THE IMPACT OF TEACHER SPATIAL ABILITY ON GEOMETRY INSTRUCTION: GESTURES, RICHNESS OF MATHEMATICS PRACTICE AND PICTORIAL REPRESENTATIONS

by

BERYL ANN OTUMFUOR

(Under the Direction of Martha Carr)

ABSTRACT

The overarching goal of this dissertation was to examine the relationship between spatial ability and geometric reasoning as it is evident in teacher instruction. A review on the roles of teacher spatial ability, mathematics content knowledge and gesture use during instruction was presented. In addition, an empirical study examining the relationship between teacher spatial skills and spatial instruction was also conducted. Fifty-six in service teachers from middle schools in the Southeast were assessed on their nature of instruction through their use of gestures, pictorial representations and overall richness of mathematics practice. A significant moderator effect for teacher instruction was found suggesting that when teachers have stronger spatial skills, they are more likely to use different strategies like gestures and pictorial representations during mathematics instruction and also draw more explicit connections to mathematics concepts and operations. Overall, the results suggest that if we are to improve mathematics achievement, then more studies need to be done to explore these relationships. Together these findings contribute to the spatial ability research by expanding our understanding on strategies like gestures and pictorial representations that can essentially improve mathematics achievement. Implications of these findings for mathematics instruction are discussed further.

INDEX WORDS: spatial ability, mathematics content knowledge, gestures, geometric reasoning, mathematics instruction, pictorial representations

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REPRESENTATIONS

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DEDICATION

To my awesome husband, *Frederick Dzabaku Otumfuor* – your commitment to your graduate education has been the source of my strength during this writing period. Your unwavering support and encouragement throughout my undergraduate and graduate education has been such a blessing. I thank the Almighty God for placing you in my life and I am excited about the journey we have started and I look forward to our next steps. You make my life so worthwhile and exciting.

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Only a wise person can solve a difficult problem (Akan Proverb)

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If you educate a man you educate an individual, but if you educate a woman you educate a family (Fante Proverb)

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Education is our passport to the future, for tomorrow belongs to those who prepare for it today (Malcom X)

My community of scholars did not begin with my graduate education. I have been surrounded with lots of great mentors who have shaped my scholarship. I developed a strong love for research during college when I first met Marité Haynes and Jeanne Slattery. Marité – your commitment to psychological research opened my eyes to explore research in cognition and child development. You served as a role model and I thank you for mentoring me throughout college. Jeanne – Thank you for giving me an opportunity to explore clinical psychology research. It was through this experience that opened my eyes to engage in other areas of psychology. Susan McHale – you taught me the importance of collaborative work ever since I first met you. It was because of you that I figured out my path for graduate school. My work with you after college opened my eyes about the world of research in human development. I thank you for your words of wisdom which continues to guide me.

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The Family is like the forest: if you are outside, it is dense; if you are inside you, you see that each tree has its own position. (Ghanaian Proverb)

They say it takes a village to raise a child and I am a true example of that. My family has been an anchor of support throughout my life. First, I thank my parents – Mary & William for their immeasurable sacrifices and for helping me to appreciate mathematics. My dad was the first person I learned from that, hard work and perseverance can provide a lucrative career. I thank my mother for her dedication and teaching me that a woman is capable to do what a man can do. Also to my stepmom, Margaret for helping me realize the beauty of education and the impact educators can have on students. Big thanks to my grandparents for paving the way of success in such uneasy times.

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Show me your friend and I will show you your character (African Proverb)

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If you want to go quickly, go alone. If you want to go far, go together (African Proverb)

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Love never gets lost it's only kept (African Proverb)

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The fear of the Lord is the beginning of knowledge; fools despise wisdom and instruction (Proverbs 1:7)

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Lastly, I note that this dissertation reflects my opinions and I ask all readers to please forgive me for any grammatical errors and misspelled words I may have made.

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DISSERTATION INTRODUCTION & LITERATURE REVIEW

Experiences in visualization and rotation are key components in the development of spatial ability (Hegarty & Waller, 2005; Newcombe & Huttenlocher, 2000; Linn & Petersen, 1985; Lohman, 1996; Voyer, Voyer, & Broyden, 1995). These skills are important for success in everyday problem-solving tasks. In addition, these skills have implications for success in school (Ceci, Williams, & Barnett, 2009; Kozhevnikov, Motes, & Hegarty, 2007; Wai, Lubinski, & Benbow, 2009; Uttal et al., 2013). Given the impact of these skills on the lives of school-age children and adults, specifically teachers of mathematics, researchers have sought to understand factors that influence, or are influenced by, the development of spatial skills.

Extant research (Cheng & Mix, 2012; Sorby, 2009; Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008) indicates that spatial skills can be improved through training and these improvements can have significant gains on mathematics achievement for students, but these research studies continue to remain methodologically limited. With spatial ability becoming increasingly important for class instruction, teacher's own mathematics knowledge can be the key to students' mastery of mathematics content and subject matter.

There is also evidence that gestures play a role in communicating mathematical ideas during instruction (Alibali & Nathan, 2012). Research studies (Cook, Duffy, & Fenn, 2013; Goldin-Meadow, Cook, & Mitchell, 2009) have also found a direct correlation between gesture and student learning in mathematics. Because gesturing conveys spatial information to students during instruction, we can argue that if we are to improve mathematics achievement, it is vital that we investigate how teachers' knowledge and spatial skills interact with their use of gestures during instruction. We know that when teachers gesture, they are able to better communicate complex and abstract information that can essentially improve student understanding of mathematics concepts (Alibali, 2005; Alibali & Nathan, 2007; Alibali, Nathan, & Fujimori, 2011; Goldin-Meadow, Kim, & Singer, 1999). Furthermore, when teachers have strong spatial skills, they tend to provide spatial tasks that are linked to mathematics tasks (Battista, Wheatley, & Talsma, 1982; Harle & Towns, 2011; Mohler, 2008; Newcombe, 2010; 2013). In this current work, a review and an empirical study are presented.

Chapter 2 includes a review of literature examining the roles of spatial ability, mathematics content knowledge and gestures during instruction. The information provided in the literature review adds to our understanding of the nature of mathematics instruction by synthesizing research on the nature and impact of relationships between teacher and student spatial ability, content knowledge and mathematics achievement. Using a quantitative design, Chapter 3 uses correlational analyses to investigate the relationship between teacher spatial skills and mathematics instruction. In this empirical study, I ask, "does middle school teacher's spatial ability predict their nature of instruction?" Of particular interest was whether teachers with strong spatial ability were more likely to provide spatial gestures or use spatial gestures, draw connection to other mathematics areas and use more pictorial representations such as pictorial figures and pictorial symbols during instruction to enrich student understanding of geometric concepts.

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CHAPTER 2

THE ROLES OF SPATIAL SKILLS, TEACHER MATHEMATICS KNOWLEDGE AND GESTURE USE DURING INSTRUCTION[1]

[1] Otumfuor, B.A. and Carr, M.M. To be submitted to *Educational Psychology Review*

Abstract

For this literature review, I asked, "What role does spatial ability, gestures and mathematics content knowledge play during instruction and what have scholars learned about the nature of instruction in middle school in-service teaching at this time? It has been found that spatial ability can be improved through instruction or training in both children and adults. In addition, strong mathematics content knowledge and increased use of gestures during instruction can enhance student understanding and ultimately improves achievement in mathematics. The findings from the current literature indicate that more work needs to be done to explore the relationships between spatial ability, mathematics content knowledge and gestures. An in-depth investigation into these relationships can ultimately provide information on effective strategies needed in the classroom to improve mathematics achievement.

Introduction

Mathematics achievement has been linked to students' spatial ability, teachers' use of gesture during instruction and teacher mathematical knowledge (Battista, Wheatley, & Talsma, 1982; Erlich, Levine, & Goldin-Meadow, 2006; Grouws & Schultz, 1996; Hill, Rowan, & Ball, 2005; Ma, 1999). Spatial ability has been positively related to problem solving skills in mathematics and geometry (Battista, 1994; Casey et al., 2008; Fennema & Tartre, 1985; Turgut, 2007). Furthermore, children's spatial ability and mathematics achievement is influenced by teachers' use of gestures during classroom mathematics instruction (Alibali, 2005; Alibali & Nathan, 2007; Ehrlich et al., 2006). When teachers talk and gesture they may be communicating both spatial and verbal information about mathematical concepts thus supporting better mathematics achievement. The purpose of this review is to describe the research on teacher and students' spatial ability, teacher mathematical knowledge and gestures as they influence mathematics achievement. It will be argued that improving spatial ability in teachers will allow students to take better advantage of instruction.

Discussed first is the current literature on instruction to improve spatial ability and mathematics achievement. Second, the research on teacher mathematical knowledge as it relates to mathematics achievement and spatial ability will be reviewed. Next, the connection between gesture, as used during instruction, teacher mathematical knowledge and mathematics achievement will be reviewed. Using what is learned from these three areas of research, recommendations are made for future research on gestures, spatial ability and teacher knowledge in science and mathematics.

For the purpose of this review, spatial ability is defined as "a combination of spatial relations tasks which involve two and three-dimensional rotations and cube comparisons and spatial visualization tasks which requires a mental manipulation and integration of stimuli with one or more movable parts" (Olkun, 2003, p. 1-2).

Improving Spatial Ability and Mathematics Achievement

Studies have shown that spatial ability can be improved in young children (Ben Chaim, Lapan, & Houang, 1988; Cheng & Mix, 2012; Ferrini-Mundy, 1987; Guay & McDaniel, 1977) and in adults (Baki, Kosa, & Guven, 2011; Piburn et al., 2005; Small & Morton, 1983; Stransky, Wilcox, & Dubrowski, 2010). The bulk of the research on adults has focused on students in engineering programs (e.g. Alias, Black, & Gray, 2003; Hsi, Linn, & Bell, 1997; Sorby & Baartmans, 1996; 2000) so it is not clear that programs designed to improve spatial ability improves performance outside of engineering. Only four studies were found that examined whether children's spatial skills could be improved through instruction. The bulk of these studies indicate that children's spatial skills can be improved through explicit instruction.

In examining the research with adults, a number of studies have compared manipulatives to paper-based instruction of spatial skills. Baki et al. (2011) found significant improvement on spatial visualization test after training. For this study, ninety-six teachers received instruction on solid geometry using 3D Cabri software, physical manipulatives (treatment groups) or traditional instruction (control group). Participants were pre- and post-tested using the Purdue Spatial Visualization Test (PSVT). This study demonstrates that both physical and virtual manipulatives have a significant effect on improving spatial skills in adults through instruction. Other research has likewise indicated that the use of manipulatives results in comparable results to strictly computer based instruction. Ferrini-Mundy (1987) found that two forms of spatial instruction (audiovisual or audiovisual-tactual) resulted in improved spatial ability as measured by the Spatial Relations Subtest of the DAT. Further analysis conducted also revealed that students in the spatial training groups improved their ability to visualize solids after training than the control group. Alias, Black, and Gray (2002) evaluated the efficacy of a structural design instruction on student performance on spatial visualization tasks among civil engineering students and found that the scores of students in the experimental group were higher than those of the control group at posttest, however the lack of pretest makes it difficult to conclude that the results were due to the treatment. Assignment was not random in this study and a lack of a true experiment cannot warrant the conclusion of a causal link because of other confounding factors.

More recently, as technology and virtual environments are been incorporated into classroom teaching to enhance student exploration of 3-D representations, researchers have explored the relationship between spatial ability, instruction and technology (Baki et al., 2011; Hannafin, Truxaw, Vermillion, & Liu, 2008; Liang & Sedig, 2010; Price & Lee, 2010; Rafi, Samsudin, & Ismail, 2006; Sorby & Baartmans, 2000; Unal, Jakubowksi, & Corey, 2009). Rafi, et al. (2006) examined the impact of using computer-mediated engineering drawing instruction to improve spatial ability among undergraduate students enrolled in a computer-aided design course. Students were randomly assigned to either an experimental or control group. The treatment conditions for the experimental group were computer-mediated instruction and video-enhanced conventional instruction. The computer-mediated instruction contained engineering drawing tasks developed to include “2-D orthographic views with isometric representations” (p. 151), while the video-enhanced conventional instruction used printed materials augmented with digital video clips. The control group received the conventional instruction but only printed materials for instruction and no video enhancements. Spatial visualization test and an online mental rotations test were administered to the students as both pre and post-test measures before and after the training. The authors found that students in the treatment group performed better than the control group, with the computer-mediated instruction group performing better than both the conventional instruction and conventional instruction with video-enhanced clips groups.

Other studies have indicated that adults' spatial ability can be improved through instruction. These studies, however, frequently lack random assignment to group, a control group or pretesting making it difficult to make a causal claim. Sorby and Baartmans (2000) developed a computer-based instructional course designed to improve academic achievement and spatial skills of first year engineering students. Students with low scores on the Purdue Spatial Visualization Test (PSVT: R) were given the opportunity to take the course so the treatment group included students who self-selected into the course and the control group consisted of students who opted out. Students in the treatment group were found to have significantly improved their spatial scores as compared to the control group and training also helped to improve retention rates in various engineering classes. In a study by Piburn et al. (2005) two geology classes were given two additional lessons on topographic maps and interactive 3-D geological blocks in a computer-based module as spatial training. Students in the treatment classes showed improved performance on measures of spatial visualization test (Surface Development) and spatial orientation test (Cubes Rotation) and a paper and pencil content-based geospatial assessment in comparison to the students in the control classes. As with previous study, however, there was no random assignment to conditions.

Stransky et al. (2010) conducted two experiments to examine the effects of spacing of mental rotation training on student performance. All participants received training on laparoscopic surgery; one treatment group received spatial instruction in one time block whereas the second treatment group received the same amount of training over a period of eight days. Although participants were pretested on spatial skills, pretest scores were not included in the analysis. Mann Whitney and Wilcox tests revealed a better posttest performance on surgical tasks that require mental rotation after students had received training in mental rotation.

While a number of research studies have clearly linked improvement in spatial ability to instructional interventions among the adult populations, very few studies have found supporting evidence for spatial ability and instruction in younger children. More recently, Cheng and Mix (2012) investigated the impact of spatial training on mathematics achievement. Fifty-eight first and second-grade students were randomly assigned to an experimental (spatial training) group or a control (no-training) group. All students were assessed on both pre- and posttest measures of spatial ability (Mental Rotation Tests and Spatial Relations Subtest of the Test Primary Mental Abilities) and a standard-based Michigan state Mathematics exam (A set of 27 problem solving tasks such as "multi-digit calculation, missing and number fact problems" p. 4). The experimental group completed a 40 minute training session which involved practice on mental rotation tasks while children in the control group completed crossword puzzles. The mental rotation tasks required students to imagine and visualize how an object would look like in a different and/or new orientation. MANCOVA analysis revealed significant differences between experimental and control groups, with the experimental group performing better on both the mental rotation and math tests. Further paired samples t-test analysis for the specific component of math tests also revealed significant differences between the experimental and control group. The spatial training group improved significantly on both the missing problems and the multi-digit calculations while no significant improvement was found with the no-training control group. These findings are consistent with previous studies that have found that learning mathematics with spatial tools can lead to improvement on quantitative tasks.

Ben-Chaim et al. (1988) evaluated the impact of a 3-week instructional intervention unit on middle school students' spatial visualization ability and found significant differences between pre and post-test scores on the spatial visualization test. In another research, Hannafin et al. (2008) compared the effects of two instructional resources – Sketchpad[2] and Tutorial programs on middle school students' spatial ability and geometry achievement and found no significant differences between treatment groups. However, the results revealed that high spatial ability students in both treatment groups performed better than low spatial ability students on the spatial tasks. Unfortunately, no control groups were used in either studies and hence, we cannot conclude that the improvement on the test is clearly linked to the training.

Another form of instruction adopted by teachers that shows significant impact on young children's geometric knowledge and spatial visualization is the origami. To compare the effects of origami math lessons and traditional geometry instruction on males and females, Boakes (2009) sampled 56 seventh grade students into experimental and control groups and pre and post-tested them on a national mathematics assessment and a set of spatial tests (Paper-Folding, Surface Development and Card Rotation). Students in the control group received traditional instruction on geometry from a geometry textbook during the geometry unit while, students in the experimental group received a combination of origami mathematics lessons and traditional instruction. Instruction lasted approximately 80 minutes for a period of one month. The author found that students in the experimental group had slightly higher scores on spatial tests in comparison to students in the control group, but the blending of the origami lessons with traditional instruction did not yield any significant differences between the experimental and control groups suggesting that both traditional math lessons and origami lessons equally

[2] Geometer's sketchpad is a type of software developed to support geometry instruction by creating geometric constructions and dynamic transformations of geometric objects and figures.

improved children's spatial ability and overall geometry knowledge. These research studies show that instruction can be effective in improving spatial skills and emphasizes the need to examine the nature of instruction and how it relates to student performance on spatial and mathematical tasks.

In general, these research studies have shown that spatial abilities are not immutable and can be improved through instruction. However, the majority of studies that have been done to date have limitations. Some of these studies have failed to use random assignment, pretesting or control groups, which lends itself to possible threats of validity. Another concern for this existing research is the type of inferential statistics used and the conclusion drawn – some studies failed to use a covariate for their pre and post-test design and others used non-parametric tests to draw inferences about their study, making it difficult to conclude if the improvements were due to intervention and/ or training.

If teacher spatial skill is important for the instruction of mathematics then we may want to target teacher spatial skills as well as mathematical content knowledge. The research to date suggests that spatial skills can be improved but it is not clear that improving spatial skills will result in improved mathematical knowledge. Another option for improving mathematics achievement is through teacher mathematical content knowledge, spatial ability and gestures. Those studies still need to be done.

Mathematical Content Knowledge, Teacher Gestures and Achievement

It is assumed that classroom instruction and student achievement is better when teachers' possess a strong conceptual understanding of mathematics (Fennema & Frank, 1992). Teacher mathematical content knowledge influences instruction and mathematics achievement (e.g. Franke, Kazemi, & Betty, 2007; Kersting, Givvin, Thompson, Santagata, & Stigler, 2012; Ma, 1999; Ng, 2011; Shectman, Roschelle, Haertel, & Knudsen, 2010). It can be argued that when teachers possess such strong mathematical content knowledge and strong spatial skills their use of gestures during mathematics instruction will result in better teaching and enhance student achievement in mathematics.

Mathematical content knowledge

Mathematical content knowledge for teaching consists of subject-matter knowledge and pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Shulman, 1986). Shulman (1986) also included curricular knowledge as a component of mathematical content knowledge. For the purposes of this review, the recent theory proposed by Ball et al. (2008) would be used.

According to Ball et al. (2008), subject-matter knowledge consists of *common content knowledge* (CCK) and *specialized content knowledge* (SCK), whereas pedagogical content knowledge consists of *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* (KCC). CCK refers to mathematical knowledge that is needed by teachers for any assignments they may provide to students and can also be used in a variety of settings. This includes recognizing inaccurate terminologies and using the correct mathematics notations during an instruction. SCK refers to the mathematical knowledge that goes beyond knowledge of content and is unique to specific teaching moments. This includes teacher's ability to provide explanations about a particular mathematical content during instruction that demonstrates their deeper understanding of the concept as well as making horizontal connections with other mathematics elements at that level. KCS refers to a type of mathematics knowledge that combines both knowing about students and knowing about mathematics. This type of knowledge focuses on teachers understanding of students' ability to learn a particular content area, specifically – student's conceptions and misconceptions of that content area. KCT refers to the combination of teacher's knowledge of teaching and mathematics. It also involves the planning of the instruction and the instructional decisions taken by the teacher such as assigned mathematical tasks for their students. KCC refers to the type of mathematics knowledge that teacher's possess when teaching a particular content. This type of knowledge focuses on teacher's use of a variety of instructional strategies and materials.

A number of studies have confirmed that teacher mathematical knowledge is vital for proficient instruction and student achievement (Baumert, Kunter, Blum, Brunner, Voss, Jordon et al., 2010; Hill & Ball, 2004; Hill, Ball, & Schilling, 2008; Marshall & Sorto, 2012; Ng, 2011). Using longitudinal data from Guatemala, Marshall and Sorto (2012) sampled elementary and middle school teachers and students to investigate the effects of teacher mathematics content knowledge (CCK and SCK) on student achievement and found that students overall mathematics achievement was positively linked to teachers with higher levels of mathematical knowledge. Although this study supports the relationship between teacher mathematics content knowledge and student achievement, there are some few problems with this body of research which poses some serious threat to the conclusions drawn. For instance, the entire study was conducted in Guatemala making is difficult to generalize the results to the population in the United States because of the teaching dynamics and cultural differences of the two countries. In addition, the observational protocols and the items used to assess teacher's SCK were developed by the researcher, who also failed to report the process used to validate the measure.

Interventions designed to improve teacher content knowledge have not provided evidence that improving content knowledge improves student outcomes. Shectman et al. (2010) found that the students of teachers who were given professional development designed to enhance teacher mathematical content knowledge did no better on proportionality and linear functions assessments than students of teachers who did not receive the professional training. In another research investigating the relationship between teacher mathematics knowledge, student learning in mathematics and teaching practice, Kersting et al. (2012) examined the relationships between teacher mathematical knowledge, their instruction and student fraction knowledge. Teacher mathematics knowledge for teaching fractions was correlated with classroom video assessments of fraction lessons; however, there was no significant relationship between teacher mathematical knowledge and student fraction knowledge.

Even with mixed result findings, Teacher mathematics knowledge can play an important role in students' mathematical learning because effective instruction of mathematical ideas and problems can enhance student mathematics achievement. If we study the nature of teacher mathematics content knowledge, we can understand how teachers express their mathematics content knowledge during instruction by their use of gestures (Alibali & Nathan, 2012). Similarly, if individuals with stronger spatial skills are more likely to gesture than those with weaker spatial skills (Hottstetter & Alibali, 2007), then we can argue that understanding gestures can help to explain how mathematics instruction can improve student mathematics performance.

Gestures

Gestures by teachers during mathematics instruction can enhance children's understanding of the mathematics concepts and also improves spatial ability (Alibali & Nathan, 2007; Alibali et al. (under review); Erhlich, Levine, & Goldin-Meadow, 2006; Hostetter, Alibali, & Kita, 2007; Newcombe, 2010). There is mounting evidence that gestures are effective in reducing the "cognitive load" of learners (Alibali & Nathan, 2012; Mayer & Moreno, 2002). We can therefore argue that limited gesturing from teachers during mathematics instruction, may not necessarily aid in student comprehension of mathematics concepts. If gestures are important for mathematics instruction, then more studies need to be done to investigate its role in teaching and learning of abstract information.

McNeil (1992) categorized gestures into: i) *iconic gestures* which are “gestures that bear a close relationship to the semantic content of speech”; ii) *Metaphoric gestures*, “gestures not limited to just concrete event but also depicts semantic content via metaphor and expressions”; iii) *Deictic gestures* are “pointing movements, which are prototypically performed with a pointing index finger, or any other body part as well as manipulated artifacts” and iv) *Beat gestures* are “ motor or rhythmic movements that do not present a discernible meaning but are aligned positively with their prototypical movement characteristics” (p. 80). More recently, Alibali and Nathan (2007) have characterized gestures into the following: *pointing gestures* – “using fingers to indicate objects, location or inscriptions”; *representational gestures* – indicates “when the motion trajectory of the hand represents a concept or relation”; and *writing gestures* – involves: “the writing produced when a teacher’s speech is integrated with their hand movements” (p. 353). For the purpose of this review, the above recent definition of Alibali and Nathan (2007) would be used because of the emphasis placed on movements of hands or arms to aid comprehension of concepts during mathematics instruction.

Gestures as Simulated Action (GSA) Framework

Hostetter and Alibali (2008) developed a theoretical framework to explain the emergence of gestures from language and mental imagery as a result of perceptual and motor simulations. The main thesis of this framework is that an individual’s thinking and speaking is grounded in their cognitive system which activates the production of mental images and language through the “sensorimotor” process. The authors stated that “when speakers activate concepts in order to express meaning, they are in essence activating perceptual and motor information, just as comprehenders do when they activate meaning from language inputs" (p. 498-499).

The GSA framework also states that gestures are produced when simulations are activated in the pre-motor area and spread through the motor area. If the activations are strong enough, they produce a *gesture-threshold* which varies per individual and depends on factors such as strength of neural connections between pre-motor and motor areas, cognitive effort and social situation. According to the GSA framework, gestures are elicited when neural activations with strong connections are able to spread from one area to another (pre-motor to motor). For instance, a weak activation will still produce gestures because of the tight connections which can prompt a smooth transmission from the pre-motor to the motor areas. The authors also state stronger activations at a given time elicit speakers to gesture more and depending on the situation, speakers are able to adjust their gesture threshold. For example, during a class instruction, teachers may gesture more when they encounter a concept that is difficult for children to easily comprehend. Finally, the third factor asserts that people gesture during their speech because the neurons in their brains are activated when they perceive some information and this causes the neurons to potentially spread from one pre-motor area to another motor area. These activations are also responsible for speech production and language acquisition.

This framework provides explanation about how gestures stem from spatial representations and mental images by explaining how gestures arise from our embodied cognitive system. If gestures are simulated actions then we can argue that teachers in particular, communicate information through gestures. The question is whether teachers' gestures support learning in mathematics.

Gestures and Spatial Reasoning

There is a growing interest linking spatial reasoning to use of gestures as this can influence student achievement. Very few studies have been conducted on gestures but what has been done have shown that gesturing during instruction impacts children's spatial reasoning (Alibali, 2005; Alibali & Nathan, 2007, Chu & Kita, 2008; 2011; Cook, Mitchell, & Goldin-Meadow, 2008; Ehrlich et al., 2006). Ehrlich et al. (2006) found that overall performance on spatial transformation tasks was improved in kindergarten children regardless as to whether the students observed movement, imagined movement, or simply practiced. They also found that boys had an advantage both pre- and post-training and that gesturing was linked to better spatial task performance. Another study by Cook et al. (2008) investigated the effect of gestures on eighty-four elementary students' mathematics learning. Students were pre-tested on six math equivalence addition problems and randomly assigned to three treatment conditions –instruction on math problems that included either speech only, gesture only, or both gesture and speech. Children were then instructed to mimic teacher's behavior as seen during instruction on practice mathematical equivalence problems. Children were post-tested on items similar to the pre-test. A follow-up posttest was conducted four weeks after the experiment. The results found that students from each of the treatment condition – speech only, gesture only and both gesture and speech improved with instruction. Furthermore, regression analysis revealed instruction that encouraged the use of gestures was more effective in facilitating better insight into new mathematics concepts than instruction that did not use gestures. Chu and Kita (2011) assigned 189 undergraduate students to one of three experimental conditions (gesture-encouraged, gesture allowed and gesture prohibited groups) to investigate performance on spatial tasks like mental rotation and paper folding. They found that in comparison to the "gesture-allowed" and “gesture-prohibited”, students in the “gesture-encouraged” performed better on spatial visualization tasks and solved more mental rotation problems. This suggests that explicitly telling students to use gestures may activate spatial skills in problem solving, but exactly how the use of gestures helped improve learning remains to be seen.

In regard to improvements in math, gesture in instruction improves mathematics achievement in younger and older students. More recently, Cook, Duffy, and Fenn (2013) investigated the effects observational gestures on student initial and subsequent mathematics learning among 22 classrooms with 188 elementary school children. Students were pre and post-tested on mathematical equivalence. Eleven classrooms with students were randomly assigned to a speech and gesture condition and the remaining eleven classrooms to the speech-alone condition. Students in the speech and gesture condition were provided with six training videos in which an instructor was gesturing and solving an equivalence problem. Students in the speech-alone condition were also provided with six similar videos but the instructor in these videos did not gesture. A multilevel logistic regression performed revealed that students in the speech and gesture condition improved and performed much better than the speech-alone condition on both posttests. These findings are consistent with previous studies that have found that gesturing can enhance student mathematics learning.

Other research among the adult populations has also indicated that gestures can enhance performance on spatial tasks. In another study, Alibali, Spencer, Knox, and Kita (2011) randomly assigned 88 undergraduate students to either gesture-prohibited or gesture-allowed condition to determine whether spontaneous gestures allowed for better learning of mathematics. All participants were administered six mathematics problems involving the rotation of gears in a horizontal line. Students were then instructed to provide information about the type of strategies they used to solve these math problems. These strategies were classified as either perceptual-motor or abstract strategy. Students who failed to solve the problem correctly initially were given another opportunity to resolve it before the next problem was provided. The authors found that students encouraged to use gestures were more likely to use perceptual-motor strategies to solve problems involving rotations than students forbidden from using gestures. They concluded that gestures do play a role in the strategy choice students' use when solving mathematics problem. Alibali and Nathan (2007) used a video analysis of classroom instruction to investigate the role of teacher's gestures on student understanding of middle school algebra content and found that teachers produced more gestures and utterances during the algebra instruction when the content was perceived as a difficult task for students. This served as a method to scaffold students learning of the algebraic content.

These aforementioned studies highlight the importance of gestures during instruction of mathematics because complex mathematics content can easily be explained with gesturing. However, more research needs to be conducted to explore the relationship between spatial ability, teacher mathematical knowledge and gestures because these areas can ultimately help to improve student performance on abstract mathematics tasks.

Conclusion

This purpose of this review was to describe the research of teacher and students spatial ability, teacher mathematics knowledge and use of gestures and its impact on mathematics achievement. This current review suggests that spatial ability and mathematics content knowledge can play an essential role in enhancing student achievement in mathematics.

Further, if gestures prove to be effective in improving mathematics achievement then students can benefit from instruction that includes gesturing. Gesturing can provide students with some spatial information that can increase their conceptual knowledge of mathematics concepts. While there is little evidence supporting the role of spatial ability, mathematics knowledge and gestures during instruction, we know that these areas are necessary for improving student performance in mathematics. Hence, tremendous amount of work needs to be done to explore these relationships.

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CHAPTER 3

THE RELATIONSHIP BETWEEN TEACHER SPATIAL SKILLS AND SPATIAL INSTRUCTION[3]

[3] Otumfuor, B.A., Carr, M.M., Beckman, S., Neuharth-Pritchett, S., and Schwanenflugel, P. To be submitted to *Journal of Research in Mathematics Education.*

Abstract

The purpose of this study was to examine the relationship between teacher spatial ability, mathematics content knowledge and use of gestures during mathematics instruction. Over nine months, 56 middle school teachers were observed and administered spatial measures on mental rotations and spatial visualization. Correlational analyses revealed that spatial ability is a significant predictor of teacher's use of gestures, pictorial figures and overall richness of mathematics practice during instruction in geometry. Partial correlations also computed found that gestures and pictorial figures were significantly correlated to spatial ability. Implications for teaching young children are discussed.

Introduction

Geometry is an important part of the kindergarten through grade 12 mathematics curriculum. It is through geometry that children begin to develop an understanding of geometric shapes and how also analyze the characteristics and relationships of these structures (NCTM, 2000). According to the National Council of Teachers of Mathematics (2000), because geometry links mathematical thinking to spatial reasoning, students need to improve both their spatial and geometric skills in order to improve mathematical thinking.

Geometric reasoning is complex and rich in that it represents mathematical concepts using physical, visual and spatial environments (Battista, 2004; Clements, 1998; Clements, Wilson, & Sarama, 2004). Geometric reasoning is defined as "the invention and use of formal conceptual systems to investigate shape and space; these conceptual systems use angle measure, length measure, congruence and parallelism to abstract spatial relationships within and among shapes" (Battista, 2001, p. 843). Such abstract spatial relationships include an ability to transform and identify shapes, manipulate and turn angles into two-dimension and three-dimension figures, and ability to compare and contrast whole or parts of a shape (Clements, 2003).

Although extensive research has been done to examine young children's numerical thinking (Baroody, Lai, & Mix, 2006; Bjorklund, Hubertz, & Reubens, 2004; Geary, 2006; Jordan, Kaplan, Nabors, & Locuniak, 2006; Mix, 1999; 2002; National Research Council, 2009; Siegler & Booth, 2004; Booth & Siegler, 2006), geometric reasoning has not been as well explored. We need to investigate geometric reasoning because geometry has been linked to every strand in the mathematics curriculum and also sets the foundation for learning other Science, Technology, Engineering and Mathematics (STEM) disciplines (NCTM, 2000). For example, spatial reasoning is used to represent and manipulate information in learning and problem solving and hence, essential for scientific thought (Clements & Battista, 1992). Spatial skills are also required in many subjects such as physics, mathematics and engineering (Pellegrino, Alderton, & Shute, 1984). Because spatial skills have been hypothesized to support mathematics achievement by helping students mentally organize information during problem-solving, Clements (1998) suggested that mathematics classroom should promote development of spatial sense.

The goal of this dissertation is to examine the relationship between spatial skills and geometric reasoning as it is evident in teacher instruction. Discussed first is the current literature on geometric reasoning of students, and then the research on the literature on the development of spatial skills is reviewed. Next, the research on teacher mathematical content knowledge will be discussed, with a special focus on geometry and spatial skills. An argument will be made that mathematics education and educational psychology would benefit from an understanding of how and whether teacher spatial skills affect their mathematical content knowledge for geometry and how they instruct their students in geometry.

Geometric Reasoning of Students

van Hiele proposed five qualitatively different and hierarchical levels of geometric thought that children progress through when aided by instruction (van Hiele, 1984). At the first level called *visualization*, students identify geometric figures as visual wholes rather than through their geometric properties. In the second level, *analysis*, students describe and characterize shapes in detail by their properties. At the third level, *abstraction*, students can identify the hierarchies of geometric figures, class inclusion and can also deduce properties of figures. With the fourth level, *deduction*, students have a conceptual understanding on how to construct proofs, and the significance of the relationships between axioms and definitions. Finally, at the fifth level, *rigor*, students have moved beyond informal and formal deductions and have mastered advanced techniques to abstract deductions within a mathematical system. Research studies (Abdullah & Zakaria, 2013; Usiskin, 1982; van Hiele, 1986; Van de Walle, 2004) have found that lower secondary school students are only able to reach the third level of thought, abstraction. These levels of geometric understanding are used as the foundation for teaching children in schools (Senk, 1989; Usiskin, 1982). Clements and Battista (2001) suggested that the van Hiele levels of geometric reasoning may develop simultaneously though not necessarily at the same rate; and teachers play a significant role in helping children advance to the next level (Fuys, Geddes, & Tischler, 1988).

A number of empirical studies have examined children's knowledge of shapes and the development of these geometric concepts. Using clinical interviews to examine the criteria 97 preschool children use to distinguish members of a class of shapes from other geometric figures, Clements, Swaminathan, Zeitler, Hannibal, and Sarama (1999) found that children had less difficulty in identifying circles, some difficulty in classifying squares without horizontal sides and the most difficulty in recognizing triangles and rectangles. Clements et al. (2004) also assessed the developmental learning trajectory that preschool through second grade children used in composing geometric shapes. They found that when given a geometric task that required decomposition and composition of 2-D figures, older children synthesized and evaluated mathematical arguments using geometric relationships, while younger children combined shapes using a trial and error method and described shapes using attributes of the shapes.

Very little research has been done to examine the impact of linear measurement on geometric reasoning; however this is an important principal application of mathematics because it connects three important areas of mathematics: geometry, spatial sense and number operations. Piaget and Inhelder (1967) found that students were able to conceptualize simple 2-D figures if they had developed an understanding of linear measure concepts and properties.

To investigate students' development of length concepts in a unit on geometry, Clements, Battista, Sarama, Swaminathan, and McMillen (1997) collected paper-and-pencil pre- and posttest assessments, interviews and case studies on 38 fourth graders. They found that students with an advanced knowledge of spatial and numeric representations used effective strategies to internalize measurement concepts. For instance, they noticed that three levels of strategies were used by students to solve different length problems. Some of these included "visual guessing of measures, drawing hatch marks, dots or line segments, drawing proportional figures and, using mental computations to connect length of segments" (p. 91). This research suggests that students may be predisposed to choose certain strategies that may or may not be effective in connecting spatial and numerical schemas. Nevertheless, teachers can observe student's interpretations of tasks and engage them in activities to improve their measurement and geometric skills.

Interventions to Improve Geometry: Computer-based Manipulatives

Research studies have found that within a spatial environment, young children can begin to mentally manipulate and represent objects (Casey et al., 2008; Clements, Sarama, & Wilson, 2001; Clements & Sarama, 2007; NRC, 2009). This therefore can be essential in building their geometric concepts through instruction. Teachers can essentially assist children consistently in connecting both real-world objects and representational symbols (NRC, 2009). Within the elementary school classrooms and curricula, less emphasis is placed on understanding of geometric concepts. However, we find that very few teachers have extensive knowledge and experience with geometry and as such fail to provide students in the classroom with spatial activities that can enhance spatial skills. This lack of spatial understanding is continually reflected in their assessments as they move from elementary to middle school. In the middle grades, the mathematics curriculum undergoes dramatic change, becoming less algorithmic and more dependent on interpretation and comprehension to solve word problems. Research on interventions indicates that middle-grades teachers can help students experience success by controlling the complexity of problem-solving tasks by using activities that promote active student involvement; for instance, use of intervention strategies like informal and formal assessments, differentiated instruction, multiple representations and real-life applications. For this section, I will highlight the different techniques adopted and used empirically to gauge student understanding and monitor student thinking and reasoning.

Research by Clements and Battista (1992), Grant, Peterson, and Shojgreen-Downer (1996) and Clements (1998) indicated that pictorial and computer generated representations are useful in developing young children's geometric thinking. For example, young children as early as five or six years of age can use information in pictures to build 2-D and 3-D figures (Murphy & Wood, 1981).

Sarama (2004) hypothesized that technology in the early childhood classroom will enable children to build solid content knowledge and develop higher-order critical thinking in mathematics. One such approach is the use of the *Building Blocks program* which is a computer based curriculum to teach spatial, geometric and numeric concepts to preschool children. Some of the activities children perform in *Building blocks* provide representations that seem like real physical manipulatives. For instance, in one *Building blocks* activity children can fill puzzle outlines using a set of pattern blocks. With this activity, the children can combine two green triangles by gluing and then duplicating the unit to fill the outline (Clements & Sarama, 2007).

Such an activity helps children to understand different ways to select fewer blocks to fill an outline and in doing so improve their geometric and spatial skill. Furthermore, the use of computers allows children to manipulate and develop new shapes (Clements, 1998).

The Logo program has been hypothesized to provide meaningful context for formulating ideas and systems of thinking about geometry (Clements & Battista, 1990). The programming involves turtle geometry, which produces a Logo command – "a set of primitive graphic commands that control the displacement and rotation of display screen cursor called a turtle" (Cuneo, 1985, pp. 1-23). The Logo program teaches Euclidean geometry when students direct the computer generated turtle to draw shapes or geometric objects based on complex instructions (Abelson & diSessa, 1986).

Clements and Battista (1990) investigated whether the logo programming experience facilitated children's development of geometric concepts and the transition from van Hiele visual level to analytic level of geometric thinking. Twelve students from a middle school class were matched on pretreatment mathematics achievement (using problem-solving subtests and mathematical concepts from the Iowa Test of Basic Skills (ITBS)) and randomly assigned to either a logo programming group or a word processing group. Students were placed in 40 minutes sessions of computer treatments for instruction. The Logo programming group activities included turtle graphics, and construction and manipulation of simple geometric figures. Each student was individually interviewed post- instruction to investigate his or her level of geometric reasoning. Although initial pretest indicated all children had very shallow understanding of angles, the results indicated that children in the logo programming group developed a more accurate concept of angle. They also gained a better ability to correctly represent problems involving rotations necessary for problem solving. The authors concluded that logo improved student’s geometric learning of concepts because a logo curriculum helped children discover geometric ideas. The above research shows that we can improve geometric reasoning in young children through the use of logo program.

Clements (1998) emphasized that using multiple modes of representation prepares children for school tasks that involve spatial and geometric skills. One such instructional approach, *The Agam Program*, was found to be extremely effective in promoting geometric and spatial thinking, as well as arithmetic and writing readiness, among young children between the ages of three and seven (Eylon, Rosenfeld, & Agam 1990). The structure of this program requires teaching activities to “begin with the building of a visual alphabet and then presenting the visual alphabet with ideas that combine to form complex and symmetric patterns, for example, introducing horizontal lines in isolation, and then teaching relations such as parallel lines” (p. 20).

When instructing children in mathematics, it is important that teachers understand that mathematical language should not be used too early without connected mathematical knowledge. This is because children’s concepts that underlie language may be vastly different from what teachers think (Clements, 2003). Therefore, teachers cannot rely solely on textbooks when teaching and representing geometric figures and objects to students. To that end, some effective instructional tools that can be adopted include diagrams, manipulatives and pictures. Clements and Battista (1992) emphasized that when perceiving a diagram for a proof problem, a student needs to focus on what is essential and dismiss what is inessential but this process has been found to be very difficult for most. For that reason, the researchers acknowledge that instructional attention to diagrams such as multiple drawings for a proof problem and discussing such problems explicitly can be very helpful.

Spatial Skills

Spatial ability has been hypothesized to support mathematics achievement by helping students mentally organize information during problem-solving and by facilitating their discovery of mathematical patterns and relationships (Tarte, 1990). Clements (1998) asserts that mathematics classes should promote development of children's spatial skills, because, according to NCTM (1989) spatial skills are necessary for "interpreting, understanding and appreciating our inherently geometric world" (p.48).

Defining Spatial Skills

Although spatial ability may seem straightforward and simple because it focuses on objects and space around us, it is important to recognize that spatial ability can be complex. What then is spatial ability? Dixon (1983) argues that "spatial ability depends on grasping the consistency in relationships between objects in our world when these relationships occur in the context of fluid, changing patterns. The fluidity represents infinite possibilities like a face seen from different angles; however, grasping the relationships allows people to recognize that they are seeing different instances of the same thing within a fluid pattern while the relationships remain consistent. Understanding fluidity allows a person to anticipate instances in the infinite flow of possibilities, even though the person may never actually have observed that given instance before" (Dixon, 1983, p. 27).

Spatial ability consists of two major categories: *spatial orientation* and *spatial visualization*. Spatial orientation involves "knowing where you are and how to get around the world; that is, operating and understanding relationships between different positions in space especially with respect to your own position; whereas, spatial visualization is understanding and manipulating imagined movements of two-and three-dimensional objects" (Clements, 1998, p.

11, 16). Spatial orientation is when young children are able to understand and navigate maps, while spatial visualization is when children create and manipulate mental images of an object. Linn and Petersen (1985) conducted an extensive meta-analysis to review and classify the different categories associated with spatial abilities since it is not a singular trait. The authors divided spatial skills into three main categories: *spatial perception, mental rotation,* and *spatial visualization*. Spatial perception which uses both cognitive and psychometric rationale requires "estimating the spatial relationship in respect to the orientation or position of one's own body" (p. 10). Mental rotation, which uses the same rationale, is the ability for an individual to manipulate data presented spatially using complex rotations. Spatial visualization, on the other hand, involves the use of spatial perception and mental rotation to distinguish potential multiple solution strategies in multistep process.

Piagetian Theory and Spatial Skill Development

According to the Piagetian theory (Bishop, 1978), spatial skills are developed in three stages. In the first stage, topological skills, which are primarily two-dimensional, are acquired by the age of five for most children. These acquired topological skills can help children recognize relationships among objects and order of objects in a group. Children who can solve puzzles have acquired these skills.

With the second stage that emerges in adolescence, children acquire projective spatial ability, which involves visualizing three-dimensional objects and perceiving what they look like when rotated or transformed in space. Piaget argues that this skill is acquired by adolescents through their everyday life experiences with familiar objects. However, with unfamiliar objects, even college students may have difficulty visualizing and transforming the object at this stage. Some studies do not support this theory because it has been found that elementary school children can perform such rotations. For instance, Dickson, Brown, and Gibson (1984) found that two-year old children show some evidence of elementary spatial visualization. Finally, in the third stage, a person is able to combine measurement concepts with their projective skills resulting in the ability to visualize concepts of area, volume, distance, translation and reflection. The research on mental rotation and spatial visualization aligns with Piaget's spatial ability theory on projective space (Caplan, MacPherson, & Tobin, 1985).

According to Piaget and Inhelder (1956) spatial ability consists of several components: topological space, Euclidean space and projective space. By assessing these different interrelated spatial structures, one can assemble a clearer picture of where an individual stands in terms of their hierarchical spatial development (McArthur & Wellner, 1996). Various studies have not supported the proposed developmental hierarchy or the interconnections of spatial structures. Tversky (1999) emphasized that spatial knowledge is not Euclidean like actual space and methods to model this knowledge has relied heavily on topological and qualitative models. Similarly, studies using single test measures such water level tasks and mental rotation tasks to assess spatial ability (Liben, 1991; Vasta, Lightfoot, & Cox, 1993) only represent a small piece of Piaget's work on conception of space.

Spatial Ability and Mathematics Achievement

There is a significant link between spatial ability and mathematics achievement. A number of studies have established a link between spatial ability and mathematics achievement – children and adults with better spatial abilities tend to have higher mathematics scores. Using four spatial ability measures, Guay and McDaniel (1977) investigated the relationship between elementary mathematics achievement among males and females with differing spatial abilities on 90 elementary school children. They found a positive correlation between mathematics achievement and spatial test scores. Battista, Wheatley and Talsma (1982) used Purdue-Spatial Visualization Test and Longeot test of cognitive development to investigate the interaction between spatial ability, cognitive development and mathematics learning on 82 pre-service elementary teachers enrolled in a geometry course. They found that the spatial visualization scores of the teachers improved significantly by instruction at the end of the semester. In another study, Ganley and Vasilyeva (2011) used spatial tests and curriculum measures to examine the relationship between math performance, spatial skills, and sex differences. The authors found that boys' performance on spatial tasks significantly predicted their math achievement in course grades and national assessments. Because of the clear relationship between mathematics achievement and spatial skills, the National Council of Teachers of Mathematics (2000) has acknowledged the importance of teaching of spatial skills in the K-12 classrooms (Casey, Nuttal, & Pezaris, 2001).

Mathematical Knowledge for Teaching

Effective instruction of mathematical ideas and problems requires a teacher to have knowledge and skills instrumental to envision and foster students' opportunities to learn. Teacher's mathematical knowledge plays an important role in students' mathematical learning (Ball, 2003). Carpenter, Fennema, Peterson, and Carey (1988) examined the relationship between teachers expectations and student performance and found a significant relationship between teacher's predictions of their students' success and actual student achievement, however, there was no significant relationship between either teacher's general knowledge of problem, awareness of problem difficulty, students strategy use and student achievement. Contrary, Hill, Rowan, and Ball (2005) found that teachers' performance on specialized and general assessment on mathematical knowledge predicted a significant gain in students' test scores. The authors suggested that effective teaching requires more than teacher's own mathematical knowledge and that teachers mathematical knowledge for teaching a specific content can help to minimize the achievement gap of disadvantaged students. Because there is little research done to examine teacher mathematical knowledge in the context of their classroom instruction, it is important that we investigate how teacher mathematics knowledge correlates with their instruction of mathematics, specifically in geometry.

Teacher mathematical knowledge can be divided into three components: subject matter knowledge, pedagogical knowledge and curricular knowledge (Shulman, 1986). Teacher subject matter knowledge is defined as "the amount of organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). Thus, understanding of the subject matter does not only include awareness of the "facts or concepts" of the domain but goes beyond to include an understanding of the structure. Subject matter knowledge therefore consists of what Schwab (1978) termed as substantive and syntactic knowledge. Substantive knowledge refers to "how to organize the basic concepts and principles of a subject, whereas, syntactic knowledge concerns the processes by which theories and models are generated and established as valid" (pp. 229-272). An example in mathematics will be constructing of proofs or in science the formulation of generalizations (Petrou & Goulding, 2011; Schwab, 1978; Shulman, 1986).

Teacher pedagogical knowledge is defined as the subject matter knowledge for teaching (Shulman, 1986, p.9). This type of knowledge includes the common topics of a particular subject, efficient ways to represent ideas, and the powerful analogies, illustrations, examples, explanations and demonstrations that make it easy for students to understand. Pedagogical content knowledge also involves understanding why some subjects are easy or difficult to learn, conceptions and preconceptions that students of different ages and backgrounds may have about a topic, and an understanding of student misconceptions that can influence their learning trajectories (Shulman, 1986).

Teacher curricular knowledge is defined as “the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serves as both the indication and contraindications for the use of particular curriculum or program materials in particular circumstances” (Shulman, 1986, p.10). It also includes an understanding of alternative curricula for instruction and requires teachers to associate the content of a particular subject with ideas in other subjects. Hence, teachers should be able to choose tools of teaching that represents a particular content very well and tools that help them to assess students’ achievement (Shulman, 1986).

Student learning however is largely influenced by the type of instruction received. In an effort to improve instruction, Swafford, Jones, and Thornton (1997) discovered that there is a common belief that the more a teacher knows about a particular mathematical concept such as rational numbers, the more effective they will be in nurturing mathematical understanding of their students. However, with geometry, very little is known about the content knowledge of in-service teachers. Also, research that has been conducted over the years has focused on students’ content knowledge in numbers and operations and instructional practices but little is known about teacher’s knowledge of students’ cognition in geometry or the impact of such knowledge on instruction especially in middle school.

Statement of Problem

Researchers have found that when American students are compared to their counterparts in other western societies, students from the U.S. have lower mathematics achievement scores.

The difference in math achievement has been attributed to the significant cross-national differences in informal mathematics knowledge that appear as early as four-years-old and at different levels of school readiness for a standards-based mathematics curriculum (Klein & Starkey, 2003). According to evaluations of mathematics learning through National Assessment of Educational Progress (NAEP), students are failing to develop adequate geometric problem-solving skills (Battista & Clements, 1998). Specifically in the U.S., students have not been prepared to learn and perform well on advanced geometric concepts (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Stigler, Lee, & Stevenson, 1986). This is because the current geometry curriculum emphasizes on the identification of figures and use of geometric terms, providing students very few opportunities to develop the spatial skills of importance for the geometry curriculum (Battista & Clements, 1988; Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988).

Spatial skills, the ability to understand, visually represent and represent problems involving physical space, shapes and forms (Tarte, 1990) have been shown to be essential for success in engineering and mathematics (NRC, 2006). Geometry, which clearly involves spatial skills, is linked to multiple other areas of mathematics. Student spatial skills are enhanced through classroom instruction and these skills are imparted on by effective teaching (Halpern, 1992; Merill, Devine, Brown, & Brown, 2010; Howarth & Sinton, 2011; Park, Kim, & Sohn, 2011). In order to be effective, teachers must have good mathematical content of knowledge and provide learning environments in which students are challenged to engage with mathematical concepts and extend their understanding in meaningful ways (Chinnappan & Lawson, 2005). Teachers' spatial skill, therefore, may influence the type of instruction provided to students and can impact student mathematics achievement, particularly in the learning of geometric concepts.

A number of studies have focused on student's spatial skills and how those skills are related to mathematics achievement (Carroll, 1993; Guttman, 1954; Hegarty & Waller, 2005; Mohler & Miller, 2009); however there's still a dearth of research that documents the impact of teachers' spatial skills on their classroom instruction. In addition, there is no research examining whether and how teacher's instruction is influenced by their spatial skills. This study will fill the gap in the research literature by investigating the role of teacher spatial skills and knowledge in the middle school classroom instruction.

Present Study

The purpose of this study is to determine whether teacher spatial skills predicts their mathematical knowledge, with a special focus on their geometric skills; whether teacher spatial skills predicts the way they teach middle school geometry; and the materials they provide to their students. It is hypothesized that teachers with high spatial skills will have better mathematical knowledge and will provide and use more spatial gestures during instruction.

The current study focused on in-service teachers, as opposed to pre-service teachers, because these teachers have developed skills and expertise as a function of their work. Pre-service teachers, in contrast, may not have a good understanding of the challenges and opportunities of the classroom. In-service teachers will provide a much more valid assessment of the role of teacher spatial skills in mathematics learning. In middle school, the difficulty of mathematics content increases rapidly and hence students without sound foundational skills can get lost and may require remedial instruction (AMLE, 2011). Therefore, this current study focused on how middle school teachers' knowledge and skills play a role in their classroom instruction particularly on the learning of mathematical concepts, specifically geometry. Accordingly, the present study addresses the following research question:

1) Do middle grade teacher spatial skills predict their nature of instruction? Specifically

i) In the form of the use of more gestures–including hands and arms.

ii) In the richness of mathematics practice – the mathematical procedures and facts, in-depth explanations of a geometry problem, type of mathematical language - the use of precise terms and vocabulary to describe complex geometric concepts, and the explicitness of instruction.

iii) Use of pictorial representations – these are tools to help with geometric concepts such as diagrams, graphs, scales and sketch figures/maps. It also provides examples that allow students to investigate 2-D and 3-D figures.

Operational Definition

1) *Spatial skills*: involves the ability to understand physical spaces by differentiating such spaces in relation to your own position in space. It also involves an ability to mentally create images from an object and manipulating such objects.

2) *Geometric skills*: involves the ability to use geometric language to describe and name shapes and identify basic properties and figures. It also involves the use of transformations to create movement of objects and combining two and three dimensional figures to create new figures.

3) *Gestures*: Based on Alibali's categorization of gestures which includes the following: representational, pointing and writing gestures. Representational gestures refer to "when the motion trajectory of the hand represents a concept or relation"; Pointing gestures includes the "use of fingers to indicate objects, location or inscriptions" and Writing gestures involves "the writing produced when teacher's speech is integrated with their hand movements during an instruction" (Alibali & Nathan, 2007, p. 353).

4) *Spatial gestures* – use of analogies to aid in comprehension of abstract relations, for example, hands and arms gestures, diagrams, mental images and visualization. This information or material is presented visually in a productive way to allow students recognize or identify information presented.

5) *Mathematical content knowledge*: Based on Shulman's dimensions of content knowledge, which includes the following three components: subject-matter, pedagogical and curricular knowledge. "Subject matter refers to knowing beyond knowledge of facts or concepts of a particular subject area and understanding the structures of the subject matter in substantive (how to organize the basic concepts and principles of a particular subject) and syntactic (for instance, are the set of rules on which mathematics builds truth or false, valid or invalid) ways simultaneously". "Pedagogical knowledge goes beyond the subject matter knowledge. It includes common topics in a particular subject, the efficient representation of ideas and the powerful analogies, examples, illustrations and demonstrations to help students learn easily". "Curricular knowledge is the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indication and contraindications for the use of particular curriculum or program materials in particular circumstances" (Shulman, 1986, pp. 9-10).

6) *Mathematical knowledge for teaching*: Based on the Ball and Hill's domains of content knowledge which consists of subject- matter knowledge – common content knowledge and specialized content knowledge and pedagogical content knowledge – knowledge of content and students, knowledge of content and teaching and knowledge of content and

curriculum. Common Content Knowledge – mathematical knowledge used in a variety of settings by teachers. Specialized content knowledge – mathematical knowledge that is specific to teaching and goes beyond the content. Knowledge of content and students – mathematical knowledge that combines mathematics content and knowledge of student ability. Knowledge of content and teaching – type of mathematical knowledge that involves an extensive knowledge of teaching and mathematics. This includes instructional decisions teachers make when preparing for lessons. Knowledge of content and curriculum – mathematical knowledge that teacher's possess when instructing a particular content. (Adapted from Ball et al. 2008, pp. 399-403)

7) *Mathematical quality of instruction:* Adapted from "Elements of mathematical quality of instruction" (Hill et al., 2008, p. 437) which includes the following:

a. *Mathematics errors* – presence of computational, linguistic, representational errors in instruction.

b. *Richness of mathematics* – use of multiple representations, linking among representations, mathematical explanations and justification around mathematical practices like proofs and reasoning.

c. *Mathematical language* – the density and accurate mathematical language in instruction to clearly convey mathematical ideas and any explicit discussion of the use of mathematical language.

8) *Nature of instruction:* refers to the following four components

a. Incorporating a variety of lesson types, that is, problem-based lesson, games and investigations in which there's still a guided, shared and independent approach to support student learning.

b. Providing a variety of representations of geometric concepts such as concrete materials that students can manipulate, helping students relate new materials to previous lessons and introducing abstract ideas such as two and three dimensions.

c. Use of examples and non-examples to introduce orientations and configurations for students to investigate 2-D and 3-D.

d. Use of geometric terminology, that is, precise terms and vocabulary to describe complex mathematical ideas.

e. Use of technology such as virtual manipulatives like geometer's sketchpad or dynamic geometric tools to assist children in learning geometric concepts.

Methods

Research Design

This study examined the relationship between teacher spatial knowledge and their classroom instruction and employed both spatial measures and classroom observations for data collection. The independent variables were the two spatial skills measures (*Mental Rotations and Hidden Figures Tests*) which consisted of 52 items respectively. The dependent variable was the teacher spatial skills checklist which contained a total of 15 items. These items were divided into three main variables - *spatial gestures* with 2 items, *richness of mathematics practice* with 5 items and *pictorial representations* with 8 items. These variables assessed the overall nature of instruction.

Participants and Settings

Sixty-two in-service middle grade school teachers were recruited from middle schools in Metro Atlanta and surrounding county school systems but a total of 56 teachers participated in the study. For this particular study, teacher racial composition was as follows: 24% Caucasian, 71% Black and 5% Asian with 42 females (75%) and 14 males (25%). There were 19 sixth grade teachers, 18 seventh grade teachers and 19 eighth grade teachers; ranging from the ages of 21 to 58 (M = 1.73, SD = .84). Demographic information including degrees, mathematics background, school district, type of community, certification, current enrollment in a degree program and years of teaching were also collected (see Table 3.1; 3.2. 3.3). Primary data collection took place over a nine month period lasting from September, 2012 through May, 2013. During this time period, data was collected extensively using the teacher instruction checklist, videotape recordings, as well as spatial measures and demographic data. Letters highlighting the research were distributed to principals and teachers of the county school systems once the Department of Research and Evaluation of the County School System Review Board and the University of Georgia's Institutional Review Board had formally approved the research. All teachers were informed that the research is voluntary and at any time, if they have a change of mind are allowed to leave without penalty.

Table 3.1

Demographic Characteristics of Participants (N = 56)

Characteristic	N	%
Age (at time of study)		
21-35	27	48.21
36-46	19	33.93
47-57	8	14.29
58-70	2	3.57
School Type		
Rural	6	10.90
Urban	35	63.63
Suburban	14	25.45

Table 3.2

Educational Characteristics of Participants (N = 56)

Characteristic	N	%
Highest level of Education		
Bachelor's	22	39.28
Master's	24	42.86
Education Specialist	9	16.07
Doctor of Philosophy	1	1.79
Current Enrollment		
Yes	14	25.45
No	41	74.54
Teacher certification		
Undergrad math education	4	7.14
Undergrad mathematics	3	5.36
Undergrad middle Edu.	14	25.00
Graduate education	15	26.79
Other	20	35.71

Note: Undergrad = undergraduate. Edu. = education. Math = mathematics

Table 3.3

Classroom Characteristics of Participants (N = 56)

Characteristic	N	%
Classroom Type		
Regular	40	71.43
Gifted	9	16.07
Special Education	5	8.93
Alternative School	2	3.57
Teaching Grade		
6^{th}	19	33.93
7^{th}	18	32.14
8^{th}	19	33.93

Materials and Procedures

Two instruments were employed to assess teacher spatial skill. Each teacher was administered a packet that included demographic questionnaire and two spatial measures (*Vandenberg Mental Rotations task and Kit Referenced Cognitive Hidden Figures subtest*). Spatial measures were administered after the observation of the class instruction by the researcher. The entire measures took approximately 30-35 minutes to complete.

The Vandenberg Mental Rotations test. The items from this battery were pictorial representations of three dimension (3-D) objects (see Appendix A). Participants selected from four rotated forms, two rotated forms that they believed to be similar to the target item. This test's reliability among adolescents and adults were found to be .83 and .88 respectively (Kuse, 1977; Vandenberg & Kuse, 1978; Wilson, DeFries, McClearn, Vandenberg, Johnson, Mi, & Rashad, 1975). Participants were given 2 points when both choices were correct and no credit otherwise. This eliminated the issue of guessing. The test contained 20 total items and took approximately ten minutes to complete.

The Kit Referenced Cognitive Tests. Participants were also administered the Hidden Figures subtest on spatial orientation and visualization. The test (see Appendix B) had 32 items and participants had 24 minutes to complete the measure. Participants were presented with five simple geometrical figures and then instructed to decide which one of them is embedded in the complex pattern. Scoring for this test was a point for the number marked correctly and no credit otherwise for incorrect answers. This minimized guessing. Internal consistency for this test was found to be .82 for high school males, .83 for college males and .80 for female college students (Ekstrom, French, Harman, & Dermen, 1976).

Observation of Classroom Instruction

To assess overall nature of instruction as well as teacher spatial knowledge, teacher participants were observed once in their classroom during a math instruction period that focused mainly on geometry. Observations were completed by the researcher. The consent forms and informational letters were distributed to participants during a staff meeting as determined by the principal. Once the consent forms were collected, correspondence was mainly done with the teachers via email regarding their teaching schedule to determine the most convenient day and times to visit their class. Most of the classroom visits involved both lecture-type instruction and classroom interactions with other students.

During the classroom observation, I sat at the back of the class watching the teacher as he or she interacted with his or her students. Each observation lasted the entire mathematics class period (ranging from 10 minutes to approximately 80 minutes and varied per each school). The observations involved observation of instructional methods employed in the classroom, classroom materials and classroom environment. The lesson was also videotaped. The video recording instrument was positioned off the side so as not to interrupt the flow of instruction. In order to calculate reliability, videotaped observations were also scored by another rater. The teacher spatial skills checklist developed by the researcher was used for scoring (appendix C).

This checklist was developed by the researcher and was based on two existing measures: Ball and Hill's Mathematical Knowledge for Teaching and Robert Pianta's Classroom Assessment Scoring System. The 15-item checklist was used to code teacher behaviors into four categories: *Spatial gestures* (2 items), *Richness of Mathematics Practice* (5 items), and *Pictorial Representation* encompassing *Pictorial Figures* (6 items) and *Pictorial Symbols* (2 items). The *Spatial gestures* category included both pointing and representational gestures. Teachers were coded for using representational gestures when they used their hands/arms to specifically depict an image of a geometric object (e.g. acute angle – using their arms to represent an angle less than 90º). The *Richness of Mathematics Practice* included items that evaluated the depth of the mathematics content provided to the students during instruction such as the explicitness of the mathematics instruction (e.g. the geometric terminology, the connections to other mathematics areas, real-life applications, and the overall mathematics practice). The *Pictorial Representations* category evaluated both the pictorial symbols and figures used during instruction. Teachers were coded for using pictorial symbols when they explicitly highlighted the conventional symbols and/or standard mathematics notations during instruction. Pictorial figures were coded when teachers drew connections to mathematics concepts by drawing figures (either scale drawing or PowerPoint drawing) and/or when teachers represented two-dimension objects in three-dimension figures.

Reliability Analyses. Cronbach's alpha were conducted and used to evaluate the reliability of the four teacher instructional variables of the current study with this sample. The overall alpha obtained for the measure used to assess overall nature of instruction was 0.61. According to Nunally (1978) and Garson (2006), an adequate cut off for internal consistency in any exploratory research is 0.70. However, a cut-off of between 0.50 - 0.60 has also been deemed acceptable (Baumgartner & Jackson, 1999; Crocker & Algina, 1986). The lowest measure of reliability was associated with the Pictorial Symbols measure (r =.20) which was a two-item scale. Spatial gestures (r =.52) and pictorial figures (r=.53) had moderate internal consistency. Richness of math practice (r = .48) was below the moderate internal consistency. However, a careful review of the items was conducted but I chose to keep all the items because deleting one did not significantly improve the overall internal consistency of the measure.

Inter-Rater Reliability. Measurement instruments can lend itself to measurement error if it requires raters to make subjective assessments. Hence, it is necessary to estimate the extent of measurement error that exists within an instrument so that interpretations made are considered reliable (Shrout & Fleiss, 1979). Maxfield and Babbie (2005) argued that consistency among raters can be achieved when raters independently code the same sample of the phenomenon of interest. One such measure for correlating the scores independently assigned by two raters to the same sub-sample is the intraclass correlation coefficient. The overall intraclass correlation coefficient computed as agreement among the two raters was $\alpha = .99$. Also, Pearson-product correlation revealed a Cronbach's alpha of .97. According to Landis and Koch (1977), values greater than .80 are considered outstanding and indicate a good level of agreement among raters.

Results

Preliminary Analysis

The *Statistical Program for the Social Sciences*, version 21.0 (SPSS, Inc., 2013) was used to compute descriptive statistics, correlations and mean comparisons among the variables. An independent samples *t*-test was also conducted to evaluate the hypothesis that male teachers are more likely as opposed to female teachers to possess stronger spatial skills which enable them to be more explicit with their explanation of mathematical concepts, and use more spatial gestures, pictorial representations, and pictorial symbols during their instruction in geometry. There was no significant difference in the spatial gestures of the males ($M = .19$, $SD = .22$) and females ($M = .22$, $SD = .19$), $t(20.02) = -.50$, $p = .75$, or the richness of math practice of males ($M = .61$, $SD = .22$) and the females ($M = .49$, $SD = .23$) $t(23.52) = 1.72$ $p = .17$. There was also no significant difference between males ($M = .02$, $SD = .06$) and females ($M = .01$, $SD = .02$), $t(13.80)$, $p = 3.96$. Finally, there was no significant difference in the use of pictorial representations among males (M = .11, SD = .11) and females (M = .14, SD = .12), $t(23.82) = -.82$, $p > .05$ during instruction, specifically geometry. The results indicate that teacher gender does not play a significant role in the type of strategies teachers use during instruction to provide cues that can increase student conceptual understanding in mathematics as well as their spatial skills.

Description of Data

Evans (1999) stressed that outliers can exist in datasets for a number of reasons, and hence, understanding the cause of these outlying data is important in the decision of retaining, eliminating or recoding observations in a question. Different sources of outliers arise and some include population variability, execution, measurement or recoding errors, incorrect distributional assumptions (Ancsombe, 1960; Iglewicz & Hoaglin, 1993). In order to screen for outliers, information about the skewness and kurtosis of each item was considered, and the normality of individual cases were tested by using the Mahalanobis distance (D) statistic, cooks distance and Leverage values provided in the SPSS output. As a general rule of thumb, researchers often designate |2.0| as the cutoff for normal levels of both skewness and kurtosis for Mahalanobis distance. However, more lenient criteria have also been proposed. Some researchers consider kurtosis values greater than |7.0| to be extreme (Kline, 2005). The Mahalonobis distance statistic tests for the non-normality of individual cases by measuring how far the set of scores is from the sample means for the set of all variables.

The descriptive statistics provided by SPSS shows one variable with skewness and kurtosis values above the cut-off criteria. For instance, pictorial symbols had skewness and kurtosis value of |4.84| and |28.15|. The following values are viewed as problematic by both the conservative cut-off criteria of |2.0| and liberal cut-off of |7.0|. An influential point causes substantial changes in the fitted model. Deletion of a point will in general cause large changes (Chatterjee & Price, 1991). In detecting significant outliers, critical values were examined using both the Mahalonobis distance and Cooks distance. With Mahalonobis distance, the maximum distance should not be greater than the critical chi-squared with degrees of freedom equal to the number of predictors and alpha of .001. On the other hand, Cook's distance should not be greater than 1. For the observation of Cook's Distance, no value was greater than 1. However, the maximum Mahalonobis distance was |42.093| which may suggest an outlier. Although, this value is a bit far off from the significance levels, I decided to retain this value but award caution during my interpretations. Another method used to detect if any outlier exist is the Variance Inflation factor (VIF) which if >= 4 suggests the existence of multicollinearity in the data. The results from different iterations showed no existence of multicollinearity when both dependent variables were controlled. We also examined the distribution of the variables. These analyses revealed that the hidden figures variable was not normally distributed (see Figure 3.1).

Table 3.4[4]

Measures of Central Tendency and Spread of Observed Variables

Measure	*Mean*	*SD*	*Skewness*	*Kurtosis*
Mental Rotations	17.79	11.73	.32	-1.16
Hidden Figures	13.13	10.76	.83	-.86
Spatial Gestures	8.59	7.85	1.32	1.20
Math Practice	20.41	8.84	.69	-.21
Pictorial Figures	5.45	5.05	1.26	1.91
Pic. Symbols	.50	1.36	**4.83**	**28.15**

Note: Pic. Symbols = Pictorial Symbols

[4] The mental rotations and hidden figures tests mean's and standard deviations are out of the total number of items on the tests. The other teaching instructional variables are based on the entire instruction period and so mean and standard deviation are based on the total number of instances divided by the total instructional time.

A Histogram of Normal Distribution of Hidden Figures and Mental Rotations

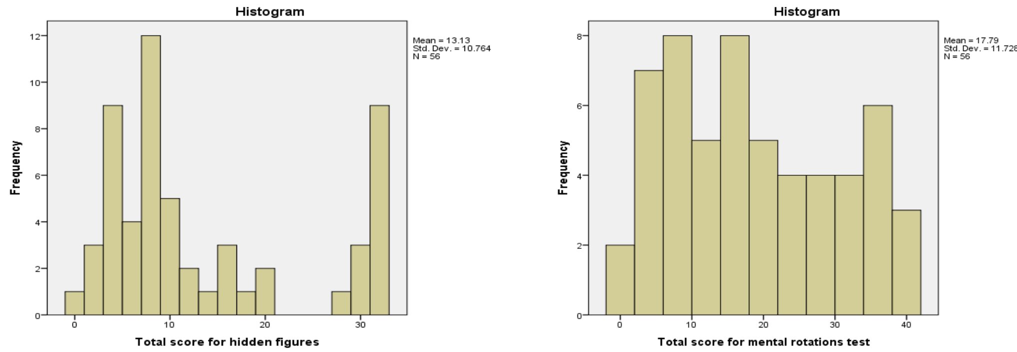

Figure 3.1[5]

Factor Analysis. Costello and Osborne (2005) emphasized that either maximum-likelihood or principal axis factoring are the best recommended methods of extraction. For the purposes of this study, Principal Axis factoring would be used to conduct a factor analysis to identify the salient single factor that measures the same underlying dimension of spatial ability during instruction. According to Gorsuch (1997), use of the common factor method is based on the assumption that the measure of variables is most likely error free and also assumes a unique factor is associated with the variables. To ensure that data is suitable for factoring, I examined the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy which yielded a value of .500. The KMO and MSA measures examine the strength of the relationship and the degree of common variance among the six variables. Bartlett's test of sphericity was also significant (chi ^2 = 22.19, p = .000) indicating that the correlation matrix was not equivalent to the identity matrix. Both of these methods also indicated that the correlation matrix was suitable for factor analysis.

[5] The histogram displays a bi-modal distribution for the hidden figures test. To estimate linearity, data can be normalized using lognormal. However, I chose not to use this procedure because normalizing the data can also skew the interpretations of my data.

Determining the Number of Factors

In determining which factors to retain, I evaluated multiple criteria including: eigenvalues (>1), scree plots, parallel analysis and factor interpretability. Items with loadings greater than .30 were used as the cut-off value for the salient loadings and hence, were considered significant. In terms of interpretability, the matrix of the factors loadings for each variable is shown below in Table 3.6. The two interpretable factors initially accounted for 79.13% and 20.87% of the variance in the data yielding a total of 100% before rotation. Because only one factor was extracted, the solution could not be rotated. The extraction of one single factor also indicates that both variables measure one specific factor – spatial ability. All the variables loaded onto one and only one factor with a 0.40 or greater factor loading. Further examination of both the eigenvalues and the scree plots indicate that I retain one factor after extraction. The factor extracted from the factor analysis was used to generate a factor score which was used to examine the relationship between the demographic variables and the teacher instruction variables.

Table 3.5

Initial Eigenvalues, Percentages of Variance and Cumulative Percentages of Factors of the Spatial Checklist

Factor	Initial Eigenvalue	% of Variance	Cumulative %
1	1.58	79.13	79.13
2	.42	20.87	100.00

Table 3.6

Factor Matrix

	Factor
	1
Mental Rotations Score	.76
Hidden Figures Score	.76

Scree Plot of the Spatial Measures

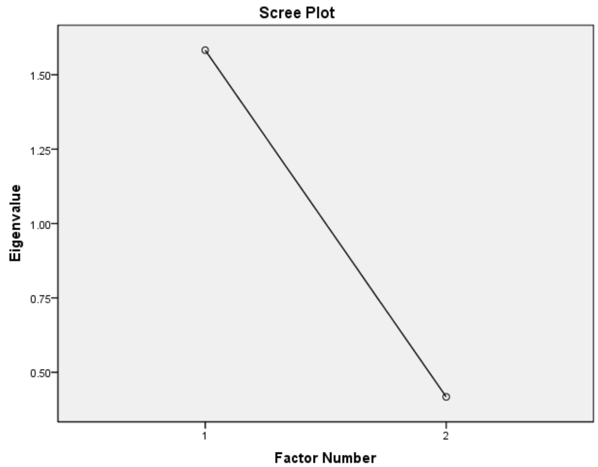

Figure 3.2

Correlations. Correlations were run to address the first question (Do middle school teacher's spatial ability predict their nature of instruction in the form of the use of gestures, in their richness of mathematics practice and in their use of pictorial representations?). Correlation analyses were computed for the four instruction measures and the factor score generated as a result of the factor analysis extraction. The results are presented in Table 3.7. There was a significant positive correlation between the spatial factor score and spatial gestures (r = .39), and the spatial factor score and richness of mathematics practice (r = .27). Thus, the use of gestures and richness of mathematics practice indicates spatial skills are necessary for mathematics and instruction.

Among the teacher instruction variables, There were no significant correlations between the spatial gestures and pictorial symbols during instruction (r = .01), richness of mathematics practice and pictorial symbols (r = .06), and pictorial figures and pictorial symbols (r = -.01). However, there were significant correlations between richness of mathematics practice and spatial gestures (r = .39) indicating that teachers with strong mathematics content knowledge may make this knowledge evident through the use of gestures. There was also a significant correlation between spatial gestures and pictorial figures (r = .51), indicating that teachers who gesture a lot also are more likely to use pictorial figures. Interestingly, the lowest correlation was found between the pictorial symbols and pictorial figures (r = -.01) variables. This near zero correlation indicates that the use of mathematical symbols is not linked to the use of figures in instruction.

Table 3.7

Pearson Product Correlations among Teacher's Instructional variables and Spatial Factor

	SFG	SG	RMP	PS	PF
SFG	-				
SG	**.39****	-			
RMP	**.27***	**.39****	-		
PS	.25	.01	.06	-	
PF	.25	**.51****	.25	-.01	-

**.Correlation is significant at the 0.01 level (2-tailed)

*. Correlation is significant at the 0.05 (2-tailed)

Note: SFG = Spatial Factor Score. SG = Spatial gestures. RMP = Richness of Math Practice. PS = Pictorial Symbols. PF = Pictorial Figures.

To find out which variables of the demographic factors have important effects on teacher instruction, Correlations were also computed between teacher instruction variables (spatial gestures, richness of mathematics practice, pictorial figures and pictorial symbols) and five demographic variables (number of teaching years, current enrollment, initial teacher certification, teaching grade and highest level of education). A p-value less than .05 was used to test significance among the variables. The results of the correlational analyses are presented in the Table 3.8 below. There was a significant moderate correlation between gestures and teaching years (r = .35), gestures and current enrollment in a program (r = .32) and gestures and teaching grade (r = .28). Also, number of teaching years and highest education level were significantly correlated (r = .27) and highest education level and teaching grade was also significantly correlated (r = .28).

Table 3.8

Bivariate Correlations of Teacher Instruction and Demographic Variables

	Teaching Yrs	Education lev	C. Enrollment	Certification	Grade
Teaching Yrs	-				
Education lev	**.27****	-			
C. Enrollment	.21	.17	-		
Certification	.13	-.01	-.00	-	
Grade	.23	**.28****	-.05	-.02	-
Gestures	**.35****	.16	**.32***	-.23	**.28***
Math Practice	.02	-.16	.13	-.07	.07
P. Symbols	-.03	-.15	.02	.15	.03
P. Figures	.26	.03	.25	.06	.10

**.Correlation is significant at the 0.01 level (2-tailed)

*. Correlation is significant at the 0.05 (2-tailed)

Note: Teaching yrs = number of teaching years. C. Enrollment = current enrollment in a program. Education lev = highest level of education received. Certification = initial teaching certification. Grade = grade teacher teaches currently

Partial Correlations. It is suspected that demographic variables might also explain the relationship between teacher spatial ability and their use of spatial gestures, richness of mathematics practice, use of pictorial symbols and pictorial figures during instruction. To assess the relationship between these variables, partial correlations were computed to control for differences that can arise from other confounding variables. Partial correlations therefore explain the unique association between two variables not accounted for by the overlap between the constructs. Partial correlation coefficients were computed among the teacher instruction variables (spatial gestures, richness of mathematics practice, pictorial symbols and pictorial figures) and demographic variables, holding constant or controlling for initial teacher certification, number of teaching years, current enrollment, and highest level of education and grade of teaching. However, upon controlling for initial teacher certification, spatial gestures and richness of mathematics practice was significant, $r = .39$, $p < .05$ and pictorial figures was also significant at $r = .55$, $p < .01$. Similarly, when number of teaching years was controlled, spatial gestures and richness of mathematics practice ($r = .41$) and spatial gestures and pictorial figures ($r = .47$) were significant. Teacher's current enrollment was also controlled and the results revealed a significant correlation between spatial gestures and richness of mathematics practice ($r = .37$) and spatial gestures and pictorial figures ($r = .52$). Upon controlling for teaching grade, pictorial figures and spatial gestures was significant, $r = .51$, $p < .01$ and spatial gestures and richness of mathematics practice, $r = .39$, $p < .01$. When highest level of education was also controlled, significant correlations were obtained between spatial gestures and richness of mathematics practice ($r = .43$) and spatial gestures and pictorial figures ($r = .51$). The spatial factor was also positively correlated with gestures when demographic variables (number of teaching years, current enrollment, initial teacher certification, highest level of education, and teaching grade) were partially controlled. The results are presented in Table 3.9. Thus, even after controlling for these demographic variables (teacher certification, current enrollment, teaching grade, and number of teaching years) teachers were still likely to gesture and use pictorial figures during instruction, indicating that the relationship between these variables are not explained by a third factor.

Table 3.9

Partial Correlations among Teacher Instruction Variables with Teacher Certification Controlled

	Partial Correlations controlling for initial teacher certification				
	Gestures	Math Practice	Pictorial Figures	Pic Symbols	Factor
Gestures	-				
Math Practice	**.39****	-			
Pic Figures	**.55****	.07	-		
Pic Symbols	.05	.26	-.02	-	
Spatial Factor	**.38****	**.26***	.27	.26	-

** $p < .01$

* $p < .05$

Table 3.10

Partial Correlations among Teacher Instruction Variables with Level of Education Controlled

	Partial Correlations controlling for highest level of education				
	Gestures	Math Practice	Pictorial Figures	Pic Symbols	Factor
Gestures	-				
Math Practice	**.43****	-			
Pic Figures	**.51****	.03	-		
Pic Symbols	.04	**.26***	-.00	-	
Spatial Factor	**.43****	.24	.23	.26	-

** $p < .01$

* $p < .05$

Table 3.11

Partial Correlations among Teacher Instruction Variables with Current Enrollment Controlled

	Partial Correlations controlling for current enrollment				
	Gestures	Math Practice	Pictorial Figures	Pic Symbols	Factor
Gestures	-				
Math Practice	**.37****	-			
Pic Figures	**.52****	.05	-		
Pic Symbols	.00	.24	-.00	-	
Spatial Factor	**.40****	**.26***	.26	.22	-

** $p < .01$

* $p <.05$

Table 3.12

Partial Correlations among Teacher Instruction Variables with Teaching Years Controlled

	Partial Correlations controlling for initial teacher certification				
	Gestures	Math Practice	Pictorial Figures	Pic Symbols	Factor
Gestures	-				
Math Practice	**.41****	-			
Pic Figures	**.47****	.06	-		
Pic Symbols	.02	.26	.00	-	
Spatial Factor	**.38****	**.27***	.26	.24	-

** $p < .01$

* $p<.05$

Table 3.13

Partial Correlations among Teacher Instruction Variables with Teaching Grade Controlled

	Partial Correlations controlling for initial teacher certification				
	Gestures	Math Practice	Pictorial Figures	Pic Symbols	Factor
Gestures	-				
Math Practice	**.39****	-			
Pic Figures	**.51****	.06	-		
Pic Symbols	.00	.25	-.01	-	
Spatial Factor	**.41***	**.27****	.25	.26	-

** $p < .01$

* $p < .05$

General Discussion

This discussion will highlight some of the major findings from this study, address some of the limitations and discuss the implications for education and offer suggestions for future research. The aim of the current study was to examine the relationship between teacher spatial ability and teaching skills, including gestures, and mathematics content knowledge, linked to student achievement. I hypothesized that teachers with strong spatial ability are more likely to use gestures and pictorial representations like symbols and figures when teaching geometric concepts during mathematics instruction. Furthermore, teachers' with strong content knowledge and strong spatial skills are more likely to engage their students in spatial activities by using both pictorial representations and gestures to enhance student's spatial skills and mathematics knowledge. The discussions of the results are organized below by the main research questions for gestures, richness of mathematics practice and pictorial representations.

Pearson product correlations were conducted to answer the research question, Do middle grades teacher's spatial ability predict their nature of instruction in the form of the use of gestures? The spatial ability factor correlated with teacher gestures. In examining the individual tests, teachers who performed well on the two spatial tests (Mental Rotations and Hidden Figures) used more gestures during their mathematics instruction. This finding supports the assumption that gestures emerge out of spatial ability (Hostetter & Alibali, 2008). This study is also consistent with research showing that individuals with stronger spatial skills are more likely to gesture than those with weaker spatial skills (Hostetter & Alibali, 2007).

Do middle grades teacher's spatial ability predict their nature of instruction in the form of the richness of their mathematics practice? Bivariate correlations indicated that teacher spatial ability was statistically correlated with richness of mathematics practice. This is in line with past research that found that teacher's with a deeper understanding of fundamental mathematics used variety of strategies, including pictorial representations and manipulatives, to aid in student comprehension of the mathematics content (Ball et al., 2008; Fennema & Franke, 1992; Ma, 1999). This research goes further to show that these instructional strategies may be supported by strong spatial ability. These findings suggest that we should look at basic skills, such as spatial skills, when assessing teachers. Future research is needed to clarify the role of spatial skills in instruction, the impact of strong spatial skills on student outcomes, and whether it is appropriate to improve spatial ability in teachers with the goal of improving instructional outcomes.

Do middle grades teacher's spatial ability predict their nature of instruction in the form of the use of pictorial representations? Bivariate correlations did not yield significant results between the spatial factor and pictorial symbols or pictorial figures. However, bivariate correlations of the two spatial tests (Mental Rotations and Hidden Figures) were significantly correlated with pictorial figures. The results indicate that spatial ability might be associated with pictorial representations and hence, teachers with low spatial ability found it difficult to mentally manipulate images or visualize objects from two-dimensions to three-dimensions. This finding supports prior studies that high spatial ability students easily construct schematic spatial representations than low spatial ability students who find it difficult to create spatial relations from pictorial images (Hegarty & Kozhevnikov, 1999). This suggests that visualization can be important for instruction in that teachers will be better able to construct and use pictorial representations during instruction. These pictorial representations support student comprehension (Lean & Clements, 1981; Newcombe, 2010).

Bivariate correlations were also computed to examine the relationship among gestures, richness of mathematics practice, pictorial symbols and pictorial figures. A significant correlation was found between gestures and richness of mathematics practice and between gestures and pictorial figures. Pictorial Symbols was not correlated with any other factor indicating that the use of symbols is not evidence of richness of mathematical content knowledge, spatial ability, or spatial gestures. This may be because the teachers, while using mathematical symbols, did not refer to them as much during explanations. Pictorial symbols were to be learned and used but were not highlighted during instruction. These findings are consistent with the work of Flevares and Perry (2001) who found that teachers used both gestures and pictorial representations along with speech to convey mathematical concepts to students during explicit instruction.

Finally, the impact of teacher demographic data on their instruction was examined. A number of research studies have explored the relationship between teacher's level of education and classroom quality and found a positive relationship between classroom quality and effects of teacher qualification (Early et al., 2006; Tout, Zaslow, & Berry, 2005; Zill & Resnick, 2005), therefore, these variables were controlled to determine whether spatial ability remained a significant predictor of teacher instructional variables. Bivariate correlations indicated that only teaching years, current enrollment and teaching grade were significantly correlated with spatial gestures. Additional analyses examining the correlations among spatial ability, gestures, richness of instruction, the use of pictorial symbols and pictorial representation controlling for teacher demographic variables indicated that these variables had little impact. These findings are surprising given that teacher certification has been linked to teaching quality.

Limitations

There are several limitations with this current study, and hence caution should be taken for any interpretations of the results. Of particular interest is that of the sample size. While, we had a 90% response rate from our recruited sample, the initial sample of 62 participants was not strong enough to elicit a higher power. Despite the limitation with the sample size, the significant correlation findings make a key contribution as one of the few studies to explore teacher spatial ability and other teaching skills in the middle school classroom and provided information for future studies.

Even though the spatial checklist provided us with important information regarding instruction and how these variables correlate with teacher spatial ability, many steps were taken to ensure that the checklist was reliable. But, the developed checklist reliability was still not above the moderate level (0.70 and above) as required by most statisticians. With only 15 items on the checklist, there is likelihood that some aspects of teacher instruction may not have been measured. For instance, the items for richness of mathematics practice could be viewed by some mathematics educators as narrow in assessing teacher instruction. In addition, while there was a great amount of agreement between the two raters, one limitation that could be emphasized is the fact that both raters are trained educational psychologists and may have a more subjective view of the mathematics instruction provided by the teachers in the study in comparison to the deep foundational knowledge, experience and objective view that experts from mathematics education and mathematics may possess.

Another limitation is the length of period teachers were observed for instruction. As much as a large number of varied teachers were observed on their instruction in mathematics, many mathematics educators could argue that observing an instruction once may not provide all the information about teacher's ability and instructional methods especially if in particular some teachers had a "bad day". Future research needs to observe teachers during the entire geometry unit to provide better insights into their teaching.

Recommendations for Future Research

While this study contributes to the current literature, there are several other areas that future research can focus on. A large sample size should be used to further examine the research questions above. A priori investigation for higher power indicated that a sample size of 115 participants can likely reduce the margin of error and produce a much better effect size than what our smaller sample size achieved. While the spatial checklist developed for this study was used specifically for instruction in the middle school classroom, it was only tested on a small sample size. I would recommend the checklist be also used with both elementary and high school teachers instructing geometry. Future studies can also investigate the usefulness of the items for the differing grade levels. In addition, the items from this checklist should be expanded to include a wider variety of items that assess instruction in both geometry and other areas of mathematics – statistics and measurement, and proportional reasoning.

CHAPTER 4

DISSERTATION CONCLUSION

The pressing need to reform mathematics has brought a call to action by many educators and researchers including such organizations like the National Council for Mathematics Teachers (NCTM). To reform mathematics, also requires a different approach to instruction – such as diverse instructional strategies for the changing population. If an early foundation is laid for geometric learning, we find that as early as infancy, children can recognize basic shapes and structures and make sense of these relationships. But these experiences vary with time from child to child because of children's encounter with two-dimensional and three-dimensional figures. This is why it is imperative that research on teaching examines relationship between mathematics content knowledge and spatial ability.

Our study contributes significantly to the current literature by providing additional reasons on the need to improve spatial ability as well as the use of gestures during instruction. We find that even with such small sample size, a relationship exists between mathematics content knowledge and teacher's use of gestures during instruction. If our goal is to improve student achievement, then we have to find ways to also improve teacher's mathematics knowledge and spatial ability which ultimately can influence their use of gestures and help teachers enrich student understanding of abstract concepts. Tremendous work still needs to be done on effective curriculum for middle schools. Clements and Sarama (2007) have developed the "*Building Blocks*" curriculum for pre-school and elementary students. A similar curriculum that takes account of student's cognition and teacher knowledge of spatial and geometry reasoning will not only be effective in improving student's spatial thinking but can also enhance teacher's strategy use of gestures and pictorial representations during instruction if adopted for use in middle schools and even high schools.

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APPENDIX A

VANDEBERG MENTAL ROTATIONS TEST

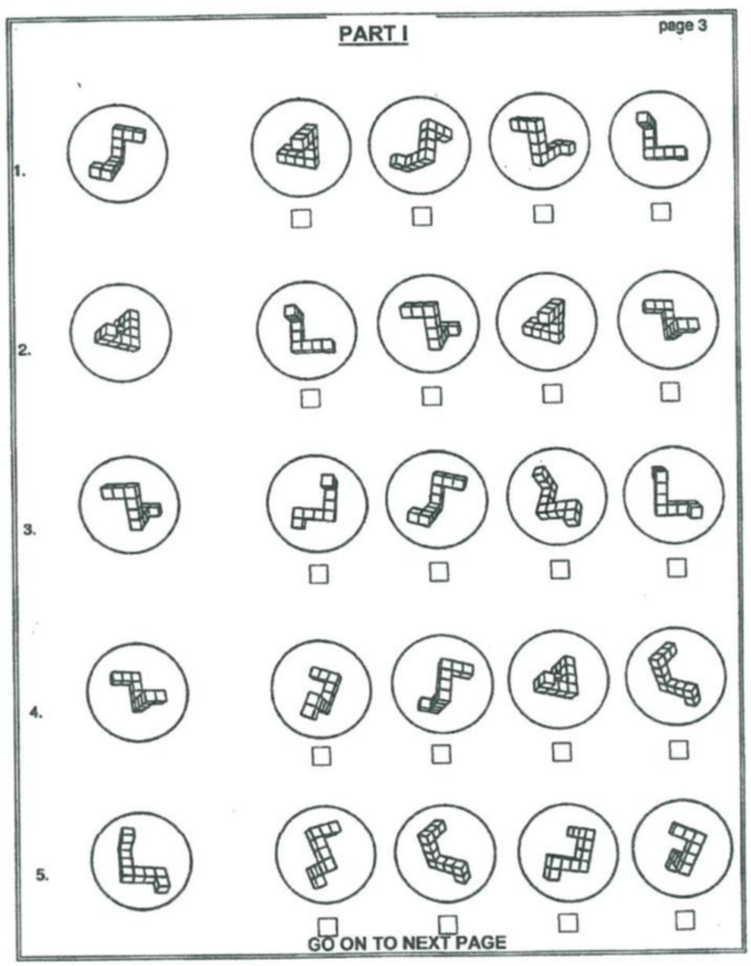

Figure 1 (continued)

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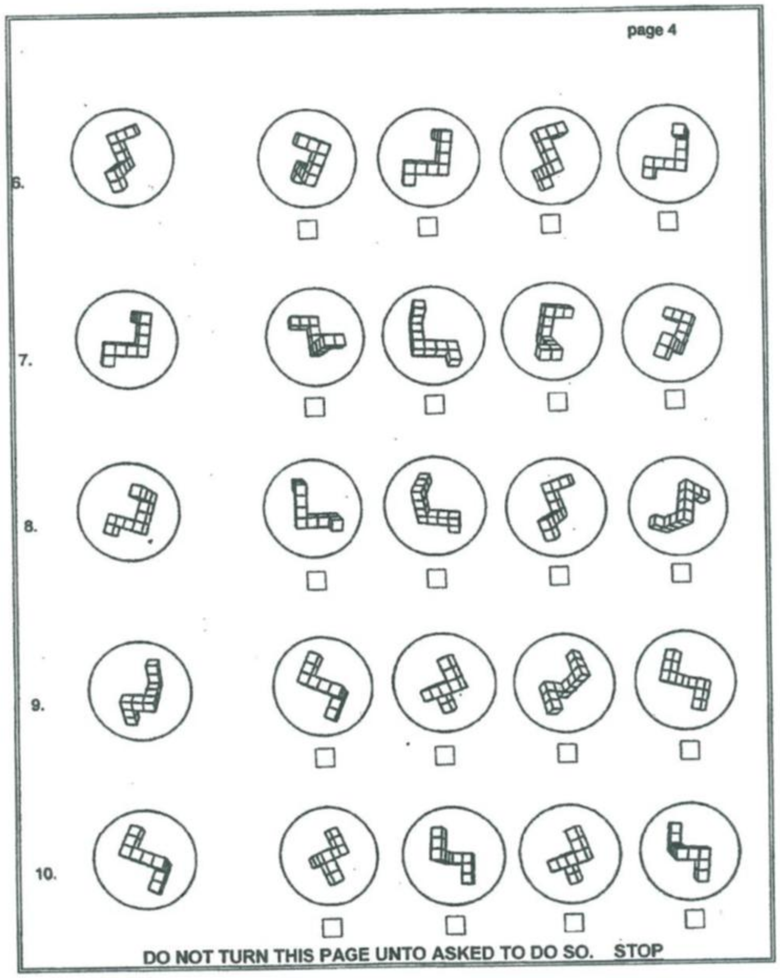


Figure 1 (continued)

70 78

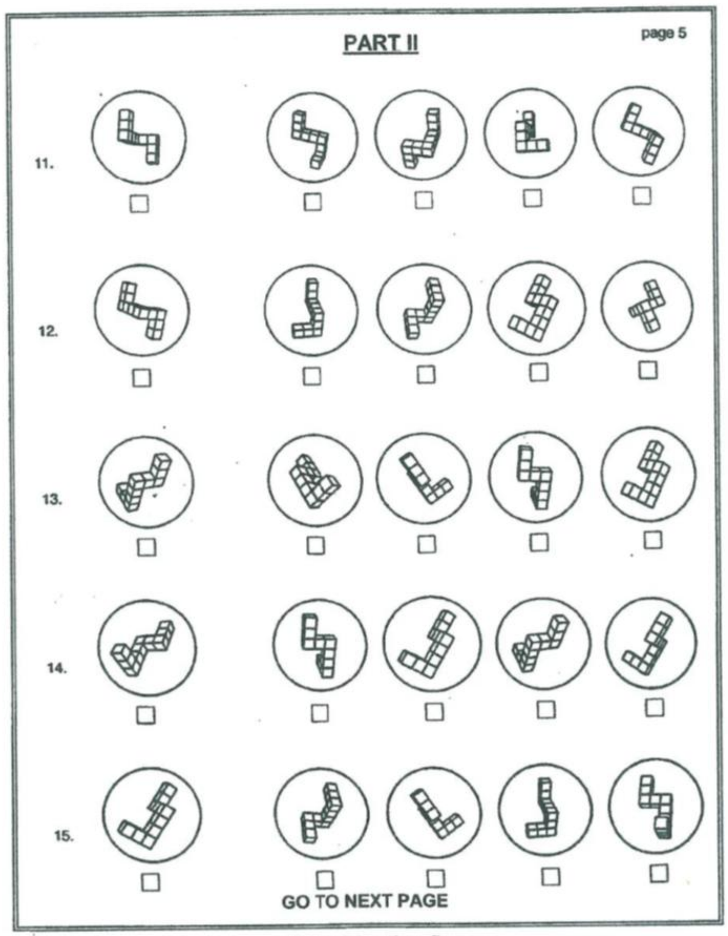


Figure 1 (continued)

71 79

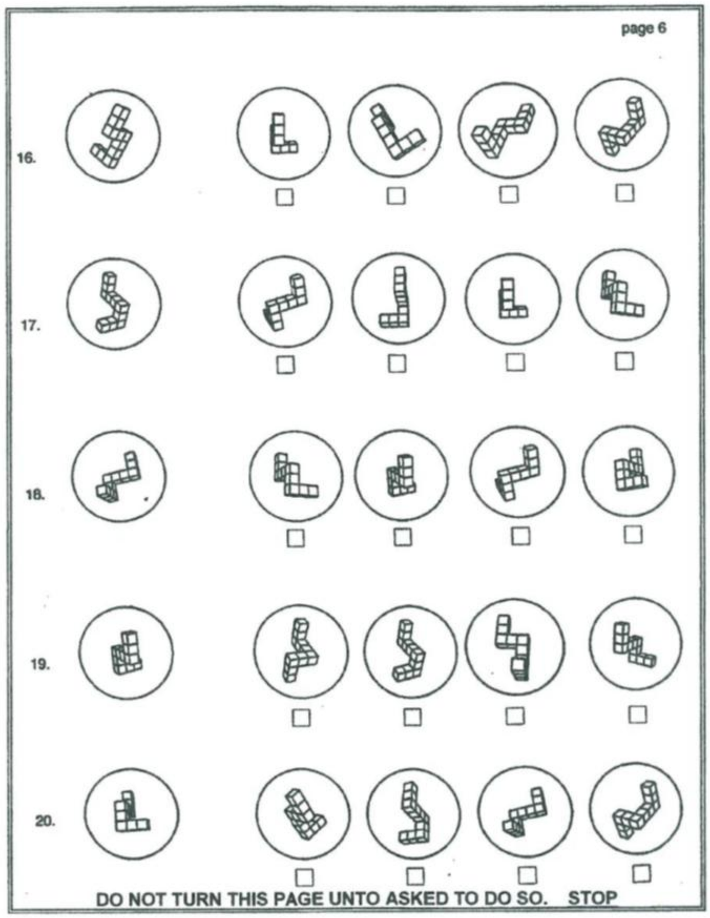


Figure 1 (continued)

72 80

APPENDIX B

HIIDDEN FIGURES TEST

Name ______________________________

HIDDEN FIGURES TEST — CF-1 (Rev.)

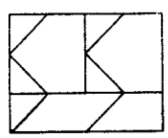
This is a test of your ability to tell which one of five simple figures can be found in a more complex pattern. At the top of each page in this test are five simple figures lettered A, B, C, D, and E. Beneath each row of figures is a page of patterns. Each pattern has a row of letters beneath it. Indicate your answer by putting an X through the letter of the figure which you find in the pattern.

NOTE: There is only one of these figures in each pattern, and this figure will always be right side up and exactly the same size as one of the five lettered figures.

Now try these 2 examples.

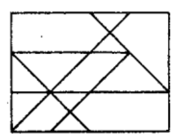
A B C D E

I



A B C D E

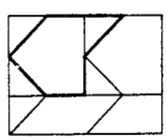
II



A B C D E

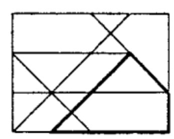
The figures below show how the figures are included in the problems. Figure A is in the first problem and figure D in the second.

I



A B C D E

II



A B C D E

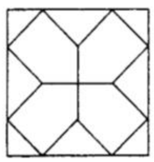
Your score on this test will be the number marked correctly minus a fraction of the number marked incorrectly. Therefore, it will not be to your advantage to guess unless you are able to eliminate one or more of the answer choices as wrong.

You will have 12 minutes for each of the two parts of this test. Each part has 2 pages. When you have finished Part 1, STOP. Please do not go on to Part 2 until you are asked to do so.

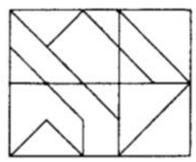
DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

Part 1 (12 minutes)

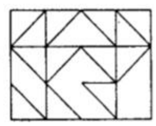
A B C D E

1. 

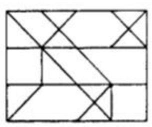
A B C D E

2. 

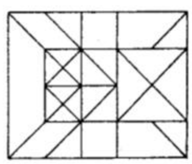
A B C D E

3. 

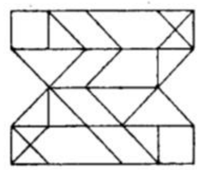
A B C D E

4. 

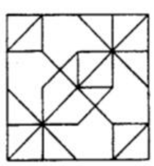
A B C D E

5. 

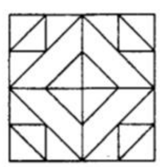
A B C D E

6. 

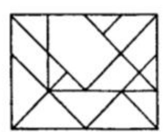
A B C D E

7. 

A B C D E

8. 

A B C D E

9. 

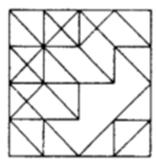
A B C D E

GO ON TO THE NEXT PAGE

Part 1 (continued)

A B C D E

10.



A B C D E

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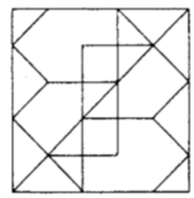
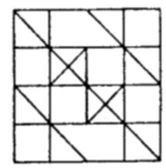


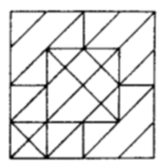
A B C D E

12.



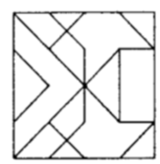
A B C D E

13.



A B C D E

14.



A B C D E

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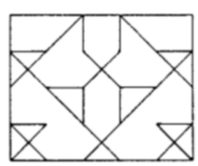
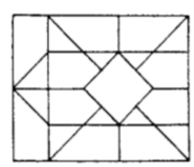


A B C D E

16.



A B C D E

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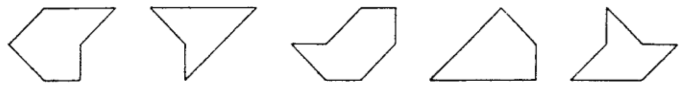
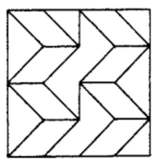
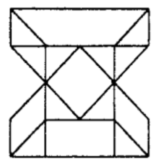
Part 2 (12 minutes)



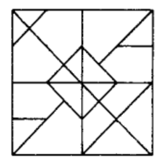
A B C D E

17. 

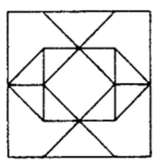
A B C D E

18. 

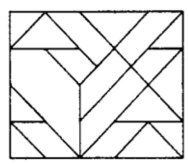
A B C D E

19. 

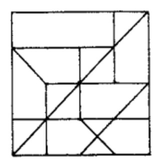
A B C D E

20. 

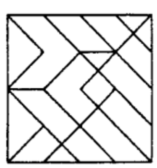
A B C D E

21. 

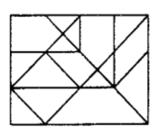
A B C D E

22. 

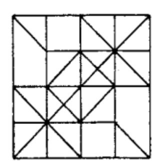
A B C D E

23. 

A B C D E

24. 

A B C D E

25. 

A B C D E

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Part 2 (continued)

A B C D E

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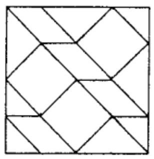
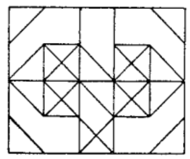
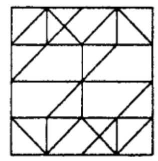
26. 

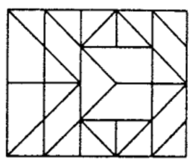
A B C D E

27. 

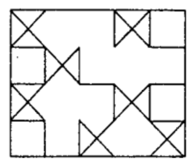
A B C D E

28. 

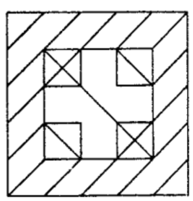
A B C D E

29. 

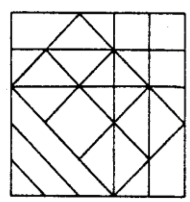
A B C D E

30. 

A B C D E

31. 

A B C D E

32. 

A B C D E

DO NOT GO BACK TO PART 1, AND
DO NOT GO ON TO ANY OTHER TEST UNTIL ASKED TO DO SO.

STOP.

APPENDIX C

TEACHER SPATIAL SKILL CHECKLIST

NATURE	CODES	Total Number of instance	Total Instruction Time	Geometry Content	Specific Examples
Spatial gestures					
Representational Gestures	RG-S				
Writing/Pointing Gestures	PG-S				
Richness of Math Practice					
Draws connections to other math areas	CM-R				
Uses multistep facts to solve simple questions	MS-R				
Connects formulas to math operations	FO-R				
Essential mathematical vocabulary	MV-R				
Elaborate complex ideas with real life applications	CA-R				
Pictorial					
Uses conventional symbols and math notations	CS-P				
Draws a math figures	DM-P				
Draws math figures and Connects to math concepts	MC-P				
Draws math figures and Connects formulas to drawings	FD-P				
Formulates problem with standard math notation	PN-P				
Represents 2-D in 3-D figures	RF-P				

APPENDIX D

DEMOGRAPHIC QUESTIONNAIRE

Age: ◯ 21 – 35 years ◯ 36-46 years ◯ 47 – 57years ◯ 58+

Gender: Male ◯ Female ◯

Race: ◯ Caucasian

◯ Black/African American

◯ Latino/Latina

◯ Asian/Pacific Islander

◯ American Indian/Alaskan Eskimo

◯ Other

Percent of students receiving free or reduced lunch at your school

Types of community in which your school is located:
◯ Rural ◯ Suburban ◯ Urban

Part 2: *Background Information*

1. Which geometry courses have you thought previously and are scheduled to teach?

2. How long have you been teaching?

3. In what school districts have you been teaching in the past 5years?

4. What is your highest degree of education? Are you currently enrolled in any advanced courses?

5. What type of initial teacher preparation or training did you complete?

   a. Undergraduate mathematics education program
   b. Undergraduate mathematics program
   c. Undergraduate middle school education program
   d. Graduate Education program ________________________
   e. Other please describe ____________________________

APPENDIX E

TEACHER CONSENT FORM

I, ________________________ agree to participate in this research study titled "THE RELATIONSHIP BETWEEN TEACHER SPATIAL SKILLS AND SPATIAL INSTRUCTION" conducted by Ms. Beryl Ann Otumfuor from the Department of Educational Psychology and Instructional Technology at the University of Georgia (678-860-0667) under the guidance of Dr. Martha Carr, Advisor, Department of Educational Psychology and Instructional Technology, University of Georgia (706-542-4504). My participation in this project is voluntary and I can refuse to participate or stop taking part at any time without giving any reason, and without penalty or loss of benefit to which I am otherwise entitled. I can ask that all information that can be identified as mine is returned, or destroyed and not used in future research.

The purpose of this study is to examine the link between teacher spatial skills and spatial knowledge as evident in their instruction. This study will help researchers and educators understand the importance of implementing geometry instruction. If I am selected to participate in this study, I would be asked to do the following:

1) I will also provide researchers basic demographic and background information about myself (5minutes)

2) I will complete two spatial measures that assess my spatial mathematical knowledge (Vandenberg Mental Rotations Test, and Kit Factor-Referenced Cognitive Tests) as administered by the principal investigator (30minutes).

3) I will also allow researcher to observe and videotape my math class period with specific content in geometry. During the classroom observation, the overall nature of my classroom instruction will be evaluated using a developed checklist (an entire math class period, varies per school: approximately 90minutes)

I understand that an incentive of $5 Starbucks gift certificate will be provided for my participation in the research study. The benefits for being this research project would be to gain in-depth understanding on how to promote quality mathematics instruction in the school which in turn will increase student's geometric and spatial skills. There are no risks or discomforts expected. However, if any discomforts are experienced by being video recorded, an audio recording will be used instead and I will be probed further regarding an observed behavior.

No individually-identifiable information about me, or provided by me during the research will be shared with others without my written permission, or if required by law. I will be assigned an identifying number and this number will be used on all of the measures I fill out. The videos will also be locked securely in researcher's office. The video recordings will be coded and destroyed after data collection is complete. The master key linking names, ID and videos will be deleted and destroyed after data collection is completed.

The investigator will answer any further questions about the research, now or during the course of the project.

I understand that by signing this form I am agreeing to participate in the study and I would receive a copy of this form for my records.

______________________________   _______________________

Researcher Name Date   Participant Name Date

______________________________   ________________________

Researcher Signature Date   Participant Signature Date

Additional questions or problems regarding your rights as a research participant should be addressed to The Chairperson, Institutional Review Board, University of Georgia, 629 Boyd Graduate Studies Research Center, Athens, Georgia 30602; Telephone (706) 542-3199; E-Mail Address IRB@uga.edu

APPENDIX F

INFORMATIONAL LETTER

Date:

Dear {Title} {Last Name}:

I am a graduate student under the direction of Dr. Martha Carr in the Department of Educational Psychology & Instruction Technology at the University of Georgia. I invite you to participate in a research study entitled ***"The Relationship between teacher spatial skills, and spatial instruction".*** The purpose of this study is to determine whether teacher spatial skills predicts their mathematical knowledge, with specific focus on their geometric skills, whether teacher spatial skills predicts the way they teach middle school geometry; and the materials they provide to their students during instruction.

All teacher participants should be 18years of age or older and should be certified mathematics teachers specifically with experience teaching geometry in middle school.

Your participation will involve providing basic demographic and background information, completing two spatial measures (Vandenberg Mental Rotations Test and Kit Factor-Referenced Cognitive Test), and being videotaped during an entire mathematics class period on geometry ***(varies per school schedule).*** The spatial measures should take approximately 35minutes to complete. Two spatial measures that assess spatial mathematical knowledge will be collected during a staff or professional development meeting. With the classroom observation, you will be given the opportunity to select the best day and time convenient to your teaching schedule. During the classroom observation, the overall nature of your classroom instruction will be evaluated using a developed checklist.

Your involvement in the study is voluntary, and you may choose not to participate or to stop at any time without penalty or loss of benefits to which you are otherwise entitled. All individually-identifiable information will be kept confidential and can only be identified by an ID number. The video recordings will be coded and destroyed after data collection is complete. The master key linking names, ID and videos will be destroyed after data collection is completed. The results of the research study may be published, but your name will not be used. In fact, the published results will be presented in summary form only. Your identity will not be associated with your responses in any published format.

The findings from this project may provide information on how to promote quality mathematics instruction in the school which in turn will increase student's geometric and spatial skills. There are no known risks or discomforts associated with this research. However, during video recording if any discomforts are experienced by you, the researcher would opt to use an audio recording and take explicit notes for scoring and analysis. You will be given a $5 Starbucks gift certificate for your participation in the research study.

If you have any questions about this research project, please feel free to call me Beryl Otumfuor at (678) 860-0667 and Dr. Martha Carr at (706) 542-4504 or send an e-mail to bbray@uga.edu. Questions or concerns about your rights as a research participant should be directed to The Chairperson, University of Georgia Institutional Review Board, 629 Boyd GSRC, Athens, Georgia 30602; telephone (706) 542-3199; email address irb@uga.edu.

Thank you for your consideration! Please keep this letter for your records.

Sincerely,

Beryl Otumfuor

APPENDIX G

TEACHER INTERVIEW GUIDE

1. Tell me a bit about yourself - What is your philosophy on teaching mathematics – specifically geometry? How is teaching geometry different than teaching other mathematical concepts?

2. Would you tell me what you think the differences among algebra, arithmetic and geometry from a content perspective is? Do you see all of them as separate or somehow tied together? Why do you believe so?

3. What do you believe is the most effective teaching techniques for geometric concepts?

4. What kind of experiences do you think would help students make the connection between concrete and symbolic representations especially when teaching geometry?

5. Do you have a special way of teaching geometry other than just teaching mathematics? I mean do you teach it differently from how you teach other math concepts – what special cues do you use (refer to gestures)?

6. What do you think are student's difficulties in geometry? And what kinds of experiences have you encountered that has made you approach teaching this content area differently – what specifically have you done – some general strategies?

7. Additional questions/comments/concerns

APPENDIX H

SPATIAL STUDIES TABLES

Study	Design	Training	Outcome Measure	Sample Size
Alias, Black & Gray (2003)	Quasi-Experimental Post-test only Post-tests –Spatial Visualization Test + Structural Design Test	Experimental Condition – Received instruction on structural design +spatial activities (generic and concrete) No control group	Experimental group performed well after intervention	138 civil engineering students
Baki, Kosa & Guven (2011)	Quasi-Experimental Random Assignment Pre & Post-tests – Purdue Spatial Visualization Test (PSVT)	10 weeks (2 hours per week) Instruction on solid geometry Three conditions Treatment (Cabri 3D & Physical Manipulative) Control (Traditional Instruction)	ANOVA ANCOVA – pretest as covariate T-test – PSVT scores were significantly higher with the treatment groups after instruction	96 pre-service teachers
Ben-Chaim, Lapan & Houang (1988)	No control group Pre & Posttest – untimed Spatial Visualization Test Retention Test (238 students randomly selected from subsample)	3 weeks MGMP Spatial Visualization Unit	ANOVA – students performed significantly well on spatial tasks after spatial training	1000 middle grade students (6th grade students)
Boakes (2009)	Quasi-Experimental No random assignment Pre & Posttest – Geometry Knowledge (subset of 27 NAEP Math questions) and Spatial Visualization Tests (Paper Folding, Cards Rotation & Surface Development)	80 minutes instruction daily over 2 months on a geometry unit Treatment – Traditional Instruction + Origami lessons (3 days a week for 30 minutes prior to instruction) Control – Traditional instruction only	ANCOVA No statistically significance	Convenience sample of 56 students
Cheng & Mix (2012)	Random assignment 3 pretest – 2 spatial + 1 math 3 posttest	40 minutes training sessions Control – crossword puzzles Experimental (Spatial Training Group) – mental rotation	MANCOVA Significant difference between training and no training group	58 elementary school students
Ferrini-Mundy (1987)	Random Sample selected from 1054 students	8 week spatial training modules Treatment – Audiovisual +	ANOVA - Training was not effective in improving spatial	334 registered calculus students

	Random Assignment Pre & Posttest – Calculus + Spatial visualization (Spatial Relations subset of DAT)	Audiovisual tactile No control reported	visualization scores	
His, Linn & Bell (1997)	No control group No random assignment Pre assessment – students with scores 2 standard deviations below received special invitation to participate in spatial intervention	Spatial strategy instruction – computer + paper based activities 3 hours of instruction on weekend No control group	T-test – significant gains in overall performance Females performed equally like their male counter parts after the spatial instruction	132 engineering students
Piburn et al (2005)	Quasi Experimental design Unequal sample group in gender No random assignment Posttest only – Geospatial Assessment + Spatial orientation measure (card rotations) & Spatial visualization measure (surface development)	4 sections of Geology 103 were taught by a TA and assigned to experimental and control groups Experimental - Instruction included traditional content of lab & 2 additional computer models on topics such as topographic maps and interactive 3-D Control – Regular traditional instruction	ANOVA – experimental treatment improved their scores on SV after the instruction	103 subjects
Rafi, Samsudin & Ismail (2003)	Quasi-Experimental Random assignment Pre & Posttest – Mental Rotation test and Spatial test	5 weeks of instructional intervention – drawing exercises 2 treatment – computer mediated + video enhanced conventional instruction Control – conventional instruction 2 levels of spatial experience – low & high. Low scored below 42 on the SEQ – self report questionnaire on spatial experience	ANOVA & Independent t –tests Students with high spatial experience received high scores on spatial visualization test	138 engineering undergraduates
Stransky, Wilcox & Dumbrowski (2010)	2 experimental designs Random Assignment Pretest & posttest on Mental Rotation	Mental Rotation Training (40minutes) Experiment 1 - 3 training groups (one day training, spaced training & no training) Experiment 2 – 3 training groups (Full MR training, MR & FLS group; MRT & FLS group & FLS-only group)	Wilcox Signed test and Mann Whitney Test – substantial group differences. Students with training performed well on laparoscopic assessment	61 undergraduates

Sorby & Baartmans (1996/2000)	Pretests- PSVT, Mental Rotations Test & background questionnaire 96 students were randomly selected (39% females initially failed & 12% males) but no random assignment to group	Instructional course on orthographic sketching, pattern development Experimental group – enrolled in course & also failed initial tests Control group – did not enroll in course and failed	Experimental group performed better than control group	535 engineering students