MECHANISMS OF EMERGENCE: INFLUENCE OF OPEN STUDENT RESPONSES COLLECTED USING AN ONLINE SURVEY TOOL IN MATHEMATICS CLASSROOMS

by

PIERRE SUTHERLAND

(Under the Direction of Patricia S. Wilson)

ABSTRACT

This study makes two contributions to mathematics education: a study of teachers and technology and a methodology that investigates technological interventions. The study included (a) the discussion of contextual problems and design principles, (b) the design of an online survey tool (i.e., Student Response Form), (c) the implementation of the tool in a variety of mathematics classrooms, and (d) a description of teachers' perspectives on the impact of using the tool on their practice and their students. In general, teachers perceived the Student Response Form to amplify existent practices due to increased student feedback. In addition, teachers reported an increase in classroom interactions. This study explored the use of a non-conventional method of inquiry called *method assemblage* that was significantly influenced by the philosophical work of Gilles Deleuze and Félix Guattari. The study is offered as a basic prototype for investigating innovative uses of technology. This method attempts to maintain the tensions of rigor and unexpected outcomes.

INDEX WORDS: Mathematics Education, Technology, Google Forms, Classroom Interaction, Gilles Deleuze, Félix Guattari, A Thousand Plateaus, Assemblage, New Materialisms, Complexity Theory, Swarm Intelligence, Emergence, Student Response Form, Metamodeling, Method Assemblage, Variation Map

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DEDICATION

Unfortunate I will never be; I had a mother who read to me.

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I am very thankful for many things at the end of this journey. I think I have experienced a significant portion of the available spectrum of human emotion. I often think of flying trapeze artists and the dual commitment needed for a good trick: I feel as though I was willing to jump, and that I found people who were willing to catch me. I am genuinely grateful and humbled.

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I thank my participants and recruiters of participants, with my mother being chief among the latter. After working with a large range of teachers, I have been inspired to do more and to work harder. I called the website "teachers are valuable" because this quickly became so painfully obvious to me. It was a privilege to work with you, and I sincerely thank you for letting me into your worlds.

Thank you, UGA and Athens. I have been able to work with and socialize with faculty and students across many departments. I have also been able to meet an incredible variety of people downtown and across town (especially the bartenders, waiters, waitresses, and cooks that work so hard). I have spent countless hours in good company, and I am thankful for it. Thank

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CHAPTER 1

INTRODUCTION

The teacher's day has little flexibility left despite significant advances on various fronts of technology and research. Unfortunately, simply bringing technology into the classroom does not imply its use, and when technology is used, it is often used to sustain old practices (Cuban, Kirkpatrick, & Peck, 2001). Technology is even sometimes used to "teacher-proof" curricula (Ferster, 2014, p. 89) despite over four decades of research that empirically states that teachers make a difference in student learning (Charalambous & Pitta-Pantazi, 2015, p. 44). Technological advances like the Internet are causing remarkable shifts in our capabilities and, in a sense, dissolving the classroom (Borba, 2009). These advances offer new potentials and raise new questions. These new capabilities may allow us to reconsider assumptions about our traditional educational settings, like the need to come together in one central building to interact and receive information, or to have set class sizes and courses for fixed periods of time.

Keeping this potential for experimentation in mind, Borba, Clarkson, and Gadanidis (2012) caution that information and communication technologies (ICT) usually find their genesis and function inspired by problems posed by business, science, or engineering sectors with education only as a secondary "market" (p. 693). Moreover, the authors emphasized that in working through how to "use ICT in mathematics education, researchers and curriculum developers should employ investigative techniques that do not lack *rigor*, but at the same time are *designed to capture unexpected outcomes* (pp. 693-694, emphasis added)." This dissertation aims to contribute two things to mathematics education literature: a study about teachers and

technology, and a methodology that investigates technological interventions while maintaining the tensions of rigor and unexpected outcomes. This methodology focuses on the capabilities of the technology, the context in which it was deployed, and in mapping the resulting variation in uses.

Educational initiatives in the 21st century should take advantage of the power of scalable, robust tools. Current technology enables opportunities to both increase and harness diverse student input in mathematics classrooms. To be more specific, for my study I have customized an online survey tool called a Google Form. This customized Google Form is called the Student Response Form. The Student Response Form was designed to help high school mathematics teachers manage diverse, open responses (e.g., responses to "How did you get 2 + 2?", rather than "What is 2 + 2?"). Student responses are automatically organized in a spreadsheet linked to the Student Response Form. Teachers were encouraged to take advantage of this capability by using those responses to influence classroom interactions. For example, a teacher might ask students to complete a *ticket-out-the-door* response. Students then submit a response that names one thing they learned that day or struggled with. These responses may then influence the teacher's planning for the next day, help them respond more strategically to some students, or even have a discussion during class time. The rest of this introduction discusses the role and nuances of philosophy and theory in addressing this kind of inquiry and finishes with stating the purpose and research questions of this study.

Technology, Mathematics Education, and Values

There are currently 7.3 billion mobile phone subscriptions in the world as of February 2016[1], which is more than the 7.125 billion humans on the planet. I use this vast proliferation of mobile technology to pose some questions: If information and communication technologies have become commonplace, and if education is largely spending time on transmitting information, then why are we still going to a large building to receive this information in real-time? Did this routine not gain momentum long before the advent and proliferation of such technologies? Why are we sitting together in one room if our society has become so well connected? I will argue that these provocative questions are mostly flawed as teachers fulfill more duties than transmitting information. In fact, this argument will frame information transmission as an impoverished state of a classroom environment which decreases opportunities of more worthwhile interactions and events. I do, however, raise the question to encourage dialogue in explicitly stating what we value about sitting together in the same room and about the role of the teacher.

I define terms like transmission, synchronicity, optimization, modality, and duration to help frame how such values might be explicitly stated. For example, a teacher has a class of 30 students in front of her (modality: whole class, synchronicity: real-time) to explain an algorithm (transmission: lecture). Later, the teacher decides to make a video of the lesson (optimization of transmission) and have the students comment on it (synchronicity: asynchronous). Students then return to class the next day to discuss in groups the video and comments (modality: overseen group work; synchronicity: synchronous). One purpose of this framing of interactions is to question whether instructional decisions were made for pedagogical reasons or out of habit due to the momentum of historical educational routines. With this in mind, the next section outlines a

[1] Ericsson mobility report, 2016

specific technological intervention that aims to take advantage of the current educational setting. Rather than trying to optimize an existing facet of the classroom alone, this study intends to not only increase diversity through allowing for open responses but also to aid the teacher in harnessing this increased variation.

Customizing a Google Form for Classroom Discussions

Smith, Hughes, Engle, and Stein (2009) propose five practices of anticipating, monitoring, selecting, sequencing, and connecting student responses, designed to orchestrate student responses into discussions to engage higher cognitive demand problems. The authors also make a simple point: while orchestrating diverse responses can be beneficial to learning, it is hard to manage. A standard Google Form is useful for collecting responses to a single prompt, but it requires customization to be able to handle various student responses to various prompts over weeks or months. This customized Google Form is called the Student Response Form. It was made available to any teacher who was interested in using it. Each student is issued a three-digit identification number to keep track of student responses in a spreadsheet. Teachers are now able to pose prompts via the Student Response Form. Responses can be submitted inside or outside of class time. Both the teacher and the students can access the Student Response Form anywhere there is an internet connection. The major aim of this study is to understand how teachers harness and take advantage of this increased diversity in student responses when equipped with a tool that monitors and organizes diverse student responses more easily.

Importantly, this study is designed to be able to capture unexpected outcomes while retaining some sense of rigor. It is with this in mind that the study does not set out to find or define an ideal implementation of this Student Response Form. The aims of this study are to be explorative in intervention, and descriptive in documenting a variety of implementations across

different contexts. Also, attention is paid to emerging practices and events as well as the thresholds at where these practices or events either occur or collapse. This is to say that attention is not only paid to what happened, but also to the conditions under which novel events emerged. To accomplish this, the choice of philosophy and theory frames the study in terms of problems, design principles, a technological intervention, a metamodel, and a variation map. These five components together constitute a method assemblage and each of these components will be discussed in more detail.

Drawing on Theories of Complexity

Jacques Derrida (1988) eloquently states why such arduous work might be pursued: "If things were simple, word would have gotten around" (p. 19). Deleuze and Guattari's philosophical work, specifically from their book called *A Thousand Plateaus* (1987), is used with various complexity theories. I did this because I found several useful analogs between philosophical concepts in *A Thousand Plateaus* and current applications of complexity theories. In the second chapter, more time is spent on discussing philosophy and theory as well as the rationale for its appropriateness. This section, however, serves to outline it briefly and motivate the choice of conceptual tools. I state two important cautionary statements concerning accountability. First, when concepts (e.g., swarm intelligence) are being taken from the hard sciences and put to use in the softer, social sciences[2]. This usage carries with it the risk of sloppy scholarship by using the concept as little more than a metaphor or "exotic pet" (Massumi, 2002, p. 19). Second, coordinating several concepts across disciplines only compounds this risk leaving a reader wondering whether this is a method or just some anti-scientific rambling. I argue the merits of the specific theories employed throughout the dissertation by not only pointing out its

[2] After having worked in both, I would rather prefer "difficult sciences" for the latter.

strengths and limitations but also in observing limitations of more conventional theory and methods used in current social scientific research.

With these qualifications in mind, the guiding concept throughout the theoretical and methodological work is the philosophical concept of Deleuze and Guattari (1987) called assemblage. An assemblage is a collection of heterogeneous elements that somehow function together. With this concept in mind, ideas are taken from complexity theory. Specifically, complexity theory from:

- mathematics education (Davis & Simmt, 2015);
- public policy (Geyer & Cairney, 2015);
- law (Webb, 2015);
- planning (Roo, Hillier, & Wezemael, 2012b); and,
- swarm intelligence (Eberhart & Shi, 2011).

These theories contribute richly to ambitious projects that engage philosophy, methodologies, modeling, policies, and real-world case studies at local, national, and international levels (Geyer & Cairney, 2015, p. 1). A key current of thought running through all these fields is that "a complex system is greater than the sum of its parts; those parts are interdependent — elements interact with each other, share information and combine to produce systemic [behavior]" (Geyer & Cairney, 2015, p. 2).

To summarize, Deleuze and Guattari's concept of assemblage is used throughout the design, implementation, and documentation of this study to account for some consistency in the decisions made in bringing together concepts from several disciplines. The main reason for this choice of theory and philosophy is that it is designed to be more sensitive to novelty or behaviors caused by the interaction between components which do not necessarily belong to either

component. For example, a teacher will not know ahead of time what course a given discussion may take even though she is familiar with each student and what they might say.

Purpose and Research Questions

The purpose of this study is to investigate how teachers perceive the use of the Student Response Form influence their practice. Although the Student Response Form affords many possible practices, the investigation is particularly focused on teacher-student interactions such as discussions. Smith et al. (2009) recommended five practices for orchestrating discussions that were used to analyze the influence of the Student Response Form on discussion. In addition to this analysis, attention is given to perceived factors that either allowed a teacher to incorporate the Student Response Form into their practice and routine, or mitigated attempts to do so. In addition, if a teacher used the Student Response Form in a novel or unexpected way then this is also documented. These factors and events are framed using the terms emergence, thresholds, and collapse.

Research Questions

1) How do teachers perceive using a Student Response Form to influence their practice?
    a) What influence on practice do teachers report or imply?
    b) With respect to their particular context and routines, what capabilities and potential uses do teachers perceive the Student Response Form to afford them, if any?
    c) How do teachers perceive using a Student Response Form to influence their practice of orchestrating discussions?
    d) What were specific instances of influence on practices or interactions, potentially brought about by the Student Response Form?
2) When teachers used the Student Response Form, what events did teachers report and under what conditions?
    a) What influence, if any, did the Student Response Form have on student participation in class activities?
    b) What factors or circumstances or conditions, if any, mitigate or encourage the use of the Student Response Form?
    c) How did the teachers' use of the Student Response Form vary across contexts?

CHAPTER 2

PART I: PROBLEMS AND DESIGN PRINCIPLES

This part is the first of four in Chapter 2. The first part discusses what are called problems and design principles. Problems and design principles influence the technological intervention. An example of a problem might be the difficulty with managing diverse student responses compared to uniform responses. An example of a design principle might be "The intervention must be non-prescriptive and exploratory." Problems and design principles are the first two of five components describing the proposed methodology. The second section transitions the reader to the denser philosophical and theoretical work in the third part. Fictions and examples largely make up this transitional part. The third part explicates various philosophical and theoretical concepts (e.g. complexity theory, the concept of assemblage, status interventions, etc.). These heterogeneous concepts are then fitted together into a metamodel. A metamodel coordinates diverse theories into a functioning patchwork with some consistency. The fourth, final part offers a rationale for the proposed methodology and a brief overview of that methodology. This methodology is called a method assemblage.

A Preamble to the Problems and Design Principles

This preamble is written to attempt describing elements of intent and method in this study without relying on dense theoretical and philosophical work. Hopefully, much of this can be read productively by keeping in mind the technological intervention is aimed at emphasizing, or amplifying, interaction in classrooms. This interaction is intended to go beyond what I refer to as a mostly transmission-based model of education. This study does not seek to optimize any aspect

of current educational practices or routines (e.g., doing *more* homework in *less* time, or grading *faster*). It is a deliberate rearrangement aimed at taking advantage of a teacher and students working together in the same room at the same time.

To make the intentions of the study clearer, I will be referring to five ideas or tendencies that I discuss in two parts. The first part discusses *transmission*, *optimization*, and *synchronicity*, and the second part discusses *modality* and *duration*. I will discuss what I mean by each word and employ this vocabulary in building an initial rationale for this study. A final purpose of this vocabulary is that I have found it useful to employ new words to describe the potential interactions among several components of the study rather than trying to force each component's vocabulary onto new interactions. In other words, tailored means may be useful in discussing the peculiar behavior beyond the metrics given by each element or component. By analogy, a water molecule behaves differently than the two hydrogen atoms and the oxygen atom that fused to constitute it, and therefore stands to gain from a reassessment of how this new molecule's behavior might be described.

Transmission, Optimization, and Synchronicity

Transmission refers to some completed idea or information transported through some medium. Optimization is, broadly speaking, the increasing or decreasing of some constraints that result in a perceived net gain (e.g., energy or money, some capacity increased, time reduced, output increased). Synchronicity refers to asynchronous or synchronous exchanges (e.g., text messages between two parties may be an example of asynchronous communication, and dialogue between a teacher and a student in a classroom may serve as an example for synchronous communication). By example, if I opened a restaurant and told someone about it, then that idea is coded into language, which can then be *transmitted* in a variety of ways. Ways like standing

within a few feet and saying "Come to my restaurant on Broad Street!" Alternatively, it could be shouted, written down, placarded, emailed, called, tweeted, or advertised. Each of these is seen as some *optimization* of something that is valued, such as the number of people reached (e.g., advertising), costs decreased (e.g., shouting), time saved (e.g., tweeting), and so on. Similarly, educational concerns such as the amount of content in a year's curriculum, class size, class time, or the amount of homework can be seen as values that are optimized in some way (e.g., lowering class size, increasing class time, increasing access to technology, etc.). The larger point that I am making is that optimization does not necessarily consider the interaction of various components or underlying assumptions and intentions.

Synchronicity refers broadly to synchronous or asynchronous communication or interaction (e.g., using Skype, texting, mailing a letter, or speaking in person) where synchronous refers specifically to communication between two or more people that offers capabilities like immediately responding to, or even interrupting, a speaker. Asynchronous communication allows parties to access an utterance (e.g., voicemail, email, a letter) after it was uttered and to respond after some delay.

Modality and Duration

Modality describes the various modes of working. For example, working alone, working with other students, or working with a teacher and other students[3] are different modalities. Often, the predominant mode of a classroom may often be in the modality of whole-group instruction. This dominant modality might temporarily transform into modalities like students working alone, working in groups, or working one-on-one with a teacher. Broadly speaking, different modalities of interaction offer different qualitative experiences. For example, a student working alone on a

[3] I will discuss the role of material objects, like blocks, technologies or a laptop, in the later sections that deal with the concept of assemblage.

task has certain potentials and capabilities. Similarly, a student working with another student and some writing pads or a calculator offers different potentials and capabilities. These capabilities may be as simple as the ability to have a dialogue, calculate a root, copy a strategy, or even feel intimidated or supported by the sheer presence of another student or teacher. Whatever the arrangements of students, resources, and teachers may be, I will argue that each arrangement is qualitatively different from every other arrangement.

I use duration in two ways. Duration refers to the amount of time spent in some modality, and it refers to the cumulative effects accruing over time spent in various modalities[4]. For example, the duration of grouping Sarah and Margaret together refers to the amount of time spent together (e.g., 12 minutes), and each student's duration also refers to the cumulative effects of being grouped with the other student for some time (e.g., two months). In other words, grouping two students together again is qualitatively different from grouping them together the first time. My use of the term duration not only considers the appropriateness of some modality, but also introduces elements of context and historicity.

To see how these terms may be useful, consider some public high school and a Montessori school in the same town. In the public school, the teacher would primarily look to the period length and class size when deciding on modalities and durations. In the Montessori school, the teacher would primarily observe or describe a particular student's activity regarding modalities and durations. As many schools are more like the public high school than the Montessori school, the primary unit of analysis is the classroom. I argue that this tendency of taking the classroom as a primary unit of analysis significantly shapes ways of inquiry, influences ways of thinking, and helps define measures of progress. If more importance is given

[4] Duration, in this sense is positioned to draw some parallels with the property of historicity in complexity science.

to modality and duration, then some focus might shift to questioning the purpose of a given arrangement. For example, a teacher arranges the 25 students in her class by five students to a group. Here, the class size is taken as primary, and the grouping is derived from it. However, if the modality is taken as primary then one can ask why five students were grouped together and not four, or three, or seven, and, why these seven specific students? Moreover, if duration is taken into account, a group of students is less considered in general (e.g., "What is the best group size?") and more considered regarding specific students and previous groupings. In this way, these five terms (i.e., transmission, optimization, synchronicity, modality and duration) are intended to bring emphasis to considering what might be done in classrooms, not simply what should be done.

Using These Five Terms

These terms are meant to be useful precursors to discussing the concept of assemblage. I state that these terms are more useful when no hierarchy is imposed (e.g., "Is duration more important than modality?"). These terms may be thought of as processes to be applied to some context in various intensities and sequences. Overall, school classrooms are largely synchronous during class time regarding classwork and largely asynchronous outside of class time regarding homework. During class time, a teacher may do several things:

- change modalities by regrouping students;
- change duration by not allowing one student to work with a close friend more than once a week;
- transmit information to students synchronously (speaking to the class during class time) or asynchronously (posting to a website); or

- optimize some transmission by recording a lecture about classroom norms to video for assigned viewing.

That being said, if someone said to a child "Go to school," and the child asked "What for?" then this confrontation might be met with some frustration, but I argue that it has now become more than rebelliousness. It may have become a good question. This rebelliousness is even more relevant depending on the conception of what a mathematics class "is." If a mathematics class is the transmission of efficient strategies and procedures, then technological advances can collapse this emission onto video to optimize its transmission. I would argue, in this case, the potential of a mathematics class has been collapsed with modality and duration sacrificed for the sake of optimizing transmission. I would argue similarly against a peer advocating a procedure to another peer based on how quickly it is executed. However, if a teacher can orchestrate various student conjectures, even partial ones, into meaningful discussions that can help students engage more difficult problems (Stein, Engle, Smith, & Hughes, 2008), then I would say the classroom dynamic is leveraged beyond transmission[5]. To conclude, this study assumes that class time can be used better when some portion of that time is used for more meaningful interactions, (e.g., discussions, more student feedback, and potentially engaging higher cognitive demand tasks) rather than mainly transmitting strategies or optimizing such transmissions.

The Rationale for Using Problems

The problems discussed here brought about the necessary frustration that made me think, long before I had any words to describe or frame them. Alternatively, a problem brings about frustration with the status quo; it is something "…that forces us [to] search, that robs us of

[5] In addition to this, see Chapin, O'Connor, and Anderson (2009) for literature advocating benefits of using 'talk moves' to change the types of classroom discussions taking place. This kind of work among students and teachers benefits from real-time, physically proximate interaction of which there are certainly many opportunities in the current educational paradigm.

peace…" (Deleuze, 2000, p. 15). I take this to be perhaps as simple as being stuck in a traffic jam or empathizing with the frustrations of students and teachers. For instance, Félix Guattari was frustrated by institutions like schools, trade unions, political parties, and research groups because they were "prone to the problems of rigid hierarchizing, segregation, and inertia" (Watson, 2009, p. 22). To take one of these problems as an example, I discuss rigid hierarchizing. Consider the different roles in some school: principal, first-grade teacher, second-grade student, coach, custodian, department head, etc. These roles could likely be organized into some treelike diagram, and movement among these roles (e.g., from first-grade teacher to department head or principal) may take years to occur if at all. The treelike diagram would represent the "hierarchization" of the system. The slow movement among roles or rare interaction between roles speaks to the rigidity of the system. Félix Guattari's work and conceptual contributions, specifically those in collaboration with Gilles Deleuze, will be discussed in more detail later because of their significant influence on this dissertation.

For this study, I would like to distinguish between two kinds of problems. These are the problems of blockages and the problems of inertia. I also distinguish between artificial blockages and natural blockages. Finally, the use of problems does not imply that the goal of the dissertation is to solve these problems. The aim is rather to engage with these problems through some intervention and describe the resulting spectrum, or variety, of interactions.

Finally, it is important to note that while these problems are important, they are not taken as absolutely primary to inquiry. By that, I mean that I did not identify problems and write them in stone, with all inquiry referring back to some absolute or general problem. Instead, while these problems do seem to initiate inquiry, they were influenced by theoretical considerations, available technologies, research questions, and interactions with teachers and students. An

example of this is given in the next section discussing the four problems selected for this study. I would like to make the point that none of the components of this study took on an absolutely primary role or hierarchy. Any number of aspects of the study (e.g., design, reading, research questions, data collection, discussions with teachers and professors) were able to influence any other component[6].

Two Types of Problems in This Study

Blockages as problems. Google Maps is a free web service or application that allows people access to maps as well as navigation functionality using their current location. And even though this capability solves many problems it also creates new ones. For instance, say several thousand people request and then drive the best route from Athens, GA to Atlanta, GA. This, of course, may very well create a traffic jam and hence cease to be the best route. This is an example of how some value (taking the best route) meets with specific populations (cars), resources (roads), and other contextual factors (getting to work at 9am). This resulting bottleneck, or problem, requires serious consideration[7]. I would refer to this bottleneck as a blockage and further distinguish between a natural blockage and an artificial blockage. A natural blockage might be the case where the road has reached its physical limit of traffic, whereas an artificial blockage would be a bottleneck brought about by inefficient road markings (e.g., allowing only the use of some small fraction of its capability, for whatever reason). I use the concept of blockages to describe specific, contextual problems that some educator may experience. For example, an educator might want students to do more open response work, but this teacher is unable to do so because of time constraints. The teacher is naturally forced to be

[6] For example, discussions with teachers made me shift emphasis among problems and design principles, readings in complexity theory influenced the metamodel, and data collected influenced the method, and variation map. Research questions influenced data collection and data collection influenced research questions.

[7] The issue of context and the tendency of theory to reluctantly engage with it, will be discussed in more detail later.

more selective in assigning this type of work. More broadly, getting students to a centralized location to receive an education was a response to a transmission problem of knowledge and resources. This was a natural blockage of getting access to education for many people by physically moving them around. However, now with the advent of the internet, I argue this arrangement is becoming an artificial blockage to accessing resources and information.

Inertia as problem. "Where there is no energy, there is inertia…" (Watson, 2009, p. 116). I use this concept of inertia, in a social scientific context, to refer to the constant effort or energy required to avoid natural tendencies towards equilibrium or decay. For example, it requires effort, or energy, from a teacher to maintain classroom discussion that continuously incorporates and tries to reconcile various points of view from many students. Should this effort or energy cease to be invested, classroom discussion may tend towards fewer or even no students speaking[8]. I will use the problem of inertia, rather than blockages, to focus on creative efforts or potentials of what might be or might have been in some classroom assemblages. For example, one teacher mentioned that a student using this Google Form was able to finish typing a thought even if it was incorrect whereas that student might have been cut off by another student if they were saying it out loud rather than typing it. In this case, the cutting off is seen as a return to a previous state of inertia, or collapse, and the student uttering their response as an investment of energy into the classroom assemblage against the inertia.

[8] To briefly remark on the theoretical influences, inertia points come from early work in thermodynamics that considered closed systems, like heat engines. This worked well for engaging the problems of heat engines but was unable to transfer effectively to biological systems due to "living matter [evading] the decay to equilibrium (Schrodinger, 1967)." The evolution of this concept profited from theories like cybernetics and information theory which Guattari was keenly interested in at the time (Watson, 2009, pp. 116–120).

Problems Influencing This Study

This section discusses the problems perceived to be influential in this study. Four problems are outlined, and each is briefly discussed regarding the intervention. Once again, these problems can be profitably viewed as processes rather than separate components. For instance, the problem of managing diverse responses and the problem homework feedback were difficult to separate into distinct categories. The four problems are respectively referred to as:

- centralized architecture blockage;
- homework feedback blockage;
- diverse responses blockage; and
- discussion inertia.

The Blockage of Centralized Architecture

Over a hundred years ago, in the US, there were initiatives based on "education for all" rhetoric. People lived scattered across great areas and this caused a geographic, or physical, blockage to getting "education to all." It was not until well into the 20th century that most of the children population could routinely go to a central location and interact with teachers, students, and resources (Clements, Keitel, Bishop, Kilpatrick, & Leung, 2013). I would frame this encounter as a value system (e.g., "education for all") that passed some threshold and became widely implemented, which in turn created new problems. For instance, advances in technologies have posed questions about this arrangement and started to turn this once natural blockage to education into an artificial blockage. What I mean by once natural and now artificial is that students needed to go to a building to receive education, but now this arrangement may be counterproductive in cases where technology makes this arrangement redundant to some degree. Examples of such technologies include educational radio (the 1920s), educational TV (1950s),

mailing audio and video like CDs and DVDs (1990s), internet downloads, and now internet streaming like Kahn Academy, MIT OpenCourseWare, and TED-Ed (Ferster, 2014, pp. 34–45).

Schools were not the only institutions that encountered these technologies; the library is a good example of how an institution has dissolved into the town or city and its defining feature is not necessarily a building with books in it. I can access library resource while I am sitting on a bus or in a coffee shop. In this case, it might be easier to talk about a process rather than a product (i.e. *library-ing* as process rather than library as product). This alleviates binding this collection of components of resources, services, and people to a physical location and defining it rather regarding interactions among these components. Discussions of process and interaction among heterogeneous elements will also be developed later in this chapter.

While it was the original purpose of this study to propose ambitious ideas for decentralized educational settings, the focus has shifted to exploring the use of technology in a way that takes advantage of the given centralized setting by placing more emphasis on capabilities afforded thereby. For example, if we are all sitting in a room together, why not interact in a meaningful way such as a discussion[9]? So, in this sense, the centralized architecture blockage was not seen as a target, but rather just recognized as context and the decision made to try and create conditions that may take advantage of it.

The Blockage of Homework Feedback

One of the identified blockages from a teacher's point of view lies in getting feedback on a student's progress on the assigned homework. Homework is assigned, and the teacher does not get feedback until the next class starts. In general, a teacher could anticipate what students might struggle with, but would not know until the class begins the next day or later. Also, due to time

[9] This change of focus was brought about during the first study iteration after teachers made it clear that using a customized Google Form for homework feedback would be more expedient currently.

constraints, the teacher might rely mostly on a performance indicator, like graded homework, rather than assessed student understanding gained through discussions or from journal entries. To be more explicit about the vocabulary, a teacher is blocked from student responses to a given homework problem at the moment that students encounter it because they are not physically proximate. This has at least two consequences: the student may not remember, or be less able to articulate, their difficulties experienced with the homework the previous day; and, a teacher does not have access to students' homework difficulties until class begins, thereby mitigating the ability to assess and prepare a response in the same way they might have with more time available to them. So, technology affords opportunities for a teacher to be aware of how students are doing on homework before both the teacher and the students are in the same room the next day. As mentioned before, this problem came to the fore with an earlier iteration of the customized Google Form. When I explained the capabilities of the form, some teachers immediately exclaimed that they could use it for homework.

The Blockage of Managing Diversity

This problem requires little explanation: it is harder to manage more student responses rather than fewer; and, it is even harder for a teacher to manage diverse responses and ideas from students rather than similar responses (Stein et al., 2008). The section on design principles and the five practices for orchestrating discussions will discuss how teachers are aided through the intervention in managing not only more but also diverse responses from students.

Inertia Points in Discussion

Without appropriate norms being continuously enforced, the total amount of talking in a classroom tends to settle on a few people in the classroom. Or at least this is taken as an assumption. For instance, a teacher would often speak the most, and some students might tend to

contribute more than others. To situate this tendency within the inertia rhetoric, one could say that when a few or no students dominate discussions, the classroom discussion state is decaying or close to equilibrium. The aim of engaging this problem is to overcome such inertia and generate new possibilities that involve more students and hence move the classroom situation away from this inert state.

Tying the Problems Together

As will be discussed later in the design principles, the study does not want to optimize, solve or address a single problem; rather it is aimed at being responsive to the intersection of several problems. In fact, optimization is seen to be at odds with resilience and robustness and not necessarily a desirable outcome. By example, should the vehicles in a traffic jam all nudge forward to be only one inch away from the car ahead, then this would be an optimization in one sense, but the larger interaction has been rigidified and is less responsive to novelty, like an ambulance that needs to pass. More specifically, say that some technology is able to reduce all mathematics homework on earth by 20 minutes, then I would speculate that, in many cases, an additional 20 minutes of mathematics homework might simply be assigned. My point with this hypothetical is that I think there is no revolution to be had in the optimization of any aspect of this educational paradigm due to the inadvertent blockages that will always follow and absorb any introduced flexibility[10]. The three blockages and one point of inertia have been identified as the most influential problems in decisions made throughout the study. For instance, as the teachers pointed out a potential use with student homework for the earlier version of the customized Google Form, the decision was made to incorporate this capability. This decision satisfied a design principle of not adding anything to the teacher's day and instead aiding them to

[10] I discuss this in more detail on research in social sciences in a section called incompleteness.

free up a time to consider or attempt new practices (addressing the homework feedback blockage). The leeway created by this capability is then intended to be taken advantage of by emphasizing student-teacher interactions, specifically discussions (taking advantage of the centralized architecture blockage while targeting discussion inertia). In other words, if I wanted to only save a teacher time, I would have designed a homework digitization application that allows a teacher to review homework before the next class begins. However, the customization of the Google Form is designed to elicit open responses to introduce more varying, qualitative information in the classroom and to see how that changes the interactions that take place. This is one of the major examples of the interplay between the problems, design principles, theory, philosophical concepts, and fieldwork that resulted in this document.

Design principles

The purpose of this section is to discuss and state simply what the values are that drive this technological intervention. Figure 1 shows how the problems, design principles, and technology contribute to the study. The rationale for this section is twofold. Firstly, to encourage stating the consequences of "almost offensively obtuse" (Young, 2013, p. 3) theoretical and philosophical work plainly. And secondly, to provide a framework for readers to judge emerging or alternate technologies as and when they become available in comparison to the Google Form used in this study (i.e. Is there a new technology more aligned with these principles than the customized Google Form?).

The rationale for including a design principle section was influenced by my work in software development where the process and guiding principles are more important than specific products partly due to the risk of any technology quickly becoming obsolete, the need to communicate with various stakeholders plainly, and the need to be responsive to context. The

design principles concern the teacher, the students, the software and devices, the context, and the student responses.

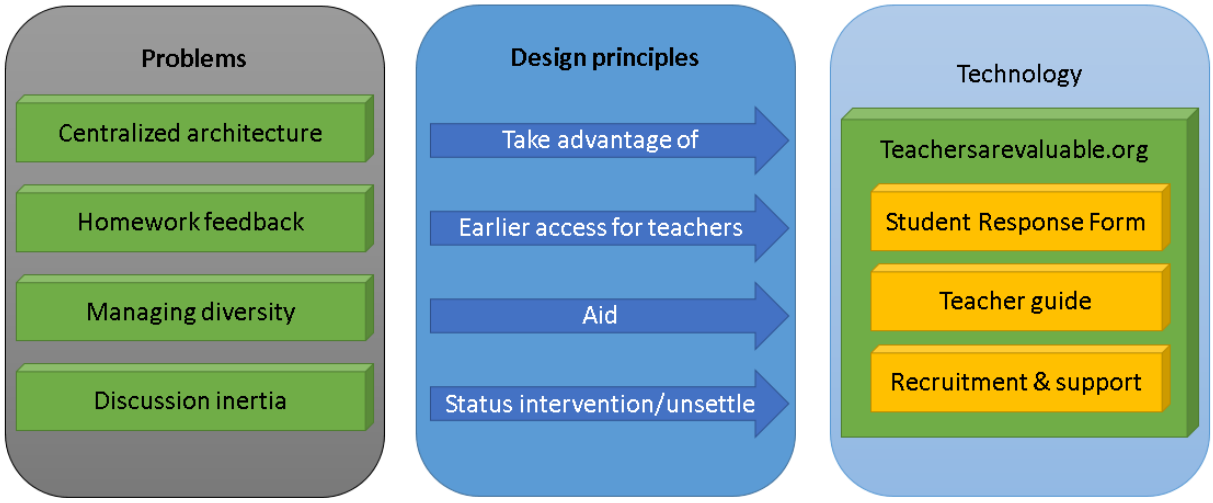

Figure 1. Relating problems and design principles to the Student Response Form

Concerning the Teacher

The technological intervention should not introduce significantly more workload than the teacher already has. It should also not simply be an optimization of an existing practice but rather create opportunities for more meaningful instruction. The technology should be straightforward and quick to be set up (between 10 and 60 minutes, once per semester or school year) by teachers working by themselves or helped by someone familiar with the project. It should only have a very basic digital literacy requirement (viewing a spreadsheet through an app or browser).

Concerning Students

The technological intervention must provide an alternative communication channel for students who are reluctant to speak up during class time or in front of peers. It needs to be more efficient than collecting responses using hand raising (this may depend on class size). It can only assume a very basic digital literacy either when working alone or aided by someone else (access a link, type a response, and click submit). It is intended to create opportunities for the teacher to

acknowledge a student's contribution and incorporate it into the lesson. The Student Response Form may be used more often if each student only requires a short identification number, access to the internet, and a short web link.

Concerning the Software and Devices

The technological intervention should be free, scalable, and robust. It should not assume one-to-one technology. It is rare for a single device to be used 24 hours a day, so by allowing for asynchronous submissions devices can be shared more easily. It does not require software purchase or licensing nor does it require students to have any online accounts like a Google account. A student should be able to use any device that can access the Internet to access the web link, enter their assigned identification number, and enter and submit their response.

Concerning Context

The introduction of the technology can be accompanied with recommendations and communication of intent, but it must be clear to the teacher that the intervention is non-prescriptive and exploratory. A teacher guide is provided and it makes recommendations from mathematics education literature on orchestrating discussions and from complexity instruction literature on status interventions should the teacher choose to use it. The bulk of the recommendations emphasize capabilities of the Student Response Form and give examples of how other teachers have used it.

Concerning Student Responses

The technological intervention must not only increase the diversity of responses by asking for open responses (e.g., "How did you solve 2+2?") rather than closed responses (e.g., "What is 2+2?"), but also aid the teacher in managing this increased diversity. The teacher should be provided appropriate recommendations and encouragement to take advantage of this

technology of using open responses beyond the sustaining of old practices. However, the teacher is free to reject any and all recommendations and use it however they please.

Tying Together the Design Principles by use of a Counterexample

The design principles are discussed by using the counterexample of the flipped classroom with the assumption that highlighting tensions between ideas may often bring out more concrete outlines, claims, or limit points than perhaps when defined well in isolation. Also, while I was describing the Student Response Form with a potential participant, she thought it sounded like the flipped classroom and I wanted to make it more clear that it is not. The flipped classroom is defined by Bishop and Verleger (2013) as "a new pedagogical method, which employs asynchronous video lectures and practice problems as homework, and active, group-based problem-solving activities in the classroom." An example of the flipped classroom, from the student's point of view, might be going to class and working on problems based on some video lecture watched the previous day, or sometime before class. The classroom is called flipped because the classroom explication is replaced with a video lecture that is viewed during what would have been homework time and group-based problem solving based on the assigned explication is done during class time.

As stated before, this study takes as its point of departure the assumption that mathematics education benefits significantly from creating opportunities for students and teachers to interact in various modalities over time without prioritizing some particular modality in general but rather in considering a given context critically and being appropriately responsive. For instance, I would regard the flipped classroom as an optimization of an educational context that is largely lecture based with teacher-student or student-student interaction mostly concerned with the transmission of efficient strategies rather than the cultivation of, say, individual and

collective conjectures. In this case, the flipped classroom seems to make better use of the time shared by teachers and students in classrooms. I would describe this as a rearrangement aimed at optimizing the diagnosis of student strategies (as opposed to, for instance, its cultivation) with the goal of realigning or correcting that student strategy with some existing strategy judged to be more efficient or ideal. By way of analogy, the flipped classroom risks being like a stock market where students trade in strategies seen as most efficient, or simplest.

I would contrast the proposed use of the customized Google Form with the flipped classroom in four ways. First, the educator does not have to alter the way they instruct. Second, some class time is made available by relocating a portion of monitoring student homework to the outside of class hours. Third, opportunities are created for every student to respond to questions asked by the instructor during class time, ask questions themselves, and voice concerns or difficulties with homework or classwork. Fourth, the section of class time that was saved by receiving student feedback before class begins is deliberately redirected to focus on classroom discussions that incorporate student conjectures and feedback. So in closing, this intervention is not aimed at the optimization of the transmission of some information but rather aimed at creating a space where students and teachers can take advantage of being physically proximate to each other by engaging in synchronous dialogue in various modalities. A major assumption for the potential of taking advantage in this way is largely fueled by the student's ability to communicate with the teacher during or outside of class without fear of peer pressure or ridicule.

Conclusion

This section addressed problems, vocabulary, and values, or, principles that influenced the intervention. The most significant problems were outlined as well as their role in the study. A simple vocabulary was introduced to outline and precede the more dense philosophical terms and

concepts to follow. Finally, design principles were addressed that influenced design decisions. The following sections are dense theoretically and philosophically. The purpose of the chapter is to attempt to explain how philosophical concepts significantly influence inquiry and pose questions of orthodox methods in mathematics education research. Stated very briefly and simply, it is very hard to say how you are going to do something new or even what it might be — for if you know how to do it, or exactly what you are looking for, how new can it truly be? The philosophical concepts of Deleuze and Guattari are, among many potential things, aimed at arranging encounters with the new, whatever that might be. I will argue that this dissertation contributes to the field of mathematics education not only in terms of its findings but also as a simple prototype, or the outline of processes, of a methodology capable of dealing with and strategically intervening with technologies and education in the 21st century.

CHAPTER 2

PART II: TRANSITIONAL FICTIONS AND EXAMPLES

This preface offers the reader two fictions which have been developed and refined over two years to allow someone who is not familiar with the work of Deleuze and Guattari an entry point using some context and fictions. The use of fictions is useful in this case because I have found it to be a reasonable compromise between speaking overly technically or simplistically. The examples are of swarm intelligence and an application of a philosophical concept to a river. I have also included some historical selections from the life of Félix Guattari that serve as early precursors to the concept of assemblage, which is used in this study. The swarm intelligence example serves as context to discuss how tendencies, like separating and joining, can be maintained in tension[11] that can have emergent effects. The example of the river serves as an instance of how a philosophical concept influences how something is seen and thought of. Another reason for the two fictions is that the philosophical concepts put forth by Deleuze and Guattari (1987) are usually discussed as processes applied to some event or age rather than implying a hierarchy or chronology (e.g., "587 B.C.-A.D. 70"; "November 28, 1947"; 1440). Finally, I would like to state that I offer these fictions and examples only as a beginning context with the caveat that there are limitations to these stories and recommend only using them to catch a glimpse of something and then continuously trying different ways of understanding this dense work better.

[11] The choice of words is deliberate and used similar to Adkins, (2015, p. 12) where he describes the *tension* between ratios of stability and change.

The first fiction about dogs and wolves concerns method. The second fiction about a mountain excursion concerns emergence and the mode of organization which fosters emergence. The first fiction highlights the significance of the assumptions we make regarding some inquiry (i.e. assuming some wolf is a dog), the stark differences between the types of questions we ask (i.e. "What is it?", "What does it do?", or "What might it do?"), and the resulting practices we undertake. This example is important because the technological intervention, the Student Response Form, is argued to be better studied regarding its various perceived capabilities across a variety of contexts. And, what it *might* be capable of, rather than trying to find an ideal implementation, or essential characteristics, to frame as a model and have other educators try to imitate it. The second example concerns a mountain excursion and highlights the importance of the inexact, experimental practices that occur between the moments of departing from the status quo (i.e. the norm of holding the children's safety as the only priority) and when a new practice emerges (i.e. exploring off the path, but with some risk). This example is important because it discusses the nuance involved in dealing with novelty and what experimental yet rigorous methods might look like.

First Fiction: Method, and Dogs and Wolves

Consider a boy who is familiar with dogs and sees a wolf for the first time, an abandoned wolf pup. The boy decides not to ignore it, but to intervene in some way. I use this fiction to discuss four ideas: universals, metaphor or analogy, questions, and variation mapping. The first idea of universals is partly adapted from Adkins (2009, p. 42)[i].

Universals, and the "Universal Dog"

A boy has spent much time with many different kinds of dogs and has probably assembled a dog concept, or his universal concept of dog, based on impressions from many

different dogs. This concept was not necessary when he only knew one dog as a child, but he grew up seeing and interacting with many dogs, and so he generalized to compensate. His universal dog is no specific dog, but saying the word "dog" might prompt him to think of a big dog with short hair whereas for other people it might invoke other impressions. Now, this boy sees a wolf for the first time – a two-week-old pup abandoned by its family – and decides that it needs rescuing and follows through, well-intending. To the best of his knowledge, the boy decides to treat the wolf like a dog.

Creating universals moves away from any particular context, from the specific to the general – this may also be called a type of abstraction. I would like to briefly make the following point concerning theorizing in the social sciences. I quote DeLanda in saying that "…there is no need to be… committed to the existence of 'hydrogen in general' but only to the objective reality of large populations of hydrogen atoms" (DeLanda, 2006, p. 28). This can be said equally well for students, teachers, schools, etc. I can confidently state that I have never met a student in general, only specific students and populations of students. Furthermore – and I will elaborate on this point in more detail later – a theory's maturity need not be measured in terms of its generality in the social sciences. Specifically, Deleuze (1987) stated that "the abstract does not explain, but must itself be explained" (p. vii). This suggests that we might look to the process of abstraction itself before taking its results necessarily as given or true. In closing, the methodology used in this study attempts to portray how inquiry might be rigorous *and* contextually sensitive.

Metaphor and Classification

If the boy decides that the creature *is* a dog, then I would call that an act of classification. If the boy decides that the creature is *like* a dog, then I would say that he is operating on

metaphor, resemblance, or analogy. I argue that both of these acts tend toward ideal models and copies. These acts also assume that novel phenomena can be traced back onto existing phenomena. An example of an act of classification from the classroom might be when a teacher observes a student's performance on various tasks and claims: "This student *is* a visual learner." An example of invoking, or operating by, analogy would be to say – as was done in the 18th and 19th century – that the brain is *like* a muscle (Michalowicz & Howard, 2003, p. 81), or more recently, that the role of the *Journal of Research in Mathematics Education* is *like* the growth of a tree (Ellerton, 2014)[12]. Zourabichvili (2005) describes the limitations of analogy, resemblance, and metaphor as follows:

> [The] concept of metaphor doubly restricts transport: by seeing in it the trajectory of a single direction [*sens unique*] from a domain of proper designation to a domain of designation via figures, and by imposing on that trajectory the condition of resemblance or analogy (p. 6).

This kind of classification tends to be a one-way street. There is a proper, or ideal object and then there is the object under judgment. The imposing of this trajectory relies on setting one object as prior or primary and the other as secondary, and the relation is based on judgments of resemblance, analogy, or imitation. In the case of the boy and the wolf, I argue that there are at least three things taking part at this intersection: the wolf, the boy, and the boy's universal concept of dog. The dog concept is interacting with the wolf in a very real, material way: it affects, through interacting with the boy, the wolf's diet, the range of motion, discipline, etc. It affects the boy's ability to judge the wolf's observed tendencies as requiring interventions like amplification, mitigation, or remediation. One can think about several kinds of concepts, like the

[12] This particular analogy is strongly opposed in the first chapter, or plateau, of A Thousand Plateaus with another concept that emphasizes connections and decentralization more like crabgrass or the internet.

dog universal, in a material way that exists along with the boy and wolf and exercises affect. Examples of other concepts are mathematical (e.g., power law distribution), statistical (e.g., class average), philosophical (e.g., assemblage), educational (e.g., ADHD, or gifted), psychological (e.g., Oedipus complex), or sociological (e.g., Pareto effect) and can also be considered in this material way[13]. This is why I say that the choice of concept, or act of classification, of saying "Look at this strange *dog*," and thus treating it like one, has ontological implications because the wolf's reality has been changed by assumptions of the dog concept[14]. Similarly, this is commonplace in classrooms and mathematics education literature (e.g., classifying an elementary student as a reciter, producer, or counter as noted by Van de Walle, Karp, and Bay-Williams (2016, p. 146))[15]. More importantly, classification requires any phenomenon to be described in terms of already existing things and so it is limited in dealing with novel phenomena. For example, classifying animals based on a treelike hierarchy cannot account for a virus transporting genetic code between species. Deleuze and Guattari discuss such a case of cats and baboons (1987, p. 10). A later section discusses the concept of variation mapping that I will use to oppose classification.

Questions: Two Kinds of False Problems

Let us imagine that the boy chose not to classify the wolf or perhaps gave up on imposing the dog concept after much frustration for both him and the wolf. Now, he might pose a problem about this thing that is not a wolf, and he wonders to himself: "What *is* this thing?" and he might quietly smile to himself in satisfaction of posing such a good and eternal problem. This kind of

[13] For more examples: Manuel DeLanda discusses the political concept of refugee applied to a woman and the medical concept of hyperactive applied to a child (DeLanda, 2006, pp. 1–2).
[14] Some authors may use the term "onto-epistemological'.
[15] To move beyond critique, I speculate that a process-based observation might refer to a student as "currently counting during this problem", instead of "This student is a counter."

essentialist thinking runs strong in Western thought. If Plato and Aristotle had been alive they might have nodded approvingly of this noble inquiry into the essence (e.g., Platonic Idea) and essential attributes or components (e.g., Aristotle's organismic metaphor) of a thing, those pure things that are "unaltered by earthly change and decay" (Adkins, 2009, p. 84). This event, of the question being posed, would also be significant to Deleuze and Guattari but for different reasons. Firstly, this is because other questions could also have been asked. Secondly, this question relies on certain assumptions that significantly determine the types of inquiry that may follow. To explain this, Deleuze highlights Henri Bergson writings about two kinds of "false problems":

> COMPLEMENTARY RULE: False problems are of two sorts, "nonexistent problems," defined as problems whose very terms contain a confusion of the "more" and the "less"; and "badly stated" questions, so defined because their terms represent badly analyzed composites. (1988, p. 17)

False problems of the "more" and the "less." By "confusion of the 'more' and the 'less'" I mean questions that try to impose hierarchies such as "Is student-centered learning more important than teacher-centered learning?" or ideal instances such as "What is the best class size?" Specifically, "Can a hierarchy of practices be developed that rank orders these practices in terms of their relative contribution to teaching quality and student learning?" (Charalambous & Pitta-Pantazi, 2015, p. 30). Questions of the more and the less seek to establish a general order of priority (e.g., "What is more important: algebra, or geometry?"). I will argue that this is a false problem largely because the specific context of any classroom may compromise any general theory's efficacy. Or, I see a similar opposition to certain types of questions in mathematics education literature drawing on complexity theory: "Was this lesson teacher-centered or learner-centered? …we regard this particular dichotomy as a distraction" (Davis & Simmt, 2003, p. 160).

To conclude: instead of establishing a hierarchy or priority list, I draw on an assumption of complexity that any element, very simply, may have a large or negligible effect on any given system (Cilliers, 1998, p. 4).

False problems of "badly analyzed composites." I take badly analyzed composites to refer to inquiry that looks for the essential components of a system so as to trace emergent phenomena back onto those component parts in a deterministic way. For example, one might argue that since "90% of engineers were observed assembling and disassembling clocks as children, therefore we need more clockwork in the curriculum[16]". In this case, the composite would be the engineer (or, final form) and bad analysis would be to attribute mechanistic causation to working on clocks as a child. A question from mathematics education literature (Charalambous & Pitta-Pantazi, 2015, p. 30) that moves away from this tendency and toward emergent and interactional tendencies might be "Do generic and content-specific teaching practices contribute individually to instruction or is their effect synergistic/interactive?"

Overall, I follow Buchanan and Collins (2014, p. 17) in claiming Deleuze and Guattari's work is more interested in process-oriented questions like "What does [mathematics education[17]] do?" and "How does it work?" rather than product-oriented questions like "What does it mean?" or "What is [mathematics education]?" The prior questions are more interested in the interaction of components, or composites, rather than essential and non-essential components. Furthermore, this interactional character of this type of inquiry functions through conjunction and redundancy: rather than asking this *or* that, it investigates this *and* that *and* that *and*…(Deleuze & Guattari, 1987, p. 98). In this sense, nothing is taken off the table so to speak: it is non-reductivist. Relating this to the theory used, badly analyzed composites stand at odds with the concept of

[16] Taken form a comic strip by Zach Weinersmith, available at http://www.smbc-comics.com/index.php?id=3978
[17] For example

emergence which speaks to capabilities that cannot be traced back onto an object's parts. For example, compare a water molecule's attributes and capabilities with those of either oxygen or hydrogen respectively when interacting with fire. The concept of emergence will be discussed several more times in this document.

Variation Mapping: Capabilities and Potentials, Emergence and Collapse

A further point I would like to make here is that the wolf might benefit more from a coordination of various concepts, i.e. a type of elementary metamodel that highlighted various tendencies rather than essences. For example, this creature tends to sleep like a dog but tends to form packs more like a hyena[18]. Keep in mind, that even all of these different concepts employed still do not add up to the novel phenomenon, or in this case, the creature under inquiry. And so – following on the idea of "badly stated" questions – the questions relating to what something does and what *might* become of something represent inquiry that emphasizes the exploration of problem spaces (or even phase spaces). Relating this to mathematics education, a better question than "What kind of student is this?" might be "What is this student currently capable of?", and "What might this student possibly be capable of, and in combination with which resources and people?" These types of questions can already be seen in complexity theory infiltrating policy (Geyer & Cairney, 2015) and planning (Roo, Hillier, & Wezemael, 2012a) among other disciplines. Once questions like these concerning possibility are posed, then Deleuze's comment (1988b, p. 125) on Spinozist influence is quite applicable:

> That is why Spinoza calls out to us in the way he does: you do not know beforehand what good or bad you are capable of; you do not know beforehand what a body or a mind can do, in a given encounter, a given arrangement, a given combination.

[18] And, should it show signs some novel phenomena then this could simply be called "wolfing."

What Spinoza has called ethics is concerned with looking for what something is capable of, or might be capable of, what connections and arrangements it can enter into and what its capacities and capabilities are in different contexts.

I argue that inquiry influenced by the work of Deleuze impacts several things, and I emphasize some of them here: Inquiry is shifted from questions of essence or attributes (e.g. "What is it?") to questions concerning interaction (e.g., "What does it do?", or "What might it do?"). This kind of inquiry is not as concerned with what the creature that the boy saw is called (classification) or what its essence is; it is interested in finding out what it can do, might be able to do, might be hooked up to, what are its capacities and points of connection[ii]? Deleuze and Guattari refer to this type of inquiry as cartography. This line of reasoning will be significant throughout the rest of the theoretical and methodological explication.

To draw this contrast by drawing on current literature, consider the following problem: Why do dogs and wolves differ so much in their behavior despite significant genetic similarities? Table 1 has two extracts, one from a journal of ethology and the other a journal of neurobiology. I liken the first to having elements of variation mapping and the second to classification: the ethologist is more interested in capabilities, context, and duration whereas the neurobiologist is more interested in making predictions based on neurological and genetic features or components. In other words, the neurobiologist is akin to someone who took photographs at two different points in time and looks for a theory to map one onto the other through prediction or causation (discontinuous), whereas the ethologist was interested in what happened between those two points in time (continuous).

Table 1

Two journals considering the differences between dogs and wolves

Ethology (parallels with variation mapping) Lord, 2013, *emphasis added*	Neurobiology (parallels with classification) Saetre et al., 2004, *emphasis added*
The results suggest that wolves and dogs both develop olfaction by 2 [weeks], audition by 4 [weeks], and vision by 6 [weeks] on average, despite the 2-[week] shift in their ability to explore. This means that when wolves begin to explore at 2 [weeks], they are blind and deaf, and must rely primarily on their sense of smell. *Thus, there is a significant alteration of how these subspecies experience their environment during the critical period of socialization*. These findings lead to *an alternative explanation* for the difference in dogs' and wolves' abilities to form interspecies social attachments, such as those with humans.	Domestication has led to dramatic changes in dogs as compared to their ancestors. Domestic dogs were (and still are) subject to selective forces very different from those in wolves. Our results suggest that changes in the level of expression in a limited number of genes in the hypothalamus may be responsible for changes in the regulation of multiple brain functions. *However, the observed changes in expression could also be the result of environment and life history instead of just genetics*. To determine which expression changes have a significant genetic component, *further research* comparing wild and domestic or tame conspecifics bred under identical condition *should be carried out*.

Table 1. Contrasting variation mapping with classification

For an example, consider the difference between the Common Core's standard that classifies in terms of "Can count to 20" compared to a more "mapped" assessment of "Peter counted to 17 and then said 19, eleventeen, twenteen on Monday". Note that the latter does not require an external thing to be measured against (i.e. the Common Core standard), or at least creates a temporary buffer against classification for a little while. An example from mathematics education literature might be Ginsburg (1997) with interview protocol principles like "Don't discourage the child's way of solving," "Don't assume that a wrong answer is wrong and right answer is right," or "Don't ask leading questions." This might be an entry point into more ethological work or variation mapping although I would position the building of a model to represent the student's thinking more within a psychology or constructivism and probably more prone to structuralist tendencies, and so in contrast to the work of Deleuze and Guattari[19].

[19] See Deleuze on structuralism (Deleuze, 1973), or Clark (1997) for a more recent, and secondary source.

To conclude, the wolf may represent any phenomenon (e.g., a student, a classroom, a study, or even a culture), and the point of the argument is that variation mapping is built to assume and work with novelty, and the conditions under which it emerged, rather than trying to compare it to something that already exists. The next section deals with the concept of emergence.

Second Fiction: Variation Mapping and a Mountain Excursion

A teacher at a school has been able to get enough support to let a class of students go on a mountain excursion. Arrangements are made, and once they get to the mountain, rules are set: "Each student must hold the hand of the student in front of them and the student behind them. Do not let go and do not go off the path." The "Safety *Über alles*[20]" policy was a roaring success with no injuries, lost children, delays, or casualties. This campaign has clear goals that I would call pure negation from some ideal instance: it can only tell you what it is not (e.g., *not* injured, *not* late, *not* lost, and *not* dead). All these ideas, of course, polled well with parents, principals and focus groups; you must, after all, be a monster to want children late, lost, injured, or dead. What is more, this status quo can be quantified regarding timeliness, efficiency, predictability, and control. It can be replicated and rolled out to significant scale with only a few rules. Deviance can be detected with ease – a child is not on the path, a hand is not held, an involuntary dash is made after a field or unusual insect – and behavior can be isolated and remediated.

Now, say that there is a growing number of alumni, parents, teachers, and politicians that decide to oppose this policy. This may result in a myriad of alternatives proposed including perhaps a movement called *anti-safetyism* with heated rhetoric that ridicules these "mediocre ways of thinking that undermine our children's freedom and choice." This movement wants

[20] Safety 'above everything else'

every child to be free and in defiance of the oppressive regime of the blind disciples of safety; let children do anything and everything that they please! Unfortunately, this results in significant tardiness of field trips, injuries, lost children and some tragedy. Now the *safeties* and *freebies* are in an escalating stalemate with the safeties setting up fences, making child harnesses, painting lines, building guard towers, installing video cameras everywhere through generous contributions from well-intending businessmen with the freebies following suit in their own way. Other oppositions may include more prescriptive movements like tree climb-ism, survivalism, dance-ism, geologism, etc. each with their own ideas of successful mountain field trips. Eventually and as per usual, it is the teachers that come up with threshold-based policies that allow for significant variation while maintaining some degree of order: "No matter what program you are enrolled in, I must be able to see you, and you must be able to see me at all times. You cannot go if your parents did not sign the permission slip. You must have a buddy, and each of you must have a whistle to blow when you need help. You must be back at 4 pm. Earlier is fine, but the bus leaves at 4:15 sharp, and we're not waiting for anyone!"

I use this context to introduce three concepts: the molar mode of organization, the molecular mode of organization, and the line of flight. Briefly put, I would refer to the molar line as a rigid mode of organization, and I would refer to the molecular line as a supple mode of organization. An assemblage may also be described in terms of a ratio of rigidness to suppleness, or, a ratio of stability to change. To oversimplify – and this may be a less orthodox interpretation because of the oversimplification – the molar is the hand-holding status quo, a line of flight may be anything that gains its own consistency outside of the molar even if it is still not yet clearly defined, and does not collapse back onto the molar due to this consistency. I note that this appropriation of the concept of line of flight is for my own purposes. I use it in the narrow sense

of referring to some consistency gained outside of the molar line that persists in its existence for some time. The molecular mode of organization lives between these two concepts. It is not molar, and it is not a line of flight although it will soon collapse onto either one because of its brief existence (i.e. It could be described as "the in-between"). This may include any movements, utterances, or explorations that are not quite molar and not quite something that has decided on its own rules or consistencies. Also, these concepts do not refer to the relative size of structures (i.e. molar aggregate versus a molecule), but rather to modes of organization (Bogue, 1989, p. 103). This is not to say that a particular context is purely molecular or exclusively molar, it should be stressed, and this is the case for all of Deleuze and Guattari's concepts; each context is rather an "impure mixture" (Deleuze, 1988, p. 19) of the molar and molecular and that neither will ever occur in a pure state, similar to the way that space and time is entangled and does not exist as pure time or pure space. In other words, a single individual has both molecular and molar capabilities and the same can be said for large institutions comprised of many people[21].

The molar mode of organization is rigid and captures or collapses any deviances or molecular movements back onto its line. Whereas the molecular mode of organization is fluid and allows for emergent or unique events, new connections and actions. That is not to say that molecular is better than, or the savior of, molar structures – these are simply modes of organization that can have good or bad consequences judged in retrospect. For example, a molar diet may very well be more efficient for weight loss than a molecular, more impulsive diet. One could also think of the molar line and line of flight as hard lines and the molecular more like a supple thread that is bobbing and weaving for a moment between these two before collapsing onto either one (see Figure 2).

[21] Molar and molecular is largely applied to human interactions in this essay, but need not be.

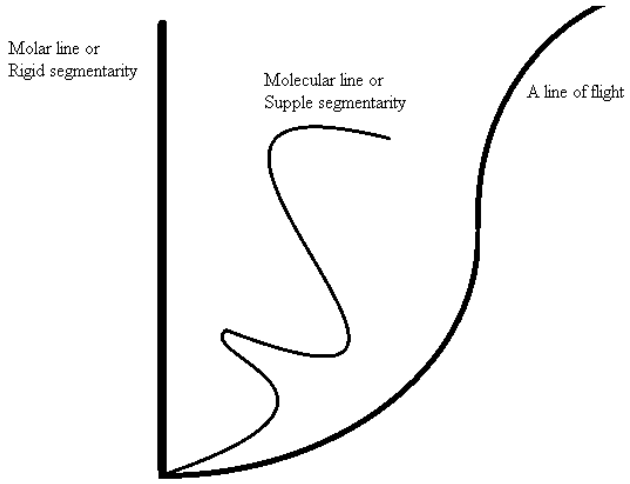

Figure 2. Molar line, molecular line, and a line of flight

Properties of Variation Mapping: Tensions, Modes of Organization, and Collapse

The following three examples are used to discuss important aspects of, what I refer to as, variation mapping. The first example is taken from 19th-century mathematics education literature and was chosen to show a tension between two views of what mathematics school curricula should look like. The second example is a discussion of what a molecular mode of organization might look like in classroom discussions. The third example uses an extract from recent news discussing a policy that would ban hand raising in classrooms, and it is used as context to explain what is referred to as sources of collapse. To relate variation mapping to the mountain excursion, I would say that I am particularly interested in the threshold-based rules the teachers came up with (e.g., "Maintain a line of sight with me at all times!"), the events that emerged under these conditions (e.g., some variation of hide-and-seek, or perhaps rock collecting), and conditions that caused a restriction or denial of potential emergent events, sources of collapse (e.g., a bruised leg, bullying, a storm, or a wag of the finger)[22].

[22] Sources of collapse are not meant to sound negative. Disciplining a student for running out of sight might avoid great risk.

Example 1: Maintaining tensions. During the nineteenth century, there was a tension in rhetoric in whether mathematics should be taught from rule to example or from example to the discovery of a rule. The prior was called synthetic and the latter, inductive. Warren Colburn's work exemplified the inductive approach (from example to rule) whereas the critique of the inductive method in favor on of the synthetic is made by Taylor Lewis. Cohen (2003, p. 59) described how a positive review, written in 1827, of the inductive approach, favored it because the synthetic approach "required heavy rote learning and succeeded only in paralyzing students' interest in mathematics, the reviewer claimed, whereas the new method created disciplined, rational minds." These ideas were widely adopted for the next four decades (p.60). But this approach did meet with its share of critique —take for example the following review from an orthodox religious journal:

> The pretense that mathematics lies out there to be discovered by each student falsely suggests that no other mind before has grasped it. A student is encouraged to believe he is a discoverer, and all things are to be taken as yet unsettled and unknown. It is made a merit in the student that he thus regards it. All his studies are to proceed upon such a supposition of fancied independence. Other minds have discovered nothing-at least nothing for him.... He grows up with this wretched conceit of thinking for himself, and despising all authority. (Lewis, 1851, p. 274)

I use the original rhetoric because I think it is important to note the political, even moral, elements that are often entangled into any curricular decisions. Interestingly, this tension draws some parallels with the problematic and axiomatic as outlined by a Deleuze scholar in

mathematics Education, Elizabeth de Freitas[23][iii]. The purpose of this example is not to identify what is molar and what is molecular but to discuss how this tension might be managed using the theory and philosophy of Deleuze and Guattari. I note that this is purely speculative and greatly simplified.

Instead of thinking that mathematics curriculum should be inductive or synthetic, these ideas could be thought of as processes which may be applied to some curricular item (i.e. an item might be synthesized or "inductivized"). Also, the curriculum might be shaped conjunctively rather than disjunctively, meaning that all items or topics will have synthetic and inductive versions. Note how efficiency is sacrificed, but the redundancy offers more choice or possible paths through a curriculum. Finally, these two processes may be placed in a relation through some ratio like 80:20 but this would not necessarily have to be enforced on all students as each student might have their own ratio (I would refer to this as a policy). For instance, a teacher or guidance counselor might urge a student to obtain a specific ratio based on some career path that the student is interested in. In summary, this hypothetical was more process oriented, conjunctive, and non-universal. This relates to variation mapping because, despite the lack of a "happy medium" between these divergent educational philosophies, they can be held in a productive tension through thoughtful policies and experimentation. A variation map would note the divergent tendencies, experimental ratios (synthetic:inductive), and events that occurred within the bounds of that policy.

Example 2: A molecular mode of organization in classroom discussions. The molar mode of organization has rigidifying tendencies whereas the molecular has supple, fluid tendencies that continuously escapes molar organization, or, "leaks from its classes or

[23] Elizabeth DeFreitas makes this case very recently and argues that this not only manifests in Mathematics Education curriculum but Mathematics itself. More detail is given in an endnote for interested readers.

categories" (Deleuze & Guattari, 1987, p. 213). As discussed in the inertia of discussions: classroom discussions tend to become rigidified, in other words, without norms or practices in place to counteract this tendency, the number of people who generally speak up during class tends to a minority rather than a majority. Now, there are many ways to counteract this depending on how the problem is formulated. For example, if I decide that being quiet is bad and speaking is good, then I could call on students who have been quiet for a while. This may have numerous consequences like affecting the student's confidence or attitude toward the class in various ways. Another example is *equity sticks* where each student's name is written on a stick and those sticks are kept in a container that the teacher selects from randomly. In this way, all student will physically say something in class and the problem, as formulated to get all students to talk in class, is solved.

But this policy of equity sticks, I argue, has substituted one large inertia point for several little inertia points, one for each student. This is because the blockage of diverse responses comes into play when all students need to contribute: it is harder to manage several diverse responses, especially open responses. One way to deal with the new problem is to pose simpler, closed questions to the class and in this case the spectacle may well become more performative, or ceremonious, rather than aiding critical thought or dialogue that requires reconciliation of various points of view. In this way, I would say that one molar mode of organization has been exchanged for another one with little molecular potential. By molecular potential, I mean that a discussion is capable of taking on various paths, making various connections among responses, or producing a novel argument or statement. That being said, the molecular is not meant to sound like a savior or force of righteousness, just as letting kids run around on a mountain is not necessarily good or bad in principle. Nor do we need to be saved from the molar by the

molecular, as illustrated by the mountain excursion example. Each mode of organization has advantages and disadvantages.

A sophisticated policy that allows for molecular organization might be that a teacher walks around the room and checks on various student work before deciding which students to call on. This selection might include factors like holding off on a complete and correct solution until later, and first hearing from various students that had misconceptions representative of the classroom, unusual responses, or simply for the reason of calling on a student because their solution is acceptable and because they have not spoken up in a while and this might increase their social or academic status in the classroom, or their territory. In addition to these practices, a teacher might also ask students how they thought about their solution and find the student had a different way of thinking than anticipated; this could in turn be met by another student commenting on it. Over time, the teacher might become more seasoned in dealing with unexpected interactions and events in her classroom. She might very well have implicit or explicit rules and norms (policies) in place like "Listen to other students", "Compliment, then critique", or "No speaking while another student is speaking." I would say that the potential for molecular movement in discussion is greater than in either a lecture oriented lesson (pure verticality), a lesson where few regulars participate (inertia point), or a lesson using equity sticks (pure horizontality). The ratio of diverse responses to closed responses might be higher from one class to the next or even from one moment in some class to the next. A variation map would attempt to incorporate various aspects of these policies and ratios into its description while maintaining strong ties to that particular context.

Example 3: Collapse in classroom discussions. One of the teachers in the study said an advantage of the Student Response Form is a student cannot be interrupted by another student

while they are typing a response. A student is not able to say "No, that's wrong!" while a student is uttering an incorrect response. I take two important points from this comment: Firstly, the teacher seems to value the ability to complete a thought rather than have it interrupted or stopped; and secondly, an incorrect response, while valued by this teacher, can easily be interrupted and collapsed by another student in conversation. By identifying a source of collapse of what might have been, I am afforded a glimpse of some potential event along with its limit or threshold (in this case the very low threshold of a single interruption), or collapse. Now, I do not specify an ideal utterance, I do not work backwards from some ideal event, nor do I prescribe norms for discussion; I can, however, speak to the potential in a specific classroom for a student to complete a thought, especially if their logic is flawed. Another way to phrase this is that the (implicit or explicit) policy of the right to complete an incorrect thought can be seen as a "mechanism of emergence" (DeLanda, 2011, p. 7). I am not able to describe what would happen when this policy is in place, but I can speak to some potential of emergence possibly being collapsed[24].

To take this problem of molecularity further, consider an article (Brennan, 2015) that describes a push for banning hand raising in a school in Australia with this rationale:

> "The same students are putting their hands up: the outgoing ones," Mr. Albiston said. "There could be half the class without their hand up, so these are the students that lack the confidence to contribute to the discussion." Teachers will instead randomly select students to answer a question, he said." This new strategy is all about the teacher firstly

[24] It is also quite possible that a student interrupting another student may invoke a meaningful discussion and thus not be a source of collapse but rather acted as a mechanism of emergence, or event that caused other events rather than collapsing them.

asking a question, and then pausing," Mr. Albiston said. "Students then learn to sit up and listen because they don't know which student is going to need to respond to the question." As discussed earlier, this may very well make students more attentive, but I interpret this as expanding the molar mode of organization, and creating a new status quo, with a tendency toward its new inertia points like a lack of spontaneity, and perhaps a lack of critical thought in favor of memorizing what the teacher wants them to say. For this example, I want to draw emphasis on the modes of organization regarding capability: what are students capable of doing? Raising their hand is not allowed in this extreme example but in similar ways, students are not contributing because of micro-political elements causing a similar effect or inertia. In the findings, I focus on sources of collapse and glimpses of fluid, or molecular, modes of organization. I would call the ban on hand raising a policy decision with more molar potential than molecular. An example of policy decisions that are more molecular may be Ginsburg's (1997) previously mentioned interview protocols although the context is different.

In closing this section, I used the mountain context for two main reasons relating to variation mapping: first, I want to highlight negation such as *anti-safetyism* as a naïve opposition to some molar mode of organization that has little substance, and this is not the purpose of this study or rhetoric[25]. Second, I use the molecular mode of organization, between the molar line and the line of flight, as a space of exploration that I navigate with threshold-based rules like the teachers made use of and I describe the variation that took place within this emergent space. In particular, attention is paid to anything new that gains its consistency in these interactions. I am not going in with an *ism* (e.g., dancism, geologism, survivalism, artism); I am interested in the creation and assembly of what we, in retrospect, might end up calling *isms*. I am interested in

[25] I am also not advocating defeatism ("Why even try?") or relativism ("Let's just do whatever we want and see what happens.") (Law, 2004, pp. 7, 62–63).

inventions, connections, circumstances that occur in this *in-between* space. Specifically, this variation map documents how the technological intervention allowed for some freedom in response and how teachers harnessed this increased variability through formal or informal policies. To tie it all together, the variation map documents the range and spectrum of variability across eight classroom implementations noting not only context, capabilities, and potentials, but also emergent events or practices, thresholds, and (potential) sources of collapse. The reader is invited to hold on to these two fictions throughout the rest of the explication and discussion should they find it useful. The following section discusses the example of what computer scientists call *swarm intelligence* or *particle swarm optimization*. The purpose is to give an example of how these ideas of maintaining tensions, creating policies, and emergent events can come together.

An Example of Swarm Intelligence

An early entry point, for me, into complexity theory and Deleuze and Guattari's work was exemplified in the field of particle swarm optimization[26]. I invite the reader to draw parallels with ideas of variability, policies, collapse, and emergence. If pressed, I would also say that this is an example of molecular organization, rather than molar. This field, hereafter referred to as swarm intelligence, is an instance of agent-based modeling and also of artificial intelligence. I discuss an example of what a swarm looks like in Computer Science as it has been somewhat successful in offering a context that people from various fields could make use of.

[26] See Cilliers (1998) for an example where another branch of artificial intelligence, called neural networks, are used in discussing links with post-structural work. Specifically, aspects Derrida's work is argued to be related to complexity theory through drawing comparisons with these neural networks.

Inspired by the flocking of birds, *particles* (sometimes called *agents*, or even *boids*[27]) are used to simulate birds flocking together, with each particle representing a single bird. The computer scientists that looked at flocks of birds decided to describe their interaction regarding three tendencies: cohesion, separation, and alignment. More specifically, flocking is described as a ratio among these three tendencies. For example, a flock of birds crossing the Atlantic would have a ratio with high alignment and high cohesiveness factors – a migrating policy; a flock that is looking for a good spot to roost would have a high separation factor so that a larger area can be surveyed – a roosting policy. In this example, an emergent behavior might be a sudden swooping movement of a flock of birds or particles to avoid a predator or settle down onto a desirable area. In algorithmic terms, both scattering into many directions and collapsing unto a single point are undesirable outcomes. To be more precise, and what makes this more complex, is that collapse is ultimately a desired outcome (i.e. finding a solution, like the equivalent of a roosting spot), just not too soon because that limits the solution space. The particle's movement is based on several factors that result in some direction at some velocity. Specifically, if I were a particle, then for every iteration, or move that I make, I could communicate with all other particles, or some neighborhood around me to find out what their best roosting spots seem to be at that moment. I would also take the average direction of all particles (alignment) as well as a random number generated between 0 and 1, like 0.24 and use that to calculate my new direction. If the weight of the random number is too high, then the flock risks scattering. Similarly, if the weight of the alignment factor is too high, then the flock risks collapsing onto a single point almost immediately.

[27] See https://en.wikipedia.org/wiki/Boids for more information.

Additionally, I would like to highlight two ideas from swarm intelligence: satisficing[28] (as opposed to ideal outcomes) and emergence (as opposed to cause-and-effect predictability). Satisficing refers to the idea that there exists no ideal or perfect swoop that the flock attempts to imitate or can be compared to. At most, two observers might only be able to agree that some movement of the flock was "satisfactorily swoopy," rather than having some external, ideal swoop to which this one may be compared (e.g., an 80% swoop, 6.5 swoop). Regarding emergence, this example shares some commonalities with DeLanda in arguing for a move away from cause-and-effect reasoning and linear progression. One can only speak of "increasing the statistical probability" (DeLanda, 2006, pp. 20–21) of an effect (a swoop in this case) and experimenting with different ratios of parameters to do so[29]. In this example, the effect relies on two important aspects of not only introducing variation (the particle's rule to deliberately move randomly) but also harnessing that variation by creating a policy (setting parameters of cohesion, separation, and alignment), to avoid collapsing too soon or simply scattering. To summarize, a swarm needs a component that deliberately introduces a degree of variability, a threshold-based policy ("Too close!", "Too far!") that can harness the introduced variability. In addition to the variation and policy, some threshold defines results that are "good enough" or that *satisfices*, for example, an expression and a data point like *find a point greater than (10, 12, 8) on this sphere*. One of the principal rationales behind even investigating a field like swarm intelligence is that these inexact yet rigorous models can find good or even excellent results now rather than perfect results later. An example of this may be in establishing radio communication among soldiers

[28] Herbert Simon (1947)
[29] More discussions of complexity and emergence specifically in education can be read in Osberg & Biesta, 2008; Osberg, Biesta, & Cilliers, 2008; Osberg et al., 2008.

during a battle where some level of communication is deemed to satisfice and preferred over better communication later[30].

To tie this example directly to the study, a classroom might be thought of as a type of swarm only far more complex because we are not just dealing with birds and food sources anymore. One way this could be thought of is that the curriculum represents the alignment factor, with the openness of items (i.e., items with more than one solution path representing a degree of separation), whereas a closed item with a particular method would represent a high cohesion factor. Also, a Montessori school, I would say, is marked by high degrees of separation where students can exercise choice in what they would like to work on, with moments of cohesion such as students coming together to work on a play. In terms of discussions, there might be a general alignment present as the class moves closer towards a satisfactory solution of a problem. This may be marked with clustering or movements of separation and cohesion as agreements and disagreements move across the class. Later, I will discuss how susceptible discussions are to sources of collapse, like a rude interruption, or decay, like the lack of encouragement by a teacher. The next, and final, section of this part is an example of how I use a philosophical concept to think with.

The Concept of Assemblage and Historical Examples

Using a philosophical concept of Deleuze and Guattari in the social sciences is complicated in various ways. In particular, trying to reconcile this work with that of qualitative or quantitative empirical research may very well be "incommensurable"; for instance, should I use assemblage as a concept to "code" the collected data would "indicate a misunderstanding of that concept" (St. Pierre, 2016, p. 11). What I have attempted here, and in the next chapters, is to

[30] See Potter (2009) for examples of several optimization techniques applied to many real-world problems like the U.S. Army Mobile Subscriber Equipment network configuration problem.

be as candid as possible with the reader in documenting how my thinking, and so too this inquiry, is influenced by this concept. Next is a brief preamble by Deleuze on the relation of these concepts to science. Then, I discuss selected biographical examples from Félix Guattari's life and work. My focus is on early precursors of the assemblage concept in Guattari's group theory and its molecular mode of organization, followed by his concept of machines and how that concept might promote inquiry without structuralist or phenomenological tendencies. Finally, I offer an example of how a river might be thought of in terms of the concept of assemblage.

A Brief Comment by Deleuze on Philosophical Concepts

Doing philosophy is the creation of concepts, according to Deleuze and Guattari (1994). In this study I am not creating a concept, I am using one to think and act with. It is not a metaphor. I do not have an ideal instance of an assemblage that I hope classrooms might aspire to, nor can I clearly state that some object is or is not part of an assemblage or what function it is performing. It is, however, also scientific but in a nuanced way, as Deleuze explains:

> A Thousand Plateaus does indeed use a number of concepts with a scientific resonance, or correlate even: black holes, fuzzy sets, neighborhoods, Riemannian spaces… I'd like to reply by saying there are two sorts of scientific notions, even though they get mixed up in particular cases. There are notions that are exact in nature, quantitative, defined by equations, and whose very meaning lies in their exactness: a philosopher or writer can use these only metaphorically, and that's quite wrong because they belong to exact science. But there are also essentially inexact yet completely rigorous notions that scientists can't do without, which belong equally to scientists, philosophers, and artists. They have to be made rigorous in a way that's not directly scientific, so that when a

> scientist manages to do this he becomes a philosopher, an artist, too. This sort of concept's not unspecific because something's missing but because of its nature and content. (Deleuze, 1995, p. 29)

I will argue later that these philosophical concepts may be very useful to social science researchers for this precise reason that is not entirely scientific, and I will argue that this may be part of their strength.

Félix Guattari: Some Background and Selected Biographical Examples.

> This was the period when Felix would say, "The world is at La Borde."
>
> - Dosse, 2011, p. 50

Félix Guattari's work and his life labors deserve some mention in this document due to its effect in this study and for one more reason. Compared to the literature on the work of Gilles Deleuze, the secondary literature on Félix Guattari is largely absent. My main resources were Dosse (2011), Genosko (2001, 2002, 2003), Watson (2009), and of these Watson is especially indispensable regarding Guattari's notion of metamodeling. Furthermore, Watson (p.4) echoes this issue of neglect in calling for "much more work to be done on Guattari, who has too often been dismissed, and occasionally even ignored outright."

I have found the practices of Félix Guattari, and his colleagues at the La Borde clinic (Clinique de La Borde), very instructive in considering contemporary issues in schools and classrooms, especially as regards implicit institutional assumptions about normality, remediation, ability grouping, language, specialization, teacher-student ratios, period length, lesson structure, etc. In this section, I discuss aspects of the La Borde clinic and its day-to-day administration, as well as aspects of Guattari's intellectual influences and endeavors.

The concept of assemblage is one point, not the final destination, on an evolutionary path of Guattari's work. Guattari initially did much work on a theory of patient groups while at the La Borde psychiatric institute which he ultimately abandoned, but which proved a "crucial conceptual precursor of the machines in Anti-Oedipus, of the assemblages in A Thousand Plateaus, and of the singularizing processes in Chaosmosis (Watson, 2009, p. 10)." So, while the concept of assemblage belongs to both Guattari and Deleuze, they do not necessarily have identical ways of thinking about it (DeLanda, 2012). More specifically, Guattari's contributions to the concept were more applicable and useful, for me, to the field of education as I was able to follow historically through his experimentation and evolution in working with contexts of institutions, psychoanalysis, patients, schools, and even the military. I will briefly touch on some of Guattari's background and these two prior states in Guattari's evolution, namely group theory and machines.

Guattari Background

Félix Guattari was born in 1930 and grew up outside Paris and showed political and scholarly promise early. He was politically active and engaged from the age of 15, going to meetings of the Communist Party and joining a few years later. During his pharmacology studies, Félix wrote for a dissident newspaper under an alias, made pilgrimages to Yugoslavia and China and organized protests in Vietnam and Algeria (Shatz, 2010). His intellectual encounters centered on Marx, Freud, and Lacan. During the 1950's, Félix Guattari was fascinated by Lacan's work. He would learn much of the work by heart – his friends even nicknamed him "Lacan" (Genosko, 2002, p. 111).

Much of this interest in Marx, Freud and Lacan played out at the La Borde clinic where Guattari lived on the property, for several years, and devoted himself to the institution which

seemed to be a microcosm of much of his philosophy: “The world is at La Borde…”, Felix Guattari would say (Dosse, 2011, p. 50). La Borde was a vibrant, heterogeneous assortment of people, disciplines, and philosophies. ‘Barbarians’ avoiding the draft and deployment to Algeria might be sleeping in the attic, but also involved in day-to-day activities of printing, pottery and ceramic workshops. There were frequent meetings of doctors, nurses, staff, and patients in distributing duties as well as discussions held at length and in depth on Marx, Freud, and the most recent work by Lacan. Various fields including psychiatry, sociology, political sciences and psychoanalysis were represented by scholars, students, doctors, staff, etc. La Borde seemed to have its finger on the pulse of contemporary psychoanalytic and institutional issues in addition to its own work at the forefront of institutional psychoanalysis.

Molecular Organization in Guattari’s Group Theory

Guattari invested much effort in a novel approach that disrupted many common sense notions about institutional life and work, specifically the one-on-one, one-way relationship between the analyst and the analysand. It is not just a child or a patient on a couch; there is an unspoken background of institutional context, compartmentalization, organization, practices, etc. Guattari had, in his critique, developed a concept that troubled psychotherapy not by working from within the reduced context of the analyst-analysand on the chair and the couch – forgetting the institutional context and its segregation, hierarchies, surveillance – but by changing the environment. “In short: *the group* is how one gets at the institution” (Genosko, 2002, p. 69, emphasis added); by grouping people across roles and hierarchies, one can start to see a supple mode of organization. It was called the grid.

The grid was a table of rotating work schedules and groupings, called Base Therapeutic Units, with an emphasis on separating the function from the person, diluting specializations, and

traversing hierarchies such as getting the cook out of the kitchen (Dosse, 2011, p. 46). Tasks like washing dishes, answering the telephone, administering shots, or even making music, were shared among everyone at La Borde. This required great coordination as it was a living document that caused the institution to bend and change: "The grid changed over time, displaying periods of centralization and decentralization, and was modified to maximize its therapeutic effects in response to changing conditions in the clinic from which were extracted whatever displayed the greatest … potential" (Young, 2013, p. 321).

Once the classical doctor-patient relation of traditional psychiatry has been disrupted, the practice was made significantly more complex by the heterogeneous and dynamic qualities of the Base Therapeutic Units, which created several, diverse subjects interacting (not just patients and doctors). These interactions immediately introduced and required a broader perspective and methods for experimentation, drawing on political, cultural, philosophical, ethnological, linguistic, and even architectural considerations. To put it another way, Guattari looked to overcome two impasses (Young, 2013, p. 321), or problems: a pure verticality (e.g., a strict tree-like hierarchy) and a simple horizontality (e.g., no explicit roles or responsibilities).

Application to this study: An example of a molecular mode of organization. This served as an example of a molecular mode of organization at an institutional level. I see such pure verticality and pure horizontality as functioning in a similar way to the mountain excursion's safe and free modes of organization respectively. In a school, a molecular mode of organization would consider what connections might be made, rather than the molar reinforcing and maintaining of the existing compartmentalization and hierarchies among all people at the school. For instance, the ability of grouping students with those of other classes, or grades would speak to its molecular potential. This could occur through some special project arranged for the

students, or simply by itself if some students from different grades sneak off to smoke together in secret. A simple example of when classroom organization was more molecular might have been in the one-room schoolhouse where students of different ages worked on various subjects together. Also, this example points to the significant amount of energy that went into maintaining the tension between these two impasses. I would say that if people were to stop attending the meetings that decided on the groupings at La Borde, or meetings started to get canceled more often for whatever reason then this would be a source of decay or collapse of the molecular potential.

Machine Against Structure

> In 1969, when Guattari addressed the Freudian School of Paris, he had already rejected Lacan's tendencies toward formalism and logic. His topic that day was "Machine and Structure," but it might just as well have been entitled "Machine Against Structure." He was no longer the master's designated successor.
>
> - Dosse, 2011, p. 223

I found Guattari's concept of machine interesting, and it influenced my understanding of assemblage. Like the comparison between ethology and neurobiology in the "Dogs and Wolves" fiction, I would like to contrast two domains of work to show the significant difference between machines and structures in table 2. I invite the reader not to judge these concepts to look for the best one, but to observe how they function; what claims are seen as legitimate to each. The first extract is from radical constructivism in mathematics education, adapted from von Glasersfeld by Steffe and Olive (2009, pp. 21–24). It takes an infant sucking on its mother's breast as an example of the concept of a scheme, namely the sucking *scheme*. In the second extract, Bogue (1989, p. 91) takes the same example to discuss the sucking *machine*. The difference in the type

of statements that are seen as legitimate regarding these respective fields of inquiry might seem of little consequence initially, but I discuss how the concept of machine may be a significant blow to structuralism.

Application to this study: an example of the machines of an assemblage. Table 2 shows two different accounts of the same phenomena. Each engaging the phenomena with its own set of conceptual or theoretical tools. I ask the reader to consider the difference in the legitimacy of claims made in each (e.g., scheme theory looks at activity, goals, and schemes, whereas the machine concept emphasizes feedback loops or fluxes and couplings).

The difference may not be obvious, but I argue there is a significant change in the unit of analysis. I would describe scheme theory roughly as a way to create a theory of a representational model of cognitive functioning. The unit of analysis has the child's mind at the center of inquiry with theorizing tending towards mapping phenomena onto the purely cognitive. I argue that the machine concept differs starkly with this orientation and affords one a glimpse into different ways of thinking about and interacting with things. I would say that the machine concept allows for a shifting unit of analysis and that flows and fluxes are key to its consistency. In this example, attention is given to the couplings that produced flows of matter, energy, and information. These flows have a very real, material footprint and do not map back onto a model of cognition or deep structure.

Table 2

Comparisons of two different concepts used to discuss breastfeeding

The sucking scheme Steffe and Olive (2009, pp. 21–24)	Anti-Oedipus'(1983) sucking machine Bogue (1989, p. 91)
One may regard the activity of the sucking scheme as being involved in assimilating objects in that case where the sucking action is driven by a sensation of hunger rather than by some sensory experience like touching the infant's cheek. In the case of the sensation of hunger, the activity of sucking is activated and the baby searches for something on which to suck, and often it is a part of the baby's hand. Here, the baby establishes a possible situation of the scheme by means of the activity that is driven by the gnawing sensation of hunger.	'Everything is a machine' (pp. 2, 8), a part coupled to a second part, coupled to a third part, and so on, in a binary, connective synthesis, forming chains of machines through which pass flows or fluxes. Every machine 'is related to a continual material flow (hyle [Greek: matter]) that it cuts into', and 'each associative flow must be seen as ideal, an endless flux' (pp. 36, 43–4) or universal continuum of unceasing production. A flow of milk between a breast machine and a mouth machine, or a flow of words between a mouth machine and an ear machine, the fluxes that pass through machines may be actual flows of physical matter, flows of energy, or flows of information (in a very loosely cybernetic sense).

Table 2: Contrasting concepts from radical constructivism and anti-Oedipus

For another example, consider a child diagnosed with selective mutism (i.e. a student who can speak but is unable to do so to a teacher or some group of people). Now, say that this child is able to speak to the teacher while it sits on its mother's lap as she comforts it. Thinking of this with Guattari's machines allows me to make statements about couplings that allow a flow of communication (child-mother-teacher) and couplings that collapse it (child-teacher). I am able to make a legitimate claim regarding the child's function, and I have not speculated or theorized about any underlying or deeper structure or motivations. I did not say that the teacher represented authority and the child reacts in this way to authority due to some event with its father. I only stated what couplings created a flow, or consistency (e.g., call it communicating, in this case). Compare this to the early group theory work. The therapeutic units were able to be arranged and changed with little notice: there was extensive experimentation in trying to create new flows or fluxes for patients. This is not to say that inquiry into possible causes of the selective mutism are

necessarily misguided, but it offers different methods of inquiry and experimentation when contrasted with the traditional analyst-analysand-couch coupling. The following section is an example of the assemblage concept used in thinking about a river.

A River Assemblage

This example is adapted from Manuel DeLanda (2000, p. 38) aimed at bringing together several ideas from the philosophical concept of Deleuze and Guattari (1987) called *assemblage*. Consider a river in some natural setting where a body of water is flowing across some bed of sand, rocks, pebbles, and other sediments. The word *river* does not refer to a specific set of elements as we could easily remove a bucket of water, a pebble, a handful of sand, and it could still be termed river to the satisfaction of observers. Moreover, if you were to look away for a moment and look back, you would be seeing an entirely different population of water particles. More importantly, the word river can be thought of as describing a process that requires the interaction of many heterogeneous elements like the sand and rocks, the geometry of the riverbed, the velocity and amount of water flowing, etc. Moreover, these elements have histories. By that I mean if we look at one of the elements, like the water molecule, it requires a fusion of hydrogen and oxygen, each of which, in turn, requires their fusions of particles in distant stars. These histories allow one to let go of speaking of these parts in generality (e.g., *the* water molecule, *the* river in general), but rather speak of specific populations of water molecules, pebbles, etc. Similarly, the velocity and direction of the water flow is intimately related to specific mountain ranges, plateaus, geological factors, planetary gravitational pull, ocean streams, condensation etc. allowing one to be less concerned with the identity or essence of these individual elements than with the relations among actual populations of interacting things and the affects, or emergent affects, these things are capable of in a given context or arrangement. In

using the word *affect*, I follow Deleuze and Guattari in defining it in the broad sense of power of affecting and being affected (1987, pp. 283 – 284).

An example of an emergent affect or capability in this river is how a single drop of water itself is unable to move a pebble, but the collection of water drops in the larger body of water can move the pebble easily. These collective affects and capabilities belong to the assemblage of elements working together to create a whole that emerges as greater than the sum of its parts (e.g., if there were 1000 rocks in the river, then a rock removed from the river would not contain one 1000^{th} of the river in its isolation, it would need to be part of the *interaction* to be part of the river). This arrangement further allows for types of emergent behavior like the water moving larger pebbles more slowly than it moves smaller pebbles and both moving far more slowly than grains of sand or dust. These parts, in turn, can cause friction for the water molecules which changes their behavior slightly in return. Furthermore, big pebbles are sorted out from smaller ones as they create a new formation made up of similar sized pieces clustering together and further cemented together over time by substances dissolved in the water that penetrate the pores between the pebbles. The pebbles can be thought of as being transported or selected out of their territories (deterritorialized) and organized into new territories that again affect other processes in return. Now it is possible to think of this arrangement not as a river in general, but rather as various specific populations of elements intermingling in such a way that we might call it a *rivering* of these elements and processes, or a *river-machine*. These processes can then be said to select, organize, and alter parts of itself, and so changing its own composition (e.g., the sedimentary rock bed created by the pebbles), in turn giving rise to different behaviors or potentials. These processes can then be further accelerated or hindered during times of flooding or draught. In this manner of speaking this rivering arrangement is undoing, or disassembling,

parts of itself while simultaneously assembling other parts: it is simultaneously an assemblage of elements as well as the act of assembling and disassembling.

In Closing

These examples and fictions are offered as context for readers to hold onto during the dense discussion of theory and philosophy to follow. The choice of this format was influenced by discussions with people from many different fields, and I have found it to be useful in communicating across several disciplines. However, I ask the reader to keep in mind that once concepts and ideas are coordinated across disciplines, there is no longer any one idea that stays primary throughout. And, even its own system of measurement might not be adequate in engaging with and describing these new interactions.

CHAPTER 2

PART III: METAMODEL COMPONENTS AND CONNECTIONS

I was struck by Félix Guattari's comment in an interview about being a thief of ideas (Guattari, 1980). It appealed to me because I was fascinated by emergence, especially after learning about swarm intelligence in computer science and also Deleuze's concept of difference-in-itself, unsubordinated to a primary concept of identity. These idea influenced my thinking and made me wonder about what an educational setting might look like that allows for some degrees of self-organization or even decentralization, but I was also wary of how concepts from scientific or mathematical disciplines are often imported into social sciences but end up ultimately being little more than a metaphor, a kind of "exotic pet" (Massumi, 2002, p. 19). The concept of assemblage, in coordination with other concepts, from *A Thousand Plateaus* is useful to me in coordinating this patchwork, or bricolage, of ideas – indeed, more so than simply a metaphor. This section has two parts: I will firstly highlight features of assemblage related to emergence, and thresholds; then, I will discuss how this concept guides the decisions I make in selecting and combining theories and concepts from other disciplines.

The Metamodel for This Study

Figure 3 is an attempt to show how concepts and theory from various fields are coordinated into this specific metamodel. The reason that I am calling this a metamodel rather than a model is that several of these components could be referred to as models in their own right, and so this is better described as a model of models, or, a metamodel. The most significant components worth mentioning initially might be stated as follows:

- From Deleuze and Guattari's work, I use a concept called assemblage, as used in their collaboration on the book *A Thousand Plateaus: Capitalism and Schizophrenia* (1987).

- From complexity theory, I use a specific kind of agent-based modeling called swarm intelligence (Eberhart & Shi, 2011).
- From mathematics education literature, I make use of the five practices (Stein et al., 2008).
- From educational literature, I make use of status interventions (E. G. Cohen, Lotan, Scarloss, & Arellano, 1999).
- From technological resources, I selected the Google Form and customized it to cater to needs of the design principles.

This section will discuss the first three of these components in this chapter, leaving the fourth component, namely the customized Google Form, to the next chapter. This section also finishes with connections made between these diverse fields using the words intensity, affect, and emergence. The discussion is sequenced to finish with the densest work: starting with mathematics education, then complexity theory, and finally the work of Deleuze and Guattari. Briefly stated, swarm intelligence provides an instance of using policies to harness increased variation (e.g., emergence) and vocabulary (e.g., policies, variable movement) used to discuss important interactions investigated in this study. This interaction works from two directions: at one end the Student Response Form prompts students for open responses which amplify or increases diversity and student responses (by counterexample, I take a multiple choice problem as a tool used to *decrease* diversity among student responses). Moreover, on the other end, the teacher's guide suggests that teachers consider practices such as status interventions, and the five practices for orchestrating discussions, to harness this increased diversity into more worthwhile classroom interaction. This is, of course, only suggested literature for teachers as they were welcome to use the Student Response Form and the resulting responses in any way they liked.

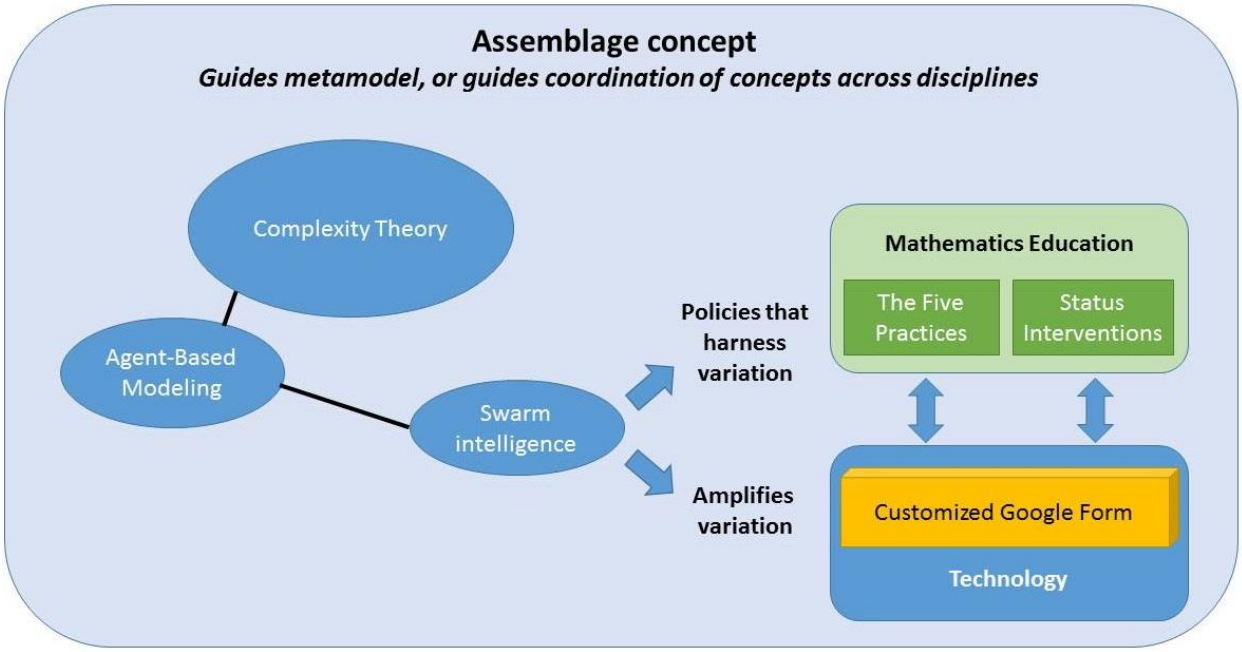

Figure 3. A metamodel using the assemblage concept

Components from Education Literature

The two main theoretical contributions are referred to as status interventions and the five practices. Both of these will be elaborated on in this section and what is meant by *open responses* is also discussed in more detail.

Status Interventions

The word status is used to refer to a student's social standing among their peers in a classroom in a given context. A status intervention broadly relates to the act of improving the students' social standings and so potentially empowering them to participate in and contribute to the class more easily. A specific example is quoted here from Shulman, Lotan, and Whitcomb (1998, pp. 39-40):

> After [the] conversation, I decided I needed to pay extra attention to the group's interaction. I began noticing that most often when Sam tried to explain something, the others would have a difficult time understanding him. Over and over again Sam would

> repeat, "no, no, I mean ...," but the others [invariably] lost patience and simply left him out of the conversation. It appeared that Sam's inability to be understood lowered his status in the group. Over time, a "pecking order" emerged and Sam was at the bottom.

This kind of dilemma found in group work is discussed in more detail elsewhere (E. G. Cohen, 2014; E. G. Cohen & Lotan, 1997; E. G. Cohen et al., 1999). Theoretically, I would refer to this pecking order as a state of the group becoming more inert, or rigidifying, and the act of leaving him out of the conversation as a source of collapse. Concerning the technology, I state that the Student Response Form was able to afford a teacher more opportunities, through receiving more responses from a student by sidestepping some of these sources of collapse. And to incorporate that student's contribution into discussions or simply opportunities for the teacher to acknowledge their contribution and raise the social status of the student in the classroom.

The Five Practices

From mathematics education literature, Smith et al. (2009) describe five practices that aid teachers in facilitating whole class discussions. This model was published in a practitioner journal and has the central thesis that conceptual understanding can be promoted by having students engage in higher level cognitive demand tasks (ibid. p. 548). Importantly, these kinds of tasks can elicit diverse student responses which can effectively be managed by using the five practices called anticipating, monitoring, selecting, sequencing, and connecting. Theoretically, I would refer to these practices as a policy intended to harness diverse student responses into what these authors refer to as productive discussions. Concerning the study, these practices are used in two ways: firstly, the teachers were encouraged to consider using these practices to deal with the increased diversity in responses through the teacher guide. Secondly, these practices were used

to frame part of the discussion on how teachers perceived their practice to be influenced by using the Student Response Form. Each of these practices are discussed in more detail here.

Anticipation. Anticipating refers to the practice of thinking about what diverse responses the particular students of a teacher might come up with when engaging in some mathematical task; essentially, this teacher will try to think of as many ways to solve a particular task as possible. By using the Student Response Form, the teacher has access to student thinking before a class discussion and has a sense of the diverse responses.

Monitoring. Monitoring refers primarily to the practice of paying attention to, and likely documenting, student solution strategies as they are working on a task. During this monitoring process, the teacher may be reconciling the student strategies with her anticipated responses and aligning it with the mathematical goals of the lesson; monitoring also includes questioning students to understand their strategies better and also to clarify their thinking.

Selecting. The results from the monitoring practice now allow the teacher to pick particular strategies or ideas from what she has observed by engaging with students and aligning it with the mathematical goals of the lesson. The teacher will likely select a subset of the entries that she has made by determining which are perhaps representative of several students' thinking (this can also include incorrect solutions or common misconceptions) and which are most aligned to her goals for this lesson. The Student Response Form can display many responses in spreadsheet format allowing the teacher to select from a larger variety of responses more easily than calling on students individually.

Sequencing. The sequencing practice draws on that set of selected entries and orders them according to some particular purpose. The authors propose presenting a strategy of the majority of students first before bringing forward a strategy that was used by only a few in

attempting to validate and engage as many students as possible; another way of sequencing student strategies might be from strategies that are incorrect followed by correct but partial strategies. The Student Response Form is customized to prompt for open responses aimed at better understanding a student's particular strategy or finding out where they got stuck, if at all.

Connecting. Finally, the teacher makes use of the various student solutions by drawing connections not only between the solutions but also with the principal mathematical ideas of the lesson; now, students have the opportunity to compare solutions and strategies gaining access not only to various ways of approaching a problem but also ways in which to discern and develop understanding of characteristics like accuracy or efficiency of some particular strategy.

Open Responses

The phrase open responses was chosen deliberately to emphasize some potential that this form may offer beyond the mere digitizing of student homework, which would only be an optimization of existing practices. The form allows students to engage in a type of asynchronous dialogue with a teacher that is not assessed and graded, as might have been the case with homework or classwork. For instance, in addition to students completing a homework task and having it graded the following day, a student may be encouraged to use the form to report how they made sense of a problem, which items or tasks they have particular difficulty with, what they have considered trying and perhaps their reasoning for it. These are only some examples of how a student might be encouraged to make use of the form and may, of course, simply use it to capture their solutions to some posed problems. Teachers are encouraged to emphasize the fact that students may use the form to voice their thinking knowing that their response will not be graded or influence their grade in any way. In addition, it allows them to share their thoughts

without fear of, say, peer ridicule or other factors that might normally inhibit more open communication.

A distinction can also be made between two different kinds of open responses based on the time of their submission: during class time, or out-of-class time. The distinction is made because responses submitted outside of class time require less coordination by the teacher and the students as to when and what kind of responses are required. For instance, responses required during homework time can be completed by the students at their pace and on their initiative. Whereas responses required during class time would require giving students time to think about the prompt that has been posed, giving them time to type their response, and making time for the teacher to read through and make decisions regarding the collective responses.

This concludes the discussion on mathematics education literature that was most influential in the metamodel. The next section discusses complexity theory, agent-based modeling as an instance of complexity theory, and swarm intelligence as an instance of agent-based modeling.

Complexity Theory

A discussion of complexity theory is offered to the reader for various reasons of which I would like to highlight two. First, complexity theory can "be read profitably" (Massumi, 1995, p. 226) along with work of Deleuze and Guattari, like *A Thousand Plateaus* (1987), by drawing on a wealth of examples and vocabulary that offer the reader entry points into this dense philosophical work. Second, complexity theory serves as a fertile ground to contrast its methods, concepts, and limitations with those of conventional qualitative and quantitative research. As an aside, I use the phrase complexity theory whereas others might use the phrases like complexity science and complex systems. That is not to say that I hold these terms to be interchangeable, I

simply find the phrase “complexity theory” to suffice in referring broadly to complex phenomena.

Simple, Complicated, and Complex

Figure 4 gives three examples taken from Bleakley and Cleland (2015, p. 82) to help distinguish complex problems from simple problems and complicated problems respectively. Briefly defined, using Johnson (2015, pp. 150–151), if a system is considered to be simple then it might be investigated with assumptions of limited interaction among variables, limited feedback loops, few variables, centralized decision-making, and the ability to predict and control outcomes with a high success rate. If a system is considered to be complicated, it might be investigated with assumptions of more elaborate interconnecting parts that make up a whole. Also, the ability to predict is challenging at the level of an agent, for example, a person, but macro behaviors can be described using statistical, probabilistic, or qualitative means. This kind of investigation was also a hallmark of policy research in the 20th century.

Following a recipe	Sending a Rocket to the Moon	Raising a child
• The recipe is essential • Recipes are tested to assure easy application • No particular expertise is required. But cooking expertise increases success rate • Recipes produce standardized products • The best recipes give good results every time • Optimistic approach to problem possible	• Formulae are critical and necessary • Sending one rocket increases assurance that the next will be OK • High levels of expertise in a variety of fields are necessary for success • Rockets are similar in critical ways • There is a high degree of certainty of outcome • Optimistic approach to problem possible	• Formulae have a limited application • Raising one child provides experience but no assurance of success with the next • Expertise can contribute but is neither necessary nor sufficient to assure success • Every child is unique and must be understood as an individual • Uncertainty of outcome remains • Optimistic approach to problem possible

Figure 4. Simple, complicated, and complex examples (Bleakley & Cleland, 2015)

A complex system differs critically from both the simple and the complicated. Defining complexity is challenging because context plays a crucial role and may span a variety of disciplines depending on the problems it is used to engage with (Hillier, 2012, p. 39). These various theories of complexity are usually defined regarding properties with definitional work

done for each property as influenced by the disciplines and context involved. I offer several examples to the reader: Figure 5 shows a comparison of traditional research with research informed by complexity. Alluding to the prior comment on efficiency versus robustness in traffic jams, notice how optimization is not stated as a defining property of complexity theory. Additionally, Figure 6 offers several overarching properties listed by Cilliers (1998, p. 257) in describing a "view from complexity." And Figure 7 offers three examples from different fields of research where the authors state the principles in their definition of complexity. The purpose of Figure 7 is to show how different properties of complexity are coordinated in response to the problems that they deal with.

Traditional Research	**Complexity**
• Reduction • Repeatable on same system • Precise • Static • Optimizing • Homogeneous agents	• Whole system • Rerun computer simulations • Flexible, versatile • Dynamic, process, networked • Adapting, emergence • Heterogeneous agents

Figure 5. Comparing traditional research with research informed by complexity

Twelve properties of complex systems
1. Complex systems are open systems. 2. They operate under conditions not at equilibrium. 3. Complex systems consist of many components. The components themselves are often simple (or can be treated as such). 4. The output of components is a function of their inputs. At least some of these functions must be non-linear. 5. The state of the system is determined by the values of the inputs and outputs. 6. Interactions are defined by actual input–output relations and they are dynamic (the strength of the interactions change over time). 7. Components on average interact with many others. There are often multiple routes possible between components, mediated in different ways. 8. Some sequences of interaction will provide feedback routes, whether long or short. 9. Complex systems display [behavior] that results from the interaction between components and not from characteristics inherent to the components themselves. This is sometimes called emergence. 10. Asymmetrical structure (temporal, spatial and functional organization) is developed, maintained and adapted in complex systems through internal dynamic processes. Structure is maintained even though the components themselves are exchanged or renewed. 11. Complex systems display [behavior] over a divergent range of timescales. This is necessary in order for the system to cope with its environment. It must adapt to changes in the environment quickly, but it can only sustain itself if at least part of the system changes at a slower rate than changes in the environment. This part can be seen as the 'memory' of the system. 12. More than one description of a complex system is possible. Different descriptions will decompose the system in different ways. Different descriptions may also have different degrees of complexity.

Figure 6. Twelve properties of complex systems (Cilliers, 1998, p. 257)

Healthcare education research Bleakley and Cleland (2015)	**Policy research** Mitleton-Kelly (2015)	**Mathematics education: Learning theory** Davis and Simmt (2003, p. 147)
1. Adaptation through change 2. Emergent properties 3. Butterfly effect: small changes can produce big results 4. Attractors 5. Nested and interacting systems and fuzzy boundaries	1. Interconnectivity 2. Interdependence 3. Feedback 4. Emergence 5. Self-organization 6. Exploration of the space of possibilities 7. Co-evolution 8. Historicity 9. Far-from-equilibrium 10. Creation of New Order	1. Internal diversity 2. Redundancy 3. Decentralized control 4. Organized randomness 5. Neighbor interactions

Figure 7. Three instances of principles of complexity

By way of example, I view the way in which a teacher and students talk to each other to reach a type of equilibrium, or more specifically, inertia. Inertia would be when the communication channels seem to settle on a few students almost in the same way a marble dropped in a bowl settles at the bottom, or runs in a small, relatively stable path before coming to a halt. I then assume that the events that bring more students into discussions require a destabilization of this inertia similar to nudging the marble in the bowl or swirling the bowl. More specifically, if a teacher calls on a student who does not contribute regularly, then this causes a destabilization, or disequilibrium, of the authority in the classroom by explicitly valuing their contribution in front of all the other students. This event may be marked by a sense of unease or uncertainty as no one, perhaps even including the student called on, knows exactly where this discussion might go. In this way, classroom discussions can be seen to be enhanced by the continuous effort of disturbing these inertia points on whom the conversation settles and thus operates *far from equilibrium*. This dynamic may, in turn, create more opportunities for the *emergence* of new interactions among students or even new norms such as critiquing the reasoning of others or making sense of someone else's strategy. It may also be the case that, similar to the mountain excursion fiction, other events like ridicule, embarrassment, or fighting, are also risked in this disequilibrium. In addition to this example of an application to mathematics education, I offer the "Practice points" of Bleakley and Cleland (2015) in Figure 8 as potentially instructive in experimenting with complexity theory, especially from the viewpoint of research and educational contexts. A key motivation for these authors is to define and introduce core concepts to healthcare education researchers so that they may see "how several methods can be productively combined without losing focus." I emphasize the third and fifth points regarding research questions and more orthodox research respectively.

Five practice points Bleakley and Cleland (2015)
1. When dealing with complex situations, researchers must shift focus from discrete phenomena and activities to interactions/connections between phenomena and activities. 2. Complexity theory, and an understanding of its key features and core concepts, can provide a conceptual framework through which to view and understand such contexts. 3. Complexity theory also informs and shapes research questions and research designs – how studies are conceived, utilized and developed, and how several methods can be productively combined. 4. Complexity of context should not present an obstacle to research, but rather a challenge to meet complex situations through thinking complexity in research designs. 5. Working with complexity may require a shift in thinking for the researcher.

Figure 8. Practice points for complexity-informed research

Agent-Based Modeling as an Instance of Complexity Theory

A branch of complexity theory is known as agent-based modeling. I would like to use one of the simpler, yet quite useful, instances of agent-based modeling to offer the reader a practical example. Crooks (2012, p. 385) makes use of agent-based modeling to investigate emergent structures of cities. He states that "[the] agent-based modeling paradigm provides a mechanism for understanding the effects of interactions of individuals and through such interactions emergent structures develop, both in the social and physical environment of cities." The example that I use is related to a physical environment, and it is called *escape dynamics*. Escape dynamics simulate humans exiting a building (Figure 9). Each one of the yellow dots in Figure 9 is an agent, or actor, which in this case would represent a human. These agents have been programmed to operate on very basic rules of interaction such as the physical consequences of not being able to run through a wall or another agent and perhaps a rule stipulating the decelerating effect of bumping into something. This model can now run thousands of simulations

under various conditions and constraints to find useful (even unexpected) results. As shown in Figure 9, it was found that a circular barrier placed a few feet in front of an exit, can relieve the flow of bodies trying to leave a building as quickly as possible (Astill & Cairney, 2015, pp. 144–145)[31]. Some of the core properties that may be observed include *interaction* among agents, *no centralized control* for policies, the *emergence* of a bottleneck effect, the experimental nature of the simulations and not simply trying to understand the bottleneck effect, but actively trying different – even counterintuitive – scenarios to change the dynamic.

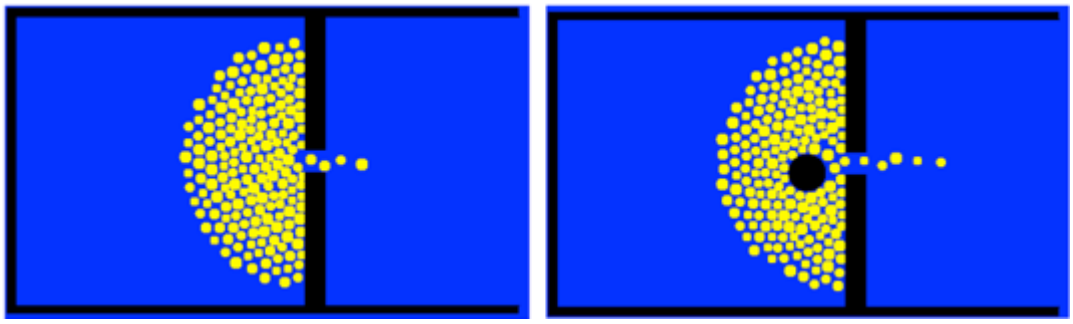

Figure 9. Escape dynamics.

This final example, along with the previous example of swarm intelligence concludes the discussion on complexity theory. The following sections will take Deleuze and Guattari's work as the primary focus and starting point. It is hoped that these examples and discussions will aid the reader through the rest of this document.

The Concept of Assemblage in *A Thousand Plateaus*

To readers familiar with Deleuze and Guattari's work, this section may serve in giving insight into my understanding of the concept of assemblage. And, to readers not familiar with Deleuze and Guattari, it may serve as definitional work while staying close to primary sources.

[31] The original source for this example is Open ABM, and is available at www.openabm.org/book/3138/64-escape-dynamics

The use of assemblage in Gilles Deleuze and Felix Guattari's *A Thousand Plateaus: Capitalism and Schizophrenia* (1987) is described by Zourabichvili (2012) as "vast and indeterminate" (p. 96). This is partly due to references to assemblages being scattered across several chapters (or plateaus) in a patchwork-like manner, and also to the application of this concept to a wide variety of topics including but not limited to geology, literature, evolution, capitalism, painting, tools and weapons, music, architecture, modern science, warfare, and mathematics.

Simply put, an assemblage is a collection of heterogeneous elements somehow functioning together. The authors distinguish between two main kinds of assemblages (1987, p.23): *machinic assemblages* and *collective assemblages of enunciation*. Machinic assemblages refer to the actual content or elements comprising the assemblage (e.g., man-horse-spear assemblage, or judge-court-legislation-constitution-lawyer-accused-prison assemblage); the collective assemblages of enunciation refer to the expressions, acts, or statements that an arrangement allows or makes possible (e.g., the man-horse-spear assemblage has the capability of lancing, the judicial assemblage allows a judge to speak the words "You have been found guilty" which, in turn, transforms the accused's life). I will now discuss both these aspects of assemblage in more detail.

Machinic assemblage (from the French *agencement machinique*) requires consideration of both of these words for a better understanding: *machinique* plays on the word 'machine' in reference to utility or function, but also on word *machin* referring more to the ad hoc nature of its assembly, as in a Rube Goldberg machine, a bricolage, thing, or even whatchamacallit (Bogue, 1989, p. 174). *Agencement* is more difficult to translate as it is literally closer to agency-ing but

often translated as assemblage[32]. Setting translations aside, the key features in addition to those already mentioned are that an assemblage is not only an assembled product but also the process of assembling or the bringing-into-existence of itself (Bogue, 1989, pp. 145–146) – an apparent need or desire to assemble and construct itself (Zourabichvili, 2012, p. 147). Finally, I emphasize that an assemblage is something that changes not only quantitatively if more elements are added to it but that it changes in nature, or qualitatively, as its connections increase or decrease (Deleuze & Guattari, 1987, p. 8); for example, working on a task by yourself has different goals, potentials and capabilities than if you were working on it with another student – the difference is subtle but significant: there are now opportunities for dialogue, negotiation, hybrid strategies, etc.

Uses in *A Thousand Plateaus*

Assemblage is used in a variety of applications throughout *A Thousand Plateaus*. Deleuze and Guattari (1987) are able to use this concept in discussing something as temporary and seemingly inconsequential as an ox plowing a field (p. 399) or something as considerable as a culture or an age (p. 406 – 407). I will give a few brief examples of how the authors use the concept of assemblage, how assemblages regulate or transform themselves, and how elements can move from one assemblage to another.

Take the example of a male wren (p. 323 – 324); this bird makes a significant effort in marking off and preparing his territory. A territory, in this case, refers to the elements and processes that were selected, organized, and displayed in a performance that cordons off a region where it will build its nests. These elements might include flowers, grasses, twigs, or even odors selected by the male along with his performances of chirping, singing, hopping, or posing that

[32] Francois Dosse (2011, p. 527) prefers ‘arrangement’ where many authors use ‘assemblage’.

together begin to create a consistency that holds together his territorial assemblage, or home (*chez moi*) (p. 319, 504). This territorial assemblage has now set the scene for a female wren to approach and show interest. The male, shifting his focus to the female, begins to change his behavior slightly: he is not simply outlining a territory anymore since he must also court the female, and so he changes his posing, dancing, and the intensity of his song. In this way, a territorial assemblage is now opening up to a courtship assemblage which, should it prove successful, will become a social assemblage with its own autonomy. The female wren also moves out of her own assemblages and connects to this new assemblage, changing her own goals, potentials, and capabilities accordingly.

Another example concerns weapons and tools (p. 398 - 404): the authors maintain that one cannot speak of a weapon or a tool before defining the assemblages that they enter into. For a simple example, the difference between a hammer and a war hammer comes not from intrinsic or essential qualities like size, material, or shape but rather the assemblages it enters into, as either one would likely serve equally well in the other's stead (or one hammer could serve equally as well for carpentry as for fighting). In both cases, there is a strong dependence on the organization of work. Take plowing for instance: a human, an animal, and a tool are combined into an assemblage. A farmer might use an ox with a light scratch plow to plow his land if it is a small area, whereas he might prefer a workhorse along with a heavy plow for larger, longer fields – the plowing assemblage is able to disassemble and reassemble itself according to different needs or desires. Interestingly, using assemblages is less concerned with the ability to classify a thing in accordance with internal properties or essential characteristics of, say, a genus or a species, but rather more concerned with the affects it is capable of in a given assemblage. In this case, when part of a plowing assemblage, the workhorse has more in common with the ox

than with the racehorse, even though it shares a species with the latter. Likewise, the Bronze Age saw many weapons come into being and change because of the man-horse assemblage. A soldier was now moving at new speeds and facing new challenges in this assemblage: he needed to protect his horse while fighting foot soldiers and mounted soldiers. This assemblage saw the emergence of the lance – a lengthening of the dagger and pike – in response to new goals and capabilities of the assemblage[33].

Finally, an important characteristic of assemblages is the point at which it either begins again or is forced to restructure itself – this refers to its *limit* and *threshold* respectively (p. 437 – 438); two situations may be provided as examples. The first case considers the alcoholic and the next-to-last, or penultimate, drink that he can have that would allow him to begin again on the next occasion – in this case, he has reached his limit. If he drinks one more drink than his limit allows then he has reached his threshold which forces him to change assemblages such as drinking other kinds of drinks or at different places or times of the day or enter a medical-, hospital-, or even suicidal assemblage. The second situation concerns two groups of people who are in a trading relationship: a farmer-gatherer group that produces seeds and a manufacturing group that produces axes. Each group has a limit point, or penultimate (as in before the ultimate) amount of stock at which the exchange of seeds for axes (or axes for seeds) is still acceptable and allows the exchange to occur and the assemblage to begin again to reproduce stock. If the threshold beyond this penultimate number of axes or seeds is reached, the trade becomes unattractive to either or both of the groups and forces them to modify or change their assemblages (e.g., change the way in which the axes are made or produce something altogether different or engage in trade with another group altogether).

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[33] Similarly, the stirrup enhanced the man-horse assemblage significantly in terms of weaponry and maneuvers as the soldier was able to stand as if grounded while moving at galloping speed.

This concludes the section on assemblage and so all the components of the metamodel. The following section is an attempt to discuss how these components connect with each other within the work of Deleuze and Guattari.

Connecting the Components using Intensity, Affect, and Emergence

In the following quote, Massumi – the English translator of *A Thousand Plateaus* – discusses a connection between Deleuze and Guattari's work with theories like those of complexity. I cite this quotation at length and use some of the vocabulary to connect my use of the assemblage concept, this study's metamodel, mathematics education, and the method of variation mapping (in my limited use of the term):

> It is all a question of emergence, which is precisely the focus of the various science-derived theories that converge around the notion of [self-organization] (the spontaneous production of a level of reality having its own rules of formation and order of connection). Affect or intensity in the present account is akin to what is called a critical point, or a bifurcation point, or singular point, in chaos theory and the theory of dissipative structures. This is the turning point at which a physical system paradoxically embodies multiple and normally mutually exclusive potentials, only one of which is "selected." "Phase space" could be seen as a diagrammatic rendering of the dimension of the virtual. The organization of multiple levels that have different logics and temporal organizations, but are locked in resonance with each other and recapitulate the same event in divergent ways, recalls the fractal ontology and nonlinear causality underlying theories of complexity. (2002, pp. 31–32)

I also follow this example by using the three words intensity, affect, and emergence for my purposes. So, here will follow three sections (named intensity, affect, and emergence

respectively) representing three different approaches on the interaction between the various components of the study.

Intensity

I use the word intensity in two ways. Firstly, intensity describes the amount of energy invested in moving away from a state of equilibrium or inertia (e.g., Predators, like the lion, necessarily maintain their assemblages with spurts of maximum intensity in capturing prey). Secondly, intensity describes the size of the phase space, or potential, of an assemblage (e.g., If a student threw something at a teacher, that moment may be very intense in the sense that it could play out in a myriad of different ways). From a living organism's point of view, inertia naturally tends toward decay and death whereas life is the deliberate movement away from inertia.

Take, for example, a balloon popping at a children's party. The loud noise might result in an entire spectrum of responses such as laughing, crying, curiosity, etc. For the sake of the argument, say that one child hears the loud noise and looks to its parents for cues, crying if they seem frightened or laughing if they seem excited. Regarding intensity, I would say the act of inflating the balloon was an investment of energy and the inflated balloon changed the phase space of the party by allowing for new events like bouncing, floating, and popping compared to a deflated balloon. These events, like popping, can also, in turn, have ricochet effects through the party goers and other objects. On this point, I would also say the question "Is balloon popping good or bad?" is a false problem, or a distraction, due to badly analyzed composites because good and bad are seen as preexisting components of the balloon composite.

Intensity and Mathematics Education: IPI and *Doing Mathematics*.

Consider the following two extracts from mathematics education literature. The first extract is from the Individually Prescribed Instruction (Glaser, 1965), or IPI as it is better known.

The second is from Smith and Stein (1998) and their work on classifying the cognitive load of tasks. These two extracts are used to discuss the two aspects of intensity, as I use the word, in this study. I use Individually Prescribed Instruction, as an example of a low-intensity arrangement. Glaser describes four important aspects of IPI as follows (Fey and Graeber, 2003, p. 542):

(a) analyzing subject-matter content and pupil behavior;

(b) diagnosing each pupil's strengths and weaknesses prior to instruction;

(c) carrying out the instructional process; and

(d) evaluating learning outcomes.

I describe this as a method of classification followed by a corresponding treatment, or instructional process. The low-intensity describes the size of the phase space allowed by the instructional process. Curricular units are broken down into smaller items and administered to a student based on their classification. By and large these items have one solution and a preferred solution path that the student is guided towards by the text (e.g., some arithmetic items might have part of the solution displayed with a dotted line, leading the student to follow suit). In terms of energy invested, significant energy is sidestepped by this arrangement in favor of a high level of autonomy for the student: "It also seems probable that some of this management process can be transferred to the student so that he can practice being a self-resourceful, self-editing learner" (Glaser, 1966, p. 7). I refer to this as a teacher-proofing tendency, which can be seen as an optimization of sorts due to the energy saved in excluding the teacher. From an emergent perspective that values teacher-student interaction, I would rather refer to this as a potential source of collapse or decay.

In contrast, I selected Smith and Stein's fourth[34], and final, level of cognitive demand of tasks called Doing Mathematics. These levels of cognitive demand form the grounds from which the authors advocate the importance of "high-level, cognitively complex tasks if the ultimate goal is to have students develop the capacity to think, reason, and problem solve" (ibid. p.344). Two properties of the fourth level are described as:

- Requires complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- [Requires] considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required" (Smith & Stein, 1998, p. 348).

This level of cognitive demand is contrasted to the instructional assumptions of IPI as high-intensity regarding energy invested and also in its phase space. More energy is required in several ways, but specifically because an item with multiple solution paths would require more preparation for the teacher. The phase space is also enlarged for similar reasons: for instance, the conversations and multiple student pathways may populate a large potential space of interactions and events. Also, the presence of some level of anxiety within students serves to increase rather than decrease this phase space. That being said, this example is not given as the savior and 'good' way to teach mathematics. For instance, anxiety introduces additional complexity: it is not a simple 'more is better' property, as great anxiety may have some short-term or even long-term consequences for a student.

Affect

[34] The first three levels are called: "Memorization", "Procedures without connections to concepts or meaning", and "Procedures with connections to concepts and meaning".

I quote Deleuze at length in the following passage as I found it to be useful in how Deleuze and Guattari speak of affect:

> A distant successor of Spinoza would say: look at the tick, admire that creature; it is defined by three affects, which are all it is capable of as a result of the relationships of which it is composed, nothing but a tri-polar world! Light affects it and it climbs [onto] the end of a branch. The smell of a mammal affects it and it drops down on to it. The hairs get in its way and it looks for a hairless place to burrow under the skin and drink the warm blood. Blind and deaf, the tick has only three affects in the vast forest, and for the rest of the time may sleep for years awaiting the encounter. What power, nevertheless! (Deleuze & Parnet, 2007, p. 60)

In this passage, Deleuze describes the tick in terms of its capabilities or affects. Some reference is also made to the connections it can make, or, combinations it can enter into. This is similar to the work of the ethologist mentioned earlier in discussing the differences between wolves and dogs. Deleuze and Guattari define affect as powers of affecting and powers of being affected. This definition is useful as it is more machinic due to the allusions not only to functions but also couplings and connections. Affect, as I use it, is analogous to an open system (in contrast to a closed system), a process that may interact with any environment, or context, resulting in a large spectrum of variation. Also, I use affect to discuss capabilities within some context as a type of phase space in itself. Consider the example of La Borde's grid where Guattari and various other people would consider the vast possible ways in which people and things can be arranged. This is quite an inversion of institutional arrangement: consider that a classroom with 20 students and

a teacher can be arranged into $2^{21} - 1$, or over two million arrangements[35]. If I add one iPad, a textbook, and a whiteboard, there are over 16 million arrangements possible and every arrangement is qualitatively different from every other.

These open systems can also be seen as processes, or loosely coupled systems (Law, 2004, pp. 141–142) that interact, and may be described in terms of bifurcations, inertia, blockages, etc. In this latter case, the inquiry would be non-reductive and non-representational. In other words, the purpose is not to identify or separate essential qualities and elements, nor does it try to represent some objects of inquiry in the way that a parabolic function might represent an object interacting with earth's gravitational force. The purpose is rather to experiment strategically and map the resulting variation of interactions. To take this even further, a human would not be seen as a discrete entity but rather as intricate and ongoing couplings of machines. For instance, once a human's inextricable connections with the world are acknowledged with concepts like the machines of an assemblage, the stable ontological unit of a person, and concepts like internal and external, are problematized. As Ansell-Pearson explains:

> The matter becomes even more complicated if one takes [Deleuze's position seriously] that the 'human' constitutes not so much a compound form, but is the site for the *transmutation of forces* (p.46) …it is rather the surface of a 'skin' that acts as a membrane which allows for an interior and exterior to take shape and communicate, transporting potentials and regenerating polarities: 'Thus, even biologically, it is necessary to understand that 'the deepest is the skin' (1999, p.86, emphasis added).

Ansell-Pearson's phrase "the deepest is the skin," I read as a strong advocacy for process-based, machinic inquiry. And in this sense – echoing Protevi's previous discussion of open, random

[35] Let one state of the classroom be represented by 10000000000000001000, where a 1 represents a student or teacher being present in an arrangement (there are two people present in this example, person 1 and person 17).

systems – a variation map would not be geared towards making statements at the level of individuals with the assumption that these observations may aggregate into a stable pattern of generalization. But rather, it would make claims of patterns observed at the macro-level behavior about consistencies achieved in some interaction between heterogeneous elements.

Relating Affect and Intensity to this Study

I now connect my use of intensity to the various components of this study. The Student Response Form can be seen as a machine that plugs into a classroom's machinic assemblage and so changes the utterances the students are capable of (e.g., what might be said or typed). Regarding intensity, this Student Response machine may require energy additional to that which the teacher is already investing in the assemblage. This would not be the case where the Student Response Form carries out existing practices more efficiently, as in the case of notecards for instance. Furthermore, the potential range of expression is amplified because students may submit more detailed, or simply different, responses than when elicited through speaking in front of students or directly with the teacher. In this sense, I would say the Student Response Form generates a type of phase space increase. Also, should a student submit a novel response and a teacher incorporate it into her lesson, then this act may open up the range of possible interactions of the classroom assemblage in response to this novelty.

Regarding the concept of assemblage, I use the word affect to refer to the capabilities and potential of some classroom assemblage. This could also be seen regarding content and expression: given this content, what range of expression is possible? And how might these expressions again act back on that specific classroom assemblage? The student is now capable of relaying an open response to a teacher without the rest of the classroom seeing it. A teacher is potentially more capable of eliciting and managing many diverse responses through the

spreadsheet. The student is potentially more capable of expressing some aspect of their thinking while they are not in class, like when they are doing homework. The way in which a teacher makes use of these responses may potentially influence classroom discussions. A student who is simply made aware of this capability might speak up more in class. So when using the word affect, I am interested in capabilities and potential of a specific, given classroom assemblage.

Emergence and Thresholds

Emergence is a consistency that "holds things together" but cannot necessarily be defined in a formalized manner (Deleuze & Guattari, 1987, p. 328). Put another way; it is a departing from common sense notions without giving in to a naïve relativism or an excuse for sloppy scholarship. Emergence requires the definition of concepts, or mechanisms of emergence (DeLanda, 2006), that can continuously operate "far-from-equilibrium" while producing a relatively stable system (Bonta & Protevi, 2004, p. 15). It requires an environment to be parameterized by threshold-based policies (too much…, not enough…, good enough etc.) and interactions rather than ideals, averages, and equilibrium. Assemblages are compatible with ongoing processes of the holding together of heterogeneous elements while allowing for emergence. In this way, there is less emphasis on cause-and-effect relations and more focus on highlighting or parameterizing the elements and interactions of assemblages that improve a statistical probability of emergent features occurring.

Relating emergence to this study. A key interaction and focus of the study are the elicited student responses and how teachers responded to these responses. The theory that most directly influences how this interaction was engaged is swarm intelligence, which is a type of agent-based modeling which in turn can be categorized under complexity theory. However, I argue that, had I only imported swarm intelligence into mathematics education by way of

metaphor, then I would not have been able to account, in any reasonably sophisticated way, for decisions made during this study. The work of Deleuze and Guattari and the later focus on the concept of assemblage aided this inquiry significantly in many aspects including decisions such as those regarding the identification of problems, recommendations made to educators, data collection, data analysis, or the adaptations made throughout the lifetime of the study. In other words, I was able to make nuanced decisions that the capabilities of a single metaphor imported from the hard sciences could not offer.

The metamodel largely describes the interaction between responses and teacher policies to harness those responses. Regarding emergence, I would call this a way of maintaining the tension between the diverse responses problem and the discussion inertia problem. Another way to think of it might be to get unstuck from the inertia problem without letting the diverse response problem overwhelm the teacher's ability to tend to the responses well. Specifically, the complexity theory influence allows a degree of variation in particles while also supplying a policy that harnesses that variance into emergent events that satisfy some criteria. I draw a parallel between this random movement of the particle and the property of the complexity-based learning theory of Davis and Simmt (2003) called *organized randomness*[36]. They describe this organization of allowing some natural diversity into a learning environment while imposing rules that define the boundaries of allowed activity (ibid. p. 154). Specifically, diversity is amplified through allowing students to submit open responses while giving the teacher a tool to manage these responses, and then document how the teachers perceived the use of this tool to influence their practice. Also, while the teachers were able to manage the responses through a spreadsheet, policies were still needed to incorporate these responses into practice. An example of a policy is

[36] As an aside, I contrast my work to those of Davis and Simmt in that I am not explicating a theory of learning but rather using a philosophical concept to guide my method and inquiry.

Smith and Stein's five practices for orchestrating discussions. While this was recommended, teachers were encouraged to incorporate the responses into their practice however they preferred to do so. This tension between variation and imposed policy creates a space where some interactions might emerge. In the following chapter, I will describe how I documented the spectrum of resulting interactions, as reported by the teachers. I refer to this method of inscription as a variation map.

Conclusion

Now, all the main components of the metamodel have been discussed separately. The next section will discuss how these parts are coordinated into a metamodel. This discussion will be framed within what is called New Materialisms, and specific references will be made to mathematics education, the metamodel, and this study throughout. As previously mentioned, the Student Response Form has not been discussed here and will be dealt with at length in Chapter 3.

CHAPTER 2

PART IV: RATIONALE AND METHOD ASSEMBLAGE

> For me, [this] will only be meaningful if it works. Which is to say quite clearly if the different theoretical arguments that I propose here help people.
>
> - Félix Guattari, December 9, 1980

Introduction

This is the final part of this chapter dealing with philosophy and theory. It has two main sections. The first section discusses the New Materialisms, which is a phrase used to refer to work published in the last decade that draws on the work of Deleuze and Guattari in some manner (e.g., Bennett, 2009; Coole & Frost, 2010; De Freitas & Sinclair, 2014; Dolphijn & van der Tuin, 2012; St. Pierre, 2004, 2011, 2016). This section contains a preamble and rationale for why this work may be useful to mathematics education research. The second section attempts to tie together the references made to complexity theory, mathematics education, and assemblages. I would like to state clearly that, once a reader is finished with this part of the chapter, the terms and vocabulary will still *not* have exact claims or mappings onto real world phenomena (e.g., "Equity sticks is a molar mode of organization", or "The discussions in which a student partakes outlines their verbal territory"). This vocabulary and these concepts are applied only in terms of harnessing a tendency, and that tendency may be invoked in any number of contexts and may very well be used in a variety of ways other than those specified here. This last part of the fourth section in Chapter 2 also contains some historical background into Félix Guattari's concepts and their evolution. This was included to show more direct links and examples from one institutionalized environment to others like schools and classrooms.

A Preamble

I would like to discuss what I mean when I use the word materialism, specifically New Materialisms, in prefacing this section. In mathematics education, there is a manipulative called a Unifix cube that is often used by elementary school children in modeling addition, subtraction, division, or multiplication problems. While this cube may have been created for the purpose of teaching mathematics, it does not belong to the field of mathematics education or any other discipline for that matter; mathematics education does not own it. That cube can be drawn in an art class, it can be dropped in a physics class, and it can be melted in a chemistry class, and so on. The material world, like this cube, moves among disciplines blindly and with no pre-established function or significance. Similarly, two researchers with vastly different conceptions of education might very well end up using this manipulative in a similar way or two people with very different ideas about politics might end up voting for the same candidate. I hold mathematics education to be part of this material world or plane. It is as material as that cube, and does not exist prior to it, or above it, or before it. mathematics education is a material-discursive practice that lives in articles, books, manipulatives, discussions, classrooms, utterances, concepts, neurons, sound waves, incomplete thoughts, chatting in a bar at a conference, and so on. It has a material footprint just as real as that of the cube. In fact, the literature generated in journals might rather be seen as the fossil record of mathematics education, far from the cutting edge (Barton, 2005). This preamble is offered to the reader as an example of what might be called a more immanent conceptualization of mathematics education, in that everything is seen as interacting on a flat plane rather than a hierarchy ranging from perhaps the pure to the most applied.

On What the New Materialisms are not, and on Getting Unstuck

To elaborate further on a key aspect of Deleuze and Guattari's work, it is a firm rejection of any transcendent notion or Platonic idea. There is no God's eye-view or universal principle to be sought that explains everything else, but it is rather the idea of a universal, or the "abstract, itself that must be explained" (Deleuze & Guattari, 1994, p. 49). That being said, this is not a phenomenology where a researcher might base qualitative claims on the assumption that a phenomenon can be perfectly preserved in a "careful word-for-word transcription of interviews" (St. Pierre, 2016, p. 6). Nor is it a "quasi-spiritual embrace of the great web of life," forming a "seamless tapestry of existence" (Castellani & Hafferty, 2009, p. 18). Nor is this a break with epistemology as an anti-scientific return to myth (Clark, 1997). This philosophy is not an empiricism that seeks to explain, predict, or generalize (e.g., Schoenfeld, 2002) though this empirical literature may be profitably read to appreciate the "new empiricisms/materialisms" (St. Pierre, 2016). Importantly and in contrast to these ideas, a New Materialism might rather detect, amplify, or harness tendencies (Law, 2004, p. 116). Or, as Adkins (2015, p. 4) puts it: "one [does not] extract timeless truths … one intervenes strategically."

Finally, I think it worth mentioning that it is not to be seen as a harsh criticism[37] for its own sake, for both Deleuze and Guattari often admire those they target [38,39]. These concepts, like assemblage, are ways of problematizing the given, or getting unstuck in our ways of thinking. To tie this to the mountain excursion, *A Thousand Plateaus* offers concepts to think with in order to think something else, or more simply, to get unstuck from some dominant mode of organization. These concepts, however, are non-prescriptive, meaning that it does not recommend what should

[37]Deleuze, (1987, p. 119) writes: "My ideal, when I write about an author, would be to write nothing that could cause him sadness, or if he is dead, that might make him weep in his grave."
[38] Deleuze on Kant: "Admiration leads you to the real critique."(Dosse, 2011, p. 123)
[39] Guattari's nickname while at La Borde was 'Lacan', a main proponent of psychoanalysis (Genosko, 2002, p. 111)

be done once having gotten unstuck. Also, these concepts are useful in attempting to parameterize – or identify and amplify – conditions under which new things may occur, similar to the teachers setting out the rules of the mountain excursion without telling the students exactly what to do.

A Rationale Regarding Mathematics Education Research

In making my argument for the practical use of work and concepts like those of Deleuze and Guattari, I would like to briefly discuss three tendencies in social science research, and so too, of mathematics education research. To be clear, I would never argue for the abolishment of one practice for another, but I do think we may profit from acknowledging limitations and ignorance. These three tendencies concern context (Dreyfus paradox), interactions (Cronbach's hall of mirrors), and incompleteness (Campbell's law).

Context

In the first case of context, I quote Flyvbjerg discussing Hubert Dreyfus' (1929 -) paradox of context:

> If Dreyfus is right he has identified a fundamental paradox for social and political science and social science theory of the kind which imitates the natural sciences, that is, a theory which makes possible explanation and prediction, requires that the concrete context of everyday human activity be excluded, *but this very exclusion of context makes explanation and prediction impossible*. (2001, p.40, emphasis added)

This can also be referred to as the Dreyfus paradox: empirical research favors reductivism in that, if your theory can predict more phenomena with less context, then it is more mature. This is a fine work ethic when engaging the inorganic. However, it is context that often compromises these theories in the social sciences. In this sense, theories and models that tend toward context-

free theory as outputs risk undermining their own endeavors. I will discuss how Deleuze and Guattari's 1987 work relates to context in the historical background section on Félix Guattari.

Interaction

In the case of the second tendency, a similar tendency of generalization suffers in the face of interaction among other variables (e.g., the effect of smaller class sizes on…, the effect of iPads on…, etc.). The interactions among variables (i.e. first-order, second-order, third-order interactions) might always be weakened at the next higher level: "Once we attend to interactions, we enter a hall of mirrors that extends to infinity" (Cronbach, 1975, p. 119)[40]. The problem of interaction will be discussed in more detail in the section dealing with affect. The chief argument I will make concerns the difference between closed and open systems, where the hall-of-mirrors effect belongs to closed systems. The latter, open systems, according to Bonta and Protevi, have self-organizing patterns and non-linear interactions that make it impossible to predict outcomes based on the starting states of the individual elements:

> Open random systems are those in which no patterns emerge in our state space representation; by contrast open, self-organizing systems are those in which patterns do emerge. In this case, we have a qualitative knowledge of the emergent properties of the system, the patterns of the macro-behavior of the system, but no quantitative knowledge of the micro-behavior of the system arrived at by analysis of the actions of the elementary particles of the system followed by aggregation of the results. (2004, p. 19)

Specifically, the concepts of molar and molecular modes of organization are mentioned as a significant blow to reductive tendencies of closed systems (Protevi, 2001).

Incompleteness

[40] Cronbach also comments on how generalizations decay (p. 122) over time due to contextual variations and shifts.

Here Goldstein discusses Campbell's law:

> These trends proved the wisdom of "Campbell's law," the oft-quoted social scientific rule named for the educational psychologist Donald Campbell: "The more any quantitative social indicator is used for social decision making, the more subject it will be to corruption pressures and the more apt it will be to distort and corrupt the social processes it is intended to monitor." (Goldstein, 2014, p. 209)

As an exaggerated example consider the situation where mathematics education research "solved" the teaching of school mathematics once and for all, and all students receive full credit on all tests and assignments[41]. Then universities and companies would still have bottleneck-like problems based on their capacity: Who gets to go to college? Who gets the job? Industry would still have problems of recruitment because of some brand new technology with novel requirements of skills and thinking. And competition among schools would cause improvisations in a variety of ways to differentiate their students from those of other schools.

I mean incompleteness here in the sense that looking for a perfect system of values or final product (e.g., This is the ideal mathematics curriculum/test) might perhaps largely be a distraction. Rather, as is the case with Deleuze and Guattari, we are locked in tensions between irreconcilable, irreducible, and changing forces. We cannot control or predict once and for all, but we can intervene strategically (Adkins, 2015, p. 5). We can rather adjust the ratio of rigidity to fluidity, or stability to change, and engage the consequences (Adkins, 2015, p. 13). Interestingly, Adkins refers to this ratio of stability to change as an assemblage. Put in another way, "we are forever rearranging the furniture" (Deleuze & Guattari, 1987, pp. 21). To say it more directly, the concept of assemblage does not work toward some end state or final product

[41] I would say this argument is somewhat similar to that of "vampires" always returning from the dead in mathematics education (Artigue & Kilpatrick, 2008, p. 5).

but focuses on the functioning and consistencies between thresholds, between beginning states and end states, or between emergence and collapse.

A Potentially Useful Flank for Mathematics Education Research

> In 1905, C.E. Comstock made the following statement about the study of mathematics: "We begin to learn that the real worth of mathematical study is not the general training of the mind, but the training of the mind in mathematical thought, *to the end that our ignorance of the phenomena about us may be lessened, and our impotence in the face of the forces which surround us may be reduced.* (1905, p. 791)

My rationale for using New Materialisms stems from, what I argue to be, a need for a similar statement about mathematics education research and the ability to react and intervene with greater potency in the face of forces that surround us within and outside this research domain. Some examples from mathematics education history include the advent of Sputnik on American curricula, or reactions to world wars, international trade, politics, or simply a Facebook post raging against the Common Core. The same might be said about any number of technologies and movements like the Khan Academy, or students' use of the impressive computational knowledge engine wolframalpha.com, any of which might influence mathematics, mathematics education, and mathematics education research.

The point I would like to make here does not require a lot of real estate in the document. I am not saying that the mathematics education research community is unable to deal with contemporary issues; I am saying that methods of inquiry that focus on "What *is* it?" or "What *should* it be?" differ starkly both in their processes and products or claims when compared to methods of inquiry that focus on "What does it *do*?" or "What *might* it do?" These New Materialisms offer concepts and methods to engage complex phenomena even before they are

perfectly understood. I mean this in the same sense as the argument for agent-based modeling: these methods offer a rigorous set of tools for good solutions today, rather than perfect solutions tomorrow.

Salt for the Constructivist, and the Scope of Guattari's Interests

The final argument that I advance in this rationale is that Deleuze and Guattari's work tends to engage context whereas empirical social scientific research tends to move away from context as explained in the Dreyfus paradox earlier. I oversimplify, for the sake of brevity, in saying that Félix Guattari is different than other scholars I have read due to the amount of context he let into his work. This is what I mean by "letting in context": Should I ask a constructivist about some constructivist theory, say "What is a scheme?" and then hear their response, I could ask something like "What role does salt play in this theory?" I might then cite some relevant literature about a minimum level of salt needed for good cognitive functioning. This is likely to be noted but ultimately dismissed because any number of things could be cited as having an effect whatever theory is under discussion. Or, if this element is let into the theory then surely there are other things that we may also let in, ad infinitum. Compounded with this tendency, runs the hall-of-mirrors problem of analysis when tending to interactions. This is a vital difference between closed systems and open systems: a closed system selects some components from an environment and tends to work towards representational theories about causality or prediction among these elements. I refer to this as empirical work or empirical social scientific research. Other tendencies of empirical work include descriptive power, explanatory power, predictive power, rigor and specificity, replicability, triangulation through multiple "sources of evidence to judge theories and models" in mathematics education (Schoenfeld, 2000, p. 646).

To contrast Guattari's work with this tendency away from context, take the following quote from Watson (2009, p. 32) to appreciate the scope of Guattari's endeavors:

> Guattari … [accounts] for much more than philosophical concepts, and for more than psychic entities. As evidence of the scope of his ambitions and the range of his concerns, I list here examples (some of which he mentions only in passing) that he gives in discussing various aspects of his [work]. I've arranged the examples into categories to make the list more readable: sociopolitical entities (the Versailles court, Capitalism, the Christian and Muslim worlds); the mental phenomena of daily life (dreams, car driving, hallucinations, the ego, Sartre's nausea, the effects of TV-watching on subjectivity); psychotherapy (the La Borde kitchen, a grieving singer who loses her voice, Freud's fort-da game); science and technology (NASA's moon program, TM machines, a hammer, the particle-accelerator, steam engines, thermonuclear weapons, airplanes); nature and the cosmos (organic systems, the Big Bang, species individuation); art and culture (musical compositions, totemic icons, photography, literature, architecture).

The purpose of this quote is to show how Guattari, along with Deleuze, engaged context rather than move away from it: they would consider anything "that came within range" (Deleuze & Guattari, 1987, p. 3) in their work. They considered all of these loosely coupled systems to interact with any others in any combination on a flat plane of immanence, where anything can interact with anything else in any order, to any degree, and at any speed. I contrast this tendency with that of dominant, empirical social scientific research.

A Brief Overview of the Method Assemblage

I use the phrase *method assemblage* to refer to the main components of this type of inquiry (Figure 10) that I propose as a methodology for designing and investigating technological interventions in mathematics education[42]. The five main components were listed in Chapter 1 and are listed again here in the figure below.

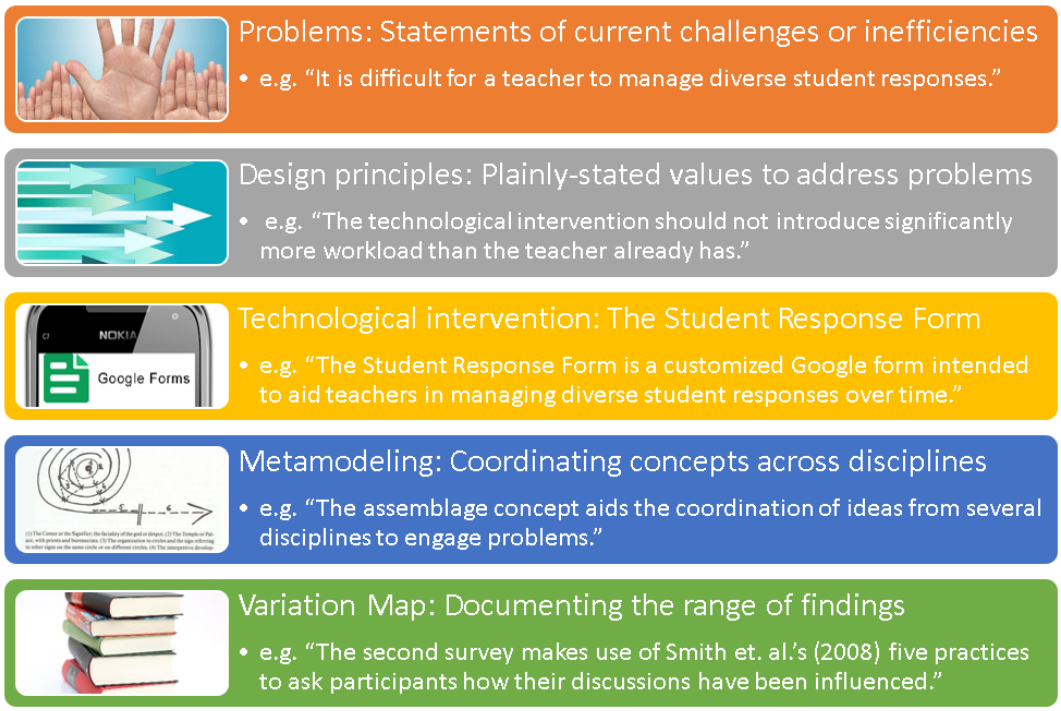


Figure 10. Components of a method assemblage.

Each of these components can, and often did, influence any other component at various times throughout the study. I think allowing this interaction was vital in shaping this study and even in what I ended up calling a method assemblage. The following three figures are shown to recap important aspects of the method.

[42] I took this phrase from Law (2004)

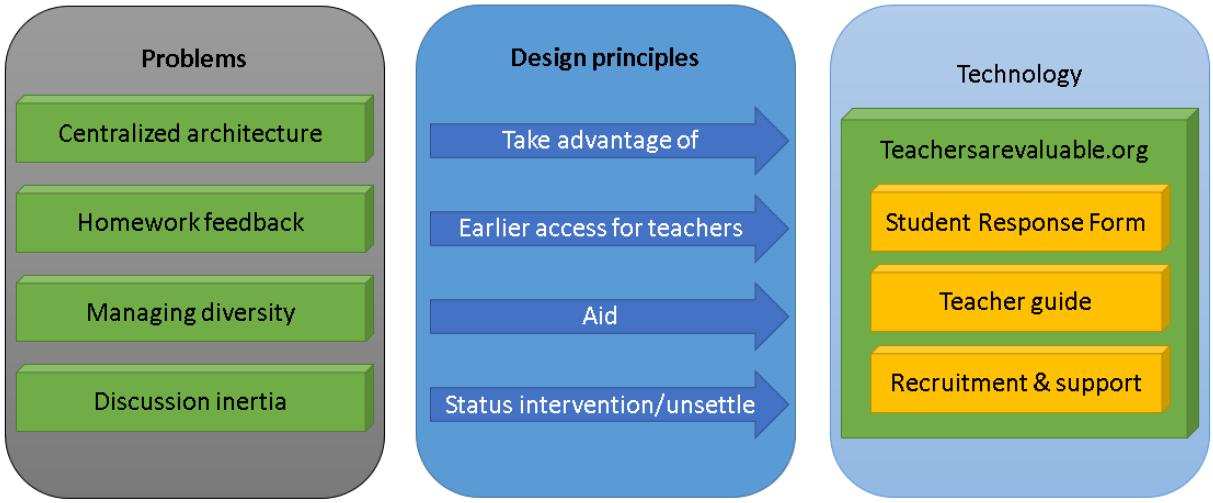


Figure 11. Problems, principles, and intervention.

Figure 11 shows some relationships between the identified problems (e.g., the traffic jam), and the technological intervention (i.e. the Student Response Form, or customized Google Form).

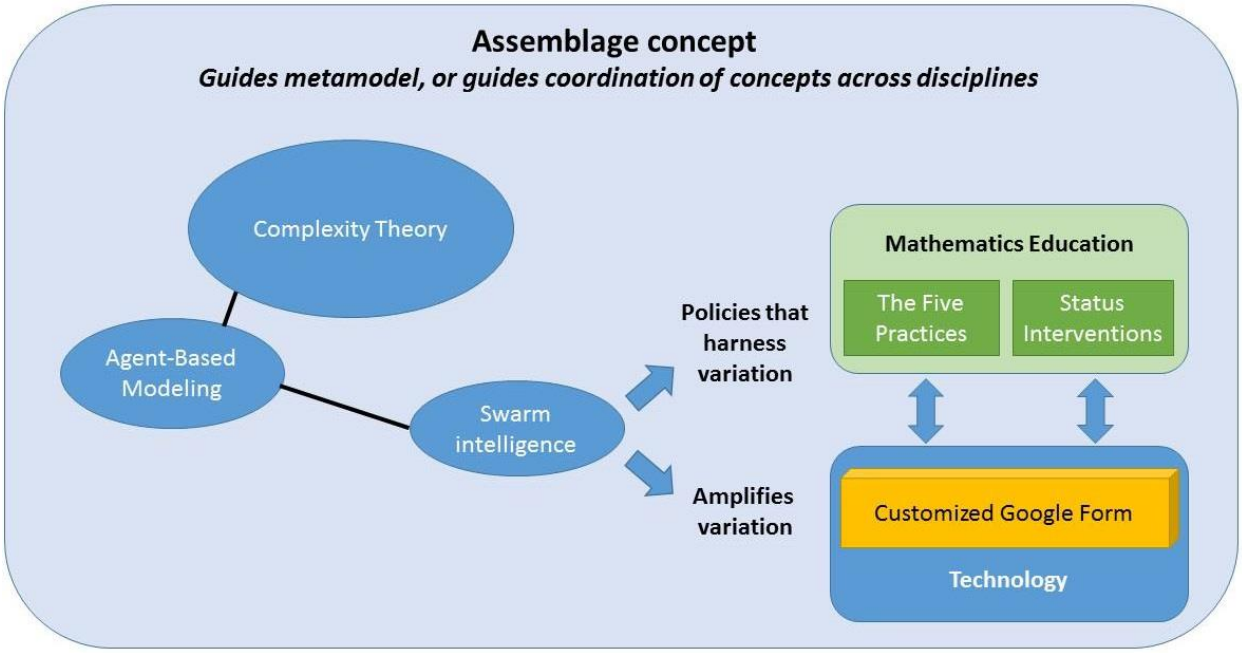


Figure 12. A metamodel for this study.

The metamodel (Figure 12) is a patchwork of concepts and theory coupled together to address problems while keeping design principles in mind. Technically stated, the metamodel is

aimed at inducing a more molecular mode of organization of classroom interaction. This mode depends on not only introducing variability into the classroom assemblage but also recommending, or documenting, the policies implemented to harness the responses into more worthwhile classroom interactions, like discussions.

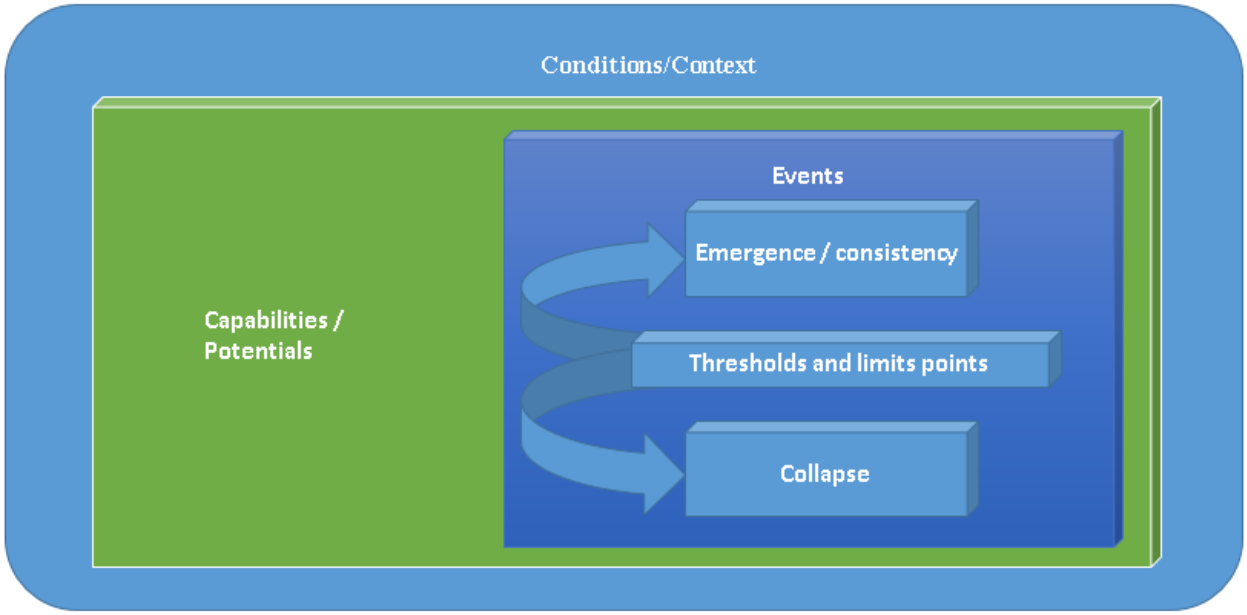


Figure 13. Variation Mapping.

The variation map[43] (Figure 13) attempts to document what happened in the classrooms in a way that is not only consistent with concepts like emergence and collapse, but that also stands as an instance of a process of inquiry that is designed to be iterated (as opposed to replicated), connected (as opposed to isolated), or modified as required. The variation map is also designed to be brief and modular for each classroom. What I mean by iteration, connection, and modification is that another researcher might use the problems, principles, technological intervention, metamodel, and variation map of this study by modifying any, none, or all of the components and iterating the inquiry again. The variation map is tied to all of the components

[43] I chose the phrase "variation map" to acknowledge the influence of Deleuze and Guattari's concept of a map. I use this phrase to indicate that I am trying to document what happened in the classrooms regarding concepts like emergence rather than, say, a rich, qualitative description.

which may give other researchers a sense of the context that brought about this inquiry, as opposed to producing some context-free solution or generalization.

CHAPTER 3

METHOD

This chapter discusses the technological intervention in more detail. The components of the intervention are the Student Response Form, the website www.teachersarevaluable.org, the participant recruitment, and the surveys and interviews. Both the Student Response Form and study evolved significantly over time. The first two sections address this evolution and the resulting version of the Student Response Form. Both sections note rationales for design decisions. The third section details the website teachers used to sign up for the study, generate their Student Response Form, get support, and access surveys and information. The fourth section details how the Student Response Form was designed to scale, and the fifth section overviews participants and their recruitment. The final two sections discuss surveys and interviews, as well as how findings were documented.

Evolution of the Study Over Time

> The original aim of this study was to produce an alternative to the classroom as the dominant unit of analysis for educational settings and educational research. This unit was called a lesson map. It was intended to allow more flexibility in the sequencing and pacing of content, and in the grouping of students, teachers, and curricula. This effort resulted in a customized Google Form similar to the Student Response Form. This original customized form was piloted with three teachers in South Africa. While the teachers responded positively overall, they said that such a form would be more useful to them if it allowed students to give feedback on homework.

This feedback from the teachers was taken seriously and incorporated into the Student Response Form. And, in keeping with the design principle that this study would not optimize an

existing practice alone, the Student Response Form was given additional capabilities. Specifically, the Student Response Form included capabilities of letting students submit open responses. These responses included asking the teacher a question, contributing feedback on assigned homework, and responding to teacher prompts during class time. In addition to the Student Response Form, teachers also received a one-and-a-half-page document called the teacher guide. This teacher guide drew on existing educational literature and provided recommendations on taking advantage of the collected student responses. This guide briefly summarized research from mathematics education literature on promoting mathematical discussions (Chapin & Anderson, 2013; Margaret S. Smith et al., 2009) and complexity instruction (E. G. Cohen & Lotan, 1997).

This final version of the Student Response Form intended to honor teacher needs noted during the pilot study, and to honor design principles like (a) taking advantage of working together in one room, (b) introducing technology that moves beyond old practices, and (c) in being mostly nonprescriptive in its implementation to allow for unexpected outcomes. I think it is this ongoing interaction between study problems, design principles, and teacher problems that allowed the intervention some robustness in honoring both academic and practical needs.

The Student Response Form

The Student Response Form is a customization of an online survey tool called a Google Form[44]. Figure 14 shows an example of a Google Form designed for a Fall camping trip. Anyone with a Google account can create a survey, like the camping trip example, and share it with other people. Teachers may create Google Forms for their classes for purposes of polling, registration, quick feedback, or short essays. The Google Form also allows for a variety of response types

[44] For more information on Google Forms, see https://www.google.com/forms/about/

such as short answer, paragraph, multiple choice, or tick boxes. The Google Form is configured to keep survey responses as part of the Google Form document, or to be captured to a separate spreadsheet. In this study, all student responses were set to be captured to spreadsheets. Although teachers have been using Google Forms for some time, the organization of the spreadsheet is not well-suited to collecting different kinds of responses over weeks or months. For these instances, teachers would often need to create multiple Google Forms.

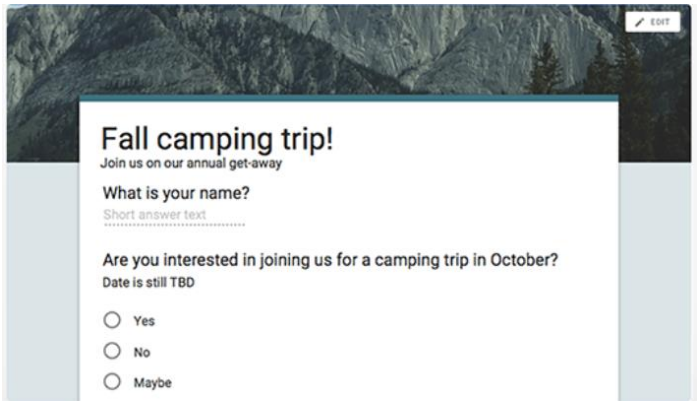


Figure 14. A Google Form.

The customizations made in the Student Response Form allowed teachers to create a single form for several classes with a variety of open response questions (e.g., "How did you decide what information was important in solving this problem?", "Ask the teacher a question."). The two most important customizations were creating a form simple enough for a student to use, and creating a spreadsheet robust enough for the teachers to access various responses over weeks or months.

The first customization required issuing each student a three-digit identification number and storing it in a separate sheet of the spreadsheet alongside that student's name (teachers were

advised on student privacy rights[45]). The reason for this customization is a student only needs to remember a web link and a three-digit number to enter a response. A student does not need to own a Google account, a special application, or even a device to submit a response. A student can access the link from any device with internet access, enter their three-digit number, enter the appropriate date, select a prompt to respond to, enter a response, and click the "Submit" button (see Figure 15). The second customization required adding several extra sheets to the spreadsheet. Due to the first customization, each of these extra sheets was able to look up and display a student's name, rather than an identification number, next to a student's response. Also, each of these extra sheets was sorted according to some rule (e.g., by submission date, by student name, by response type).

Three attributes uniquely determine each submitted response: identification number, date, and response type[46]. And, each spreadsheet can contain tens of thousands of these responses. So, when compared to the traditional Google Form, the Student Response Form lets a teacher manage a variety of student responses more efficiently over months or even years.

[45] Teachers are cautioned to be careful with how they use the spreadsheet and are strongly encouraged to use only first names and the automatically generated ID rather than entering full names and using school assigned IDs especially if the educator is uncertain about the school's policies regarding the use of technologies like Google Forms to store student information. For US educators, the FERPA act is mentioned.

[46] A later iteration of the Student Response Form included the teacher's classes (e.g. "1st Period", "2nd Period", etc.) allowing teachers to sign up all of their students with one Student Response Form. Previously, they needed to create a Student Response Form for each individual class.

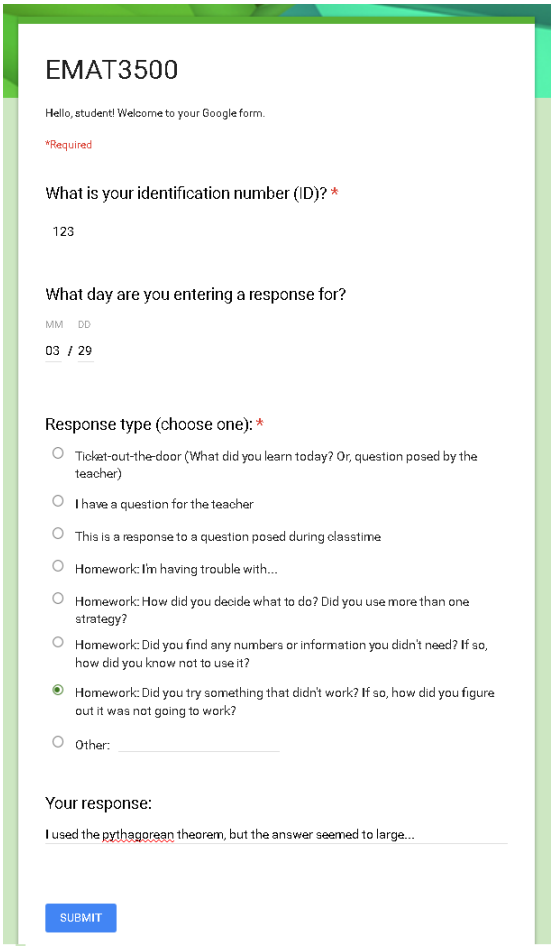
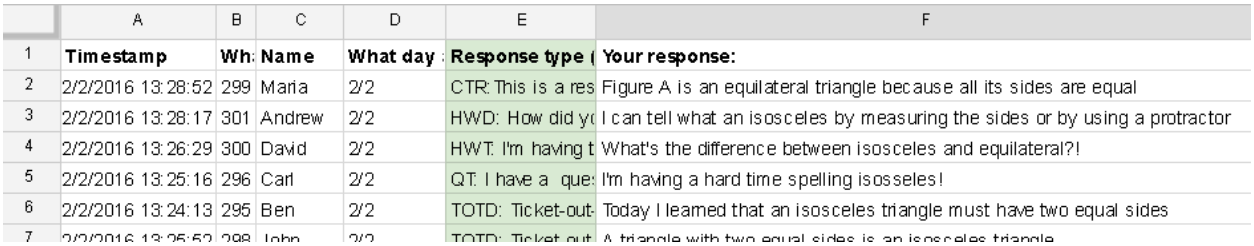


Figure 15. An example of a student completing a Student Response Form.

The Spreadsheet

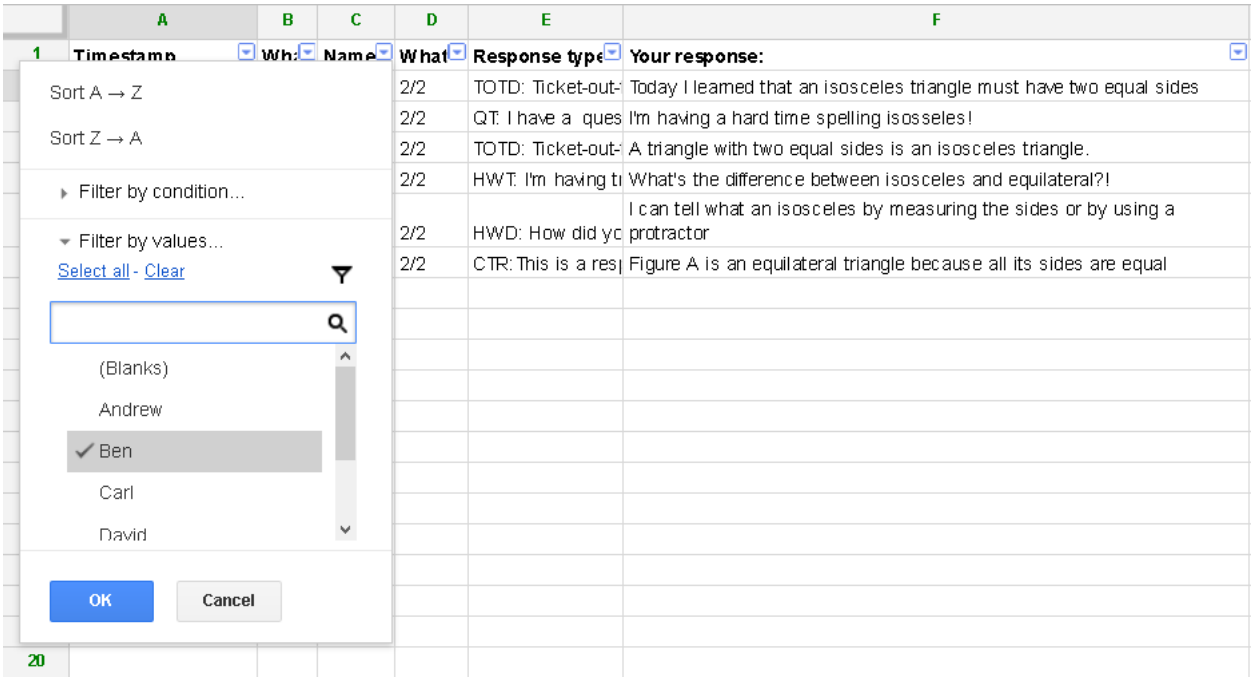
Once a student clicks the submit button on their class' Student Response Form, the response is appended to a spreadsheet in the teacher's Google Drive. Figures 16 and 17 show what this looks like from the teacher's point of view. Figure 16 shows one of the sheets in the spreadsheet sorted by response type. In this case, a teacher might select the "By Response Type" sheet, scroll to the response type "QT: I have a question for the teacher", and see if any students have recently posed questions.



	A	B	C	D	E	F
1	**Timestamp**	**Wh**	**Name**	**What day**	**Response type (**	**Your response:**
2	2/2/2016 13:28:52	299	Maria	2/2	CTR: This is a res	Figure A is an equilateral triangle because all its sides are equal
3	2/2/2016 13:28:17	301	Andrew	2/2	HWD: How did yo	I can tell what an isosceles by measuring the sides or by using a protractor
4	2/2/2016 13:26:29	300	David	2/2	HWT: I'm having t	What's the difference between isosceles and equilateral?!
5	2/2/2016 13:25:16	296	Carl	2/2	QT: I have a ques	I'm having a hard time spelling isosseles!
6	2/2/2016 13:24:13	295	Ben	2/2	TOTD: Ticket-out-	Today I learned that an isosceles triangle must have two equal sides
7	2/2/2016 13:25:52	298	John	2/2	TOTD: Ticket-out-	A triangle with two equal sides is an isosceles triangle.

Figure 16. The spreadsheet as seen by the teacher, ordered by response type.

Similarly, the teacher might select the sheet with the filtering option, shown in Figure 17. Here, the teacher can select any of the categories shown in the columns (e.g., student, identification number, date, and response type) and choose one or several of the values available in that column (e.g., "Ben" in the name column). For example, a teacher might choose to view all the TOTD (i.e., ticket-out-the-door) responses from Ben and Carl. So, a teacher is now able to draw on more student responses, and have more opportunities to notice changes in student thinking over time.



	A	B	C	D	E	F
1	**Timestamp**	**Wh**	**Name**	**What**	**Response type**	**Your response:**
2				2/2	TOTD: Ticket-out-	Today I learned that an isosceles triangle must have two equal sides
3				2/2	QT: I have a ques	I'm having a hard time spelling isosseles!
4				2/2	TOTD: Ticket-out-	A triangle with two equal sides is an isosceles triangle.
5				2/2	HWT: I'm having t	What's the difference between isosceles and equilateral?!
6				2/2	HWD: How did yo	I can tell what an isosceles by measuring the sides or by using a protractor
7				2/2	CTR: This is a res	Figure A is an equilateral triangle because all its sides are equal
20						

Figure 17. Sheet in spreadsheet with filtering option.

A Website for Recruitment, Data Collection, and Support

The centerpiece of the study as far as recruitment, communication, information, and data collection is concerned, is hosted at www.teachersarevaluable.org. This website displays the introductory video on the landing page as well as links for signing up, contacting me, and completing surveys, giving consent for the study, opting out of the study, and finding information (see Figure 18). This online infrastructure offered several advantages: participants can access the website at their convenience, participants can sign up with or without contacting me, and I can view recruitment and survey information as it becomes available in real-time.

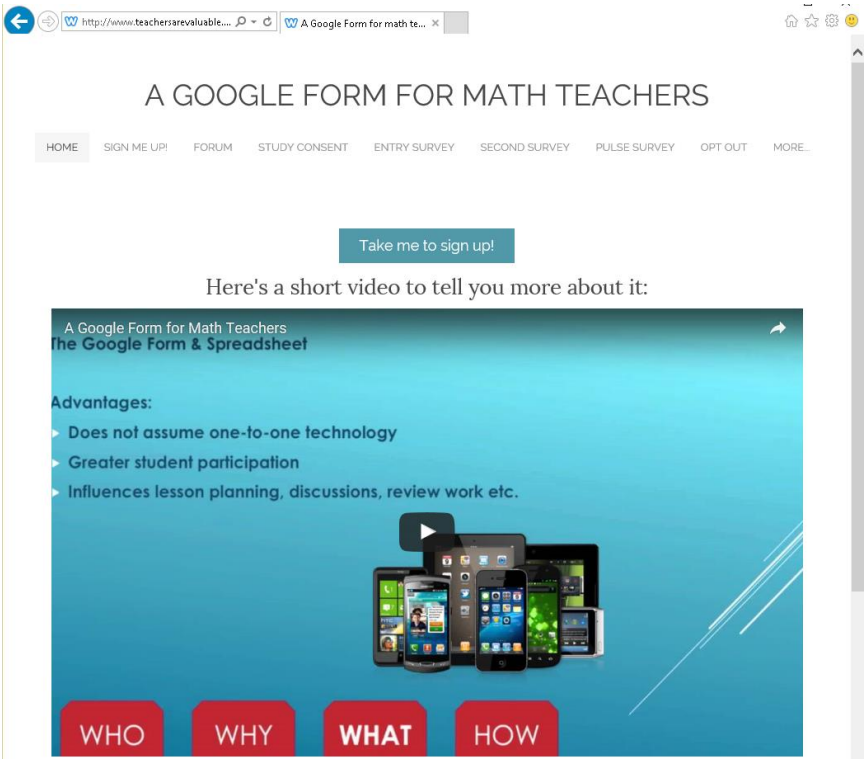


Figure 18. The landing page of teachersarevaluable.org.

The Sign-Up Script and Privacy Concerns

Once a teacher has clicked the "Take me to sign up!" button, the teacher is prompted to authorize a Google script. Figure 19 shows an example of someone using the script. Once the teacher clicks the "Create form" button, the script copies the customized Google Form template,

generates the accompanying spreadsheet, and places these files in the teacher's Google Drive. As the Google script was made available to the public, anyone with a Google account can run it as an "owner". This means not even the original creator of the script is aware of someone else using the script, or able to access the documents created by the script.

There are additional reasons for granting teachers this level of privacy. Should I have wanted access to all student responses, then this would have required getting consent from well over 1000 students in two countries. Moreover, based on pilot information, teachers preferred this lack of interference at least initially. Also, since this study was interested in the teachers' *perceived* influence of the Student Response Form, and not a researcher's *observed* influence. I argue that even with access to student responses, I could not have said what the teacher found useful and integrated into their practice. However, during the interview, I did ask teachers to give examples of student responses they perceived as useful. Should a researcher want to gain access to student responses, then the researcher must obtain student and parent consent, and adhere to information sharing policies of the school, state or province, and country (e.g., FERPA[47] in the US). I also speculate that a teacher might alter their practice, or prompts, if they are aware that a researcher can access their student responses.

[47]Family Educational Rights and Privacy Act available at http://www2.ed.gov/policy/gen/guid/fpco/ferpa/index.html

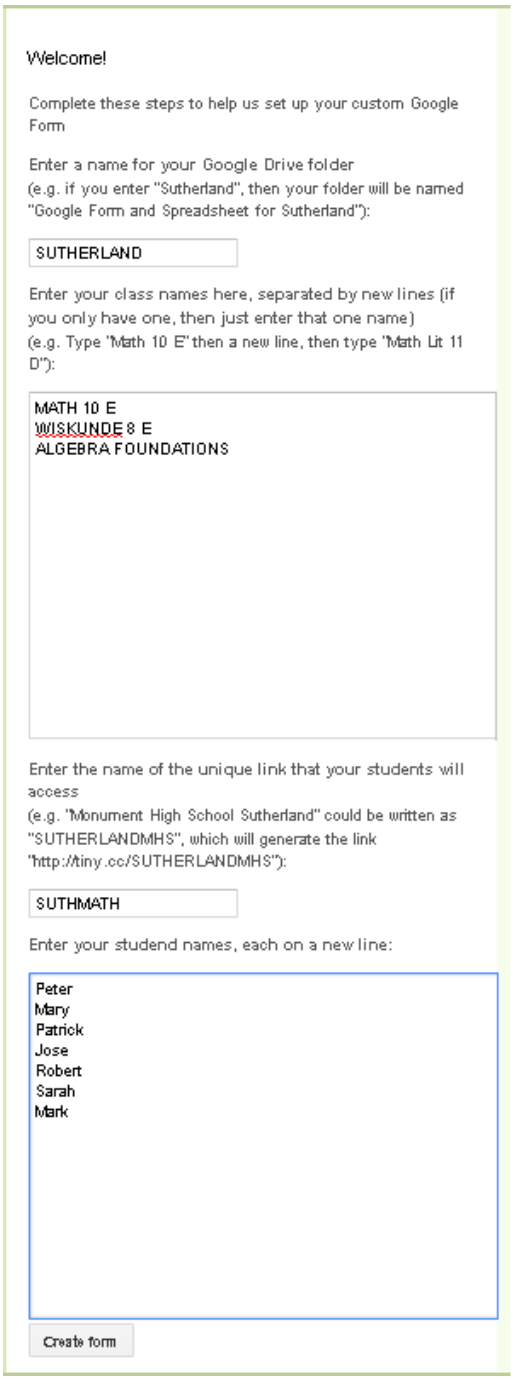


Figure 19. The complete signup process for a teacher.

The Teacher's Google Drive and Assigned Documents

The Google Script creates four documents: A Google Form, its corresponding Google spreadsheet, a teacher guide, and a document with student instructions (see Figure 20). The first

PDF document is the teacher guide with recommended practices for taking advantage of the Student Response Form, and the second document is a "Student Instructions" page generated for each student name entered during the sign-up process.

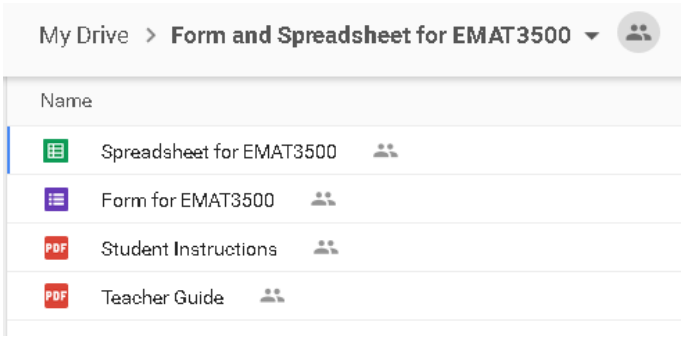


Figure 20. The four files placed in a teacher's Google Drive.

Potential to Scale

The Google script created in this study has potential to scale well. This section gives more technical detail on Google scripting and on the particular script used in this study. Once a person has signed up for a Google account, they are afforded many free services like email, a calendar, file storage, spreadsheets, word documents, etc. A person with a Google account is also able to create and execute Google scripts. Google scripts can automate tasks across these services, and even create web applications. As scripting is a free resource that requires processing power (e.g. a processor on a server owned by Google), restrictions are put in place to avoid exploitation of processing resources. For example, a script with the single function of continuously creating Google spreadsheets and storing that spreadsheet in a Google Drive can create 150 Google documents in one 24-hour period. Creating a Google document is also penalized with a processing delay of about 20 seconds. The Google script that creates the Student Response Form copies runs for about a minute to set up initially, and then no scripting is required afterwards.

Teachers are encouraged to create a new Student Response Form every semester or academic year even though a Student Response Form should last several years. For example, I received roughly 2000 responses from 24 students in four months. Also, a Google Sheet[48] can contain about 2 million cells of data, where one student response requires one cell. This means that I could have used a single Student Response Form for several classes over several years. Added to this large capacity of a Google Sheet, one Google account can create roughly 40 Student Response Forms *per day*. Therefore, one Google account offers enough free processing and storage capacity in one day to span an entire teaching career.

What makes the Google platform particularly attractive is that Google has around 1 billion active users of which the majority use mobile devices. Hypothetically, should all 33 million secondary teachers in the world decide to sign up for a Student Response Form, then they can do so simultaneously, and it should take about a minute. To take this hypothetical further, say that only one student makes use of this form per teacher, then several million students may be more likely to contribute to their classes. In other words, generalizations from ideal implementations or circumstances (e.g., here's the model, make a copy) are not necessary for this study to contribute useful findings to literature. Rather, this intervention quickly enhances the capabilities of actual populations of students and teachers and does so with almost no prescription. Moreover, the findings of this study can make statements regarding the probability, and spectrum, of potential classroom interactions.

Participants

The study originally targeted Grade 10 Mathematics teachers in South Africa. This choice was deliberate due to my experience in this context. Also, the 10th-grade student

[48] https://support.google.com/drive/answer/37603?hl=en

population struck a balance between having reasonable access to technology compared to lower grades and having less curricular pressure compared to higher grades. “Grade 10 Mathematics” refers to both the subjects called Mathematics and Mathematical Literacy. The first course is a college-bound curriculum for potential mathematics majors and the second a mathematical literacy curriculum. The literacy course is compulsory should a student elect not to take, or drop out, of the mathematics course in Grades 10 through 12. The original target population had a significant influence on later iterations of the Student Response Form. Also, pilot work seemed to indicate the Student Response Form was more sensitive to contextual and personal factors more so than any particular grade or content. These early findings indicated that recruitment may benefit from casting a wider net.

Recruitment

Once the decision was made to expand recruitment, the Student Response Form was configured to automate several parts of recruitment and data collection. In fact, any teacher who was able to participate would not be denied access to a copy of this form nor denied access to participate in the study. The initial recruitment focus was on Mathematics, Science, and Technology Coordinators (MST Coordinators) of the South African National Research Foundation (NRF) of each of the nine provinces in South Africa. These coordinators, in turn, contacted various teachers at their discretion. Unfortunately, the target range of 10 to 20 participants did not seem promising in South Africa, and recruitment expanded to the US, specifically the surrounding areas of where I was based in the Southeastern United States.

Overall, teachers were either contacted directly by me or through a mutual contact. The website and introductory video were circulated using social media sites like Facebook, Twitter, LinkedIn, and YouTube. These recruitment attempts were unsuccessful regarding recruiting

teachers outside of one degree of separation from myself. Typically, I contacted personal acquaintances by phone, email, or social media with links to the website and the introductory YouTube video[49]. Generally, a potential participant showed interest or recommended someone else to contact. Potential participants were taken through the recruitment script and if they showed interest I would explain the sign up process, or help them to sign up. Teachers are clearly informed that they may use the Student Response Form as they please and they do not need to participate in the study to use the Student Response Form.

Towards the end of the recruitment period, I was invited to address a professional development group of teachers at one of their meetings. I discussed the Student Response Form, and aided any teachers who were interested in signing up. This latter recruitment was significantly more effective than previous recruiting initiatives. This improved recruitment episode may be due to the evolution of the talking points and the live demonstration that used the teachers as "students." However, I speculate getting the professional developer on board was crucial in granting me the consideration of the teachers.

Script and Recruitment Evolution

The recruitment process itself also evolved over several months in response to the needs of potential participants. For example, I can walk a teacher through the sign-up process over the phone, in person, through a fellow teacher, or by personally setting up a Student Response Form and then transferring ownership to a teacher. In addition, the talking points of the recruitment script evolved to include any useful feedback or comments from other using the Student Response Form. For example, some teachers were interested in managing homework feedback

[49] It would be remiss not to acknowledge that my mother was able to recruit far more teachers than me in South Africa: she reached out to veteran teachers in her book club who were able to recommend teachers at their schools and other schools.

while other teachers were interested in encouraging more students to contribute to the class. Perhaps the most practical tips came from a public high school teacher in Georgia. His feedback included:

- combining several classes into one form;
- having clear goals (e.g., one ticket-out-the-door per week); and
- using the Student Response Form as a "literature connection" for Mathematics.

Feedback from other teachers included:

- adding a prompt to allow the student to ask the teacher a question;
- requiring all students to respond to a simple question during the first use (e.g., "What did you do over spring break?");
- giving a student the responsibility of reminding the teacher to use the form; and
- using students' school-assigned identification numbers in "Google for Education" schools.

In this way, the recruitment process and the Student Response Form continuously integrated teacher feedback while tending to design principles.

Tying the Theory to Participant Selection

Although the participant pool was initially limited to 10^{th}-grade Mathematics teachers in South Africa, it was soon expanded to include a variety of grade levels, courses, locations, and experience. There are two reasons for the expansion: first, the prompts of the Student Response Form are open and encourage students to discuss their thinking, so the prompts are not content specific. Secondly, I suspected that perceived contextual factors beyond specific content or grade level may be crucial in implementing the Student Response Form or not. Additionally, the

relative ease of an automated sign-up and survey process made it easier to recruit teachers in various locations. Therefore, participants were recruited from a variety of contexts ranging:

- middle school, high school, and college teachers;
- small to large class sizes;
- various Mathematics courses;
- various levels of experience;
- various levels of digital literacy; and,
- various degrees of access to technology.

This variety of contexts sustains the variation map in guarding against overgeneralization or over-theorization across similar cases. The variation map aims to make claims regarding context, capabilities, and the thresholds at which events emerge or collapse. For example, during the pilot study, one teacher noted in one of her classes with eight students it takes longer for them to submit responses than to speak to her directly. In this case, the number eight is not as important as the idea that a teacher might perceive some lower bound on class size at which the Student Response Form is not useful to them. Therefore, including participants from a variety of contexts, such as a class with fewer students, helped identify a source of collapse that might not necessarily have been apparent otherwise.

The Twenty Participants

As shown in Figure 21, the recruitment expanded beyond 10^{th}-grade teachers in South Africa. The different content areas were Algebra, Geometry, Calculus, and Mathematics with the latter referring to a South African course with both algebraic and geometric components. Participants 19, 20, and myself taught college-level, mathematics education courses.

	6	7	8	9	10	11	12	1st year	2nd year	3rd year		Algebraic	Geometric	Calculus	Math	College Ed course		USA	SA		Years Teaching	Teaching Math
Participant 1	1	1	1												1			1			5	5
Participant 2		1	1	1	1	1									1			1			5	5
Participant 3			1												1			1			27	27
Participant 4				1								1						1			2	2
Participant 5				1								1						1			9	9
Participant 6				1								1						1			8	9
Participant 7				1								1						1			17	10
Participant 8					1										1			1			17	17
Participant 9					1								1					1			3	3
Participant 10					1		1						1	1					1		7	4
Participant 11					1										1			1			5	5
Participant 12					1							1	1						1		25	25
Participant 13					1										1				1		33	28
Participant 14					1										1			1			3	3
Participant 15						1	1					1							1		14	14
Participant 16						1									1			1			10	10
Participant 17						1									1			1			1	1
Participant 18							1								1			1			13	13
Participant 19									1	1						1			1		3	3
Participant 20									1	1						1			1		5	3
Researcher										1						1		1			6	4
	1	2	3	5	8	4	3	0	2	3		6	3	1	10	1		14	6			

Figure 21. Participant information.

Due to the spectrum of participants, the study can investigate variation in grade level, content, and experience. The decision was made not to include gender or age but rather focus on a participant's overall digital literacy and the extent to which the technology used in the study was novel or not. The rationale for this decision is that this study would rather contribute to the literature regarding capabilities, or level of digital literacy, that may promote the use of the Student Response Form rather than qualitative or quantitative claims on success rate predictors concerning gender or age. Experiences with teachers spanning a range of backgrounds formed the basis of this decision.

Surveys and Interviews

The data collection for the study consists of an entry survey, then a second survey some time later, and potentially an interview based on the information gathered from the surveys

(Appendix B). Briefly put, the survey methods evolved to allow participants to start participating at any time and for any length of time within reasonable confines of the study period. Participants could also give feedback whenever it suited them and in any of several available formats (e.g., email, pulse survey, website contact form, instant messaging, texts, or phone calls). All surveys were administered using Google Forms and are available to participants online to complete at their convenience. The entry survey collected information on a participant's country, subjects and grade level taught, experience, digital literacy, and intent for using the Student Response Form. The second survey was originally called the exit survey and was intended to be taken one month after starting to use the Student Response Form. The design decision was made to rename it to the "second survey" and to allow participants to take it whenever it suited them. Participants were encouraged to complete the survey even if they were not able to get it to work, stopped using it, or used it for less than a month. The decision to remove the one-month timeline supports data analysis because the study is more concerned with specific steps teachers were taking to make the Student Response Form part of their practice rather than, say, the effect of a treatment for the period of a month. The pulse survey was an additional, single-entry survey introduced to be more responsive to teachers who had limited time or wanted to contribute any insights to the study that were not addressed in the surveys.

The second survey targeted both teachers who were able to use the Student Response Form and teachers who were not able to do so. In this latter case, participants were directed to a different part of the survey that focused on the substantial obstacles that mitigated the teacher's efforts in using the form. Among participants who were able to make use of it, a distinction is made between participants who continued to use the Student Response Form and participants who stopped using the Student Response Form after the study. If a participant reported continued

use of the Student Response Form after the study, then this was viewed as a consistency that had been achieved. The participants who were able to use the form are asked how the use of the Student Response Form influenced their practice not only regarding routines but also in their ability to orchestrate discussions as analyzed through the lens of the five practices (Stein et al., 2008). For example, the second survey asked participants to rate the influence of the Student Response Form on their ability to *select* student responses (the third practice of orchestrating discussions) from 1, for *no influence*, to 4, for *strong influence*, and then explain their rating.

A subset of participants was selected for interviews based on the survey data (Appendix A). The purpose of these interviews is to describe each participant's particular context better. Specifically, the interviews were used to discuss:

- teaching routines and practices;
- the practices in using the Student Response Form;
- the frequency of and types of prompts posed;
- the types of responses used; and
- significant factors that helped them integrate this into their practice or make them unable to incorporate it into their practice.

To summarize, the entry, second, and pulse survey was used to select participants for interviews. In other words, the data collection aimed at finding out how teachers intended to use the form (entry survey), how teachers did use the form (second survey), and gathering more detail on selected cases (interview and pulse survey) with a focus on classroom interaction.

Documenting Findings with Variation Mapping

A principal aim of documenting findings in this study is to be descriptive. The survey and interview data were analyzed to document the ways in which teachers made use of the Student

Response Form, inside or outside of class. Classroom interactions across a variety of contexts were a focus on the analysis. In addition, in keeping with the exploratory nature of the study, attention was given to notable influence on the practices of teachers. For instance, a student entering a response while doing homework (out of class) is an event brought about by the Student Response Form, so too is a student entering a response during class time. These types of events create opportunities that may influence planning and instruction. In other words, the Student Response Form both amplified the amount of diverse student responses and aided the management of these responses, and this study documented teachers' perceptions of how this may have influenced their practice in a descriptive and exploratory way.

Linking the Data Collection to the Theory

The main components of the theory that directly drives the documentation of findings are:

- Context (the teacher's routine and environment) and capabilities afforded;
- Emergence, collapse, and thresholds of teacher practices;
- The five practices (to frame the description of student-teacher interaction); and
- Status interventions (Did more students contribute than before?).

Context and capabilities afforded. Variation mapping is interested in new capabilities offered by introducing the Student Response Form into a classroom, and in tying these capabilities directly to that context. For example, one teacher could only afford internet access during certain months and only had access at home. These two restrictions then limit this teacher's capabilities regarding the kind and frequency of prompts that can be assigned and when the responses can be accessed. The context helps frame a teacher's capabilities and also the thresholds or cutoff points at which they may decide for or against some course of action

regarding the Student Response Form. Additionally, teachers were asked about potential capabilities that they perceived the Student Response Form to have. Therefore, variation mapping focused on context as well as perceived current and potential capabilities afforded within that context.

Emergence and thresholds. This section reiterates ideas from Chapter 2 with more direct application to this study. The concept of emergence is used to focus on the perceived influence of the Student Response Form on teacher's practices. Here is one definition of emergence I find applicable (Webb, 2015, pp. 50–51, *emphasis added*):

> At its core, emergence is the idea that the whole is greater than the sum of its parts. More specifically, it explains how it is the *contextually situated interactions* between the parts of the system which generate its character, not the individual nature of the parts themselves.

This definition emphasizes the need to acknowledge context as well as those practices that have emerged but cannot necessarily be traced back to its constituent parts in a cause-and-effect manner. This may refer to a few isolated events (e.g., a discussion or novel student response), or some practice sustained over time (e.g., a ticket-out-the-door every day). For instance, positioning a discussion as an emergent event rather than an effect brings both more emphasis on context and less emphasis on cause-and-effect relationships. For example, even if a teacher knew what every student thought, they still would not know which direction a discussion might take or whether it would even happen at all. This idea of looking beyond constituent components is important to the study because, even if teachers' descriptions of their environments are similar, the emergent practices might differ significantly. Variation mapping, therefore, identifies some practice or practices as having emerged (a consistency was achieved), or not (collapsed or

decayed over time). If some practice did emerge, then I classified this as an expected or unexpected event or outcome.

Regarding the theory, the concept of thresholds describes the point at which assemblages change. Like the tribes as mentioned earlier, that would change a trading assemblage based on some perceived threshold of acceptable seeds to be exchanged for axes. In this study, the Student Response Form is a deliberate investment of effort to unsettle the status quo, or inertia in a classroom with the aim of creating some new consistency, be it anticipated or novel. Thresholds are used to describe the perceived points at which some consistency (new practice or type of interaction) is collapsed, or decays over time, back to the status quo. For example, particular types of responses might be more manageable for small classes than large classes due to the brute number of words a teacher would need to read. Another example might be the perceived point at which a teacher judged a prompt to be more appropriate for one situation rather than another. Other examples of thresholds might be the number of students, the length of responses, the number of replies, fine motor skills, the type of homework assignment, or the length of the class period. These thresholds are relevant not only to calibrate a teacher's use of the Student Response Form but also to describe variance across multiple contexts.

The five practices. The five practices were designed to prescribe five different components that may promote effective orchestration of discussions namely anticipating, monitoring, selecting, sequencing, and connecting. These five words are used in a more general sense to aid participants in describing the perceived influence of the Student Response Form. Figure 22 is an example of a survey item regarding the practice of monitoring. The item asked the participant what influence the Student Response Form had on their ability to monitor student responses.

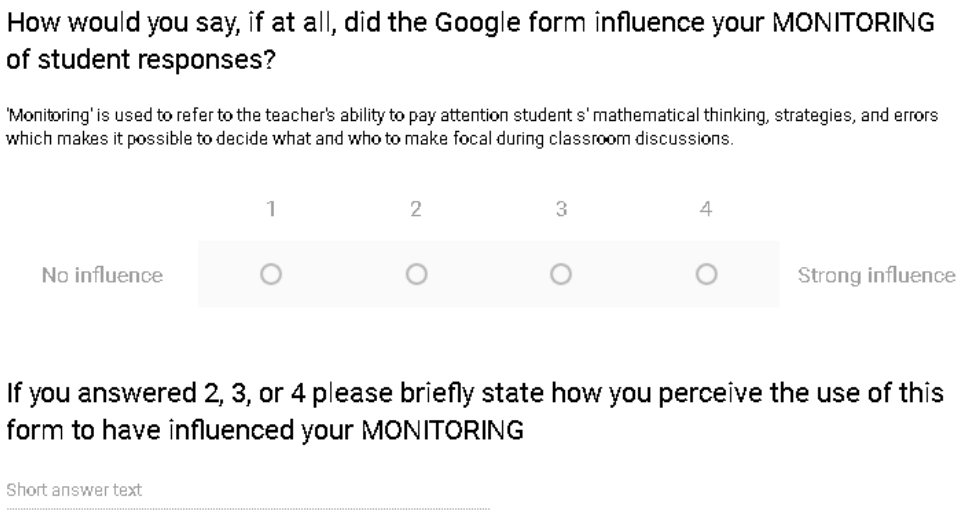


Figure 22. A survey item is asking about the practice of monitoring.

In addition to these prompts in the second survey, the five practices are also used in the interview to frame a discussion of the Student Response Forms influence on classroom interaction.

Points of inertia. As described in Chapter 2, a point of inertia is used to describe how communication in a classroom tends to settle on a few people in a room. This problem was engaged by recommending participants to use student responses for status interventions (E. G. Cohen et al., 1999) with the aim of getting more students involved in lessons and to introduce some disequilibrium to unsettle this point of inertia. The impact of this consideration on the interview protocol is straightforward; participants are asked the following question: *Is there any difference to who contributes to classroom discussions? In other words, many conversations often tend to settle on a few students in the classroom, has this unsettled that dynamic in any way by bringing more students into the conversation?*

Limitations

No data collection is conducted within any classrooms. I was not able to observe how a teacher's perception of the influence of the Student Response Form translates into their actual practices and discussions. Also, as the teacher is the sole owner of the spreadsheet containing

student responses, I only have access to anonymous student responses volunteered by the teacher. I do argue that, even if I had observed every class and read every student response that was submitted, my perception of the Student Response Form's influence might very well differ from the teacher's perceived influence. Finally, as emphasized throughout, the study only reports on teachers' perceived influence of, and capabilities afforded by, the Student Response Form.

CHAPTER 4

CONTEXT

This chapter reports extracts from the surveys and interviews conducted with teachers who used the Student Response Form as part of their practice. This chapter was called context rather than data. This decision was made to emphasize the importance of staying close to the particular circumstances under which events occurred rather than tending to generalize away from it. Of the initial 32 participants that gave consent for the study, seven participants were available for interviews. The eighth case is my own use of the Student Response Form. The reporting of data is framed by the research questions and guiding concepts of variation mapping. Findings are reported on how teachers perceived open student responses, collected using an online survey tool, to influence their practice and classroom interactions. Table 1 gives a brief outline of all eight cases. Participants were given pseudonyms based on the region followed by a letter (e.g., US – A, US – B, US – C, and so on). Each participant's case in the table lists basic information regarding the level, subject, years of experience, the number of classes and students, etc. as well as some unique context or example that is offered to aid the reader in differentiating among these several cases.

Some differences need to be noted regarding the two countries. One noteworthy difference between the students of research participants in South Africa and the United States of America is that the students in South Africa have *capped* data plans. This means that, many students of the participants were without data and so unable to access the internet for prolonged periods. The tendency across the board seems to be that students would use up the bulk of their available data for social communications leaving the teacher with frequent cases of students not able to submit responses due to a lack of data. That being said, students in these cases would

simply use a parent or friend's device or go to a location that has free Wi-Fi, like a McDonald's restaurant. Also, South African public schools do not have middle schools like in the United States of America — elementary schools cater for pre-K through seventh grade and high school for eighth grade through 12th grade. Finally, all the South African teachers chose to respond to the interview questions, in Afrikaans and so the translated transcript extracts, on some occasions, include the original Afrikaans word where a direct English translation did not convey the intended meaning of the teacher as judged by myself, a native speaker of Afrikaans.

Table 3

The eight cases of teachers using the Student Response Form

Case	Country	Level	Subject	Experience in years	Student age	Classes	Students	Responses	Unique context or example
Mr. SA-A	South Africa	High school	Mathematics	28	Grade 10	1	30	3	Moved to another school during study
Mrs. SA-B	South Africa	High School	Mathematics	20	Grade 8	1	23	6	Exponent example
Ms. SA-C	South Africa	High School	Mathematics & Mathematics Literacy	4	Grade 8 & 11	6	161	10	Value-added tax example
Mr. US-A	USA	High School	Advanced Algebra	3	Ages 15 to 18	3	~70	~220	Also used it for baseball athletes
Dr. US-B	USA	Middle and High School	Geometry and Algebra	5	Grades 7 & 8, 9, 10, 11	4	5 - 11 students per class	~200	Had an email practice
Mrs. US-C	USA	High School	Analytic Geometry	17	Grade 10	2	21 and 24 students	~500	Clearest and muddiest point
Dr. US-D	USA	College	Content course for pre-service teachers	5	3rd year	1	10 students	~170	Office hours problem
Researcher	USA	College	Methods course for pre-service teachers	7	2nd & 3rd years	1	24 students	1929	Assignments broken into smaller pieces

Table 3. Eight cases of the study

* This implementation was most closely aligned with researcher expectations.

Eight cases

Each of the following cases reiterates some of the information in Table 1 and supplements it with additional context as well as extracts from transcripts. The substantial use of transcript extracts was deliberate in showing the reader close ties to the framing of the findings and the claims made in the following chapter. In other words, this chapter serves to provide the conditions, or context, to support the variation map of Chapter 5.

Case 1: Mr. SA-A

Mr. SA-A taught at a public high school in South Africa and has taught mathematics for 28 years. He only implemented the Student Response Form in one of his 10th-grade classes, and this class had 30 students. This class was his homeroom class, and the class' average grade in mathematics was significantly above the average for the entire grade. Mr. SA-A reported that all of his students had access to the Internet either through their own mobile devices or a mobile device of a friend or parent. Students would have paid plans or access free Wi-Fi should they reach their data cap. While this use of technology was completely new to him, it was not outside of his technological range. In total, he received three student responses that were unprompted. He emphasized that forgetfulness due to a pressured routine caused him to check the spreadsheet less often than he would have liked.

He reported that he received, "without exception, responses from students who were very quiet in class and serious about their studies." He saw these responses as an "*aansluiting*," or segue, which he elaborated as "a little something, like even if it is a little story or something interesting, etc. that enters the experiential field of the child that which matters to the student is made a reality." Mr. SA-A noted that "there [were] definitely students that previously were quiet and introverted… [who] later popped out as students who wanted to participate, which [I] was

very proud of. Whether this is due to the Google Form is not something [I could] say for sure – how can you prove that? However, [I do] believe that perhaps it is possible". The interview took place two days after Mr. SA-A had accepted another position at a different school and he clearly stated the Student Response Form could not be implemented before issues of discipline and norms were first addressed. He was very sorry to part ways with his students and believed there was a kind of bonding, "…something special that I have not really experienced from another class before" although he maintained that he cannot comment on the potential influence of the Student Response Form on the unique classroom atmosphere.

Case 2: Mrs. SA-B

Mrs. SA-B also taught at a public high school in South Africa and has taught mathematics for 20 years. She implemented the Student Response Form in one 8^{th}-grader class of 23 students. Two students in her class submitted all six responses that she received. All students had access to mobile devices but also had problems concerning limited data plans as mentioned before. She mentioned that maintaining the Student Response Form requires some commitment in remembering to check the spreadsheet often for any new responses, as she did not access the Internet every day. She mentioned a sense of urgency in this regard because it felt as though "[I'm] actually kind of dropping [the student who submitted a response] by not going to have a look… every night."

The students who submitted responses were also the students who "constantly had their hands up in class" rather than the students who were not performing well in class, which "was a pity". Mrs. SA-B said that the Student Response Form made her more attentive to the actual effectiveness of communicating some idea, which a teacher might otherwise assume obvious. She gave an example of how a student response made her realize that her example of $b^{-1} = \frac{1}{b}$

does not reveal properties of exponents as well as $b^{-2} = \frac{1}{b^2}$ as some student thought that $b^{-2} = \frac{2}{b}$ when using the prior example rather than the latter. She mentioned that one of the two students who used the Student Response Form was an interesting case to her because he entered the class with a failing grade but a recent test showed he was passing and doing significantly better. She talked to him and "he said he just wasn't interested in the subject in elementary school (the previous grade). [And now] he was one of those who would often ask questions… so with him I can specifically see that he asks questions frequently in class… and suddenly became more interested in the subject. I don't say it's just the spreadsheet but overall his attitude changed towards the subject. I do think something like this might be able to change a student's attitude because it is something interesting. It is something different, another subject does not offer him that and I kind of want to use the word 'thrill' but that's probably not the right word…"

Case 3: Ms. SA-C

Ms. SA-C taught at a high school in South Africa and has taught mathematics and for four years. She introduced the Student Response Form to all six of her classes, 161 students in all. She received a total of ten responses and mostly within the first week. As she made use of the Student Response Form after teaching these classes for a couple of months, she was also able to comment on perceived indirect influences of the form. She mentioned that due to the low response rate it was easy to become forgetful and not check the spreadsheet on some days.

Ms. SA-C perceived one of the main capabilities of using the Student Response Form was that it allowed her to "approach students in a slightly different way if [I knew] what the need is for the students instead of just speaking in general with 40 different personalities every day." For example, one of the responses was "Ma'am, how does value-added tax work?" This surprised Ms. SA-C ("I did not expect kids [to] know this little about the topic") because she

assumed the concept of value-added tax to be a prominent topic in past work. This response later led the teacher to revisit the topic during class. She noticed that some students were multiplying by 14 rather than 14% and "…they were getting these crazy big answers, and I was like 'Where's your percentage button?!' and the student said 'Must I press the percent button?', so definitely in the future I will explain the percent [button]…" She added that once the student started to ask questions "…it started helping and we could make some progress [instead of] most students… sitting back thinking… 'I don't know how to get it but it's okay because she said it is so, so it *is* so.'"

In addition to reporting that she got a better idea of her student needs, she added that, after the Student Response Form was implemented, the "shy kids… felt a little bit more '*vrywillig*' [candid/frank/free-willing] to ask questions once they knew, above and beyond the form itself, once they knew that they can ask me questions." Specifically, she was able to name one student who "… sits right in front of the class and if I stand close to him then he becomes all jittery and stuff. But once he knew that he could use the form, even though he hasn't sent a response yet, he just felt that he can be a little different, he felt a little bit more at ease…"

Case 4: Mr. US-A

Mr. US-A has three years of experience and taught at a public high school in Georgia, USA. He made use of the Student Response Form in three Advanced Algebra courses with students aged 15 to 18. The three class sizes ranged from "low 20s to low 30s," and students had submitted 220 responses by the time of the interview. Mr. US-A's school was transitioning to providing each student with a personal computer. Because this school was a "Google for Education" school, students and teachers were familiar with Google services like Gmail, Google Docs, and Google Drive. He reported that not all of his students had mobile phones so he was

concerned that there might be some "socioeconomic bullying," but fortunately "there was nothing like that." This teacher also had a co-instructor, and they would often switch roles during class allowing one to teach upfront and the other to walk around and monitor student work. The Student Response Form was mostly used for "a closing question towards the end of class. A few times, we use that like a little halfway point in class to introduce an error like a common error… I think it's 'Answer a question posed by the teacher' and then I just give them free response." He might also provide an example of student work and have students submit a response to comment on the correctness of the work. Mr. US-A also reported that he and his co-instructor would actively and continuously try to involve as many students as possible in classroom discussions.

One example of how the Student Response Form was used in introducing the concept of extraneous solutions. Mr. US-A said "We thought a lot of students wouldn't know what the word extraneous meant… but the answers that we got showed a lot of kids have an idea of what the word extraneous meant… so somebody said 'Put a lot of effort in', or 'irrelevant'… well, that's pretty close… that's good… that'll probably fit when you define certain things as extraneous but we have to… stretch your definition of extraneous a little."

Mr. US-A customized the functionality of a separate Student Response Form to aid him in monitoring academic performance of the students for whom he was the baseball coach. The students were required to self-report their academic progress at regular intervals with the aim of Mr. US-A being able to provide timely support to students should they require it. He reported this practice to be highly successful and recommended it as being able to make a "significant difference" in an athlete's academic performance.

Case 5: Dr. US-B

This teacher had five years of experience and taught at a private school in Florida. The school emphasizes project-based learning and values the use of technology as an integral part of the students' lives. For instance, every student is required to have a computer and the school does not have any policies against the use of mobile phones in class. Students also have a lower "seat time" when compared to students in a public school, as more time is allocated to project-based learning and individual catering to students. His routine consisted of teaching geometry and algebra to four classes. Two of these classes were mixed seventh and eighth graders. They meet four times a week, spending one or two days on an activity or some problem-solving tasks, and two days using a flipped classroom model. This teacher received roughly 200 responses from all four classes during the time of the study.

The most significant advantage afforded by the Student Response Form for Dr. US-B lay in helping him "to focus on providing closure to lessons… and this was a kind of structured way for me to… [get] some indication of what [they] know." While it did somewhat inform the direction which a lesson might take. The teacher noted "[I do] not think that [I] did a very good job of incorporating… the responses into [my] practice. That is something that [I] would've liked to do more." He gave an example of the use of the Student Response Form where the teacher prompted the students to submit a response where they explained to another student how to get the equation of a line. He mentioned that responses stood out that had "proper Mathematical vocabulary" and ones that did not simply state something like "I don't get this" but rather offered an "honest assessment" of what they understand. For an example of an honest assessment, he shared a response that stated: "Do the change in Y over the change in X so you get the slope, but then you plug that into… but then I really don't know how to get the B…" The teacher also

noted that he had an email-based practice of students working until they got stuck and then sending him an email with a screenshot and a brief description. In addition, Dr. US-B expressed the need for being able to use mathematical symbols in the responses.

Case 6: Mrs. US-C

This teacher had 17 years of experience and taught at a public high school in Georgia, United States. She used the Student Response Form with two grade 10 classes for analytic geometry, of which one was a support class. She received roughly 500 responses from 45 students during the time of the study and the only instance where a teacher's use of the Student Response Form increased over time. She reported that the Student Response Form was complementary to existing practices that she had "because we just kind of slid it right in there… it doesn't take long… that's the beauty of it." The first existing practice required students to report the clearest point and the muddiest point of a lesson as a ticket out the door. Mrs. US-C initially made use of notecards that were passed out to students on which they would write the responses whereas she is now using the Student Response Form. The second existing practice was called *My favorite no* where she noted that she "[likes] to have the students tell [me] something right about what we did and maybe an error in understanding… they have to mention something good and then something that was a mistake." This practice was initially carried out through discussion or notecards during class time whereas she is now using the Student Response Form. The Student Response Form was used a couple of occasions a week.

One example of how Mrs. US-C made use of the Student Response Form was a ticket-out-the-door where she prompted the students as follows: "Tell me what is clear to you from today's lesson. Tell me what is still muddy." One student response was "Help! I don't get any of this!" and another response was "I get the SOHCAHTOA, but I'm still shaky on how to decide

which one to use when solving a right triangle." The teacher also reported that "I was very pleased at how much the students wrote especially from the students who don't like to speak up during class… I think [that] was my biggest pleasure, that 'Okay!' they're really taking the time to answer the questions even more so than they did on notecards…"

Case 7: Dr. US-D

This instructor had five years of experience and used the Student Response Form for a content course for pre-service teachers with a focus on trigonometry and more specifically the covariation of variables in a function; this class also had a strong focus on mathematical thinking and students are expected to voice their struggles or difficulties with tasks. The class had ten students of whom most were 3rd-year students. Common practices of the class included regular group work where students work on large whiteboards with dry erase markers or using dynamic geometry software; the instructor and teaching assistants would walk around the class facilitating discussion either within the groups or with the class as a whole. The Student Response Form was used from the onset of the semester and received a total of around 170 responses. The Student Response Form for his students "as part of their homework assignment to say… [which] questions were the most confusing, what was the most difficult, what questions they have about the homework… that sort of stuff… that I would want them to come to me within office hours." The Student Response Form was customized according to problem sets and the respective questions of a problems set.

Dr. US-D said that one of the significant struggles for him was getting "students to come to office hours and ask questions outside of class." He added that: "That's a very… I don't know if it's a power structure that exists with faculty and students… I know as a student I was always afraid to approach faculty so I can see [why] students are uncomfortable doing that… So I think

in terms of changing the practice it created an environment for them to feel… safe, comfortable, just sharing…" One of the examples that he offered was a response by a student who stated: "I found question three the hardest to explain because I never thought of how to explain arc length to someone who does not understand the concept." The instructor went on to say that he "never thought about how, to them, that might be such a foreign concept — to think about how somebody else is thinking… so that was pretty cool…" The instructor noted that student responses tended to change over time from problems with questions to more content-focused problems. He gave an example of the student response that was more focused on content: "I think number seven was the most confusing. I am not sure we ever defined what is a function or determined what is not a function, but I figured that most graphs were functions because they showed a relationship between two variables."

Case 8: The Researcher

This instructor had seven years of experience and taught a college level methods course for 24 preservice teachers who were in their second or third years. The course was focused on teaching methods to teach K – 5 Mathematics, and it had a field component where students worked with first graders at a nearby elementary school and reflected on their planning and teaching throughout the course. The Student Response Form was a central part of the course as students were required to submit between one and five responses before each class. The required submissions ranged from responding to an assigned reading, commenting on an assigned short video, or simply whether they would like to share out the responsibility of bringing food to class and how much they would be willing to spend. Several assignments were broken down into 10 to 20 responses on a given topic and combined into a single submission. For example, students had to review all their submitted responses after watching short videos of teachers working on

problems with elementary school students, and the pre-service teachers had to try and notice changes in their own responses over time. Submissions were graded by the teaching assistant with zero points for no submission, one point for an incomplete submission, and two points for a complete submission. By the end of the semester the Student Response Form had nearly 2000 responses. The Student Response Form was customized by response topic and number (e.g., CML video, task 7).

One example of a student response submitted after watching a video is: "Some of the children understood relationships between numbers and thinking strategies. These children were able to see the 3+3=6 and find that 3+4=7 since 4 is after 3 and 7 is after 6. This demonstrated their understanding of number relationships." Another response required the student to discuss a strand of Mathematical Proficiency (Kilpatrick, Swafford, Findell, & Committee, 2001) called Productive Disposition: "I think that the last student also showed productive disposition. Productive disposition was evident because he shows that Mathematics is useful by using known Mathematics facts to solve a more challenging Mathematics problem". These student responses were also incorporated into discussions by printing off all the student responses for some task and selecting a few students to talk about their responses in more detail. They were handed the printout to familiarize themselves with their response if they were not able to recall what they submitted. A class list was maintained to keep track of making sure that each student was called on at least a couple of times a semester, although this practice only persisted for the first two months of the course.

CHAPTER 5

VARIATION MAP AND DISCUSSION

This chapter offers a variation map of what the teachers reported during the surveys and interviews. The focus in Chapter 4 was on documenting context, and the focus in Chapter 5 is on documenting the variety of reported implementations across various contexts. The variety of implementations are framed in three ways. First, I report teachers' perceived influence on practice. Second, I report findings as framed through Smith et al.'s (2009) five practices for orchestrating discussions. Third, I report findings as framed through emergence, collapse, and thresholds. Chapter 4 and Chapter 5 jointly address the research questions posed in Chapter 1. Chapter 4 addresses the conditions and context of each of the eight cases. In response to the first question, Chapter 5 addresses teachers' perceived influences on practice and perceived capabilities offered by the Student Response Form. In addition, this chapter also offers a synthesis of findings regarding classroom discussions framed within the five practices (Smith et al., 2009). In response to the second question, Chapter 5 also offers a variation map to situate and describe events across a variety of contexts. The variation map frames this description using the terms emergence, collapse, decay, and thresholds. This chapter ends with thoughts on conclusions, implications, and future research.

Varying Capabilities and Influences

Perceived Influence on Practice

The Student Response Form was perceived to have influenced practice in a variety of ways with impact ranging from large to negligible. The most direct influence is suggested as enhanced learner feedback (both regarding quantity and candidness) which influences planning in turn, whereas more indirect influences point toward influence at individual (e.g., attitude of a

single student) and collective levels (e.g., overall class participation). To summarize, the chief capability of the Student Response Form is to collect many open student responses quickly when compared to other monitoring strategies like walking around the class to assess student work, getting verbal feedback from students, from discussions, or from using notecards (US – A, US – C). Opportunities afforded by this capability include but is not limited to

- hearing from more students in a class (SA – A, US – B);
- giving students more opportunities to write about their thinking (US – B);
- creating a communication channel where students might feel "safe… [or] comfortable" when posing questions (US – D);
- monitoring when students complete a task by having them submit some response when they are finished, and catching a common mistake quickly (US – A); and,
- breaking large assignments into smaller tasks to avoid large workloads at the end of a course (researcher).

Specifically, these are the influences reported by teachers:

Mr. SA-A. "The small amount of data, the kids' reactions, that did actualize, and I was able to read, did give me some insight into what students are struggling with… these few responses that I did get from students led me to go to the class the next day in a different way… It gives you… A little something, like even if it is a little story or something interesting, etc. that enters the experiential field of the child. That which matters to the student is made a reality."

Mrs. SA-B. "I do think the one or two times that they did respond, . . . It makes you attentive to do something that you might have thought was communicated well, that you assumed as obvious… And then you realize that this specific concept would have to be explained in a different way because the child does not understand it."

Ms. SA-C. "I think I can just learn a little bit more of how to approach students in a slightly different way if I know what the students' needs are instead of just speaking in general with 40 different personalities every day."

Mr. US-A. "So how did it change my practice? It made me think more deliberately about the technology and how… well, whatever technology I am going to implement… [and] how the kids are going to receive it because I'm pretty sure gone are the days… that just using technology engages them… if it's just another way of doing what you always do… [then] it's not going to work with them… It more influenced what I did next, or what we did next, I should say because I was always collaborating [with another teacher]. [For example,] I might have said: 'Okay, well I've been seeing a lot of this, why is this wrong?'"

Dr. US-B. "When I was trying to use that in class, it helped me to focus on providing closure to lessons... It was a… structured way for me to say, before the kids leave the class… to describe to me what they learned [that] day… The big take away from the lesson… I didn't do a very good job of incorporating exactly the responses into my practice, that is something that I would've liked to do more."

Mrs. US-C. "It helped me to better do a quick assessment on where they were right after a lesson, during the lesson, before the lesson… What I was doing before is they would answer on a note card and I would have to read through the notecards so this was much, much quicker."

Dr. US-D. "One of the things I really struggle with… is getting students to come to office hours and ask questions outside of class… stuff I would want them to come to me within office hours, I use the form to have them answer that. So I think in terms of… changing the practice, it created an environment for them to feel… safe, comfortable, just sharing…"

The researcher. The Student Response Form was used to break large, higher-stakes assignments into smaller pieces which allowed a more even work distribution throughout the semester and more opportunities to gauge how the students are grasping the content. I also use the Student Response Form when I pose a question that might be answered differently in person rather than in front of a group, such as: "Please give me some feedback on the field experience component.", and "How much money do you think is fair to expect a student to spend on buying breakfast for the rest of the class?"

Influence on Practice Using Smith et al.'s (2009) Five Practices

Before reporting on the influence of the Student Response Form using the five practices, it is important to note that these five practices were interpreted in a broad sense and aimed more at adding redundancy to the interview protocol to create more opportunities for teachers to talk about different aspects of their practices. For instance, the original intent of the anticipation practice refers to a teacher's ability to anticipate specific student responses to a task which the teacher is planning to pose. However, in this study, the word anticipation was used to refer more broadly to a teacher's ability to know, in advance, how students might respond to some topic in general.

Generally speaking, I claim the following: The Student Response Form tends to enhance a teacher's monitoring practice by simply gathering many student responses with relative ease. This amplification of monitoring may influence, or add to, a teacher's repertoire of anticipated student responses although this tendency was less pronounced with more experienced teachers. Selecting is simply influenced to the extent that a teacher might now have more responses to select from, whereas a teacher's sequencing practice seems unaffected other than perhaps having to sequence *more* selected responses. The connecting practice is influenced in a similar way as

the selecting practice in that a teacher might have more student responses to draw on when attempting to make connections among student responses. However, if a connecting practice is not in place, then the presence of the Student Response Form does not seem to promote or induce it. More specifically, I report the perceived influence of the Student Response Form using broad implementations of each of the five practices in referring to related instances from the eight case studies.

Anticipation. The Student Response Form influenced teachers' anticipation by providing student feedback which resulted in teachers being able to use a student response as a segue (SA – A); getting feedback on the effectiveness of an example (SA – B); becoming more aware of students' background knowledge (SA – C); quickly judge students understanding of some vocabulary word (US – A); getting an honest assessment of student understanding and problem areas (SA – A, US – B); getting a more accurate idea of student perceptions of homework difficulty (US – D, researcher); better understanding student needs that are not easily detected through grading homework (US – D, researcher).

Monitoring. The Student Response Form influenced teachers monitoring by being able to gather "mass responses quickly" (US – A) and was mostly effective where similar practices (e.g., notecards, ticket-out-the-door, or a class discussion of a solution) were already in place (US – A, US – C). The Student Response Form is also effective as a persistent monitoring practice if its use was explicitly part of the course requirements (US – D, researcher). The Student Response Form was not as effective when used in an instance where an email-based practice was in place, serving a similar purpose (US – B). Also, the Student Response Form's use decreased rapidly when teachers do not explicitly and continuously require the students to submit responses (SA – A, SA – B, SA – C).

Selecting, and sequencing. These two practices are treated together as both relied mostly on questions of lesson goals and teaching style of individual teachers. This section is only referring to instances where the entire class or large groups of students were engaged in a selected response and not instances where a teacher would follow up with individual students (SA – B, US – A). Also, these practices were of limited use in describing the influence of the Student Response Form where response rates were very low (SA – A, SA – B, SA – C) as these teachers were able to address every response. Reasons for selecting responses were varied and mainly included addressing common errors or problem areas (US – A, US – B, US – C, US – D, researcher). The sequencing of responses might be "on the fly" (US – A), "in terms of their cognitive difficulty" (US – B), moving from common errors to unexpected as well as good responses (US – C). Alternatively, in the case that there might be two identified problem areas there are not any "conscious strategic decisions" made about which might be addressed first or last (US – D, researcher).

Connecting. The Student Response Form influenced practices of connecting student responses by "adding to the repertoire" where teachers "had these practices already stored away and ready to come out at any given time" (Mr. US – A). In addition to Mr. US – A having the practice of discussing common errors or misconceptions, Mrs. US – C had a practice called *My favorite no,* where students had to look at a student response and "mention something good and then something that was a mistake." Both Dr. US – B and Dr. US – D, when asked about connecting practices, speculated that a future implementation of the Student Response Form might involve showing a student response to the students to have them discuss it.

Emergence, Collapse, and Thresholds.

Claiming that some event was a source of collapse, a source of decay, or emergent, is only done after the fact. These claims are made from some perspective (e.g., increased interaction between students and teachers). Similarly, something that is claimed to collapse interaction may just as well be perceived as emergence from some other perspective (e.g., the teacher taking a stronger leadership role, or bringing focus to a specific strategy).

Thresholds and Sources of Collapse

The Student Response Form is most easily incorporated into a teacher's routine if it optimizes existing practices like a notecard system (US – C). When similar practices are not in place (SA – A, SA – B, SA – C, US – D), then two likely sources of collapse are forgetfulness and the amount of time it takes to submit responses and read them. For example, 15 out of 50 minutes took up too much time and was reported to mitigate the form's use (SA – C, US – B). Forgetfulness was less of a factor when a teacher has a co-instructor (US – A) or a teaching assistant (US – D, researcher[50]). I could also refer to another teacher or co-instructor as a type of coupling that supports the circulation, or flow, of open student responses in the class.

Regarding how the Student Response Form might influence what students say during class time, Mr. US – A brought up two points I would like to emphasize. Firstly, if students are able to type responses, they are able to complete their thoughts even if they are incorrect. If they say it out loud there is a risk of being interrupted or cut off with a response like "No, you're wrong!" interrupting the related thought processes. Secondly, now that a student requires access to a device and Internet to submit a response it creates a greater risk of "socioeconomic bullying", although fortunately this was not the case in this particular class. Also, I take the event

[50] I would describe my forgetfulness more as decay, as the frequency of discussions decreased over time before collapsing, or, stopped occurring.

of a student saying "No, you're wrong!" as an enhancement of the intensity of that moment, meaning that it is not necessarily a source of collapse although it has increased the phase space of potential events that might follow be it a student keeping quiet, a rebuttal, or something else.

The amount of time it takes to submit a response depends on the familiarity students have with the technology and whether they are allowed to use technology at school. For example, US – B's seventh graders took 25 minutes to submit a response, but it is unclear whether this was due to the difficulty of the question or fine motor skills. It does not seem to be straightforward to estimate the length of time based on student age. Mr. US – A's high school's freshmen were issued laptops, as will be the next year's freshman and so on, whereas the current sophomores are not issued laptops. Thus, the sophomores are less likely to have the same opportunities in learning how to use the technology at a practical pace. Also, the amount of time it takes to take out and use the technology is important (e.g., often laptop carts are not used for this reason (US – A)); also, Internet access through Wi-Fi or mobile phone carrier may be limited in some classrooms of a school (US – A). Class size may also influence the use of the Student Response Form as smaller classes can be monitored easily by a teacher walking around and checking on students' work instead of using the Student Response Form (US – B) whereas with larger classes this monitoring strategy was less efficient.

Emergence

I cannot attribute causation of any event to the Student Response Form, or generalize across cases in this analysis. So, I have chosen to discuss two specific classroom events, one example of variation across classroom cases, and two unexpected events to give the reader a sense of what I would claim to be emergence in this study. Overall, I note these instances as events that have been made more likely with the introduction of the Student Response Form into

these classrooms. In other words, it can be seen as a mechanism of emergence that increases the probability of certain events that take place. It can also be seen as an adjustment of ratios between certain kinds of events. For example, I claim that the "No, you're wrong!" can be lessened through allowing typed responses. In other words, the event of completing an incorrect thought may occur more often, and the event of a student saying "No, you're wrong!" may occur less often. Due to this change in ratio, single instances of events and possibly cumulative effects can be noted as an emergent event or tendency.

Example 1: A better example. Mrs. SA – B gave one example of how a student response made her change her usual example for negative exponents. This is a significant example for me because this teacher has 20 years of experience in teaching Mathematics. Granted that this teacher had not taught Grade 8 mathematics in a while, it seems the Student Response Form may still have fast-tracked her ability to draw on a different example from her existing repertoire. So, I claim that this capability of students being able to give this teacher feedback in a different way, has created a new encounter for the teacher with her example which, in turn, influenced her practice an explication of exponents.

Example 2: The percentage button. Ms. SA – C gave a specific example of a student who used the Student Response Form to indicate that they did not understand the concept of value-added tax. This prompted the teacher to review it in class the following day, which in turn, caused another student to ask whether they should be using the percentage button for this. From the interview, I interpreted that the teacher was surprised by this question which she may have presumed to be obvious previously. So, I claim that the likelihood of this chain of events was increased, or amplified, due to the inclusion of the Student Response Form in the classroom.

Example of variation: The "type of student" responding. All three South African implementations of the Student Response Form did not require the students to submit responses during class or after class, so all the responses that they received were unprompted, or volunteered. Furthermore, each teacher speculated on the "type of student" that used the Student Response Form. I would like to note all three instances here taken from Chapter 4 (emphasis added):

Mr. SA-A. He reported that he received, "without exception, responses from students were *very quiet in class* and *serious about their studies*." Also, "there [were] definitely students that previously were *quiet and introverted*… [who] later popped out as students who wanted to participate, which [he] was very proud of."

Mrs. SA-B. The students who submitted responses were also the *students who "constantly had their hands up in class" rather than the students who were not performing well* in class which "was a pity." She also mentioned that one of the two students who used the Student Response Form was an interesting case to her because he *entered the class with a failing grade* but a recent test showed he was passing and doing significantly better.

Ms. SA-C. "…sits right in front of the class and if I stand close to him then he becomes *all jittery and stuff*. But once he knew that he could use the form, even though he hasn't sent a response yet, he just felt that he can be a little different, he felt a little bit more at ease…"

From these three cases, and especially the first two, I would reject claims of this technology necessarily being suited to a particular type of student. I claim that this type of framing of these events may be a distraction: the range from merely three cases indicate that students with academic classifications ranging from weak to strong, and social classifications

ranging from quiet and introverted to significantly involved in discussions can benefit from the Student Response Form.

Two Unexpected Events

The use of the Student Response Form with baseball players (Mr. US – A) was an unexpected but welcomed event. He reported significant potential, as well as several positive results, in using this type of customized Google Form to help student athletes monitor their academic performance. Furthermore, Mr. US – A is continuing his use of this form in the new semester and has also created a similar form for another sport at the school. In this sense, I would speak of this as a consistency that has been achieved. Again, this does not require a significant argument for me other than Mr. US – A has been able to create a consistency out of an adaptation of this form that has continued to persist in its existence.

The second unexpected event that I would like to note can be mentioned quite briefly. I found it very easy to ask students questions about topics that they might be reluctant to discuss in front of the class like snack preferences[51], or what they considered to be a reasonable amount of money to spend on those snacks. I think that this capability has significant potential to allow students to respond candidly, and even anonymously, to potentially difficult questions like those mentioned here and other questions regarding the syllabus and course feedback.

Conclusions, Implications, and Future Research

This study was an attempt at reconciling dense philosophical work with a technological intervention in mathematics education. I claim to make two contributions. The first contribution is a study of teachers and technology. The study documented the perceived impact of the Student Response Form on the practice of a variety of mathematics teachers. The second contribution is a

[51] I think questions like these that require students to disclose their favorite food, music, TV shows, etc. are likely to elicit very different responses depending on who is being spoken to/in front of.

proposed method for investigating technological interventions in mathematics education. The proposed method is an attempt at reconciling diverse theories and concepts, transdisciplinary work, changing technological landscapes, and social scientific research. The method has tendencies that differ from conventional, social scientific research. In this final section, I note limitations of this study. After this, I reflect on the method assemblage used in this study. The purpose of this reflection is to show readers how I would consider beginning this assemblage again, and to reemphasize in what ways each component was useful in conducting this study. After the reflection, I discuss theoretical, methodological, and practical implications regarding the study. Finally, I discuss potential avenues for future research.

Limitations

The three most significant factors that limited this study were the lack of observation, lack of professional development, and the lack of on-site support. I did not observe any teachers using the Student Response Form in their classrooms. Chapter 3 discusses the rationale for this decision. Teachers did not receive significant professional development. Professional development was restricted to hearing talking points during recruitment, and receiving the teacher guide. With the exception of a few teachers, I was not able to offer teachers on-site support. Recruitment was generally more successful when I was able to sit next to a teacher, help them sign up for the Student Response Form, use the Student Response Form, and talk about strategies for using it in their practice. Such personal attention contributed significantly to recruitment, and the use of the Student Response Form.

Reflections on This Method Assemblage

I have identified five parts of this method assemblage: problems, design principles, technological intervention, metamodel, and variation map. This is the most transparent account I

am able to offer the reader in describing how this inquiry manifested, evolved, and took shape over time. Instead of assigning this section of the document as the endpoint of this inquiry, I would rather end with some brief reflections on how I would modify this assemblage should I want to iterate the study.

Concerning the problems. I would deemphasize the centralized architecture blockage should I want to investigate classroom interactions more. Alternatively, I would emphasize this blockage more should I want to experiment more with decentralized educational settings. The idea of a problem of inertia, and the idea of feedback loops or flows, has become more valuable as time progressed in describing the status quo and rigid modes of organization. Also, pursuing interventions that improve classroom discussions may profit from a substantial professional development component for teachers who do not already have norms and practices that support classroom discussions with varying points of views. Finally, I emphasize that my initial formulation of problems and eventual reconciliation with the respective teacher's contextual problems has been very helpful. Problems gave me a practical way of outlining intentions of my own research while becoming more mindful of those of the teachers. I think that the resulting encounter produced something that belonged both to academia and practice without neglecting either significantly in favor of the other.

Concerning the design principles and resulting intervention. I would deemphasize recommendations for orchestrating classroom discussions and place more emphasis on a faster setup and implementation. For instance, teachers would be able to set up much faster if the students could to register themselves. Also, while the interface was simple, it was not intuitive enough and might benefit the teacher more if it were available as a phone application or Google script rather than a spreadsheet. I also think that the recruitment would have been better served if

teachers were demonstrating their own uses of the form, rather than my academic arguments. Finally, I speculated that a teacher might be more invested in the project if they have more say in the types of questions that can be asked. Specifically, I think that Mrs. US – C's "What is your clearest point? What is still muddy?" and "My favorite no", and Dr. US – D's "Ask the teacher a question" would have been far more useful to teachers than the van der Wall prompts for open responses.

Concerning the metamodel. I would have preferred, in retrospect, to tie the blockages and points of inertia to the metamodel (Figure 3) but I was concerned that an overly complicated figure might not serve its purpose of communicating the coordination of theories well. Regarding theoretical choices, swarm intelligence seems to remain a strong conceptual metaphor and, interestingly, allowed me to create, among others, the Google alert (automated search query) "complex adaptive systems Deleuze" that ultimately led me to several contemporary and useful sources of complexity theory being used in public policy and law outlined in Chapter 2. In addition to this, swarm intelligence better exemplified the coordination of increased variation with policies that harness that increased variation. For instance, Davis and Simmt (2003) outlined several useful properties in their complexity theory like *organized randomness* but did not coordinate this property as explicitly with a policy that might harness it. Finally, using the concept of assemblage, I was able to conceptualize swarm intelligence, the Student Response Form, status interventions, and the five practices on a flat, material plane – so to speak – with various possible, machinic couplings.

Concerning variation mapping. I have attempted to offer a subtle shift from conventional qualitative or quantitative claims. Specifically, the description tended to move away from generalization, describing causality, or using people as discrete units of analysis (e.g.,

the type of student). The concept of assemblage functioned largely in two ways for me. Firstly, it helped me conceptualize and think about interaction among the five identified components of the *method assemblage*. Secondly, I was able to conceptualize each teacher's implementation as a *classroom assemblage*. Once this assemblage is identified I was able to become less concerned with attributing causal relations between component parts and more concerned with different couplings of people, resources, norms, etc. and the resulting capabilities, and flows or feedback loops. Also, these flows or feedback loops were thought of as a type of consistency that emerged, or decayed or collapsed under certain conditions and modes of organization. In this sense, description was largely framed regarding probabilities under given conditions and capabilities.

Implications

Three parts make up this section. Each part discusses theoretical, methodological, and practical implications respectively.

Theoretical implications. This dissertation attempted to draw some attention to concepts and ideas that may aid thinking about and experimenting with new methods of inquiry. I agree with Bleakley and Cleland (2015) that these methods require a shift in thinking from the researcher. Furthermore, the kind of questions asked largely binds the researcher epistemologically and ontologically. In other words, the assumptions, methods, and findings that inquire into what something *is* differs starkly from the assumptions, methods, and findings that inquire into what something *does* or *might do*. Loosely speaking, the theory and philosophy employed in this dissertation regard phenomena in terms of open systems rather than closed systems. Specifically, I think open systems are concerned with

- processes more than products;
- thresholds more than averages;

- states more than types;
- interactions more than essential components;
- robustness more than optimization;
- probability more than predictability;
- strategic intervention more than representational modeling;
- connections more than hierarchies; and
- specifics more than generalities.

In summary, the philosophical work of Deleuze and Guattari influenced the way I thought, the questions I asked, the methods I employed, and the findings I reported.

Methodological implications. Regarding this study, the Student Response Form can be seen as a mechanism of emergence under certain conditions, which may increase the probability of certain classroom interactions. Similarly, and I can speak of forgetfulness as a potential source of collapse of these potential classroom interactions. Regarding the method assemblage, the five components (i.e., problems, design principles, technological intervention, metamodel, and variation map) discussed in this dissertation serve as one possible way of putting diverse theoretical and philosophical concepts and ideas to work. Importantly, it was difficult to draw a clear line where theory ends and method begins. It was useful to regard attempts to cleanly separate theory and method as a distraction. As mentioned in the theoretical implications section, this method is concerned with experimentation. This experimentation can be described as shifting the ratio of stability to change in some given context. Supple modes of organization rather than rigid modes of organization create this change in ratio. The findings of this experimentation are then framed regarding emergence, collapse, and thresholds. The findings seem to be more probabilistic, macro-level claims rather than predictive, individual-level claims.

Practical implications. In general, it was much harder than anticipated to get teachers to use the Student Response Form. There was no exponential takeoff that is so sought after in technological realms. Simply, I think I was not able to observe the design principle that the intervention would not add significantly to the teacher's existing workload. It seemed to be the case that a technological intervention is unlikely to sustain a consistency in a teacher's practice if it does not optimize some existing part of their practice or introduce something new that is seen as desirable. In addition, I was unable to support teachers as much as I wanted to. This was largely due to the distributed nature of the study. Generally, the most successful implementations of the Student Response Form came from people who lived and worked close to me. In these instances, I could visit their classrooms and provide any support they asked for.

Another two practical considerations were recruitment and intuitiveness of design. Regarding recruitment, I would prefer to show teachers a variety of short videos of other teachers successfully using the Student Response Form. In fact, I would prefer if teachers were able to show other teachers the Student Response Form and help them sign up without my intervention or guidance. In this regard, the technological intervention was not intuitive enough. Also, the Student Response Form may benefit significantly from more development. I spent roughly 100 hours developing the Student Response Form and roughly 400 dollars on freelance development. Doubling either or both of these quantities would likely have produced a more intuitive tool.

Potential Avenues for Future Research

There are several potential avenues for future research. I single out potential theoretical, methodological, and technological extensions. Regarding philosophy and theory, I think there is still much work to be done in making dense theory and philosophy accessible to an academic audience, especially graduate students. For example, Deleuze and Guattari have created a wealth

of concepts, like assemblage, that may be beneficial to the mathematics education community to aid thinking about existing problems in new ways. In addition, I think that carefully thought-out metamodels may help guard against using metaphor for the raw import of scientific concepts into the social sciences.

Regarding method, the method assemblage was designed with the intention of being modular and non-prescriptive in its reuse, iterations, modifications, or couplings with other studies. Potential next iterations and adaptations are considered in the section discussing reflections on the method assemblage. I also note that a researcher might find their method assemblage requiring four or six components rather than the five mentioned here, or even different components entirely. This dissertation attempted to offer the reader a prototype of what such inquiry might look like, and the method assemblage helped to articulate my most sincere account of this process.

Regarding technological innovation, the findings from this study indicate teachers are unlikely to adopt some technology if it does not optimize an existing aspect of their practice. I do not oppose the concept of optimization but there is no revolution to be had in merely optimizing any existing aspect of this educational paradigm. Generally speaking, technological innovation beyond the status quo seems highly susceptible to collapse without support and professional development. Based on the findings of this study, I speculate technological innovation may do well to have two kinds of capabilities. The first kind of capability aids teachers in a way that encourages the technology to be initially used, and the second kind of capability creates opportunities for emergence beyond the status quo.

More specifically, this study investigated ways of introducing supple modes of organization to increase the probability of classroom interactions that take advantage of

gathering together in a room. More ambitious, future studies might investigate supple modes of organization that allow students, teachers, and curriculum to move beyond period lengths, classes, grades, and subjects, ultimately allowing schools to develop new structures and practices.

In Closing

This dissertation aimed to contribute two things to mathematics education literature: a study about teachers and technology, and a methodology that investigates technological interventions while maintaining the tensions of rigor and unexpected outcomes. This methodology focuses on the capabilities of the technology, the context in which it was deployed, and in mapping the resulting variation in uses.

The study reports findings from implementing the Student Response Form across eight cases. The eight implementations of the Student Response Form varied across contexts. In general, teachers perceived the Student Response Form to amplify existent practices related to classroom interaction. Teachers also perceived the Student Response Form to increase the probability or frequency of certain kinds of events and interactions. Overall, the amount of implementations that collapsed outweighed the amount of implementations where new practices or consistencies emerged.

I offer this study as a basic prototype of a method of inquiry that I call a method assemblage. The philosophical concept of Deleuze and Guattari called assemblage influenced this method of inquiry. I follow Deleuze in claiming that such philosophical concepts are inexact not because something is lacking but because of their very nature. Moreover, this inexactness may offer robustness in serving as a useful flank to mathematics education research in the face of

various forces affecting it. I advocate the legitimacy and potential usefulness of this kind of methodology in social scientific research.

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APPENDIX A

INTERVIEW PROTOCOL

IRB verbal consent script: All information that I receive from you during this phone interview, including your name and any other information that can possibly identify you, will be strictly confidential and will be kept secure. Remember, your participation is voluntary; you can refuse to answer any questions, or stop this phone interview at any time without penalty or loss of benefits to which you are otherwise entitled.

Do I have your permission to ask you these questions?
[If no, stop interview. Proceed to "Thank you…"]
Do I have your permission to record this interview?

Preamble: These questions are aimed at understanding how the Student Response Form (that's how I refer to the customized Google Form that you used) influenced *your* practice. So I would like to emphasize that I have do not have some ideal implementation in mind that I am comparing you to, nor am I hoping that you to say any specific phrases. A focus of the study is to see how the Form functioned in various contexts. As a developer, I am often surprised by the ways in which my software is used and I am almost never as good at using my software as other people who try it. I am genuinely interested in *your* specific context, what it made *you* capable of, and how *you* used it.

- **Routine and context:**
    - Could you give me an idea of your routine in teaching the class or classes that you use the Student Response Form for?
        - What kind of technology do you and your students have access to?
        - Overall, in referring to the technology that you were exposed to in the study, how much of this was new to you? Or, how significantly was this a departure from your use of technology?
        - In general, do you think you have any flexibility in your schedule that allows you to try out something like this Google Form in your classroom?
- **Use of the Student Response Form (influence and capabilities):**
    - How do you think this Student Response Form to have influenced your practice?
        - If at all, how do you perceive this customized Google Form to have influenced your discussions?
    - What capabilities, if any, does the Google Form actually afford you or your students now that you did not necessarily have before? Or, what is more easy to do now than before?
        - What capabilities, if any, do you think the Google Form might afford you? Or, could potentially afford you?
        - Has this Google Form aided students who are generally reluctant to speak up during class?
        - Is there any difference and who contributes to classroom discussions? In other words, most conversations often tend to settle on a few students in the classroom, as this unsettled that dynamic in any way by bringing more students into the conversation?

- Has this contributed anything unexpected or out of the ordinary to your class or classes?

- **Five practices:**
  - ***Anticipation***: Has the Student Response Form in any way influenced your ability to anticipate student responses? In other words, did you receive student responses that you might not have gotten otherwise?
    - If so, could you give me an example?
  - ***Monitoring***: Was the Student Response Form useful to you and monitoring the student responses?
    - Have you used any other monitoring strategies? How do they compare?
  - ***Selecting***: How do you select which student responses to attend to? Did the Student Response Form facilitate your selection and use (e.g., planning, discussions, interactions with students)?
    - Could you give me some examples?
  - ***Sequencing***: Are there any student responses from the Google Form that you decided to talk about before or after feedback you got from students during class?
    - For example, would you start off homework feedback with some responses on the Google Form?
    - How did you decide which student responses to use and when to use them?

  - ***Connecting***: Was there at all any sense of connections being drawn between student responses?
    - In other words, did you mention a student response that some other student might've agreed with disagreed with, or,
    - that some student talks about something and then you would draw connection to the responses of the Google Form?
- **Use of the spreadsheet:**
  - How many student responses that you get in total?
  - What was you rate and diversity of student responses with the Student Response Form? More, less, same, different?
  - How did you use the spreadsheet?
    - Before class? After class? During class?
    - How much time did you spend on it then? Seconds? Minutes?
- **Kinds of responses:**
  - Which student responses to stood out to you? E.g., what kind of prompts did you pick out that were useful to you? Could you show me any of those responses?
    - What kind of responses would ideally be useful to you?
    - One educator noted that some responses are "too silly" (I'm guessing that a student might have shared some 'math joke'). Are there points at which you would say some responses are 'too silly' that you ignore them or too plain that it does not really help you more so than grading homework?
  - How did you manage all the student responses?
    - How many of what kind of student responses can you manage? E.g., were there prompts that had too much detail for the amount of student you had?
  - Were there any noticeable changes in student responses over time?
- **Thresholds:**

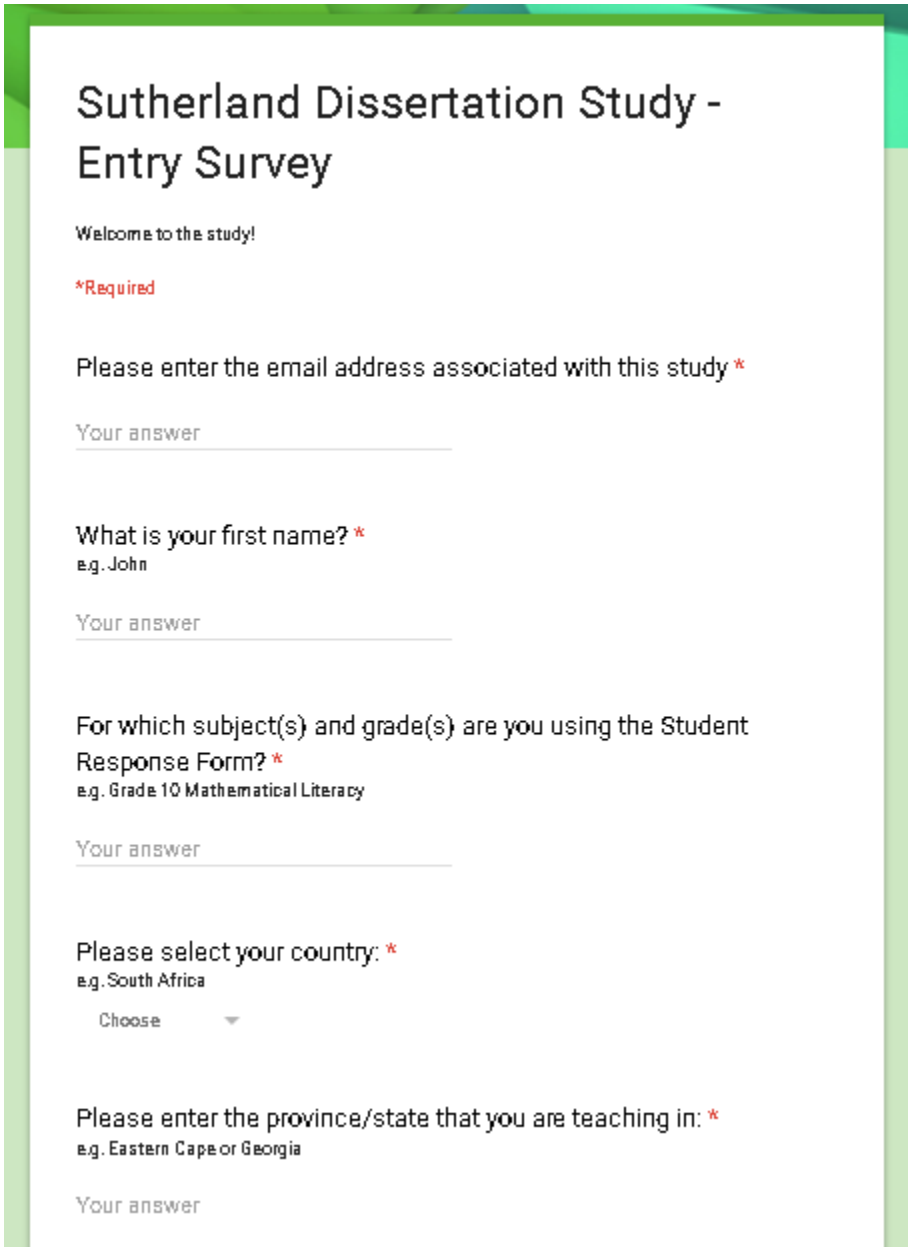
- Teachers have to make judgment calls every day. Describe the times when you decided to use the Student Response Form and times when you decided not to use the Google response form.
    - Where were the cutoff points for you where you decide against using the Google Form regarding
        - time,
        - types of responses,
        - technical issues, or
        - other contextual factors?

- **Feedback:**
    - This is the final section of the interview. Do you have any feedback for me in general?
        - modifications,
        - different uses,
        - potential uses etc.
    - Would you mind responding to these quotes? The purpose is to get a better idea of what you think of potential roles and uses of technology in education:
    - "The most dangerous experiment we can conduct with our children is to keep schooling the same at a time when every other aspect of our society is dramatically changing." (Chris Dede; American computer educator and futurist; from written statement to the PCAST panel, 1997.)

    - "Nothing could be more absurd than an experiment in which computers are placed in a classroom where nothing else is changed." (Seymour Papert; South African/American mathematician, computer scientist, and educator; 1928-.)
    - That's it! Is there anything else that you would like to add?
- Thank you for participating in my study!

APPENDIX B

SURVEYS



Sutherland Dissertation Study - Entry Survey

Welcome to the study!

*Required

Please enter the email address associated with this study *

Your answer

What is your first name? *
e.g. John

Your answer

For which subject(s) and grade(s) are you using the Student Response Form? *
e.g. Grade 10 Mathematical Literacy

Your answer

Please select your country: *
e.g. South Africa

Choose

Please enter the province/state that you are teaching in: *
e.g. Eastern Cape or Georgia

Your answer

How many years have you been teaching?
e.g. If you have been teaching for 5 years, enter '5'.

Your answer

If at all, how many years have you been teaching mathematics?
Enter 0 if this does not apply to you

Your answer

Please select every item that applies to you:
'A Spreadsheet' refers to document used in applications like Microsoft Excel, Google Sheets etc.

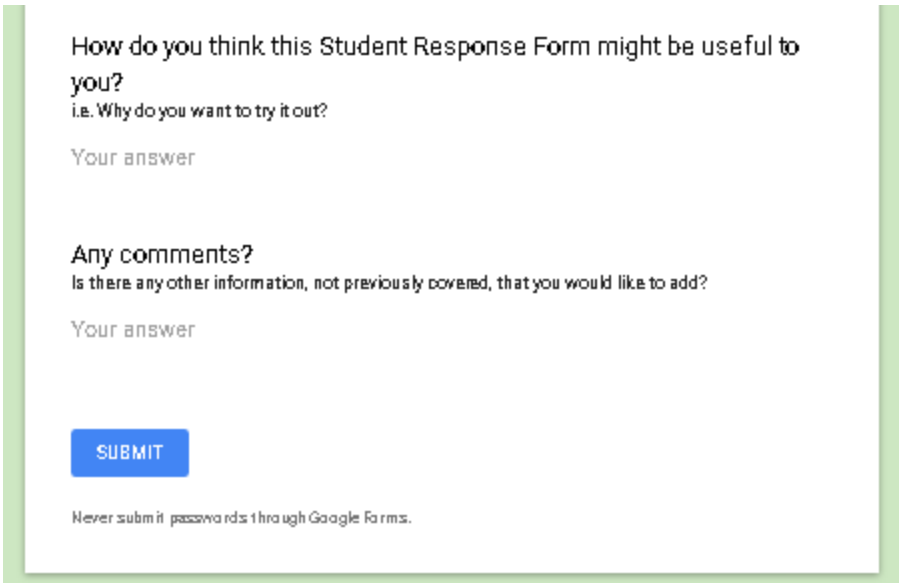
- [ ] I am familiar with using an internet browser (e.g. IE Explorer, Firefox, or Google Chrome)
- [ ] I am familiar with using spreadsheets
- [ ] I can order/sort a column in a spreadsheet program
- [ ] I can (or, I could probably learn how to do it if I was shown how) apply a 'filter' in a spreadsheet

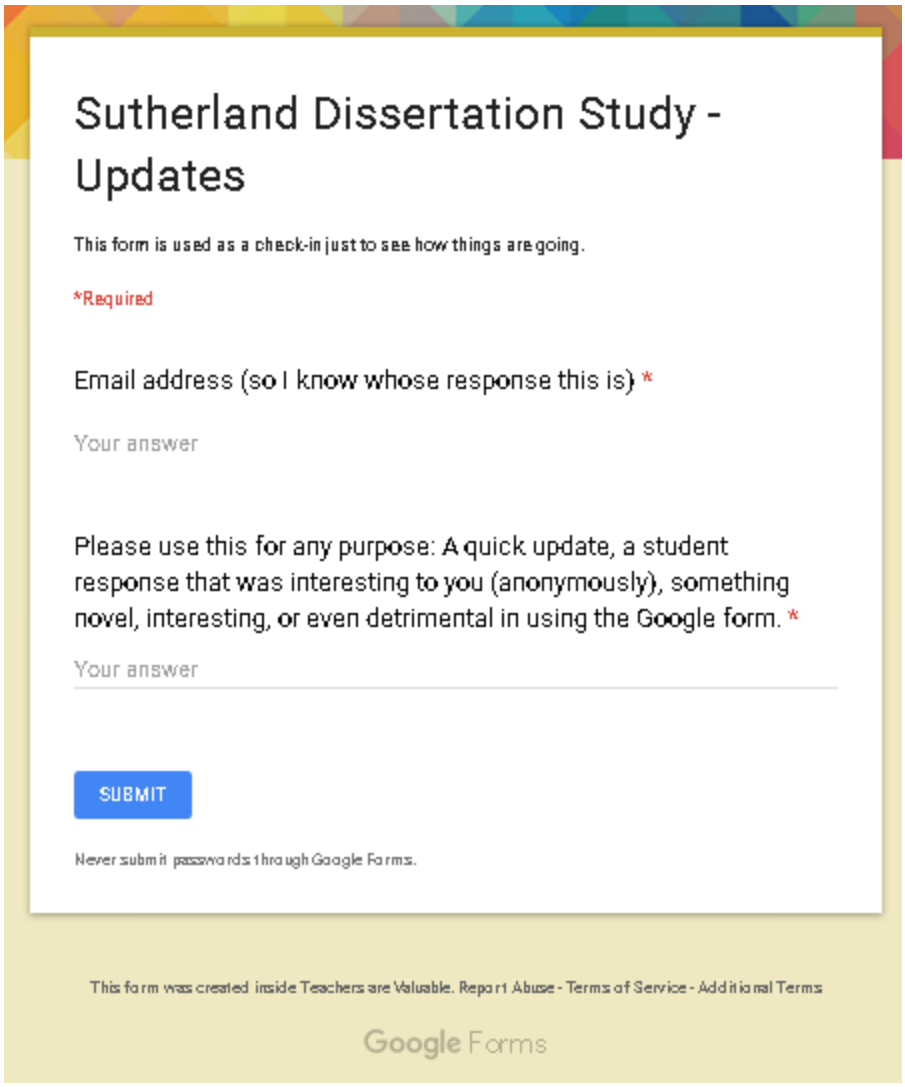
I have read the Teacher Guide (in your folder on drive.google.com)

- ( ) Yes
- ( ) No, and I probably will not
- ( ) No, but I intend to read it later

Please select every item that applies to you:
'SRF' is an abbreviation for Student Response Form

- [ ] I intend to use the SRF for student feedback on homework
- [ ] I intend to use the SRF mainly for homework but I might use it in class too if there is time/on occasion
- [ ] I intend to use the SRF for classwork
- [ ] I intend to try and make use of student feedback in promoting classroom discussions
- [ ] Other:


How do you think this Student Response Form might be useful to you?
i.e. Why do you want to try it out?
Your answer
Any comments?
Is there any other information, not previously covered, that you would like to add?
Your answer
SUBMIT
Never submit passwords through Google Forms.



Sutherland Dissertation Study - Updates

This form is used as a check-in just to see how things are going.

*Required

Email address (so I know whose response this is) *

Your answer

Please use this for any purpose: A quick update, a student response that was interesting to you (anonymously), something novel, interesting, or even detrimental in using the Google form. *

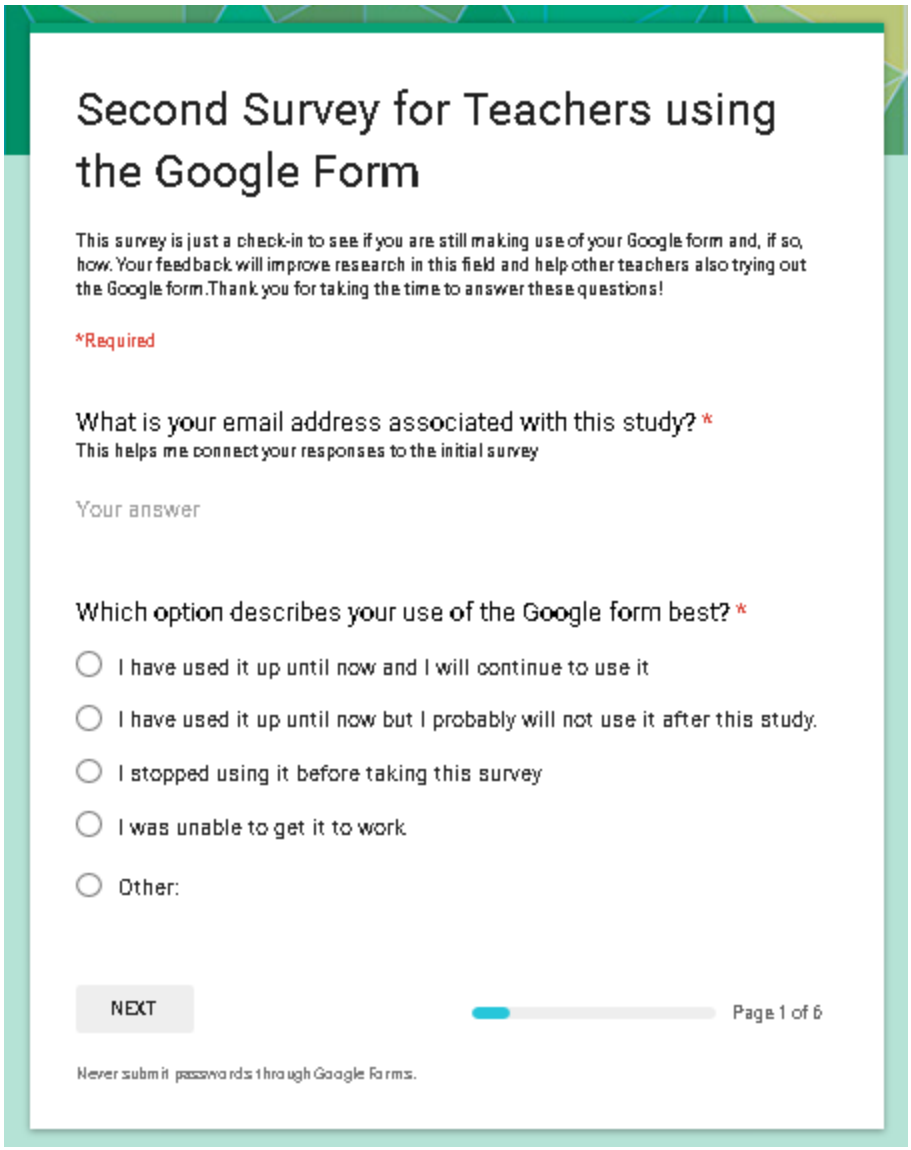
Your answer

SUBMIT

Never submit passwords through Google Forms.

This form was created inside Teachers are Valuable. Report Abuse - Terms of Service - Additional Terms

Google Forms



Second Survey for Teachers using the Google Form

This survey is just a check-in to see if you are still making use of your Google form and, if so, how. Your feedback will improve research in this field and help other teachers also trying out the Google form. Thank you for taking the time to answer these questions!

*Required

What is your email address associated with this study? *
This helps me connect your responses to the initial survey

Your answer

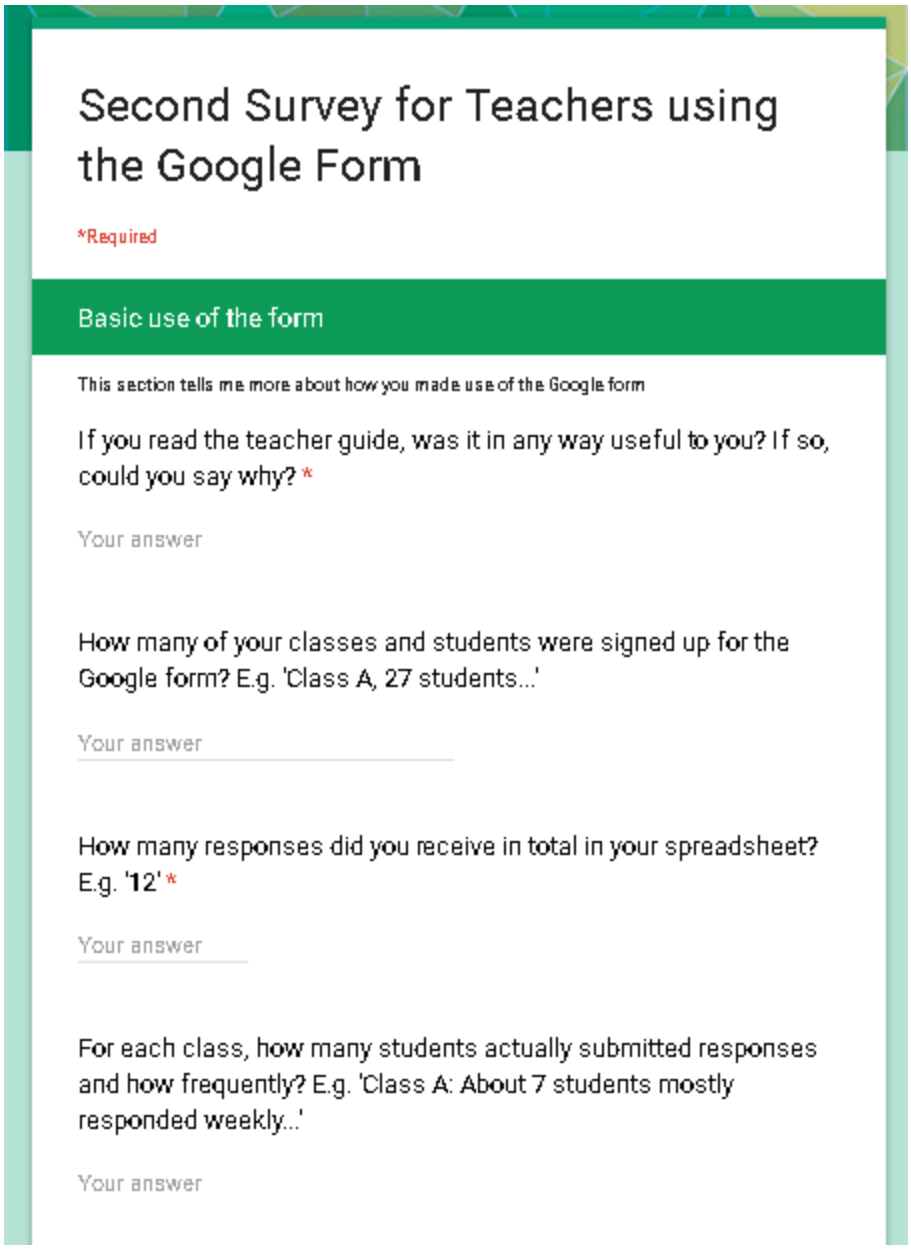
Which option describes your use of the Google form best? *

- I have used it up until now and I will continue to use it
- I have used it up until now but I probably will not use it after this study.
- I stopped using it before taking this survey
- I was unable to get it to work
- Other:

NEXT

Page 1 of 6

Never submit passwords through Google Forms.



Second Survey for Teachers using the Google Form

*Required

Basic use of the form

This section tells me more about how you made use of the Google form

If you read the teacher guide, was it in any way useful to you? If so, could you say why? *

Your answer

How many of your classes and students were signed up for the Google form? E.g. 'Class A, 27 students...'

Your answer

How many responses did you receive in total in your spreadsheet? E.g. '12' *

Your answer

For each class, how many students actually submitted responses and how frequently? E.g. 'Class A: About 7 students mostly responded weekly...'

Your answer

How often did you use the Google form?

Your answer

What kinds of prompts/questions did you use mostly and when? E.g. 'I required all students to respond to one homework prompt once a week and...'

Your answer

How long have you been using the Google form up until now? E.g. 'I used it for two weeks in January but then stopped...'

Your answer

If at all, how did you get your students to start using it? In other words, how did you introduce them to the Google form and get them comfortable with using it?

Your answer

What kind of student responses were useful to you and why? You may paste specific student responses or speak in general, without identifying students directly.

Your answer

BACK NEXT Page 2 of 6

Never submit passwords through Google Forms.

Second Survey for Teachers using the Google Form

*Required

The Five Practices

This section contributes to research on teachers perception of the influence of this Google form on their practice, specifically student-teacher interaction like classroom discussions.

How would you say, if at all, did the Google form influence your ANTICIPATION of student responses? *

'Anticipation' is used to refer to the teacher's ability to foresee how students might solve a problem, interpret a problem, or what difficulties or strategies your students might encounter or use.

	1	2	3	4	
No influence	○	○	○	○	Strong influence

If you answered 2, 3, or 4 please briefly state how you perceive the use of this form to have influenced your ANTICIPATION

Your answer

How would you say, if at all, did the Google form influence your MONITORING of student responses? *

'Monitoring' is used to refer to the teacher's ability to pay attention student s' mathematical thinking, strategies, and errors which makes it possible to decide what and who to make focal during classroom discussions.

	1	2	3	4	
No influence	○	○	○	○	Strong influence

If you answered 2, 3, or 4 please briefly state how you perceive the use of this form to have influenced your MONITORING

Your answer

How would you say, if at all, did the Google form influence your SELECTING of student responses? *

'Selecting' is used to refer to the teacher's ability to decide which student responses to incorporate into classroom discussions.

	1	2	3	4	
No influence	○	○	○	○	Strong influence

If you answered 2, 3, or 4 please briefly state how you perceive the use of this form to have influenced your SELECTING

Your answer

How would you say, if at all, did the Google form influence your SEQUENCING of student responses? *

'Sequencing' is used to refer to the teacher's ability to decide on the *order* in which to make use of selected student responses during classroom discussions..

	1	2	3	4	
No influence	○	○	○	○	Strong influence

If you answered 2, 3, or 4 please briefly state how you perceive the use of this form to have influenced your SEQUENCING

Your answer

How would you say, if at all, did the Google form influence your CONNECTING of student responses? *

'Connecting' is used to refer to the teacher's ability to draw connections between student responses in classroom discussions.

	1	2	3	4	
No influence	○	○	○	○	Strong influence

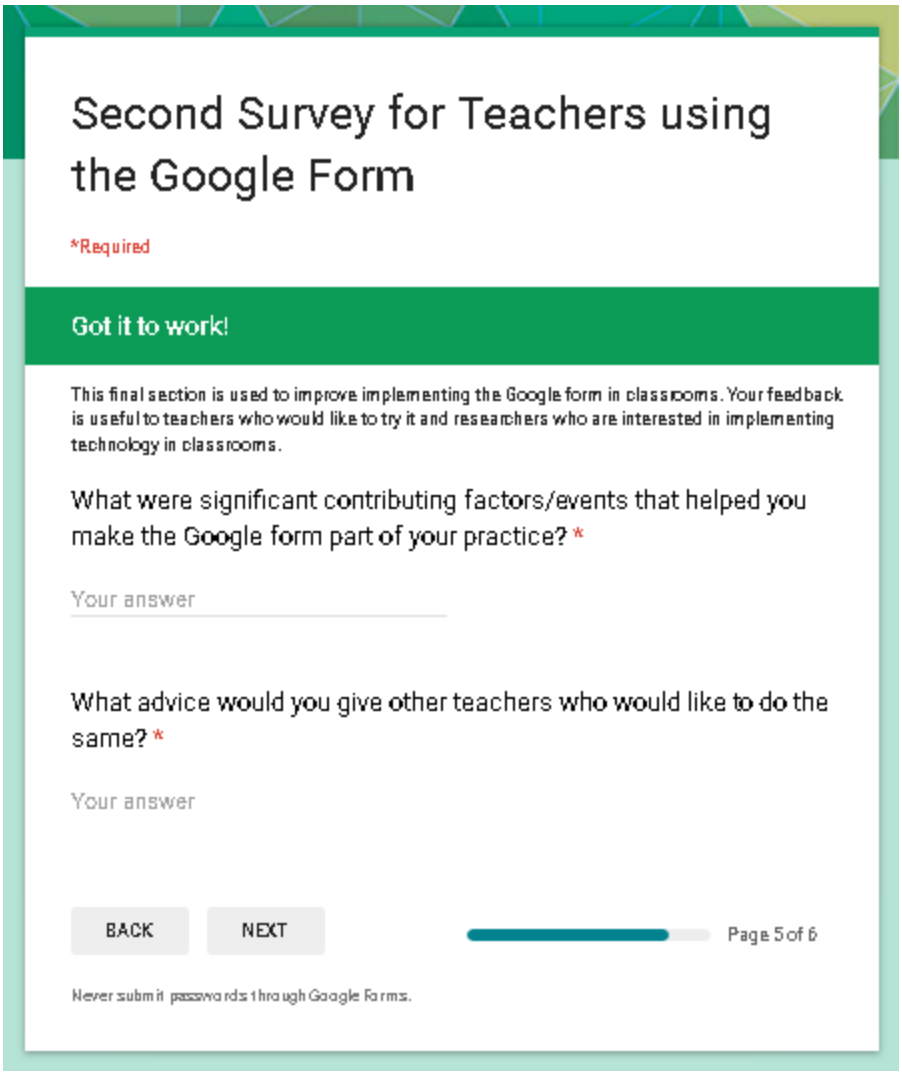
If you answered 2, 3, or 4 please briefly state how you perceive the use of this form to have influenced your CONNECTING

Your answer

BACK NEXT

Page 3 of 6

Never submit passwords through Google Forms.



Second Survey for Teachers using the Google Form

*Required

Got it to work!

This final section is used to improve implementing the Google form in classrooms. Your feedback is useful to teachers who would like to try it and researchers who are interested in implementing technology in classrooms.

What were significant contributing factors/events that helped you make the Google form part of your practice? *

Your answer

What advice would you give other teachers who would like to do the same? *

Your answer

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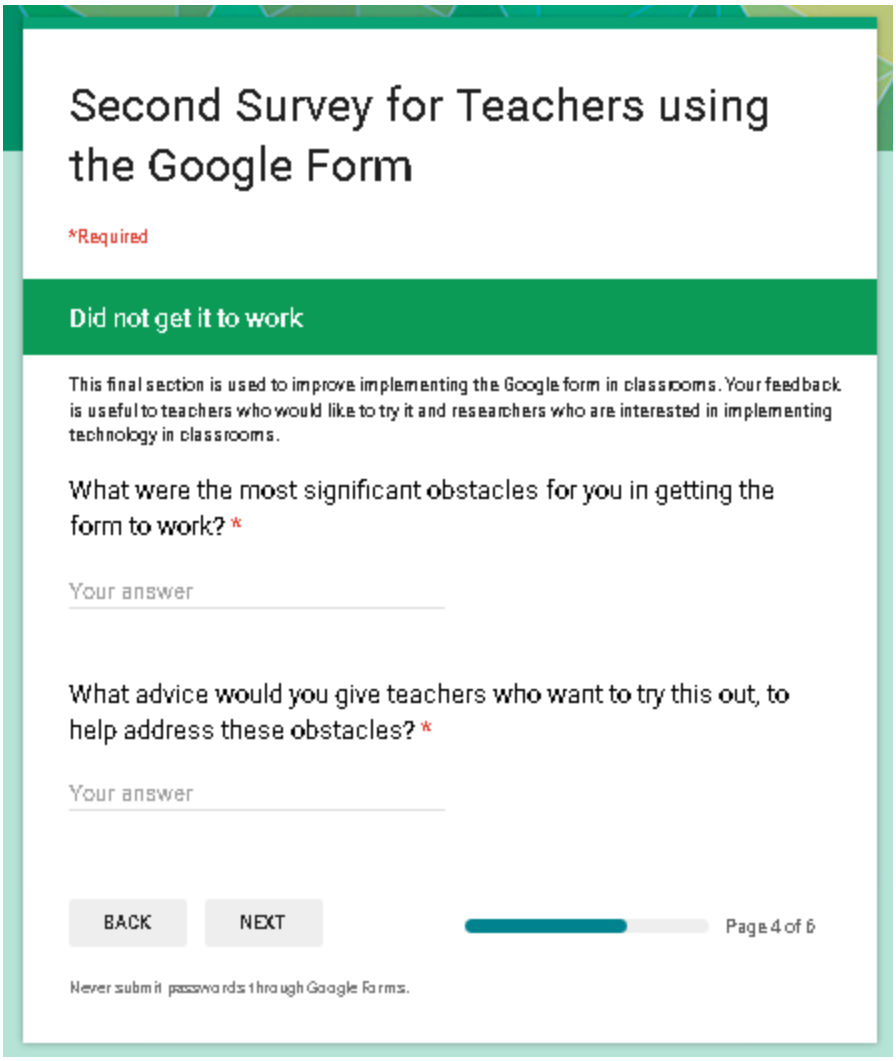

Second Survey for Teachers using the Google Form

*Required

Did not get it to work

This final section is used to improve implementing the Google form in classrooms. Your feedback is useful to teachers who would like to try it and researchers who are interested in implementing technology in classrooms.

What were the most significant obstacles for you in getting the form to work? *

Your answer

What advice would you give teachers who want to try this out, to help address these obstacles? *

Your answer

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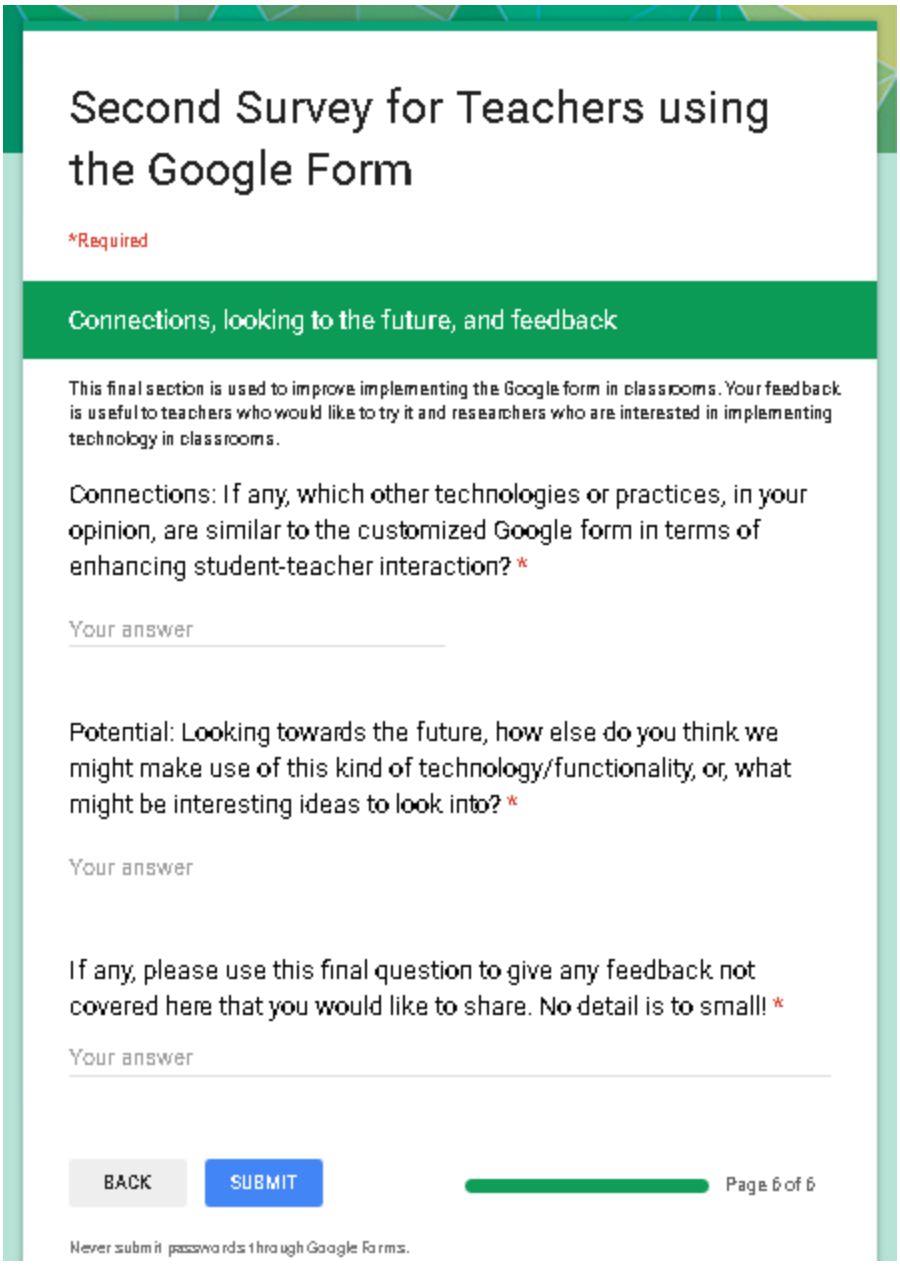

Second Survey for Teachers using the Google Form

*Required

Connections, looking to the future, and feedback

This final section is used to improve implementing the Google form in classrooms. Your feedback is useful to teachers who would like to try it and researchers who are interested in implementing technology in classrooms.

Connections: If any, which other technologies or practices, in your opinion, are similar to the customized Google form in terms of enhancing student-teacher interaction? *

Your answer

Potential: Looking towards the future, how else do you think we might make use of this kind of technology/functionality, or, what might be interesting ideas to look into? *

Your answer

If any, please use this final question to give any feedback not covered here that you would like to share. No detail is to small! *

Your answer

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APPENDIX C

PARTICIPANTS REPORTED INTENTIONS

- Improve justification of responses
- Possibility on feedback from pupils
- Immediate feedback answers from students
- To help the silent learner to ask questions
- Getting every student involved in a safe way.
- More efficient way to analyze formative assessments
- to see if I can get more communication going from my students
- It will provide immediate feedback to me on my student's progress.
- In situations where teachers do not have a lot of administration work to do
- It seems like a great way to get students talking about their Mathematical reasoning.
- I think it will promote student feedback as well as provide timely responses to be used in the moment.
- I think it will be useful to get answers from the quieter students who don't normally speak up in class.
- To award an opportunity for learners to receive additional support as well as increase productivity of contact time
- To improve communication between teacher and student, to assist learner who do not have confidence to ask questions in class
- To identify misconceptions of concepts, need for revisit of a topic or acceleration, see if the class as a whole is prepared to move on.
- I would like to get feedback on what topics the students are struggling with in class and on their homework to promote more beneficial class discussions.
- I think the students will be more apt to provide questions and comments. Doing so will also provide me more of an opportunity to use questions from HW in class or through email.
- Giving learners who are too shy or scared to ask questions in class the chance to ask questions and building a better individual relationship with students and to improve on lesson planning when knowing specific needs of students.
- My students always seem to enjoy using their phones during class, so I think that incorporating their cell phones in a useful way will increase student involvement and hopefully that will increase student understanding and mastery of material.
- I teach a remedial Mathematics class and any chance I can take to engage my students in authentic writing and reading when it comes to Mathematics I try to take advantage of. I believe that this could be a beneficial part of my classroom that will help students ask questions in a comfortable atmosphere as well as write about their thought process.

[i] Adkins on the universal dog: "If I see a particular dog, its uniqueness stands out. After seeing five dogs, the uniqueness of each begins to fade. At a hundred dogs, it becomes exceedingly difficult to talk about individual dogs. In order to compensate for my limits here I invent the universal "dog," which refers to all dogs indeterminately. There are two problems with this. First, the universal lies. It cannot capture the uniqueness of individual dogs, and the fact of the matter is we do not imagine a universal dog; we only have images of particular dogs. Second, our universals are dependent on our bodily disposition. If I am inclined to like bigger dogs with short but not curly hair, my universal will reflect that. If another person is inclined to fear dogs and think them vicious and dirty, his universal will reflect that. The fact that universals obscure what they claim to explain and that different people produce different universals is the reason why so much controversy exists. We argue by defending our universal against others. But, even if we win the argument, we've gained nothing. We've only proved that one figment of the imagination is superior to another figment of the imagination."

[ii] This tension can also be seen between Spinoza and Descartes: Cartesian dualism is concerned with identity, and that in a manner that proceeds in classification through binaries: a thing is either A or B, and if it is B then it is of B1 or B2, and so on. This imposition of taxonomies and hierarchies of classification is dependent on being able to say what a thing is; Spinoza never cared much for what a thing is, he cared for what a thing does. From this point of view one might be more ignorant of classification of genus and species and more concerned with affect: a workhorse does not have the same capacity to be affected as a racehorse, but rather has affects in common with the ox (D. Smith, 2012, p. 154): Spinoza insists that we know not what a body can do, what it is capable of; Deleuzian philosophy asks not what a thing is, but what does it do? And, more importantly, what might become of it (Holland, 2013, p. 53)?

[iii] Smith (2006) offers the example of the line, suggesting that ‘‘Euclidean geometry defines the essence of the line in purely static terms that eliminate any reference to the curvilinear (‘a line which lies evenly with the points on itself). Problematics, by contrast, found its classical expression in the ‘operative’ geometry of Archimedes, in which the straight line is characterized dynamically as ‘the shortest distance between two points’’’ (Smith 2006, p. 148). We can see that the Archimedean definition allows for a multiplicity of curvilinear paths between the points, and defines straightness, rather than line, in terms of distance travelled or movement (de Freitas 2012a). Smith suggests that such a definition marks the line as a continuous operation and a ‘‘process of alignment’’. Similarly, the circle is a process of rounding, the square a process of quadrature, and so on. ‘‘In problematics, a figure is defined dynamically by its capacity to be affected—that is, by the ideal accidents and events that can befall the figure (sectioning, cutting, projecting, folding, bending, stretching, reflecting, rotating, and so).’’ (Smith 2006, p. 149). Axiomatics, however, prevailed over problematics (through Euclid) as a ‘‘triumph of the rectilinear over the curvilinear’’, and a triumph of essence over event, the latter being seen as the deterioration of the essential form. Desargues’ attempt to develop a Mathematics of the problem-event, for instance, in his Draft Project of an Attempt to Treat the Events of the Encounters of a Cone and a Plane, was opposed and marginalized by the algebraic or analytic geometry of Fermat and Descartes (Smith 2006) (Freitas, 2012, p. 583).