The Implications of Using Models of Direct Democracy for Cases of Representative Democracy.

by

Robert A. Ragan

(Under the direction of Robert Grafstein)

Abstract

Representative democracy translates the preferences of the electorate into policy outcomes. Individual voters do not directly vote on policy; rather, their elected representatives create and establish policy. How well does representative democracy translate the preferences of the electorate into policy? Is there any systematic bias in a representative democracy system? I examine the policy implications of a representative democracy system itself absent these other effects. To explore this question, I have formulated a series of computational models that calculate policy outcomes of both a direct democracy and a representative democracy system. The results allow me to isolate any systematic deviation between the two systems. I find that there are two main factors that cause the policy outcomes of a representative democracy system to deviate from direct democracy outcomes. The first is the distribution of preferences across the general population of voters. The second is the degree to which legislative districts are "gerrymandered". When population preferences are normally distributed and there is little gerrymandering of districts, there is little difference between the policies predicted under representative democracy and direct democracy. However, when the preferences in the population are calibrated using the distribution of income in the U.S. and a moderate degree of gerrymandering is allowed then the representative and direct democracy outcomes diverge substantially.

INDEX WORDS: political science, congress, computational models, redistricting, gerrymandering, direct democracy, representative democracy, redistribution

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Robert A. Ragan

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Robert A. Ragan

Approved:

Major Professor: Robert Grafstein

Committee: Keith Dougherty
Jamie L. Carson

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
December 2007

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Chapter 1

Introduction

Representative democracy translates the preferences of the electorate into policy outcomes. Individual voters do not directly vote on policy; rather, their elected representatives create and establish policy. How well do the mechanisms of representative democracy translate the preferences of the electorate into policy? Is there any systematic bias in a representative democracy system? There are several lines of research that examine the degree to which various components of a representative democracy system might move policy outcomes away from those policies preferred by a majority of the population. Research on special interest groups examines how groups lobby to move policy toward their ideal points and away from the electorate's (Ainsworth 2002, Grossman & Helpman 2001). Another line of research examines how intra-legislative dynamics and the interplay between legislatures and executives may influence policy outcomes.[1] There is work in the congressional literature that emphasizes the role of parties in influencing policy outcomes (Cox & McCubbins 2005, Aldrich 1995). This paper has a different focus. I examine the policy implications of a representative democracy system *itself* absent these other effects.[2] To explore this question, I have formulated a series of computational models that generate policy outcomes of both a direct democracy and a representative democracy system. The results show that it takes only a moderate level of gerrymandering (with respect to income) to generate representative democracy policy outcomes that deviate significantly from the direct democracy outcome.

[1] See for example Weingast & Marshall (1988), Cox & McCubbins (2005) and Krehbiel (1998).

[2] It should be noted that there are possibly "distortions" in the direct democracy outcome itself. If the voting population is not a random sample of the entire populace, then the policy outcomes of direct democracy may not conform to the preferences of the median member of the population. For more on this, see Boix (2003) p.172-200.

The findings are important because a direct democracy model is often used as the model of policy formation in social science research. This modeling convention is common within the political economy literature, beginning with Romer's (1975) work on the political determination of the parameters of an income tax. Modeling representative democracy systems using direct democracy was firmly established in the literature with Meltzer & Richard's (1981) model of the "rational" size of government. The Meltzer and Richard model assumes a direct democracy median voter model of policy making. The further below the mean income that the median voter finds themselves, the more redistribution they prefer. The model assumes that the policy making system translates the preferences of the median voter into policy in an unbiased way. However, when one introduces some of the basic machinery of representative democracy, such as single member districts, the preferences of the median voter may not always be reflected in the policy outcome.

A brief example will help illustrate the phenomenon. Assume that the entire voting population consists of 9 voters with policy preferences with respect to the level of income redistribution on the line [0,100]. Their policy preferences are {25, 32, 33, 44, 49, 51, 56, 66, 67}. Suppose these voters are divided into three congressional districts. Table (1.1) shows assignment of the voters to three districts. The median of each of the districts is also displayed.

	District 1	**District 2**	**District 3**
	33	44	67
	25	32	49
	51	66	56
Median	33	*44*	56

Table 1.1: Example Districts

Each district elects a representative to the legislature whose policy preferences correspond to the ideal point of the median voter in that district. This mini-congress contains 3 members with ideal points at 33, 44 and 56. If the legislature uses a majority rule and open rule system, the policy outcome will be 44. Now compare this result to a typical model of direct

democracy. Using a majority rule system, the policy outcome would be 51.[3] The magnitude of this difference and some of the conditions under which it occurs will be explored throughout this paper.

The implications of this deviation can be best seen when examined in light of a substantive issue. Meltzer and Richards' "Rational Theory" of the size of government, the preferences of the voters, with respect to income redistribution, are derived from their relative position in the income distribution. The median voter in the income distribution has an income below the mean. Meltzer and Richard use a direct democracy model, and a majority rule decision rule is applied. The level of redistribution is set at the median voter's ideal point. As seen in the example, this does not always occur in a representative system; the median voter's ideal point does not always become the policy. Rather, the direct democracy median voter votes for a representative to represent his or her district (as do all the other voters). The direct democracy median voter may not even be the median voter within their own district (like the voter at 51 in the example). Further, the elected representative from this median voter's district may not be at the median of the legislature. If the policy outcomes from the representative democracy model differ substantially from the direct democracy approach, then models like Meltzer and Richards' may benefit from including more institutional detail. In the remainder of this paper I seek to explore some of the conditions that can lead to the type of deviation seen in this example. What are the determinants of the deviation of the direct democracy and representative democracy policy outcomes? What institutional features and preference distributions would cause the direct democracy outcome and the representative democracy outcome to diverge? The computational results demonstrate that under certain stylized distributions of preferences and degrees of gerrymandering, the direct democracy and representative democracy outcomes diverge substantially. However, empirically it is difficult to find real deviations. This may be due to fact that the sufficient conditions for deviation found in the models are not present in the U.S.

[3]This also assumes single peaked preferences.

Using the ideal point of the median member of the population as the prediction of a policy outcome was probably never intended to represent reality. Rather, it was used as a simplifying assumption. This assumption lends a great deal of parsimony to the models using it. However, it may be clouding their predictions. The Meltzer and Richards' model has been found to predict policy outcomes that differ substantially from those empirically observed (Holsey & Borchering 1997, Gouveia & Masia 1998).[4] One source of the model's failure may be the assumption of a direct democracy policy formation rule.

The paper proceeds as follows: first, I review previous work examining the deviation of representative democracy outcomes from direct democracy outcomes. Next, I provide more examples of research analyzing representative democracies that use a direct democracy model of policy formation. I then formally present two computational models based on stylized preference distributions (uniform and normal). After discussing the results of these models I present implications for a particular policy area, income redistribution. I then discuss a set of empirical results and present their implications for the findings of the computational models.

[4]It should be noted that Meltzer and Richards' own empirical test of their theory finds evidence that supports their hypothesis (1983). Tullock (1983) claims that their empirical results are spurious.

Chapter 2

Literature

I will briefly review the existing literature that deals with the causes of divergence between the policy outcomes predicted by direct democracy and those observed in representative democracy. Additionally, I will discuss several papers that assume a direct democracy policy formation process while examining a representative democracy.

2.1 Causes of Divergence

Black's (1948) Median Voter Theorem[1] assumes that, under certain assumptions, electorally motivated candidates will represent the preferences of the median voter. When using the theorem as a policy formation model, researchers are implicitly assuming that the direct democracy outcome and the representative democracy outcome are the same. This does not follow from the theorem. The same forces that drive outcomes toward the median in the voting population also drive representatives toward the median of their legislative districts and outcomes in a legislature toward the legislative median. There is no *a priori* reason to expect that the population median (direct democracy) and the legislative median (representative democracy) will be the same.

There are works that examine the deviation of legislative outcomes from the preferences of the electorate. Wittman (1977), Calvert (1985), and Alesina (1988) cite policy-oriented candidates as the source of such divergence. Erikson & Stimson (2002) argue that the policy-oriented goals of legislators is tempered by the public's "policy mood." Legislators must

[1]This theorem is often attributed to Downs (1957). Indeed the framework which gives rise to the theorem is often called the "Downsian" model. The theorem pre-dates Downs' work by almost a decade. It should also be noted that Black's model is simply the application of Hotelling's (1929) work on spatial location of firms.

weigh their approximation of this "policy mood" against their own (often more extreme) preferences.[2]

Beasley and Cote's "citizen candidate model" (1997, 1998) takes a different approach to explaining the deviation of representative democracy outcomes from direct democracy outcomes.[3] Representatives are elected by the voting population as a whole, and for any number of "citizen candidates" greater than one, they find that the policy outcome deviates from a median (direct democracy) outcome. This deviation is not a central concern of their work. Their model also does not take into account any affect of the filtering of preferences through legislative districts.

2.2 The Use of Direct Democracy Models of Policy Formation

How prevalent are models dealing with representative democracies that assume a direct democracy policy formation process? Within the political economy literature they are quite common. The aforementioned models by Romer (1975), Roberts (1977), and Meltzer & Richard (1981) were the first. In addition, the direct democracy model of policy making has been used to examine several other policy areas. Boadway & Wildasin (1989) use a direct democracy median voter model to analyze the determination of the level of social security. Perotti (1993) uses a median voter direct democracy policy making process in a two period non-overlapping generations model to examine the effects of redistribution on human capital investment. Mayer (1984) uses a direct democracy majority rule model to examine the formation of tariffs, with voters aligned along a single dimension according to their ownership of the factors of production.

[2]Erikson, MacKuen and Stimson argue that "people who think about public controversies for a living are more likely to arrive at relatively extreme positions than is the amateur electorate."

[3]Beasley and Cote refer to this as the Downsian model. They are referring to a median voter model of the general population, which is equivalent to a direct democracy model of policy formation.

Chapter 3

The Computational Models

Instead of using a purely mathematical approach to solving this problem I have taken a computational approach. There is no reason that the models could not have been formulated using generalized proofs. However, rather than proving that there is one sufficient set of parameters that can generate a deviation between representative and direct democracy, I am able to explore the entire parameter space of the models. In this case there are three parameters that can vary across and within the models. I am able to examine every possible combination of these three parameters. Taking one (or two or three) parameter vectors and solving for the deviation of the representative and direct democracy outcome would suffer from the "curse of dimensionality".[1] The model would only be telling us something about the particular vectors of parameters chosen. There are 3 parameters in this model. The first has 3 possible values, the second has 435 possible values and the third has 50. This means that there are 65,250 possible versions of the model. Following the prescription put forth by de Marchi (2005) for overcoming the "curse of dimensionality", rather than picking one or a few of these vectors I compute them all.[2]

Each of the first two models represents the underlying distribution of voters in a different way (uniform and normal). In each model, a single dimensional policy space $X \in \Re^1$ on the interval [0,100] is assumed. The interval reflects the set of feasible alternatives. Each of the voters in the model v_i has an ideal point with symmetric[3] and single peaked utility. This implies that they prefer alternatives closer to their ideal point to alternatives further

[1] This is a concept first put forth by Bellman (1961).

[2] Even with the powerful computers available today each model took over a week to run.

[3] $U(x+l) = U(x-l)\ \forall l$.

away. Voters are assumed to vote sincerely for their preferred alternative. In the Mayhewian tradition, I have assumed that the representatives are single minded seekers of re-election (1974) and have no policy preferences of their own. Representatives also vote for alternatives closest to their ideal point when choosing a policy.[4] There is no uncertainty for the voters or representatives in any of the models.

This set of assumptions may seem overly restrictive. In seeking to explain the divergence between the direct democracy and representative democracy outcomes due to the representational mechanism itself, it is necessary to rule out utility functions that are not well behaved as the source of the divergence. Allowing for utility functions that violate these assumptions would make it easier, not harder, to find divergence between the two systems. In addition, factors such as lobbying and other special interest pressure are not included in the models.[5]

The goal of this exercise is to determine the conditions under which the mechanics of a representative system will produce policy outcomes that differ from outcomes generated by a direct democracy system. There will be three parameters that vary across the models. The first parameter is the distribution of policy preferences among the population. The second is the process used to assign voters to legislative districts, and the last is the number of districts into which the population is divided.

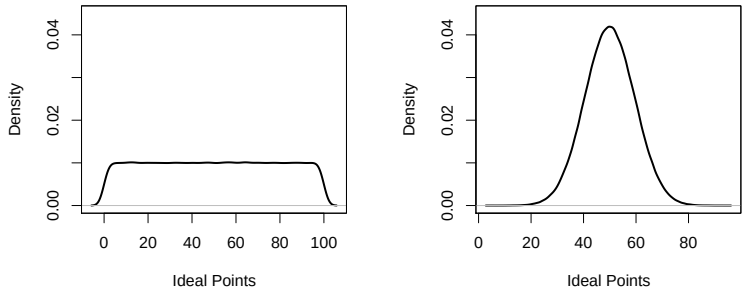
3.1 Distribution of Preferences

Each of the models will use a different distribution of voter preferences. The uniform (Figure 3.1a) and normal (Figure 3.1b) distributions will serve as the baseline models. In the absence of any information about the true distribution of preferences, these distributions seem like

[4]Allowing policy motivated candidates is possible; however, it would make it much easier to find divergence.

[5]The effect of these pressures on the deviation of the direct democracy and representative democracy outcomes is likely mixed. In some cases, these pressures may serve to bring the representative democracy outcome closer to the direct democracy outcome, and, in some cases, these pressures may exacerbate the difference.

reasonable choices. When I apply the models to income redistribution policy, I will use a more empirically informed distribution of preferences.



(a) Uniform (b) Normal

Figure 3.1: Distribution of Voters

3.2 District Formation Process

In order to explore the degree to which gerrymandering may affect the deviation of the representative and direct democracy outcomes, a district formation process is constructed. In the baseline case, voters are assigned to districts randomly. However, Congressional districts are not drawn randomly, and their construction is subject to the political process. Political parties attempt to construct districts in order to maximize the representation of voters predisposed to vote for them (Cox & Katz 2002). Parties do this by creating legislative districts that have a solid majority of "their" voters and then filling the remainder of the district with other voters (Erikson 1972, Owen & Grofman 1988). To model the effect that gerrymandering has on the deviation of direct and representative democracy outcomes each of the three models is run over a continuum of district formation rules ranging from "no gerrymandering" to "extreme gerrymandering". In the "no gerrymandering" case, voters are assigned to legislative districts in a random manner. In the "extreme gerrymandering" case, the districts are built as to maximize the influence of one of the parties in the legislature.

The direction of gerrymandering will benefit the liberal party (the one who prefers more redistribution).[6] The results from this continuum of district formation rules can give some insight into the effect of gerrymandering on policy outcomes.

3.3 Number of Legislative Districts

The number of legislative districts may affect how far the direct democracy outcome deviates from the representative democracy outcome. In order to explore the effect of the number of districts on the deviation of direct and representative democracy outcomes, each of the three models is run for all possible numbers of districts between 1 and 435.

3.4 The Model

1. A population of voters $\boldsymbol{v}$ is created.
2. The population of voters is then split into D districts using a gerrymandering algorithm (described in detail later).
3. The districts "elect" a representative using majority rule. r_i =median(d_i) is elected as the representative.
4. The set of representatives is assembled into a legislature: $\boldsymbol{l} =< r_i ... r_D >$, and vote on the policy.
5. Using a "majority rule" voting rule, the representative democracy outcome is: p_{rd} =median($\boldsymbol{l}$).
6. Steps 2-5 are run for 1,000 iterations, and the average policy outcome is found $\bar{p}_{rd}$
7. The direct democracy policy outcome is calculated as: p_{dd} =median($\boldsymbol{v}$)

[6]The normal and uniform distributions are symmetric, and therefore a version of the models with a conservative bias (less redistribution) is unnecessary.

3.4.1 The Gerrymandering Algorithm

Each of the models is run over a continuum of parameters for the degree of gerrymandering in the model. This is operationalized by designating 50%+1 of the districts as the "minimum winning coalition" for the legislature. For the "extreme gerrymandering" case, 51% of voters in each of the minimum winning coalition districts are partisans. The remainder of the voters in those districts are randomly assigned. In addition, the remainder of districts are constructed by randomly assigning voters. The degree of gerrymandering is determined by how many partisans are packed into the minimum winning coalition districts. The "no gerrymandering" case is simply random assignment of voters to these districts. In the other cases, the districts are rigged with increasing severity, in order to give one of the parties an advantage.

The annotated psuedocode for this algorithm is as follows (the actual code is presented in Appendix 1):

1. Determine how many districts constitute a minimum winning coalition in the legislature based on the current parameters of the model.

   C=roundedup(D/2)

2. Determine how many voters will be in each district given the current parameters of the model.

   S=N/D

3. Determine how many voters are needed to "control" a district.

   K=ceil(S/2)

4. The number of voters needed to control the minimum winning coalition in the legislature is the product of the two values found above.

   F=C*K

5. Now I create the districts. For the each of the minimum winning coalition districts (C), I draw $g * K$ voters from the liberal side of the distribution of voters.

   (a) g here represents the degree of gerrymandering in the current model. For the "no gerrymandering" case, $g = 0$. In the next case after "no gerrymandering", $g = 1/100$, and in the next, $g = 2/100$ and so on. For the "extreme gerrymandering" case, $g = 50/100$, half of the voters in each of the minimum winning coalition districts are partisans.

6. The remainder of the voters in the minimum winning coalition districts are assigned by random draws from the distribution of voters.

7. The remainder of districts (the non minimum winning coalition districts) are created using random assignment.

An example of the gerrymandering algorithm appears in Figure (3.2).

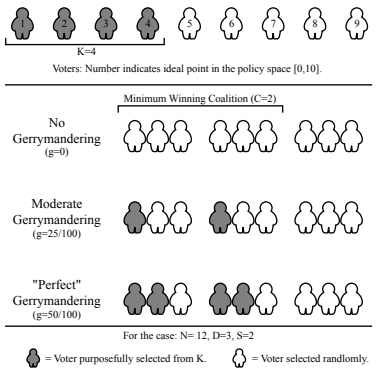


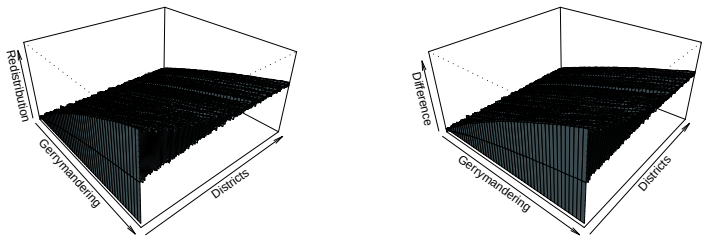
Figure 3.2: Example of Gerrymandering Algorithm

In this example, there is a set of twelve voters who are allocated among three districts. Following the algorithm, the number of voters needed to execute a "perfect" gerrymander is found (K=4). In the no gerrymander case, the voters are all allocated randomly. In the moderate gerrymander scenario, for each of the minimum winning coalition districts, one

($g * K = 25/100 * 4 = 1$) of the voters in each district is purposefully selected from the K section of the distribution of voters. In the "perfect gerrymander" scenario, for each of the minimum winning coalition districts, two ($g * K = 50/100 * 4 = 2$) of the voters in each district is purposefully selected from K. For a given N and D the variable g in the gerrymandering algorithm dictates how many of the voters in a given minimum winning coalition district are from one of the extremes.

3.4.2 Uniform Model

The Uniform model uses a uniform distribution of voter preferences $\sim U[0, 100]$. The direct democracy model returns the median of the distribution of voters, which is 50. The results of the representative democracy model can be seen in figures (3.3a) and figure (3.3b).

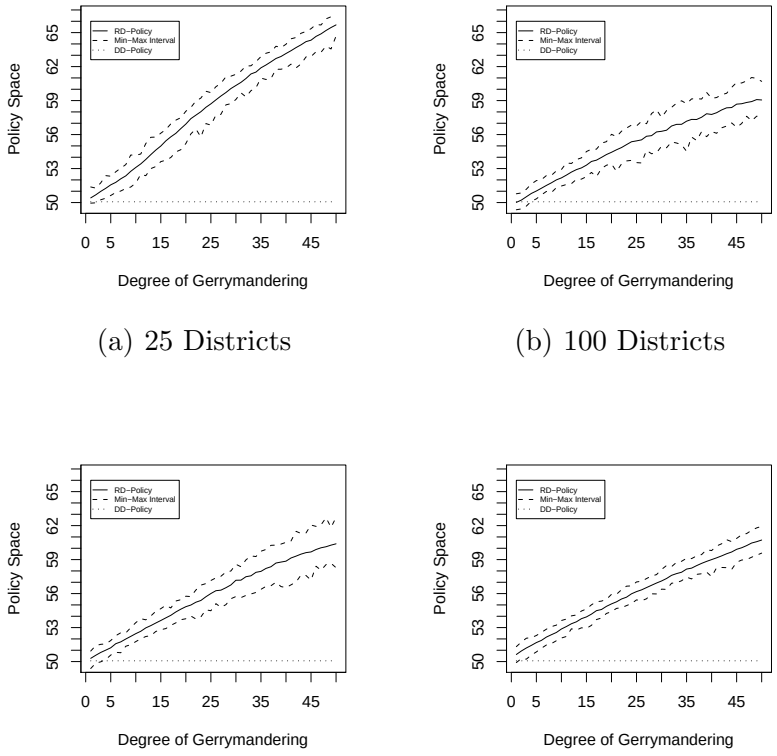


(a) Rep. Dem. Policy (b) Deviation of Rep. and Direct

Figure 3.3: Uniform Model Results

In figure (3.3a) we can see that the policy predicted by the representative democracy model is greatly affected by the level of gerrymandering in the system. Figure (3.3b) graphs the differences between the direct democracy outcome of 50 and the representative democracy policy predictions.

To better illustrate the effect of gerrymandering and the number of districts and separately, several cross sections are taken from the graph above. First, the effect of gerrymandering is examined. The cross sections in figures (3.4a), (3.4b), (3.4c) and (3.4d), look at the effect of gerrymandering on the system for 25, 100, 200 and 435 districts respectively.



(a) 25 Districts

(b) 100 Districts

(c) 200 Districts

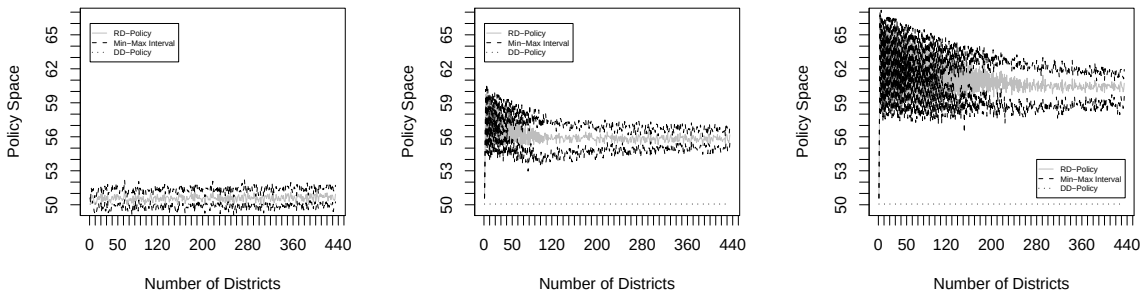
(d) 435 Districts

Figure 3.4: Effect of Gerrymandering on Uniformly Distributed Voters

Each figure also includes a confidence interval based on the maximum and minimum values found for each instance of the model.[7] The results demonstrate that for all but the lowest levels of gerrymandering, the representative democracy generates a higher policy outcome than is generated by the direct democracy model.

[7]Recall that each permutation of the model was run 1000 times, and the value reported as the representative policy outcome is the mean of these 1000 iterations. The maximum and minimum values reported are the maximum and minimum values found in these 1000 iterations.

To examine the effect of the number of districts, I take cross sections at three levels of gerrymandering. Figure (3.5) shows the effect in the presence of low ($g = 2$), medium (g=25) and high (g=50) levels of gerrymandering).



(a) Low Gerrymandering (b) Medium Gerrymandering (c) High Gerrymandering

Figure 3.5: Effect of the Number of Districts on a Uniform Distribution of Voters

The number of districts seems to have little effect on the policy outcome for any of the levels of gerrymandering. As D becomes large the difference between d and d+1 becomes infinitesimally small. The median of the median moves very little for a sufficiently large D. The "jagged" nature of the graphs are due to the representative democracy outcome "jumping" as the legislature size switches from even to odd numbers (Dougherty & Edward 2007). The "jumping" becomes less pronounced as the number of districts increases.

The effects of gerrymandering and the number of districts on the representative democracy outcome are summarized in Table (3.1).

Going across the rows in the table it is clear that there is very little movement in the representative democracy policy outcome. The difference between the representative democracy and direct democracy outcome also changes little looking across a row. This reflects what was seen in Figure (3.5). Looking down the columns, a different pattern emerges. Taking the 435 district case, we see that for low levels of gerrymandering the representative democracy

		25 Districts	100 Districts	200 Districts	435 Districts
Low Gerrymandering	Direct	50.06	50.06	50.06	50.06
	Representative	50.68	50.22	50.56	50.93
	Difference	0.62	0.15	0.49	0.87
	(Min,Max)	(49.98,51.28)	(49.44,50.85)	(49.86,51.41)	(50.26,51.72)
	Std. Dev.	0.32	0.29	0.31	0.29
Medium Gerrymandering	Direct	50.06	50.06	50.06	50.06
	Representative	58.71	55.44	56.01	56.18
	Difference	8.65	5.38	5.94	6.11
	(Min,Max)	(56.83,59.7)	(53.53,56.76)	(54.48,57.16)	(55.45,57.03)
	Std. Dev.	0.49	0.62	0.44	0.31
Extreme Gerrymandering	Direct	50.06	50.06	50.06	50.06
	Representative	65.68	59.05	60.39	60.73
	Difference	15.62	8.99	10.33	10.66
	(Min,Max)	(64.7,66.61)	(57.58,60.7)	(58.28,62.73)	(59.57,61.94)
	Std. Dev.	0.41	0.61	0.95	0.55

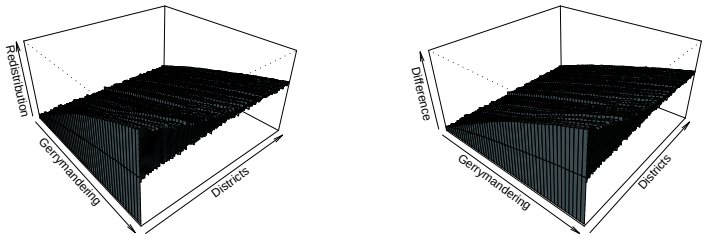
Table 3.1: Results of the Uniform Model

outcome begins close to the direct democracy outcome. However, for moderate levels of gerrymandering the policy moves a full 6 points (100 point scale) from the direct democracy outcome. For extreme levels of gerrymandering the policies diverge a full 10 points from the direct democracy case. These results correspond to the results seen in Figure (3.4).

3.4.3 Normal Model Results

The Normal model uses a normal distribution of voter preferences $\sim Normal(\mu = 50, \sigma^2 = 9.5)$. As with the Uniform model, the direct democracy model returns the median of the distribution of voters, which is 50. The overall results of the representative democracy model can be seen in Figure (3.6a) and Figure (3.6b).

In figure (3.6a) we can see that, as with the uniform model, the policy outcome is greatly affected by the level of gerrymandering in the system. The number of legislative districts again proves to have little impact on the outcome. The difference between the direct and representative democracy policy outcomes is graphed in Figure (3.6b).



(a) Rep. Dem. Policy (b) Deviation of Rep. and Direct

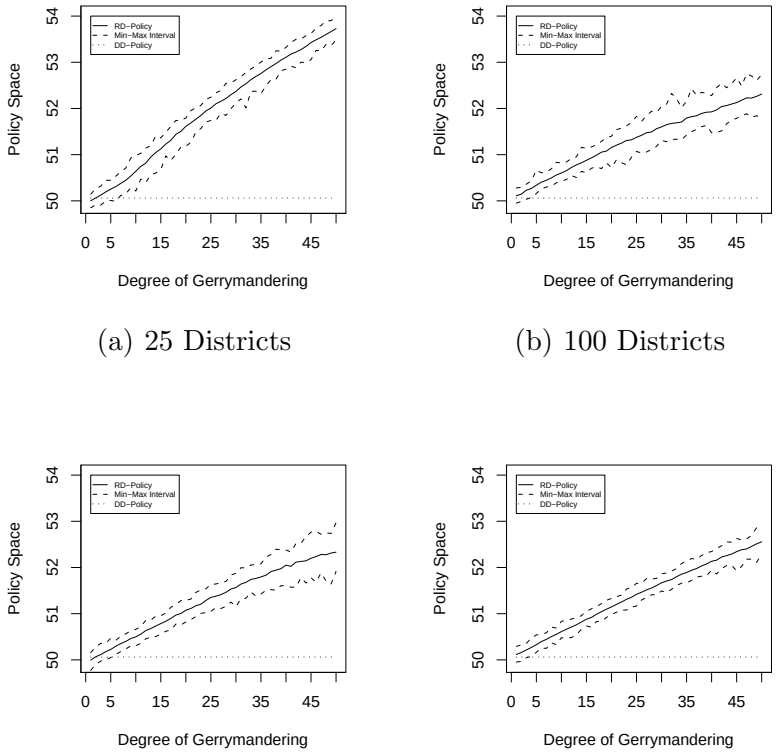
Figure 3.6: Normal Model Results

Again, several cross sections of the three dimensional graph are presented to examine the effects of the number of districts and the degree of gerrymandering separately. The cross sections in figure(3.7) look at the effect of gerrymandering on the system for 25, 100, 200 and 435 districts.

As with the cross section from the Uniform model, each figure also includes a confidence interval based on the maximum and minimum values found for each instance of the model. Again, for all but the lowest levels of gerrymandering, we find that the representative democracy generates a higher policy outcome than is generated by the direct democracy model.

To examine the effect of the number of districts, I again take cross sections at three levels of gerrymandering. Figure (3.8) shows the effect in the presence of low, medium and high levels of gerrymandering.

The results of the Normal model are similar to those from the Uniform model. The number of districts seems to have little effect on the policy outcome for any of the levels of gerry-



(a) 25 Districts

(b) 100 Districts

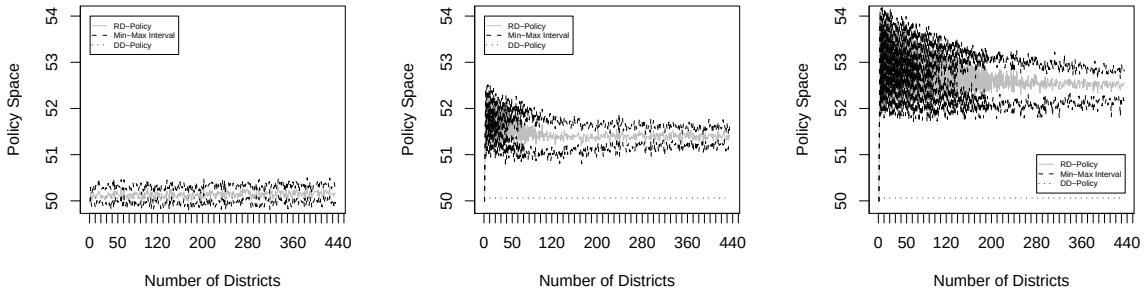
(c) 200 Districts

(d) 435 Districts

Figure 3.7: Effect of Gerrymandering on Normally Distributed Voters

mandering. The effects of gerrymandering and the number of districts on the representative democracy outcome are summarized in Table (3.2).

Looking down the columns, the effect of gerrymandering is seen, although the results are less pronounced than in the Uniform model. For the 435 district case at low levels of gerrymandering, the representative democracy outcome is close to the direct democracy outcome (a difference of .1). For moderate levels of gerrymandering, the policy moves one point from the direct democracy outcome. Lastly, for extreme levels of gerrymandering, the policies diverge about 2.5 points from the direct democracy case. These results correspond



(a) Low (b) Medium (c) High

Figure 3.8: Effect of the Number of Districts on a Normal Distribution of Voters

to the results seen in Figure (3.7). Going across the rows in the table, it is clear that, just as in the uniform case, the number of districts has little effect on the representative democracy policy outcome. This reflects what was seen in Figure (3.8).

		25 Districts	**100 Districts**	**200 Districts**	**435 Districts**
Low	Direct	50.06	50.06	50.06	50.06
Gerrymandering	Representative	50.06	50.15	50.07	50.16
	Difference	0	0.08	0.01	0.1
	(Min,Max)	(49.91,50.26)	(49.99,50.29)	(49.9,50.28)	(49.97,50.33)
	Std. Dev.	0.07	0.07	0.08	0.07
Medium	Direct	50.06	50.06	50.06	50.06
Gerrymandering	Representative	52.01	51.38	51.35	51.42
	Difference	1.95	1.31	1.29	1.36
	(Min,Max)	(51.74,52.25)	(51.07,51.83)	(51.05,51.64)	(51.17,51.66)
	Std. Dev.	0.11	0.14	0.1	0.08
Extreme	Direct	50.06	50.06	50.06	50.06
Gerrymandering	Representative	53.73	52.31	52.33	52.56
	Difference	3.67	2.25	2.27	2.49
	(Min,Max)	(53.48,54)	(51.79,52.73)	(51.91,52.96)	(52.3,52.81)
	Std. Dev.	0.11	0.18	0.2	0.12

Table 3.2: Results of the Normal Model

Chapter 4

Income Redistribution

The above results are general and a bit abstract. What would the results look like if the models are used to look at a particular policy area? As previously discussed many formal models of redistribution use a median voter direct democracy framework. This policy area seems like a good choice for applying the model. All of the assumptions of the previous two models remain, save one. In this version of the model, empirical data is used to calibrate the distribution of preferences. Many models of redistribution that use a direct democracy policy formation process assume that voters' policy preferences are determined by their location in the income distribution.[1] If this assumption is retained, then a more empirically informed distribution of preferences is possible. To this end, I construct a distribution of preferences calibrated from the U.S. income distribution. Data on the U.S. income distribution was obtained from the U.S. Census Bureau's Current Population Survey for 2006.[2] In keeping with the literature (specifically the Meltzer and Richard framework), voters with more income prefer less redistribution, and voters with less income prefer more redistribution. The resulting distribution of preferences can be seen in Figure (4.1).

The gerrymandering algorithm remains the same; however, this distribution of preferences is not symmetric. As such, the effect of a conservative party (less redistribution) controlled gerrymandering algorithm will not be the mirror image of a process controlled by the liberal party (more redistribution). In order to take this into account, the gerrymandering algorithm is run both ways. First, the liberal party controls the creation of the districts (this is the

[1] Among these are the aforementioned Meltzer and Richard model.

[2] Table HINC-06

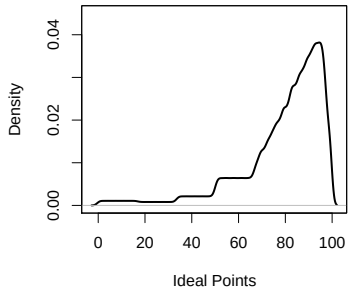
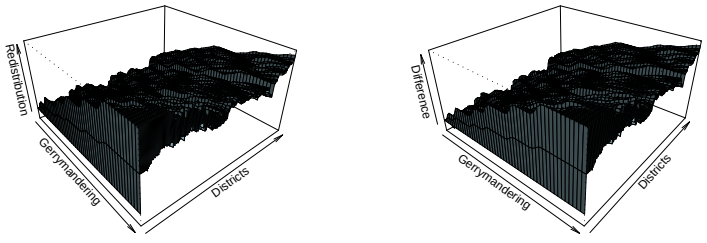


Figure 4.1: Distribution of Voters

analog of the gerrymandering used in the previous two models). Then the model is run with the conservative party controlling the process.

4.0.1 Liberal Gerrymander Results

The direct democracy model predicts a policy outcome of 83.47. The predictions of the representative democracy models can be seen in Figure (4.2a) and Figure (4.2b).



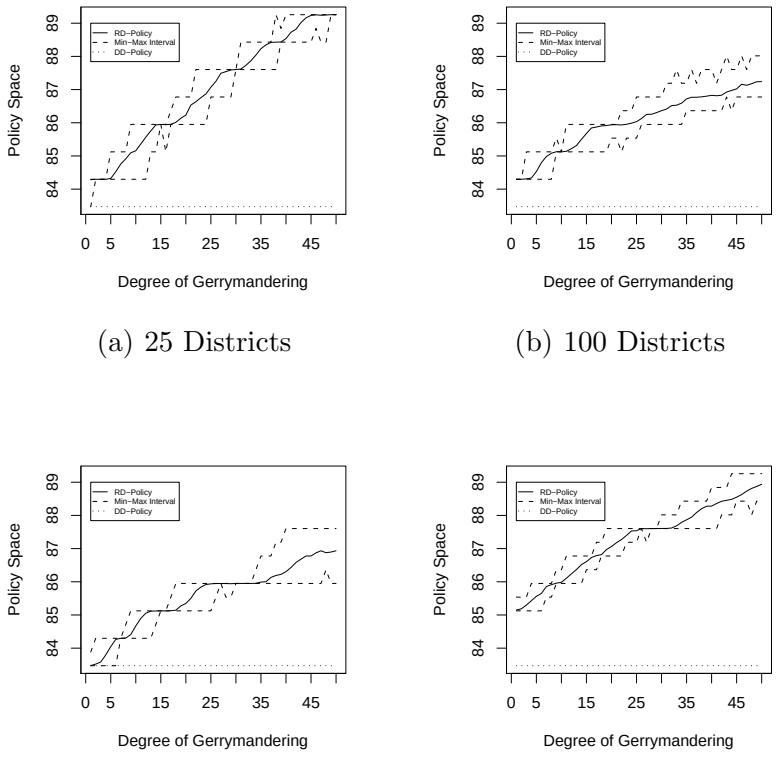
(a) Rep. Dem. Policy

(b) Deviation of Rep. and Direct

Figure 4.2: Liberal Gerrymander Model Results

In figure (4.2a) we can see that, as with the previous models, the policy outcome is greatly affected by the level of gerrymandering in the system. The number of legislative districts again has very little impact on the outcome. The difference between the direct and representative democracy policy outcomes is graphed in Figure (4.2b).

Cross sections of the three dimensional graph are again presented to examine the effects of the number of districts and the degree of gerrymandering separately. The cross sections in figure(4.3) look at the effect of gerrymandering on the system for 25, 100, 200 and 435 districts.



(a) 25 Districts

(b) 100 Districts

(c) 200 Districts

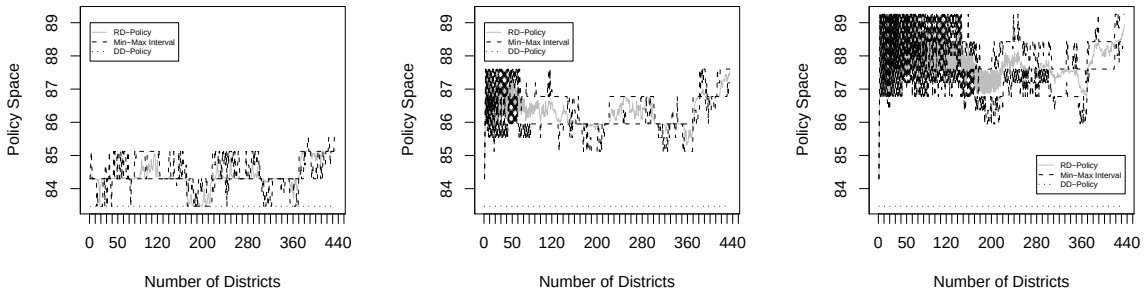
(d) 435 Districts

Figure 4.3: Effect of Liberal Gerrymandering on Income Calibrated Voters

As with the previous cross sections, each figure also includes a confidence interval based on the maximum and minimum values found for each instance of the model. For all but the

lowest levels of gerrymandering, we find that the representative democracy generates more redistribution than is generated by the direct democracy model.

To examine the effect of the number of districts, I again take cross sections at three levels of gerrymandering. Figure (4.4) shows the effect in the presence of low, medium and high levels of gerrymandering.



(a) Low (b) Medium (c) High

Figure 4.4: Effect of the Number of Districts on a Income Calibrated Voters

The number of districts seems to have no effect on the policy outcome for any of the levels of gerrymandering. A more detailed look at the effects of gerrymandering and the number of districts on the representative democracy outcome is presented in Table (4.1).

Looking down the columns, the effect of gerrymandering is seen. For the 435 district case at low levels of gerrymandering, the representative democracy outcome is already a full point higher than the direct democracy outcome. For moderate levels of gerrymandering, the representative policy is 4 points higher from the direct democracy outcome. Lastly, for extreme levels of gerrymandering, the policies diverge about 5.5 points. These results correspond to the results seen in Figure (4.3). Going across the rows in the table, it is clear that, just as in all the previous cases, the number of districts has little effect on the representative democracy policy outcome. This reflects what was seen in Figure (4.4).

		25 Districts	100 Districts	200 Districts	435 Districts
Low	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	84.3	84.3	83.52	85.19
	Difference	0.83	0.83	0.05	1.71
	(Min,Max)	(84.3,84.3)	(84.3,84.3)	(83.47,84.3)	(85.12,85.54)
	Std. Dev.	0	0	0.19	0.15
Medium	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	87.07	86.03	85.94	87.54
	Difference	3.6	2.56	2.47	4.07
	(Min,Max)	(86.78,87.6)	(85.54,86.78)	(85.12,85.95)	(87.19,87.6)
	Std. Dev.	0.39	0.19	0.09	0.15
Extreme	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	89.26	87.24	86.93	88.94
	Difference	5.79	3.77	3.46	5.47
	(Min,Max)	(89.26,89.26)	(86.78,88.02)	(85.95,87.6)	(88.43,89.26)
	Std. Dev.	0	0.18	0.31	0.29

Table 4.1: Results of the Income Calibrated Model with Liberal Gerrymandering

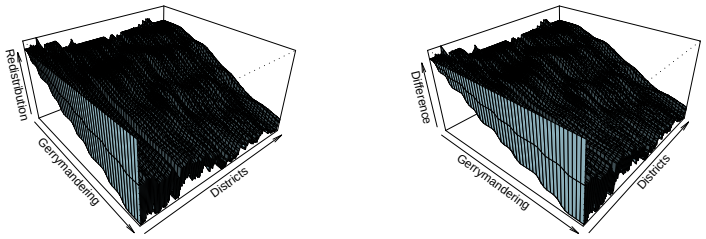
4.0.2 Conservative Gerrymander Results

The direct democracy model policy outcome remains the same as in the liberal gerrymandering case,83.47. The predictions of the representative democracy models can be seen in Figure (4.5a) and Figure (4.5b).

In figure (4.5a) we can see that, as with the previous models, the policy outcome is greatly affected by the level of gerrymandering in the system. However, this time the gerrymandering leads to less redistribution than the direct democracy model predicts. The number of legislative districts again has very little impact on the outcome. The difference between the direct and representative democracy policy outcomes is shown in Figure (4.5b).

Cross sections of the three dimensional graph are presented to examine the effects of the degree of gerrymandering[3]. The cross sections in figure(4.6) look at the effect of gerrymandering on the system for 25, 100, 200 and 435 districts.

[3]The district effect cross sections are omitted here, as the are similar to all the others in the paper. The Table (4.2) still reports the values for the district effect

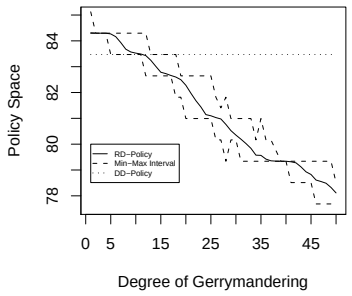


(a) Rep. Dem. Policy (b) Deviation of Rep. and Direct

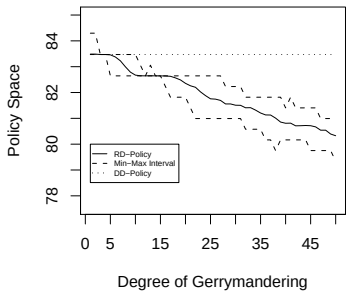
Figure 4.5: Conservative Gerrymander Model Results

Each figure includes a confidence interval based on the maximum and minimum values found for each instance of the model. For all but the lowest levels of gerrymandering, we find that the representative democracy generates a lower level of redistribution than is generated by the direct democracy model. A more detailed look at the effects of gerrymandering and the number of districts on the representative democracy outcome is presented in Table (4.2).

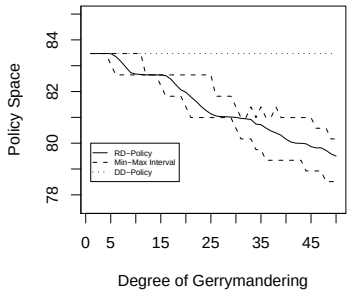
For the 435 district case at low levels of gerrymandering, the representative democracy outcome is identical to the direct democracy outcome. For moderate levels of gerrymandering the representative policy is 2.2 points lower (less redistribution) from the direct democracy outcome. Lastly, for extreme levels of gerrymandering, the policies diverge about -5 points. These results correspond to the results seen in Figure (4.6). Going across the rows in the table, it is clear that, just as in all the previous cases, the number of districts has little effect on the representative democracy policy outcome.



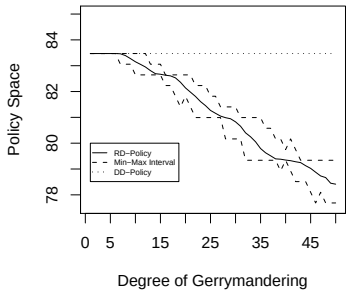
(a) 25 Districts



(b) 100 Districts



(c) 200 Districts



(d) 435 Districts

Figure 4.6: Effect of Conservative Gerrymandering on Income Calibrated Voters

		25 Districts	**100 Districts**	**200 Districts**	**435 Districts**
Low	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	84.3	83.49	83.47	83.47
	Difference	0.83	0.02	0	0
	(Min,Max)	(84.3,84.3)	(83.47,84.3)	(83.47,83.47)	(83.47,83.47)
	Std. Dev.	0	0.12	0	0
Medium	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	81.1	81.76	81.14	81.27
	Difference	-2.37	-1.71	-2.33	-2.2
	(Min,Max)	(80.99,82.64)	(80.99,82.64)	(80.99,82.64)	(80.99,81.82)
	Std. Dev.	0.3	0.4	0.31	0.27
Extreme	Direct	83.47	83.47	83.47	83.47
Gerrymandering	Representative	78.11	80.33	79.5	78.41
	Difference	-5.36	-3.14	-3.97	-5.06
	(Min,Max)	(77.69,78.51)	(79.34,80.99)	(78.51,80.17)	(77.69,79.34)
	Std. Dev.	0.4	0.4	0.47	0.47

Table 4.2: Results of the Income Calibrated Model with Conservative Gerrymandering

Chapter 5

Empirical Implications

The results of these computational models are in and of themselves interesting for social choice reasons. Changing the preference aggregation mechanism from one of direct democracy majority rule to representative democracy majority rule has the potential to significantly change the policy chosen. What are some of the empirical implications of these models? How likely are we to see the conditions that led to the policy deviation found in the models? Direct democracy models are often used as the model of policy formation in social science research. If the conditions for policy deviation are found empirically, then models like Meltzer and Richards' would benefit from including aspects of the representative democracy system in their models. In order to approach this question empirically, I will adopt a similar framework as Meltzer and Richard. I will make the assumption that a voter derives his or her preferences about income redistribution from his or her position in the income distribution. I will explore three different empirical implications of the computational models. The first will be to examine Congressional districts and see what the level of "gerrymandering" is within them with respect to income. Secondly, I will look at the impact of the number of districts on the deviation by looking at state legislatures. Lastly, I will compare the national median income in the population as a whole with the median income of the median Congressional district within the U.S. House of representatives.

5.1 "Gerrymandering"

All three of the models demonstrate that the level of gerrymandering in a system can affect the degree to which the direct democracy and representative democracy outcomes deviate

from one another. What level of "gerrymandering"[1] do we see with respect to income in U.S. congressional districts? In order to examine this, I look at the overall percentage of congressional districts that are primarily made up of voters drawn from one side of the income distribution (both high and low).[2] For the 108th Congress, 225 of 435 congressional districts have a majority of voters who's income falls below the national median family income). This of course implies that 210 of 435 districts have over half of their voters falling above or equal to the median family income. Figure (5.1a) shows the districts by the proportion of voters which fall below the national median income. There are enough districts that have half their voters coming from the lower half of the income distribution to form a minimum winning coalition in Congress.

Another way to think about the degree to which congressional districts are gerrymandered with respect to income is to examine the variance (standard deviation) of income within each district. Figure (5.1b) shows the districts by the standard deviation of income within each one.[3] The standard deviation of income within districts is on average $16,047.01. This is quite high considering that the median national family income for this time period is $50,046. This is evidence that many districts may "gerrymandered" with respect to income.

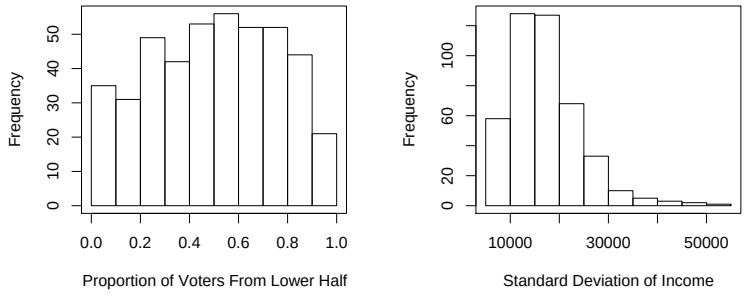
5.2 "Number of Districts"

The number of legislative districts does appear to have a small effect on the deviation of the representative and direct democracy outcomes in each of the models. In order to test the empirical implication of this model, a data set of state legislatures was assembled. The data

[1]It is not necessary for Congressional districts to have been intentionally drawn with respect to income. However, income is often correlated with other factors that parties do use when gerrymandering. Additionally, Tiebout sorting could lead to districts that appear to be gerrymandered (Kollman, Miller & Page 1997).

[2]U.S. Census tract level data on Family Income for the district boundaries for the 108th Congress are used.

[3]Census tract level data for 108th Congress.



(a) (b)

Figure 5.1: Empirical Measures of Income Gerrymandering

is at the state legislative district level and is compiled from the U.S. census bureau.[4] The following ordinary least squares regression was run.

$$Difference = \beta_0 + \beta_1 Districts + \epsilon \tag{5.1}$$

Here *Difference* is the difference between a state's median family income and the median income in the median legislative district. The state's median family income is a proxy for the preferences of the voter who would obtain their ideal point under direct democracy. The median family income of the state's median legislative district is a proxy for the voter who would obtain their ideal point under representative democracy. The *Districts* is the number of legislative districts that a state legislative chamber has. In Table (5.1), the results are presented for three specifications of this model. The first is a basic bivariate OLS; the second uses a Huber-White Heteroskedastic Robust Covariance Matrix to calculate the standard errors; and lastly the model is run with state level dummy variables to control for any

[4]Data is from the 2000 Census using current state legislative boundaries and 1999 dollars.

unobserved heterogeneity that may be present in a state's districting process that may be correlated with both *Districts* and *Difference*.

	OLS	**Huber-White OLS**	**Huber-White OLS**
Districts	-7.67 $*$	-7.67$*$	-4.84$*$
(t)	(-2.49)	(-3.00)	(-2.44)
Intercept	-611.48$*$	-611.49$*$	-1830.14$*$
(t)	(-2.54)	(-2.66)	(-4.83)
State Dummies	No	No	Yes‡
N	102	102	102
Adj, R^2	0.05	0.05	0.65
Significance levels :	† : 10%	$*$: 5%	$**$: 1% ‡ : 24 of 50 are significant at 5%

Table 5.1: Effect of Number of Districts on Difference between Legislative and State Medians.

The results indicate that ceteris paribus an increase of one legislative district increases the difference between the state's median income and the median income of the median district falls by between $4.84 and $7.67. The result is statistically significant at the 95% confidence level.

5.3 Median of the Medians

Another empirical test of the implications of the computational models would be to simply compare the national median family income and the median family income of the median congressional district. National median family income serves as a proxy for the preferences of the median voter, whose ideal point would become policy under direct democracy. Median family income of the median congressional district serves as a proxy for the preferences of the median Congressman, whose ideal point would become policy under a representative democracy. In table 5.2, the national median family income and the median family income of the median congressional district are reported. There seems to be little difference in these values for most of the years examined. If these proxies are indeed good measures of voters' preferences for redistribution (as Meltzer and Richard assume), then the level of

redistribution under our representative system does not deviate from the level that would be seen under direct democracy.

Congress	**83rd**	**88th**	**93rd**	**98th**	**103rd**	**108th**
Census	1950	1960	1970	1980	1990	2000
Median of Median	3047	5518.5	9555	19739	34105	49168
National Median	2619	5,660	9,586	19707	35225	50,046
Difference (Dollars)	428	-141.5	-31	32	-1120	-878
Percent Difference	16.34	-2.5	-0.32	0.16	-3.18	-1.75

Table 5.2: Comparing the National Median Family Income to the Median Family Income in the Median Congressional District

Chapter 6

Discussion

The focus of this essay was exploring some of the conditions under which the policy outcome from a representative democracy system would deviate from that of a direct democracy system, owing to the system itself. After formulating a series of computational models it appears that the degree to which legislative districts are gerrymandered with respect to preferences about the policy is a major determinant of policy deviation. Household income was used as a proxy for voter's preferences with respect to redistribution. The most interesting finding is that even when the majority of voters are in favor of redistribution, if districts are constructed with a sufficient level of conservative gerrymandering, the policy outcome under representative democracy will favor far less redistribution than the policy outcome under direct democracy.

When using income as a proxy for voter's preferences for redistribution, there is some empirical evidence that a sufficient level of "gerrymandering" may be present to cause policy divergence. There also appears to be some empirical support for the weak effect that the number of districts has on policy divergence. When looking at the U.S. overall, the median congressional district has a median family income that is not significantly different from the median family income for the country as a whole. If the models here are correct this suggest that in the U.S. there is not a sufficient level of gerrymandering with respect

Chapter 7

Future Research

This paper has assumed that the pivotal actor in the legislature is the median member and that policy is set at this member's ideal point. For the U.S. case, there are a number of models of policy making that contain more institutional detail. The distributive/committee model (Shepsle & Weingast 1987) (Weingast & Marshall 1988) holds that the median of the "relevant" committee is the member. The partisan model (Cox & McCubbins 2005) (Aldrich & Rohde 2000) assumes that it is the median of the majority party who is the important actor in Congress. Lastly the informational model (1996, 1998) model finds members at the 3/5ths and 2/3rds members in the legislature to be pivotal. It is unclear at the outset if this intuitional detail will cause more or less deviation of the representative and direct democracy outcomes. If, for example, the median of the majority party is closer to the median of the electorate as a whole than is the median of the chamber then deviation of the representative democracy outcome will not be as great. On the other hand, if the median of the majority party is even further form the median of the electorate than is the chamber median then the deviation will be even greater.

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Appendix

Sample R Code

```
# DEFINE VARIABLES AND VALUES
    # Number of loops in the simulation
    L <- 1000
    # Number of voters in the population
    N <- 43500
    # Number of legislative districts:
    D <- 435
    # Steps of Gerrymandering (needs to be even)
    gSteps <- 100
    gDenom <- (gSteps/2)

# SET UP MATRICES TO HOLD THE RESULTS OF THE ANALYSIS. #
    #Matrix of RD policies
    RDpolicyMatrix <- matrix(NA,gDenom,D)
    #Matrix of Mins from calculation of RD policy
    RDpolicyMins <- matrix(NA,gDenom,D)
    #Matrix of Maxs from calculation of RD policy
    RDpolicyMaxs <- matrix(NA,gDenom,D)
    #Matrix of SDs from calculation of RD policy
    RDpolicySDs<- matrix(NA,gDenom,D)

# LOOP OVER THE NUMBER OF DISTRICTS
    #Set up a vector to hold policys for each loop
    policyd <- vector(mode="integer",gDenom)
    policydMins <- vector(mode="integer",gDenom)
    policydMaxs <- vector(mode="integer",gDenom)
    policydSDs <- vector(mode="integer",gDenom)
    #LOOP
    for (d in 1:D){
    #To avoid fractional people the number of voters must be adjusted slightly each iteration
    rawS <- (N/d)
    s <- floor(rawS)
    v <- s*d
#Create the vector of voters
    voters <- rnorm(v, mean=50, sd=9.5)
    voters <- sort(voters, decreasing = TRUE)
    #The direct democracy outcome can now be calculated
    DDpolicyMatrix <- matrix(median(voters),gDenom,D)

# LOOP OVER THE DEGREE OF GERRYMANDERING
    #Set up vector to hold the congress for each degree of Gerrymandering
    congressg <- vector(mode="integer", d)
    #Loop
    for (g in 1:gDenom){
```

```
    divPoint <- ceiling((d*s)/2)
    minWinCo <- ceiling(d/2)
    LHSize <- ceiling((g*s)/gSteps)

#SIMULATION LOOPS
    #Set up vector to store the policy from each iteration
    policyl <- vector(mode="integer",L)
    # LOOP
    for (l in 1:L){

# BUILD THE DISTRICTS AND CREATE A CONGRESS WITH A LOOP
    #Set up a vector to be each district
    districti <- vector(mode="integer",s)
    #LOOP for minWin districts
    for (i in 1:minWinCo){
    districti[1:LHSize] <- sample(voters[1:divPoint],LHSize,replace=FALSE)
    districti[(LHSize+1):s] <- sample(voters,(s-LHSize),replace=FALSE)
    #Elect the members of the legislature and place them in the Legislature
    congressg[i] <- median(districti)

} # ends the minWin districts loop

    #LOOP for the remaing districts
    for (i in (minWinCo+1):d){
    congressg[i] <- median(sample(voters,s,replace=FALSE))

} # ends the "other" districts loop

policyl[l] <- median(congressg)

} # ends the simulation loop

# Summary Statics for the policies generated by the simulation
policyd[g] <- mean(policyl) policydMins[g] <- min(policyl)
policydMaxs[g] <- max(policyl) policydSDs[g]   <- sd(policyl)

} # ends the loop over g

    RDpolicyMatrix[,d] <- policyd
    RDpolicyMins[,d] <- policydMins
    RDpolicyMaxs[,d] <- policydMaxs
    RDpolicySDs[,d] <- policydSDs

} # ends the loop over d
```