Effects of Capacity Constraints on Bundling and Investment Decisions

by

Derek Ponticelli

(Under the Direction of John Turner)

Abstract

This paper develops a model that describes the incentive structure of cable and internet providers acting as geographic monopolists. The model demonstrates that for service providers facing capacity constraints, marginal returns from capacity expansion decrease as technology improves, in part explaining a reluctance on the part of providers to invest in additional data capacity. This paper further shows that bundling television and internet is not always an optimal strategy given the substitutability of the two products and the capacity constraint faced by the provider.

Index words: Multi-product Monopolist, Capacity Constraint, Bundling, Capacity Investment
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by

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Chapter 1

Introduction

Since the days of dial-up, internet use has grown at an unrelenting pace. According to the Pew Research Center, the percentage of American adults who use the internet has grown from 14% in 1995 to 85% in 2013 (Pew Research Center, 2013). With such rapid growth, it would seem rational for internet service providers to invest in broadband internet capacity to provide a sufficient supply of data. However, every internet consumer has at some point experienced slower download times or a streaming video that takes an eternity to buffer. These negative experiences reflect the effects of a clogged pipeline that cannot carry enough data to every device all the time. If each individual is free to use as much data as they like, then everyone is subjected to these negative externalities.

This paper seeks to answer the questions of how substitutability and capacity constraints affect multiproduct monopolists’ investment and bundling decisions. While I exclusively investigate these questions in the context of the internet and cable television industry, these questions may apply to several other industries presently and in the future as well. For example, technological change has driven increased substitutability among cell-phones, cameras, and music devices, and in 2012, more than one third of all international phone calls were made via the internet, highlighting another industry experiencing increased competition due to growing substitutability (Tosie, 2013). I consider a strict case of two-product monopolists in a market with a fixed number of consumers, but the results have broad applicability and address a current void in economic theory (as I describe in the literature review).

Perhaps counterintuitively, I find that the technological improvement of the internet from an e-mail service to the current multi-faceted web is in part causing this lack of investment. In this
paper, I develop a model that demonstrates how internet service providers’ marginal returns from
capacity expansion decrease as technology improves and substitutability between television and
internet products increases. I also show that bundling television and internet is not always an
optimal strategy given the substitutability of the two products and the capacity constraint faced
by the provider.

Cable and internet service providers can often be characterized as geographic monopolists selling
substitutable products, television service and internet access. As video-streaming resources such as
Netflix or Hulu become more prominent, it is plausible that consumers will receive less utility from
owning television when everything they want to watch can be found online. However, this increase
in internet use is exactly what will cause internet overcrowding.

Cable and internet data flow through the same pipeline, but the way consumers source the two
products is fundamentally different. Using what is known as a multicast scheme, cable providers
constantly carry television data packets from program vendors through an internet backbone net-
work to households. Under this scheme, every household receives its television programming from
the same content provider at the same time, allowing firms to provide television to all customers
with a single stream of data from the content vendors. As long as customers have complete freedom
to choose video suppliers from anywhere on the internet, however, internet providers will have to
use a unicast scheme to deliver the video content. Whereas a multicast format ensures that a single
stream of data packets provides content to many customers, a unicast format means that each
video streamed by a consumer must be delivered through the internet backbone using a separate
stream of packets (Clarke, 2009). Typically packets can be split so that many users may access
the broadband backbone at once with varying individual connection speeds. Currently, users make
requests to the internet, sending groups of packets to the internet backbone, and the TCP/IP
system in place manages these requests by processing the individual packets and responding with
the desired webpage or content. As requests are processed, a larger amount of data per second
is allocated to the consumer, but as the consumer reads the webpage or responds to the loaded
e-mail, the system efficiently provides less data per second to that consumer, freeing space for more
active users (MacKie-Mason & Varian, 1995). In this way, firms provide an average connection
speed of 15 MBPS by delivering between 25MBPS and 5MBPS. However, as Clarke describes, an
average throughput achieved by providing such drastically varying speeds is entirely inadequate for
streaming SDTV or HDTV. For these purposes a continuous stream of packets and hence a minimum data per second threshold must be allotted to the user (Clarke, 2009). Since these internet backbones cannot currently simultaneously deliver a separate stream of packets to each customer, the internet service providers are capacity constrained, especially during peak usage hours.

Due to the multicast structure of television provision, firms view the marginal costs of providing an additional hour of content on a television channel as zero. However, cable providers recognize that as internet utilization increases, performance eventually decreases. The effects of increases in utilization do not affect performance until consumers are using nearly the full capacity of data that firms can provide. Firms want to prevent consumers from utilizing too much traffic at one time to guarantee everyone a reasonable experience and preserve customer loyalty. Otherwise, providers risk losing dissatisfied customers to the limited alternatives that they can find. Therefore, cable providers view their internet data marginal cost curve as flat until the utilization is near the full capacity level and then increasing very steeply as it approaches that level. Since it is not well known precisely how the marginal costs increase near the constraint, in this paper I will abstract away from attempting to perfectly characterize the costs and will assume a vertical marginal cost curve at the constraint. The substitutability of the products and this unique cost structure create a challenging investment decision for firms. Investing more in broadband can shift providers’ marginal cost curves to the right, allowing them to sell more internet, but firms worry that increased internet sales come at the price of cannibalized television sales.

Quarterly data collected by Nielsen substantiates cable providers’ fears. Over the past two years, from Q2 of 2010 through Q2 of 2012, the number of Americans who watch television at home decreased by 1.2%. In that time, the number of internet users increased by 9.3% and the number of people who watch video on the internet increased by 16.2%. However, these raw numbers regarding the number of consumers do not tell the entire story. Not only has the number of internet and video users been increasing at a much faster rate than the number of TV users but the amount of time people spend on the internet has been growing quicker than the amount of time people spend watching TV. Television users increased the average amount of time spent watching TV by 0.9% while the time spent on the internet by users increased by 13.4% from Q2 2010 through Q2 2012. Lastly, online video consumers increased their average video consumption by 51.3% over that same time span. As video streaming websites have become more prominent, individuals have devoted
more time to enjoying video over the internet, and television consumption has been crowded out as a result. Furthermore, in Q2 2008, only 6.2% of time spent on the internet was spent watching videos. That number more than doubled by Q2 2012, when watching videos accounted for 15.8% of internet usage. The data leave little doubt that consumers are shifting their video consumption to the internet as technological improvements allow the internet to crowd out television sales. Furthermore, many experts from Nielsen agree that these trends depicting the growth of internet video consumption will continue (Nielsen, 2013). Nielsen recently went so far as to alter its definition of television viewing to account for the growing number of viewers who connect broadband internet to their television sets to stream TV shows (Stelter, 2013). These numbers point to the increasing substitutability of internet and television.

This trend has direct effects on the sale of television and internet as well as firms’ investment strategies. To manage consumption, providers may implement quotas on individual consumption, but those that do seldom enforce them (possibly fearing a negative public response). Most providers instead manipulate consumer choices through pricing strategies. By offering bundled packages, internet providers can shift demand from pure internet services to television service and hence preserve internet capacity for other users. By introducing usage based pricing tiers or fees through some form of 3-part tariffs, companies may further guide consumers toward moderate data consumption while simultaneously maximizing their own profits. Likewise, by limiting investment in capacity, providers can maintain the appeal of television. This paper concentrates on bundling and investment decisions for providers in this challenging environment.

In the next section, this paper reviews relevant research into cable and internet bundling and pricing. In the third section I develop a model based on McAfee, McMillan, and Whinston’s reservation price model and discuss the model’s intuition through two special cases. In the fourth section I use the model to reach general conclusions regarding bundling and investment decisions. By replicating the strategic choices available to providers, section four shows that the marginal profit of investing in additional capacity decreases as technology improves and internet and television become closer substitutes. Thus, I develop an economic model capable of describing the perverse incentive structure of the providers in a monopolistic environment. I argue that providers acting as geographic monopolists with a fixed number of consumers are subject to incentives which encourage them to postpone investment for as long as possible. Furthermore, this paper shows that once
substitutability increases beyond a threshold level, providers never offer to bundle services. Lastly, this paper concludes by looking toward the future of the cable and internet industry and considering the long-run viability of current bundling and investment strategies.
Chapter 2

Literature Review

Adams and Yellen first demonstrate the possible optimality of bundled pricing structures, by providing examples when pure or mixed bundling strategies outperform separate pricing models (Adams, 1976). McAfee, McMillan, and Whinston, however, provide a deeper theoretical basis for the profit maximizing potential of bundling. First of all, they develop a reservation price model, which has become the primary framework used in this area, for a 2-product monopolist or oligopolist where consumers value each good independently. They also find sufficient conditions to guarantee mixed bundling optimality while using minimal assumptions regarding consumer probability distributions to reach their conclusions (McAfee, McMillan, & Whinston, 1989). McAdams extends the model of McAfee et al. to an N-product monopoly and identifies conditions which determine when a monopoly finds it profitable to introduce a new bundle consisting of some subset of the N-products to the market (McAdams, 1997). Venkatesh and Kamakura extend McAfee et al. in a different direction by considering special cases of 2-product monopolists selling complements or substitutes (Venkatesh & Kamakura, 2003). This paper builds on the McAfee et al. framework’s restriction of independently valued products to model substitutability and extends Venkatesh and Kamakura’s findings by examining the effects of the capacity constraint on firms’ profit maximizing strategies.

A usage-based fee designed to restrict internet consumption will likely take the form of a three part tariff, involving a positive service fee, a usage fee of $0 up to some cap, and a positive fee for each unit of data consumed beyond the cap. Calem and Spulber explore two part pricing strategies for 2-product monopolists and draw several relevant conclusions (Calem & Spulber, 1984). They find that firms maximize profits by charging per unit fees above marginal costs when the two products
are substitutes and solve for the optimal service fees and per unit prices for each good up to some specification of the consumer’s utility function. They also develop a limited model incorporating consumer heterogeneity by allowing consumer preferences to take on two forms and reach similar conclusions as in their homogenous consumer model (Calem & Spulber, 1984). Gao expands upon their analysis to account for a larger number of products. However, in his model, a firm charges a single service fee to access any of the N products that the firm has to offer (Gao, 2010). Finally, in an empirical study, Barrientos, Tobon, and Bedoya examine the three part tariff pricing strategy put in place by a local telephone company. The company allows consumers to choose from a variety of packages with differing initial service fees and usage caps. Higher initial fees correspond to larger caps, and Barrientos et al. find a number of results implying short-term irrationality in consumer choices. Specifically, consumers appear to make decisions based on the price and minutes cap and routinely surpass the usage limit (Barrientos, Tobon, & Bedoya, 2010). These economists have already made significant progress exploring optimal pricing strategies for multiproduct monopolies when implementing two part tariffs. The problems posed by the capacity constraint are inherent in any two or three part tariff pricing research yet absent from most bundling research to date. This paper will address this current void in economic theory by focusing exclusively on the effects of the existing capacity constraint on bundling and investment decisions.
Chapter 3

Model Specification

The model is based on the reservation price framework developed by McAfee et al. In this paper, cable and internet service providers act as a multiproduct monopolists selling two substitute goods, internet service and television service, as well as a bundle of the two goods at prices $p_i, p_t,$ and $p_b$ respectively. As in McAfee et al., consumers’ valuations, $(v_i, v_t)$, for the two goods fall on independent continuous uniform distributions from 0 to 1. This structure creates continuums of consumers with varying preferences. Consumers may purchase 0 or 1 units of each good. To introduce substitutability into the model, marginal utility for each good $j$ is defined in the following way:

$$mu_j(v_j) = \begin{cases} v_j & : q_k = 0 \\ \beta \cdot v_j & : q_k = 1 \end{cases}$$

The parameter $\beta$ falls in the interval $[0, 1]$ and reflects the decline in utility from an individual good resulting from purchasing both goods. As later sections describe in detail, $\beta$ varies with technological change and indicates the substitutability of television and internet. The case when $\beta = 1$ reflects the early stages of the internet when no crowd out effects between television and internet were present, nesting McAfee et al.’s independent goods model within this framework. The case when $\beta = 0$ may reflect the state of technology in the future by demonstrating a world with complete crowd out effects and perfect substitutability between television and internet.

Based solely on that definition of marginal utility, consumers fall into five different groups defined by relationships between consumer valuations and firm prices. Figure 3.1 shows how customers...
segment themselves based on the technology level, $\beta$, and the prices of the goods, and Table 3.1 specifies the valuation inequalities defining each segment. Individuals in regions A and D purchase solely television or internet respectively. The consumers in regions B, C, and E all purchase the bundle; however each of these individuals would act differently in the absence of a bundle. The customers of region E would purchase nothing if firms didn’t offer a bundle, whereas customers in regions B and C would purchase television or internet respectively if a bundle weren’t available.

The most novel element of the model involves the restrictions of the capacity constraint. If the average amount of internet consumed is normalized to one, then the combined area in B,C,D, and E in Figure 3.1 represents the total amount of internet consumption. In all situations, the monopolist maximizes profits while constraining this area below some cap, $c$, which will reflect the cable company’s capacity constraint.

Figure 3.1: Internet and TV Customer Segmentation when $p_i = 0.5, p_t = 0.5, p_b = 0.7, \beta = 0.85$
Table 3.1: Consumer Valuation Inequalities Defining Figure 2 Regions

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<th>Figure 2 Region</th>
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<td>A</td>
<td>$v_t - p_t \geq \beta \ast (v_i + v_t) - p_b, v_t - p_t \geq v_i - p_i, v_t \geq p_t$</td>
</tr>
<tr>
<td>B</td>
<td>$\beta \ast (v_i + v_t) - p_b \geq v_t - p_t \geq v_i - p_i, \beta \ast (v_i + v_t) - p_b \geq 0$</td>
</tr>
<tr>
<td>C</td>
<td>$\beta \ast (v_i + v_t) - p_b \geq v_i - p_i \geq v_t - p_t, \beta \ast (v_i + v_t) - p_b \geq 0$</td>
</tr>
<tr>
<td>D</td>
<td>$v_i - p_i \geq \beta \ast (v_i + v_t) - p_b, v_i - p_i \geq v_t - p_t, v_i \geq p_i$</td>
</tr>
<tr>
<td>E</td>
<td>$\beta \ast (v_i + v_t) - p_b \geq 0, p_i \geq v_i, p_t \geq v_t$</td>
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3.1 $\beta = 1$ Case

Recall that when $\beta = 1$, consumers treat the two products as independent goods as in McAfee et al. so that no crowd out effects appear. Consumers get the same amount of utility from an individual good regardless of whether they own the other good or not, reflecting a past state of the internet and television industry.

First consider the differences in customer segmentation caused by offering a bundle. In Figures 3.2a and 3.2b, regions A and D indicate consumers who purchase just television or internet respectively. As before, region E in Figure 3.2b represents consumers who purchase a bundle rather than nothing. In Figure 3.2a regions B and C depict people who purchase both internet and television. They are essentially purchasing a bundle even though the provider is not directly offering one. In Figure 3.2b, those same consumers who create artificial bundles (regions B and C) now purchase the bundle and the regions B’ and C’ represent consumers who previously purchased television or internet respectively and purchase a bundle once the provider begins offering one.
Initially consider the case when the provider does not offer a bundle. To find when the capacity constraint begins to bind, I first develop general area equations for the regions A, B, C, and D in Figure 3.2, multiply these by the appropriate individual prices, and sum the results to reach a revenue equation. I then maximize this revenue equation without the capacity constraint to determine the level at which a capacity constraint would begin to bind. In this case, the area of internet consumers (region B+C+D) is \((1 - p_i)\) and the area of television consumers (region A+B+C) is \((1 - p_t)\). Then, the total revenue for the firm is:

\[
R(p_i, p_t) = p_i \cdot (1 - p_i) + p_t \cdot (1 - p_t).
\]

Since there is no crowding out in this situation, before taking a capacity constraint into account, the monopolist acts as a single product monopolist in both markets by selling internet to \(\frac{1}{2}\) of the population, selling television to \(\frac{1}{2}\) the population, and charging the monopoly price of \(\frac{1}{2}\) for both goods. Therefore when \(\beta = 1\), I only need to consider the affects of the constraint when the capacity constraint is less than \(\frac{1}{2}\). Since the monopolist treats the markets separately, and the amount of internet sold is independent of the price of television, intuitively the firm should always charge the
monopoly, profit maximizing, price for television. However, the lower the maximum capacity, the more the provider should charge for internet service.

At the monopolist prices, the area of internet consumption is $\frac{1}{2}$. Therefore, whenever the capacity constraint is above $\frac{1}{2}$, the monopolist can maximize profits with symmetric prices of $\frac{1}{2}$ and will never sell internet to more than $\frac{1}{2}$ of the population. However, once the capacity constraint falls below $\frac{1}{2}$, the firm sells as much internet as possible without going over the constraint. Intuitively, if the cap is 0, and the firm cannot sell any internet, then the provider must set an internet price that is prohibitively expensive. As the cap decreases, the provider should increase the price of internet above the optimum price of $\frac{1}{2}$ to restrict consumption. However, if the internet provider ever sets a price that is too high, such that it sells even less internet than the cap allows (with the cap below $\frac{1}{2}$), then the provider could move closer to the unconstrained maximum by charging less and selling more internet until it runs into the constraint. Therefore the monopolist’s problem is to solve (where $\lambda$ is a Lagrangian multiplier and $c$ is the capacity constraint the firm faces):

$$\max_{p_i, p_t} \pi = p_i^* (1 - p_i) + p_t^* (1 - p_t) + \lambda^* (c - (1 - p_i)).$$

Solving that profit maximization problem leads to the following optimal price equations, holding when $c$ is between 0 and $\frac{1}{2}$:

$$p_i(c) = 1 - c$$
$$p_t(c) = \frac{1}{2}$$
$$\pi(c) = \frac{1}{4} + c - c^2$$

The analytic descriptions of the optimal prices and profit, as plotted in Figure 3.3, confirm previously developed intuition. In section 3.2 I’ll compare these profits with the case when $\beta = 0$.

Now, consider the case where the firm offers a bundle for the price $p_b$. In this McAfee et al. setting, the area of consumers who will purchase the bundle (regions B+B'+C+C'+E in Figure 3.2b) is $1 + p_i + p_t - 2p_b - \frac{p_i^2}{2} - \frac{p_t^2}{2} + \frac{p_b^2}{2}$, the area depicting internet consumption (region D in Figure 3.2b) is $(1 - p_i)(p_b - p_i)$, and the area showing television consumption (region A in Figure 3.2b) is
Figure 3.3: Optimal TV and Internet Prices and Profit when $\beta = 1$ without Bundling

\[(1 - p_t)(p_b - p_t)\]. Based on these equations, the monopolist’s revenue equation is:

\[
R(p_i, p_t, p_b) = p_i(1 - p_i)(p_b - p_i) + p_t(1 - p_t)(p_b - p_t) + p_b(1 + p_i + p_t - 2p_b - \frac{p_i^2}{2} - \frac{p_t^2}{2} + \frac{p_b^2}{2})
\]

\[
= p_b(1 + 2p_i + 2p_t - 2p_b - \frac{3}{2} p_i^2 - \frac{3}{2} p_t^2 + \frac{1}{2} p_b^2) - p_i^2 - p_t^2 + p_i^3 + p_t^3
\]

By maximizing the revenue equation with respect to $p_i$, $p_t$, and $p_b$ without considering the capacity constraint, the monopolist can earn a maximum profit of $\frac{12 + 2\sqrt{2}}{27} \approx .5492$. To reach this optimum profit, the provider charges $p_i = p_t = \frac{2}{3}$ and $p_b = \frac{4 - \sqrt{2}}{3} \approx .8619$ and provides internet (through bundling or pure internet subscriptions) to an area of $\frac{4 + \sqrt{2}}{9} \approx .6016$. So, the firm sells internet to just over sixty percent of the market through pure internet subscriptions or the bundled package as long as its capacity constraint is above $\frac{4 + \sqrt{2}}{9}$. However, when the constraint is below that level, as before, the monopolist will maximize profits by selling internet or bundle packages until the constraint binds. Therefore, when the capacity constraint is accounted for, the monopolist’s problem is to solve the following:

\[
\max_{p_i, p_t, p_b} \pi(p_i, p_t, p_b) = R(p_i, p_t, p_b) + \lambda \left[ c - 1 - p_i + p_b + p_ip_t - p_ipb + p_i^2 \right], c \in (0, \frac{4 + \sqrt{2}}{9})
\]
The first order conditions for the optimal prices are then given by the following equations:

\[
\begin{align*}
\frac{\partial \pi}{\partial p_i} &= p_i(-2 + 3p_i - 3p_b) + 2p_b + p_i \lambda = 0 \\
\frac{\partial \pi}{\partial p_t} &= 3p_t^2 + p_b(2 - \lambda) + (p_i - 1)\lambda + p_t(2\lambda - 3p_b - 2) = 0 \\
\frac{\partial \pi}{\partial p_b} &= 1 + (2 - 1.5p_t)p_i + p_b(1.5p_b - 4) + p_t(2 - 1.5p_t - \lambda) + \lambda = 0 \\
\frac{\partial \pi}{\partial \lambda} &= -1 + c + p_t(-1 + p_i + p_t - p_b) + p_b = 0
\end{align*}
\]

While it is possible to find some required relationships between prices and the capacity constraint such as \( p_i(p_t, p_b, c) = p_b - \sqrt{-2 + 2c + 2p_b - 2p_t + p_t^2} \) analytically, this system of equations is not tractable in general, as solving for optimal prices as a function of \( c \) reduces to solving a quintic equation with general coefficients for each term. However, Figure 3.4 depicts numeric plots of the solutions (a description of the numeric methods used is in the appendix). Figure 3.4 shows that with an extremely restrictive cap, the monopolist sets \( p_i \) and \( p_b \) at such high prices that very few consumers purchase internet or the bundle. In this case, the monopolist charges \( p_t = \frac{1}{2} \) and acts as a single product monopolist, but as the firm invests in capacity such that the constraint relaxes, \( p_t \) increases while \( p_i \) and \( p_b \) decrease toward their optimum levels.

**Figure 3.4: Optimal Prices and Profit when \( \beta = 1 \) with Bundling**
3.2 $\beta = 0$ Case

When $\beta = 0$, we observe perfect crowd out in the market for internet and television since marginal utility from owning either good drops to 0 once a consumer purchases the other product, indicating that the goods are perfect substitutes, a possible future state of the industry. Figure 3.5 shows that in this situation no consumer finds it optimal to purchase the bundled package. The bundle consumers from regions B and C in Figure 3.2a and 3.2b purchase just television or internet, joining segments A and D respectively when $\beta$ declines to 0. Meanwhile, the customers from region E leave the market entirely. Therefore, in this section there is no need to consider separate cases with and without bundling since consumer behaviors are not affected by that offering.

As before, I first determine when the capacity constraint begins to bind. Define the area for internet consumers (region D) as $\frac{1}{2} - p_i + p_t + \frac{p_i^2}{2} - p_i p_t$ and the area for television consumers (region A) as $\frac{1}{2} + p_i - p_t - \frac{p_t^2}{2}$. Using these formulas for the consumer areas, the monopolist’s revenue equation is the following:

$$R(p_i,p_t) = \frac{p_i}{2} + \frac{p_t}{2} - p_i^2 - p_t^2 + 2p_ip_t - \frac{3}{2}p_i^2 p_t + \frac{p_t^3}{2}.$$ 

Without considering a capacity constraint, the monopolist’s goal is then to solve:

$$\max_{p_i,p_t} \frac{p_i}{2} + \frac{p_t}{2} - p_i^2 - p_t^2 + 2p_ip_t - \frac{3}{2}p_i^2 p_t + \frac{p_t^3}{2}.$$ 

By maximizing this, the monopolist earns a profit of $\frac{2}{3\sqrt{3}} \approx .385$ by charging $p_i = p_t = \frac{1}{\sqrt{3}} \approx .577$. Initially these optimal prices appear counterintuitive in that they are higher than the independent monopoly levels. Venkatesh and Kamakura achieve similar results showing that when selling strong substitute goods the monopolist sets prices above the optimal independent goods monopoly levels (Venkatesh & Kamakura, 2003). Since consumers only purchase one product, by charging a lower price in either market, the provider loses consumers from the other market who previously would have been willing to purchase a more expensive good. So, by raising the prices slightly, the provider loses the consumers with low valuations from both markets, but the revenue gained by charging higher prices to the remaining consumers far outweighs the value of those lost consumers.
Figure 3.5: Pure Internet and TV Customer Demographics when $\beta = 0$

Incorporating the capacity constraint, the provider solves the following equation to maximize profits when the capacity constraint is between 0 and $\frac{1}{3}$:

$$\max_{p_i, p_t} \pi(p_i, p_t) = \frac{p_i}{2} + \frac{p_t}{2} - p_i^2 - p_t^2 + 2p_ip_t - \frac{3}{2}p_i^2p_t + \frac{p_t^3}{2} + \lambda \left( c - \left( \frac{1}{2} - p_i + p_t + \frac{p_i^2}{2} - p_ip_t \right) \right).$$

The first order conditions for the optimal prices are then given by the following equations:

$$\frac{\partial \pi}{\partial p_i} = \frac{3}{2}p_i^2 - 2p_i + 2p_t - 3p_ip_t + \frac{1}{2} + \lambda (1 - p_i + p_t) = 0$$
$$\frac{\partial \pi}{\partial p_t} = -\frac{3}{2}p_t^2 + 2p_i - 2p_t + \frac{1}{2} + \lambda (p_t - 1) = 0$$
$$\frac{\partial \pi}{\partial \lambda} = c - \frac{1}{2}p_i^2 + p_i - p_t + p_ip_t - \frac{1}{2} = 0$$
As before, a solution to this system of equations reduces to finding a general solution to quintic equations. The charts in Figure 3.6 show that when the capacity constraint is near 0, the monopolist acts as a single product monopolist by charging $\frac{1}{2}$ for television and a very high price for internet. As the constraint is relaxed, the cable provider lowers $p_i$ and increases $p_t$ to sell more internet. By increasing $p_t$, the firm loses low television valuation customers but also induces some consumers, those on the border between regions A and D, to switch from buying the low priced television to the much higher priced internet. The increased revenues from the switching consumers outweigh the lost revenues from the consumers at the bottom edge of region A who leave the market. Both prices converge towards $\frac{1}{\sqrt{3}}$ until the capacity constraint is greater than $\frac{1}{3}$, at which point the provider sets the unconstrained optimum prices found above.

![Figure 3.6: Optimal Prices and Profits when $\beta = 0$](image)
Chapter 4

General Results

4.1 Signing the Mixed Partial Derivative of Firm’s Profits

This section shows that as substitutability of television and internet increases, the marginal return to investing in capacity declines.

A comparison of pricing and profits between the $\beta = 0$ case and the $\beta = 1$ case without bundling first develops intuition for this result. Changes in the profits of the firm can come from two markets, the internet market and the television market. That means there are four ways in which strategy can differ between the $\beta = 0$ and the $\beta = 1$ case that would lead to differences in marginal profits: marginal changes in television prices, marginal changes in television penetration, marginal changes in internet prices, and marginal changes in internet penetration. Figure 4.1b shows that when the goods are independent, investment in capacity will only affect the internet market. So, the television price and penetration level will stay constant at the monopoly levels regardless of the capacity level. In this case, an investment in additional capacity allows the monopolist to lower the price of internet and thereby sell internet to a greater proportion of the population. While investing, the firm steadily decreases the internet price and sells a greater quantity until it reaches the monopoly level in that market as well, when capacity is greater than or equal to $\frac{1}{2}$.
When the goods are perfect substitutes, however, very different dynamics are at play. Just as when $\beta = 1$, investing in capacity allows the firm to sell internet to more people by decreasing internet prices. But unlike in the independent goods case, the television market is not static. As the firm invests in additional capacity, it encourages people to purchase internet in two ways, by lowering the price for internet and by increasing the price for television.

The firm attracts people who previously purchased neither good by lowering the price of internet. In both cases of Figure 4.1, the price for internet is always greater than or equal to the price for television. However, in the $\beta = 0$ case, the price for internet declines to the optimum pricing level of $\frac{1}{\sqrt{3}}$ at a quicker rate than the internet price declines to $\frac{1}{2}$ in the $\beta = 1$ case. When $\beta = 0$, the average slope of the price of internet is $\Delta p_i / \Delta c = \sqrt{3} - 3 \approx -1.27$, while when $\beta = 1$, the average slope of the price of internet is $\Delta p_i / \Delta c = -1$. So, when the cap is near 0 and the firm invests in additional capacity, it drops the price of internet further when the goods are perfect substitutes because it must attract consumers away from purchasing television. The firm has to incentivize them above and beyond their desire to purchase internet at a particular price, to compensate them for the loss of utility they’ll incur by giving up television in exchange for internet. That tactic alone would already tempt some television consumers who have very high internet valuations and fairly low television valuations to switch their purchases from television to internet.

The firm further induces television consumers with high internet valuations and lower television valuations to purchase the more expensive internet instead of television by increasing the price of television, causing to the shifting penetration rates shown in Figure 4.2.
This action leads to a decreasing penetration rate in the television market which is not entirely compensated for by the increased television price. So, as the firm invests in capacity, the marginal profit from the television market is slightly negative. This loss drags down the new marginal profits from the internet market so that while the cumulative marginal return to investing in capacity is clearly positive, it is always less than or equal to the marginal return when the goods are independent.

The plots in Figure 4.3 show the firm’s optimal profit when $\beta = 0$ and when $\beta = 1$. As the monopolist invests in capacity, the marginal return when $\beta = 1$ is always greater than or equal to the marginal return when $\beta = 0$. Figure 4.3a highlights the case when the capacity constraint is binding whether $\beta$ is 0 or 1, while Figure 4.3b displays the profits in both cases over the entire range of $c$. Both curves clearly show that the marginal return to investing in additional capital, $\frac{\partial \pi(1,c)}{\partial c}$ and $\frac{\partial \pi(0,c)}{\partial c}$, is greater than or equal to zero for all values of $c$ between 0 and 1. Together these cross sectional graphs imply that the mixed partial of the optimal profit function, $\frac{\partial^2 \pi(\beta,c)}{\partial \beta \partial c}$, is positive, which can be interpreted as saying that as substitutability increases, the marginal return of investing in capacity decreases. Table 4.1 displays numeric estimates for $\frac{\partial^2 \pi(\beta,c)}{\partial \beta \partial c}$ for the case without bundling and likewise confirms that $\frac{\partial^2 \pi(\beta,c)}{\partial \beta \partial c}$ is non-negative over the domain. However, when the goods are not substitutes, the marginal return is always greater than or equal to the marginal return when the goods are perfect substitutes. Connecting these general results with the cable and internet industry, we see that as technology improves, (and television and internet become closer substitutes), a firm’s incentive to invest in additional capacity actually diminishes.
Lastly, notice that for all cap values greater than $\frac{1}{2}$, the mixed partial is 0. Once the firm invests enough to move the constraint beyond $\frac{1}{2}$, the constraint no longer binds, and $\frac{\partial^2 \pi(\beta, c)}{\partial \beta \partial c}$ will always be 0. Therefore, the marginal return to capacity will not change as $\beta$ varies, explaining why the lower half of Table 4.1 is constant.

Table 4.1: Numeric Estimates for $\frac{\partial^2 \pi(\beta, c)}{\partial \beta \partial c}$ without Bundling

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Figure 4.3: $\beta = 0$ and $\beta = 1$ Profit Comparisons without Bundling ($\beta = 0$ Case Dashed and $\beta = 1$ Case Dotted in Both Graphs)
While the extreme cases considering internet and television as perfect substitutes and as independent goods illustrate why $\frac{\partial^2 \pi(\beta,c)}{\partial \beta \partial c}$ is positive, the argument above does not necessarily imply that this is true for all levels of $\beta$ between 0 and 1. To this end, numeric estimates of firms’ optimal profit functions at various values of $\beta$ illustrate how the marginal return of investment changes for intermediate $\beta$.

The numerical analysis provides further evidence that the mixed partial of the profit function is positive. Figure 4.4 shows that as $\beta$ decreases, the level at which the capacity constraint begins to bind decreases as well. While $\beta$ is less than $\frac{1}{3}$, the slope, $\frac{\partial \pi}{\partial c}$, increases as $\beta$ increases. Then for levels of $\beta$ between $\frac{1}{3}$ and $\frac{1}{2}$ the capacity constraint binds for some $\beta$ levels, and profit curves corresponding to higher $\beta$ levels have a positive slope while profit curves corresponding to lower $\beta$ levels have already reached their maximum values. Lastly, once the firm reaches capacity levels above $\frac{1}{2}$, the constraint no longer binds for any level of $\beta$, and the marginal return of investing in additional capacity is zero regardless of the level of $\beta$.

Figure 4.4: Profit Cross Sections without Bundling
When $\beta = 1$ and the capacity constraint does not bind, one quarter of the population purchases both internet and television. As $\beta$ decreases, that proportion shrinks as well, and that relationship further describes why the mixed partial derivative is positive. At any general level of $\beta$, an investment in capacity will allow the firm to charge less for internet and sell more internet to two groups of consumers, people who previously purchased neither product and people who previously purchased television. The action of the second group is dependent on the level of $\beta$. When $\beta$ is near 1, almost all of the consumers who previously purchased television and are now interested in buying internet at the lower price purchase internet as well as television. However, a small proportion of these new consumers switches their consumption from television to internet and no longer purchases television. As $\beta$ declines, the firm further encourages buyers to switch their purchases from the relatively cheap television service to the more expensive internet service by increasing the price of television service. Thus, as $\beta$ decreases, the proportion of new internet purchasers who previously consumed television increases so that the increase in profit from new internet purchases is increasingly counterbalanced by the corresponding loss in profit from switching television purchasers. The numeric plots in Figure 4.4 leave little doubt that when the firms do not offer bundled packages, as technology improves and $\beta$ decreases, the firm’s incentive to invest in additional capacity decreases as well.

Before moving on, consider the case when the firm offers a bundled package. Again, first consider a comparison between prices, penetration percentages, and profits, for the $\beta = 0$ and $\beta = 1$ cases.

McAfee et al. describe how a monopolist selling independent goods and a bundle can maximize profits by increasing the prices for the individual goods to levels above the monopoly prices of one half. In this way, the monopolist can incentivize more consumers to purchase the more expensive bundle and gain profits that outweigh the loss of the consumers who no longer purchase either product (McAfee et al., 1989). By charging higher prices, for internet and television, the monopolist sells each good to approximately 6.5% of the population as opposed to 50% of consumers in the absence of a bundle. Instead, the provider sells a bundle package to over 53% of people, which demonstrates how drastically the monopolist will alter its strategy when selling a bundle along with the individual goods.

Figures 4.5 and 4.6 depict how as the capacity constraint is relaxed, the provider raises prices on individual goods to incentivize consumers to purchase the bundled package instead as well as
lowering the price of the bundle itself. As before, when $\beta$ is extremely low, fewer consumers are willing to purchase both goods or in this case the bundle. Consumers’ reluctance to purchase the bundle as technology improves indicates that the mixed partial of the monopolist’s profit function, $\frac{\partial^2 \pi(\beta,c)}{\partial \beta \partial c}$, is positive when incorporating a bundling option for firms as well. The numeric analysis depicted in Figure 4.7 and Table 4.2 verify this intuition by showing that slopes of profit functions, $\pi(c)$, do increase as $\beta$ increases and mixed partial values are always non-negative. It is worth noting that in Figure 4.7, the lowest curve, representing the case when $\beta = 0$, is the exact same as in Figure 4.4. The profit curves corresponding to higher levels of $\beta$ shift up, but the base case remains the same regardless of the firm’s bundling decision. Both sources confirming that as technology improves, firms’ incentive to invest in capacity diminishes.

![Figure 4.5: Pricing Comparison between $\beta = 0$ Case and $\beta = 1$ Case with Bundling (p_i Medium Dashed, p_i Dotted, and p_b Large Dashed in Both Graphs)](image)

![Figure 4.6: TV and Internet Penetration Rate Comparisons with Bundling ($\beta = 0$ Case Dashed and $\beta = 1$ Case Dotted in Both Graphs)](image)
Figure 4.7: Profit Cross Sections with Bundling

Table 4.2: Numeric Estimates for $\frac{\partial^2 \pi(\beta, c)}{\partial \beta \partial c}$ with Bundling

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4.2 Customer Categorizations and Effects of Product Substitutability on Firms’ Bundling Decisions

In the previous section, each of the numeric analyses omits cross sectional profit functions for $\beta$ values between 0 and $\frac{1}{2}$. This section explains why substitutability forces the monopolist to pursue the same optimal strategy regardless of the level of $\beta$ within that interval. In other words, when $\beta$ is less than $\frac{1}{2}$, reflecting a high-substitutability future state, the provider never sells a bundle. First, I’ll further discuss the effects of substitutability on the buying patterns of potential customers. Then, I’ll discuss numeric evidence supporting my claim that $\beta = \frac{1}{2}$ is a bundling decision threshold.

When $\beta$ is free to take any value between 0 and 1, we find that consumer segmentations can take a variety of forms. The value of $\beta$ as well as the relative price of the bundle compared with the individual prices will dictate how consumers react to prices and segment themselves. When $\beta > \frac{1}{2}$, and $\beta(p_i + p_t) \leq p_b < (1 - \beta)p_i + \beta p_t + 2\beta - 1$, (case 1, Figure 4.8a), a slim outward facing triangular region depicts the region of consumers who purchase the bundle. Alternatively, when $\beta > \frac{1}{2}$ and $p_b < \beta(p_i + p_t)$, (case 2, Figure 4.8b), bundle purchasers take the form of a truncated outward facing triangle. Lastly, when $\beta > \frac{1}{2}$ and $p_b > (1 - \beta)p_i + \beta p_t + 2\beta - 1$, (case 3, Figure 4.10b), consumers act as if $\beta = 0$ by purchasing only television or internet. However, when $\beta < \frac{1}{2}$, only low-valuation consumers are interested in purchasing a bundle. In that case, when $\beta < \frac{1}{2}$ and $\beta(p_i + p_t) > p_b \geq (1 - \beta)p_i + \beta p_t + 2\beta - 1$, (case 4, Figure 4.9a), a slim inward facing triangle indicates bundle purchases. When $\beta < \frac{1}{2}$ and $p_b < (1 - \beta)p_i + \beta p_t + 2\beta - 1$, (case 5, Figure 4.9b), a thicker inward facing, truncated triangle indicates those consumers, and when $\beta < \frac{1}{2}$ and $p_b > \beta(p_i + p_t)$, (case 6, Figure 4.10b), the $\beta = 0$ case occurs again. Lastly, when $\beta = \frac{1}{2}$ and $\beta(p_i + p_t) > p_b$, (case 7, Figure 4.10a), three parallel lines designate the region of consumers who purchase the bundle. In the final case, when $\beta = \frac{1}{2}$ and $p_b \geq \beta(p_i + p_t)$, (case 8, Figure 4.10b), consumers again act as if there is complete crowding out.
(a) Case 1: $p_i = 0.5, p_t = 0.5, p_b = 0.85, \beta = 0.8$

Figure 4.8: Relatively High $\beta$ Consumer Segmentations

(b) Case 2: $p_i = 0.5, p_t = 0.5, p_b = 0.7, \beta = 0.85$

Figure 4.9: Relatively Low $\beta$ Consumer Segmentations

(a) Case 4: $p_i = 0.5, p_t = 0.5, p_b = 0.35, \beta = 0.4$

(b) Case 5: $p_i = 0.5, p_t = 0.5, p_b = 0.25, \beta = 0.4$
(a) Case 7: $p_i = 0.5, p_t = 0.5, p_b = 0.4, \beta = 0.5$

(b) Case 3, 6, 8: $p_i = 0.5, p_t = 0.5, p_b = 0.95, \beta = 0.7$

Figure 4.10: Consumer Segmentation when $\beta = \frac{1}{2}$ and when Consumers Avoid Bundles

These graphs already hint at why bundling is not an optimal strategy when $\beta$ is less than $\frac{1}{2}$. When $\beta$ is close to 1, representing the technology level from the past, the intuitive graphs in Figure 4.9 show that consumers who highly value both goods purchase the bundle, and the relative price of the bundle just dictates a threshold level for the sum of the individual valuations. The lower $p_b$ is, the lower the threshold is. If a consumer’s valuation for the bundle of goods surpasses that threshold, then he or she will purchase the bundle.

However, when $\beta$ is lower than $\frac{1}{2}$ (Figures 4.9a,b and Figure 4.10a), the consumers who purchase the bundle are the individuals who do not value either product too much. In the low $\beta$ environment, the future state of internet and television technology, consumers lose at least half their marginal utility from their preferred good by purchasing the other good as well. Consequently, high-valuation consumers do not sacrifice a portion of the high utility they receive from one good by purchasing the other as well. To offset this utility loss, the monopolist must charge a very low price, lower than the average price of television and internet, to attract new consumers to the bundled package. At such low prices, the monopolist convinces many low-valuation consumers who would purchase internet or television in the absence of a bundle to alter their purchasing decisions. Since a binding capacity constraint forces the monopolist to charge a higher price for internet, the monopolist makes less
money from each consumer who switches from purchasing internet to purchasing the bundle than if it did not offer the bundle. Due to the low bundle price necessitated by a low $\beta$ value and the high internet price guaranteed by a restrictive capacity constraint, the additional revenue earned from new consumers is less than the revenue lost from former internet and television consumers who purchase the cheaper bundle.

The curve in Figure 4.11 represents the provider’s bundling decision boundary. For pairs of $(c, \beta)$ above the curve, the monopolist would choose a mixed bundling strategy, but for points below the curve, the firm would sell its goods independently without offering a bundle. For values of $\beta$ less than or equal to $\frac{1}{2}$, the firm would never bundle products. Regardless of the constraint, $c$, as technology improves and $\beta$ shrinks, the provider eventually must charge such a low bundle price to attract low valuation consumers that the opportunity cost of selling the bundle is more than the profits generated by the bundle.

![Figure 4.11: Internet Service Provider Bundling Decision Boundary](image-url)
Chapter 5

Conclusion

In this paper I develop a model that explains cable and internet provider’s reluctance to invest in increased capacity. However, not investing is a strategic choice for the current monopolists. While I assume that service providers are local monopolies, in the future this assumption may be less reasonable, as delaying investment could open the market to entrants. Just in the past year, new competition promising to provide faster service, such as Google Fiber, has successfully prodded service providers to adapt their strategies and consider further investment. Once Google announced that the company would provide internet service through Google Fiber in Austin, Texas, AT&T released a press statement promising to provide equally fast service to the city once Austin signs a similarly lucrative deal with the firm. If AT&T does eventually sign contracts in Austin or other cities to offer significantly faster internet, the firm will have to incur the costs of dramatically increasing capacity (Oreskovic & Carew, 2013) (AT&T, 2013). While my model showed that delaying investment and bundling products may be the optimal choice for providers in the current environment, as $\beta$ declines and competition improves the optimal dynamic strategy will likely shift.
Chapter 6

Appendix

This section describes the methods I used to reach each of the numeric results in the order that they appear in this paper.

When $\beta = 1$ and the monopolist bundles its products, the firm’s maximization problem breaks down into an intractable system of equations. First, I restrict $p_i$ and $p_t$ to be between 0 and 1, inclusive, and $p_b$ to be between 0 and 2, inclusive, and depending on the case, $p_b$ is restricted further with relationships to $\beta, p_i$, and $p_t$. These restrictions are reasonable because if the monopolist set prices outside of those ranges, then no consumers purchase the product corresponding to the irrationally high or low price, and it is just as if the monopolist set the price at the high or low extreme of the price’s range. $p_b$ is further restricted because each of the three cases from section 4.2 with $\beta$ greater than $\frac{1}{2}$ has unique area equations identifying the amount of television, internet, or bundle purchased and hence a unique profit function. Therefore, I treat each case separately and use Mathematica’s Nmaximize function for constrained optimization to numerically find the global maximum at each cap level for each of the cases. Nmaximize is a Mathematica implementation of the Nelder-Mead direct search method which also uses random initial values to insure that it finds a global maximum. By repeating trials with varying initial seeds, I find the optimal price values corresponding to each case and cap. Then, by directly comparing the profits earned through each case’s pricing strategy at each cap level and taking the prices corresponding to the maximum profit, I create the optimal price functions, $p_i(c), p_t(c)$, and $p_b(c)$. With these price function, I can create the optimal profit, $\pi(c)$ plot as well as the penetration rate plots by substituting the prices into the appropriate profit or area equations.
When searching for the optimal price solutions for the $\beta = 0$ case, it is possible to reduce the system of first order conditions to a single quintic function of $p_t$ and $c$. In order to reach the quintic function of $p_t$, I make substitutions for $p_t(p_t)$ that assume $p_t \neq 1$ and $p_t - 1 \neq p_t$. Neither of these assumptions eliminates plausible solutions, as $p_t = 1$ implies the provider never sells internet and $p_t$ and $p_t$ can never have a difference of 1 due to their ranges. These substitutions create the following quintic:

$$3p_t^5 - 13p_t^4 + (20 - 6c)p_t^3 + (18c - 12)p_t^2 + (1 - 22c)p_t + (1 + 10c - 8c^2) = 0.$$ 

Solving this with Mathematica gives us 5 roots as functions of $c$, which Mathematica can plot numerically but not express analytically. Plotting each of the 5 reveals that two roots are two roots are imaginary for all values of $c$ between 0 and 1/3, one is less than 0 for all values of $c$ within 0 and 1/3, and another is greater than 1 for all values of $c$ between 0 and 1/3. The last remaining root is the only possible solution for $p_t(c)$ given the initial constraints and, as shown in Figure 3.6, makes intuitive sense. Using this solution I can create plots of $p_t(p_t(c)) = p_t(c)$ and profit and penetration plots for analysis as well.

I create Figures 4.4 and 4.7 by repeating the process described for $\beta = 1$ for each level of $\beta$ between .05 and .95 incrementing by .05 each time. Rather than save prices to plug into profit equations and area equations, I save maximal profits for each $(\beta, c)$ pair for each case, compare profits for each pair across cases, and save the maximum profit for each pair. Then, these I used these three-tuples, $(\beta, c, \pi(\beta, c))$, to create the figures and Tables 4.1 and 4.2.

To determine the mixed partial values in Tables 4.1 and 4.2, I first use finite differences to estimate the marginal return to capacity at each $\beta$ level as follows:

$$\frac{\partial \pi(\beta, c)}{\partial c} \approx \frac{\pi(\beta, c + .05) - \pi(\beta, c)}{.05}.$$ 

This process yields marginal returns to capacity for each capacity level between 0 and .95. Then, I repeat the procedure to estimate the mixed partial as follows:

$$\frac{\partial^2 \pi(\beta, c)}{\partial \beta \partial c} \approx \frac{\partial \pi(\beta+.05, c)}{\partial c} - \frac{\partial \pi(\beta, c)}{\partial c} \frac{.05}{.05},$$

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which determines the mixed partial derivative values shown in the tables. Lastly, due to the numeric calculations Nmaximize uses, some results are machine numbers such as $-2.7438 \times 10^{-23}$, which we recognize as 0 but initially appears negative. I compare the absolute value of each estimated mixed partial derivative against .000001 as an arbitrary threshold and replace the estimate with 0 if the absolute value is less than that. Otherwise, I round each result to the nearest .001 to create Tables 4.1 and 4.2.
Chapter 7

Bibliography


