part-whole analysis plan (Vagle, 2010b, p. 18)
5. Craft a text that captures tentative manifestations of the phenomenon in its multiple, partial, and varied contexts (Vagle, 2010b, p. 9; 2014)	a. Re-state the multiple and varied contexts b. Brainstorm potential forms (Vagle, 2010b, p. 21; 2014)

Even though I used Vagle's (2010b, 2014) research methodology, it is important to note that most phenomenology methodologists do not see the methods as fixed. Van Manen (2000) writes that "phenomenological inquiry methods cannot be formalized into a series of technical procedures" (p.24). He does add, however, that activities exist which can assist the phenomenological researcher, including empirical and reflective methods. He continues:

> From a phenomenological point of view, we are less interested in the factual status of particular instances: whether something actually happened, how often it tends to happen, or how the occurrence of an experience is related to the prevalence of other conditions or events. For example, phenomenology does not ask, "How do these children learn this particular material?" but it asks, "What is the nature or essence of the experience of learning (so that I can now better understand what this particular learning experience is like for these children)?" The essence or nature of an experience has adequately been described in language if the description reawakens or shows us the lived quality and significance of the experience in a fuller of deeper manner. (p. 10)

Van Manen's phenomenology reflects the title of his book, *Researching Lived Experience: Human Science for an Action Sensitive Pedagogy* (1990). He attempts to understand the nature of learning phenomenologically, as it is lived by learners. Although Vagle's method was used, a commitment to "maintaining a strong and orientation relation" (p. 135) in a pedagogical sense is a lens used throughout each stage of the project. For this project, a strong pedagogical orientation refers to reflecting on pedagogical situations as a teacher researcher and bringing these to bear in activities of design and theorizing.

Post-intentional phenomenology investigates phenomena as humans in the world experience them. The post indicates a rejection of universal truths and grand narratives and a rejection of anyone's ability to take a perspective outside of the human condition. Researchers in this tradition assume meanings and phenomena shift, are partial, and resist centering. It may seem contradictory, conceptually, to study phenomena that shift and change over time, but from a post-structural standpoint, this is the complex nature of phenomena. In this study, the phenomenon of interest is moments of shift in one's perspective. A post-intentional

phenomenological lens allows the shifts and changes to be taken up more fluidly across time and contexts, acknowledging the complex nature of altering perspectives. Post-intentional phenomenology offers the researcher a method for interrogation, interpretation, and ways to account for the multiple complexities in ill-structured research domains. Vagle's (2010b, 2014) research methods, described below, suggest strategies for conducting this type of phenomenological research.

Before explaining my methods in detail, I present key notions in post-intentional phenomenology (lifeworld, natural and phenomenological attitude, intentionality), and a brief history of the three major forms of phenomenology showing how each of the methods and philosophies connect and attempt to address the issues of previous perspectives. This tracing ends with post-intentional phenomenological methods, the most recent methodological variation, adding post-structural commitments to open up the shape of experience. This tracing aims to support the reader to understand how the underlying philosophy informs the methodological activities and commitments.

Operationalizing Key Phenomenological Terms

In this section, terms specific to phenomenology are explicated. These include lifeworld, natural attitude, phenomenological attitude, and intentionality.

Lifeworld. The term 'lifeworld' is a part of any philosophical and methodological description of phenomenology, usually presented with notions of the natural and phenomenological attitudes. It is also considered a central theme of phenomenology (e.g., Dahlberg et al., 2008). Husserl (1901/1970b), attributed as the founder of phenomenology as a philosophy, saw science belonging to the world rather than something to be abstracted and construed as *more* precise. This lifeworld is not something humans can escape or see from the

outside, as all subjects and objects reside here. The natural and phenomenological attitudes are two ways humans might find themselves experiencing the lifeworld.

Natural Attitude. Drawing on Husserl, Dahlberg, Dahlberg, and Nyström (2008) describe the natural attitude as "the everyday immersion in one's existence and experience in which we take for granted that the world is as we perceive it, and that others experience the world as we do…we do not critically reflect on our immediate action and response to the world, we just do it, we just *are*" (p. 33). As a researcher of complex, connected phenomenon, one must be aware of this "natural" and unreflective or unnoticed lens. Dahlberg et al. warn "the things that we are closest to are the things that are most hidden from us" (p. 34).

Phenomenological attitude. The phenomenological attitude, on the other hand, is sometimes referred to as a scientific or open attitude. I choose to present an understanding of the phenomenological attitude as one of openness, not of phenomenological reduction described by Sokolowski (2000), Moustakas (1994), and Giorgi (1997), who use transcendental notions of bracketing and Epoché. Instead, the phenomenological attitude is an "open attitude" described by Dahlberg et al. (2008) as "having the capacity to be surprised and sensitive to the unpredicted and unexpected" (p. 98). Bridling, described later, is one way both Dahlberg (2008) and Vagle (2010b) suggest remaining open to the phenomena under investigation. Vagle (2014) furthermore advices the researcher to be open to "connections/disconnections, assumptions of normality, bottom lines, and what shocks you" (Vagle, 2014, p. 133).

Intentionality. The term "intentionality" is easy to confuse in the everyday English usage of the term, such as "I *intend* to pick up more rice at the store tomorrow." Rather, intentionality in the phenomenological sense describes an active relationship, as Dahlberg et al. (2008) indicate, "in which we experience the things and events of our world as endowed with

meaning, as meant" (p. 49). Although the aims and methods of phenomenology are construed differently in descriptive, interpretative, and post-structural traditions, a commitment to investigating intentionality remains central. In phenomenology, intentionality is where one looks to find meaning. This "where" isn't a place one can point to – it's doesn't merely reside within people and things – intentionality is more the relations running through the lifeworld, and probably most analogous to the traditional Chinese *ch'i,* or life force. This is the reason phenomenology investigates intentionalities of human experience – the experience that Husserl demonstrates even allows science to mathematize the world. *To the things themselves* signifies these connections between humans and the world.

Philosophical Claim

Phenomenological research should make a philosophical claim to help orient the research project and those taking it up. In this section, phenomenology is traced to show the way phenomenology as a philosophy guides this investigation. Phenomenology is the study of phenomena as humans in the world experience them. It is rooted in Husserl's reaction to the traditional Western philosophy of his time, one of impersonal, "scientific" reason. According to van Manen, "Phenomenology attempts to explicate the meanings as we live them in our everyday existence, our lifeworld" (1990, p. 11). Investigating intentionality illuminates these meanings. A brief tracing starting with Husserl will provide the necessary background to the way phenomenology is interpreted for this project.

Husserl: Transcendental, descriptive phenomenology. The crisis of modern science for Husserl (1936/1970b) is its substitution of an ideal for the real world. He accuses the sciences of forgetting their grounding in the lifeworld and subsequently making the lifeworld "off-limits" and a "veil of appearances" (Russell, 2006, p. 189). He offers phenomenology as a way to

reconnect what science has separated, a connection to its roots in human experience. Husserl argues that "any consciousness is a consciousness of something, and the modes of consciousness are highly diversified" (1913/1982, p. 234). For Husserl, we should go *to the things themselves*, which he considered "essence" (1901/1970b). For Husserl, getting to the essence of things requires a focus on consciousness as experienced rather than on abstract theorizing.

Phenomenology then becomes a way of seeing that does not elevate objective science as that which more closely represents reality or offers a better way to understand the world. Husserl argues that science and philosophy are human activities in the lifeworld and secondary to human experience. Instead, human experience in the lifeworld is the source, or analogously, the umbrella of all science and philosophy. He considered phenomenology the a priori grounding for all other sciences, sometimes referred to as the first philosophy. In this sense, phenomenology is characterized as an attitude, which does not take theory as a given. Scientific approaches are not thrown aside, but are not construed as the sole, or superior, way of understanding. Husserl developed a descriptive method, where the investigation focuses not on the psychological or logical, but rather "the *relation* between the two" (Russell, 2006, p. 42). Important to his method, was the notion of intentionality.

Husserl modified the idea of intentionality he learned from his teacher, Franz Brentano. For Husserl, consciousness was not "a self-enclosed room or box," a position held by representationalists (Russell, 2006, p. 80). Rather, consciousness "reaches out," it is conscious *of* something, just as remembering is remembering something. According to Husserl, this directed act is intentionality and is found in the meanings that present themselves in human consciousness through lived experience – "any consciousness is a consciousness of something" (1913/1982, p. 234). Yet, not all phenomenologists share the same focus on consciousness as Husserl.

Heidegger: Hermeneutic, existential phenomenology. Heidegger, a student of Husserl, investigated the notion of being and saw phenomenology as the appropriate method. Although Heidegger draws on Husserl's phenomenology, he questions the epistemological focus on intentionality (consciousness of something), recognizing that experience is already situated in the world of being. Throughout his major work, *Being and Time* (1927/2008), he uses the phrase, *Being-in-the-world*, to indicate the ontological phenomenological starting point. The hyphens are included in this German translation of *Inderweltsein* to emphasize "the unity of the concept" (Matthews, 2006, p. 12). In this way, Heidegger's phenomenology is one of interpretation (experience of being in), rather than description as emphasized by Husserl. *Being,* for Heidegger, is already in the world and affected by its history. Being is hermeneutic and interpretative. He feels one cannot transcend being, which contrasts with Husserl's phenomenological reduction that such transcendence may exist. According to Heidegger, phenomenology's task is to question what it is to be in the everyday world.

Heidegger feels strongly that how things manifest and matter must be the focus (Cerbone, 2008, p. 31). He writes of "grasping and explicating phenomena" interpretatively and sees phenomena as "that which shows itself be seen from itself in the very way in which it shows itself from itself" (Heidegger, 1927/2008, p. 58-61). Intentionality for Heidegger is "letting the meaning of being reveal itself" (Matthews, 2006, p. 82) or more simply, "that which becomes manifest for us" (Vagle, 2010b, p. 4).

Merleau-Ponty: Hermeneutic, existential phenomenology. The French philosopher Merleau-Ponty agrees with Heidegger's notion of Being-in-the-world and makes embodiment a central theme in his work – a subject embedded in a certain position and time in space. This is exemplified in his work with the primacy of perception. He sees this as a way of becoming

directly involved in the world, an active engagement. This is not the perception of empiricists, indirect and passive, but rather the world we perceive as a whole, always affected by a reflexivity between foreground and background.

Like Heidegger, he does not believe a complete phenomenological reduction (Husserlian) is possible. Rather, he speaks of relaxing our ties to the world so that the world is viewed as strange and unfamiliar, contrasting with our natural, everyday attitude. He describes this type of reflection:

> Reflection does not withdraw from the world towards the unity of consciousness as the world's basis; it steps back to watch the forms of transcendence fly up like sparks from a fire; it slackens the intentional threads which attach us to the world and this brings them to our notice…it reveals that world as strange and paradoxical. (1945/2002, p. xv)

Merleau-Ponty shows the embodied nature of phenomenon, setting him apart from Husserl with regards to epistemological concerns.

Merleau-Ponty's embodied focus on the way we experience the world also adds to our understanding of intentionality. He reformulates Husserl's phenomenological reduction by writing not of a transcendent withdrawal from the world, but rather of a reflection that "slackens the intentional threads which attach us to the world" (1945/2002, p. xv). Merleau-Ponty felt strongly, as Husserl did, that science could not answer all the meaningful questions of the world. The major limitations of science for Merleau-Ponty is that the objective viewpoint of science comes from humans, and that a "complete picture of the world cannot leave out experiencing subjects" (p. 77). He writes, "Because we are in the world, we are *condemned to meaning* and

cannot do or say anything without its acquiring a name in history" (p. xxii). These "intentional threads" best describe Merleau-Ponty's understanding of intentionality, showing the ch'i quality mentioned earlier.

Ihde and Vagle: Post Structuralism and Phenomenology. Phenomenology has continued to progress and develop since the mid-1900's, although it is sometimes "conflated" with Husserl's concern with essences (Vagle, 2010a, p. 3). Ihde (2003) shows pragmatically how postphenomenology can support research and development efforts, especially imaging technologies and their various ways of being embodied. He argues for a postphenomenology that draws on Merleau-Ponty's notion of embodiment, while at the same time expanding it to include gendered and cultured embodiments. In addition, postphenomenology values not only invariants of experience, but also variants – bringing the "multidimensionality, multistability, and the multiple 'voices' of things into account" (p. 25). Rather than try to describe or interpret a phenomenon's essential structure, Ihde finds value in viewing phenomena with a poststructuralist lens to attend to the many ways phenomena vary across contexts, cultures, genders, discourses, etc. He considers "variational theory its most important methodological strategy," especially focused on perceptual and cultural variation (p. 7).

Ihde describes the notion of intentionality as "the directional shape of experience" (1977/2012, p. 24). Similar to Merleau-Ponty, he envisions the embodiment of humans in this intentional relationship expounding, "far from being self-evident or initially transparent, the "subject" is enigmatic for phenomenology. It is known only *reflexively* from which phenomena and how these phenomena are made present to it" (Ihde, 1977/2012, p. 11). Ihde also investigates variants of embodied experience – a central move in his postphenomenology – and writes,

"every dimension of intentionality displays a possible field" (p. 111). In trying Husserl's method of variations, Ihde failed to find essences, but instead "discovered multistabilities" (p. xiv).

Drawing on Ihde's postphenomenology, Vagle (2011) constructed a postphenomenology research method, called *post-intentional phenomenological research*. His first post commitment is viewing "knowledge as partial, situated, endlessly deferred, and circulating through relations" (2011, p. 3). He shares Ihde's practice of focusing on embodiment, rather than subjectivity. With these as backdrop, intentionality becomes central in Vagle's work, interpreted post-structurally. Adding to Merleau-Ponty's "intentionality" as connected threads of "meaning that runs through relations," Vagle adds "those threads…are constantly being constructed, de-constructed, blurred, disrupted…intentionality is running all over the place, all the time" (p. 8). In posting intentionality, Husserlian essences are replaced with "tentative manifestations" – recognizing that intentionality manifests (ontologically as Heidegger implies), but not as a stable structure. Instead, a tentative manifestation "resists centering and embraces contexts, situations, and the partial" (p. 10). In this reframing of phenomenology, Vagle remains vigilant with the original intent of phenomenology – opening up phenomena and incorporating the multiplicity of ways this might happen.

Vagle identifies three roots of post-intentional phenomenological research: "Heidegger's manifestations, the philosophical notion of intentionality, and post-structural commitments to knowledge always, already being tentative and never complete" (Vagle, 2010b, p. 6). These all affect how Vagle (2010b) comes to describe intentionality, which is:

> Whatever understanding is opened up through an investigation will always move with and through the researcher's intentional relationships with the phenomena – not simply in the researcher, in the participants, in the text, or in their power positions, but in the

> dynamic intentional relationships that tie participants, the researcher, the produced text, and their positionality together. In this way, intentionality is always moving, is unstable and therefore can be read *post-structurally*. (p. 5)

Thus, instead of trying to find essences as in a descriptive method, Vagle speaks of tentative manifestations of phenomena. Tentative manifestations "signifies a move away from essence and towards contexts, situations, and the partial…flexible and malleable…permeable…multiple and more temporary...move[s] and shift[s]" (p. 7) The activity of researching then, is not an investigation about individual subjects, objects, or the meaning attributed by subjects onto objects, but the meaning residing in these intentional relationships which is not a singular meaning.

Distinguishing phenomenologies. An important distinction exists between transcendental and existential phenomenologies as described above, most notably a subjective versus embodied view respectively. When these different views are translated to research methods, issues of description (Cilesiz, 2011; Giorgi, 1997; Moustakas, 1994) versus interpretation (Smith, Flowers, & Larkin, 2009; van Manen, 1990) come to the fore, again respectively. Postphenomenology (Ihde, 1993) and post-intentional phenomenological research (Vagle, 2010b) draw on many of the existential, embodied views, and integrate "post" work; most notable is attention to the multiple, partial, decentered, and fleeting view of intentionality. Where traditional transcendental phenomenological research advocates methods for describing invariants of phenomena (Cilesiz, 2011; Giorgi, 1997; Moustakas, 1994), postphenomenologies and even phenomenography (Larsson & Holmström, 2007; Marton & Pang, 2008) turn their attention to variants and value multiplicity of meaning. In postphenomenology and post-intentional phenomenology, the researcher is interested in intentionality – "the [various] ways

meanings "come-to-be" in relations…those in between spaces where individuals *find-themselves-intentionally-in* relations with others in the world" (Vagle, 2011, p. 4). For phenomena such as shift in perspective, a post-intentional phenomenologist asks, 'what is it to live through shifts in perspective' (while assuming this is an embodied experience).

Unit of analysis. The philosophical nature of post-intentional phenomenological research, rooted in the notions of embodiment, including perspective and perception, is well suited for understanding what it is like for learners to live through moments where their perspective shifts. Phenomenology is concerned with investigating experience as it is lived. The unit of analysis resides in the intentional meanings of the learner's experience, embodied in the lifeworld. Analysis of these intentionalities is *tentatively and partially captured* throughout the research process.

Step 1: Identify a Phenomenon in its Multiple, Partial, and Varied Contexts

Step one of Vagle's post-intentional phenomenological method includes identifying a phenomenon in its multiple, partial, and varied contexts. He suggests several systematic steps as part of this identification process. Writing resulting from the first three sub-steps (stating the research problem, completing a partial review of the literature, and making a philosophical claim relative to the research problem) were addressed earlier. This section will focus on the other steps: articulating the phenomenon and the accompanying research questions, situating the phenomenon in the multiple, partial, and varied contexts in which it tends to manifest, and describing research participants.

Statement of the phenomenon (research questions). This investigation did not attempt to look at the whole of perspective, but rather examined perspective taking and perspective creating with regards to space and dimension cases in a middle school geometry classroom.

Perspective is a constant current running through much of human action (and probably non-human) – reacting to an emotionally moving book, engaging conversation, etc. However, by situating perspective as a part of geometry instruction, this discussion is narrowed for the moment. The phenomena of interest in this study aimed to articulate moments of shift in perspective.

Theses moments when a perspective changes can be found when contemplating modern art like a Picasso cubist painting of a distorted, yet multi-perspective image or using a camera/microscope to create a magnified field. What is the experience of that moment and how does it change the way we experience subsequent moments (or reflect on prior ones)? This phenomenon, when viewed as embodied, can help researchers understand the intentionality that is constantly running through the subject and world with all the space between. In this sense, perspective implies a connective tissue with everything in the world. The research questions below include a primary question about the learners' experience and a secondary question to focus the work. The secondary question seeks to inform future iterations of the learning environment design.

The primary question concerns the learners' experience and asks, "What is it (like) for learners to find themselves perceiving space that is problematized?" The secondary question asks, "What role does the learning environment play in the experience for students?"

Contexts where the phenomenon resides. The phenomenon of a changing spatial perspective most likely inhabits all humans (and even animals), not only in places, but also across time. The simple motion of stepping a foot to the side necessarily changes one's perspective – their distance and point of view in relation to objects shift. As humans age, they grow taller and relate to the world differently. The perspective of a chair and even the way one

sits down changes over time. It seems that every moment is a change in perspective, even if it is mostly unconscious and not reflected upon. This led to a focus on contexts in which humans' perspective shifts suddenly, or can at least be compared to an earlier, different perspective. A few examples are articulated below that contain some of the contexts and relations considered for this project.

Non-seeing perspectives. Although it is impossible to consider all the lifeworld in this one investigation, the phenomenon of shifts in perspective resides everywhere. Non-seeing perspectives, such as those in literature and conversations, are likely contributing events that change one's future visual perspective. The nature of being a body in space that interacts with others, things, and ideas inevitably means perspective (whether in a natural or phenomenological attitude) is always circulating through everything.

Multiplicity. After reading Ihde's (1993) chapter on literate and illiterate cultures and their art, I was struck by the images of art from differing perspectives that seemed related to a perspective one might take when reading a book (angled, slightly from the sky) or an undefined perspective (in aboriginal, non-literate cultures). Egyptian art seemed to show multiple perspectives in one painting, similar to cubist modern art that endeavors to capture multiple moments of time (girl walking down staircase) or Picasso's faces (multiple perspectives of the nose). Being embodied in the world does not allow us to take on multiple perspectives simultaneously, but paintings, multiple exposure photographs, film, and other technologies do. Humans do of course have two eyes that create a sense of depth, so perhaps there is a multiplicity of perspective on some level.

Taking on (perspective) and creating (perspective) in art. An artist creates perspective as Merleau-Ponty describes in *Phenomenology of Perception* (1945/2002). The photographer and

filmmaker do the same. Anytime humans (or animals, trees, the sun, etc.) create something, a perspective is created for others. Humans are both *taking on* and *creating* perspective. In terms of taking on perspective, I wonder how openness allows us to accept multiple perspectives. What about problematizing what is taken for granted? In the southeastern U.S., there is a plethora of folk art and something similar in the perspective being portrayed. The folk art lacks depth, appears "flat" in a sense, and pieces such as cut out "Blow Oscars" by R.A. Miller portray a side view. Why? What might have caused this to be a default perspective for so many folk artists? Is it similar to the cubist movement where artists influenced each other? Is there a connection to the perspective of aboriginal cultures Ihde describes? Maybe it relates to aesthetics. Either way, part of the phenomena of a changing perspective can be seen to reside in the relationship between creators and viewers of these art forms.

Architecture. Being an enthusiastic fan of Buckminster Fuller and Frank Lloyd Wright, I have visited exhibitions and spaces they've both created. Although awkward to put into words, I am very aware of my preference for their designs. Buckminster Fuller's work is geometric and ideal, in the Husserlian (1936/1970a) sense of the word. He uses geometric shapes to create geodesic domes and the Diamatrix home/car. Wright's designs are full of lines that relate to each other and the landscape in ways that propel me into action. I remember trying to photograph his homes and running out of camera space – I didn't know where to focus my lens – a million angles and photos could not seem to capture all the intricacies. In fabricating their designs, the artists create many experiences for others. In addition, I consider the homes I have lived in and where I currently reside. It appears that any designed/created space affects our perspective, even if we don't problematize or take time to reflect on it with a phenomenological attitude.

Place. I live and grew up in the southeastern U.S., which has a very luscious, almost jungle of overgrowth during the summer months. The canopy of trees I find myself moving under can hide the horizon and night sky for a full year before I find myself somewhere else that changes how *far* and *wide* I can see. I never really problematized the limiting nature of sight in the southeast until traveling outside of this landscape at the age of 20. I remember taking a trip to Utah with my new 35mm Canon Rebel. During the drive west, I was most astonished when I saw the night sky in Texas – I could see stars from horizon to horizon. During the day, the colors of the desert literally overwhelmed me – I didn't know red dirt could come in so many shades. I remember seeing shadows in the way the sun fell on the mesas and how this changed throughout the course of a day. Similar to my experience with the Frank Lloyd Wright houses, I found that I could not possibly capture the vistas that I was taking in with my camera. The photos were all distorted when I had them developed, but again, it was interesting that the camera portrayed that landscape differently from my memory. Maybe it attests to the different ways we perceive landscapes and mediating technologies.

Most notable is after coming back home, my perspective of the sky and landscape in Georgia was something to ponder – before it was the way the world, my world, was shaped. Studying comparative religion at the time, I started to draw connections between place and beliefs – really place and EVERYTHING! Skin and eye color, monotheistic/polytheistic, bland/spicy food, individual/communal living conditions. It all started to relate and it seems to originate in the place we happen to be born.

One colleague told me she paints "bottom-heavy" because of the open sky she grew up with in the Midwest. So then she becomes the creator of a perspective for others drawing on her consumption of a perspective of place. I can see this cycle ebbing and flowing out of one

another. I wonder what Matisse or Picasso did to both create perspectives for each other, while at the same time receiving perspectives from the other? I also wonder how the world has changed for humans where we are not "stuck" in a canopy for 20 years, but rather flooded with images from other spaces, especially since the proliferation of the Internet. Have we evolved to take on multiple perspectives?

Other contexts that might cause a perspective shift. There is probably an infinite array of experiences that might cause perspective to shift. The context is everywhere, at every moment, and most likely changes through time. For now, I have started a running list in a post-reflexion journal. Many of these ideas have been developed into investigations and can be found on the *Space and Perspective* project site used with learners (http://spaceandperspective.com/).

Participants. It is probably accepted that all humans experience the phenomena of a changed perspective of space at some point in their lives, even if the experience is not a conscious and reflective one. A newborn for instance is believed to focus best at 8-12 inches, the distance from eye to eye of their caregiver when being held. Change in the baby's ability to focus probably impacts their perspective of space. For vision-impaired individuals, there is a change in ability to focus when wearing glasses or contacts. From personal experience, I remember this change in perspective at the age of 12, the first time I could make out distinct leaves on a tree. Along the same lines, it may be possible that someone who is blind experiences a change in their relationship to space as they grow, even though they are unable to use the perception of sight.

There are probably an infinite number of everyday examples like these, some natural (seeing on Earth compared to seeing under water) and some created (like an artist's optical illusion painting, such as Dali). Although I believe everyone experiences these changes in perspective, this project attempted to investigate the ways this phenomenon manifested in middle

school geometry classrooms, where learners intentionally investigated spatial perspective and dimensional relations. Many of the experiences above were drawn on and used as points for investigating and reflecting on the phenomenon (see investigations and cases at http://spaceandperspective.com/). A prior investigation sought to understand fifth and sixth graders' experience investigating space and dimension concepts after watching the film *Flatland: The Movie* (Travis & Johnson, 2007) and creating straw and bubble models of 4D shapes (Valentine & Kopcha, 2014). This age seemed well suited for investigating the phenomenon. I intentionally started this study with participants attending an independent school in the Northeastern U.S. The reason for this specific selection was two-fold. One, I have a connection as a former teacher in this school and have been given permission to conduct research. Second, the curriculum was not constrained in this setting like most public schools' with intense focus on standardized test preparation and preset curriculum maps, but rather welcomed new ways of engaging students with mathematical phenomena. Twenty-one eight-grade math students were included in the study. Table 4.2 shows participants split among the two classes (8a and 8b) and their assented level of involvement in the study. All learners engaged in the project as part of their normal mathematics curriculum. The Institutional Review Board, the school, the parents, and the learners permitted artifacts and audio recordings from each class session. However, learners were given the option to allow researchers to use their blog postings, photographs, and video from the class sessions. In addition, they had the choice to contribute a lived-experience description, participate in an interview, and share their final projects.

Table 4.2

Participants by Class Section and Data Contribution

Class Section (N = number of students)	Audio/ Artifacts	Blog Postings, Photos, Video	Lived-Experience Descriptions	Interviews	Final Projects
8a (N=9)	9/9	3/9	1/9	2/9	2/9
8b (N=12)	12/12	10/12	3/12	3/12	3/9

Five participants agreed to interview nine months after the *Space and Perspective* project. At the time of the interview, all participants were in ninth grade. Of the five interviewees, four experienced *Flatland: The Movie* and associated learning activities in fifth grade. Two of the five participants were interviewed in the pilot study as well. Table 4.3 summarizes prior experiences and involvement with the project's previous iteration.

Table 4.3

Participant's Gender and Involvement with the Previous Project Iteration

Participant	**Former fifth grade experience (Yes/No)**	**Interviewed for pilot study (yes/no)**	**Gender (Male/Female)**
Albus	Yes	Yes	Male
Jack	Yes	No	Male
Alistar	No	No	Male
Lynn	Yes	No	Female
Beck	Yes	Yes	Male

Step 2: Data Collection Process

The second step in Vagle's (2010a) method includes devising a clear and flexible process for collecting data appropriate for the phenomenon under investigation. In this section, data sources related to the primary and secondary research questions are described with attention to the data's role in opening up, or supporting our understanding, of the phenomenon under

investigation. Table 4.4 below details the data sources by collection date related to the primary and secondary research questions. Following is a description of each data source's role for opening up the phenomenon. Data sources include: lived-experience descriptions, open-ended interviews, semi-structured interviews, observational data (audio, video, and photos of the investigations), lesson artifacts, students' blog postings, and post-reflexion journal entries.

Table 4.4

Data Sources and Collection Date

Primary Research Question:
What is it (like) for learners to find themselves perceiving space that is problematized?

Secondary (Design) Research Question:
What role does the learning environment play in the experience for students?

Data Source	**Collection Date**
Lived-experience descriptions	June – December, 2013
Interviews	December 2013 – January 2014
Follow-up questions via email	December 2013 – February 2014
Observational data (audio, video, photo)	February – May, 2013
Artifacts from the lesson	February – May, 2013
Student blog postings	February – May, 2013
Post-reflexion journal	Throughout the entire research process

Lived-experience descriptions. After learners completed their investigation of *Space and Perspective*, those agreeing to write lived-experience descriptions responded to this prompt:

> Please write a direct account of your experience investigating geometry (specifically perspective) as you lived through it during our *Space and Perspective* investigations. Additional directions:

- Describe the experience as much as possible as you lived through it. Avoid casual explanations, generalizations, or abstract interpretations.
- Describe the experience from the inside, as it were-almost like a state of mind: the feelings, the mood, the emotions, etc.
- Focus on a particular example or incident of the geometry learning experience: describe specific events, an adventure, a happening, a particular experience.
- Try to focus on an example of the experience which stands out for its vividness, or as it was the first time.

I used van Manen's (2000) suggestions, which included: asking participants to describe the investigation of space as they experienced it, avoiding "casual explanations, generalizations, and abstract interpretations," including "feelings, moods, and emotions," focusing on "particular examples," and choosing an example "which stands out for its vividness, or as it was the first time" (p.27). The lived-experience description served as a starting point for later interviews.

Interviews. The next data source includes 5 transcripts from open-ended interviews with learners through Skype, recorded as a video and audio. Using the lived-experience descriptions as a point of reference when available, the initial interview aimed to help students articulate their experiences more fully. There was one instance where a student was interviewed who chose not write a lived-experience description.

Students were asked to engage in a conversation about their lived-experience description in an attempt to help him or her articulate more aspects of their manifest experiences concerning the phenomenon. According to Denzin, "Open-ended interviewing assumes that meanings, understandings, and interpretations cannot be standardized: They cannot be obtained with a formal, fixed-choice questionnaire" (Denzin, 1989, p. 42). I started all interviews with a similar

question, “talk about your experience investigating *Space and Perspective*.” At this point, I listened and took short notes in order to remind myself of follow-up questions focused on opening up the phenomenon. This open-ended format allowed students to recount their experience freely. For students who wrote a lived-experience description, I asked questions that built off their earlier description, again taking notes for follow up questions.

Because I interviewed former students, I intentionally used a conversational approach (Denzin, 1989, p. 43) to help alleviate any possible power structures. According to Denzin, interviewing “should *not* be a relationship where one party does all the talking and the other only asks questions. When interviews turn into this form, they become asymmetric, authoritarian social relations in which the power of social science determines the information given” (Denzin, 1989). At the same time, I allowed space for students to describe their experience in detail, with the goal of capturing the multi-faceted aspects of the phenomenon.

In addition to the open-ended interviews, specific questions were asked to help open up experiences pertaining to the secondary design and technology research questions. Although these questions are labeled as “specific,” they are still open-ended in nature. For example, students were asked to talk about their experience blogging with the aim of opening up this particular aspect of investigating space. One helpful strategy for interviewing middle school learners was sifting through a learners’ blog posts, surveys, and transcripts of class discourse in addition to the lived-experience description. Although the goal is to remain open to the way learners talk about the phenomenon, it’s helpful to support their recall of particular moments to gather more information on the way it was lived by learners.

Follow up questions via email. After transcribing the interviews and reading over the transcripts, I organized follow-up interview questions for each participant via email. These responses were incorporated into the data set as text documents.

Observational data (audio, video, and photos of the investigations). During the project, student participation and work was captured through whole class and group microphones with intermittent video and photographs. Video and photos included students working through hands-on aspects of the investigations, such as creating straw and bubble models of four-dimensional shapes. Table 4.5 below shows the time length of audio recording for the two classes (transcribed), totaling 11 hours, 10 minutes and 12 hours, 23 minutes respectively (denoted as 8a and 8b below). Table 4.5 also provides information relating the class meeting date and title of the investigation and artifacts from class (discussed in the next section). The audio (and to a lesser extent photos and video) served two purposes: a record of all class interactions and as a source for interviews with students. Before interviewing, the transcripts of lessons were reviewed and follow up questions were created to help learners reflect on particular aspects of their experience.

Table 4.5

Information Relating Class Meeting Date, Investigation, Audio Length, and Artifacts

Class Meeting Date and Investigation	**Audio (hr:min:sec)**	**Artifacts**
Class 8b (2/27/2013) **Class 8a (2/28/2013)** *[Inv. 1: Norms and Setting up Student Blogs]* *[Inv. 2: Considering the Dimensions of Space, Flatland]*	1:36:51 1:28:17	• Norms, About me page (blog site), Edmodo, cases • Homework: Blog assignment (*Flatland*), Chapter 1-2 *Flatland*
Class 8a and 8b **3/1/2013** *[Inv. 3: Considering the Dimensions of Space, Dimensional Mapping]*	2:58:25	• Pre-concept map, pre-survey (Google form), Student blog sites, Revisit norms, 25 minute *Flatland* movie discussion, Bubble and straw models, Cases • Homework: *Flatland* house floor plan, Flatland Chapters 3-7, Blog prompt
Class 8b (3/4/2013) **Class 8a (3/5/2013)** *[Inv. 4: Considering the Dimensions of Space, Additional Cases]*	1:53:22 1:54:35	• Blog site feedback groups, Cases • Homework: *Flatland* Chapters 8-12, Blog prompt
Class 8b ***3/12/2013*** *[Inv. 5 Shortened: Capturing and Representing Space]*	1:09:25	Cases
Class 8a (3/14/2013) **Class8b (3/15/2013)** *[Inv. 5 Shortened: Capturing and Representing Space]*	1:45:03 1:41:57	• Cases, Google survey (mid), *Flatland* group book discussion, Photographing space activity • Homework: *Flatland* Chapter 13-17, Blog prompt, Finish posting photos
Class 8a (4/30/2013) **Class 8b (4/30/2014)** *[Mathematization, Flatland, and Introduce Project]*	00:43:22 00:28:42	• Google PowerPoint, Revise norms, Padlet *Flatland* wall, sections 13-16, Project description • Homework: Update blogs, Blog prompt, Brainstorm projects, Finish *Flatland*
Class 8b (5/8/2013) **Class 8a (5/9/2013)** *[Inv. 6: Representing Impossible and Invisible Spaces]*	1:45:42 1:36:53	• Share blog posts, Geometry quiz, Book share for projects, Idea maps, Group project discussions, *Flatland* progression discussions (arithmetical and geometrical)
Class 8a (5/10/2013) **Class 8b (5/10/2013)** *[Inv. 7: Culminating Project, Innovative Uses of Flexible Space Concepts]*	00:42:46 00:46:46	• Post-concept map, Survey, Final projects
Total Meetings: **Class 8a** = 7 **Class 8b** = 8	***Total Time:*** ***Class 8b** = 12:23:10* ***Class 8a** = 11:09:21*	

Artifacts used in the lesson. In Table 4.5 above, the third column indicates artifacts generated by learners over the course of the project. These include blog postings, projects, pre/post concepts maps, idea maps, Padlet online discussion board, Google documents (surveys, norms of engagement document, presentations), straw/bubble models, Flatland floor plans, and occasional photographs of student work on the whiteboard. The hypermedia site (created in Wordpress) served as the class website. Figure 4.1 shows the homepage indicating the 16 themes organizing the 70 cases and mini-cases in the form of blog posts. There is also a tab for the 7 investigations listed in Table 4.5. Cases and mini-cases are made up of text and symbols, visual imagery, animations, video, models, simulations, and hyperlinks. According to Jacobson's (2008) hypermedia design principles, these make up the representational affordances of cases. These resources on the hypermedia site made up part of the learners' experience and were used to help learners recall class activities during the interview. Other artifacts from the lesson included lesson plans and student blog postings. The students' blogs in particular helped to capture their experience, especially perceptions of space over the course of the investigations. Blog postings helped guide questions for interviews with learners.

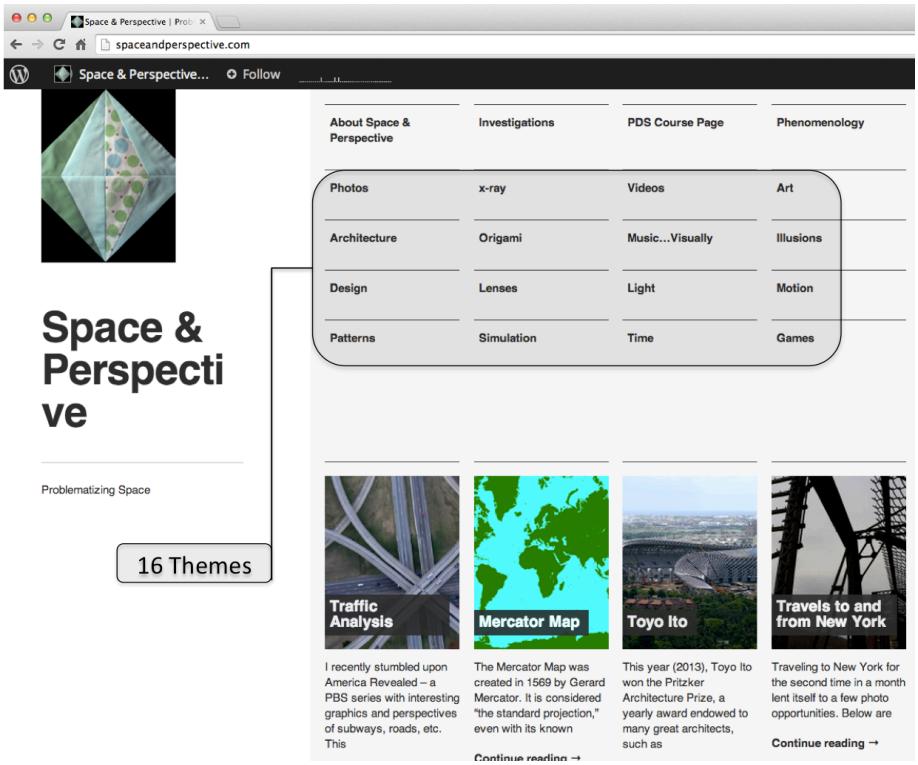

Figure 4.1. Space and Perspective hypermedia site homepage with 70 cases (coded across 16 themes) and 7 investigations.

Student blog postings. The students' blogs were an especially rich data source for answering the research questions. Learners did not always reflect on the lived quality of their experience, but their postings did help frame the semi-structured interview questions.

Step 3: Post-reflexion Plan

In interpretative and post forms of qualitative inquiry, validity is still as important a concern as with descriptive qualitative or quantitative traditions, but reality is not seen as an objective truth. In post-intentional phenomenology, depicting reality is not attainable in any complete or final way. As researchers, we are embodied with gender, culture, history, relationships, experiences – when we investigate a phenomenon, it is never in and of itself. This is similar to Heidegger's "recoil of the question 'why' upon itself" (1935/2000, p. 5). Reflexivity

refers to the practice of simultaneous inquiries – the phenomenon, the research process, and ourselves. Post-reflexion, as a research activity, reimagines the phenomenological practice of bridling. Bracketing is a common practice in descriptive, transcendental phenomenological approaches and is practiced as a way of suspending one's pre-understandings from influencing one's understanding of a phenomenon. Bracketing, or Epoché, in this sense allows the researcher to claim validity because as assumption is made that prior knowledge as an influence is taken care of (Husserl, 1901/1970a; Moustakas, 1994). Dahlberg et al. (2008) found this practice troubling because bracketing only dealt with understanding as pointing backwards, implying the researcher can somehow bracket at the start of the project and not return. They offered bridling as an ongoing activity of restraining one's pre-understandings, not making definite what is indefinite, and pointing forward, allowing the phenomenon to "present itself" (p. 129-130). Bridling is a way to acknowledge the researcher's role in the investigation. From a post-intentional phenomenological stance, bridling can sometimes be practiced as a peripheral way of dealing with reflexivity (Vagle, 2014). However, post-reflexion as an activity imagines reflexivity as a "destabilization source" (Lawson, 1985, pp. 68–69), "an aggressive [move] for bringing more of an unsettled field into view" (Macbeth, 2001, p. 37), and "seeing what frames our seeing" (Lather, 1993, p. 675). Phenomenology as a post-structural inquiry is not intended to cast declarations of self-standing truths, but rather invites others to connect with the phenomenon. Heidegger writes, "the difficulty of this kind of research lies in making it self-critical in a positive sense" (1927/2008, pp. 60–61). Post-reflexivity, as a productive disruption, should attend to the following according to Vagle (2014): moments of "connection" and "disconnection," "assumptions of normality," "bottom lines" consisting of beliefs, etc. that we can't shake, and moments where we find ourselves "shocked" (p 132-133). With attention to

these moments, assumptions, and beliefs, post-reflexion is continuous throughout the research project. Vagle (2010b) recommends three main actions. The first is creating a journal. The form is unimportant (digital, audio, hand written), but each entry should include a date and entries should be captured often throughout the research process. Second, the research should create an initial post-reflexion statement, including one's role as researcher, assumptions, beliefs, perspectives, background – all in relation to the phenomenon. In post-reflexion, this is merely the beginning of an ongoing process of post-reflexion. Vagle writes that "examining your assumptions gives you a better chance of taking hold of them, rather than the assumptions taking hold of you and in turn the phenomenon under investigation" (Vagle, 2010b, p. 17). Lastly, post-reflexion continues throughout the data collection and analysis phase and in a sense becomes "a data source for the crafting of your text" (p. 17).

I started journaling as part of a *Practicing Phenomenology* course during the fall of 2011. I wrote frequently, at least weekly, in response to readings, conversations, and every facet of designing, developing, implementing, and analyzing the study. I found myself writing most often when I was at the peak of making connections, envisioning future trajectories, and questioning my assumptions. I enjoyed reflecting and felt free to write openly and honestly, not perceiving an audience. I would read and re-read these entries, especially during the analysis phase, to further connect and problematize my assumptions. I used this space to write specifically about my program of inquiry as a researcher – a more long-term outlook. I've noticed that simple life occurrences, such as observing teachers at a local elementary school would cause me to question what is important in my field and the connections I hope to make to practice. I created entries in five different formats: (1) as comments inserted alongside lived-experience descriptions and transcripts, (2) as individual, dated Word document files, (3) as notes inserted in books and

articles, (4) as audio recordings on my computer and phone, and (5) as video/face-to-face conversations with others – I usually used these as a springboard for writing in my journal. Later in the process, I started using the Day One app on the iPhone. The app has changed over time, but allows me to incorporate images, weather data, location, and tagging features. An example of my early post-reflexion concerning the intersection of the pilot study (Valentine & Kopcha, 2014), my personal life, and reaction to readings about phenomenological research can be found in Appendix A.

Step 4: Read and Write your Way through the Data in a Systematic, Responsive Manner (or Analysis Plan: Whole-part-whole)

Both Vagle (2010b) and Dahlberg et al. (2008) guided the analysis as well as interpretation techniques. Vagle suggests a whole-part-whole analysis of the data and also explicating "how you will analyze data within and across each data source" (p. 42). Dahlberg et al. add, "[W]hen analyzing a text for meaning, it is imperative that each part is understood in terms of the whole, but also that the whole is understood in terms of its parts. It is always a question of seeing the relationships in the text and carrying on a dialogue with it" (p. 236). Analysis in lifeworld research is ongoing, dialogic, and seeks tentative manifestations of the phenomenon. Tentative manifestations appear in the relationship between the parts and the whole of the research. The reason for intentionally attending to the "whole" reflects the idea that "meanings belong to the lifeworld and come to be in the relationship between subject and object, i.e. between researcher and phenomenon" (p. 233).

Vagle (2010b) describes a process for whole-part-whole data analysis, which was used in this project:

- The first reading focused on the "whole data collection event," where all data pieces are

brought together and read (not analyzed) as a whole.

- This was followed by multiple "line-by-line readings" with "careful note taking" and the "marking of excerpts."
- Follow-up questions were created for each participant.
- The second line-by-line reading was focused on "articulating meanings" and took into consideration any notes, markings, follow-ups, and post-reflexion up to this point. This ended with a saved document for each participant.
- The third line-by-line reading focused on "articulating the analytic thoughts for each part for each participant."
- Subsequent readings "involved reading across individual participants' data, with the goal of looking for beginning tentative manifestations."
- It is important to note that post-reflexion occurred throughout every step of analysis. (p. 18-20)

Instead of merely listing quote after quote under tentative manifestation headers, I engaged in an interpretative hermeneutic process (Gadamer, 1975/1994), with the goal of helping open up the phenomenon for the readers. The data sources (lived-experience descriptions, interview transcripts, observational data, and lesson artifacts) were analyzed *within and across* as described below.

Preparations for analysis of the whole data collection event. Although mentioned in the data collection section, it is important to reiterate that interviews for each learner included attention to the data already collected for each learner in the form of lived-experience descriptions, blog postings, class discourse, and artifacts (projects, photos, etc.). In preparation for each interview, I created a file for each student that included their lived-experience

description, class discourse transcripts, blog postings in chronological order, surveys, pre/post concept maps, projects, and any other artifacts for the learner. This preliminary analysis was attended to as follows: (1) initial reading of all data for the learner with no mark-ups or analytic notes, (2) journal entry for each learner after reading through all data, (3) subsequent readings with mark-ups, questions, and a summary – saved as a Word file for each participant, (4) A refined Word document with semi-structured interview questions.

I conducted the interview and transcribed these within a week. After each interview, a post-reflexion entry was created and pasted at the end of each transcript. This contained my overall sense of the interview, structurally and personally. Sometimes I wrote comments with ideas for opening up the phenomenon in future interviews, where meanings might be better drawn out. This process continued for subsequent interviews. I would print out the transcription and include it in the file already created for each learner.

Observational data, including photographs of students' investigating space and video of the face-to-face and online lessons were saved for each class session by date and were accompanied by a Word document transcribing audio from the class. These transcripts were included as part of the whole data collection event.

Lesson artifacts were numerous including: all blog posting for each student, the class blog site and all the artifacts contained in or linked from the site, including lesson plans, surveys, etc. These documents were a mix of text, video, and images. In order to create text documents from videos used in the lesson, pertinent sections of the video were transcribed that learners drew on in their blog postings and class discussions. In addition, a written description of each artifact was created.

After the interviews were complete and all the data was organized in the folders, I stepped away from the data for almost a month. During this time, I wrote frequently in my post-reflexion journal, worked on other papers, read books I felt might help me better understand the phenomenon. After the month passed, I brought out all of the data and made sure I had large pieces of poster paper to jot down assumptions, connections I thought existed, and most importantly, kept writing down the phenomenon over and over. It was after writing the phenomenon down multiple times, that I realized the wording wasn't capturing the phenomenon as I felt it. Rather than living in space that is being problematized and mediated by technologies, it was more simply about the moment when a perspective shifts, whatever one attributes to the shift – technology, person, activity. I was less concerned with what was causing it than what it was to experience a shift in one's perspective of space. I was still interested in the secondary question about the role of the learning environment, but the phenomenon had shifted slightly for me as I started the next step – my first reading of the whole data set.

First reading of the data (whole). All of the data, including post-reflexion entries were first read as a "whole data collection event" (Vagle, 2011, p. 42). I spent the most time in this phase of analysis and wrote in my journal weekly at minimum. As I started to glimpse various manifestations of the phenomenon, I would write in my journal or on the large sheets of chart paper, still attuning to the whole. The journal itself was also used as an accompanying data source, read as part of the whole.

First line-by-line reading (part). I started to carefully read and annotate line-by-line on the computer and sometimes by writing on printed data sources. I also highlighted sections of text that appeared to contribute to the meaning of the phenomena. Other times, I inserted comments, questions, and connections. Throughout this process, I continued to journal, reacting

to the questions and comments posed. Sometimes, these writings were inserted at the end of the document, and other times dated in the larger journal. I ended this line-by-line analysis by organizing follow-up interview questions for each participant, incorporating these responses into data set as text documents.

Second line-by-line reading (part). I organized the second line-by-line reading by participant. First, I consolidated lived-experience descriptions, interview transcripts, observational data, student blog postings, and associated post-reflexion excerpts into one document, retaining all comments and mark-ups from the previous analysis. I continued to insert comments and questions, highlight meanings regarding the phenomenon, and mark through parts that seemed unrelated to the phenomenon. I continued this process for each participant. At this stage, overlapping data chunks were combined within the document. For example, if a transcript and lived-experience description addressed a similar experience, they would read one after the other in the document. In addition, the post-reflexion process continued.

Third line-by-line reading (part). The third line-by-line reading focused on organizing meanings, or "chunking" narratives, transcripts, and observations by participant. At this point, I started creating analytic "labels" or "meaning sentences" as a header for each chunk. In addition, I wrote comments and questions about these headers in the margins, coming back to them frequently as I reflexively continued my way through understanding. I did this for each participant and ended up with 5 documents "articulating the analytic thoughts for each part for each participant" (Vagle, 2011, p. 42). At the end of each document, I wrote about the experience for each learner as a whole as it relates to the phenomenon, and then my own wonderings, reflections, and insights.

Reading across individual participants' data (whole). To prepare for this reading across all participants data, I printed out a hard copy of everything I had up to this point. With scissors and tape, I started to cut chunks from each of the documents that seemed like they fit with other documents and continuously arranged and rearranged with tentative terms. I was able to copy and paste within Word to create one document articulating "tentative manifestations" (Vagle, 2011) of the phenomenon under investigation. This process was long and carefully crafted with many revisions. I wanted to make sure I was not overlooking meanings that may be connected in future analysis, but also needed a way to reflect back on my decisions to combine excerpts under a tentative manifestation header. This newly combined document was then printed out and bound in a folder. At this point, I continued to organize and structure the manifestations and the data that articulate them.

Tracking Data Analysis

This section describes additional research techniques (e.g., memos, analytic files, and reports) and their use in tracking the origins of data and documenting the flow of analysis. Before starting the project, all data sources and their approximate date of collection was organized into a table (see Table 2). The organizational plan for organizing the multiple sources consisted of sections and more specific headings for each file folder (both digital and traditional paper folders). The preliminary sections included: post-reflexion journal, student blog postings, investigation bank and lessons, quotations from readings, tentative manifestations, communications with school, and reports/log of research events. These sections contained multiple folders, similar to subheadings. The main intent was to create a space that was flexible and allowed ideas/thoughts to be placed alongside each other. Electronic files were somewhat harder to move around but the physical file method allowed ideas, thoughts, and data to be

arranged and moved around as the project progressed. The particular files for the post-reflexion section consisted of dates, allowing the entries to be organized sequentially. However, I also include tags for each entry to allow for easier navigation of what became several hundred pages of journal entries.

I used the filing system described above to store additional research sources, such as memos, analytic files, and frequent summary reports. These were organized sequentially by date or by theme – depending on what made sense in relation to the entire data collection. Memos, analytic files, and the reports are described in more detail below.

Memos. During the data collection, analysis, and even during the manuscript writing phases, I kept a research log. This log resembled a composition notebook in which all data collection and analysis activities were organized by date as a running record. In addition, memos were be integrated into the log. Memos consisted of preliminary thoughts and insights within and across data sources. These memos were written after face-to-face meetings with students, after an interview, even during a car ride when connections formed – no matter how preliminary. These memos were captured through audio recordings, written notecards, and as formal journal entries. These were compiled and organized into a research log by transferring audio, notecards, and handwritten journal entries into text files by date. In this way, the memos become a way to trace data source origins, connections, insights, and a place to remind myself of important questions/wonderings needing attention. I read and reread the entries – clarifying and adding to them as often as possible. Rather than artificially separate journal entries related to post-reflexion from the log, these entries were included sequentially alongside the memos.

Analytic Files. According to Glesne (2011), analytic files "provide a way to keep track of useful information and thoughts" (p. 190). The files consist of thoughts belonging to the

researcher and others. In addition, Glesne recommends specific files: a reflexivity file, title file, introduction/conclusion file, and quotation file. The post-reflexion journal served as a space to write down reflexive ideas, so it is not discussed in this section. The title file served as a place to try out titles during the data collection and analysis phase in order to begin reflecting on the eventual narrative. Glesne also writes about files related to the introduction and conclusion, helping focus the researcher towards the scope of the research study. The value in reflecting on context, conclusions, implications, and questions "alerts you to what you might otherwise miss in the course of your study; they stimulate you to notions that, like your titles, are candidates for inclusion in your forthcoming text" (p. 190). Similar to the reflexivity file, post-reflexion seemed a more appropriate activity to capture the activity of reflecting on implications and further questions. The quotation file provided a place to organize quotations from readings that seemed useful. According to Glesne, the researcher "acknowledges that the world has not been born anew on your terrain" (p. 191). This file sought to inform the study as literature was continuously integrated.

Weekly/Monthly Reports. Glesne suggests creating monthly reports as a "way to examine systematically where you are and where you should consider going" (p. 192). I created dated reports concerning progress, problems, and plans as part of the research log in conjunctions with documented research activities.

Tentative Manifestations of the Phenomenon in its Multiple, Partial, and Varied Contexts (Findings)

In this section, I explicate the tentative manifestations (sometimes referred to as findings) with the understanding that moments of shift in perspective as a phenomenon is not one thing but multiplicities, partialities, and highly contextual. This section shares some of the possible ways shifts in perspective in the context of middle school mathematics may transpired for learners.

Vagle (2010b) advises researchers to "restate the multiple and varied contexts" (p. 21) where the phenomenon in focus resides. The phenomenon, moments of shift in perspective, is most likely experienced by all humans and even animals across time and place as expressed in the methods section. Contexts identified include: non-seeing perspectives (i.e., related to conversation and literature that change one's future visual perspective), multiple perspectives (i.e., bird-eye view, horizontal view, lateral view of same object), taking on and creating perspective (i.e., the artist and musician), architecture (these designed/created spaces affects our perspective), place (i.e., living under a canopy of trees versus the desert), and even across time as with the process of aging (i.e., growing taller, eyes weakening). Although some of these contexts are addressed in the study, an eighth grade learning environment (learners, materials, classroom, activities) and learners' experiences outside of the classroom (i.e., home, dance class, art museums, movies, 9th grade classrooms, prior memories) form the boundary and context for this investigation. An important consideration concerning boundaries from a complex systems perspective shows how boundaries are never objectively separate. Davis and Sumara (2006) write:

> The critical point here is not that researchers must define boundaries of the phenomena that they study (although this is a vital point). Rather, the main issue here is that

> complexity thinking compels researchers to consider how they are implicated in the phenomena that they study – and, more broadly, to acknowledge that their descriptions of the world exist in complex (i.e., nested, co-implicated, ambiguously bounded, dynamic, etc.) relationships with the world. (p. 15)

Even though the phenomenon, a shift in perspective, is articulated as being bounded by a middle school classroom and learners' experiences outside, we (the learners and I) bring our own histories, cultures, experiences, etc. with us, making this boundary ambiguous at times. This crops up in the findings throughout this section, for example, in the form of childhood memories.

According to van Manen (1990), "the aim of phenomenology" is to create, similar to the act of a painter, an action on the part of the reader (or viewer) whereby "the effect of the text is at once a reflexive re-living and a reflective appropriation of something meaningful: a notion by which a reader is powerfully animated in his or her own lived experience" (p. 36). The presentation of phenomenological data can take many forms. There is not a singular, accepted technique. Rather, the researcher, according to Vagle (2010b), "should feel free to play with form, bringing all that you have (from the data, the [post-reflexion] journal, other readings, other theories, other philosophies) to bear" (p. 22).

Considering the intent (to support an active reading) and freedom of form, the tentative manifestations in this section are organized around/through/with two intense manifestations. The term, "intense manifestation," indicates a connective tissue that runs through and relates to every other manifestation with an intensity for me and my "pedagogical orientation" with children (van Manen, 1990, p. 135). The term, "intense manifestation," is not meant to prioritize one manifestation over another, as much as to indicate that they feel intense, revolutionary, and fluid. Thus, in this particular explication, they are brought to the fore and in dialogue with other

manifestations, philosophies, and notions of learning and living as humans. Figure 4.2 shows the intense manifestations: (1) learners' realization that their current perspective (or concept) is incomplete and (2) problematization, the act of troubling one's own experiences in order to account for complexities and attempt to understand the world more fully. These manifestations are intense because coming to recognize an incomplete understanding and troubling one's previous unreflective position is key to how we open ourselves to learning - learning in ways that makes our currently held concepts revisable, flexible, adaptable, and usable. Yet, these moments are fluid, even tentative, because this moment is brought on by other manifestations of experience, such as encountering elusiveness, feeling provoked, coming to a place of impasse, and feeling entangled with emotions of discomfort, frustration, acceptance, etc. Furthermore, these manifestations are mediated by cases as alternative perspective (especially the pivotal case, *Flatland*) and various means of discourse and reflection. Problematization runs through all of these moments and feelings and represents the action-oriented compulsion on the part of learners. Rather than something being done to them, they are iteratively cycling between being triggered by problems and problematizing their world. This is similar to John Dewey's (1929) notion of "reflective inquiry" as operationalized in Hiebert et al. (1996): "Familiar objects, including subject matters in school, are treated as "challenges to thought…They are *to be* known, rather than objects of knowledge…[t]hey are things *to be* understood (Dewey, 1929, p. 103, emphasis in original)" (p. 15).

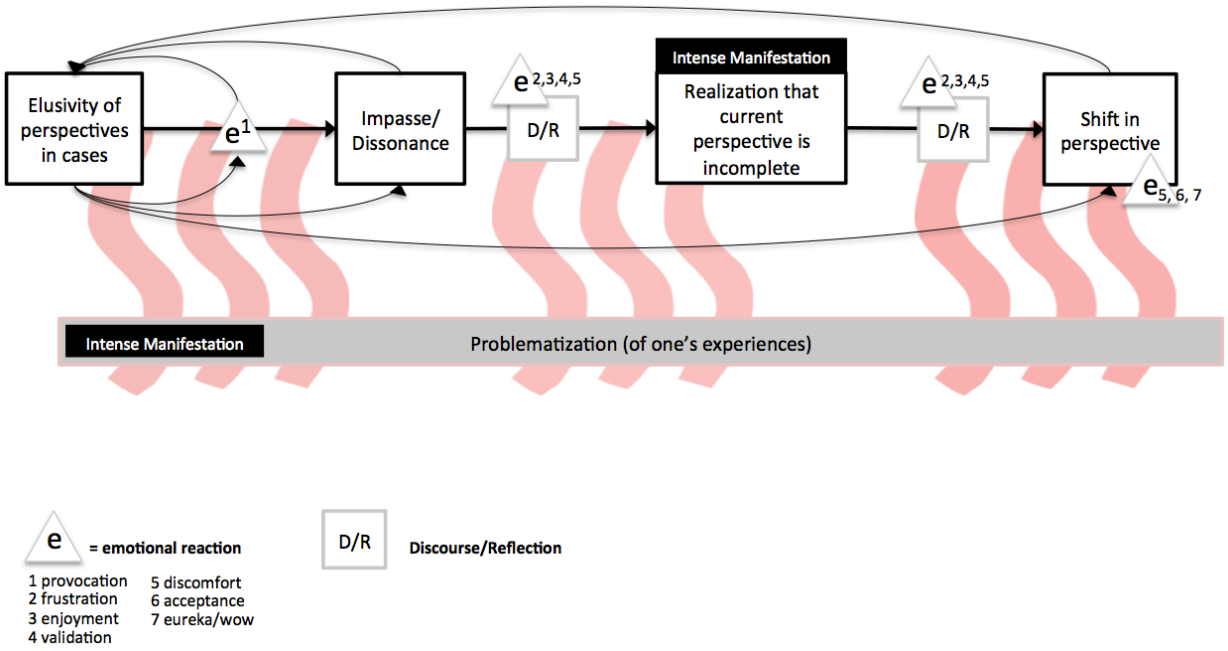

Figure 4.2. Relationships between intense and tentative manifestations.

Figure 4.2 is a representation of relationships between the manifestations as lived by learners in the *Space and Perspective* project. The representation falls short by the limits of this static, two-dimensional page. Tentative manifestations are tentative because of their partial, fleeting, and intimately intertwined nature with people, contexts…the world. Motion (time) and space are two directions this figure is unable to portray. Still, I chose to present this one particular way we might conceive of a shift in perspective drawn from the learners' experiences. At the bottom of the figure, problematization is represented as permeating through every other tentative manifestation. This intense manifestation is a way of being, an attitude or lens, from which all the other emotions and provocation tools (such as cases as alternative perspective) are considered.

The elusivity of perspective in cases is shown on the left side of the figure. For learners, the role of the cases affected their experience from the onset and remained a trigger throughout,

even after formalized investigations into space, perspective, and dimension. The seventy cases on the *Space and Perspective* hypermedia site varied in their importance for learners. Where one student related more to cases as alternative perspective in video game play another student related better to the Eames and Eames video, *Powers of Ten* (1977). Either way, certain cases and activities from the class sessions were pivotal, such as the book, *Flatland* (Abbott, 1884/1992) and *Flatland: The Movie* (Travis & Johnson, 2007).

The elusivity of *Flatland* and other relatable cases provoked learners as indicated by the triangle + e, representing emotion. In addition, the superscript 1 indicates that this provocation led to the recognition of more elusivity from the cases and/or brought about feelings of impasse or dissonance, represented with arrows to show the multiple paths. The impasse, or stuck feeling, either led to a discussion and/or reflection on the part of learners, as denoted by the D/R box. Again, the triangle + e includes superscripts 2, 3, 4 and 5, representing the emotions related to discourse (frustration, enjoyment, validation, and discomfort). Also indicated by the cyclical arrows, the impasse may have caused learners to problematize cases further to uncover more elusive aspects and even contribute similar cases that exude elusive qualities.

Discussions didn't follow all cases as alternative perspective – only cases that created dissonance or impasses for learners. Discourse followed either as a blog posting or as a class conversation or debate when learners tried to come to terms and agreement on what something might be and how we as humans should be conceptualizing our world epistemologically, ontologically, and cosmologically. Throughout the class and continuing after the project ended, learners experienced shifts or changes in the way they saw their world, usually marked by a realization that their current perspective was incomplete and didn't explain, for example, what it might be like to see and experience gravity in another dimension. In this way, the experience was

similar to Valentine and Kopcha (2014), where they were named "ontological 'trying-on' of dimensions." Again, this 'trying-on' or shared recognitions and shifts occurred throughout discourse and reflection, shown as D/R in the figure.

The far right box in the figure, shift in perspective, is marked by the emotions: discomfort, acceptance, and eureka/wow. The circular arrows moving through this shift and back to the elusivity of perspective in cases attempts to show the iterative relationship between a moment of shift and this new lens for revisiting prior cases. The new lens allowed for more discrimination in the same case (and the new impasse, discourse, recognition) or as applied to the many cases on the hypermedia site or as contributed by the community of learners.

To help you better relate to the experiences of middle school learners, the findings – from now on called tentative manifestations – will be linked showing the various ways this story transpired. Sections explicating the tentative manifestations are organized as follows: of the nature of *Flatland*, concerning the elusiveness of space, impasse/dissonance, concerning the nature of discourse, of the nature of shift. Throughout the next sections explicating these manifestations, I wrote my way through the data to portray the shape of the experience illustrated in Figure 4.2. In addition, Table 4.6 shows the tentative manifestations related to the research questions. The table, as a more linear representation, helps to organize the structure of this section. It is important not to confuse the structure with a phenomenon that is somehow stable or generalizable. Vagle (2010b) is clear that manifestations "are de-centered as *multiple, partial, and endlessly deferred.* A post-intentional phenomenological research approach resists a stable intentionality, yet still embraces intentionality as ways of being that run through human relations with the world and one another" (p. 6).

Table 4.6

Tentative Manifestations Related to Research Questions

Phenomenon: The phenomena of interest in this study aims to articulate moments of shift in perspective

Research Question	Tentative Manifestations (Findings)
Design: What role does the learning environment play in the experience for students?	Ia. **Of the Nature of *Flatland*:** *Flatland* serves as a "pivotal" case, providing a trigger for student discourse and thinking. It is a case that provokes and creates feelings of impasse. It helps learners realize the elusive nature of dimension.
	Ib. Learners seek out cases on the hypermedia site related to their interests, such as art, photography, and video gaming. Sometimes, learners' generate/contribute cases to the class. Some cases, such as gravity, serve as familiar objects. As Dewey conveys, "They are *to be* known, rather than objects of knowledge" (Dewey, 1929, p. 103), meaning they use these cases from experience as new sources to problematize.
Primary: What is it (like) for learners to find themselves perceiving space that is problematized?	II. **Concerning the Elusiveness of Space:** Investigating the elusive nature of ideas and concepts investigated is fun to discuss, frustrating, something to come to terms with, scary, and an inability to undo (ones 3D way of being). Elusiveness was communicated as the notion of not having mathematical and bodily access.
	III. **Impasse/Dissonance:** Learners experience impasse and dissonance, but this is not regarded negatively. The elusive nature of ideas and concepts (see II) brings about impasses that vary in quality and how they are experienced across learners. For many, this is communicated as a type of productive struggle.
	IV. **Concerning the Nature of Discourse:** Learners have strong emotions related to discussing ill-structured (elusive) concepts in the cases as alternative perspective (e.g., *Flatland*, Cubist art). This emotion ranges from enjoyment to frustration. Some learners talk about feelings of validation when able to convince others of their way of thinking about an idea (and discomfort when others take a different stance). However, discourse is perceived as benefiting growth of one's own point of view.
	V. **Of the Nature of Shift:** Learners express a shift (change/distinguishing activity) and feel "compelled to act." Cases and discourse contribute to shift. These moments are characterized in variant ways, lived-out in different ways, and lead to different foci for the learners. One living out is that of ontologically 'trying-on' dimensions.

Another method that is central to this research project concerns my serious commitment to "maintaining a strong and oriented [pedagogical] relation" (van Manen, 1990, p. 135). Van Manen writes:

> As Nietzsche (1984) said about the art of reading (and writing), "every strong orientation is exclusive" (p. 164). A strong pedagogic orientation requires that one reads any situation in which an adult finds himself or herself with a child as a pedagogic situation, as an answer to the question of how we should be and act with children. (p. 152)

When writing through the manifestations, it is important to preface this pedagogical orientation of manifestations. I chose this very project because of my experience teaching these very kids and because of my practice as a teacher and a guide to the exploration we all embarked on together. I saw this particular learning experience as different from most others – learners investing in something for no real reason other than to wonder about something so extremely elusive. It is this reason that I, as the researcher and teacher, invested time into opening up this phenomenon for others. Van Manen's (1990) remarks weigh heavily on me:

> To be unresponsive to pedagogy could be termed the half-life state of modern educational theory and research which has forgotten its original vocation: that all theory and research were meant to orient us to pedagogy in our relations with children. (p. 135)

Of the Nature of Flatland

The third day of class, both eighth-grade classes (21 learners) came together for close to three hours. We continued establishing norms for engagement from the previous day, set up blog sites, and finished watching the end of *Flatland: The Movie* together. When the classroom teacher Kevin asked, "Do you have a minute for me to point out a problem I had with *Flatland*," three students blurted out words like "light," "color," and "projections" before he continued with

the problem of "implied thickness." This became the start to an hour-long debate, conversation, and conjecturing session, or as one student aptly called it, "chucking out ideas." *Flatland*, as a pivotal case, served as the main reading material for book discussions. In addition, learners problematized the film adaptation of the book, most notably taking issue with a film that can somehow portray a dimension without height.

We are probably aware as humans that our "reality" often times varies and sometimes clashes with other humans inhabiting the same space as us. Sometimes we ignore differences, celebrate them, but at other times we can use these moments of "noticing" to check in with our epistemological, ontological, and cosmological assumptions. *Flatland* is a case that begs one to consider held assumptions about space, place, society, racism (a.k.a., shapism), sexism, fairness, etc. Positioning myself with a strong pedagogical orientation, I see *Flatland* as a pedagogical tool with young learners, especially fitting for adolescents who are experimenting with many coats of reality. Even the author, Edwin A. Abbott, in his third person preface to the second edition, "begs his readers" to be open-minded in reading and:

> not to suppose that every minute detail in the daily life of Flatland must needs correspond to some other detail in Spaceland, and yet he hopes that, taken as a whole, his work may prove suggestive as well as amusing, to those Spacelanders of moderate and modest minds who – speaking of zthat which is of the highest importance, but lies beyond experiences – decline to say on the one hand, "This can never be," and on the other hand, "It must needs be precisely this, and we know all about it." (p. x)

It is precisely these elusive, yet familiar aspects coursing through the book and the movie that make *Flatland* a pivotal case when investigation space, perspective, and dimension. To show the way *Flatland* as a case affected learners and contributes to understanding the phenomena of a

shifting perspective, the following section will share learners' remarks concerning: a "fondness" for the story, *Flatland* as food for fodder (provokes), and differences between the movie and book. *Flatland* is also a case that creates feelings of impasse and helps learners realize the elusive nature of dimensionality. However, this particular aspect is discussed later in sections dedicated to exploring these feelings.

Fondness for Flatland. One student, Lynn, recounts her fondness for *Flatland* during the first day of class. In a later interview she attributes these feelings to the novel ideas brought about in the story:

> It was in 5th grade when we watched it, first of all I liked just being able to watch a cool movie – you know in class. And I also think it was something that I hadn't really thought about ever. Like just all of these concepts had never crossed through my mind. And so I just remember being so infatuated with it and being like what! This is like a thing that happens! I just remember thinking that it was super cool and interesting.

Lynn's fondness for *Flatland* is linked to it provoking her awareness of "concepts" she never considered. I can't help but wonder why new material in a textbook doesn't typically elicit this reaction. This seems to indicate that there is something more about the case of *Flatland* than simply just being new. Her blog post following her viewing of the film in eighth-grade helps show that it is indeed more than just the newness of *Flatland*, because this writing is three years after her first viewing:

> The movie *Flatland* is an extremely informative and thought provoking movie. The beginning makes it clear what it would be like to live in a 2D world. The way that the government and society is set up was extremely interesting to me and made me personally think about how it is similar to society in our 3D world. As the movie progresses through the story, the concept of what a 0D and a 1D world is made very clear, a topic that I had never personally thought about. Then when the main character discovers a 3D shape, it showed me what 3D is in comparison to 2D. The brief thought at the end of the movie about a 4D world made me feel similar to the 2D shapes. The concept is so abstract it is almost scary. However, it is very much possible that a 4th dimension does exist.

In the blog post, we can start to see that emotions are tied to these ideas of dimension, abstract = almost scary and possibility of a 4th dimension. She even writes about the zero dimension (0D) and first dimension (1D) as "a topic that I had never personally thought about." At first, I was confused because I knew we watched the film in fifth grade and talked about these various dimensions. However, from her point of view:

> I think that in 5th grade I didn't fully like get it (the movie, *Flatland)*. Because I mean I was like eleven then and I think I didn't really have as clear of a concept of like infinite space and like just space and like shapes and stuff like that. I feel like I just didn't really get it and I finally got it last year when we watched it…so like the different dimensions and stuff.

In later sections she continues to talk about the elusiveness of infinite space, which she identifies as an important aspect of learning about space, perspective, and dimension concepts.

Food for fodder. *Flatland* as a pivotal case (Linn, 2008), is full of content to discuss, argue, ponder, and even leaves room for imaginative additions. Learners describe the book and movie versions of *Flatland* as mysterious, bringing about more questions than answers, and the movie version filled with problems of translation. In an interview, Alistar's description of the book is telling, "With that book, there's a lot to be discovered in between the lines as kind of a metaphor." He talks more about this "in between" space in his lived-experience description:

> During the time we were reading *Flatland*, I started to realize that parts of the book that were not explained might not have been possible. If everything was flat, would laws like gravity and density actually apply? The comprehension of questions like these compelled me to read the book.

He articulates more nuances about this connection between things that seemed impossible and how they motivated him to read the book. In this sense, what was hidden or out of reach for him (a.k.a., mysterious, elusive), was motivating or "compelling" for him:

> Well, I think that, the idea of why I had those questions, was just thinking about the world. Since it was flat, like, if it rained, where would the rain go? Or like, if there was gravity, why aren't they like – are they stuck to the world or are they just always falling

> or something – I don't know. Is the world really flat or like propped up like this? That's what I could never really think about - because that wasn't really ever explained too much. So that's what I had questions about. And, I think that – those are the things that, I felt like - I kept reading because I wanted to know if it ever explained that. And while they might not of, I still thought it was a good book, for sure. Even though I feel like there are – some things that I would have liked to know more about it, for sure.

Wonderings like Alistar's, concerning the orientation of the flat plane, the way objects are acted on by gravity – these were barely mentioned in the book (actually, orientation was not ever mentioned). It's almost as if he is noticing the "gaps in the literature," which is a characteristic of someone with an investigative attitude. It is in this sense that *Flatland* becomes a case that provides food for fodder.

Alistar is not the only learner to express this feeling concerning *Flatland*. In a joint interview with Jack and Albus, they both talked about problems in *Flatland* similar to Alistar, and also indicative of the classroom discourse that transpired. It is in identifying and working through these problems that *Flatland* shows itself to be a pivotal case that is food for fodder:

> **Jack**: I was trying to figure out how the problems would work with paint. I tried to figure out how paint would work and how water would work and gravity would work.
> **Albus**: That's interesting
> **Jack:** I kind of explained them I think.
> **Albus**: I mean paint wouldn't make any sense, but you'd be able to – I mean cause you can't hold anything, so in a 2-dimensional space
> **Jack:** Yeah, well I said that if this was like a flat surface, paint would just be around it and that would be it and then because they only see the sides of things so that would look like it was paint.
> **Keri**: Right, I remember you actually talking about that. I think what you were saying is that it just wouldn't have any area – maybe?
> **Jack:** The shape – I don't know. I said that it would have area, just that area wouldn't have paint on it. Things couldn't be on top of each other – it would just be around it

As a result of reading *Flatland*, Jack is oriented towards thinking through paint in *Flatland* (or on a plane). In this particular discussion, it is apparent that he has developed a sophisticated concept of the plane (2D). In considering paint (which Flatlanders apply to themselves during the color revolution), in the book version he has imagined the impossibility of paint "on" the surface

of Flatland, recognizing "things can't be on top of each other" but rather "around it," which is the notion of perimeter. This does indicate that *Flatland*, as food for fodder, plays a role in the experience for learners that even allows them to express/contemplate notions of planes, space, and perimeter. As a pedagogue, *Flatland* as a case served a special role for learners.

Book vs. movie. Movie adaptations of books are usually a flop (in terms of having to delete characters, ideas, and conversations from the book). Although *Flatland: The Movie* suffers a similar fate, the reason is more nuanced than time restrictions. Although this issue was not raised in the pilot study with fifth graders, eighth graders were quick to point out the problems with portraying a two-dimensional world for three-dimensional humans. The problems in translation most likely benefited the discourse, raising a lot of issues and allowing learners to address complex notions of dimensions. Although the book could describe elusive concepts (such as gravity and paint in the second dimension), the movie had to be "tweaked" and was actually 3D. In a conversation with Jack and Albus, both address these differences:

> **Albus**: The movie was less realistic I think.
> **Jack**: Is it, the movie had to show it, but the book could only talk about it theoretically, but the movie had to show it or else it wouldn't actually – it would just be a book.
> **Albus**: Yeah, and if things were constantly falling through space, I feel like the movie would have been less entertaining to watch. It would have been like nauseating. So I think that the book had the freedom to explain it without having to tweak anything.
> **Jack**: Or without actually having to show it. It did use diagrams though but it didn't really have to show it all.
> **Albus**: Yeah, it could just explain things that people – like it's easier to understand something when you're shown it, but I feel like it's harder to show something that's unshowable.
> **Keri**: That's true, yeah. And you had talked about that. Like *Flatland: The Movie* was almost 3-dimensional in the sense that you have this top-down perspective.
> **Jack**: Yeah, also there was different layers and stuff.
> **Keri:** Oh right, with the background
> **Albus**: Stuff was moving on top of the background and that doesn't make any sense. Like that would have to be moving too with them, like they'd have to be on the same plane cause they can't have multiple.
> **Jack**: Yeah

In the interview, I asked a follow-up question concerning the differences between the movie and book:

> **Keri:** You said that the reason that the movie was different than the book was because they had to sort of – I mean you had to be able of see it [**Jack:** Yeah], so they had to make these choices in developing the film. But, did you think it changed the concepts? You think the concepts were represented differently?
> **Albus**: I think slightly [**Jack:** yeah] but only, only in order to show it in the movie – I think because I mean they probably wanted to keep it the same concept but they wanted it to have characters and they wanted it to be simpler for probably younger kids to understand – to like give them a broader explanation of it for people who haven't really talked about perspective that much so like it was a good intro movie to watch – which is, I mean, that's how we used it in our class. And then we kind of used it to talk about and then as we got older, we used the book in the 8th grade and I think that the book is like kind of a secondary thing to go on to after *Flatland*, like to enhance your understanding of what would actually be going on in Flatland.
> **Keri**: Okay, so do you think that that order was helpful?
> **Jack**: Yeah definitely [**Albus:** Yeah] because in the movie there were a lot of little things that were wrong but you didn't really notice them until you actually dissected the movie. But the main thing was accurate and that led you into thinking about 2-dimensional space and 4-dimensional space. And the book actually helped you understand it.

Again, I invoke my strong pedagogical orientation to take notice of Jack and Albus's reflection on *Flatland* (the movie and book) and the way their relationship with it has changed from fifth to eighth grade. This is important concerning the events of the learning space and the way they perceived *Flatland* differently at different grades. The guiding theory for *Space and Perspective*, Cognitive Flexibility Theory and the associated cases as alternative perspective seek to "support learners in this type of differentiated noticing. Jack and Albus are able to see the complexities of a 2D plane and the misrepresentation of the plane because of a second variation of the case *Flatland*, the book.

Provokes. *Flatland* as a case garnered reaction from everyone who watched it and acted as a provocation tool. The case has qualities of a discrepant event (Posner, Strike, Hewson, & Gertzog, 1982) meant to cause a perturbation, or disruption thinking. It also acts similar to a pivotal case (Linn, 2008) in the sense that it supports learners' ability to make comparisons

between situations and draw on everyday experiences. In addition, *Flatland* is similar to a video anchor, but the characters are not solving problems as is typical with video anchors (Cognition and Technology Group at Vanderbilt, 1992; Zech et al., 1998). In this particular instance, it makes sense to view *Flatland* as a case of alternative perspective (Jonassen, 2011). *Flatland*, both the movie and book, are aimed at providing the reader or viewer with an alternative perspective, each in a variant way. This alternative perspective is foreign to a three-dimensional way of seeing and perceiving the world.

An interesting phenomenon that followed watching *Flatland* and that occurred during book discussions, included learners bringing their own ideas from everyday experience to try and reconcile these in *Flatland*. For example, Beck brought up the idea of digging during the interview:

> I remember the topic of the digging. I can actually see how – how it could make more sense now how if there's these big piles of dirt or matter that they could push and move around and make stuff with. Wouldn't that be cool if they had had that in the book?

Alistar expresses a similar sentiment concerning shadows:

> **Alistar:** I feel that in the story of *Flatland*, if they did discover that actually the shadow is not just part of the thing that is creating the shadow, I feel like that kind of would have been cool if there was some kind of discovery that was talked about in the book
> **Keri**: Oh you could put it in your game
> **Alistar**: Yeah, I can make extra levels and talk about those questions I have and make up my own answers.

What Alistar is suggesting here, is a modification or addition of shadows to the story of *Flatland*. One of the things Alistar talked about in his book group, was not knowing where light was coming from, if the surface of *Flatland* was part of something bigger (he suggests a cube), or where gravity might be centered. If a Flatlander did indeed decide to investigate shadows in the book, they would realize that something is causing shapes to have a shadow (a light source), indicating a higher dimension.

Rather than share the specific ways learners felt provoked to question, conjecture, extend ideas, etc., the particular feelings of elusiveness and impasse are detailed in sections that follow. In addition to *Flatland's* provocation, learners talk about other compelling cases. The five interviewed students chose variant cases/activities that usually connected to their personal interests. For example, Alistar connected most to the various perspectives created in video games. Lynn connected most to the photographing space activity, Jack to video game design, Beck to Cubist art, and Albus to *Flatland*. Their discussion of these cases will be integrated in the sections that follow.

Concerning the Elusiveness of Space (Mathematically, Bodily, and a Different Reality)

The idea of elusivity started showing up as a repeating theme in post-reflexion entries during the pilot study (2011). I first shared the story of a pivotal walk with my husband at a presentation describing the post-reflexion (a.k.a. post bridling) activity (Valentine & Gardner, 2013). I asked him to tell me his thoughts concerning the fourth dimension seeing as I talked about it all the time. His response startled me. "I don't even know what you mean when you say two-dimensional." He did not feel he could understand a dimension he doesn't have access to physically. I remember clearly his saying, "I can't touch something that has no height – where would it be?" This was the first moment I remember problematizing to the elusiveness of lower dimensions, or the fact that we as three-dimensional beings don't really have access to the second dimension (a height-less object would be invisible or more coordinate mapping directions than a thing). There is no point, line, or polygon available to our vision or touch.

The more I thought about the second dimension as elusive, I started to think about how we present the second dimension as a substructure of our 3D world to learners. In a similar way, this is how we talk about water – 2 hydrogen atoms for each oxygen atom. Yet, I experience

water, not the substructures of water. Sometimes, especially in mathematics, curriculum designers (including teachers) make the assumption that substructures would be easier for learners to grasp. A huge turning point for me was the interrogation of this position pedagogically. According to Cognitive Flexibility Theory (Spiro et al., 1987, 1988), misconceptions are compounded when complex concepts are reduced or simplified.

The idea of a thing (concept, idea, object, picture, etc.) being elusive makes space, perspective, and dimension worth investigating. This sort of characterizing of what it is to be elusive and the benefits of investigating something elusive motivates sustained inquiry. It begs for attention to see how other aspects of the world may be brought to bear. It is what draws me to theories of ill-structuredness, cognitive flexibility, complexity theory, and emergence – even post-intentional phenomenology. All these areas are not out to settle or finalize anything, but accept not being able to see all sides of something at the same time.

"Elusive" is defined as "hard to find or capture," "tending to evade grasp," "hard to comprehend or define" and "hard to isolate or identify" ("Elusive - Definition," 2014). Related words and synonyms that indicate the nuances of the term include: slippery, shifty, fleeting, impermanent, momentary, passing, transient, transitory, inaccessible, unapproachable, unattainable, unavailable, unobtainable, unreachable, and untouchable. These tentative ways in which elusivity manifests (temporally, physically, and cognitively) in the terms above is in line with the ways learners expressed this feeling in their own experiences. An example from the book *Flatland*, includes A. Square dreaming of travel to the zero and first dimension. During his travels he talks to the Monarch of Lineland where many ideas concerning elusive concepts arise for A. Square, the Monarch, and also for those reading the story. Words like "assuming, ignorant, what I called…, but what he called…, all was a blank to him; nay, not even a blank, for

a blank implies Space; say, rather, all was non-existent" (p. 44-45). This pattern relating varying perspectives of dimensional beings repeats throughout the dream and throughout the rest of the book. In order to bring all to bear on this particular manifestation, I started looking for some other perspective on elusiveness. Yi-Fu Tuan (1977), a human geographer, adds a valuable ontological perspective, writing:

> Above all, we are oriented. This is a fundamental source of confidence. We know where we are and we can find our way to the local drugstore. Striding down the path in complete confidence, we are shocked when we miss a step or when our body expects a step where none exists. Eventually what was strange town and unknown space becomes familiar place. Abstract space, lacking significance other then strangeness, becomes concrete place, filled with meaning. Much is learned but not through formal instruction. (p. 199)

If orientation creates a sense of confidence, then disorientation may be viewed as leaving one feeling unsure. In the case of reasoning spatially, this uncertainty may actually help one remain open to considering multiple perspectives. At least this is how it seemed to manifest for learners. Below, learner's express the various ways they found themselves encountering elusivity. In some cases, concepts become more familiar ("filled with meaning"). At other times, a concept remains beyond grasp. In this spectrum, there seems to be an acceptance over time. The elusive nature of concepts played a large part in learners' experience during the *Space and Perspective* project. Rather than view elusivity as something to avoid, learners talk about the various ways it was felt and moments where it shifts for them.

Jack and Albus discuss two types of elusiveness as it relates to perspective and dimension – one mathematical and one bodily. The first example addresses the mathematical elusiveness:

> **Keri**: Is there anything about perspective/dimension that you still wonder about?
> **Jack**: I guess, just how it works cause you can't really think of it at all.
> **Albus**: Yeah. It's like I have a conceptual understanding of it, but mathematically I feel like – I mean cause there isn't – I mean it's very advanced so I don't really – cause I haven't even covered 3D space that much and then if you talk about higher dimensions

> and then it's like 4-dimensional space. I really have no mathematical capability to understand how that moves.

Having "no mathematical capability to understand how that moves" – at least in a way that captures in a definite way is beyond Albus's capability. However, there is a way to mathematize the fourth dimension – it's not completely elusive mathematically. Still, this type of mathematics is advanced and even I feel naïve about this mathematization beyond patterns and progressions. Immediately following the discourse above is this variant view of a bodily elusiveness:

> **Jack**: Yeah, it doesn't actually make sense unless we actually see it and we can't actually see it
> **Albus**: Yeah, like 3D space – when you're talking about the x, y, z graphing stuff – I understand it more and I understand cause we've solved equations like that and we've done less graphing but we've done – we've solved things that are like that so it's where it's coming from – you know that you're going to have 3 roots to it and that's how you're going to configure it. So, we have a greater understanding of that than a 4^{th} dimension because the numbers – we haven't even talked about them at all.

Because Jack is unable to "see" a fourth dimension, he equates this with not making sense. It is at this point that Albus starts to bring the conversation back to the mathematical elusiveness. I decided to prod Jack and Albus to see how they felt learning about something without a definitive answer:

> **Jack**: It wasn't – I mean you can't think of it really, so there would have had to be things that doesn't make sense.
> **Albus:** Things that are impossible are fun to discuss. I think. Because it's like there's no right or wrong but there is convincing, so it's like – it's more fun – like you said some things you can look up, like you can Google when you're having an argument about something. It's different when it's impossible because there's nothing that someone can say that is just wrong, but you can disagree with it, which makes it more interesting because then it's all about opinion and it's all about knowledge of other concepts and how you're applying them to the situation that can't ever happen.

When ideas are "impossible," they are also "fun to discuss" according to Albus. Discussions and arguments for these two learners was "fun" in part because of the elusive nature of the ideas

being discussed. I don't believe personally that the fourth dimension is impossible, but certainly impossible to investigate (in a bodily way).

Lynn also added a similar sentiment about the fun nature of discussing elusive concepts, "I was pretty curious. Just like the whole concept of different dimensions and space and stuff is always really like boggled me. And I like thinking about it." However, later in the interview with Lynn, she talked about class conversations around non-definitive ideas in a different way:

> **Lynn** – I think it is definitely frustrating. Because I want the answer. I want to know what this it. But I think that I also kind of - at least for the infinite space [an idea she talks about earlier in the interview] I have kind of come to terms with the fact that there really isn't because that's what infinite is. There isn't an end – so there isn't an answer. And I think that is just something that I have accepted and there's not going to be an answer because infinite doesn't end; therefore there can never be an answer. So, I don't know. I've just kind of been okay, this is the way it is.
> **Keri** – Have you noticed ideas like that in other subjects? Like where there is not really an answer?
> **Lynn** – I think that like at least in math now that I am getting into like imaginary numbers and stuff that I don't know, like problems where there is no real solution - like there is just no solution. You know. We just got into those like negative square roots and stuff and a whole like 'i' thing.

In this particular excerpt, Lynn talks about the multiple feelings (frustration, coming to terms with/acceptance) when dealing with something that has no definite answer or where there is no end and no solution. She even connects this to current mathematical concepts of imaginary numbers and algebraic equations with no solutions. Lynn helps us understand what it is to engage with elusive concepts, or in her words, "something so out of reach…many complexities." She reacts in a similar way to Albus and Jack, coming to accept the ill-structured nature of space and dimension concepts. In addition, they both talk about enjoying these non-definitive ideas. Lynn also adds another emotion especially pertaining to dimensionality – that of being scared. To her it was scary that someone/thing could have a different reality than hers and perceive theirs as normal. I wonder if this is a characteristic of adolescents or something that runs across all age

groups of humans. In reflecting on this manifestation, it seems like researchers with their different views of validity and reliability may sometimes suffer a similar fear that someone's reality does not match their own in terms of what it means for something to be valid and reliable. In this next excerpt, Lynn adds a different dimension to elusiveness, that of differing realities:

> **Keri** – In your blog post reaction to *Flatland: The Movie*, you said, "The brief thought at the end of the movie about a 4D world made me feel similar to the 2D shapes. The concept is so abstract it is almost scary. However, it is very much possible that a 4th dimension does exist." How did you feel similar to Flatlanders?
> **Lynn** – Well just the concept of like something so out of reach and like not something that I feel that many or enough people are exposed to, it's like something that's just something so unknown to us that like it's kind of scary. It's like everything that I know is different to someone else. It's kind of like the concept that there is someone out there who has as many thoughts that I have. And have many complexities, or whatever to their life. It's kind of like there is another dimension or there could be another dimension out there that just experiences things so differently than me. But it's normal to them kind of. I don't know. It's hard to explain.

This particular part by Lynn seems to link scary to different or unknown - the idea that someone out there can have a different reality and that it can be normal for them. Really, this exists in our own shared third dimension. Part of growing up is realizing that we all actually view the world with different realities. Although this might be attributed to adolescence, this seems like something we do as humans throughout our lives. This line of conversation continued with an interview question concerning the elusiveness of the second dimension:

> **Keri** reading **Lynn [from class]**: we can't really like imagine – well obviously we can imagine what their perspective is like – but we can't really. I mean they don't see up or down. I know that I personally can't think of not being able to see up or down.
> **Lynn** – Exactly. Like, yes. I like – just like even to like I said being in the second dimension like I think of things like I'm putting my arm up and below and stuff and I just have this one like – not even left to right – because it's like I'm going left to right, right now, but that's in the third – I don't know. It's really crazy.

This idea of not being able to undo what you know was a new consideration for me as a pedagogue. Lynn brings up this idea that the second dimension is as elusive to her as the fourth dimension for the very reason that she can't, in a sense, forget what she knows or how she moves

in the third dimension. It brought me back to a warning from the author's preface of *Flatland*, the Second Edition (Abbott, 1992). He writes, "He begs his readers not to suppose that every minute detail in the daily life of Flatland must needs correspond to some other detail in Spaceland; and yet he hopes that taken as a whole, his work may prove suggestive as well as amusing" (p. ix-x). His work was definitely suggestive, but did in a sense ask us to forget what we know about our own dimension and see through a two-dimensional reality. Yet, this reality left us with more questions than answers. Lynn talks more about wanting to know about the ontological nature of the 4th dimension, but admits it's elusiveness will not be solved in answer form:

> I don't think the questions I have can be answered because my questions are like, "What would it be like being in the 4th dimension?" and "what does that feel like?" Do you feel like you are constantly moving or is that just the norm and you are like OK. Is there some other direction that exists? Like ewwww, what!
> **Keri** – Yes. And you had made the conjecture in class that if the direction from the 2nd to 3rd dimension is height, maybe the direction from the 3rd to 4th dimension is *in*?
> **Lynn** – Yes. Like because I feel like to us *in* is, I don't have anything around me, but like in is my finger is going into my t-shirt kind of. Or if I were to take a needle like the needle is like going into a sponge. Like we can't even like think about what in is to the 4th dimension - because I don't know.

The ontological nature of the *Space and Perspective* investigation is discussed in more detail following this section, but it is important to note that learners mainly saw dimensions as elusive when they imagined them bodily (in terms of seeing, perceiving, moving, hearing). For example, when I asked Lynn to explain the distinctions in a comment she made – "I understand *Flatland*, but I couldn't comprehend it – she responded:

> I think that there's a big difference between understanding something and comprehending it. I think understanding it is knowing what they are talking about and getting the gist of it. Comprehending is really getting it and really knowing in-depth about something. And I think the 4th dimension – because we don't live in the 4th dimension – we can't comprehend it. We can't kind of fully know what it is because we have never experienced it.

Lynn doesn't believe one can comprehend something that is experientially off limits. Although we conjectured as a class, viewed animations of rotating tesseracts, reasoned by analogy about properties of a hypercube, Lynn equates the elusiveness (bodily) of the fourth dimension with an inability to comprehend, which she interprets differently than understanding.

Impasse/Dissonance

The elusive nature of ideas and concept brought about impasses for learners, a type of productive struggle. Positioning impasse after elusiveness is intentional. It is the elusive character of a concept or idea that brings one to an impasse. Coming to an impasse in one's inquiry is akin to feeling stuck - like things are not quite fitting together in a way that makes sense (dissonance). This feeling has been described in many learning theories (e.g., cognitive perturbation by von Glasersfeld (1991) and Steffe (1991); cognitive disequilibrium by Piaget (1977); discrepant events in conceptual change theories (e.g., Posner et al., 1982)). This feeling can be traced back to Aristotle in his writings concerning Metaphysics (2002). For Aristotle, an impasse provides the entry point into inquiry. He writes, "it is profitable for those who want to get through something well to do a good job of going over the impasses…it is not possible to untie a knot one is ignorant of" (Aristotle, 2002, p. 35). I contend that the name indicating disequilibrium, perturbation, and impasse is less important than paying attention to the origins of these moments and how learners navigate them. It is important to create opportunities for productive struggle (National Council of Teachers of Mathematics, 2014) and cognitive demand (Stein, Engle, Smith, & Hughes, 2008). These are the moments where learning happens, where learners have an opportunity to notice where their current operations fall short and where shifts in perspective have an opportunity to emerge. To better understand what it is for a learner to have a shift in perspective, these moments of impasse help bridge the cases with their elusive

nature to discourse and subsequent shift. In this sense, the impasse allows an individual or group to make distinctions in the knot so as to "untie" it.

Before indicating particular impasses, their qualities, and how learners experience them, it is helpful to go even further back than Aristotle – to Plato's (2002) dialogue between Meno and Socrates. They are discussing the nature of virtue. Although Meno gives talks to the masses on virtue and considers himself an expert, Socrates starts to confuse Meno, or in Meno's words, "I think you are betwitching and beguiling me, simply putting me under a spell, so that I am quite perplexed" (p. 69-70). Later on Socrates responds:

> I myself do not have the answer when I cause perplexity in others. So now I do not know what virtue is; perhaps you knew before you contacted me, but now you are certainly like one who does not know. Nevertheless, I want to examine and seek together with you what it may be. (p. 70)

Meno then puts forward what Socrates labels "a debater's argument," which is:

> How will you look for it, Socrates, when you do not know at all what it is? How will you aim to search for something you do not know at all? If you should meet with it, how will you know that this is the thing that you did not know? (p. 70)

It's at this point that we are brought to an impasse about impasse. Socrates then replies:

> We must, therefore, not believe that debater's argument, for it would make us idle, and fainthearted men like to hear it, whereas my argument makes them energetic and keen on the search. I trust that this is true, and I want to inquire along with you into the nature of virtue. (p. 71)

What is worth noting however, is that this impasse (or aporia) allows Meno to admit his shortcomings concerning virtue – whereas before their talk he considered himself an expert on

the matter. In a similar vein, my pedagogical stance with learners sought to help them question their current view of the world in terms of space, perspective, and dimension.

The impasses arising during the *Space and Perspective* investigations include those related to problems inherent in viewing a 2D film, inability to escape one's body, and many other ideas related to complexities of ideas like partial and higher dimensions and animal seeing. Two data sources inform this particular manifestation more than others: blog postings and class discourse.

The following example concerns a twenty-minute conversation with both classes following a viewing of *Flatland: The Movie*. Table 4.7 summarizes the ten main impasses arising in the discourse. Although the nature of these impasses were important to the discussion, I will rather point out the patterns in Table 4.7. Of the ten impasses, the first three point to problems inherent in a 3D rendition of a 2D world. The rest of the impasses however, represent phenomena that learners don't have access to, such as the fourth dimension, animal seeing, and even the concept of a negative dimension.

Table 4.7.

Ten Main Impasses That Arose After Viewing the Movie, Flatland

Impasse	**Description**	**Example**
1. Implied thickness of Flatland	Flat objects moving over a planar background cannot happen if both part of the same plane. Problematic that we can view a film taking place on a 2D plane.	The cloudy background moves independent of Hex. Something with height would be invisible.
2. 2D characters performing 3D activities	Eating and dragging objects, 3D activities, don't make sense in *Flatland* film.	2D characters in film dragging objects
3. A. Square and his wife seeing and speaking through prison wall (line)	Is a line in 2D like a wall in 3D – can sound travel through it?	Class argues about the nature of a line in Flatland
4. 2D just as elusive as 4D space	Limited understanding of other dimensions not 3D	If the direction up describes 2D to 3D, what is the direction from 3D to 4D? How can you see 2D without height?
5. Gravity in 2D plane	Standing in 2D is hard to conceptualize	Gravity not the same in 2D
6. Are negative/partial dimensions possible?	If there are infinite positive dimensions, can it work in a negative direction	One student asked this question and the class tried to find examples, like the zero gravity ride.
7. Motion of 4D animations	Is motion part of the structure of a hypercube or 4D object?	Maybe time and space are made up constructs – more about perspective
8. 4D as time	Conjecturing that 4D is time	Learners have hear this concept in the past
9. What is it to see in 4D	Not able to see 4D	4D seeing may be like seeing all sides of a 3D object at once – maybe we see 2D and synthesize 3D
10. Animal seeing	Can we really know what animals see?	Class discussing animal seeing and human's seeing color

**Impasse 7 and 9 start to combine towards end of discourse*

Although this example only shows the impasses relating to *Flatland*, many other impasses arose for learners connected to the elusivity of cases, ideas that propelled them to talk, etc.

Concerning the Nature of Discourse

Discourse played a central role not only as a data source, but as a manifestation of the *Space and Perspective* experience for learners. It's through discourse that we philosophize, problem solve, negotiate meaning – even if in writing to ourselves. Discourse was not only the means of confronting impasses, but also the vehicle for shift. A pattern emerged where discourse arose naturally, usually following a case that presented some aspect of elusiveness and subsequent impasse to work through. Debate was common, and although little was settled, much was brought to bear on the matter of space, perspective, and dimension epistemologically, ontologically, and cosmologically.

All the learners interviewed gave reasons for sharing ideas and joining in debates/arguments in class. These reasons are varied and multiple even for the same person. The three that resonated most include: (1) strong emotions related to discussing ill-structured (elusive) concepts, ranging from enjoyment to frustration, (2) validation when able to convince others of one's way of thinking about an idea, and (3) the benefit to one's self and others from access to multiple perspectives, sometimes leading to shift in one's own point of view.

Emotions related to ill-structured concepts. The ill-structured, elusive nature of concepts from cases like *Flatland* provided the stimulus for classroom discourse and debate as described in the previous section. For students, these conversations were either frustrating, satisfying and enjoyable, or a combination of emotions.

Frustration was described as manifesting from conflicting points of view and discourse about an elusive concept. For example, talking about dimension and perspective (both of these especially) is frustrating for Albus and Jack because of the elusive nature of these ideas. They found it frustrating because of the differing perspectives. When everyone thinks about something

differently, it's harder to explain your point of view. Frustration also arose because the second and fourth dimensions are elusive (can't be seen) and when discussing the ideas with others, it is common to picture them differently, making it hard to talk about. In the transcript below, Albus and Jack talk about this aspect of discourse when asked what it was like to engage with other people talking about space, perspective, and dimension:

> **Jack**: It was kind of frustrating in a lot of points because [**Albus:** yeah] people don't really have, share the same perspectives on perspective – everyone thinks of it really differently, so it's very hard to explain what you're trying to say.
> **Albus**: Yeah, especially when you're talking in terms of like a 2-dimensional or a 4-dimensional world because you can't really – since you have no way of seeing it yourself, you don't – it's hard to kind of portray what you're trying to think of on to someone else. So it's just kind of frustrating, like Jack said. Just like talking to another person about 2-dimensional space is really difficult because you're both picturing it in very different ways. And since we don't really know what it's like very much, it's hard to talk about.

For Albus and Jack, this frustration was alleviated when common cases, such as *Flatland*, became the topic of conversation, or a discursive tool. Albus shares:

> I think that watching *Flatland*, put us all on the same level cause that kind of helped us all think of it in the same terms because we all pictured it similarly, because when we were talking about 2-dimensional space, I think that we all kind of thought of those characters and we all kind of pictured it the same way as that was and I think that that made it a lot easier.

Flatland created homogeneous visuals, for example, helping picture characters as Flatlanders. This common ground lessened the elusively and in turn created a productive conversation. In a sense, *Flatland* as a case provided learners with a similar experience.

When Lynn was asked what that process of conjecturing with her classmates was like, her response was:

> I like – I think that is one of the things I liked about the class is that we kind of – just like experimenting with all the 'what ifs' of dimensions and like I don't know I just really like 'what ifs' and I feel like that is what I do a lot all the time is like I – well this is the scenario that's going to happen, but what if this happened? Like that's just something that I do. And I like it. Yeah, I mean it is not always a good quality because I am like what if I go skiing and I die but in terms of math and science I definitely like it.

The emotions of frustration, freedom to wonder, and enjoyment continue to be expressed in the next section, but are more indicative of discourse aimed at validation.

Validation. When learners talk about their reasons for engaging in discourse, they talk about seeking validation for their way of thinking and the joy of convincing others (especially when there is little or no evidence). The following excerpt below concerns why Albus and Jack want people to think the same way as them. It's less about the moments of discourse and more about what motivates them to share/argue about ideas. They talk about 3 reasons: (1) validation for their way of thinking, (2) convincing others without evidence – feels good winning, not about right [like a game/play], and (3) helping others and in turn yourself to understand. Emotionally, they express discourse around ill-structured concepts as "frustrating" and also "more satisfying":

> **Keri:** you said it's kind of hard to talk about your perspective when everybody has a sort of different perspective. [**Both:** agree] Talk about that, because you guys didn't give up - why do you feel like you kept trying anyways?
> **Jack**: Well I don't know – I felt very strongly kind of about how other people should perceive it. I wanted other people to perceive it how I thought of it and I think that's what kind of made me keep going
> **Albus**: Yeah, I think that's just kind of – a characteristic that both of us share that we want people to think of it the same way that we do cause it makes it easier [for/to live our lives – can't decipher] knowing that other people are thinking the same way
> **Jack**: We don't like other people not liking how our ideas work
> **Albus**: Yeah

They both have strong feelings about how others should perceive space, dimension, and perspective. This seemed to motivate their persistence - wanting others to accept a particular point of view. In these next two passages, Albus continues talking in more depth about the satisfying feeling of persuading people, especially concerning elusive or conceptual ideas:

> I like more conceptual conversations cause like there's no proving it – it's more frustrating but it's more satisfying when you make a point. I think it's more like just me wanting other people to think the same way cause I think that made it easier on me, to like be able to think about myself as being right. But when other people - but like when other people think differently than you, there's kind of always this lingering thought in

> the back of your mind that you're wrong about it, which isn't like always fun. So, it's nice to like– it's so satisfying when someone else thinks the same way as you do – about something. Especially something that can't be thought of in like – something that can be seen by many perspectives.

This particular passage comes across very powerful and raw to me. Wanting others to see your perspective is such a common emotion we probably all share (or have at multiple points in our life). If others don't share your point of view, there's this "lingering thought in the back of your mind that you're wrong about it." Albus and Jack talk more about the ways they avoid being wrong:

> **Keri**: You guys had a lot of classmates that were willing to throw themselves out there – put ideas out there or be wrong in a way…[**Albus:** Maybe] You hate being wrong?
> **Albus**: I don't think I was one of those people. Um, I definitely throw ideas out there, but like sometimes I try to use big words so that other people won't correct me, like for fear of them being wrong themselves. [**Jack:** Yeah] I do that often. So it probably wasn't actually me being brave.
> **Jack**: Sometimes if there's a chance that I'm going to be wrong, I don't say it cause I don't want to be wrong, it's easier to wait. [**Albus:** yeah, or like I wait for someone else to say it and then I piggyback] yeah
> **Albus**: Yeah, I'm not a very brave conversation-haver, I don't think
> **Keri:** Really?
> **Albus**: Yeah, I mean, in class – yeah, like between us Keri – no sometimes I'll make up words to make my point [laughing] like sometimes I'll use adjectives that don't really exist or don't apply at all to the situation in order to make it sound smarter so that other people don't question it and they're just like yeah, yeah – and then I'm like, "Yes!"
> **[Keri:** laughs] And I'll just use – I think I use um, I use carcinogenic a lot to explain things in science that don't [**Jack:** Yeah]– that are not carcinogenic, but it sound like
> **Keri**: Carcinogenic – that sounds like cancerous or something
> **Albus**: Yeah, it is – but a lot of people don't know – but, a lot of people know that word because they hear it a lot in terms of like food, like and in terms of technology – like oh yeah, microwaves are carcinogenic, but you don't actually know what it means. Like I know what it means because my mom is like, "Oh, you can't get that – it's carcinogenic" – "You can't use that soap, it's carcinogenic" but like, a lot of people don't know, so I use words that people like know exists but they don't know what they mean.
> **Jack**: Yeah, but they're used to hearing it with smart things
> **Albus**: Yeah [laughs]

If being a "brave conversation-haver" is putting oneself in a position that may be realized to be wrong, does seeking validation in discourse (such as Jack and Albus describe), indicate something apart from brave – maybe fearful? This particular passage makes me laugh (and saddens me) each time I read it. I would never have perceived either of these students being afraid of being wrong. From my perspective, they were always "chucking out ideas." Still, I was happy to see them talk this openly during the interview. When prodding them further this transpired:

> **Keri**: But you guys are really – I mean, if someone else threw an idea out there, you guys felt comfortable pointing out their flaws…[**Both:** Yeah]
> **Albus**: No, I definitely – sometimes I can be brave and argue it and just like completely disregard it, especially if I'm trying to like egg someone on, which is something that I do a lot. Especially with our new math teacher – like I've been like very brave with arguing with her, like I'll just – like I know she knows more about math than I do but I try to – put it out there anyway.

It seems that the fear of being wrong does not apply to all situations and is highly contextual.

As this particular line of conversation continued, Albus and Jack start to elucidate another reason for engaging in discourse with classmates – that of convincing others. In this first excerpt, they talk about the special case of convincing someone without evidence:

> **Jack**: It's nice having the thought that you made someone think something even with no evidence [**Albus:** Yeah] – just your own thoughts.
> **Albus**: Yeah, cause it's like that all came from you. [**Jack:** Yeah]

In the passage that follows, Albus and Jack talk more about what it is to argue. Not as much about coming to a conclusion or trying to increase understanding of the group, but of taking multiple other sides for different reasons. Even in this transcript, the end comes back to this idea of wanting to be right:

> **Albus**: It always depends on the person that you're arguing with, like sometimes I'll take a different side, like, I'm sorry I keep going back to this but– cause it just happened like we were arguing about cell phones [**Jack:** yeah] I feel like if I was having that same argument – like I was saying that they were harmful and Jack was saying that he believed

> our science teacher that they weren't – or that there wasn't enough radiation – the radiation was harmful but it wasn't enough to actually do anything to our cells. Which is like, it could be considered true but we don't really know. So um, it's, so like – but I feel like that if I was having that same argument with say my parents and they were telling me that my cell phone was dangerous for me, I'd probably take the other side. And that's just because like I'm trying to prove 2 different things in there. Like with my argument with Jack, I was trying to prove that our science teacher doesn't know everything because I don't really believe him with a lot of stuff. Like with my parents, I would be trying to prove that they worry too much – that they're too protective of me and that I know what I'm doing with a cell phone and everything.
> **Jack**: Also, I think when I take arguments, I don't really – as long as I kind of try to take all the sides so even if I'm wrong I'm still kind of right
> **Albus**: Yeah, you do that a lot
> **Jack:** Like I was saying – I was saying no one knows [**Albus:** Yeah, yeah, yeah] cause I knew, I didn't really – I didn't know, but I didn't really think that we'd come to a conclusion.
> **Albus:** Yeah, no, yeah, I definitely know that you do that, yeah
>
> **Keri**: So what I'm hearing you guys say then is that you both like have the qualities of people that like to argue and you're not – it's not just about being right then.
> **Albus**: No, it's not always [**Jack:** yeah] just about being right. I really like to argue.
> **Jack:** I don't even really like to argue. Sometimes, I just try to say something because I think it's right and then I just get sucked into an argument and then I'm like, all right, I might as well keep trying. I can't back out of an argument once I'm in an argument.
> **Albus:** Yeah, especially if it means you're going to look like you were wrong all along. [**Jack:** Yeah] Like I never give up on an argument – like I might get out of an argument if it means that I can just walk away, but if it looks like I was wrong, I can't just like leave – I have to like finish it.

Although they admit that being right isn't their only motivation, this discourse also ends with an aversion to seeming wrong – "if it looks like I was wrong, I can't just leave." Most people have probably felt this way at some point. All one has to do is look towards religion, politics, Facebook debates, etc. to see that some arguments are had more for the sake of proving oneself correct than to advance one's own ideas or those of others. I wonder how many discourse studies in K-12 classrooms have opened up this nuance of learners' intention when joining in an argument, especially with adolescents. I am unable to find a study, but see this as a potential area

that may inform the way teachers consider orchestrating class conversations. Maybe it is as simple as talking about the purposes of discourse and argument when establishing norms for learning as a community.

Benefit to self and others. All of the learners interviewed talked most about discourse as beneficial – helping themselves and others in turn. Sometimes it would shift their own perspective or that of classmates. In the following passage, there is the recognition that others contribute to understanding. At first these two students seemed very egocentric (convincing others to make themselves feel right), but here you see they value peers that will throw ideas and challenge them in order to leave with a better idea:

> **Keri**: Do you guys feel like you convinced anybody of your ideas?
> **Albus**: Um, I don't really remember specifically [**Jack:** yeah] but I think that I did. I kind of remember being satisfied after one of those classes – satisfied and frustrated – I kind of remember talking about *Flatland* I really helped other people understand it, which helped me understand it too, which I liked…My class was full of big personalities, so a lot of people were kind of yelling and stuff I think. Like I think Kim was probably against what I was saying cause she usually is. I kind of remember being able to talk to Lynn – so I think that, I think that we kind of helped each other explain it and I remember liking that – that was satisfying.

In this passage, Albus talks about a specific discussion in class about *Flatland* and remembers it helping him and others. Below, Beck explains what the class was like from his perspective. He says rather than passive, he felt "chucking stuff out" was beneficial in a contributory way. "It is better than like just sitting at your desk and copying down stuff. Actually contributing ideas to the class instead of memorizing stuff. When you start contributing stuff you obviously have a better understanding of it." It may not be that contributing ideas means you "obviously have a better understanding," but I believe it say's something about one's inquisitive nature (and the desire to come to understanding).

A great example of "chucking stuff out" occurred several times throughout the class as learners were discussing the book, *Flatland*. This particular excerpt from the book conversation shows the way discourse may progress a groups' understanding. They are discussing aspects they like and dislike about the book and Bilbo states that he didn't like the start of the book:

Bilbo: Yeah, I didn't like how he started off the book actually.
Alistar: I thought it was cool – it was like a guidebook
Bilbo: Yeah, yeah, that's what I didn't like about it – how it was kind of like a tour. Um,
Alistar: For humans
Bilbo: Right, like bringing you through it. I didn't really like that. When it started off and I read the first few chapters, I was like – oh this is kind of cool – then I thought about it
Alistar: Then it kept on going on – it kept going, it kept going
Bilbo: Kind of – like it didn't stop and I didn't like that about it.
Alistar: Although I think it is interesting that the guidebook part really does reflect that society
Bilbo: Yeah, I did, I thought it was interesting that, you know, he had to, that he wanted to write a book and he did it in that way and he chose to do it in that way – I just didn't like it
Alistar: Hey **Keri**, how old is the book *Flatland*, is it old?
Keri: The second edition came out in 1884
Alistar: Okay
Bilbo: Really
Alistar: See yeah,
Kim: It's that old! WOW!
Bilbo: Okay, so, then you're definitely right (to **Alistar**)
Alistar: That means that we're on to something
Bilbo: Definitely right
Keri: Victorian era
Alistar: Definitely out there – religion was much bigger back then – religion was a big factor in that time
Kim: And I think race was too
Bilbo: Yeah
Alistar: Right, so I think Flatland, the polygons, the triangles
Kim: And that also explains why they would be, as **Keri** says, shapist – like cause that was kind of what was going on
Bilbo: Yeah – so it all connects to the book
Alistar: Yep – it's all connected – the whole world's connected to it
Kim: I think…
Bilbo: Just not literally, I love how the author did that though. It wasn't very literal – you kind of had to think about it and connect to it, which is what I did so

This particular excerpt shows quite a few phenomena, but was included to show the nature of young learners helping a group and themselves through discourse. Alistar talks about the benefit of others in helping bring about "a Eureka moment" and specifically refers to the book conversation above:

> **Keri:** Can you talk about that experience of conjecturing about the book [*Flatland*]? **Alistar**: When me and Bilbo finally got together and talked about, there was stuff he realized I didn't think about and stuff I realized he didn't think about. And kind of putting it altogether made that experience in that one time – it kind of – it was a Eureka moment. Putting all the things that we both know about the book so far together to make a bigger picture.

For Alistar, a Eureka moment is operationalized as realizing something that you didn't know or consider prior to the moment. He talks more about this moment throughout the interview as a type of shift. This shift will be detailed in the next section.

When we talk with others using "an open attitude," as Socrates talks to Meno about the nature of virtue, we learn about the parts we didn't consider until that moment (Plato, 2002). Additionally, we contribute our ideas for others to in turn consider as alternative points of view (or confirm an already existing notion). A discourse in this manner benefits both parties (and possibly the silent ones like June benefit as well).

This next group of excerpts still relate to the idea that discourse is perceived as helping yourself and others. These share the common feature of giving/receiving multiple perspectives. The following excerpt from Albus's lived-experience description in conjunction with a follow-up interview, shows the way he perceived these multiple perspectives as helpful:

> **Albus's Lived-Experience Description:** This experience was interesting. Reading and watching *Flatland* brought me back to the fifth grade, which was fun. There was so much more for my brain to comprehend because I was older. When we discussed *Flatland* in groups, I felt that my brain was actually getting bigger.
> **Keri:** you said when we were having the class discussions - You said you could feel your brain getting bigger. **Albus:** [laughs] So I just wondered if you could talk about what it feels like for your brain to get bigger?

> **Albus**: Yeah, I mean obviously like I meant less in like a physical sense and more in like an actual like thinking sense.
> **Keri**: Sure
> **Albus**: It kind of feels like someone says something and it like unlocks this door to like a whole other room in your brain – this whole other concept that you couldn't even like fathom before that because what they said triggered it, so I think that's what happened in that class a lot – there were a lot of those moments, especially when it was you and Kevin teaching together because you both kind of would – like maybe Kevin would explain something and then you would say something about it and I was like, "Oh! Okay, now I understand the whole thing" and – like that's how a lot of teachers work weirdly like where they'll say like, like um – our math teacher now will explain something and take like an hour to explain the whole topic and then someone will be like, "I didn't understand it, can you explain it to me again" and she'll take a sentence to explain it and they'll be like okay, well now I get the whole thing and that was all a waste of time. Like some people use that differently, but I think that it worked a lot better last year because we had 2 teachers and we had the ability to like have 2 different perspectives even in the teachers [can't decipher – faces, ?] which is why I think that 2 teachers works better because it just makes more sense because one will see it a different way than the other.

Albus talks about the benefit of multiple perspectives – be it from multiple teachers, classmates, even the various cases. These various points of view for Albus, "unlocks this door" to a whole other room in his brain. Because Albus was interviewed with Jack, Jack adds to this particular conversation.

> **Jack**: Yeah, it's all about like the different perspectives and seeing different ways [**Albus:** Yeah] of thinking about a certain thing [**Albus:** Yeah], even if you both kind of figure the same thing, it's slightly different and that helps you understand it [**Albus:** Yeah] helps you see all the different sides of the situation.
> **Albus**: Yeah, because like if you don't understand something and then someone says it a different way, even though you might not understand the way they said it either, you're kind of like, by the common denominator, you're like, "Okay, so that can't be it, that can't be it, maybe its this"
> **Jack**: Yeah, if you see that they're both saying one thing, it's probably right. And if they're both saying different things, they're probably not right.
> **Albus**: Yeah [**Jack:** Yeah]

Although I find myself laughing and slightly disagreeing with Albus and Jack's conclusion concerning agreement or disagreement indicating "truth" – it is interesting that various perspectives are again brought up as a way towards understanding. They value various

perspectives regardless of if they are deemed correct or incorrect. This is actually in line with Cognitive Flexibility Theory and the cases as alternative perspectives, which advocates for multiple perspectives to develop a more flexible concepts.

So far we have seen the various manifestations of discourse from the learners' perspectives: (1) strong emotions related to discussing ill-structured (elusive) concepts, ranging from enjoyment to frustration, (2) validation when able to convince others of one's way of thinking about an idea, and (3) the benefit to one's self and others from multiple perspectives, sometimes leading to shift in one's own point of view. But what do we value as pedagogues in terms of classroom discourse. In particular, what does the literature indicate in terms of designing for argument/debate as part of productive learning? According to the literature, Kuhn (1991) has explicated five skills (qualities) of a strong argument that have been used by others in empirical studies (Jonassen & Kim, 2009, p. 441). These include (1) the ability to generate causal theories to support claims (supportive theory), (2) the ability to provide evidence to support these theories (evidence), (3) the ability to generate alternative theories (alternative theory), (4) the ability to come up with counterarguments (counterargument), and (5) the ability to rebut alternative theories (rebuttal).

The excerpt below illustrates two students argumentative disposition. They both want to argue and have strong feelings about how others can and should engage in argument:

> **Albus**: I like arguing a lot. [**Jack**: Yeah] I think I bring that into all of my classes kind of, except like sometimes – it depends on the teacher too – like sometimes I'll be in a class with a teacher like Mandy, last semester was our English teacher and I didn't really like arguing with her because it wasn't really satisfying for me because it kind of felt like, I don't know [**Jack:** Yeah]
> **Jack**: Like she just kind of like accepted it and then just sat there
> **Albus**: Yeah, and having like a really fun time kind of like – not like fighting with her, but like kind of disagreeing with and of - hashing it out with our geometry teacher just kind of like – because I think of it as like 2 minds kind of working together to like find the answer to something in like an argumentative way, which is a way that I prefer to do

> it [**Jack:** yeah] because – that's just the way my mind works is like I'm trying to win at knowing something, but um, I mean I don't know
> **Keri:** Well and that's a very like mathematical and scientific way of thinking – getting an idea out there until someone says, no and I can prove why that can't work
> **Albus**: Yeah, it's like ideas are just like any other commodity – it's just like you're fighting over them. Like and you want to get your stake in [can't decipher] ideas. [**Jack:** Yeah]

For Albus and Jack, they did not find arguing with their English teacher satisfying because she didn't join in. I can imagine this is like talking to a wall or trying to engage in improv according to Tina Fey's recommended *and*, where your partner never joins in. Arguing with oneself is not really going to work out just as arguing with someone who "accepts it" or "sits there" is not an argument. Most salient for me from this passage is "ideas as commodity" – something to fight over.

Final thoughts. One last aspect of discourse occurred outside of the learning environment, in the interviews and with myself as post-reflexion entries. During the actual interview process, both Albus and Jack continued the conjecturing process that was common in class. At first I was worried, but started to see these feelings differently as indicated in a post-reflexion entry from February, 2014:

> I'm feeling a little worried about this interview at this point because more than an interview to figure out the phenomenon, the phenomenon is taking place. It's like conducting an interview on smoking cessation as someone's quitting smoking. Anyhow, the more pages I flipped that have this sense of "conjecture, mathematizing," I'm realizing that I am a teacher and these were my students and this was/is our relationship. We debate, argue, conjecture, problematize the world for fun together every time we've had the chance – what would make the interview different? The beginning almost alludes to this type of "way and mean" they like to engage. Rather then dismiss this as a bad or failed interview, I need to realize that they are showing me the phenomenon the way they know how – through debate and conjecture.

This particular entry was written following the transcription of the passage below. In this excerpt, Albus and Jack are talking about a "major shift" that occurred (and still seems to be occurring during this moment). The reason for including this particular excerpt here and not with

the shift manifestation, is that this is in fact discourse happening in the moment, showing the shift manifest:

> **Albus**: Yeah, and like [can't decipher] would cite work in the movie, they could see things but I don't really understand like cause seeing things I guess if you have one eye – you could see 2-dimensional space right? But, then – I mean we can't cause it's all – everything's moving all the time like everything is always do we can't really see in 2-dimensional space if we just shut one eye
> **Jack:** But you could also argue that that is 2 dimensions and with 2 eyes that's 3 dimensions. And then if you have like 3 eyes that would be 4 dimensions and you would actually just see the 4^{th} dimension. You would just see another perspective of the object.
> **Albus:** That's true
> **Keri**: I didn't think about that. 3 eyes. Well let's think about this, so a spider has what, 7 eyes, 8 eyes?
> **Albus**: 6 or 8 eyes
> **Keri**: Yeah
> **Albus:** 6, I don't know
> **Keri:** So do you think they see higher dimensions?
> **Jack**: I don't know
> **Albus**: Yeah, they could – I mean that makes
> **Jack:** If they have an object, like right in front of them, they could see all of the sides of the object
> **Albus:** Yeah
> **Keri**: Yeah, it would definitely be a different image they'd get, right?
> **Jack**: I mean it must be just - like that part of the brain that figures out where things are must be higher, highly developed.

Of the Nature of Shift

It may appear odd to have a tentative manifestation section for shift, when shift is the phenomenon under investigation. Shifts are brought about by the impasse, the discourse, etc., but they manifest in many complex ways, not just in relation to triggers. Choosing to investigate the phenomenon of shift and change is rooted in a belief that learning in a deep and connected way manifests as a shift or change in perspective. Learning in this sense is an increased ability to notice and make distinctions, to connect prior experiences, and to remain open and aware of a constant state of incomplete knowing. This section is about shift, change, adaptation, and making distinctions, or problematizing, in the world – all actions that encompass the activity of learning.

During the *Space and Perspective* project, learners were confronted with cases that intended to expose them to alternative viewpoints and perspectives about their world, their senses, and their space. Inciting a shift or change was intentional and was done by introducing learners to complexity and ill-structuredness early on. As part of this inciting activity, learners were asked to make distinctions between what they see and perceive, what is made clearer and at the same time more hidden (such as looking in a microscope), distinguishing between capturing and representing space, and even considering impossible and invisible spaces. According to Aristotle (2002):

> All human beings by nature stretch themselves out toward knowing. A sign of this is our love of the senses; for even apart from their use, they are loved on their own account, and above all the rest, the one through the eyes. For not only in order that we might act, but even when we are not going to act at all, we prefer seeing, one might say, as against everything else. And the cause is that, among the senses, this one most of all makes us discover things, and makes evident many differences. (p. 1)

In the *Space and Perspective* project, learners senses, especially that of sight, was problematized as was the space their eyes sought to capture. In this "problematizing" activity, learners would frequently experience shifts, ontologically, epistemologically, and cosmologically. In this section, I will show the ways shifts relate to Aristotle's (2002) notion of distinguishing, while also drawing on more current theories of learning from embodied cognition where the notion of adaptation and making distinctions are central (e.g., Gibson, 1986; Reyes & Zarama, 1998).

The shift in one's perspective as the phenomenon under investigation seeks to understand the ways learners live out their shifts in perspective. So far, a shift in the *Space and Perspective* context has been shown to originate from engagement with cases, such as *Flatland*, video games,

etc. Cases that convey elusive qualities, such as the second dimension, impossible perspectives, and infinity become fodder for discourse with others. It's through this discourse that learners at times experience a shift in perspective. Before sharing learners experiences, it may help to operationalize shift as relating to making distinctions (Reyes & Zarama, 1998) in one's environment or even a process of adaptation from the ecological perspective (Gibson, 1986). When conceiving the study, a shift was operationalized as a movement from a natural/normal seeing to a phenomenological/meta/open seeing. The phenomenon of a changing spatial perspective most likely inhabits all humans (and even animals), not only in places, but also across time. The simple motion of stepping a foot to the side necessarily changes one's perspective – their distance and point of view in relation to objects shift. As humans age, they grow taller and relate to the world differently. The perspective of a chair and even the way one sits down changes over time. It seems that every moment is a change in perspective, even if it is mostly unconscious and not reflected upon. In this study, the particular focus sought to understand moments when learners' perspective shifted suddenly, or could at least be compared to an earlier, different perspective. The shifts described in this section seem to have been brought on by cases as alternative perspective from the *Space and Perspective* hypermedia site.

Learners talk most about their perspective and shifts in perspective as part of the interview, class discussions, and their blog postings. In their talk, they focus on three main parts of a shift: (1) contributing factors to the shift, (2) the lived-quality of the shift, and (3) perceived consequences of having a shift. All learners discuss a particular shift. In addition, some learners talk about multiple shifts.

Contributing factors. Learners talk about three main contributing factors for bringing about a shift. First, the classroom discourse exposed learners to multiple points of view (as

discussed previously). Second, cases used in the project caused perturbation/impasse in the environment. Third, class activities are credited for contributing to their shift, such as making bubble and straw models of hyper shapes and photographing space.

Discourse. Although discourse as a contributing factor and tentative manifestation of experiencing a shift was discussed earlier, one more example that shows a strong connection between discourse and shift adds meaning to the way these relate. One way as pedagogues that we're able to tell that someone learned something is when they shift the way they talk about, write about, or enact an idea/concept. This section talks in more depth about the particulars of shift for learners, and the following string of passages below will provide that transition, showing the way discourse in particular brings about shifts from the learners point of view.

In the interview with Alistar, he talked about debates, conversations, and arguments contributing to a shift in the way he conceptualized dimension. After a particular class session (inserted below) his inquiry continued at home where he looked up, for example, a rotating tesseract. When a concept comes up in class that he doesn't understand or had never heard about, he found himself propelled to learn about it after the conversation (at home). He didn't really talk about the nature of these arguments, debates, and conversations, just that they helped bring about shift. Following is a shift that Alistar experiences related to hearing about a fourth dimension for the first time. At first he questions seeing in three dimensions and later sought out animations of higher dimensional shapes. This section ends with him taking about changes to his concept map concerning dimension:

> **First time hearing about the fourth dimension**
> **Corey:** Four-dimensional I remember is something to do with like inside.
> **Albus:** It goes, it's like, it has like a cube inside of it too. It goes in and out of itself – so the little cube…
> ***Alistar:*** *Do we see in three dimensions?*
> **Keri (to Alistar):** Do you think we see in three dimensions?

Corey: Well, I can see **Dante** in three dimensions. [**Alistar:** Yeah]
Keri: You can, can you see the back – the other side of him. [**Alistar** and **Corey**: No]
Jason: So 4D is like see through, like a computer – you can't see the back of a computer (something about 3D – if its 3D). But if it was see through, I could see through it to the other side.

March 1, 2013: Whole Class Conversation
Bilbo: Are there negative dimensions?
Keri: Um – good question. What do you guys think?
Albus: Wouldn't that be kind of like a shadow? Cause it's like negative shape, but it would be negative shape.
Keri: Well, I think before we could answer that question, "Is there a negative dimension?" I think one of the kind cool things to think about is, you know, are there partial dimensions? I mean, what is a dimension? If you have a negative one, what is that telling us?
Albus: I mean there can't be really like partial dimensions, because as soon as a second dimensional thing has any height at all, it becomes three-dimensional. It's not like there's like a certain height it needs – it's like any height at all. So it's not like there's any leeway there.
Keri: I'm going to go to Adrian and then you because he's had his hand up forever.
Jerome: Well, from fourth dimension, if as it shows in *Flatland*, so if a cube's in a cube that's moving [**Keri:** okay] then wouldn't maybe our perception of the fourth dimension be say we would be looking at something yet we'd be able to see something else inside moving at the same time. Say we were in here and then all the sudden the room starts moving around, and we see other things that weren't here before.
Keri: Hum, I think movement's going to be a key to help us understand.
Ian: Well, I mean there probably is some kind of negative dimension or something like that, because there kind of has to be like a law of nature, there's yin and yang. So there's going to be another like side or something. And also since there are a number - because dimensions are numbers or they are like 1, 2, 3, 4, 5 – they also – there's an infinite amount of them, so
…(a few minutes later)
Alistar: I found this animation – I'm not really sure what dimension, whether it's all dimensions, or if it's the third, or if this is…
Keri: Oh yes, [**Jason:** Oh, I saw that] um, that would be like probably fifth or sixth – it depends on what shape it is, but yes…actually you can – you guys, and I encourage you do this, you can go online and, and look at YouTube animations of different dimensions.

Alistar talks about this moment above in an interview about his "revelation":

Um, so I don't think actually at first I really caught on in the class, like cause I always thought that this was kind of a Hollywood thing. Cause in like all, you know, the blockbuster sci-fi movies – dimensions to me, never actually meant 2D/3D – it meant like other universes or parallel universes – that's kind of what I was always taught to think, like through movies. And at first I didn't get it cause that's what I thought that really was.

> So I think kind of the revelation when I, during some of these like, um, you know debates and conversations we had when I realized that that's actually not really what it's all about - helped me realized that in fact, dimensions are not exactly the same thing as like other universes. I mean they are because – but also at the same time they are more than just that.

Alistar was among many students who talked about outer space and science fiction notions of space and dimension He attributed class debates and conversations as being central to helping him realize that there was more to dimension than the "Hollywood" science fiction movie notions. He continues to talk more about this moment:

> **Keri:** So can you talk about a key moment when you realized something was shifting about your perspective?
> **Alistar**: Oh yeah, so I think the moment I realized this was – I think it was a conversation between Jason, Neal, and Ian. I don't remember exactly what they were arguing about, but then at one point, um, Neal had hinted at this tesseract thing. I didn't know what it was. I went home, looked up what this was, then I was like whoa! And then I was like, well where would this ever appear in our world and that's kind of when I realized well maybe - well there goes more beyond our world. There must be more.
> **Keri**: Ah, and then I remember because your blog post – you posted that – it was a rotating tesseract, but I think it was 5 or 6 dimensions.
> **Alistar**: It was like this cube thing where it would, side of it would flip and it would be this never ending cube inside of a cube inside of a cube. It was pretty cool and strange. I want to find that thing again actually. It's kind of a cool animation.

Throughout the interview, Alistar describes a shifting perspective in this way – the realization that there is more – that something may be hidden from our world. For Alistar, the elusive nature of a new idea or thing provokes him to find out more, in this case, search YouTube for animations. This is the second time discourse supporting multiple perspectives manifests in this study. Investigating discourse further may be helpful for future investigations

Cases that cause perturbation/impasse in environment. Learners talk about specific cases (e.g., *Flatland*, Top 100 Video Games) from class that caused a perturbation, or some sort of discrepancy in their current way of conceiving of space, perspective, and dimension. Earlier, *Flatland* as a special case was discussed as one that provides food for fodder, provokes and

creates feelings of impasse, and a case that helps learners realize the elusive nature of dimension. Other cases are similar in this way. For example, Alistar, a self-identified gamer, experienced a shift in perspective related to video games (as a case):

> *The Top 100 Games* I remember a lot – just the diversity in each one. And that kind of helped me at first figure out what it was about because I didn't really get it and then after looking at *Flatland* and that, I was like, oh wait a minute, I guess that Mario or Pong isn't really 3D and that kind of helped me figure it out through something I could relate to. So having played those games, I think that helped me get it along with *Flatland*.

In this passage, he talks about this idea of a particular case ("Top 100 Video Game of All Time") helping him relate games he is used to playing (e.g., Pong, Mario Brothers) to *Flatland* and dimensions. Although these are games he has played in the past, he never seriously considered that they are not "really 3D." Although this quote is specifically trying to show cases that contribute to shifts, he also helps us understand that part of a shift is making distinctions about phenomenon in the world that were previously untroubled, or not interrogated for multiple facets. Additionally, Alistar talks about criss-crossing the variant cases (games and Flatland). This is what Spiro et al.'s (1988) theory of cognitive flexibility suggests happens when juxtaposing cases that show the variant, complex nature of concepts. During the interview, Alistar continued to criss-cross cases, especially video games. For example, he starts to draw on a video game not part of the hypermedia site:

> If you look at things differently, you can manipulate if it's 2D or 3D or whatever. I think Ortho Robot [a perspective video game] really showed that to me too. Because if you're looking down it's 3D and when you look to the side, it's not. I think that perspective does change with space.

Lynn, in a similar way, drew on an idea outside the hypermedia site concerning finite to infinite space. Although she credits class conversations for this shift, there is not a recorded conversation where she speaks about the idea. It is possible that *Flatland* propelled her to think about infinity

because she talks about lines going on forever being the "thing" that caused her to first consider infinity.

> **Lynn** – I remember we were talking about the concept of infinite space one time and that has just always been kind of something I tried to grapple with as much as possible. Did you see the movie *Gravity*?
> **Keri** – No. But people keep telling me to.
> **Lynn** – Oh it is so good. But like I actually thought a lot about like that conversation we had in class one time while watching the movie *Gravity* because it really touches on the concept of like space - like it's out there, like it's infinite. Just like the idea of something being infinite and going on FOREVER. Like I just can't get my hands around that. Or get my mind around that. I thought that was really cool, especially the infinite lines and just like line segments. That was something that I kind of had to be like – what? So like what? OK.
> **Keri** – So was there a conversation in class where that was the first time you really thought about infinite space?
> **Lynn** – I think that it was really when I first learned it was probably in your class. That like lines go on forever in Flatland. Like I think that that was the first time that I was like WHAT? Like that's a thing that WHAT! Like so – yeah

I find it interesting that this idea was later retrieved by her when watching *Gravity*, the movie. For Lynn, *Flatland*, or really the concept of something being infinite, portrayed in *Flatland*, is identified as a contributing factor for her shift. The concept of infinity was a perturbation or impasse, or really a new idea she hasn't considered. Similar to Alistar, she too pulled in an outside case and criss-crossed it with *Flatland* to consider the idea of space as infinite.

Class Activity. When students talk about the source of a shift, they sometimes talk about class activities, such as making bubble and straw models and photographing space. In the excerpt below, Alistar discusses an activity where students created three-dimensional solids with Zome tools that they dunked in bubble solution. When a straw is used to blow a bubble in the center of the solid, a four-dimensional model is created. For Alistar, the see through "glass" effect of the bubbles were mind blowing:

> So I remember that we had these straws and we would have to submerge them in this bubble fluid and it would make bubbles but with certain geometric shapes to them. And I think, what was kind of cool about that was that, so technically if you think about it

> something like glass is kind of 4th dimensional cause you could see through it and that the bubble helped me realize that, so that kind of blew my mind.

The Bubble and Straw activity contributed to the shift for Alistar (similar to *Flatland*). Where *Flatland* helped him consider dimensions beyond his perception, the bubble and straw models gave him a 3D object to experience the fourth dimension through – he says "glass is kind of 4th dimension".

Lived-quality of shifts. Learners all give a personal example of when they remember a shift in perspective happening for them. In discussing the lived-quality of the shift, four emotions/activities are described: (1) discomfort, (2) acceptance, (3) a eureka/wow feeling, and (4) making distinctions. These four will be discussed in this section, followed by specific shifts for each student interviewed. The first example from Alistar describes all four (discomfort, acceptance, eureka, and making distinctions) as he reflects on a particular moment of shift where he extends his current concept of dimensionality:

> **LED:** I had a *feeling of eureka* in that class. I never actually knew that dimensions could go beyond what we can perceive.
> **Keri:** Talk to me about what that felt like – that moment.
> **Alistar**: So I think that *Flatland* explained this to me – I didn't really get before – but I think that's just because as a human being I can't perceive - or to live in the 4th dimension, so I never actually thought of it's existence – until *Flatland* of course. So I feel like after reading through *Flatland*, I was like – wait a minute! There's more that I can't see, why can't I see this? What is this? And that's kind of what the – the I was like, well – I just figured out why - I mean [laughs] – that's kind of, that's kind of what the feeling was like. I felt like at first it was uncomfortable, but then I just kind of came to accept it. This is what I can perceive, what I can see, but there is more out there for sure.

This passage is a great example of the moment of shift at a particular moment, and what it felt like for Alistar. He uses the phrase, "feeling of eureka." Notable is the impetus for this feeling – "never actually knew that dimensions could go beyond what we can perceive." He credits *Flatland* for spurring this moment (or prodding the "explanation") – there is more that I can't see. He's uncomfortable and then comes to accept it (Lynn noted this too in the Elusive section).

The moment of shift is marked by a realization that one's current concept is incomplete or that there are aspects ones has not considered before. In this particular explanation of the moment, he says "at first it was uncomfortable, but then I just kind of came to accept it." What might not be as clear from this excerpt is that his realization "there is more out there" is similar to making distinctions. Before this moment, dimension and space were not really problematized and distinctions were unnecessary. Flatland served as an event that mediated his perception of space and dimension, causing his to start making distinctions in his environment. For Alistar, it's a "feeling of eureka." This particular aspect of the shift is considered an intense manifestation because these particular moments are those of vulnerability, openness, and the realization that something is not stable in one's current concept.

In the example above, Alistar described a moment of shift resulting from activities in the learning environment. When Albus and Jack talked about shifts in perspective from their past, they reflected back on events as a young child. In the example below, Albus describes his memory of seeing trees through his bedroom window versus seeing them from outside. He talks about the realization of these being the same trees as one of amazement. It is important to note that this amazement was brought on by the activity of distinguishing:

> When I was little, I would look out my bedroom window and I saw like all these trees, like the tops of trees and I never put together that those were the same trees that I saw when I was outside looking in the same spot. Like it always – to me, because I was so small, like I couldn't see like down out the window, I could see up – so I would see this whole forest and I was like this is so amazing – and I think I must have been like 4. And then I would go outside and I would see the same trees and then, I guess I slowly realized something – those are the same exact trees I'm seeing when I'm inside. I just kind of remember being amazed every time I saw those trees from the inside – I was like, those are so cool. I wish I could see them from the outside.

Albus's description of seeing and cognizing as young child spurred Jack to offer up a moment of shift from his childhood as well. Jack and Albus also give another example from their childhood

that helps show this moment of shift as one of making distinctions and for Albus, "a really big deal." This example involved playing with his fingers close to his face.

> **Jack**: I was in like 1st or 2nd grade and I like looked at my finger and I closed one eye and closed the other eye and kind of just like figured out how eyes work, that's when I kind of figured out like what perspective was – and like how it's 3 dimensional stuff and how it just works
> **Albus**: Yeah, I remember when I was little, I thought I had magical powers cause I could see through my hand when I looked at it from like a distance. And then like if I shut one eye, I couldn't see through it and then when I had both my eyes opened and I kind of blurred, I could see through my hand and then I realized that I was seeing around my hand and it was kind of similar to what Jack's saying. [**Jack:** Yeah] But ah, I remember it was just a really big deal – I was like oh, so it's both of my eyes kind of converging on it. That's when I realized – when you shut one eye, you see one thing and when you shut the other eye, you kind of get the larger span on that side

The particular phenomenon they describe is similar to the peek-a-boo phenomenon, where young children are caught by surprise at a person's presence and absence. This playing with space, our eyes, and our bodies is probably something that most children engage in, even if adults can't recollect these moments. I didn't intend to talk to Jack and Albus about childhood shifts, but they were what came to them during the interview. It may be that these natural, informal, and playful childhood activities can somehow be intentionally designed at more complex levels to help learners understand dimensionality.

Albus did offer another shift taking place in the learning environment. He discussed the gravitational pull in the second dimension as marking a shift for him.

> For me I think it was the talk of the gravitational pull into 2-dimensional space that like was really an a-ha moment for me because like it just really like opened my eyes about it. Cause that makes a lot of sense to me now more than it did before cause like in Flatland it doesn't show that necessarily so just like they're moving through the space, but it makes sense that there would be some sort of pull because there's always a pull in every dimension, right, like there would have to be some sort of gravity.

This "a-ha moment" for Albus relates to a particular conversation from class where learners were conjecturing about gravity in two-dimensional space. From his memory of this particular time, it

is apparent that a shift includes an opening of one's eyes to another way of conceptualizing – in this case, something "that makes a lot of sense to me now more than it did before." Albus makes another distinction concerning gravity during the interview concerning human-centrism, this time with Jack:

> **Jack**: Oh yeah, the book when it said that everything was pulled down to the south that – I fell like that was just because it was written in the 1800s and people didn't really understand how gravity worked and that it wasn't – I mean, obviously they didn't think the Earth was flat but they didn't really understand. Like it was still a really human centric way of saying it – that everything's going down. Because in Flatland, it wouldn't really be like that. Like in the universe, it's not like that – not everything's pulled down because it's all pulled towards [**Albus:** something] yeah, towards a large object
> **Albus**: It's very egos – yeah, it's not even like human-centric, it's like egocentric because it's like down, but down for someone in another part of the world isn't down for us. Not in the same – it's not all south. It's like we're a circular sphere so it's going to go towards something else.

In the excerpt with Jack and Albus, there is a nuanced type of distinction making happening that is beyond the scope of considering space, perspective, and dimension. Here, they are acutely aware of the possible biases in the book concerning gravity, direction, and the relationship to being.

The examples so far have illustrated the connection between a eureka/wow/amazement moment in connection to making distinctions. In these next few excerpts, making distinctions seems to be key to these moments of shifts for learners. The idea of making distinctions can be found in Aristotle's writing about inquiry and Reyes and Zarama's (1998) writing concerning learning as "embodying distinctions." After sharing a few more examples, Reyes and Zarama's (1998) learning theory is connected to the possible relationship between shift, distinguishing, and learning. In the excerpts below, two particular distinctions are made that cropped up in the classroom discourse throughout the project: motion related to seeing and problems with a film taking place in the second dimension. The first, motion related to seeing, was discussed in whole

class and small group conversations. For Albus in particular, his position shifted concerning motion related to seeing. Originally he stated that for a Flatlander to see a 3-dimensional solid, they'd have to move around it and for a 3-dimensional being "we'd have to somehow like be like be able to move in order to see like 4th dimension." This in part helped explain that whenever we look at 4-dimensional animations, they're always in motion to "help us have a different point of looking at it." Yet, Albus shifted this position stating, "if we were really like 4-dimensional beings, we shouldn't have to move." As Albus talks more about this shift in relating motion to seeing, he starts to make further distinctions concerning the problems in *Flatland* along with Jack:

> **Jack:** we don't have the capability of seeing 4-dimensional space because – it depends because if you're saying we could see it like – 2-dimensional objects, I don't think would ever be able to see 3-dimensional space. Like that's why *Flatland: The Movie* didn't really make sense. At the end of it, it's kind of confusing because they can kind of see and that's because it's like brought into another world.
> **Jack:** Yeah, all they had to do was just rise above a tiny bit and that didn't really make any sense.
> **Albus**: Yeah, cause they're still 2-dimensions. So in order to actually see the next dimension, I think you'd have to be that – you'd have to be an object in that dimension, which means that we can never have the capability of truly seeing one because we could see a 2-dimensional object if we could have like an actual plane and there were 2-dimensional beings on it and I think that we could – we'd be able to see them but we'd be able to see them from above and we wouldn't actually be able to see them from the 2nd dimension. We'd still be seeing in 3 dimensions because we'd see other things going on too cause we'd be seeing them from above and they couldn't see us because we're here out of the space. And then a 4-dimensional object – I feel like we couldn't be able to see truly because we could see them from our 3rd dimension perspective and they have like this whole other side that's happening.
> **Jack**: Just how like in *Flatland*, when Spherius moves through the Flatland and it – he just changes size, that's all it looks like. That's the only way for him to actually see 3-dimensional objects when they move and they seem just like slices of it. [**Albus**: Yeah] So we would see slices of it but it's in 3-dimensional space - slices
> **Albus**: Yeah, so we'd see – it's hard to tell because like our, since our world is 3-dimension, where would that other side be? Would it like be within that or would it be on the other side of it and we just could never see it cause it's constantly in motion there or something?

> **Jack**: That's how when we see the hypercube or whatever it's called, it's always moving – it's through 3-dimensional space and that's – it just looks like its changing size [possibly sides] for us

According to Reyes and Zarama (1998) there are 4 steps to learning as "embodying distinctions": (1) declaring a break (perturbation in environment), (2) drawing a distinctions, (3) grounding the distinction, and (4) embodying the distinction (like ontological 'trying on'). Winn (2002) echoes this perspective writing, "the student makes distinctions with more certainty" (p. 18). In this passage, the break, or perturbation in the environment is the impossibility (for Jack and Albus) of cross-dimensional seeing. They draw several distinctions, including: within and between dimensions, motion and seeing, problems with inter-dimensional travel/seeing, why hypercube animations are always in motion. They ground these in examples from *Flatland* characters, their own experiences, and phenomena they have access to, such as the hypercube animation. The embodiment in this passage is similar to learners' ontological 'trying-on' from the pilot study (Valentine & Kopcha, 2014). For example, "we'd be seeing them from above" and the many other times in this passage they insert themselves into the other dimensions, even contemplating their own.

Ontological 'trying on.' In a previous study related to space, perspective, and dimension with fifth and sixth graders, an experiential manifestation for learners occurred that I labeled, "ontological 'trying-on' of dimensions." The learning experience only included watching *Flatland: The Movie*, classroom discussions, and learners making models of 4D shapes with straws and bubbles (Valentine & Kopcha, 2014). They distinguish between seeing and perceiving from various dimensions towards other dimensions (e.g., 1D to 2D, 3D to 4D, 3D to 2D). Related to this study, the 'trying-on' of dimensions manifested differently for learners and is better reflected by the phrase, 'ontological trying-on' without specifying dimensional seeing

and perceiving. Instead, learners consider more diverse 'trying-ons' – two-dimensional gravity, motion, animal seeing, etc. not necessarily related to dimension. It is important to note that 4 out of 5 students interviewed were former fifth graders. Alistar is the only student interviewed that was not exposed to *Flatland: The Movie* prior to the current study. Former 5th graders had plenty to say about ontological trying-on of dimensions in the pilot (2 out of 4 former participated), but this manifestation wasn't so focused on dimension this time.

In this section, ontological trying-on as a tentative manifestation of the experience for learners takes on three variant forms: (1) trying-on or conjecturing other dimensional being/living, (2) technologically mediated seeing and the sight of other animals, and (3) other ontological trying-on, such as paint and gravity in second dimension. As mentioned earlier, the ontological trying-on of dimensions was the most satiable manifestation of learners experience in the pilot. Learners still talk about this particular experience of moving, seeing, perceiving from and to the various dimension. In the excerpt below, Lynn talks about the bubble and straw activity leading to her wondering what a 4D person might look like:

> **Lynn:** The experiment we did with bubbles was cool. And kind of just with the 4th dimension. Something that I always think about it, is like when we did the bubbles and we kind of tried to experience the 4th dimension. Like we just did it with a square. But how would it look like with a person or like the shape of a person. Like ahhh, what?!?

Even though discourse is not as prevalent in these interviews as the pilot, every student wondered this – what it would be like to be in the fourth dimension. Even Alistar and his game design plans imagines a 4D game world. Also, Jack and Albus conjecture two-dimensional life (rather than focus on four dimensions) concerning seeing, gravity, paint, etc. in two-dimensions. Beck, included below, talks about his continued wonderings concerning the fourth dimension:

> **Keri** –you said in the survey that one of the things you would have liked to learn more about was like the first dimension and fractal dimensions?
> **Beck** – Yes. Like creatures swimming around in the flat land probably.

> **Keri** –There was something like you felt yourself wanting to know more about the dimensions – was it a particular one or how they relate? What was it about it?
> **Beck** – It's just intriguing. I was kind of just wanting to think about stuff like that. Just trying to understand it. Because I like the other dimension. Others like what their perspective would be and what it would be like living in one. Like polygons and stuff.
> **Keri** – Yes. I remember you wrote that story about the dog – the girl and the dog.

When illustrating the tentative manifestation concerning elusivity earlier, Lynn talked about questions she felt like could never be answered. In particular, "what would it be like being in the 4th dimension?" She continues:

> What does that feel like? Do you feel like you are constantly moving or is that just the norm and you are like OK. Is there like some other direction kind of that exists? Like ewwww, what! Like because I feel like to us in is, I don't have anything around me, but in is like my finger is going into my t-shirt kind of. Or if I were to take a needle the needle is going into a sponge. Like we can't even think about what in is to the 4th dimension - because I don't know.

In these examples, the fourth dimension is elusive but learners still try to conjecture what it might be like to see, perceive, move, live, imagine, etc. a 4D existence and relates strongly to an ontological 'trying-on' of dimensions.

Concerning our own third dimensional existence, learners also talk about trying-on, but this instance is characterized more by mediated seeing – where technology and tools allow us to see something that would otherwise have been impossible. The telescope and microscope are two common examples of mediated lenses that alter what we are able to see. It's not trying on a dimension, but trying on different lenses (that of the camera) and juxtaposing it with her natural sight. In the examples that follow, learners talk about their experiences with mediated seeing. As Lynn talks, notice how she prioritizes seeing from her eyes at first:

> **Lynn:** I think that technology - because I don't know I've been thinking about this a lot actually. Like how we see things is it's kind of I just think that technology doesn't portray how we see things as accurately I feel like. Because I feel like our eyes almost can pick up on things like a camera can't. So like – like shadowing and dimension so I feel they – like cameras kind of only show the second dimension. And you can show depth but not as much as like the human eye can.

> **…**
> **Keri** – What have you noticed, as a photographer, how the camera is different than your eye in particular?
> **Lynn** – I've noticed that to kind of get like – get the angle of something that I want. So like if I'm looking at like a box straight-forward and I can kind of see partially into the box, I have to move my camera up a little bit more in order to see what my eye can see. Sort of. So you have to get different angles with the camera to get the same as your eyes. Eyes are more powerful because of shadows and just like depth. I don't know how to describe it.

As you can see from the two excerpts above, Lynn contrasted seeing from her own eyes to that of a camera. In her comparison, she seems to prioritized the capability of her eyes as being able to pick up depth in ways that a camera lens cannot. As she continues, she starts to talk about the advantages of cameras, such as panoramic capabilities:

> I think that even though cameras can't pick up on things that we can see – like they can't pick up like depth and stuff as much, but I think they can also pick up on things that we can't see. Like panoramas – I took a panorama of my room that went from like my door to like right next to my door and it – I just feel like I can't look at everything in my room all at once. But the camera can pick up on it. Which I think is so cool because I think that's just a new way of seeing things.

The lenses of a still camera, moving camera, and even the cameras views in a video game allow us access to another view. One more point that Lynn makes in talking about this mediated seeing stands out as being different than her classmates concerning whether we see in 2D or 3D. Lynn believes we see in three dimensions:

> **Lynn** – I think we can see in 3 dimensions because I can see depth and height and length. And so I think that the 2nd dimension doesn't have like height but I can see height and I can see depth so I think that is why we can see in third dimension.
> **Keri** – OK. So you think that you have access to all those different ways
> **Lynn** – Yes. My eyes can pick up on the fact that like there is space and matter underneath that slanted picture like so.

Although not included here, several more examples are shared by learners where their sight was mediated by technologies.

Consequences of shift. Learners talk about three main consequences from a shift in perspective, although they don't necessarily refer to it as "consequence." They talk more about how the *Space and Perspective* project is affecting their current work or current view of the world. Four main consequences include: (1) an ability to be flexible/apply, (2) seeing something different than before, (3) a shift in being/preference, and (4) a more robust concept.

An ability to be flexible/apply. When designing the cases used for the project, Cognitive Flexibility Theory theorized that criss-crossing a variation of real-world instances concerning space, perspective, and dimension would allow learners to use the concept more flexibly in the future. Alistar began his lived-experience description talking about how he views video game and movies differently. His description along with an interview excerpt shows this flexibility:

> **LED:** Because of the class, I look at movies, and mainly video games differently. A 2d world versus a 3d world is now a totally different idea for me because I understand how the dimensions worked.
> …
> **Keri**: What is it that you would attribute to the class that caused you to look at those things differently?
> **Alistar**: Well so I think that this being in filmmaking class [current high school course] with shots and cinematography, you can set things up more to have the illusion of 3D or 2D, when really all it is, is just a screen. So I've been using that to make shots. So I tried out a concept where I tried to do a movie about kind of what it would be like if the camera was the only thing you ever got to see in real life and how you can change it to move around in the real world. So like if the camera's flat against something. Kind of like in *Fez* where the camera's just flat, but you can switch it around. So like if the camera was 2D, but then you would lift it up, and it would suddenly reveal a whole 3D world. So I feel like that concept I would have never thought of had – and cinematography – had I never taken this class…just the concept of changing 2D and 3D and how to manipulate it pretty much.

This particular passage describes Alistar's shift in his concept of changing dimensional perspectives in film, but it doesn't talk about that shift happening. It's more about the consequences of the shift – the outcome of understanding a concept with more flexibility and his new way of being able to use/apply it. For Alistar, the shift led to his flexibility in manipulating

2D/3D shifts with video clips. He continued talking about the impact of *Flatland* and "realizing there is more out there," by describing the way he is applying space, perspective, and dimension concepts in his current projects.

> **Keri**: So now that like you've seen that and you realize there's more out there, what does that feel like now?
> **Alistar**: Um, well now I think that um, well I don't necessarily apply it to everything I look at. It's helped me a lot with like creative thinking. Just because like most of like my favorite classes in school are based on you know, creativity. Like programming and filmmaking. So that's kind of helped me apply those kind of things to objects in a film or characters or scenes in a game. So like I think that um, being able to have control of the dimensions through a camera or through Blender has made making games much more interesting.

In this passage, he talks about the impact of realizing more (or the impact of the new, shifted concept) for him. It helped him "have control of the dimensions" as he created video/film and games.

Beck, Albus, and Jack also talk about their ability to use space, perspective, and dimension concepts with flexibility. Below Beck talks about the future actions he takes in his film work as a result of his "shift" in perspective. He talks about "playing mind games" with film techniques and attributes it to activities with photography.

> **Keri** – Do you feel like investigating space and perspective has affected anything that you have done since?
> **Beck** – Filming, yes. I really like filming stuff and directing and like the way shots are set up and the way the camera moves. Yes, and when I make little movies I try to you know like make the whole – I don't know how to explain it but – I try to capture you know like the space like the hall scenes kind of that stuff – the room space and like you went through the space and just cutting away when the character is walking and kind of tracking the character through space like keeping a wide view so you can get by. Have you seen the action movies like the little clip shots of the character and spaces? But usually better directors have wider views so you can see the same space and kind of get a – sometimes it's a good technique to have like really close up shots and see the characters face and really their emotions but lots of times the directors don't have kind of a wide view so they can see.
> **Keri** – OK. Oh, that's cool. So like do you remember any particular examples that made you think about that?

> **Beck** – I remember the stuff about pictures – the camera taking pictures and stuff. And my – through that I like all the different perspectives you can like you can make something look like kind of a mind game – play mind games with people where you…oh, I have a good example too. It isn't really – it's more like an artsy fartsy but – you know lots of times directors will have a scene where the angle of the – the wide angle of the bright light of the camera where you stand in a doorway or something. Where it's kind of like attracting – they do that a lot where they have two characters like a pole in between characters where they shoot like they were separated.

At first, this next example from Albus seemed far-fetched to me for showing the ability to use a concept with more flexibility. However, the more I read this particular excerpt, the more I started to realize that Albus's concept of the second dimension is quite advanced and very different from the way he talked about it at the beginning of the project. Below, Albus reacts to a class conversation about *Flatland* not needing to be flat:

> **Keri:** Your class came up with the idea that maybe Flatland isn't really flat. Like maybe it's on a Mobius strip, maybe it's on a cube, like something like that. I just wondered have you thought about it since?
> **Albus**: I mean I definitely think that it could be like on any shape as long – cause for it to be 2-dimensional it doesn't necessarily mean that it has to be a flat plane. It could be on anything cause there's like 2-dimensional space. Like the way I picture 2-dimensional space is that it's on – it's kind of like a coating on everything but that we can't – we just can't pick it up because it's like, it has no depth at all in our world.

The idea of 2D being a "coating" is quite advanced, but even more complex is the idea that it can't necessarily be picked up. Once something has thickness and an ability to be picked up, it is no longer two-dimensional.

Seeing something difference than before. The second way that learners talk about the consequences of a shift is that they see something different than before. This probably relates strongly with making distinctions described above. Part of having a shift then, is not seeing the world, or some aspect of the world, the same way as before the shift occurred. In Albus's lived-experience description, for example, he writes:

> I have to bring up a trip that I took with Jack and my family. For my whole life I have taken the ferry to Block Island, and every year I have taken it from the same point A to

> the same point B. But when I got on the ferry with Jack and we started to near the island, we started talking about what we saw, how we could jump from the cliffs into the water, how maybe if we designed a wind surfing board that had the ability to steadily let us down to the water from the cliff like a hang-glider, so that we could jump off right into windsurfing. I will never look at the island the same way again because my perspective has been completely shifted.

Lynn talks about her shift as it relates to the realization that things in her world are infinite. Earlier she talked about this idea of infinity being elusive (hard to grasp). Here she talks about it in terms of changing her perspective (the way she now thinks about the world):

> I was opened up to so many new concepts and like new shapes and things that I just didn't know about so I think because of that definitely my perception was definitely changed. And like circles and how like circles are infinite kind of and that was definitely something that I was opened up to. So I think that kind of changed my perspective about some things a little bit.

Here Lynn talks about shift as being exposed to new concepts she "just didn't know about" and says, "my perception was definitely changed." She considers her perspective changed, but I am left wondering what it is for her to have a shift in perspective.

When interviewing Beck, I asked him about a shift he mentioned in class, that from space as outer space:

> **Beck** – At first space was kind of a word describing outer space the birds and stuff and all that out there but then after the class I just started thinking of space as more just as space when we move through and like everything. And I actually had this kind of like space and time. There's not really time there's just space and then there's movement through space and stuff happening that changes it and that's movement but there's no real – time does not really exist. I don't know.
> **Keri** –So it really kind of changed your concept of time too?
> **Beck** – Yes. Space doesn't really change it is the stuff in it.

Shift in being/preference. A third consequence of shifts for learners is an actual shift in their being or preferences. Alistar, the gamer, experienced an especially strong shift in his preference for video games that he enjoys. This may not seem like a huge shift to a non-gamer, but games are an identity of sorts, similar to music. Just as one might identify with The Cure and

a Goth-like style of clothing, one can also identify as an indie gamer like Alistar. This particular excerpt relates to another passage (about games). Because most of the interview with Alistar has been dominated with his talk of game experience, and basically himself as a gamer, it made sense to ask him if his life as a gamer had been affected by *Space and Perspective*. In this part he talks in detail with specific examples – ending with preferences for indie games because of the way they play with perspective:

> **Keri**: Have the kind of games that you're drawn to changed at all since *Space and Perspective*?
> **Alistar**: Yes. So when I was little, since I was about 8, I only played shooters. I just played loud, obnoxious, over the top shooters. Like before then, I play - you know Quake, that series or Wolfenstein, or Zoom?
> **Keri**: Oh Wolfenstein – that sounds familiar, but
> **Alistar**: It's like a WW2 shooter but it's really over the top and crazy and awesome. I would only play Wolfenstein or any games like that and then I'm going to say when I was like 10-13, I got into more adventure/RPG [role-playing] games. And then after this class, it's all been indie games for me, like I don't care what they're about. There are so many indie games that have to do with like perspective. There's um, well there's another one I haven't told you about – it's called Window Sil: W-i-n-d-o-w-S-i-l. And there's another one called Lume: L-u-m-e. So these are both games by one developer, where they've experimented with putting real life objects into a game, so like some of the objects are real things, like they're not animated. They've filmed things with a camera and put it in the game. These are all 2D games but have real 3D objects in them.
> …
> **Keri**: You say that after investigating space and perspective you are more drawn to Indy games. Do you think that would have happened anyways or is there something about it that made you look at games differently?
> **Alistar**: No. I think that this happened actually because of this class. Because before then, I just liked games that were big, you know were fun, interesting – I didn't really care if they did anything new. And with indie games, a lot of them with their art styles and their like mechanics like Fez kind of feel like they're almost taken from like different perspectives in space where it feels like every game feels different in that regard. And um, so I feel like I would not have picked up indie games had I not taken this class. Because before, I was like eew, what? – these indie games look really stupid – they're just like really colorful and short and dumb. And then I played one and it was actually really fun.

I remember being exhilarated when Alistar talked to me about his shift from shooter games to indie games like *Fez* and *WindowSil*. Before the interview, I downloaded the game *Fez* because

he told me about his redesign project, where he was taking the book, *Flatland* and also *Flatland: The Movie* as the narrative and skin for a game similar to *Fez*. *Fez* is a unique game with a game mechanic that allows the player to shift the play space from 2D to 3D in order to move the character (see Figure 4.3). If you'd like to see the trial game play for *Fez*, this video will help you understand the unique mechanic of the game (http://vimeo.com/38466193).

Figure 4.3. Four views of the same space in the game *Fez* created by moving the world, not the character.

The last example concerning consequences of a shift involves Beck and the Cubist art. In this excerpt, Beck talks about all three consequences of a shift: application, seeing something different than before, and a shift in his preference/being.

Keri – So did it make you look at the painting differently?

> **Beck** – Yes. I started like kind of I mean just looking at it as a whole. I kind of like looked around and then I usually just stepped back and look at altogether and kind of looked at it all together.
> **Keri** – Have you been to a museum since?
> **Beck** – Yes. I've been to the MOMA. I don't remember if I saw any Picasso there.
> **Keri** – Did you see any like Cubist like paintings or sort of modern art that's a little more abstract?
> **Beck** – Yes. Like Ralph Goya and stuff. I saw stuff like Picasso and it helped in like appreciating the painting. When you know more like about it and what the artist is trying to do and convey. In the they try to – yes.
> **Keri** – Do you think it will change the way you look at art in the future?
> **Beck** – That kind of art, yes.

For Beck, an understanding of what an artist is trying to 'play around with' regarding space and the viewer, helps Beck appreciate the art in a more nuanced way.

Concept more robust. After a shift in perspective, there are indications that a concept becomes more robust for some learners. For example, Alistar created the blog post below concerning the light/shadow concept. There isn't a before concept to compare this to, but he seems to connect ideas from *Space and Perspective* to write a blog post applying these light and shadow ideas in a real-world context that is familiar to him, *Darwinia*, the game.

> When making a movie, animation, game, or any kind of visual media, lights and shadows are an important aspect. A light and colorful film makes the film look more vibrant and happy. A very dark game makes the feeling very dreary and dark. Light and shadow can also factor into space. If shadows are all together gone, then from some camera angles there it may appear that whatever is being viewed is not there at all. If something flat is being viewed from a birds eye view without any shadow than it is very hard to see. A great example is a game called *Darwinia*. *Darwinia* is a game about a scientist who creates a computer program that evolves into sentient life and can carry out tasks. The inhabitants of the program look like crosses with little legs and they are flat. In game changing the camera shows the Darwinians in new light. When on rocky mountains the characters can sometimes be rendered invisible to the eye because the lack of light and shadow dilute the image. When looking down from the ground and the characters are on hills they are shimmering in light but take the camera and put it in the sky and you will no longer see the Darwinians. This is not because of height but because of light and shadow and there role in changing the players vision. There are many other examples but I feel as if *Darwinia* is a prime one.

In the interview, I was able to ask him about this particular post:

> **Keri**: You had written on a blog post – that the light and shadow in – is it *Darwinia*? Am I saying that right?
> **Alistar**: Yes! OH! I forgot about that game – that is an amazing game!
> **Keri**: Yeah, you said it changes a player's vision
> **Alistar**: Yeah, so its kind of like the light and shadow of that game – that game's a strategy game it's where you're inside of a computer program infected by this weird virus that's taking over it, but the program's a whole world. It's like some guy built the world in the computer and the way you have to find the enemies, is you don't really get to see them, but if you look up like bird's eye view and you see like shadows, you can kind of figure out where things are and figure out the map. And so using different light, have like some guys in some areas, uh so that way you kind of – as like just detector guys to have some guys in other areas just looking and scouting. It's kind of cool because you can figure out where things are on the map just by using light.
> **Keri**: So would the light source be in one place or could it be in multiple places?
> **Alistar**: It would probably be in one place but if you have guys all fanned out – looking – you can kind of catch things that you wouldn't see.
> **Keri**: Oh, so in a way, you have multiple perspectives?
> **Alistar**: Yeah
> **Keri**: That come together to show you like what you're trying to look at?
> **Alistar**: Yeah, sort of. It's pretty cool.
> …
> **Keri:** You talked about light and shadow earlier. How does light and shadow play into all of this for you?
> **Alistar**: Yes, so I think that um, one way to kind of discern one dimension from another is if there is lig – like any sort of shadow – because if something is flat, it won't really have any shadow – maybe a very little amount to it. Which I feel like was part of the reason why– was one of my questions in *Flatland* – like were – like was there any source of light that was creating shadow and could that have possibly hinted at the 3rd dimension or not? And um, so I feel like if you can take something and use light to it, you can kind of – like at the same time create the illusion of having more than 2 or 3 or 1 dimension to it.

It is an interesting way Alistar has come to talk about light and shadow. This particular passage makes me think about an experiment that could become part of the next *Flatland* iteration where figuring out matching shadows or experimenting with 4D shadows may be possible.

Summary/Implications. To summarize, a shift usually occurs when learners are confronted with something new, in these examples cases for *Space and Perspective*, or moments from childhood – both marked by a perturbation or break in everyday operating/being. The shift

evolves as learners draw distinctions and "try on" or apply these to real-world phenomenon. This type of shift is similar to the way Reyes and Zarama (1998) conceptualize learning as embodying distinctions and also similar to Dewey's notion of "reflective inquiry" (1933). If we are interested in investigating learning, these particular ways that shifts seem to manifest may be helpful to guide the way we theorize, talk about, and assess ill-structured, complex learning of this sort.

Lasting Resonance (Residue)

Lasting resonance, or the "residue" (Hiebert et al., 1996), from the phenomenon is not presented as a manifestation, although it helps us understand the lasting significance for learners who are given opportunities to problematize and experience shifts in perspective. The term "residue" refers to "understandings that remain after an activity is over" (p. 17). Reside is described in more detail as related to problematization in the implications section, but the term seems appropriate here for sharing those articulated by learners.

Residue is expressed as learners finding themselves thinking about space, perspective, and ideas long after instruction, sometimes affecting their current work (e.g., game design), and as inspiration for current projects. The phrase "finding myself thinking about these concepts at odd times" (e.g., middle of night) is one way resonance manifests for Lynn. A post-reflexion journal entry struggling to name this activity follows:

> Continued Learning – No, but what? Maybe this is more *elusive*. Seems like elusive is a better fit. I'm having mixed feelings about this passage. It doesn't really get at continued learning – better phrased maybe as "keeps coming up in life and I can't stop thinking about it." I mean for the idea of INFINITY to actually "wake" her "in the middle of the night" is very intense in my book. My dissertation wakes me, but I can't imagine anything but love lost that woke me at her age. Rather than continued learning maybe it's more the effect of pondering elusive concepts.

Albus and Jack help add another perspective of resonance as thoughts that creep into one's contemplation:

> **Albus**: I mean yeah, I mean like it – in most classes it just comes up like [**Jack**: yeah], also I feel like I just learned more about like the way the world works in that class so it kind of made more sense to me – like it just kind of carries on through everything.
> **…**
> **Jack:** Sometimes I just kind of think about it like – I don't even know how I start thinking about it but I just start thinking about it and then I can't get it out of [can't decipher – my mind?]

The way I've come to understand this particular aspect of resonance is that it creeps into current thinking and is more like the progression of learning. Maybe these ideas express more flexibility of concept or that its complexity keeps it in our thoughts and "carries on through everything."

A second way these residual qualities appear is by affecting learners' current work (e.g., game design). This is especially true for both Jack and Alistar as they are designing video games. For example, Jack talked about concepts from *Space and Perspective* in relation to camera settings when designing video games:

> In the engine that I was making games with, for the camera, there's one setting that makes it so it's wider or thinner and I even kind of like play with that and I was just thinking about how the edges work and like how it's not accurate so I was [can't decipher] if you could get a huge screen and increase the field of view immensely, it would - I was thinking like would that actually make it so that you feel like you're completely surrounded by it? Or would you have to have the screen surrounded by – surround you?

Most poignant for me is the phrase, "I even kind of like play with that." This idea of play, tinkering, visualizing – these are all considered aspects of someone with "mathematical power" (Goldenberg et al., 1998). Goldenberg et al. argue about the importance of geometry instruction in schools, specifically visualization and visual thinking, "which are at the heart of what makes geometry a special case within mathematics" (p. 5). They believe visual approaches contribute to access, which attracts students' attention, and for some learners, this becomes their first

opportunity to participate in mathematics. Further, they write that if visualization is ignored, "curricula not only fail to engage a powerful part of students' minds in service of their mathematical thinking, but also fail to develop students' skills at visual exploration and argument" (p. 6). In addition to advocating for visualization with natural objects, they write about people with mathematical power as those that tinker with real and imagined machines, invent items, visualize things (even when the "things" are not inherently visual), seek to explain *why* things are as they see them, and argue passionately about intellectual phenomena. When mathematical power is interpreted as involving this type of visualizing, providing opportunities where students can investigate their perception of space becomes an important goal in geometry education.

Alistar is another learner who "tinkers," or as he calls it, "programs." He talks about registering for classes in programming the year following *Space and Perspective* and his project to create a game like *Fez*, but also incorporating *Flatland*. He talks about the continued effects of *Space and Perspective* in his work.

> **Alistar:** Programming – yes! Which has been helping me out a lot with my modding and stuff.
> **Keri:** When you say modding, do you mean like creating things you can use in the game or actually kind of like redesigning?
> **Alistar:** So when I first bought *Fez,* this was right after the *Space and Perspective* class cause I was interested and I had heard of this game before and I bought it. I played through the entire game and I thought it was great. Then I came across this guy on YouTube who had taken another game – like a space fighting game and made a modification where he actually converted the entire game and redesigned all the skins for everything – made all the different weapons for everything. And he made it a Star Wars game, which was kind of interesting. So what I've been trying to do is redesign *Fez* to kind of pretty much be *Flatland*. That was my plan.

Modding here refers to customizing a game in existence, like *Fez*, and changing the story of the game, the way the characters look, and the way the game environment looks. However, the mechanic of the game (the ability to move the character and rotate the world) remains the same.

Although modding is the focus of this passage, two other phenomena inform design implications of cases as alternative perspective. One, Alistar talks about seeking out the game *Fez* after *Space and Perspective* "cause I was interested." In particular, he was interested in the dimension and perspective ideas from *Flatland*, a pivotal case for Alistar. *Fez* is a video game of a flat (2D) character who finds a Fez hat, making it possible for him to rotate his cube-like world, even though his motion is limited to two dimensions (see Figure 4).

Another important phenomenon is Alistar's activity of connecting cases from class to cases outside of class in ways that not only criss-cross, but constructively mesh the two together in his current video game project. He talks about this project in depth and expresses his enthusiasm:

> One day I was on the Fez website waiting for them to release the source code and a message pops up. It says, *Fez 2* has been cancelled and then two paragraphs written by this guy [creator of *Fez*] of just him cursing at everyone – it's really interesting like to look at the whole spiral down …
> And um, but so, what I'm trying to do now is to just figure out the tricks that they use to make Fez – so that way I can try and convert it cause there's no source code anymore so I can't directly take what they made and use it, so I have to try and figure it out kind of in reverse by playing the game
> **Keri:** Oh, it sounds like there would be a lot of code in there too
> **Alistar:** Oh yeah, no, there – I mean, like I figured out, um, the animation, that's fine, but it's like, I have to figure out the mechanics and how they coded how you switch the sides of the world. I need to figure that out. I have to figure out how they coded that, so with one button it would input so that way the world would flip and then the animations would change.

I am struck by Alistar's persistence and "need to figure out" a mathematical relationship such as programming an interactive environment. He is invested in trying to figure out the mechanics, mathematizing the way phenomena work in the world in order to produce the "mechanic" in the game world. This is the main "outcome" or activity Freudenthal (1973, 1983) supports in Realistic Mathematics Education. As a mathematics teacher, it's outside of the norm to hear a learner utter, "I need to figure that out." In constructionism and design, this is a normal feeling,

but in mathematics it's usually a begrudging activity of "having to figure it out" so as to complete a homework assignment of answer a test question. Alistar credits *Space and Perspective* in his lived-experience description:

> It [*Space and Perspective*] actually inspired me to learn animation in programs like Blender and start making my own projects. I have been making modifications for a game called *Fez*. If you are willing to spend five dollars on the game, I am sure it will interest you. The game deals with a 2d world that can be manipulated to change the 2d space inside and make it 3d. It is a very cool concept, and a very good game.
> …
> Going back to *Fez*, that game is so much more interesting after learning about different dimensions. I would highly recommend giving it a try.

Alistar is a learner on this current trajectory with game design primarily because of *Space and Perspective*. To see how highly related his current project is to the case *Flatland* and the way this compelled him to find other relatable cases, this section ends with Alistar's current struggle and way of dealing with the lack of available source code for *Fez:*

> So, yeah so *Fez* this, the whole source code thing's a little bit troublesome. So I've been looking at other games that have ideas of dimensions and space and I've been trying to figure out or maybe some that I could also try and mod with that have modding capability. And I found nothing…I found one game that's really cheap. It's not really like a story. It's kind of like a mobile game, but it's interesting where you play it – you're kind of stuck to this hexagon shape thing and you're this little plane and you have to spin around it and these obstacles, kind of try and dodge them. But as the game ramps up, difficulty is measured in dimensions and dimensions are added, which makes it harder and harder as you go on. So it starts out as points and points really simple. And then it just sends little things at you. Then it goes to line. And then line speeds up. And then it goes to shapes – so then it goes to triangle, so it's not really dimension, but it is still a shape and it is…And then it gets 3D. At first it's just flat, then it gets 3D and the stuff in the middle starts spinning around kind of like in a cyclone tornado shape and it gets much more hard to see. Then it goes to square…And then it ends with hexagon. And if you can beat hexagon, there are 6 difficulty levels: they go hard cause that's the easiest one but it's still very hard, then harder, then hardest, then hardester…then harder than hardester, and then hardester-er-er … And I've been looking at this because I feel like, I wouldn't do a total conversion cause this [*Fez*] game on it's own is great. But um, I would maybe do graphics, changes – I would maybe add more, like a 4th dimension maybe. I would try and see if I can figure something out. Or a tesseract level!

Throughout the interview, he continued to talk about the various games he has played since *Space and Perspective*, even sending a follow-up email with twenty games related to *Space and Perspective*, such as Ortho Robot. He admits, "I've just been putting a lot of time into that and it's been very time consuming, but I think it'll pay off."

Implications

In these two classes of 9 and 12, every learner shared an experience concerning a moment of shift, either in book discussions, whole class conversations, in their blog postings, and/or in their lived-experience descriptions and interviews. Designing learning experiences to support a shift in perspective is possible as shown with these learners. I suspect that knowing the shape of this shift, as shown in Figure 4.2, is important for informing our understanding about learning, problematizing, hypermedia design, including designing cases as alternative perspective. This section focuses on four poignant implications stemming from this post-intentional phenomenological analysis. The first concerns cases as alternative perspective and their relationship in supporting learners' shifts in perspective. Cognitive Flexibility Theory (CFT) recommends that the multiple perspectives shaping ill-structured domains (and concepts) needs to be conveyed to learners. However, the literature is inconclusive concerning learners ability to consider multiple perspectives related to hypermedia cases (Strobel et al., 2008; Zydney, 2005, 2010). Understanding learners' experiences with multiple perspectives and the ways these create shifts in seeing, being, etc., especially in a relatively new context (math) with young learners, is a step in addressing this gap. Considering the role of pivotal cases, such as *Flatland,* may be a promising strategy for conveying multiple perspectives with hypermedia. The second discussion point concerns the connections between shifts in perspective and learning. The notions of shifting perspective and learning share many parallel activities (e.g., provocation, impasse). The

phenomenon opened up in this study informs our understanding of one way we might conceptualize learning, that is, as a shift in perspective. A third implication concerns the activity of problematizing in the learning environment as described by Hiebert et al. (1996). Similar to the connection to learning, the phenomenon of a shifting perspective as it manifested in this study aligns with notions of problematizing from mathematics education. Allowing *Space and Perspective* to be problematic and encouraging learners to problematize their experiences in the world, may be a promising strategy for conveying multiple perspectives related to hypermedia. This implications section ends with a fourth consideration and concerns the mediating technologies necessary to make this project possible and considers promising trends in this area.

Cases as Alternative Perspective Supporting Shift

Cases as alternative perspective (CAPs) have the capability to support productive struggle. According to the National Council of Teachers of Mathematics (NCTM), effective mathematics teaching "embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (2014, p. 48). The tentative manifestations in this study indicate learners experienced this type of opportunistic struggle, part of the shift phenomenon.

A shift in perspective, drawn from the learners' experiences in this study, is triggered by the elusive qualities inherent in the cases as alternative perspective. Certain cases and activities from the class sessions were pivotal, such as the book, *Flatland* (Abbott, 1992) and *Flatland: The Movie* (Travis & Johnson, 2007). The elusivity of cases provoked learners, leading to recognition of more elusivity from the cases and/or bringing about feelings of impasse/dissonance. Usually these feeling would lead students toward discussion or reflection.

Discussion was associated with feelings of frustration, enjoyment, validation, and discomfort. The impasse may cause learners to problematize cases further to uncover more elusive aspects and even contribute similar cases that exude elusive qualities. Throughout the class and continuing after the project ended, learners experienced shifts or changes in the way they saw their world, usually marked by a realization that their current perspective is incomplete. Sharing these recognitions and shifts again occurred throughout discourse and reflection. In terms of the shift in perspective, learners talk about this with the emotions of discomfort, acceptance, and eureka/wow. There was an iterative relationship between a moment of shift and a new lens for revisiting prior cases. Throughout this experience, problematization permeates as a way of being, an attitude or lens, from which all the other emotions and provocation tools (such as cases as alternative perspective) are considered.

Understanding how shifts in perspective manifest for learners is helpful for adding to our understanding concerning CFT and the design of (CAPs). Although this study did not aim to test one design principle over another, this study helps us understand a possible shape of the shift experience and the related activities that contributed to a shift in this particular context. A large gap in the hypermedia design literature concerns learners' ability to consider and take on alternative perspectives. Strobel, Jonassen, and Ionas (2008) show learners' difficulty accommodating and creating multiple perspectives in a hypermedia system in their design-based research study. However, learners' thinking about concepts from multiple perspectives is shown to improve in Zydney's (2005, 2010) two design iterations with high school learners Specifically, learners did well in both studies considering multiple perspective of a science problem after using the *Pollution Solution* hypermedia software exposing learners to 4 different perspectives on an environmental issue (economic, legal, technical, and environmental).

According to Spiro et al. (1988), ill-structured domains are shaped by multiples perspectives and these need to be conveyed to avoid reductionism and possible misconceptions. However, the current research is inconclusive concerning the benefits of hypermedia, the sorts of shifts in perspective that might manifest, and design strategies for conveying multiple perspectives.

Although this study does not claim that hypermedia systems support multiple perspectives, learners attribute part of the shift experience to cases on the *Space and Perspective* hypermedia site, especially *Flatland: The Movie.* In a pilot study (Valentine & Kopcha, 2014) using the pivotal case, *Flatland: The Movie,* learners experienced similar shifts, expressed as an ontological 'trying-on' of dimensions, analogous to the characters of *Flatland*. This pattern across two iterations suggests that conceptual change strategies, such as using a pivotal case (Linn, 2008) and supporting learners to develop bridging analogies (Clement, 1993, 2008) may support the development of shifts on the part of learners.

A second consideration when designing hypermedia systems concerns the context-specific, complementary theory to guide the design of cases. CFT is a meta-theory that informs advanced knowledge acquisition in ill-structured domains and relies on domain-specific theories. Mathematics is a relatively new domain for the application of hypermedia and specifically CAPs. Each domain has specific processes that guide the way learners engage in activity (e.g., inquiry in science, debate in political science). Regardless of the strategies considered, it seems an important consideration when designing CAPs for the hypermedia site. For this project, Realistic Mathematics Education (RME) aligned well with the principles of CFT. For example, cases were developed with real-world phenomena as the starting points for investigations. More specifically, CAPs illustrated and conveyed the complex nature of space, perspective, and dimension, designed to that illustrate connections between domains (geometry, algebra, number,

engineering, etc.). These cases formed part of more encompassing investigations on the hypermedia site asking learners to engage in activities with the phenomenon. More details about the ways CFT and RME merged to create the *Space and Perspective* hypermedia site is articulated in *Chapter 3: The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site.*

Connections between Perspective Shifts and Learning

This investigation sought to understand what it is like for middle school learners to live through shifts in perspective as part of mathematics learning. Two intense manifestations were highlighted and include: (1) the moment of realization that one's understanding is incomplete and (2) feeling compelled to act (problematization). Figure 2, described earlier, shows these intense manifestations related to other tentative manifestations. To understand what this phenomenon of a shifting perspective may mean for various fields in education, this section will compare Figure 2 to models of learning in mathematics education and more broadly. First, it will help to see the connections purported between perspective and learning in the learning sciences related to complex systems from Goldstone (2006):

> Complex systems are exciting developments in the learning sciences because they are not standard ways of approaching the world, but once understood can become a conceptual tool with the potential to dramatically transform one's perception of this world. How can the pedagogical promise of complex systems best be realized? One key element is suggested by taking literally the notion that learning involves *perceiving* the world in new ways. Thomas Kuhn (1962) described how scientists, when exposed to a new theoretical paradigm, see physical phenomena in new ways: "Though the world does not change with a change in paradigm, the scientist afterward works in a different world" (p. 121)

> and "During [scientific] revolutions, scientists see new and different things when looking with familiar instruments in places they have looked before" (p. 111). (p. 39)

This notion, "that learning involves *perceiving* the world in new ways" manifested in this project with learners. This sentiment connecting learning to shifts in perceiving is echoed by Jacobson and Kapur (2012), Jacobson and Wilensky (2006), Jonassen (2011), with others, usually accompanied by notions of complexity, context, and ill-structuredness.

The phenomenon of a shift in perspective may also be thought of in relation to "folding back" (Pirie & Kieren, 1994), which is the activity of revising former informal noticings mathematically. Figure 4.4 shows a model of folding back (indicated by the non-linear line) adapted from Pirie & Kieren's (1994) model for the dynamical growth of mathematical understanding. Most notable from Figure 4.4 is the portrayal of non-linear growth folding back to earlier knowings and noticing (e.g., from image having to primitive knowing).

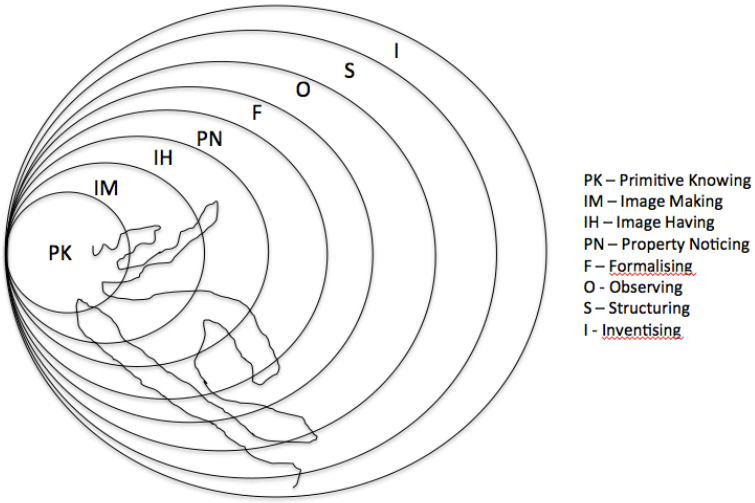

Figure 4.4. Model of folding activity from Pirie & Kieren's (1994) model for the dynamical growth of mathematical understanding. Adapted from "Growth in Mathematical Understanding: How Can We Characterise It and How can We Represent It?," by S. Pirie & T. Kieren, 1994,

Educational Studies in Mathematics, 26, p. 187. Copyright 1994 by Kluwer Academic Publishers.

Within mathematics education, another related perspective from constructivism portrays the equilibrium model (e.g., Piaget, 1977; Steffe, 1991; von Glasersfeld, 1991), where learners act to dissipate perturbations in the environment. This is portrayed as containing a goal, activity, feedback, and results. In this model, as shown in Figure 4.5 below, a learner has the goal of dissipating a perturbation. Through cognitive activity and feedback, the perturbation is either alleviated (reaches equilibrium) or the cycle continues until equilibrium is reached.

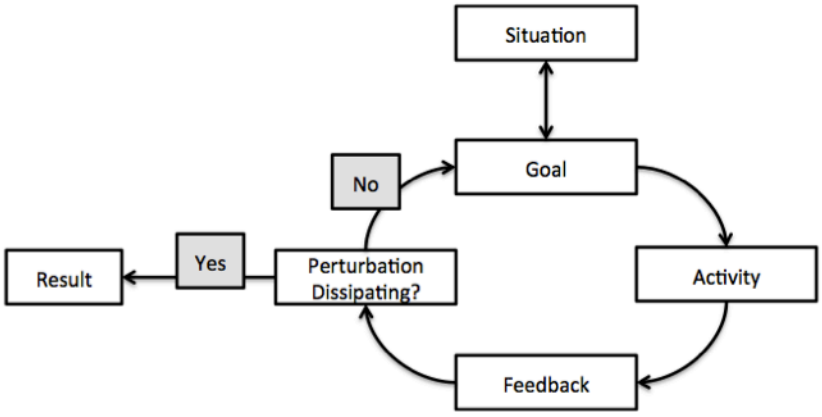

Figure 4.5. Equilibrium model.

One other theory about learning that is considered in relation to the manifestations from this study comes from an embodied cognition perspective, where learning is conceptualized as "embodying a distinction" (Reyes & Zarama, 1998, p. 28). Reyes and Zarama created a model with "four stages: declaring a break; drawing a distinction; grounding and embedding a distinction; and embodying a distinction" (p. 28). In their model the last three stages equate with their operationalization of knowing, understanding, and learning respectively.

Learning is not a static notion, but a fluid activity that exhibits patterns we might want to consider. In the section below, each of the perspectives described above are situated side by side to show the variant ways learning is conceptualized, including the figure created from this study. In this juxtaposition of variant conceptualizations of learning with the phenomenon of a shifting perspective, I contend that this study gives us a lens to see overlaps in the theories and sheds light on the connections between shifts in perspectives and learning more broadly.

1. The environment (cases). The environment, or world, consists of phenomenon. The hypermedia site for *Space and Perspective* includes 70 cases as alternative perspective, drawing on real world phenomenon for learners to consider in their investigations with space, perspective, and dimension. Of all the models mentioned above, the equilibrium model is the only one that identifies an outside influence, labeled "situation," as part of the phenomenon of learning. Still, Reyes and Zarama's (1998) model implies an environment as the very thing being distinguished throughout each phase as does Pirie & Kieren's (1994) model. This similarity is not that surprising, but does indicate that the environment, or world is not theorized (or experienced) separate from the activity of learning. One element that seems missing from the models shared is the social aspect of learning. For learners in this study, their peers contributed to their shifts through various forms of discourse and reflection.

2. Provocation. Feeling provoked stimulates an active stance on the part of learners. *Flatland* is a prime example of a case as alternative perspective that incites learners to ask questions, conjecture, extend ideas, etc. In Figure 2, the circular arrows indicate that sometimes this spurs learners to seek out other cases, such as animations of hypercubes. Even still, the elusivity of complex phenomenon cannot always be easily integrated by learners, leading to impasse and dissonance. The models of learning juxtaposed lack this feeling except maybe the

equilibrium model, labeled "activity." Still activity doesn't seem to capture this particular manifestation when considering a shift in perspective. Even Pirie and Kieren's (1994) model doesn't indicate the provocation. Stilll, in order to have a reason to fold back on an informal, primal knowing with a new, mathematical lens, the learner must feel provoked in some way. The same can be said for Reyes and Zarama's (1998) model of making distinctions in the world.

3. Impasse/dissonance. This particular facet of discourse around learning can be traced back to Plato (2002) and Aristotle (2002) and I assume even further. Aristotle writes about the importance of impasse for helping learners identify what they don't know. Learners talked about impasses they experienced as being caused by the elusive nature of the cases. Impasse is not represented in Pirie and Kieren's (1994) model of dynamical growth of mathematical understanding, but can be found in the equilibrium model as "perturbation" and in Reyes and Zarama's (1998) model as "declaring a break" (a.k.a., "interruption of flow") (p. 28-29). Although not represented in the models being juxtaposed, many models of conceptual change, from revision to radical, identify the importance of learners "confronting" or recognizing the limits of their current reasoning. It is this recognizing, this impasse, that gives learners an opportunity to move forward.

4. Discourse/reflection. Discourse and reflection was a critical part of the experience of shift for learners. Not only is discourse a way of exploring impasse, but it supported learners in their recognition and articulation of their current perspective, including its blind spots. In Figure 2, discourse/reflection surrounds the intense manifestations of realizing the incompleteness of perspective. The emotions attributed to discourse/reflection in Figure 2 are frustration, enjoyment, validations, and discomfort. In the models of learning being juxtaposed, discourse and reflection are not indicated as such. It is possible that discourse relates to "feedback" in the

equilibrium model, although feedback in the cognition model does not imply discourse. Reflection most likely is a part of the Pirie and Kieren (1994) model, allowing the folding activity to take shape.

5. Compulsion to act. The compulsion to act, marked by the activity of problematizing, indicates moment of 'trying-on,' envisioning, or taking some sort of action. In Figure 2 this is represented as permeating every aspect of shift for learners. This particular action is represented with more specificity in Pirie and Kieren's (1994) model with activities of image making, image having, and property noticing. In Reyes and Zarama's (1998) model of learning, a compulsion to act, or problematize ones' world, is similar to the specific activity of drawing a distinction. The equilibrium model includes "activity" and "feedback," which is highly related to being able to distinguish. However, the goal-oriented aspects of the equilibrium model doesn't seem to capture the compulsion to act and problematize in situations where one is simply curious and is not so focused on an end goal, such as with the *Space and Perspective* project.

6. Shift. The shift in perspective for learners manifested with accompanying feelings of discomfort, acceptance, and a eureka/wow moment – meaning it's not necessarily a great feeling to notice phenomenon in new ways or to have your world "shattered" in a sense. It may help to consider this in line with Reyes and Zarama, who distinguish between "breakthrough" and "breakdown" (p. 29). This shift for learners manifests after the realization that something about their current concept isn't helping them integrate a case, a discourse, or any number of phenomenon. Sometimes this shift can motivate one to seek out more elusive aspects of cases. In other models, shift is portrayed as "grounding and embodying a distinction" (Reyes & Zarama, 1998, p. 28), a folding activity (Pirie & Kieren, 1994), and a "result" as with the equilibrium model (e.g., Piaget, 1977; Steffe, 1991; von Glasersfeld, 1991).

These connections between the shift experience and models of learning are important for informing CFT and the way cognitive flexibility, especially in terms of conveying multiple perspectives, may be theorized and this design to be brought about.

Problematization and the Phenomenon of a Shifting Perspective

Problematization is a notion building on John Dewey's (1933) idea of "reflective inquiry" as put forward by Hiebert et al. (1996). They consider problematizing in mathematics education "one principle for reform in curriculum and instruction" (p. 12) and a shift away from discourses attending to dualisms (e.g., acquisition and application, functional and structural mathematics, cognition and affect, etc.). They operationalize problematization:

> Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas, and questions for students. We do not use "problematic" to mean that students should become frustrated and find the subject overly difficult. Rather, we use "problematic" in the sense that students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills. (p. 12)

The reason dualisms lose their charge when the focus shifts to problematization, is that priority is not given to the task, to the functional versus structural concern, but to "notions of reflective inquiry and mathematical residue" (Hiebert et al., 1996, p. 20).

Hiebert et al.'s (1996) conception of problematization emphasizes the notion of problems as triggers for "reflective inquiry" (see Dewey, 1929, p. 189) and the iterative problematizing of one's experiences "in order to understand them more fully" (p. 15), and in this way, a seeker of

problems. This activity, although cyclical, also results in "radical reorientation," which Hiebert et al. (199) described as:

> Familiar objects, including subject matters in school, are treated as "challenges to thought…They are *to be* known, rather than objects of knowledge…[t]hey are things *to be* understood" (Dewey, 1929, p. 103, emphasis in original). "The subject-matter which had been taken as satisfying the demands of knowledge, as the material with which to frame solutions [becomes] something which sets *problems*" (Dewey, 1929, p.99, emphasis in original). (p. 15)

Learners engaged in reflective inquiry are simultaneously triggered by problems and actively seek them out. Hiebert et al. (1996) write, "they problematize their experiences in order to understand them more fully" (p. 15) or:

> When we treat an object as a problem to be solved and examine it carefully, said Dewey (1929), we begin to understand it, to gain more control over it, and to use it more effectively for our advantage. (p. 15)

This problematization, or reflective inquiry, stance towards the world and towards learning encompasses several activities, "action, overt doing, that changes something about the problems and/or the situation in which the problem is embedded. Activity is central to the process" (p. 15). In addition, this stance to living and learning involves perturbation, dissonance, and impasse. In Dewey's words as cited in Hiebert et al. (1996), it involves a "willingness to endure a condition of mental unrest and disturbance" (Dewey (1910, p. 13)" (p. 15).

So what does problematization as a primary concern, as argued by Hiebert et al. (1996) add to our understanding of CFT, RME, and the design of the hypermedia site and cases as alternative perspective? Mathematics education is oriented towards supporting learners to solve

problems, invent strategies, and construct understandings about relationships in the world. Cases (as alternative perspective) and CFT with the associated hypermedia will need to consider these domain-specific concerns in application. A prime example where confusion may exist in the design of CAPs involves the differing perspective of cognitive apprenticeship models (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991), where the focus is on learning from an expert. In case-based reasoning (Kolodner, 2006), expert scenarios provide content for the case, usually text or video-based, although Jacobson (2008) and Spiro et al. (1988) do not make expertise a tenet of CFT or hypermedia design. Cognitive flexibility is different than expertise. Cognitive flexibility is characterized by constructing multiplicities and complexities. Mathematics is most similar to inquiry, where problem solving, not mastery of expert skill, is the focus. Where learners most likely produce a product in apprenticeship models, problem solving (and problematization) is visible in the residue of discourse, sharing strategies, and writing.

In this project with *Space and Perspective*, the activity of problematizing is evident in relation to several of the design strategies, residual outcomes, and activity in the learning environment. For example, the cases as alternative perspective intentionally "triggered," or propelled learners to act. This design strategy in addition to establishing norms for engaging in conjecture, discourse, etc. allowed for a sustained reflective inquiry. An important consideration when developing and implementing the cases with learners that Hiebert et al. (1996) make clear, is gathering knowledge of student thinking in order to select tasks (in this project, cases) that connect to their experiences and that are relevant. Teachers also need knowledge of the subject to select "tasks" that encourage cognitive demand and opportunities for learners to struggle with meaningful concepts and relationships. This is an important consideration because "[t]asks are inherently neither problematic nor routine" (p. 16).

Another residue from *Space and Perspective,* relating to problematization as explicated by Hiebert et al. (1996), includes learners problematizing their experiences of seeing, moving, and their ontological limits (dimensionally and pertaining to infinity). In order to support this kind of learning, the classroom teacher will need to allow troubling and unresolved concepts/problems/ideas to remain troubled. Sometimes a natural tendency among mathematics teachers is to reduce cognitive demand. However, the act of engaging in "reflective inquiry" (Dewey, 1929), requires that teachers sustain, rather than reduce, cognitive demand (Stein et al., 2008). Attending to and revisiting norms of engagement is one strategy for creating a learning community that sets expectations for problematization. In *Space and Perspective*, norms were established and revisited throughout the project as well as many opportunities for reflection (individually and as a group).

Additional parallels between problematization and shifts in perspective relate to the actual shift moment and Dewey's (1929) notion of "radical reorganization" (described above). In order to support these moments (and the process of) shift, learners will need to be given opportunities to seek out problems in their world and experiences. In addition to attending to norms of engagement that encourage this shift, or reorganization, in the learning environment, it may be helpful to consider Pirie and Kieren's (1994) "folding back" notion, especially in the context of mathematics classrooms. The idea of revisiting prior experiences in order to mathematize (Freudenthal, 1973), notice mathematical relations in previous informal experiences (Pirie & Kieren, 1994), radically reorganize (Dewey, 1929), and problematize (Hiebert et al., 1996) all support the activity of troubling, questioning, inquiring, conjecturing, etc. the everyday phenomena in the world in order to understand it at a deeper level. Supporting learners to develop multiple perspectives purports the same aims.

Technology as Mediating Tool

The primary research question asks, "What is it (like) for learners to find themselves perceiving space that is problematized?" Technological affordances change the phenomenon that we as humans can access. *Flatland* as a book, unaccompanied by the film adaptation, would most likely not be as pivotal of a case because the book version doesn't contain the animations that supported learners' visualization. A rotating tesseract and fractal are difficult to represent with three dimensions, but again, computer animations make these invisible phenomena, visible. Near the introduction, I wrote about the microscope and telescope already impacting the way scientists (and humans) have come to know our world. These affordances give us access and the ability to manipulate speed, time, distance, and foreground/background. In this way, they mediate our access to phenomenon. The hypermedia site used for the project acted as another mediating technology, connecting cases in criss-crossing patterns, learners' blog sites, digital artifacts, and acted as a place to connect both in and out of class. Technology permeated the tasks, communication, and ability to collaborate. Although coursing through every aspect of *Space and Perspective*, mediating technologies presence seemed invisible.

Future research may explore the ever increasing mediating nature of technology and the implications for shifting "core curriculum" foci. With greater access to phenomena (and ideas), it seems a core makes less sense than identifying a range of phenomena to investigate. For example, what if learners were asked to mathematize (at appropriate levels) the game mechanics of video games (e.g., perspective of character relative to world, control of movement, object manipulation, etc.)? Technology affordances offer a rich array of "manipulatives" for learning and extend beyond apps and Web 2.0 communication and collaboration tools. They are hiding in

the functionality and the programming layer as phenomenon to investigate. This trend is already evident in the makers movement, coding activities, and game design camps.

Limitations

Every investigation is limited by the questions asked and methods used. By choosing to investigate experiential facets related to a learning environment design, we better understand a particular way shifts in perspective can manifest for learners. However, measures of transfer, cognitive flexibility, and mathematization were not directly measured. This makes it impossible to draw causation claims between learner outcomes in relation to CFT and the use of CAPs. However, the exploratory nature of this phenomenological investigation informs design to some extent with regards to designing CAPs as discussed in the implications section. Additional limitations concern my role as a teacher researcher, adolescent age participants, and other particulars affecting this study. These are addressed below in addition to a discussion concerning notion of validity and four related "validation strategies" (Creswell, 2013, p. 250) carried out in the study.

My Role as a Teacher Researcher

I integrate limitations throughout this paper, such as the role of the researcher and the commitment to post-reflexion at all stages of the research. To recap, a post-reflexion practice finds ways to articulate connections and disconnections, assumptions of normality, bottom lines, and shocking moments throughout the research process (Vagle, 2014). A few themes that cropped up in my post-reflexion journal and acted as a lens for my noticing of the phenomenon include my experience as a teacher, specifically in the school where I conducted this research. Although I was not the teacher at the time of this study, I chose to implement the *Space and*

Perspective project with learners and also research their experiences. Although I contend that this position benefits what we are able to glean about experiences of middle school learners, it was certainly a lens that framed my seeing of the phenomenon throughout.

Adolescent Age Participants

A second theme in my post-reflexion journal entries concerns the learners (a.k.a., participants) themselves. As a former middle school teacher, I tout having some understanding of the complexities of adolescence and its very real implications for teaching, learning, and research. Adolescence is a period of human growth and development marked by an assertion of independence, possibly related to actively defining oneself. Sometimes this manifests as asserting authority or questioning authority, testing boundaries, taking risks, and increased self-consciousness. This particular age seems ideal for problematization investigations, but also makes me wonder how this might manifest in similar and different ways for learners of various ages. The problematic aspect of adolescent learners as participants for this study concerns the interviews. Although the learners I talked with were very verbose and appeared comfortable to talk freely, there are many learners that did not agree to be interviewed. Of the one's I did interview, I had the sense that some feelings and ideas could not be gleaned from the 1:1 interview that seemed to arise naturally in classroom discourse with peers as captured in the audio. There are many complexities concerning the adolescent learner that will not be reconciled in these pages, but it is a definitely a limitation and at the same time an addition to the literature where young learners' phenomenological data are scant in comparison to adults.

My suggestions do not lean towards avoiding this particular group of learners, but I do recommend that researchers seriously consider innovative ways to work with this age group, to understand more about their potential to engage with complex concepts while at the same time

recognizing that traditional research techniques may not be optimal. Traditional techniques of attempting to remove oneself as the researcher from the context seems like it would block access to open discourse with adolescents. And thus we come full circle to my earlier limitation as teacher researcher. I do not see it as only a limitation, but also as a promising strategy for understanding the lived nature of phenomenon from the perspective of adolescents.

Validation Strategies

The aim of the study is not to generalize to the population. "What motivates the researcher to consider a large number of cases is the idea of *generalizability*, a term that holds little meaning for most qualitative researchers (Glesne and Peshkin, 1992)" (Creswell, 2013, p. 101-102). In the exploratory work of phenomenology, the aim is rather to understand the lived qualities of a phenomenon, requiring a strong commitment to contexts and the humans inhabiting them. Two other traditionally quantitative terms concern validity and reliability. Validity is not conceptualized as less of a concern in post-intentional phenomenological work, but rather is a concern that the method embraces through strategies such as post-reflexion. Creswell (2013) conceptualizes validity as a process rather than a way of verifying:

> I also view validation as a distinct strength of qualitative research in that the account made through extensive time spent in the field, the detailed thick description, and the closeness of the researcher to participants in the study all add to the value or accuracy of a study. (p. 250)

Even still, he suggests using at least two different "validation strategies." (p. 250).

In this study, I used four validation strategies and these include: "*Prolonged engagement and persistent observation* in the field," "*Peer review or debriefing*," "*Clarifying researcher bias*

from the outset," and "*Rich, thick description"* (p. 250-252). In this section I will describe each of these strategies in detail.

Prolonged engagement and persistent observation. "Prolonged engagement and persistent observation in the field includes building trust with participants, learning the culture, and checking for misinformation" (Creswell, 2013, p. 250). I intentionally spread the *Space and Perspective* project over a one year period: 2.5 months with learners in the classroom and the rest of the year for follow up data collection (lived-experience descriptions, interviews). Using audio recorders, video, photographs, artifacts, field notes, voice memos, etc., I tried to capture multiple perspectives (observations) of the learning space and my reflections engaged in the research project. My role as a former teacher in the school and specifically as a former mathematics teacher with 50 % of the learners participating in this study, contributed to building trust with participants. In the case of new participants, I found former contacts (learners from my former 5th grade class) pivotal for helping me construct a space where trust was easier to build. As a former teacher, I was also able to recall the nuances of the school, teaching community, and student culture. Of course, each classroom is different and I was mindful of picking up cues from those around me.

Peer review or debriefing. My research collaborator (advisor) and I came together weekly to share understandings about the method (before collecting data), to talk through data collection in the field, and during the data analysis process. We asked hard questions about the data and spent much time agreeing and disagreeing. Although consensus in an ultimate sense was not achieved (or the goal), the space to sound off ideas was a useful strategy for post-intentional phenomenological research, helping me confront my blind spots.

Clarifying researcher bias. Clarifying researcher bias is addressed in the activity of post-reflexion described earlier. The example of my role as teacher researcher comprised an ongoing theme in the journaling activity (also shared previously).

Rich, thick description. Creswell (2013) writes, "Rich, thick description allows readers to make decisions regarding transferability" (p. 252). Post-intentional phenomenology naturally lends itself to a rich, thick text, although notions of description are problematic with assumptions of a transcendent view point (see earlier discussion in methods section). Still, the presentation of data, retaining the voice of learners, allows the reader to engage in a simultaneous process of engagement where "decisions regarding transferability" are made available. It is important to understand however, the results from this study seek to open up our understanding of what it is like for learners to experience shifts in perspective related to concepts of space, perspective, and dimension and in relation to hypermedia cases that intentionally seek to bring about this experience.

Concluding Remarks

This study investigated the ways learners experienced shifts in perspective. The hypermedia site, *Space and Perspective,* designed by building on the success of a pilot study (Valentine & Kopcha, 2014), continued to use *Flatland: The* Movie as a pivotal case. The pivotal case provoked learners to contemplate space, perspective, and dimension concepts. In addition, the hypermedia site included 69 more cases and minicases guided by Cognitive Flexibility Theory (CFT) and the domain specific theory of Realistic Mathematics Education. The addition of cases, such as "The Top 100 Video Game of All Time," also provoked learners to problematize everyday activities in order to understand aspects of their world with more depth.

The learners articulated their experiences through several manifestations, including: moments of realization that their current perspective (concept) was incomplete and a propensity to problematize their world. These moments were brought on by other manifestations of experience, such as feeling provoked by the elusiveness of cases, encountering impasse and dissonance, and feeling discomfort and acceptance. These manifestations were triggered by cases as alternative perspective (especially the pivotal case, *Flatland*) and various means of discourse and reflection.

The ways shifts in perspectives manifest for learners are helpful to informing the ways designers of hypermedia sites or software may consider conveying multiple perspectives through cases as alternative perspective. There is little research in hypermedia design or CFT concerning learners' perspectives, an important consideration for flexible and usable knowledge construction. Parallels between learners' shifts and problematization and between learners' shifts and several models of learning from this study may spur future questions and research to understand this possible connection.

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CHAPTER 5

CONCLUSION

This dissertation project sought to investigate an innovative learning environment design and the related ontological experiences of learners' shifts in perspective. The three articles that comprise the bulk of this dissertation (Chapters 2, 3, and 4) share the context of a middle school mathematics classroom and the goal of conveying the complexity of space, perspective, and dimension. The exploratory nature of post-intentional phenomenological research methods set a foundation for future educational design efforts involving hypermedia design, in particular, design seeking to develop cases that support learners' shifts in perspective. This conclusion looks across all three articles and summarizes the implications of the larger body of work. In this conclusion chapter, I first discuss how the articles in this multiple-article dissertation fit within the context of mathematics education.

The Context of Mathematics Education

The National Council of Teachers of Mathematics (NCTM, 2014a) just released *Principles to Actions: Ensuring Mathematical Success for All*. Two parts of this title are terms repeatedly used by NCTM for over twenty years (2014b): *success* and *all*. In this conclusion, I connect not only the articles represented in this multiple-article format dissertation, but argue that these articles are highly related to issues of access and equity noted by NCTM.

Throughout the course of developing these articles, I have presented my work on *Space and Perspective* at a number of professional organizations (i.e., Association for Educational Communications and Technology, International Society of the Learning Sciences, etc.).

Colleagues repeatedly ask me to articulate how my work will improve math achievement. If math achievement is defined as doing well on a standardized test, then I'm not sure I can say it will. However, there may be other outcomes associated with *Space and Perspective* that are equally if not more valuable. Engaging with cases as alternative perspective, considering higher dimensions, ontologically 'trying-on' dimensions – these are experiences that provoke and awaken the mathematical mind. I feel no hesitation asserting that these are meaningful and worthwhile experiences in the context of mathematics that will affect learners on a deep level, and most likely resonate for a lifetime. Still, I have no information about whether these experiences will help a learner do well on a standardized test. So what is "success" and more specifically "mathematical success"? And what does success look like "for all"?

According to NCTM, *access and equity* is one of the five "essential elements" of successful mathematics programs (2014a, p. 59). *Principles and Standards for School Mathematics* states, "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12). Flores (2007), adds another dimension to this view of access and equity concerning "opportunity gaps":

> Often, inequalities in achievement are perceived as the result of a hierarchy of competence. When the very students who have been given more opportunities to learn show higher achievement than students provided fewer opportunities to learn, they are perceived as more capable or having more aptitude. This manner of talking about achievement gaps without mentioning opportunity gaps that cause them invites a focus on deficit models to "explain" low performance in terms of factors such as cultural differences, poverty, low levels of parental education, and so on. (p. 40)

Flores' opportunity gaps help me better explain the "so what" of this project. This project is about an *opportunity* to conjecture in mathematics class, an *opportunity* to ask questions about our world, an *opportunity* to consider the ill-structured aspects of a typically routinized domain, an *opportunity* to talk and reflect with each other, and mostly, an *opportunity* to be mathematicians. This is different from the many mathematics classrooms I've observed where learners are simply repeating a procedure that others spent years to discover. According to Ellis (2008) and Ellis and Berry (2005), an obstacle to access and equity is instruction that barely focuses on meaningful learning but rather foregrounds rote skill and procedural learning.

Space and Perspective encouraged learners to problematize their space in a way that helped them develop multiple perspectives of space and dimension. Exploring a concept from multiple and varied perspectives are the building blocks of developing a flexible, usable concept that transfers to future situations. Findings indicate that learners' varied perspectives persisted months after investigating space and perspective. For example, one student (Alistar, in Chapter 4) applied the concepts from *Flatland*, a pivotal case, to game design. Another (Beck, Ch. 4) expressed his shift in viewing Cubist art at museums. These particular manifestations indicate that concepts from the cases can be related (transferred) to future experiences.

Still, *Space and Perspective* is not intended to substitute for standards-based mathematics programs. But the content, as examined using a phenomenological method, helps us see what is possible when learners are free to conceptualize mathematics apart from procedures, rules, and static terminology. The concepts of space, perspective, and dimension, as experienced in this project, seem to align to the views of the International Commission on Mathematical Instruction's (ICMI) *Perspectives on the Teaching of Geometry for the 21st Century.* As part of the series, Hansen (1998) writes, "the necessary and sufficient criteria" for the "selection of new

matters to include in a core curriculum" are explicated as follows: (1) "it contributes in a significant way to the grasping of space" and (2) "it is learnable (not the same as teachable)" (p. 241). *Space and Perspective*'s central aim is to support learners' "grasping of space," an idea expressed in Realistic Mathematics Education (Freudenthal, 1973, 1983).

In the sections that follow, I briefly synthesize the three articles. This is followed by a discussion of the unifying themes of the articles and implications for learning, design, and technology. Next, I take up the issues of access and equity in mathematics education by considering the notion of success. This chapter ends by highlighting future directions for research and parting thoughts.

Looking Across the Articles

Each article contained in this dissertation sought to highlight an inherent problem concerning education in general, one of reducing complexity. Reduction in education manifests as a focus on basic, sometimes disjointed parts of a domain (i.e., facts, skills, etc.) that lend them to ease in teaching and assessment. Cognitive Flexibility Theory (Spiro, Coulson, Feltovich, & Anderson, 1988; Spiro, Vispoel, Schmitz, Samarapungavan, & Boerger, 1987) was constructed by Spiro and his colleagues along with recommendations for hypertext systems (now hypermedia) to allow learners access to the complexity in ill-structured domains. This focus in their work was a reaction to a large number of reductive biases Spiro et al. (1987) identified when learners simplified complex concepts. An example of this simplification includes the complex concepts of area and volume being taught as an algebraic formula, common in many curriculum materials used in schools. This can create mathematics learning situations that feel rigid and decontextualized for learners. According to NCTM (2014), "we must change a range of troubling and unproductive realities":

- Too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures.
- Too many students are limited by the lower expectations and narrower curricula of remedial tracks from which few ever emerge.
- Too many teachers have limited access to the instructional materials, tools, and technology that they need.
- Too much weight is place on results from assessments – particularly large-scale, high-stakes assessments – that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning.
- Too many teachers of mathematics remain professionally isolated. (p. 3)

My hope is that this start of my research agenda will help others seeking to support complex concept development on the part of learners. I chose a multiple article dissertation format in an effort to disseminate findings to the fields of mathematics education, learning sciences, and instructional design/educational technology. This work represents four years of design-based research and post-intentional phenomenological research examining learning experiences in middle grades classrooms that support flexible thinking, deep understandings, and mathematical dispositions of conjecturing, reflecting, and arguing. The goals are twofold: to explicate the characteristics of innovative design strategies in the context of practice and to understand learners' lived sense of emerging and shifting perspectives.

Chapter Two: Middle School Learners' Ontological 'Trying-on' of Dimensions: A Phenomenological Investigation

This first article shares the embodied ways that learners consider space, perspective, and dimension. The study suggests that *Flatland: The Movie* (Travis & Johnson, 2007) operated as a

pivotal case (Linn, 2008) by giving learners a provoking starting point to conjecture about dimensional relationships. Learners' concepts persisted a year later, suggesting that sustained learning may be supported by engaging learners with ontological, embodied experiences in middle school geometry. *Flatland: The Movie* served as a powerful analogy for learners, propelling them to "try-on" other dimensional seeing and perceiving. The study portrays middle school learners as highly capable of reasoning deeply about space, perspective, and dimension and helps open up manifestations of a possible geometry experience.

Chapter Three: The Embodiment of Cases as Alternative Perspective in a Middle School Geometry Hypermedia Site

This second article describes a framework for designing cases on a hypermedia site for middle school geometry (see http://spaceandperspective.com/). This design framework focuses intentionally on developing cognitive flexibility and alternative perspective-taking through criss crossing real-world cases. These cases are intended to support investigations with space, perspective, and dimension. Major theories informing the framework include Cognitive Flexibility Theory (CFT) and Realistic Mathematics Education (RME). CFT is a learning theory that recommends introducing complexity early in instruction to avoid reductive biases and misconceptions in later learning, while at the same time promoting usable and flexible concept development. RME is a mathematics education theory that advocates mathematizing real-world phenomena as starting points in mathematics teaching and learning.

The article draws on the design-based research strategy of conjecture mapping (Sandoval, 2014), which is a method for mapping design and theoretical conjectures in order to make explicit these mostly hidden facets of design-based research projects. In this article, a description of the design of cases as alternative perspective housed on the hypermedia site include embodiments of the learning environment (e.g., tools, materials, structures, practices), mediating processes these embodiments are conjectured to support, and outcomes relating to these processes.

The framework organized by the conjecture map is presented as a single article because it serves as a tool for design and research, both within this project as well as for future scholars. Sandoval (2014) writes that designing learning environments is a theoretical activity and "as researchers (and not just designers) we have an obligation to be as explicit as possible, *in advance"* (p. 20) about how learning happens or can be designed to happen. The conjecture map presented in this article helped organize conjectures about the ways multiple theories and design principles (CFT, RME, and hypermedia design) inform design decisions related to the processes and outcomes they are expected to bring about. Of particular interest to other designers, is the integration of CFT (Spiro et al., 1988). As a meta-theory, CFT will need to be "custom fit" around the nuances of specific content areas. The map itself offers a rich exemplar of CFT principles as they are integrated within a mathematics context; this has the potential to offer researchers in other content areas a model for integrating CFT. The map also offers a powerful tool for research. By mapping the design, I am now able to make adjustments based on the actualized trajectory. This allows for an intentional iterative design process while informing practice and theory elements that are essential in design-based research.

Chapter Four: Productive Problematization: Moments When Perspectives Shift in Middle School Mathematics

This last article comprising the dissertation investigates the ways learners experience shifts in perspective, in large part relating to the *Space and Perspective* hypermedia site containing the cases as alternative perspective. This third article lends itself nicely to targeting three potential communities: learning scientists, instructional designers, and mathematics educators. This study built on the success of the design noted in the pilot study (Chapter 2), which drew heavily on conceptual change as a theoretical perspective. In that study, *Flatland: The Movie* served as a pivotal case. The pivotal case provoked learners to contemplate space and dimension concepts from an embodied perspective. Drawing on that pilot study, the current study continued to use *Flatland: The Movie* as a pivotal case but added 69 more cases and mini-cases to the hypermedia learning environment. As such, the current study examined CFT as it emerged in the design of a hypermedia environment that provided cases as an alternative perspective (Jonassen, 2011). Guided by CFT, the large number and variety of cases as alternative perspective are provided to support learners' flexible concept development and alternative perspective taking with space, perspective, and dimension concepts. The study investigated the primary research question, "What is it (like) for learners to find themselves perceiving space that is mediated through technology"? In addition, a secondary research question asked, "What role does the learning environment play in the experience for learners"?

As part of the phenomenological method, the learners articulated their experiences through a variety of sources (e.g., lived experience descriptions, interviews, conceptual maps of their knowledge, etc.). Several manifestations of the phenomena emerged, including moments of realization that their current perspective (concept) was incomplete and a propensity to

problematize their world. Problematizing refers to being both triggered by problems (cases) and the act of troubling one's own experiences in order to understand the world's complexities more fully. These moments were brought on by other manifestations of experience, such as feeling provoked by the elusiveness of cases, encountering impasse and dissonance, and feeling discomfort and acceptance. These manifestations were triggered by cases as alternative perspective (especially the pivotal case, *Flatland*) and various means of discourse and reflection. By better understanding the ways shifts in perspective manifest for learners, we see the possible ways theory from CFT and the design of cases as alternative perspective (Jonassen, 2011) plays out in a middle school mathematics learning environment. To date, little research on these theories has occurred in a mathematics context or with middle school learners.

Unifying Themes and Implications for Learning, Design, and Technology

The field of Learning, Design, and Technology is interdisciplinary in nature, working across domains such as K12 mathematics education, medical education, mediation training, etc. The unifying themes indicated in the name represent learning sciences (how people learn), design (design to support learning or performance), and technology (technology integration, adoption, affordances, etc.). I share implications for the three articles concerning learning, design, and technology in the sections that follow.

Implications for Learning Mathematics

A shift in perspective manifests in similar ways as general and mathematical theories of learning (e.g., Piaget, 1977; Pirie & Kieren, 1994; Reyes & Zarama, 1998; Steffe, 1991; von Glasersfeld, 1991). In *Productive Problematization: Moments When Perspectives Shift in Middle School Mathematics*, Figure 5.1 represents the complex and possible ways that shifts in perspective manifested for learners. In this figure, the elusivity of the differing perspectives

inherent in the cases provoked learners, sometimes creating impasse and dissonance. Throughout the figure, discourse and reflection played a prominent role for supporting learners to realize that their current perspective is incomplete. The waves bubbling from beneath in the figure suggest that problematization of one's experiences weaves in and out of these shift moments.

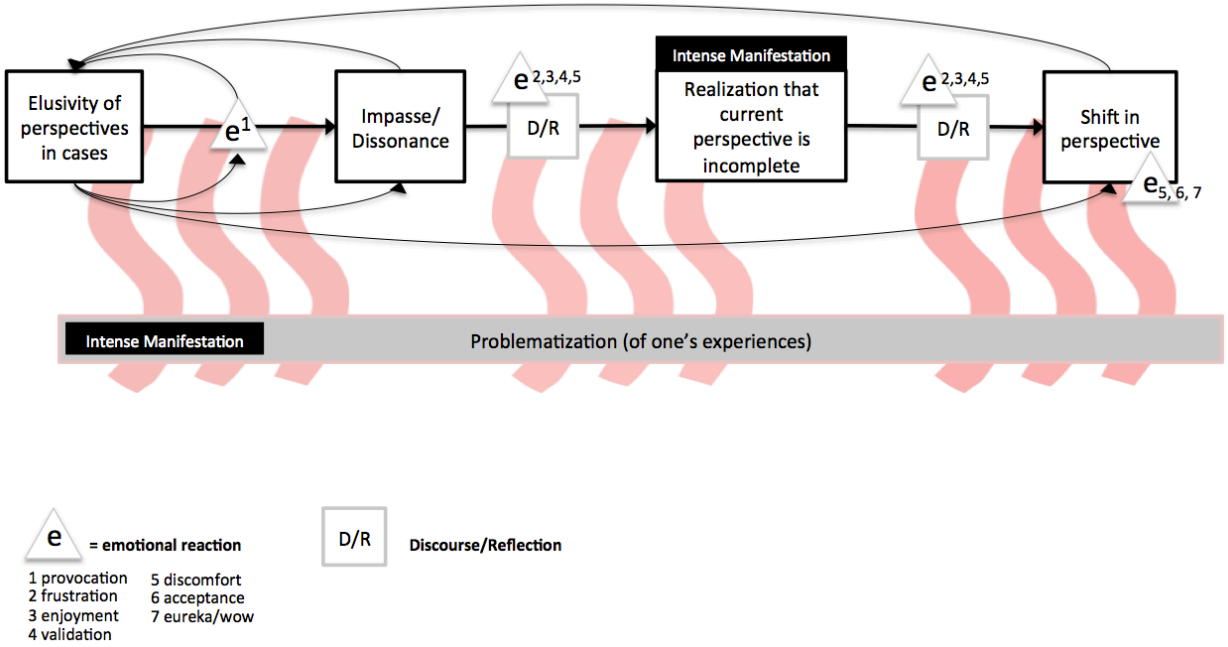

Figure 5.1. The figure shows the relationship between intense and tentative manifestations.

When juxtaposed with models of learning, the shape of a shift in perspective shares six broad characteristics. These include: the environment (cases), provocation, impasse/dissonance, discourse/reflection, problematization (or action), and shift. For learners, a shift in perspective manifested as a fluid activity that exhibited patterns in line with many learning theories. In this sense, the phenomenon opened up in this study informs our understanding of possible ways learning can be conceived and the associated design of learning experiences supporting perspective shifts.

Implications for Design

The ways shifts in perspectives manifest for learners are helpful to understanding CFT and the design of cases as alternative perspective. Both stress the importance of learners considering multiple perspectives, indicative of ill-structure domains. However, there is conflicting research about the effectiveness of hypermedia design cases to support multiple perspectives (Strobel, Jonassen, & Ionas, 2008; Zydney, 2005, 2010). After two iterations of design-based research, the pivotal case *Flatland: The Movie* remained an effective case for supporting learners 'trying-on' of dimensions and perspective. It guided them to reason about sight, perspectives, and motions in each of the dimensions. In addition, *Flatland* provoked them to problematize their own three-dimensional space. It is possible that certain features about *Flatland* made it a pivotal case, such as the animation or the characters analogous reasoning between Flatland and Lineland, Flatland and Pointland, etc. One learner indicated that the characters were relatable and allowed him and his classmates to have a similar base in which to have a discussion about space, perspective, and dimension.

In addition, learners did not indicate feeling overwhelmed by the hypermedia site, but rather reported (by survey and interview) that they liked having everything in one place. Although the hypermedia site integrated Jacobson's (2008) hypermedia design framework for creating representational affordances (e.g., text, images, simulations, video), other scaffolds were immersed in the design as part of the mathematics focus. These included blog prompts, chunks of cases organized by investigation (sometimes 30 cases in one investigation), directives to click on certain links, and text explicating information about a link. Shapiro (2008) describes many of these scaffolds in her design framework, HAL. There is research indicating negative learning effects with multiple scaffolds (Zydney, 2005, 2010). Although learning was not assessed in this

study, learners did not indicate a negative association concerning the multiple hypermedia scaffolds. It's possible that the embedded nature of the scaffolds (Shapiro, 2008) as part of a common Wikipedia blog design, did not appear out of the ordinary for learners. They did not consider the site to be a "software program" or an educational site, but a website with all of the materials, links, videos, etc. that they would need for the investigations. It's possible that the learning environment, a 1:1 laptop school with projectors in every classroom and teachers who are encouraged to integrate multiple Web 2.0 tools in their teaching, shapes learners' perceptions about hypermedia sites. If it is simply perceived as another website, then it's possibly just another tool they access and interact with each day. It is also possible that the nature of the learners in this school – exposed to non-traditional curriculums, smaller class sizes, and highly interactive, dialogic learning spaces – perceive hypermedia and scaffolds differently than learners in more traditional school environments. This is an area where future research may be warranted.

Implications for Technology

Mediating technologies change what is visible to the eye – the telescope and microscope comprising some of the earlier examples in history. Technology in this case is not always synonymous with digital devices. However, mediating technologies such as slow motion cameras, gyroscopic cameras, magnetic resonance imagine (MRI), and x-rays all mediate the natural way humans see the world (with their eyes). The hypermedia site, *Space and Perspective*, contained many cases characterized by these mediation affordances. The site provided opportunities for learners to criss-cross these cases to explore the many facets of space, perspective, and dimension. The opportunity to see from a different perspective, visualize an impossible space, and slow down time affords learners an opportunity to revisit taken-for-

granted phenomena. This is the revisiting activity advocated for by both CFT and RME. This activity is conjectured to help learners revisit the multiple perspectives of a concept as well as revisit phenomenon when mathematizing the world. With the increasing affordances of mediating technologies, it is possible that other concepts in mathematics, geography, and art/music, to name a few, may benefit from a similar criss-crossing of mediated cases.

In addition, technologies such as blogging, collaborative Google documents, and Skype allow for increased paths for communicating and collaborating about ideas. Although *Space and Perspective* used these communication and collaboration technologies, they did not seem to take center stage. Yet, they were part of the learning environment and learners used them heavily throughout discussions, reflections, and collaborative activities. This suggests that they may have operated in the background. The implications from this research suggests that technologies have the capability of changing the types of phenomena learners mathematize in the classroom as well as supporting many of the 21st century standards including communication, collaboration, creativity, and critical thinking.

Access and Equity in Mathematics Education

I opened this conclusion acknowledging the need to address discrepancies of access and inequities for opportunities in mathematics education classrooms in the U. S. as articulated by NCTM (2014a), Gutiérrez (2010), and Flores (2007). Definitions of success, according to Gutiérrez (2010) are: "largely driven by discourses of achievement and proficiency on standardized exams, tangible outcomes that can be measured in some way" (p. 7). She takes issue with this reality and suggests that self-actualization may be an indicator of success that is currently lacking in the discourses:

> Rarely do our definitions of success include self-actualizing – the idea that we should be allowed to become better people by our own definitions, not just those prescribed by schooling…the goals we have for students may be disconnected from the ways in which they see themselves now or in the future. (p. 7)

Defining success myopically with standardized tests has been problematized by NCTM (2014a) as well in their explication of "troubling and unproductive realities," shared in a previous section (p. 3).

In this project, *Space and Perspective* was developed in an effort to support learners' reasoning in geometry - space, perspective, and dimension particularly. By designing and researching learning experiences of the type described in these articles, it is possible that future policies and initiatives can point to the value added, especially with deep descriptions of the learners' experiences, adding a sorely missing voice to the conversation. I acknowledge that mathematics is foremost a "human practice" and "teaching and learning mathematics are not politically neutral activities" (Gutiérrez, 2010, p. 4). A first step may be to shift our discourse about success from one of "proficiency on standardized exams" to one about becoming "better people by our own definitions" (Gutiérrez, 2010, p. 7)

The exploratory results in this dissertation suggest a shift in how success might manifest for learners. The *Space and Perspective* project, and learners' experiences engaged with the investigations, allowed them access to complex concepts in a way similarly described by NCTM (2014a), Flores (2007), and Gutiérrez (2010). The learners in this study showed success in non-standardized ways, such as conjecturing about physical phenomenon like gravity in a two-dimensional world, considering geometry and perspective as it relates to informal experiences at an art museum, and integrating cases from class to construct projects like a video game.

Opportunities to "grasp space," to visualize, to conjecture, and to consider the ill-structured, complex nature of mathematics seems to capture a non-standardized type of success. As Jörg et al. (2007) eloquently wrote, "Rather than being framed in ends-oriented terms, education might become possibility-oriented" (p. 152). It is the possibility of what could be that drives the exploratory research in this project.

Future Directions

I am preparing for the next stage in my journey as a tenure-track Assistant Professor in Science, Technology, Engineering, and Math (STEM) Education in the Curriculum and Instruction Department at West Virginia University (WVU). This research-intense position requires a commitment to trans-disciplinary approaches to increase STEM proficiency for all learners through iterative design research, P20 partnerships, and excellence in teaching. The relatively new focus on STEM as an integrated field is promising for the exploratory work I hope to continue at the beginning stages of my career. I have already started to seed conversations about a potential STEAM (adding the art) focus as we start this new initiative at WVU.

Becoming a STEM trans-disciplinary researcher reminds me of the importance of complexity thinking (Davis & Sumara, 2006) in educational research. Complexity thinking means attending to "multiple levels of interest (neurological, subjective, interpersonal, cultural, etc.)" (p. 3). I am planning to work with those from arts and sciences, agriculture, and engineering as part of the Flexible Education Research Network (FERN). Although this is a new initiative and I'm not certain what all of this will mean in practice, I feel strongly that it will be important to view problems at many levels, from diverse perspectives, and simultaneously work with future teachers to imagine possibilities. In addition, I intend to continue exploring phenomena that are inadequately understood.

Research and Writing In the Works

An article in the works, but not part of this dissertation, seeks to inform the intersection of design-based research and qualitative inquiry, specifically post-intentional phenomenological research. As designers working in complex spaces, we can benefit from post-intentional phenomenological methods to better understand how phenomena of interest are lived out in the spaces we impact. This particular methodological tool is well positioned to support iterative design work in educational research, especially in spaces integrating emerging technologies and innovative learning strategies. I am currently writing an article with Mark Vagle and Theodore Kopcha titled, *Offering Post-Intentional Phenomenology as a Way to Investigate Complex Phenomena in Educational Design Research*, to further conceptualize phenomenological research in fields that conduct design research.

Plans for Near Future Research

One of the original research questions I planned to address in the final article concerns communication technologies and asked, "How do the various forms of communication (Skype, face-to-face, blogs, photo diary, etc.) unfold"? There is a tremendous amount of data available to answer this question. However, the data informing the phenomenon of moments of shift in perspective were so dense and focused that these seemed like two different studies, with a different lens on analysis. Although discourse and communication played a large role in the shift experience for learners, this question is better answered in another study. Examining how technology and face-to-face discourse supported student communication and collaboration would help the scholarly community better see the relationship between discourse and advanced knowledge acquisition. The blogging activity in particular may help refine the prompts embedded in the hypermedia site and possibly indicate a supportive structure for learners.

This dissertation presents two iterations of design-based research that occurred over a four-year period. It is not clear, however, if the shift in perspective is entirely due to the design or the content itself. Future iterations of this work could examine the application of cases as alternative perspective to other complex mathematical concepts, such as measurement in geometry or integrals in calculus and may also consider additional contexts (e.g., science education) and learners (e.g., elementary age). This would offer educational designers integrating cases as alternative perspective more examples to see the nuanced ways context specific considerations and hypermedia design work together. In addition, multiple iterations across contexts may start to reveal patterns similar to the importance learners' attributed to the pivotal case.

Parting Thoughts

This entire project has intentionally focused on opening up what is possible with learners – creating a space where educators, researchers, and policy makers can reconsider what is valuable and possible in a mathematics classroom. We may find it beneficial to consider the role of complexity thinking in our work with learners. With a complexity lens, we can find ways to work collaboratively across disciplines and contribute to many ways and levels of understanding. Jörg, Davis, and Nickmans (2007) convey the complexities of research:

> We must first become aware that we tend to take complexity of practice for granted. Only after recognizing this tendency will we be able to recognize the complexity of reality of education. To escape old ways of thinking, we need to face the complexities of educational practice by starting "to think in complexity" (Mainzer, 2004). Consequently we should develop new tools of thoughts, of thinking beyond dualism, reductionism, the

> calculable, and the strong wish of being in control of what happens in practice (Prigogine & Stengers, 1984). (p. 151)

By increasing our awareness and developing multiplicities in perspective, we are not condemned to solipsism, but we are made aware of blind spots and possibly find ways to open up important phenomena for others. Reeves (2011) remarked that the "continued failure of educational researchers to have meaningful impact on real world educational problems" is "heartbreaking" and "deplorable" (p. 15). I agree.

When I first started the Learning, Design, and Technology (LDT) program, I was hesitant. Michael Hannafin asked me why LDT and not mathematics education. I remember telling him that mathematics education was certainly my experience, but I wanted to be able to work across fields in order to view mathematics education from a different perspective, in this case, as a learning scientist, a designer of learning experiences, and someone who is able to best leverage technological affordances. After a year in the program, I started to question some of the engrained discourses in LDT. I was accepted to the Mathematics Education Department, but decided to stay in LDT. This was the right decision. Just today, at the end of my dissertation, I realize why.

My passion lies with children. Every question I ask and every effort I extend is to better the lives of children. I don't see myself passing along knowledge or information or even desire for that to happen. Rather, I want to provoke them to question, to act, to rebel, and to take control of their blind spots. I realize this is a reflection of me, but it is at the core of why I entered the field of education. Mathematics seems an unlikely field to provoke these tendencies, but it is not impossible. In an effort to challenge norms operating in school mathematics, to deconstruct what

mathematics thinks it is and what it could be – these are my daily exercises. Gutiérrez (2010) captures this poetically:

> This move to challenge what counts as mathematics is driven not from a perspective that assumes certain students cannot be motivated by abstract versions of mathematics (Dowling, 1998) or that all mathematical practices should relate to the "real world" in a concrete sense, but rather from a perspective that assumes that mathematics as a human practice can become more just. (p. 19)

I now return full circle to my confidence in staying in the LDT program. I am not sure I would have had the freedom to investigate mathematics philosophically, practically, as a designer, or any of the various lenses I had the freedom to wear during this program. These lenses gave me an opportunity I didn't recognize until recently – a position to merge discourses and adopt a "sociopolitical stance" (Gutiérrez, 2010), which is rare in a discipline like mathematics, mostly regarded as "devoid of human influence" (p. 21). Gutiérrez writes that challengers of this view "come from researchers with one foot in mathematics and the other in philosophy, sociology, science studies, or anthropology" (p. 21). I conjecture that I have feet in mathematics education, philosophy, learning sciences, and design/technology. It is with this foundation that I, in a similar vein to A. Square from *Flatland*, plan to "stir up a race of rebels who shall refuse to be confined to limited Dimensionality" (Abbott, 1884/1991, p. 102).

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APPENDIX. KERI VALENTINE BRIDLING 10-12-2011 (A.K.A. POST-REFLEXION)

Mark's first four articles have helped me see a sort of bridling in action. Reading about it in Dahlberg's book was descriptive and full of relational ties to the lifeworld, but I don't think the full importance was sinking in. I also didn't find it problematic because I feel like I'm reflecting all the time on my study, my personal life, course readings - everything. I try to write about the times I feel compelled or feel like I'm going to forget a connection I'm making.

The struggle in relation to the dimensional study, is that each reading, each conversation, each re-reading of the transcripts, my comments and notes, each time I edit the class video and photos to remove students that aren't part of the study - each of these moments and more, cause a shift in my own perspective concerning the phenomena. It's not that it changes in a way that makes me disregard everything I thought before - sometimes it strengthens notions of meaning. This is when it seems like a good time to bridle.

I don't feel the phenomenon is as open as it could or should be. I know I need to talk to students again and I plan to use the questions and comments I generated on the transcripts. I wonder if I can ask questions about aspects of the experience that a student may not have talked about that others remember? Is this leading? I think it is, but in a way if the goal is to open this up, then maybe its a part of the experience that can be attended to - maybe they haven't talked about it because I as the interviewer didn't know it needed tending to. I can see parts of their dialogue where, if I could do it over, I would have tried to seek more depth. I can see the importance now of multiple interviews. I wonder with children though, how long an interview can really go on? Not that I don't think they have as much, if not more to say, than adults - there's just something I can't point at yet. Since I'm interested in learning and learners, it might be helpful to look for some readings regarding children and open-ended interviewing.

For now, I want to remember a conversation I just had with my husband - one he viewed as an interview. It started with me telling him about a grant for doctoral students focusing on improving programs and instruction for low SES/high achievers. At first, I almost stopped reading because I realize I'm not really interested in one type of learner over another...I'm just interested in learning that engages one so deeply at the core, they are compelled to explore, and in a sense become "life long" learners. The type of learning without single answers - sort of like an ill-structured problem, but really an ill-structured concept. It can apply to anyone and I believe everyone deserves a chance to tap into this part of themselves.

Anyhow, I started thinking how I could take this current study (I guess this is a pilot?) and create a similar experience for low SES/high achieving students in Clarke County. In fact, I don't see why it has to be limited to high achieving students at all. This is when my husband chimed in and said that he thought what was happening in the learning of dimensionality with the current study, was that kids were being helped to be learners. I was struck and a little confused by this. I never thought about that as something that might have been happening. I just thought they were so engaged with the idea that their perspective about space and the world they lived in was "troubled" and caused some type of cognitive dissonance. It seems like they "just had to know" as one student describes. But I couldn't just tell them - I didn't know. I had ideas about space, but I was also finding myself in disequilibrium. I also wanted to know about this new way of imagining the world - along side them. It's like contemplating what had always seemed so secure and stable. In a way, I think this is what happens with our thoughts - and again why I'm seeing the benefit of bridling - they are always in a state of flux. Sometimes just a slight sway in which you choose to roll through a right on red and other times a jolt, where red starts to take on the qualities of green.

So in this "interview" with Pat, where he was trying to explain learning from his perspective to me, he talked about his "low achieving" status and how he considered himself a learner, just not at school. He started talking about how he would talk to his mom in the car about very precise and expansive knowledge of cars - he would point out all the specs of many odd car types. Yet, in school he did poorly. He said he wasn't “focused in”, it just wasn't what drove him. I asked what this was like - not achieving in school, not feeling driven or focused in. He said, “I was just disinterested. I cared, but not enough to drive me.” I found myself wanting to know more. What is it that makes learning disinteresting in one context and not in another? Are these the secondary questions that Mark writes about? I'm confused about that part of his writing - what are secondary questions?