LEIBNIZ'S LABORATORY OF CONCEPTS: THE STATUS AND STRUCTURE OF INFINITESIMALS AS METAPHYSICAL LABORATORY

by

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(Under the Direction of O. BRADLEY BASSLER)

ABSTRACT

In this dissertation I take up a reading of Leibniz's metaphysics from the perspective of his mathematical reflections on the nature of infinitesimals. A longstanding tradition of interpreting Leibniz's treatment of infinitesimals has focused on the problem of its logical consistency as the key to unlocking its relation to the rest of his philosophy. Scholarly inquiry around the status of "infinitesimals" throughout Leibniz's work has thus tended toward logical reduction. By pointing to some key moments where this approach runs into difficulty, I develop an alternative where the speculative consequences of Leibniz's infinitesimals can be viewed through the lens of the mathematical, rather than logical, concepts produced through his mathematical work. The issue of the status of infinitesimals thus provides a means by which to understand important aspects of his metaphysics which is often misunderstood. In turn, this exploration allows a rereading of important developments in Leibniz's treatment of the structure of reality as an actual infinitely divided one. This rereading provides a different means to understand the role that the reality of bodies and their motion plays in his mature metaphysics. As such, I suggest that Leibniz's mathematical reflections serve as a laboratory of concepts which not only have profound consequences for his own means of thinking through problems but can also guide us in understanding aspects of his method and scope of his work. The first chapter sets the stage for the rest of the dissertation by a cursory examination of the short-comings of logical reduction in order to underline the nature of the speculations afforded by Leibniz's lifelong engagement with the infinite and infinitesimal. The second and third chapters treat the problem of the status of infinitesimals in Leibniz's work through a critique of contemporary interpretations and underlining the importance of his mathematical reflections through epistemological and metaphysical lenses. The fourth and fifth chapters treat the development of Leibniz's commitment to the reality of corporeal bodies and motion in the light of his mathematically embedded assertions about the actual infinite dividedness of reality.

INDEX WORDS: Leibniz, infinite, infinitesimal, indivisible, syncategorematic, metaphysics of motion, corporeal substance, labyrinth of the continuum.

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DEDICATION

To my grandmothers 涂陳璧人 and 杜劉月嬌

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CHAPTER 1

MOTIVATION AND INTRODUCTION

"The mathematicians have as much need to be philosophers as philosophers have of being mathematicians." –Leibniz, Letter to Malebranche, 13/23 March 1699[1]

1. Some mathematical and philosophical motivations

The dissertation below developed from a deep interest in the relation between mathematical concepts and philosophical practice. Since the founding texts of Plato and Aristotle, we have seen a deep influence of mathematics in philosophy, constituting at various times a paradigm of clarity and certainty, at other times a marker of mind independent reality and at yet other times a field that generates the very terms for explaining the ordered nature of the world. At some of the most critical junctures in the history of philosophy, the very proofs and concepts developed in mathematics have served as positive inspiration for alternative concepts that philosophers thought could break beyond sedimented arguments concerning questions about what is and what is not, unity and multiplicity, part and whole, ordered and unordered, limited and unlimited and the like. At the same time, philosophers have also attempted from the time of Plato and Aristotle to stand outside of mathematics in order to give this activity and the sort of knowledge that it produces a classification, a proper place in the diversity of knowledge disciplines and human activity. Here the technicality involved in mathematics has often limited

[1] Leibniz, *Phil* I, 356. Please see Appendix A for abbreviations in citing Leibniz's work.

the degree to which philosophers could grasp the most advanced aspects of mathematical work, and as such mathematics has often superseded the characterization of its activity at the very moment of its classification by philosophers. Despite this philosophical limitation, however, mathematicians have nonetheless benefited from this sort of mutual appreciation. Some of the epoch making mathematicians in history were often keen readers of philosophical texts and were not ashamed to cite philosophy as inspiration. Indeed the various reciprocal relations between mathematics and philosophy mentioned above can be used as themes in the study of any sequence in the history of philosophy. For example, the problem of interminable divisibility of continua and the emergence of irrational numbers both circulate throughout ancient Greek philosophy. In the modern period, the idea of a mathematical physics and the development of the infinitesimal calculus posed challenges and opportunities for rationalists, empiricists and transcendental thinkers alike. In contemporary philosophy, Cantor's transfinite numbers and Gödel's incompleteness theorem provide a multitude of philosophical problems concerning multiplicity, the possibility of knowledge and the status of science that is still to be fully resolved.

As such, the figure of Leibniz is of particular interest in that he was both a groundbreaking mathematician as well as a major philosopher. He was certainly not the only philosopher during this early modern period to have practiced both mathematics and philosophy, yet Leibniz does represent a figure where the longstanding relation between mathematics and philosophy intersects in a very intimate way. Leibniz lived at a historical crossroad, a time when the very relation between philosophy and mathematics was being recast. Between the increasing work on mathematical physics in the late 16th and 17th centuries and the philosophical contributions of Descartes, Hobbes, Gassendi, Spinoza and others, Leibniz entered into a historical context where traditional philosophical claims were being disputed and reconfigured. In this context, some of the central philosophical problems concerned the status of mathematical knowledge itself and how deeply it related to an account of reality. This age of discord is certainly reflected in his work ranging from juridical and theological texts, writings on logic, mathematics and physics, to reflections on metaphysics and epistemology. Far from claiming any of these areas as the "heart" of Leibniz's philosophy, one can nonetheless assert without much controversy that the mutual relation between his mathematics and metaphysics constitutes at least a key dimension of his thought. Read against the larger historical background of the relation between mathematics and philosophy, Leibniz is particularly interesting insofar as the expression of this relation in his work centers on the problem of the infinite. In this, one can simply reference his famous remark, "In truth there are two labyrinths in the human mind, one concerning the composition of the continuum, the other concerning the nature of freedom. And both of these spring from exactly the same source – the infinite."[2] His treatment of the infinite, greatly informed by his work on the infinitesimal calculus, is one that greatly impacted how the concept would be understood in the following centuries.

Moreover, Leibniz's work provided a wealth of resources for thinking about the relation between mathematics and philosophy especially as it pertains to the infinite. It is perhaps the status of the infinite/simal caught between a negative notion of something that is indefinite, not limited, not finite, and its possibilities for a positive designation that first opened up this field of investigation and invention. While Leibniz was not the first to enter into a mixed context of metaphysical, mathematical and perhaps also theological indetermination and speculation, he certainly represents one of the figures who changed the very nature of this field of investigation.

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[2] Leibniz, *PE*, 95.

In relation to the imaginary root (√-1) that the infinitesimal is often compared to in Leibniz, he remarked in 1702:

> [N]ature, mother of the eternal diversities, or rather the divine spirit, is too jealous of these marvelous variations for permitting a single model to depict all things. This is why she has invented in this elegant and admirable shorthand, the miracle of analysis, prodigious in the world of ideas, an object almost amphibious between being and non-being, that we call the imaginary root.[3]

Indeed this amphibian nature of the imaginary number, inhabiting some space between being and non-being, is equally applicable to the infinite/simal. Equally tied to the method (or "miracle") of analysis, the infinitesimal is, like the imaginary number, something that is at the limits of conventional determinations: between a negative designation (in-finite) and the possibility of a positive conception. It is precisely in this problematic and indeterminate field that Leibniz produced, in his mathematical thought, a space of invention.

Indeed, perhaps Leibniz's greatest single impact on future generations has been his invention of the infinitesimal calculus. While both Newton's and Leibniz's work on the calculus, as well as the work of many intermediate mathematicians who have systematized and developed these initial contributions, have been synthesized in contemporary textbooks, Leibniz's notation for the differential (dy/dx) has remained canonical. Yet Leibniz's most renowned invention has also been the site of the greatest amount of controversy. We can mention but leave aside the charges of plagiarism brought against Leibniz by Newton and his supporters. What is more important for us is that even during Leibniz's lifetime, debates raged concerning the reality and legitimacy of the use of the infinite and infinitesimal in calculation and their possible relevance

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[3] [Author's translation] Leibniz, "*Specimen novum analyseos pro scientia infiniti circa summas et quadraturas*" in *Naissance du Calcul Differentiel*, ed. and intro. by Marc Parmentier (Paris: Librairie J. Vrin, 1995), 396.

for interpreting reality. In the contemporary period of mathematics, opened by Dedekind and Cantor's work on the infinite, the reevaluation of Leibniz's mathematical reflections has only intensified. Yet in response to these skeptical accusations concerning the rigor of infinite series and infinitesimal quantities during Leibniz's lifetime, he provided a powerful response in 1701 that gives us something of an interpretive orientation:

> [I]n lieu of the infinite or the infinitely small, we take quantities as great or as small as it is required so that the error would be less than the given error such that we do not differ from the style of Archimedes except in the expressions, which is more direct in our method and is modeled more on the *art of invention*.[4]

Here Leibniz asserted an art of invention that allows us to provide expressions that can more directly intervene on sedimented problems (the Archimedean problem of proportions filtered down through the centuries). Certainly, more than an art of invention within the confines of mathematical thought itself, Leibniz's larger philosophy can also be read through a spirit of invention that extends this attitude into metaphysics. As a key to unlocking this method of invention, we can trace how the infinite/simal itself extends beyond mathematics to inform some of the fundamental bases of Leibniz's metaphysics. It is through the force of this conjunction of remarks and considerations that I entitled this dissertation project "Leibniz's Laboratory of Concepts". Leibniz's relationship with mathematics provided him with a wide resource of concepts that were constantly put to work in a reinvention of traditional metaphysical problems evident in his novel and often counterintuitive solutions.

Much of the contemporary philosophical reevaluation of Leibniz's work on the infinitesimal calculus has been articulated in a way that is fundamentally different from what informs my investigation. In the contemporary evaluation, most of the emphasis has been put on

[4] [Author's translation] Leibniz, *Math* V, 350.

the problem of mathematical foundations, that is, the foundations of Leibniz's use of the infinite/simal in mathematics. Much of this approach is informed by the early twentieth century debate over the foundations of mathematics itself. Beginning with the work of Frege, Hilbert and Russell, this question of foundations concerned all mathematical entities. What constitutes the objectivity of mathematical entities? What are numbers? This problem of mathematical foundations is however equivocal. One could interrogate the question from a philosophical or mathematical perspective. From a mathematical perspective, problems of foundation concern the foundational methods and elements that allow a particular field to be mathematically rigorous according to current or canonical modes of evaluation. Here the foundational question concerns a problem of rigor that remains intra-mathematical. From a philosophical perspective, however, problems of foundation concern the relation of mathematics with natural language, logical determination, psychological activity or metaphysical reality, relations that provide mathematical entities with an extrinsic basis. Both types of foundational questions are present in Leibniz's vast work, but due to the differences in the nature of posing these types of question, the various sources for responding to them in Leibniz correspond to very different solutions. Indeed, Leibniz was faced with an intra-mathematical problem of the rigor of infinite/simal quantities and he was also faced with problems concerning the status of mathematical entities themselves. At times he was faced with both problems at once. Since the infintite/simal is problematic as a mathematical term insofar as there is no infinite/simal entity or number, it presents a difficult limit case for the problem of accounting for numbers in general. It is from this crucial but equivocal question of foundations that my research began on the laboratory of the relations between mathematics and metaphysics in Leibniz. That is, it is from this reflexive question concerning the metaphysical status of mathematical entities and the mathematical conditions of metaphysical conceptions that

I began an intensive look at how to separate and relate the mathematical and metaphysical in Leibniz.

Stemming from this basic motivation, a fundamental critique of Leibniz's contemporary reception underlies the whole of the dissertation that follows. In the contemporary evaluation of Leibniz's use of the infinite/simal, either as philosophical concept or mathematical terminology, one often finds the unsatisfactory idea that Leibniz's use of these terms should be reduced to a less problematic field of terms or objects. This aim in interpreting Leibniz effectively undoes the problematics that Leibniz himself carefully constructed around the problem. There are roughly three ways in which this occurs. The first version of this reductive reading is a "logicist" one. The logicist reduction takes Leibniz's confrontation with the problems of the infinite/simal as something ultimately reducible to a standard set of logical elements. As such, this sort of response tries to legitimate Leibniz's use by making it compatible with other standard sorts of mathematical entities and in turn, justifiable as consistent in terms of a standard logical evaluation. This position is best represented by B. Russell who in his 1900 publication, *The Philosophy of Leibniz*, attempted to employ this logicist approach not only to Leibniz's use of the infinite/simal and mathematical ideas but also to his metaphysics.[5] Indeed, the fact that Russell was a logicist about mathematics as such, the idea that the entire field of mathematics could ideally be put in terms of logic, should not surprise us. A second problematic reading of Leibniz is a kind of "philosophy of mathematics" approach. This approach aims to reduce Leibniz's use and conception of the infinite/simal to a problem of the sort of entities they represent inside the realm of standard mathematical entities. As such, this approach tends to be both anachronistic and narrow. By assuming that the dispute over the very nature of mathematical entities can be bracketed and separated into a different debate, this reading of Leibniz renders the problem of

[5] Bertrand Russell, *A Critical Exposition of The Philosophy of Leibniz* (Oxon and New York: Routledge, 2002).

infinite/simals resolved once it can be put on par with other standard entities. This approach is often anachronistic in the sense that the very "crisis" of foundations that treats mathematical entities in this manner had only arisen in the late 19th century, and this way of posing questions was not Leibniz's own. While I agree that one should make clear Leibniz's explicit reflections on the relation between the infinite/simal and standard mathematical entities both in the context of the 17th century and in a contemporary context, this approach often reduces the problem to an intra-mathematical one. This reduction tries to legitimate Leibniz's employment of these infinite/simal terms in the context of the history of mathematics and the calculus, ignoring and dissolving the larger interrelation that Leibniz saw between mathematics and his philosophical concerns. A third unsatisfactory approach is the merely developmental one. Reading the development of Leibniz's views requires an appreciation for detail insofar as he often changed his mind on a large number of issues, often within the same period. From the period before and around 1671 and the period around and after 1676, Leibniz definitively shifted from admitting infinitesimals as indivisibles to rejecting indivisibles and developing a new concept of infinitesimals. In this extremely productive period, Leibniz may have taken, explicitly or privately, every position in between. My research on this problem owes a great deal to the developmental approach but is unsatisfied with the fact that it often focuses on the resolutions that Leibniz produced at various important junctures. Indeed, it is necessary to see how Leibniz justified his various positions in his development, yet these changes in position are accompanied by a larger set of philosophical concerns that his mathematical views would condition and constrain. As such, without understanding how Leibniz constructed the problematics that would provide the very frameworks and motivations for his various changes of position, this developmental approach will remain merely a catalogue of different views without penetrating the metaphysical and epistemological means by which he understood the stakes of his arguments.

The dissatisfaction with the contemporary evaluation of Leibniz's treatment of the infinite/simal led to a reconsideration of how one should understand Leibniz's metaphysics. Insofar as Leibniz employed the idea of the infinite in many aspects of his work (logic, metaphysics, epistemology and the like), this led me to consider how changes in his treatment of the infinite itself could engender different results in these other philosophical and extra-philosophical (or para-philosophical) domains. Indeed I wondered how Leibniz's mathematics might have constituted a laboratory for his invention of metaphysical concepts. In this, the basic critique of the influential logicist position provided a key. In the logicist vision of Leibniz's metaphysics, the conception of substance is intimately tied to Leibniz's development of the subject-predicate logical model. In his metaphysical conception of individual substance, most canonically given in the *Discourse on Metaphysics*, the nature of substance is tied to the idea of truth.[6] That is, what is true is the correct identification of a predicate that belongs in the subject of a proposition. As such, a true proposition is the correct inclusion of a predicate in a subject. To use Leibniz's own example, "Julius Caesar crossed the Rubicon" is true insofar as "crossed the Rubicon" is included in the subject "Julius Caesar". More than a logical model, this is also a metaphysical model, Leibniz's way of thinking through the nature of substance in the *Discourse*. Indeed Leibniz's modal theory, his distinction between the necessary and the contingent, the idea of possible worlds, the status of the compossible as well as important metaphysical elements (such as the celebrated principle of sufficient reason) are all connected by this intimate link between the metaphysical and the logical. Based on this idea, a contemporary logicist position has attempted to read all of Leibniz's metaphysics into this logical configuration. More precisely,

[6] The *Discourse on Metaphysics* is hereafter abbreviated as *Discourse*.

this position attempts to read all of Leibniz's metaphysics through the idea of a substance based on the logical model of the subject-predicate theory of truth.

Following the work of M. Fichant, I maintain that this interpretation of a logical background as invariant between the metaphysics of the *Discourse* and the *Monadology*, two of Leibniz's landmark metaphysical works, cannot be maintained. Indeed Fichant has argued that the two texts represent a major transformation on the level of the model of substance itself. In the later *Monadology*, the monadological substance does not resemble the individual substance of the *Discourse* insofar as the monad is absolutely simple. The monad is an entity, or "entelechy" to use Leibniz's terminology, which does not fundamentally serve to subsume or include predicates but rather plays the role of an absolute simple from which other complex things can be composed. This simple observation requires a longer explanation, one that I will enter into in the following chapters, but its implications are potent. If we can accept that Leibniz turned from a model of logical truth to a model of simplicity and aggregation, we can also understand that the later monadological view took a metaphysical problematic that is more closely related to a problem of points, parts and continuity. One should be careful not to equate this metaphysical transformation with a mathematical or geometric turn, but there does seem to have been a transformation in terms of the problematics that they engender. Hence, despite the mediating complexity in interpretation, one can at least claim in the most general terms that it is the intervening problem of the relation between points, parts and continuity that informs, constrains and conditions the very ground from which the models of substance in the *Discourse* and the *Monadology* are to be distinguished.

The distinction of metaphysical vision between the *Discourse* and the *Monadology* serves to provide a symptom of a deeper unresolved question that I believe persisted in Leibniz's thought from his earlier period. Here I read this difference as a sign of the interrelation between mathematical, physical, epistemological and metaphysical concerns that were already present in his early work. Though I cannot address all these issues, I do believe that by taking some of the central elements concerning the problem of infinite/simals at work in the transition between the *Discourse* and the *Monadology*, I can at least provide grounds for developing some insight into Leibniz's laboratory of invention. Here I aspire to provide the grounds for a larger investigation that will be able to more fully answer the question of how changes in Leibniz's work on the mathematical conception of the infinite/simal impacted and conditioned his larger philosophical project. My contribution to this question in the present dissertation is in the development of a reading of Leibniz that locates some of the problematic areas that can open up this approach, a reading that is critical of some contemporary research on the same field and that focuses on this crucial passage between the *Discourse* and the *Monadology*.

Without yet entering into my interpretation and criticism, I wish to provide a little more depth to my basic inspiration. I will present two short arguments as an introduction to what I aim to accomplish. These arguments are neither developmental nor fundamental to what I will argue in the following chapters. They will rather be a means to pointing at the gaps in our understanding of Leibniz that indicate the need for the sort of research that I hope to develop in the dissertation. They provide the means to concretely fill out and give sense to the abstract motivations and unresolved questions with which I approach the following chapters.

2. Entering the laboratory: A comparison of arguments concerning the infinite

My first argument consists in showing that the problem of the infinite/simal circulates throughout Leibniz's metaphysics in a way that, even while allowing the problem itself to be somewhat independent of his metaphysical position, enters into his metaphysical reflections as a kind of problematic articulation. That is, rather than proceeding in such a way as to borrow from a mathematical solution to answer a metaphysical question, the two dimensions are joined through a problematic ground. In the following, I will present side-by-side two of Leibniz's arguments that employ the infinite. Chronologically speaking, my presentation proceeds in a historically inverse fashion. The first argument is from the period of the *Discourse*, a text dated to 1685-1689 concerning infinity as it relates to the inclusion of predicates in a subject. The second argument is from an earlier period 1678-1681 concerning the multiplicity of the world.

I pull the first argument from the text entitled "On the source of contingent truths". Before entering into a reading, I will provide a brief sketch of Leibniz's modal theory of this period, drawing from some of the resources in this text. This theory is perhaps best known today in a form that has filtered through the theory of possible worlds represented most notably by the work of David Lewis. This is the idea that possible things that are not actual in this world can be identified by reference to their actuality in another world. Whether other philosophers take these non-actual possible worlds to be real, in the sense that they exist as parallel universes, or as a simple thought experiment that allows us to represent a model of possibility, Leibniz himself had a very different idea. For Leibniz, contingent things are things that could have been otherwise for the simple reason that God could have created the world differently. For example, God could have *not* created Adam and insofar as Adam was *not* created, the murder of Abel by Cain would not have occurred. In our actual world, however, God did create Adam and thus allowed the causal chain that followed this act to take place. Conversely, if God had not created Adam but rather some other individual or a being with another nature, this would constitute another possible world. In short, Leibniz's idea of possible worlds is both different from its contemporary interpretation and is itself a complex theory in terms of its structure (theological, metaphysical and logical) and consequences. My aim here is not to enter into these complications but simply to provide a background. The more crucial point concerns the status of the necessary and the contingent.

What Leibniz was concerned with in his modal theory corresponded most fundamentally to the status of necessary and contingent propositions. Following a logical model of truth where what is true is the correct identification of predicates in a subject, Leibniz sought a means by which to separate necessary truths from contingent truths. In both necessary and contingent truths, something is true by virtue of the inclusion of the predicate in the subject. From a Kantian perspective, all truths in Leibniz's thought are in this sense analytic. Of course, Leibniz did not hold the same distinction between analytic and synthetic as Kant. Leibniz did hold an uncontroversial view of necessity and contingency, however, at least in terms of the meaning of these terms. As we discussed above, what is contingent and possible are things that can be, but can also be otherwise. In these conventional terms, a statement like "a triangle has internal angles that measure 180 degrees" is necessary and "Caesar crossed the Rubicon" is contingent. As we mentioned, God could have *not* created Adam, and he also could have created a world where Caesar did not exist or where he did not cross the Rubicon. The precise difference between necessary and contingent truths is not the analytic logical form of truth that they share but rather the number of steps necessary to demonstrate them. That is to say, a necessary truth, according to Leibniz, can be demonstrated in a finite number of steps but a contingent truth would require an infinite number. This distinction is in contemporary terms quite problematic and would certainly strike some readers as strange. I will not enter into the difficulty of fully justifying the argument. However, an explanation of this difference is certainly needed. How is it that necessary statements can be proven in a finite number of steps? The measure of the internal angles of a triangle can be proven in a finite number of steps in Euclid; there are six steps to be exact.[7] One should note that from a contemporary point of view, this is not always the case.[8] Why, for Leibniz, were an infinite number of steps needed for demonstrating a contingent truth? Contingent facts require a demonstration that must consider the chain of causes that bring them into actuality. Even if one could, with a super-human intelligence, identify the entire chain of causes that result in a contingent actuality, this chain could never be distinctly accounted for in a finite number of terms. For example, not only does each individual have for a cause their entire ancestry up toward the creation of Adam, but the multiple and various causes that intervened at every moment up to the point of some individual's birth intertwine to produce the precise context of that birth. Needless to say, adding to this complexity of each individual's existence, there are complex lines of environmental, historical and other causes that engender the cause of a particular event or the precipitation of an actuality in the existence of any particular individual. There is then no finite way of demonstrating the contingent inclusion of a predicate in a subject.[9]

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[7] Euclid, *Elements*, trans. Thomas L. Heath, ed. Dana Densmore (Santa Fe: Green Lion Press, 2002), 24-25.

[8] For a proposition like the Goldbach conjecture, where a prime integer greater than 2 is argued to be the sum of two other primes, one cannot refute, once it is proven, that it is necessary. At the same time however, there is as yet no direct way of proving it, and indeed, one must proceed step by step, evaluating every prime integer. Insofar as there are an infinite number of them, it is a proof with an infinite number of steps.

[9] Confusion may arise here concerning what precisely is infinite or infinitely complex. Is Leibniz invoking the infinitude of substance, the set of its true predicates or the world? While the infinite applies to all of these, the level at which Leibniz is treating the irreducibility of the infinite logical analysis of contingent truths to finite demonstrations in this context concerns the subject-predicate relation in each individual substance. No doubt the infinite complexity of any causal series in a world and the inherent infinite complexity within an individual substance itself are directly related in Leibniz's metaphysics. However, these different levels of Leibniz's treatment

Since this contingency could have *not* taken place, its very actuality is due to some chain (or even web) of causes that is infinitely complex. Again here, in contemporary terms, the mathematical distinction between very large numbers and the infinite may allow us to interject by arguing that, while this model of the contingent may present a chain of causes too extended and immense to humanly account for, it might not actually be infinite. This is a cogent counter-argument but I will leave it aside in the interest of pursuing Leibniz's own problematic. The important point between necessity and contingency in Leibniz's distinction of modality hinges on the means by which he employed the infinite. Even if we can, with a finer mathematical vantage, introduce a difficulty into Leibniz's position here, the point is finally that contingency and necessity share a common logical root but differ precisely in the complexity that they engender. With this short sketch, I will enter into the argument itself.

In this text, "The source of contingent truths", Leibniz made an explicit comparison, line by line, between the necessary and contingent distinction in the analysis of propositions and the difference between commensurable and incommensurable magnitude in geometric structure. The schematic form that the text takes highlights a mirror relation between the two logical and geometric sides which can be read analogically. The text is divided into two columns. On the left side, Leibniz distinguished between necessary and contingent truths. On the right side, he distinguished between commensurable and incommensurable magnitudes. In this he hoped to bring these two sets of differences into analogy in order to argue that the indemonstrability of contingent truth lines up with the infinite number of steps needed to find a common factor for

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can de distinguished. Through the method of logical analysis, Leibniz's argument here produces a convenient way to treat a number of different expressions of infinitude through a focus on true predication. Ultimately, the demonstration of any *one* contingent truth requires the demonstration of its causes and this is where infinitude is invoked in the argument. Here I try to emphasize Leibniz's remarks in the context of the argument without any assumptions about the larger metaphysics of modality at work.

two incommensurables magnitudes.[10] To begin, Leibniz lined up the notion "Truth is containment of the predicate in the subject" with the notion "Proportion is containment of a smaller quantity in a larger or of an equal in an equal."[11] With the link between the subject's containment of predicates and the proportion's containment of commensurable (smaller or equal) quantities made, Leibniz compared necessary truths to "the discovery of a common measure or a commensuration, and the proportion is expressible."[12] A line of four units goes into a line of eight units twice, this proportion is expressible, and the demonstration requires a finite number of steps. This, Leibniz held, can be compared to the demonstration of a predicate's inherence in a subject by a finite number of steps, the analysis of a truth to an identity in the context of necessary propositions. The difference consists in the fact that, as Leibniz explained, "if the analysis proceeds to infinity and never attains completion then the proportion is unexpressible, one which has an infinite number of quotients."[13] This is just the relation that holds between incommensurable lines; the search for a common root will involve an infinite (interminable) number of divisors with a remaining quotient. In the domain of worldly events in the left column, the comparison is made to a contingent truth that "involves an infinite number of reasons, but in such a way that there is always something that remains, for which we must, again, give some reason."[14] As we discussed above, one way of understanding this is that the complexity of causes for any mediate worldly event requires us at each stage of accounting to include the analysis of at least one other cause. This compounding of events, the irreducibility of a contingent truth to an

[10] The sort of relationship that Leibniz drew between geometrical demonstration and the analysis of contingent truths into identity is one that we can see in the second proposition of the tenth book of Euclid's *Elements*. There, Euclid presented the method by which two magnitudes are continually subtracted from each other in such a way that if the two magnitudes were commensurable, there could be a last possible subtraction that would put them in a definite ratio with each other. In the case of incommensurable magnitudes, such a mutual subtraction does not have an end. Euclid, 238-239.
[11] Leibniz, *PE,* 98.
[12] Leibniz, *PE,* 99.
[13] Leibniz, *PE,* 99.
[14] Leibniz, *PE,* 99.

identity in a finite number of steps in comparison to the infinite series of quotients in geometric proportion "yields", as Leibniz explained, "an infinite series".[15] This infinite series of evaluating worldly events, like the infinite number of steps required to find a common divisor for incommensurable magnitudes, renders these two distinctions analogical.[16]

The symmetry that Leibniz saw between God's perfect knowledge of the infinite and the domain of incommensurable proportions in geometry hinges on the latter's treatment of the notion of "irrational number".[17] Leibniz added that this field of irrationals is "distinct from common arithmetic", just as God's intuitive knowledge of the infinite (absolute and immediate) is distinct from "knowledge of simple understanding". Here Leibniz made an explicit reference to the tenth book of the *Elements* where Euclid developed a series of demonstrations concerning incommensurable lines. A number of propositions make use of the fourth definition in the first set of definitions of book X, that the irrational is defined as anything that is incommensurable with the rational. [18] Leibniz however understood this aspect of the *Elements* in a peculiar way. What Leibniz affirmed, quite rightly, is that for the nature of irrationals to be incommensurable to rational factors, their reduction into a rational proportion always produces a quotient. Yet Leibniz analogized the definition of this incommensurability with the naming of the "irrational number" in God's supposed intuitive knowledge of the infinite. Here he explained that "both are a priori infallibles and known each according to its kind." Knowledge of irrationals is gained

[15] Leibniz, *PE,* 99.
[16] Leibniz, *PE,* 99.
[17] An irrational number is related to incommensurable magnitudes by thinking of one of the magnitudes as a "unit" to which the incommensurable magnitude relative to it will be expressed as an irrational number of the "unit". No doubt, issues of infinitesimals are not far away here, as Leibniz seems to have suggested. In some ways, one of the historical roots of the idea of the infinitely small comes from the idea that the indefinite carrying out of the search for the least common measure between two incommensurable magnitudes could be arrived at if the process were to be carried out infinitely. As such, a fictional "infinitesimal" magnitude could possibly stand in for the common measure of incommensurable magnitudes. Cf. Euclid, *Elements*, 237.
[18] Euclid, 237.

"through necessary demonstrations known to geometry." This analogizes the necessity of God's knowledge of contingent truths with the necessity of the knowledge of irrationals in geometry.

In view of this odd use of Euclid, caution should be injected in underlining the fact that Leibniz's argument tempts a misunderstanding that incommensurables can be made commensurable, as it were, at the ideal infinite step of demonstration. This would not only be an erroneous position insofar as it misunderstands Euclid, but it would also misunderstand Leibniz. The distinction that Leibniz made between the knowledge of commensurable and incommensurable magnitudes is marked by the inapplicability of an "arithmetic" notion in understanding incommensurables. This problem of the incommensurable, Leibniz argued, is impossible "to be understood arithmetically, that is, they cannot be explained through the repetition of a measure". This insight, Leibniz argued, is parallel with the conclusion and main aim of his entire argument that "it is impossible to give demonstrations of contingent truths".[19] We should pay attention to the distinction between geometry and arithmetic here. Geometry functions by the comparison of magnitudes and proportions. In this, magnitudes are not "in themselves" commensurable or incommensurable. They are only so by their comparison with other magnitudes. In arithmetic, on the other hand, the unit is the starting place and one proceeds by the structure that unfolds from the operations on this unit, by the succession of units, and by the structure of wholes and parts within them. In short, arithmetic concerns numbers and numbers are themselves units and identities. The application of arithmetic to geometry then can only deal with the unities that occur in the latter. Avoiding the possible misunderstanding mentioned above, geometry is in this sense superior to arithmetic in that it has the capacity to treat the incommensurable relation between two continuities that are not reducible to a common divisor. In this, the validity of geometrical demonstrations in their affirmation of the positive

[19] Leibniz, *PE,* 100.

existence of incommensurable proportions stands in analogical relation to contingent truths. In other words, geometry has the capacity to treat the irrational and the incommensurate, but more than this it can demonstrate their existences. These sorts of demonstrations of course make full use of the negative fact that no commensurate measure can be gained through the comparison between two commensurate measures. Analogically, contingent truths can exist even if one cannot fully analyze them into their distinct causes known only by the superior intuitive knowledge of God. The negative approach to the existence of contingent truths is held to be justifiable by reference to the negative approach to the incommensurables.

The point that Leibniz strove to make is the conclusion that "it is impossible for irrational proportions to be understood arithmetically, that is, they cannot be explained through the repetition of measure."[20] Leibniz held this to be parallel to the idea that despite God's immediate grasp of the causal series behind contingent truths, "it is impossible to give a demonstration of contingent truths."[21] What should be highlighted here is that the introduction of God's intuitive knowledge runs parallel to the argument with the introduction of irrational numbers as a naming of incommensurable relations in geometry. The analogy at this stage of the argument functions by placing the geometrical grasp of irrational proportions in parallel with God's grasp of contingent truths. These are both ways of cutting the knot of the infinite at work on either side. The designation "irrational" allows us to understand the interminable division that is required in seeking a divisor for incommensurable magnitudes without actually carrying out an infinite number of steps. Similarly, God's intuitive grasp of an infinite chain of causes behind a contingent truth allows us to understand the interminable analysis of the truth without actually carrying out this analysis. There is no mediate relation that can bridge the gap between human

[20] Leibniz, *PE,* 100.
[21] Leibniz, *PE,* 100.

and divine understanding, just as there is no mediate relation between mathematical rationals and irrationals.

In the conclusion of this argument which textually follows the termination of the argument by analogy, Leibniz argued that the reality of contingent truths can alternatively be understood by "the fact that there is actually an infinite number of creatures in any part of the universe whatsoever, and each and every individual substance contains the whole series of things in its complete notion, and harmonizes with everything else, and to that extent contains something of the infinite."[22] The idea here, present also in the *Discourse*, is not only that it is impossible to give a diachronic demonstration of contingent truth, but that it is impossible to give a synchronic demonstration since the actual relation between beings in the world is infinite at any given state. The bursting infinity of the world implies a veritable vertigo for the thinker who attempts to analyze it into unities. Yet while an interminable task faces anyone who takes up such a project, the very infinity of the world is not incomprehensible. Just as the infinity implied in the grasp of the incommensurable can be circumvented by the understanding of its interminable nature, the infinity implied in contingent truths can be circumvented by understanding that their complexity does not entail their incomprehensibility.

The main point that I wish to draw from the analysis of this argument, beyond the powerful circulation of the infinite throughout the text, is that a mathematical engagement with the problem of the infinite informed and conditioned Leibniz's metaphysical thought. The explicit use of the relation between two incommensurable magnitudes in geometry and their relation to the rational and irrational indicates the presence of a mathematical idea that legitimated as well as helped Leibniz articulate the modal distinctions at the heart of his metaphysics. As such however, the infinite circulates throughout this argument while remaining

[22] Leibniz, *PE,* 100.

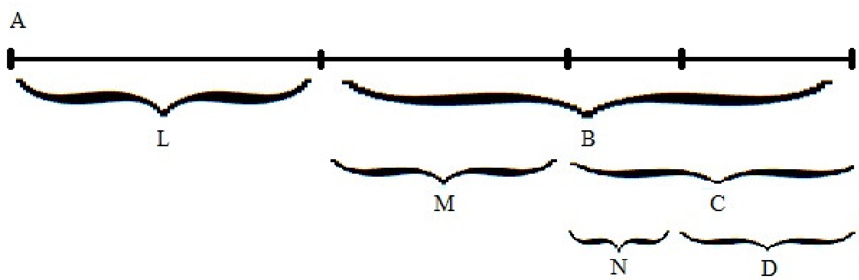
somewhat independent of it. That is to say, the nature of the infinite, directly involved in showing the indemonstrability of contingent truths, is imported into the argument as a standard upon which the argument is founded. Nothing has been said about the infinite itself. It remains a necessary condition deriving from a mathematical resource.

My use of this argument aims to provide an example for the "mathematical philosophy" at work in Leibniz's thought. Demonstrating more than a "philosophy of mathematics" that aims to philosophize about mathematical methods, entities and activity, Leibniz reasoned *through* mathematics. By underlining the immanent mathematical dimension of Leibniz philosophy, I do indeed constrain my investigation in the following in emphasizing the influence of mathematics on philosophy. This is clearly only one direction in the mutual inflection in Leibniz's work between mathematics and philosophy. A more complete picture of this mutual influence will have to await a different project but this incompleteness should not take away from the profundity of this immanent condition of mathematics for philosophy in the work of Leibniz and the lessons it can provide for philosophical practice today. This immanence of mathematics is also evident in the argument that I will analyze in the following. Yet while the argument in "On the source of contingent truths" is framed around a background of his theory of truth as predicate-containment, we can see in the argument below how Leibniz employed this philosophical immanence of mathematics in a different manner.

The argument that we now turn to corresponds in some ways to the conclusion of the previous argument. It concerns the actual infinity of the world. The argument is deductive and requires little introduction. Leibniz wrote, sometime between 1678 and 1681:

> Created things are actually infinite. For any body whatever is actually divided into several parts, since any body whatever is acted upon by other bodies. And any

part whatever of a body is a body, by the very definition of body. So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number. The inference is obvious; for if we suppose any division to be made into only two parts, neglecting the others; and we suppose only the second of these two parts to be subdivided, at least as many parts will be produced by this subdivision as there are divisions made: for example, if A=L+B, B=M+C, C=N+D, it is obvious that from the three divisions at least three different things are produced, L, M, and N.[23]



[Figure 1][24]

Certainly this argument echoes the conclusion of the last argument that we visited. Here however, we do not find any reference to the logical theory of truth. Evaluating this argument alone, we find a direct relation between a mathematical concept of the infinite progressing through the process of infinite division and its implication of the actuality of the infinity of the world. In many of the other texts that we will analyze in the following chapters, things are not so simple and the status of interminable division as well as the nature of body employed here will be given a more complex treatment. Furthermore, the directness of the correlation between the divisibility of any extension whatsoever and the nature of body would be deeply problematized by Leibniz himself. Indeed a significant portion of what follows in this dissertation will aim at

[23] Leibniz, *LC*, 235-237.
[24] Figure created by author.

showing how Leibniz complicated and problematized the very assumptions that allowed him to put forward such an argument.

My aim in presenting this text is to underline the nature of the infinite that Leibniz employed here. Infinity, even "actual" infinity, here is defined through an excess of unity. That is to say, insofar as there are more bodies than unities, bodies are actually infinite. Infinity is understood in this context as an interminable excess of multiplicity over unity. This notion of infinity is based on a syncategorematic infinite. The categorematic infinite by distinction is a quantity, a unity or mathematical entity, which is the infinite number. This categorematic infinite is contradictory insofar as once it is posed, there will always be a number that can be added to produce a greater sum. The syncategorematic infinite is an infinite that rejects an infinite number or totality. To put this in terms of a definition, I will draw from R. Arthur's description that "to assert an infinity of parts syncategorematically is to say that for any finite number x that you choose to number the parts, there is a number of parts y greater than this: $(\forall x)(\exists y)Fx \rightarrow y>x$, with $Fx=x$ is finite, and x and y numbers."[25] What Leibniz posited in the passage above is precisely this. The actuality of the infinite in the world is precisely demonstrated in the fact that there are more bodies than their possible divisions. Referring to the expression in first order logic above, we might say that given some X number of division, there is a Y number of parts that is greater than the division. No doubt, much more has to be analyzed before we can fully grasp what Leibniz meant by the "actual" infinity of the world, but this argument does give an image of how mathematics constituted a condition of his metaphysics. The distinction between a categorematic and a syncategorematic infinite may sit somewhere between the conceptual (or philosophical) and mathematical, but we do see it at work in this metaphysical argument. A mathematical

[25] Richard T.W. Arthur, "'A Complete Denial of the Continuous?' Leibniz's law of continuity," in *Synthese* (forthcoming): 7.

constraint is at least present insofar as Leibniz did not introduce any sort of infinite totality into his argument. It is present in terms of the exclusion of a categorematic infinite and the insistence on a syncategorematical infinite. Here mathematical thought is at work at the very basis of a metaphysical argument concerning the actual infinity of the world. Indeed we see a very direct example of the immanence of mathematics in Leibniz's metaphysics.

The difference between the two arguments is clear. In the first, the infinite circulates in a framework that is established by a logical theory of truth. It is this framework that employs the infinite to distinguish between what is demonstrable and indemonstrable in the two kinds of truth, necessary and contingent. In the second argument, the framework is nothing but the multiplicity of the world itself, an employment of the process of division that delivers an argument about the actuality of the infinity of the world. Without directly bringing them into explicit relation, we can grasp a vision of the way in which mathematics permeates Leibniz's philosophy as resource and condition. Neither argument engenders a philosophy of mathematics despite the fact that both represent a mathematical philosophy; mathematics is at work in Leibniz's philosophy immanently as resource, constraint and condition.

Having briefly taken a look at this way in which the mathematical circulates throughout Leibniz's metaphysics, I turn toward the danger of ignoring this aspect of his philosophy. In this, I take up Russell's influential reading of Leibniz's work as a symptomatic example of the consequences of rejecting this immanent condition.

3. The ignorance of mathematics: Russell and the priority of logic

Russell's landmark monograph on Leibniz straightforwardly entitled *A Critical Exposition of the Philosophy of Leibniz*, published at the turn of the century, marks not only an important moment in contemporary philosophy but also in the history of Leibniz's reception. Russell's important contributions to the neo-positivist movement in the form of a reevaluation of the status of logic made its mark around this time in his devastating letter to G. Frege in 1903, transforming not only the logical constraints of mathematics but also setting down important consequences for philosophical practice. His writings on Leibniz might be seen in this explosive context as a critical homage to a forebear who had, as he saw it, placed logic at the center of philosophy.

Conceived first as a series of lectures for the 1898 academic year at Cambridge University, Russell sought, as he put it, to dispel the notion that Leibniz's philosophy was merely a possibly coherent but hopelessly arbitrary "fantastic fairy tale."[26] Russell's way out of this metaphysical fiction was to reinterpret Leibniz's theses in light of five basic premises. From these premises Russell intended to save Leibniz's relevance, despite sustained criticism of the latter's dogmatism, through positive demonstration of logical clarity in his work. Russell's fundamental intention was to show that an ultimately logical foundation to Leibniz's metaphysical thought would emerge by reading him through these five premises.

In Russell's second preface to the text, written roughly thirty years later in 1937, he maintained that his guiding logicist mode of reading Leibniz had only deepened in the years. This preface was written especially in light of Louis Couturat's 1903 re-edition of Leibniz's important logical and metaphysical manuscripts unknown to Russell at the time of his writing. In

[26] Bertrand Russell, *A Critical Exposition of the Philosophy of Leibniz* (Oxon and New York: Routledge, 2002), xxi.

this preface, Russell indicated how far Couturat went in solidifying his interpretation. "The Principle of Sufficient Reason, he [Couturat] maintains, asserts simply that every true position is analytic, and is the exact converse of the Law of Contradiction, which he asserts that every analytic proposition is true. The Identity of Indiscernibles, also, is expressly deduced by Leibniz from the analytic character of true propositions."[27] Leaving aside the inferential relationship that Russell held to exist between these important principles, Couturat shared the fundamentally logicist approach of Russell. Couturat, for his part, presented an equivalent attitude, "The monad is the logical subject erected as substance."[28] Without reviewing the entirety of Russell's interpretation, we will take up this issue of the analytic in order to show what is symptomatic in a global sense for Russell's logicist reading of Leibniz's philosophy.

Through the correlation between the logical analysis of truth into subject and predicate and the metaphysical relation between substance and its properties and events, Russell attacked what he took to be Leibniz's fundamental metaphysical position. If substance is what underlies change, then all the changes in physical or metaphysical states are to be attributed to substance. This is held to be parallel with the number of predicates that are contained in or imputed to a subject. The analysis of the subject unfolds each of these various states of change. Among other theoretical developments that Leibniz built around this relation between metaphysics and logic, a fundamental principle is the identity of indiscernibles. The idea here is that the identity of each substance implies that, between two different substances, there must be a difference of at least one predicate. More precisely, this means that if two sets of predicates for any two individual subjects are identical, it must be concluded that they are the same substance. Absent this principle, one would be saddled with a problem of individuation, a situation where two

[27] Russell, xiv.
[28] [Author's translation] Louis Couturat, "Sur la Métaphysique de Leibniz," *Revue de Metaphysique et de Morale*, 10 (1902): 10.

substances could share every possible predication except existence. This situation is also something that Leibniz sought to avoid. In this way, Russell thought that Leibniz attempted to close the gap between essential and existential claims by tying these two aspects together through a principle deeply steeped in the logical apparatus. From this observation Russell reasoned that if the identity of indiscernibles governs the distinction between individual existences, what would then be the role of positing an additional term, that of substance? What is the use of asserting a substance beyond merely a sum (or bundle) of differentiable predicates? Here Russell asserted, "The ground for assuming substances – and this is a very important point – is purely and solely logical."[29] Russell's understanding of Leibniz's metaphysical dogmatism, that is, the latter's attachment to the reality of substances, seems to have been generated from the latter's position on the rationality of the world, a form of rationality expressed through the principle of sufficient reason as well as the identity of indiscernibles, principles that for Russell implied a direct correlation between logical form and metaphysical reality. In his understanding of the logical framework undergirding this metaphysics, Russell argued:

> [P]redications concerning actual substances would be just as analytic as those concerning essences or species, while the judgment that a substance exists would not be one judgment, but as many judgments as the subject has temporal predicates. Confusion on this point seems, in fact, to be largely responsible for the whole theory of analytic judgments.[30]

For Russell, Leibniz's confusion consisted in the insufficient distinction between judgments concerning the properties of substances and judgments concerning the existence of individual substances. As such Russell pointed to the inadequacy of this logical model to provide for the

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[29] Russell, 58.
[30] Russell, 59.

reality of substances. There would not only be the impossibility of distinguishing between necessary and contingent claims, but a problem concerning existential claims as such. Indeed, in Russell's imposition of a Kantian analytic/synthetic distinction in understanding Leibniz's analytic theory of truth, the former applied this conception to the inadequacy of a distinction between necessary and contingent claims. In Kant's use of the analytic/synthetic distinction, contingent claims can only be expressed in synthetic judgments. Further, in Russell's view, existential claims cannot be made inside this analytic mode insofar as the limits of analytic judgments remain within the making explicit of properties of a subject and are not about the existence of the subject itself. It seems however that it was Russell himself who had been mistaken in treating the meaning of Leibniz's analytic theory of truth.

There have been myriad responses to Russell's reading of Leibniz. H. Ishiguro's text *Leibniz's Philosophy of Logic and Language* is one that responds most directly to Russell's mistaken reading of Leibniz's analytic theory of truth. Ishiguro points to an obvious anachronism. For Leibniz, the analytic was a form of analysis that went from the complex to the simple. She notes that "at one point Leibniz uses the word 'analytic' in the *New Essays on Human Understanding* in connection with 'practical' to refer to the technical methods used to dissect a problem in order to bring about a given goal, in contrast to 'synthetic' which is used in connection with 'theoretical' and means something like systematic."[31] Further, Ishiguro notes the difference between Leibniz and Kant on the question of the analyticity of mathematical claims. She underlines the fact that despite the agreement of both thinkers in holding mathematical (arithmetical) truths to be a priori and necessary, Leibniz thought of them as being true by definition while Kant held them to be (synthetic) intellectual constructions.

[31] Hide Ishiguro, *Leibniz's Philosophy of Logic and Language*, 2nd edition (Cambridge, UK: Cambridge University Press, 1990), 173. *New Essays on Human Understanding* hereafter abbreviated as *New Essays*.

Ishiguro suggests that the superposition of Kant's distinction between the analytic and synthetic on Leibniz's logical framework obfuscates what is at stake. This is one of the fundamental ways that Russell misunderstood Leibniz. As Ishiguro remarks, Russell's problems with Leibniz were highlighted by the latter's own work. Leibniz wrote:

> There is something which has perplexed me for a long time. I did not understand how the predicate could be in the subject, without the proposition thereby becoming necessary…. But the knowledge of geometrical things and especially that of infinitesimal analysis showed me light, so that I came to understand what it is for concepts to be resolvable in infinity.[32]

Leibniz was in fact quite cognizant of the sort of criticism that Russell leveled against him. Leibniz saw the distinction between the necessary and the contingent as occurring within the context of what Russell understood as the "analytical" method. The distinction as we saw in our analysis of the essay "The source of contingent truths" is the difference between logical analyses that terminate in finitude and those that are resolvable only in infinity. Against Russell's faulty reading, true predications of the contingent sort begin with the analytic determination of their possibility. As such, whether Caesar crosses or does not cross the Rubicon does not imply any sort of contradiction. Hence both predications are possible. The fact that God created a world which unfolds in such a way that includes a Caesar who does cross the Rubicon in no way denies the possibility that he could have just as easily not crossed it. As such, Leibniz's use of an analytical form of truth does indeed account for a distinction between the necessary and the contingent that is independent of the Kantian distinction between analytic and synthetic. Likewise, the distinction between essential and existential claims does not fall under the distinction between the analytic and synthetic nor the necessary and contingent. Though Russell

[32] Ishiguro, 175.

was correct in noting that the only necessary existential claim that Leibniz made is with respect to God while all else fell under the criterion of created being, this by no means reduces the division to the sort that Russell made. Instead, as Ishiguro explains:

> [I]n asserting that in all true subject-predicate propositions the predicate is included in the subject, Leibniz certainly did not mean to imply that all subject predicate propositions were necessary. He explains that the required link between subject and predicate exists only *a parte rei* (from the side of the thing). This is obviously contrasted with a link which exists *a parte dicti* (from the side of what is said), or *ex termini* (due to [general] conceptual links). A predicate that is true *a parte rei* is always in some manner contained in the nature of the subject.[33]

It appears that the primacy of logic in Leibniz's metaphysics that Russell diagnosed as the source of his dogmatism is merely an apparent one. In this instance, Russell imported an erroneous division of the analytic and synthetic onto his reading of Leibniz. As Ishiguro points out, Russell's reading not only enacted an anachronism of attributing Kant's use of the analytic to Leibniz, but it also misunderstood the latter's distinction between logic and metaphysics. For Leibniz the distinction between the subject and its predicates depended on a prior metaphysical reality.

In light of this, how then do we assess Russell's accusation of the gratuitous metaphysical supplement of substance? The primary problem here is that Russell's analytic/synthetic distinction divides Leibniz's use of essential and existential claims in a misguided way. This division separates the existential and the essential such that Leibniz's reliance on a theory of substance appears gratuitous and a mere reification of a logical model. Ishiguro remarks that the

[33] Ishiguro, 172-173.

artificial evaluation of essential and existential claims according to the Kantian analytic/synthetic division mistakenly obfuscates the status of existential claims in Leibniz.

The crux of Russell's criticism is that Leibniz's predicate inclusion form of truth constrains us to understand an individual as a concatenation of a series of predicates. Ishiguro points out that the proposition "Adam sins" would then be true on the account of the accurate predication of "sins" on the subject "Adam". Propositions like "The present king of France is bald" or "Zeus is strong" are difficult to assess since there is no present king of France and Zeus does not exist.[34] Echoing some of Russell's own work, Ishiguro points out that the immediate consequence of this is that such a proposition might be counted as contingent if we were actually in a possible world where Zeus exists and is strong. Without a more substantial treatment of Leibniz's theory of possible worlds, we can nonetheless understand what is erroneous in Russell's account. What is assumed in this basic reading of the problem of the contingency of an existential claim is that a concept like "Adam" is reducible to the notion of the name "Adam". For Leibniz, propositions expressing the predication of "sins" to the subject "Adam" are truthful because the concept "Adam" includes that quality, and not vice-versa. As Ishiguro points out, "It is the nature of individual substances (that is to say, individuals in the actual world) to have complete individual concepts – corresponding to the particular '*haecceitas*' (thisness) of the individual. We cannot, however, get at the '*haecceitas*' of things which exist in other worlds."[35] Against Russell's criticism, we see that the analysis of propositions is anchored in the metaphysical consistency of individual substances. These sorts of analyses are essentially linked to actual substances that provide the pre-conditions for the sort of truths one might utter. While an actual individual may be shown through logical analysis to be existentially contingent insofar

[34] Ishiguro, 187.
[35] Ishiguro, 188.

as its predicates are not necessary, a non-actual individual like “Zeus” cannot be submitted to an analysis of its possible predicates. Conversely, a certain complete concatenation of predicates is nothing without its subject designated as individual.

If we put aside the anachronisms of a Kantian-style analytic/synthetic division and the imposition of a theory of names and definite descriptions on Leibniz, how can we understand the relation between logic and reality? Russell was right in focusing on the peculiarities of the articulation of contingent truths in Leibniz’s philosophy, for it was a problem that troubled Leibniz himself. Russell’s mistake was in staking his interpretation of Leibniz’s metaphysics on a logical primacy of the analysis of propositions. Leibniz’s approach was quite different. Indeed as Leibniz remarked, it was the “light” sparked by the infinitesimal calculus that allowed him insight into the distinction between the contingent and the necessary. How does this use of the infinite/simal in Leibniz then respond to Russell’s criticism?

The kind of infinitesimal analysis (analogized between the logical and mathematical) implied in the essay “On the source of contingent truths” refers to the impossibility of resolving incommensurable proportions by finding a smallest common part. Incommensurable magnitudes bring out this difficulty in a particular way insofar as they demonstrate, at least in Euclid, that a finite number of steps can never bring these lines into a rational or commensurate relation. The search for a common factor between incommensurable magnitudes will always produce a remainder. For Leibniz, this was one way of expressing the coherence of the idea that an infinite number of predicates actually cohere in an individual concept even if our analysis of its completeness is unachievable. What then about the actual consistency of this infinite regression? If we take Zeno’s paradoxes seriously, the infinite decomposition of the finite extension implies the contradiction of the composition of the finite extension from points. Leibniz’s understanding of infinite decomposition differed from Zeno's in that he did not take the relation between the division of the continuum and composition to be symmetrical. The composition of a line from points is not possible because a point has no dimension and cannot compose even the smallest part of the smallest division. This fact does not however prohibit a line from being infinitely divided in a syncategorematic sense. As we mentioned above, the infinite division of a line is actually infinite because there will be parts that are greater than any finite number. To give some insight into the status of the infinite at work in Leibniz, we quote the clarification written in a 1676 article "Infinite numbers": "In the continuum, the whole is prior to its parts; the absolute is prior to the limited; and so is the unbounded prior to that which has bound, since bound is a kind of addition."[36] While Leibniz's treatment of the infinite and infinitesimals is an important question to be treated in more detail in what follows, the notion that the whole is prior to its parts serves, at this mediate juncture, to allow us some insight into the problem of substance that we have been examining.

How does this insight add to our refutation of Russell's interpretation of the equivalence between the sum of the predicates that is contained in an individual concept and the substance that this concept in turn expresses? Even with the background of the principle of the identity of indiscernibles, the fact is that Leibniz saw the coherence of an infinite multiplicity of predicates as possible in analogy with the idea that in the analysis into infinitesimals, the "whole is prior to the parts." This priority of pre-given unity can be seen to operate in logical analysis as well. Privilege is given to the real metaphysical unity of a substance, and the decomposition of this unity into an infinite number of corresponding predicates is coherent only with respect to this prior unity. The relation between division and composition is not symmetrical. The use of the subject-predicate logical form does not exhaust the full content of the concept of an individual

[36] Leibniz, *LC*, 97.

substance. In fact, the series of predicates determined as true for any subject does not exhaust the identity of the subject (for any series of predicates, there are always more). The identity of indiscernibles should then be understood as a principle that expresses the relation between a substance and its predicates. It does not allow us to take the sum of its predicates, accidental properties, events and the like as a metaphysical determination of the being of this prior unity. The doctrine that a substance is not exhaustible by its predicates is in fact one of the consequences of Leibniz's account of contingent truths.

The ignoring of the interwoven filiations between the mathematical and the metaphysical along with the insistence on an anachronistic reading of the analytic form of Leibniz's logic pushed Russell to hastily conclude that the fundamental motivation of Leibniz's philosophy was logical. This may have indeed represented the saving grace of an otherwise "fairytale" philosophy for Russell, a light in an otherwise fantastical account of a fictional reality. Our analysis of Russell is not meant to show anything fundamental about Russell's larger philosophical work but rather to demonstrate the incomplete relationship between the logical and the metaphysical in Leibniz's thought. In this, Russell represents a logicist turn that is regrettable. In this position, gaps emerge that can only be filled by a reckoning with the role of the mathematical.

It is from this underestimation of the role of the mathematical that we can more closely comment on a further aspect of Russell's systematic error in interpreting Leibniz. Here we take a step closer into the specific themes of the direct confrontation between mathematics and metaphysics.

In one of his deepest, cutting criticisms of Leibniz, his discussion of the labyrinth of the continuum, Russell aimed to obliterate Leibniz's usage of the mathematical infinite in his metaphysics of the *Monadology* and its supposed compatibility with the latter's reflections on the mathematics of the continuum, concluding, "The boasted solution of the difficulties of the continuum is thus resolved into smoke, and we are left with the problems of matter unanswered."[37] In this passage, Russell's strongest claim is that "in spite of the law of continuity, Leibniz's philosophy may be described as a complete denial of the continuous."[38] Referring here to Leibniz's law of continuity that stated in various forms that "no transition in nature ever occurs by a leap", Russell attempted to show the deep contradiction that must follow if one were to hold both the principle of continuity and the theory of monads.[39]

Russell began by underlining the unique position that Leibniz held about the infinite. For the latter the infinite was actual, a thesis that we have seen in various forms in the preceding parts of this chapter. This is the position that it is not only that every part of nature is infinitely divisible, but rather that every part of nature is infinitely divided. At the same time, Leibniz held that the infinite could not be a totality and hence there could be no infinite number. Russell turned to the case of the metaphysical picture developed in the *Monadology*, where Leibniz argued for "metaphysical" atoms that underlie all reality as the ultimate substance composing larger aggregates. These ultimate unities, monads, are no doubt discrete, that is, atomic. From this basis, Russell understood the continuum or any other case of the continuous as ideal rather than actual. Russell here cited relevant passages from the *New Essays* and also added statements of the same effect drawn from Leibniz's letter to Volder in January 1706:

> [I]n actual things, there is only discrete quantity… But continuous quantity is something ideal, something that pertains to possible and to actual things considered as possible. The continuum of course, contains indeterminate parts.

[37] Russell, 137.
[38] Russell, 129.
[39] Leibniz, *Math* VII, 25.

> But in actual things nothing is indefinite; indeed every division that can be made has been made in them […] but in confounding ideal things with real substances, such that we look for actual parts in the order of possible and indeterminate parts in the aggregate of actuals, we have ourselves introduced the inextricable contradictions in the labyrinth of the continuum.[40]

From this Russell asserted that Leibniz rejected any actuality of the continuous and further pointed to Leibniz's own warning that the traps in the labyrinth of the continuum consist precisely in confounding the actual with the ideal.

From here Russell easily aligned discreteness with the actual and continuity with the ideal. Thus the sort of actual infinity that Leibniz is said to have held becomes problematic. Since discreteness is aligned with the actual, it seems that by implication a discrete actual infinity must also be held. Russell pointed out that this implies that an infinite number of entities in reality must be held, since there is not merely an indefinite number but an actual infinite number of entities. Granting Leibniz a mediate resolution to this difficulty, Russell argued, "To evade this argument, Leibniz makes a very bold use of his principle that … nothing is absolutely real but indivisible substances and their various states….. Aggregates, not having unity, are nothing but phenomena, for except the component monads, the rest (the unity of the aggregate, I suppose) is added by perception alone."[41] Here Russell saw a resolution by turning to a phenomenalist reading of all composed or aggregated things such that Leibniz did not fall into the implication that extended things (corporeal existences), insofar as they are continuous, imply an infinite number of constituent partless elements. Following out Russell's alignment of the discrete with the actual, the ultimate reality of the monad can have a relation with plurality in its

[40] Leibniz, *PE*, 185.
[41] Russell, 134.

various sorts such as bodies, aggregation, infinitely divisible matter and the like, only through a "phenomenalization" of them understood as monadic self-perception. Russell concluded his argument here by stating:

> Now this position is a legitimate reduction from the theory that all propositions are to be reduced to the subject-predicate form. The assertion of a plurality of substances is not of this form – it does not assign predicates to a substance. Accordingly, as in other instances of a similar kind, Leibniz takes refuge, like many later philosophers, in the mind – one might almost say, in the synthetic unity of apperception. The mind, and the mind only, synthesizes the diversity of monads each separate monad is real apart from the perception of it, but a collection, as such, acquires only a precarious and derived reality from simultaneous perception. Thus the truth in the judgment of plurality is reduced to a judgment as to the state of every monad which perceives the plurality. It is only in such a perception that a plurality forms a whole and thus perception is defined by Leibniz as the expression of a multitude in a unity.[42]

Great care needs to be taken in understanding Leibniz's use of the infinite between the global structure of monadic reality and its relation to plurality. Here Russell was certainly wrong in holding the actuality of the infinite in Leibniz as being available only through a reduction to the phenomenalization of multiplicity. It seems that in reading the caution that Leibniz pronounced to Volder, Russell missed the key nuance that Leibniz aimed to express. If we read the text closely, we see that Leibniz argued that continuous quantities are ideal in the sense that they are understood in the context of the possible. An extended body, for example, is divisible without end. The continuous is expressed as the possible indefinite division of such an actual thing.

[42] Russell, 136.

However, Leibniz also said in the same breath, “But in actual things nothing is indefinite; indeed every division that can be made has been made in them.”[43] The actual infinity in things arises from the fact that every division that *can* be made *has* been made. Leibniz articulated this by saying in an earlier letter to Volder:

> But in real things, namely, in bodies, the parts are not indefinite … but are actually assigned in a certain way, in accordance with how nature has actually instituted division and subdivisions as a result of various motions; and although these divisions might proceed to infinity, nonetheless, everything results from certain first constituents, that is, real unities, though infinite in number.[44]

Leibniz was clear here in saying that the sort of infinity that is to be found in reality is determinate and not merely possible as in the case of the continuous. What we can draw from this is that real and actual infinity cannot be simply reduced to the phenomenal representation of shape, size and magnitude. Leibniz’s reasoning about the actual infinite then involved an understanding of the infinite that escapes the contradictory idea of the infinite number while maintaining the real infinite division and subdivision in actual things. I will venture to give a more rigorous account of what this amounts to in the later chapters of this dissertation, but in our examination here I wish to clearly point to the dangers of Russell’s approach. The path that Russell pursued in his text too quickly takes the rejection of an infinite number to mean that all non-contradictory senses of the infinite are to be aligned with the continuous, and thus to the ideal. This finally implies, for Russell, that the infinite has meaning only in the continuous and the phenomenal. Rather than taking a more focused look at what the infinite meant for Leibniz, Russell immediately skipped to recasting the entire debate in terms of the fundamental status of

[43] Leibniz, *PE*, 185.
[44] Leibniz, *PE*, 178-79.

the subject-predicate relation for all of Leibniz's work. This sort of reduction, as we can already see in this very cursory glance, occludes a deeper understanding of a number of fundamental distinctions in Leibniz: the ideal and actual, unities and aggregation, discreteness and continuity. Indeed even the genuine role that phenomena played in Leibniz's monadic vision is overshadowed by a logicist framework. As such, we can see how a clearer conception of the complex reflections on the infinite and the infinitesimal can in turn clear up what these other terms mean and thus understand what Leibniz attempted to accomplish with his metaphysics.

Having seen the insufficiency of the logicist model for reading Leibniz in the context of both the problem of substance as well as in the treatment of the discrete and continuous, we will turn in the following chapters to seeing what a focus on the mathematical can in turn bring to the discussion. Even though the problems with the logicist model of reading Leibniz have been indicated symptomatically here in Russell, the central problem is systematic, the wholesale importation of a reductive basis for understanding the thinker. Not only is the content of this reading erroneous, the form that it takes is equally problematic. By rejecting the logicist approach, I will also reject any pre-given global basis for interpretation, building my reading from local configurations of problems in Leibniz's texts.

4. The immanence of the mathematical as a laboratory of concepts

In the two arguments presented above, the first concerning the comparison of two representative arguments in Leibniz's corpus and the second concerning Russell's logicist interpretation, I have provided some motivations for maintaining a position that can both reject a current contemporary approach for Leibniz interpretation and provide the grounds for an alternative. In this, I began with a perspective of mathematics as the immanent condition of

Leibniz's metaphysics. This approach does not equate Leibniz's mathematical reflections with his metaphysical ones but presents mathematics as a condition that not only constrains, by providing conceptual limits, but also provides resources for how the complexities of treating the infinite can inform a metaphysical project. In turn, the mathematical reflections on the infinite can be seen as providing the conditional sources for interpreting the various solutions that Leibniz provided for his metaphysics across his long period of work, but can also be read as a key to understanding the different problematics that he constructed around these changes. We have also seen in Russell how leaving aside the immanence of the mathematical in Leibniz's metaphysics can engender misconceptions. Indeed Russell's mistakes were not simply those of the ignorance of a mathematical dimension in Leibniz's metaphysics but more importantly an anachronistic misreading of fundamental distinctions even in Leibniz's logic itself. However, a closer attention to the mathematical aspects, such as the presence of a sophisticated approach to infinity in Leibniz's logic, could have provided a key for Russell to reexamine and rearticulate the central claims he aimed to make about Leibniz's philosophy. In this symptomatic reading of Russell, I hope to have made the dangers of ignoring the mathematical condition of Leibniz's philosophy sufficiently clear.

In what follows, I will take up the central themes addressed in this introduction by treating the status and structure of the infinite in both the mathematical and metaphysical senses in Leibniz's philosophy. I will treat the problem of the infinite as a means to interpret the immanent mathematical condition of Leibniz's work as it circulates throughout his metaphysics in two senses: in terms of the status of the infinite/simal and in terms of the structure that this infinite/simal engenders. In this I hope to lay the foundation for a larger project for interpreting

Leibniz through this immanent mathematical condition. That is to say, I aim to demonstrate how Leibniz employed his mathematical reflections to provide a laboratory of philosophical concepts.

PREFACE TO CHAPTERS 2 AND 3

THE STATUS OF THE INFINITE AND INFINITESIMAL

"[I] do not speak of infinitely small magnitudes any more than of infinitely large, nor more of infinitesimals than of infinituples. For I treat them all *per modum loquendi compendiosum* for fictions of the mind, useful for calculation, like imaginary roots in algebra."[45]

"To tell the truth, I myself am far from convinced that our infinites and infinitesimals should be considered as anything other than ideals, or well-founded fictions."[46]

What is the problem of the status of infinite/simals?

In the previous chapter, we saw that Leibniz rejected the existence of an infinite totality and treated infinitesimal quantities as "fictions", "*façons de parler*", "*compendium loquendi*" and the like. This understanding of infinites and infinitesimals (infinite/simals), however, leaves much to be (mis)interpreted, insofar as it is far from obvious how these terms (and quantities), understood as fictions, ways of speaking, or abbreviations circulate throughout Leibniz's larger philosophical work. One question that immediately arises brings us toward a seeming contradiction. That is, if Leibniz held a fictional determination of the infinite/simal, how can it also be the case that he understood the world as having an "actual" infinite nature (an "actual"

[45] Leibniz, *Phil* II, 305.
[46] Leibniz, *Math* IV, 110.

subdivision of things into infinite parts)? Here for example, the metaphysical picture presented in the *Monadology* that places such heavy emphasis on the actuality of the infinite would seem to be contradictory to the designation of these infinite/simal quantities as fictions.

On the other hand, Leibniz qualified his use of these quantities in his method of the infinitesimal calculus as “well-founded”, such that they are not only fictions but more precisely “well-founded” fictions. Would this well-foundedness of the fiction of infinite/simals then lend itself to an understanding of the nature of the actual infinite that Leibniz brought to his metaphysical vision?

Understanding the “status” of infinite/simals -- their fictionality, actuality or well-foundedness -- is perhaps the most direct way of confronting their role in Leibniz’s philosophy. The problem of infinite/simals is only a “problem” most immediately in terms of their dubious status. Quantities (magnitude, number-entities) of a standard type can be understood as obeying the Archimedean principle. This principle, in its canonical form in Book V of Euclid’s *Elements,* states that quantities “have a ratio to one another which are capable, when multiplied, of exceeding one another.”[47] The positing of infinite/simal quantities (magnitudes, numbers, sums, etc.) is problematic precisely in the sense that they break with this notion of comparability. They are either too great or too small to be put into a definite ratio with other “standard” quantities. As Zeno’s paradoxes quite intuitively demonstrate, the moment that the calculation of motion and change involves infinite division, the battle against the reality of these features is already won. In Leibniz it is also within these problems of reality -- the different modalities of establishing the actuality, reality and admissibility of things involving the infinite -- that the problem emerges. If, for instance, the reality of motion required the positive notion of an infinitesimal quality to come into play, one would certainly have to offer some means of establishing motion’s reality. Hence,

[47] Euclid, 99.

to focus on the status of the problem of infinite/simals is to focus on the heart of the "problem" of the infinite/simal itself.

Given the apparently contradictory uses of infinite/simals, we shall turn to Leibniz's nuanced understanding in order to dissect the problem. By deploying the distinction between a syncategorematic and a categorematic notion of the infinite/simal, Leibniz provided a concept of the infinite/simal that was compatible with the Archimedean principle, and as such, an unequivocal resolution to one important dimension of the status problem. By positing the idea of a syncategorematic infinite/simal where the infinite and the infinitesimal are respectively greater or lesser than any given finite number, Leibniz attempted to clarify his concept of the infinitely large and small, and insisted that he did not depart from the Archimedean principle but for the "expressions" that he chose. This is one important aspect of how he understood these mathematical quantities as well-founded fictions.

With the distinction made by the syncategorematic in mind, in the following two chapters I will treat the problem of the status of the infinite/simal by thematically dividing the expression *well-founded fictions* into a question concerning "fiction" and a question concerning "well-foundedness". The next two chapters will thus be organized under the title, "The status of the infinite and infinitesimals" and individually entitled, "The status of fiction" and "The status of the well-founded" respectively.

§

In the following I will give an overview of the aims of the two chapters comprising the second section.

Overview of Chapter 2: The status of fiction

In order to treat the fictional status of the infinite/simal, I will begin by examining how *not* to understand this well-founded fiction. It appears that what Leibniz understood as the fiction of the infinite/simal departs quite a bit from the contemporary evaluation. As it is often read, this fiction designates the need for the reduction of the term to a more rigorous (and more actual) determination. This rigor is, as in the case of contemporary commentators like Ishiguro, founded on the level of a linguistic-logical form of reference compatible with Fregean and Russellian ideas about references to mathematical objects. I will show that this connection cannot be successfully made and thus requires us to look further to understand in what sense infinite/simals are fictional. No doubt, this fictionality will lead us to a more thorough understanding of the infinite/simal as syncategorematic rather than categorematic.

Employing the distinction afforded by the syncategorematic infinite/simal as a bridge between critical remarks on Ishiguro and a reading of the major text (*New Essays Concerning Human Understanding*) where it is fully employed by Leibniz, I will look at how Leibniz himself used the term in the context of a larger philosophical (metaphysical and epistemological) dispute with Locke. In this, I will underline the fact that the Leibnizian sense of the infinite was not merely used as a justification of the mathematical operations in the differential calculus, but as a positive term in the context of his philosophical arguments against Locke, as well as in his larger metaphysical work. In turn, using the context of the *New Essays*, I will provide a reading of Leibniz's own understanding of the syncategorematic sense of the infinite/simal. In this first section treating the status of the infinite/simal, I will proceed from the contemporary problematics surrounding the understanding of the infinite/simal as fiction to its positive deployment in Leibniz's wider philosophical thought. This will demonstrate that the sense of fiction in Leibniz's treatment of the infinite/simal is not merely a correlate which can be more "rigorously" reduced to a "non-fictional" level but constitutes a concept that is positively at work in his thought. In this, the fictional status of the infinite/simal can be understood not only in the negative sense of its Latin root as *fictum* and *fingo*, to "dissemble in order to deceive", but also in its positive sense, "to mold and devise".

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Overview of Chapter 3: The status of the well-founded

In treating this second dimension of the status of infinite/simals in Leibniz's philosophy, I aim to unravel the meaning of the explicit relation that he used to identify the well-foundedness of infinite/simals. Moving from the previous chapter where I argued for a non-reductive deployment of the fiction of infinitesimals, I will turn to the basis of the well-foundedness of infinite/simals in order to establish the necessary links that Leibniz made between these terms and his philosophical works. I will begin by addressing the fundamental obstacle that he identified in thinking about the infinite/simal. That is, he noted that the obstacle to grasping the status of infinite/simals lies in confounding the ideal and the actual, which has led some commentators to think of infinite/simals as merely fictional (a dissemblance) or imaginary, objects akin to unicorns or mermaids, the ideal seeking anchor in the actual. By taking a look at important instances where Leibniz treated the distinction between the ideal and the actual, I will argue that far from being separate domains, it was in their interrelation that Leibniz recognized the very context where knowledge is constituted. As such, far from separating the two dimensions, Leibniz saw the possibility of scientific knowledge in the very interaction between the imaginary, the understanding and the sensible. In the brief sketch of Leibniz's epistemology, I aim to provide a vision of the well-foundedness of knowledge in the context of understanding the well-foundedness of infinite/simals. Here, far from forcing us to distinguish between the imaginary and the actual, the problem of the status of infinite/simals corresponds to a dynamic of thought **-** and knowledge **-** formation as an on-going process.

In order to illustrate the inadequacy of the standard reductive reading of what constitutes the well-foundedness of the infinite/simal, I will turn to a short reading of Leibniz's transition from holding the position of infinitesimals as actual , under the form of indivisibles in motion, to his renunciation of this position and his arrival at the resources for formulating his mature, syncategorematical view of infinite/simals. Here, my aim is to demonstrate that this status problem should be read together with his larger metaphysical reflections rather than constituting a separate aspect of his thought to be resolved by means of a reduction of terms. By showing Leibniz's own conflict with the problem of status, I aim to show that Leibniz's metaphysics and larger metaphysical vision cannot be adequately understood without attention to this mathematical dimension of his thought.

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CHAPTER 2

THE STATUS OF FICTION

1. The rhetoric of "fictions"

As laid out in the preface to the next two chapters, in this chapter I wish to underline the important role that mathematics played in Leibniz's philosophy by looking at the problem of the status of infinite/simals through the lens of their fictionality. A major obstacle in taking this approach is the implication that infinite/simals are "merely" ways of speaking that require reduction to a more rigorous level. I aim to show that reading Leibniz under this reductive comprehension of his fictions not only leads one down an erroneous path of interpretation, but also obfuscates the very problematics that Leibniz himself confronted. Against reduction, I hope to use the lens of fictionality in order to unfold the complex set of problematics that intertwine the various dimensions at work in his philosophy.

Before I begin, I would first like to clarify the context for the term *well-founded fictions*. Leibniz communicated to P. Varignon in 20 June 1702:

> Between you and me, I think Fontenelle ... was joking when he said he would derive metaphysical elements from our calculus. To tell the truth, I myself am far from convinced that our infinites and infinitesimals should be considered as anything other than ideals, or well-founded fiction.[48]

[48] Leibniz, *Math* IV, 110.

Bernard le Bovier de Fontenelle and Pierre Varignon were both major figures in the French intellectual context of the late seventeenth and early eighteenth centuries as members of the French Academy of Sciences and early enthusiasts of Leibniz's methods of the calculus. The use of the expression here is situated in this context of Leibniz's late exchanges with those already familiar with the calculus -- Nieuwentijt, Rolle, Pinson, Varignon, L'Hopital and others affiliated with the Paris Academy of Sciences. In this late period of letters and reflections, we find a wide-ranging rearticulation of well-established positions in Leibniz's mind, which sought to rein in heterodoxy and deviations from the "Leibnizianism" that was then emerging. Here Leibniz often played a very conservative role, prudentially maneuvering between defending the method of the infinitesimal calculus that had come under significant attack and playing down its apparently revolutionizing novelty. In this period we see that the fictional status of the infinitesimals formed a "camp". Indeed, partisans of the infinitesimals like L'Hopital and others tended to argue for a more "realist" notion of the infinitesimals, a tendency that Leibniz sought to contain. By comparing infinitesimals with the imaginary root, the number *pi* and so forth, although not at all new in his writings, Leibniz wished to cast off any suspicion that he took these infinitesimal quantities as anything more than a "means of speaking".[49]

Despite the conservative tone of his letter to Varignon, a fictional notion of the infinitesimals had been part of Leibniz's conception for quite some time and had been employed in a number of ways. The negative sense of fiction in this correspondence was rhetorically directed toward the ways in which he felt his theory was over-extended into domains where it did not belong. While one can certainly sense the deflationary meaning of Leibniz's use of *fiction* here, this is far from being the whole story.

[49] In order to prevent any confusion, I wish to underline that Leibniz's comparison here is couched on the accepted use of terms like the imaginary root without the danger that they imply an independent claim to reality.

2. The contemporary status of the status problem: Robinson and Ishiguro

In contemporary research on the problem of the infinite/simal, the method of Leibniz's calculus, and more generally his views on the role of the infinite in logic and metaphysics, the starting point is often Leibniz's remarks in these late correspondences. These texts are in many ways the origin of the contemporary problem of the status of infinitesimals. This interpretive tradition, in conjunction with the preoccupations of the philosophers who in the early twentieth century led the renaissance of Leibniz research, probably accounts for most of the "logicist" tendencies that I remarked on in the first chapter. In the context of the problem of status of infinite/simals, this reading has focused on how a proper understanding of the foundational arguments for these quantities provides a basis for the consistent concept of infinite/simals in the other, non-mathematical domains of their use. This is simply a general application of the "logicist" reading of Leibniz, one that sees the sorting out of problems in the mathematical domain as separate, and the relation between the mathematical domain and the metaphysical or otherwise as being regulated by a wider logical reference. Under these conditions, the problem of status would be solved insofar as it stands up to a critique of logical consistency aimed at the foundations and consistent usage of infinite/simals in Leibniz's larger philosophical work.

To begin to demonstrate the inadequacy of this position, I will return to Ishiguro's reading of the status of infinite/simals in her 1990 second edition of *Leibniz's Philosophy of Logic and Language*. What is particularly important in treating this text as in some ways highly symptomatic of contemporary readings is that the first edition of 1972 did not contain chapter five, a separate chapter entitled "Leibniz's notion of infinitesimals". Outside of the influence of her first edition, this additional chapter has been cited by leading scholars in their turn as to the problem of infinite/simals, their status and foundations as a major dimension of Leibniz's philosophy.[50]

In Ishiguro's treatment of the problem, she begins her discussion by referring to Abraham Robinson's seminal *Non-Standard Analysis*.[51] Robinson's project for a notion of a non-standard, that is, non-Archimedean concept of an infinitesimal quantity, represents perhaps the most far-reaching attempt to combine the elements of the Newton-Leibniz calculus with the transfinite revolution of Cantor: the development of a strong concept of the infinitesimal as, formally speaking, a number less than the absolute value of any real number ($n<|R|$ where n is a number and R is a real number), a project that has, in turn, revived new reflections on Leibniz's legacy. In Robinson's own book a certain number of pages are devoted to a reading of Leibniz on numbers and infinitesimals and an interpretation of his own contributions along these lines.

What is curious in this intersection between the development of mathematical concepts driven by Robinson's work and Ishiguro's reinvestigation of Leibniz's philosophical legacy is that there is something of a misunderstanding of the former by the latter. Robinson's contribution, according to his own vision, is the development of the concept of infinitesimals in the Leibnizian fashion that had been superceded in the early nineteenth century by Cauchy's re-foundation of the interpretation of infinitesimals in the differential calculus by means of limits rather than as mathematical entities.[52] That is, according to a basic understanding of Leibniz's calculus, whereas the dubious status of infinitesimal qua mathematical entity had always cast a

[50] Richard T.W. Arthur notes that it was Ishiguro who first highlighted Leibniz's "syncategorematic infinite" in her 1990 second edition of *Leibniz's Philosophy of Logic and Language* and thus set the tone for the discussion of infinitesimals in contemporary Leibniz literature. Cf. Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 10.
[51] Abraham Robinson, *Non-Standard Analysis* (Amsterdam: North-Holland Publishing Company, 1970).
[52] The definition and use of limits by Cauchy got rid of a number of ambiguities surrounding the infinitesimal by relating it to the idea of a function that indefinitely approaches zero. Employing this idea, Cauchy removes the need to appeal to geometric intuitions where confusions about the infinitely small magnitude often invited confusing appeals to an "infinitesimal" entity. Cf. Carl Boyer, *The History of the Calculus and its Conceptual Development* (New York, Dover Publications, 1959), 272-274.

shadow on the entire method, the notion of the limit proved to provoke less skepticism.[53] In turn Robinson, using the sort of mathematical reasoning made standard by the Cantorian revolution and the renaissance of axiomatic reasoning in mathematics, returns to the original project of Leibniz as interpreted by L'Hopital by providing a rigorous method of cashing out the infinitesimal as entity (and not merely as limit) in his "non-standard" approach to arithmetic, topology and analysis. Robinson's ultimate justification, outside of the demonstration of his method in roughly two hundred pages of mathematical argument, is a direct quotation of Leibniz. Citing Leibniz's letter to Pinson in 1701, Robinson notes that Leibniz held to a wholly traditional or Archimedean conception of infinitesimals:

> Car au lieu de l'infini ou de l'infiniment petit, on prend des quantités aussi grandes et aussi petites qu'il faut que l'erreur soit moindre que l'erreur donnée de sorte qu'on ne diffère du style d'Archimède que dans les expressions, qui sont plus directes dans notre methode et plus conformes à l'art d'inventer.[54]

In fact, the recognition of a distinction between limit and entity is at the center of Robinson's justification of his own contribution. After the inspiration of Cantor's revolution, which had introduced a method for the introduction of a hierarchy of infinities beyond what is denumerable, Robinson introduces infinitesimals that run against the standard notion of what counts as a difference between two mathematical entities. Under this mode of legitimation, citing the formalist camp, Robinson argues, "from a formalist point of view we may look at our theory syntactically and may consider that what we have done is to introduce new deductive procedures

[53] In his well-known attack on the notion of infinitesimals, George Berkeley balked at the idea of the infinitesimal quantity or increment, an attack launched equally against Leibniz and Newton. He famously characterizes these terms as "the ghosts of departed quantities". As Abraham Robinson and other contemporary mathematicians in the domain of infinitesimals remark, Berkeley's criticism was aimed at a theory of infinitesimals that implied, directly or indirectly, the idea of an infinite/simal number or entity. As Robinson notes, Leibniz was often careful to avoid this implication, but he did not lay any clear basis to sort out the issue. Robinson, 265-266.
[54] Robinson, 262. Cf. G.W. Leibniz, *Math* V, 350.

rather than new mathematical entities."[55] In this, Robinson places his argument about infinitesimals squarely in the modern, that is, post-Cantorian framework, in the debate about the status of entities introduced by and justifiable through an emphasis on formal or axiomatic means. Hence Robinson's use of Leibniz's statements above fully acknowledges the relation Leibniz made between his own introduction of infinitesimal sums and other mathematical entities. Yet there is a central difference between Robinson and Leibniz here that is crucial. Insofar as Cantor and the "crisis" of foundations that was provoked by the development of the transfinite had focused on the globally logical character of the issue of mathematical objectivity, the reality and objectivity of mathematical entities and their surrounding methodology are treated as a whole and not separated between "normal" mathematical entities like the natural numbers (1,2,3…) and numbers with special status like Π, irrational numbers and even infinites and infinitesimals. Thus, Robinson argues,

> [W]hatever our outlook and in spite of Leibniz' position, it appears to us today that the infinitely small and infinitely large numbers of a non-standard model of Analysis are neither more nor less real than, for example, standard irrational numbers. […] For all measurements are recorded in terms of integers or rational numbers, and if our theoretical framework goes beyond these there is no compelling reason why we should stay within an Archimedean number system.[56]

Here, quite deftly, Robinson returns to his citation of Leibniz's 1701 letter to Pinson, "On ne differe du style d'Archimède que dans les expressions….", implying that even if the non-standard infinitesimals go beyond standard expressions, they are in no sense in conflict, nor do

[55] Robinson, 282.
[56] Robinson, 282.

they cover the same field of application.[57] What he then makes clear regarding his relation to Leibniz is that the debate of the status of "his" infinitesimals should, on the one hand, be held in common with the debate on the status of real numbers themselves and, with respect to methodology, with the debates surrounding the foundations of arithmetic. In view of Robinson's treatment of Leibniz in his book published in 1966, where he already understood Leibniz's own treatment of the infinite by a syncategorematic interpretation, it seems that any subsequent worry of a non-standard interpretation of Leibniz's infinitesimals would be deeply anachronistic.

As Robinson himself explains, even if Leibniz was clear on this particular aspect of the logical status of infinitesimals, the story of the calculus and the legitimacy of infinitesimals remain far from concluded. As he continues, he touches on Lagrange, d'Alembert and finally Cauchy's deepening of the calculus project by means of limits and analysis. In this, though Cantor himself rejected the possibility of infinitesimals qua number, the Cantorian revolution of method for entering into these problems provided the grounds for Robinson's own infinitesimal quantities. The irony of comparing Robinson to Leibniz commentators is twofold. First, while the philosophical commentators continue to emphasize the clarification of Leibniz's understanding of the infinitesimal, attributing more and more importance to the "enlightening" discovery of Leibniz's syncategorematic infinite, it seems that Robinson's rather superficial reading of Leibniz's late correspondences on the issue had already in 1966 grasped the unequivocal baseline of Leibniz's treatment of the issue. Second, not only does Robinson grasp this fundamental fact of Leibniz's position, he is also able to distinguish this from the contemporary debate on the status of infinitesimals, arguing that the post-Cantorian debate on the status of infinitesimals, along with all numbers, had dramatically shifted since Leibniz's time. In this, Robinson uses the reference to Leibniz to establish a historical understanding of some of the

[57] Robinson, 282. Cf. Leibniz, *Math* IV, 95.

continuities, but also more importantly the discontinuities, that exist between his project and its Leibnizian legacy.

This second point is worth stressing. Despite the fact that Leibniz's syncategorematic infinite was announced clearly in his *New Essays*, it seems to have taken philosophical commentators quite some time to center on this qualification. If we understand the contemporary discussion of Leibniz to have been in some ways reinvigorated by Russell's monograph of 1900, we can see that even at the start of this renaissance, Russell did not mistake Leibniz's explicit view and had an accurate definition of it, the maintaining of both a belief in the actual infinite and the rejection of an infinite number or whole.[58] Yet despite Russell's being convinced by the arguments in the *New Essays* from which he correctly appraised Leibniz's views, he spends the rest of the chapter on the "Labyrinth of the Continuum" moving on to issues of aggregation, motion and space/time without stopping to appreciate the implication of Leibniz's literal statements about the syncategorematic infinite in the *New Essays*, concluding that Leibniz's philosophy suffered a deep inconsistency between his law of continuity and his "complete" rejection of the continuous in the aggregation of bodies. More recently, the tide against the Russellian reading of Leibniz has focused on the compatibility of the law of continuity and the actual infinite division of things, and it seems that among others in this recent tide, Hide Ishiguro's chapter-length treatment of the syncategorematic infinite has been recognized as one of the earliest of the philosophical commentaries to have sufficiently treated this issue.[59] What is not surprising in Ishiguro's reading is how this issue of the syncategorematic interpretation provides us with some subtlety in the interpretation of Leibniz. However, what becomes surprising is Ishiguro's aim in proposing such a reading, that is, to read Leibniz as compatible

[58] Russell, 128.
[59] Arthur, "'A Complete Denial of the Continuous?' Leibniz's law of continuity", 10.

with Russell's (and by extension Frege's) views on the contextual definition of numbers. In stating her aims in reading Leibniz's syncategorematic infinite, she writes that "we shall see however that for certain expressions like that of 'infinitesimal', Leibniz took a position closer to that of Bertrand Russell in his theory of description, a theory which Russell claimed was a theory of contextual definition".[60] As "*compendium loquendi*", Ishiguro ultimately goes on to claim:

> Reference to mathematical fictions can be paraphrased to talk about standard mathematical entities. One can be a monadologist while remaining a finitist in mathematics, or one could be a realist about infinitesimals and also be a monadologist […] Frege… and Russell…both of them were interested in the logical priority of different kinds of numbers and the problem of contextual definition of differentials and infinitesimals […] It is therefore singularly ironical that Russell misunderstood Leibniz's thoughts on infinitesimals in the traditional manner and attacked Leibniz.[61]

For a closer look at Ishiguro's reading of Leibnizian infinitesimals, we should look at the problem raised earlier in Robinson's invoking of the two paths of the development of infinitesimals: the path of entity and the path of limit. What is interesting about Ishiguro's reading is that her argument involves a distinction between her reading and Robinson's, namely that her reading demonstrates Leibniz as a proto-Cauchyian theorist (tending toward the limit concept) about the infinitesimal rather than a proto-Robinsonian (tending toward infinite/simal entities).[62] Her argument stakes its claim precisely on the use of a syncategorematic interpretation of infinite/simals literally stated in Leibniz's writing. It is, however, unclear if

[60] Ishiguro, 82.
[61] Ishiguro, 100.
[62] Ishiguro, 83.

Ishiguro's own understanding of the syncategorematic infinite/simal in Leibniz can successfully establish such a dichotomy.

Ishiguro distances her reading of Leibniz from Robinson by making a clear distinction of camp. In her view, Robinson, by the standards of modern mathematics, successfully argues for non-Archimedean infinitesimals. Yet even if we accept the results of his work, "his belief that, even if Leibniz considered infinitesimals to be an ideal fiction which can be dispensed with, these infinitesimals for Leibniz were fixed entities with non-Archimedean magnitudes, the introduction of which shortens proofs, seems unwarranted."[63] We should note that this is a gross misreading of Robinson, for whom Leibniz served only as a forebear of his method, as the latest development of the entity path of the infinitesimal. Nonetheless, what Ishiguro hopes to argue is that a proper reading of the notion of the infinitesimal puts us in a theoretical continuity with the limit path. The main reasoning for this is that a proper understanding of Leibniz's syncategorematic infinitesimals will place it in continuity with traditional or standard Archimedean sums that follow the principle of the whole being greater than a part.

Here Ishiguro's argument begins with an (incorrect) construal of Robinson's view of Leibniz where infinitesimals are fixed, categorematic entities. She then argues against this by arguing for their having a syncategorematic status in Leibniz's work. From here, she proceeds to show that this implies that Leibniz's treatment of syncategorematic infinitesimals is effectively what gives the entity character of infinitesimal magnitudes a fictional status. This fiction does not allow for any true entity but rather, through Leibniz's use of mathematical symbolism, only captures the relationship between two finite but variable points that ultimately obey the Archimedean principle of the whole-part relation.

[63] Ishiguro, 83.

Despite these stated misgivings in her reading of Robinson, Ishiguro does provide one very pertinent assessment of how one is to understand the syncategorematic character of infinitesimals in Leibniz. First, the distinction that she employs, the distinction between entity and limit, is based on the idea of real or actual infinitesimals as a special, non-Archimedean type of number, the very type of number that Robinson himself aims to develop in *Non-Standard Analysis*. Her argument is obviously that there is no such special status given to the infinitesimal in Leibniz's work. But in Leibniz's own words, as Ishiguro is keen to point out, "This is why I believed that in order to avoid subtleties and to make my reasoning clear to everyone, it would suffice here to explain the infinite through the incomparable, that is, to think of quantities incomparably greater or smaller than ours."[64]

Here, it is only apparent that Leibniz leaned toward just the kind of incomparability that would give infinitesimals this "incomparable" status, that is, non-standard character. It is precisely this sort of claim that would have given rise to the varying interpretations of Leibniz's infinitesimals even in his own day. As Ishiguro points out, however, there is no great mystery about how one is to understand these magnitudes or quantities termed "incomparable", since the relation between the side of a plane to its area (a one dimensional comparison with a two dimensional) is the standard sort of relationship referenced by the incomparable. The term *incomparable* distinguishes two quantities and two dimensions but does not give any of these a non-Archimedean status. The incomparable is thus a neutral term in this debate about the status of infinite/simals. As Ishiguro continues to argue, the supposedly infinitesimal magnitude in question, the differential dy/dx is not infinitesimal but rather a finite relation between differentials (unless dy and dx are infinitesimals of different orders). Here, the incomparability in question concerns the relationship between the magnitudes themselves and not, as it might seem,

[64] Ishiguro, 86. Leibniz, *Math* IV, 91. Leibniz, *PPL*, 542-54.

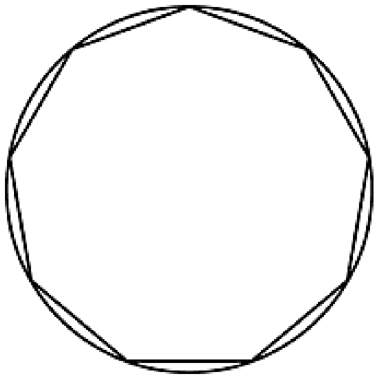
the relation between finite and infinite/simal magnitudes. Though she does not give an example, this neutral notion of the relation of incomparability can be said of the relation between the side of a polygon and its area.

From this clarification of the equivocation of the use of (in)comparability, Ishiguro's argument turns to introduce the syncategorematic character of the infinitesimal to "save" Leibniz from misreadings that identify "incomparable" with the "non-standard". She quotes Leibniz's argument to Varignon where he argued for the non-rigorous identification of rest as a kind of motion and a circle as a kind of polygon by virtue of the fact that,

> [R]est, equality and the circle terminate the motions, inequalities and the regular polygons which arrive at them by a continuous change and vanish in them. And although these terminating points [terminaisons] are excluded, that is, are not included in any rigorous sense in the variables which they limit, they nevertheless have the same properties as if they were included in the series, in accordance with the language of infinites and infinitesimals, which take the circle, for example as a regular polygon with an infinite number of sides.[65]

From this, Ishiguro argues that here we find Leibniz describing the infinite/simal problem in many of the same ways that contemporary students of calculus would, that is, with the terminating points treated as limits that are approached by a function (like a regular polygon approaching circularity by virtue of increasing its number of sides) without rigorously taking the last point into the function itself.

[65] Ishiguro, 90. Leibniz, *PPL*, 546.



[Figure 2][66]

She adds to this, however, a caution:

> I think Leibniz is misleading when he writes to Varignon that truthfully speaking he himself is not sure whether he shouldn't treat infinitesimals as ideal things or as well-founded fictions. The limit may be a well-founded fiction, but talk of infinitesimals is, as he says, syncategorematic and is actually about 'quantities that one takes… as small as is necessary in order that the error should be smaller than the given error.'[67]

According to the idea of the infinitesimal's syncategorematic status, Ishiguro's interpretation of Leibniz naturally leads to favor the reading of Leibniz's infinitesimals as a proto-limit, that is, a proto-Cauchyian idea. In this, she strongly identifies the syncategorematic understanding of infinite/simals with the limit concept. Here Ishiguro argues that the combination of the infinitesimal with the notion of the limit allows us, as it does in common usage today, to speak of the variables on a function approaching zero as a limit. As such, Ishiguro remarks:

> [W]e have seen that Leibniz denied that infinitesimals were fixed magnitudes, and claimed that we were asserting the existence of variable finite magnitudes… we

[66] Figure taken from Educational Technology Clearing House, accessed March 21, 2011, http://etc.usf.edu/.
[67] Ishiguro, 90. Leibniz, *PPL*, 543.

> could say that Cauchy claimed that limits existed whereas Leibniz wanted to say that they were a well-founded fiction.[68]

The difference between Leibniz and Cauchy would then be "very little" according to this reading.[69]

This interpretation of Leibniz's infinitesimal as a proto-limit is of course quite convincing. In retrospect, Leibniz's notion of the syncategorematic infinite/simal is not so far from the idea of the limit. Insofar as Leibniz did not commit to an actual infinite/simal entity but proposed a syncategorematic one, Ishiguro's proto-limit interpretation of Leibniz is perhaps all too obvious. However, she is only half right. That is, the larger aspect of her argument seems to skew the many different attempts to both put the idea into question in Leibniz's own reflections and the different ways in which the attempts to defend his conception provide opportunities for him to reinvent or make nuances in his use and significance of the term. An obvious mark of this is how her argument turns on the false dichotomy between a Robinsonian, non-Archimedean or non-standard "entity" nature of the infinitesimal, and a proto-Cauchyian limit or finite variable nature of the infinitesimal. Indeed, before Cantor's and Dedekind's revolutions in the field of mathematical objectivity, Leibniz did not view this alternation between a fictional entity and a limit as a problem for developing a new class, type or field wherein these quantities exist. The baseline criterion for Leibniz was rather the question of the legitimate use of the term given the centrality of the Archimedean principle. These constraints seemed to have already been foundational before any questions came up about the status of infinitesimals, the relationship they expressed, or their status in the space of concepts.[70] In turn, Leibniz's problem seems to

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[68] Ishiguro, 90. Leibniz, *PPL*, 92.
[69] Ishiguro, 90. Leibniz, *PPL*, 92.
[70] H.J.M. Bos's landmark work in evaluating Leibniz's calculus is one of the many sources that one might cite in holding a position that Leibniz never held a limit concept of the infinite/simal. While I do not employ Bos's

have been how one is to understand the status of infinitesimals given the baseline condition that they are not in any manner fixed quantities, unities, wholes or numbers.

In other words, from a developmental point of view, Leibniz had already rejected actual infinite measure since his writings before his work on the calculus. In his notes on Galileo's *Discourses and Mathematical Demonstrations Relating to the Two New Sciences* in 1672, Leibniz drew the following conclusion from Galileo's insights, one that he thought the latter should have been forced to accept given his commitments. Leibniz remarked, "Infinity itself is nothing, i.e. that it is not one and not a whole."[71] Although Leibniz did not always hold an unequivocal rejection of the actuality of infinite/simals, a theme we will visit in the next chapter, this position certainly characterizes Leibniz's mature work. The gap between Leibniz's own early realizations and the problems that Ishiguro highlights at least opens the possibility that what Leibniz saw as an issue was far from the problem of non-Archimedean quantities, but rather the further problem of what sense these fictional infinitesimals had after the definite exclusion of non-Archimedean possibilities. To be clear, Leibniz's own problematic of the status of infinite/simals concerned their status as Archimedean quantities and not the issue of judging

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comparison of Leibniz's calculus and modern calculus undertaken in "Differentials, higher-order differentials and the derivative in the Leibnizian Calculus", he has argued here and elsewhere that Leibniz's calculus stands on its own and does not require later mathematical developments to either vindicate or reappraise it. In this same article, he attempts to provide two different ways of reading "foundation" in Leibniz's methods for the calculus. He distinguishes between the use of the differential algorithm through the infinite series and the use of infinitesimals as entities. While I do hold these two sorts of justifications or "foundations" apart, Bos's fundamental argument in this text aims at arguing that Leibniz's development of the *method* of the calculus is rooted in the "extrapolation" of the infinite/simal from the finite case. While Bos does note that Leibniz's legacy left the status of the terms of his calculus open to misinterpretation in its lack of a foundation, he also notes that Leibniz was careful not to treat his infinite/simal terms as independent definite entities. He warns, "The common concern of historians with the difficulties connected with the infinite smallness of differentials has distracted attention from the fact that in the practice of the Leibnizian calculus, differentials as single entities hardly ever occur. The differentials are ranged in sequences along the axes, the curve and the domains of the other variables; they are variables, themselves depending on the other variables involved in the problem, and this dependence is studied in terms of differential equations." H.J.M. Bos, "Differentials, Higher-order Differentials and the Derivative in the Leibnizian Calculus," *Archive for the History of the Exact Sciences* 14 (1974): 17. Cf. H.J.M. Bos, "The Fundamental Concepts of the Leibnizian Calculus," in *Lectures in the History of Mathematics* (Providence, RI: American Mathematical Society, 1993), 83-99.

[71] Leibniz, *LC*, 9.

between Archimedean and non-Archimedean magnitudes or quantities. As such, Ishiguro poses her question incorrectly.

To put a final punctuation on Ishiguro's argument in her otherwise concise chapter, we shall consider another, and perhaps the ultimate intention of Ishiguro's argument. In the final section of her chapter, Ishiguro moves on to examine the relation between the association of the infinitesimal in Leibniz and Cauchy's limit, and a meta-mathematical discussion of the status of infinitesimals. Citing Frege and Russell, Ishiguro argues that in order to understand the status of infinitesimals in Leibniz, we should not hesitate to import the idea of contextual theory of meaning in Frege and the idea of definite descriptions in Russell, two fundamental principles of the contemporary philosophical approach to mathematical objectivity. For Ishiguro, the Fregean idea of *salva veritate* applies entirely to Leibniz's use of infinitesimals in calculation, and thus there should be nothing, in terms of the conservation of truth values, to mark Leibniz's use of the infinitesimal as inconsistent with contemporary evaluations of mathematical propositions. In this, she also notes that Frege explicitly remarks on the differential, "It is enough if the proposition taken as a whole has a sense; it is this that confers on its parts and also their content. This observation is destined, I believe, to throw light on quite a number of difficult concepts, among them that of the infinitesimal."[72] On the other, Russellian side, she remarks that as a special example in his section on definite descriptions in the *Principia Mathematica*, "Russell…gave the sign for the differential d/dx as his first example of a sign with which we are familiar which has a meaning only contextually."[73] Noting that Russell's theory of definite descriptions is aimed at unique existentials, she invokes the syncategorematic status of infinitesimals as that which determines a limit. This determination, through the use of a symbolic notation dy/dx, puts the

[72] Ishiguro, 96.
[73] Ishiguro, 98.

Leibnizian determination of the infinitesimal in consistency with the modern, that is, post-Russellian view of the logical status of mathematical objects. In the end she argues, despite the differences between Frege and Russell, Leibniz's views on infinitesimals can be consistent with both theories.[74]

In Ishiguro's larger argumentative goal in this chapter, her ultimate intention is to demonstrate the compatibility of Leibniz's views on infinitesimals with a contemporary one, the Fregean and Russellian orthodoxy with respect to the legitimacy of mathematical entities. In this, she is correct to argue against Russell's prejudice against Leibniz in saying, "It is therefore singularly ironical that Russell himself misunderstood Leibniz's thoughts on infinitesimals."[75]

While I agree with Ishiguro's remark against Russell, the irony here is much deeper than Ishiguro suspects. The fact is that Robinson, as Ishiguro also claims, had already laid the foundation for a rigorous, albeit non-standard use of infinitesimals qua entity. Thus Robinson's infinitesimals, according to the standards of modern, post-Cantorian means of argumentation, fulfill both the Fregean and Russellian conditions of mathematical legitimacy raised in her evaluation of Leibnizian infinitesimals. Ironically, her evaluation seems to be lacking in the very places where Leibniz's views would require the most explanation, in the deployment of infinite/simal terms in his account of motion as well as in his metaphysical work. In this sense, her justification of the Leibnizian infinitesimal seems historically superfluous and thematically irrelevant. The irrelevance I refer to here lies in that the difficulty Ishiguro attempts to resolve is neither Leibniz's nor Robinson's. Indeed, Leibniz's difficulty with the status of infinitesimals lies not in his already formed idea of a fundamental criterion of the exclusion of non-Archimedean wholes, but rather in its conceptual coordination with his other ideas of the infinite

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[74] Ishiguro, 100.
[75] Ishiguro, 100.

and infinitesimals. That is, it lies not only in the status of bodies, motion and divisibility but also in the status of the infinite in his account of contingency and divisibility, and the status of the distinction between imagination and the actual. In this sense, even if we accept her argument about the syncategorematic status of Leibniz's infinitesimals as entirely correct, her fundamental application of this status into her more global reading of Leibniz's philosophy is ironically out of joint.

3. Infinite/simal fiction and the false dichotomy of the sensible and the ideal

This basic irony of misconception pushes us toward the main component of Ishiguro's larger argumentative strategy. This is her intent to show that Leibniz's reflections in the mathematical realm allow us another view of his theory of ideas. While I agree with her here in the most thematic, abstract sense, the concrete content of her analysis falters precisely on the false understanding that the problem of infinite/simal fictions can be resolved by recourse to a distinction between direct and indirect means of reference, a criticism that I elaborated above. It is because she sees the problem of fictions in this way that her reading of the exchange between Locke and Leibniz reduces the problem of the infinite/simal fiction to a distinction between ideas and sensibility (or intuition). Before taking a look at Leibniz's responses to Locke, I will employ a further criticism of Ishiguro's understanding as a point of entry.

Here we can begin with Ishiguro's point about the distinction between the sensible and the ideal in the context of mathematics:

> Leibniz's claim that ideas are explained through propositions and truths, has all too often however been examined by his admirers only in the context of his

> disagreement with the empiricists and the ideas of sensible qualities. Whereas it seems to me that this view is also important in understanding his mathematical ideas.[76]

To be more precise, the asymmetry between idea and representation, Ishiguro thinks, goes far beyond the basic "rationalist" repertoire of the field of language and logical relations. The field of mathematics and geometry becomes, in this view, another means to see the extent of Leibniz's anti-empiricism. Indeed she states that, "Leibniz wanted to free mathematics from geometrical intuitions."[77] With this, Ishiguro implies that the notion of the infinite/simal in Leibniz should be detached from its intuitive connection with the geometrical notion of division and the convergence between the infinitely-sided polygon and the circle, and should rather be read in light of Leibniz's logic of propositions. In this, the counter-intuitional notion of the infinitesimal, ontologically designated as "fictional", more conveniently expresses proximity with the logical notion of Leibniz's theory of propositional truth such that, "reference to these ideal entities or fictions can be paraphrased into propositions and equations in which such entities are not designated, which have strict truth values."[78] Hence in Ishiguro's interpretation, Leibniz's infinitesimals are to be considered as rigorous not simply because they avoid the sort of contradictions Leibniz himself was keen to avoid but also, in her reference to Frege and Russell, because they measure up to contemporary notions of paraphrase, propositions and truth conditions. For Ishiguro, all this attests to the notion that Leibniz's theory of ideas was "so ahead of his time" that it was inevitable that it would be misunderstood and misconstrued.[79] In turn, all mathematical ideas are to be reduced to a common level of "abstract ideas", ideal entities which

[76] Ishiguro, 97.
[77] Ishiguro, 88.
[78] Ishiguro, 93.
[79] Ishiguro, 96.

in Leibniz's general division between the ideal and the actual, do refer, albeit not always, to sensible correlates. Infinitesimals, on the other hand, are secondary terms that are to be reduced to the status of mathematical references; or rather, to be more precise, propositions containing the use of infinitesimals reduce to the status of the propositions themselves. Hence, for Ishiguro, the true distinction between ordinary mathematical entities and infinitesimals is the one that exists between the ideal and the fictional. She writes:

> When we refer to numbers or to relations which are ideal entities, we are referring to something…. Fictions, on the other hand, are not entities to which we refer. They are not abstract entities. They are correlates of ways of speaking which can be reduced to talk about more standard kinds of entities. Reference to mathematical fictions can be paraphrased to talk about more standard kinds of entities.[80]

Again, what is problematic here in her distinction between ideal and fictional entities is that, without denying the difference, the relationship is made into a reductive one. Even if, from a contemporary perspective, one can make Leibniz compatible with a Russellian standard of propositions that definitely describes, and a reductive reading of the distinction may be perfectly sound, this intra-mathematical distinction between ideal and fictional reference nonetheless reveals much more than its comfortable lodging in a Russellian universe. In fact, the fictional status of infinitesimals plays a larger role as a problematic in the articulation of Leibniz's larger metaphysical project beyond merely a theory of ideas or the rigorous foundation of the method of the calculus. In this, the partial correctness of Ishiguro's evaluation of the syncategorematic status of infinitesimals is not faithful to Leibniz's own lifelong reflections. With the syncategorematic qualification and the fictional status of these terms, Leibniz did not wish to

[80] Ishiguro, 100.

escape with a theory that merely satisfies the basic conditions of consistency, nor did he (nor could he) intend to intervene in a debate on mathematical objectivity centuries after his time. That is, Leibniz did not seek to refer to infinitesimals as fictional in order to reduce the impact of their metaphysical meaning in such a way that they would be reduced to a standard for all mathematical entities suitable in the contemporary context and use of terminology. Much to the contrary, when Ishiguro reads her citation of Leibniz's letter to Varignon, "My intention was to point out that it is unnecessary to make mathematical analysis depend on metaphysical controversies"[81], she understands Leibniz as saying that, facing two choices, one that reduces infinitesimals to metaphysical entities or one that reduces them to logical ones, he refused to reduce them to the metaphysical, but rather to the logical. I would suggest that the refusal to reduce the problem of infinitesimals to the metaphysical does not in any sense imply a reduction to the logical. Instead, a clear reading of Leibniz's work discerns a refusal of both these reductions.

In sum, the error of plucking Leibniz's reflections on infinitesimals from their historical context and inserting them into contemporary quarrels in the "philosophy of mathematics" forces Ishiguro's main argument to default both on contemporary problems in this domain and on providing any insight on how Leibniz saw these problems. This approach hardly corresponds to the importance that Leibniz gave to this set of problems in his philosophical development.

Advancing onward from here, we can nonetheless find in the very arguments that Ishiguro cites to establish her reading something else at stake in Leibniz's thought. I turn to the very sort of debate cited in Ishiguro to open this problem. While she argues that one should develop a view of the infinitesimal debate that goes beyond Leibniz's debate with the burgeoning empiricism brought to the fore by Locke, I think, on the contrary, that it is precisely in these

[81] Ishiguro, 86. Leibniz, *Math* IV, 91.

debates that we are able to get the true face of what the question of the status of infinitesimals means for Leibniz. Returning to the 1702 letter to Varignon that Ishiguro cites, Leibniz argues:

> Although it is not at all rigorously true that rest is a kind of motion or that equality is a kind of inequality, any more than it is true that a circle is a kind of regular polygon, it can be said, nevertheless that rest, equality and the circle terminate the motions, the inequalities and the regular polygons which arrive at them by a continuous change and vanish in them. And although these terminating points [*terminaisons*] are excluded, that is, are not included in any rigorous sense in the variables which they limit, they nevertheless have the same properties as if they were included in the series, in accordance with the language of infinities and infinitesimals, which take the circle, for example as a regular polygon with an infinite number of sides.[82]

It is clear what Ishiguro would draw from this dense passage. The limit is an ideal entity that puts the polygon in relation to the circle or rest in relation to motion precisely by distinguishing the sequence of "standard" terms from the terminating point. As we examined in the above, the many-sided polygon is not a circle and the decelerating body is not at rest. Variables that tend toward a limit are rigorous but the terminating term is not to be rigorously included, as he explains above. Despite this, they can be taken "as if" they were included. That is, it is through this distinction between variables at the limit and the other variables that we can nonetheless hold that they possess the same properties as the continuous approach that they terminate. Ishiguro draws this discussion close to the notion of the limit and reasons that these terminating points are non-rigorous, ideal entities which nonetheless refer in the Fregean/Russellian sense. Hence the result of the continuous approach of a regular polygon with multiplying sides toward a

[82] Ishiguro, 90. Leibniz, *Math* IV, 91.

circle would not, rigorously speaking, "be" a circle since the arrival at the termination would be a "fictitious" one. Ishiguro takes this to mean that there is nothing lacking in rigor in saying that *the limit* of this same infinite multiplication of sides for any polygon is the circle. From this, Ishiguro makes the further association with Frege that "the problem is not, as might be thought, to produce a segment bounded by two distinct points whose length is dx, but rather to define the sense of an identity of the type df(x)= g(x)dx."[83] The consequence is clear: infinitesimal quantities are a correlate to an ideal entity which actually refers. Hence Ishiguro argues that:

> An idea of a cube is, for Leibniz, not a visual or factual representation of it, but understanding truths about it. Leibniz's claim that ideas are explained through propositions and truths, has all too often however been examined by his admirers only in the context of his disagreement with the empiricists and idea of sensible qualities.[84]

That is, for Ishiguro, the ultimate reduction of the status of infinitesimal quantities, along the axis of the problem of their rigor, provides another avenue from which to view Leibniz's rationalism. Not only is this position applicable to the relation between the logical analysis of ideas in the propositional sense, but also equally applicable to mathematical ideas. In this, she asserts a strong continuity between Leibniz's logic and mathematics, absorbing both into a larger rationalist mantle which frees "mathematics from geometrical intuitions."[85] Consequently, what Ishiguro fails to mention is the importance of Leibniz's argument with Locke's view along these lines. While it might be true that the analysis of the argument between Leibniz and empiricism is too often staked on the relation between ideas and truth in the logical or propositional dimension, often excluding the mathematical, this in no way means that the argument between Leibniz and

[83] Ishiguro, 97.
[84] Ishiguro, 97.
[85] Ishiguro, 88.

the empiricist does not hinge in some way on the mathematical. By presenting a reading of Leibniz's exchanges with Locke, I will show that what Ishiguro misses in her reading of the same text is precisely how the content of Leibniz's argument against Lockean empiricism brings out the very basis on which Leibniz thinks of the status of the infinite/simal.

The mathematical content of the exchange between Leibniz and Locke, if we take his major work in this genre dedicated to a criticism of Locke's *Essay Concerning Human Understanding*, speaks directly to the content of Ishiguro's commentary on the 1702 letter to Varignon. In 1700, Locke's *Essay Concerning Human Understanding* was translated into French by Pierre Coste, and in 1703 Leibniz began to write and edit a lengthy response, now edited and published as the *New Essays on Human Understanding*. Indeed, this text is important for us in the sense that it is where Leibniz unequivocally defined the infinite as syncategorematic.
The issue here is clear. In his section "On the infinite" in this text, Leibniz aimed to sidestep a direct confrontation with Locke on the relation of the infinite/simal and its relation with sensible qualities or intuitions. Although the issue of innate ideas dominates the first section of the book, these questions do not take center stage concerning the existential status of the infinite/simal. Instead, Leibniz focused on the idea of the infinite itself. The two major arguments against Locke concern the status of its absoluteness and the second, the status of its non-compositeness. While it appears that the conflict between Leibniz and Locke is one of the sensible or innate origin of infinity, a classic battle between rationalism and empiricism taught in introductory classes, the point Leibniz made here is rather indifferent to this genetic problem. Leibniz first cited Locke as saying that the infinite arises from modification of expansion and duration. That is, according to Locke, since the human mind gains access to finite modes of quantity in expansion and duration, it is through their imaginary concatenation, repetition, and multiplication

that one gains the idea of infinity.[86] Leibniz responded that there would be no way to get an idea of the infinite from mere repetition. True infinity is not a modification; it is absolute.[87] The case is directly inverse for Leibniz, where the finite is gained through the limitation of the absolute. Why is this? Leibniz agreed with Locke that, following the idea that the repetition of a duration or expansion of a limited extension, a line can be extended into infinity, but this dimension of the infinite can only come from an exterior idea of the absolute. Nonetheless Leibniz argued, “But it would be a mistake to try to suppose an absolute space that is an infinite whole made up of parts. There is no such thing: it is a notion that implies a contradiction.”[88] Now Leibniz took into account that Locke did not hold infinity as an “all” totality or as a number, and in this he was in agreement. Yet Leibniz pointed out that their agreement on this point existed despite having completely different reasons for believing such. While Locke argued that the idea of the infinite cannot be a totality or number because the repetition that gives rise to it can never be actual, Leibniz argued that, far from facing any problems concerning its inactuality or its sensible genesis, the idea of the infinite as a number was rejected because it is a contradiction.

Whether this is a fair assessment of Locke or not, Leibniz clearly saw his argument concerning the infinite as being staked on terms other than its origin. In this, Leibniz argued that the infinite is of the same origin as necessary truths, linked to the debate concerning innate ideas already launched at the beginning of the book. Yet if we look closely at the start of his argumentation in the chapter “On infinity”, Leibniz’s central point against Locke had little to do with the sensible or innate quality of the idea of the infinite, instead allowing this issue to flow

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[86] Leibniz, *Nouveaux Essais sur L’Entendement Humain*, ed. Jacques Brunschwig (Paris: GF Flammarion, 1995), 124. Leibniz, *New Essays on Human Understanding*, trans. Peter Remnant and Jonathan Bennet (Cambridge, UK: Cambridge University Press, 1981), 157.

[87] The notion of the infinite as absolute is a major theme that will be treated in what follows in this chapter and again in the following chapters. It is sufficient at this point to note the fundamental disagreement concerning the basic nature of the infinite between Leibniz and Locke.

[88] Leibniz, *Nouveaux Essais*, 125. Leibniz, *New Essays*, 158.

out as a consequence. What Leibniz held on to is the problem concerning the *concept* of the infinite itself. That is, what Leibniz is invested in is the rationality and not the actuality of the infinite. In this, he held on to the consequences that followed from the absoluteness and not the constructive status of the infinite.

Keeping this in mind, we are in a better position to understand Leibniz's comment on Locke's argument for why the idea of infinity is not applicable to certain other ideas like sweetness and whiteness. This was a response to Locke's position that, along a basic distinction between quality and quantity:

> For those ideas, that consist of parts are capable of being augmented by addition of the least part…. Those Ideas that do not consist of parts cannot be augmented to what proportion men please, or be stretched beyond what they have received by their senses; but space, duration, and number, being capable of increase by repetition, leave in the Mind an idea of an endless room for more…. And so those Ideas alone lead our minds towards the thought of infinity.[89]

For Leibniz, the usage of this distinction between quantity and quality, that which admits to parts and that which does not, would not do at all for clarifying the nature of infinity. In response, Leibniz argued:

> I do not understand the force of this reasoning for nothing restricts us from being able to receive a perception of whiteness more clear than that which we actually conceive. The real reason for why we might believe that whiteness could not be augmented to infinite is because it is not an original quality; the senses do not give anything but a confused understanding; and when we have a distinct

[89] John Locke, *An Essay Concerning Human Understanding*, ed. and introduction Peter H. Hidditch (Oxford and New York: Oxford University Press, 1975), 213.

> [understanding], we will see that it comes from structure and is bounded on the organ of sight. But with regard to original or directly understandable qualities, we see that there is some way to go to the infinite, not only where there is extension or if you like diffusion, or that which the schools call *partes extra partes*, as in time or space, but also where there is intension and degrees, for example with regard to speed.[90]

Leibniz's point here, refuting Locke's distinction between quality and quantity and replacing them with the distinction between extension and intension, or more precisely, putting the issue of the infinite in terms of the discrete and continuous, was that the problem of the infinite is indifferent to the mode of perception by which Locke supposed the idea of the infinite is gained. Here, Leibniz did not resort to a contestation of the sensible origin of the idea of the infinite or of the disagreement about innate and sensible ideas. Instead Leibniz argued, on the one hand, that the problem of quality concerns the distinction between confused and distinct ideas, and on the other hand, that the infinite applies equally to both extension and intension or discreteness and continuity. In this, Leibniz's argument hinged not on any contest of the sensible or innate origin of the infinite but rather on the idea of infinity itself, regardless of the problem of its origin or perception. The refutation of the sensible origin of the infinite follows as a consequence since it results in an account of infinity that is inconsistent on the one hand, and insufficient to handle the application of the infinite to the intensive or continuous.

The heart of Locke's mistake, in Leibniz's view, was that he held the finite and infinite as modes of quantity. Again, arguing from the aspect of the concpet of the infinite alone, Leibniz put it very clearly in the beginning of his commentary on Locke's infinity:

[90] [Author's translation] Leibniz, *Nouveaux Essais*, 125.

> Properly speaking, it is true that there is an infinity of things; that is to say that there is always more than can be assigned. But there is nothing such as an infinite number or a line or other infinite quantity if we take them as true totalities, as it is easy to show. Due to this, the schools wanted to admit a syncategorematic infinite and not a categorematic infinite. The true infinite, in rigor, is nothing but in the absolute, what is before any composition, and not formed by the addition of parts.[91]

Here the primacy of the absolute infinite, from whose limitation we get the finite, is based on an argument about the concept of the infinite itself and not its origin. It is precisely on this point that Leibniz's debate with Locke can be viewed as more than a contestation over the sensible or innate origin of ideas, but the concept itself. Indeed in Leibniz's arguments here, an important conceptual distinction between three senses of the infinite corresponding to different status was mobilized to help clarify the framework of the debate. The first two, the syncategorematic and the categorematic infinite, have already been discussed briefly above. The idea that finitude is achieved by a limitation of the infinite leads us to characterize another sense of the infinite: the hypercategorematic infinite. This is the infinite in its absolute sense, the infinity of God or an infinity without parts. As Leibniz argued in the above, the syncategorematic sits somewhere between the false infinity of the categorematic and the "true" and absolute infinite that is without parts. Now, since Leibniz rejected both the categorematic infinite as contradictory and the infinity of repetition as impossible, the syncategorematic infinite must however also be distinguished from the absolute, that is, hypercategorematic infinite. As we shall see, this three-fold distinction is important precisely because it allows us to prevent the development of the syncategorematic infinite from falling into two types of error. The first error is to explain the

[91] [Author's translation] Leibniz, *Nouveaux Essais*, 124.

syncategorematic infinite by means of a "fictionalization" of the categorematic infinite, to make the mediated and non-absolute identical to the immediate and the absolute, that is, to model Leibniz's infinite on the fictionalization of a contradiction. This is no way to read Leibniz's understanding of fiction. The second error is to explain the syncategorematic infinite by means of a reduction to a "potential" infinite by means of division or repetition. Since what the syncategorematic infinite aims to express, distinguished from the contradictory categorematic infinite and the absolute hypercategorematic infinite, is an actuality that, despite being derived from an origin in the absolute, cannot easily be reduced to other terms.[92] The consequences of this irreducibility will be explored in what follows. Without having yet unfolded all the details of the syncategorematic infinite, we are nonetheless in a position to see that Leibniz's deployment of the syncategorematic infinite/simal concerns more than a debate about the empiricist position. That is, it is the very content of his argument over the *concept* of the infinite (and corresponding status) that provides the real field of disagreement, one which delivers, only as a result or a consequence, an implication that refutes the empiricist theory of the sensible origins of the infinite.

Far from any dogmatism about innate ideas, Leibniz delivered here an argument that hinged on a disagreement with Locke about the nature of the infinite itself. Leibniz's insistence on the status of the mathematical infinite as syncategorematic thus allowed him to displace Locke's arguments into a different register. Whereas Ishiguro thinks that the reference of Leibniz's problems with the infinite to his debates with Locke elides the propositional status of

[92] A treatment of the relation between the three senses of the infinite is undertaken in O. Bradley Bassler's "Leibniz on the indefinite as infinite". While I do not follow him in adopting a terminology of the "indefinite" or "parafinite", I do share in his general framework of reading Leibniz's introduction of the syncategorematic infinite in the *New Essays* and elsewhere. In this discussion, I merely aim to show that the distinction between the three senses of the infinite exists, and that the syncategorematic infinite cannot be reduced to either of the two other senses. Cf. O. Bradley Bassler, "Leibniz on the indefinite as infinite," *Review of Metaphysics* 51 (June 1998): 856-857.

his claims, it is in fact here that we find a precise point where Leibniz's mathematical reflections exerted their force into a metaphysical terrain. The key question here is then whether, as Ishiguro indicates, the fictional status of infinitesimals or their syncategorematic status only leads to rigor via a reduction to referring terms. In order to evaluate this problem, we should turn to the problem of the status of the infinite/simal in its syncategorematic interpretation. What does this term mean, and how did it constrain Leibniz's larger philosophical project?

As our examination of Ishiguro has shown, if we take the contemporary standard of a "philosophy of mathematics" approach to understanding Leibniz's problem of the status of the infinite/simal, we will always be constrained to a project of a reduction of the fictional terms to the non-fictional. The argument about rigor, or Leibniz's foundations for the calculus and the infinite/simal term, will remain within the horizon of demonstrating the compatibility of his view with some standard, non-offensive vision of mathematical objectivity. While, to be fair, this spirit of conservatism is certainly present in Leibniz's own writings, especially in his later correspondences, the approach will always file the issue of the infinite/simal under its broader categories, to resolve the issue under the propositional analysis of truth and its correlates. In this, the encounter with Locke as we read above would be devoid of Leibniz's real confrontation with the nature of infinity itself. Two aspects of this encounter between Locke and Leibniz are thus critical in our consideration of the infinite/simal. First, with respect to the mathematical dimension of this issue, Leibniz and Locke agreed that there is no infinite number (no infinite totality), but their reasons were different. Locke's infinity by repetition produces no final stopping number and no totality, meaning that the idea of infinity is given in the absence of a last finite number. It is a negative one. Leibniz held that there is no infinite totality since it would result in contradiction, but by maintaining that, bracketing a hypercategorematic infinite in the idea of God, nothing forbids us from viewing the infinite as syncategorematic. The major addition that Leibniz provided to the discussion was the removal of the negative or merely potential nature of infinity. Second, Locke argued that the idea of infinity is related to the idea of the "duration and ubiquity" of God or the absolute. Leibniz agreed with this. Yet far from starting with the innateness of the idea of infinity given or communicated from God to mind, Leibniz argued that Locke was wrong in simply conceptually opposing the "incomprehension" of this infinity to the comprehensibility of the finite.[93] It is for this reason that Leibniz confidently opened his refutation of Locke in this chapter by invoking his thesis of the actual infinite division of the world.

4. What is the syncategorematic infinite/simal? A closer look.

From his response to Locke, the status of Leibniz's infinite/simals, that is, their syncategorematic status, constitutes the primary focus for commentators who wish to demonstrate its mathematical rigor. Among contemporary commentators (including Ishiguro), Richard T.W. Arthur has devoted a lot of work in expounding this position and its consequences. In original articles and the editing of Leibniz's own writings, the meaning of Leibniz's syncategorematic infinite proves to be itself a labyrinthine topic. In a recent article, "A complete denial of the continuous? Leibniz's law of continuity", Arthur provides a succinct reading of Leibniz's view. How can we evaluate Leibniz's fundamental thesis that "infinitesimals are fictions for whose difference is as small as one can take"? To set up the terms of his analysis, the distinction between syncategorematic and categorematic infinite should be understood in the following way. Using standard (first order) predicate logic: "To assert an infinity of parts

[93] Locke, 210.

syncategorematically is to say that for any finite number x that you choose to number the parts, there is a number of parts y greater than this: $(\forall x)(\exists y)Fx \rightarrow y>x$, with Fx= x is finite, and x and y numbers."[94] In turn, for a categorematic infinite, to see the precise difference, we should understand an infinite number of y parts where: "$(\exists y)(\forall x)Fx \rightarrow y>x$".[95] The difference is precisely this: one can assert an infinite divisibility of parts for any given (continuous) extension without asserting a corresponding number to these parts, since for any x, there is a larger number in the syncategorematic infinite. For the categorematic infinite, one asserts the existence of a number of parts which is greater than any number x. If this formalization is faithful to Leibniz's conception, two points are immediately evident. First, Leibniz's assertion of the infinite/simal as syncategorematic puts him outside any post-Cantorian or Robinsonian problematic. There is nothing "non-standard" or "indenumerable" about the syncategorematic infinite/simal; it obeys the Archimedean principle. The syncategorematic infinite does not posit an entity, number or extension that would be in violation of the Archimedean principle; its infinity is expressed through boundlessness or interminability. More precisely, this means that there is no particular number in the syncategorematically infinite expansion of quantity that could not, by itself, be taken in comparison with standard, that is, Archimedean numbers. This is precisely what Leibniz meant when he communicated to Varignon in 1702, as we examined above, "Although these terminating points [*terminaisons*] are excluded, that is, are not included in any rigorous sense…they nevertheless have the same properties as if they were included in the series."[96] Just as in our figure above of the comparison between an infinitely sided polygon and a circle, there is no real "terminating" final step of adding sides that will turn a polygon into a circle. For the sake of rigor, that is, for the sake of the Archimedean property, one excludes the ultimate or

[94] Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 7.
[95] Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 7.
[96] Leibniz, *Math* IV, 91. Leibniz, *PPL*, 542-54.

penultimate additions of sides. Even so, all of the intermediate additions, syncategorematically infinite in number, toward this ultimate step, where there would be no difference between the two figures, should be treated “as if” one were simply the adding of another side. Secondly, this characterization allowed Leibniz access to the infinite/simal while overcoming the basic Aristotelian and Galilean (and certainly Lockean) characterizations of the infinite.

This second point requires some explanation. Aristotle viewed the infinite as potential. Drawing from the constraints imposed by the Eleatic paradoxes, the infinite divisibility of extension was potential but not actual. In the *Physics*, Aristotle framed the discussion of the infinite from the model of divisibility. As such, “A quantity is infinite if it is such that we can always take a part outside what has been already taken. On the other hand, what has nothing outside it is complete and whole. From thus we define the whole – that which nothing is wanting.”[97] The infinite is that which has an inexhaustible degree of division and separation, “but in the direction of largeness it is always possible to think of a larger number: for the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, and never actual: the number of parts that can be taken always surpasses any assigned number.”[98] Leibniz’s position concerning the syncategorematic infinite differs from Aristotle’s model precisely in the account of the evaluation of its status. Against Aristotle, Leibniz argued for the characterization of the world as actually, and not merely potentially, infinitely divided. In turn, Leibniz’s syncategorematic notion of the infinite/simal itself was then based on understanding actual infinite divisibility. Since the distinction between the categorematic and syncategorematic is a conceptual one, his disagreement with Aristotle on this point does not simply reduce to a simple overturning of the Aristotelian notion of the infinite, a simple displacement of the infinite from

[97] Aristotle, 207a 5-10. Aristotle, *The Basic Works of Aristotle*, ed. Richard Mckeon (New York: Random House, 2001), 266.
[98] Aristotle, 207b 10-15. Aristotle, 267-268.

the potential to the actual. The difference made by the deployment of the syncategorematic infinite is one based on the reconstruction of the framework itself. In the first place, the understanding of a syncategorematic infinite renders a non-contradictory notion of the infinite. Secondly, its status in the actual plays a different role; the infinite is no longer something that is merely abstractly grasped from the idea of a possible further division that lies in the nature of the continuous, but the actual infinite (syncategorematic) division of the continuous.

Here, we can also take a quick look at Galileo in order to see the difference that Leibniz introduced to the discussion. Galileo's basic conception of the infinite is given in the *Discourses and Mathematical Demonstrations Relating to the Two New Sciences,* where he first provided a demonstration of the impossibility of an infinite number, such that the number of squares, the number of square roots could all be put into an isomorphic relation with the number of all numbers and stated,

> I do not see how it is possible to come to any distinction other than to say that all numbers are infinite, the squares are infinite, and their roots are infinite; the multiplicity of squares is neither less than that of all the numbers, nor is the latter greater than the former. And in final conclusion, the attributes of equal, greater, and less have no place in infinites, but only in bounded quantities.[99]

While Galileo will respond to this problem by concluding that infinity is to be understood through the number one, Leibniz, as I mentioned above, made use of this argument for his own ends. Galileo's reasoning was that only the number one contains all of its powers ($1^2=1$).[100] Leibniz's own understanding of Galileo's posing of the problem here implies that, "Infinity itself

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[99] Leibniz, *LC*, 356.
[100] Cf. Bassler, "Leibniz on Indefinite as Infinite", 857.

is nothing, i.e. that it is not one and not a whole."[101] Using the following chart, we can reproduce a simpler version of Galileo's problem. We arrange the series of squares such that they are "numbered" by the number of which each is a square. Evidently, the series of the squares of n is a "part" of the natural numbers. There are numbers in n that do not belong in the series of squares, but every square is (eventually) going to be found in the series of natural numbers. In other words, there are many numbers in n that do not belong to n^2, but every number in n^2 belongs to n. Yet arranged in this way, the number, or "multiplicity", of n's and the number of n^2's appears to be equal. As such, Galileo reasoned that evaluations of greater and lesser seem to be inapplicable to infinite expansions. The obvious and direct implication of this is that infinite terms break with the fifth of Euclid's common notions outlined in Book I, that "The whole is greater than the part".[102]

Natural Numbers (n)	1	2	3	4	5	6	7	…	n+1
Squares (n^2)	1	4	9	16	25	36	42	…	$(n+1)^2$

[Figure 3]

Galileo's basic position, as Leibniz remarked in *Pacidius Philalethes* dialogue of 1676, was highly problematic, and he claimed that Galileo had "abandoned it [the status of infinite/simals] as hopeless".[103] Leibniz thought of Galileo as abandoning the problem since what Galileo did was to disassociate the standard features of finite numbers and extension (equal, greater and less)

[101] Leibniz, *LC*, 9. Bassler notes that in an earlier text, *Accessio ad Arithmeticam Infinitorum* of 1673, Leibniz argued that if we employ Galileo's reasoning concerning the infinite, it should not be the number one but rather zero that is infinite. Like the number one, Zero contains all the powers of itself, but beyond that, zero also contains all multiples of itself. According Galileo's parameters, zero is the infinite number. From this, Leibniz holds that there is no infinite number since zero is no number at all. We might further remark that the infinite is thus *literally nothing*. Cf. Bassler, "Leibniz on Indefinite as Infinite", 858.
[102] Euclid, 2.
[103] Leibniz, *The Labyrinth of the Continuum*, p. 173.

from the infinite/simal dimension, leaving a minimalist conclusion that merely excludes these infinite/simal calculations from standard, part-whole relations. The syncategorematic infinite in Leibniz can no doubt be read as compatible with Galileo's position, except that the latter had not taken the extra step to qualify these infinite terms with the status of the syncategorematic.[104] It is this "hopelessness" or paradoxical unintelligibility that Leibniz aimed to combat in Galileo. Of course, the subtlety of Galileo's own resolution of the problem was not investigated by Leibniz in this text and one could object by remarking that he failed to do justice to Galileo's response to the problem. In reading the *Pacidius Philalethes*, I simply aim to see how Leibniz constructs the problematics of the infinite that he borrows from Galileo. Indeed, what Leibniz borrows from Galileo (and Aristotle) is not a solution but the problem.

However, if we were to gloss Leibniz's rejection of Aristotle and Galileo and to accept an actually infinitely divided extension, much less an infinitely divided world, it would seem to imply the fundamental discreteness of the extension in question or of the world, forcing us into two consequences that Leibniz was keen to reject. The discreteness of infinite divisions in the world seems to impose a categorematic infinite, such that if there were an actual infinite division of the world, there would be an actual number of divisions and hence of parts. Secondly, if we reject that there is an actual infinite number of parts of the world, then it seems that we might have to turn to an incomprehensible infinity as a recourse. Both of these results were unacceptable to Leibniz. In turn, given the Galilean problem of assigning measure between the infinite and a "part" of it, what results is to return to an idea of the infinite as an undifferentiated

[104] O. Bradley Bassler argues a variant of this position by remarking that the difference between Leibniz and Galileo on the status of the infinite/simals is that the former understood the "indefinite" of the continuum to be "infinite". But in this, Leibniz focuses on the subtlety that lies in the treatment of the term. Bassler remarks that, "Accepting an infinity of things, but not infinite wholes, amounts… to accepting syncategorematic infinite but not a categorematic one." O. Bradley Bassler, "Leibniz on the indefinite as infinite", 854.

expanse. This would not only refute Leibniz's idea of an actually infinite universe, but also push us back into the Eleatic paradoxes and reestablish Aristotle's potential infinite division as the more appealing option.

To avoid this merely apparent loop, we can turn to see precisely how a syncategorematic understanding of the infinite allowed Leibniz to maintain the thesis of the infinite divisibility of the world that is both actual (against Aristotle) and intelligible (against Galileo). Leibniz's thesis of actual infinite division aimed away from a categorematic infinite insofar as it is not resolvable into infinitesimal points. This involves a dynamic notion of what "actually divided" means. If some finite thing is divided, this division into smaller parts is given only insofar as the thing to be divided is given. The primacy of the unity before division thus governs the meaning of "actually divided". How does this then imply an actual infinity of division and not merely a potential? Leibniz's idea was that for any division, another division of the parts is not merely possible but actual. What the syncategorematic infinite posits is that this further division is both actual and intelligible.

If the problem of the status of infinite/simals is reflected in the dissymmetry between the infinite divisibility of a finite extension and its composition from these infinite divisions, it seems as though Leibniz's strategy for arguing for an actual infinite consists primarily in displacing this difficulty. That is, in Leibniz's syncategorematic infinite, it is only if we begin with a given bounded unity that we can proceed into a division of an infinite number of parts. In turn these parts, while themselves discrete from one another, do not end in a final part nor is there a final sum of divisions. As such, the configuration of problems that lead to a negative or potential infinite remains present within the syncategorematic interpretation. The key here is that under a syncategorematic interpretation, the division into infinity which produces both a syncategorematic infinitesimal parts and a syncategorematic infinite quantity of parts, eliminates the contradiction that this dissymmetry between infinite divisibility and given finite extensions tends to produce. That is, with no commitment to the infinite number or totality, this dissymmetry becomes much more subtle. The infinitesimal, produced by the infinite division of a discrete unity, can only hold given this prior unity. This displacement of the terms however, far from giving a simple resolution, provokes the appearance of a larger and more difficult problem.

According to the syncategorematic interpretation of the infinite/infinitesimal, we seem to lose the ability to maintain a simple distinction between discreteness and continuity. If the discreteness of a multiplicity means the actual "dividedness" of the thing into infinity, then the syncategorematic interpretation also implies that this actual dividedness of the thing implies its continuity. In Leibniz's conception, this actual infinity rests on the fact that there is always another division given beyond any division. How do we then reconfigure the relation between discreteness and continuity?

Leibniz's view on discreteness and continuity is a point on which Russell has been adamant in pointing out inconsistency. Russell points out, while Leibniz argued for a law of continuity, "In nature everything happens by degrees, and nothing by leaps, and this rule regarding change is part of my law of continuity."[105] The obvious question here is how Leibniz can hold to a law of continuity and an actual infinite division of things at the same time.

The account usually made for this is as follows. Leibniz argued for different levels at which discreteness and continuity apply. From a metaphysical level, especially with regards to the late Monadic metaphysics, the infinite division of the world produces a discrete reality, the infinity of monads as distinct unities. On the level of empirical reality however, as it relates to

[105] Russell, 264. Russell's quotation is drawn from Alfred Gideon Langley's 1896 edition of the *New Essays on Human Understanding*. Leibniz, *New Essays*, 374.

perception, things are continuous; waves of “modification” move one frame of experience into the next. This might roughly be the interpretation that could resolve our problem by reading the following explanation in his correspondence to De Volder in 19 January 1706:

> [I]n actual things, there is only discrete quantity… But continuous quantity is something ideal, something that pertains to possible and to actual things considered as possible. The continuum of course, contains indeterminate parts. But in actual things nothing is indefinite; indeed every division that can be made has been made in them.[106]

In this formulation, it seems that what Leibniz offered is a peculiar form of dualism that distinguishes a level of unity pertaining to “actual things”, unities, substances, monads, etc., and another level pertaining to continuity of perception, the imaginary and the like. We should note here that the syncategorematic interpretation of infinitesimals is entirely indifferent to this “dualist” solution to the configuration between continuity and discreteness.

Taking this dualist solution, however, focus is drawn away from the role of the syncategorematic infinite and turns Leibniz’s response to the problem of status into an assigning of different forms of the infinite to different domains, where the question of the actuality of the infinite would be dependent on the realm of which one is speaking. In this reading, the problem of the status of infinitesimals is entirely absorbed by the level of the continuous, pertaining to perception, imagination and the like. Hence, the problem of infinitesimals is best seen as a problem of the different elements that inhabit this level, removed from anything actual or metaphysically pertinent. This falls roughly in line with the contemporary “philosophy of mathematics” approach we criticized in Ishiguro. All mathematical elements are ideal, and the

[106] Leibniz, *PE*, 185. Leibniz, *Phil* II, 282.

question of their rigor is relative to the other elements in the ideal. The status of the infinite/simal would then be a subordinate case of the general status of mathematical elements.

If this were the case, then Leibniz would seem to be stuck giving a dogmatic response to the reality of infinite discrete parts in the "actual" metaphysical case. As such, we would have to understand the admission of "infinitely small spaces and times in geometry, for the sake of invention, even if they are imaginary…"[107] in the most prudential sense. What then does one do with the actually infinite number division of the world? That is, if we admit fictional infinitesimals of the syncategorematic sort on the side of mathematical, or geometrical elements, what is left to account for the sort of infinity that applies to the actual case? As we shall see the deployment of the fictional status of the infinite/simal loses its meaning when we neatly cut it away from its role in the actual. It is only in the actual that a fictional status counts as an explanation.

I think a closer look at the syncategorematic infinite itself will demonstrate the error of this dualist approach as well as allow us to have a fresh look at the relation between Leibniz's thoughts on the infinite and metaphysics.

5. The syncategorematic reframing of the discrete and the continuous

As we saw briefly in the above, one way of reading the syncategorematic infinite resolves a line of questioning with respect to the rigor of Leibniz's infinitesimal. Although the clarification of this logical consistency of the status of the infinitesimal is necessary, the reduction of the question concerning this status to a question about its logical status is unjustified. The mathematical status of this syncategorematic can help us widen the scope of this

[107] Leibniz, *LC*, 207.

question. With the aid of Arthur's work in discerning the meaning of this syncategorematic infinite, I hope to see how the mathematical status of such elements can help resolve our problem of interpretation above.

Leibniz's commentary on Galileo's reasoning on the square of infinites resulted in his firm conviction of the application of the Archimedean principle to the infinite and infinitesimals. Here, the syncategorematic interpretation allows us to see how such a designation of the infinite can help Leibniz both go beyond the idea of an infinite/simal whole as well as use it in calculations of the standard or Archimedean sort.

How would these calculations work? Arthur refers us to Leibniz's use of the infinite series. Taking the kind of division that Zeno made famous, the division of a line into two, we can produce the sort of concatenation of parts characterized by what is called a *dichotomy series*.

$$S_n = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \ldots + (\tfrac{1}{2})^n$$

The calculation of this series can be given if we take:

$$\tfrac{1}{2} S_n = \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \ldots + (\tfrac{1}{2})^{n+1}$$

and subtract it from the Sn series, giving us:

$$S_n - \tfrac{1}{2} S_n = (\tfrac{1}{2} - \tfrac{1}{4}) + (\tfrac{1}{4} - \tfrac{1}{8}) + (\tfrac{1}{8} - \tfrac{1}{16}) + \ldots + (\tfrac{1}{2})^{n+1}$$

or in shorter form:

$$S_n - \tfrac{1}{2} S_n = \tfrac{1}{2} - (\tfrac{1}{2})^{n+1}$$

Hence,

$$S_n = 1 - (\tfrac{1}{2})^{n+1}$$

As Arthur explains, from this kind of relation between two infinite series, we have a rather transparent idea that "the greater the number of terms one takes, the closer the sum of the resultant series is to 1".[108] This is a concrete example of Leibniz's notion of "unassignable" difference. What is unassignable is the gap of this syncategorematically small difference: $(1/2)^{n+1}$.

Here we can notice both a discrete and continuous dimension to this explanation of the unassignable. For any assignable difference, there will always exist an unassignable one. Hence the discreteness of the terms in the infinite series displays its capacity to unfold the inexhaustiblity or continuity of the unassignable terms. Yet the sum of the infinite series in the arithmetic operation is treated "as if" it were a whole. This however, under the application of the syncategorematic understanding of the infinite now applied to the infinite series, is secured not by the taking of an infinite whole sum or an infinitesimal smallest unit, but rather by the cancellation of terms between the two series, a principle that Arthur coins the "difference principle".[109] This principle or general orientation sheds light on what Leibniz explained to Bernoulli in Feb. 1699:

> [S]ince an infinite plurality does not constitute a number or a single whole, it follows that even given an infinite series, there need not be an infinitesimal term. The reason is that we can conceive an infinite series consisting merely of finite terms ordered in a decreasing geometric progression.[110]

Leibniz's explanation here consists in determining a concrete mathematical infinite, a series or plurality that could nonetheless avoid the implication of a "number" or "single whole". That is, we can avoid the infinite/simal term while making clear its infinite plurality. Through the expounding of this series and other series like it, Leibniz demonstrated the possibility of moving

[108] Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 19.
[109] Richard T.W. Arthur, "The remarkable fecundity of Leibniz's work on infinite series", accessed January 4, 2008, http://www.humanities.mcmaster.ca/~rarthur/articles.arthur.htm, 2.
[110] Leibniz, *Math* III, 575.

beyond the difficulty that Galileo posed: the vertigo of the non-Archimedean infinite. Leibniz showed that the treatment of infinite series can be handled coherently without giving up the commitment to the Archimedean principle. The operations involved are the very same as those which apply to finite numbers. In preserving a dynamic domain of the unassignable in this context, Leibniz's syncategorematic in fact demonstrates the false dichotomy between the concept of the infinite and the Archimedean principle. Instead, only by preserving the applicability of the Archimedean principle do we gain access to an appropriate treatment of infinite series. More precisely, without the constraint of the Archimediean principle, the infinite series as Leibniz lays out would fall prey to inconsistent ultimate terminating entities.

This coherency of treating infinite series has a larger application, one that will resolve the only apparent incompatibility in Leibniz's statements about discreteness and continuity. The above general treatment of infinite series turns out to be quite central to the development of the calculus. As Leibniz wrote in 1702:

> For example, $1/3 + 1/8 + 1/15 + 1/24 + 1/35$ etc. or $\int dx/(xx-1)$, with x equal to 2,3,4, etc. is a series which, taken wholly to infinity, can be summed, and dx is here 1. For the case of numbers the differences are assignable. […]But suppose that if x and y are not discrete terms but continuous ones, i.e. not numbers that differ by an assignable interval but the abscissas of a straight line, increasing continuously or by elements, that is, by unassignable intervals, so that the series of terms constitutes the figure. It is clear that in the same way, the sum of rational

> fractions… can either be established completely or treated by the quadrature of the hyperbola."[111]

The quadrature of the hyperbola can be understood as one of the fundamental methods of the infinitesimal calculus. Without losing ourselves in the details of method, we can understand this term as indicating one of the fundamental methods of the calculus. Since I will not deal directly with the mathematical methodology, I will note that in the equation of the infinite series with the sum $\int dx/(xx-1)$, the x and y, the variable and the sum, are not taken to be applicable merely to discrete, but to continuous terms as well. Thus, whatever the pre-assigned difference being stated, there is always an error that designates an unassigned difference that is smaller. Each term, each assignation of difference is discrete, but the coherence of the infinite series attests to the presence of an "unassigned" term and thus to the validity of a continuous understanding of these series. As Arthur succinctly puts it:

> That is, even though the terms or states are discrete, so that their differences are too, there are always differences so small as to be smaller than any given. Consequently the process is continuous…. In reality there are only finite differences. This is not a different conception to the one that says that infinitesimals are fictions whose use can be justified. They are two sides of the same coin.[112]

The demonstration of a continuous process holds because of the discrete terms and differences, not despite of them. This is the fundamental result of Leibniz's interpretation of the infinite as syncategorematic.

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[111] [Translation modified by Author] Leibniz, *Math* V, 356-357. Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 20.
[112] Arthur, "'A complete denial of the continuous?' Leibniz's law of continuity", 32.

What is highlighted here is that Leibniz's syncategorematic infinite is important, not only as a new conception of the infinite that corresponds to a logical ambiguity concerning the in/consistency of the infinite number, but also in that it allows us to reconfigure the status of the relation between continuity and discreteness. Here the actuality of infinite division is not merely a dogmatic assertion. Thus instead of a dualist reading of statements like "In real things, unities are prior to a multiplicity, and multiplicities exist only through unities…"[113], what we can understand is something like a mathematical priority of unities in the sense of terms, elements and the given figure that is to be divided. With each element of division or term of the series, the infinite and inexhaustible multiplicity of the continuous becomes expressed. Leibniz's mathematical reinvention of the cohesion of these relations gives rise to a structure wherein continuity and discreteness form two sides of the rule of the priority of unity over multiplicity. At the heart of this reconfiguration is a very elaborate metaphysical structure, but it also corresponds to two issues of metaphysical status that gain traction because of this solution. The first is the direct relation between the ideal and the actual. The second is the understanding of motion, substance and substantial forms.

Before addressing these two domains in the following chapter, we can give some clarity to our reading above of Leibniz's relation to Locke and the issue of the absolute. In his response to Locke, we see that Leibniz's issue of the infinite is articulated through two aspects. The first is that the infinite cannot be formed by the addition of parts. Leibniz disagreed because of the priority of the whole over parts, unity over multiplicity. The assignment of parts is a result of taking this priority as a guiding principle. Hence in a very mathematically literal sense, the finite is reached by a "limitation" of the infinite, not by the concatenation or repetition of the finite.

[113] Leibniz, *Phil* II, 276.

The infinite is then not a mode of quantity insofar as it is not a modification of extension or duration. Both are already at play whenever there is multiplicity and length.

What is implicit in these two responses is that Leibniz's reconfiguration of continuity and discreteness plays a greater role in his argument against Locke than the consideration of the origin of the infinite or the mere "requalification" of the metaphysical status of infinite/simals themselves. Our closer look at the meaning of the syncategorematic infinite allows us to see the displacement of Leibniz's argument with Locke, from speaking directly about the infinite as real, ideal or actual to the reconfiguration of the relation between continuity and discreteness. This reveals that any merely prudential understanding of Leibniz's invocation of the syncategorematic status of the infinite as fictional and its reducibility to the general status of the ideal cannot do Leibniz's thought justice. Leibniz's response to Locke on the status of the infinite then should not be read as an opposition between a rationalist and empiricist stance, but an opportunity to understand both how the infinite functions as fictional within its terms as an ideal and, on the other hand, its role in the actual infinite which Leibniz is committed to maintaining right at the beginning of his response to Locke. In other words, while this is not the place to give a full account of Leibniz's complex relationship with Locke, it should be sufficiently clear that Leibniz saw the *New Essays* and his arguments with Locke as a chance to demonstrate what is wrong with the usual connections made between the part-whole relation, the infinite and the finite. It is perhaps owing to the over-determination of the context of Leibniz's statements here by a dogmatically abstract confrontation between empiricism and rationalism that the interpretation of the syncategorematic infinite has always gone towards the sorting out of a "rigorous foundation" for the infinite/simal in terms of the calculus or Leibniz's more general use of the notion. Instead, what we should be more keen on analyzing is the way the syncategorematic interpretation circulates around the many different domains and axes on which the problem of terms like part and whole, divisibility and variable, assignable and unassignable, continuous and discrete come to bear. In the context of the *New Essays*, I would suggest that the reduction of the context of disagreement to the problem of origin of ideas and sensible qualities is a mistake. This mistake in turn reduces the problem of the infinite into a problem of ideas and propositions and quickly files away the syncategorematic infinite under the heading of "rigorous foundation" concerning the infinite/simal in mathematics as well as in metaphysics.

CHAPTER 3

THE STATUS OF THE WELL-FOUNDED

1. Against reduction and dissolution: Preserving the power of fiction

In the last chapter, I illustrated Leibniz's use of the term "fiction" in his the treatment of the status of infinite/simals by criticizing a reductive interpretation of it. In so doing, I pointed to the positive deployment of the fiction as a means for Leibniz to re-invent some fundamental notions at work in his thought. In this chapter, we will venture into some aspects of this method, namely in epistemology and physics, where the development of the syncategorematic infinite, as fiction, expresses its profundity. In order to do so, I have borrowed the first half of the expression *well-founded fiction* to orient my argument. This usage of the expression may be misleading since I do not aim to develop an argument for the well-foundedness of the infinite/simal in the calculus. To be clear, I wish to understand the status of the well-foundedness of the infinite/simal in the following sense. In the context of correspondences concerning the discrete and the continuous in physical phenomena and reality, Leibniz communicated to De Volder in 1706, "[T]he science of continua, that is, the science of possible things, contains eternal truths, truths which are never violated by actual phenomena, since the difference [between real and ideal] is always less than any given amount that can be specified."[114] Leibniz insisted that while confusions arise when there is a confounding between the ideal and the actual, there was a link between the actual or real and the science of the continua, that is, the investigation of the domain

[114] Leibniz, *PE*, 186.

of the continuous and the infinite/simal. That is, if we might extrapolate a little further, it is precisely through fiction that these "eternal truths" are grasped. The expression *well-founded fiction* then forms a reflexive duo. Only insofar as the infinite/simal is a fiction is well-foundedness needed, and only insofar as it is well-founded can we grasp the power of the fiction.

In order to remain within this difficult tension of the fictional and the well-founded, the actual and the ideal, we must neither reduce the one to the other, nor dissolve its status as problem. Having laid out some objections to a reductive reading, we can also remark on how the dissolution of the problem of the status of the infinite/simal can be an all too simple escape. If one were simply interested in knowing what Leibniz thought of the problem of the status of infinite/simals without asking how he thought about infinite/simals themselves, his distinction between the actual and ideal would be, as it were, without any reference to his more subtle qualification, enough to diffuse any deeper problems concerning consistency. Leibniz related to Volder in 1706, "In confounding ideal things with real substances, such that we look for actual parts in the order of possible and indeterminate parts in the aggregate of actuals, we have ourselves introduced the inextricable contradictions in the labyrinth of the continuum."[115] Here, the problem of the status of infinite/simals, insofar as it is related to the labyrinth of the continuum, is self-inflicted and based on a failure of method, the confounding of ideal and actual things. That is, by the possible division of a continuum into parts, we should not then directly relate this to actual extension, and unassignable remainders should not be read into the composition of actual multiplicity. Elsewhere, Leibniz admonished this confounding by remarking, "Let philosophers, in their turn, stop referring everything to the imagination and figures . . . they will recognize . . . motion itself is not subject to the imagination."[116] That is, the

[115] Leibniz, *PE*, 185. Leibniz, *Phil* II, 282.
[116] Leibniz, *LC*, 219.

continuity seemingly implied in motion should be distinguished from the continuity in the imaginative figures of geometers and mathematicians. From these remarks, it appears that the problem of the status of infinitesimals is not due to the infinite itself but rather an error in confounding the actual, ideal and imaginary, a problem of method and not a real problem of things themselves.

This interpretation which aims at dissolving the problem of the infinite/simal has been taken up by some recent commentators. In a recent article C. Marras argues that "Leibniz himself gives one of the most important indications about the Unicursale labyrinth: the continuum is only apparently a labyrinth, because it is grounded in a false problem."[117] Here, she interprets the problem of the continuum as a methodological confusion inspired by an encounter with the unknown frontiers of mathematical discovery. In turn, this labyrinth is understood as the problems concerning the epistemological conditions of mathematics itself and not in reality. Marras writes, "We know the Unicursale is a complex labyrinth, and sometimes we are unable to count all the involution[s] made by the thread. But we do have a thread to follow . . . . That leads to the solution."[118] What she implies here is that, since the labyrinth of the continuum is a unicursal one, that is, capable of being traced in one continuous stroke, its complexity is merely apparent; there is no deeper problem if we are capable of tracing the "thread". This thread, she indicates, "is the line in geometry and the calculation in mathematics, permitting us to not get lost. There is a possibility to solve the problem . . . . The thread as *ars inveniendi* . . . and as infinitesimal calculus is what allows us to proceed step by step, without wandering, and without fear of being unable to leave the labyrinth."[119] This characterization of the problem, however,

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[117] Cristina Marras, "Leibniz and his Metaphorical Labyrinths: The Manneristic and the Unicursale," in *Leibniz et les Puissances du Langage*, ed. Dominique Berlioz and Frédéric Nef (Paris: Librairie J. Vrin, 2005), 297.
[118] Marras, "Leibniz and his Metaphorical Labyrinths", 298.
[119] Marras, "Leibniz and his Metaphorical Labyrinths", 298.

does not explain much. I suggest that instead of the dissolution of this problem through the pursuit of a thread of mathematical discovery, what we encounter is a much more complex interrelation between the actual, the imaginary and the ideal. I maintain that the complexity of the labyrinth, whether unicursal or not, cannot be dissolved into a wider epistemological question. It is this epistemological context that will explain the former "thread" of mathematical discovery rather than the reverse.

2. Ideal, actual and imaginary

It does not take much to overturn this apparent and perhaps too simple impression that Marras tries to sketch. We recall that in the letter to Volder cited above, a letter that Marras herself cites, Leibniz claimed, "However, the science of continua, that is, the science of possible things, contains eternal truths, truths which are never violated by actual phenomena, since the difference [between real and ideal] is always less than any given amount that can be specified."[120] How should we interpret this? Even though we should not confuse the ideal with the real or the actual, their difference is less than any assignable amount. Here Leibniz added not just a cautionary distinction but a theoretical one, insofar as the separation between the actual and the ideal does not merely distinguish levels of discourse or ways of speaking about things like motion, substance and geometrical figures, but shows them to be directly pertinent to each other by the use of assignable difference and the notion that truth in one, namely the science of continua, applies to the truth of actual aggregation and extension. Hence while one should be cautious in conflating the two levels of actual and ideal, this does not entail a theoretical separation but rather a relation.

[120] Leibniz, *PE*, 186.

Bracketing for a moment the idea of the gap between the real and ideal as always less than any given amount, let us pay attention to the idea that the science of continua is the science of possible things. Perhaps the best place to start in giving an account of Leibniz's theory of ideas and the ideal is to return to Leibniz's theory of truth and propositions. As we have encountered in the first chapter, Leibniz's theory of truth is centered on its method of analysis, the way in which truths can be analyzed into the subject-predicate relation in reducing them into identities. From this we saw that necessary truths are the sort of propositions for which the predicate can be shown to be contained in their subjects, or reduced to a sort of identity given a finite number of steps. Contingent truths, on the other hand, are the sort that involves an infinite number of steps. From this, Leibniz concluded that there is no possible complete demonstration for contingent truths. Given this, Leibniz was committed to a distinction between two forms of knowledge, a blind form and an intuitive form. That is, in an infinite analysis of terms in contingent truths, the direct knowledge of this truth can only be an intuitive one. This intuitive knowledge has two aspects.[121] One is the intuitive knowledge of God who knows, in one divine instant, all the intermediate terms of analysis between a contingent truth and its identical, that is, primary identity definitions. The other aspect can only be a human one, confused and indistinct. If human knowledge were to be distinct, then it would have to concern necessary truths, truths that are reducible by demonstration into containment relations, or with regard to complex truths (truths for which we are incapable of reducing to their primary identities), blind or symbolic

[121] For the purposes of clarification, I shall state Leibniz's general *tableau* of the various forms of knowledge that is developed in "Meditations on Knowledge, Truth and Ideas". I will treat some of the key forms in what follows but a global picture of their organization may be helpful. Leibniz writes, "knowledge is either obscure or clear, and again, clear knowledge is either confused or distinct, and distinct knowledge either inadequate or adequate, adequate knowledge either symbolic or intuitive: and, indeed, if knowledge were, at the same time, both adequate and intuitive, it would be absolutely perfect." Leibniz, *PE*, 23.

identities. These identities, complex as they are, do not merely drown in incomprehensibility but indeed provide some degree of knowledge. As Leibniz remarked:

> However, we don't usually grasp the entire nature of a thing all at once, especially in more lengthy analysis, but in place of the things themselves we make use of signs, whose explicit explanation we usually omit for the sake of brevity, knowing or believing that we have the ability to produce it at will . . . [I]n my mind I use these words (whose sense appears only obscurely and imperfectly to the mind) in place of ideas I have of these things, since I remember that I know the meaning of these words, and I decide that explanation is not necessary at this time. I usually call such thinking, which is found both in algebra and in arithmetic and, indeed, almost everywhere, blind or symbolic.[122]

The nature of symbolic or blind knowledge is such that we take terms as they are. While they are not analyzed into their primitive elements, they nonetheless give us a form of understanding or knowledge relative to their appropriate field or scope of applicability. Symbolic knowledge gives us a form of knowledge by means of its clarity and distinctness, that is, the clear means to recognize something and further its distinction from other things. The basic capacity to recognize a representation allows us to attribute clarity. As an additional condition, a distinct knowledge obtains when we have the distinctive means to what is recognized from others. A composite notion like a chiliagon, a regular polygon with a thousand sides, can be clear and distinct but its knowledge remains blind or symbolic; we cannot really grasp all the properties (such as the relation between the sides) of such a thing except as separate or symbolic, that is, an algebraic or geometric understanding.[123] This sort of knowledge is of course not as complete as an intuitive

[122] Leibniz, *PE*, 25.
[123] Leibniz, *PE*, 24-25.

grasp of all of the notion's complexity, but it does not disqualify it from being a real instance of knowing. Symbolic knowledge, though blind to the multifaceted details that it subsumes, does constitute real knowledge insofar as it can be clear and distinct. But at the same time, Leibniz cautioned that the very elements that symbolic knowledge subsumes can often lead to error and falsity. He explained:

> For, often, we do understand in one way or other the words in question individually or remember that we understood them previously. But since we are content with this blind thinking and don't pursue the resolution of notions far enough, it happens that a contradiction that might be included in a very complex notion is concealed from us."[124]

As an example of this, Leibniz noted that the notion of a "fastest motion" conceals a contradiction. This would apply equally to an infinite number. What this indicates, however, is not a denigration of symbolic knowledge but the importance of analysis. In some respects, the inaccessibility of intuitive knowledge for the most complex notions is an insurmountable limitation of finite human capacities. Yet to rest too comfortably with a "blind" knowledge cripples the ability to gain what knowledge we can have within this limitation.

As in the case of the distinction between necessary and contingent truths, just because contingent truths cannot be resolved into their infinite detail does not mean they are less truthful. They remain known to a degree, even if they do not register the infinite number of details and causes within them. Yet since intuitive knowledge is available to human beings only with respect to the simplest recognitions, the importance of analysis, or the investigation of a notion into their component parts, is highlighted. The task of knowing then rests on the basis of the disposal of this blind form of knowledge. Our access to ideas then is given by the mediating role of symbols

[124] Leibniz, *PE*, 25.

and the means of their analysis. In turn, our blind "access" to complex truths is not always perfectly executed. Given any complex notion, the insufficient analysis of its component elements can lead us to believe in the truth of a contradictory notion, while at other times, a demonstration of necessity can help us ascertain the completeness of an idea. In the 1680 "On first truths", Leibniz noted that one aspect of this mediating function of analysis is the thesis that what is possible, that is, what is without contradiction, can exist.[125] The positive result of analysis is some notion's possibility. In this text, Leibniz equated this possibility with a thing's essence. With the exception of God, via the ontological proof, a thing's essence, that is the component and non-contradictory parts of the constitution of a thing's idea, does not guarantee its existence. This essence, Leibniz remarked, has a "certain inclination to exist, or else nothing would exist."[126] That is, given that creation is rationally and consistently ordered, what actually exists must first be possible. This notion of creation implies that the nature of existent things can be understood essentially through the analysis of their possibility, their logically consistent nature. As such, a major part of knowledge, that which interrogates the essence or nature of things, occurs on the level of investigating their possibility or the analysis of their notions. The method of knowledge then, as we shall see, hinges on the mediating role between ideas and the realm of symbolic reasoning.

Here I recall Leibniz's 1706 correspondence with Volder cited at the start of the chapter. With respect to understanding the science of continua as a science of "possibles", we see that the ideal realm of science, under which geometrical and mathematical reflections operate, is formed

[125] Leibniz, "Sur les Vérités Premières" in *Recherches générales sur l'analyse des notions et des vérités. 24 thèses métaphysiques et autres textes logiques et métaphysiques*, ed. Jean-Baptiste Rauzy (Paris: Presses Universitaires de France, 1998), 447.
[126] [Author's translation] Leibniz, "Sur les Vérités Premières", 448.

under a notion of consistency that would be the key to an understanding of essence and possibility (a thing's "inclination to exist").

Leibniz's remarks here present an alternative to the contemporaneous Cartesian view of the relation between geometry and reality. Taking a little step backwards, we should first look at how Leibniz distanced himself from Descartes in terms of their different analyses of the famous ontological proof for the existence of God. To take one instance of this, in Leibniz's 1684 "Meditations on Knowledge, Truth, and Ideas" that we have been considering above, he introduced this difference in the context of a justification of blind symbolic knowledge. Lebiniz wrote:

> [O]ne must realize that from this [ontological] argument we can conclude only that, if God is possible, then it follows that he exists. For we cannot safely use definitions for drawing conclusions unless we know first that they are real definitions, that is, that they include no contradictions, because we can draw contradictory conclusions from notions that include contradictions.[127]

Here, the objection to Descartes' employment of the ontological argument stems from a methodological or epistemological worry. They do not disagree on the outcome of the argument but rather on the need for the analysis of the notion "God" itself. Another aspect of this same criticism can be seen from his argument against the Cartesian notion of the division of matter into actually indefinite particles.[128] Descartes' position was that "it must be admitted, however, that in motion something is found which our mind perceives as true, even though it does not

[127] Leibniz, *PE*, 25.
[128] Here one faces the equivocal usage of the term *indefinite*. While *infinite* might render Descartes' position clearer, I use the term *indefinite* so as to be consistent with the translation I am using.

comprehend how it occurs: namely, a division of certain particles into infinity – that is to say, a division that is indefinite."[129] To give a justification of how this is the case, Descartes added:

> [A]lthough we are unable to comprehend in thought how this indefinite division occurs, we must not for that reason doubt that it does occur: for we clearly perceive that this necessarily follows from the nature of matter that we know most evidently, and we also perceive it to be one of those things which our mind, inasmuch as it is finite, is unable to grasp."[130]

Leibniz reasoned that Descartes here laid too much emphasis on a unrefined qualification of clear and distinct knowledge. Leibniz's further classification of degrees of knowing indicates that clear and distinct knowledge could also be blind and inadequate. Just like his criticism of Descartes' use of the ontological argument, Leibniz's methodological position implied that emphasis must be placed on the analysis of this "indefinite division". The clear and distinct knowledge of the division implied here by Descartes is due to a lack epistemological analysis, a context where the terms are unfolded despite its infinite and complex nature.

The key epistemological difference that this contestation with Descartes makes is in Leibniz's understanding of how knowledge circulates between ideas, imagination and the actual. That is, the mediating role of symbolic knowledge reflects on Leibniz's views on the status of the idea itself. In the undated manuscript "What is an idea?", what Leibniz meant by idea is not an act of thought.[131] Thoughts, perceptions, affectations are all mental processes, but none of these qualify as ideas. Leibniz explained:

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[129] Descartes, "Principles of Philosophy" (excerpts) in *Labyrinth of the Continuum*, ed. Richard T.W. Arthur (New Haven and London: Yale University Press, 2001), 358.
[130] Descartes, 358.
[131] [Author's translation] Leibniz, "Quid sit idea" in *Recherches générales sur l'analyse et des verités*, 446.

> [I]f for example I enumerate, in an ordered manner the sections of a cone, it is certain that I would arrive at the understanding of the opposed hyperbola, even if I do not yet have the idea. It is thus necessary for there to be something in me that is not merely directed to the thing but also expresses it.[132]

In this context, Leibniz insisted on the relative nature of expression. That is, what expresses and what is expressed do not form any fixed necessary relations. He continued to explain, "These expressions are varied, the model expresses the machine, the perspective drawing expresses the volume of a figure, discourse expresses thoughts and truths; characters express numbers."[133] Perhaps written to refute the simplicity of certain contemporaneous views on the relation between sense impressions and ideas, Leibniz's explanation nonetheless honed in on the problematic relationship between mind, idea and reality. Knowledge operates between ideas and things, and the knowledge of things does not necessarily imply the possession of an idea. Ideas then, for Leibniz, were conditions for the possibility of thoughts, but this does not equate the two. Hence the relation between the object and idea is mediated by other mental activities that serve as the expressive medium of their correlation. Ideas then are "in" a capacity or faculty of thought, the dynamic intersection between thoughts and things.

According to this reading of ideas, expressions constitute the recognizable surface of the relations between things and ideas. Perception, in this case, would be understood in terms of expression. Leaving the larger problems of perception aside, it is clear that for Leibniz the level of ideas was not to be separated from the actual but constituted the primary condition for which knowledge of the actual is possible. Here we can recognize Leibniz's conception of the relation between the ideal and the actual, that the results of ideal mathematical calculation are applicable

132 [Author's translation] Leibniz, "Quid sit idea", 445.
133 [Author's translation] Leibniz, "Quid sit idea", 445.

to actual things despite the existence of unassignable errors. How then does mathematical thought fit into the picture of this mediation between ideas, knowledge and actual things?

To add this other piece of the explanation, we can turn to Leibniz's views on the status of symbols or signs. No doubt, this issue was central for Leibniz. In his complex relationship with Hobbes, Locke and the empiricist tendency of the time, Leibniz had already developed a nuanced view on the status of symbols. On the one hand, he agreed with Hobbes and Locke on a certain kind of "arbitrariness" of the nominalistic relation between the symbolic and things, consonant with what we saw in the above, but on the other hand, he insisted on a genuine epistemological positivity of this symbolic realm. That is, though he argued that words and definitions are arbitrary in the sense that they are established by convention, he nonetheless argued for their ability to make essences and the possibility of things known to us. As he put it quite concisely in the *New Essays*, "I think that the arbitrariness lies wholly in the words and not at all in the ideas. For an idea expresses only a possibility…."[134] This is consonant with what we have seen in Leibniz's view on the use of symbolic knowledge to understand possibility and essences. However not all symbols are equal. Leibniz made a special case for mathematical knowledge. Leibniz's point against Locke in this contest over the status of language came down to their disagreement about nominal and real essences. In this disagreement, Leibniz argued that the distinction between nominal and real can only be applied to definition, that is, what is nominal about words and what is conventional about symbols concerns the means by which we express the possibility of things, not the things themselves. Leibniz remarked, "Something which is thought possible is expressed by definition; but if this definition does not at the same time express this possible then it is merely nominal, since in this case we can wonder whether the

[134] Leibniz, *New Essays*, 233.

definition expresses anything real –that is, possible ….”[135] The ambiguity of language and indeed the use of signs and symbols do, however, fulfill a positive function in Leibniz's epistemology. This is due to the ambiguity of symbolic knowledge, being both too underdetermined to directly instruct us on the actual, and open to the results of logical and propositional analysis by which it serves the kind of role it does in the development of human knowledge and the sciences in general. In this, the mathematical use of symbols converges on a point of necessity and arbitrariness, the analysis of which brings out the methodological importance of the realm of the symbolic. What distinguishes the symbolic of mathematics and the symbolic of words is the exceptional orderedness of mathematics or geometry. The blindness of mathematical symbolism is then paradoxically poised on this epistemological edge of symbolic knowledge.

This powerful conjunction of blindness and order had long been a fundamental aspect of Leibniz's thought. Leibniz's on-and-off project for a *mathesis universalis* hinged on the basis of a "universal characteristic" and is a well-known part of his early philosophy, often seen to have been largely abandoned in his later years. The project, generally speaking, concerned the creation of a symbolic language that would transcend linguistic and ethnic barriers, and ultimately aimed at resolving any dispute by the sole means of manipulation of symbolic combinations. While I will not venture to enter into the details of this project, a few very pointed aspects of it serve to indicate Leibniz's views on the status of the symbolic nature of mathematics. First, Leibniz's project concerned the combination of two aspects of what might be called today "formal" logic. He explained:

> [N]o one has put forward a language or characteristic that embodies, at the same time, both the art of discovery and the art of judgment, that is, a language whose

[135] Leibniz, *New Essays*, 293.

> marks or characters perform the same task as arithmetic marks do for numbers and algebraic marks do for magnitudes considered abstractly.[136]

That is, an art of judgment, like that of traditional forms of syllogism, allows us to formally represent the terms of argument split between subject and predicate, and demonstrates the validity and soundness of arguments. On the other hand, there is something in the art of discovery that allows for the manipulation of symbols to uncover what was not initially represented by the terms. Here Leibniz pointed to the discoveries of geometry and algebra. The combination of these two forms of reasoning, he argued, should be accomplished by something analogous to the model of mathematics, and more precisely arithmetic and algebra. In algebra and arithmetic, both sorts of reasoning are able to be accomplished with a single formal system. He continued in arguing, "But now our characteristic will reduce them [controversies and disputes] all to numerical terms, so that even reasons can be weighed, just as if we had a special kind of balance." Now we should be clear, without entering into a full discussion of the universal characteristic, that Leibniz was not advocating a reduction of the entire domain of disputes and reason to the domain of numbers. Rather, he sought to employ the clarity of numerical combination and arithmetic calculation as a means to model and thus provide a formal way to demonstrate and reflect on more general disputes. Much of the "specimen" (samples) of the universal characteristic was comprised of the attempt to use arithmetical operations like addition and subtraction, factors and exponents to establish this link between a logical calculus of subject and predicate relations with the familiar operations of arithmetic, but the evaluation of this project is not our concern here. What is striking is the special status that Leibniz gave to mathematics as a model for reasoning in general. Following this path, much of classical formal logic leading to the groundbreaking projects of Boole, Frege and Russell would be an attempt to

[136] Leibniz, *PE*, 6.

symbolize propositions in terms of subject and predicate and make heavy use of *salva veritate* as it operates in algebra. Yet at least for Frege and Russell, it seems that what resulted was a formal symbolic logic that would come to schematize mathematics, not the other way around. In this, Leibniz was certainly a key figure of the development of modern formal logic, yet the direction would be reversed. The reason for the special case of mathematics in Leibniz's own project however, despite being influential, is nonetheless enigmatic.

It is clear that a formal system does not have to be one based on numbers. It could be any arbitrary system of marks, given enough distinction between the marks and a set of operations to define their relation. Nonetheless, for Leibniz, it was clear that "we must go beyond words . . . I have contrived a device . . . by which I can show that it is possible to corroborate reasoning through numbers."[137] That is, words occurring in natural language are too ambiguous to give us the means to create a distinct enough set of signs and operations by which clear and distinct reasoning can be modeled.

In the *Dialogus* of 1677, Leibniz stated that actual thoughts can develop without words, but not without any kind of sign at all. As Leibniz put it, "Indeed, if characters were lacking, we would never distinctly know or reason about anything."[138] Given the distinction between intuitive and symbolic knowledge, signs or characters are crucial to any human scientific endeavor; without them, no clear and distinct thoughts actually develop. Here the expressive dimension of symbolic thought played a central role. As Leibniz argued in "What is an idea?", "Characters express numbers, the algebraic equation expresses the circle or any other figure; from the examination of the relations of the expression, we can arrive at the knowledge of the

[137] Leibniz, *PE*, 7.
[138] Leibniz, *PE*, 271.

corresponding properties of the thing expressed."[139] Thus the circle that we examine in geometry is not a real circle, in the completeness of its idea, but in the expression of the circle by figural and algebraic means, we find the relations between the corresponding properties in the model that we organize in a symbolic dimension. To this Leibniz added, "[W]e also see that, among the expressions, certain of them have a foundation in nature, and the others have in part been founded arbitrarily, as they are in expressions produced by sounds or characters."[140] Words, it seems, being too connected to the contingencies of the human capacity of making sounds or the historical transformations of inscribing their sounds into images, are not endowed with a natural foundation. But on the other hand, there are symbols that express a certain form of "similitude" with nature. Leibniz remarked here:

> As for those that are founded in nature, they propose rather a certain similitude, such as that which exists between a big circle and a smaller, or between a region and the geographic map of the region; or in any case the connection like that which exists between a circle and an ellipse that represents it optically, in the sense where all the points of the circle correspond according to a determinate law to a point on the circle.[141]

What is interesting about this passage is the correlation between the "natural" and "similitude". The relation here that Leibniz qualified as "founded" in nature has little to do with the natural givenness of the representation but rather their coordination according to a "determinate law". Here Leibniz made explicit a point-wise correlation between the circle and its optically projected ellipse. In fact, what would usually go under the name *natural language* would be that which least qualifies for this foundation in nature.

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[139] [Author's translation] Leibniz, "Quid sit idea", 445.
[140] [Author's translation] Leibniz, "Quid sit idea", 446.
[141] [Author's translation] Leibniz, "Quid sit idea", 446.

One way to read this idea of a foundation in nature, putting certain of our intuitive notions of representation aside, is to revisit the idea of continuity discussed in the last chapter. In a response given in 1687 to Bayle's comments on Descartes' physics, Leibniz wrote about a "principle of general order" where he noted:

> [W]hen two instances or data approach each other continuously so that at last one passes over into the other, it is necessary for their consequences or results (or the unknown) to do so also. This depends on a more general principle: that, as the data are ordered, so the unknowns are ordered also.[142]

Following this, Leibniz went on to explain, apropos of theorems about ellipses and hyperbolas, such results can be equally applicable to parabolas:

> We know that a given ellipse approaches a parabola as much as is wished so that the difference between ellipse and parabola becomes less than any given difference, when the second focus of the ellipse is withdrawn far enough from the first focus, for then the radii from that differ from parallel lines by an amount as small as can be desired. And as a result, all the geometric theorems that are proved for the ellipse in general can be applied to the parabola by considering it as an ellipse one of whose foci is infinitely far removed from the other.[143]

Normally this relationship between the ellipse and the parabola can be read as a version of the principle of continuity. Indeed, Leibniz continued after this citation to remark that this principle in physics can be seen as the consideration of an infinitely slow motion as rest. What is relevant to us here is that the qualities of an ellipse (by an argument concerning the continuous transformation of an ellipse into a parabola) can hold for the parabola. This no doubt echoes, in a

[142] Leibniz, *PPL*, 351.
[143] Leibniz, *PPL*, 352.

more clarified way, the relation of an expression "founded in nature" that exists between a circle and an ellipse. What does this foundation then refer to? This explanation of the principle of general order in fact highlights that what Leibniz held as something to be founded in nature is that which follows a general rule of ordering.

With this general relation of order in mind, we can see that the arbitrariness of symbols is not necessarily more or less natural according to some *ad hoc* notion of natural "givenness" or intuition but rather the availability of some determinate relation between an expression and the thing expressed. In this sense the region and the map, the circle and the ellipse, the ellipse and the parabola served as primary examples for Leibniz. It is also in this sense that we can describe Leibniz's motivation to seek a model for the universal characteristic in the arithmetic qualities of numbers. On the side of the expressions being numbers, Leibniz explained:

> [B]y using these numbers I can immediately demonstrate through numbers, and in an amazing way, all of the logical rules and show how one can know whether certain arguments are in proper form. When we have the true characteristic numbers of things, then at last, without any mental effort or danger of error, we will be able to judge whether arguments are indeed materially sound and draw the right conclusions.[144]

The reason for privileging numbers in his universal characteristic project was the internal, unequivocal, clear order of numbers, the order of expressions themselves. Now the work of the project itself would still have to correlate (as if by a point-wise correlation) the characteristic numbers with the concepts and notions they would express. Whether this is possible remains an open question but Leibniz seems to have had a strong vision of its possibility and his specimen (samples) of the numerical characteristics demonstrated a serious attempt at doing so.

[144] Leibniz, *PE*, 10.

Nonetheless, what becomes clear here is that the term *expression* is a relative one in Leibniz's use. The "foundedness" of expressions does not imply a direct correlation between thing and representation as one would find in a form of naïve empiricism. Instead the rigor of expression and thing expressed is to be founded in the ideal of order. It is in fact this ideal of order that guarantees the *telos* of symbolic knowledge in a *mathesis universalis*. As such, any correlation between expression and expressed will hide the complexities of the expressed. Yet the principle of general order, as we discussed, provides the illumination of those hidden complexities by the investigation of the expressions under which these complexities are subsumed. The circle examined in geometry is never a "real" circle, but what can be proved from it, with increasing complexity, nonetheless applies to a real circle. This relation triangulates idea, symbol and thing. The circle represents an ellipse by a certain principle of order, but the ordering itself belongs to the ideal and is not itself "given" by the thing. Hence the idea of expression in Leibniz does not have an absolute basis but is the very paradigm of a relational term. The realm of the symbolic may be arbitrary in the sense that its orthographic or typographic actuality is a result of human will, but the relation that these express can be highly rigorous with respect to the order that they come to correlate and organize. Furthermore, the difference between expressions and things expressed is not inscrutable. That is, the difference between these two realms is not discontinuous but can be evaluated according to degrees of order. As we commented earlier, linguistic expression is perhaps most contingent and arbitrary, given to historical transformation and the limits of human pronunciation. In turn, mathematical expressions are less arbitrary because the internal order of numbers and the rigor of geometrical method in dealing with figures provide a clearer and more distinct means of ordering the thing expressed by them. In the case of mathematical expression, like the expression of a parabola by an ellipse, the correlation made between expression and thing expressed is precisely ordered. In this sense, we can gain some further understanding about the depth of what Leibniz meant when he argued that "the science of continua, that is, the science of possible things, contains eternal truths, truths which are never violated by actual phenomena, since the difference [between real and ideal] is always less than any given amount that can be specified."[145] The science of the continua, the ideal and representational schema used to approach the mysteries of the actual continuum, does not differ from its object of knowledge as a difference of kind but a difference of degree. The difference between the real and the ideal, owing to the precision of mathematics, is actually and precisely less than any given amount that can be specified.

In order to more closely appreciate this relation between the ideal and actual, we must take a further step. The unexamined notion thus far in my analysis of Leibniz's distinction between the ideal and the actual is the role of imagination. This epistemological feature of Leibniz's theory of the relation between the symbolic and actual is in fact not an explicit aspect of the relation between the domain of the possible and the actual. That is to say, the analysis of the internal consistency of a symbolic field like that of numbers in mathematics or the relation of curves in geometry does indeed bring forth a necessary condition for the understanding of the actual, but there remains a gap to be resolved between the field of necessary conditions and the field of the actual. The key here is to understand a glaring difference between the autonomously consistent dimension of the mathematical that we find in the Cartesian position and the Leibnizian conception of the mathematical. If we now take up the second part of the quote, that "the difference [between real and ideal] is always less than any given amount that can be specified"[146], we find that there is a relation between the actual and the ideal insofar as a

[145] Leibniz, *PE*, 186.
[146] Leibniz, *PE*, 186.

difference between them is undetermined. This indetermination can be taken as a fixed relation, in which a symbolic understanding of the continuum will always be different from an actual one, or we can take this indetermination as a dynamic relationship wherein the increasing degree of our understanding of the continuum brings to the fore a real understanding, one that is in a differential relationship with the actual.

Perhaps Leibniz's clearest explanation of the role of imagination in mathematics was given in his letter to Queen Sophie Charlotte of Prussia in 1702. He argued here:

> Yet we must do justice to the senses by acknowledging that, besides these occult qualities, they allow us to recognize other, more manifest, qualities which furnish us with more distinct notions. These are notions we attribute to the common sense because there is no internal sense to which they are particularly attached and belong. It is here that definitions of the terms or words we use can be given. Such is the idea of number, which is found equally in sounds, colors, and tactile qualities. It is in this way that we also perceive shapes which are common to colors and tactile qualities, but which we do not observe in sounds. However, it is true that in order to conceive numbers, and even shapes, distinctly, and to build sciences from them, we must have recourse to something which the senses cannot provide and which the understanding adds to the senses. Therefore our soul compares the numbers and shapes that are in color, for example, with the numbers and shapes that are in tactile qualities, there must be an internal sense in which the perceptions of these different external senses are found united. This is called imagination, which contains both the notion of the particular senses, which are clear but confused, and the notions of the common sense, which are clear and

> distinct. And these clear and distinct ideas, subject to imagination, are the objects of the mathematical sciences, namely arithmetic and geometry, which are pure mathematical sciences, and the objects of these sciences as they are applied to nature, which make up applied [*mixtes*] sciences.[147]

Here then, the objects of "pure" mathematics are not the result of the pure generation of ideas but are themselves the product of a process between the sensible and the soul's internal operations. In turn, three epistemological levels are required to categorize the "objects" of reflection. Leibniz said that "beside the sensible and the imaginable, there is that which is only intelligible, the object of understanding alone."[148] The objects of mathematical thought then are products of thought which are clearly given an intermediate place between sense or perception, and ideas or the intelligible. Yet what Leibniz clearly avoided doing here was to hierarchically organize them according to priority. In this, Leibniz, somewhat echoing Descartes, argued, "It is worth observing that, if in dreaming, I should discover some demonstrative truth, mathematical or otherwise (as, in fact, can be done), it would be as certain as if I had been awake."[149] This certainty in dreams suggests that the triangular relationship between the actual, the imaginary and the ideal does not have a static hierarchy of priority. Rather concepts (or proofs or analyses of notions) circulate between them, taking on different modes of investigation which constantly inform and reorganize the problematics of thought. Hence in his letter to Sophie Charlotte, he remarked that the understanding adds something to sense; in turn, it is through the uniting of senses in an "inner" or "common" sense that the objects of mathematical inquiry and science are produced.

[147] Leibniz, *PE*, 187-188.
[148] Leibniz, *PE*, 188.
[149] Leibniz, *PE*, 189.

This dynamic, multilateral relation between the senses, imagination and understanding takes us back to the problem of the idea in "What is an idea?". As we remarked earlier, an idea is not a certain activity of thinking but a "facility" or "capacity" of circulation of thoughts between the senses, imagination and understanding. The symbolic dimension, in its "expressive" function, is thus crucial to this triangular relationship of thought in these expressions, and is not simply a passive reception of the given but the active "making" of relations, the coordination of the relations between things and the relations between ordered symbols. The objects of mathematical thought are then the complex circulation between these relations.

What emerges from this trilateral epistemology is the means by which the status of infinitesimals becomes a problem. That is, the problem of the infinite/simal is not solely a mathematical problem; it is not a problem simply by virtue of being an object of imagination. As Leibniz put it in proposition ten of the *Discourse*, "A geometer does not need to burden his mind with the famous labyrinth of the composition of the continuum . . . since the geometer can achieve all his demonstrations . . . without entering into these discussions."[150] In turn, the emergence of the problem of the status of infinite/simals through the labyrinth of the continuum, circulating between sense and thought, arises precisely as a discord within the treatment of these constitutive elements in imagination itself. It is perhaps due to this distinction between the object of science and the status problem that Leibniz designated an important part of his "method of invention" to the creation of problems or questions (*arte inveniendi quaestiones*), a role he gave to combinatorics.[151] This trilateral epistemological picture sheds light on how infinite/simals become the "name" of a problem in two senses. First, it is the name of the gap between the ideal and the actual whose difference is smaller than any assignable amount. This gap designates a

[150] Leibniz, *PE*, 43.
[151] Leibniz, *Opuscules et fragments inédits de Leibniz*, ed. Louis Couturat (Paris: Felix Alcan Editeur, 1903), 167.

discord in the space of imagination, and hence it is the space of the emergence of the problem insofar as it is the place of the discordance between the ideal and the actual. Second, this gap formulates itself as a methodological problem, one that seeks to clear up the proper relationship between mathematics and its approach to the actual, its expression of the actual and its being expressed by the actual. We will attempt to gain some resolution of these two problems, or rather the formulation of the problematics, in the following section. More precisely, in examining the context of Leibniz's move toward the stability of the syncategorematic interpretation of the infinite/simal, we will try to track the implications of this epistemological and methodological picture. Here, the syncategorematic infinite/simal will prove itself to be more than the neat resolution of a reducible or dissoluble "false problem" but itself the exercise of the problematic that emerges in the interrelation between the actual, ideal and imaginary. In other words, the status problem of the infinite in Leibniz's thought cannot be filed away once designated syncategorematic. Instead, this questioning of the status is itself an ampliative, experimental and inventive space that disrupts and intervenes in Leibniz's thought on multiple levels.

3. Nothing without motion: Leibniz's indivisible as infinitesimal

In the previous sections, we have seen how some contemporary attempts in understanding Leibniz's views on the status of the infinite/simal imported a distorted view of the problematics that Leibniz imposed on himself as he attempted to resolve them. As such we saw that the dissolution of the problem into a clean division between ideal and actual, or representation and represented serves only to obfuscate Leibniz's grasp of what is at stake. As an alternative, I propose that more time should be taken in reflecting on how Leibniz himself might have constructed these problematics in which the status of the infinite/simal comes into play. By examining the status of infinitesimals through Leibniz's distinction between the actual, the ideal and the imaginary, we have seen how the neat separation between the ideal and the actual, the dissolution of the problem of the infinite/simal into a field of abstract entities, occludes the sort of dynamic and experimental role that Leibniz gave to the complex practice of mathematical inquiry. In this, not only mathematics but the problematic philosophical status of the infinite/simal plays a central role in constituting the general gap between knowledge and the real. In the proceeding, we shall follow Leibniz's own path: tracing the problem of the status of infinitesimals along the lines of the account of the movement of bodies.

In Leibniz's own accounts of his journey as a philosopher, he often remarked on this aspect of metaphysics as the guiding thread of his development. As a young man, he was enamored with the new sciences of Galileo and in turn sought to understand metaphysical reality in light of this. Along with these autobiographical admissions, he remarked that as a youth he was attracted to atomistic explanations via the neo-atomism of Gassendi and the *conatus* theory of Hobbes and was also for a while interested in Cartesian dualism.[152] The tracing of this developmental path is not my aim here, but this is certainly one way for gaining a comprehensive view of what Leibniz meant when he said that the labyrinth of the infinite characterizes one of his two main philosophical occupations.[153] In his development, the transformation of his metaphysical views seems to be deeply correlated to his different approaches to the problem of the infinite and infinitesimals as they relate to the problems of motion.

We can recommence here by returning to a familiar passage from his correspondence with Varignon on June 20, 1702. Here Leibniz remarked, "Between you and me, I think

[152] Leibniz, *Phil* III, 205.
[153] Leibniz, *Essais de Théodicée*, edited by Jacques Brunschwig (Paris: GF Flammarion,1969), 29.

Fontenelle . . . was joking when he said he would derive metaphysical elements from our calculus. To tell the truth, I myself am far from convinced that our infinites and infinitesimals should be considered as anything other than ideals, or well-founded fictions."[154] This joke, in reading Leibniz's earlier work in the period that preceded his four-year stay in Paris from 1672 to 1676, could be told in reverse. In this earlier period, we see a major investment in the actuality of infinitesimals understood as "indivisibles", culminating in an ambitious two-part work, the *Hypothesis Physica Nova* of 1671, divided between a "Theory of abstract motion" and a "Theory of concrete motion" where, as our analysis will show, he attempted to flesh out a metaphysical commitment to the reality of motion and the actual infinity of reality, by means of a mathematical or geometrical explanation of motion. Moving forward from this, we will see that while in many ways Leibniz's commitment to these fundamental metaphysical propositions did not change very much, the terms through which he argued for them, in part concerning the status of the infinitesimal, transitioned from actual to fictional. As we know, Leibniz's qualification of the infinitesimal as fictional was already well established by 1675 in his mathematical treatise "*De quadratura arithmetica circuli ellipseos et hyperbolae cujus corollarium est indivisibilium firmissime jacidenda*". In the scolium following the seventh proposition, Leibniz remarks on the invocation of "fictive quantities, that is to say, the infinite and infinitely small".[155] Following this mathematical synthesis, Leibniz composed in 1676 a philosophical synthesis of a theory of motion in the dialogue *Pacidius Philalethes* where the explicitly fictional infinitesimal comes to play a significant role. In considering the transition between these two texts, we will see how the problem of infinite/simals provides a laboratory in which a general metaphysical direction is set into motion. In this sense, while the idea of metaphysical elements being implied by the methods

[154] Leibniz, *Math* IV, 110.
[155] Leibniz, *Quadrature Arithmetique du Cercle, de l'Ellipse et de l'hyperbole*, ed. Eberhard Knobloch, trans. and intro. Marc Parmentier (Paris: Librairie J. Vrin, 2004), 69-71.

of the infinitesimal calculus might seem to be a joke in 1702, it is precisely in the dangerous relation between these two dimensions (the metaphysical and mathematical) in Leibniz's work that one of the central aspects of Leibniz's philosophy crystallizes. Here I would argue that in this transition, the relation between mathematics and metaphysics is surely not one of implication; his mathematical views do not imply a corresponding metaphysics. Instead, what remains constant is a mathematical constraint, the development of mathematical figures that set constraints on metaphysical thought but also supply it with models and relations. Thus, while the joke might have been that metaphysical elements could be directly drawn from mathematics, the fact is that mathematics served a powerful conditioning role for Leibniz's philosophy.

The research on the transformation of Leibniz's views on infinite/simals constitutes a field all by itself. In the contemporary research, a large catalogue of different positions that Leibniz held from his earliest developments to his late mature work constitute a spectrum of widely differing views.[156] In his longer and more polished work, Leibniz can be seen as proclaiming strong landmark positions by which we can detect some theoretical decisions. In his sketches and drafts scattered among these pronouncements, evidence shows that Leibniz held, at one time or another, many positions in between. A comprehensive developmental treatment of this spectrum of positions will not be attempted here. Far from it, I wish to mark only one significant difference by treating these two texts, the *Theory of Abstract Motion* and the *Pacidius Philalethes*, as the markers of a shift. At first glance, the shift between the *Theory of Abstract Motion* and the *Pacidius Philalethes* is a shift in solution. In the *Theory of Abstract Motion*, Leibniz treated the infinitesimal as an actual indivisible, an entity that accounts for the reality

[156] Richard T.W. Arthur lays out four distinct periods of treating infinite/simals beginning in 1669 in his article "Actual infinitesimals in Leibniz's early thought". Cf. Richard T.W. Arthur, "Actual Infinitesimals in Leibniz's Early Thought" in *The Philosophy of the Young Leibniz*, ed. Mark Kulstad, Mogens Laerke and David Snyder (Stuttgart: Franz Steiner Verlag, 2009), 11-28.

and “smallest part” of motion. In the *Pacidius Philalethes*, Leibniz treated the infinitesimal as fictional and employed this fictionality as a positive aspect of the nature and reality of motion. Yet merely showing that Leibniz changed his mind, a task easily done, does not at all demonstrate how this problem of the infinite/simal provided a laboratory for Leibniz’s philosophy. Upon closer examination, the shift between the *Theory of Abstract Motion* and the *Pacidius Philalethes* resides in the different constraining and conditioning role that mathematical thought plays in the understanding of motion. This shift in the role of mathematics not only provides a picture of the different positions concerning motion in Leibniz’s philosophy, but more importantly registers the shifts in the mutually inflecting relation between metaphysics, mathematics and physics in his thought. In what follows, we thus take a forward step from Leibniz’s epistemology to an application of this methodology, from Leibniz’s abstract conception of the relation between ideas, imagination and knowledge to his practice of this relation.

What the examination of this period of transition aims to show is the following. The transition between the *Theory of Abstract Motion* and the *Pacidius Philalethes* marks more than a transition between two different solutions to the relation between the infinitesimal and the structure of motion. From the *Theory of Abstract Motion* to the *Pacidius Philalethes*, the difference that I aim to make explicit is that Leibniz’s position concerning the nature of motion changed radically in the span of a few years due to the transformation of the role of the mathematical. In this transformation, not only does a designation of a “fictional” account of the infinitesimal emerge, but a conceptual space opens up wherein the metaphysics of motion is constrained more determinately by the mathematical reflections on the continuum. That is, between the two texts, Leibniz’s metaphysics of motion shifted from being merely constrained by the geometrical account of the continuum to being a space where speculation is conditioned by the very problematics of the continuum itself.

Unfortunately, without the further investigation of this fine conceptual distinction, I cannot make these aims more clear. Awaiting further clarification, I can only suggest a direction at this point by saying that the tracing of this difference between the two texts will demonstrate that, far from allowing either a reduction or a dissolution of the labyrinth of the continuum, the problem of the infinite/simal provided a laboratory for Leibniz's reflections on motion. As such, the real stakes for the metaphysics of motion that emerge from this transition can be understood through the shift in the role of mathematics in Leibniz's thought. Rather than merely a collection of shifting positions, this implies that Leibniz's encounter with the labyrinth of the continuum and the status of the infinite/simal was indeed a laboratory of invention.

The *Hypothesis Physica Nova* of late 1670 and 1671 consists of two parts, the *Theory of Concrete Motion* and the *Theory of Abstract Motion*. Insofar as we are focused on the status of infinitesimals in this text, we will look primarily at the *Theory of Abstract Motion*. This text came at the end of a long engagement with atomism and Cartesian views on the metaphysics of motion, and both of these influences are felt within it. Despite having these explicit critical engagements with contemporary theories (Galileo, Descartes, Hobbes, etc.) about motion and the nature of infinitesimals, our reflections on the arguments concerning the status of infinitesimals and its relation to motion will begin with Leibniz's reading of Aristotle and the Eleatic proofs.

In reading the *Theory of Abstract Motion*, we are reminded that Leibniz's interest in the problem of infinitesimals was not purely mathematical or epistemic but rather one rooted in philosophical problematics from his earliest works on substance. In this, Leibniz's references to the Eleatics, Aristotle and Galileo concerned not mathematical or geometrical method but rather their direct implication for an account of motion. For Leibniz the problem of infinitesimals, their structure and their status were more often than not tied to the issue of motion. Recall that Zeno's arguments, in some ways the starting point for any discussion of the infinite, divisibility and the miniscule, were tied to the reality of motion. For Zeno motion was unreal and inactual insofar as there is no rational account that can resolve the problem of divisibility of continuous extensions. In turn, Leibniz's grounding argument of the *Theory of Abstract Motion* should be read through an Aristotelian influence.

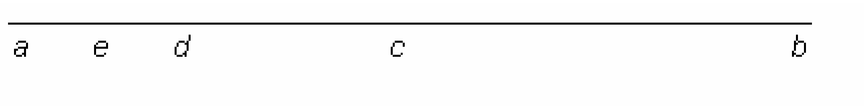
In the *Physics*, Aristotle gave this firm response to the Eleatic paradoxes:

> What is continuous is divisible *ad infinitum*, but there is no infinite in the direction of increase. For the size which it can potentially be, it can also actually be. Hence, since no sensible magnitude is infinite, it is impossible for it to exceed every assigned magnitude; for if it were possible there would be something bigger than the heavens.[157]

Having made this distinction, Aristotle continued to remark that motion can be saved from the paradoxes insofar as the application of divisibility in a continuum is secondary to the relation of greater and smaller sensible magnitudes. Here, Richard T.W. Arthur suggests that we can characterize Aristotle's criticism as Zeno's failure to distinguish between an "infinite by divisibility" and an "infinite by extent".[158] To be clear, if we hold motion to traverse a given length, any part of this length can be, according to Aristotle, (potentially) infinitely divided, but this is only if the "extent" is already given. We can allow this criticism to guide our reading of Leibniz's central argument, proposition 4, in the "Predemonstrable foundations" of the *Theory of Abstract Motion* for the existence of actual indivisibles. Leibniz argued:

[157] Aristotle 207b 16-21. Aristotle, 268.
[158] Leibniz, *LC*, 350.

There are indivisibles or unextended things, otherwise neither the beginning nor end of a motion or body is intelligible. This is the demonstration: any space, body, motion has a beginning and an end. Let that whose beginning is sought be represented by the line ab, whose midpoint is c, and let the midpoint of ac be d, that of ad be e, and so on. Let the beginning be sought to the left, on a's side. I say that ac is not the beginning, since dc, can be taken away from it without destroying the beginning; nor is ad, since ed can be taken away and so on. Therefore nothing is a beginning from which something on the right can be taken away. But that from which nothing having extension can be taken away is unextended. Therefore the beginning of a body, space, motion, or time (namely, a point, an endeavor, or an instant) is either nothing, which is absurd, or is unextended, which was to be demonstrated.[159]



[Figure 4.][160]

In brief, this was the core of Leibniz's argument for the actuality of infinitesimals as indivisibles in 1671. The implicit assumption, coming from an Aristotelian influence, seems to be the distinction between the infinite by division and the nature of pre-given extension. Now at the end of the argument, the absurdity of there being no end or finite extent of a "body, space, motion or time" is something that assumes the acceptance of Aristotle's response to the Eleatics, but as Arthur has noticed, the positive argument here for the actuality of indivisible as infinitesimal

[159] Leibniz, *LC*, 339.
[160] Figure taken from Leibniz, *LC*, 13.

hinges on the form of argumentation given by Zeno himself, an "inverted Zeno argument".[161] The inversion is that, far from demonstrating the impossibility of motion, Leibniz understood the problem of the infinite divisibility of continuity as imparting to the "beginning" of motion the same status as the motion itself. Since this beginning of motion, analyzed through the Eleatic style, brings us to a convergence on an instant, the actuality of the whole extent of motion must be imparted to this point, endeavor or instant. Failing this, there is no motion, which Leibniz took to be "absurd".[162]

The argument for the actuality of the indivisible hinges on the idea that any motion, however small, must have a beginning, and if this motion is real, then its beginning is real. It is however not yet clear what this argument means in terms of infinitesimal entities. In the third, that is, preceding proposition of the *Theory of Abstract Motion*, Leibniz argued that there is "no minimum in space or body, that is, nothing which has no magnitude or part."[163] Again, following Greek thought, a minimum cannot be taken in terms of space since it has no "situation". Having no situation means that it has no possible relation or ratio with any magnitude, as defined in Book V. of Euclid's *Elements* as something which "measures the greater".[164] It has no place and no measure and thus no situation. From this, any discussion of a point or smallest part must take place via a displacement into the context of a given body or motion. This puts us in an uncomfortable position. We can see that even as Leibniz attempted to argue for the actuality of the infinitesimal as indivisible, this status was designated with the awareness of its problematic as a purely mathematical reasoning, the extrapolation of a smallest part through division,

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[161] Arthur, "Actual infinitesimals in Leibniz's early thought", 15.
[162] Leibniz, *LC*, 339.
[163] Leibniz, *LC*, 339. I should also note that while Leibniz argued in this text that there are no minima in space, he also argued that time does admit of minima. Here he remarked that "one instant is equal to another, whence time is expounded by a uniform motion in the same line, although its parts do not cease in an instant, but are indistant." Leibniz, *LC*, 341.
[164] Euclid, 99.

fictional or not. Already we can see how, even at this early stage, the construction of the problem of infinitesimals by means of reference solely to a mathematical stratum of objectivity is erroneous, a confounding of actual and ideal. Instead, Leibniz placed the mathematical consideration and the operations available to him in the context of a metaphysical foundation of motion and corporeality.

By rejecting minima and accepting this smallest part of a given motion, Leibniz argued for the actuality of the infinitesimal as indivisible. Insofar as motion is real, its beginning must be real. From this, Leibniz went on to argue for the role of the infinitesimal in the case of a change of direction when two bodies collide. Here, at the moment of collision, two bodies are momentarily joined into one body and, with respect to motion, occupy the same "point of space."[165] What occurs at the beginning of motion is reciprocal with the end of motion. When a body collides with another, these two bodies find themselves in a situation of a point. As they collide and repel each other, the motion begins again, starting from that point of collision outward. Here, Leibniz's central argument concerning the status of infinitesimals rests on this relation between the reality of motion and the inference of that actuality to the infinitesimal as indivisible. In addressing its status in proposition 5, Leibniz argued:

> A point is not that which has no part, nor that whose part is not considered; but that which has no extension, i.e. whose parts are indistant, whose magnitude is inconsiderable, unassignable, is smaller than can be expressed by a ratio of another sensible magnitude unless the ratio is infinite, smaller than any ratio that can be given.[166]

[165] Leibniz, *LC*, 341.
[166] Leibniz, *LC*, 340.

Here Leibniz was addressing a number of issues simultaneously. The most important of these is his maintenance of a rejection of minima in space and the holding of an infinitesimal part. He began by arguing that these physical points have parts, but that these parts are indistant and do not have an assignable ratio. Leibniz's physical indivisibles are not partless points. They are indivisible in the mathematical sense; their parts have no assignable distance. Yet they are far from being points in the geometrical sense. This is the main distinction that Leibniz made in this text, perhaps the theoretical lynchpin of his entire argument for distinguishing extension and magnitude. The point or endeavor, Leibniz's reinterpretation of Hobbes' *conatus*, has magnitude but no finite or assignable extension. This tenuous distinction between magnitude and extension is the key distinction that results from his "inverse Zeno" argument. It is only with a healthy dose of philosophical generosity that we might see what he is aiming. As long as the infinitesimal point is arrived at by the process of infinite division, the point will remain in a "situation" and thus can be analyzed from this process of division. On the other hand, insofar as the point is arrived at with this infinite division, when this division is carried to an "unassignable" degree, the result is a magnitude that will be, by definition, without finite extension. Thus when Leibniz arrived at proposition 18 of the *Theory of Abstract Motion*, he used the example of angles of contact to argue that points can be without extension but nonetheless have magnitudes. That is, Leibniz argued:

> [W]hence the unassignable arc of a bigger circle is greater than that of a smaller one: and any line whatever, drawn from the center to the circumference, commensurable with the circle, that is, the line by whose rotation the circle is

generated, is a perpetually increasing minimum sector, but extensionless within."[167]

From this discussion of angles, we can also understand why Leibniz claimed that "this is the basis of the Cavalierian Method, whereby its truth is evidently demonstrated, inasmuch as one considers certain rudiments, so to speak, or beginnings, of lines and figures smaller than any that can be given."[168] Without going into detail, Cavalieri's method proceeds by bringing different figures into a ratio by means of mapping them onto a series of parallel lines intersecting these figures. The basic example of the method is that the proportion of the area of a cone and a cylinder can be compared and put into a ratio in terms of the parallel planes that intersect them. These planes will be themselves without any height or thickness, hence without extension. However, Cavalieri's method does not prevent these parallel intersections from ordering the area of the three dimensional figures that they intersect. The lesson that Leibniz drew from this is that indivisible parts can be conceived in calculation without themselves having to be put in a ratio with the figures that they cross section.

In this further explanation by means of angles, and from Leibniz's positive reference to Cavalieri, the difficulty of a magnitude without an extension can be understood insofar as this early infinitesimal conception is dependent on a pre-given extension, within which an extensionless part can be situated. This "situation" which allowed Leibniz to sustain an actual indivisible as infinitesimal was itself revised in the period immediately following the *Theory of Abstract Motion*, but we should not fail to notice the construction that Leibniz built around this actual infinitesimal.

[167] Leibniz, *LC*, 342.
[168] Leibniz, *LC*, 340.

This exposition of the fundamental argument in the *Theory of Abstract Motion* is aimed at two points. The first aim is simply to mark a reversal of views. For at least a period in his early work, Leibniz held, as exemplified in this text of 1670-1671, a position of actual infinitesimals as indivisibles. This is a position that he eventually gave up with a rejection of indivisibles and a fictionalization of infinitesimals. Secondly, we see that this position of actual indivisibles was argued for through the correlation of the actuality of motion with the geometrical extension. The inverse Zeno argument employed the division of the geometrical extension such that the attainment of the "unassignably small" measure of the indivisible beginning of motion does not equate to a "partless" point but rather a point where no definite or assignable difference can be taken. This latter point demonstrates one way in which the labyrinth of the continuum constituted a laboratory of invention.

In this early period, Leibniz invented a distinctive hybrid, a three-headed monster of heterogeneous parentage. The response of the Aristotelian tradition to the Eleatic paradox provided the basis for understanding the fundamental problem of motion and divisibility. Descartes provided the idea of the division of a motion into an infinite number of indivisibles. Hobbes, through Cavalieri, supplied the provocative idea that points can be compared in terms of greater and smaller in analogy with angles. Leibniz's invention was however not simply a version of these positions. While borrowing from these different discussions, he constructed the indivisibles in view of a "situation", that is, through his own construal of the pre-given extension of the interval of motion.

I will not enter into the nuanced difference between these other positions and that of Leibniz's in the *Theory of Abstract Motion*, but one point needs to be explicitly underlined. The *Theory of Abstract Motion* constitutes one way in which Leibniz brought together the reality of motion and the nature of the continuum. The unending division of the continuum posed, for Leibniz, not an obstacle to grasping the reality of motion, but its very structure. In the *Theory of Abstract Motion*, Leibniz's inversion of Zeno's argument laid the basis for understanding the fundamental elements of motion: the "beginning" and "end" of motion constitute undivided points (with parts) that serve to account for the extent of motion. The possible adequacy of this inversion of the Eleatic argument is, however, based on Leibniz's introduction of a difference between a purely geometric argument and an argument based on the reality of motion. That is, a clarification for the potential "confounding" of the levels of the ideal and the actual must be sought. In particular, in order for the fundamental argument in the *Theory of Abstract Motion* to be successful, the indivisible as infinitesimal must first be anchored to the reality of motion and at the same time be distinguished from the purely geometric or mathematical understanding of the continuum.

From the *Theory of Abstract Motion*, we can already grasp one way in which Leibniz attempted to accomplish this distinction. The metaphysical commitment to the reality of motion employed mathematical thought to account for the necessary difference between motion and continuum by inventing such things as points with parts or magnitude without extension. As a laboratory, mathematics constrained Leibniz's inventions but also served to provide, with the notion of angles of contact, the very elements that he recombined to form new concepts. At the same time however, Leibniz applied the distinction between the mathematical and the physical to construct a space of speculation. That is, by distinguishing the realms of geometry and the structure of motion, Leibniz was able to distinguish between the mathematical or geometrical constraints that he avoided violating while inventing new figures to account for actual physical or metaphysical realities.

In order to more coherently understand Leibniz's response to these questions of the period, we can also take a brief look at his conception of a level of primary matter in his text "On primary matter", written during the same period of the *Theory of Abstract Motion*, the winter of 1670-1671. In this text, primary matter roughly followed its Aristotelian designation, an undifferentiated state of "matter" (insofar as it is undifferentiated by form). In turning to this discussion, we shall see that Leibniz did not view the problem of the well-foundedness of infinitesimals, even in this period, as a mere geometrical consequence of contemplating space in an abstract sense. Furthermore, the well-foundedness of the infinitesimal, even in this period of holding actual indivisibles as infinitesimals, relied on an anchor in the actuality of motion. Reading the *Theory of Abstract Motion* in light of this text can help us understand what is at stake in the relation between mathematics, physics and metaphysics during this period of Leibniz's reflection.

In this text "On primary matter", Leibniz put forth the position concerning the metaphysics of motion that "primary matter is nothing if it is at rest", under the Aristotelian inspiration of a mute material level that is, as he explained, "divisible to infinity."[169] Now in the course of the *Theory of Abstract Motion*, Leibniz's position hinged on the structural constraints that a geometrical conception of the continuum, the general features of divisibility, puts on a rational account of motion. Given these constraints, Leibniz turned to other geometrical features, the dynamics of magnitude, and more precisely the possibility of a magnitude without extension to provide a geometrically modeled (but not geometrical) account of motion. What this small, uncompleted text clarifies however is the possible distinction between a continuum or a notion of primary matter that is conceived along the lines of infinite division at any point and, on the other hand, a structure of motion, insofar as it is the "actualization" of this "nothing at rest". Leibniz

[169] Leibniz, *LC*, 343.

seems to have thought that this latter, actual dimension was at least conceptually distinguishable. As he argued here in 1671, "If all primary matter were to move in one direction, that is, in parallel lines, it would be at rest, and consequently would be nothing. Everything is a plenum, since primary matter and space are the same."[170] Leibniz's argument here hinges on the problem of differentiation. A mute primary material level is as good as nothing, that is, as good as an abstract geometric manifold of space if there is no level of differentiation, or difference, introduced in it. Noting an Epicurean or Atomistic inheritance, Leibniz went so far as to note that movement in parallel, undifferentiable lines is nothing insofar as it does not make any mark of difference in its being at rest. While Leibniz here drove at conclusions basically consonant with other writings at the time, what is important to note is that the structure of motion was not itself absorbed into the problems of structure of the continuum. What Leibniz marked here is a difference between the possibility of a purely geometrical continuum correlated with something at rest and an actual extension of motion. Now the picture here is that this primary matter or what is equivalent to "space", as he explained, is nothing without motion, that is, nothing without the actuality of differentiation. If we understand what Leibniz said here, he meant that the field of the problem of the continuum concerns, insofar as it is a problem, not only its infinite divisibility, but also the problem of not letting this division ruin the actuality of motion. Hence this level of zero motion, primary matter, must be a "nothing" that is only brought into view by means of motion. This 1671 text does not fully respond to all the consequences that this position provokes. However in 1672, in the text "On Minimum and Maximum", he became less ambiguous in remarking that "there is no space without body, no body without motion."[171] Now this 1672 text is a very rich text, and I will not spell out Leibniz's transformation from the

[170] Leibniz, *LC*, 344.
[171] Leibniz, *LC*, 15.

Theory of Abstract Motion to the position held in this text here. However, what I wish to note for a moment here is the idea of a primacy of motion over body and body over space. In this movement, the abstract uniform divisibility of a continuum receives a distinction from real motion, one that only arises insofar as there is a motion to provide the needed situation of differences and differentiation.

Building a bridge between the *Theory of Abstract Motion* and the idea that there is "no space without body and no body without motion", I think we can recast the question of well-foundedness in infinitesimals. Borrowing the principle of transitivity, we can shorten the above expression as "there is no space without motion". Reflecting on Leibniz's argumentative strategy in the *Theory of Abstract Motion*, the idea of an indivisible and its accompanying idea of a magnitude without extension are "not nothing". They are not nothing because the argument begins with the metaphysical thesis of the actuality of motion and a physical reasoning that works backward from the extensional properties of motion. This strategy makes use of a distinction between an abstract geometrical continuum and the reality of motion. The former helps to unfold the properties of the latter without being identical. They are not identical precisely due to the fact that there is no space without motion. Yet the geometrical elements that were at Leibniz's disposal can help in understanding that very reality that makes space itself real. Motion activates space, and the epistemological consequence of this is that the investigation of motion activates mathematical and geometrical thought. As such, the idea of well-foundedness does not simply imply the idea that certain geometrical figures or mathematical arguments represent actual motions in a reliable way. We can see that, at least in this period around 1671, Leibniz thought of the activation of mathematics in the theoretical exposition of motion as involving a separation between geometry and actual motion that nonetheless forms a distinct relation of “activation”.

This clarification allows us to understand the nature of Leibniz’s use of mathematical thought in the *Theory of Abstract Motion*. In this early period, Leibniz was clear in his understanding that the problems of the continuum and the related problem of the status of infinitesimals are not the same as the problems involved in the account of the reality of motion. As we have seen, this account of motion makes use of a wide range of resources, and here Leibniz’s work can be characterized as one of invention: recombining earlier notions into a synthesis of his own conception. Mathematical thought, in this context, served to constrain the speculations of this period but also provided the very elements that constituted the key aspects of his argument. Leibniz’s conception of the indivisible was an attempt to strike a balance between a number of different constraining factors and speculative aims. Two of these are the Greek notion of “situation”, a constraint, and a notion of magnitude without extension, of Hobbesian and Cavalierian inspiration. The idea of a point with parts is not obviously a mathematical concept, but Leibniz attempted to deploy it by a reference to angles of contact and furthermore by trying to show that the nature of motion, in his use of the inverse Zeno argument, would require just such a notion.

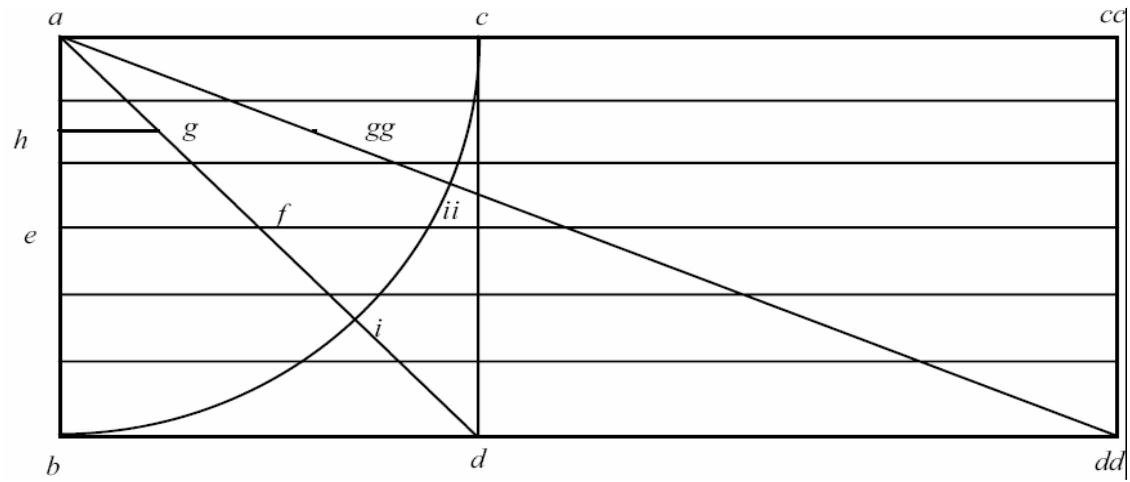
Moving forward to compare the *Theory of Abstract Motion* to the period around 1676, what I aim at in the following will use the examination of this early position as a marker of difference. I hope to use this marker to clarify the difference that his mature position made. More precisely, I will illustrate that the shift toward holding a fictional status of infinitesimals and the rejection of indivisibles is accompanied by the closer proximity of the problematics involving the labyrinth of the continuum and the account of motion. Holding to the idea that it is the problems of motion that activated the problems of space and the continuum in this latter period, I will illustrate how mathematical thought no longer merely constrained and lent solutions to the problems of an account of motion. Instead, the problems of the continuum were directly involved in the problems of the account of motion. Leibniz's inventions in this later phase involved a different theoretical strategy.

4. From indivisibles to actual fictions

Jumping forward about five years, in the texts that Leibniz wrote in 1676, he definitively turned to an explicit definition of the infinitesimal as fictional. This is the focus of the following section, the dialogue *Pacidius Philalethes* of 1676. With the *Theory of Abstract Motion* in mind, this turn, after his four-year stay in Paris between 1672 and 1676, involved a revision of his earlier distinction between magnitude and extension with respect to infinitesimals. In an intermediate text, one that we have visited briefly above, "On Minimum and Maximum; on bodies and minds", written between November of 1672 and January of 1673, Leibniz entered into a transitory stage where he made one major distinction. "On Minimum and Maximum" retains most of the central arguments from the *Theory of Abstract Motion* except that here, all the while maintaining the idea of a minimum without extension, he dissociated the beginning and end of motion and body from the idea of an indivisible. In this 1672-73 text, Leibniz readopted the arguments of the beginning of motion from a line [fig a], the magnitude without extension of an angle, and the identity of the point of motion with *conatus*. What changed however took the form of an argument that recurs throughout his writings from this point. I quote his argument at length:

There is no minimum, or indivisible, in space and body. For if there is an indivisible in space and body, there will also be one in line ab. [fig. b] If there is one in line ab, there will be indivisibles in it everywhere. Moreover, every indivisible point can be understood as the indivisible boundary of a line. So let us understand infinitely many lines parallel to each other, and perpendicular to ab, to be drawn from ab to cd. Now no point can be assigned in the transverse line or diagonal ad which does not fall on one of the infinitely many parallel lines extending perpendicularly from ab. For if this is possible, let there exist some such point g: then a straight line gh may certainly be understood to be drawn from it perpendicular to ab. But this line gh must necessarily be one of the parallels extending perpendicularly from ab. Therefore the point g falls –i.e. any assignable point will fall – on one of these lines, nor can one parallel fall on several points. Therefore the line ad will have many indivisible points as there are parallel lines extending from ab, i.e. as many as there are indivisible points in the line ab. Let us assume in ad a line ai equal to ab. Now since there are as many points in ai as in ab (since they are equal), and as many lines in ab as in ad, as has been shown, there will be as many indivisible points in ai and ad, namely, in id, which is absurd.[172]

[172] Leibniz, *LC*, 9.



[Figure 5][173]

This argument, as Arthur remarks, might seem illegitimate to contemporary readers in the sense that Leibniz conflated "the set of points in a line with the number of points contained in it".[174] Yet regardless of contemporary standards of evaluating this argument, it certainly speaks to the unraveling of Leibniz's self-understanding of his previous position. That is, his argument about indivisibles from the *Theory of Abstract Motion* might stand as an argument for the reality of the beginning of motion. It does not, in turn, as Leibniz realized in "On Minimum and Maximum", account for any sort of point-wise comparison of indivisible points. Hence what changed from the *Theory of Abstract Motion* to the text "On Minimum and Maximum" was the status of the infinitesimal even as the underlying account of motion stayed the same. In turn, the inverse Zeno argument took on a new valence. In "On Minimum and Maximum", this argument still reserved a special status for the beginning of motion. Leibniz here reaffirmed the theory of *conatus* or endeavor and the comparability between different points as being greater and lesser in terms of magnitude. This however was given a shift in register in "On Minimum and Maximum". Leibniz

[173] Figure taken from Leibniz, *LC*, 11.
[174] Arthur, "Actual infinitesimals in Leibniz's early thought", 17.

explicitly argued against any thesis of indivisibles by introducing a comparison with the problems of the divisibility of the continuum:

> For a line, however infinitely small it is, will not be the true beginning of body, since something can still be cut off from it, namely the difference between it and another infinitely small line that is still smaller; nor will this cease until it reaches a thing lacking a part, or one smaller than which cannot be imagined, which kind of thing has been shown to be impossible. But if a body is understood as that which moves, then its beginning will be defined as an infinitely small line. For even if there exists another line smaller than it, the beginning of its motion can nonetheless be taken to be simply something greater than the beginning of some slower motion. But the beginning of a body we define as the beginning of motion itself, i.e. endeavor, since otherwise the beginning of body would turn out to be an indivisible. Hence it follows that there is no matter in body distinct from motion, since it would necessarily contain indivisibles, so that there is even less ground for a space distinct from matter. Hence it is finally understood that to be body is nothing other than to move.[175]

Leibniz here remained committed to the metaphysical principle that he once motivated to argue for indivisibles in the *Theory of Abstract Motion*. In his realization of the impossibility of indivisibles, Leibniz's basic view of the infinite divisibility of bodies and of motion did not change but in turn became more invested. That is, whereas the inference toward an infinitesimal as indivisible in the *Theory of Abstract Motion* was made on the basis of the parts of motion, the exclusion of indivisibles in this text was made through Leibniz's appeal to a hypothesis of an extra-geometrical form of motion's consistency. We have briefly commented above on the

[175] Leibniz, *LC*, 17.

separation of “mere” space, an abstract geometrical continuum, and its activation through bodies and motion. Here, the metaphysical actuality of motion brings about an effect on the continuum but is not explained nor founded in it. This is related to the means by which Leibniz will account for a metaphysical foundation of bodies and motion, “For the existence of bodies, it is certain that some mind immune from body is required, different from all the others we sense.”[176] How then should understand this existence of body, suspended between metaphysical and mathematical problems? His metaphysical commitment to the actuality of motion is what provided the backdrop for the position of indivisibles in the *Theory of Abstract Motion*. Yet whereas the account of motion in the *Theory of Abstract Motion* separated the levels of motion and geometry by using the geometrical elements as a repository of figures that allowed Leibniz to construct surprising solutions, here the argument drew on geometrical arguments concerning indivisibility to reject such constructions. The strengthening of mathematical constraints in this context required Leibniz to shore up an account in terms of metaphysical reality. That is, in “On Minimum and Maximum”, the lack of an explanation for the beginning of motion by means of mathematics alone occasioned the introduction of a metaphysical concept. The change of status for the infinitesimal then closely corresponds to the change in the role of mathematics in Leibniz’s philosophy. Mathematics was no longer adequate for a direct account of the metaphysical reality of motion but rather furnished the background of an epistemological reconfiguration of the role assigned to each domain.

As if giving a summary of his passage from *Theory of Abstract Motion* to “On Maximum and Minimum”, he wrote:

> So look how many things we have accomplished in philosophy in how few lines!
>
> We excluded indivisibles, i.e. minima, from nature just as we also excluded

[176] Leibniz, *LC*, 17.

> maxima; then we excluded space distinct from matter, and matter distinct from motion, from which we inferred indivisibles; finally we vindicated the necessity of minds.[177]

While I hesitate to give a definitive reason to the shift in perspective in this argument of 1672-1673, this text does indeed represent the tendency of Leibniz's trajectory in reflecting on the relation between mathematics and metaphysics. In particular, the shift in the relation between mathematics, physics and metaphysics clarifies how the status of infinitesimals in his account of motion served as a privileged laboratory in this transition.

The dialogue *Pacidius Philalethes* was written during Leibniz' voyage back, after four very productive years in Paris, to Germany. Recently hired by the new Duke of Hanover to assume archival duties, Leibniz decided to leave Paris by way of England and the Netherlands, using the opportunity to discuss his recent works with luminaries scattered in these countries. On his way to the Netherlands and his famous encounter with Spinoza, he found his trip delayed due to stormy weather on the English Channel. Cut off from the continent, he composed this highly synthetic text on which many have remarked. Biographical context aside, this text synthesized Leibniz's reflections of the previous years, reviewing his positions on atomism, Cartesianism, his readings of Galileo, his previous position on indivisibles and the results of his work on the infinitesimal calculus. It is also in this text that Leibniz gave an unequivocal assertion of infinitesimals as fictional in the context of an examination of the metaphysics of motion. Though brilliantly written and both dramatically and theoretically complex, I will only focus on the central points of his argumentation in order to present the main conclusion that he arrives at by the end of the text.

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[177] Leibniz, *LC*, 17-18.

Starting at the end, the main conclusion of the *Pacidius Philalethes* concerns the reality and structure of motion (a theme that I will investigate in the following chapter in more depth). Leibniz concluded here that motion is a change of place. Motion is the change of the position of a body (x) from one place (A) to another (C). With the help of the figure below (figure c.), we see that this change in position does not involve just any position. A body in motion does not occupy two spaces or any intermediate space. It is also not a change in position that involves a leap. It is a contiguous change in position (A to C) where there is no gap in between, a change to a *locus proximus*. Since motion is "situated" at the edge of two contiguous points, motion occurs at neither and hence there will be no need to summon up the smallest part of an extension of motion that would have to account for the rest of the extension of motion. Motion refers to the juxtaposition of the two positions of a body at two moments, one before and one after. As Leibniz explained in the text, "It cannot be said that something is moving now . . ." but it can be said that something has moved.[178]

[Figure 6][179]

To provide a metaphysical supplement to this, Leibniz argued that God "trans-creates", annihilating body at the "before" position (A) and recreating it at the "after" position (C). The juxtaposition of these two moments (AC) would then translate to the "situation" of motion discussed above. This short resume of the general conclusion of Leibniz's argument is meant to

[178] Leibniz, *LC*, 167.
[179] Figure taken from Sam Levey's discussion of this passage in the dialogue *Pacidius Philalethes* in "The Interval of Motion in Leibniz's *Pacidius Philalethes*". I also owe the framing of this argument to Levey's work in this paper. Sam Levey, "The Interval of Motion in Leibniz's *Pacidius Philalethes*," *Noûs* 37, no. 3 (2003): 380.

help guide us to his arguments toward this point in the *Pacidius Philalethes* since it is not exactly clear how this account would shed light on Leibniz's position on infinitesimals. In the following, keeping this conclusion in mind, we shall see how he developed this conclusion from the argument in the dialogue.

At the start of the dialogue, Leibniz introduced the problem by considering some problems offered by the Sorites or "heap" paradox. This sort of paradox is expressed by the following sorts of problems. If something goes from living to dead, or if a man goes from poor to rich, is there a precise point when the living becomes dead or the poor man becomes rich? Which additional cent turns a poor man into a rich one? Leibniz argued here that the idea of an "intermediate" stage between alive and dead is not sustainable since it would lead to a point where a thing is simultaneously both or neither which is contradictory. Introducing (via reference to Aristotle's *Physics*) the idea of contiguity, Leibniz argued that there are two stages, alive and dead, the extrema, that is, the last point of being alive and the first point of being dead, and they are not the same point, but contiguous ones. This solution helped dissolve the problem of there being a point where something is both alive and dead. They meet without sharing any point in common; hence they meet without being continuous. Leibniz then applied the idea of contiguity to the problem of motion. If becoming dead and becoming rich are changes, these examples are relevant insofar as motion is the change of place. The content of Leibniz's argument for explaining change of place, or motion, by this idea of contiguity reversed the central argument of the *Theory of Abstract Motion*. In the *Pacidius Philalethes*, he questioned whether the change of place can be ascribed to the "last part" or a minimum of the motion. Whereas in the *Theory of Abstract Motion* he employed the same argument for the actuality of indivisibles, here, after having collapsed the distinction between indivisibles and magnitudes without extension, Leibniz supplied a reversal of the terms. If we assume the minimum as the state of the beginning of motion, it would be this part where the state of change, the becoming-moving, occurs. Yet suspending for a moment the earlier arguments about the absurdity of such a state, Leibniz asked, “Now can a minimum in place be completed in anything other than a minimum of time?” The answer was simply, “No, otherwise in part of this time part of the place would be completed, but a minimum has no part.”[180] This in turn suggested that a change in place, or motion, occurs not at the minimum place but at the moment marked by the two contiguous points “before” and “after”. Following this, Leibniz introduced a few other distinctions concerning his difficulties with the continuity of motion. At this point, however, about a third of the way through this text, Leibniz made a provisional conclusion that grounded the rest of his arguments about the continuum. He argued:

> [M]otion is a state composed of the last moment of existing in some place and the first moment of existing, not in the same place, but in the next, different place. Therefore the present motion will be nothing but the aggregate of two momentaneous existences in two neighboring places. So it cannot be said that something is moving now, unless this now is interpreted as the sum of two neighboring moments or the point of contact of two times characterizing different states.[181]

Having arrived at this provisionary conclusion which can be read as clearing the ground of some of the tendencies he had adopted in his earlier work, Leibniz jumped head-on into the problem of the continuum.

[180] Leibniz, *LC*, 157.
[181] Leibniz, *LC*, 167.

Leibniz reasoned that whether or not we see motion as continuous or composed of these contiguous aggregates of positions across time, the question of the composition of a line or continuum by points will be raised. Leibniz's employment of contiguity aimed to untangle the account of motion from the problems of continuity. Here the problem is that even if we might grant that Leibniz had provided a coherent account of motion as the change of position *locus proximus*, he might have escaped Scylla just to encounter Charybdis. Do these contiguous points really add up to the sort of motion that moves across an extension? Would this not mean that a prolonged path of motion would imply the aggregation of contiguous points into a continuous path of motion? Here Leibniz turned toward the tradition of this problem of composition by looking at Aristotle, Galileo and Descartes' responses to it. As we saw in earlier sections, Aristotle posited a potential infinite, Galileo remained ambiguous, making the infinite/simal incomparable to finite ratios, and Descartes held a realm of indivisibles incomprehensible to finite minds. Hence Leibniz remarked, "Neither Aristotle nor Galileo nor Descartes was able to avoid this knot, although one of them pretended not to see it, one abandoned it as hopeless, and the other severed it."[182]

It is at this point that Leibniz offered his hypothesis, a daemonic moment (in the Platonic style) in the dialogue where, without fully answering the previous questions about the continuum, he proposed to cut off the tricky knot of the continuum. As an alternative to Descartes' perfect fluid composed of points that are, in their final division, not merely unassignable spaces but in themselves indivisibles incomprehensible to finite minds, he posited a perfectly pliant continuum, one where points are folds rather than fluid or powdery parts. He argued:

[182] Leibniz, *LC*, 173.

> Accordingly the division of the continuum must not be considered to be like the division of sand into grains, but like that of a sheet of paper or tunic into folds. And so although there occur some folds smaller than others infinite in number, a body is never thereby dissolved into points or minima. On the contrary, every liquid has some tenacity, so that although it is torn into parts, not all the parts of the parts are torn in their turn; instead they merely take shape for some time, and are transformed; and yet in this way there is no dissolution all the way down into points, even though any point is distinguished from any other by motion. It is just as if we suppose a tunic to be scored with folds multiplied to infinity in such a way that there is no fold so small that it not subdivided by a new fold: and yet in this way no point in the tunic will be assignable without its being moved in different directions by its neighbors, although it will not be torn apart by them.[183]

By employing this "folds" metaphor, Leibniz undertook a major reconstruction not only of the problem of the continuum but also the relation between the metaphysics of motion and the mathematical problem of the infinite divisibility of the continuum. A more careful investigation of the structure of the continuum that this conception leaves us with will have to wait until the next chapter, but it is nonetheless crucial for our purposes here to examine a few of its central features. I say *conception* because what Leibniz presented here is neither a systematic response to the many pressing questions, nor does it constitute a real argument. What Leibniz was doing here was inviting us to reconsider the nature of the problem by reorganizing our usual views on the relation between the problem of motion and the problems of continuity that it engenders.

First, let us see how the "folds" argument avoids the problem of the decomposition of the continuum. Like pinching two parts of a napkin together, the points held between one's fingers

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[183] Leibniz, *LC*, 186.

are contiguous points that do not presuppose that every other part of the napkin is pinched together. This tunic-fold allows us to "pinch" or "fold" wherever we want, as small as we desire and in as many places as we desire, without any further implication that this commits us to generalize that very folding for every part of the tunic as a fold. What this allows us to separate is the operation of division or determination of a set of contiguous points from the structure of the continuum itself. That is, just because we can fold the tunic at one place does not mean that it can be generalized to exhaust the continuum itself. This pinching or folding is the actualization of a real division or determination, the points that constitute two moments of a motion and the like (points between living and dead, poor and wealthy, etc.). This continuum-tunic is not something like a separately or abstractly conceivable geometrical continuum. Every fold is a determination of points. Infinite folding does not dissolve the continuum into points, nor can the juxtaposition of folds reconstitute the tunic itself. Leibniz noted that "the tunic cannot be resolved all the way down into points; instead, although some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain mere extrema."[184] As such, on the one hand, a tunic that is folded to infinity does not exhaust the fabric of the continuum; there will be another smaller fold. On the other hand, any two points that determine the contiguously marked change of place will be determinable since this folding or pinching will not incur the problem of composing a continuum out of points. This conception allows the infinite divisibility of the continuum to be intelligible without falling prey to the idea that a continuum could either be composed of points or dissolved into indivisibles.

Secondly, we can see how Leibniz reconceived the general relation between motion and the problem of the continuum. In an earlier text like the *Theory of Abstract Motion* or in the earlier hypothetical constructions of the *Pacidius Philalethes*, the nature of motion seemed to

[184] Leibniz, *LC*, 185-187.

have been constrained by the abstract and mathematical nature of the continuum, the problems of its composition and decomposition. If we take the sort of argument used in the *Theory of Abstract Motion*, the temptation to argue for something like an indivisible seems to be tied to the correlation between the beginning of motion and something like the smallest indivisible part at the start or end of a continuum. With the tunic-continuum however, the actuality of motion, taken as primary, does not confer its structure onto the continuum itself, and hence there is no inference of the smallest part from the extension of a finite motion. Motions can be larger or smaller, down to unassignable differences without dissolving or decomposing the continuum-tunic, since in a manner of speaking, there is nothing to dissolve. That is, motion does not dissolve the continuum because it is constituted by the extrema of two contiguous edges. Like folding a tunic once, the fold itself separates two continuous parts of the fabric. Thus, instead of seeing the account of motion as being constrained by the problem of the dissolution of the continuum, Leibniz used the account of motion as a means to understand the continuum. Whereas in the *Theory of Abstract Motion* Leibniz attempted to give an account of motion by means of geometrical figures that tempted contradiction (the figure of the indivisible), here he attempted to clarify the nature of the continuum by his theory of motion. Of course, the separation of a purely abstract geometrical continuum and the actuality of motion in both was made clear. Yet in the earlier period, Leibniz viewed the continuum problem and its dissolution as a constraint. Here Leibniz actively transformed the nature of the continuum by placing the reality of motion at the edges (and hence outside) of continuity. As such, Leibniz reserved a place for the abstract mathematical understanding of the continuum outside of reckoning with the actuality of motion. At the same time however, Leibniz, by deploying the tunic-folds metaphor, introduced the two-fold notion that an infinite division of the continuum does not exhaust it, and an infinite aggregation of separate folds does not make up a tunic. As Leibniz reasoned through his account of motion, what developed was the idea that a continuum can be infinitely divided without being dissolved.

5. Consequences of the syncategorematic infinitesimal

With these two points in mind, we move on to the final series of arguments in the *Pacidius Philalethes* that will take us to the conclusion. Putting aside continuous motion insofar as it would reintroduce the problem of decomposition of the continuum into the reality of motion, Leibniz faced the problem of having to explain motion by means of a discontinuous and non-uniform structure. The problem here was that of leaps. Although treated in earlier sections of *Pacidius Philalethes*, Leibniz seemed to be saddled with the existence of leaps if motion was to be both discontinuous and without little, interspersed rests.

To this problem, Leibniz proposed a solution that is bracketed off in the manuscript. In this section, he proposed that there are discontinuous leaps in motion across an extension but that they are “indistant”. That is, leaps occur insofar as the body in motion is “transcreated” at the next contiguous point. In this bracketed section however, Leibniz raised another suggestion and considered the following possibility:

> [P]erhaps leaps through infinitely small spaces are not absurd and nor are little rests for infinitely small times inserted between these leaps. For assuming the spaces of the momentaneous leaps to be proportional to the times of the rests, they

will all correspond together in the same way as the leaps and rests through ordinary times and lines that we expounded above.[185]

What Leibniz considered for a moment here is one attempt to resolve the heavy consequences of his account of motion through contiguous points. What if this very change in position that constitutes motion could occur in a dislocated manner? What if the change in position could make leaps that are infinitely small and hence possess an unassignable measure? This possibility would begin to resolve the problem of how his non-continuous and non-uniform account of motion presented here could form a path of motion.[186] But here he argued:

> I would indeed admit these infinitely small spaces and times in geometry, for the sake of invention, even if they are imaginary. But I am not sure they can be admitted in nature. For there seem to arise from them infinite straight lines bounded at both ends . . . Besides, since further infinitely small spaces and times can also be assumed, each smaller than the last to infinity, again no reason can be provided why some should be assumed rather than others; but nothing happens without reason."[187]

Here we can take the opportunity to underline the reference to infinitesimals as both imaginary and for the sake of invention. However, as Leibniz remarked, he is not sure that they can be

[185] Leibniz, *LC*, 207.
[186] One major problem left unresolved in the *Pacidius Philalethes* dialogue is the issue of how to understand the path of an extended motion given the premises that Leibniz develops along the lines of the contiguity of positions. With Leibniz's deployment of the "folds" metaphor and the affirmation of a transcreation thesis, a concrete grasp of how motion travels across an extension becomes all the more difficult. I raise the problem here without any attempt to resolve it. My aim is to use this unresolved problem as a means to understand Leibniz's construal of these problems in this period. Among others, Samuel Levey has provided a critical commentary of this argument in terms of contemporary mathematics. Other than providing a clear exposition of the problematic, he argues that this conception of motion through the "fold" renders an account that can be compatible with a fractal view of infinitesimal scales, where complexity occurs at every level and is never resolved at any level of reduction. By using Helge von Koch's formalism of Koch's curve, Levey attempts to provide a contemporary account and critique of Leibniz's conception of the structure of motion in this 1676 text. Samuel Levey, "The interval of motion in Leibniz's Pacidius Philalethes", 371-416.
[187] Leibniz, *LC*, 207.

admitted in nature. The consequences, Leibniz noted, seem to end in contradiction: infinite straight lines bounded at both ends. In this, the appeal to the principle of sufficient reason is clear. The fear is that if we assume leaps across infinitesimal spaces, then these could in turn be generalized into perceptibly distant leaps. Or on the other hand, the danger could be that the relation between extrema could again bring about the decomposition of the continuum into points.

This argument allows us to clarify the conceptual background informing his final justifications of the "indistant leaps" in explaining the actuality of motion as the aggregate of two extrema. Here Leibniz attempted to explain the idea of an infinity of non-continuous and non-uniform moments of transcreation. The tunic-continuum conception allowed Leibniz to argue that even if "the motion of a moving thing is actually divided into an infinity of other motions, each different from the other, and that it does not persist the same and uniform for any stretch of time."[188] This understanding does not bring about the problem of the composition and decomposition of the continuum. At any contiguous conjunction of two points, a motion is defined. This contiguous conjunction and any other change of the position of a body are only due to another act of transcreation (by God) which is equally singular. In generalizing relation into a consideration of the extended path of motion, Leibniz elaborated:

> Accordingly I am of the following opinion: there is no portion of matter that is not actually divided into further parts, so that there is no body so small that there is not a world of infinitary creatures in it. Similarly there is no part of time in which some change or motion does not happen to any part or point of a body. And so no motion stays the same through any space or time however small; thus both space and time will be actually subdivided to infinity, just as a body is . . . . This does

[188] Leibniz, *LC*, 207.

> not mean, however, either that a body or space is divided into points, or time into moments, because indivisibles are not parts, but the extrema of parts. And this is why, even though all things are subdivided, they are still not resolved all the way down into extrema.[189]

This division and subdivision into infinity of the world is not by any means a new position, but here we can underline, in this context, how Leibniz unequivocally brought the position in line with a syncategorematic interpretation. This position can be understood in the context of the *Pacidius Philalethes* as a consequence of Leibniz's tunic-fold conception of the continuum and the account of motion. In this text, Leibniz developed a new model for motion by a series of rejections. He rejected motion as continuous, rejected it as uniform and rejected it as involving leaps (over even infinitesimally unassignable intervals). Yet while the basic model of motion as change of the position of a body between contiguous points is clear, much is left unanswered. Most importantly, given this model, how can we concretely grasp extended motions? A non-uniform, non-continuous account of motion may be theoretically appealing in view of the many unsavory consequences that Leibniz indicated would result if things were otherwise. Yet a fully positive account of what this means for an extended path of motion still seems far from reach. Be this as it may, as we can see in this citation above, his work in the *Pacidius Philalethes* allowed him to arrive at some important conclusions about the structure of bodies and motion. This is the idea that an actual infinitely divided nature of the world is neither a contradictory notion nor that which would incur the dissolution of the continuum into points or the Galilean worry of unintelligibility. This deployment of a syncategorematic infinite in Leibniz's reflections allowed the development of a new space of problematics. He distinguished between an imaginary, abstract geometrical continuum and the non-uniform, non-continuous series of folds or extrema

[189] Leibniz, *LC*, 211.

that constitute motion. Further, through the complexities of motion, he made explicit some consequences that would reflect back onto the merely geometrical continuum. That is, motion allows us to understand that the continuum can be actually infinitely subdivided without dissolution. In the *Pacidius Philalethes*, Leibniz's laboratory of invention did not merely take mathematical thought as a constraint to be overcome but actively rethought the framework of the mathematical basis for his reflections. As such, while making a clear separation between mere geometrical figures and the "actual" problem of motion, this framework in fact pulled them closely together, reinventing both dimensions by bringing their problematic aspects into convergence.

In addition to this powerful laboratory, we find that Leibniz's examination of the consequences of an actually infinitely divided reality also involved an experiential reflection. Leibniz's argument here implied a notion of scale into a reflection on the experience of these metaphysical realities and geometrical contractions. On this point, the problem of non-continuous and non-uniform motion was translated into the vertigo of infinite divisions in experience. Beings of any size will always fail to sense the smaller divisions that occur in the micro-regions of their capacities, while what might appear to be a unity by a larger entity will be divided for a lesser one. As he put it, "For just as we contended that the leap would not occur among us but only among certain much smaller bodies, by the same right these same more minute bodies, if we imagined them reasoning about these things, would relegate this same disproportion to other still smaller things."[190] There is hence a proper dimension of the larger argument that concerns perception or coordination of the perspectives of beings of different scales along syncategorematic lines. Every scale of perception implies an even smaller one. Part of the results of this reflection will also provide a clue to the methodological separation that

[190] Leibniz, *LC*, 199.

underlines Leibniz's developments beyond 1676. As we know, by the time of the *Discourse* and the subsequent correspondences with Arnauld and beyond, the successive order of motion in the metaphysical account relied heavily on a theory of perceptions invested in the intensive features and infinite character of a substance's internal modification or apperception. As I will argue in the following chapters, this phenomenal feature of the infinite/simal at work in his philosophy told only part of the story. However, there is no doubt that in this later period, the treatment of the status of infinitesimals was dealt with in one important respect through the experience of different scales: what appears to be continuous may actually be discrete if analyzed far enough, what appears to be a unity may indeed turn out to be multiple, and what appears finite may turn out to be infinite. This experiential component was already present in the *Pacidius Philalethes*.

In this examination, the transformation of the status of infinitesimals at work in Leibniz's account of motion, marked roughly between 1671 in the *Theory of Abstract Motion* and 1676 in the *Pacidius Philalethes*, from actual indivisibles to fictional entities corresponding to folds in the tunic-continuum, exhibits more than a difference between the holding of a "naïve" notion of the indivisible part of motion and a "mature" syncategorematic one. It corresponds to a transformation of philosophical method where the general relation between the role of mathematics, physics and metaphysics is rethought. In particular, the role that mathematics played in his thought, as a constraining and conditioning dimension, underwent a transformation in his laboratory of invention. We can note that, in general terms, prior to his intensive work on mathematics and the development of the methods of the infinitesimal calculus in Paris during the mid 1670's, Leibniz's use of infinitesimals as indivisibles was already developed under mathematical constraints. Regardless of the inadequacies of the notion of the infinitesimal as indivisible in the *Theory of Abstract Motion*, Leibniz separated his physical account from an abstract geometrical discussion and developed his indivisibles through the notion of a magnitude without extension, deploying geometrical figures without making a geometrical argument. In this period, mathematics served as a constraint. Leibniz displayed an unwillingness to overstep the bounds of mathematical consistency, constructing his concept of the indivisible with a finesse that made use of certain gaps in the geometrical and metaphysical discussion of that period in his thought. In his mathematical maturation, Leibniz remained insistent on the distinction between a purely geometrical level and the complex level of physical reality or actuality. In the *Pacidius Philalethes*, having already developed the initial conception of a syncategorematic infinite, Leibniz attempted to provide a different account of motion. While the account made a great stride in his maturation by denying the reality of indivisibles, it was, as we have seen, far from a complete account but nonetheless an inspired sketch, a powerful and synthetic thought experiment. More than a change in position, what I have shown is the development of a new role that mathematical thought played in his argumentative strategy. By drawing the problem of motion out of the paradoxes of continuity, Leibniz untangled the question of the actuality of motion from the ground level of the problem of the compostition (and dissolution) of the continuum. In so doing, the account of motion as non-continuous and non-uniform allowed Leibniz to develop a concept of the continuum where the infinite foldings and divisions do not exhaust it into indivisible points. While Leibniz's account of motion remains unsystematic and incomplete, the separation between an abstract geometrical continuum and the actual path of motion was accomplished. As such, Leibniz enabled the construction of a new laboratory. At the same time as holding the paradoxes of continuity and the account of motion apart, Leibniz brought them more closely together, using the infinite, non-continuous and non-uniform nature of motion to understand the consistency of an actually infinitely divided continuum. These two dimensions mutually implicate without being identified with each other. As Leibniz continued after 1676 to rework and reconstruct his basic account of motion, the continued examination of the problem of the continuum, in the mathematical sense, received a solid and positive foundation. What are the consequences of an actually infinitely divided continuum? This syncategorematic organization of infinitesimal parts of the continuum had major consequences, some that we have visited in the previous chapter. In turn, in the realm of physics, the idea of infinite division of body, motion and space no longer forced Leibniz to contend with such inconsistent notions as indivisibles. It is not only that Leibniz demonstrated a rejection of them in the transition from the *Theory of Abstract Motion* to the *Pacidius Philalethes*; it is the case that Leibniz no longer required them. The actuality of motion had become invested in the exposition of the metaphysically charged idea of a change in place at the extrema of extensions. While Leibniz employed the argument of transcreation in the *Pacidius Philalethes* to account for the reality of this process, it is clear that much more needs to be said in order to fully flesh out the reality of motion. Indeed, the solution of transcreation disappeared from his account of motion after this period, but in order to fully address these issues, a number of geometrical and mathematical elements had already been put in place. The infinite division of motions and bodies provided a consistent basis here, and the syncategorematic infinitesimal was able to positively serve as a resource against the false solutions of indivisibles, all the while effectively handling small unassignable magnitudes without confusion. The problems of the continuum and difficulties of the account of motion converged. Not only do we see that they reflected one another, but in this transformation from 1671 to 1676, Leibniz constructed the grounds of a problematic for the account of motion that not only become more heavily infused with mathematical constraints but treated the inventive reframing of the infinitesimal as a condition for attaining the new horizons of physical and metaphysical invention.

In our examination of the transition from the *Theory of Abstract Motion* to the *Pacidius Philalethes* from 1671 to 1676, we have seen how Leibniz's transformation exemplified his remark years later to De Volder in 1706, "In confounding ideal things with real substances, such that we look for actual parts in the order of possible and indeterminate parts in the aggregate of actuals, we have ourselves introduced the inextricable contradictions in the labyrinth of the continuum."[191] It seems that Leibniz had always been cautious of this labyrinth in his method without having always seen the ultimate consequences of his arguments. Yet without confounding these two realms, the abstract geometrical and that of actual motions and substances, he always sought to remain in the labyrinth without reducing or dissolving them, and in so doing he raised these problems in the process of reframing his laboratory of invention. Hence, far from having a concise narrative of what a "well-founded fiction" might be, in Leibniz's work we encounter a proliferation of half-solutions, sketches and inspired dialogues. A facile understanding of the well-founded might be gained from seeing Leibniz's mathematics as an "Ariadne's thread" through the labyrinth of the continuum. The problems that Leibniz took upon himself are, however, far from being so decisive. Indeed, Leibniz did not wish to be lost in the labyrinth, and hence held onto the thread, but he also wished to remain in the labyrinth for the sake of what he might uncover, treasures that are, as he put it, "eternal truths".[192]

[191] Leibniz, *PE*, 185. Leibniz, *Phil* II, 282.
[192] Leibniz, *PE*, 186.

PREFACE TO CHAPTERS 4 AND 5

METAPHYSICAL STRUCTURE AND THE INFINITE AND INFINITESIMAL

In the previous chapters two and three, I have developed a reading of Leibniz's laboratory of invention through a consideration of the status of the infinite/simal and the role it played in his philosophy. This problem of status is no doubt the most direct way of seeing his explicit treatment of infinite/simals as well-founded fictions. Turning now to another approach in which the problem of infinite/simals was central, I will take up in chapters four and five the issue of structure. As we saw in chapter three, Leibniz's conception of the nature and structure of the continuum had intimate connection with his views on physical and metaphysical reality but often not in straightforward ways. Even as he maintained a conceptual separation between a merely geometric continuum and the reality of motion, the problematic framework with which he interrogated these issues interrelated in complex ways. In this discussion, we saw that a deep appreciation of the relation between mathematics and metaphysics in Leibniz's work must take into account his various arguments concerning the structure of the continuum, of motion and of multiplicity in reality itself. In what follows, I will turn to look at the problem of structure in the same spirit. That is, I will take up the interrelated issue of structure and the infinite/simal by considering Leibniz's working through of the problem of the infinite/simal from within his own construction of the problematic. I aim to do this by entering this domain through an explicitly metaphysical framework.

At the end of the third chapter, we visited some of the basic elements of Leibniz's future mathematical, physical and metaphysical commitments. To put these commitments simply, Leibniz articulated a view of reality (including matter, bodies and motion) as actually infinitely divided while holding at the same time a syncategorematic infinite/simal (well-founded fictions). We also saw that Leibniz motivated his metaphysics as an explanation of the consequences of this view. By entering into the problem of structure by means of metaphysics, I will attempt to understand how this problematic expressed itself in the context of Leibniz's reflections on substance and how his metaphysical trajectory from the *Discourse* to the *Monadology* expressed the way in which mathematics served as a condition for his philosophy.

Just as the two previous chapters were oriented around the axis of the "well-founded fiction", the two new chapters will be organized around another axis, the axis of structure. The next two chapters will then be organized under the title, "Metaphysical structure and the infinite and infinitesimals" and individually entitled, "The structure and reality of body" and "Motion and metaphysical structure" respectively.

Unlike the previous section, the two following chapters will not be framed by a precise expression like "well-founded fiction", but rather an observation. This observation is the transformation of Leibniz's position concerning the metaphysical structure of reality. That is, after the *Discourse* Leibniz was motivated by an interest in establishing the reality of corporeal substance (the possibility of a form of body outside of soul) such that he began to move away from using the notion of individual substance as the organizing principle of his metaphysics and moved toward an eventually monadological one. I will not trace all the multiple influences that led to this change but I will underline one condition of this transformation along the path of the increasing role that a mathematical understanding of body and motion played in his framing of metaphysical problems. As such, rather than interrogating Leibniz's metaphysics by treating the structure of the continuum and the infinite/simal as an extrinsic field, I will try to uncover Leibniz's reorganizing of the structure of reality by seeing how the problem of the infinite/simal allowed an internal reconfiguration of metaphysical problems as constraint and condition.

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Overview of Chapter 4: The structure and reality of body

The aim of the fourth chapter is to draw out the complex relation that exists between Leibniz's metaphysical and mathematical reflections by underlining some of his hesitations concerning the reality of bodies. This approach will allow us to bracket his positive systematic articulations about metaphysical reality and its relation to the infinite/simal by drawing some insights from the fault lines of his thought, how he worked through positions to which he was not fully or not yet fully committed. As such, my goal is to develop an interpretation of the problematics that Leibniz constructed for himself concerning the metaphysical structure of reality.

In order to do this, I will first take a look at Michel Fichant's observation of a change in metaphysical structure that took place between the *Discourse* and the *Monadology*. Following Fichant's work, I will pinpoint one central issue that guided Leibniz's transformation during this period. The most fundamental changes between the *Discourse* and the *Monadology* followed from Leibniz's turn toward singularity and simplicity as the marks of substantiality in contrast to the earlier reliance on identity through the model of the subsumption or containment of predicates in a subject. This shift can in turn be traced to a fundamental change in his position on the reality of bodies. By turning to Leibniz's correspondence with Arnauld in the period immediately after the *Discourse*, I will sketch a conceptual development that will demonstrate the complex relations between Leibniz's metaphysical, mathematical and scientific concerns that contributed to this shift. By taking a look at these correspondences, I will underline how a turn toward corporeal substantiality produced a focus on singularity and simplicity. In doing so, I will clarify some of the stakes involved in his eventual commitment to the position of corporeal substances.

In the final parts of this chapter, I will argue that as the "individual substance" model retreated into the background of his thinking, there remained a lack of a new model to replace it. That is, with the foregrounding of corporeality over predicate inclusion and singularity over identity, Leibniz faced the need for a new model that would not only integrate a commitment to corporeal substance but also articulate his shift in metaphysical vision. Fichant interprets this new need as being filled by the foregrounding of the organic body at the center of Leibniz's later metaphysical system. In developing a picture of the vitalist model based on the notion of the organism, Fichant underlines the use of the concept of force that would allow Leibniz to reconstruct a feature of his earlier position, the relationship between subjective experience and the hierarchy of soul over body. While I do not disagree with Fichant's interpretation, I believe that the centrality of the concept of force that remained a problematic throughout this transformation could serve to extend the consequences of Leibniz's commitment to corporeal substances. I will treat this problem in the following chapter.

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Overview of Chapter 5: Motion and metaphysical structure

Following from my arguments in the previous chapter, I will develop a more focused reading of the problem of force in the fifth and final chapter. The aim will be to extend Fichant's argument in order to understand the important role that the problem of the infinite/simal and its connection with Leibniz's development of a mathematically informed interpretation of corporeality played in the restructuring of metaphysical reality away from the subject/predicate model to its new basis on singularity and body.

Beginning with a look at the problems concerning Leibniz's refutation of Descartes in the period of the *Discourse*, a refutation that contributed to his turn toward the substantialization of bodies, I will illustrate how Leibniz developed an idea of an inherent character of body that does not reduce to phenomenal or purely extensional features. The idea that bodies have an inherent nature of force irreducible to phenomenal effect however required Leibniz to develop a larger project in order to systematically account for corporeal motion and interaction. I will hence turn to a representative sketch of this project in the *Specimen Dynamicum* where Leibniz systematically presented the basic elements and distinctions of this account. Although I will not attempt to produce a comprehensive account of Leibniz's dynamics, by looking at the *Specimen Dynamicum* I will illustrate that Leibniz did not simply correct what he considered to be mistaken in the Cartesian view of *res extensa*. Rather, I will show that he provided an alternative systematic framework. In this framework, Leibniz deployed many of the conceptual and mathematical distinctions developed through the problematic of the infinite/simal discussed in previous chapters. That is, in his formulation of a dynamics, we find Leibniz actively interlacing metaphysical, mathematical and scientific concerns, a concrete image of Leibniz's laboratory of invention.

As I have already insisted in the previous chapters, the aim of the dissertation is to show how Leibniz's mathematical thought conditioned and circulated through his philosophical and metaphysical work. Here in the final chapter, my reading of the *Specimen Dynamicum* will illustrate that the use of infinitesimals, despite their fictionality, and the methods of the infinitesimal calculus were crucial to his development of the dynamics and the concept of force. Being thus so crucial to his concept of force, and this concept in turn being central to his commitment to corporeal substances, and his commitment to corporeal substances being key to his transformation from the conception of metaphysical structure in the *Discourse* to that of the *Monadology*, it appears that what lay at the root of this series of conditions is a mathematical basis, a chain ultimately tied to the problematics of the infinite/simal. What then can this chain of conditions tell us about the nature of infinitesimal fictions?

To answer this question, in the final part of the chapter, I will mount a criticism of Daniel Garber's reading of the role of the infinitesimal in the *Specimen Dynamicum* by arguing against the idea that the fictional infinitesimal played a "representative" role insofar as the mathematics of the calculus represented an actual reality where infinitesimals do not exist. There is no doubt that infinitesimals are not actual entities in the sense that they are not unities or totalities. However, infinitesimals contribute to a larger theory of dynamics that does not simply represent phenomena but accounts for their causes. As such, the infinitesimal is one of the key elements of a dynamics that allows us access into the very actuality of the metaphysical structure of bodies and explains the causes underlying their motion. The interrelation of mathematical, metaphysical and physical reflections on the problem of infinite/simals in Leibniz thus constituted a laboratory where the mathematical conditions of his thought were not only to be understood as a constraint, but also a space of invention. It is this inventive dimension that will give a deeper understanding of what Leibniz meant when he wrote to De Volder in 1706 that "the science of continua, that is, the science of possible things, contains eternal truths".[193]

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[193] Leibniz, *PE*, 186.

CHAPTER 4

THE STRUCTURE AND REALITY OF BODY

1. What is structure?

Before we begin our investigation concerning the reality of bodies, I hope to fix some coordinates for what the question of metaphysical structure was for Leibniz. I will shortly discuss the nature of the observation that I mentioned briefly above, but before this, I hope to remark on a canonical way in which Leibniz tied together his reflections on the infinite/simal and the problem of metaphysical structure. For this we shall take a look at a short and clear text dated to 1683-1685 (before the *Discourse*). As we have seen in the earlier parts of this dissertation, Leibniz had maintained since his earlier writings, such as the physical manuscripts of 1671, a commitment to an actual infinity of the world that goes beyond Aristotelian notions of potential infinity. In later key texts like the *Discourse* and the *Monadology*, this actual infinity of the world metaphysically manifests itself as expressions of plenitude (in the case of creation) and perfection (in the case of the divine). In this, the *Monadology* is perhaps more explicit, continuously employing the language of an inexhaustible division of the parts of nature in any part of nature. The *Discourse* is perhaps less explicit but no less clear. The logical model on which individual substance was accounted for in this work necessitated that every individual substance express the infinite complexity of the whole world. As we know, the appellation *infinite* is equivocal and can refer to a categorematic, hypercategorematic or syncategorematic sense. As we have argued in previous chapters, the infinity of God refers to its absoluteness or being beyond measure, a hypercategorematic sense. Attributing the mathematical infinite to the nature of God would in any case be missing the point. While the consistency of reference in Leibniz's use of the term *infinite* in his various works can be questioned, he did from time to time indicate the mathematical sources of his use of the term as we see in the following text. We know that Leibniz rejected the idea of an infinite totality, whole (entity) or number. In turn, his references to infinity with respect to creation were expressions of modes of analysis or division. They concerned the infinite complexity of a substance, the infinitude of predicates in a subject, or in the later work, the infinite division of parts of the world into an infinity of extra parts. In this short text entitled "God is not the soul of the world", Leibniz laid down an argument in which a mathematical structure of infinity is directly related to the structure of creation and distinguished from God. He argued:

> It can be demonstrated that God *is not the soul of the world*, for the world is either finite or infinite. If the world is finite, then God, who is infinite, certainly cannot be said to be the soul of the world; but if the world is assumed to be infinite, it is not one being, i.e. one body in itself (just as it has been demonstrated elsewhere that the infinite in number and in magnitude is neither one nor whole). Therefore no soul can be understood in it. Certainly an infinite world is no more one and whole than infinite number, which Galileo has demonstrated is neither one nor whole.[194]

In this short argument, Leibniz shifted from the impossibility of an infinite whole to the disassociation of totality from the world. The world is not *ens per se*, not a totality. More

[194] Leibniz, "God is not the soul of the world", trans. Lloyd Strickland, accessed May 10, 2008, http://www.leibniz-translations.com/worldsoul.htm.

precisely, what Leibniz put into words is a disjunction. If the world is finite, then God is not the soul since God is infinite. On the other hand, if the world is infinite, then it is not a totality and thus has no soul; hence God is not the soul of the world. In either case, God is not the world-soul. While in this particular text Leibniz did not argue for the infinity of the world, the guiding idea through which Leibniz posed his argument made use of an important distinction between the syncategorematic and hypercategorematic infinite. Leibniz first used this distinction of senses of the infinite to separate God's infinity from the infinity of the world or creation. God is absolute and is thus infinite in a different sense than the world. In turn if one holds the actual infinity of the world, it must be a syncategorematic infinite, and cannot be a totality. As such, distinguishing between two senses of the infinite, the world cannot be identical to the unity of God. Given that we understand Leibniz as having been committed to some form of an infinite created world, we can read Leibniz as having demonstrated one of the consequences of holding such a position. This consequence is that the infinity of the created world must be of such a structure that there is no infinite totality but rather an infinity without unity, a sprawling infinity that is absolutely multiple, albeit with intermediary stages of finite unities.

My intention in highlighting this short argument is to point to Leibniz's conscious distinction between different senses of the term *infinite*. No doubt given the nature of Leibniz's vast unpublished manuscripts, this position can neither be taken as definitive in his work nor even in this period of writing. Despite this, the text does demonstrate the explicit relation that Leibniz saw between a firm understanding of the nature of the infinite and the sort of conclusion about the world that can be metaphysically drawn from it. The provisional lesson that we can draw from this text is that Leibniz deeply anchored his metaphysics to a distinction that his earlier mathematical reflections made possible, at once separating the infinity of the world from that of the absoluteness of God and also firmly rejecting the application of a certain notion of infinite totality or unity to the nature of the world. While this text does not give us any further details about what Leibniz thought about the structure of metaphysical reality, it does provide a glimpse into what Leibniz thought, in general, about the relation between the mathematical notion of infinity and God's absolute infinity. It briefly demonstrates at least one of the structural consequences in holding the position of an actual infinite world: there is no infinite totality of the world. That is the mathematical sense of the term *infinite*, one that rejects any infinite unity or infinite term and results directly in a basic principle for understanding the metaphysical structure of reality. While this remark is too general to give us a concrete understanding of the relation between mathematics and metaphysics in Leibniz's thought, it does however set up a general framework for understanding the relationship between mathematics and metaphysics that is generally held in Leibniz's method. In no way am I implying that Leibniz imported mathematical structure directly into a metaphysical one, but as we have seen here in a very general way, the mathematical structure of the infinite served as a necessary condition or a fundamental constraint upon which his metaphysics was built up.

Now even if we have been commenting on the structural influence of Leibniz's mathematical reflections on his metaphysics, the question of structure is nonetheless still unclear. In our look at this brief text, the question of structure pertains to the general relation between the notion of totality or a whole and its possible subsuming of an infinite complexity. The mathematical constraint in this case is the impossibility of an infinite whole of the world or creation. This general figure of constraint does not correspond to Leibniz's problem of metaphysical structure and the fundamental transformation that it undergoes from the *Discourse* to the *Monadology*. In what immediately follows, I will try to address what a structural transformation means by taking up the idea of structure from a general metaphysical perspective, proceeding from Leibniz's metaphysical transformation to the mathematical conditions that subtend this movement.

To address this structural transformation, I begin without assuming any pre-given solution of what "structure" as a metaphysical question might have been for Leibniz. That is, without a strong thesis about what structure was or what might be read as a structural argument in his more programmatic texts such as the *Discourse* or the *Monadology*, I wish to open with a very minimal observation on the metaphysical, touched on in the first chapter and in the preface to these two chapters. This is the observation mentioned in the above that Fichant makes in the introduction to his edition of the *Discourse* entitled "*L'invention métaphysique*".[195] The aim of Fichant's lengthy introductory essay is simple and clear: The *Discourse* and the *Monadology* should not be read together as a single work or quoted from without qualification for a simple reason: a structural reversal occurred in Leibniz's metaphysical vision between the *Discourse* and the *Monadology*. I will employ this simple observation of a reversal to build up a concept of structure in Leibniz's metaphysics.

Fichant indicates that the reversal between the two texts took place through the very different valence that the notion of substance took between the two works. Holding the term *substance* as a placeholder that occupied an organizing role in Leibniz's metaphysics, *individual substance*, the central term in the *Discourse*, corresponded to a problem of individuation. This concept organized reality through a model inspired by Leibniz's analytic treatment of truth via the logical principle *praedicatum inest subjecto*. In this context, individual substance is understood through its grammatical or epistemological counterpart, the "individual notion" as the

[195] Michel Fichant, "L'Invention Métaphysique" introduction to G.W. Leibniz, *Discours de Metaphysique et Monadologie*, ed. and intro. Michel Fichant (Paris: Editions Gallimard, 2004), 7-140.

unifying index, as a proper name which subsumes a multiplicity of events, predicates and attributes under an individual. For example, Julius Caesar is a subject under which a multiplicity of attributes, predicates and events are united. Each substance is then understood through its "individual notion" whose completeness is a sum grouping, an inventory of predicates that will either be necessarily or contingently true of it. The global metaphysical view that these local or particular substances constitute is then expressed in the twin principles of non-contradiction and the teleological principle of harmony (an expression of the principle of sufficient reason). Outside the basic set of necessary predicates whose non-existence would constitute a contradiction in the world, created substances cohere in the world by a rational interrelation understood through a model of logical analysis. The mutual existence of necessary states constrains each one in a strict way: the substances that exist are compossible or logically mutually compatible in the sense that the truths that constitute one substance do not violate the truth of some other predicate's inclusion in a subject. Although things could have been otherwise and contingent truths are not necessary, they do nonetheless conform to a principle of harmony that expresses God's rational creation and ordered unfolding of the world. The global interrelation between substances then is constituted through a divine mediation whose main work might be seen as something like a universal filing system, the distribution of predicates and combination of predicates into different folders whose actual state neither violates the principle of contradiction nor the inclusion of predicates in other folders.

Turning from the *Discourse* to the *Monadology*, Fichant admits that it might be tempting to say that nothing much about the global metaphysical picture changed since many of the same principles remained invariant. However, Fichant highlights a major transformation: the problem of individuation, which was of central importance in the *Discourse*, slipped into the background.

Fichant remarks on the fact that the *Monadology* demonstrates a very different understanding of the singular or unity. From the start of the *Monadology*, the first proposition states that monads are simples and not aggregates; they are without parts. More specifically, the second proposition states that "there must be simple substances, since there are composites…."[196] In this latter text, metaphysical simples exist because composites exist, a consequence of the reality of multiplicity and aggregation. These simple things will be referred to as monads and in turn, these "true atoms of nature", not to be taken on par with the atoms of the atomist tradition, are absolutely simple and singular.[197] In contrast to the logically extensional differences between individual substances, Leibniz assigned qualitative differences to monads, intensive features of these singular beings. In turn an organism such as Julius Caesar is then, in the *Monadology*, taken to be a composite of bodily organs with a soul-like monadic hegemon. Here Leibniz understood a living being as a composition, a temporary and constantly mutating configuration of monads, each with its own singularity. At the same time, the composition of monads constitutes an organism, an aggregate with an internal hierarchy of soul and body that we may recognize historically and indexically as Julius the Caesar.[198] As Fichant remarks, however, this aggregation of monads into the animal is only superficially analogical to the individual substance of the *Discourse*:

> [B]y its universality and naturalness, the Monad cannot then be employed except in an anonymous way […] and nowhere will Leibniz write something like 'individual monad', not only because the expression would be redundant, but rather because the monadic conception of reality dissolves the problem of individuation. The Monad can no longer be exhibited as the referent to a proper

[196] Leibniz, *PE*, 213.
[197] Leibniz, *PE*, 213.
[198] Leibniz, *PE*, 221.

> name, and this is then why the 'monad of Caesar' is a poorly formed expression, which the metaphysical language of Leibniz does not permit us to give any meaning.[199]

It is clear that despite certain expected conceptual continuities between the *Discourse* and the *Monadology*, the dominant role played by the logical model of the individual notion slipped into the background in the later monadological vision of Leibniz's metaphysics. In the *Monadology*, from propositions thirty-one to thirty-seven, we find a discussion of a logical universe that reminds us of the *Discourse*, but the heart of the metaphysical universe described in the *Monadology* no longer begins from the basis of predication and instead turns to focus on the relation between simplicity and aggregation. Leibniz remarked:

> Thus each organized body of a living being is a kind of divine machine or natural automaton, which infinitely surpasses all artificial automata […] natural machines, that is, living bodies, are still machines in their least parts, to infinity […] From this we see that there is a world of creatures, of living beings, of animals, of entelechies, of souls in the least part of matter.[200]

Admiring the surprising and swarming universe that this passage sketches for us, I would like to highlight the very different role that substance played in this latter global metaphysical picture. Whereas individual substances in the *Discourse* were unities that subsume differences and thus constitute a network of unities whose individual relation with God constitutes a global metaphysical picture, through the later model of the monad Leibniz constructed a substance that would fulfill a multiplicative role whose aggregated unities correspond to the infinite divisibility of common extended things. In analogy with atoms, monads qua substances are the singularities

[199] [Author's translation] Fichant, "L'Invention Métaphysique", 136-137.
[200] Leibniz, *PE*, 221.

at the basis of the divisibility of concrete reality. Monads, rather than responding to the problem of individuation, could more accurately be said to respond to a principle of identity. Thus Fichant argues:

> In the time of the *Discourse on Metaphysics* and in connected texts, it was a question of the individual and its events; in the *Monadology*, it is a question of the composed and its elements, with the effacement of the problematic of individuation. […] We might say in addition, if we like, that the general architecture of the Leibnizian universe conserves its great equilibriums, even in welcoming new transformations at its interior; but the passage from individual substance to the monads means something else.[201]

Our clue for this "something else" is the reversal of the means by which metaphysical unity is given between the two texts. In the *Discourse* there is a "top-down" constitution of metaphysical unity. A substance subsumes and unites the multiplicity of predicates and attributes that lie beneath it and to which it gives a degree of reality. In the *Monadology*, unity is a precondition of any aggregation, it is the result of a "bottom-up" vision that sees aggregates as real insofar as their parts (infinite in number) are real unities. Whether this difference in presentation and argumentative strategy accounts for a thoroughgoing revision of Leibniz's metaphysics will require a different study, but what is at least clear from Fichant's suggestion is that Leibniz developed a different way to frame his approach to the fundamental features of metaphysical reality. Instead of holding substances as unifying, he saw them primarily as composing or constituting. The infinite multiplicity of substances thus became identified with the infinite multiplicity in reality. This starting point for the development of the very elements of his

[201] [Author's translation] Fichant, "L'Invention Métaphysique", 137.

monadological metaphysics did mark a radical difference with the idea of individual substances whose starting point is unification through subsumption.

Fichant remarks on the multiple and varied influences of this inversion but nonetheless points to a decisive moment that, for him, symbolizes the moment when the vertigo of the infinite inflicted itself on Leibniz. Fichant marks this shift in Leibniz's exchanges with A. Arnauld, in a series of correspondences that occurred just after the writing of the *Discourse*. Fichant highlights the occasion when Leibniz argued to Arnauld in the correspondence dated 30 April 1687, "To put it briefly, I hold this identical proposition, differentiated only by the emphasis, to be an axiom, namely, that what is not truly *one* being is not truly one *being* either."[202] The convertibility of one and being, Fichant argues, takes on the role played by the principle of *praedicatum inest subjecto* as the basic touchstone of substantiality.[203] Hence, despite remaining committed to many common metaphysical features such as the principle of harmony, the non-interaction between substances, the analytic structure of truth, the models through which the two metaphysical visions were constructed are radically different. A structural reversal took place despite the maintenance of certain global metaphysical principles.

What I have up until this point omitted in my discussion of both Leibniz and Fichant's commentary is the link which may illuminate the nature of the transformation. This is the problem of the relation between body and substance. I will focus on this in what follows. However, what Fichant highlights on a structural level, even at first glance, should be sufficient for posing a question. Beyond general issues surrounding the sort of differences between the two texts, we can notice that Leibniz turned from the problem of individuation and subsumption to that of aggregation from absolute simples. This no doubt prompts the question of composition

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[202] *Philosophical Essays*, p. 86. Cf. Fichant, "L'Invention Métaphysique", 90.
[203] Fichant, "L'Invention Métaphysique", 90.

and aggregation that recalls the labyrinth of the continuum in its many formulations. What sort of structural problems did Leibniz face in this transformation?

The sort of *structural* transformation that we have only cursorily outlined here was not only Leibniz's shift in emphasis between a logical model to a constitutive vision, but was one wherein the metaphysics of motion, the relation between body, motion and matter, emerged not as a subordinate problem in metaphysics but a central one. That is, we can begin to appreciate the centrality of the problem of body and motion for metaphysics only by recognizing this structural transformation. In other words, Leibniz sought in the idea of the aggregation of monads another basis not only for expressing the infinite multiplicity of metaphysical reality but also the concept of substance itself. What is more, it is not simply that Leibniz, in the twenty-some years between the *Discourse* and the *Monadology*, provided a supplementary treatise on bodies and motion that would accompany his understanding of substance in the *Discourse*, but rather that his continued reinvestigation of body allowed him a means to reinvent a theory of substance. The transformation of the structural elements of his late metaphysics was the result of serious considerations about the structure of body, which began to take center stage in his correspondences with Arnauld, and was in turn manifested in the systemization of these reflections in the *Monadology*, and thus ultimately tied to turning his focus on the nature of body away from his earlier reliance on the sufficiency of the model of logical analysis. In this, Leibniz invited into his metaphysical program the sort of structural problems that he was forced to confront in his early accounts of motion, that is to say, the problematic that he encountered surrounding the continuum. In this way Leibniz, despite being the very target of the later "critical" program of Kant and the Neo-Kantians, can be read as being far from dogmatically fixated on a logical program, and the stability of the metaphysical picture of the *Discourse* was deliberately overturned by the author himself. No doubt the roots of this problem were already prefigured in Leibniz's dedication of substantial sections of the *Discourse* to his rearticulation of the "new science" against a Cartesian vision of the relation between metaphysics and physics. Nonetheless, we encounter here a two-fold structural problem. The first problem that faces our reading of Leibniz in this transformation is that we find Leibniz attempting to provide an account of similar metaphysical principles between the *Discourse* and the *Monadology* with radically incongruent counterparts. While maintaining similar metaphysical principles, Leibniz reinvented the content of these principles, developing between the *Discourse* and the *Monadology* different components that populate metaphysical realities. This constituted a change in model, the transformation away from a logical model of substance conceived through a grammatical subject and completed through a logical analysis, toward a model of constitution where absolutely simple and singular atoms or soul-like unities build up everything from the absolutely simple to infinitely complex animated bodies.

In the above I have, with the aid of Fichant's observations, pointed to a general structural reversal in Leibniz's transformation from the *Discourse* to the *Monadology*. The movement from a logical model to one of composition leads us to pose questions invoked by the labyrinth of the continuum with respect to bodies and motion. Given a correlation between the nature of bodies and motion and the labyrinth of the continuum, what then does this correlation imply for the structural transformation that we have thus far been tracking? At the start of this chapter, we visited some straightforward consequences that directly followed from holding the position of an actual infinity of the world. I held that these remarks, however preliminary, nonetheless implied an awareness of the mathematical conditions at work in his reflections on metaphysical reality. Against the backdrop of this general correlation, I will construct in the following a more concrete framework where Leibniz's structural considerations of metaphysical reality brought together the mathematics of a theory of body and motion with the reinvention of substance. The question no longer concerned the correlation between a general principle of mathematical infinity with a general metaphysical structure but situated the relation between mathematics and metaphysics with the problem at the very heart of Leibniz's later metaphysical vision: the reinvention of substance as a constitutive element of reality.

2. The problem of body

Leibniz concerned himself with the metaphysics of body quite early on, decades before his statements in the *Discourse*. We know from autobiographical accounts that the various Aristotelian, neo-Atomist and Cartesian studies on the nature of body and substance were part of his youthful philosophical preoccupations. To the English clergyman Thomas Burnett, he wrote that he was not yet fifteen years old when he would spend whole days meditating in the woods, trying to decide between Aristotle and Democritus.[204] Evidently, for many readers of the history of philosophy of the 17th century, Leibniz represents a figure who attempted a last-ditch effort to strike a balance between Aristotelianism and the new sciences. Read through this lifelong process of synthesis and reflection, Leibniz's many attempts to address the issue of body and substance constituted a multi-level approach, from the theological to the scientific and directly metaphysical paths. This eclectic approach has at times been judged with mockery. Russell famously remarks that Leibniz's tendency for a "middle position" made him fall into "not two but three stools".[205] Against these scatological appraisals, C. Mercer argues that one cannot understand Leibniz's trajectory and motivations without understanding the synthetic

[204] Leibniz, *Phil* III, 205. Cf. Fichant, "L'Invention Métaphysique", 21.
[205] Russell, 106-107.

methodology constrained by these vast conditions historically imposed on the thinker.[206] I cannot address all these different constraints here. I will instead attempt to use the observation of a structural reversal discussed in the previous section as a means to provide an interpretive thread which will land us in a more focused discussion of the structural influence of Leibniz's reflections on mathematics in his metaphysics. That is to say, I will argue that the decisive role played by Leibniz's rethinking of the nature of body produced a transformation from the *Discourse* to the *Monadology*, a transformation wherein a theoretical gap opened that Leibniz responded to by a mathematically charged model of metaphysical structure.

In order to show the precise shape of the gap that this mathematically charged model would have to reorganize, we will turn away from more general discussions about Leibniz's reflections on the nature of mathematics and metaphysics and instead focus on the problem of body as the main axis on which his transition toward his late metaphysics turned. Here I think we find a concrete point from which to understand mathematics as a condition for Leibniz's metaphysical project. As such, I will first turn toward the special place given to the discussion of bodies in the *Discourse*. Two things in particular stand out. The first is the hypothetical relation between bodies and their possible substantial form used throughout the *Discourse*. In the famous proposition nine of the *Discourse*, where Leibniz gave his basic formulation of singular substance through the use of the complete notion and its sequence of events and predicates, where he argued that things cannot differ only by number, the final draft excluded the line, "Also that if bodies are substances, it is not possible that their nature consists only in size, shape and

[206] Mercer writes, "Not only did Leibniz use major parts of the history of philosophy without citation or explanation, he thought that it was a good thing to combine ideas taken from the great philosophical systems. Because many of his contemporaries shared a similar conception of philosophical history, they could grasp what he meant when he used terms like 'World Soul' and 'seminal causes.' One of the main reasons that it is so difficult for the twentieth-century scholar to recognize the borrowed doctrines and transformed assumptions in Leibniz's writings is that he made such abundant use of the entire history of philosophy as it was understood in the seventeenth century." Christa Mercer, *Leibniz's Metaphysics: Its Origins and Development* (Cambridge, UK: Cambridge University Press, 2004), 13.

motion, but something else is needed."[207] Understood through the logically modeled complete notion, this early formulation of the Leibnizian principle of the identity of indiscernibles had the consequence that no "two" substances are identical and that their differences are to be marked by the different set of predicates that are subsumed under their complete notion. To this he added that there is a specific difference in seemingly identical substances, a difference that could be marked "as the geometers do with respect to their figures."[208] This specific difference then was connected to geometry as a paradigm of clarity. It is at least possible to understand in theory how Leibniz's use of truth analysis would be able to provide the grounds for such an account. Leibniz explicitly made the connection between the logical analysis of substances and Euclid's geometry in both "On contingency" and "The source of contingent truths". In turn, it is clear why the nature of body would be at least problematic since it remains a mystery how bodies are to be understood as substances if the latter are to be analyzed according to this logical model. A block of marble and a historical human actor are both bodies (or can be said to have bodies), but the analysis of their bodily nature would be subsumed under the larger or significant subject. For a block of marble, Michelangelo's *David* has a corporeal element that would not be its substance, and for a human subject, Julius Caesar is hardly analyzable according to his body. Here with respect to the deleted sentence cited above, Leibniz, referring to the principle of the identity of indiscernibles, argued that "if" bodies were substances, the Cartesian position according to which *res extensa* would be accounted in terms of size, shape and motion would not fit the bill. Descartes' notion of difference between extended things would only imply a difference *solo numero* or "in number only", since size, shape and motion reduce to a quantitative difference. This hypothetical or conditional mood returned in the later propositions. The key discussion in

[207] Leibniz, *PE*, 42n.
[208] Leibniz, *PE*, 42.

proposition thirty-four takes us from Leibniz's discussion of substances through a logical model, one that focused on the difference between substances, to a discussion of the immanent identity of substances, their internal coherence and unity.

> Assuming that bodies that make up an *unum per se*, as does man, are substances, that they have substantial forms, and that animals have souls, we must admit that these souls and these substantial forms cannot entirely perish, no more than atoms or the ultimate parts of matter can.[209]

Again, Leibniz made no commitment to bodies themselves being substances. Yet Leibniz's point in this proposition was quite different from his deleted line in proposition nine. In this context, he wished to highlight the indestructibility of moral souls by underlining the fact that "the immortality required in morality and religion does not consist merely in the perpetual subsistence common to all substances, for without the memory of what one has been, there would be nothing desirable about it."[210] Hence the essential condition for the moral feature of souls is their memory, an identity that persists beyond their eventual separation from embodied life. The invoking of the possible substantiality of body itself here produced a difficulty concerning the moral preconditions for understanding the nature of soul qua substance. That is, if these moral or mnemonic aspects do not exhaust the nature of substance, then Leibniz would be required to make a distinction between two levels of substances, a higher level of moral beings to be contrasted with a lower, corporeal one. At least in the context of this text cited above, however, the tone of the hypothetical substantiality of bodies then should be understood in the sense that "even if" bodies had substantial forms, and were hence imperishable, this did not make them equivalent to souls in the moral sense. We can thus underline the hypothetical distinction

[209] Leibniz, *PE*, 64.
[210] Leibniz, *PE*, 66.

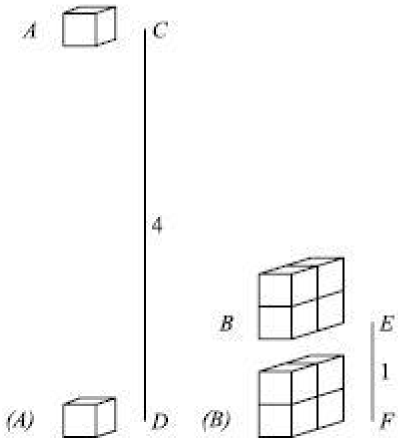
between substantiality tied to the moral and mnemonic senses of soul and another, corporeal, sort. Yet, without a theory of substance that can incorporate this distinction, this "other sort of substance" remains indeterminate. In addition we could note that in an earlier draft the same proposition began with the following sentence, "I do not attempt to determine if bodies are substances in metaphysical rigor or if they are only true phenomena like the rainbow and consequently, if there are true substances, souls, or substantial forms which are not intelligent."[211] It is sufficiently clear that despite Leibniz's uncertainty about the substantiality of bodies and the way in which this substantiality could be made consistent with to the moral dimension of souls, he nonetheless could not resist raising the idea in a hypothetical way. This attests not only to the experimental nature of Leibniz's response to the problem at this stage but also provides a clue to the sort of metaphysical problems, ones that Leibniz felt he still had to confront, that lie in the background of his remarks here.

A second interesting feature of the *Discourse* pertaining to the problem of bodies is the significant portion of the text dedicated to criticism of Descartes and the Cartesians. This problem centers on the understanding of bodies and motion. It is a criticism that Leibniz had long held against Cartesianism, that the account of bodies cannot be reduced to a mere mathematical or geometrical account of size, figure and motion. We have visited this issue in the last chapter beginning with his two part *Hypothesis Physica Nova* of 1671. In the *Theory of Abstract Motion*, we saw that in this earlier period, despite holding on to the uneasy consequences of an "extensionless part" for motion, Leibniz resisted a direct extensional account of motion for the very reason of the infinite divisibility of a given extension: the labyrinth of the continuum. There the attempt to provide a metaphysical foundation for the nature of corporeal motion in order to overcome a purely extensional account of motion relied on some dubious

[211] Leibniz, *PE*, 65n.

applications of geometric figures.[212] Even if Leibniz's account of motion and bodies took many turns during its path toward maturity, he saw a Cartesian extensional explanation as insufficient for most of his life. In the context of the *Discourse*, the most sustained critique of Descartes begins from proposition 17, in his argument against the Cartesian thesis on the conservation of the quantity of motion. His text here is a shorter presentation of the argument published in *Acta Eruditorum* in 1686, around the same time as the *Discourse*. This proposition centers on the refutation of the position that God (or natural physical systems) preserves the quantity of motion in the context of locomotion. In mechanical terms, what Leibniz rejected is that the (scalar) product of speed (scalar velocity) and the size of the body (mass), MV, is the quantity preserved in a motion. I will call this product, MV, Q. It is the quantity of motion which the Cartesian hypothesis asserts is preserved. This quantity Q is what Descartes maintained in his argument about the conservation of motion and what Leibniz disputed. What Leibniz rejected is that motion preserves Q=MV in the Cartesian interpretation. We should leave the Newtonian standardization of the terms aside for the moment. Leibniz reasoned that if we maintain that Q is the quantity preserved in a motion, then this same quantity would remain constant in the rising and falling of a body. Leibniz's argument was that, roughly, if we take two bodies A and B, one of unit mass (A) and the other of four units (B), then the quantity that is expressed in a motion for raising the first unit mass up four units of distance (Q=1x4=4) and of raising the second up one unit (Q=4x1=4) will equal [Q(A)=Q(B)]. This initial part of measuring Q is consistent with the Cartesian account.

[212] Cf. Leibniz, *LC*, 342.



[Figure 7][213]

If Descartes was right in equating the quantity conserved in a system with the conservation of the quantity of motion, then this should allow us to draw an equation, or conservation, between the quantities of motion expressed by the falling of each of these bodies from their respective heights to the ground. Leibniz remarked that Galileo had already demonstrated in his famous episode from the tower of Pisa that bodies of unequal masses fall at the same rate, or to put it more concisely, that the speed (V) of a falling object is independent of its mass but correlates to the time it takes to fall. In Leibniz's way of using this example, we know that given that the rate of a falling body (V) is proportional to the square of the duration of the fall (acceleration as dd/dt^2), the mass raised up four units will acquire twice the falling rate of the mass raised up one unit, regardless of the two masses. The respective products of mass and velocity in bodies A and B will thus be different.

This demonstration can appear confusing from the context of standard Newtonian mechanics, in the sense that we can very easily separate two kinds of conserved quantities in motion today. On the one hand, the designation of the product of mass and velocity can be seen as an earlier version of Newton's second law of motion. The product of mass and velocity (MV)

[213] Figure taken from Leibniz, Leibniz, *PE*, 50.

gives us the quantity of momentum (P=MV). On the other hand, $1/2MV^2$ is the quantity of the kinetic energy of the system. Now Leibniz's example here is in part a conservation of energy problem. A raised mass stores potential energy, and when it is dropped, it converts this potential energy into kinetic energy ($1/2MV^2$). In modern terms, bodies A and B in the example have the same potential energy at their respective heights - the product of height, mass and gravitational pull. Since energy is conserved, they will thus have the same kinetic energy when they fall. The fact that they have the same kinetic energy, however, does not in any sense mean that they have the same falling velocity. As such, the conservation of energy between potential and kinetic is not equivalent to the Cartesian idea of the conservation of the quantity of motion. At the same time, Leibniz's discussion of the difference of the velocities of the two falling bodies also resembles the problem of the conservation of momentum. The product of mass and velocity (P=MV) expresses the quantity of momentum but this would have been more appropriate in a different sort of demonstration such as the quantity of momentum conserved in the collision of bodies. As such, the fact that Leibniz distinguished between the two quantities in motion is correct. The energy stored in the raised masses A and B does not correspond to the quantity expressed by the product of mass and velocity. Although Leibniz did not have a precise way of following this criticism by supplying a correct alternative theory his criticism of Descartes is nonetheless correct.

Indeed, all that Leibniz really needed for his argument here is the idea that since gravitational acceleration works on the two bodies through different durations of fall, the velocities of the two bodies will be different at the end of their fall. In turn, the product of these velocities and their masses will hence be different. The consequence is that the presumed quantities of motion (MV) in their fall will be different from the quantity of motion in lifting up these two masses. The Cartesian proposal that quantity of motion is what is preserved is then not the case. Since Leibniz's argument here is not complete in its account of motion, we will stick merely with his understanding that there is a difference, that the quantity of scalar motion (MV) is not the quantity that is preserved in a motion. This difference means:

> [T]he force or proximate cause of these changes is something more real, and there is sufficient basis to attribute it to one body more than to another. Also, it is only in this way that we can know to which body the motion belongs. Now, this force is something different from size, shape and motion, and one can therefore judge that not everything conceived in body consists solely in extension and in its modifications, as our moderns have persuaded themselves. […] And it becomes more and more apparent that, although all the particular phenomena of nature can be explained mathematically or mechanically by those who understand them, nevertheless the general principles of corporeal nature and of mechanics itself are more metaphysical than geometrical, and belong to some indivisible forms or natures as the causes of appearances, rather than to corporeal mass or extension.[214]

The significance of Leibniz's rejection of the Cartesian position of the conservation of motion should not be taken at face value. No doubt Leibniz for many years held the position repeated in the *Discourse*, that the mechanistic explanation of bodies in motion is inadequate. This should however not be taken to mean an outright rejection of any geometrical explanation of motion. Here in the context of the *Discourse*, Leibniz took the chance to assert a middle ground between traditional or scholastic insistence on substantial form and the new mechanistic explanations. Leibniz underlined the need to

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[214] Leibniz, *PE*, 51-52.

understand force as something more real than what is given in the phenomenal nature, that is, more than what can be grasped through size, shape and magnitude. This still undeveloped concept took geometrical aspects of motion into account but also superseded them through metaphysical speculation. Leibniz opened up an experimental site where metaphysical, mathematical and physical ideas become entwined. The understanding of force at this point remained a provocation, an "invisible" form or nature of moving bodies that can express itself in terms of the measurable extensional aspects of motion but at the same time indicates something inherent and non-extensional about body. With this refutation of Descartes, Leibniz opened a situated context where the mixture of metaphysical reflections and geometrical understanding would come together to provide a foundation for understanding the conserved quantities of motion.

To this distribution of tasks between geometrical and metaphysical in the explanation of the structure and status of motion he added, "This is a reflection capable of reconciling the mechanical philosophy of the moderns with the caution of some intelligent and well-intentioned persons who fear, with some reason, that we are withdrawing too far from immaterial beings, to the disadvantage of piety."[215] What results from following Leibniz's redrawing of lines between the mechanistic account of the moderns and the traditional metaphysical domain of the immaterial anchoring of substances? Leibniz's rebalancing of the scales, all the while holding Descartes' dualism at bay, came with a price. The strict implication of any rejection of a mechanistic explanation of motion placed the status of body in a precarious position. As Leibniz maintained, corporeal motion does not have a nature that is adequately explained by geometry and in terms of extension. Between mechanistic explanations and traditional substantialist accounts, Leibniz seems to have had to provide a third option. The "something else" that the

[215] Leibniz, *PE*, 52.

notion of force pointed to landed on the soul as one of the means by which the counterintuitive or even outrightly oxymoronic notion of the "immaterial" nature of bodies was to be explained (a position that Leibniz ultimately abandoned). Before looking at how Leibniz eventually used the notion of force and this "something else" to pose the problem of corporeal substance in a different way, I wish to highlight the partial solutions given in the *Discourse* and the limitation of Leibniz's solution at this time. In the *Discourse*, Leibniz's retreat from the view of corporeal substance implied a necessity of having to face the sort of problems implied by a Cartesian framework. While Leibniz strongly disagreed with the Cartesian views on the nature of motion itself, his own view implied the need to reconcile mind and matter, soul and body. Leibniz answered:

> [W]e have said that everything that happens to the soul and to each substance follows from its notion, and therefore the very idea or essence of the soul carries with it the fact that all its appearances or perceptions must arise spontaneously from its own nature […] But they correspond more particularly and more perfectly to what happens in the body assigned to it, because the soul expresses the state of the universe in some way and for some time, according to the relation other bodies have to its own body. This also allows us to know how our body belongs to us, without, however, being attached to our essence.[216]

This statement of essential distinction between soul and body did leave a place for the relevance of body in metaphysics. Yet in this response to the Cartesian problem, Leibniz still declined a direct foregrounding of the nature of body and used the notion of a disembodied phenomenon to supplement his theory of substance. We see this clearly in proposition 34, where he began with a sentence that clarified what he meant by the preceding statements.

[216] Leibniz, *PE*, 64-65.

> Assuming that the bodies that make up an *unum per se*, as does man, are substances, that they have substantial forms, and that animals have souls, we must admit that these souls and these substantial forms cannot entirely perish, no more than atoms or the ultimate parts of matter can, on the view of other philosophers.[217]

By referring to "other philosophers", he draws our attention to a particular problem that is still unresolved in his attempt at synthesis. If Leibniz attempted to explain the nature of bodies by way of the disembodied phenomenal effects registered on souls, then what could be said about the nature of bodies themselves? In other words, if the metaphysical reality of bodies, regardless of their substantiality, is to be explained by reference to a metaphysical level of souls, what then is the status of the bodies themselves? This question is an open one since in the previous proposition he invoked a real separation between a body belonging to a soul and its "essential" being. As we noted in the final draft of this proposition 34, Leibniz eliminated the following sentence, "I do not attempt to determine if bodies are substances in metaphysical rigor or if they are only true phenomena like the rainbow and consequently, if there are true substances, souls, or substantial forms which are not intelligent."[218] At this point in the *Discourse*, the question of the body was an unsettled one for Leibniz. The question of whether a body can be a unity was at the heart of this confusion. Leibniz seems to have considered this possibility while resisting it. In the finished version of the *Discourse*, Leibniz identified the problem of the substantiality of body with its possible claim to unity, and this precisely became the very benchmark of substantiality itself in Leibniz's exchanges with A. Arnauld. Indeed, the considerations surrounding the substantiality of body became the very model for considering substantiality as such. But at this

[217] Leibniz, *PE*, 65.
[218] Leibniz, *PE*, 65n.

crucial moment of Leibniz's reflection, the nature of body still remained squarely a sort of "true phenomenon", granted reality only through its possession by and expression in the soul.

The status of body (and bodily motion) as phenomenal has received a wide set of readings. It is here that I think Fichant allows us to cast light on the stakes of this indecision concerning body in Leibniz's *Discourse*. Citing some influential readings of the ambiguity of the *Discourse*, notably Catherine Wilson's separation of Leibniz's metaphysics into three distinct systems that she deems only partially compatible, Fichant introduces his own position that reads the ambiguity of the *Discourse* retroactively by highlighting the central transformation that took place on the path toward the M.[219] The two readers share a similar problematic. For Wilson, Leibniz's metaphysical project at the time of the *Discourse* can be organized into three distinct levels. At the first register, Leibniz pushed for a theory of individual substance based on a logical model, from the global consistency of reality based on the relation between subjects and the predicates they contain. At a second level, there was a project for developing the concept of corporeal substances based on the notion of force and the science of dynamics. Thirdly, there was a project based on the concept of harmony, a conception based on the unfolding of a divine ordering within the consciousness or "I" of a subject.[220] Fichant remarks approvingly of Wilson's judgment, agreeing that while the first and the second of these partial systems can be at least compatible, the second and third cannot. Fichant explains, "For if bodies are coherent phenomena, there would no longer be the question of the sense in which they would be substances."[221] That is to say, if bodies and their motions are to be metaphysically reducible to the level of coherent phenomena, which would in turn receive a metaphysical explanation by

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[219] Fichant, "L'Invention Métaphysique", 74.
[220] Fichant, "L'Invention Métaphysique", 74. Cf. Catherine Wilson, *Leibniz's Metaphysics* (Manchester: Manchester University Press, 1989), 80.
[221] Fichant, "L'Invention Métaphysique", 74.

way of either a logical or providential model, then there is no longer the need to dwell in the problem of the metaphysical significance of the body as substance. We shall see below how this problematic reduction of bodies to phenomena, even "true" phenomena, is precisely where the trouble lies.

In a later text, Fichant sets up the stakes of this reduction very clearly. Referring to A. Robinet, he writes:

> André Robinet has also established the inarguable manner in which, read through all the strata of writing in their genetic states, the text of the *Discourse on Metaphysics* is traversed with a tension (a "disjunction") between two interpretations: on one hand, if bodies are substances, and since extension is not adequate, contrary to what Descartes held, to contribute to substances, we should then return to substantial forms rehabilitated by the notion of force in order to give an account of the persistence of the identity and reality of bodies; but on the other hand, the formula remains conditional and if bodies are not substances, then they are only true phenomena like a rainbow.[222]

We shall investigate the notion of force with more focus later. Before that, we should first underline the depth of confusion that mired Leibniz's earliest reception in this phenomenon-oriented reading of corporeal reality. Since the early reception of Leibniz took up merely one side of this "disjunction" that Robinet had subsequently formalized, Fichant argues that Leibniz's early reception was shot through with absurdities concerning the compatibility of bodies and substances. Without yet entering into the details, I should first signal that this position of phenomenalism about bodies is a problematic that was not necessarily resolved even for

[222] [Author's translation] Michel Fichant, "La Dernière Métaphysique de Leibniz et d'Idéalisme," in *Bulletin de la Société Française de Philosophie* 100, no. 3 (2006): 18.

Leibniz himself in the *Monadology*. In the *Monadology*, where everything that is composed is made up of simple monads, these very monads, whether they are our own human souls or non-human entelechies, do experience a succession of perceptions according to an internal principle that is created by the divine and then unfolded according to a divine providential plan.[223] In turn, our empirical access to bodies and their motions would seem to be a strictly phenomenal one. I will complicate this view in what follows, but while keeping this in mind, I wish to enter into the difficulty by beginning with a contemporary reference. Remarking on R.M. Adams, who has taken a strong phenomenalist reading of bodies in Leibniz, Fichant notes a fundamental difficulty. He cites Adams' interpretation of Leibniz's letter to De Volder in 1704. Adams remarks:

> The most fundamental principle of Leibniz's metaphysics is that 'there is nothing in things except simple substances, and in them perception and appetite'. It implies that bodies, which are not simple substances, can only be constructed out of simple substances and their properties of perception and appetition.[224]

Fichant draws our attention to two points. First, what Leibniz called the phenomenon given to perceivers is matter and movement, as Adams rightly notes, but not bodies. These two levels, the "properties" of motion and matter, and the nature of bodies, are not equivalent. Fichant explains that if these two levels were equivalent, "Leibniz would have considered matter and movement to be sufficient constituents of the nature of bodies, which is not the case."[225] Secondly, Fichant notes that Leibniz did not argue for the phenomenal reduction of the properties of matter and movement as following from the consequences of establishing simple substances. It was rather a

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[223] Leibniz, *PE*, 212.
[224] [Author's translation] Fichant, "La Dernière Métaphysique de Leibniz et d'Idéalisme", 12. Cf. Robert M. Adams, *Leibniz: Determinist, Theist, Idealist* (New York and Oxford: Oxford University Press, 1994), 217-218. Cf. Leibniz, *Phil* II, 270.
[225] [Author's translation] Fichant, "La Dernière Métaphysique de Leibniz et d'Idéalisme", 13.

complementary or independent proposition, one which would require separate justification. This difficulty that Fichant sees in Adams, he notes, can be attributed to the history of Leibniz's reception. Here Fichant raises a series of problems which followed from Christian Wolff's attempt to make sense of the differences between the various schools of philosophical positions in his 1734 *Psychologia Rationalis*, distinguishing monists from dualists, idealists from materialists, and the like. The key point that Fichant wants to make can be brought to Wolff's translation of Leibniz's *Monadology* from French to Latin in 1721. Fichant explains:

> It is true that Wolff… was not bothered by the forcing of the text in the direction of his own interpretation of the physics of the simple and composed, in consistently translating "*les composés*" by "*substantiae compositae*", in the place of what he should have designated as '*composita*'.[226]

In Wolff's translation, the *Monadology* would receive a very different meaning. That is, instead of a composite that reduces to a simple and thus an ultimate monadic level, both simples and composites shared an equivocal substantial quality. According to Wolff's translation, both metaphysical simples and at least some composites were considered substantial. Fichant judges that this was not entirely the fault of Wolff. In texts preceding the *Monadology*, like the *Principles of Nature and Grace*, Leibniz did indeed employ the term *composite substances* when saying "A composite substance is a collection of simple substances, or monads."[227] Here, even though followed by a sentence explaining that there are no composites without simples, Leibniz seems to have given an equivocal status of "substance" to both simple and composites. But here Fichant points us to a further distinction that will allow us to make sense of this confusion. Fichant notes:

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[226] [Author's translation] Fichant, "La Dernière Métaphysique de Leibniz et d'Idéalisme", 21.
[227] Leibniz, *PE*, 207.

> [I]n a letter of 1699 to Thomas Burnett, where he outlined a very precise approach of the reality of corporeal substance according to a monadological foundation, Leibniz employed a rigorous distinction, in body, 'between corporeal substance and matter', and he 'distinguishes primary matter from secondary'. With this distinction, he introduces the traditional notion that requires us to analyze the meaning that Leibniz gives to it: what is 'secondary matter'?[228]

This distinction concerned precisely the difference between mere aggregates and that which has an animate unity (hence an animate body), themes that we will look at more closely in the following. Yet without entering into this further discussion that will take us into the correspondence with Arnauld, we can at least clear up both issues with Wolff's translation and Fichant's argument against Adams' strong phenomenalism. In Wolff's case, the ambiguity or the equivocal nature of substance with respect to body pointed to a real distinction between body and primary matter that had yet to be fully recognized in this early reception. It is this equivocation, symptomatic in Wolff and the early reception of Leibniz that Fichant uses to highlight the problem with Adams' overly abrupt reading. Despite the real existence of a phenomenal register of matter and motion, body remained something inadequately circumscribable in terms of perception and appetition. For Fichant at least, the stakes are then clear: the metaphysical explanation that is left without answer in the *Discourse* became the central problematic to be addressed in his exchanges with Arnauld in the immediately following years. Following Fichant's critical remarks on Adams, it is clear that if Leibniz wanted to radically reduce the reality of bodies to phenomena, he would not have bothered with the distinction between primary and secondary matter. More importantly, Leibniz would not have placed so much focus on the relation between simples and aggregates in the *Monadology* if substance were to have been, as in

[228] [Author's translation] Fichant, "La Dernière Métaphysique de Leibniz et d'Idéalisme", 24.

Wolff's understanding, equivocally attributed to the two levels. How then did Leibniz finally take on a commitment to a notion of corporeal substance consistent with the notion of simplicity? How did this reconsideration then transform Leibniz's understanding of the connection between mathematics and metaphysics? We turn to Leibniz's correspondences with Arnauld for some clarifications.

3. The correspondence with Arnauld and the new models of substance

Following Fichant's suggestion, let us maintain that the question left open in the *Discourse* is one concerning the ambiguity surrounding two incongruent (and perhaps also incompatible) models of substance: one following the logicist model of the subject and predicate and the other one following unity as pure simples. The reading of a structural reversal suggests that there was an earlier commitment to a logical model centered on the subject-predicate distinction wherein the grammatical subject was imbued with soul-like qualities, and predicates unfolded in the analysis as events or perceptions. The second model, already present in the *Discourse* but remaining a subordinate one, was constructed after the relation between the one and the multiple where an "*ens per se*" served as the expression of substance and where its aggregation was the ground of multiplicity. In the course of the correspondence with Arnauld, the former model slowly ceded priority to the latter. That is, the individual substance, based on the subject-predicate relation, was displaced by a model of unity and aggregation, the path toward a monadological substance. In turn, the ambiguity posed above regarding the equivocity of substance ascribed to primary and secondary levels of substance, that is, to substance at the level of both simples and aggregates, was revised, and a turn toward a more systematic conception of the relation between bodies, their phenomena and substance would, through a space of contestation and reinvention, find a more coherent articulation.

Leibniz's correspondences with Arnauld were an immediate and natural outcome of the *Discourse*.[229] Before the period of serious correspondence, Leibniz had sent Arnauld a summary of the *Discourse* containing the thesis statement at the heading of each of the propositions without the elaboration that followed. Passing this text through a common contact and authority, the Landgrave Ernst von Hesse-Rheinfels, Arnauld did not initially respond directly back to Leibniz, passing instead through the aristocrat. The debate quickly took off from there, with Arnauld's keen comprehension that everything hinged on Leibniz's logical understanding of the individual notion and subject-soul conception. Focusing on this central theoretical proposal in the *Discourse*, Arnauld set his sights on proposition 13.

Before entering into the key transformations that occurred during the course of this series of correspondences, I hope to provide some context by showing the progression to these later debates starting from the dispute over proposition 13, a framework that will shed light on how Leibniz's turn toward corporeal substances responded to ambiguities within his earlier metaphysical arguments.

Proposition 13 reads:

> Since the Individual notion of each person includes once and for all everything that will ever happen to him, one sees in it the a priori proofs of the truth of each event, or, why one happened rather than another. But these truths, however certain, are nonetheless contingent, being based on the free will of God or of his

[229] Leory E. Loemker, "A note on the origin and problem of Leibniz's Discourse of 1686," in *Journal of the History of Ideas* 8, no. 4 (Oct, 1947): 449.

> creatures, whose choice always has its reasons, which incline without necessitating.[230]

Arnauld's initial responses, first to the Landgrave and then finally in the letter of 13 May 1686 to Leibniz himself, reflected his difficulty in sorting out Leibniz's counterintuitive theory of contingency and its implications for the free will of God and creatures. Arnauld's sharp mind (at age 74) is evident in his highlighting this proposition out of the thirty-six others as both the most problematic as well as the one heaviest with implication. For Arnauld, the theory of the individual notion seemed to collapse the distinction between the necessary and contingent, where the latter would seem to be pre-determined in such a way as to absorb them both into the creation of an individual. This would then imply a collapse of distinction between necessary truths, which would be dependent on God's allegedly "free" creation, and contingent truths, which would be "once and for all" determined since creation.[231] This collapse cut both ways. It cut against the nature of necessity, which is independent of God's free decrees, as well as against the contingency of the free acts of created beings which should be outside of this a priori configuration. To this challenge, Leibniz responded by pinpointing what he considered to be Arnauld's major confusion: the confounding of absolute and hypothetical necessity, *necessitatem ex hypothesi* with *nécessité absolue*.[232] Here Leibniz delivered a traditional defense of his theory of the individual notion. Hypothetical necessity, the unfolding of events from an individual's created notion was necessary only in a secondary sense of the word. That is, it was necessary with regard to the consequences of what was created but contingent in that these entities could also not have been created. They were hence contingent in the absolute sense but necessary in a

[230] Leibniz, *PE*, 44.
[231] G.W. Leibniz, *Discours de Métaphysique et Correspondence avec Arnauld*, ed. Georges Le Roy (Paris: Librairie J. Vrin), 95-96.
[232] Leibniz, *Correspondence avec Arnauld*, 95.

"hypothetical" sense.[233] A contingent event could have been otherwise in an absolute sense but not in view of the active force or decree (in some past event of creation) of God. Arnauld replied to this that he had not conflated the two. For, the absolute necessity of certain truths notwithstanding, it was the ambiguous nature of these hypothetical necessities that was at the heart of his difficulty. It seems that Arnauld underlined two problems that would swiftly force Leibniz to provide deeper clarifications. Arnauld argued that, from the perspective of hypothetical necessity, God knows that when he creates an individual like Adam, he is simultaneously choosing all the events that would unfold from this choice, not only in the very individual "Adam" but also for the world that he is a part of and acts in. Hence had Adam not existed, all the other figures and events that followed from this individual would have happened quite differently (the entire set of Adam's descendants, for example). In turn, the allegedly "free" creation of Adam and the "necessary" events that followed from this creation can be taken on the same meaning. That is, if the creation of Adam was free, then the events that followed from this creation had the same status and if the events that followed from this creation were necessary, then the creation of this Adam would be also.[234] With this, Arnauld highlighted the intrinsic connection between "a" creation and its posterior consequences, pointing once again to the difficulty of distinguishing between necessity and contingency by the intrinsic connection between them. Secondly, Arnauld raised the question of the individual, the "*moi*", "I" or "me" that is the "subjective" and singular experience of the world. A "visible contradiction", Arnauld wrote, results from considering that as many "me's" exist as possible. "The reason is that these different me's [*divers moi*] would be different from one another, or rather, they would not be many me's [*plusieurs moi*]. It would be necessary then that there be some of these many me's

[233] Leibniz, *Correspondence avec Arnauld*, 106-107.
[234] Leibniz, *Correspondence avec Arnauld*, 96.

who are not me: a visible contradiction."[235] Arnauld drew out a contradiction meant to show what contemporaries might call "trans-world identity". Yet Arnauld pointed to a deeper problem with this. With Descartes' cogito in the background, this reflexive self-experience of the "I" in the "I think" should be epistemologically detached from a God's-eye view. The individual experience of oneself is not adequately explained by the unfolding of a notion's internal contents whether or not created by God's free or constrained will. Here Arnauld underlined two important points that Leibniz would develop more deeply and that would constrain the arguments in the following correspondences. Both of them dealt with the structural ambiguities that followed as a consequence from his theory of the individual: in the first place, the nuancing of the global structure between necessity and contingency, and in the other, the problem of individual experience of the "I" that is "me".

Leibniz answered these objections in a piecemeal fashion. For the first problem, Leibniz returned to the insistence on the separation between necessary truths, the free will of God and the hypothetical necessities through which the world unfolds after God's creation. Having before himself a range of possibilities, God chooses the best order and it is this best, articulated by Leibniz here as the "principle of general order", that enters into the creation of Adam as one part of the intrinsic relations within the created world. God's choice is a choice of one universe among others. The creation of Adam the individual is part of this choice of an interlaced worldly order. For Leibniz this spelled out the distinction between God's free will and the intrinsic and global connection of the events enclosed in it.[236] This also spelled out the rejection of the Cartesian idea of the dependence of necessary truths on God's will. There was a real distinction between necessary truths and God's freedom since the former do not logically or actually depend

[235] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 97.
[236] Leibniz, *Correspondence avec Arnauld*, 116-117.

on the latter. In this, Leibniz highlighted the global sense of the concept of individual substances. "Each individual substance envelops the whole universe of which it is a part according to a certain relationship, through the connection which it has to all things by virtue of the coherence of the decisions or purposes of God."[237] For the second problem, Leibniz responded by highlighting the necessary continuity within a "me" that subtends all the events that would happen. Whereas Arnauld posed the problem of the identity of many "possible" me's, Leibniz responded by focusing on the identity of a substance as a problem of intra-world identity. Leibniz argued that to give coherence to the identity of any "me" experience, such as the one central to Descartes' meditations, a further step is needed; the "me" right now and the "me" that will take a trip at a later date must be tied to the same continuous substance. It is this continuity that gives us an actual individual substance, and all other comparisons should be excluded. Leibniz added to this by saying that he did not hold any world to exist other than the one actually created by God. However this does not forbid us from distinguishing possible events and actual ones. Leading up to something of a climactic thesis, Leibniz argued that the decisive formulation of these metaphysical statements can be tied to the principle *praedicatum inest subjecto*, a formulation that allows us to access the general order of the global structure of God's creation, the intrinsic connection in creation as well as the identity of the "subject" within each world which would explain even our subjective experience of the contingent nature of events and perceptions. It is with this rounding out of the argument that Leibniz posed what would become the central theme of their future correspondences. That is, after having underlined the central thesis of his response with his discussion on the theory of the individual substance, Leibniz entered into the explication of what he considered to be one of the main implications of this

[237] Leibniz, *PPL*, 333-334. Cf. Leibniz, *Correspondence avec Arnauld*, 117.

thesis, aimed no doubt at Arnauld's critical Cartesianism and the famous conflicts with Malebranche, of the relation between soul and body. Leibniz wrote:

> Hence, each individual substance or complete being is like a world apart, independent of every other thing than God. There is nothing stronger for demonstrating not only the indestructibility of our soul, but even if it holds always in its nature the traces of all its previous states with a virtual memory independent from body that can always be activated, since it has consciousness or knows in its self what everyone calls "me". […] But this independence does not forbid the commerce of substances with each other. For since all created substances are a continual production of the same sovereign being according to his own designs, and expresses the same universe and the same phenomena, they are exactly in accord with each other, and this makes us say that one acts on the other, since one expresses more distinctly than the other the cause or reason of change… […] It is thus that we should understand, in my view, the commerce of created substances with each other, and not by influence or real physical dependence, that we can never distinctly conceive.[238]

Here Leibniz stated a very clear general claim about the implication of the theory of the individual notion for an account of causality. Causal relations are global and structural in nature and cannot be put into terms of particular relations between substances. This theory of causality of course would allow Leibniz to reject occasionalism, the discrete and punctual intervention of God at every moment of causal interaction, explaining that "this introduces a sort of continual miracle, as if God at every moment changes the law of bodies, at the occasion of the thoughts of

[238] Leibniz, *Correspondence avec Arnauld*, 122.

mind."[239] Leibniz's presentation here, however, of the internal accord between substances, led to a general problem of the relation between soul and body, more specifically of the reality of bodies. Excluding occasionalism, causality is a structural principle. With this, Leibniz needed to account for the nature of causality as it applies to bodies. Can the internal experience of a substance account for everything implied here in the nature and movement of bodies just as it accounts for the unfolding of events in the life of a substance? Leibniz closed this letter of 4/14 July 1686 by posing a general question that remains unresolved.

> If the body is a substance and not a simple phenomenon like the rainbow, or a being united by accident or aggregation, like a pile of stones, it cannot consist in being extended, and we must necessarily conceive of something which we may call its substantial form and which corresponds in some way to soul. After having long held otherwise, I have finally been convinced of this, almost in spite of myself. […] We must always explain nature mathematically and mechanically, provided we keep in mind that the principles themselves, or the laws of mechanics or of force, do not depend on mere mathematical extension but on certain metaphysical reasons.[240]

I wish to highlight the nature of Leibniz's confession to Arnauld here, a confession "despite himself". Whereas the consideration of body as a substance was not something Leibniz was committed to in the *Discourse*, here in this correspondence he set up a genuine difficulty that was far from being resolved and on this occasion was moreover being put in an explicit connection with the implication of one of his central metaphysical theses in the *Discourse*. The letter finally closed with a summary of proposition 17 in the *Discourse*, the demonstration of the

[239] Leibniz, *Correspondence avec Arnauld*, 123.
[240] Leibniz, *PPL*, 338. Cf. Leibniz, *Correspondence avec Arnauld*, 123.

falling masses against the Cartesian idea of the conservation of quantity of motion. Leibniz made the suggestion that his turn toward corporeal substance held off any Cartesian shortcut by making body substance as *res extensa*. Here with this scientific hypothesis of a “something else” that is conserved in motion and a critique of Descartes, Leibniz introduced a place-holder for an alternative that posits, albeit only negatively, the possibility of a term that allows us some insight into the substantiality of matter. That is, by resisting at once a Cartesian, mechanistic and phenomenalist explanation, Leibniz set out on a course to construct a “something else”.

When Arnauld finally responded to Leibniz in the landmark letter of 28 September 1686, about three months after Leibniz’s long response, he seems to have given way to Leibniz’s insistence. “I was above all struck by this reason, that all affirmative true propositions, necessary or contingent, universal or singular, the notion of attribute is comprised in some ways in that of the subject: *praedicatum inest subjecto*.”[241] The reasons for Arnauld’s acquiescence are not at all clear, but it does allow us to see the correspondence as taking a different turn from this point on. That is, the discussion between the two thinkers stopped being centered around proposition 13 and turned to the supplementary explanations that Leibniz had given as implications, focusing on the relation between body and soul. Arnauld wondered if the nature of causality could really be explained as an internal and structural feature of substance itself. Is it not the body that causes the soul to feel pain at the point where the body is injured, and if not, how does this relation occur? In turn, if one wishes to initiate a bodily movement, something must account for this relation between will and movement. Arnauld’s further question took this general critique of the relation between soul and body deeper. If body or matter is not a simple phenomenon like a rainbow, then in what positive sense could this reality be grasped? Arnauld formulated these

[241] [Author’s translation] Leibniz, *Correspondence avec Arnauld*, 133.

objections in a very precise way. The substantiality of body would be either unextended and indivisible, or extended and divisible. Arnauld pointed out:

> If we say the first, it seems that it would be as indestructible as our soul. And if we say the latter, it seems that we do not gain anything in making body *unum per se*…. For it is in the divisibility of extension into infinite parts that we have trouble conceiving of its unity. Hence this substantial form does not at all help in this if it is just as divisible as extension itself.[242]

Arnauld pointed out the difficulty that the designation of body as substantial brings it either too close to the nature of the soul, making it simply collapse into soul-like qualities, or too close to the nature of extension and thus indeterminate. Arnauld's criticism was very keen on this point. It is the body's infinitely divisible nature that opens up the problem of its unity. If one holds to either of these cases, on the one hand the divisible nature of body, and on the other the unity of bodily substance, then the problem of a body's substantiality evaporates. Holding this question in mind, Arnauld wondered to which case a truly substantial body could be attributed. Is a piece of marble a body? Is the Earth? Is the Moon? "Milk is composed of serum, cream and that which coagulates. Are they three substantial forms or are they one?"[243]

The strength of Arnauld's questions cannot be underestimated. What started as a back and forth on Leibniz's defense of the metaphysical meaning of the *praedicatum inest subjecto* transformed into the development of a new idea. In a sense, Arnauld's explicit acquiescence to the original metaphysical principle allowed the two to sidestep their opening disputes and open up a deep reservoir of unfinished excavation. The question of the union between soul and body, a locus of contestation for Western philosophy since its Platonic inception, did not seem to trouble

[242] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 135.
[243] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 135.

Leibniz in the way that one would perhaps expect. Even as Arnauld posed the question, the answer seems to have already been in sight. It was Arnauld himself who posed the solution given by Augustine that a bodily pain is to be explained by a certain "sadness" of the soul. Leibniz seems to have been generally content with such a description as well as the general Cartesian solution of a concomitance between soul and body, even as he held the occasionalist position at bay. In any case, this problem seems to have caused Leibniz little worry. Keeping this in mind as if he was waiting for a more synthetic answer, Leibniz moved toward the question of the composition of bodies. We encounter here what Leibniz seems to have been more anxious about. He remarked, "The other difficulty concerning the substantial forms and the soul of bodies is much greater and I admit that I have not been satisfied at all."[244] The occasion for this dissatisfaction was also the occasion for expressing in the most explicit terms the fundamental problems that would occupy his metaphysical project in what followed. Leibniz began by repeating his position that bodies are not merely "true" phenomena like rainbows. What followed then was the insistence, marked by his argument against the Cartesian explanation of motion, on the idea that the substance of body is not to be found in extension and divisibility. In turn, the bodies that one comes in contact with are all composed: blocks of marble, a liter of milk, a heavenly body. He explained, "Thus we will never find a body for which we might say that it is truly a substance. It will always be an aggregation of many. Or rather, this would not be a real being, since the parts that compose them are subject to the same difficulty."[245] Here Leibniz reasoned in the sort of fashion that would echo in his later works. Whereas in the earlier letters the issue of body was treated through the general conception of substance as subject, now Leibniz turned to the language of aggregation and "true" unity by which substance was to be

[244] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 141.
[245] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 141.

understood. Priority was given to the search for a “real unity” through which bodies can be given reality. “It follows that the substance of body, if there is one, should be indivisible; what we call soul or form, but I am indifferent to this.”[246] Here still, Leibniz fell short of a satisfactory explanation, but the models by which he spoke of substances began to change. In what follows, Leibniz invoked two models for body, that of the machine and that of the microscopic world. These models or illustrations showed Leibniz’s turn toward considering the body itself, apart from the individual notion. In the first model, the body is taken apart from soul: what accounts for the type of aggregative unity that is given in a functional body? There is a sort of consistent unity taken apart from soul; a cadaver or body without life demonstrates a sort of functionality and aggregation that a machine exhibits. “I respond that, in my opinion, our body in itself, the soul held apart, or the cadaver, cannot be called a substance except by abuse, as a machine or the pile of stones, that are not but by aggregation; for the regular or irregular arrangement does not make a substantial unity.”[247] As for the second, Leibniz invoked Leeuwenhoeck’s development of the microscope and the discovery of small, animated organisms. This discovery, enacted through the endeavors of the new scientific spirit in which he viewed himself as being a cautious but supporting member, implied the animated nature of the smallest piece of organic nature. As such he argued, “Since all generation of an animal is nothing but a transformation of an already living animal, there is reason to think that death is nothing but another transformation.”[248] This implied the attribution of soul-like natures for all animated beings, many of which are invisible to the naked eye, existing at levels beneath (and perhaps also beyond) the “medium-sized” organisms that we normally come across. For Leibniz, the animated nature of these little organisms was the major implication for the defensibility of corporeal substance, an implication

[246] [Author’s translation] Leibniz, *Correspondence avec Arnauld*, 141.
[247] [Author’s translation] Leibniz, *Correspondence avec Arnauld*, 144.
[248] [Author’s translation] Leibniz, *Correspondence avec Arnauld*, 141

that he seems to have been ready to accept if he were to commit to the thesis. By obliquely putting these questions at the forefront, rather than the problem of the union of soul and body, Leibniz avoided the traps of the mind-body question raging in the philosophical community of his day. The experimentation with these models did not immediately set up a satisfactory solution. What is more important to note here is this experimentation itself. This subtle transformation was perhaps more explicit when, given all that remained undecided in his thought, he conclusively stated:

> But at least I can say that if there were no corporeal substances … it follows that bodies would be nothing but true phenomena like the rainbow. For the continuous is not only divisible to infinity, but every part of matter is actually divided into other parts just as different between them … we will never arrive at something where we might say: *voilà*, a real being .[249]

I should note that what lay in the background of this exclamation was the syncategorematic infinite: every part of nature is *actually* infinitely divided but we cannot ever arrive at a unity, a real being or *unum per se*. The turn toward an acceptance of the substantiality of bodies would have to take this problem of the actual infinite division of nature as a basic condition. Taken in the context of the correspondence, these statements amounted to a real transformation in his thinking-through of the consequences of the reality of corporeal form. That is, without making the error of affirming the consequences (that is, affirming the substantiality of bodies by affirming the various models he turned to), this way of making explicit the choices that he faced points to the centrality of this problem for Leibniz at this time. It is clear that though Arnauld did not force Leibniz to accept any particular solution, the former had been thoroughly convincing in persuading

[249] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 146.

Leibniz to at least accept the rigidity of the disjunction: either bodies are phenomenal like rainbows or they are substantial and face the problem of divisibility, courting the dangers of the labyrinth of the continuum.

Leibniz's invocation of the models of machine and microscopic animated bodies were attempts to frame this disjunction into a structure compatible with a larger theory of substance. Indeed, it seems clear that at this point of Leibniz's articulation, the idea of a substantial body did not conflict with the subject-substance modeled on the *praedicatum inest subjecto*. The only trouble was that this issue of the body was orthogonal to it. In an earlier letter, Leibniz dealt with the problem of substance from the point of view of the global cohesion of events starting from God's initial creation by trying to make it consistent with the internal experience of the singular "me" or "I". This problem returned here in a different way. If bodies are substances, then they must have a reality beyond that of phenomena and thus an external means by which to account for their reality. The emerging criteria of *unum per se* for substance would then replace that of the theory of the individual notion. In the same way, Leibniz would prioritize a non-individual frame of reference, holding the union between soul and body as a secondary level of the account and privileging the position of oneness as the point of entry. Similarly, the turn toward corporeal substance had to imply a global structure, and the two models of the machine and the microorganism indicated his inquiry into two different domains for articulating unity. In the first case, we have a consideration of body without soul-like nature and in the second case, we have the panpsychic consideration of nature full of different registers of hylomorphic substances.

In the following letter that Arnauld composed, he addressed Leibniz's many proposals by testing the assumption that there really is a corporeal substance. Outside of questions that Arnauld continued to pose concerning the relation between body and soul and the true nature of composed things, he devised a way to question the consequences of admitting corporeal substance according to the criterion of *unum per se*. This consequence was something that Leibniz would either have to accept or let the entire project go. Arnauld asked:

> But what would you conclude here? That there is nothing substantial in the body which is not simply the soul or substantial form? […] I call substance and the substantial that which has a true unity. But as this definition has not yet been received, there is no philosopher that has more right in saying: I call substance that which is not simply modality or a manner of being, and it follows that we could not sustain the paradox of saying that that there is nothing substantial in the block of marble since the block of marble is nothing but simply another substance's manner of being and that all that we could say is that it is not a single substance but many substances joined together in a machinelike way. […] But you put matter and its modification in the place of what is not soul or substantial form, indivisible, indestructible and ungenerated: and it is only with animals that you admit these sorts of [substantial] forms. You are then obliged to say that the rest of nature is simply imaginary or only apparent .[250]

Arnauld posed a challenge here. If one wanted to sustain a theory of corporeal substance, one could not simply enlarge the field of substance to some forms of organized bodies. Rather, it would have to apply to all reality and barring that, only to the human form, sustained by the form of the thinking subject, to which his previous theory seemed to be limited. This globalization of corporeal substances would have to be hinged on the problem of unity or true unity, a position that would have to find its way into accounting for the real aggregation of certain sorts of unities like a block of marble. That is, Arnauld indicated that Leibniz had to be saddled with the task of

[250] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 156.

articulating a sort of substantial univocity or universalization if he continued on the path of corporeal substantiality.

In Leibniz's response of 30 April 1687, a little more than a year since the correspondence first started, he showed the courage of biting this rather explosive bullet. In his clearest statement toward a thesis on corporeal substances yet, Leibniz remarked:

> It also seems that what makes the essence of a being by aggregation is nothing but a manner of being for that which it is composed of; for example that which makes the essence of an army is nothing but a manner of being for the men which compose it. This manner of being thus supposes a substance whose essence is not in turn a manner of being for another substance. All machines presuppose some substance in the pieces from which it is made and there is no multitude without true unities. To cut it short, I hold as an axiom this identical proposition that does not distinguish itself except by an accent: that which is not truly *a* being is not truly a *being*. [...] But the plural presupposes the singular, and where there is not [even] one being, it is even less [likely] that there would be many beings [*là où il n'y a pas un être, il y aura encore moins plusieurs êtres*].[251]

While the central focus of this passage is the "axiom" identifying unity with being or substantiality, Leibniz actually said much more in this remark. What Leibniz committed to was not corporeal substance per se, but a framework for drawing the implication of unity from the nature of multiplicity. Much like his eventual foundational remarks in the opening statement of the *Monadology*, an aggregation implied the unities from which it was aggregated. More than this, Leibniz placed this underlying unity as a part of the aggregate: the man in the army, the mechanical part in the machine. Leibniz's formulation of this new starting point as an "axiom"

[251] [Author's translation] Leibniz, *Correspondence avec Arnauld*, 165.

was very apt in many regards, as if drawing up something that might be equivalent to his earlier logical model but would not be deduced from it. Yet it is also a position that was at this point in Leibniz's reflections rather empty and without any real content. He was, however, quite insistent on using this starting point to respond systematically to Arnauld's challenge. The admission of corporeal substances did not simply supplement a preexisting theory of substance. Substance itself had to undergo a different structural reconfiguration.

4. From well-founded phenomena to well-founded reality: Body and life

With this "axiom" of the identity between unity and being, Leibniz responded to Arnauld's earlier comments in three distinct ways. In the first place, the problem of aggregation and unity in body would find a direction in the orientation charted out by this new axiom. At this point, Leibniz had already rejected the reduction of body to mere extension. Here the concept of true unity would be the guiding thread for fleshing out how a corporeal substance could exist. From this, Leibniz hammered down the criteria with which to brush aside any account of body that is not founded on unity. While he still held open the possibility that bodies might be nothing but true phenomena, he could nonetheless put the status of aggregate bodies to rest. From this configuration, the sort of reasoning that would eventually constitute the first two lines of the *Monadology* became apparent. "[W]here there are only beings by aggregation, there aren't any real beings."[252] Secondly, this position would in turn give a distinctive philosophical meaning to his argument against the Cartesian reduction of body to extension.

> On my view, the notion of singular substance involves consequences incompatible with a being by aggregation. I conceive properties in substance that

[252] Leibniz, *PE*, 85.

> cannot be explained by extension, shape and motion, besides the fact that there is no exact and fixed shape in bodies due to the actual subdivision of the continuum to infinity, and the fact that motion involves something imaginary insofar as it is only a modification of extension and change of location […] I confess that we can explain the particularities of nature mechanically, but that can happen only after we recognize or presuppose the very principles of mechanics, principles which can only be established a priori by metaphysical reasonings. And even the difficulties concerning the composition of the continuum will never be resolved as long as extension is considered as constituting the substance of the bodies, and as long as we entangle ourselves in our own chimeras.[253]

Just as the rejection of the Cartesian position of the conservation of motion pointed, in Leibniz's earlier statement, toward a metaphysical level, the metaphysical position of the equation between unity and reality now pointed back toward the problem of body and motion considered mechanically. It turned out that the reason why the problem of the continuum could not be resolved came from an inadequate reckoning of the principles regarding unity in body. The imaginary character of extension and location has us "tangled" up in the phenomenal dimension of corporeal reality, and here Leibniz attempted, with this unity-reality equation at hand, to untangle us. Leibniz wrote:

> Everything is strictly indefinite with respect to extension, and the extensions we attribute to bodies are merely phenomena and abstractions; this enables us to see how easily we fall into error when we do not reflect in this way […] [[…It is as great an error to conceive of extension as a primitive notion without conceiving the true notion of substance or action as it was to be content considering

[253] Leibniz, *PE*, 87.

> substantial forms as a whole without entering into the details of the modifications of extension.]][254]

Leibniz here turned toward a critical position and aimed to reorganize, on the one hand, the "proper" use of ideas with a geometrical apparatus to deal with the "particulars" of nature, and on the other hand, the foundational or metaphysical understanding of the phenomenal nature of this use. This account no doubt pointed to a still-to-be-understood substantial background for the meaning of motion, but nonetheless allows us the possibility of recognizing a real substantial nature of body tied to the formula or axiom "unity=reality".

In a third aspect, this turn toward unity made the proliferation of substance throughout the non-human an almost obvious consequence. Although Leibniz held some of the same ideas in his earlier work, he now definitively turned from privileging a soul-like nature to a criterion of unity for thinking about substance. He reasoned, "I also think that to want to limit true unity or substance almost exclusively to man is to be as shortsighted in metaphysics as were those in physics who wanted to confine the world in a sphere."[255] The existence of unities that composed and were implied by aggregation provided a global distribution of these unities. Wherever there were unities, there was substance. The consequence that these substances were soul-like was an implication rather than a starting point. With the image of infinite subdivision, Leibniz remarked:

> We find that there is a prodigious quantity of animals in a drop of water imbued with pepper, and with one blow millions of them can be killed… […] I confess that the body by itself, without the soul, has only a unity of aggregation, but that

[254] The double brackets indicate sections of correspondences that were drafted but not included in the correspondence that Arnauld received. Leibniz, *PE*, 87.
[255] Leibniz, *PE*, 87.

> the reality inhering in it derives from the parts composing it, which retain their [[substantial]] unity [[through the countless living bodies included in them.]][256]

Here Leibniz seems to have committed himself to the next major model that would complete or rival that of the logical one, a vitalist model based on microorganisms. This model would allow Leibniz to found a new metaphysical vision of the animated body (with the microorganism as a guide) that would lead him in the following correspondence to claim that it is not merely matter (body without substance) that is divisible into infinity as he had long claimed, but that bodies everywhere (with substance) are themselves divisible and infinite in number.

> For I believe rather that everything is filled with animated bodies. And in my opinion there are incomparably more souls than there are atoms according to Mr. Cordemoi [Cordemoy], who makes their number finite, while I hold the number of souls, or at least of forms, to be infinite. And since matter is infinitely divisible, no portion can be designated so small that it does not contain animated bodies, or at least bodies endowed with a primitive entelechy or, if you permit me to use the concept of life so generally, with a vital principle; in short, corporeal substances.[257]

Explicitly then, there are as many or more substantial unities as there are parts in the division of merely extensional matter. Infinite division neither exhausts matter nor takes us to a smallest animated body.

I pause a moment to return to my remarks on Fichant's interpretation above. For Fichant, Leibniz's turn from the individual toward unity left many traces. I have already mentioned the

[256] Again here, the double brackets are portions of Leibniz's draft that were not included in the letter that Arnauld received. Leibniz, *PE*, 88.
[257] Leibniz, *PPL*, 343. Cf. Leibniz, *Correspondence avec Arnauld*, 186.

obsolescence of the "proper name" for the later conception of substance, which in the face of infinite swarming microorganisms seems obviously insufficient. A clue more apropos to the content of the correspondence with Arnauld concerns the early question posed concerning the union of soul and body. Fichant argues, "It is no longer the question from this point on to correlate *a* body with *its* unique substantial form, but to climb up from the multitude that envelops each aggregate-body to the true unities that they suppose."[258] The entire question of union of soul and body would be sidestepped in favor of a search for a model of unity. As Fichant points out, through the results of his correspondence with Arnauld, there were four basic models for these unities on the table for Leibniz. There was unity conceived first through unextended points, secondly through extended but indivisible atoms, thirdly through the "me" or "I" of subjective experience and finally through the organic. As we might already guess, the first two solutions, taken literally or simplistically could be ruled out. Unities were not points for they could not compose anything, a trap of the continuum problem. Unities could not be atoms either. In the form posed by Leibniz's contemporaries, these indivisible atoms were contradictory notions (as we examined in the previous chapter). The third, the "me" or "I" of the perceiving and acting subject would not be entirely discounted of course since Leibniz still held that substances do express within themselves the entire universe, but it would become limited. Since Leibniz discounted the anthropocentric perspective of substance in his turn toward animated bodies, the experience of the "me" that was previously privileged could not be fully generalized for the smallest bit of matter containing the swarm of animated bodies. Only the fourth vitalist position focused on the model of the organism seemed to fit Leibniz's future trajectory. For Fichant, this turn from the individual substance to the singular and unity constituted the turn from a logical model to an organic, a turn grounded in the idea of microscopic organisms.

[258] [Author's translation] Fichant, "L'Invention Métaphysique", 91.

The task that Leibniz faced, half-way through his correspondences with Arnauld up to the writing of the *Monadology*, was a formidable one. With the turn toward the criterion of unity, Leibniz would be saddled with the task of reorganizing the features of his previous philosophical positions into one systematic exposition starting from the distinction between the simple and the complex. Fichant's interpretation of this path is that Leibniz allowed a vitalist model to guide him through this organization, a model that would allow Leibniz to reintroduce the features of his earlier accounts under a different mode. For Fichant, the structural turn from individual substance to unity did not by itself entail any definite metaphysical picture. Rather, it was by the turn toward a vitalist or organic model that the destitution of Leibniz's original metaphysical vision reorganized into his later metaphysical system. It is clear that Leibniz would have to fully explain what he meant by unity. Thus far, the idea of the division of aggregates which reveal smaller and smaller animated bodies does not suffice since, as we know, and certainly Leibniz was aware, the division of aggregates would never bring us to any unit unless we were to literally identify them with points or atoms which Leibniz would have no doubt refused. In other words, the idea of unity by itself is too simple to render any real explanation for the multitude of questions that a metaphysical structure must respond to. Hence, even while declaring to Arnauld the impossibility that corporeal substance could be thought of as many, he added:

> [A]ssuming that there is soul or entelechy in beasts or in other corporeal substances, we must reason in this matter just as we all reason in regard to man. Man is a being endowed with a true unity given him by his soul, in spite of the fact that the mass of his body is divided into organs, ducts, humors, and spirits

> and that these parts are undoubtedly filled with an infinity of other corporeal substances endowed with their own entelechies.[259]

A pre-systematic anticipation of the idea of the dominant monad was given here, that is, despite the universalization and proliferation of animated bodies down to the smallest part of matter, some temporary unities come together in the functional sense of an organism and can be considered rigorously even if it is ultimately divisible into parts or will become (decay into) those parts at death. The model of the organism then does not only operate through proliferation but also through organization. As Fichant understands it, the new life breathed into the significance of body and corporeal reality, no longer merely a "true phenomenon", would lead Leibniz to enact a convergence between his thoughts on force and the organic. The implications of Leibniz's earlier meditations on the conservation of force already tended toward the identification of a "deeper" metaphysical level of bodily activity that the Cartesian *res extensa* concept could not be adequate to. In turn, when Leibniz gave this concept of force a vitalist implication, Fichant remarks that Leibniz had finally found an expression of the "something else" that the Cartesian account of bodies and motion lacked. As Fichant notes, in the *Specimen Dynamicum*, Leibniz introduced precisely the sort of division that allowed him to draw out the features of body that would have major implications for metaphysics: the division between active and derivative force.

> Active force… is two fold, that is, either primitive, which is inherent in every corporeal substance per se…, or derivative, which, resulting from a limitation of primitive force through the collision of bodies with one another, for example, is

[259] Leibniz, *PPL*, 344. Cf. Leibniz, *Correspondence avec Arnauld*, 187.

> found in different degrees. Indeed, primitive force (which is nothing but the first entelechy) corresponds to the soul or the substantial form.[260]

Here substance was given a nature which the previous logical conception could not have, a full participation in the physical world. In turn, there was a second, passive force:

> Similarly, passive force is twofold, either primitive or derivative. And indeed, the primitive force of being acted upon…. Or of resisting constitutes that which is called primary matter in the schools, if correctly interpreted. These forces are that by virtue of which it happens that a body cannot be penetrated by another body, but presents an obstacle to it, and at the same time is endowed with a certain laziness, so to speak, that is, an opposition to motion.[261]

Although I believe there is much more to say about this division, Fichant interprets this distinction as a means to situate an organic model, one that would allow the organization internal to a substantial unity to be articulated by an "active force". This is something that Fichant interprets in Leibniz's letter to Fremont as a conception that allowed the unity of substance to be "something vital".[262] For Fichant, what results would then be a metaphysical hierarchy, a structure, where an organic model would find two articulations, two "functional sides" of the monadic conception.

Reading the letter to De Volder in 1703, Fichant finds Leibniz arguing:

> If we take the mass [*massa*] to be an aggregate containing many substances, you can, however, conceive in it one substance that is preeminent, if that mass makes up an organic body, animated by its primary entelechy. Furthermore, along with entelechy, I don't put anything into the monad or the complete simple substance,

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[260] Leibniz, *PE*, 119.
[261] Leibniz, *PE*, 120.
[262] Fichant, "L'Invention Métaphysique", 103. Cf. Leibniz, *Phil* IV, 483.

> but the primitive passive force, a force corresponding to [*relatus ad*] the whole mass [*massa*] of the organic body. The remaining subordinate monads placed in organs don't constitute part of the substance but yet they are immediately required for it, and they come together with the primary monad in a corporeal substance, that is, in an animal or plant. Therefore I distinguish: (1) the primitive entelechy or soul; (2) the matter, namely, the primary matter or primitive passive power; (3) the monad made up of these two things; (4) the mass [*massa*] or secondary matter, or the organic machine in which innumerable subordinate monads come together; and (5) the animal, that is, the corporeal substance, which the dominating monad in the machine makes one.[263]

Hence every level of vital or active force organized matter relatively, a hierarchy which did not bottom out into a primary matter unless all unity was removed from consideration. For Fichant, this completed the basic elements for a monadological "tableau":

> The joining of entelechy or primitive active force and primary matter or primitive passive force properly constitutes *the* monad. But the multitude of monads, greater than any number, designates their secondary matter as the organic machine which, under the domination of the preeminent monad, designates the animal. This double employment, plural and singular, of the notion of monad relative to the same body, might suggest a duality of signification, installing a new disequilibrium or a new ontological dichotomy. But Leibniz unites these two functional sides of the monad through a theory of what he calls the dominant or central monad: the monads of a body A that grounds the derived reality of the

[263] Under the third distinction here, we can interpret Leibniz's expression that a monad is "made up" of two things as the distinction between two aspects of the monad. Leibniz, *PE*, 177. Cf. Fichant, "L'Invention Métaphysique", 127.

> composed [body] are hierarchically subordinated to the dominant monad of A which makes unity [out] of composition.[264]

This internal organization of the new monadological reality then found a new equilibrium by a stratified relative ordering of unities. At a later point, Leibniz would argue in a similar way:

> [I]n reality where there is nothing but actually made divisions, the all is nothing but a result or assemblage like a troupe of sheep; it is true that the number of simple substances that enters in a mass, however small, is infinite, since other than the soul which makes a real unity of the animal, the body of a sheep (for example) is actually sub-divided. That is to say, there is also an assemblage of invisible animals and plants, composed of many others …. and even if this goes to infinity, it is manifest that at the end we [must] return to its unities , the remainders or the results being nothing but well-founded phenomenon.[265]

Leibniz's introduction of unity as the new criterion for the realization of substance threatened to undo the reality of substance by privileging the reality of the indivisible which one could never reach by subdivision. However, this new reorganization by means of force, or even vital force, situated unity within the metaphysical reality of this concept of force. In turn, the association of substantial unity with active force allows us to systematically conceive of degrees of the division of active and passive force as relative levels by which any animal can be conceived as a substance without reducing its reality to an infinitely divided one, taking us back to the labyrinth of the continuum. As Fichant explains:

> The monad thus ensures two modes of integration: as central or dominant monad it confers 'from on high' the unity of the aggregate of the composed and gives it

[264] [Author's translation] Fichant, "L'Invention Métaphysique", 128.
[265] [Author's translation] Leibniz, *Phil* IV, 492. Cf. Fichant, "L'Invention Métaphysique", 123.

> its corporeal reality in an organic body. At the same time, the aggregate itself does not draw its derived reality except from "below", the monads which are its constitutive unities. The idea of a hierarchy or the unifying monad implied in the aggregate enters here to resolve the tension between two functions, unifying and realizing, in the very same concept of the monad.[266]

For Fichant, this organizational structure drawn from the distinction between active and passive forces became the key to realizing an organic model in which the universalized distribution of animated bodies qua substance arrested the slide of the new metaphysical picture into its dissolution into "points". A second articulation of the organic model came to fill out the internal structure of the animated body by the concept of force which would fill in the necessary levels of scale with dominant monads exercising organizational "force" over its subordinate matter. For Fichant, the primacy of this organic model is capital. For him, a mathematical model could not fulfill this role in place of an organic model since the problem left open by Leibniz's turn to unity is precisely the problem of the vertigo of infinite division. The discussion of body in terms of point, extension and place had no doubt been productive in the development of a fuller concept of body, but remained the trap that Leibniz did not wish to fall into, the trap of the atomists on the one hand and the Cartesians on the other. In any case, infinite subdivision did provide a way to account for the relative unities that distinguished the hierarchical relations between intermediate "dominant" monads and the functional bodies that they reigned over. For Fichant, it is precisely here that the model of organic bodies fulfilled a philosophical and illustrative role that a mathematical model could not.

For Fichant, a number of other features of substance also became reorganized through the model of the organic. The most important of these was the reconstitution of subjective

[266] [Author's translation] Fichant, "L'Invention Métaphysique", 129.

experience. As Fichant explains, “Life, in the sense that Leibniz intends, consists in perception and appetite, it is the ‘perceptive principle’; a living being is ‘gifted with perception.’ ‘There is life and perception everywhere’, and thus since ‘there is no perception without organs’, ‘all is full of organic bodies’.”[267] The organic model thus allowed Leibniz to reintroduce a feature of perception in substances, an internal subjective experience which every organic unity, at every level of its subdivision into smaller animated bodies, would have in its possession. Yet this element of substance, which it shared with the description of substance in the *Discourse*, would be introduced under a different character, that is, no longer under the logical model of the subject encountering its events. Rather, as the *Monadology* would report:

> Just as the same city viewed from different directions appears differently and, as it were, multiplied perspectivally, in just the same way it happens that, because of the infinite multitude of simple substances, there are, as it were just as many different universes, which are, nevertheless, only perspectives on a single one, corresponding to the different points of view of each monad.[268]

The issue of the “me” that entered into the qualification of substance in his correspondences with Arnauld would be preserved in a totally different framework. It is not merely the human being or the “I” of a proper name that experiences. Everything, or at least every substance, experiences. An organ experiences, a microorganism experiences. But this was no longer the “I” of Cartesian philosophy, neither would it be that of the subject of the *Discourse*. All substances, great and small, insofar as they are living, perceive (to a greater or lesser degree). In this, Fichant quotes Leibniz in saying:

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[267] [Author’s translation] Fichant, “L’Invention Métaphysique”, 129.
[268] Leibniz, *PE*, 220.

> [I]f there are in nature other organic bodies than those of animals, as it appears, plants then seem to provide us with an example. These bodies would also have their simple substances or Monads which give them life, that is to say, their perception and appetite.[269]

Here Fichant pushes toward the full implication of Leibniz's vitalism. Indeed perception was not merely a feature of substance but rather the image of life itself, which in turn implied substance. As such, Fichant remarks:

> We understand also why Leibniz, here no longer associates a proper name to the designation of monads and never writes something like 'the monad of Caesar' (but he continues of course to speak on occasion of the 'soul of Caesar'): he thus preserves the universal character of monads as the ultimate elements of the world, and as such we can never speak of individual substances in the sense of speaking of them as the elements of worlds.[270]

With the structural transformation that occurred after the *Discourse*, the dependence of the position of the subject on a logical model would be loosened. Rather, subjectivity would be articulated through perception in a vitalist model. The proliferation of subjectivity in turn implied its anonymity from the point of view of metaphysics. This anonymity was in direct equilibrium with the disembodiedness of the subject in the *Discourse*. Whereas in the metaphysics of the *Discourse*, Arnauld raised the problem of the embodiment of a soul concerning the adequacy to the subject-predicate model, in the later work, the problem seems almost to have been the opposite. Since soul would become defined by perception and appetition, articulated as the active force of every infinitely small body, everywhere present, the embodied character of soul-like

[269] [Author's translation] Leibniz, GP VII, 535. Cf. Fichant, "L'Invention Métaphysique", 129-130.
[270] [Author's translation] Fichant, "L'Invention Métaphysique", 130.

substances returned to pose problems concerning the distinction of the soul and body, almost as if the soul were *too* embodied. In this, it was life itself that had to provide the fundamental metaphor for the immaterial nature of the active principle in bodies. With this, the language of atomism returned in a surprising way. In the double articulation of the vitalist model, not only would the internal organization of the hylomorphic organism articulate the relation between the dominant monad and the secondary matter that it organizes, but it would also develop the internal subjective life of the monad, a life force that is perceptive, appetitive and punctuated like a point, a partless index at the base of the universe. We read in the third proposition in the *Monadology*, “These monads are the true atoms of nature and in brief, the elements of things”.[271] No doubt, the strongest sense of this proposition concerned the definition of the monad as the constitutive element of metaphysical reality, but it also emphasized the elemental and immaterial nature of the active force that constitutes the global active force of all reality, the swarming clamor of life which reduces to partless and “metaphysical” atoms. Through the adoption of the organic model and the rejection of each of the three other models initially posed, Leibniz reintroduced under the heading of the “vital” something of an idiosyncratic atomism. Equally as metaphysical points (not equated with a geometric point) and as the “I” of the subject, the partless designated the singular being of substance.

[271] Leibniz, *PE*, 213.

CHAPTER 5

MOTION AND METAPHYSICAL STRUCTURE

1. Force and substance

In Fichant's vitalist reading of Leibniz's transition between what is known as the "middle" and "late" or "last" metaphysics, he produces something that rivals that of the early twentieth century reading by Russell and Couturat. Despite never having published a book-length treatment of this reading, in this long introductory essay that I have been amply referring to, "*L'invention métaphysique*", and in other shorter works, the systematic reach of his treatment should at least convince us that Leibniz did not remain fixed on a logical axis for developing his metaphysical thought. Rather, the suggestion of a strong vitalist model, much of which I have left uncommented and uncriticized, itself gives way to a mode of reading early modern metaphysical projects as a context where the emerging scientific models, many of which did not fully constitute themselves as strictly independent fields or "sciences" per se until the 18th and 19th centuries, came to influence the very means by which these thinkers attempted to clarify their problems and provide solutions. While a certain French reception of Leibniz, from Maine de Biran to Bergson and the Bergsonian tradition (in Merleau-Ponty, Simondon and Deleuze), followed a vitalist reading, none of these readings confront the logicist interpretation in such a contextual and straightforward fashion. Despite this, Fichant's research project nonetheless leaves many questions unanswered. Outside of Leeuwenhoek's microscope, what were the other concrete sources of this vitalist model? How did Leibniz's metaphysics influence later thinkers of the organic and the vital? Did the metaphysical aspects of Leibniz's notion of the organism dictate the path of later scientific inquiries? Any of these questions would require book-length treatment. Fichant's lesson for us in the present context however is the suggestion that to read Leibniz as a metaphysical inventor whose models of investigation borrowed from diverse fields affords us an insight into this process that the mere clarification of the thinker's positive solutions cannot. That is to say, the experimental model was a tool by which elements of a systematic structure could be composed without either reducing the resulting structure to the model or allowing the model to dictate what the final structure would look like. In the case of the organic or vitalist model, the richness of the microscopic organism not only provided an imaginative depth but also a concrete model of the minuteness of animated bodies from which Leibniz drew a new organization of unity and multiplicity, forces and bodies. It is this imaginative richness and concreteness that allowed Leibniz to recast previous problems in a different light. Reading a few centuries later, it is this same richness that allows us to grasp the depth of the problems that troubled Leibniz. In this same sense, however, my reading of Fichant's invocation of the organic or vitalist model is not a reductive one. By rereading the very same transformation that Fichant ties into his vitalist account, I believe that the "architechtonic disjunction" that Fichant highlights (drawn from Robinet's work) can be organized around a different axis. That is, during the period that followed Leibniz's letters to Arnauld in the 1690's, Leibniz attempted to reconstitute his earlier reflections on the continuum and bodily motion into a complete account of dynamics, an attempt that produced a series of works. A summary and preliminary publication of this larger project was published in the *Acta Eruditorium* in 1695, the *Specimen Dynamicum*. Some of the main positions of this text and its surrounding work are remarked on by Fichant in the explanation of his vitalist reading in the above. Yet against Fichant's exclusion of a strong mathematical dimension during this period of the reorganization of Leibniz's metaphysics, I will argue that the metaphysical construction site, opened up during this period of reflection and reinvention, cannot be completed without seeing how Leibniz's attempt to reconfigure his position of the structure of the continuum constituted a necessary dimension of his final metaphysical vision couched in terms of the monad.

As I noted in the previous chapter, Fichant treats the main picture of the *Specimen Dynamicum* in terms of the distinction between primitive, secondary, active and passive forces in his construction of the organic model. What he misses however is that these very distinctions were drawn from a mathematically informed model that allowed Leibniz to make such distinctions. In particular, the issue of force, which was central to Leibniz's move away from his previous logical model of substance, finds itself treated in a non-vitalist way in these texts of the 1690's. In this disagreement, I will remain in accord with Fichant that the adoption of singularity and simplicity as the central criteria would serve as the fundamental principle against which Leibniz would test his results. Yet to remain close to a reading that holds Leibniz's own problematics in the *Specimen Dynamicum* as a priority, I think that the first moment when the issue of singularity and aggregation met the problem of active and passive force, an important landmark that allows Fichant to demonstrate the richness of the organic model, was a problem internal to dynamics above all else. Here Leibniz would definitively leave aside his considerations of the logical model of substance in an explicit way and take up the contexts of propositions 17 and 18 of the *Discourse* as a starting point of rearticulation. That is, whereas Fichant sees the absence of the organic model in the *Discourse* and its presence in the later writings leading up to the M as a reason to use it as a guiding thread for understanding Leibniz's transformation in this period, I will use the very gap between substance and body in the correspondences with Arnauld to build up an alternative model that constitutes an extension to the vitalist one. Fichant himself relies on the importance of Leibniz's early rejection of both the atomist and Cartesian account of body and motion to motivate the adoption of corporeal substance during his correspondence with Arnauld. My intention is to show that the field which Leibniz would ultimately come to call dynamics, a field of investigation that Leibniz had invested so much in during his earlier work (from at least the Paris period of 1672-1676), a field that for Leibniz meant the rejection of a major part of Cartesian metaphysics and that deals specifically with the structure of bodies and motion, is not a detail to ignore. As we saw in the previous chapter, the rejection of the Cartesian position was two-fold. On the one hand, Leibniz rejected the idea that bodies can be accounted for in terms of geometrical features alone. On the other hand, he posited a "something else" that would account for the quantity conserved in a motion. These two aspects were a minute portion of what Leibniz only began to develop in his correspondence with Arnauld.

My motivations for developing a mathematical model for the later monadological metaphysics straightforwardly follow from the discussion in the previous chapter. While I do not deny the presence of a strong vitalist aspect of Leibniz's laboratory of metaphysical invention, Fichant did not however fully underline its important mathematical dimension. As we saw, the relation between unity and aggregation of body were, in Leibniz's thought, necessarily related to the problem of the mathematical account of body and motion. A vitalist coup de grâce, such as is developed by Fichant, is of course not rejected, but this hardly tells the full story. In addition, with the numerous texts that prefigure this late period of Leibniz's thought, the issue of force, an issue on which Leibniz wrote feverishly, made a determinate and coherent impact on the structure of metaphysical reality. The earlier account of causality, such as the one that appeared in the middle of his correspondences with Arnauld, remained strongly abstract and disembodied, articulated through a global structural interaction mediated by the divine. The later metaphysics would also place God as the mediating center of a global holistic relation of causality, but here he allowed a substantialized body to stand as a bearer of these interactions without sacrificing his strong anti-Cartesian and anti-atomist metaphysical intent. In turn, Leibniz would also enact a rethinking of the phenomenal, a restaging of the experience of motion and causal interaction in a more subtle way, one that more fully harmonized his faith in the mathematical explanation of motion with his substance-oriented metaphysics.

In order to begin building the mathematical model of Leibniz's transformation, I will return to an earlier point in this chapter concerning Leibniz's adoption of the reality of bodies. At what point did bodies become real for Leibniz? Put in the language of the correspondence with Arnauld, at what point did bodies, necessarily multiple, meet up with the criteria of unity and simplicity? Leibniz's earlier solutions to the problem of bodies and substance are at least theoretically coherent. Bodies and their motion are mostly phenomenal in the *Discourse*. However, as we argued in the previous chapter, Leibniz also strongly considered that there might be corporeal substances. Even if he held the hypothesis of a corporeal substantiality, Leibniz did not think that the success or failure of this hypothesis should worry scientists and geometers. Considered as phenomenal or imaginary, the scientific aspect of thinking about motion could be done through geometric and mathematical expressions. Indeed, Leibniz held that there is something metaphysical underlying this field, but geometers and physicists could carry on with their investigations accurately without worrying about this deeper reality. This separation between the "technical" aims of geometers and scientists, concerned with exactitude and rigor, and the more “fundamental” aims of philosophers is an old trope. This attitude toward bodies is one way of reading his argument against the Cartesians about the conservation of the quantity of motion. According to this reading, Descartes was wrong, but it was an error that could be sorted out on a purely scientific level. Hypothetically, Leibniz could have left the discussion at this point, debating with physicists and geometers on one level and developing his metaphysics on another. An imaginary level of the *experience* of bodily interaction could have nicely mediated between these two levels of reality. Given the availability of such a solution, Leibniz seemed to have taken an alternative route and provoked problems for himself without much prompting. As we have seen, the development of corporeal substance was at once instigated by the inadequacy of the scientific account of bodily motion but nonetheless remained something that could easily have been reabsorbed into a phenomenal level. Leibniz did not take this route. Faced with the disjunction posed by Arnauld, that either body remains phenomenal and accountable by relative and shifting registers of unity and composition or that its proper description would require a deep metaphysical account of unity, to a depth that no scientific account could be adequate, Leibniz took neither approach. The turn to unity as a metaphysical criterion constituted a metaphysical account of the undivided elements of reality, but here it would take on a corporeal sense, a register where mathematical physics would meet metaphysical description. This turn put Leibniz on the path of reconstituting the metaphysical concept of body from the starting point of its physical explanation.

Our discussion in the following of Leibniz’s theory of dynamics will focus on his 1695 *Specimen Dynamicum*. The text represents an attempt at recasting the problem of corporeal reality and shows how Leibniz overcame both sides of the famous disjunction between body as phenomenon and body as metaphysical reality. The phenomenal level of physical explanation did in fact meet with corporeal reality in Leibniz's mature work. Part of this argument has been made in the previous sections concerning the centrality of active and passive forces, a theory that was to be systematically treated in the *Specimen Dynamicum*. However we saw that Fichant's reading ties this conceptual distinction between active and passive to a vitalist model that sits at the heart of the M. I will not refute this position outright but will claim that it is incomplete. Far from vitalism as a fundamental model, we will find a metaphysics in the *Specimen Dynamicum* that sheds light on the nature of Leibniz's development of the concept of force (living force) that takes into explicit account the problem of infinitesimals, a problem that allows us to see a deeper continuity of problematics between Leibniz's earlier and late works, a continuity built not from the transformation of Leibniz's positive solutions but rather by unresolved problems.

Let us make our fundamental problem clear. The issue that arises from treating Leibniz's metaphysics from the point of view of the labyrinth of the continuum is that, from an early stage of his development, Leibniz rejected the idea that mathematical or geometrical descriptions could adequately capture the nature of body. In turn, when questioned on the nature of infinitesimals, Leibniz could always maintain the position that geometrical or mathematical truths pertained to a phenomenal or "imaginary" treatment of bodies, shapes and such. As we saw in previous chapters, when questions pertained to the status of "imaginary" entities such as infinitesimals, Leibniz used the terminology of the imaginary, especially in cases such as physical accounts, to correlate a rigorous relation and distinction between the immanent order of mathematical or geometrical figures and the order of actual things. The term *imaginary* was not synonymous with *fictional*. As I highlighted in the third chapter, the relation of imaginary and actual, with regard to the infinite/simal, differed by "degree".[272] Understanding this issue in the context of the *Specimen Dynamicum*, we see that on the one hand, this position was formulated

[272] Leibniz, *PE*, 186.

explicitly against the neo-atomists who read indivisibles directly into nature (as Leibniz once did but had since rejected). On the other hand, this position resisted the emerging influence of Cartesianism, a position that separated body from mind and held the adequacy of natural description in terms of extension alone, that is, with geometrical and mathematical terms alone. One might say that Leibniz held a "critical" position against the philosophies of his day, "critical" in the sense of the rectification of the "correct usage" of ideas. This "critical" position however, did not result in a merely negative position. It was not simply that the atomists were wrong, or that the Cartesian position was wrong, or that once they put the geometrical explanation "in its place," aware of the fact that it pertained only to a phenomenal level of description, that the metaphysical and the mathematical physicists could go their separate ways. Or, to articulate this negativity a little differently, it was not that Leibniz simply wanted to point out the inadequacy of the geometrical account of bodies in order to demonstrate a gap in scientific explanation in which a metaphysical explanation would have to intervene, an argument from ignorance. Far from this, Leibniz did not develop his theory of mechanics simply to put things in their proper place or to underline our ignorance of the physical world. It is possible to see Leibniz developing, independently from any vitalist considerations, a theory of bodies that had direct implications for his later metaphysics, a dynamical laboratory for metaphysics. In short, I will describe the difficulty as one concerning the problem of reduction. That is, if the science of corporeal reality could really be reduced to phenomenal laws, then there would be no reason to adopt the position of corporeal reality. In turn, it was in the very investigation of corporeal reality in terms of dynamics that Leibniz laid down one of the foundations for his monadological edifice.

As a way to refresh the stakes of this text, we can begin our look at the *Specimen Dynamicum* with a facet of Leibniz's "critical spirit". Here we can turn to look at how Leibniz was critical of his previous work. Halfway through the first part of the *Specimen Dynamicum*, Leibniz described his earlier mistakes in thinking about the nature of body and motion. Remarking on his "New Physical Hypothesis" (*Hypothesis Physica Nova*), he admitted that in this early text, he had not recognized the idea that a stationary body resists a moving body that strikes it. His earlier position was that impact consisted in the transferring of "*conatus*" of the moving body into the stationary body, causing the latter to move. The absurd result of this position was:

> I showed that it ought to follow that the *conatus* of a body entering into a collision, however small it might be would be impressed on the whole receiving body, however large it might be, and thus, that the largest body at rest would be carried off by a colliding body however small it might be, without retarding it at all, since such a notion of matter contains not resistance to motion but indifference.[273]

The main content of the *Specimen Dynamicum* spelled out the consequences of the turning away from this position of bodies and motion in a systematic way, with a comprehensive treatment to replace his earlier error. The main consequence of this revision of position was crystallized in the concept of force. Leibniz described his maturation from the earlier position:

> Therefore, I concluded from this that, because we cannot derive all truths concerning corporeal things from logical and geometrical axioms alone, that is, from large and small, whole and part, shape and position, and because we must admit something metaphysical, something perceptible by the mind alone over and

[273] Leibniz, *PE*, 124.

> above that which is purely mathematical and subject to the imagination, and we must add to material mass [massa] a certain superior and, so to speak, formal principle. Whether we call this principle form or entelechy or force does not matter, as long as we remember that it can only be explained through the notion of forces.[274]

What Leibniz said here seems to be a little different from what I would like to establish, but I think we should hold the meaning of the difference between the "metaphysical" and the "mathematical" in suspense for a moment. The distinction that he made here between mathematical and metaphysical seems to have been in reference to a dogmatic separation between a typically Cartesian idea of the adequacy of mathematical description of nature and the "something else" in body and motion that is at a superior level to this descriptive stratum. In any case, what Leibniz saw as his metaphysical insight did not directly invoke bodies. Instead, what it concerned was the "addition" of a concept of material mass [*massa*] into his consideration of force and corporeal interaction. What this "metaphysical perception" added is a notion of matter considered by itself (understood distinctly from body), a consideration that included the element of the resistance of mass beyond that of Cartesian extension. If we understand this "mental perception" of matter (distinct from geometrical reasoning) not as the epiphany of a completed concept of corporeal structure but rather the recognition of an additional element that had not yet been fully accounted for in his earlier formulation, then we can take the distinction between mathematics and metaphysics invoked in this passage in a different way. The introduction of matter reconfigured the mathematical account of motion rather than replacing it with a metaphysical one. As we shall see, there is nothing un-mathematical about the turn to force in his mature position on physics. Indeed, as he put it, the development of force involves a relation of

[274] Leibniz, *PE*, 125.

quantity, contra Descartes, the quantity preserved in a motion, and it is "…necessary that it follows from something else inherent in bodies, indeed from force itself, which always maintains its same quantity."[275] In addition, Leibniz noted that geometry was not a mode of investigation through which force could be discovered, but this does not mean that the discovery or its articulation did not involve mathematical reflection. Leibniz helped to clear these ambiguities up in the paragraphs that follow. First he argued against an "argument from ignorance" perspective, remarking that he resisted the injection of God's intervention in the place where mere mathematical description of motion and bodies would suffice. Further, he argued that a mechanistic explanation of bodies, though yet to be perfected, is adequate to the phenomena of bodies and their motions. As long as one keeps in mind the necessity of a metaphysical layer for the account of nature that is superior to a mathematical one, then problems should not arise. Leibniz went further to distinguish between two "kingdoms", one of physical interaction, the "kingdom of power" and the other of ends, the "kingdom of wisdom". Where the "laws of size or mathematics" would govern the former, the "laws of goodness or moral laws" would govern the latter. This distinction assured the independence of a mechanistic explanation from the direct intervention of divine or metaphysical realities.

Citing this separation between metaphysics and mathematics alone, however, cannot fully capture the complexity of Leibniz's systematic presentation of force. The key notion here lay in the remarkable role that the infinitesimal played in this text. As we have already argued, a new notion of substance would have to pass through the construction of the concept of bodies and motion, but such a task could not be completed without the construction of a mathematical scaffolding for the elements that would ultimately constitute what he understood as a science of mechanics which he called a *dynamics*. Keeping this in mind, we turn to his presentation of force

[275] Leibniz, *PE*, 125.

itself in the *Specimen Dynamicum*. Again, the *Specimen Dynamicum* was a presentation intended to introduce the basis of this new orientation rather than be a comprehensive account of it. This project continued until his death.

Having seen Leibniz's presentation of the motivation for his project in the *Specimen Dynamicum* as a combination of his anti-Cartesian position and his rejection of his earlier views on motion, we have something of a framework in which to situate the following examination of the elements of force. As we saw briefly in the last chapter and in the above, Leibniz divided force in the *Specimen Dynamicum* into two types, active and passive. Each of these in turn was subdivided into two types, primitive and derivative. We thus have a four-fold basic configuration of the dynamics. It would be a mistake to take the elements of this configuration as independently existing entities. Taken independently they are notions that allow the dissection of bodies in various states. The distinction between active and passive correlates basically to the distinction between agent and patient: an impacting body and a body being impacted. In each of these states, there is a primitive level of the body's "per se" state and a derivative level of a body's state when its force is limited by another body. To use a simple example, when one moving body A strikes another stationary body B, the active primitive force is the force of that body A in motion, a sort of *conatus* or striving, which is not to be equated with the Cartesian conservation of the quantity of motion. In turn, the passive primitive force is the force held in B before being struck by A. It is the force of a body in place or resistance. When A strikes B, the active derivative force in A is the way in which the force of A is limited by the body B's resistance. As patient of the action, the passive derivative force in B is the force of resisting the impact of body A. This passive derivative force limits its primitive but passive force from remaining in position and playing a role in determining the kind of motion that it will have after impact. In this four-fold configuration, Leibniz laid down the fundamental schema by which he would move away from his earlier position on motion and bodies. The new position would take the patient body, in our case the stationary body B, as having a resistant force that not only counteracts the primitive active body A but also contributes to the resulting motion that follows from this interaction. In the text of the *Specimen Dynamicum*, Leibniz immediately turned to identify the "derivative" character of bodies in collision with a notion of primary matter. He reasoned:

> [T]he primitive force of being acted upon [*vis primitiva patiendi*] or of resisting constitutes that which is called primary matter in the schools, if correctly interpreted. This force is that by virtue of which it happens that a body cannot be penetrated by another body, but presents an obstacle to it, and at the same time is endowed with a certain laziness, so to speak, that is, an opposition to motion, nor further, does it allow itself to be put into motion without somewhat diminishing the force of the body acting on it.[276]

The derivative forces are forces in collision. On the one hand the active body is limited by its striking of the passive body, resulting in an active derivative force, and on the other, the passive body both slows the active body down and is set into motion by being acted upon, a passive derivative force. Now this initial presentation in the *Specimen Dynamicum* is an entry into a more complex discussion of dynamics and should not be taken to have an absolute frame. In actual interactions, motions have a relative framework and hence the moving and stationary bodies in this example provide only a characterization of the basic relations between active and passive forces which become complicated when we consider two moving bodies in collision. In this initial example, the primitive characters of the active and passive bodies are their forces

[276] Leibniz, *PE*, 120.

considered “in themselves”. The primitive active force is the moving body considered per se, and the primitive passive body is the stationary body considered per se. The two together, insofar as primitive active force is “entelechy” or action, and primitive passive force is “matter” or resistance, compose the two sides of a corporeal substance that we discussed in the previous chapter as the outcome of Leibniz’s metaphysical laboratory in his correspondence with Arnauld and more generally during this period. This elementary distinction should be highlighted as the basic element upon which Leibniz would add another layer of interpretation, a distinction between living and dead force. The four-fold configuration above only set up the general description of the elements of forces, forces that are relative with respect to interacting bodies. In what Leibniz continued to explain, this four-fold led to a more important distinction, the difference between living and dead force. This difference followed from the initial four-fold elements and their interaction. Dead force is force without motion, living force is force in motion. Through the comprehensive relationship between force and motion, Leibniz would constitute the question that dynamics as a science sought to account for. We will first need to understand what force without motion is. That is, how can we conceive of force outside of motion?

Now to put the terms in order, Leibniz gave a basic lexicon that summarized his earlier conflicts with the Cartesian position on motion. Maintaining a fundamental notion from his earlier positions, motion is nothing but the change of position of a body in time. No doubt, this is a position that we have visited a number of times in earlier chapters. A *conatus*, in turn, the elementary character of an active primitive force, is a primary “striving” expressed as the momentaneous velocity with direction, which together constitute what we now refer to as a vector in Newtonian physics. Impetus, on the other hand, is the product of “bulk” and velocity.

Here we roughly translate "bulk" into mass, and impetus will be the product of mass and velocity (MV). Leibniz introduced a refinement of the terms through the concept of impetus. That is, strictly in terms of calculation, the motion of a mass across an extension is the summation of many little motions across the continuum. With the results of the infinitesimal calculus, the quantity of mass and velocity can be determined at a moment. Leibniz did not take the capacity to determine mass and velocity at a moment to mean that motion is actually composed of discrete "points" of motion since, as we argued earlier in our discussion of *Pacidius Philalethes*, this would engender the problem of the dissolution of motion into stationary positions, or the continuum into points. However, insofar as this quantity of motion can be determined at a moment, Leibniz would term this product of mass and velocity at a moment "*motio*". As Leibniz put it, "The quantity of a motion, which exists in time, of course, arises from the sum over time of the impetus (equal or unequal) existing in the mobile thing, multiplied by the corresponding times."[277] This was no doubt made possible by the calculation of instantaneous change at a point in the infinitesimal calculus. In turn, he characterized the quantity of motion across time and extension by saying:

> [T]he numerical value of a motion [*motus*] extending through time derives from an infinite number of impetuses, so, in turn, impetus itself (even though it is something momentary) arises from an infinite number of increments successively impressed on a given mobile thing.[278]

From a perspective of merely putting the terms in order, we have three distinct levels of mathematical terms involved in the account of motion. A motion across an extension through time is made up of the summation (or integration) of the instantaneous products of mass and

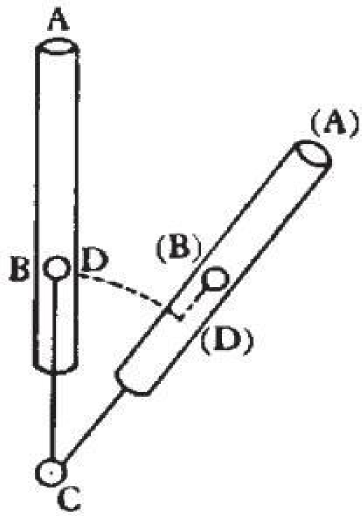
[277] Leibniz, *PE*, 120.
[278] Leibniz, *PE*, 121.

velocity, or impetus, and in turn, impetus is made up of the summation of infinite "increments" or inflections in the motion that determines its state at any point. There remains a final level here, that of living and dead force. We shall see how Leibniz cleared it up in the following.

Leibniz put these new distinctions into immediate use to deliver the third level. He gave the following landmark example:

> Consider tube AC rotating around the immobile center C on the horizontal plane of this page with a certain uniform speed, and consider ball B in the interior of the tube, just freed from a rope or some other hindrance, and beginning to move by virtue of centrifugal force. It is obvious that, in the beginning, the *conatus* for receding from the center, namely, that by virtue of which the ball B in the tube tends toward the end of tube, A, is infinitely small in comparison with the impetus which it already has from rotation, that is, it is infinitely small in comparison with the impetus by virtue of which the ball B, together with the tube itself, tends to go from place D to (D), while maintaining the same distance from the center. But if the centrifugal impression deriving from the rotation were continued for some time, then by virtue of that very circumstance, a certain complete centrifugal impetus (D)(B), comparable to the rotational impetus D (D), must arise in the ball. From this it is obvious that the *nisus* is two-fold, that is, elementary and infinitely small, which I also call solicitation, and that which is formed from the continuous or repetition of elementary *nisus*, that is, impetus itself.[279]

[279] Leibniz, *PE*, 121.



[Figure 8][280]

This case of centrifugal force allowed Leibniz to build a bridge between the account of motion and the nature of bodies. In the first place, the rotation of the tube results in a change in place which is the movement of the ball from position or place D to (B). This result is the combination of the rotation of the tube, equidistant from a center C, and the centrifugal force that brings the body further away from the center. This sum of motion however can be effectively divided into two distinct forces, the force of rotation and centrifugal force. The two together, centrifugal impetus and rotational impetus, constitute the actual motion of the ball away from the center. The difference is that centrifugal force acts on the body in an infinitesimal solicitation to motion, and the *conatus* here is infinitesimal relative to the impetus, which can be highlighted if we consider the ball remaining tied to the string at a fixed rotation from the center C. The solicitation does not actually move the ball away from this point of reference, the center C. Rotational force on the other hand is simply the motion away from a point in the circumference, the product of extension and time. To explain this distinction, Leibniz argued:

[280] Figure taken from Leibniz, *PE*, 121.

> One force is elementary, which I call dead force, since motion [*motus*] does not yet exist in it, but only a solicitation to motion [*motus*] as with a ball in the tube, or a stone in a sling while it is still held in by the rope. The other force is ordinary force, joined with actual motion, which I call living force.[281]

Dead force then, like centrifugal force or the force of gravity, is a *conatus* that does not necessarily produce motion, even if it gathers the solicitation for motion in the many moments (a duration) wherein that force is compounded. Living force, on the other hand, is force understood through the sum of its momentaneous *motio*. In turn, Leibniz explained that just as in centrifugal force, the case of gravitational force is one of dead force:

> [W]hen we are dealing with impact, which arises from a heavy body which has already been falling for some time…the force in question is living force, which arises from an infinity of continual impressions of dead force. And this is what Galileo meant when he said, speaking enigmatically, that the force of impact is infinite in comparison with the simple *nisus* of heaviness.[282]

Leibniz explained it quite clearly: The infinite compounding of force in a falling body from the solicitation of gravitational pull or gravitational acceleration is not identical with the force of gravitational pull in the body manifested as weight (the *nisus* of heaviness). This presentation no doubt refers us back to Leibniz's argument against the Cartesians. Having distinguished these different modes of force, we can look back at his refutation of Cartesian quantity of motion from a different perspective. A body of mass 1 and the body of mass 4 take an equal number of work (as we would call it today) to raise the former four units and the latter 1 unit. This work is due to the counteracting of the "dead force" exerted by gravitational pull. The motion of both of these

[281] Leibniz, *PE*, 121.
[282] Leibniz, *PE*, 122.

bodies when dropped will be different since the first body of mass 1 will fall 4 units which, due to the accelerating solicitation of gravity, to use Leibniz's vocabulary, will acquire a faster velocity since its duration of fall is longer than the second body that falls only 1 unit. Hence, since the solicitation of gravitational force is constant in both bodies but the duration of fall of the two bodies is different, the respective compounding of impetus will be different since the former falls for a longer duration than the latter. The respective velocities at the end of their fall are hence different. Again, some of Leibniz's terms here are convoluted in Newtonian terms, but Leibniz did make use of a distinction of two kinds of forces (dead and living) to correctly separate two kinds of quantities. On the one hand, the accumulation of the potential energy of a body at rest being lifted up against the constant dead force of gravity conserves energy that will be released at fall. On the other hand, a falling body accelerating at the rate of gravitational attraction will increase in velocity proportional to the time of its duration.

Having defined the importance of dead force, like the weight of a suspended mass and centrifugal force, and having seen a simple example of the difference between dead and living force as it relates to Descartes' account of motion, Leibniz turned to provide a more definitive understanding of living force, the force of bodies in motion. Leibniz distinguished living force from dead force by distinguishing between static bodies and moving bodies. He argued that in dead force or in static bodies:

> [W]e treat only the first *conatus* of bodies acting on one another, before those bodies have received impetus through acting. And although one might, in a certain way, be able to transpose the laws of dead force over into living force, great caution is needed... […] For example if different heavy bodies are falling, then at the very beginning of their motion, at least, the very descents or the very

> quantities of space traversed in descent, though, at that point, infinitely small or elementary, would be proportional to the speeds or to the *conatus* of descent. But once they have made some progress, and once living force has arisen, then the speeds acquired are no longer proportional to the spaces already traversed in descent… but are proportional only to the sum of their own elements.[283]

This comment should no doubt remind us of the problems of motion and the continuum treated in the earlier chapters. The sorts of arguments that Leibniz put forth in 1671 relied on couching the metaphysical foundations that he deemed necessary for the rational account of motion in the geometrical features of extension itself. In the earlier context, Leibniz tried to develop the concept of an extensionless magnitude from which the starting point of motion and its end would not become tangled with the problems of indivisibility and the continuum. The recasting of the problem in terms of force in the *Specimen Dynamicum* allowed Leibniz to treat the extensional problems of motion within another framework. The impetus of a falling body at the moment of fall correlates to the "start" of a motion but will be disentangled from extensional properties arising from a geometrical account. The downward attraction acting on a suspended mass, as Leibniz explained in this passage, will be transformed into the living force of the downward motion and this, in turn, will be transformed into an active derivative force that will be imparted on what it collides with at the end of its fall. In contemporary terms, we would say that the downward gravitational attraction acting on a mass, together with height and mass, determine its potential energy. As it begins to fall, it converts this potential energy into downward motion. Yet the duration of its fall will determine the velocity it has at the end of the mass's fall. In Leibniz's conception of this initial conversion of potential and kinetic energy, the difference between a suspended deadweight and the first moments of its fall is expressed in the conversion of dead

[283] Leibniz, *PE*, 122.

force into living force. At the initial stage of falling, the physical properties of the body remain in the infinitesimal neighborhood of the initial solicitations to motion. As the body continues to fall, this living force gains velocity through gravitational acceleration and expresses the dynamics of living force. Finally, as the body hits the ground or another body, the very same body is understood as imparting an active derivative force on a receiving body which resists it (a passive derivative force): the nexus of the two resulting in a situation of impact. Just so that we do not confuse our terms, in Newtonian physics the "impact" imparted at the end of the fall (or any other collision) is described in terms of the dual conservation of momentum, the product of mass and velocity (MV), and energy ($1/2mv^2$). Both quantities are conserved in an elastic collision. But without taking away from our discussion of Leibniz, we can see from his discussion that while he saw the difference between the two sorts of quantities, he did not provide a complete account of their conservation here. As such, in Leibnizian terms, motion will then be dissected into a series of intervening complexes of forces. In other words, forces will come to explain the properties of motion rather than vice versa. As such, the geometrical properties of moving bodies will be explained by the calculation of forces (living and dead) at work. We should note here that while Leibniz did substitute a geometric understanding of motion for the concept of force, in no sense would this mean the overlaying of a metaphysical plan from without. Rather the development of the concept of force was precisely given through an explicit mathematical meaning with heavy metaphysical repercussions. To give an account of these consequences, I will return to the question of how the *Specimen Dynamicum* provided a model for the emerging monadological vision.

The concept of force would allow Leibniz to situate non-moving bodies in order to rethink the concept of body. Two dimensions of this force without motion arise in the *Specimen*

Dynamicum. In the beginning sections where Leibniz introduced the four-fold structure of active, passive, primitive and derivative forces, he commented on the "crude notion" of corporeal substances developed by the Cartesians. A stationary body, in turn, has a primitive force of resistance that Leibniz thought of as an inherent character of bodies. Moreover, as we remarked in the above, he saw our access to this nature of resistance as a metaphysical insight rather than a geometrical or mathematical one. When a body is acted upon, this inherent resistant force will both limit the body that acts on it and participate in the resulting motion insofar as it characterizes the reception of the action. Passive force then acts by limiting without necessarily passing over into motion itself. A second dimension of this non-moving force is that of dead force, where a body is solicited to move without actually moving. Gravitational pull solicits a raised body to fall even if the body is not (or not yet) in motion. Equally, centrifugal force will solicit movement from a body circling around a fixed point. This dead force of the body demonstrates the way in which a body exhibits a degree of force whose qualities (mass) proportionally relate to constitute a measurable physical quantity without the extensional measure of motion. That is to say, passive force and dead force are "in" a body and not an issue of extensional, geometrical measure. While we have remarked that some of these problems may be more accurately and perspicuously described by the dual quantities (energy and momentum) conserved in Newtonian mechanics, for Leibniz these non-moving characterizations of body would allow him some insight in the search for an inherent nature of body that he saw as more than sufficient to substitute for the Cartesian account of corporeal reality. As such, he gave bodies a fundamental dimension of materiality, a resistance, which he correlated with the "primary matter" of Scholastic philosophy. This is what he termed *primitive passive force*. This primary matter however gives us no access to its reality without its expression in terms of force, a certain "primitive force of being acted upon", which gives bodies their barest substantial marker, that of resistance. Insofar as resistance is highlighted in the very context of an action on a body and its resulting resistance, two sides, active and passive constitute the primary context of a new substantial model. The active side of this primary context, which Leibniz did not hesitate to correspond to "soul or substantial form", is also what he associated with an account of entelechy.[284] The very transformation marked by Fichant in his account of the movement away from individual substance towards simple substances is given another positive expression here. Bracketing Fichant's organic or vitalist model, we find a different expression of the substantiality of bodies in terms of the concept of force. Here the question of simplicity and aggregation was synthesized by a concrete organizing principle realized in the fundamental elements of dynamics. Not only do we find that this characterization of the inherent quality of bodies differs from the Cartesian notion of extension and the indivisible in the Neo-atomist conception, Leibniz also laid this reorganization down as the missing account in previous attempts, like the *Discourse*, of a "something else" missing in the account of bodies and motion. Indeed the "something else" that was lacking in both the extensional accounts of motion as well as in his earlier theories (like the *Theory of Abstract Motion*) found a positive expression in the concept of force that reunites positive elements of corporeal substances (analyzable into relative parts) and a rectification of the account of motion. We can thus see this new concept as the solution, or at least a model that tends toward a solution, to our original problem. That is, couched in terms of his argument against Descartes, Leibniz did not simply want to reject the conservation of the quantity of motion. The dispute was not simply over the better mathematical formula. It was rather a dispute over the very relation between mathematics and metaphysics.

[284] Leibniz, *PE*, 119.

The development of the concept of force and the corresponding substantialization of body in his metaphysics should be understood as a reorganization of the relation between the principles of motion, its phenomenal dimension and the fundamental reality of body that undergirds it.[285] As such, the centrality that Leibniz placed on force as the fundamental reality accounting for the nature of motion would provide a different model for distinguishing the phenomenal level of the laws of nature from their metaphysical implications. In previous accounts, the dichotomy of the phenomenal effects of motion and its rational account placed the entire field of the scientific understanding of motion on the side of phenomenal or imaginary reality. The introduction of the substantiality of body reconfigured this dichotomous distribution. The concept of force sat precisely halfway between a metaphysical reality and a physical one. It was both the true or real element in a physical account as well as the reference that indicated a superior reality. With the positive development of force beyond its existence as a negative reference that subtends motion in extension, a redistribution of the fields of mathematical physics and metaphysics occurred in Leibniz's philosophy. As Leibniz explained:

> Therefore we have shown that there is a force of acting in every substance, and that there is also a force of being acted upon [patiendi] in every created substance, and that the notion of extension is incomplete in itself, but is relative to something

[285] There is no complete consensus on how closely Leibniz held the thesis of the phenomenalization of body and at what periods in his various stages of development. Between 1678 and 1682, Leibniz wrote radically divergent texts where he asserted the substantiality of bodies in one and rejected it in another. In his "Metaphysical definitions and reflections" of the period, he directly stated that "body is extended substance", while in another text of the same period, he remarked that "a body is not a substance". R.T.W. Arthur notes R.M. Adam's suggestion that Leibniz made a decision in favor of corporeal substance in the summer of 1678. On the other hand, after the 80's, the arguments in the M and the *Theodicy* may seem to indicate that Leibniz shifted toward a radical phenomenalization of body in his last period. However, given my arguments in the two preceding chapters, one can at least hold that Leibniz entertained the hypothesis of the substantialization of body late into his life. His distinction between, on the one hand, the phenomenal nature of the extensional aspects of body and motion and, on the other hand, force, an inherent nature of body, can provide a clue to understanding two different aspects of the reality of bodies. While I disagree with the phenomenalist reading, even if this reading of late Leibniz is correct, I believe that the reasons that Leibniz developed for understanding a substantialization of body indicate a fully coherent commitment to this thesis. That is, even if Leibniz ultimately decided against the thesis, he nonetheless produced a number of coherent arguments that can represent one cogent aspect of his decades-long effort. Cf. Leibniz, *LC*, 416n.

> which is extended, something whose diffusion or continuous repetition extension indicates; further, we have shown that the notion of extension presupposes the substance of body, which involves the power of acting and resisting, and exists everywhere as corporeal mass [massa], and that the diffusion of this substance is contained in extension.[286]

This signification of force as a means of indicating (by means of reinventing) the reality of corporeal substance equally indicated that "it follows that motion taken apart from force, that is, motion insofar as it is taken to contain only geometrical notions (size, shape, and their change), is really nothing but the change of situation, and furthermore, that as far as phenomena are concerned, motion is a pure relation."[287] The moment that the reality of motion was understood by an account of the interaction of forces, that is, the moment when Leibniz found a way to explain motion as a relation between forces, he could designate the extensional features of motion on a phenomenal level without sacrificing their reality, a reality anchored by the substantiality of bodies and their forces. The primary way that Leibniz demonstrated this was through the relativity of motion, motion as a phenomenal feature of bodies relative to the bodies themselves. Thus the re-organization of the relation between metaphysics and mathematical physics was also a reorganization of the hierarchy of reality pertaining to the means by which we have access to them.

We have noticed in the above how Leibniz qualified the metaphysical status of forces even as he imbued them with mathematical sense. Given our discussion above, we are in a better position to see the precise role that mathematics played in this reorganization. Access to the concept of force was not something that extensional measure or geometrical figures could, by

[286] Leibniz, *PE*, 130.
[287] Leibniz, *PE*, 130.

themselves, give us. This does not discount the mathematical formalization of force that was necessary to the completion of the fundamental elements of this theory. Indeed, if we hold up the Cartesian approach as an exemplary version of what it means to derive truths (of motion and bodies) from geometry alone, we find a position that is not only metaphysically “imperfect” (as Leibniz would have put it) but also, perhaps more seriously, mathematically inaccurate. Leibniz’s remarkable insight was to trace this inaccuracy back to a question of principle, an investigation that obtained a metaphysical result in the inherent properties of matter, a result that in turn reorganized the mathematical formalization of motion. One of the fundamental difficulties and an important factor of Leibniz’s transformation during his “middle” metaphysics of the *Discourse* was precisely the overcoming of his treatment of bodies and motion as merely “true phenomena”, however true they might be. Force then, insofar as it did not reduce to the register of phenomena, was the superior concept by which bodies and motion could then be treated in a non-phenomenal way. Part of this treatment or account was precisely a mathematical one, developed throughout Leibniz’s work on the field of dynamics, some elements of which we have taken a look at in the *Specimen Dynamicum*. The general picture of the relation between metaphysics and mathematics with respect to reality and phenomena that this leaves us is the following one. As exemplified by the concept of force, the mind has access, through metaphysical speculation or invention, to an inherent reality of bodies that anchors it to metaphysical structure in a way that mathematical thought cannot have by mere imagination. From this, the mathematical formalization of force is a formalization of a reorganization based on the introduction of this concept of force, an understanding of the perception of bodies and motion in a better way, that is, in an extra-phenomenal way. The mathematical account of force would then be the mediation between the conceptual development of the notion of force and its explanatory power for both the phenomenal effects of bodies and motion and their inherent reality. Indeed, we have seen in the previous chapter and the preceding sections how the development of the concept of force was not due to a single moment of illumination but rather a series of reflections where different factors such as the internal hesitations in the *Discourse* itself, the long standing argument against the Cartesian account of motion, the inadequacies of a logical model for metaphysics, the new microscopic revolution in biology and the like, played important roles in the development of the concept. In this, if Leibniz's turn toward the later monadological metaphysics was constituted by this multi-leveled push away from a merely phenomenal reality of bodies, then this mathematical model for force, its formalization and its explanation of the phenomenal effects of motion would constitute a central source through which he developed this later philosophy.

2. Return to the reality of infinitesimals: From force to motion (and back)

In order to put a finer point on the importance of this mathematical condition for the later metaphysics, I will return to the problem of infinitesimals especially as it emerges in the *Specimen Dynamicum*, a dimension of the text that I have not yet treated at length. Indeed, if the mathematical characterization of force served the role of mediating between a metaphysical reality and its phenomenal effects, then it is fitting that infinitesimals seem to have emerged precisely in this place. Indeed what role would "fictitious entities" play other than mediating the role between fiction, the imaginary register of phenomena, and entities, *ens per se*? As Leibniz explained:

> [J]ust as the numerical value of a motion [motus] extending through time derives from an infinite number of impetuses, so, in turn, impetus itself (even though it is something momentary) arises from an infinite number of increments successively impressed on a given mobile thing. And so impetus too has a certain element from whose infinite repetition it can only arise.[288]

In this lead-up to Leibniz's presentation of rotational and centrifugal force discussed above, we encounter some different registers of the infinitesimal. The first register is the most elementary level of deploying the infinitesimal as momentaneous rate. Motion across extension is an integration of all the intermediate moments of the path traversed. Here, the infinitesimal sum would be the momentaneous element of a continuous motion at a point, what Leibniz called an impetus. The summation of these impetuses integrates into a coherent continuous motion. This is perhaps the most elementary problem that the infinitesimal calculus corresponds to. In mathematical terms, if we were to take one moment of this impetus, it would be to take a first derivative of the motion at an instant, the momentaneous rate of change at an instant. A second register occurs at the level of impetuses themselves. As Leibniz explained, the state of an impetus at any given moment is an interaction of different forces that, due to the initial force of motion, forces of friction, the compounding solicitations of gravity and a number of other factors, modify the impetus itself. These different factors inflect motion in such a way that they determine the way in which change (or motion) itself changes. This observation of the changes in motion (a change of change), as well as the nature of *conatus*, corresponds to what we call higher-order differentials today. Outside of marking a difference between impetus and the changes in impetus, the mathematical details here should be reserved for a different context. In Leibniz's terms, this second register of the infinitesimal concerns infinite repetition (like that of a

[288] Leibniz, *PE*, 121.

"solicitation" to motion such as the centrifugal force) which, in the case of a ball tied to a fixed center in a cylinder, remains constant and infinitely impresses this force without actually causing any change in the velocity of the ball.[289] In view of the fact that Leibniz did invoke what would later be more rigorously formalized as higher-order differentials, we should look at how Leibniz understood the entities that they imply. In Leibniz's conclusion to this paragraph, we find a familiar remark on these mathematical quantities. "Nevertheless, I wouldn't want to claim on these grounds that these mathematical entities are really found in nature, but I only wish to advance them for making careful calculations through mental abstraction."[290] Let us then make no mistake in reading what Leibniz meant by his explanation here. The infinitesimal entities (even if we could place them in different registers) involved in the measure of the interaction between force and motion were advanced for calculation's sake and through mental abstraction. These entities do not exist in nature. What then were their roles in the development of the concept of force?

This rejection of infinitesimals as entities in nature would serve to cement the role of mathematics in Leibniz's later theory of substance. We will build toward this account by understanding what this theory of force implied for the reality of infinitesimals. To answer this question I address a recent article by Daniel Garber who has attempted to take this question head-on in his discussion of the *Specimen Dynamicum*. In reading the presentation of dead force in the *Specimen Dynamicum*, Garber notes the use of infinitesimal quantities as I have in the previous paragraph. He remarks:

> As Leibniz presents it in the *Specimen Dynamicum*, dead force would appear to be a real instantiation of an infinitesimal quantity, an infinitesimal magnitude that

[289] See figure 7.
[290] Leibniz, *PE*, 121.

> really exists in nature. But, of course, Leibniz is not inclined to take a realistic view of infinitesimal magnitudes. Is the reality of dead force consistent with the very skeptical attitude that he takes to the reality of infinitesimal magnitudes?[291]

Indeed, to follow Garber's prompting we should pose the question: How can the metaphysical reality of force be sustained if infinitesimal quantities are not? As Garber explains further, this question is key. He continues:

> Now, the physical and metaphysical reality of force in general, and dead force in particular seems evident: these are important constituents of Leibniz's world. But Leibniz wants to build a mathematical physics. That is to say, he wants to subject these physical magnitudes to mathematics. It is not surprising that when he does so, he makes use of notions from his calculus.[292]

The use of notions from the calculus demonstrated a constitutive mathematical dimension to this metaphysical reality insofar as they would constitute the basic elements for a renewed metaphysics. Without having answered the question of the role of infinitesimals, however, this relation remains deeply ambiguous.

The way that Garber responds to this question is ultimately by maintaining the distinction between a level of reality and a level of mathematical description or representation. He begins by referring back to Leibniz's own distinction between reality and imagination that had been persistent as a central distinction since the latter's early writings on infinitesimals. Garber explains that this distinction allows us to grasp a first clear resolution of the problem. Remarking on Leibniz's correspondence with Foucher, Garber notes:

[291] Daniel Garber, "Dead Force, Infinitesimals and the Mathematicization of Nature," in *Infinitesimal Differences*, ed. Ursula Goldenbaum and Douglas Jesseph (Berlin and New York: Walter de Gruyter, 2008), 280.
[292] Garber, 289.

> Leibniz draws a clear distinction between the world of mathematical entities (lines, surfaces, numbers), and the world of concrete things. The problem of the composition of the continuum is concerned with the parts from which continua can be constructed. Leibniz's point is that the mathematical continuum does not have such parts, nor does it need them: its parts come from the division of the line, and these parts are not properly elements of that line. However, in real *concreta*, the whole is indeed composed of parts, though those parts don't make up a genuine mathematical continuum. The problem of the composition of the continuum is thus solved: the objects of geometry, which exist in the realm of the ideal, are continuous, but not composed of parts; the real objects that exist in the physical world are composed of parts, but they are not continuous.[293]

At this fundamental level, Leibniz made the philosophical distinction that reality cannot be reduced to mathematical entities. As such, the objects of geometry and the entities of reality are to be distinguished. In turn, the problem of the continuum is one rooted in the danger of the misuse of ideas. The importance here resides in resisting the reduction of reality into geometrical objects. What I take Garber to say with this remark is that this distinction between mathematical entities and concrete reality does not entirely separate the realm of reality from that of geometry as if one has nothing to do with the other. Garber formulates this by saying:

> Geometry in this way can be said to represent something that is really in body, even if it has properties that the concrete body it represents does not, such as continuity: mathematical representation is not identity. Indeed, this is one way of putting Leibniz's point, and this is exactly where Descartes erred, in confusing the

[293] Garber, 294.

> mathematical representation of bodies in geometrical terms with their concrete reality.[294]

Mentioning Descartes, Garber explains the mathematical sense of Leibniz's critique of the Cartesian position as well as the development of the concept of force by showing the real distinction between the reality of force and its mathematical formalization. The reality of force positively designates a metaphysical reality to bodies that is inherent in them, a reality that allowed Leibniz to disentangle it from the ambiguities of corporeal extension and the extension of motion. This distinction gives the relativity of motion a non-relative foundation in the immanence of force in bodies. Here Garber quotes proposition 18 of the *Discourse*:

> For if we consider only what motion contains precisely and formally, that is, change of place, motion is not something entirely real, and when several bodies change position among themselves, it is not possible to determine, merely from a consideration of these changes, to which body we should attribute motion and rest […] But the force or proximate cause of these changes is something more real, and there is a sufficient basis to attribute it to one body more than to another. Also, it is only in this way that we know to which body the motion belongs.[295]

This anchoring of the relativistic interchange of bodies in motion to a metaphysical core of force grants a fully analyzable and positive dimension as its basis. This core of force is a basis for motion insofar as it anchors it to a corporeal reality more real than motion itself. As we mentioned earlier, Leibniz's fundamental aim in the *Specimen Dynamicum* was to understand the properties of motion, which are ultimately relative, as being analyzable according to a more basic level of the reality of force and not vice versa. For Garber the mathematical dimension of the

[294] Garber, 295.
[295] Garber, 51.

development of force was the attempt to measure the nature of this force with a mathematical apparatus, that is, in terms of motion and mass. From the various quantities that Leibniz provided in the *Specimen Dynamicum*, the derivation of instantaneous change at a point (momentaneous change) did indeed imply an infinitesimal differentiation and, in turn, an integration of infinitesimal moments. This term followed directly from the presentation of the exchange between motion and force. Following Leibniz, Garber does not take this as a cue to take infinitesimals as real quantities. He explains:

> With this I think that we have resolved the question that we originally posed about the dead force and infinitesimals. Mathematics can represent physical phenomena without being identified with it; even a good mathematical representation is going to have features that don't correspond to the features of concrete reality, and vice versa.[296]

In turn, Garber will understand these two levels, "concrete reality" and "mathematical representation" (which possesses its own ideal sphere of entities like derivatives and higher-order derivatives), as distinct levels coordinated by a relationship of "representation". Garber concludes this analysis by characterizing the critical approach with which Leibniz treated this issue -- mathematics on one side, concrete entities on the other. He explains:

> [W]e use mathematical representations of these notions, then we can express in a rigorous way the relations between the two, the fact that living force arises from an infinite repetition of dead forces. This is not to say that dead force is literally an infinitesimal, any more than living force is literally mv^2 or impenetrability is literally extension. But representing them in that way allows us to treat them and their relations in a systematic and rational way, to state the laws that they observe

[296] Garber, 304.

> and the relations between them in an exact way. And as long as we don't try to impose every feature of the mathematics that we use to represent reality onto that reality, then we shouldn't get into trouble.[297]

I will characterize this explanation of the gap between mathematical representation and concrete reality as "critical" in the sense that what guides Garber's reading of Leibniz's treatment of this difficulty is centered on the latter's criticism of the "misuse" of concepts. No doubt, this is warranted and a large part of the *Specimen Dynamicum* is dedicated, as I mentioned in the above, to a critical review of his contemporaries and his own previous writings on the same subject. This perspective alone, however, is not enough. There is a positive dimension of his formalization of force in mathematical terms that escapes Garber's reading.

Garber's reading of this text and the problem of infinitesimals in Leibniz's account of motion is very precise in many respects. However, in his diagnosis of the problem above, the conclusion that he gives is inadequate insofar as he steps away from affirming the central role that mathematics played in Leibniz's development of the concept of force by neatly separating the two levels, represented and representational. As I have commented in previous chapters, the means of resolving the problem of infinitesimals by putting them into the register of representation or sequestering them into a separate mathematical level is mistaken. As I have tried to show, the status of infinitesimals in Leibniz's mind remained problematic even if they remained on a mathematical level. The issue of the correlation between infinitesimals and dead force, and the mathematicization of nature in general could not be resolved by simply putting all the concrete elements of the *Specimen Dynamicum* on one side and all the mathematical entities on the other. The represented/representation dichotomy simply does not answer the question. In the context of the *Specimen Dynamicum*, the critical place to see this conflict, a counterargument

[297] Garber, 305-306.

against Garber's reading, is where Leibniz invoked the principle of continuity or the principle of general order, a crucial part of the *Specimen Dynamicum* that remains unanalyzed in Garber's reading. Here in the second part of the *Specimen Dynamicum*, Leibniz wrote, "If one case continually approaches another case among the givens, and finally vanishes into it, then it is necessary that the outcomes of the cases continually approach one another in that which is sought and finally merge with one another."[298] This argument about continuity arrived at the moment when Leibniz aimed to justify his claim that the laws governing motion and rest are to be understood consistently with one another.[299] That is, the laws governing rest should be understood in terms of the limit of motion. As such, in the case of the collision between two bodies, A and B, moving toward each other, the point of their collision would be understood as the momentaneous limit of motion, the point of rest for both bodies A and B. At the moment after this collision, the two bodies move apart, traveling away from each other after the mutual effect of forces in that moment of collision. Leibniz understood this mutual effect through the mutual deformation of their elastic bodies, and in turn, the restoring of their bodies from deformed shapes as they move away from one another. One of the main conclusions that Leibniz wished to draw from this model of collision and elasticity was the following:

> [I]t is already obvious how no change happens through a leap; rather, the forward motion diminishes little by little and after the body is finally reduced to rest, the

[298] Garber, 133.

[299] Caution should be taken in reading Leibniz's comments about continuity here together with the separate but related problem of infinitesimals in their mathematical use. As I have remarked in the second chapter, the principle of continuity cannot be straightforwardly read as a mathematical (or even meta-mathematical) principle. As H.J.M Bos has argued in his celebrated article "Differentials, higher-order differentials and the derivative in the Leibnizian Calculus", Leibniz provided two sorts of foundations for his mathematical use of the infinitesimal concept, one based on the infinite series and the other on the infinitesimal as entity. While Leibniz's interpretation of infinite series is related to the principle of continuity, this is not an adequate account of Leibniz's development of the calculus. What I analyze in this section does not make this mistake but rather tries to show the complexity of how Leibniz employed the concept of the infinitesimal by using this problem of continuity as a point of entry. Cf. Bos, "Differentials, higher-order differentials and the derivative in the Leibnizian Calculus".

> backward motion at last arises.[…] And so, rest will not arise from motion, much less will motion in an opposite direction arise, unless body passes through all intermediate degrees of motion.[300]

The idea that rest is a limit of motion is of course a position that Leibniz developed as early as 1671, and this relationship between motion and rest played different roles in his arguments throughout the years.[301] Here however, Leibniz used this relationship to reconfigure the nature of motion and collision in the *Specimen Dynamicum*.

Leibniz had for a long time rejected the idea that the rebounding of two bodies in collision could be explained by a reduction to an atomistic model where two absolutely hard bodies repel each other after having come to a full stop. In the case of the *Specimen Dynamicum*, this rejection of the atomistic model was rearticulated along with the idea of a force inherent in bodies which allows for a continuous change (a change without leaps) in the context of collision. In this context, the eventual rebounding of the two bodies is preceded by their elastic deforming in a continuous way. In turn, Leibniz underlined the fact that the momentary limit of rest is nothing but the limit of motion. Hence, as Leibniz explained, "Therefore, the case in which body A collides with the moving body B can be continuously varied so that, holding the motion of A fixed, the motion of B is assumed to be smaller and smaller, until it is assumed to vanish into rest, and then increase once again in the opposite direction."[302] The point that Leibniz wanted to make in the relation between force and motion is clear. Motion is an effect that is relative to the underlying reality of the interplay of forces. It is then only a phenomenal appearance that colliding bodies stop before repelling each other. In fact, as Leibniz explained, "By that very

[300] Leibniz, *PE*, 132.
[301] We assessed in the third chapter some of his early conceptions of rest as a limit of motion in the *Hypothesis Physica Nova* as the foundation of the "reverse Zeno" conception of the smallest part of motion.
[302] Leibniz, *PE*, 133.

circumstance the motion itself is weakened, the force of the *conatus* having been transformed into their elasticity, until they are altogether at rest. Then, finally restoring themselves through their elasticity, they rebound from one another."[303] The interchange between the *conatus* of moving bodies, force expressed in body becomes converted according to an interplay of forces into a sort of tension within the shape of the body itself. This conversion of forces comes to a limit where elasticity or tension gets re-converted into motion. The Cartesian notion of a quantity of motion (MV) would then vary while force remains constant in the interchange. This conservation was for Leibniz the mathematical expression of a substantial reality that underlies phenomena (of the extension of body and motion) and constitutes the inherent qualities of a body accountable in a mathematical manner. Leibniz's expression was that "what happens in a substance can be understood to happen of that substance's own accord, and in an orderly way."[304] By *accord* we may understand *conatus*, but we can understand it more appropriately as the force inherent in body, which allows us to distinguish body as a singular thing.

With this closer analysis of Leibniz's argument about the reality of force, we can correct Garber's statement that "mathematics can represent physical phenomena without being identified with it; even a good mathematical representation is going to have features that don't correspond to the features of concrete reality, and vice versa."[305] For Leibniz, mathematics could no doubt explain the features of phenomena that involve quantity or extension, as they belong homogenously together. What is more precise however, contra Garber, is that mathematics can also describe the reality of the causes behind phenomena. That is, it can correspond to the concrete reality of continuity in corporeal interaction that remains hidden from mere phenomena. Here Garber's distinction between concrete reality and mathematical representation is not only

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[303] Leibniz, *PE*, 132.
[304] Leibniz, *PE*, 131.
[305] Garber, 304.

unclear but misleading. From Leibniz's perspective, there were no doubt elements of concrete reality that did not involve force, that is, the "kingdom of wisdom" governed by God's final causes imbued in the universe in creation. There may also have been mathematical entities or mathematical features that did not have any concrete meaning. No doubt, the infinitesimal entities of geometric thought do not directly apply to reality. In this case however, with respect to the reality of force, Leibniz's sketch of a general systematic account of bodies and motion through force would be entirely impotent if it did not include the mathematical account of force that does not merely represent phenomena, but rather gives us access to the non-extensional "dynamical" reality underlying them and, to use Leibniz's language, causing them. The alignment of infinitesimal quantities with the crucial aspects of dynamics is one that should not simply be separated into represented and representation. It is precisely the use of these problematic entities that allowed Leibniz to model these realities, a model without which bodies and motion would remain accountable only on a purely phenomenal level.

To be clear, in my analysis I am not suggesting that infinitesimals understood mathematically are real entities in the physical or metaphysical realm. Further, the argument concerning continuity is one among many other arguments that follow from his model of collision and rebounding. Employing a critique of Garber's analysis, I wish however to point to the extra-representational dimension of Leibniz's use of mathematics. Of course a mathematical account of force, however precise, is not force itself. To then separate mathematics from concreteness by placing it in a domain of representation, however, would be simply a tautology: A mathematical account of force is not force itself. Thus a mathematical account of force is a mathematical representation of force. However, its role is explanatory in a sense that calling it a mere representation does not illuminate. When Leibniz put forth the idea of the principle of continuity and designated the moment of rest between two colliding bodies a limit of motion, the concrete understanding of this principle clarified by the concept of force could not be understood without the robust mathematical understanding of a momentaneous situation of two colliding bodies and that of the role of the infinitesimal within it. That is, Leibniz's argument that "no change happens through a leap" becomes actualized and operative only through the mathematical model of continuity and its use of the concept of the momentaneous through the derivative. Pushing the language of representation to its limits, we might say, in generosity to Garber, that Leibniz's mathematical formalization "represents" the non-phenomenal interchange between force and motion. That is, it "represents" the non-phenomenal. This would however be an abuse of the term "representation" in the sense that it fails to grasp Leibniz's use of the mathematical terms in a way that oversteps the pre-given domain of what he considered to be mathematical or geometrical, the relation and proportions of size, shape and magnitude.

It is of course no coincidence that this example of force and continuity converged on a mathematical structure where the infinitesimal was at issue. As a mediate conclusion to this chapter, we can clarify the nature of infinitesimals in this context. The nature of continuity in the nature of motion allows us to understand it beyond its phenomenal appearance. In this, the mathematical concept of the derivative is needed as a rigorous means of measure and, along with that, fictional infinitesimal quantities allowed Leibniz to provide an access to the structure for the reality of bodies through the concept of force. As such, what enabled Leibniz to turn a metaphysical hypothesis concerning the inherent reality of bodies into a concrete account of their activity owes a great deal to the mathematical concepts and methods developed in the infinitesimal calculus. However, the infinitesimal part of the continuous collision (and deformation) between bodies would then neither be a "true" part of nature nor a quantity in a categorematic sense. The syncategorematic infinitesimal at work in the *Specimen Dynamicum* is something that played a role in the larger mathematical and scientific theory of the dynamics, which achieved an account of the causes of motion that in turn constituted an account of the motions and interactions of bodies. Without this mathematical grounding, Leibniz would have simply been left with purely extensional figures, which would be a mere modification of the Cartesian position. In this case, a mathematical domain of explanation would indeed be, to use Garber's words, a representation of the physical phenomenon. Indeed, Leibniz's project toward a maturation of the concept of monadological substance included within it an attempt to give an account of motion rooted in a metaphysical understanding, an account of motion that stepped beyond a purely extensional or geometrical account. What Leibniz presented in the *Specimen Dynamicum* concerning living and dead force is, by modern standards, insufficient for a complete understanding of the conservation of energy for classical (Newtonian) physical systems. However, Leibniz fully extended his argument with the Cartesians far beyond a quarrel about numbers or mathematical formula and brought these issues into a full blown account of the nature of bodies and motions themselves. In Leibniz's turn to the concept of force, this "something else" in motion marked by his early dissatisfaction with the Cartesian theory of motion was brought to actualization. At the heart of this alternative, Leibniz constructed the elements of a substantialized body with the full range of mathematical resources available to him. In turn, mathematical expressions such as infinitesimals, used to explain continuity and the minute structures of the interchange between force and motion, were not to be understood as mere representations of a phenomenal level. These elements were the account itself, irreducible to any phenomenon and serving as the explanation of their cause.

3. Mathematical and metaphysical structure: From phenomenon to reality

In this chapter, building from the resources of the previous one, I have argued that the heart of Leibniz's transformation between the *Discourse* and the M constitutes a discontinuity, a structural transformation that cannot be fully reconciled. This is seen in the transformation from the centralization of the individual substance as a complex entity that unites a series of infinite predicates through identity, to that of the singular substance, the simplicity of monads that aggregate to form larger and more complex entities. Following Fichant's argument, I have argued that this transformation is due to a conflict, already present at the heart of the *Discourse*, concerning the reality of bodies and their irreducibility to a phenomenal level. We have also argued that, beyond this discontinuity, there is continuity on the level of problematics. In tracing out the developments in Leibniz's correspondence with Arnauld, which are inseparable from the *Discourse*, we see the problematic configuration of issues surrounding corporeal substance came to dominate Leibniz's metaphysics and became central to Leibniz's later metaphysical project. In turn, one characterization of the resulting metaphysics given by Fichant is that the emerging substantialization of body should be read as a progressive adoption of the model of life and organism which constituted the double articulation of vitalism at the heart of Leibniz's monadic metaphysics. A first level was articulated as the global structure, the structure of infinite complexity of life where every part is divided into infinite parts bursting with the clamor of vitality. A second level was articulated at the level of singularity where the simplicity of substance is the singular life that every monad lives, indivisible and indestructible. In turn, these two levels combine to give us the sort of aggregation where a dominant monad ranges over the infinite number of less dominant monads that constitute its organic machine. The confrontation between this new synthesis of the conception of substance and the former model of substance based on the subject-predicate model found in Leibniz's logic of truth had to pass through a necessary, mathematically charged stage. That is, it had to pass through a stage that would allow Leibniz to reorganize the elements of this new model in a way that concretized the reality of corporeal substance. In this, I have argued that it is through a reconception of bodies and forces, made possible through and conditioned by Leibniz's mathematical work, that Leibniz provided the fundamental schema where this new conception of corporeal substance could stand. The problem of providing an account of motion in mathematical terms had no doubt already been a constant part of Leibniz's early work. This commitment to a mathematically informed physics was perhaps most pronounced in the very means by which he refuted the Cartesian principle of the conservation of motion and the metaphysical characterization of *res extensa*. In the *Specimen Dynamicum* and related works of the same period and after, however, we see a return to this arena of research in his concrete development of the concept of force and its relation with motion, a return that was mediated by a number of metaphysical principles that reorganized the understanding of phenomenally apparent behavior of bodies in motion and the search for their causes. My attempt was thus to identify the development of the mathematically informed account of force as the nexus where the metaphysical requirements of a corporeal substance, given its structural transformation away from the logical model, met the phenomenal appearance of bodies and motion without being reduced to either. Of course, without the phenomenal nature of bodies, no mathematical account would have been necessary since no quantitative elements would have to be accounted for. And on the other hand, without the metaphysical nature of bodies, this mathematical account would have remained purely on the level of imagination which would amount to nothing but a modification of the Cartesian position, a strict representation of extensional appearance which Leibniz was never satisfied with. Furthermore, in terms of Fichant's reading, which I have taken as a guiding thread through Leibniz's revision of metaphysics between the *Discourse* and the M, I argue that if the argument from the position of structural transformation is to be complete, the vitalist reading is not enough. As a necessary aspect for the very narrative that Fichant hopes to establish about Leibniz's structural transformation, I have argued that Leibniz's association of corporeal substance with the concept of force could not have overlooked the mathematical account of corporeal substance developed in the context of the *Specimen Dynamicum*. Here Leibniz developed a systematic account of force that, whether or not it had achieved the standard of scientific rigor in that text or in Leibniz's lifetime, did indeed trace out the metaphysical consequences of the idea of an inherent reality of body with a mathematical apparatus which took full advantage of the concepts developed in the infinitesimal calculus comprising the use of infinitesimal quantities and momentaneous change. In turn, I have argued that without this mathematical register, the relation between motion conceived as phenomenal effect and the reality of the causes of motion could not have fully overcome the Cartesian paradigm of a purely geometrical understanding of motion and bodies. As such, between the real causes of motion rooted in a metaphysical reality and motion understood as the phenomenal effects of these causes, a mathematically charged account of forces intervened in the gap of a reality which did not merely represent but provided the very access into the inherent reality of corporeal substance.

Leibniz's path toward corporeal substances brought his metaphysical insights to converge with mathematical formalization, a mathematical account of force by which the concrete character of the inherent (metaphysical) reality of bodies could be filled out. The mathematical structure developed in the infinitesimal calculus, the coherence of the relation between infinitesimal quantities and their sums, then provided the model for a positive articulation of the coherence of the interchange between forces and the motions they subtend, a scheme from which Leibniz was able to reorganize the elements of his metaphysics. In turn, Leibniz's structural transformation that we have traced here not only required a necessary stage of an encounter with the mathematical, but we find that the mathematical invention of the infinitesimal calculus provided a major component for the metaphysical structure to come.

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APPENDIX A

NOTE ON ABBREVIATIONS

Some references to collected editions of Leibniz's work are abbreviated in the footnotes. Following standard conventions in Leibniz scholarship, CI. Gerhart's 7 volume edition of Leibniz's philosophical writings (*Philosophische Schiften*) will be abbreviated by "*Phil*" followed by volume number indicated by Roman numeral and then page number indicated by Arabic numeral. I follow the same convention for Gerhart's 7 volume edition of Leibniz's mathematical writings (*Mathematische Schiften*), abbreviated by "*Math*" followed by volume number indicated by Roman numeral and then page number indicated by Arabic numeral. References to R. Ariew and D. Garber's edition *Philosophical Essays* are abbreviated by "PE" followed by page number. Reference to L. Loemker's edition *Philosophical Papers and Letters* are abbreviated by "PPL" followed by page number. R.T.W. Arthur's selection *The Labyrinth of the Continuum* is abbreviated by "LC" followed by page number.