

MENTORING PRACTICES WITH STUDENT TEACHERS OF MATHEMATICS

by

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(Under the Direction of James Wilson)

ABSTRACT

This study examined the relationship between three pairs of mentors and student teachers in middle and secondary mathematics. Each of the participants completed a survey and was interviewed prior to the experience to examine her beliefs about learning to teach, mentoring, expectations, and prior experiences with student teachers. I observed each pair for three 1-week cycles and interviewed each participant upon completion of her experience. The data were categorized into themes and constantly redefined as additional data were collected. Whereas some themes that emerged for each pair were unique - such as a focus on technology, timing in lessons, and classroom management - other themes coincided such as questioning and other formative assessments. The data illustrated how each student teacher was influenced by the mentor's practice and the foci of the feedback she received. Other factors like class schedule were also considered and discussed as features that shaped the participants' experience.

The results of the study indicate that student teachers model their mentors' practices closely. Each of the pairs discussed lesson planning and the student teachers adopted the same format as their mentors. When the student teachers assumed responsibility of the instruction, they implemented traditional practices and developed more student-centered approaches with the guidance of their mentor. In each case the mentors aided in the student teachers' development of using formative assessments and questioning strategies to gauge students' understanding. The mathematical conversations tended to be limited in terms of their depth and because less frequent as the experience progressed.

INDEX WORDS: Mentoring, cooperating teacher, mentor teacher, student teaching, preservice education, mathematics teaching

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I find it interesting how certain conversations and experiences have stayed with me years after they happened. When I was young my dad took my brothers and I to the baseball field to practice pitching, however, I just wanted to be an outfielder and catch fly balls. When I told him I was not good at pitching and I did not want to do it anymore he said to me in frustration, “you can do anything if you really want to.” I did not pitch very much after that, but my dad’s support never wavered whether I told him I was going to study in England for a year or move to Georgia for a doctoral degree. I want to thank my father for the confidence to do anything I really want.

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CHAPTER 1

BACKGROUND AND RATIONALE

The field of education values mentors for their role in training novice teachers. A capstone experience in training teachers in most teacher education programs involves a practicum where a university student is placed with an inservice teacher for a period of time (10–15 weeks typically). During this practicum, the student teachers are given the opportunity to implement what they learned in the teacher preparation program and experience many realities of the profession as they perform the daily requirements of teachers alongside their mentors. Throughout my life, mentors have made significant contributions to my career in different ways. Whereas some mentors provided a supportive environment, others presented challenges that pushed my ways of thinking about mathematics and teaching, and there were those who offered pragmatic advice that reflected years of experience.

Beginning as early as the 17th century, a similar pattern to this capstone experience has been used in the training of teachers where a student teacher works closely with a mentor teacher within the context of the mentor teacher's school and classroom (Johnson, 1968). Research on mentoring in mathematics education has grown especially over the past three decades. As Feiman-Nemser (1996) warns, however,

enthusiasm for mentoring has not been matched by clarity about the purposes of mentoring. Nor have claims about mentoring been subjected to rigorous empirical scrutiny. The education community understands that mentors have a positive effect on teacher retention, but that leaves open the question of what mentors should do, what they actually do, and what novices learn as a result. (p. 2)

Since beginning my graduate studies, I have had the opportunity to work with student teachers and mentor teachers of secondary mathematics who vary greatly in style and ability. Much as teachers have differing styles of instruction, the mentor teachers I have worked with took a variety of approaches to give feedback and advise the student teachers. In some cases the

mentor teacher was very directive in supplying the content and instructional approach, whereas in other cases, the mentor supplied very little material or instructional advice but provided an environment where the student teacher could experiment and the mentor asked questions to promote reflection. I have observed numerous mentors who have taken a variety of approaches to guide a student teacher that was learning to teach, and like teachers in a classroom, many mentors were effective regardless of the approach they took. There is no approach that works for every mentor.

Novice teachers commonly report student teaching as one the most influential experiences in an undergraduate teacher-education program. Little is known, however, about its effects (Llinares & Krainer, 2006; Wilson, Floden, & Ferrini-Mundy, 2001), or the influence of the mentor teacher (Borko & Mayfield, 1995; Wang & Odell, 2002). Bullough, Young, and Birrell (2003) claim that “the typical pattern of student teaching has remained little changed for 50 years: A teacher education student is placed in a classroom with a single cooperating teacher for varying lengths of time, a term or perhaps a semester” (p. 57).

Researchers (Borko & Mayfield, 1995; Glickman & Bey, 1990; Llinares & Krainer, 2006; National Council of Teachers of Mathematics [NCTM], 2007; Wang & Odell, 2002) have identified student teaching and the mentoring of novice teachers as an area in need of research in a number of different ways. After their extensive review on mentoring student teachers, Wang and Odell (2002) concluded that research on mentoring is limited in the following ways:

1. Although reformers in teaching and teacher education identify important conceptions of teaching and learning to teach, these conceptions are not often used as the basis for studies of teacher mentoring and novices' learning to teach. Without such a conceptual base, it is very hard to identify and contrast in useful ways the findings on mentoring and novices' learning.

2. With such conceptions as bases, we need more creative ways to capture relationships between mentors' preparation, their knowledge of teaching and mentoring, their mentoring practice, and the quality of novices' learning to teach. As reflected in this review, many studies capture such connections by relying on fragmented information, inferences, and self-report.

3. We need studies to examine these connections, if there are any, in various program and school contexts and with mentors and novices who have different beliefs and experiences of teaching and mentoring. (p. 535)

The literature reviewed in this document support these limitations. One apparent trend is that mentor teachers take a variety of approaches, and each party involved may have different goals for the student teaching experience. Some mentors may view their role as socializing the student teacher into the school community, whereas others may take a collaborative approach to give the student teacher access to their knowledge of students and planning. At the same time, the student teacher may be looking for advice on how to effectively organize discussions in his or her lessons. In many cases, this situation leads to a lack of common goals for the student teaching experience between the student teacher and the mentor.

Researchers have identified models of mentoring. These models are typically based on survey data (Saunders, Pettinger, & Tomlinson, 1995), alternative mentoring programs (Bullough et al., 2002; Feiman-Nemser & Parker, 1992), or trends that are identified throughout the current literature (Wang & Odell, 2002; Zeichner, 1983). Missing from these accounts are analyses of how mentoring relationships develop, characterization of the relationship between the mentor and student teacher, and implications for the student teachers learning to teach.

Research Questions

This study examined the feedback given by mentors in a traditional model of student teaching as well as nature of the relationship between the mentor teacher and student teacher. Specifically, this research addressed the following questions:

1. What is the nature of the mentoring relationships between mentor teachers and student teachers of middle and secondary school mathematics?
2. What influence does the mentoring relationship have on the practice of student teachers of middle and secondary school mathematics?

CHAPTER 2

LITERATURE REVIEW

The present study considered the relationship between mentor teachers and student teachers of middle and secondary mathematics. Given this context, the following review of literature begins with an overview of research on student teaching and then becomes more specific by summarizing research on lesson planning, discourse, classroom management, and mentoring. These sections focus primarily on research pertaining to student teaching; however, I also consider learning to teach and beginning teacher literature in areas where research particular to student teaching is limited. In the overview I examine literature describing a traditional student teaching experience with the perceived benefits and limitations. The section on planning considers factors such as mathematical knowledge, pedagogy, and formative assessments. The section on discourse includes a summary of student teachers' struggle with leading mathematical discussions as well as questioning. Since classroom management is a problematic area for many student teachers, I consider it as a factor that contributes to, or hinders, the success in a student teacher's experience. The last section is on mentoring student teachers and I look at the roles and limitations that are thematic in education literature.

Overview

Student teaching is seen as a central component of virtually all preservice teacher education programs (Kagan, 1992; Borko & Mayfield, 1995; Wang & Odell, 2002). In a traditional experience, the student teacher assumes responsibility for classroom instruction and management while "practicing" teaching (Bullough et al., 2003). "Traditionally student teaching has been seen as the bridge between the theory, knowledge, and skills gained at the university and their application in the classroom" (Wideen, Mayer-Smith, & Moon, 1998, p. 152). Student

teaching is a place where novice teachers learn, “(1) how to facilitate student learning and (2) run a classroom” (Leatham & Peterson, 2010, p. 100).

Guidance for the student teacher is typically shared between the mentor teacher and university-appointed supervisor who acts as a liaison between the university and school setting where the student teaching takes place (Borko & Mayfield, 1995). The combination of the university supervisor, student teacher, and mentor teacher is often called the student teaching triad. A central part of the supervisor’s guidance is the conferences that take place before and after lessons. Supervisors generally use some version of the clinical model of supervision where there is a sequence of preconference, observation, post observation analysis, post conference, and critique by the university liaison (Glickman & Bey, 1990).

Borko et al. (2000) point out that the overarching goal for these novice teachers, that they assume the professional responsibilities of a teacher and teach competently, is shared by members of multiple communities involved in mathematics education. This study examined the nature of the relationship between two of these members, the mentor and student teacher, in a traditional student teaching setting. I considered the nature of both mathematical and pedagogical discourse and analyzed its impact on the student teacher’s practice.

Student teaching is often seen as one of the most influential components of teacher education programs (Borko & Mayfield, 1995; Leatham & Peterson, 2010; Wilson et al., 2001); however, integrating coursework with field experience has proven to be complex work. “Universities want to honor the knowledge of experienced teachers, yet there are often differences in views across schools and universities that are difficult to resolve” (Wilson et al., 2001, p. 18). An example of these differences may be when a student teacher wants to implement a cooperative learning activity and the mentor teacher believes that having tight control of the classroom is an essential first step before working with groups. Kagan (1992) offers some

insight into the influential nature of the experience by claiming, “student teachers approach the classroom with a critical lack of knowledge about pupils. To acquire useful knowledge of pupils, direct experience appears to be crucial, particularly extended opportunities to interact with and study pupils in systematic ways” (p. 142).

Despite the little change to student teaching that has occurred during the last 50 years, little is known about the influence of either the supervisor or the mentor teacher (Borko & Mayfield, 1995). Borko and Mayfield assert, “at their best, student teachers' relationships with both cooperating¹ teachers and university supervisors can provide feedback about specific lesson components, suggestions about new ways to think about teaching and learning, and encouragement to reflect on one's practice” (p. 515). The mentor teacher has traditionally been seen as offering a pragmatic view of teaching, whereas the university supervisor provides knowledge of the theory espoused in the teacher education program, and the role of the student teacher is to negotiate the two (Glickman & Bey, 1990; Wideen et al., 1998). Bullough and Draper (2004) put the situation this way,

a closer look at the politics of mentoring, particularly within the triadic relationship that is common to teacher education, reveals a much more complicated story than is typically told: a tale of power negotiation and of positioning and being positioned to influence learning, preserve one's sense of self, and achieve or maintain a measure of control over one's situation. (p. 418)

Bullough and Draper go on to say that student teaching triads are inevitably hierarchical and promote “shifting alliances” (p. 406) between the members of the triad where conflict is a potential consequence.

Whereas these researchers focused on the triad, the present research offers a different perspective by providing an in-depth analysis of the relationship between student teachers and

¹ *Cooperating teacher* and *mentor teacher* are both used in educational literature to refer to the

their mentors, who are the members of the triad who spend the duration of each school day together. Kagan (1992) highlighted the importance of the relationship between these members in her review of student teaching literature:

Whether completed as a practicum, internship, student teaching, classroom practice teaching is affected by many school and classroom variables: the nature of pupils, principals' beliefs, parental attitudes, availability of materials, communication between school and university personnel, attitudes of teachers in a school, and the personal relationship that develops between a novice and his or her cooperating teacher. (pp. 149–150)

One possible conflict is when there is philosophical difference between the supervisor and the student teacher or mentor. This was the case in the failed triad Bullough and Draper (2004) described where the supervisor observed a lesson and gave feedback to the student teacher. The purpose of this account was to illustrate how lack of communication or lack of common goals led to a breakdown in the student teacher's experience that was not due a lack of performance, but rather was due to efforts to appease everyone in the triad. Based on the feedback, the student teacher changed the lesson and taught it later the same day while the mentor observed. The mentor "ripped her just up and down about how awful her lesson was" (p. 413). The mentor apparently disagreed with the flow of the lesson because it was changed from the teacher doing examples and the students practicing exercises similar to the examples, to a class discussion of the mathematical theory followed by exercises where the students applied the theory. The student teacher thought the lesson went well the second time until she received feedback from the mentor. This episode affected the remainder of the student teacher's experience because she was constantly trying to please her mentor *and* supervisor, who had different philosophical views on teaching. "Beginning teachers sit in the middle and expend energy learning how to 'manage mentors' (Maynard, 2000) as they strive to balance sometimes

conflicting demands to maintain desired relationships and to obtain positive teaching evaluations” (p. 408).

There are other issues that potentially arise when student teachers attempt to bridge the cultures of the university and the school where they teach. Wideen et al. (1998) claim the practical pressures of student teaching limits the ability of preservice teachers to do anything but survive. This assertion is consistent with Franke et al. (1997) and Cady et al. (2006), who found that beginning teachers go through a period where they reverted to traditional practices before they were able to teach in ways that were consistent with their teacher-training program. In the present study I considered factors that potentially contribute to this reversion and how student teachers progress past it.

The practical pressures of the experience appears to further limit student teachers’ ability to do anything other than survive (Wideen et al., 1998). This situation is not surprising since, prior to student teaching, few preservice teachers have been asked to fulfill the daily requirements of an inservice teacher, such as planning lessons for multiple classes, grading for each class, instructing lessons on different topics, and helping with responsibilities outside the classroom such as bus duty.

From the perspective of the student teacher, another limitation of the traditional experience is lack of control over the curriculum. In a study by Bullough et al. (2002), the participants claimed that lesson plan topics almost always came from cooperating teachers and that student teachers rarely sought to influence those topics. Once the topics were assigned, student teachers were given flexibility for planning and presenting the lessons. The student teachers in that study reportedly saw their role as “fitting into the mentor’s program with minimal disruption, which was a source of disappointment for some” (pp. 73–74). One influential factor that contributes to this situation is that schools have worked hard to develop

pacing guides for each course to ensure that different teachers address the required curriculum, thereby limiting the student teachers' choice of curriculum topics. This situation provides more insight into the reason mentor teachers have a large impact on student teachers.

In their summary of research on student teaching, Wilson et al. (2001) described their findings as “sobering”:

- √ Several studies found that field experiences were often disconnected from other components of teacher preparation, and prospective teachers had difficulty applying what they had learned in those other components when they entered their practica.

- √ In one study, researchers found that student teachers' experiences in classrooms were limited in range, tending to focus on mechanical aspects of teaching and dominated by worksheets and workbooks.

- √ Some university programs do not coordinate student teaching experiences with the university coursework. Other researchers have found that university courses and student teaching experiences can work together to maintain the status quo.

- √ In one study, researchers found that when the student teachers become overwhelmed with the challenges of learning to teach, they revert to the norms of the schools in which they were taught, which sometimes means that they teach in ways quite different than those envisioned by university instructors. (p. 18)

This list succinctly reiterates many of the findings discussed above. To summarize, the limitations in student teaching experiences are characterized by lack of common goals and expectations, connection to coursework, and curricular freedom. The present research contributes to the current literature base by providing insight into the influence mentors have on student teachers' practice, a focused analysis of the nature of the communication between two members of the triad, and the degree to which student teachers have the opportunity to make curricular decisions.

Lesson Planning

Research specific to student teachers' lesson planning is limited; however, it provides some insight into difficulties that student teachers face. Reynolds (1992) claimed that novice

teachers typically take longer to plan lessons and tend to “focus mostly on the development of concrete strategies and activities for involving students with the content” (p. 9). In a descriptive study, Eisenhart et al. (1993) concluded that student teachers acknowledged the importance of procedural and conceptual knowledge; however, they consistently showed more attention to procedural knowledge. Brown and Borko (1992) also supported this view by claiming preservice teachers have trouble making connections between concrete or semi-concrete representations and the underlying mathematical ideas. Instead, the preservice teachers in their study tended to focus on procedural aspects of activities.

Content knowledge is one factor that is a common explanation for student teachers’ apparent inability to teach conceptually and make mathematical connections. Reynolds (1992) surmised that lack of content knowledge contributed to difficulty in planning in a number of ways:

Beginning teachers appear to understand the need for creating lessons that are appropriate for the subject matter and students, but they seem to accomplish this task only in superficial ways. They often do not know their subject matter in a way that allows them to explain it to students. They also have difficulty seeing the pedagogical implications of student differences, even though they may be able to detect overt student differences. Thus, they are often unable to tailor materials and instruction to individual students. (p. 10)

Brown and Borko (1992) claimed content knowledge influences student teachers “ability to select and structure content for teaching, activities and assignments for students, and use textbooks and other curriculum materials” (p. 217).

Another factor in student teachers’ difficulty with planning lessons is that without guidance they find it challenging to make pedagogical decisions (Brown & Borko, 1992; Wang & Odell, 2002). Reynolds (1992) offered one explanation for the struggle with pedagogical decisions beginning teachers have a difficult time taking students prior knowledge and learning experiences into account when planning.

Campbell and Evans (2000) suggested another deficiency in student teachers' ability to plan lessons was the absence of a direct link between instructional goals and an assessment of those goals. They analyzed 309 lessons from 65 student teachers and found that 59 had no reference to any assessment and of the remaining lesson, less than half required students to construct a response to demonstrate learning. Only 13 of the lessons included a rubric to add validity to the student teachers' assessment of their students' understanding.

The present study can contribute to the literature on student teachers lesson planning by providing detailed descriptions of the feedback from mentors and its influence on the practice of student teachers. More specifically, this research sheds light on the nature of the mathematical and pedagogical conversations between mentors and student teachers, as well as their influence on assessing student learning.

Discourse

Research on the discourse in student teachers' practice is also limited. The NCTM (2007) suggests that teachers of mathematics should orchestrate discourse by

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listen carefully to students' ideas and deciding what to pursue in depth from among the ideas that students generate during a discussion;
- asking students to clarify and justify their ideas orally and in writing and by accepting a variety of presentation modes;
- encouraging and accepting the use of multiple representations;
- making available tools for exploration and analysis;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let students wrestle with a difficulty; and
- monitoring student's participation in discussions and deciding when and how to encourage each student to participate. (p. 45)

However, in Reynolds' (1992) review of literature, she concluded beginning teachers "are less effective [than experienced teachers] in their questioning strategies about children's performance;

thus, they are less proficient at using this information about student performance to change their own teaching in response to the students' understanding” (p. 22).

Borko et al. (2000) describe a student teacher, Ms. Savant, who found it “difficult to engage students in such inquiry-oriented discourse in her own classroom. She had developed a commitment to this type of discourse, however, and her ability to engage students in mathematical discussions improved during the student teaching experience” (p. 202). They also addressed the issue of content knowledge and claimed,

When teaching topics for which [student teachers] had high subject-matter knowledge, they asked fewer questions but a greater proportion of higher order questions, and they tended to talk less of the time and for shorter periods of time. Students talked more, asked more questions, volunteered to speak more, and spoke in longer discourse sequences. Another especially salient task for the novice teacher is acquiring the knowledge and skills for managing a classroom. (p. 204)

The researchers in this study were testing the impact of teacher education courses where the preservice teachers were given tasks to gauge their mathematical knowledge and the instructors modeled inquiry-oriented discourse. The present study is different from Borko et al.’s study in the sense that they focused on the impact of preservice education courses rather than the mentoring relationship.

Nicol (1999) tested a theoretical framework used in a preservice education course called Question, Listen, and Respond, which was based on the work of Ball (1990). There were 20 prospective elementary teachers and 14 prospective middle school teachers who were videotaped in field experiences over the course of a semester. One interesting result that came from this study is that despite training in their class, the participants felt tension “with the kinds of questions posed and the reasons for posing them, with what they were listening for, and with how they responded to students' thinking and ideas” (p. 52). In interviews after the teaching episodes, the participants seemed to struggle to determine the purpose of the questions they

asked; however, “as prospective teachers, for example, began to consider students' thinking and create spaces for inquiry through the kinds of questions posed, they began to question, listen, and respond to mathematics differently” (p. 62).

Studies specific to student teachers' questions are absent from the educational literature, which provides relevance for the studies from Borko et al. (2000) and Nicol (1999) because they give some insight into preservice teachers' questioning. They highlight that preservice teachers have difficulty leading inquiry-oriented discussions, posing questions and responding to students' answers. The current study provides a different perspective by analyzing the practice of student teachers, including their classroom instruction where questioning and leading discussions are pivotal.

Classroom Management

Classroom management is one of the largest concerns for prospective teachers entering their student teaching experience (Davis, 1990; Kagan, 1992; Tully & Chiu, 1995). In this section, I review the findings from three sources including Kagan's oft-cited synopsis of literature on preservice and beginning teachers. I then consider two studies that deal specifically with student teachers' classroom management.

In her model of learning to teach Kagan (1992) suggests that prospective teachers need to focus on matters of management before engaging in issues of teaching subject-matter content and of considering alternative approaches to teaching. She claims,

Once in the classroom, novices first seek to confirm and validate their self-images; gradually, given the appropriate conditions, novices begin to use their growing knowledge of pupils and classrooms to modify, adapt, and reconstruct their images of self as teacher. Thus, in a very real sense, the initial focus of novice teachers is inward. (p. 147)

The “appropriate conditions” she was referring to largely dealt with creating an environment where teacher can experiment with different teaching approaches and learn from their efforts.

Without fluency with classroom management, this environment is highly unlikely because salient features of the experience are confounded by disruptive behavior. Another implication of beginning teachers' need to validate their self-image has an influence on their pedagogical decisions. It seems reasonable that teachers need to have some degree of order in their classroom before they can attend to their students' thinking and understanding of mathematics.

Studies focusing on classroom management have largely used surveys and self-report data. Davis (1990) used questionnaires with 43 secondary school student teachers in the final week of their placements. The participants in that study reported that they most frequently encountered discipline problems in low-level classes and that their mentor teachers had only a moderate influence on their ability to manage a class, which was a disappointment for some since the student teachers and mentors were in daily contact. Davis hypothesized,

Some cooperating teachers may have employed an indirect approach in working with their student teachers, avoiding direct suggestions so that their student teachers would make decisions and learn from the outcomes of them. Other cooperating teachers may have offered suggestions which did not completely solve the problems, leaving the student teachers less than satisfied. (p. 50)

Tulley and Chiu (1995) pointed out that preservice teachers have consistently ranked discipline as one of their greatest sources of anxiety and uncertainty. They had 135 undergraduate elementary- and secondary-level student teachers describe successful and unsuccessful situations. Based on the responses they developed five categories of discipline problems and 91% of the reported issues dealt with three of the categories: disruptions, defiance, and inattention. One interesting result from this study is that the participants reported that positive reinforcement was the most effective strategy for achieving desired behavior and among the least effective strategies were the use of threats and warnings. The relevance of these studies to the current research is that classroom management is a serious concern and logistical obstacle that student teachers have to address before attending to areas of teaching such as

leading discussions and investigating student thinking. The present study can provide a detailed description and analysis of how student teachers of mathematics attend to classroom management issues.

Mentoring

In this section I discuss literature that approaches the practice of mentoring from a number of perspectives. I begin by considering Wang and Odell's (2002) review of the literature on mentored learning, and then address four descriptive studies that offer different contributions. I conclude by discussing how the current study offers a perspective that is absent in other models of research on mentoring.

Wang and Odell (2002) concluded that none of these conceptions of mentoring are adequate for helping student teachers to learn to teach in ways that are consistent with current reforms.

They do not expect to support novices by posing problems for them or by uncovering assumptions that underlie existing practice. They have no intention of engaging novice teachers in developing a deeper understanding of subject matter and connecting that knowledge to a diverse student population. They do not see their role as helping novice teachers to understand the relationship between theoretical knowledge as a basis for standards-based teaching and relevant teaching practice. Nor do they see their role as supporting novices in acquiring more knowledge about teaching through systematic reflection and analysis. Instead, they focus on providing emotional support and technical assistance in their collaboration with novice teachers, just as many novice teachers expect. (p. 511)

Other researchers have done descriptive analysis of mentor teachers' practice that support, contradict, and shed light on these claims.

Borko and Mayfield (1995) studied the impact of the supervisor and mentor teacher and found discouraging results. The student teachers' beliefs in their study did not change in ways envisioned by the researcher and the conversations between the mentors and student teachers tended to focus on emotional support rather than "in-depth exploration of issues of teaching and

learning” (p. 515). Borko and Mayfield went on to say that mathematical conversations were at a superficial level, even when the student teacher’s treatment of mathematics was problematic in a lesson. Borko and Mayfield’s study was important because it drew attention to the type of conversations between mentors and student teachers and highlighted the lack of focus on content and student learning.

Hawkley (1998) did interviews and analyzed audio recordings of conversations between two mentors and their student teachers. One of the mentors tended to be directive, whereas the other was more collaborative. In the case of the directive mentor, the student teacher developed a wide variety of teaching techniques to add to her repertoire, but encountered some frustration because she felt reluctant to approach the mentor with general concerns or issues where they disagreed. In contrast, the student teacher with the collaborative mentor was able to discuss situations they encountered in more depth; however she developed fewer techniques for teaching. Hawkley’s study illustrates that different approaches to mentoring offer different advantages; however, neither of the student teachers was able to develop a diverse teaching repertoire and the ability to analyze complex classroom situations.

Awaya et al. (2003) viewed mentoring as a dynamic journey where the role of the mentor changed as a result of new encounters rather than remaining a static approach that the mentor took. They claimed that at different times mentors act as a practical guide, a moral support, and as someone to whom student teachers prove their worth. Awaya et al. point out that at the beginning of the student teaching experience, mentors need to introduce their student teachers to school and classroom routines. As student teachers begin to assume responsibility for planning and teaching lessons, they often encounter anxiety, in which case they need encouragement and advice. Mentors also share the responsibility for preparing student teachers to be autonomous teachers who are prepared to enter the workforce, in which case they need to show they are

capable teachers. The importance of this study is that it highlights that mentoring student teachers is highly contextual and mentors fulfill a number of roles. Not all student teachers require the same amount of practical advice or emotional support, and hence it seems inappropriate to posit one overarching approach to mentoring.

Leatham and Peterson (2010) collected surveys from 78 mentor teachers for secondary mathematics education majors that inquired into their views of the purposes for student teaching. The purposes the mentors reported were teacher interaction, real classroom experience, classroom management, student interaction, proving ground, affective development, and enculturation into school culture. The mentors viewed their role as one of providing experiences (and environment), modeling, facilitating reflection, and sharing knowledge. Absent from the majority of the responses was any specific reference to mathematics or helping student teachers develop content knowledge. “This result seems to imply that mathematics is somewhat removed from the [mentor teachers’] perceived roles” (p. 113). Leatham and Peterson’s study supports that mentors view themselves as having a number of roles in helping student teachers learn to teach, which further questions a systematic approach to mentoring. This study also supports previous results that mentor teachers tend not see themselves as having a role in the development of content knowledge.

There are themes that emerge from these accounts of mentoring, including that mentors take a variety of approaches, they view their role largely in terms of emotional support, and there is a lack of focus on mathematics in their conversations. Research on mentoring is largely descriptive and tends to use interviews and analysis of post-lesson conferences as sources of data, with the exception of survey responses in Leatham and Peterson’s (2010) case. In the present research I used many of the same techniques employed by previous researchers; however I also conducted classroom observations to add a different perspective to current mentoring

literature. Absent in each of these studies are classroom observations, which provide an important context for the conversations between mentors and student teachers. Furthermore, classroom observations provide insight into the impact that the mentoring relationship has on the practice of the student teacher.

CHAPTER 3

METHODOLOGY

In my experience as a university supervisor with approximately 30 student teachers, I have taken an interest in the variety of foci that student teachers and mentors choose to pursue and the contexts that cause the foci to emerge. I believe being a student teacher is a unique experience in a teacher's development because it is rare to have an opportunity for feedback and communication with an experienced teacher on a daily basis, or even several times a day. Student teaching is a time when preservice teachers observe an experienced teacher as he or she plans for different levels of students, implements the lessons, and changes his or her practice based on what occurs in the classroom and throughout the day. Student teachers can model their mentor's techniques, experiment with different approaches to teaching, and receive feedback that develops as an ongoing conversation because of the extended nature of the student teaching experience.

I chose to take a different approach from other studies of student teachers' experience, which have typically considered survey responses or interviews that focused on changing beliefs. To study the mentoring relationship I used a qualitative research design with case studies to do an in-depth analysis of three pairs of mentors and student teachers. Data were collected from a variety of sources including surveys that gathered background information, interviews that focused on mentoring rather than beliefs about teaching and learning in mathematics, field observations, lesson plans, and evaluations. These data were analyzed using constant comparison techniques to develop themes that emerged during each pair's experience.

Design of the Study

The present study used case studies to explore the methods mentor teachers use with their student teachers and their influence on the student teachers' practice. Yin (2003) described case

studies as empirical inquiry that considers contemporary phenomena in a real-life context where the boundaries and context are not clearly evident. He went on to say that case study methodology is appropriate when there are many more variables of interest than data points and, hence, the study requires multiple sources of data that converge in a triangulating fashion. The present study was field-based research that attempted to analyze human relationships. The relationships between the student teachers and mentors changed and evolved over time as the student teachers gained more experience, their teaching ability became more dynamic, and new issues arose in the classroom. Through this process, there was a constant need to redefine the role of each person involved, which made the interaction between the mentor and student teacher highly contextual.

Given this complex situation, a number of sources of data were necessary to paint an accurate and honest picture that resembled the reality of mentoring relationship between two people. The data were consequently collected for the duration of the student teaching experience. Prior to the student teaching experience, I provided each participant with a survey to gain some insight into her beliefs and expectations. Based on their responses, I conducted a series of semi-structured interviews to inquire into their beliefs about mentoring student teachers and provided an early indication of the mentors' approach and the student teachers' expectation for their mentors. Observations, recordings of oral feedback on planning and instruction, and written feedback from the mentor also provided data for the evolving relationship.

Participants

The participants in this study were three preservice teachers completing student teaching and the mathematics teachers who were their mentors. Student teaching was a 12-week experience in which the student teachers spent the full school day observing, planning, and teaching, as well as responding to other requirements of the mentor teacher. The student

teachers were also required to participate in a university-based seminar that met periodically while the student teachers were at their school placements and for two consecutive weeks after their placement. Because this research focused on the relationship between the mentor and student teacher, no data were collected from this seminar. The student teachers were assigned a supervisor from the university who gave feedback on lesson plans, maintained e-mail contact to discuss issues that arose, conducted 3 observations of the student teachers implementing lessons, and took part in the final evaluation of the student teacher. Each of the participants had a different university supervisor and I did not supervise any of the participants.

All three of the student teachers in this study; Lisa,² Meg, and Judy, were Caucasian females in their senior year at the University of Georgia. Lisa was paired with Marge, Meg with Lois, and Judy with Jane. Prior to student teaching, they had completed three semesters in the mathematics education program where they took many of the core courses in curriculum, instruction, learning, and technology, with one another. Thus, the preservice participants had a similar preparation for learning to teach. All three were highly regarded by teacher educators in the mathematics education department and excelled in subject-specific courses. The semester before student teaching the preservice participants took part in a field-based course I helped instruct where they conducted focused observations (e.g., teacher questioning, task implementation), worked with small groups, and taught at least one lesson in local schools. No data were collected during this course; however, I was able to observe the participants and considered them to have strong content knowledge and their reflection papers showed they were able to critique teachers' practice, including their own. These were important considerations

² All names in this study are pseudonyms.

because I wanted participants who were able to communicate effectively and handle a variety of situations that arose in teaching.

This research took place in an urban school district in the southeastern United States. The research sites included one middle school and one high school. The middle school fed into the high school and hence had similar demographics of approximately 50% African American, 25% Caucasian, 15% Hispanic, and 10% from other backgrounds. The middle school made Adequately Yearly Progress in accordance with the No Child Left Behind Act of 2001 (NCLB, 2002) the previous year; however, the high school did not.

Purposeful selection was used to provide different cases of mentoring approaches and levels of mathematics. I had prior relationships with each of the mentors in graduate classes or as a student-teaching supervisor. Marge was an experienced educator who had taught high school for 7 years, middle school for 5 years, and worked with 3 teachers prior to this research. She completed her bachelor's degree in psychology, a master's degree in mathematics education, specialist degree in mathematics education, and was gifted-certified. Marge was one of two eighth-grade mathematics teachers in the school and taught one on-level section of Math 8, three sections of Advanced Math 8 (Georgia Department of Education, 2005), and one section of a remediation class. Judy was an African American female who had taught for 23 years at all levels including 16 years in high schools and she had mentored more than 10 student teachers. She had a master's degree in education, was gifted-certified, and working towards a specialist degree in mathematics education. During this study Judy taught three blocks of Accelerated Math I (Georgia Department of Education, 2005). Lois, at the same high school as Judy, taught one block of Accelerated Math III, on-level Math III (Georgia Department of Education, 2005), and a Support class that was intended for additional remediation. Lois had taught in middle schools for 8 years and at this high school for 2 years. She had completed a master's degree in

education as well as a specialist degree in education. Lois had mentored three student teachers while in middle schools and one at the high school level.

Data Collection

Data collection for the case studies was conducted for the duration of the student teaching experience as well as interviews conducted before and after the experience. Table 1 provides a timeline for the different data sources.

TABLE 1: Timeline of Data Collection

December 2011	<ul style="list-style-type: none"> • Surveys sent and collected from student teacher and mentor • Initial interviews
January – March 2012	<ul style="list-style-type: none"> • Conduct observations for 3 one-week periods for each pair • Audio record discussions between mentors and student teachers • Collect written feedback from mentor • Collect documents developed by student teachers including lesson planning notes, Smartboard files, and classroom activities
April 2012	<ul style="list-style-type: none"> • Final interviews • Collect final evaluations

Surveys. The initial surveys for the mentors and student teachers (see Appendix A and Appendix B, respectively) were based on the work of Leatham and Peterson (2010). The first nine items of the mentor survey and five items of the student teacher survey gathered data on the participants' education and teaching experience (e.g., classes taught and number of years teaching). The other items asked the participants to consider the most significant contribution the mentor would make during the experience and to describe the approach to mentoring and giving feedback (in the case of the mentor) or receiving feedback (for the student teacher) that was expected.

The mentor-student teacher relationship was dynamic and changed over time. Therefore, it was important to capture the expectations and beliefs about the experience that was to come for the participants. These surveys helped identify significant features in the relationship and interactions between mentors and student teachers. Furthermore, the responses from these surveys were used to develop the interview protocol.

Observations. During the course of the student teaching experience there were three 1-week observation periods. I attended each day and took field notes and audio recordings of the conversations that took place between the mentor and student teacher throughout the day. There were a number of purposes for this data source including triangulation between the survey and interview responses, providing a context for the feedback, and observing the nature of the feedback over a continuous period of time.

As the student teacher was instructing classes, I looked for evidence of the feedback in the student teacher's actions. As the researcher, I observed the cycle of the mentor providing feedback prior to implementation, the lesson as the student teacher enacted it, and the conversation afterwards where the pair discussed the lesson. As themes emerged from discussions between the mentor and student teacher or developing struggles for the student teacher (e.g. timing of lessons), they became foci for classroom observations.

Interviews. Each participant in the study took part in two interviews: one prior to the experience, and the other upon completion of student teaching. The first interview focused on the participants' expectations for the experience, including expectations for giving or receiving feedback on planning and instruction. The final interviews reviewed the experience and attempted to capture how the feedback changed and the themes that developed during the time that the student teacher and mentor spent together.

The protocol for the first interview (see Appendix C) with the mentor teachers was generated from responses I received while doing pilot work during the previous semester and with the survey responses from the participants. The purpose of these interviews was to add specificity to the answers the participants reported on the survey and to inquire into the expectations for the student teaching experience. In the case of the mentors, this data was important to determine if the experience was typical compared to their work with other student teachers. The student teachers were asked similar questions (see Appendix E), and their responses were important for establishing whether their expectations were similar to those of their mentors. In both cases the student teachers and mentors were asked about their views on the purpose of student teaching, lesson planning, feedback based on lessons, differentiating instruction, and how they expected the feedback to change over the course of the experience.

The protocol for the final interviews (see Appendices D and F) asked the participants to discuss themes that emerged and how the pairs decided which aspects of teaching to focus on. The participants were asked about how the mentor assisted in planning and evaluating lessons as well as classroom management issues that arose. The participants were asked to discuss how the nature of the feedback changed over the course the experience and what the mentors did that was both effective and ineffective.

The questions in each protocol were intended to be guiding questions rather than an exhaustive list. Semi-structured interviews allowed for flexibility to following up on the responses of the participants when I wanted to clarify something that was confusing or lacked detail (Seidman, 2006).

Written feedback. Each pair in this study was provided with a composition notebook to write down informal observations while the student teacher or mentor was instructing the class. These informal observations were used as prompts for later discussions as well as a reference for

the pair. Both the mentor and student teacher made notes in the notebook, and both had access to read the observations the other made. At the end of the semester, I collected the notebooks as a data source that chronicled the feedback throughout the experience.

I also collected the final evaluation submitted to the university, which was a collaborative assessment by the mentor, the student teacher, and the university supervisor. The evaluation included items on curriculum and content, knowledge of students and their learning, learning environments, planning and instruction, assessment, and professionalism.

Data Analysis

The data were analyzed using constant comparative techniques (Glaser & Strauss, 1970; Strauss, 1987). In constant comparison analysis the researcher codes initial data for themes that are supported or come into question as more data are gathered. Some themes were combined to create new categories as supporting data were collected and they became intertwined. Other themes were divided as divergent data emerged, and some were discarded if data failed to support the theme. Each theme was related to the research questions, then compared to existing theory to question, support, and expand upon previous results. Constant comparison is particularly useful in studies such as this one where there were multiple data sources gathered over a long period of time.

The initial interviews were coded for themes with particular attention to items that the mentors and student teachers discussed in a similar manner or had differing views on. I used these themes as foci when I was initially conducting observations. During the observations some themes were condensed and other themes emerge. In the case of Marge and Lisa, timing was identified as an area of concern early in the student teaching experience and it continued to play an important role in the pair's experience.

When the participants had completed the student teaching experience I transcribed the final interviews and used the existing categories to code the transcriptions. I began to build each case with the observation data from my field notes and the notebooks from each pair. I then used the data from the coded transcriptions to support or contradict the categories that were emerged from the notes. I reviewed the audio recordings and final evaluations for relevant data. I then framed the themes within the research questions and compared the cases with current research to discuss how this research was related to other researchers' findings.

After each case was developed, a cross-case analysis was conducted to look for common themes and patterns in the case studies. During this time the participants were given an opportunity to read their cases to see how well my interpretations represented their perceptions of the experience.

CHAPTER 4

THE CASE OF MARGE AND LISA

Marge and Lisa taught different levels of eighth grade mathematics in an urban middle school. Their schedule consisted of five periods lasting 70 minutes each and one remedial class lasting 55 minutes. They had planning first block, on-level Math 8, two periods of Advanced Math 8, Extended Learning Time, and one more period of Advanced Math 8. Throughout their time together the schedule played an important role in the mentoring relationship.

Prior to the experience, Marge and Lisa had similar expectations for the Marge's role as a mentor and foreshadowed some of the issues that became themes for the pair. In her survey, Lisa reported the following:

I feel the most significant contribution my mentor teacher will give me is the confidence in planning a lesson, executing it with my students, and then reflecting on how successful it was... I expect that my mentor teacher will continually be giving me feedback and criticism. If she has helpful tips I would hope to receive them before executing a lesson. In addition, whenever I teach, I would expect immediate feedback so I could have something fresh in my mind to reference and make connections with.

Marge had similar views regarding her role as a mentor and commented in her initial interview that student teaching was, "a place to make lots of mistakes with support, ... where someone can give you immediate feedback of how you could do something different and it's not just self-reflection, but you have someone to talk to" (p. 2).

Lisa was an independent individual who saw Marge as someone "that I can look to for guidance and support, and also not telling me what to do and not doing it for me... Let me learn on my own, and then be there to help me if I need it, and giving me feedback by saying 'this was good, this wasn't good'" (Initial Interview, p. 1). Marge was willing to "give her as much freedom as she can handle and to intervene when I feel like I need to. Not that I will let her flounder, but it's pretty shocking when you figure out all the stuff you need to do to teach"

(Initial Interview, p. 2). Lisa expected Marge to ask her to reflect on how her lessons went, and Marge also expected to talk about the lessons that Lisa taught as opposed to giving written feedback.

Questioning became a theme for this pair, as I discuss in detail in a following section. Before the experience Lisa anticipated one of her strengths would be coming up with questions to ask when the students were having trouble. During the preplanning days before classes started, the pair met and discussed the Connected Mathematics Project (Lappan, Fey, Friel, Fitzgerald, & Phillips, 1995) curriculum they would use. Marge commented, “The questions they have in there are great. That’s one of the things I said to Lisa this morning; you read these questions every time” (Initial Interview, p. 6). Marge said she used to put the suggested questions from CMP in her Smartboard files to make sure they were addressed, but she had been teaching with these materials for five years, so “I don’t rely on those questions very much any more because I know where they’re going” (Initial Interview, p. 6).

There were other issues that the pair anticipated in the initial interviews that became themes for the experience such as timing and formative assessments. Lisa was conscious of her role as a teacher and said, “Probably a weakness would be doing too much, like showing the kids too much. Another weakness is probably timing. I have no idea how long things are going to take, but I think that’s something that will come with practice” (p. 3). In her experience with previous student teachers, Marge said that one thing that young teachers usually struggle with is “false affirmation of success or understanding” (p. 14) because a few students are answering the questions and “they didn’t assess enough, or they didn’t see the terror on three of their faces” (p. 14). Young teachers have other issues as well: “Time management is huge, and not allowing enough time for a summary” (p.14). One of Marge’s goals when planning her own lessons was to do a Ticket Out The Door (TOTD) each day as a formative assessment. A TOTD is a

formative assessment where teachers have the students answer a question on a piece of paper before leaving the classroom after a lesson. The TOD is an exercise or open-ended question that gives the teacher an indication of what the students learned from a lesson.

The research questions for this study dealt with the nature of the relationship between student teachers and their mentors and the influence that relationship has on the student teachers' practice. The following sections are organized in a fashion similar to a teacher's practice. The first section deals with planning and gives a broad description of how the pair began planning together and the nature of the relationship as Lisa took over. The next four sections—questioning, timing, formative assessment and modeling—deal with classroom instruction, and give a more detailed look at Lisa's practice and the influence Marge had on it. The final section is on reflection. Much like Marge's feedback with other student teachers, which typically became "less specific and more generalized" (Initial Interview, p. 16), the nature of their relationship changed as Lisa gained more experience and new challenges arose. The following analyses attempt to capture the dynamics of those changes.

Planning

Given that Marge and Lisa had three sections of Advanced Math 8, they decided Lisa would begin teaching the last section of the day so she could watch Marge teach the lesson two times and then model Marge's instruction. Marge described the situation: "She would go from watching what I would do and try to replicate, which I don't know if that's the best way or not, but that's what we did" (Final Interview, p. 4). At the beginning of the experience, Marge did the pacing guides that were required by the school district and the pair discussed the lesson plans beforehand. Lisa said, "We talked about how she would present the information and how she had seen it work best, like teaching a new concept and going through one of those examples in the book" (Final Interview, p. 3). Two aspects of the lessons they focused on were the questions that

were provided by CMP and adding additional questions, which became important in Lisa's practice. Marge complimented the questions in CMP and said,

I think that's what I said to Lisa, just cut and paste those questions. That's what I did in the beginning. I would write all those questions down on sticky notes and I would pick the ones that I wanted to do and order them out. (Final Interview, p. 5)

Lisa reported a similar process by saying, "We would talk about what parts we were doing and how she introduced them and what questions she used and which ones she edited and include some supplemental stuff..." (Final Interview, p. 7). She would write additional questions in her notes because

I would forget about them... I had to hand-write things. So I had to hand-write every lesson for every day colored, so I could quickly look down and find my questions, which were a certain color. So I would just look down and say 'Ok, I want to ask them that question.' If they're struggling, I could ask them another question. (Lisa, Final Interview, p. 8)

When Lisa began to take responsibility for teaching the other sections of Advanced Math 8, and Marge gave Lisa less direction and advice on how she had approached the topics in the past, Lisa's reaction was as follows, "At first I was like 'What?' I had no idea, but then it made sense why. How else am I going to learn how to teach?" (Final Interview pp. 3-4). Lisa also became more involved in long-term planning with the pacing guides where Marge would show Lisa the pacing guides from previous years and "towards the end, we were more involved in a conversation like, How long did this take last year? Can we still take that amount of time? Do we need to shorten something up" (Lisa, Final Interview, p. 6). These were common conversations between the two because Lisa would not get through her entire lesson plan and they would have to adjust the pacing of a unit, which occurred "quite often" (Lisa, Final Interview, p. 4). These timing issues had a large influence on the planning decisions and later formative assessment played a role. The following quote from Marge in her final interview illustrated these points succinctly:

So we would have the conversations, or try to, then tweak things. I would try to give some leading questions like, 'What do we really want these kids to know?' A lot of things were about timing. That's the hard part... Her timing would get off, which I think is normal, but she'd have to cut the lesson back, and that is hard to do when you put all this energy into making a great lesson. Then to cut the lesson back and figure out where to go from there and pick out what the kids really have got to know... The other thing we got into is how do you know that they know? How do we make sure the kids know the material? (p. 3)

In the next section I consider Lisa's questioning, which was a focus from the beginning of the experience for Marge and Lisa. Marge emphasized the CMP questions as they went through the material together and discussed the lessons, and questioning continued to play an important role throughout their time together.

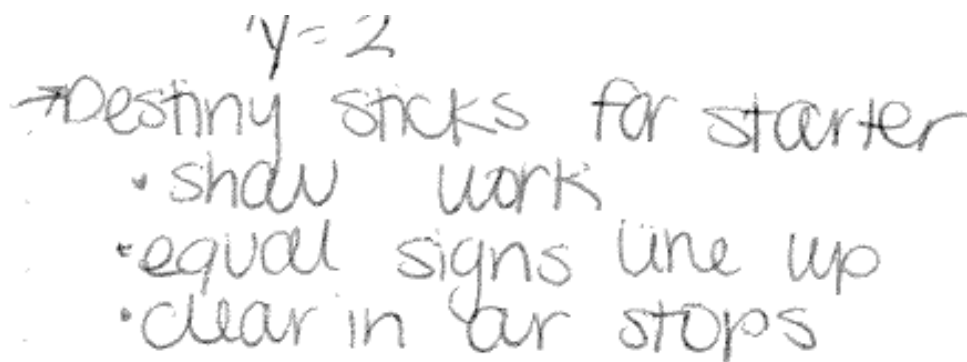
Questioning

The focus on questioning was not only due to the way that Marge and Lisa planned, but also something they observed in each others' practice. During the first week I was doing observations of the pair (this was the second week of the semester), I asked Lisa for three things she noticed and would like to emulate in Marge's teaching. She responded that Marge "creates discussions between students through her questions" (Observation Notes). Lisa expanded on this idea in her final interview when she explained:

One big thing that I took away was her questioning. It got them thinking in the right direction. It created a really comfortable environment, and she would have these conversations or discussions going on in class that were awesome. A lot of the times, after a few questions, she would sit back, and these kids would be talking about math in a way that I didn't anticipate. So she could get them to have conversations all together. I think that was the biggest thing, and I think that I am striving to do that. ... I think that her questions were open enough so that. ... They were very balanced between open and closed so that the kids knew where to go with it. It wasn't like "How? Why? Talk about it. ..." It made them anticipate what she was expecting and what would be good. It gave them the direction to think and the confidence so that they knew that were going to be answering in the right direction, then they would go for it. So part of it had to do with the questions and also part of it had to do with them feeling really comfortable in the classroom because it was a good environment. (p. 17)

Marge was also complimentary of Lisa's questioning ability. In the notebook that the two shared Marge made comments like "great questioning ($x^3 \neq x$) + connection to linear equations" and "the way you explain something and hear from them is great—you do not let on that they are right or wrong!!" At this point of the experience, Lisa was instructing only part of the lessons such as warmup exercises and going over homework. When I asked how Lisa did on the previous Friday, the first day she led part of the instruction, Marge further commented, "She did great. She asked great questions" (Observation Notes).

When Lisa took over the instruction of the last Advanced Math 8 class of the day, she used many of the same questioning techniques that Marge had used in the previous two classes. Both would use destiny sticks during the warm-up exercises, which were sticks that had each students' name and they would draw them to choose who would answer a question. In Lisa's notes, she had the items in Figure 4.1.



The image shows a piece of paper with handwritten notes in cursive. At the top, it says 'y = 2'. Below that, it says '→ Destiny sticks for starter'. Underneath that, there is a bulleted list: '• show work', '• equal signs line up', and '• clear in car stops'.

Figure 4.1. Lisa's handwritten notes.

Another technique that Marge modeled and Lisa used in her practice was to have a student answer another student's questions. This was typical when the question dealt with a common misconception such as when a student asked, "What is $x^2 + x^5$?" (Observation Notes), and Lisa had the student call on other students for the answer. Another way Marge dealt with common errors and Lisa used in her practice was to write a number of answers on the board and have the

class decide which answer was correct. For example, the class was discussing the area of a pool with a rectangular part and a semicircular region (Figure 4.2). Lisa asked, “What is the area of the rectangle” (Observation Notes) and received the answers, $8x$ and $8x^2$. She wrote both on the board, had three students justify which they thought was correct, and then erased the $8x$.

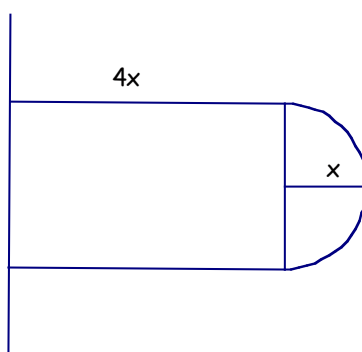


Figure 4.2. Diagram of a pool.

Like many novice teachers, Lisa had issues with listening to students as she took over more of the teaching responsibilities. She would ask a question, the students would supply answers until the correct one was said (or the answer she was looking for), and then she would highlight it. Examples of this include Lisa asking, “What properties did we talk about yesterday?” Different students said, “Addition,” “Subtraction,” and “Distribution,” at which point she pointed to the student and said, “Say it louder” (Observation Notes). During the same lesson she asked “What is the property we’re using when we have the 4 in front of the parenthesis [in the example $8 + 4(x-1)$]?” (Observation Notes). A student said “multiplication,” and she waited until someone said “Distribution,” which she acknowledged as the answer. In this case, she either did not hear the student say *multiplication* or was waiting for someone to say *distribution*. On another day, she was talking about an area model for multiplying a monomial and a binomial and asked, “What’s x times x ?” She got the answers “ x ” “ $2x$ ” and waited until someone said “ x^2 ” before filling in the appropriate area of Figure 4.3.

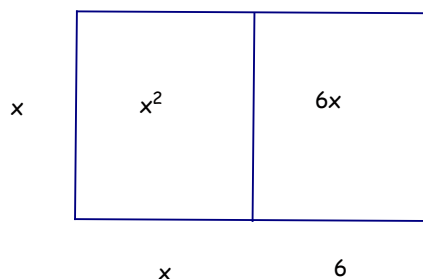


Figure 4.3. Area model for distribution.

Lisa's view on the role of questioning influenced her practice as well. In her final interview, she said, "In the beginning I was approaching it [questioning] where I would never tell. ... I never wanted to tell them anything, so I would keep questioning" (p. 10). This view eventually contributed to issues of timing and not addressing all the material in her lesson plan when she took over the planning and instruction for all three of the Advanced Math 8 classes. To address some of these issues, Marge would interject, which Lisa described in her final interview.

If I was trying to ask questions, and it wasn't going anywhere, a lot of times she would say "Can I ask a question?" So she would come in with a question that would have the thinking in it. ... Sometimes she would ask me a question as a student, and other times she would ask the class. She probably asked me more questions in the middle when I was taking over. It was like she was just another student in the class. Her asking the rest of the class was probably when I wasn't completely in control. She would interject, but in a natural way." (p. 11)

An episode where Marge interjected and added some "thinking" that Lisa was referring to occurred when they were discussing parallel lines and transversals. A student asked, "So if you have two lines that go through parallel lines, are they both transversals?" (Observation Notes). Lisa answered by returning a question to the class by saying, "Can someone tell me what the definition of a transversal is?" (Observation Notes). Another student explained that a transversal was "a line that goes through two parallel lines," and Lisa asked "What do you mean *goes through*? What is a more mathematical word?" (Observation Notes). The student replied

“Intersects,” and Marge interjected by drawing Figure 4.4 and asking, “So this isn't a transversal according to your definition?” (Observation Notes). The student changed his definition to include two or more lines. After this exchange Lisa returned to the original question, and the class was able to conclude that both lines intersecting the parallel lines were transversals.

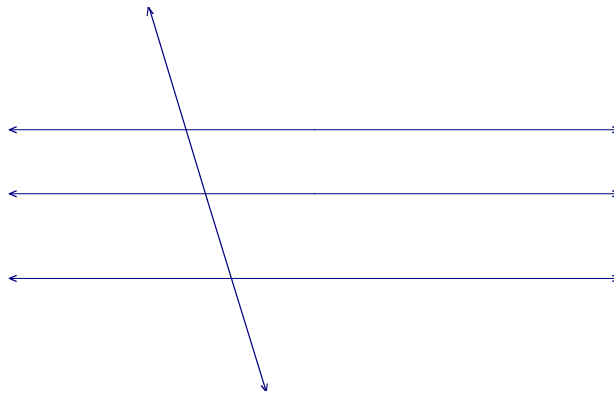


Figure 4.4. Parallel lines cut by a transversal.

During the last observation cycle, Lisa was teaching a lesson on finding the equation of a circle, and Marge interjected in a different way, which drew a connection to material the students had encountered the semester before Lisa began student teaching. The class was having discussion about how the point (x, y) in Figure 4.5 appeared to be at $(3, 4)$, and they used the Pythagorean Theorem to check if $3^2 + 4^2 = 5^2$. Marge raised her hand and said, “I have another question, and I know you [Lisa] weren’t here for this. So should I have to check that with the Pythagorean Theorem?” A student answered, “No, it’s a 3-4-5,” and Marge responded, “What’s that called?” The student followed up with the answer “a Pythagorean triple” (Observation Notes).

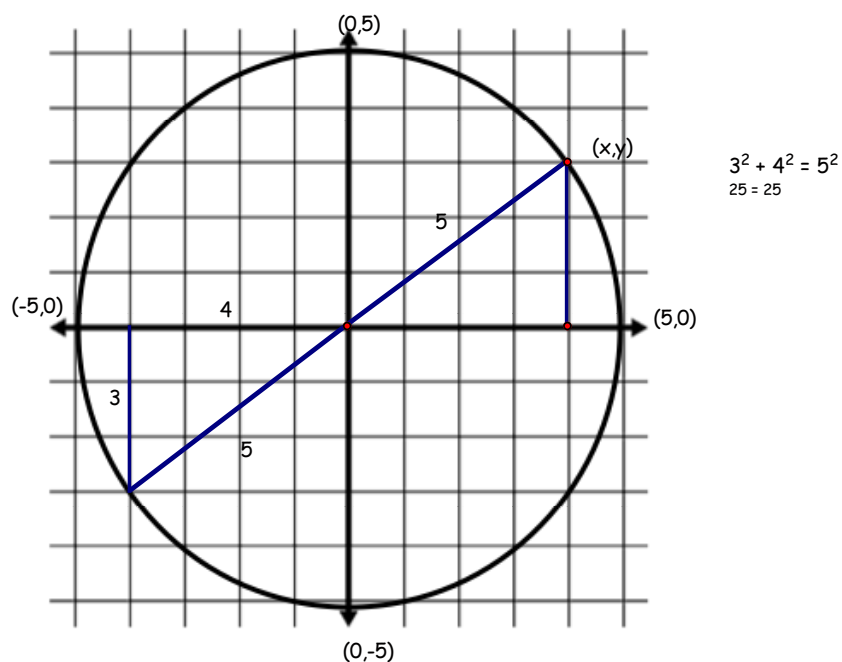


Figure 4.5. Developing the equation of a circle.

In summary, Marge and Lisa focused on questioning from the beginning of the student teaching experience. In the beginning they discussed the questions in the CMP curriculum material and noticed what they considered effective questioning by the other. Lisa also used many of the same techniques such as considering multiple answers and redirecting questions back to the class. Once Lisa assumed more responsibility for the instruction her dedication to questioning began to cause timing issues with her lessons and Marge found ways to interject and add depth to the topics Lisa was discussing and focus the students' attention on important ideas.

Timing

In this section I discuss in more detail how Lisa's questioning led to issues of timing and how the pair attempted to address this issue. The section goes beyond questioning to include other planning factors and their implications. While Marge and Lisa were each teaching Advanced Math 8 in the second week, Lisa assigned the homework with 15 minutes left in the

final period. They talked about how important the summary was since Lisa did not address it in either section she taught that day. One of the challenges for Lisa was deciding what parts of the lesson to emphasize.

When Lisa began teaching the Advanced Math 8 classes in the third week of student teaching, Marge noticed that the timing issues continued. In their notebook, Marge wrote the comments, “Time MGMT” and “Maybe get kids back in rows before the closing.” In a discussion early in the third week, I asked Marge whether she used the warmup exercises to review the previous lesson or transition into the next lesson since Lisa was using them to transition into the current lesson. Marge answered, “Well, both. You need to come back to what you don't finish. I'm not seeing enough of that. ... I feel like we're not on the same page with this” (Observation Notes). Their discussions after school centered on not spending too much time on the homework and warmup exercises.

The day after this discussion was the first time I observed Marge taking an active role to influence the timing of the lessons during Lisa's instruction. During the first Advanced Math 8 class of the day, the warmup took approximately 20 minutes to complete and discuss. When they transitioned to the homework, Lisa asked, “What ones do you have questions on?” (Observation Notes), and multiple students called out numbers. Marge responded, “We only have time to do one.” The exercise the class discussed from the homework was to factor $28ab^3c + 21ab^2c^2$. After a student provided the answer $7ab^2c(4b + 3c)$, Lisa highlighted that they could check their answer by distribution. A student asked, “How do you distribute $7ab^2c$ to $3c$ ” (Observation Notes). Lisa answered, “This [points to $7ab^2c$] is the greatest common factor” (Observation Notes). This was another illustration of Lisa's struggle to listen to student's questions at that point. Another student asked, “Can you give us an example of a word problem where we would use this stuff?” and Lisa responded “We don't really have time for that right

now, so can we get back to you” (Observation Notes). Marge followed Lisa’s answer by saying, “We’ll do a warmup with a word problem tomorrow” (Observation Notes). Marge then commented that they had to move on, and this was 17 minutes before the end of the period. As the students were working on another exercise in class, Marge and Lisa were talking on the side of the room. I observed Marge say, “It’s ok. I think you’re just going to have to start using timers, and if it goes off then you have to finish it for them” (Observation Notes).

The following period, Lisa had a 5-minute timer on the Smartboard as the students walked in. The timer was set for working on their Big 20, which was a weekly worksheet with 20 problems on it reviewing topics from previous units. Once the 5 minutes had elapsed, she told the students it was time to work on the starter exercises, which she did not use the timer for. Allowing the students too much time to work was not as significant an influence as the time spent discussing the answers. The students worked on the warmup exercises for 7 minutes. By the time the class had finished going over the warmup and homework, there was 35 minutes left of a 70-minute period. The final period of the day was very similar. Lisa used a timer only for the Big 20, and they were finished going over the warmup and homework with 40 minutes remaining for the lesson.

Later that week, the pair had a discussion, and Lisa talked about how she was feeling more comfortable in front of the class, but admitted that she did not know when to stop questioning the students and move on by either giving them a hint or telling them an answer. Marge was encouraging and said, “You’ll learn that” (Observation Notes), and they discussed having a sign (e.g., Marge rubbing her eyebrows); however, I never observed the pair use any type of sign.

The timing of Lisa’s lessons continued to be an issue for the pair. Between the second and third observation cycle, Marge wrote in their notebook, “TOO much time on starter and

homework. ... We need to figure out how to manage that.” Data further supported this struggle with timing during the lesson on finding the equation of a circle. The students were estimating the y -value as Lisa was giving them different x -values using Figure 4.5, and they were nearing the end of the period. Marge was sitting at a table in the back of the room with me as Lisa told the students, “Ok, try 2” and gave the students some time to look at the graph before calling on a student who said, “About 4.5” (Observation Notes). At that point it was 4 minutes before the end of the period, and Marge quietly said to me, “We’re out of time.” Lisa was still in the front of the class and with 2 minutes left in the period said, “Do you want to do another one? Try $y = 2$.” Marge said to me again, “We’re out of time” (Observation Notes). Marge and I discussed Lisa’s timing in the hall between periods, and Marge said, “Her intentions are great, but she lets [certain students] sidetrack her.” I reminded her of a previous conversation when we had talked about Lisa trying to question through the discussion and not knowing when to move on. Marge said, “You hit the nail right on the head” (Observation Notes).

It is important to note that Lisa was aware of the issue and making an effort to address it in her planning. She had prepared the two summaries for the lesson and they are shown in Figure 4.6. The first was intended to address the material in the lesson if they did not develop the general equation of a circle, and the second summarized the lesson if they did.

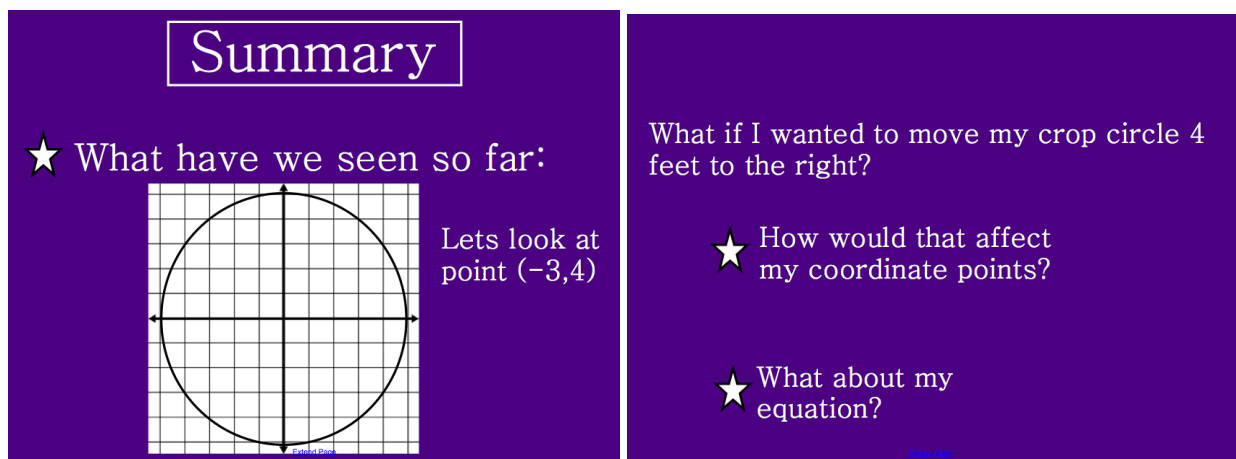


Figure 4.6. Two summaries for a lesson.

Lisa made adjustments in her instruction during the next two sections of Advanced Math 8. In both classes she asked, “What kind of triangle is it?” in Figure 4.5. The students were able to tell her it was a right triangle, and more specifically, “a 3-4-5 triangle” (Observation Notes). She continued the line of questioning with, “So do we need to do the Pythagorean Theorem?” and “What are they [the sides] called?” (Observation Notes). In both sections, Lisa was able to reach the first summary and develop the equation $x^2 + y^2 = 5^2$.

The lessons with the equation of a circle provided important data because they highlighted a number of reoccurring aspects of Lisa’s student teaching experience. These lessons happened during the third observation cycle, which supports the observation that Lisa was still struggling to determine how long different parts of each lesson would take. However, Lisa was able to make adjustments that incorporated Marge’s contribution, as well as reaching the summary. The fact that Lisa had two summaries provided evidence that the pair had made them a point of emphasis. I discuss the role of summaries further in the next section.

In her final interview, Lisa talked about the ongoing struggle with timing by saying, “I think one thing that came up a lot was the balance between how to get your students to discover math as opposed to telling them when they're not catching on” (p. 1). Marge put it as follows:

That [timing] was a hard thing for her. That's when we really started to have the conversations of "What do they really need to know and how are you going to know that they know it?" You can lecture, lecture, lecture all day, and they might not get anything, and if you don't ever ask them, then you'll never know. She was doing these great lessons, asking great questions, but would never get to the summary. So it was like "Did they get it?" I think that was one thing that towards the end, I was trying to help her focus on. We had conversations about "How do we get that to happen?" (p. 5)

An example relevant to these comments occurred in a lesson on midpoints. Marge and Lisa talked about how it was important for the students to be able to find a point given the midpoint and one other point because it was common on the Criterion-Referenced Competency Test (CRCT) that the students had to take to advance into high school. The lesson began by using a problem from the previous day, which dealt with the distance formula. In the problem, the students estimated the midpoints of different sides of a quadrilateral, and they were going to check their estimations with the distance formula and the fact that the distance from the midpoint to each vertex should be the same. Lisa planned on using these answers to have the student make conjectures about how to find the midpoint between two points, but it took longer than expected to find the distance, and she eventually said, "In the interest of time, I'm just going to show you how to find the midpoint" (Observation Notes). She then showed two examples where she found the midpoint between (6, 4) and (3, -4) and another given the point (-2, 3) and the midpoint (1, 0). The students were given three exercises to try on their own, they got to the summary, and they had time to do a Ticket Out the Door.

By the end of the student teaching experience, Marge complimented Lisa's use of timers how she promoted student engagement through questioning. In the final week of student teaching, another student teacher observed Lisa during a class on solving a system of equations that contained $8.80 = .01p + 0.05n$ and $2p = n$. During their conversation after the lesson, Lisa made an insightful comment showing her maturity in recognizing what parts of a lesson to emphasize and its influence on the timing of lessons:

We only had time to go over one, but it was a valuable experience because they got to talk about it. [A student] did it, and I asked him why he did substitution. The group did it both ways, and wanted to write both, but we didn't have time to show both.
(Observation Notes)

To summarize this section, Lisa began to have trouble finishing her lessons when she started planning and instructing all three sections of Advanced Math 8. Lisa's tendency to address every question the students asked without knowing when to move on, her desire to question the students rather than answer their questions directly, and her answering a large number of homework exercises contributed to the timing issues. The result of these tendencies caused the class to rarely address the material in the lesson or the summary. To manage these issues, Lisa began using timers to limit the time the student spent working, Marge interjected at times to refocus the students on the material in the lesson, and the pair had discussions that focused on what the most important aspects of each lesson were.

Formative Assessments

As I discussed in the previous sections, timing was a challenge for Lisa, and she was not getting through all the material in her lessons. Questioning was her primary source of formative assessment, and as Marge described in her final interview,

Especially on our schedule, if you're trying to remember informal observations in second period and you don't have a break until 5 periods later, you can't do that. You can't compartmentalize—well, I can't—all those things all day long, so you need something concrete. (p. 8)

Another informal assessment technique that the pair used was to have students raise their hands and put up one, two, or three fingers to indicate how well they thought they understood some material. I first observed this technique during the second observation cycle when the Math 8 class was starting to graph parallel and perpendicular lines. Lisa explained how to graph a line given a point and the slope, and Marge did not think that the class followed the explanation. She asked,

How many people got that? How many need to see it again? ... If we were to take a test right now on graphing, how many people would get a 100%? 50%? 0%? Raise your hand with three fingers if you would get a 100%. (Observation Notes)

Lisa explained the example again and used the same technique of having the students raise their hand with fingers to indicate their level of understanding. She used this technique regularly in her instruction after this episode.

During the second observation cycle, Lisa and Marge had a conversation that changed the nature of the mentoring relationship and Lisa's practice. In her final interview, Lisa discussed the conversation by saying,

[Marge] brought it up. ... We were getting ready for a test, and she was like, "Well, what do they know?" I said, "I taught them about the stuff." ... And she was like "Well, what do they know?" And I said, "Well, I think they know how to do this." (p. 9)

This conversation occurred in the eighth week of student teaching, and Lisa had been planning and teaching all the classes for a number of weeks. Their prior conversations that I had observed focused on different ways to approach mathematical topics and how to change the lessons to improve the timing. Marge admitted she was feeling anxious because she had not been able to look at Lisa's lesson for 4 days. She had been out one day due to sickness, and she was the coach for a mathematics competition that had occupied her schedule the previous day. The Advanced Math 8 classes were reviewing for test on properties of parallel lines and transversals, and Lisa had taught the last lesson of the unit without Marge being present. During their planning period and before the review, Marge asked Lisa to summarize what happened while she was gone and asked her what the students understood about the material in the unit. Lisa talked about how much material the class had covered the previous day and how she had difficulties assessing what the students understood and what topics they would have trouble with, as illustrated by Lisa's previous quote at the end of the last paragraph. In her final interview, Lisa continued her discussion with, "I think it was realization on my part: 'Oh, I don't actually

know what they know.’ I don’t actually know what they understood. They told me they did, but. ...” (p. 9). The pair decided to push the test back one day so they could assess what the students knew and spend an additional day targeting the topics that were the most difficult.

During the first section of Accelerate Math 8, while the class was playing the review game Lisa had planned, Marge and I sat at a table in the back of the room and discussed how it was easy for student teachers to think that the students understood the information because a few students were answering the questions. Marge went further and gave some insight into her mentoring approach and her own teaching. She said, “You can’t just give a test without knowing how they’re going to do. I don’t want them to bomb it and lose faith in her” (Observation Notes). During the review game, Lisa commented to the class, “This is not only for you to practice, but it’s also for me to find out where you are, so when you’re going through it if there’s something that you don’t understand, make a note” (Observation Notes).

The review game was called Whack-a-Math (a spinoff of Whack-a-Mole), and the students threw a soft ball at the Smartboard to choose a review exercise for all the groups to work on. Lisa used timers to limit the time the groups spent working on each exercise and discussed the answer to each exercise by asking questions and writing the answers on the board. Each class completed 3–4 exercises out of 18, and after all three sections of Advanced Math 8 had played the game, I asked Lisa and Marge if they thought the review game gave them a better assessment of what the students understood from the unit. Lisa thought it did by the end of the day because the classes had completed each type of exercise, although no class addressed them all. For the same reason, Marge did not think the game was effective for assessing the students’ understanding because each class addressed only a small number of topics.

The following day the Accelerate Math 8 classes reviewed by using a menu activity where the students had to complete an initial exercises in six sections; then they got to choose

which four sections to do the follow-up exercises similar to the initial one. This activity allowed the students to practice the topics where they felt weakest. The lesson concluded with the students doing the following TOTD:

1. Put name
2. For each number ask yourself, “If I were to take a test right now on this, would I make a 100%? 50%? 0%?”

Topics

1. Classifying and naming angles
2. Identifying angle pairs
3. Identifying angle parts
4. Proportions of segments
5. Write equations of parallel and perpendicular lines
6. Angle word problems.

At the end of the day, I asked Lisa how the menu activity compared to the Whack-a-Math, and her answer was more specific than the previous day, stating that the students were struggling with ratios, word problems, and identifying angle pairs. The combination of seeing what exercises the students chose from the menu activity and the TOTD supplied Lisa with information that improved her assessment of the students’ understanding.

After this conversation, the use of TOTD increased to nearly every lesson. The pair’s discussions began focusing on what they believed to be the most important parts of the lesson and how they could assess the students’ understanding, which is supported by Marge's comment in the previous section (p. 41) about the changing focus in their conversation. In Lisa’s final interview she talked about the increased use of TOTD as a formative assessment and said,

I just saw what the kids knew instead of waiting until a quiz or a test, or something like that and then them not knowing. So I knew right then if they understood it. It all made sense after not doing it and then doing it. (p. 9)

I pointed out that formative assessments, along with questioning, were topics that we discussed the semester before when I was an instructor of part of her methods courses. Her response was the following:

The thing that sticks out the most to me is formative assessments. We talked [about] them and that's fine, I got a definition and some examples, but I didn't know. ... We were told they were important, and we were told why we use them, but I didn't see the value in them until I didn't do them and then I was like "Ok, wait a second, this is going to help me." That was one of the biggest things that sticks out in my mind. (p. 8)

The conversation before the test was critical in Marge and Lisa's relationship because it changed the nature of the conversations and Lisa's practice. After this conversation, their discussions centered on what parts of the lesson were more important than others and determining how they (Lisa in particular) could gather evidence for what the students understood from the lesson. Lisa began using the informal techniques of having the students show a number of fingers to indicate their confidence in a topic and of using the more concrete TOD to assess students each lesson.

Modeling

In this section, I provide data on the ways Marge modeled mathematics in her instruction and subsequent similarities in Lisa's practice. From the first observation cycle, there were pedagogical techniques Marge frequently used, including consideration of a number of students' answers, using Venn diagrams, and highlighting, that Lisa used throughout the experience. During the unit on factoring, Marge instructed the first two sections of Advanced Math 8 and Lisa instructed the last period. As I discussed in the planning section, the pair was using Marge's lesson plans during this unit. They would have a conversation about how Marge would implement the lesson, Lisa would watch Marge teach the first two sections, and Lisa would instruct the last section. As a consequence, the majority of the data in this section comes from the period when both Marge and Lisa were teaching.

Factoring. As Marge and Lisa discussed distribution, they began looking forward to factoring, and Marge explained, “It’s called an area model. The inside is called the expanded form, and the outside is the factored form” (Observation Notes). Marge and Lisa began using area models to multiply binomials, and then progressed to a version of an area model called the AC method. They used that method to factor trinomials, and finally they factored by grouping.

During a lesson multiplying expressions, the class was discussing the areas of the different parts of a pool in Figure 4.7, and there were a number of instances where Lisa used techniques she had observed or had talked about with Marge. The students had been given time to work on the problem, and Lisa asked for the area of the diving part. A student said, “ s minus 10 squared.” Lisa wrote $s - 10^2$ on the board, and the student quickly followed up with, “No, with parentheses” (Observation Notes). Marge was in a similar situation in the class before this one: When a student said “ x two,” she wrote “ x^2 ” on the board, and the student said, “No, x squared” (Observation Notes). In the hall between classes, the pair discussed how this technique helped students communicate mathematically.

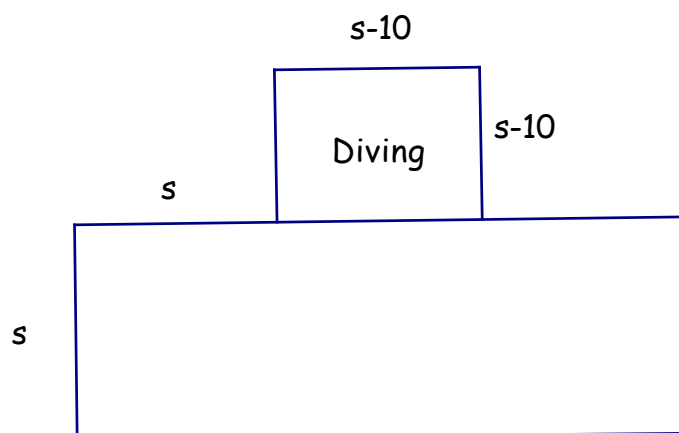


Figure 4.7. Diagram of a pool.

The lesson then asked the students to write $(s - 10)^2$ in expanded form, and one answer a student gave was $s^2 - 20s + 100$. Some students were confused by the answer, and a student said, “I see the s^2 and the 100 because it’s s squared and -10 squared” (Observation Notes). Lisa asked what it meant to square something, and the students said it was a quantity multiplied by itself. Lisa developed Figure 4.8 on the Smartboard by using the student’s answer and drawing the connection between $x \cdot x = x^2$ and a more general case where a quantity in parentheses is being squared, or $(\text{Fred})^2 = (\text{Fred})(\text{Fred})$. Lisa pointed out that it did not matter what was in the parentheses. So $(s - 10)^2 = (s - 10)(s - 10)$. Lisa asked the students for each area of the box, which they were familiar with since the previous lessons had situations where areas were represented as the product of a monomial and binomial.

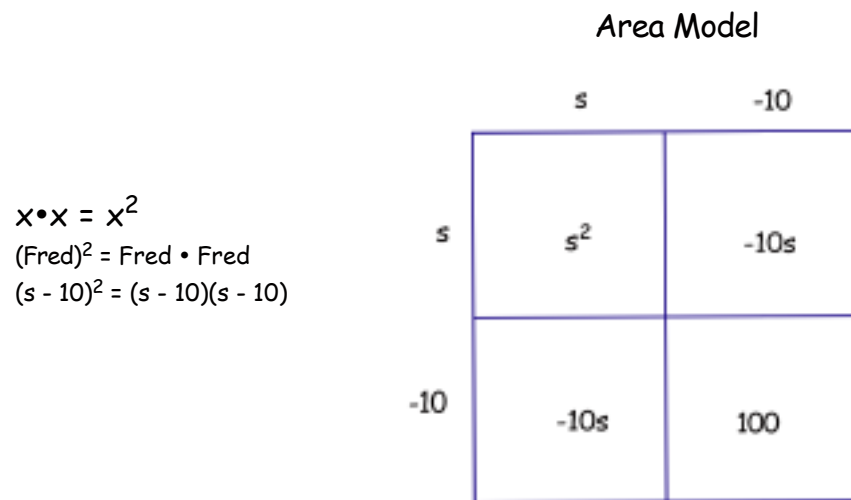


Figure 4.8. Area model used to multiply binomials.

Later in the lesson, the students were asked to multiply $30(x + 10)$ and $(x + 2)(x + 3)$. The solutions are presented in Figure 4.9. Area models were used to show the factored form (FF) and expanded form (EF) of expressions.

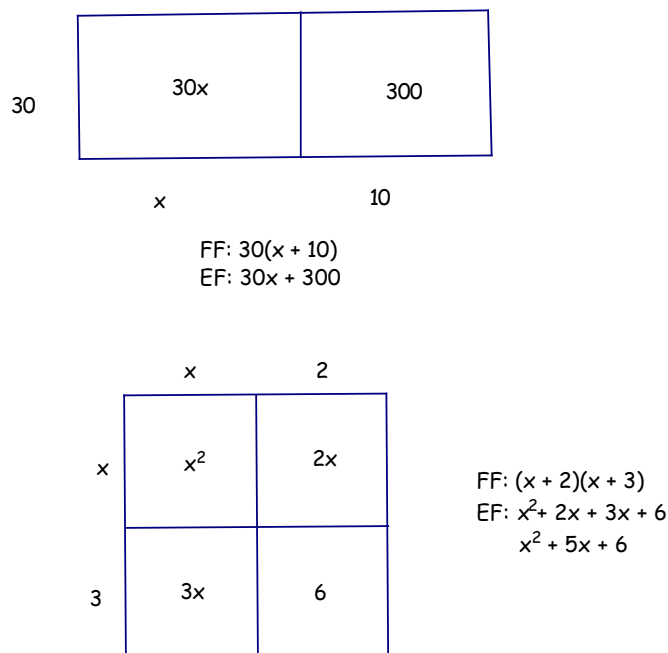


Figure 4.9. Area model solutions.

Lisa used the same techniques in Math 8 the following day when they were multiplying $6x$ and $3x - 5$. Referring to the area with dimensions $6x$ and $3x$, she asked, “What goes in this box?” A number of students called out “ $18x$,” which she wrote and followed with the question, “What’s $6x$ times $3x$?” (Observation Notes). The same students repeated themselves and said $18x$, and others said $18x^2$, which she wrote once she heard the correct answer.

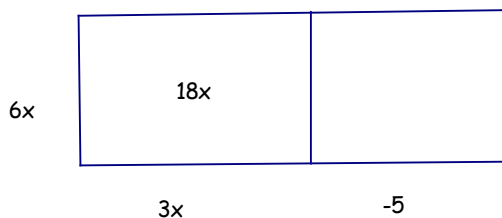


Figure 4.10. Area model

When the Advanced Math 8 classes began factoring trinomials with leading coefficients not equal to one, they were introduced to the AC method. Figure 4.11 illustrates the AC method that Marge used to factor $4x^2 + 25x + 6$. The leading term and constant were multiplied ($24x^2$) and put in a box in the top corner and the bx term was placed in the bottom corner. The ax^2 and

the constant term were placed diagonally from each other, and the process was to consider what factors multiply to $ax^2 \cdot c$ ($24x^2$) and add to the bx term ($25x$). These factors were placed diagonally from each other. The greatest common factor was taken from the areas horizontally and vertically to determine the factors of the trinomial. Marge commented, “This area model makes it a bit more clear” (Observation Notes) because it gave students a visual way to organize the factoring process that connected to previous area models.

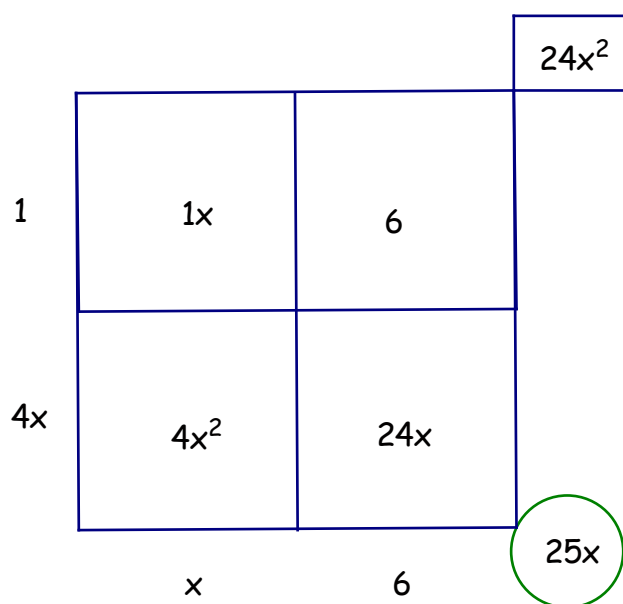


Figure 4.11. Marge’s AC method of factoring.

Lisa produced Figure 4.12 when she taught the same lesson later in the day. Lisa adapted the model slightly and wrote different factors of the $ax^2 \cdot c$ term on the side to help the students organize which added to the bx term

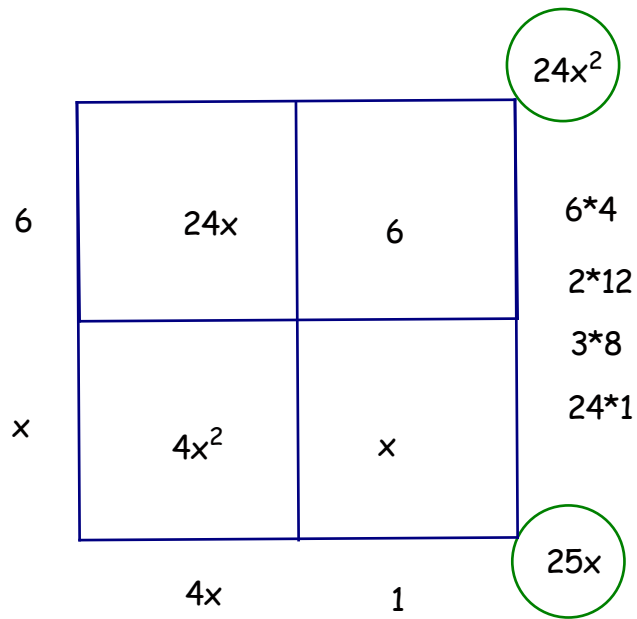
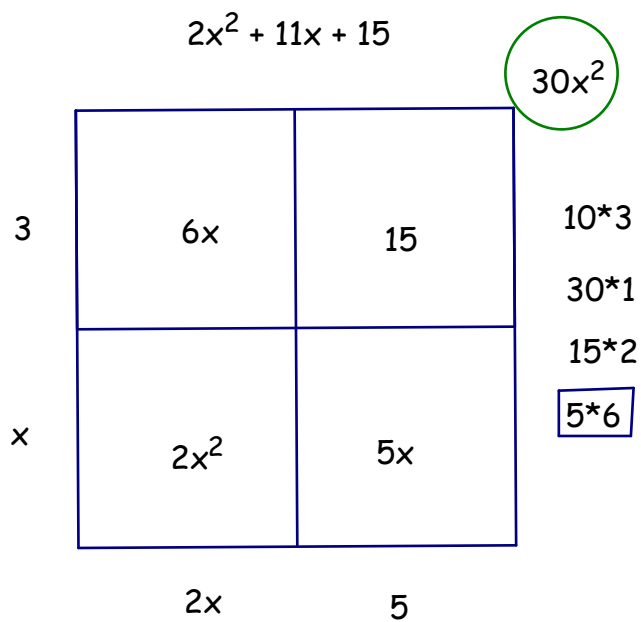


Figure 4.12. Lisa's AC method of factoring.

Figure 4.13 was from an exercise that the students did later in the lesson and illustrated the difference between Marge's and Lisa's versions of the model.



$$(x + 3)(2x + 5)$$

Figure 4.13. AC method of factoring.

The classes spent one more day with the AC method and then moved on to factoring by grouping. Marge described the process of factoring by grouping by starting in the same manner as the AC method by multiplying the ax^2 term by the constant. She wrote the $-45x^2$ on the right side of the expression $(3x^2 - 4x - 15)$ in Figure 4.14, asked what two factors of $-45x^2$ add to $-4x$ and indicated them with two lines. She then split the $-4x$ into $-9x$ and $+5x$ in the trinomial and took the greatest common factor of the first two and second two terms. She used the highlight feature on the Smartboard and asked, “What do you notice?” (Observation Notes) in reference to the $x - 3$. Marge then went to the right side of the board and considered $3x(\odot) + 5\odot$. She talked about what these expressions had in common and how to take the smiley face as a factor: $\odot(3x + 5)$.

$$\begin{array}{l}
 3x^2 - 4x - 15 \\
 \swarrow \quad \searrow \\
 3x^2 - 9x + 5x - 15 \\
 3x(x - 3) + 5(x - 3) \\
 (3x + 5)(x - 3)
 \end{array}
 \qquad
 \begin{array}{l}
 -45x^2 \\
 \swarrow \quad \searrow \\
 -9x \qquad 5x \\
 3x(\odot) + 5(\odot) \\
 \odot(3x + 5)
 \end{array}$$

Figure 4.14. Marge’s model of factoring by grouping.

Lisa’s instruction was very similar to Marge’s with some slight differences in the case of factoring by grouping as illustrated in Figure 4.15. She began by asking what the first step in the AC method was. The students were able to tell her to multiply the ax^2 and c terms. She asked what factors of $-45x^2$ add to $-4x$ (the bx term), and they gave -9 and 5 as answers. She wrote those answers and asked, “so $-9 \cdot 5$ is $-45x^2$ ” (Observation Notes), which was another case of her writing students’ incorrect answers on the board. After they corrected the answers to $-9x$ and $5x$,

she split the trinomial into four terms and highlighted the first and second two. She instructed the class to factor the greatest common factor to produce the third line and then asked, “What are we going to do next? What do you notice about these?” (Observation Notes). On the side she revisited her earlier use of Fred and asked what $3x(\text{Fred})$ and $5(\text{Fred})$ had in common, then factored Fred from each term.

$$\begin{array}{l}
 3x^2 - 4x - 15 \\
 \swarrow \quad \searrow \\
 \boxed{3x^2 - 9x} + \boxed{5x - 15} \\
 3x\boxed{(x - 3)} + 5\boxed{(x - 3)} \\
 (3x + 5)(x - 3)
 \end{array}
 \qquad
 \begin{array}{l}
 -45x^2 \\
 \swarrow \quad \searrow \\
 -9x \qquad 5x \\
 3x(\text{Fred}) + 5(\text{Fred}) \\
 \text{Fred}(3x + 5)
 \end{array}$$

Figure 4.15 Lisa’s model of factoring by grouping.

Lisa’s method of factoring by grouping stayed consistent for the remainder of the observation cycle where she would write the product of ax^2 and c on the side, use lines to indicate the factors that summed to bx , and highlighted the common binomial.

Venn Diagrams. The use of Venn diagrams provided interesting data that was unique and supported earlier analyses. One unique feature of the use of Venn diagrams was that I observed Marge or Lisa use them only in Math 8 classes and Extended Learning Time; never in Advanced Math 8. Early in the experience, while Marge was still teaching Math 8, the class was in a unit on equivalent expressions, and they produced the Venn diagram in Figure 4.16 as a class. Before crossing out the “used to solve,” she commented that “some of these are right and some are not,” and when someone brought up the “use to solve,” she followed with, “What do you mean to ‘use to solve’?” (Observation Notes). The students talked about finding solutions to

equations, so Marge crossed it out and replaced it with “cannot solve.” This was another instance where Marge considered correct and incorrect answers that were supplied by the students.

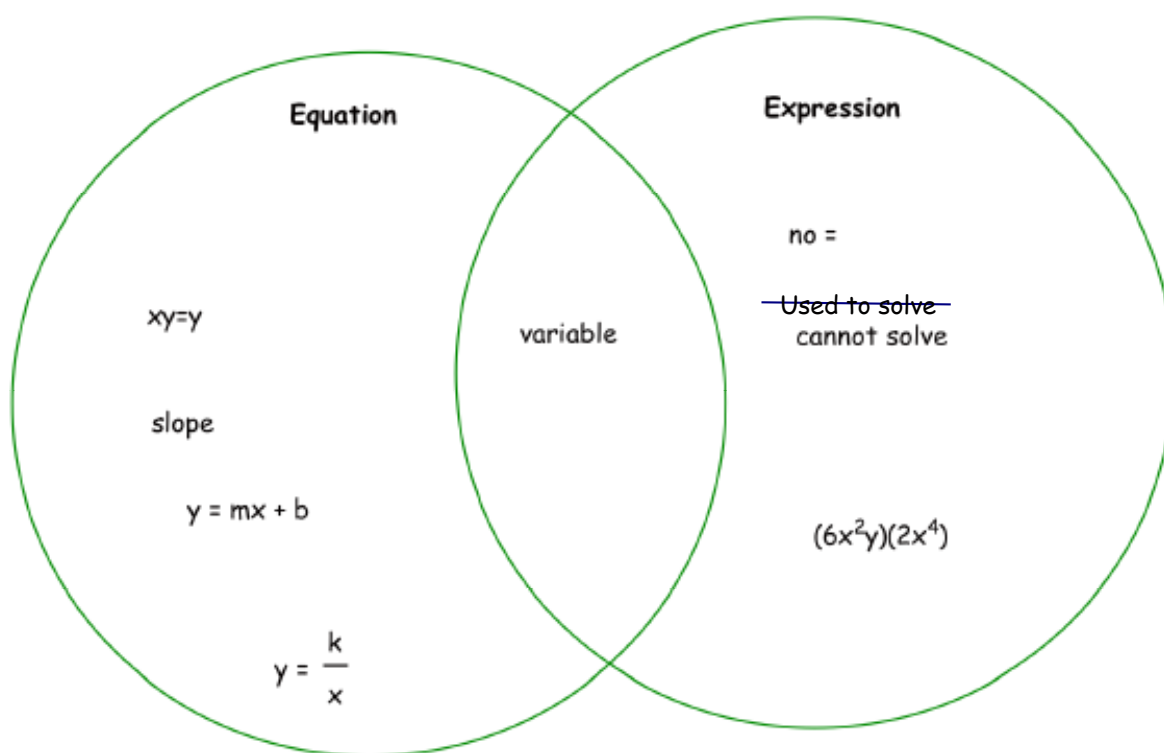


Figure 4.16. Marge’s Venn diagram for expressions and equations.

When Lisa taught Math 8, I observed her use Venn diagrams in each of the units she instructed. The first unit was on solving literal equations, and the Math 8 students supplied the answers for the Venn diagram in Figure 4.17. The ABC’s the diagram referred to were from working with the equation $Ax + By = C$, and P.O.E. stood for properties of equations. They used this acronym to justify steps in solving equations such as subtracting the same quantity from both sides.

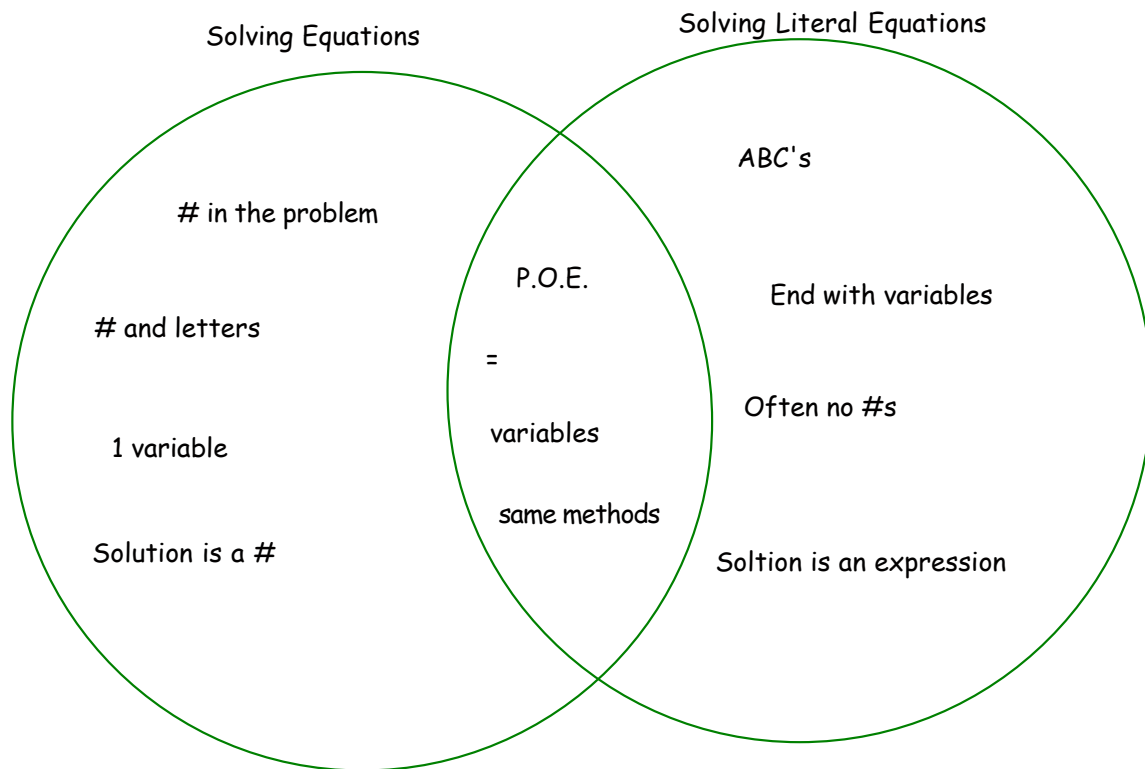


Figure 4.17. Lisa's Venn diagram for solving equations and literal equations.

The second example came from a unit toward the end of the experience on solving inequalities. The class had written inequalities and solved equations prior to the lesson on solving inequalities. The Venn diagram in Figure 4.18 came just after the starter exercise, which asked the students to provide five examples of numbers less than 7, provide five greater than or equal to -3, solve $2x + 7 = 13$, and solve $4(x - 10) = 2x + 30$. During the discussion of the Venn diagram, one student said, "You've got those backwards," referring to the greater than and less than signs. Lisa replied "No, because this is like x is greater than 9 [wrote $x > 9$]," and Marge added, "You could use some numbers" (Observation Notes). Marge then wrote "6 5" and asked the students which way the greater sign went. This was another instance where Marge interjected—however, in a different way than earlier in the experience when Lisa was beginning to teach and she asked Lisa a question.

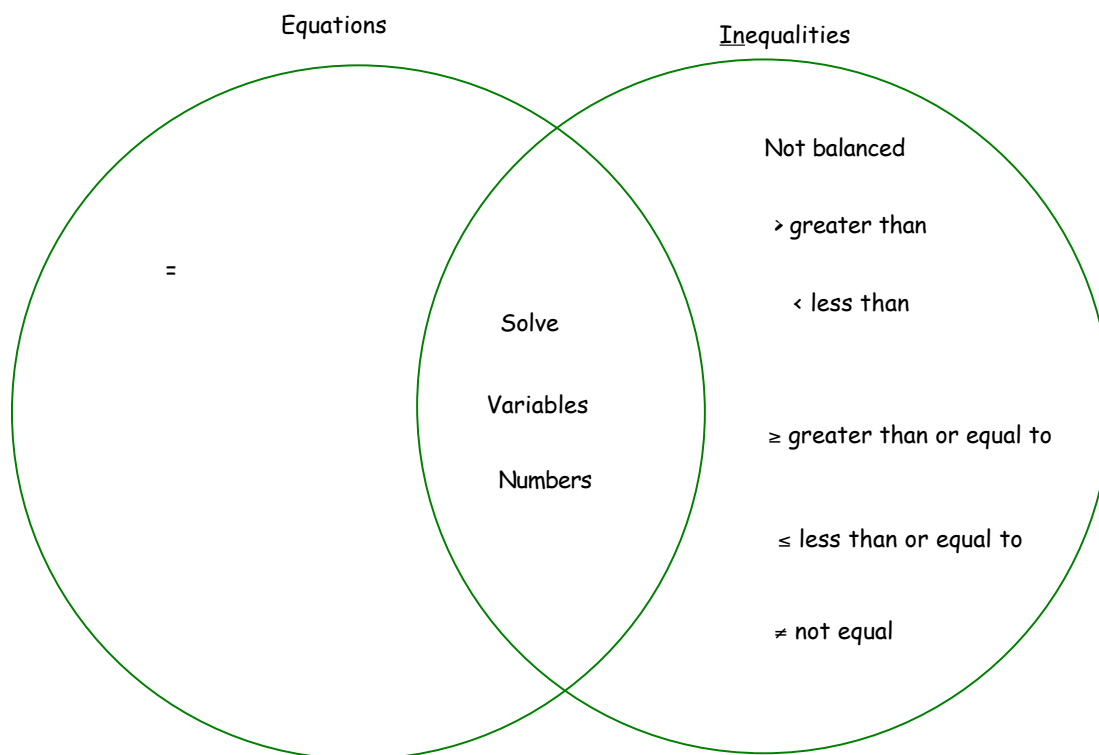


Figure 4.18. Lisa's Venn diagram for inequalities.

The examples in this section illustrate Marge's influence on Lisa's practice in terms of pedagogy and mathematical content. The pedagogical techniques that Lisa used in her instruction throughout the student teaching experience were highlighting key parts of an exercise, considering a number of answers whether they were correct or incorrect, writing exactly what a student said to stress precision (such as Lisa's factoring-by-grouping example), and Venn diagrams as a visual tool to make connections for the Math 8 students. The factoring examples provide data from the time when Marge and Lisa were teaching the same lessons and Lisa closely modeled Marge's instruction. After the unit on factoring, Lisa took over instruction for the Advanced Math 8 classes, and she was given more autonomy for way the material was being presented. The conversations between Marge and Lisa centered on what aspects of the mathematical content to focus on—rather than how Marge presented the mathematics—and having Lisa reflect on her practice.

Reflection

Lisa and Marge discussed the role of reflection in learning to teach in their initial interviews and after the experience. Before Lisa began working with Marge, Lisa anticipated as follows:

I'm going to have to look at what worked and what didn't work, so when I'm learning to become a teacher I know what is going to work for me and what won't work for me. I can't just start being the exact same teacher as Marge. However good she is, we're not the same person, so personality wise it's a learning process, and I'm going to have to reflect. [Teaching] and reflecting is how I'm going to learn. (p. 6)

Marge also valued reflection in learning to teach by commenting, "I feel like you can't be an effective teacher without self-reflection. I guess my role is to facilitate that reflection" (Initial Interview, p. 15). To facilitate reflection in student teachers, Marge approached mentoring in a similar manner to the problem-solving skills she was teaching her students. She said in her initial interview,

You get them thinking of [heuristics] themselves, and you create problem solvers within a child, so it's the same to me when you create that reflection within a student teacher. You can ask the questions and get them to think about things, then hopefully they start asking themselves and questioning themselves to get those skills. (p. 15)

Marge's and Lisa's comments in their final interviews were consistent with their initial expectations. Marge reflected on what she learned from watching Lisa as well as other student teachers and claimed,

I was learning so much about my own teaching from watching somebody and thinking about how I would do it, and then having to talk about it, and then think about how I could do that differently or better. I just think that learning and reflecting go together." (p. 1)

Marge was discussing the point in the student teaching experience when Lisa was becoming more autonomous in her planning and instruction and provided some insight into her mentoring approach and beliefs on teaching.

But by that point, she's got to make some mistakes and figure some of that stuff out by herself, I feel like. Part of teaching is that reflection. And you get up there, and you say something that you know is wrong, and part of it is figuring out how you're going to make that correction. You can either let it go, or correct it right then, or come back the next day and address that you made a mistake, or not. I feel like there's a lot of that kind of stuff that it's better to do while you have some support than when you're out there on your own." (Final Interview, p. 5)

Lisa also gave some insight into Marge's mentoring approach towards promoting reflection by beginning with questions.

It was always pretty much "How did that go?" She would wait for me to say it, and I think that was really great instead of her telling me what she thought, ... I mean, she knows how she thought it went. So if there was something that I didn't recognize, or I didn't bring up, and she felt it was important, I'm sure she had them tucked away in case I didn't bring them up." (Final Interview, p. 19)

Lisa thought this method of beginning with questions was effective because she discussed the following:

One of the biggest things I learned from [Marge] watching me was how to reflect on what I was doing and why I was doing it. ... To ask myself those questions, without having someone there. ... I don't know if that's something she taught me, but she set me up to be successful, so that when I don't have a mentor or someone else, I feel comfortable enough to ask myself those questions. (Final Interview, p. 19)

Summary

The data for the case of Marge and Lisa began with an overview of how the pair began the experience by implementing Marge's lessons and Lisa modeling Marge's instruction. Both participants were complimentary of the other's questioning ability, and questioning became a theme for the remainder of their time together. As Lisa began to take over more of the planning and teaching responsibility, she was consistently running out of time and not getting to any closure of the lesson. Their conversations changed as a result of their timing issues to focus on what parts of each lesson were the most important, which was determined largely by prominence on standardized tests. The pair had an important discussion on a day scheduled for review before a test that also changed the nature of their conversations to focus more on formative assessments

and the importance of gathering evidence based on their students' understandings prior to giving a test or quiz.

The modeling section provided examples of how Marge influenced Lisa's practice in terms of content and pedagogy. Both were teaching the same lessons during the unit on factoring; hence their methods of modeling mathematical topics were very similar. The pedagogical techniques—such as the way they considered students' answers, and organizational tools like highlighting and Venn diagrams—spanned the experience.

Finally, Marge believed it was her role to promote Lisa's reflection by encouraging her to ask self-reflective questions. Lisa supported this belief in her final interview by describing this skill as one of the most important things she had learned from Marge.

CHAPTER 5

THE CASE OF JANE AND JUDY

Jane and Judy taught three 90-minute blocks of Accelerated Math I in an urban high school. They had two blocks of Accelerated Math I followed by a 30-minute period where students either came for advisement or extra help, then one more block of Accelerated Math I that was split by a lunch break, and finally planning during the last block of the day.

Prior to the student teaching experience, Judy reported in her initial survey that the most significant contribution Jane would make was the following:

[To] give me constructive criticism, highlighting what I did well and also explaining what I can do to improve. I hope she will be encouraging throughout my experience, even when things do not go as planned. ... If she is positive and tells me what I can do to constantly improve my teaching, I think I will have a successful experience. (Survey Response)

In her initial interview, Judy expanded on her expectations for student teaching and Jane's role.

She reported that the purpose of student teaching was

to become more comfortable in front of a classroom and to have reassurance. I've got ideas, and I feel like I could go out and teach a class, but just to have someone there that's like "Yeah, that's good, or no, that's not going to work." Also to use what you've learned while it's still fresh in your mind. (Initial Interview, p. 1)

She did not expect Jane to have a large role in making the transition from being a student of mathematics to a teacher of mathematics. Rather, she expected Jane to provide resources to look at, but "I'm more on my own to figure [the transition] out" (Judy, Initial Interview, p. 4).

In her initial interview, Judy foreshadowed two themes of grouping and teaching with technology the pair encountered during their time together. When asked what areas of teaching she felt confident about, she said,

After writing my paper on differentiating instruction, I feel that like I have a pretty good idea about how to do that. ... One way in my methods class was having students work in groups. I think I had them in groups for all three of the lesson plans. One of them, they were in the computer lab. I'm trying to think ... I think I also included a couple extra

questions for the more advanced learners just to dive in a little more. (Initial Interview, pp. 3–4)

In Jane’s survey, she also talked about support and differentiating instruction. She said that her most significant contribution to mentoring student teachers was

helping him or her feel confident about their teaching ability to teach all types of students and to have fun doing it. Learning how to differentiate instruction to meet student needs is a must. ... The first week I will give verbal feedback every day. Every week thereafter I give verbal feedback at least 3 times per week. I’m always giving suggestions, praise, and voice concerns on a continual basis. (Jane, Survey Response)

Jane had mentored more than 10 student teachers in her career and developed a routine for establishing a relationship with them. Jane provided some insight into her philosophy of teaching others when she said, “In the beginning, you have to establish relationships. You don’t start teaching on the first day; you establish relationships on the first day” (Initial Interview, p.

2). When dealing with student teachers she said,

I always meet with them before the first day of school. I ask them if they've had any previous experience in the classroom. Some of them have worked with kids before; some of them haven't. Most of them are working on their bachelor's degree and I ask them: “What have you learned? What did you talk about in the classroom?” Because I want to hear what they have to say, and as they talk, I'll write it down. I'll say, “I like what you said about this because this is what we're going to be doing and this is how we're going to do it.” I like to hear what they've already started thinking about and to reinforce what they've learned in school and their expectations coming into the classroom. In receiving that first-hand experience, I always break it up with them, the beginning, the middle, and the end. I say, “How should we handle things? How do we handle things in the beginning? What kind of things do you think needs to be handled at the beginning? What kind of expectations should we have for our kids at the beginning?” In the middle I usually say, “We know what our expectations are, we know what our end result [is], and what do we want to see toward the middle that will help us toward that end result?” (Jane, Initial Interview, pp. 2–3)

This quote also suggests that Jane was open to her student teachers’ ideas and valued their contributions. She further supported this claim by saying, “When your student teacher is teaching, you get so many ideas from them. It’s like you’re always adding to your repertoire.

That’s one of the things that I like: They bring a lot to the table” (Initial Interview, p. 1). Another

insight these quotations provided was Jane's tendency to begin with questions and allowing those she mentors to talk before she offers advice and direction. Later in the interview she was discussing how she helps student teachers plan lessons, and she reiterated many of these points by saying,

I always ask them "Have you ever done this before?" and they say yes. And I say, "Can you share with me some of the things that you did? How did you learn it?" I always ask them. You've got another body in there that can help you tremendously. It's a 2-way street. I don't think student teaching is all "do this, do this..." You don't get good yourself until you learn from other people. Just because they're fresh; you know they're fresh, and I'm old school, and I think you need to bring the old with the new. (Initial Interview, p. 11)

To begin the semester, Jane and Judy gathered data about their students and did problem-solving exercises to establish classroom norms. The following sections are organized in a similar way. The first section is on grouping and how the dynamics of the groups changed as the experience progressed. The second section deals with planning and describes how the pair co-planned at the beginning as well as some activities that used groups and technology. The final section considers both formative and summative assessments.

Grouping

In this section I discuss how grouping students played a role in Jane and Judy's experience together. Grouping was a focus from the first day of classes, and each group had a specific role throughout a lesson, such as presenting answers to the warmup exercises. While Jane was teaching, these roles were an emphasis more than when Judy took over; however, by the end of the experience, the group roles were part of Judy's instruction.

On the first day of class, the students in Jane and Judy's classes filled out an information sheet they called an *interestlizer*. The interestlizer gathered data on the students' gender, desired profession, favorite class, hobbies, and three International Baccalaureate profiles (IB, 2012). The IB profiles were qualities that the students felt described themselves, such as being an inquirer,

knowledgeable, thinker, communicator, principled, open-minded, caring, risk-taker, balanced, and reflective. Gathering these data was typical for Jane, and in her initial interview she said,

What I do is an interestlizer. I found out what the likes and dislikes are of the students, and we make up nametags. The student teacher and I make nametags for the kids, and I put them in groups. I'll ask Judy, "What are the different ways that we can group students?" and she shares with me the different ways, and I'll share with her some different ways. And I'll say, "You know, I've done that too, but this is new, and I haven't tried that." So I always try to get feedback and write those down. After I do the interestlizer, and I go home and read them, and then I ask her to go home and read them. I say, "Based on the things we wrote down, how can we differentiate, or how can we break this up that we know about these certain kid?" So, based on the lesson that we're doing for the day or week, is based on how we're going to differentiate instruction. (p. 13)

After school on the first day, Jane and Judy looked through the interestlizers and came up with possible groups for two of the classes based on the students' hobbies and favorite classes. Based on the students' interests and hobbies, the themes for one class were drawing, social studies, music and dancing, science and computers, and debating and acting. The themes for the other class were builders, science and computers, social studies, sports, debating, writing, and art. These groupings were intended for special assignments such as projects versus groups for daily lessons.

Jane and Judy also went through each of their three classes and created seven groups, which were based on anticipated behavior issues and mixed abilities. Each class needed seven groups to be responsible for (1) the opening of the lesson, (2) homework, (3) summarizing yesterday's lesson, (4) introduction to the current lesson, (5) solving the day's examples, (6) summarizing the day's lesson, and (7) administrative duties. The intention was to have each group responsible for one role for a week, and the groups would change every unit. This setup was difficult for the pair to manage, however, which I illustrate in the rest of the section by describing two lessons I observed Jane teach followed by some of Judy's lessons from each observation cycle.

On the first day that Jane used the group roles, they did not get through the entire lesson plan in any of the classes. In the first two sections, they spent approximately one hour on the warmup exercises and homework. Each class was given time to work on the warmup, then a group used the document camera to show their answers to the exercise, which wanted to know how many cds and dvds a person bought if cds were \$12 each, dvds were \$20, and he spent \$136. They described each step, and Jane asked questions to expand the students' explanations such as, "What are some of the main things you do with a word problem? What do these problems deal with?" (Observation Notes), and she explained how they needed to write a sentence rather than circle the answer. In one class, a student presented the warmup exercise and had the wrong answer. She appeared to be slightly embarrassed, and when I asked Jane about the situation, she responded, "That needs to happen early so they know they need to be on their game when it's their group's turn" (Observation Notes). After school, she reiterated this point to Judy and talked about how seeing a student embarrassed will make all of the groups work together more because they do not want to be wrong in front of the class.

After the groups showed their answers to the warmup exercises, another group was called on to show their homework using the document camera, so one student showed her paper. The other students in the class were allowed to ask questions on the homework. During the lesson, the students worked on two exercises that dealt with coins. Jane gave the students some time to work, then showed a solution, without writing a sentence, for the first exercise and talked with the class about the second. The students then worked on two investment exercises in their groups, and Jane had one student who finished early put the solution to the first exercise on the board. The student wrote a sentence in their solution, and Jane had a student from another group go to the board and check the answer; hence, students were chosen to present answers to exercises rather than one group as intended. The time ran out at this point, and there was no

summary or administrative duties. The lack of a summary should be noted here because it became an issue in Judy's practice.

For the remainder of the first week, this trend continued: Some of the group roles were performed, and others were not. During this class, the group roles were used for the warmup exercises and homework. There was no introduction to the lesson, which typically meant reading the standard and essential question. One student was called on to present one of the practice exercises, a student from another group to check it, and Jane showed a solution for the other exercise the class discussed. The time spent on the warmup exercises and homework decreased as during the observation cycle as more students presented answers where they described what the exercises were asking for, had sentences in their answers, and checked their work.

By the end of the first observation cycle, Jane and Judy were co-teaching, and the roles were used in some cases, but the changing format of each day presented challenges for their use. For example, on the first day of the observation cycle, the classes took a 50-minute pretest, and Jane and Judy talked about using Frayer models (Figure 5.1; 2008) to review absolute value functions. The students worked on these models in pairs for the remainder of the block. The following day, the classes began with a 15-minute quiz, and then students got in pairs of their choosing rather than pre-assigned groups. Jane went over the homework and had students call out answers to her questions about the homework (e.g., the next step in an exercise, or how to set an exercise up). Judy took over the instruction with less than 30 minutes left and did three examples of solving absolute value inequalities, during which she asked general questions to the whole class. The final example finished near the end of the block, and again there was no summary. The following day, the classes did a carousel activity in pairs, which involves the students circulate the room and addressing different exercises on poster paper. I describe the

activity in more detail in a later section. The final day of the first observation cycle was spent testing.

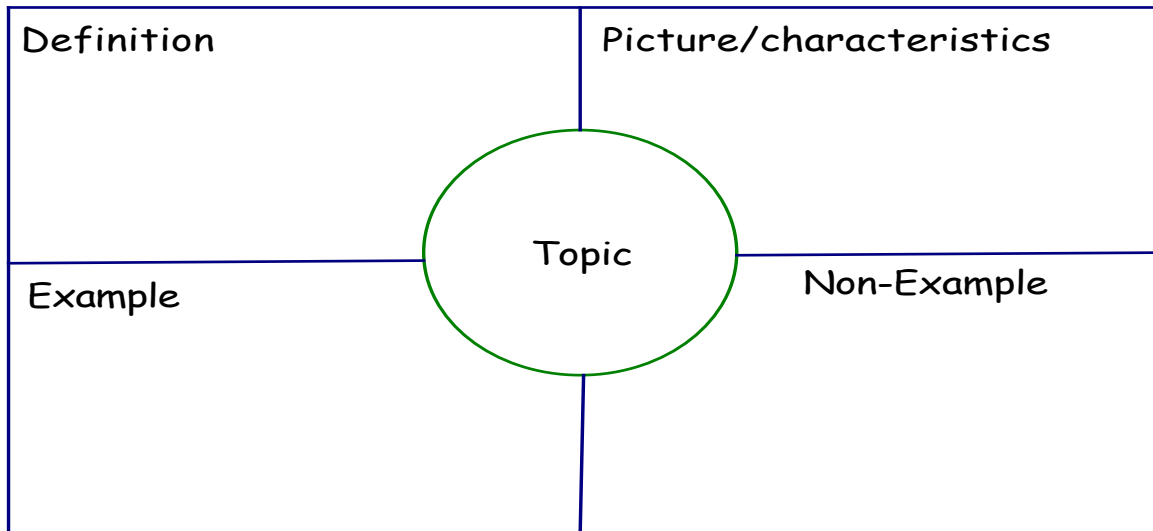


Figure 5.1. Frayer model.

Challenges to organizing the group roles continued during the second observation cycle. The classes were supposed to go to the computer lab for an activity, but Jane emailed Judy the previous night and was unexpectedly absent on the first day, so Judy had to plan a lesson for the day without very much notice. The classes went over the homework from the previous day, and Judy called on random students to provide the answers, then did a review sheet, and finally played a game for the final 15 minutes of class.

The following day, Jane was back; however, the group roles were not used. The warmup activity consisted of looking up logic terms (e.g., *inductive reasoning*, *contrapositive*, *negation*) in their textbook. Before discussing the terms, Judy called on a student to read the essential question and standard, which was one of the roles, then Judy called on random students for the definitions. The class worked on one example where they changed the statement “Everyone who works hard will succeed” (Observation Notes) to different forms using the *negation*, *converse*,

inverse, contrapositive, and biconditional, and students were then instructed to create a conditional statement and write it in the different forms. Each pair presented their statements, and the homework was assigned. In other words, no group was assigned to do the classwork. Jane, who was sitting next to me, said, “We need a summarizer” (Observation Notes) and walked up to tell Judy, who then reviewed the vocabulary from the day by calling on different students, rather than a group, to describe each form.

The third observation cycle was split by spring break. During the week before the break the class was arranged in rows, and it appeared that the group roles had been abandoned. The students were given warmup exercises, and Judy walked around and checked everyone’s homework with a clipboard. The warmup and homework solutions were discussed verbally with Judy calling on students. The students then got in pairs and did another carousel activity with quadrilaterals. After the activity Judy asked, “What three shapes did we talk about that are parallelograms today?” (Observation Notes) as a summary for the class. During that week, I asked Judy why the class structure changed, and she responded,

I guess I did change [the structure]. I like the idea of it, but there was never one week where students had the same role all week even when Jane was doing it. Maybe that’s it; maybe I never saw it done successfully. I saw bits and pieces, but it was almost like the students would come in every day and have a different role. It was too much to pass everything out between every class and get set up. She did tell me yesterday that she did want to see the summarizer of the lesson, so I will get back into that. (Judy, Audio Recording)

The week after spring break marked a return to some of the group roles. The first day back the students were in groups with name tags, and one group answered the warmup exercises, another group used the document camera to show their homework, and a third group read the standard and essential question. Judy had the first student who was finished with his class work put the answers on the board, and she had summary questions that were presented to the whole class. In first block of the second day, the students came in and sat in groups that were different

from the previous day. Judy realized, “Ms. Jane, they changed seats. We didn’t put out the nametags” (Observation Notes). During this block, no group showed their solutions for the homework; however, one group was chosen to show the class work. After these classes, I asked Jane what sparked the return to the groups, and she told me the classes took a test on Friday before spring break, they went over the test on Monday. She said, “We need to get back into the groups” (Observation Notes).

These data illustrated the struggle Jane and Judy had with their initial intention of having seven groups that would have a different role each week. They began the semester using the group roles and gradually used them less until spring break. After spring break, the pair returned to group roles, although they did not use all of the roles. I never observed all seven of the roles implemented in any lesson at a point during the student teaching experience.

In the final interviews, I asked Jane and Judy about the group roles since they were such a focus at the beginning and changed over the course of experience. Jane said,

I did the group roles a lot. I didn't get to the 7th one. The 7th one was kind of like if I have someone extra and I need an extra job, and that was administrative. It was giving out papers and collecting papers. Judy didn't do it as much. I did try to encourage her, and she started doing it more. At one point we were doing the roles every day when I was teaching, and then I think the more we got into making changes, like having to reteach a lesson made it difficult. There were some things that I told her that we had to do every day, such as making sure that they knew what the essential question was. (Jane, Final Interview, p. 9)

Judy did not see the situation the same way, particularly at the beginning of the experience. She claimed,

[Jane] had the nametags out every day and had them in groups, and she did have them do homework and warmup and then summarize [the] lesson. It was mainly just those three. Maybe introduce today's lesson, but to me, the only thing you can really introduce is really the essential question and the standard, whereas I just called on someone, “Hey, read the standard,” and I called on someone who was staring off into space or talking to their buddy rather than having a whole group stand up and read that. It was just kind of quick and to the point. (Judy, Final Interview, p. 7)

As Judy said during the third observation cycle, she had a difficult time implementing the group roles because she felt she did not see them modeled enough, and it was logistically challenging to pass everything out and get the groups set up. She supported and expanded on her statements in her final interview by saying,

That was hard for me to really divide them up like that and to give them each a job for the week. I think that because I didn't really see Jane do it that often, or maybe because she wasn't teaching that much that I saw, and it was hard to get to everything in a day. For me, it was almost easier to say, "Ok, your homework looks great. You're going to go over the homework today." I didn't like having kids who didn't do their homework to have to go sit up there and struggle through explaining the homework. I think towards the end, I did get back more into the groups and doing that. I think in the middle, it was more going up and down the rows calling on people. Towards the end, I definitely had the groups do more. (Final Interview, p. 7)

There were several possible reasons for Judy's struggle to implement the group roles. One possibility was that in the beginning of the experience Judy was helping pass out the nametags and getting the room ready for the groups, whereas she had a difficult time with this when she took over responsibility for the instruction. Another reasonable explanation was that Judy was struggling with the timing and not getting through her entire lesson, so as a result, she tried to reduce the amount of time spent with the group roles. At the beginning of the third observation cycle, I asked Jane what the pair had been working on, and she listed classroom management as a focus, which could have caused Judy to reorganize the class into rows as well. A last possibility for losing the focus on group roles was that many lessons were not conducive to the structure, such as the carousel and technology activities.

Planning

This section begins by considering Jane and Judy's expectations and how the pair began the semester, followed by a description of a co-planned lesson that highlights some key issues and elements that would play a role later in Jane's lessons. I then discuss two lessons Judy

planned and implemented to draw connections between the co-planned lesson. Finally there are descriptions of carousel and technology activities that had some unique features.

Expectations. The initial interviews provided some interesting foreshadowing for Judy and Lisa. In the previous section on grouping, one of the things that Judy felt confident about was differentiating instruction, and Jane said one of the main ways they were going to do that was with flexible grouping. Despite Judy's confidence in differentiating instruction, she was nervous about lesson planning. In her initial interview, she said,

It's really hard for me to come up with lesson, like good strong lessons that are going to work. I hope that in the beginning we'll co-plan and she'll help me go through it... The warmup and the summary are always hard for me. Just finding something that will really grab their attention, or some little activity when they first walk in. (Initial Interview, pp. 2–3)

Indeed the pair did co-plan in the beginning of the experience, and the summaries were difficult for Judy. There were different types of co-planning, including weekly and daily lessons. Judy said,

It started every Thursday; we would plan together for the next week. That was just really basic. We basically said the lessons that we wanted to cover and the main ideas of what we wanted to cover. We would think if we had an activity for a certain day and throw that in the basic lesson plan, but if we didn't, then it was, like, "Oh, well, explain this and this. ..." (Final Interview, p. 3)

The weekly co-planning was consistent throughout the experience. The pair entered the following week's lessons in a computer program called Campus, where the teachers were required to submit their lesson plans. Jane explained,

First of all, [student teachers] are used to writing 2–3 page lesson plans. I told them, that doesn't work here, we don't have time for 2–3 pages of lesson plans. I usually ask them if they can bring a copy of a lesson plan they've done, and I go to Campus and show them how we do lesson plans. (Initial Interview, p. 7)

The 2–3 page lesson plans Jane was referring to did not include Smartboard presentations that were used almost daily. Jane also talked about helping student teachers with long-term planning and organizing each unit.

What I usually do is when we sit down together to plan the next unit, I share with them what I've done in the past. Because sometimes I don't teach Lesson 1, then Lesson 2, then Lesson 3, then Lesson 4. I look at the whole unit, and that's something I tell them, "You've got to look at the whole unit. See if there's any connections you can make with those units. You may want to teach Lessons 2 and 4 together because 2 and 4 have connections. You don't want to be jumping all over the place. If you can take one thing and bring in a new word and connect it with several things in there, then kids are more likely to remember what they're doing." So what I do is sit down and look at the chapter, if you will. I may not like how Chapter 1 introduces something, I may start in Lesson 3, and I say, "This is why I do that, what do you think?" (Initial Interview, p. 11)

In her initial interview, Jane talked about how it was important to establish relationships in the first week, and particularly on the first day. For two days they did problem-solving exercises involving coins and investments. After the first two days, they continued the problem-solving exercises by including them in the homework problems. Jane talked about her approach to the problem-solving exercises at the beginning of the semester,

You want to just present the problem and see what they know and see how [the students] set it up, so you can get an idea of what their right thinking is and what their wrong thinking is. Then you can say, "What if we do this?" and present it to them this way... I think Judy learned a lot from that too... She picked it up very quickly, especially when you have to grade the papers. At the end of every week, I gave a quiz on problem solving. (p. 4)

She also said,

I set my work up with charts, and she wasn't quite familiar with that. ... She would just read it and write the equation down, but I always do the thought process. "Let's set it up. Who were they talking about? Write that. ..." Then I'd have a problem, if it's age, how old they are now? In 10 years, what will they be? Because I wanted the kids to work problems like that. That was the thinking that I wanted them to get used to. Not just thinking and writing it down, but I wanted to see the whole process. I told her that ninth graders are at a starting ground, they can [just write down the equations] a lot later, but right now you want them to start thinking about what they're doing and setting it up, and it makes it a lot easier because it is on the SAT. That's another thing, setting them up as charts, and after a while it [goes] very easy for her to grade those papers because I told her now if they're going to be worth 10 points, we want 2 points for the set up, 2 points

for the equation, 2 points if they solved it correctly, then another set of points for answering the question in a complete sentence. (Jane, Final Interview, pp. 4–5)

Grading and assessments also became a theme that Jane and Judy discussed throughout the semester, which I discuss more in the assessment section. I now consider how the pair co-planned at the beginning of the semester.

Co-Planning. On the second day of classes, Jane and Judy sat down together and planned a lesson that was an introduction to functions. To create the lesson, Judy worked on the computer, while Jane looked in their book and described how she typically creates the Smartboard presentations. Jane said the essential questions should be “how” questions as opposed to “what” or “why” questions because they are more open-ended. Jane suggested that Judy use the section headings to help create the essential questions, and for this lesson it was, “How do you represent functions as graphs?” Jane had Judy create eight slides and put the essential question and standard at the top of every slide, which was consistent throughout the experience for both Jane’s and Judy’s practice.

Jane and Judy then went to the last slide and filled in the homework assignment, and Jane said, “I think having them do all the evens or odds is too much. What about the first column?” and Judy said, “Sounds good” (Observation Notes). Next, they inserted a TOTD, which was an exercise from the textbook, and Jane said, “Always remember that you need to bring some of the old with some of the new” (Observation Notes). They inserted an opening activity next, which consisted of two coins and investment exercises on the first slide and added a timer. On the second slide, Judy typed, “Homework: Coin and Investment 5–7” at the top. The third slide consisted of the terms *domain*, *range*, *input*, *output*, *independent variable*, *dependent variable*, *zeros*, *intercepts*, *maximum value*, *minimum value*, *end behavior*, *intervals of increasing and decreasing*. The slide was labeled 1.2, and these terms were scattered across the slide, with the

intention that the students would choose which terms they already knew. Jane showed Judy where the blank graphs were in the Smartboard program, and they inserted them on the two slides before the TOTD and labeled them 1.3 and 1.4.

There were three aspects of this co-planned lesson that reoccurred in Jane's lessons and I describe in the next section. The first is that Jane consistently put the standard and essential question at the top of each slide of her lessons. I observed the beginning of two units, and each time there was a slide with a number of vocabulary terms. The last aspect to note is that there was no slide dedicated to a summary and closure, which became a theme that Jane and Judy discussed throughout the experience.

One other aspect that was not explicit in the co-planned lesson, but Jane that talked about in her initial interview was providing solutions to the examples in the lesson plan. She was talking about her work with previous student teachers and said,

They write a problem, and then they work out the whole problem. We don't do that. What we want is to know what the goal is, how are you going to reach that goal, what problems you're going to do on what pages, what questions you're going to ask the children. They might have on their long sheet, these are the possible questions we're going to pose, here are the possible answers, this is how we're going address those answers. We don't put all that. We just write the questions down. You write your questions, and you should already have an idea how you're going to address particular answers. ... The solutions are already given in the book; we have answer keys, so you don't need to do all that. It's a lot of wasted work, doing it twice. If you want to, I always have a copy of the book, and I have a copy of the teacher's guide. I always give [student teachers] their own copy because they can go home and take a look at how the problems are worked out so they know what ones to use. (Initial Interview, p. 8)

Judy's Lessons. In this section, I provide data from Judy's lessons that are relevant to the standard and essential question, emphasis on vocabulary at the beginning of units, closure, providing solutions to exercises, and grouping. I start with two episodes from lessons where Judy did not have the exercises completed. I discuss two lessons that provide data for foci of

Jane and Judy's relationship and end with an episode that provides evidence for Judy's growth in closing lessons.

During the first observation cycle, Judy was teaching a lesson on graphing absolute value functions, and she started with the example $|x| = 3$ to build on the students' prior knowledge that there were two solutions at $x = 3$ and $x = -3$. The next example she discussed was $|x + 4| = 8$, and the class was able to develop the solutions $x = 4$ and $x = -12$. She transitioned to graphs and asked how many solutions each of the graphs in Figure 5.2 had.

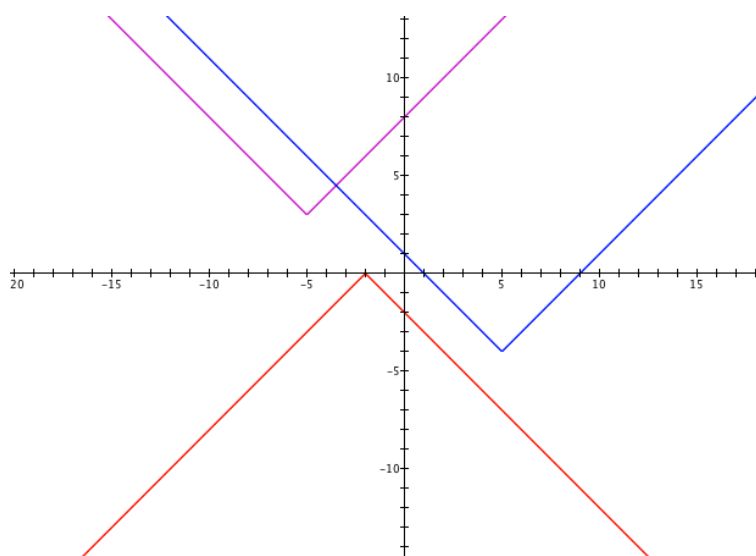


Figure 5.2. Graphs of absolute value functions.

She wrote $|x + 4| - 8 = 0$ and explained how the solutions to this equation were the roots of the graph of $|x + 4| - 8 = y$, and she said that $(-4, 8)$ was the vertex, but it should have been $(-4, -8)$. She plotted the point $(-4, 8)$ and got confused when she tried to explain that the graph opened up since the sign before the absolute value was positive, and it should have roots at $x = 4$ and -12 . She asked, “Ms. Jane, what did I do wrong?” (Observation Notes), and they were able to find the error and correct it. At this point in the experience, Jane and Judy were still discussing the examples they would use in class, and I did not observe Judy make any teacher notes beyond the

Smartboard slides. One possible reason for Judy's inability to catch this mistake was that she did not complete the exercise before the lesson.

Another episode where not having solutions to the examples influenced Judy's lesson was on a day when Jane was absent. She e-mailed Judy the previous night around 6:00 and informed her she would not be because of illness. The classes were supposed to go to the computer laboratory, but were unable to. Judy decided to go over the homework, have the students work on their review sheet, and play the skill-building game SMADness. During the first block the students were in rows, rather than groups, and she called on students to volunteer answers for the homework. She instructed the students to get out their review sheet from the previous day and start working on it. After a few minutes, she repeated the instructions and said, "If you've already done it, check your answers. If not, then start working on it" (Observation Notes). A student asked her if she was going to go over the review sheet, and she said, "Yes, I should probably make a key" (Observation Notes). She sat next to me and completed a review sheet. Later in the block, she called on volunteers to provide the answers, verified them with her key, and showed the solutions for two exercises that the students had questions on. There was no summary to this lesson.

This episode was not a typical situation in that Jane was absent; however, there were salient features to note. One feature was that the group roles were not used during the lesson. Regarding the review sheet, Jane had informed Judy she was ill early in the evening the night before, and at other times observed how Judy consistently arrived at school early, which suggests that Judy had time to complete it. Judy took time during the lesson to make an answer key, which prevented her from walking around to answer students' questions. It was also important to have the key later in the block because it allowed her to verify the answers.

During the second observation cycle, Judy was teaching a lesson on logic. The students were in rows of pairs, and the warmup activity was to look up definitions for *inductive reasoning*, *deductive reasoning*, *conditional statement*, *contrapositive*, *biconditional*, *counter example*, *negation*, *inverse*, and *converse*. This was an instance where the Judy began a unit with the definitions similar to the co-planned lesson. Judy had one pair read the essential question and standard, which was a group role. To discuss the definitions Judy had a slide with the statement “Everyone who works hard will succeed” and she asked for volunteers to read the definition of each term, rather than having a group do the class work, then the students rewrote the statement as a negation, converse, etc. Each pair was then instructed to come up with a conditional statement and change the statement using the terms they defined, as well as the truth-value. Each pair then presented their examples, and Judy assigned the homework. As I discussed in the section on grouping, Jane was sitting next to me and said, “We need a summarizer” (Observation Notes) and walked up to say something to Judy, who then called on volunteers to review the terms.

Some significant aspects of this lesson were that the group roles were being used for some parts of the lesson, the way the Judy approached the vocabulary similar to the co-planned lesson, and the episode with the closure where Jane had Judy do a summary. She used the groups only to introduce the lesson by reading the essential question and standard. She had the vocabulary terms scattered on a slide and had the students look them up rather than discuss which ones they already knew as in the co-planned lesson. The fact that Jane went up to say something to Judy indicates that she felt as though it was an important part of each lesson and an aspect of teaching that Jane was trying to emphasize.

At the beginning of the third observation cycle, I asked Jane what she and Judy were focusing on and closure was one of the things she listed. During this observation cycle, Judy

asked summary questions at the end of the majority of the lessons. One day she asked, “What’s the difference between the multiplication and addition counting principle?” and answered her own question when no one volunteered. She also asked, “What’s a factorial?” (Observation Notes). After the lesson, I asked her about the summary questions, and she didn’t think these were very good, but she couldn’t remember what she intended to ask. The following day, she asked “What’s the difference between a combination and permutation?” (Observation Notes) as a summary.

At this point, Judy was still putting the essential question and standard at the top of each slide, focusing on vocabulary, and she was getting back to the group roles. In a lesson on expected value and sampling, the students were arranged in groups of three. Judy had one group show their homework answers with the overhead camera, she had another group put their answers to the two practice problems, and she had the slide shown in Figure 5.3.

EQ: How do you use expected value? How do you identify populations and sampling methods?

Standard: MM1D2d- Use expected value to predict outcomes. MM1D3c- Understand that a random sample is used to improve the chance of selecting a representative sample.

Lesson 6.6: Analyze Surveys and Samples

Population entire group you want info about.	Sample Part of the pop.	Random Sample every member of pop. has an equal chance of being selected.
Representative Sample good representation of the pop.	Stratified Random Sample divide pop. into groups and take a random sample from each group.	
Self-selected Sample members of pop. volunteer themselves.	Convenience Sample members of pop. who are easily accessible are selected.	
Systematic Sample A rule is used to select members of pop.	Biased Sample not representative of the Population.	
Biased Question Encourages a particular response.		

[Extend Page](#)

Figure 5.3. Vocabulary slide.

There was no summary during this lesson; however, Jane was complimentary of Judy's summary the next day during the lesson on mean average deviation. Judy asked, "What two ways did we talk about describing the spread of the data [mean average deviation and range]?" and "What's the difference between range in functions and statistics?" which Jane described as, "the kicker" because Judy was "bringing in some of the old with some of the new" (Observation Notes).

In their final interviews, Jane and Judy discussed some of these issues. Jane talked about her emphasis on providing closure and said,

I just told her, please remember to do the closing, but I didn't harp on it. I just said, "If at all possible, if you could please remember to have some type of closing. These kids are jumping out of their seats, and you've got a minute of class, and you've got to work until the end of class. You've got to come up with some questions." She started doing that, she really did. Judy was very teachable. She would not get offended when I would say things to her. She really wanted to learn. (Final Interview, p. 9–10)

I did not observe Jane constantly reminding Judy to do a summary; however, there was an episode where they discussed the importance of the closing during each observation cycle. One possible reason for Judy's difficulty with the summary is that Jane would encourage her to "come up with some questions" rather than have a structured summary. During many lessons, Jane ran out of time, and at other times she forgot to do a summary or the questions she intended to ask. After discussing summaries throughout the experience, Jane complimented Judy by the end when she was consistently closing the lesson with questions.

Jane's emphasis of coming up with closure questions was consistent with her own practice when she said, "Most of my [formative] assessment is questioning" (Observation Notes). Jane also had an award on her desk for being a Promoter of Meaningful Learning for her "use of mathematical language related to the standards and higher order thinking skills," which suggests that she was good at coming up with assessment questions herself, so it was not

surprising that she suggested that Judy do the same. It was not until the end of the experience, however, that Judy was able to consistently use assessment questions in her closing. One hypothesis for this disconnect was that student teachers are so overwhelmed by the logistics of planning and implementing lessons that they need more structure and detail.

Jane also made some interesting comments regarding Judy's preparation. In her initial interviews, Jane mentioned a number of times how working the examples in the lessons took too much time and that it was unnecessary since the answers were in the book. When Judy started teaching, her university supervisor required her to work out the examples, which Jane noticed made an improvement in her teaching. In her final interview, she said,

She was great at writing lesson plans. ... Probably preparing more the night before. ... If she would have worked out some of the problems first, then she would have been more comfortable teaching it, although towards the end it got a lot better. I think with her writing those long lesson plans it got better because she had to work them out. I mean it's not a lot that she asked me to teach a lesson, don't get me wrong, and she wasn't doing lesson plans the first [days] she was there. For her supervisor, she wasn't doing lesson plans the first week she started teaching. That wasn't until later that she started doing them. (p. 14)

When I asked Judy what some of the most effective things Jane did to influence her teaching, she responded,

I think having the standard and the essential question at the top of every slide. That's something I probably wouldn't have thought about doing. Maybe because I didn't learn with standards, or at least I didn't know about them. I think really unpacking the standards and saying, "This is what we're doing today. Here's our essential question. ... Going back to the essential question at the end." (Final Interview, p. 13)

When I asked her where she could still improve as a teacher, she talked about closing lessons and one idea would be to collect a TOTD.

I probably wouldn't use it every day. ... Maybe if I asked those essential questions and have them write it down as a ticket out the door, then it would be good. (Final Interview, p. 16)

The structure of the lessons influenced Judy's view of teaching. Jane and Judy began planning together, and one of the first actions they took was to put the essential question and standard at the top of each slide in their lessons. This was consistent throughout the experience; however, coming back to them at the end of the lesson as a summary was something that they worked on in terms of quality and frequency.

Carousel. In the third week of the student teaching experience, Jane and Judy did a carousel activity in which they created six exercises on large poster paper shown in Figure 5.4. The previous day Jane had said, "We did a lot of talking today; tomorrow we need to let them put their hands to the plow" (Observation Notes), so the six posters were spaced around the room, and the students worked in pairs to answer the exercises on each poster. The students put their answers on Post-It notes, so the posters could be used for all three classes. After the students had put their answers on the posters,

the kids would then go around and look at all the posters and talk about "This person did this. ... This is wrong because they didn't do that. ... Mine's right." After they did that, then we would lead a discussion. We would go to the first one and say "Ok, what'd you find on this one? Do you still have misconceptions? Do you know how to do it now?" Then we'd go to the next one, and we'd just talk about it and make sure that everyone knew how to do it and why their answer might have been wrong. (Judy, Final Interview, p. 4)

This activity was important for Jane and Judy because it is one of the activities that made them focus on formative assessments. Judy said,

I guess they were a prompt for the discussion, but it also helped if we had one problem where everyone got it right, that was helpful to see. If there was another poster where half the kids get it right and half the kids get it wrong, then that's helpful to see. If there's one where everyone got wrong and everyone had different things, then that's something that we obviously need to go back over. (Final Interview, p. 4)

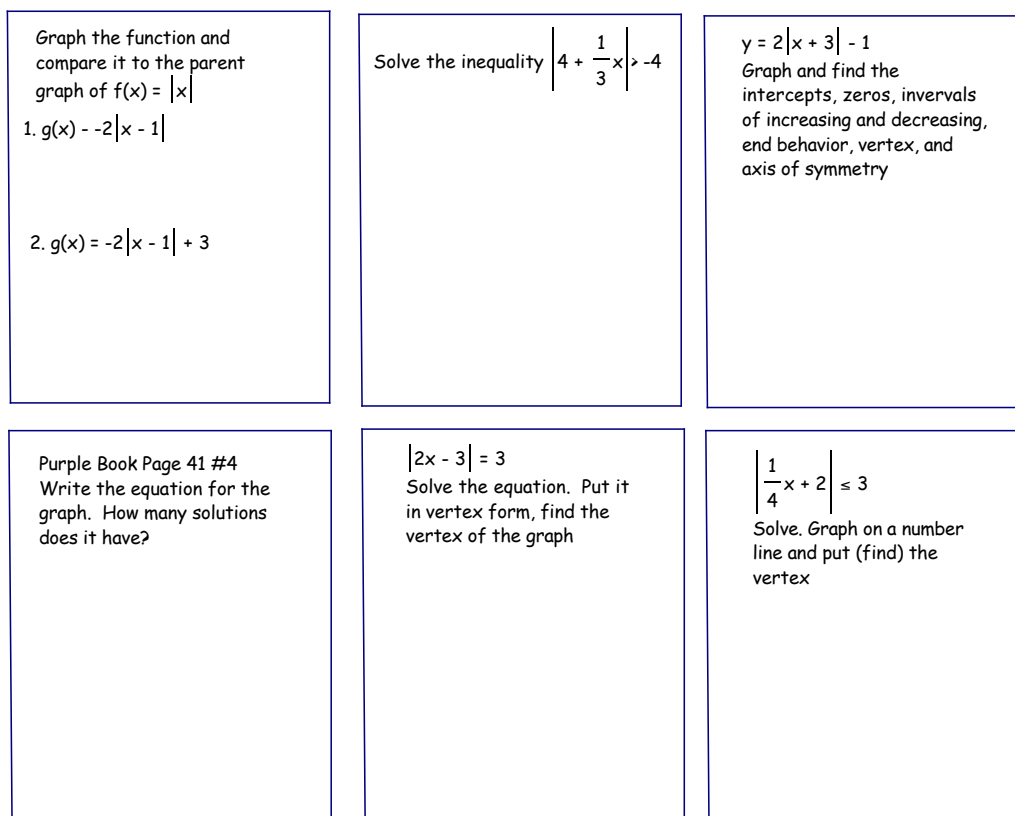


Figure 5.4. Posters for carousel activity.

Jane made similar comments regarding the activity:

[We] had the kids go around and look and make comments on what they saw. The differences in the solutions, the different ways to solve the problem, and they're all working on the same problem. "What did this person do that others didn't? What did you do different that the person over here did?" We talked about things like that. Another thing that this is the first semester that we did is we really assessed good work and unacceptable work. (Final Interview, p. 7)

When I asked Judy for an instance where the ideas for most of the projects and activities came from she said the following:

Mostly they would start with Jane, but then we'd talk about them, and we'd build ideas. ... I guess the first time we ever did carousel, ... we took our own spin and both of our ideas together, we ended up with the posters on the boards, putting them in pairs, and posted notes. ... That's good formative assessment for one. It's good to address their misconceptions. (Judy, Final Interview, p. 3)

The first carousel activity occurred on the last day of the first observation cycle, so I did not observe the following day to see how the pair used the answers; however, I did later in the semester.

During the last observation cycle there were posters with the terms rectangle, square, rhombus, trapezoid, and *kite*. The students were instructed to write down everything they knew about these quadrilaterals. Judy read through the answers, and there were overlapping properties that caused some confusion. For example, a rhombus and square both had diagonals that were perpendicular bisectors and as Judy read the responses for the rhombus, a student said, “I thought that was for a square” (Observation Notes). After school, she looked at the responses and created Figure 5.5, which she displayed on the front board the next day.

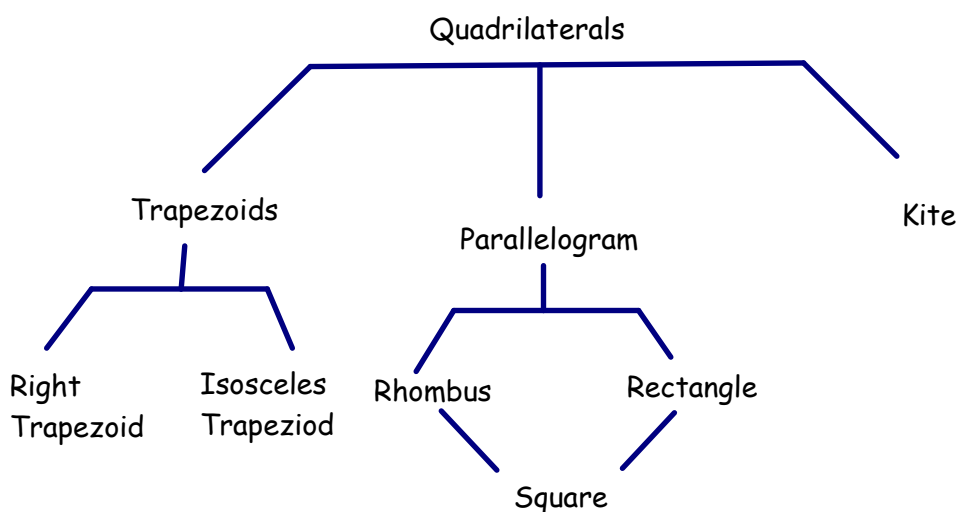


Figure 5.5. Quadrilaterals.

At the beginning of class the next day, Judy said, “I drew this diagram up on the board yesterday. So the rhombus has all the properties of a parallelogram and some of its own” (Observation Notes). This was a case where Judy was able to take the responses from the carousel activity and use them to change her plan the next day and address something that was causing students confusion.

This is also an instance where Judy used a formative assessment to influence her instruction, which is a topic I discuss further below. Before considering assessments, I examine two technology assignments that were also unique to Jane and Judy.

Technology Lessons. Jane and Judy planned two assignments using the computer program Geogebra. The students completed the activity in Appendix G in a computer lab and the other assignment dealing with centers of triangles at home. The assignment in Appendix G focused on transformations of absolute value, quadratic, cubic, and hyperbolic functions, and the students tried different values for the parameters in each equation. Jane and Judy intended for the students to have two days in the computer lab to do this assignment, but Jane was out of school, so it was shortened since Judy did not have authorization to take the class to the lab.

The technology assignments were important for this pair because they were a unique assessment for this study, highlighted their method of grading, and gave some insight into Jane's approach to working with Judy. This was the only activity I observed in all three pairs where the students went to a computer lab and used technology to enhance learning. In the following section, I discuss the grading in more detail; however, it was an important part of this activity because of the distinct way Jane and Judy made comments in the grading process using IB profiles.

The first [technology assignment] was a learning experience for both of us. We didn't even grade the first one heavily. We kind of used it as a learning experience for us too, since it was the first time. "What should we do next time? What are the kids understanding and not understanding?" So we developed the second project where we would make sure that they had a rubric, and it was more independent. The first time we took them to the computer lab and [showed] them how to do it. The second time they had to download [Geogebra] at their houses and do it on their own. So the second time was when we made comments about their IB profiles. It was really exciting to read their projects and really see how they described themselves because it all came out in their work. Actually I had them write the top 3 [profiles] that they were. We really enjoyed reading their write ups because you could just see their personalities come alive, and we made comments about it. We both got excited about that. (Jane, Final Interview, p. 8)

The technology activities, like the carousels, were new for Jane, and her quote showed how she worked with Judy to develop activities. Later in her final interview, Jane elaborated on implementing the transformations activity in the lab:

We were both learning Geogebra, and the kids were too... When a kid had a question, we would bring it up on the overhead screen because we figured other kids in the classroom had the same question. We would pull it up and literally go through it. I thought that was really effective. See, that's another thing that took up time. They had a day in the lab, then they had to go home to finish and leaving them on their own to do stuff was intense for them. (p. 16)

Judy had mixed feelings about the transformations activity. In her final interview she said,

I think [the assignment] was just very vague. All the instructions came from us talking. When we are up there just stating instructions, they can't remember them. I think making it almost more step-by-step. ... I do know where [Jane] was going with it. She wanted it to be vague and for them to come up with their own generalizations and explanations. At the same time, I think that was great for some of them and they did a great job, but others were like, "Ms. Judy, what am I supposed to write? What am I supposed to say here?" Those were not the best students in the class, but they even had questions. (p. 15)

I pointed out that perhaps the purpose of an activity like this was to make everyone in the class, not just the best students, practice communicating mathematically, and she replied, "Yeah, I think that's great, but I think that we threw it at them. We really hadn't really asked them to communicate mathematically except in class when we were asking questions. We hadn't had them really write at all" (p. 15). In my observation of previous lessons, the students were required to write their answers in complete sentences to homework exercises that dealt with a context, and they had to explain their work as part of presenting solutions; however, the technology assignments involved more detail than typical classroom situations. Judy also said,

I think I would revise the activity because I do see how it's helpful for the students to see it on the computer and for them to move it around and make their own generalizations. But at the time, they were all coming up with so many questions... (p. 15)

Each of Judy's quotes in this section makes some reference to the number of questions asked by students. A possible explanation for Judy's fixation on the number of questions was that it was difficult for the pair to anticipate the students' questions since Jane and Judy were both learning to use Geogebra, and this was the first time doing a technology-based assignment.

Assessment

In this section I discuss assessments from a number of perspectives. This was a semester in which Jane was trying many different assessment techniques for the first time. Early in the experience, the pair discussed a unique way of commenting on assignments, they talked about creating assessments in their interviews, and their methods of assessment had repercussions at the end of the experience.

In the first week of school, Jane and Judy collected the homework and graded it. The next day in class, Jane used three students' work from each class to illustrate exceptional, acceptable, and unacceptable work. In her final interview Jane explained,

It was the first time that we did that. The school wanted us to show good work and poor work, but I told Judy that we needed to take an initiative to show it to the kids, so they can strive for what we consider to be good work. We don't want to say this is good and bad without ever showing them, or giving them a mark to reach for. What makes it good work. ... So we were able to put it on the Smartboard and hide the name, and for the same homework assignment I would usually use three different scenarios, one good one, one average, and one unacceptable. She also saw how it made a difference in how the kids did their work, because one girl stopped after of class and said to her on the way out, "I want to do it that way, I like the way that was done. You're going to see one of those papers from me tomorrow." So we did see expectations rise. The quality of the work rises, and what's good about that is that when the quality of their work rises, you can see where they're making their errors because they're showing all of their work. (p. 7)

The school district where they worked went beyond encouraging the teachers to show different types of student work. For example, during the first observation cycle Jane and Judy had a conversation about grading, and Jane said to Judy, "We can't just write 'good job,' we have to say what's good about it. We need to make comments like 'very thorough work, explanations

shows great understanding, please use graph paper, and explanation without graphs”

(Observation Notes). Jane went on to describe how they also used the information from the interestlizers:

The kids knew then what we were looking for. What we also did was the school is turning into an IB school, and the schools have certain characteristics of IB. So what we also started doing, and Judy made a nice big poster board for me where we put all the different profiles, and we took the kids around the corner, and we let the kids read the profiles, and we had them put down what best describes them. They were things like inquirer, thinker, risk taker. ... So the kids put down what they thought they were, and Judy went through and she had all the different profiles and she had Block 1, Block 2, and Block 3, and then she made envelopes and she put the cards in there with their names so that when we would look at their tasks, or look at their write-ups, we could identify and say, “You were really a thinker when you [did] this. I see that you are certainly an inquirer because you went beyond what you were asked to do and found out some background information.” It helps too if you want to group kids according to the same characteristics. We just thought of that two weeks before she left. (Final Interview, p. 8)

Jane and Judy made comments from the quote on quizzes, projects, and homework. One limitation of the data I collected was that I did not collect any student work to illustrate these comments.

These comments did not pertain to tests because they were all multiple choice.

Let me tell one thing that I've done differently this semester; all the major unit tests were multiple choice. I think we did a lot of evaluating other ways and cut down on the grading time of doing those multiple-choice tests. The multiple-choice test, a lot of times, doesn't show you where they're making errors. I think what I'll do with the next student teacher, but what I would like to do is let her develop more tests than just going to a test bank and choosing questions. ... Judy and I did sit down and choose the multiple-choice questions together. We went to the test bank and looked at all the different categories, like sequences and series, factoring, imaginary numbers, and you just click on the category, and we would choose all of the questions. From that pool of questions, we would just click on all the ones that had MC for multiple choice. By the time you did that you might have 150 questions, and from that we would sit down ... and go through each question, “which ones are the same? Which ones could we eliminate? What do we want to focus on? What do we not want to focus on?” I think what I want to do with the next student teacher is have tests that more evaluate towards exactly what you do in class. We don't do multiple choice in class, so that's what I'd like to do differently. (Jane, Final Interview, p. 15)

Two of the other ways Jane and Judy evaluated the students other than tests were the carousel activity and the technology assignments, which were both new to Jane's practice that semester. Jane and Judy used the information they gathered in the carousel activity to readdress issues that the students were having, which took more time than they originally planned for. The technology assignment on transformations also took extra time since they spent a day in the computer lab. In the final interview Jane said,

I will say this, out of all of my student teachers, this is the first time I've never been finished by EOCT time. ... I asked myself, "Is she going too slow?" It was frustrating for me because we kept pushing dates back because the kids still weren't getting it. I was used to an accelerated class moving a little quicker. One thing that we had to do was there's a lot more that we had to incorporate this semester with Math In The Fast Lane. I had to show how I was using some of those strategies. The carousel was one of them. You know we also had more meetings where they wanted us to make changes and put things in, so we were constantly making changes and adding things. I think that pushed it up too, but one good thing I must say, was adding things like that did allow us to see where the kids were struggling, so we had to add days in there. I don't think it was her fault at all, I just think the more that we did to assess the kids, the more we found that they were having trouble, so we spent more time. This is the first semester that I've spent so much more time assessing kids. (Jane, Final Interview, p. 4)

This was an interesting situation because Jane was trying new projects, activities, and test formats for the first time. I asked her if she felt like her students were less prepared for the EOCT compared to previous semesters. Her response was the following: "I don't know, because my test scores were so good [in the past]. I won't know if that until next Monday. ... I think that was 99% of it; we did more assessing. We did much more assessing than I usually do" (Final Interview, p. 7). After the semester, Jane reported that in their three classes combined she had three Cs, three Bs, and the rest were As. She said this was the best her students had ever performed.

There were no data to support any statistically significant gain; however, these data suggest that using a variety of assessment techniques with Judy was an effective way to promote student learning. In Judy's case, making comments and highlighting student work, combined

with activities such as the carousel, gave her insight into her students' thinking and influenced her practice. This influence could be seen in her creation of Figure 5.5. At a minimum, using a variety of assessments was as effective as reviewing for 2 weeks, which was typical for Jane.

Summary

Early in the experience, Jane and Judy had discussions that focused on grouping and set up an ambitious format where there were seven groups, which each had a specific role during lessons. Jane was responsible for the instruction at the beginning and began to develop the roles, such as a structured way for the students to do the warm-up exercises and displaying the answers to the homework. When Judy began instructing the classes, the group roles were used sporadically and became less frequent until spring break, when she returned to them for the final two weeks of the experience. Some factors that accounted for these changes were structural, such as requirements imposed by school like the carousel task and the technology assignment, which were two strategies from a program that was adopted by the school. Other reasons were intentional, and Judy chose to change the format for reasons such as thinking that other methods were more efficient for many lessons.

Jane and Judy planned together in different ways. They did weekly lesson plans throughout the experience and created Smartboard presentations together in the first week. There were salient features in the creation of the Smartboard presentations that resonated in Judy's practice such as having the standard and essential question at the top of teach slide and the lack of a structured slide for a summary. Judy often did not have a summary, which was a theme the pair worked on throughout the experience, and Judy made improvements in the frequency and quality of her summaries by the end. While the pair created the lessons at the beginning, Jane told Judy that it was not necessary to work out the answers to the examples she used in class since they were in the book and she should know how to do them. Later in the

semester, Judy's university supervisor made her work these exercises out, and Jane noticed an improvement in her practice as a result.

Jane had previously mentored many student teachers and reported that she and Judy assessed the students in more ways than she had done with prior student teachers. One formative assessment they used a number of times was the carousel activity, which both of them thought was effective because it gave them insight into their students' thinking and where they were making mistakes. Another form of assessment that Jane had not used was the technology assignments using Geogebra. Judy had mixed feelings about their implementation and thought she it was difficult to address all the students' questions; however, she saw value in the students doing explorations and making generalizations. Jane focused on the grading of the technology activities and discussed how the pair used information they gathered at the beginning of the semester to make comments on the write-ups. Another change this semester for Jane was giving all multiple-choice tests for the first time. These changes caused some stress for Jane because she did finish the curriculum later than she typically did; however, she reported that her students did very well on the EOCT.

CHAPTER 6

THE CASE OF LOIS AND MEG

Lois and Meg taught three 90-minute blocks of Math III in an urban high school; however, the levels of Math III varied. They had one block of Accelerated Math III, then second-block planning, followed by a 30-minute period where students either came for advisement or extra help. Their third block was On-Level Math III, followed by lunch, and their last class was Math III Support, which was a remediation class for students who were struggling in mathematics.

Prior to the experience, Meg mentioned planning and classroom management as two aspects of teaching that she was concerned about. In her survey, she said,

I hope that [Lois] will set an example for me in how to manage a classroom and plan effectively for a class. I think it will be most helpful if she is willing to share with me things that haven't worked for her in her classroom, and the changes she has made to compensate for her mistakes. I am most stressed about planning so I hope that she will have good tips for how to plan and think about your lesson without spending hours writing a lesson or unit plan. (Lois, Survey Response)

These comments foreshadowed the pair's experience in both intentional and unintentional ways. Lois had had four student teachers prior to Meg, and her experience with a recent one taught her she had to be more explicit about what she expected in regard to planning than she had previously. In her initial interview she was describing a lesson the previous student teacher taught, and when I asked her if she talked to the student teacher before he taught the lesson she said, "We did! I thought we did. I think at the end when I sat down and thought about it, I think it was major breakdown in communication. I don't think he was hearing what I was saying" (Observation Notes). As a result, planning became an intentional focus for Meg and Lois from the beginning.

Lois continued talking about planning with a student teacher and said she would “not just give her a blank template [lesson plan]. I’m going to give her an example so that she can see it and just talk to her about my expectations. ...” (Initial Interview, p. 5). Early in the experience Lois admitted, “My weakness is taking things from my head and putting them on paper [when creating documents]” (Observation Notes), so she recognized she needed to be explicit in her expectations. She said that she and Meg would “sit down and outline expectations about time. What time you’re going to be here and here. ... Planning. ... Grading.” (Initial Interview, p. 9). Grading became another intentional focus for the pair.

Lois was optimistic about recent changes in the state curriculum and about working with another student teacher. She said,

I am very positive about all the change occurring in math. I think that I can help student teachers understand and apply why we need to teach math differently than we were taught. I also find that developing student relationships is important and hopefully that transfers to my student teachers... Offering criticism is hard to do. I like to couch it in between some positives, but I think it is important to be honest about the negatives because student teaching is a time for learning. (Survey Response)

One theme that emerged as a function of the circumstances the pair encountered rather than Lois’s previous experiences with student teachers was their focus on classroom management. Meg was very concerned with her ability to manage the behavior in a classroom. When I asked her what she thought Lois would give her feedback on, her response was “probably different things with classroom management” (Meg, Initial Interview, p. 1). When I asked her what some of her weakness were, she said, “I think that I need to work on. ... I guess the best way to look at it is relationships with students. ... I guess that goes along with classroom management” (Initial Interview, p. 6). I asked her what class she would like to take over first, and she said the accelerated class because she thought “they’ll be easier to manage” (Initial Interview, p. 4).

The following sections begin with a discussion of planning and instruction and how the pair began planning together. Issues began to arise when Meg started planning and teaching on her own. The final section is on classroom management, which proved to be a major focus for the pair because of the behavior of two of the three classes.

Planning and Instruction

Lois and Meg began the experience with a focus on planning. In her initial interview Meg talked about her confidence in planning. She said,

I think that I feel pretty good about my ability to plan a lesson. I feel like, for me, the lessons are organized and well thought out and flow, I guess. I feel like I've been able to think more about what could happen. ... I think I'm a good planner. (Initial Interview, p. 5)

Lois saw her role as helping Meg see the reasoning behind different parts of a lesson. She said,

I think that my role is to let her understand why you even need to think about the essential question and why you have to have an activating strategy, which is what they would call it, but I would call it a launch. ... So having her understand what the launch should look like and the explore period and the summary and how important that is to bring it all back together. (Initial Interview, p. 11)

When I asked Meg about her experience in the past with helping student teachers with their lessons, she said,

I have not [looked at student teachers' lesson plans] in the past, and it was something I have thought about. If I'm going to discuss the expectations for planning, then we need to look at them together before we get going. And hopefully that can taper off as she goes. (Lois, Initial Interview, p. 14)

Indeed this was the case. Lois and Meg began the semester by looking at some of Lois's lesson plans together and discussing them in detail. Once Meg began planning lessons, the nature of the discussions changed, just as Lois predicted.

On the first day of the semester, Lois and Meg discussed the lesson plan for Accelerated Math III on arithmetic sequences and sums in Appendix H (Georgia Department of Education, 2005). Lois told Meg that she adapted it every year and still was not happy with the ordering and

wording of the questions. Meg had completed the task the night before, so she was familiar with the answers as they went through it together. Lois highlighted that students typically struggle with identifying the common ratio in Part g and the summation notation. Lois had Meg go through the task and tally the number of exercises of each type. For example, using the equation $a_n = a_1 + d(n - 1)$, they found zero exercises that found the first term, zero that found d , four that found the last term, and one that found the number of terms. For the equation $s_n = \frac{n}{2}(a_1 + a_n)$, they found five that found n , two that found the first term, two that found the last term, and 16 that found the sum. As a result of these tallies, Lois and Meg added some exercises, for example, find the first term and common difference given $a_{18} = 3846, a_{38} = 7492$. They also discussed which exercises Lois would model and which ones the students would complete.

Lois said she did not use Smartboard presentations very often, which was the case the following day. The class spent the majority of the time on the homework, which consisted of factoring quadratic expressions with leading coefficients not equal to one. In the mini lesson, which was part of their lesson plan where they introduced a topic, Lois did three examples of arithmetic sequences where she defined the terms and drew connections with linear equations, such as in Figure 6.1.

$$\begin{array}{l} \text{slope} = \text{common difference} \\ m = d \\ \\ \text{y-intercept} = \text{zero term} \\ b = a_0 \end{array}$$

Figure 6.1. Lois's board work.

The class did not get to the task, and in their discussion after the lesson, Lois said she gave too many homework exercises, and they also did not address the essential question. The essential question was typically written on the front board, and Lois said, “We use them for a summary. Today we didn’t get the summary” (Observation Notes). The following day, the students were given the majority of the block to work in pairs on the task.

There were aspects of this episode that were consistent for Meg throughout the experience. It was typical of both Lois and Meg to use tasks for Accelerated Math III from the Georgia Department of Education. Neither used Smartboard presentations with slides made in advance; however, they would use blank slides to write on. The essential question was written on the front board, and Meg rarely addressed it, or used it only as a summary, which became a point of emphasis for the pair.

One week later, Lois was not at school because of a professional development experience, so the instruction was up to Meg. The Accelerated Math III class began with time to work on the Big 20 review sheet, and then she handed out a quiz. During the quiz, Meg sat next to me in the back of the room and made examples for her mini lesson on geometric sequences. She asked me whether she should begin with the sequence 3, 9, 27, 81, ... and show another representation of a_4 . She worked the exercise out by writing $3(3(3(3)))$ and realized that the initial term and common ratio were both 3, so she changed the first example to 6, 12, 24, 48 After the quiz, she wrote the sequence on a blank page on the Smartboard and asked, “Who can walk me through the recursive formula?” (Observation Notes). A student was able to tell her how to use the formula to find a_2 shown in Figure 6.2. Then she asked how to find a_5 , and a student said, “You need a_4 ,” so she asked for it, and the student answered 48. Another student jumped in and said, “And you multiply by 2,” and Meg replied, “Good job” (Observation Notes). Notice that she did not give the answer for a_5 , and she did not write the answer on the board.

One possible reason for this omission was that she was developing the examples at the beginning of class and did not prepare the question about a_5 .

$$\begin{aligned} \text{Recursive: } \left\{ \begin{array}{l} a_1 = 6 \\ a_n = (a_{n-1})(2) \end{array} \right. \\ \\ a_2 = (a_{n-1})(2) \\ = 6(2) = 12 \end{aligned}$$

$$\text{Explicit: } a_1 = 6, a_2 = 12, a_3 = 24, \dots$$

$$\begin{aligned} a_4 &= 6(2)(2)(2) \\ &= 6(2^{n-1}) = a_1 r^{n-1} \end{aligned}$$

Figure 6.2. Meg's board work.

A similar situation emerged when Meg was talking about the explicit form. Her aim was to use a_4 to develop the explicit formula, and her board work is shown in Figure 6.2. Again, one possibility for the incomplete answer of $a_4 = 48$ and misuse of the equals sign (there were three terms, not $n-1$) in this answer could be that she created this example during the quiz.

After the mini lesson, the students were given time to work on a task involving the Koch Snowflake. After iteration, the unit lengths of the segments on each side were 9, 3, 1, Meg asked, "What are you doing in each stage?" and a student responded, "You're dividing." Meg said, "You're multiplying. What are you multiplying by?" (Observation Notes). There was no closure or discussion of the essential question in this lesson. In her final interview Meg admitted,

I guess in the beginning I was presenting information to them. My questions were more closed. I would always— ... I guess [my supervisor] would say that I still do this. ... I

took more responsibility of the math on the board, and they just basically had to copy it down. (p. 10)

Lois also gave some insight Meg's practice early in the experience:

She started off with the easier classes. At the beginning with accelerated because the behavior was not an issue, and there were things that she wasn't doing. She wasn't soliciting enough information from the kids. ... So at first it was "Can you get through your lesson? Does your lesson make sense? Does the flow make sense?" Then it became more of "You've got to read and feel your kids. Are they with you? Are they not with you? Take your temperature. See where you've got to go." (Final Interview, p. 2)

One week later, I observed Lois and Meg discuss their lesson plans. Lois did a formal lesson plan, using the form in Appendix H, and she discussed the rationale behind each of the sections, just like she alluded to in her initial interviews. She said, "You want enough detail in this so someone else can pick it up and use it." She also pointed out that in planning, "Timing is the number one issue I see student teachers have trouble with." When they got to the section on differentiation, Lois said, "This is where you have to think like a student. Where are they going to mess up?" Finally they were discussing the TOTD, and Lois said, "Ok, so we've done one where they're given the information and have to create the graph, now let's go the other way." I asked Lois how often she did a formal lesson plan her first year, and she pointedly said, "Every day" (Observation Notes).

The pair then went through Meg's lesson plan for the Support class, which is shown in Appendix H. Lois complemented several parts of the lesson including the skills check examples because Meg had "some negatives, some fractions... good." Lois suggested that Meg always put her examples in her lesson plans. She also said, "We've got to put some formative assessments in here. Can we put 9 and 10 on a sticky note," and included this suggestion in the activating strategy. Meg pointed out where the students were going to struggle in the differentiation section, and Lois asked, "Ok, how are we going to help them?" Meg referred to the examples at the bottom of the page, and Lois added,

If this happens, the teacher will go back to the labels of the matrix and reinforce with students the idea of a directed path. She will help them “read” the matrix to find out what paths go to and from different vertices. The teacher can also have students help her come up with a context for the adjacency matrix examples, and this too will reinforce the meaning behind the numbers. (Appendix I)

Meg’s original TOTD was, “How are vertex-edge graphs useful outside of a math classroom? Give an example.” Lois said, “Here’s an idea, give them the matrices, and ask them what it could be used for. That’s higher order thinking.” Meg asked, “Should we give them choices for what it should be,” and Lois replied, “You can let them come up with something on their own and have a few things in mind if they don’t come up with anything” (Observation Notes).

There were salient features of this episode that should be highlighted. One such feature is Lois’s view on anticipating student errors and addressing them in the differentiation section. She also encouraged Meg to anticipate where the students would struggle with the matrix TOTD. The use of a TOTD and other forms of formative assessment were also emphasized, and Meg and Lois consistently used TOTDs. One more thing to note was that there was no discussion of the essential question. Later in the experience these became a point of emphasis; however, I did not observe attention to essential questions early in their conversations.

At this point of the semester, Meg was instructing the Accelerated Math III and the Math III Support classes, and this episode suggested that the pair communicated well in regard to planning. Lois shared her rationale for each section of their lesson plan template, and they were discussing Meg’s lessons. However, in their final interviews, both participants made some comments that question this claim. Lois may have been being modest when she said, “Planning for me has always been something that I do in my head, and it’s really hard to articulate it to somebody” (Final Interview, p. 1). In the case of the arithmetic sequences and vertex-edge graphs, she seemed to clearly articulate how she plans. When I asked Meg how often she and Lois planned together, she said, “Not much” (Final Interview, p. 3). She added,

Well, at the beginning of the semester, it was mostly like she would plan it, talk to me about it, and then I would do it. I never got to see ... we never got to plan one together and then watch her implement it, and then I have to copy it [in] the next class. (Meg, Final Interview, p. 1)

As Meg pointed out, the fact that they did not have two of the same class prevented them from implementing the same lesson; however, they did discuss lesson plans prior to their implementation at the beginning of experience.

By the second observation cycle, Meg was teaching full time and the nature of their conversations regarding planning had changed. Rather than being directive as in the previous episodes, Lois said,

I always wanted to make sure she was ready. At the end of each day, “What do you have on tap for tomorrow?” It became much more big-picture focused. “Ok, this is your unit. How do you plan on getting through it? What are you going to? When are you going to give tests? How are you going to give tests? What is the test going to look like?” instead of the individual lessons. (Final Interview, p. 2)

On the second day of the second observation cycle, I asked Meg what were three things they were working on, and she named classroom management twice. She also said they were working on making lessons more student centered.

I observed a lesson during the second observation cycle that encompassed many of the same issues from the previous episodes and illustrated the pair’s new challenges. The Accelerated Math III class had just finished a unit on rational functions and was working with the unit circle. This was the second lesson in the unit and began with Meg giving the answers to the homework without writing any answers, and she asked if there were any questions. There were none, so she passed out a blank unit circle and asked the students to fill it in using their notes from the previous day. After she passed it out, she changed the direction and told the students to fill in the first quadrant, put their notes away, and fill in the rest using patterns. While the students were filling it out, Meg said, “Maybe it would be a good idea to write this

somewhere on your paper ...” (Observation Notes) and wrote $(\cos \theta, \sin \theta)$ on the board. The fact that Meg changed the directions after she passed out the unit circle and wrote the coordinates suggests she was either getting more comfortable making adjustments during her lessons, or was not thinking about the details of the activities in her lesson prior to their implementation.

Before handing out a worksheet, Meg drew Figure 6.3 on the board and asked about terminal, coterminal, and reference angles, which suggests she thought these were potentially confusing and wanted to review the terms before addressing them a number of times with individual students. This provides evidence that Meg was anticipating areas of difficulty for students; however, after passing out the worksheet, she sat at a desk in the front of the room and completed it. She was also completing the homework exercises just before school started. Not completing these worksheets may have contributed to Meg doing the adjustments in the directions, such as the unit circle activity. The students worked on the worksheet for the remainder of the period, and there was no summary or review of main ideas.

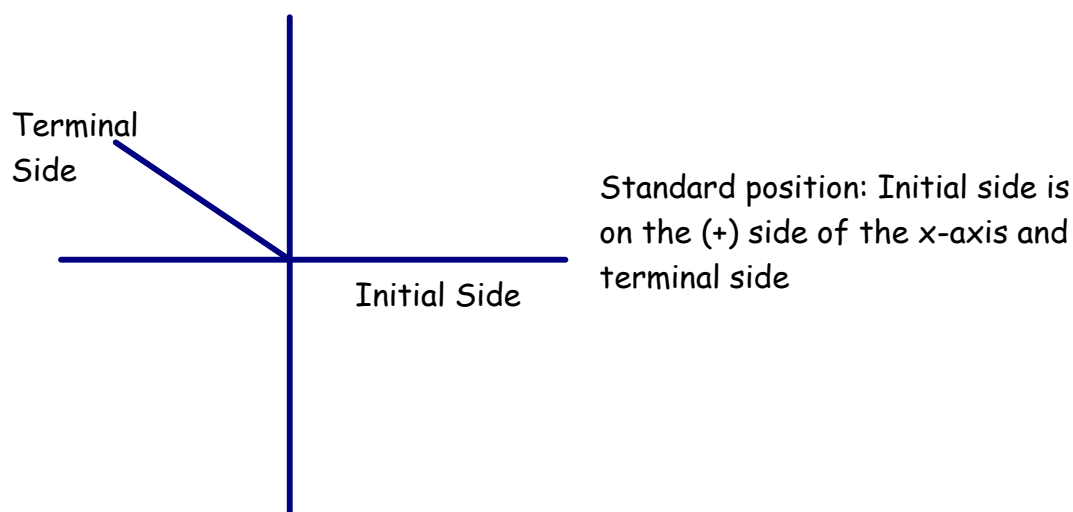


Figure 6.3. Meg's drawing of terminal side.

During their planning period, Lois told Meg that she needed to talk about reference angles more, since there were only two on the worksheet. Meg responded, “I thought that would be the easiest things,” and Lois said, “Actually, that’s where they struggle, and they’ll

need them later” (Observation Notes). This exchange seemed to indicate that Lois was not looking at, or discussing, Meg’s lessons prior to implementation at that point and her focus on planning had “tapered off.” Lois also told Meg, “The only thing I would have done differently is to go over one of the inequalities because you had time” (Observation Notes). Rational inequalities were an area that Meg had identified as a place where the students struggled on a recent test.

This episode suggests that Meg was getting more comfortable making adjustments (e.g., writing the coordinates with cosine and sine); however, her planning was missing important details, such as clear directions and what examples to emphasize (e.g., reference angles). Lois made the comment to me a number of times during the second observation cycle that Meg did not see “the big picture” (Observation Notes), and was referring to connecting lessons, the importance of talking about graded assignments, and missing opportunities to review material that the students did not do well on in previous units. One reasonable explanation for these factors was that Meg had more autonomy in the planning and was struggling to plan for three different classes for the first time.

Much like Lois, Meg used the white board and blank Smartboard slides to present mathematics during her lessons. One repercussion of this lack of structure was that Meg would rely on verbal explanations, which had consequences for the effectiveness of her lessons. In this lesson, she read the answers to the homework exercises; however, the pair did not collect it, so questions were rarely asked. One explanation for the lack of questions on the homework was that students would look at their answers (if they did the homework), and if they got one wrong, there was little motivation to correct their mistakes. Another repercussion was that the board work was incomplete at times, such as in Figure 6.3. The statement “Standard position: Initial side is on the positive (+) side of the x-axis and the terminal side” is incomplete. Meg used the

diagram to describe the terminal side, but did not write anything about it in the statement. This method of delivery had the potential to confuse students who were not paying attention during the explanation, or were not good at taking notes based on verbal explanations. This method of delivery had larger consequences in the behavior of the On-Level Math III class and the Math III Support. If the students in those classes missed some explanations, many of them would become disinterested with the lesson and act out, or disengage, rather than try to catch up by asking Meg to repeat her explanation or ask a classmate for clarification.

During the third observation cycle, I observed an Accelerated Math III lesson on graphing trigonometric functions that showed growth in many areas and gave insight into the mentoring relationship between Meg and Lois. The lesson began with Meg using the document camera to show hand-written answers to the homework, which consisted of graphing trigonometric functions given the amplitude, frequency, and so on. One of the answers had to do with a tangent function and the cotangent function, and Meg showed the graph in Figure 6.4, which became a point of discussion after the class. Then she gave an overview of the lesson by saying, “We have been drawing the graph from the equation and now we’re going backwards” (Observation Notes). This was an important statement because making connections between lessons was a criticism that Lois had of Meg during the second observation cycle in her final interview.

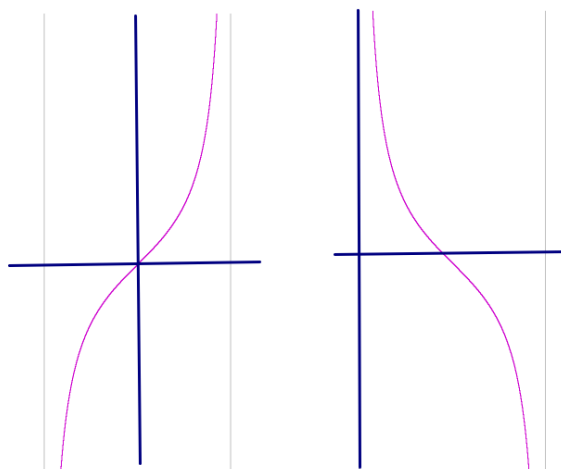
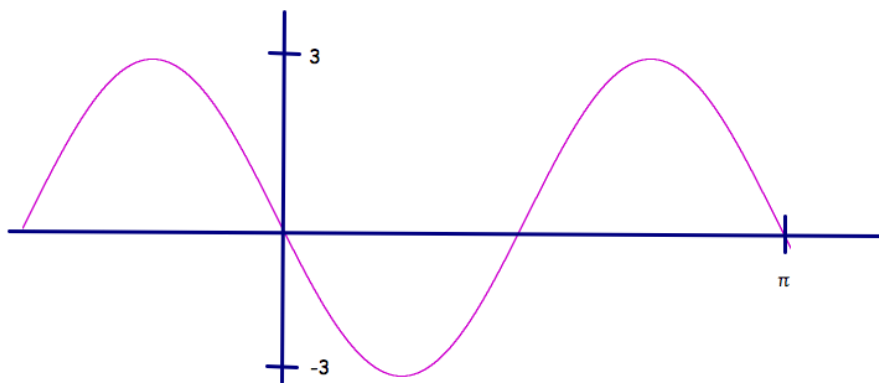


Figure 6.4. Meg's tangent and cotangent graphs.

Meg discussed three examples during the lesson where the students were given a graph and they had to develop the formula of the function. Figure 6.5 shows the first example along with a representation of how the solution was organized on the Smartboard by Meg. Note that she showed three solutions and each was organized the same way. When the class was discussing the solution involving the cosine function, a student said the phase shift (p.s.) was $\pi/3$ and Meg asked if anyone agreed. When there was no answer, so she said, “talk to your partner and see if you agree” (Observation Notes), and then the students agreed it was $\pi/4$ after discussing it with their partner. An important feature of Meg's reaction to this question was that she gave ownership of the mathematics to the students by asking them to discuss it with their partner rather than answering the question herself.



Sin (x)

$$f(x) = a\sin(bx + c) + k$$

$$a = 3 \quad r: \text{yes (reflection)}$$

$$p = \pi$$

$$p.s: \text{no} \quad \frac{2\pi}{b} = \pi$$

$$v.s.: \text{no}$$

$$f(x) = -3\sin(2x)$$

$$a = 3 \quad r: \text{no}$$

$$p = \pi$$

$$p.s: \frac{\pi}{2} \quad \frac{-c}{b} \quad f(x) = 3\sin(2x - \pi)$$

$$v.s.: \text{no}$$

$$\frac{-c}{2} = \frac{\pi}{2}$$

$$f(x) = a\cos(bx + c) + k \quad f(x) = 3\cos(2x + \frac{\pi}{2})$$

$$a = 3 \quad r: \text{no}$$

$$p = \pi$$

$$p.s: \frac{-\pi}{4} \quad \frac{-c}{b} = \frac{-c}{2} = \frac{-\pi}{4}$$

$$v.s.: \text{no}$$

Figure 6.5. Solution to first example.

The solution to the second example was shown in Figure 6.6 and the solution was organized in the same way. Having an organized structure to her board work was lacking in the first two observation cycles, however this lesson illustrated improvement in this area. When they were discussing the cosine solution as a class, she assigned all the parameters and had the students finish the equation, which was further evidence her giving ownership of the mathematics to the students. After the entire solution was on the board Meg said, “give me a thumbs up if you feel good about this and a thumbs down if you’re nah...” (Observation Notes), which was a type of formative assessment that she used more as the experience progressed. Many students had their thumbs down, so she said, “ok, we’ll do this one together” (Observation Notes) referring to the third example.

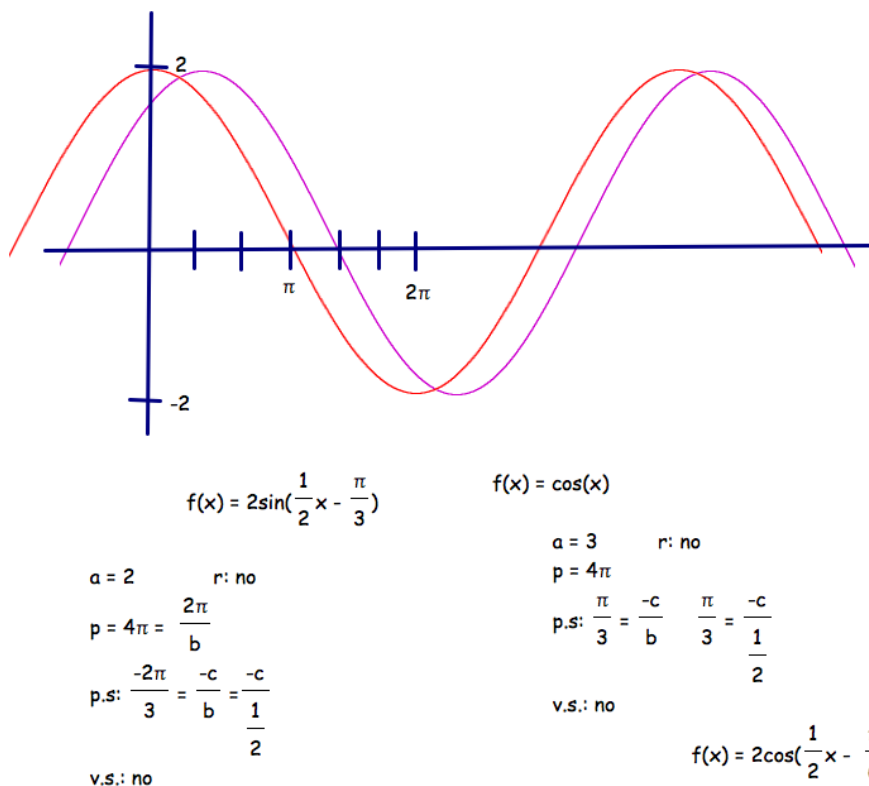


Figure 6.6. Solution to second example.

There were three important questions that were asked during the third example in Figure 6.7. The first was asked by a student when discussing the phase shift when she asked, “why did you start counting at the y-axis,” and Meg responded, “I don’t know, because it’s the easiest.” Lois interjected and said, “you can start anywhere. Were you at the $\pi/3$? Then you go to π and it’s still $2\pi/3$ ” (Observation Notes). This question was relevant due to the way she graphed the tangent function, which I discussed with Lois after the class. The second question was asked by Meg after the class had the first solution and she said, “can you write this with a phase shift?” A student asked if they would be told to write with a phase shift on the test and Meg referred to Lois who said, “we’ll take whatever answer, as long as it’s right” (Observation Notes). The relevance of the exchange was that it provided evidence that Meg was still struggling to see the “big picture” Lois referred to. Meg did not know what expectations to have for the student even

though the test was scheduled for three days after this lesson. The third important question Meg asked after the second version of the solution was provided. She was referring to the c-parameter and asked, “how do we know this is going to be negative” and a student said, “because you move to the right.” She responded, “right, it’s like transformations” (Observation Notes), which provided evidence that she was showing attention to making connections to previous material.

After these three examples were discussed, the class was given a quiz for the final 25 minutes of class. There was no closure, review of main ideas from the examples, or attention to the essential question in this lesson.

Lois had been absent the previous two days and after the lesson she asked Meg why she graphed the tangent function the way she did. Lois replied, “because that’s how we graphed it yesterday” and Lois said, “ok, but we’ve got to go back and change that. They need to see that the tangent looks like this and cotangent looks like this [draws graphs with $-3\pi/2 \leq x \leq 3\pi/2$]. They need to see the period of tangent and it goes through the origin” (Observation Notes). Lois went on to suggest that Meg show more of the negative side on the x-axis and acknowledged that phase shifts were difficult for students.

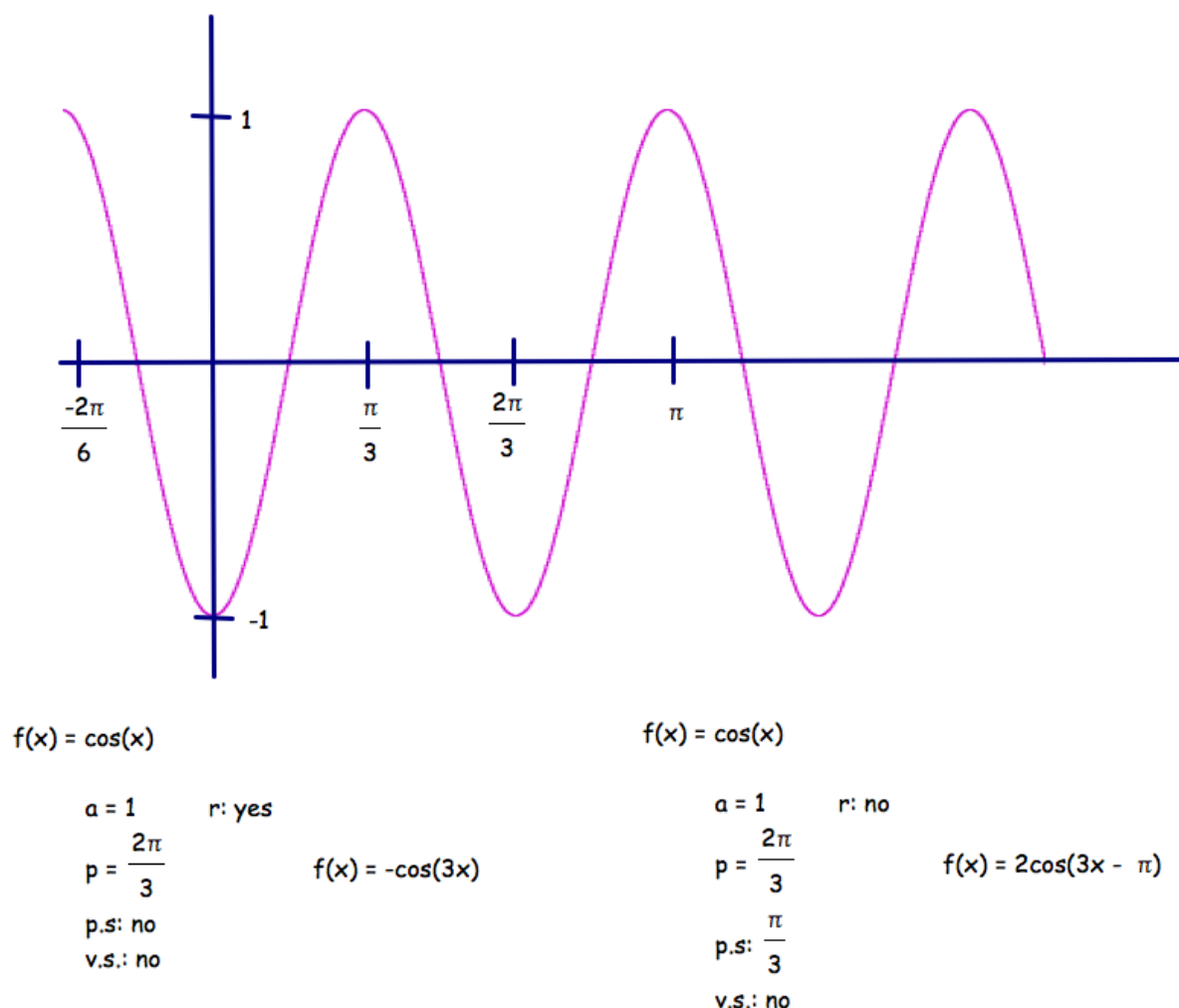


Figure 6.7. Solution to third example.

Lois and I discussed the tangent graph after Meg had left the room with $-3\pi/2 \leq x \leq 3\pi/2$. I suggested that she drew it that way because she started counting at the origin, just like in the third example. Lois countered and said that it was probably because they had just completed a unit on rational functions and they considered $\tan(x) = \sin(x)/\cos(x)$. She said they probably drew in the asymptotes and zeros, and then drew the graph. I thought this was a good possibility, so I asked Lois to say where the asymptotes were and she listed $\pi/2$, $3\pi/2$, $5\pi/2$, etc. I asked her where the zeros were and she listed 0 , π , 2π , etc. I pointed out that these were all positive values

and they were probably the same answers the students gave, which would produce the graphs in Figure 6.4 if she only considered those hypothetical answers.

This conversation during planning was important to observe because it illustrated the nature of the communication between the pair. Lois began the discussion with a question (why did you graph tangent like that), but accepted Meg's simple answer (because that's how we drew it yesterday) and quickly became directive on what adjustment she would like to see, whereas Lois and I had a mathematical discussion. Another important aspect of this lesson and conversation was that it seemed Meg was organizing the board work more effectively and asking better questions than early in the semester, but this was a missed opportunity to have a discussion of a mathematical topic. Early in the experience, Lois provided rationales for the different parts of the lesson plan and the importance of the summary and returning to the essential question, however there were still parts of the lessons Meg was struggling with. At this point of the experience, Meg had improved in many of the areas the pair discussed earlier and their discussions were much shorter and less detailed.

In their final interviews, Lois and Meg made some comments that gave some insight into the nature of their discussions. Some comments contradicted each other while others were supportive of one another. One of the most astute comments Lois made in her final interview was,

She probably a better planner than some I've had in the past, which I emphasized and was different from the past. Whatever you monitor gets done. Whatever you emphasize gets done. (p. 1)

Early in the experience Lois and Meg discussed lesson planning and focused on the rationale for each section of the lesson plan, such as anticipating student difficulties and how to address them in the differentiation section. Once Meg assumed responsibility for more of the planning and instruction they talked about what was planned for the next day and when assessments would be

given. The feedback that Lois gave was typically after individual lessons and focused on what Meg should have emphasized more.

This type of feedback may have had repercussions for Meg's ability to summarize and make connections between lessons. In her final interview Lois said,

She never linked the lessons. That's one of the most important parts. She never put the meat on the bones. She had the bones. I think she discounts how important what you say is... She would start fresh every day... Maybe she doesn't have the connections either. Maybe that knowledge wasn't as strong. I didn't ever really have huge issues other than the tangent graph. I'm still baffled by why she would graph it that way. (pp. 9–11)

One explanation for Meg's lack of growth in making connections between lessons is that it was difficult to receive feedback about the connections that should have been made and use those comments to make connections in the future.

Lois's quote also pointed out a paradox that mentors face. In her initial interview, Lois talked about how her feedback typically tapers off with the intention of giving the student teacher more autonomy. However, Lois identified two issues, connecting lessons and mathematical knowledge (e.g. tangent graph), where she saw deficiencies in Meg's practice. The paradox is that on one hand mentors want to give their student teachers freedom to plan and implement lessons so they will be able to function as independent teachers in the future, and on the other hand, interjecting when they identify areas of student teachers' practice that can be improved.

Summaries were another area where the repercussion of Lois's style of giving feedback and this paradox could be seen. When I asked Meg to talk about something she learned from watching Lois teach she said,

The way that she handled Standards in that class. She would always write the essential question and the Standard on the board... I didn't do anything with them. I would just write them up there... I guess I just would forget that they were there. (Final Interview, p. 13)

Similarly, when I asked her for one thing that they talked about that did not seem to have an effect, she continued talking about using the essential question to do a summary and said, “[Lois] would always tell me that I needed to do them, but I would always forget” (Final Interview, p. 14). One explanation for why Meg would forget to do the summary is that Lois was consistently gave directive feedback and told Meg not to forget. An alternative would be to consistently ask Meg how effective her summary was, thereby giving her ownership of the problem. After repeatedly admitting that there was no summary, perhaps Meg would have realized how important they were for Lois and started to incorporate them more often.

Lois admitted having trouble incorporating summaries in her own practice, which brought up another interesting point of analysis; how much of a student teacher’s practice is influenced by the deficiencies of their mentor’s practice? In her final interview Lois said,

Even I still struggle with pulling up a summary, just because it's really hard. I can open anything, but taking the time to close it back down is tough... It would irk me to no end that it would be 9:51 and the class ended at 9:52 and she's just talking about homework. We did talk a lot about that and it did get better. (p. 14)

Perhaps Meg’s difficulty with providing summaries was related to Lois’s struggle to do the same thing. Lois reminded Meg to do a summary and talked about their importance early in the experience, but since they were not a strength of Lois’s practice, it may have been reflected in Meg’s practice. Meg would include them in her lesson plan, however, as she said, forget about them when teaching the lessons.

When Meg and Lois talked about feedback on planning, they viewed their discussions differently. Lois said, “I would make her start with the test, which is backwards design, which is what you're supposed to do. Make sure you know where you want your kids to go” (Final Interview, p. 2). Meg on the other hand said,

A lot of times we would get through the whole unit and then at the end, Lois would... We were looking at the unit test that [another Math III teacher] made and she wouldn't send

them to us until we were in the unit. We would look at it Ms. Lois would be like, 'well you never gave them an example like this' and I would say, 'well there's not one like that in the book and I never would have thought they need this example.' It's not anyone's fault, it's just lack of experience on my part and I didn't know that they needed to see something like that. (Final Interview, p. 4)

Meg gave the example when, “one of the questions on polynomial functions, it said 'what are three different names for the x-intercepts of a graph?' We'd always called them x-intercepts and zeros” (Final Interview, p. 5). One possibility for why there were different interpretations was classroom management issues dominated their conversations for two of the three classes, hence they did not talk about lesson planning or mathematics very often in the context of Math III or Math II Support. Each participant made comments in their final interviews to support this claim. Meg was talking about planning and said, “there was nothing. We didn’t talk about Support at all” (p. 8). Lois commented that,

unfortunately in 3rd block, the behavior was such an issue, that that's what we talked about the whole time. I feel like we were able to focus in first block on the teaching part. In the beginning it really was about planning. 'Do you have a lesson that you can deliver that makes sense, gets the point across, did you hit all the standards?' Later it was like 'That could be better.' (p. 3)

To further substantiate Lois comment, notice that all the examples in this analysis were from Accelerated Math III. As a researcher it proved difficult to gather data on Math III or Math III Support from discussions based on planning and instruction that dealt with anything other than classroom management. Due to this situation, another reason Meg may have had trouble with incorporating feedback such as the Lois’s reminders to do the summary was because their conversations for two of the three classes focused on classroom management rather than reinforcing the conversation from Accelerated Math III.

To summarize this section, Meg and Lois began the experience by discussing the rationale for each part of the lesson plan template that Lois provided (e.g. Appendix I). They analyzed some of Lois’s lesson as well as Meg’s. Lois complemented Meg’s ability to plan,

however, three issues that she identified in Meg's practice that could be improved were soliciting more information from students, making connections between lessons and including summaries. Meg improved in all three areas with the most improvement in soliciting information from students. One possibility for the lack of improvement in the other areas was because classroom management dominated many of the pairs' conversations, which is focus of the next section.

Classroom Management

To discuss Lois and Meg's struggles with classroom management, I begin by considering their comments before the experience, followed by a discussion of how the pair tried to prevent behavior issues early in the semester. I then consider some reasons for the management issues Meg encountered and finally some techniques Meg and Lois used to deal with the issues.

In their initial interviews both participants talked about classroom management as a major reason for student teaching. Meg referred to a field experience she had at another school and how her lesson did not go as well as she hoped due to management issues, so she was nervous about it. She said she would prefer to take over the accelerated class first, "because I think they'll be easier to manage" (p. 4).

When I asked Lois for the top three reasons for student teaching she said, "number one is classroom management" (p. 10). She also said,

I don't know Meg and what her background is, but you've got to build relationships with kids and they've got to know that you like them and love them before they'll ever do anything for you. Period. (p. 10)

When Lois was talking about previous student teachers, she said that she learned to be clear with her expectations and, "I want to make sure she understands my routine, because when they [students] talk, I just stand there and look at them. Management kind of things" (p. 10). This management technique did come into play among others.

During the first week of the semester they were discussing the third block on-level Math III class and Lois talked about how she disliked seating charts, but they made one since it was a large class and said, “you can make huge management adjustments just by where they sit” (Observation Notes). During the semester I observed three different seating charts in this class, and I asked Meg for her rationale during the second observation cycle. She said it was to, “break up the talkers” (Observation Notes) and referred to individual students in the class who she wanted to separate.

While she was still observing Lois teach, I asked Meg which class would be the toughest class to take over first and she said the accelerated class, which was a change from her initial interview. Her answer was likely influenced by Lois who talked about how she thought the on-level students will ask more questions and accelerated students will ask more difficult questions, “because they want to show you how smart they are” (Observation Notes). Meg thought the easiest class would be the Support class since it was their smallest class with only 15 students.

Meg did take over the accelerated class first and during the first observation cycle I asked her to write down three things Lois did that she was trying to emulate. The first thing she listed was, “she respects the students, but doesn’t let them do whatever they want. She’s not afraid to discipline. I need to get better about disciplining and making sure students work on my schedule rather than theirs.” The third thing she listed also had to do with management and she wrote, “I’ve been trying to be more mindful of where I’m standing while I’m at the board. She is always facing the class. Rarely is her back to the students” (Observation Notes). These points illustrated that Meg was concerned with classroom management from the beginning of the experience, before she took over the Math III and Math III Support classes, which were where the majority of the management issues occurred.

Several times throughout the experience I made comments in my notebook that Meg had a tendency to stay at the front of the room, including observations during the third observation cycle. One contributing factor to Meg's management issues was that she would stay at the front of the room at the beginning of class when the students were supposed to be working in their Big 20 review sheets. Lois noticed the same thing and said,

It would drive me crazy that she would have them do the Big 20 and she would quietly stand there at the table. They weren't doing the Big 20 they were talking. If I was teaching, I would say a thousand times, 'do you need help, are you working, do you have questions' just to remind them that I'm in charge, to make my presence known. (Final Interview, p. 12)

Another contributing factor to Meg's management issues was the tone she took with students. In her final interview Meg said,

For a long time after class Ms. Lois's biggest criticisms would be 'you said that like you were trying to apologize to them. You're not apologizing and this is what needs to happen and she would say like...' Sometimes she would be like 'you need to call people out. Don't be afraid of them. If they're not listening to you, if they're not writing things down, call them out.' (Meg, Final Interview, p. 6)

Lois admitted that she could have done more to help and gave the following suggestions for mentoring student teachers in the future,

The behavior, because I don't think student teachers always feel as empowered to make the phone calls home that need to be made. I could have probably helped with that. So last semester I realized, 'oh my gosh, I didn't give enough explicit instruction on planning' and now I'm realizing that I maybe didn't do enough to help that discipline aspect. We probably should have gotten on the phone way earlier and I should have empowered her more, although I know for myself I was very intimidated to call parents when I was 22 or 23 years old. 'Who am I to tell you how you child should behave?' She also doesn't have the relationships with the families that I have, so probably we could have empowered her more to take a bigger stand on that kind of thing. But again, behavior is one of those things that you have to screw up enough to get pissed at yourself. (Final Interview, p. 5)

Lois brought up another factor that may have contributed to the lack of motivation for some of the Math III students, which was lack of a test to use as leverage. She explained,

In this world, juniors, they changed the legislation, so we don't have graduation test for everybody, so it's hard to focus on that because it doesn't apply to everybody. We don't have an EOCT... they did in eighth grade, but there was a lot at stake, and there's not a lot at stake here. (Lois, Final Interview, p. 3)

Lois had only taught at the high school for two years and taught many of these students while they were in eighth grade, which allowed her to compare the two situations and comment about her relationships with the families.

I also pointed to two other contributing factors in the previous section on planning and instruction. Meg had the tendency to use verbal explanations rather than writing, or writing partial explanations, such as in Figure 6.3. Meg's Math III class had approximately 30 students and many of them would not ask her to repeat something if they missed her explanation the first time and would become behavior issues if they got behind during the lessons. The other factor that was an issue early and helped the classroom climate improve was when she began to give ownership of the mathematics to the students. The students were more engaged when she began to use techniques like talking to their partner or neighbor when she asked a question and nobody responded. This technique encouraged the students to focus on the point of the lesson where they were.

As Lois talked about in her initial interview, one management technique she used was to stand and wait rather than talk over the students when they got loud. I observed Meg do the same thing in the first observation cycle and several times throughout the semester. During the first observation cycle I also asked her to write three things that she noticed about Lois's teaching in my notebook and one of them was, "don't talk while they are talking. Stand and wait. Demand their complete silence and attention" (Observation Notes). Meg wrote a list of "thing to consider while teaching" in her notebook and listed several items pertaining to classroom management techniques including ask questions while you have a student at the board,

redirect class attention, become a force in the classroom and let them know you're all on the same train, and address misbehavior (I need you to...), and get their attention. The parenthetical reference she made was to a book that Lois gave her early in the semester, which Lois talked about in her final interview.

The other book is 'How to Talk so Kids Can Learn' and I learned that you have to phrase what you say in terms of 'I'. Instead of '[student's name], you never stop talking. You're always disrupting me. You're so disrespectful...', but if I say [student's name], I got this really hard lesson to teach and I need your help' it's a totally different framework and they won't shut down on you. They're going to shut down on you as soon as you start in with 'you do this... you do that.' These books helped me change how I taught. I tried to talk about that with her, but I don't know if it was always soaked up. I asked her to read this and she said she did. (p. 7)

By the second observation cycle Meg was trying to address some of the issues she was having by using these techniques. In my observation notes for the Math III on the first day of the cycle I noted that she was moving around the room during the Big 20, she had a new seating chart, and phrased at least two directive statements beginning with 'I...' One instance was when she wanted to tell the students about a common mistake entering information into a calculator and said, "I need everyone's eyes... If you're putting the whole numerator in the calculator, you need to put the whole thing in parentheses." The other instance was during a rare occurrence when she was providing a summary and said, "before you pack up, I need everyone to write this..." and wrote, "exponential functions have a variable in the exponent" (Observation Notes). The underlined statement was blank and she asked the students to fill it in before she wrote the answer. She used the same "I need all eyes on me" technique the following day in Accelerated Math III and took it one step further. She asked for a coordinate point on the unit circle and quickly said, "If you know it, don't shout it out... Raise your hand if you have the answer... Ok, keep working, I want to see more hands" (Observation Notes).

After that class Lois complemented Meg on both the content and the management of the lesson. Meg pointed out that one student in particular did a good job and Lois said, “good, then you need to tell him that.” This prompted them to talk about other students, specifically in the Math III class that was the most challenging to manage Lois referred to two students and said, “you need to comment any time [student A] is doing something good today. [Student B] as well, if you can catch him doing something right” (Observation Notes). I did not observe this praise, however they had similar conversations later in the semester about reinforcing positive behavior.

When Meg and Lois discussed classroom management in the final interviews their comments suggested that it was not a regular focus of their conversations, despite being an influential aspect of the student teaching experience. When I asked Meg for three words or phrases that summed up her experience she said, “...we worked a lot on classroom management. That is the biggest thing that sticks out to me. What about relationships with students...” (p. 1). She continued by saying, “ with classroom management and disciplining them, [Lois] would always tell me 'you need to figure out a way to make them know that this is your world and they're living in it'” (p. 7).

Lois’s comments provided some insight into why she would encourage Meg to be in more command classroom and make it “her world” because she would avoid confrontation. Lois said,

Well she lets them talk. She would rather not have a confrontation, so she'll let them talk over her versus a potential confrontation. I've gone through enough confrontation, that's were some of it was partly my fault. I had to take ownership of that. There were times when I could have avoided it, but I got pissed off. I don't think she gets pissed off enough, and maybe that's her personality, but she would avoid a confrontation and then she would confront the wrong things. (p. 5)

The “wrong things” she was referring to were certain students whom Meg would constantly tell to be quiet and Lois thought Meg dealt with ineffectively. Lois further explained that,

I think that by the time it started getting really bad in 3rd block it was more than half way done and she could see the end of the tunnel and she was done. She was mentally done. It was really a challenge. (Lois, Final Interview, p. 6)

Lois made a similar comment at the beginning of the third observation cycle that she thought Meg saw the light at the end of the tunnel and was “cruising” (Observation Notes).

Meg claimed that she did not realize what kind of impact her demeanor had on the classroom. In her final interview she said,

I honestly learned a lot about classroom management. At first I really didn't know that it was so bad. I guess because I didn't know any different. I didn't know that I was supposed to have all of their... I mean that stupid... Not that I was supposed to have all of their attention, but... I guess I just didn't know that I wasn't doing the right things. Like I didn't realize that by talking to them like I was that I was basically letting them think that 'ok, she's a joke. She doesn't know what she's doing.' I think that I learned a lot about that because I didn't have any idea that I was coming off like that and I didn't have any idea that I needed to be like such a big personality in the room. (p. 13)

These quotes provided evidence that Meg was getting frustrated with having classroom management issues and by the end of the experience Lois and Meg were communicating sparsely about solutions. As I alluded to in the previous section, this situation likely contributed to Meg's struggle with issues like providing closure to lessons despite Lois's consistent reminders. Classroom management became such an area of frustration for the pair that it hindered their discussions.

To summarize this section, both Meg and Lois identified classroom management as an area of concern early in the experience. They talked about it in their initial interviews and had discussion about it prior to Meg assuming responsibility for instructing a class. Lois even provided Meg with literature on how to talk to students. There were several contributing factors that led to the management issues including Meg's tendency to stand up front while the students were working in the beginning, her verbal explanation and students falling behind in the lesson, the students' lack of ownership over the mathematics, her tone, and having a difficult group to

motivate due to the lack of a high stakes test to prepare for. Meg used the techniques of waiting quietly for the students to stop talking similar to Lois, phrasing statements in terms of “I...”, and reinforcing positive behavior. By the end of the experience classroom management became a point of frustration for the pair and hampered their discussions about teaching.

CHAPTER 7

DISCUSSION

The previous three chapters considered the experiences of three pairs of mentors and student teachers. Each pair had unique characteristics as well as experiences that were similar to the others. In this chapter, I compare these similarities and differences and discuss patterns across the cases. I additionally relate the findings from the three cases in this study with other research. In this chapter, I discuss decisions for curriculum topics, how the student teachers modeled their mentors' practice, the student teachers' practice as they began to take responsibility for the instruction, mathematical discussions between the pairs, and formative assessments. These further analyses provide insight into the nature of the relationship between student teachers and their mentors as I explore the impact of this relationship on the practice of student teachers.

Curriculum Topics

One feature that was consistent in all three cases was that curriculum topics came from the mentor teachers as well as other teachers who taught the same classes. Bullough et al. (2002) found a similar result, but claimed that lack of control over the curriculum was a frustration for many student teachers. There were no data to support such frustrations for Lisa, Judy, or Meg. I never commented in my notes or had audio recordings of any of them expressing frustration over the curriculum topics during the observations. Each pair was involved in some type of common planning with other teachers.

Marge and Lisa planned with the other eighth grade teacher and used the CMP curriculum to guide their planning. Jane and Judy used the pacing guides Jane had already developed with other teachers in previous years, which were adapted from those provided by the state (Georgia Department of Education, 2005). Lois and Meg planned with the other Math III

teachers and the coordination of on-level and Support classes was most apparent in their case because the group of Math III teachers had a common planning period and shared activities, worksheets, and had common tests. One reason common planning was important for the teachers was that a large number of the students who were in the on-level classes also attended the remedial Support classes. The students in the Support classes did not necessarily have the same teacher for the on-level classes; hence, the teachers needed to communicate and stay on pace with one another, or it would be difficult to plan for students who were days ahead or behind the others.

Fitting Into the Mentor's Practices

Bullough et al. (2002) also discussed how many student teachers find themselves “fitting into the mentor's program with minimal disruption” (pp. 73–74), which could be seen in a variety of instances in this study. Each of the student teachers adopted practices modeling her mentor's practices very closely. Meg and Lisa talked about the importance of questioning at the beginning of the experience, and it became a focus of Lisa's practice. More apparent modeling practices could be seen in the way Lisa displayed answers on the board during the factoring unit. An important element in the factoring unit was that Marge shared her rationale for choosing the representations. Other instances where Marge's practice was influential were the highlighting Lisa used while factoring by grouping and Venn diagrams.

Jane and Judy began the experience of making Smartboard presentations together, and two of the first things they added were the standard and essential questions at the top of each slide. From the beginning, Judy consistently had these features in her lessons; however, just as in the co-created presentations, she did not have a slide dedicated to a summary. Jane provided closure to her lessons by asking questions and encouraged Judy to do so, which Judy did not start doing until near the end of the experience. Jane talked about how sometimes she forgot the

questions she intended to ask and forgot which questions she intended to ask at other time. This case suggests that student teachers could benefit from more structured way to remember to provide closure, such as the TOTDs used by Meg and Lisa. The other feature of Jane's practice that was difficult for Judy to adopt was the use of the group roles, which was likely due to the complexity of format. Student teachers are learning about so many aspects of teaching early in the experience that it is difficult to organize and effectively implement a format such as assigning 7 groups to perform different parts of a lesson. Since Jane was able to use the groups towards the end of her experience, it implies that student teachers are capable of using this format; however, early in experience it was too complex to implement.

Meg's method of delivery modeled Lois's as well. Neither of them created Smartboard presentations and instead used blank slides to present exercises and answers. Meg's management techniques also closely followed those of Lois. One such technique was when the students were getting loud, Meg would simply stand and wait for them to acknowledge that she was waiting for them. Another technique was phrasing things in the first person by saying, "I need you to..." rather than saying, "You need to..." Interestingly these techniques were not as effective for Meg as for Lois, which was likely due to Lois's better established relationships with the students. Using a technique like waiting for the students to quiet down on their own works more effectively when the teacher has established a respectful environment where she is seen as having some authority. Establishing this environment is challenging for student teachers because the students know they do not have as much authority as the mentor teacher.

Lois made a statement in her final interview that is particularly relevant to this section: "Whatever you monitor gets done. Whatever you emphasize gets done" (p. 1). In each of these cases, the mentors emphasized aspects of their practice they felt were important and those emphases became foci for the student teachers' experience. In some cases, such as Lisa's

structure models of factoring, these tendencies could be very helpful. In other cases, such as Meg waiting for the students to recognize she would not talk over them, the methods used by the mentor were not as effective. This difference could be because of peripheral factors like established relationships or because of the ability of the student teacher. Jane won an award for her questioning, but she struggled to get Judy to provide closure to her lessons using the same techniques. It is important for mentors to recognize how much or how little influence their tendencies and habits have on student teachers.

Traditional Practices

Wideen et al. (1998) concluded that the practical pressures of student teaching prevent preservice teachers from teaching in a manner consistent with the training they receive in their undergraduate program, and the student teachers feel as though all they can do is survive. Franke et al. (1997) and Cady et al. (2006) claimed that these pressures cause student teachers to adopt traditional practices. These results could be seen in Judy's and Meg's experiences, where different practical pressures of their classrooms influenced their practice.

The grouping format that Jane and Judy devised was challenging for Judy to use for a number of reasons that I have discussed. When Judy assumed responsibility for the instruction, the logistics were too difficult for her to consistently organize the groups to use the different roles, and as a result, she began to reorganize the room in her lessons. The lessons followed a traditional format where students were in rows and the discussion was dominated by teacher-student interactions where Judy asked a question and the one student responded at a time. When Judy got more comfortable conducting lessons, Jane encouraged her to get back to using the format with the group roles, and Judy used them partially toward the end of the experience.

A large practical pressure Meg faced was managing difficult classes. The Math III class was a particular challenge because the students were difficult to motivate. Lois and Meg talked

about how there was no high-stakes test for this class and how the students knew they only had to pass the class to get a third mathematics credit to graduate. As a result, they thought many of the students only put in the effort they needed to pass rather than working to excel. Keeping the students engaged and on task was challenging for Meg, which limited the instructional techniques she tried. She typically stood at the front of the room with the students in rows, and she presented the answer to each exercise on the Smartboard while asking general questions (e.g., What is the next step? Can I just divide one side of the equation by 3? How do we add fractions?). The students rarely did group work or had class discussions where there was student-to-student interaction; however, Lois did encourage her to lecture less and solicit more information from the students in the Math III and Math III Support class.

Judy's and Meg's experiences shed light on two reasons why student teachers use traditional practices that were different from those promoted in their teacher education program. In Judy's case, the format she and Jane discussed was challenging to organize logistically, so she began to plan lessons that were more teacher-centered and traditional. Teachers attend to a large number of details beyond instructing the lesson, which is a challenge for student teachers to coordinate because they have to think about the details of the lessons they plan such as how much time they are going to spend on each section, what misconceptions the students will have, the questions and follow-up questions they will ask, and what connections they will make to other mathematics. This observation does not imply that experienced teachers do not have to consider these factors when planning. Research (Leinhardt, 1989; Reynolds, 2002) has shown, however, that experienced teachers are more efficient at doing it than inexperienced teachers are. Judy's situation illustrates how student teachers need to develop the capacity to make a classroom functional before they can attend to complex formats such as the one envisioned by Jane and Judy at the beginning of the semester.

Meg's struggle with classroom management in her On-Level Math III class was a major obstacle preventing her from experimenting with different lesson formats and discussion techniques, and instead, she experimented with different management techniques. Meg did try to use tasks and promote more student-to-student interaction in her Accelerated Math III class, where she did not have as many discipline issues. This situation supports Kagan's (1992) assertion that student teachers need to develop some fluency with managing the behavior in a classroom before they can experiment with lesson formats and techniques that are promoted in teacher education programs.

In both cases the student teachers were given some autonomy, then they implemented traditional practices, and the mentors encouraged them to change and include more group work and increase student involvement. Mentors who are working with extreme novices, such as student teachers, are faced with the situation where at some point, they have to give the student teacher an opportunity to perform independently and give them feedback on ways to improve. Since Lois and Jane attended the same university as the student teachers for their advanced degrees, both mentors were familiar with ideals that the teacher-education program promoted. The mentors gave the student teachers some freedom to plan and implement lessons and then encouraged them to develop more student-centered instruction.

Mathematical Discussions

The present study supports the work of Borko and Mayfield (1995), who claimed that the student teachers and mentors in their research had mathematical discussions at a superficial level. Another finding in this research is consistent with observations by Leatham and Peterson (2010), who claimed that the mentor teachers did not attend to mathematics even when asked specifically how they would contribute to their student teachers' ability to teach mathematics (see Appendix A, #10).

The examples discussed in Chapters 4, 5, and 6 illustrate how each of the pairs discussed mathematics, which were largely organizational and focused on ways to present mathematics. Marge and Lisa discussed representations of factoring trinomials. Jane and Judy discussed ways to present mathematics early in the experience when they were doing the problem-solving exercises using tables to develop equations. Lois was very directive when she told Meg how she should have represented the tangent graph and what range of x -values to show the next day. Classroom management likely influenced the nature of the tangent conversation because it dominated their typical conversations between Meg and Lois and limited the pairs' willingness to share ideas. In each case the frequency of mathematical conversations were higher at the beginning of the experience and tapered off towards the end. Something to consider is that mathematical conversations can be productive in a teachers' development regardless of their experience level, so discussions of mathematics should be a point of emphasis throughout the student teaching experience.

Formative Assessment

One area of improvement for all of the participants during their student teaching experience was in their use of formative assessments. The types of formative assessments typically used by the participants were questioning, Tickets Out The Door, summaries, and activities such as the carousel. Researchers (Reynolds, 1992; Borko et al., 2000) have noted how novice and student teachers have difficulty leading mathematical discussions, which was consistent in the analyses in this study. The present research gives some insight into how each participant improved in similar areas such as questioning and unique ways such as Judy's carousel activity.

Kagan (1992) concluded that student teachers go through a period where they focus on themselves before they begin to consider their students' thinking, which can help explain why

each of the participants' questioning improved. The participants tended to ask more closed questions (e.g., Is $x * x = 2x$?) and procedural questions (e.g., What is the next step?) in the beginning, and their mentor influenced each in different ways. Lisa complimented Marge's questioning techniques from the beginning and noted that this was one of the aspects of Marge's practice that she was trying to emulate. As a result, Lisa used practices such as writing a number of answers on the board, and eventually had students answer each other's questions, or show their confidence in the material using one, two, or three fingers. Jane and Judy focused on using questions in the summaries of lessons. Jane and Judy had a number of conversations about asking some questions to highlight the main points of the lesson, and by the end of the experience, Judy was asking more conceptual questions such as the difference between the range in statistics and functions, which Jane described as "the kicker." During the second observation cycle, Meg said that she and Lois had been working on soliciting more information from the students through questioning. Early in the experience, Meg would typically settle for the first answer when she asked a question and eventually she used methods such as having the students talk to their neighbor and give thumbs up or down to increase student participation.

Providing closure to lessons played an important role in the participants' experiences. TOTDs became more of a focus for Lisa as a result of a conversation before a test when she came to the realization that she had covered a number of topics without collecting very much information to assess their understanding. Providing summaries was challenging for Judy because she did not have a structured method such as a Smartboard slide dedicated to closure, and she admitted that often she either forgot the for the summary, or completely forgot to ask any closure questions. Meg was the most consistent of the three at providing closure; she typically used a TOTD. This was likely due to her capacity for lesson planning, which Lois complimented in her final interview.

Each of the participants was influenced by her mentor and improved in her use of formative assessments in her own way. Lisa and Marge had the conversation before the test that caused changes in Lisa's practice to assess the students understanding in different. Jane and Judy used the carousel activity a number of times to gather information on what their students understood and the common errors they made. Lois and Meg worked on soliciting more information from the students and encouraged their participation by using techniques such as having the students talk to one another and using their thumbs to show if they understood an example. All of these practices resulted from conversations or suggestions from the mentors. The practices were also techniques that the mentors used in their own teaching, which suggests that student teachers are influenced by the conversations they have with their mentors as well as their mentor's practice.

CHAPTER 8

CONCLUSIONS AND IMPLICATIONS

This study examined the nature of the relationship between three pairs of student teachers and mentors of mathematics in middle and high schools as well as the impact of the relationship on the student teachers' practice. The field of education values mentors for their role in helping novices learn to teach; however, little is known about the effects of mentoring (Llinares & Krainer, 2006; Wilson et al., 2001), or how mentors approach their craft (Borko & Mayfield, 1995; Wang & Odell, 2002). Research on mentoring has been limited in the analysis of how the mentoring relationship develops, characterizations of the relationship between mentors and student teachers, and the implications for the student teacher learning to teach. The following research questions guided this study:

1. What is nature of the mentoring relationship between mentor teachers and student teachers of middle and secondary school mathematics?
2. What influence does the mentoring relationship have on the practice of student teachers of middle and secondary school mathematics?

To address these questions I conducted three case studies that followed mentors and student teachers of mathematics for the duration of their experience together. One pair was at an urban middle school, and the other two were at a high school in the same district. Each of the three mentors and student teachers completed a survey; these data were used to provide information about the participants' backgrounds and beliefs about the purpose of student teaching, to examine the mentor's role in the experience, and to generate initial interview questions. Each of the participants was interviewed prior to the semester when the research took place to gain insight into their expectations for the experience, and in the case of the mentors, to inquire about their work with previous student teachers. During the semester, I observed each

pair for three 1-week cycles, collected documents such as lesson plans and activities, and obtained audio recordings of the pairs' discussions. Upon completion of the experience, I interviewed each participant to ask her about her perceptions of the mentoring relationship and its impact. The data were organized into themes for each pair and reevaluated using constant comparison techniques (Glaser & Strauss, 1967; Strauss, 1987) as additional data were collected throughout the study. Individual cases were written based in the themes that emerged, then a cross-case analysis compared and contrasted the experiences of the participants.

Marge and Lisa taught 3 sections of Accelerated Math 8, 1 section of Math 8, and 1 section of a remedial class that was split between eighth grade math and English. Questioning was a focus for this pair from the beginning and Lisa emulated many of Marge's questioning techniques, which led to issues with timing in Lisa's lessons. Lisa also closely mimicked Marge's instruction, particularly her methods of factoring. After a conversation where Lisa was unable to articulate what the students understood and where they would struggle, different methods of formative assessment became a focus for the pair.

Jane and Judy taught 3 blocks of Accelerated Math I and began their time together with a number of discussions about ways to group students and developed a system where groups had different roles during each lesson. They did not use all the roles, but implemented this system until Judy assumed responsibility for the instruction and then the group roles were used less and the class resembled a traditional classroom. During this time, the pair focused on providing closure to lesson through asking key questions, which Judy quite often forgot to ask. They also used a variety of assessments that were unique to this pair. Jane encouraged Judy to return to the group roles and by the end of the experience, she was able to implement the system they designed in the beginning.

Lois and Meg taught 1 block of Accelerated Math III, Math III, and Math III Support. They focused on lesson planning in the beginning of the experience and Lois shared her rationale for how she planned and what parts of the lesson were most important. As Meg assumed responsibility for the instruction the pair worked on soliciting more information from the students and encouraging more student involvement. Classroom management of the Math III and Math III Support was challenging for Judy and it dominated the pair's conversations for much of the experience.

The three case studies presented in chapters 4, 5, and 6 described the participants' student teaching experience and the mentoring relationship as it developed. I described the nature of each pairs' conversations, the feedback from the mentors, and how the mentors' practice influenced each student teacher. The cross-case analysis in chapter 7 discussed additional findings and themes that arose from comparing the cases. The following sections address the findings of the study, limitation, and implications for teacher education and further research.

Findings of the Study

Nature of the Interactions. The nature of the interactions between the mentors and student teachers tended to change over time as the student teachers gained more experience and developed a more dynamic teaching repertoire. As the student teachers assumed responsibility for planning and instructing lesson their practice resembled traditional models of teaching that were teacher-centered. The mentors provided feedback to that promoted more student involvement and different methods of instruction such as including group work and using various forms of formative assessments. Each of the student teachers improved in the use of formative assessments in various ways, which were largely influenced by their mentors' suggestions and practices. With the guidance of the mentors, questioning was a type of formative assessment where there was improvement in all cases. Issues with classroom

management in all cases, but caused a great deal of frustration for some and limited the scope and nature of the interactions.

One area where the nature of the interactions was consistent during the experience was that the mentors supplied curriculum topics. However, the conversations about the curriculum topics were limited and they occurred more frequent at the beginning of the experience and tapered off towards the end. The mathematical conversations varied in complexity and typically focused on ways to represent the curriculum topics. At times mentors provided a rationale for the type of representations they suggested whereas at other times they were directive and supplied the representation they wanted the student teachers to use.

Influence on Student Teachers' Practice. Mentors influenced student teachers in a variety of ways through their modeling and feedback. Mathematically the mentors influenced the topics the student teachers taught and the representations they used. The mentors also influenced the way student teachers presented mathematics, their classroom management techniques, the way they considered students' answers, and the format of their lesson (e.g. tasks, Smartboard presentations, how to address essential questions). The student teachers' use of formative assessment improved greatly and was influenced by the mentors, particularly in the areas of questioning and assessing students during the closure of lessons. The student teachers attempted to incorporate the feedback, which had varying degrees of success and was more influential when mentors shared the rationale for their suggestions.

Limitations

As I discussed in the rationale, I previously observed a number of mentoring approaches as a supervisor and researchers have characterized others (Kagan, 1992; Wang & Odell, 2002), however, the three cases presented in this study was too small of a sample to encapsulate them all. Furthermore, I observed each pair for three weeks during the semester, which represents

only a portion of their time together. The conversation Lisa and Marge had before the test was critical for the data in their case and I did not see a similar conversation for the others. This does not imply that analogous conversations did not happen, only that I did not observe them. Even during the cycles of observation the pairs would talk before and after school, or on the phone in the evenings. Marge and Lisa would regularly exchange text messages regarding lesson plans, Jane and Judy stayed late after school, as did Lois and Meg. The data for Jane and Judy were limited because the observation cycles coincided with a large number of tests, which reduced the number of lessons I observed either of them teach. In the case of Lois and Meg, the frustration with behavior issues led Meg to meet with her university supervisor outside of school to discuss options. I learned about this conversation during Lois's final interview, which was after Meg's, hence there is no data to indicate what type of influence it may have had.

Student learning is at the heart of effective teaching and collecting student work would have contributed to the findings in this study. Perhaps the best examples come from the technology assignments and method of grading in the case of Jane and Judy. The pair talked about the insight they gained into their students' thinking from the technology assignments and how to give feedback and make comments when grading, however it was difficult to document any impact of these on Judy's practice without examples of student work. In each case formative assessments from the students could have contributed to the analysis. In Marge and Lisa's case, for example, collecting students' TOTD could help to analyze the impact of the conversation they had about assessing what the students understood before a summative assessment.

Implications for Teacher Education

The results in this study show how student teachers are greatly influenced by their mentors. Student teachers emulate the practices that their mentors use and attempt to implement the feedback and suggestions that the mentors provide. Wideen et al. (1998) discussed how

student teachers implement traditional practices due to the practical pressures of the experience. The participants in this study implemented traditional practices for a period of time when they assumed responsibility of the instructions, despite the training they received in their mathematics education courses; however, the student teachers were able to improve their use of formative assessments and student-to-student interactions with the help of their mentors. These results suggest that we need to seek student teaching context where the practices that are promoted in teacher education programs are present.

Student teachers beginning with traditional practices also suggests that mentors should be cautious of implementing an instructional format that is complicated and requires a great deal of multitasking on the part of the teacher. The student teachers in this study were able to develop practices, such as differentiate group roles and implementing multiple forms of formative assessments in the same lesson, but it was after they gained some fluency with all of the things a teacher must think about during a lesson. Mentors should recognize the complexity of implementing a lesson and start small by working on a few areas at one time.

Mentors supply the curriculum topics; however, the topics are largely dictated by state standards. Bullough et al. (2002) talk about lack of choice in the curriculum is a frustration for many student teachers; however, practicing teachers also lack choice because are also under pressure to address the state-mandated curriculum. This situation suggests that teacher education should provide prospective teachers with opportunities to develop an understanding of state standards. Furthermore, student teachers are asked to teach a variety of levels of mathematics, which they rarely know in advance, so knowledge of the breadth of the curriculum is important.

Teachers plan for instruction, design presentations, and use a variety of sources of curriculum materials in their practice. The participants in this study used tasks that were supplied by the state and they adapted, the CMP curriculum (Lappan et al., 1995), and

Smartboard files that they created. Student teachers closely emulate the methods of planning and presentation that their mentors use so teacher education programs should provide prospective teachers with exposure to a variety of models of planning.

Implications for Further Research

The findings in this research lead to potential studies worthy of exploration. Each of the student teachers improved in their ability to use formative assessments to evaluate student understanding. Since this was thematic in all the cases, it would be interesting to investigate how other student teachers develop in the areas of questioning and other forms of formative assessments. In this study it was not clear how the student teachers were using the information they gathered to influence their practice, which is an important aspect of the assessment. A study that analyzes how student teachers create, implement, and use formative assessments for further instruction would contribute to an understanding of how student teachers learn to teach.

I have mentioned the lack of mathematical conversations a number of times and a variety of studies could be derived from this result. Research similar to these involving case studies could focus only on the mathematical conversations between mentors and student teachers and the influence in the classroom. One hypothesis I have is that mentors with stronger mathematical knowledge for teaching are more inclined to discuss mathematics with their student teacher. All the mentors in this study had at least one degree specific to mathematics education, which opens the door for studies such as investigating mentors with alternative preparations such as mathematics majors or second-career teachers. Additional studies could examine the frequency and nature (e.g. conceptual versus procedural) of the mathematical conversations over the course of the experience.

I do not assume that the mentors in this study capture all mentoring practices, however I believe these were typical student teaching experiences. Further case studies can be done with mentors that take different approaches such as mentors who are very hands-off or are directive.

Other researchers (Bullough et al., 2003) have begun to explore alternative models of student teaching such as pairing two student teachers with one mentor. It would be interesting to compare the nature of the relationships in such models with the findings in this study. A study of such alternative models could also focus on the discourse between the triad of the mentor and student teachers and its impact on their practice.

Another study that could contribute to the findings here is to examine the relationship between university supervisors and student teachers. The supervisors in this study had varying degrees of involvement in the student teachers' experience. These data were not part of this study, which leaves an open question regarding the influence of the other people involved in the student teaching experience.

Considering alternative methodologies can also contribute to research on student teaching. In this study I did three one-week observation cycles, which represents only a small fraction of the pairs' time together, hence, I only observed a fraction of the instruction and conversations. Conducting a single case study where I spend more time with a single pair may give a better picture of the relationship develops and its impact. Another methodological change could involve videotaping, which provides a more thorough account than observation notes.

Final Thoughts

Research on student teachers in middle school and high school mathematics classes is sparse and tends to be based on self-report data or interviews. I did case studies where I followed three capable pairs of mentors and student teachers for the duration of the student teaching experience. The findings are consistent with much of the research that has been reported

and adds to our understanding of the discourse between student teachers and mentors of mathematics.

The findings in this study are valuable, but they suggest more research is needed. The student teachers closely modeled their mentors' planning and instructional practices. We need further study of the mechanisms for this. The student teachers adopted traditional practices when they assumed responsibility for the instruction and developed more student-centered lessons with the guidance of their mentors. The mentors also influenced the nature of the formative assessment and questioning techniques the student teachers used and helped them to improve in these areas. The mathematical conversations tended to be limited. We need studies that focus on the mathematical discourse between mentors and student teachers. Issues of classroom management were present in all cases, which is typical, but I feel there was considerable information on these matters, because they were limited, may have been missed.

Student teaching is one of the most important experiences in training mathematics teachers. Research on the discourse between mentors and student teachers can lead to greater understanding of the process of learning to teach mathematics, it can facilitate better conceptualizations of the student teaching process, and it can lead to better practices in the preparation of mathematics teachers.

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APPENDIX A

MENTORS' BACKGROUND INFORMATION

Please provide the following demographic information:

1. How many years have you taught each of the following levels?
Elementary (K-5)____ Middle (6-8)____ High (9-12)____
2. Approximately how many student teachers have you supervised?
3. Indicate which of the following courses you teach or have taught:
Prealgebra____ Algebra I____ Algebra II____ Algebra III____ Geometry____
Advanced Algebra & Trigonometry____ Precalculus____ Calculus____
Statistics____ Math I____ Math II____ Math I Support____ Math II Support____
Math III____ Math III Support____
Other_____
4. Gender: Male____ Female____
5. What is degree(s) have you earned?
6. Where institution was your degree(s) from?
7. What was your major?
8. Do you have a current certification to teach? If yes, what level?
9. Do you have any other certifications (e.g. National Board, gifted)?
10. Specific to teaching mathematics, what do you feel is the most significant contribution you will make in helping a student teacher have a successful experience?
11. How do you typically approach providing student teachers with feedback and criticism for teaching mathematics?

APPENDIX B

STUDENT TEACHERS' BACKGROUND INFORMATION

Please provide the following demographic information:

1. Which of the following levels will you teach during student teaching?
Middle (6-8)____ High (9-12)____
2. Indicate which of the following courses will you teach during student teaching:
Prealgebra____ Algebra I____ Algebra II____ Algebra III____ Geometry____
Advanced Algebra & Trigonometry____ Precalculus____ Calculus____
Statistics____ Math I____ Math II____ Math I Support____ Math II Support____
Math III____ Math III Support____
Other_____
3. Gender: Male____ Female____
4. What degree are you working to complete?
5. What is your major?
6. Specific to teaching mathematics, what do you feel is the most significant contribution your mentor will make in helping you have a successful experience?
7. How do expect your mentor will typically approach providing you with feedback and criticism for teaching mathematics?

APPENDIX C

FIRST INTERVIEW PROTOCOL (MENTORS)

1. Although there are many purposes to student teaching, there are likely some purposes that you, as a cooperating experienced teacher, believe to be more important than others. List and briefly describe what, for you, are the three most important purposes of student teaching.
2. For each of the three purposes you listed in question #1, what role(s) do you, as the cooperating teacher, play in making sure those purposes are accomplished?
3. How do you typically assist a student teacher in planning lessons?
4. Do you ever discuss, and give feedback on, lesson plans before they are implemented by the student teacher?
5. How do you mentor a student teacher who is dealing with long-term planning as well as 2 or 3 different classes?
6. Do you ever give written feedback? If so, in what ways?
7. In what ways to you typically see student teachers differentiate instruction? Do you usually give feedback on how to differentiate instruction? What are some techniques you use to mentor student teachers to plan (or instruct) for students who learn at different rates or have different styles?
8. How do you assess your own teaching?
9. What advice do you usually give to student teachers that are experiencing the “realities of teaching?”
10. Do you see yourself as having a role in the student teacher’s transition from a student of mathematics to a teacher of mathematics? If so, what is that role?

11. Does the style which you give feedback change over the course of the time you are mentoring a student teacher? How does it typically change?
12. What role does reflection play in learning to teach mathematics?
13. What is the most important thing you want a student teacher to learn from his/her student teaching experience?

APPENDIX D

FINAL INTERVIEW PROTOCOL (MENTORS)

1. In your encounters with other student teachers, was this experience typical?
2. What are 3 words, or phrases, that you would use to describe your overall student teaching experience? (no classroom management)
3. Do you typically focus on the same aspects of teaching with each student teacher you mentor?
4. How did each of these themes emerge? How was the decision made on what to focus on?
5. Describe how you and ____ typically discussed planning and teaching lessons. (Focus on mathematics.)
6. What were some issues with classroom management that you ran into? How did you and ____ address these issues?
7. From the time when you first began incorporating yourself into your classroom to the end of the experience when you were teaching classes on your own, has the type of feedback you have received changed? If so, in what ways? Can you give one example?
8. Did you learn anything from watching ____ teach? If so, what?
9. What were the most effective things ____ did to influence your teaching? In what ways did it help? Example?
10. What were some things that you talked about that didn't seem to have an effect? Why do you think this was the case?
11. Based on our final evaluation, what are some aspects of teaching that ____ are still working on improving?

12. Based on your experience this semester, what are some things you will change if you have another student teacher in the future?

APPENDIX E

FIRST INTERVIEW PROTOCOL (STUDENT TEACHERS)

1. Although there are many purposes to student teaching, there are likely some purposes that you, as a student teacher, believe are more important than others. List and briefly describe what, for you, are the three most important purposes of student teaching.
2. For each of the three purposes you listed in question #1, what role(s) does the mentor teacher in making sure those purposes are accomplished?
3. How do you expect your mentor teacher to help you plan lessons?
4. How confident are you in the mathematics you are teaching this semester? What role, if any, do you expect your mentor to play in preparing you mathematically?
5. What type of feedback do you expect on your lessons before you implement them?
6. What type of written feedback do you expect from your mentor?
7. How do you expect that your mentor teacher will give feedback on your teaching?
8. What role do you see your mentor playing in your transition from a university student of mathematics to a teacher of mathematics?
9. What role does reflection play in learning to teach mathematics?
10. What is the most important thing you want to learn from the student teaching experience?

APPENDIX F

FINAL INTERVIEW PROTOCOL (STUDENT TEACHERS)

1. What are 3 words, or phrases, that you would use to describe your overall student teaching experience? (no classroom management)
2. Describe how you and ____ typically discussed planning and teaching lessons. (Focus on mathematics.)
3. How did each of these themes emerge? How was the decision made on what to focus on?
4. What were some issues with classroom management that you ran into? How did you and ____ address these issues?
5. From the time when you first began incorporating yourself into your classroom to the end of the experience when you were teaching classes on your own, has the type of feedback you have received changed? If so, in what ways? Can you give one example?
6. What were the most effective things ____ did to influence your teaching? In what ways did it help? Example?
7. What were some things that you talked about that didn't seem to have an effect? Why do you think this was the case?
8. Based on our final evaluation, what are some aspects of teaching that you are still working on improving?

APPENDIX G
TECHNOLOGY LESSON

Name _____

Functions Technology Assignment

Today you will be exploring functions using Geogebra. To download the program, go to <http://www.geogebra.org/cms>.

Explore all of the problems listed below and prepare a write-up. You are to

- a) Write the type of function and its equation.
- b) Discuss the domain and the range.
- c) Write a description of the changes that occur as it relates to its parent graph.
- d) Is this function even, odd, function or neither.
- e) Describe the end behavior.
- f) Note the intervals of increase and/or decrease.
- g) Min. or max values.

Remember that you are discussing the mathematics, not just the pictures.

Try at least 3 different values for each a, h, and k and describe what happens.

Functions:

- 1. $y = a |x-h| + k$
- 2. $y = a (x-h)^2 + k$
- 3. $y = a (x-h)^3 + k$

4. $y = \frac{a}{x-h} + k$

5. Determine a value of c such that the equation $x^3 - 4x + C = 0$ has

- a) 3 solutions
- b) Exactly 2 solutions
- c) Exactly 1 solution

6. Determine a value of c such that the equation $x^2 - 10x + c = 0$ has

- a) 0 solutions
- b) Exactly 2 solutions
- c) Exactly 1 solution

Further Applications:

1. Fire Fighting

The velocity V and maximum height h of water being pumped into the air are related by the equation $V = \sqrt{2gh}$ where g is the acceleration due to gravity (32 feet/second²).

a) Determine an equation that will give the maximum height of the water as a function of its velocity.

b) The Mayfield Fire Department must purchase a pump that is powerful enough to propel water 80 feet into the air. Will a pump that is advertised to protect water with a velocity of 75 feet/second meet the fire department's needs? Explain.

2. Electronics

Suppose the current I in an electric circuit is given by the formula $I = t + \frac{1}{10-t}$ where t is time.

What happens to the circuit as t approaches 10?

APPENDIX H

SEQUENCES AND SERIES TASK

Georgia Department of Education

Accelerated Mathematics III

Unit 2

2nd Edition

RENAISSANCE FESTIVAL LEARNING TASK

As part of a class project on the Renaissance, your class decided to plan a renaissance festival for the community. Specifically, you are a member of different groups in charge of planning two of the contests. You must help plan the archery and rock throwing contests. The following activities will guide you through the planning process.

Group One: Archery Contest¹

Before planning the archery contest, your group decided to investigate the characteristics of the target. The target being used has a center, or bull's-eye, with a radius of 4 cm, and nine rings that are each 4 cm wide.

1. The Target

- Sketch a picture of the center and first 3 rings of the target.
- Write a sequence that gives the radius of each of the concentric circles that comprise the entire target.
- Write a recursive formula and an explicit formula for the terms of this sequence.
- What would be the radius of the target if it had 25 rings? Show how you completed this problem using the explicit formula.
- In the past, you have studied both arithmetic and geometric sequences. What is the difference between these two types of sequences? Is the sequence in (b) arithmetic, geometric, or neither? Explain.

One version of the explicit formula uses the first term, the common difference, and the number of terms in the sequence. For example, if we have the arithmetic sequence 2, 5, 8, 11, 14, ..., we see that the common difference is 3. If we want to know the value of the 20th term, or a_{20} , we could think of starting with $a_1 = 2$ and adding the difference, $d = 3$ a certain number of times. How many times would we need to add the common difference to get to the 20th term? _____ Because multiplication is repeated addition, instead of adding 3 that number of times, we could multiply the common difference, 3, by the number of times we would need to add it to 2.

¹ Elements of these problems were adapted from *Integrated Mathematics 3* by McDougal-Littell, 2002.)

Georgia Department of Education

Accelerated Mathematics III

Unit 2

2nd Edition

This gives us the following explicit formula for an arithmetic sequence: $a_n = a_1 + (n-1)d$.

- f. Write this version of the explicit formula for the sequence in this problem. Show how this version is equivalent to the version above.
- g. Can you come up with a reason for which you would want to add up the radii of the concentric circles that make up the target (for the purpose of the contest)? Explain.
- h. Plot the sequence from this problem on a coordinate grid. What should you use for the independent variable? For the dependent variable? What type of graph is this? How does the a_n equation of the recursive formula relate to the graph? How does the parameter d in the explicit form relate to the graph?
- i. Describe (using y-intercept and slope), but do not graph, the plots of the arithmetic sequences defined explicitly or recursively as follows:

$$1. a_n = 3 + \frac{4}{3}n - 1$$

$$3. a_n = 4.5 - 3.2n - 1$$

$$2. \begin{cases} a_1 = -2 \\ a_n = a_{n-1} + \frac{1}{2} \end{cases}$$

$$4. \begin{cases} a_1 = 10 \\ a_n = a_{n-1} - \frac{2}{5} \end{cases}$$

2. The Area of the Target: To decide on prizes for the archery contest, your group decided to use the areas of the center and rings. You decided that rings with smaller areas should be worth more points. But how much more? Complete the following investigation to help you decide.

- a. Find the sequence of the areas of the rings, including the center. (Be careful.)
- b. Write a recursive formula and an explicit formula for this sequence.
- c. If the target was larger, what would be the area of the 25th ring?
- d. Find the total area of the bull's eye by adding up the areas in the sequence.

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- e. Consider the following sum: $S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_{n-2} + a_n$. Explain why that equation is equivalent to $S_n = a_1 + a_1 + d + a_1 + 2d + \dots + a_n - 2d + a_n - d + a_n$. Rewrite this latter equation and then write it out backwards. Add the two resulting equations. Use this to finish deriving the formula for the sum of the terms in an arithmetic sequence. Try it out on a few different short sequences.
- f. Use the formula for the sum of a finite arithmetic sequence in part (e) to verify the sum of the areas in the target from part (d).
- g. Sometimes, we do not have all the terms of the sequence but we still want to find a specific sum. For example, we might want to find the sum of the first 15 multiples of 4. Write an explicit formula that would represent this sequence. Is this an arithmetic sequence? If so, how could we use what we know about arithmetic sequences and the sum formula in (e) to find this sum? Find the sum.
- h. What happens to the sum of the arithmetic series we've been looking at as the number of terms we sum gets larger? How could you find the sum of the first 200 multiples of 4? How could you find the sum of all the multiples of 4? Explain using a graph and using mathematical reasoning.
- i. Let's practice a few arithmetic sum problems.
1. Find the sum of the first 50 terms of 15, 9, 3, -3, ...
 2. Find the sum of the first 100 natural numbers
 3. Find the sum of the first 75 positive even numbers
 4. Come up with your own arithmetic sequence and challenge a classmate to find the sum.
- j. Summarize what you learned / reviewed about arithmetic sequences and series during this task.

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3. Point Values: Assume that each participant's arrow hits the surface of the target.
- a. Determine the probability of hitting each ring and the bull's-eye.

Target Piece	Area of Piece (in cm^2)	Probability of Hitting this Area
Bull's Eye	16π	
Ring 1	48π	
Ring 2	80π	
Ring 3	112π	
Ring 4	144π	
Ring 5	176π	
Ring 6	208π	
Ring 7	240π	
Ring 8	272π	
Ring 9	304π	

- b. Assign point values for hitting each part of the target, justifying the amounts based on the probabilities just determined.
- c. Use your answer to (b) to determine the expected number of points one would receive after shooting a single arrow.
- d. Using your answers to part (c), determine how much you should charge for participating in the contest OR for what point values participants would win a prize. Justify your decisions.

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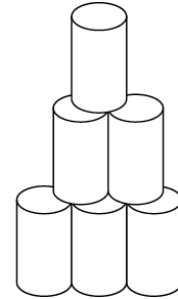
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Group Two: Rock Throwing Contest²

For the rock throwing contest, your group decided to provide three different arrangements of cans for participants to knock down.

1. For the first arrangement, the tin cans were set up in a triangular pattern, only one can deep. (See picture.)
 - a. If the top row is considered to be row 1, how many cans would be on row 10?
 - b. Is this an arithmetic or a geometric sequence (or neither)? Write explicit and recursive formulas for the sequence that describes the number of cans in the n th row of this arrangement.
 - c. It is important to have enough cans to use in the contest, so your group needs to determine how many cans are needed to make this arrangement. Make a table of the number of rows included and the total number of cans.



Rows Included	Total Cans
1	1
2	3
3	
4	
5	
6	
7	
8	

- d. One of your group members decides that it would be fun to have a “mega-pyramid” 20 rows high. You need to determine how many cans would be needed for this pyramid, but you don’t want to add all the numbers together. One way to find the sum is to use the summation formula you found in the Archery Contest. How do you find the sum in an arithmetic sequence? _____ Find the sum of a pyramid arrangement 20 rows high using this formula.

² Adapted from Manouchehri, A. (2007). Inquiry-discourse mathematics instruction. *Mathematics Teacher*, 101, 290–300.

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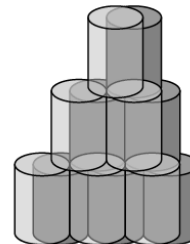
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We can also write this problem using summation notation: $\sum_{i=1}^n a_i$, where i is the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation. We can think of a_i as the explicit formula for the sequence. In this pyramid problem, we have $\sum_{i=1}^{20} i$ because we are summing the numbers from 1 to 20. We also know what this sum is equal to. $\sum_{i=1}^{20} i = \frac{n}{2} a_1 + a_n = \frac{20}{2} 1 + 20 = 210$. What if we did not know the value of n , the upper limit but we did know that the first number is 1 and that we were counting up by 1s? We would then have $\sum_{i=1}^n i = \frac{n}{2} 1 + n = \frac{n(n+1)}{2}$. This is a very common, important formula in sequences. We will use it again later.

- e. Propose and justify a specific number of cans that could be used in this triangular arrangement. Remember, it must be realistic for your fellow students to stand or sit and throw a rock to knock down the cans. It must also be reasonable that the cans could be set back up rather quickly. Consider restricting yourself to less than 50 cans for each pyramid. Describe the set-up and exactly how many cans you need.
2. For the second arrangement, the group decided to make another triangular arrangement; however, this time, they decided to make the pyramid 2 or 3 cans deep. (The picture shows the 2-deep arrangement.)
 - a. This arrangement is quite similar to the first arrangement. Write an explicit formula for the sequence describing the number of cans in the n th row if there are 2 cans in the top row, as pictured.
 - b. Determine the number of cans needed for the 20th row.
 - c. Similar to above, we need to know how many cans are needed for this arrangement. How will this sum be related to the sum you found in problem 1?
 - d. The formula given above in summation notation only applies when we are counting by ones. What are we counting by to determine the number of cans in each row? What if the cans were three deep? What would we be counting by? In this latter case, how would the sum of the cans needed be related to the sum of the cans needed in the arrangement in problem 1?



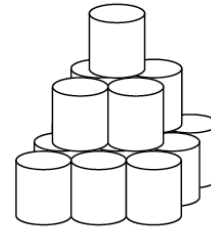
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- e. This leads us to an extremely important property of sums: $\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$, where c is a constant. What does this property mean? Why is it useful?
- f. Suppose you wanted to make an arrangement that is 8 rows high and 4 cans deep. Use the property in 2e to help you determine the number of cans you would need for this arrangement.
- g. Propose and justify a specific number of cans that could be used in this triangular arrangement. You may decide how many cans deep (>1) to make the pyramid. Consider restricting yourself to less than 50 cans for each pyramid. Describe the set-up and exactly how many cans you need. Show any calculations.
3. For the third arrangement, you had the idea to make the pyramid of cans resemble a true pyramid. The model you proposed to the group had 9 cans on bottom, 4 cans on the second row, and 1 can on the top row.
- a. Complete the following table.



Row	Number of Cans	Change from Previous Row
1	1	1
2	4	3
3	9	5
4		
5		
6		
7		
8		
9		
10		

- b. How many cans are needed for the n th row of this arrangement?

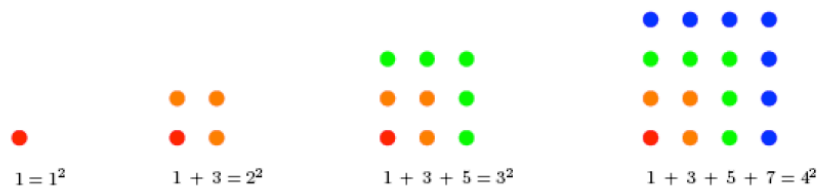
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- c. What do you notice about the numbers in the third column above? Write an equation that relates column two to column three. Then try to write the equation using summation notation.
- d. How could you prove the relationship you identified in 3c?
- e. Let's look at a couple of ways to prove this relationship. Consider a visual approach to a proof.³ Explain how you could use this approach to prove the relationship. We can represent the sum of the first n odd natural numbers as the number of dots contained in the square arrays drawn in the figures below:



- f. We know that the sum of the first n natural number is $\frac{n(n+1)}{2}$, so
- $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$. If we multiply both sides of the equation by 2, we get the sum of the first n EVEN numbers. How can we use this new equation to help us find the sum of the first n ODD numbers?
- g. Consider another approach. We have $S_n = 1 + 3 + 5 + \dots + (2n-1)$. If we reverse the ordering in this equation, we get $S_n = (2n-1) + \dots + 5 + 3 + 1$. What happens if we add the corresponding terms of these two equations? How will that help us prove the relationship we found earlier?
- h. In a future task, you will learn another way to prove this relationship. You will also look at the sum of rows in this can arrangement. Can you conjecture a formula for the sum of the first n square numbers? Try it out a few times.

³ A similar approach can be found in the August 2006 issue of the *Mathematics Teacher*: Activities for Students: Visualizing Summation Formulas by Gunhan Caglayan.

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4. Throughout this task, you learned a number of facts and properties about summation notation and sums. You learned what summation notation is and how to compute some sums using the notation. There is another important property to learn that is helpful in computing sums. We'll look at that here, along with practicing using summation notation.
- a. Write out the terms of these series.

i. $\sum_{i=1}^4 5i$

ii. $\sum_{i=1}^5 i + 6$

iii. $\sum_{i=1}^5 i^2 + 3$

- b. You have already seen one sum property. Here are the important properties you need to know. Explain why the two new properties make mathematical sense to you.

Properties of sums (c represents a constant)

1. $\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$

2. $\sum_{i=1}^n c = cn$

3. $\sum_{i=1}^n a_i \pm b_i = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$

- c. Express each series using summation notation. Then find the sum.

i. $2 + 4 + 6 + \dots + 24$

ii. $5 + 8 + 11 + 14 + \dots + 41$

- d. Compute each sum using the properties of sums.

i. $\sum_{i=1}^{20} -3i - 4$

ii. $\sum_{i=1}^{20} 4i$

iii. $\sum_{i=1}^{20} -4i$

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Dr. John D. Barge, State School Superintendent
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APPENDIX I

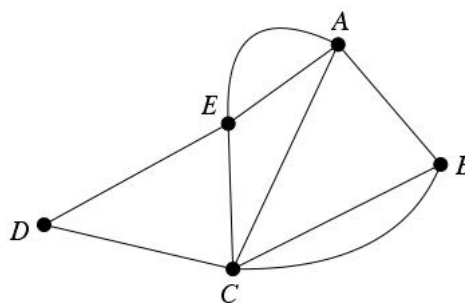
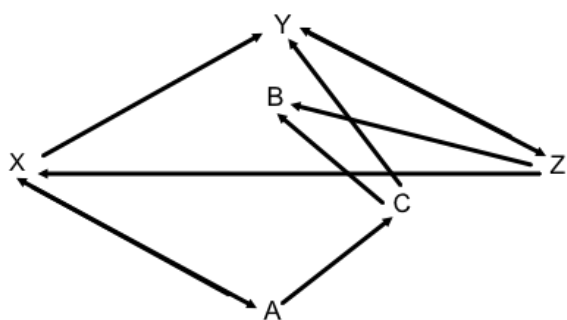
MEG'S LESSON OUTLINE

<p><u>Unit:</u> 1 – Matrices</p>	<p><u>Date:</u> January 17, 2012</p>
<p><u>Lesson:</u> Two-Step Equations, Vertex Edge, Test Review (Support Class)</p>	
<p><u>Standard:</u></p> <p>MM3A7: Students will understand and apply matrix representations of vertex edge graphs.</p> <p>a) Use graphs to represent realistic situations</p> <p>b) Use matrices to represent graphs, and solve problems that can be represented by graphs.</p>	<p><u>EQ:</u> How can we use matrices with vertex-edge graphs to represent realistic situations?</p>
<p><u>Activating Strategy:</u> Students will work on their skill checks for two-step equations. The teacher will use the first few examples to review the steps of using inverse operations to solve these equations. The students will then work on rest of the first 10 problems of the worksheet. This should take 15 to 20 min. The teacher will have the students put #9 and #10 on a sticky note so the teacher can check their progress (formative assessment). They will finish the worksheet Thursday as a starter.</p> <p>The worksheet that will be given is attached below</p>	
<p><u>Homework Check:</u> We won't have a homework check because there is no homework assigned in Support. They will have 30 minutes at the end of the period to work on their assigned homework from Math 3</p>	<p><u>Vocabulary:</u></p> <p>Vertex Edge Graph</p> <p>Vertices/Nodes</p> <p>Digraph</p>

	Adjacency Matrix
<p><u>Mini-Lesson:</u> Students did a vertex edge task in Math 3 class that introduced how to go from a vertex edge graph to a matrix and how to go represent a word problem with a vertex edge graph and matrix. In support, students will practice going from a vertex edge graph to a matrix and matrix to vertex-edge graph. Then they will be given another word problem (see below) to translate into a vertex edge graph and matrix. Teacher will guide students through one of each type of problem except for the word problems.</p>	<p><u>Work Period: Explore, Extend:</u> Students will get out white boards after they do the examples together as a class. They will have the chance to work out problems on their own and answer on their white boards so the teacher can check and make sure that understand the concept (formative assessment). Students will do both word problems without an example to follow from class to see if they can apply what we have learned. Teacher will check their answers using the whiteboards (formative assessment). The problems and examples we will use are attached below</p>
<p><u>Summarize:</u> The teacher will bring the class together to discuss vertex-edge graphs and how they can be useful in more areas of study than math and science. They will also discuss the usefulness of matrices in organizing data. The students will answer the following question for a ticket out the door: <i>How are vertex-edge graphs useful outside of a math classroom? Give an example.</i> (formative assessment)</p>	
<p><u>Differentiation:</u> I think students will struggle the most with examples that have more than 3 or 4 variables. And I think they will struggle most understanding how to translate a matrix into a vertex-edge graph. If this happens, the teacher will go back to the labels of the matrix and reinforce with students the idea of a directed path. She will help them “read” the matrix to find out what paths go to and from different vertices. The teacher can also have students help her come up with a context for the adjacency matrix examples and this too will reinforce the meaning behind the numbers.</p>	<p><u>Homework:</u> no homework</p>

Vertex-Edge Review Problems:

Graph → Matrix



Matrix \rightarrow Graph

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Real World Word Problem \rightarrow Graph and Matrix

A group of friends is trying to decide whom they should go to prom with. Alyson will go with Brian or Greg. Brian will go with Alyson or Cristina. Greg will go with Flora or Cristina. Cristina will go with Greg or Dadrion. Flora will go with Dadrion or Brian. Dadrion will go with Flora or Alyson. Represent this data using a vertex edge graph and a matrix, and then decide whom everyone should go with.

A sports announcer is trying to figure out which basketball team will win the tournament this weekend. The Hornets can beat the Warriors and Eagles. The Warriors can beat the Eagles, but last time they played the Eagles beat the Warriors, they can also beat the Wildcats. The Wildcats can beat the Warriors. Represent this data using a vertex edge graph and a matrix, and then decide which team you think will win.

Skills Check

Solve the equations below for x .

1.) $6x + 10 = 76$

2.) $-4x + 8 = 20$

3.) $\frac{x}{3} = 4$

4.) $6x - 2 = 40$

5.) $4 - 2x = 14$

6.) $1 - 2x = 17$

7.) $-8 + 4x = 16$

8.) $\frac{x}{4} - 19 = 22$

9.) $7x - 7 = -14$

10.) $5 + \frac{x}{2} = 13$

11.) $-2x + 10 = 10$

12.) $10 + \frac{x}{5} = 2$

13.) $3x + 9 = 30$

14.) $6x + 5 = -1$

15.) $-8 + 4x = -28$

16.) $3x + 10 = 40$

17.) $\frac{x}{7} - 5 = 9$

18.) $-5x - 8 = 2$

19.) $6 + 2x = 20$

20.) $-5 + 3x = 13$