IMPLEMENTING HIGHER-ORDER THINKING IN MIDDLE SCHOOL MATHEMATICS CLASSROOMS

by

EILEEN CHRISTINA MURRAY

(Under the Direction of Dorothy Y. White)

ABSTRACT

Effective professional development strategies include a focus on the cyclical process of teaching: plan, teach, reflect. This process is used to provide teachers with practice-based experiences from which they can learn about instructional strategies and student thinking. Although the use of this model has been shown to be effective, more research needs to be done to understand the best ways to support the professional development of teachers, especially those in an urban setting. This study examined the influence of reflective teaching cycles on two urban middle school mathematics teachers' selection and implementation of tasks that had the potential to facilitate higher-order thinking. Data were collected during the reflective teaching cycles, which consisted of the three phases planning, teaching, and reflecting. In planning, the teachers chose a task(s) that matched their goals for students' learning, worked to understand and identify what mathematics their students would need to know in order to solve the task(s), and considered how to implement the task(s). As teachers reflected on their lessons, they considered the type and level of thinking in which their students engaged, how their pedagogical decisions influenced students' learning, and considered alternative instructional strategies. I audiotaped my planning and reflection meetings with the teachers and used an observation protocol to record events during each teacher's instruction to help prepare for future meetings. The results of the study indicated that reflective teaching cycles focused on higher-order thinking could influence teachers' selection and implementation of tasks in many ways. The collaborative nature of the cycles and their capacity to build teachers' knowledge about mathematics and to promote reflection on pedagogical strategies affected how teachers chose tasks and executed instruction. Teachers were also influenced by pacing and assessment pressures, how they could engage their students in mathematics, their teaching experiences, and their views on what it meant to do mathematics.

INDEX WORDS: Mathematics, middle school teachers, higher-order thinking, reflective teaching cycles

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DEDICATION

This dissertation is dedicated to my boys – Chris, Maxwell, and Joseph. Chris' support and faith in my abilities helped keep me going and gave me the strength to finish. Maxwell and Joseph kept me grounded and were constant reminders that life is more than work.

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CHAPTER 1

INTRODUCTION

Over a decade ago, the National Council of Teachers of Mathematics (NCTM, 2000) published *Principles and Standards for School Mathematics* (PSSM). This document, as a continuation of NCTM's attempts to describe effective mathematical instruction of all students, has inspired years of research into the teaching and learning of mathematics. PSSM emphasized that all students should be provided high-quality instruction to help them be successful in mathematics; *high-quality instruction* was defined to be instruction in which teachers understand students' prior knowledge and children's mathematical development. Teachers should challenge and support students as they develop mathematical understanding that allows students to actively build new knowledge from prior knowledge and their experiences. In addition to effective instruction, PSSM stressed that student learning should involve conceptual understanding for students to be able to flexibly use knowledge, deal with new problems and settings, monitor their own learning, and develop higher-order thinking skills. For example, one of the standards of school mathematics in PSSM, reasoning and proof, advocates teaching that helps students learn to justify claims and results; develop and describe ideas; and make, test, and refine conjectures.

More recently, NCTM has presented a conceptual framework intended to guide secondary mathematics instruction. The *Focus in High School Mathematics: Reasoning and Sense Making* (FHSM: NCTM, 2009) incorporates reasoning and sense making into high school mathematics curricula to help students develop "the ability to formulate, represent, and solve mathematical problems and the capacity for logical thought and explanation" (NCTM, 2009, p.

2). In order for teachers to support students' progress to such higher levels of thinking, they must select and implement worthwhile tasks that engage students in reasoning and sense making.

The PSSM (NCTM, 2000) has also influenced the latest national standards, the Common Core State Standards for Mathematics (CCSSM, 2010) in many ways (Hirsch & Reys, 2009). In the introduction for the CCSSM, mathematical understanding is described in much the same language as NCTM: "Mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from" (National Governors Association and Council of Chief State School Officers, 2010, p. 4). One reason that justification and explanation are important is that students who can successfully engage in these types of higher-order thinking are more likely to be able to tackle unfamiliar or unique problems (Wenglinsky, 2000). The standards for mathematical practice outlined by CCSSM include ideas based on the reasoning and proof standards for school mathematics in PSSM. For example, students should be able to construct viable arguments and justify conclusions. Mathematical understanding should also include the ability to make and explore the truth of mathematical conjectures. This renewed emphasis on students reasoning through and making sense of mathematical concepts correlates with using higher-order thinking in the mathematics classroom.

Mathematics curriculum focused on higher-order thinking increases student achievement, especially students' problem-solving and critical-thinking abilities (Boaler & Staples, 2008; Gutierrez, 2000; Pajkos & Klein-Collins, 2001). Many schools serving high percentages of minority or low socioeconomic students do not routinely promote mathematics teaching and learning that is focused on higher-order thinking (Kitchen, 2003; Kitchen, DePree, Celedon-Pattichis, & Brinkerhoff, 2007). Instead, these students are more likely to experience instruction focused on rote learning of low-level skills. However, there are schools that support high-achievement and student learning of challenging mathematics.

Kitchen and his colleagues (2007) found that highly effective schools that served the poor held high expectations for students, provided sustained support for excellence, presented challenging mathematical content and high-level instruction, and recognized the importance of building relationships among faculty and students. Effective schools focused on developing students' higher-order thinking skills by concentrating on problem-solving and critical thinking. Unfortunately, there are obstacles for teachers to employ pedagogical strategies that promote higher-order thinking such as beliefs and expectations of students, teaching context, and teachers' mathematical and pedagogical knowledge (Beswick, 2004; Horn, 2007; Kitchen, 2003; Rousseau & Powell, 2005; Zohar, Degani, & Vaaknin, 2001; Zohar & Dori, 2003). Therefore, it is important that we understand how to help teachers overcome such obstacles so that all students can be given the opportunity and necessary support to learn mathematics for conceptual understanding (NCTM, 2000) and to develop higher-order thinking.

Higher-Order Thinking

There are several concepts associated with higher-order thinking: critical thinking, problem solving, creative thinking, and decision-making. Lewis and Smith (1993) define *higher-order thinking* as instances in which "a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations" (p. 136). When people use higher-order thinking they decide what to believe and what to do. They create new ideas, make predications, and solve nonroutine problems. Educational researchers correlate higher-order thinking with creative and abstract thinking, decision-making, analyzing theories, and active mental construction (Peterson, 1988; Raudenbush, Rowan, & Cheong, 1993; Zohar & Dori, 2003). All of these concepts are important and necessary to help students gain a deeper understanding of school mathematics.

Instruction and Higher-Order Thinking. When teachers use the type of instruction called for in the standards documents, which includes pedagogy that helps develop students' higher-order thinking skills, they are able to improve student achievement (Boaler & Staples, 2008; Franco, Sztajn, & Ramalho Ortigao, 2007; Gutierrez, 2000). Mathematical achievement improved and gaps diminished when students experienced instruction focused on problem solving, conjecturing, and explanation and justification of ideas (Boaler & Staples, 2008; Franco et al., 2007). Students had a better attitude toward mathematics, were more likely to take more mathematics and higher-level classes, and had higher test scores when they were encouraged to use multiple representations and make connections between new and previous knowledge (Boaler & Staples, 2008; Gutierrez, 2000).

Some teachers implement tasks that promote higher-order thinking with specific populations of students. Zohar (2001) found some teachers used higher-order thinking only with high-achieving students because they believed either that tasks that required higher-order thinking were too difficult for low-ability students or that such tasks would be too frustrating for these students to solve. Other teachers believed that higher-order thinking was an appropriate goal for all students, and worked to modify their instruction to provide the necessary support and guidance for low-ability students to engage in higher-order thinking. For example, these teachers reported:

> breaking up a complex task into several simpler components; leading students through a sequence of steps necessary to solve a problem; giving clues; adding more examples; and letting students work in groups of mixed ability so that peers can learn from each other. (Zohar et al., 2001, p. 479)

Although these teachers felt that higher-order thinking was an appropriate goal for low-ability students, they were actually reducing the cognitive demand of tasks by making many of these adjustments.

The cognitive demand of a task is the type and level of thinking required to solve the task (Stein, Smith, Henningsen, & Silver, 2000). High-level tasks require the use of higher-order thinking. Research has shown that students are more successful in mathematics when their teachers use high-level tasks and are able to maintain this level during the implementation of the tasks (Boaler & Staples, 2008; Gutierrez, 2000; Stein et al., 2000). Therefore, it is important to investigate how teachers select and implement high-demand tasks and maintain that demand during implementation to understand how they can facilitate students' engagement in higher-order thinking.

Teachers need to understand students' common conceptions, preconceptions, and misconceptions in order to teach higher-order thinking skills. "Many studies have shown that students often make sense of the subject-matter in their own way which is not always isomorphic or parallel to the structure of the subject-matter or the instruction" (Even & Tirosh, 1995, p. 3). Therefore, it is important that teachers understand students' knowledge construction in order to guide students toward more sophisticated conceptions of mathematics. According to Stein and Kaufman (2010), it is important for teachers to spend time understanding mathematics in order to help them be able to provide effective lessons. By understanding mathematics, teachers are better able to focus on advancing their students' conceptual understanding, supporting their students' thinking, and maintaining the level of the tasks in order to facilitate higher-order thinking. Professional development can support teachers as they learn how to implement pedagogical strategies that promote higher-order thinking and understand the mathematics they are expected to teach.

Professional Development

Specific professional development models have been shown to offer teachers a way to reflect upon their practice and implement change (Smith, 2001). Pappano (2007) found that teachers needed to engage in collaborative professional development in order to learn how to implement standards-based instruction. Teacher collaboration could include joint lesson planning, reviewing and interpreting student work together, and writing common assessments. Teachers need to have time to reflect upon what is happening in their classrooms and be provided opportunities to talk with colleagues. Teachers who are willing to collaborate by reflecting on their mathematical knowledge and instructional practice with their colleagues can ultimately transform their practice by simultaneously considering mathematical content, pedagogical issues, and student thinking (Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007). Wenglinsky (2000) found that teachers who engaged in sustained professional development geared specifically toward higher-order thinking improved classroom practices. In summary, collaborative professional development that is sustained in general and focused on higher-order thinking in particular may provide the necessary support for teachers to learn how to implement standards-based curriculum and instruction.

Many professional development strategies include a focus on the cyclical process of teaching (Loucks-Horsley, Hewson, Love, & Stiles, 1998; McDuffie & Mather, 2006; Silver et al., 2007; Smith, 2001). This process is used to provide practice-based experiences from which teachers can learn about different instructional strategies and student thinking. Smith (2001) uses a *reflective teaching cycle*, which consists of the stages of planning, teaching or acting, and reflecting. As teachers *plan*, they not only decide what to teach, but also spend time understanding the mathematics in the task, their students' prior knowledge, their mathematical goals, and how they can achieve them. Therefore, professional development should allow teachers to explore and discuss mathematical tasks they may use with their students before planning a lesson (Stein & Kaufman, 2010). Teachers should know if the mathematics in a task reflects the mathematical knowledge and processes they want their students to learn and if the tasks are building on students' prior knowledge and experience. After planning, teachers enact their idea by implementing their plans in the classroom.

During *teaching* or *acting*, teachers must make decisions about how to engage students in learning and what changes in their pedagogy, if any, should be made. They should also be continuously assessing student learning to help with these decisions and to encourage continued engagement with the mathematics. "Determining what students know and can do mathematically is a critical aspect of teaching, since it allows teachers to make appropriate instructional decisions before, during, and after teaching" (Smith, 2001, p. 13).

After the lesson, teachers *reflect* on the type of thinking in which the students engaged during the lesson and how deeply they grappled with the mathematics. Teachers can consider what students did and said to help them gain access to students' understanding of the central mathematical ideas of the lesson. Teachers can do that by remembering what happened during the lesson and by using student work. As teachers examine student work they can move from their own understandings to the "broader consideration of what students' responses might reveal about their thinking, what difficulties such tasks might present for students, and how teachers might help students address (or avoid) common misunderstandings" (Smith, 2001, p. 13).

Rationale

Although the use of models such as the reflective teaching cycle have been shown to be effective (McDuffie, Mather, & Reynolds, 2004; Silver et al., 2007; Stein et al., 2000), more research needs to be done to understand the best ways to support the professional development of teachers, especially those in an urban setting. Recent national assessments in mathematics showed no significant advances in mathematics achievement for fourth- and eighth-grade students and persistent achievement gaps between racial groups (National Assessment of Educational Progress, 2009). International assessments reported the mathematical literacy of 15-year olds in the United States as below average and lower than 17 other countries (Program for International Student Assessment, 2009). According to the Trends in International Mathematics and Science Study (TIMSS), the United States is 11th in participating countries for average mathematics scores of fourth- and eighth-grade students (TIMSS, 2007). Therefore, it remains critical that we, as a country, work towards improving mathematics education in our country.

Student learning and achievement, including students' higher-order thinking skills such as reasoning and problem solving in mathematics, are important components of educational improvements. We need to focus specifically on higher-order thinking in mathematics because of the importance of mathematics in everyday life, including economic and financial success (Peterson, 1988). Mathematics is also crucial for the ability to succeed in the workplace, which requires an increased level of mathematical thinking and problem solving (NCTM, 2000). By focusing on higher-order thinking in mathematics, students can prepare themselves to face economic and workforce challenges in an increasingly global and technological society (NCTM, 2009).

Significance

I designed this study to add to the research base of effective teaching strategies that incorporate higher-order thinking skills for all learners. I investigated how teachers working with low achieving, low socioeconomic status (SES), or minority students selected and implemented tasks that had the potential to increase students' higher-order thinking skills in mathematics. I worked within a school with a predominantly African American, Latino/a, and low SES student population. I chose to work with teachers rather than students because student achievement is directly related to the instructional strategies employed by teachers, teacher beliefs about students and mathematics, and teachers mathematical and pedagogical knowledge (Boaler, 2002; Darling-Hammond, 2004; Hill, Rowan, & Ball, 2005; Love & Kruger, 2005; Schoen, Cebulla, Finn, & Fi, 2003).

Previous work has illustrated how the reflective teaching cycle is effective when used in a one-on-one setting with a teacher and teacher educator (McDuffie et al., 2004) or infrequently in large professional development workshops (Silver et al., 2007). Therefore, this study contributes to the literature by investigating how the reflective teaching cycle could be used as a collaborative professional development model with a team of teachers. Stein and her colleagues (2000) used professional development to help teachers reflect on and discuss their instruction with colleagues through the shared language of cognitive demand. Although cognitive demand is one component of higher-order thinking, there is more to developing higher-order thinking in the mathematics classroom. Hence, this study extends this research by examining the influence of conversations with mathematics teachers about selecting and implementing tasks to develop students' higher-order thinking.

Research Questions

I conducted a qualitative study that examined how a series of reflective teaching cycles influenced teachers' selection and implementation of tasks that had the potential to facilitate higher-order thinking. Through these cycles, I engaged teachers in conversations about mathematics, pedagogy, and higher-order thinking in order to provide students with effective mathematics lessons. The research questions guiding this study were the following:

1. How does a series of reflective teaching cycles influence the way mathematics teachers choose tasks that have the potential to develop higher-order thinking?
    a. In what ways do mathematics teachers consider students while choosing mathematical tasks that have the potential to develop higher-order thinking?
1. How does a series of reflective teaching cycles influence the way mathematics teachers facilitate students' implementation of tasks that have the potential to develop higher-order thinking?
    a. In particular, how do teachers introduce mathematical tasks (i.e., problematic aspects of the task), and how do teachers use questioning to hold students accountable for higher-order thinking and skills?
    b. In what ways do mathematics teachers consider students while implementing mathematical tasks that have the potential to develop higher-order thinking?

CHAPTER 2

REVIEW OF RELATED LITERATURE

In this chapter, I review the literature about higher-order thinking and professional development that informed the theoretical framework for this study. Many studies have been conducted on teachers' pedagogical decisions, particularly how they relate to the development of higher-order thinking. More research has explored the effect of facilitating higher-order thinking and the ability of and influences on teachers to facilitate higher-order thinking in mathematics classrooms. Specific attention has been paid to teachers of minority, low SES, or low-achieving students. I also review research on the characteristics of effective professional development that cultivate standards-based teaching and higher-order thinking. Drawing on these studies, I developed the theoretical framework for my study, which incorporated the cyclical process of teaching focused on teachers' facilitation or hindrance of higher-order thinking skills in their practice.

Higher-Order Thinking

In order to help students develop higher-order thinking, teachers must select worthwhile tasks that encourage students to reason, make sense of mathematics, and figure things out for themselves (NCTM, 2009). Higher-order thinking in mathematics requires not only that students know the general patterns and principles of mathematics, but also that they are able to understand the relationships among these patterns and principles and apply such knowledge (Raudenbush et al., 1993). These skills involve good judgment, content mastery, and conceptual understanding (Peterson, 1988; Zohar & Dori, 2003). Students use higher-order thinking when they adjust their initial understanding based on new evidence, identify patterns, make and test conjectures, and propose and defend claims (Thompson & Zeuli, 1999). However, the intellectual engagement in the classroom is the responsibility of the teacher and this engagement should focus on mathematical thinking (NCTM, 2000). It is important to understand how to do this and the effects of such instruction on student learning.

Boaler (2002) compared an open, or process-based, form of mathematical approach with a traditional, content-based mathematical approach to see how these different teaching approaches created different forms of knowledge for students. People who advocate process-based approaches argue that students will benefit in a number of ways including increased enjoyment, understanding, and opportunities in mathematics. This open approach incorporates higher-order thinking because it allows students to engage in "open-ended, practical, and investigative work that requires them to make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge" (Boaler, 2002, p. 42).

To understand how process-based and traditional instruction would encourage different forms of knowledge, Boaler (2002) conducted ethnographic case studies of two schools. One school followed a traditional approach, which focused on procedures, techniques, and practice. The second school followed a process-based approach, which used projects to expose students to mathematics through "situations that were realistic and meaningful to them" (p. 49). Through engagement in the projects, which lasted 2 – 3 weeks, students were "encouraged to develop their own ideas, formulate and extend problems, and use their mathematics" (p. 49) with limited teacher guidance. If a student or students needed some mathematics that they were unfamiliar with, the teacher would teach it to them. The way students engaged in mathematics incorporated components of higher-order thinking.

Boaler (2002) collected data through observations, questionnaires, interviews, and quantitative assessments. She found that students in the process-based school underachieved in formal test situations relative to the content-based school. She concluded that this underachievement had to do with student characteristics developed at each school as a result of the mathematics environment. The students at the process-based school

> were able to use mathematics because of three important characteristics: a willingness and ability to perceive and interpret different situations and develop meaning from them (Gibson, 1986) and in relation to them (Lave, 1993); a sufficient understanding of the procedures to allow appropriate procedures to be selected (Whitehead, 1962); and a mathematical confidence that enabled students to adapt and change procedures to fit new situations. (Boaler, 2002, pp. 59–60)

Boaler concluded that the content-based approach that emphasized computation, rules, and procedures did so at the expense of depth of understanding. Because of its emphasis, it was disadvantageous to students because it promoted learning that was inflexible, was school-bound, and had limited use.

The idea that different schools offer different types of instruction that results in different learning opportunities (Boaler, 2002) was one of the components of research focused on effective schools serving the poor (Kitchen et al., 2007). Kitchen and his colleagues (Kitchen, 2003; Kitchen et al., 2007) found that many schools serving high percentages of minority and low SES students were more likely to provide instruction focused on rote learning of low-level skills rather than higher-order thinking. They were also able to identify significant characteristics contributing to the academic success, particularly in mathematics, of schools serving high-poverty communities (Kitchen et al., 2007). Kitchen and his colleagues identified themes in nine public secondary schools that demonstrated high student achievement. Data were collected from interviews with teachers, administrators, and students; an administrator survey; and classroom artifacts and observations. The classroom observation instrument was used to determine the intellectual quality in classrooms and the extent to which students experienced reform-oriented instruction. “The observation scales included: (a) intellectual support, (b) depth of knowledge and student understanding, (c) mathematical analysis, (d) mathematical discourse and communication, and (e) student engagement” (Kitchen et al., 2007, p. 169).

In this analysis, the researchers identified three major themes and areas related to each theme. The three major themes were as follows: “(a) high expectations and sustained support for academic achievement, (b) challenging mathematical content and high-level mathematics instruction that focused on problem solving and sense making (as opposed to rote instruction), and (c) the importance of building relationships” (Kitchen et al., 2007, p. xiv). The related areas for the first two major themes included characteristics of higher-order thinking such as challenging students with cognitively demanding curriculum, promoting critical thinking through problem solving, encouraging mathematical communication, and engaging students in mathematical inquiry. The related areas for the third theme included faculty having a solid and clear sense of purpose, collaborating and supporting each other, focusing on student dispositions towards mathematics, and understanding and caring for students. According to the researchers (Kitchen et al., 2007), the most important conclusion of the study was that when schools began with the belief that students have the ability to learn challenging mathematics, they could accomplish this goal. However, other studies have shown that some teachers do not believe that all students are capable.

Beswick (2004) looked at specific beliefs that affected practice, specifically in reform-oriented classrooms. The focus of her study was the beliefs and practice of one experienced mathematics teacher across classes. Beswick wanted to try to understand this teacher’s belief structure with respect to his students. She collected data from a survey and semi-structured interviews and found that the teacher held a problem-solving belief of the nature of mathematics and a constructivist view of teaching and learning. The problem-solving view of mathematics characterizes mathematics as “a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision” (Ernest, 1994). Taylor, Fraser, and Fisher (1993) said that

> a constructivist classroom environment was considered to be one in which: students were able to act autonomously with respect to their own learning; the linking of new knowledge with existing knowledge was encouraged and facilitated; knowledge was negotiated by participants in the learning environment; and the classroom was student centered in that students have opportunities to devise and explore problems that are of relevance to them personally. (as cited in Beswick, 2004, p. 113)

Beswick found the teacher held beliefs about average-ability students that contradicted his problem-solving and constructivist beliefs. He believed that average-ability students were uninterested in mathematics and that mathematics suitable for these students was boring. These beliefs about the ability of certain students affected his practice, shown by the fact that he provided limited engagement in mathematics for his average-ability high school students.

Zohar, Degani, and Vaaknin (2001) conducted a study to investigate the patterns of teachers’ beliefs about low-achieving students and how these patterns affected instruction of higher-order thinking. Data were collected from semi-structured interviews with 40 secondary school teachers from two schools. The teachers taught mathematics, science, social sciences, humanities, and foreign language. Zohar and his colleagues analyzed transcripts of the interviews and determined categories for recurring ideas. Teachers were divided into three categories: distinguishing consistently (DC), not-distinguishing consistently (NDC), and inconsistency (INC). DC teachers routinely distinguished low- and high-achieving students and believed that certain instructional techniques were more appropriate for high-achieving students. NDC teachers expressed the view that instructional techniques were appropriate for low- and high-achieving students alike. INC teachers did not have a set way of thinking.

One of the conclusions that Zohar et al. (2001) drew from this study was that one of the major factors in teachers' decisions not to use thinking-based learning with low-achieving students was that higher-order thinking was inappropriate for these students. Teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics influenced their expectations of students. In sequential, linear, or hierarchical learning theory, students move from lower- to higher-order cognitive tasks. Since higher-order cognitive tasks are more complex, they require deeper content understanding. Before students are able to tackle more advanced topics they must master prerequisite and basic skills. Understanding evolves from disjoint, smaller pieces of complex ideas. Teachers who subscribed to this mastery-learning model of instruction were likely to think that higher-order thinking tasks were not appropriate for low-achieving students. Teachers who had lower-order instructional goals for their low-achieving students believe these students were stuck at lower level thinking skills. This belief may have "serious educational implications because it undermines the goal of helping lower achieving students in closing gaps, thereby denying them equal educational opportunities" (Zohar & Dori, 2003, p. 146).

Believing that minority, low socioeconomic, or low-achieving students are incapable of higher-order thinking has far-reaching consequences in mathematics education. If the general consensus among teachers is that lower-ability students are not capable of higher-order thinking, they may deprive these students of high-level tasks. But higher-order thinking in mathematics is central because of "the importance of mathematics to an individual's performance in everyday life and to an individual's economic and financial success" (Peterson, 1988, p. 2). Higher-order thinking is important because people need to be able to use knowledge to solve new problems, which requires them to understand what they learn. Unfortunately, children in the United States are provided unequal access to curriculum and teaching that incorporates higher-order thinking, which is important because such disparities are strongly related to student achievement (Darling-Hammond, 2004a, 2004b). For example, inequalities in the allocation of resources has created and continues to create a situation in which most minority students attend schools with inadequate classroom situations, few academic resources, and unqualified, unprepared, or inexperienced teachers (Darling-Hammond, 2004a; Kitchen, 2003; Oakes, 1990).

Another reason for the limited contact with higher-order thinking skills for particular students is the continued belief that students attending certain schools do not need and will not profit from education focused on higher-order thinking (Darling-Hammond, 2004b). Instruction instead focuses on rote skills and low cognitive tasks, and is oriented towards the improvement of test scores (Darling-Hammond, 2004b; Kitchen, 2003; Rousseau & Powell, 2005). Teachers rarely give their students the opportunity to have conversations about what they know about mathematics, to conduct inquiry, or to compose and solve problems; that is, use higher-order thinking. As a result, schools are producing more unskilled workers than the economy can absorb at livable wages (Darling-Hammond, 2004b).

Teachers in urban, high minority and high poverty school districts routinely have less class and planning time, larger classes, more absenteeism, greater pressure to improve test scores, and less experience and preparation than teachers in affluent, homogeneous schools (Darling-Hammond, 2004b; Kitchen, 2003; Rousseau & Powell, 2005). These factors have a direct impact on how teachers approach instruction. Kitchen (2003) determined that teachers' overwhelming workload served as the main obstacle to reforming their practices and implementing innovative instructional strategies. Teachers in high poverty, diverse schools do not have the time or energy they need to figure out how to teach in new ways, including teaching higher-order thinking skills. They have limited planning time, poorly defined work conditions, and minimal support from administrators, colleagues, and even parents. Furthermore, these teachers are more likely to be working in districts in which mandated tests have high stakes for students and teachers. Thus, “teachers may be reluctant to take time away from skill development to engage students in activities that focus on thinking, reasoning, and problem solving” (Smith, 2001, p. 46). Therefore, those who support mathematics teachers should help them cultivate instructional practices that can advance students’ development of both high- and low-level skills.

Professional Development

In the field of mathematics education, professional development opportunities that effectively help teachers transform their practice have common characteristics and abide by similar principles.

> Effective professional development experiences foster collegiality and collaboration; promote experimentation and risk taking; draw their content from available knowledge bases; involve participants in decisions about as many aspects of the professional development experience as possible; provide time to participate, reflect on, and practice what is learned; provide leadership and sustained support; supply appropriate rewards and incentives; have designs that reflect knowledge bases on learning and change; integrate individual, school, and district goals’ and integrate both organizationally and instructionally with other staff development and change efforts. (Loucks-Horsley et al., 1998, p. 36)

Successful professional development experiences are associated with principles that are focused on the teacher and school community. Such professional development is motivated by a clear image of effective classroom learning and teaching and provides teachers with opportunities to build their content knowledge and pedagogical content knowledge. Facilitators of professional development model strategies they would like to see teachers use with students. They continually assess themselves and the program so that they can make improvements to ensure that they are having a positive impact on teacher effectiveness, student learning, leadership, and the school community. Louckes-Horsley and her colleagues described effective professional development as helping build learning communities in schools to provide teachers with the support to be able to take risks and learn, empowering teachers to become agents of change and to serve in leadership roles, and providing links to state and district goals, curriculum frameworks, and other parts of the educational system.

Pappano (2007) found that teachers learn how to implement standards-based instruction through collaborative professional development. However, collaboration is more than just getting together to talk. Teachers need opportunities to examine, critique, and support each other's work. Teacher collaboration is critical in high-minority and high-poverty schools in order to help teachers improve their practice and increase student achievement (Saunders, Goldenberg, & Gallimore, 2009). Professional development must also give teachers the opportunity to build their content knowledge and pedagogical content knowledge (Loucks-Horsley et al., 1998). Teachers can better support student learning when they have opportunities to increase their understanding of mathematics, which helps them develop ways to analyze student thinking (Sherin, Linsenmeier, & Van Es, 2009).

Teachers' understanding of mathematics, curriculum, and higher-order thinking contribute to teachers' classroom practice, professional growth, and student achievement. Stein and Kaufman (2010) sought the link between curriculum and instruction. They compared two school districts' implementation of different curriculum to determine how the curriculum (or textbooks) used influenced teachers' instruction. Data were collected from classroom observations and teacher surveys. The researchers analyzed the data by defining the quality of lesson implementation, teacher capacity, and teachers' use of the curriculum. High-quality lesson implementation meant that teachers maintained the cognitive demand of the task, attended to students' thinking, and made the intellectual authority in the classroom the mathematical reasoning of the students. Teachers' capacity included their knowledge of mathematics, education, and prior experiences. They defined teachers' use of curriculum using teachers' perceptions of the curriculum and how they prepared for lessons. After defining these three components, Stein and Kaufman determined the relationship between them. They found a positive effect on teachers' ability to support student thinking when teachers understood curriculum materials before implementing lessons. There was a strong relationship "between teachers who reviewed the big mathematical ideas in the curriculum and teachers who implemented lessons at a high level" (Stein & Kaufman, 2010, p. 686). However, there was evidence that part of this correlation was due to professional development and curricular resources. Specifically, a curriculum that offers "ample support and explanation within lessons that will help teachers present the concept to students and skillfully facilitate student thinking and discussion about that concept in the classroom" (Stein & Kaufman, 2010, p. 687) could help teachers implement high-quality lessons.

Little and Horn (2007) looked at a professional learning community in a mathematics department in which the teachers were able to engage in dialogue about classroom practice that was generative of professional learning and instructional improvement. They focused on a specific conversational practice called "normalizing problems of classroom practice" that seemed to help teachers use their conversations as places to learn about teaching and learning instead of gripe sessions. When normalizing problems of practice, teachers define a problem as a normal, expected part of their work as teachers. The others in the group then reassured and supported each other as a starting point to more detailed discussions of classroom practice. In particular, teachers took responsibility for the learning of their students, and even though they assured each other that they had common problems in the classroom, the teachers used the episodes to "communicate the inherent complexity and ambiguity of teaching while supplying themselves with the specifics needed to introduce and evaluate multiple explanations for the problems that surface" (p. 88). Little and Horn concluded that several characteristics of professional learning communities make them successful. One characteristic that helped the community normalize problems of practice in a way that was "generative of professional learning and instructional improvement" (p. 91) was a focus on mathematics. The teachers worked on problems together to unpack the mathematics (i.e. consider cognitive demand, student difficulties, conceptual understanding) and created problems together.

Wenglinsky (2000) analyzed a national database to map out the influence of teacher quality on classroom practice and student academic performance. He looked at three aspects of teacher quality—teacher inputs, professional development, and classroom practice—and found that teachers who received sustained professional development geared towards higher-order thinking helped improve their students' performance in class and on mathematics assessments.

Teachers change their beliefs when they have the opportunity to develop new understandings and experiment with new instructional strategies (Loukes-Horsley et al., 1998). Professional development should be situated in practice to help teachers make changes in their instruction because teachers thereby see how the different approaches are successful with their students and can reflect upon what they are doing (Loucks-Horsley et al., 1998; Smith, 2001). This approach allows teachers to construct their own knowledge of practice instead of having it told to them by "experts" and allows any changes to be transformative or generative; that is, changes in basic epistemological perspectives, knowledge of what it means to learn, and conceptions of classroom practice (Franke & Kazemi, 2001; Smith, 2001).

Practice-Based Professional Development Models

Professional development models that incorporate collaboration, a focus on mathematical knowledge, and higher-order thinking are based in teachers' practice. Practice-based professional development helps teachers engage in transformative learning that changes their beliefs and knowledge as well as their practice. This professional development is based on the use of *professional learning tasks*, which are "tasks that engage teachers in the work of teaching, can be developed in order to meet a specific goal for teacher learning and to take into consideration the prior knowledge and experience that teachers bring to the activity" (Smith, 2001, p. 8). These tasks provide teachers with learning experiences that are connected to and contextualized in professional practice. This connection and contextualization enables mathematics teachers to make the kinds of complex nuanced judgments required in teaching (Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007).

The cyclical process of teaching is one way in which teacher educators can create professional learning tasks that have the potential to change teachers' practice. Loucks-Horsley et al. (1998) advocate the use of the cyclical process of teaching to enhance the professional learning of teachers. Teachers need to devote time during this cycle to learn mathematical content in order to be able to teach a new curriculum, conduct activities, and understand how students will learn new material. McDuffie et al. (2004) engaged in a series of teaching cycles with a middle school teacher in order to understand how instructional materials supported or impeded the implementation of problem-based instruction. The ongoing, classroom-based support of this professional development allowed the teacher to change her practice. Initially the teacher chose low-level tasks and was not able to maintain the level required of high-level tasks during her instruction. By the end of the year, the teacher was consistently selecting high-level tasks and was able to maintain that demand.

Silver, Clark, Ghousseini, Charalambous, and Sealy (2007) used a "collaborative lesson planning and debriefing cycle" (p. 265) in monthly professional development workshops to help teachers grapple with mathematical content while considering pedagogy and student thinking. Several factors contributed to teachers' opportunities to learn mathematical content including consistent and sustained use of professional learning tasks that made mathematics salient; the ability of facilitators to focus participants on mathematics and the mathematical basis for pedagogical decisions; and "the willingness of participants to work and reflect on their mathematical knowledge and instructional practice in a public setting" (p. 276).

In summary, the cyclical process of teaching provides teachers with practice-based experiences that help them learn about mathematics, pedagogy, and student thinking. These experiences are necessary in order to help teachers learn how to implement standards-based instruction, and ultimately higher-order thinking in their classrooms.

Theoretical Framework

In this section I discuss two important elements that informed the analysis of the data. First, I describe how a cyclical process of teaching, or reflective teaching cycle, can influence teacher learning and ultimately their practice. Second, I describe how higher-order thinking can be measured using cognitive demand and the Mathematical Tasks Framework.

Reflective Teaching Cycle

The idea behind using reflective teaching cycles relies on the belief that teachers change, or try to change, when they see a problem with their current instructional techniques or knowledge (Loucks-Horsley et al., 1998; Silver et al., 2007; Smith, 2001). When teachers find conflict, such as student achievement not being aligned with expectations, and they believe that it is related to their teaching and not students' understanding, they are ready to think about changing their practice and reorganizing their knowledge of mathematics and pedagogy. The classroom becomes a learning environment for teachers as they try to work through new understandings about what it means to teach and learn mathematics. How teachers learn particular instructional strategies, whether it be in the classroom or in a professional development setting, is a fundamental part of what they learn (Borko et al., 2000). By providing teachers the opportunity to learn about a variety of pedagogical strategies in the context of their own classroom, professional developers can help teachers transform their practice toward using higher-order thinking skills with their students.

Researchers have used the reflective teaching cycle and the classroom as a learning environment for teachers (McDuffie & Mather, 2006; Silver et al., 2007). My study built on this work by looking at the reflective teaching cycle in an urban setting. Research shows that generally urban teachers are not currently using higher-order thinking with their students (Darling-Hammond, 2007; Kitchen, 2003) and that the reflective teaching cycle is an effective way to provide teachers with a way to change the way in which they select and implement mathematical tasks that have the potential to facilitate higher-order thinking (Smith, 2001; Stein et al., 2000). Because we do not know enough about how we can help teachers in urban settings address higher-order thinking, this study provided further insight into this issue by using an established professional development model.

Cognitive Demand and the Mathematical Tasks Framework

The content and pedagogy of professional development dictate whether professional development can help teachers make transformative changes in their practices toward reform-oriented teaching and learning (Thompson & Zeuli, 1999). Transformative modification "involves sweeping changes in deeply held beliefs, knowledge, and habits of practice" (Smith, 2001, p. 3). The content of professional development should focus on teaching students to think by supporting and guiding them along productive paths of inquiry (Thompson & Zeuli, 1999). One way to teach students to think is to concentrate on the selection and implementation of high level mathematical tasks (Stein et al., 2000). However, "even interventions intended to transform practice in fundamental ways seem frequently to be assimilated into the established pattern, their contents reduced to techniques and tools that expand the teacher's repertoire incrementally but leave her basic mode of practice undisturbed" (Thompson & Zeuli, 1999, p. 355). Thus, to help teachers transform their practice, teacher educators should understand that teachers may need to first experiment with the new instructional strategies and only when these become routine begin to turn their attention to students' thinking (Thompson & Zeuli, 1999).

> When teachers experiment with new activities in their classrooms, they judge the new practices according to whether or not they "work." When these new activities engage the students, do not violate the teacher's particular need for control, match the teacher's beliefs about teaching and learning, and help the teacher respond to system-determined demands for such outcomes as high test scores, they are deemed to work. If they do, they are internalized and absorbed into the teacher's repertoire. (Richardson, 2003, p. 403)

After teachers struggle with students' thinking and how they may plan and adapt lessons based on this thinking they are prepared to reconsider mathematical content and begin to identify the big ideas in the curriculum.

The QUASAR Project (Quantitative Understanding: Amplified Student Achievement and Reasoning) was a national project in the 1990s that aimed to improve mathematics instruction in urban middle schools serving economically disadvantaged communities. A major goal of the project was to provide students "with increased opportunities for thinking, reasoning, problem solving, and mathematical communication" (Stein et al., 2000, p. 24). To see whether that goal had been reached, the researchers focused on *instructional tasks* used in the classrooms. An instructional task is "defined as a segment of classroom activity devoted to the development of a mathematical idea" (p. 7). The research from the QUASAR project indicated that when teachers chose tasks that required a high-level of cognitive demand and were able to set them up and implement them such that the demand was maintained, the result was an increase in student understanding and reasoning (Stein & Lane, 1996).

To direct the analysis of the mathematical lessons in the QUASAR project, the researchers created the Mathematical Tasks Framework. This framework described three phases that mathematical tasks passed through. Tasks were written by curriculum developers and selected by a teacher, set up by the classroom teacher, and finally implemented by students during the lesson. The framework outlined two dimensions of mathematical tasks: task features and cognitive demands. The task features were "important considerations for the development of mathematical understanding, reasoning, and sense making" (Henningsen & Stein, 1997, p. 529) and included multiple solution strategies and representations and mathematical communication. As the teacher set up the task, she or he could encourage the use of various strategies or representations and ask students to explain and justify their reasoning. As students implemented the task, they could use these features to varying degrees. Cognitive demands "refers to the kind of thinking processes entailed in solving the task as announced by the teacher (during the set-up phase) and the thinking processes in which students engage (during the implementation phase)" (Henningsen & Stein, 1997, p. 529). The creators of the framework defined four levels of cognitive demand of tasks: (1) memorization; (2) use of procedures without connection to concepts, understanding, or meaning; (3) use of procedures with connection to concepts, understanding, or meaning; and (4) complex thinking and reasoning such as conjecturing, justifying, or interpreting; that is, “doing mathematics.”

Memorization and procedures without connections were low-level demands. Memorization involved “reproducing previously learned facts, rules, formulas or definitions or committing facts, rules, formulas or definitions to memory” (Smith & Stein, 1998, p. 348). Tasks requiring memorization were not given enough time to use procedures, which was why fact or formula recall was needed. These tasks had no ambiguity and no connection to underlying meaning behind the concept or facts being used. Procedures without connections focused on correct answers instead of mathematical understanding. Students needed only to describe the procedures they used or were not asked to explain anything. These tasks were algorithmic and contained little ambiguity. Particular procedures were usually asked for or were evident from the instructions.

Procedures with connections and doing mathematics were high-level demands. Procedures with connections required students to follow procedures while engaging in the conceptual ideas behind the procedures they used. This requirement allowed students to develop “deeper levels of understanding of mathematical concepts and ideas” (Smith & Stein, 1998, p. 348). Such tasks generally had multiple representations and connections among these representations were used to help develop meaning. Doing mathematics was the highest level of cognitive demand. These tasks required students to use higher-order thinking such as self-monitoring and self-regulation of their cognitive processes, nonalgorithmic thinking, and recall and appropriate use of prior knowledge and experiences. The tasks lent themselves to student exploration, which allowed students to gain understanding of the mathematical concepts, processes, or relationships present.

The level and kind of thinking involved in solving mathematical tasks in the classroom dictated the level and kind of learning that took place. If teachers focused on low-level tasks, such as memorizing procedures in a routine manner, that would lead to low-level understanding and learning of the concept. However, if the teacher focused on high-level tasks, such as ones that "demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas" (Stein et al., 2000, p. 11), that would lead to the learning of higher-order thinking skills. The cognitive demand of a task could change during the set-up or implementation phases based on different factors. Teachers' goals, knowledge of mathematics, and knowledge of the students influenced the setup of the task. Classroom norms, task conditions, teachers' instructional dispositions and classroom management, and students' learning dispositions and motivation all affected task implementation. Analysis using this framework produced two major findings:

> (1) Mathematical tasks with high-level cognitive demands were the most difficult to implement well, frequently being transformed into less-demanding tasks during instruction; and (2) student learning gains were greatest in classrooms in which instructional tasks consistently encouraged high-level student thinking and reasoning and least in classrooms in which instructional tasks were consistently procedural in nature. (Stein et al., 2000, p. 4)

Stein and her colleagues (2000) determined specific pedagogical moves associated with the decline or maintenance of high-level cognitive demand tasks. When teachers reduced cognitive demand they made the problematic aspect of the task routine by taking over or giving explicit procedures; shifted emphasis to correctness and completeness from meaning and understanding; had classroom management problems; did not hold the student accountable for high-level products or processes; did not give the students adequate time to wrestle with the task;

or gave a task that was inappropriate for their students (Henningsen & Stein, 1997; Stein et al., 2000). On the other hand, when cognitive demand was maintained, teachers used strategies that resembled the pedagogical strategies used with higher-order thinking skills. The researchers identified seven factors associated with the maintenance of high cognitive demand:

1. Scaffolding of student thinking and reasoning.
2. Students are provided with means of monitoring their own progress.
3. Teacher or capable students model high-level performance.
4. Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback.
5. Tasks build on students' prior knowledge.
6. Teacher draws frequent conceptual connections.
7. Sufficient time to explore (not too little, not too much). (Stein et al., 2000, p. 16)

In order to maintain high-cognitive demand, "teachers should be especially attentive to the extent to which meaning is emphasized and the extent to which students are explicitly expected to demonstrate understanding of the mathematics underlying the activities in which they are engaged" (Henningsen & Stein, 1997, p. 526). By being attentive in that way, teachers could expect students to develop higher-order thinking skills and be able to solve mathematical problems in powerful and appropriate ways.

Henningsen and Stein (1997) investigated classroom-based factors that influenced students' engagement in mathematics. They wanted to identify, examine, and illustrate which factors hindered or supported high-level mathematical thinking and reasoning. Data were collected from classroom observations. Observers used an observation instrument that allowed them to define the mathematical task that was the focus on the lesson, describe that nature of the task (i.e., the mathematical content, teacher's learning goals, and behavior of students while engaged in the task) and classroom events. These descriptions were coded for set-up of the task and implementation of the task based on the levels of cognitive demand. Descriptions were also coded for "*factors* associated with task implementation" (p. 532). The list of factors coders chose from represented the possible ways teachers were able to maintain or diminished the cognitive demand of the task.

The findings suggested that there was a distinct set of factors influencing students' engagement in high-level tasks. "These included factors related to the appropriateness of the task for the students and to supportive actions by teachers, such as scaffolding and consistently pressing students to provide meaningful explanations or make meaningful connections" (Henningsen & Stein, 1997, p. 546). The researchers suggested that teachers must be able to "select and appropriately set up worthwhile mathematics tasks" as well as "proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task" (p. 546). However, teachers had a hard time understanding how to do that without guidance.

Arbaugh and Brown (2005) sought to investigate how to "engage a group of high school teachers in an *initial* examination of their teaching in a way that is non-threatening and, at the same time, effectively supports the teachers' development of pedagogical content knowledge" (p. 500). They helped teachers learn to analyze mathematical tasks using the levels of cognitive demand and looked at how teachers' learning about the levels influenced their thinking about tasks and choice of tasks. This research was based on the work of the QUASAR project and part of a larger research and professional development project. The professional development project included eight of nine geometry teachers who met in a study group ten times throughout one school year. The groups' goal was to focus on the levels of cognitive demand of mathematical tasks the teachers used in their classroom. Data were collected from two audiotaped interviews, written artifacts from the interviews, mathematical tasks teachers used in their classrooms, and audiotaped study group meetings.

The research showed that most of the teachers thought deeply about how students' work was related to the mathematics tasks used in their classes. Also, several teachers altered the types of tasks they were selecting. One participant *did not* change his choice of tasks but *did* show growth in understanding the levels of cognitive demand. Arbaugh and Brown (2005) thought that this lack of change could have occurred because teachers change their practice over long periods of time and at different rates. The research also suggested that learning about cognitive demand was "an effective way to engage high school teachers in *initially* examining their teaching practice" (p. 528). However, the researchers wondered if the intervention would continue to affect teachers' thinking and practice after it was over.

McDuffie and Mather (2006) used the reflective teaching cycle along with the Mathematical Tasks Framework as a professional development strategy. Their study considered how the professional development of one middle school teacher in collaboration with a university teacher educator influenced the teacher's use of instructional materials in planning and implementing instruction and how the instructional materials facilitated or hindered the implementation of problem-based instruction. The professional development consisted of collaboration between the teacher educator (McDuffie) and the teacher through the teaching cycle of planning, teaching, and reflecting. Data were collected from 7 planning and reflective meetings and 24 classroom observations. During the data analysis, McDuffie and Mather considered patterns in the implementation of curriculum as well as communication in the classroom. They performed the instructional material analysis using the Mathematical Tasks Framework from the QUASAR project to consider how the teacher selected and used problem-based tasks in her instruction. They were interested in the initial cognitive demand of the chosen tasks as well as the maintenance or decline of high demand tasks.

The teacher initially chose low-level tasks or reduced the cognitive demand of high-level tasks. Her lessons “tended to focus on rules and following procedures, rather than developing understandings,” and when she made connections between and within mathematical content, “these connections tended to be told to the students, rather than providing opportunities for students to develop these understandings” (McDuffie & Mather, 2006, p. 444). The teacher reduced the demand of the tasks by using closed questions that required a single, correct answer or by using “a rapid series of questions without much emphasis on thinking and approach” (pp. 442–443). Moreover, the more the teacher “saw her students doing mathematical problem-solving without explicit instruction, the more she believed that the students’ thinking and ways of solving problems were important in determining how a lesson should proceed, and vice versa” (p. 456). This observation was further support for the notion that practice-based professional development provides teachers with evidence that different instructional strategies can work with their students and thus give them the impetus to change.

At the end of the school year, the teacher was consistently selecting high-demand tasks and maintaining this demand during instruction. She was able to consider how tasks met her instructional goals and her students’ needs, which are deeper considerations than she had been using previously. She asked for justifications and explanations and prompted her students to monitor their own progress. She allowed her students sufficient time to explore instead of being concerned with completing the task in a certain amount of time as she had been in the beginning of the year.

By using the professional development model of a series of reflective teaching cycles, my goal was to see how task selection and implementation could be influenced. The Mathematical Tasks Framework categorization helps determine if higher-order thinking is being addressed by looking at how teachers choose and implement tasks. Understanding how teachers implement task demand allowed me to explore how teachers introduce the problematic aspects of the task, encourage students to justify and explain their understanding of the mathematics, require students to engage in higher-order thinking and skills, and emphasize the meaning of the concepts over correctness or completeness of answers (Stein et al., 2000).

CHAPTER 3

METHODS AND METHODOLOGY

This chapter discusses the design of the study, research site, participants, data sources, and data analysis. The data were collected in the fall of 2009 and spanned seven reflective teaching cycles with a pair of seventh-grade urban middle school mathematics teachers. Within each cycle the teachers participated in collaborative planning and reflection meetings that occurred preceding and following individual classroom observations of the planned lesson.

Design of Study

The study examined how a series of reflective teaching cycles influenced mathematics teachers' selection and implementation of mathematical tasks that had the potential to facilitate higher-order thinking skills. In particular, observations of planning and reflection meetings supported by classroom observations helped me describe how teachers selected and implemented mathematical tasks. The research questions guiding this study were as follows:

1. How does a series of reflective teaching cycles influence the way mathematics teachers select tasks that have the potential to develop higher-order thinking?
    a. In what ways do mathematics teachers consider students while selecting mathematical tasks that have the potential to develop higher-order thinking?
2. How does a series of reflective teaching cycles influence the way mathematics teachers facilitate students' implementation of tasks that have the potential to develop higher-order thinking?

a. In particular, how do teachers introduce mathematical tasks (i.e., problematic aspects of the task), and how do teachers use questioning to hold students accountable for higher-order thinking and skills?

b. In what ways do mathematics teachers consider students while implementing mathematical tasks that have the potential to develop higher-order thinking?

Research Site

District. The study was conducted in an urban school district in Georgia. Kent County School District* (KCSD) served over 12,000 students in 14 elementary, 4 middle, and 2 high schools. Of these institutions, nearly 32% did not meet Adequate Yearly Progress (AYP) as described in the No Child Left Behind (NCLB) legislation for the 2008–2009 school year, including three of the four middle schools. The student population was approximately 53% African American, 21% Hispanic, 20% White, 2% Asian/Pacific Islander, and 4% multi-racial, with close to 70% of its students listed as economically disadvantaged. With respect to state testing, 30% of the students in Grades 1–8 did not meet the basic mathematics requirements for the 2008–2009 school year. For the middle schools, the AYP indicator for Academic Performance was students' performance on the end-of-the-year state-directed standardized test. The state's Annual Measurable Objective for the mathematics test was 59.5%, and according to NCLB, the percentage of students scoring proficient or advanced on the selected state assessment had to exceed this number. In KCSD, nearly 40% of the African American and 30% of the Hispanic students did not meet the basic mathematics requirement in contrast to 11% of the White students. Thirty-six percent of the economically disadvantaged students also did not meet the basic mathematics requirement.

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* All name are pseudonyms.

School. College* Middle School (College) was the smallest middle school in KCSD, enrolling close to 500 students. The school's demographics differed from those of KCSD in that College enrolled 58% African American and 32% Hispanic students, but only 7% White students. College listed over 90% of its students as economically disadvantaged. The school did not meet AYP for the 2008–2009 school year as a result of the mathematics assessment AYP indicator. Nearly 43% of the African American, 32% of the Hispanic, and 44% of the White students did not meet expectations on the test as well as 40% of the economically disadvantaged students. The school's biggest struggle was with its students with disabilities. Close to 74% of these students did not meet expectations on the mathematics test. By the 2008–2009 school year, College had state-directed status because of their Needs Improvement Year 6 status according to the federal NCLB Act.

Project ISMAC. For the 2008–2009 school year, College was the site of Project ISMAC (Improving Students' Mathematical Achievement Through a Professional Learning Community). Project ISMAC was a school-based research and professional development project designed to increase teachers' mathematical content and pedagogical knowledge, while building a mathematics education community among the entire mathematics department, a university professor, Dorothy Y. White, and two doctoral students, including myself. The overall research goal was to examine the development of a mathematics education community as well as try to understand how such a community could operate and affect mathematics teaching and learning. The professional development components of the project were created to help facilitate the creation of the community as well as to support the mathematics teachers' practice.

Project ISMAC was driven by the belief that teachers improve student learning when they plan and reflect on lessons based on the mathematical needs of their students and the curriculum. Therefore throughout the year, we facilitated monthly professional development workshops with all mathematics teachers and weekly grade-level content planning meetings. We worked in classrooms by doing demonstration lessons, co-teaching, and performing observations. We thought that the teachers needed to be aware of the cognitive demand of the instructional tasks they select and how to implement tasks to support students' engagement with the mathematics. In the weekly grade-level content planning meetings, we encouraged discussions of the curriculum by asking the teachers to predict how students would engage in the selected tasks and to focus on student thinking. We discussed how to modify mathematical tasks to best suit students' needs and to be consistent with district guidelines.

From my classroom observations and participation in planning meetings, I noticed that even though the mathematics teachers at College used the required standards-based curriculum, they generally taught in traditional ways; That is, teacher-centered lecture. I saw many teachers spending time trying to teach the students explicit rules and procedures to follow. As I spoke with the students, it was apparent that they were not remembering those rules and would try to use a rule, or a part of a rule, inappropriately. Instead of thinking about problems and trying to determine how to solve them, the students attempted to remember what the teacher had told them or wait for instructions on how to proceed.

Project ISMAC research ended after the first year, but the professional development components of the project continued. During the 2009–2010 academic year, the district hired Dorothy White and paid four doctoral students and one post-doctoral fellow to support the mathematics teachers in the district's four middle schools and one elementary school. I was assigned to work in College and did so by attending the weekly grade-level content planning meetings, doing classroom observations, co-teaching, and providing general support to the teachers in the form of curriculum modification, assessment creation, and location of alternate resources for Grades 6–8. This work was in addition to my dissertation and I conducted my study outside of the weekly grade-level content planning meetings. Part of the reasoning behind conducting the reflective teaching cycles outside of regularly scheduled planning time was the presence of Betty, a new mathematics coach in the school. Betty was hired by College to provide additional support to the mathematics teachers and the administration. Her duties included facilitating planning meetings, providing professional development, and performing administrative duties such as textbook distribution, data analysis, and test preparation. Betty was supportive of my presence in the school and allowed me to facilitate some planning meetings. There were many instances, however, in which she was required to cover administrative duties, and thus the planning time was not consistent or guaranteed.

I was assigned to work with the seventh-grade teachers the first year of Project ISMAC and as a result we had established a good working relationship. By the time I began this study, I had worked with the teachers for over a year and felt comfortable with them, which helped me gain access to their thinking.

Organization of classes. There were six periods in each school day. Each teacher was responsible for teaching five classes a day and was allotted one period for planning. The first period of the day was called “extended learning time” (ELT) and lasted 55-minutes. This period was for an additional mathematics or language arts class provided to students who required further instruction. Each mathematics teacher and several nonmathematics teachers taught ELT mathematics. ELT did not follow a prescribed curriculum but the mathematics teachers provided all ELT teachers with information about concepts being covered in their classes as well as mathematics problems to do during the ELT class.

Second, fifth, and sixth periods were 63-minute classes. The third period of the day lasted 93 minutes and included lunch. Each grade received 20-minutes for lunch on a rotating schedule. The seventh-grade teachers had their planning during second period. Two of the five planning periods each week were allotted for mathematics planning—one day for mathematics teachers to plan and the other day for them to share and modify plans with special education teachers. College ran on an alternating schedule with A and B days. ELT was always the first period of the day, but the remaining periods met in one order on A days and the opposite order on B days.

College tracked students and placed them in different types of classes. The highest-level students were in "gifted" or "advanced" classes. There were also "regular" classes that consisted of students who were considered on grade level. "Collaborative" classes had on-level students as well as students who had been identified as needing special accommodations such as additional time on tests. A special educator was typically assigned to assist the regular mathematics teacher for collaborative classes.

Instructional materials. Teachers used the *Connected Mathematics 2* (CMP2) textbook and state resources. CMP2 provides eight units for each grade level. Each unit has a separate textbook that is organized by investigations. Each investigation has a specific mathematical goal for students to work toward and there are typically 3–5 investigations per unit. The investigations include 2–5 sequenced problems, or tasks. Each investigation also includes a set of exercises that can be used for homework or additional practice. These are called ACE problems, which stands for applications, connections, and extensions. The *applications* exercises help students "solidify their understanding by providing practice with ideas and strategies that were in the Investigation" (Connected Mathematics Project [CMP], 2009). The *connections* exercises provide additional review of concepts by connecting them to different grade levels and mathematical ideas. The *extensions* exercises challenge students to think further than the problems covered in class, offer interesting detours that look at related mathematical ideas, foreshadow future mathematics, or pursue interesting applications (CMP, 2009).

In addition to CMP2, the state had produced instructional units, called *frameworks* (Georgia Department of Education [GADoE], 2009a, 2009b). The frameworks are sets of tasks organized into 9-week units that are in a “coherent and sequential pattern” (GADoE, 2011). The tasks were created as models of instruction to support teachers in their implementation of the state’s standards and include “Enduring Understandings, Essential Questions, tasks/activities, culminating tasks, rubrics, and resources for the units” (GADoE, 2011). School systems and teachers were encouraged to use these tasks as models, meaning they should feel free to “modify them to better serve classroom needs; or create their own curriculum maps, units and tasks” (GADoE, 2011).

Teachers in Kent County were required to follow district pacing guides, which outlined the content to be covered in every unit, the order in which it should be taught, the tasks from the CMP2 textbooks, and the district’s instructional materials that should be used for instruction. The teachers then chose the problems they would implement. In some cases, the teachers supplemented the guide with tasks they found in other textbooks or on the Internet. Betty reinforced the district pacing guide by requiring the teachers to inform her of their day-to-day activities. Every 2–3 weeks during the teacher’s planning time, Betty would update a calendar by listing one or more tasks for each day of the week. The teachers were expected to abide by this calendar or inform Betty of any changes that they made.

Because I began the study in September and completed data collection in early November, the reflective teaching cycles that I engaged in with the teachers spanned the second and third instructional units for the academic year. These units used the CMP2 units Variables and Patterns and Accentuate the Negative. Table 1 provides the mathematical goals of each unit.

Table 1 *Mathematical Goals of CMP2 Units Covered in Reflective Teaching Cycles*

Variables and Patterns	***Accentuate the Negative***
• Identify quantitative variables in situations • Recognize situations where changes in variables are related in useful patterns • Describe patterns of change shown in words, tables, and graphs of data • Construct tables and graphs to display relations among variables • Observe relationships between two quantitative variables as shown in a table, graph, or equation and describe how the relationship can be seen in each of the other forms of representation • Use algebraic symbols to write rules and equations relating variables • Use tables, graphs, and equations to solve problems • Use graphing calculators to construct tables and graphs of relations between variables and to answer questions about these relations	• Use appropriate notation to indicate positive and negative numbers • Locate rational numbers (positive and negative fractions and decimals and zero) on a number line • Compare and order rational numbers • Understand the relationship between a positive or negative number and its opposite (additive inverse) • Develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers • Write mathematics sentences to show relationships • Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations with missing facts • Use parentheses and order of operations to make computational sequences clear • Understand and use the Commutative Property for addition and multiplication of positive and negative numbers • Apply the Distributive Property with positive and negative numbers to simplify expressions and solve problems • Use positive and negative numbers to graph in four quadrants and to model and answer questions about applied settings

Note. The goals in column 1 are from Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006b, p. 2. The goals in column 2 are from Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006a, p. 2.

Assessments. The teachers were expected to give regular assessments, such as quizzes and end of unit tests, during each unit of instruction. Students took quarterly benchmark standardized tests administered and graded by the district as well as a standardized, state-directed test, the Criterion-Referenced Competency Test (CRCT), at the end of the year.

Participants

Clark Welling and Tess Freeman, the two seventh-grade teachers, agreed to participate in the study. The teachers began teaching at College the same year. In their first year at College, they were both assigned to the eighth grade. They moved to the seventh grade their second year. During the 2009–2010 school year, the teachers were in their second year at the seventh grade level.

Clark. Clark is an African American man in his 50s. He was in his 19th year of teaching and had experience at the elementary and middle school levels as well as with adult learning. Clark was in his third year at College. He described his experience at College as challenging because of the school's culture. He had previously taught in a school with the same demographics as College, but the students and administration had different attitudes about teaching and learning. Clark stated, "It has been a challenge moving from a poor community where the students were successful to a poor community with the challenges I am having to face" (Clark, private communication, 2/19/2010). He perceived these challenges as a lack of student accountability and administrative support. Clark was still involved in educational endeavors as he was working on completing his doctorate in education leadership.

Clark was gifted certified and taught the one seventh-grade gifted class, as well as the advanced students. His gifted class met during fourth period and his advanced students were placed in his sixth-period class. Clark taught two regular classes during the third and fifth periods. For the study, I attended Clark's fifth block class.

Tess. Tess is a White woman in her 40s in her third year of teaching middle school. For these 3 years, Tess taught mathematics at College. She has an educational psychology background with experience teaching undergraduate classes in that field. Tess taught all regular and collaborative classes. For the study, I attended Tess's fourth block collaborative class.

Clark and Tess worked together in the same grade level for 3 years, had developed a good working relationship, and enjoyed talking with each other about teaching. They were eager to engage in professional conversations in part because they were still learning about the new seventh-grade curriculum and were thus excited to participate in the study.

Observed classes. Many factors contributed to choosing the classes I would observe. I wanted to attend the same classes each week in order to establish a relationship with the students and make them feel comfortable with my presence. I also had to consider my other duties at College for the district. Since I was hired to attend all grade-level content planning meetings and perform classroom observations for all mathematics teachers, I needed to make sure that some periods of the day were free. In addition, I wanted to avoid the period in which the students went to lunch since, depending on the day, that lesson could be interrupted. Finally, I wanted to attend two classes that were similar in terms of class composition. Therefore, I chose two regular classes that did not meet during the lunch period and had mostly male students.

Regardless of my careful selection of the classes, I encountered several problems that could have affected the teachers' selection and implementation of tasks. First, Tess's class was required to have a bathroom break in the middle of the period. Second, Clark's class was cut short on several occasions because of scheduled school assemblies. These instances were sometimes made known to me ahead of time, in which cases I was able to attend other classes to observe implementation. However, Clark had advanced students in two of his other classes, so there were times that my observations were of classes that were not similar to Tess's students in terms of size and student composition.

Data Collection

The main data source was the reflective teaching cycles. Each meeting in the cycles was audiotaped and supplemented with my fieldnotes. I took notes during the meetings to record our nonverbal actions and written work or to help me remember particular events. After each meeting, I wrote a narrative of the events and recorded my thoughts concerning questions that I had not had the opportunity to ask, or new questions or ideas that I felt should be brought up in later meetings. Every cycle consisted of a planning meeting, two observations, and a reflection meeting. The cycle began with a planning meeting with the two teachers and me. This meeting was followed by separate observations in each of the teachers' classrooms. I observed Clark during his fifth block and Tess during her fourth block on the same day. The cycle concluded with a reflection meeting, which was held with the two teachers and me, as soon after the observations as possible. After the cycles were completed, I transcribed all 14 planning and reflection meetings. Table 2 shows when the cycles occurred and the units covered.

The transcripts and notes from the planning and reflection meetings helped me address the research questions by allowing me to investigate the influence of our conversations on the teachers' selection and implementation of mathematical tasks. The observations aided in the analysis by providing me with information about how the teachers introduced problematic aspects of the tasks and used questioning during instruction. This information allowed me to cater future cycles to the needs of the teachers and gave me the opportunity to observe the influence the cycles were having on the teachers' decisions and pedagogical moves.

Table 2 *Reflective Teaching Cycles*

Cycle	**Unit**	**Planning**	**Observation**	**Reflection**
1	*Variables & Patterns*	Wednesday, 9/16	Thursday, 9/17	Thursday, 9/17
2	*Variables & Patterns*	Tuesday, 9/22	Wednesday, 9/23	Thursday, 9/24
3	*Variables & Patterns*	Tuesday, 9/29	Thursday, 10/1	Thursday, 10/1
4	*Variables & Patterns and Accentuate the Negative*	Tuesday, 10/13	Thursday, 10/15	Tuesday, 10/20
5	*Accentuate the Negative*	Tuesday-Wednesday, 10/20-10/21	Friday, 10/23	Tuesday, 10/27
6	*Accentuate the Negative*	Wednesday, 10/28	Thursday, 10/29	Thursday, 10/29
7	*Accentuate the Negative*	Wednesday, 11/4	Thursday, 11/5	Friday, 11/6

Although individual meetings were slated as either planning or reflection, the nature of the conversations would regularly incorporate both aspects. Although the cycles were designed to deal with one lesson, I was open to talking about other lessons and general teaching issues because this openness helped the teachers develop their practice. For example, there were times they needed to use the cycle meetings to plan or reflect on lessons that I would not or did not observe for the cycles. The teachers sometimes needed to plan lessons that would be taught *before* the observed lesson or reflect on classroom events that occurred *after* the observed lesson in order to concentrate on the cycle's lesson. By engaging in these professional conversations, the teachers felt better prepared for lessons and were able to think about how they could do things differently in the future. I did however ask specific questions about either planning or reflection, depending on the meeting, and guided the discussions to the cycle's lesson.

During the series of cycles, the teachers and I considered ways in which they could implement higher-order thinking in the classroom through task selection and implementation. I continuously evaluated any changes in each teacher's practice by comparing task selection and implementation throughout the observed lessons. In the sections below, I describe in greater detail how I implemented each segment of the reflective teaching cycle.

Planning Meetings

In the planning meetings, the teachers and I discussed mathematics, pedagogy, and higher-order thinking. Although those meetings were intended as a time for the teachers to plan a particular lesson that I would observe, we wound up discussing several lessons each meeting. We talked about the task or tasks they would use and particular pedagogical strategies, including how they would introduce the tasks and their questioning strategies. I was an active participant in these discussions and frequently clarified mathematical content, discussed the curriculum materials, and made pedagogical suggestions.

In order to focus the teachers' attention on higher-order thinking, one method I employed was to initiate discussions about the cognitive demand of tasks and the way in which the teachers could decrease or sustain the tasks' demand through implementation. As a participant observer in the planning meetings, I asked questions to better understand the teachers' reasoning for selecting or implementing tasks in particular ways. I modeled pedagogy and questioning strategies that would facilitate higher-order thinking during lessons and asked teachers to consider how their actions affected student learning. I challenged the teachers' thinking about pedagogy and mathematical content to get them to think more deeply about their decisions and how they could facilitate or hinder higher-order thinking. We talked about classroom management and student issues including pacing, student grouping, discipline and student motivation. We discussed mathematical content in order to build a better understanding of the curriculum, student thinking, and mathematical ideas in the tasks. The teachers listened to each other and proposed ideas concerning classroom management and instruction.

My questions were dictated in part by previous meetings and classroom observations, and thus changed from cycle to cycle. I began with an outline of questions from which to start (see Appendix A), so many of my questions were constant as they were modifications of these original questions. My intention was to have the teachers work toward building instructional practices that facilitated higher-order thinking and I worked to keep the discussions focused on mathematics, tasks, and students.

In order to understand teachers' selection and implementation of tasks, I asked questions about the mathematical goals for the task or lesson as well as how the teachers planned to introduce the task, how much time the students would need to successfully engage in the task, and how the students would engage in the activity; that is, individually, in pairs, in groups, or as a whole class. In addition, we talked about how these decisions would affect students' engagement in the task. I asked the teachers to consider the mathematics in the task by considering the stumbling blocks, misconceptions, or preconceptions students might have about the mathematical ideas in the task and how they could address them in their lesson. We worked through tasks in order to get a sense of how students would go about solving the task and the methods they might generate. I used other questions to understand how teachers were considering student thinking as they planned. I asked the teachers to think about students' prior knowledge and how they would be able to access that knowledge. I asked the teachers to contemplate the types of questions they themselves could use during class to facilitate higher-order thinking. In particular, I asked the teachers to think about how they could help the students focus on conceptual understanding rather than rules or procedures, think independently about the task, assess the students' understanding of the mathematical ideas in the lesson, and encourage their students to be more precise with their own questions.

Observations

Within 2 days of the planning meeting I attended one class for each teacher in which the lesson was taught. During my observations, I focused on the cognitive demand of the mathematical tasks. I used an observation protocol (see Appendix B) to keep consistent records of the lessons, systematically record the main mathematical ideas in the lesson and evidence of student learning, and look for changes in instruction. The protocol provided me with a way to think about what the teachers did to facilitate or hinder students' learning of mathematical ideas and how the teachers reduced or maintained the level of thinking in the classroom (Stein et al., 2000).The observation protocol included scales for aspects of pedagogy that have been shown to facilitate higher-order thinking. (See Appendix C to see how cognitive demand, higher-order thinking, and classroom events are related to the scales in the observation protocol.) In addition to the protocol, I took notes that described events in the classroom that attended to the mathematical goals or pedagogical strategies that we had discussed during the prior meetings. I noted episodes that I believed reflected instances of higher-order thinking or opportunities for higher-order thinking strategies.

The observation protocol and my notes provided me with information about the teachers' practices and helped me formulate ideas about the teachers' task selection and implementation. They aided me in creating questions that pushed the teachers to think about how they could increase their ability to facilitate higher-order thinking in their lessons. The observations showed me how well the teachers' intentions matched their actions in the classroom, which allowed me to formulate questions that would challenge the teachers' ideas of how they were or were not doing what they wanted in their practice.

Reflection Meetings

After each pair of classroom observations, the teachers and I met to reflect on the lessons. I encouraged the teachers to think about what they were attempting to accomplish during the lesson and why they chose a particular course of action. We considered what each teacher learned about the students and how they planned to use that information. We frequently talked about mathematical concepts and worked to build a common understanding of the concepts and student thinking. As with the planning meetings, I was an active participant in the discussions and would try to steer the conversation towards particular events in each teacher's classroom that highlighted evidence of higher- or lower-order thinking or opportunities for the use of higher-order thinking skills. The reflection meetings also provided the teachers an arena in which they could comment on each other's practice and provided feedback on pedagogical strategies and classroom management.

As I listened to the teachers and later reviewed the audiotaped conversations of the planning and reflection meetings, I took note of patterns and worked to identify episodes of talk focused on higher-order thinking. This identification provided another lens from which the teachers examined their practice and their use of higher-order thinking skills (McDuffie & Mather, 2006). Using ideas from Smith's (2001) work, I analyzed the learning environment, what students seemed to learn, and how they learned it. This additional analysis helped me identify questions I could ask in future meetings. As with the planning meetings, I began the study with a meeting protocol (see Appendix A). These original questions intended to have the teachers reflect on the implementation of the tasks and student thinking during the lesson. I wanted the teachers to reflect upon how their selection and implementation of tasks either facilitated or hindered higher-order thinking.

As the series of cycles unfolded, I found that I started reformulating reflection questions to address four themes: questioning, mathematical goals and pedagogy, mathematical content, and student thinking and understanding. For example, I began to ask specific questions about how the teachers could use questions to challenge their students thinking without giving them the answer or support student thinking and promote student understanding. I asked the teachers to consider the mathematical goals of the tasks they selected and to evaluate pedagogical decisions they made during class. With respect to mathematical content, the teachers and I talked about the mathematical meaning behind student ideas as well as the teachers' mathematical understanding of the concepts they were expected to teach. Finally, I asked the teachers how they were or were not promoting thinking rather than procedures or rules, how they were assessing student thinking, how their actions were influencing student understanding, and what classroom norms they could work to institute that would promote higher-order thinking.

Data Analysis

After transcribing each meeting, I analyzed the data by using a qualitative research method, thematic analysis. Thematic analysis is a "common general approach to analyzing qualitative data that does not rely on the specialized procedures of other means of analysis" (Schwandt, 2007, p. 291). By using this approach I was able to code sections of the transcripts according to emerging themes.

I began my analysis by reading through all 14 transcripts from the seven cycles, which consisted of seven planning and seven reflection meetings. In this initial reading, I separated the text into *episodes*. I use this term to define sections of text in which the conversation was about a particular topic. An episode ended when the focus of the talk changed. For example, if the teachers were discussing what tasks they would be using and then began to consider the mathematical goals of the task, I would create two episodes: one for the task they would be using, and one for the discussion about the mathematical goals. During this process, I took notes about each episode that reflected general observations of the conversation as well as my initial attempt to categorize the episodes based on the research questions. This open coding allowed me to develop main themes that became organizational categories for my data (Emerson, Fretz, & Shaw, 1995; Maxwell, 2005). After several more readings of the data and close reflection on my notes and coding, I ultimately settled on four organizational categories: *task selection*, *task implementatio*n, *building knowledge*, and *students*. By separating parts of the transcripts into these themes, I was able to sort the data for further analysis concerning the facilitation or hindrance of higher-order thinking.

After settling on the categories, I separated all episodes from every transcript into separate documents, one for each main category. As I divided the data by category, I took additional notes about specific episodes if I had ideas about what they meant and how they were related to higher-order thinking. Once this step was completed I focused on the task implementation episodes. As I concentrated on these episodes, I first divided them into three subcategories: introduction, questioning, and other. These subcategories reflected my second research question in which I asked how teachers would introduce mathematical tasks and use questioning during implementation. *Introduction episodes* included discussions about the problematic aspects of the task or how it would be presented to the students for the first time. *Questioning episodes* consisted of conversations about teacher or student questioning, which included questions teachers planned to use but did not use during implementation. The episodes that did not fit in either of these subcategories were coded as *other*. Examples of these episodes include those whose focus is on student thinking, student knowledge, student actions, or teacher actions. Many of these other episodes focused on building teacher knowledge about student thinking, prior knowledge, and current understandings, the curriculum and tasks, mathematical content, pedagogical strategies, or teaching constraints. Although one of the main categories was building knowledge, these episodes were coded as task implementation because teachers and I were determining how they would implement a task, the questions that they would use, or how they could best address the needs of their students. Frequently in these episodes, I dominated the conversation and thus was concentrating on the professional development of the teachers. Nevertheless, these episodes are important because they helped create a picture of how the reflective teaching cycles influenced how the teachers facilitated students' implementation of tasks with respect to higher-order thinking.

I then developed substantive categories (Maxwell, 2005) to describe the nature of the episodes. These categories included modeling pedagogy and questioning; building knowledge; communication and language; classroom management; reflection; guiding students; student expectations; confidence and frustration doing mathematics; mimicking teachers; differentiation; moving between hindrance and facilitation of higher-order thinking; and teaching philosophy. These categories helped me develop a general theory of what was happening during the reflective teaching cycles. I also used the categories further review the other main categories: task selection, task implementation, building knowledge, and students. I looked for additional evidence of each substantive category or indications that a category did not occur in the episodes for the other main themes. This aided me in creating the stories of Clark and Tess, which are descriptions of the teachers' thoughts and beliefs about mathematics, pedagogy, and higher-order thinking.

After I divided the task implementation episodes into subcategories, I coded the episodes with respect to higher-order thinking strategies. To develop the coding scheme, I reviewed the literature on higher-order thinking, critical thinking, cognitive demand, and reform or standards-based teaching, strategies, classrooms, and curricula. I used the ideas in this research to create a list of possible ways in which I could identify whether the teachers were either facilitating or hindering higher-order thinking as they selected and implemented tasks. I began by coding episodes using 6 facilitating (F) and 4 hindering (H) codes related to higher-order thinking (see Appendix D). As I coded the first 3 cycles, I modified the codes to better capture the essence of the episodes. After coding the episodes, I created frequency tables that allowed me to look at the number of episodes that facilitated and hindered higher-order thinking. I was able to draw conclusions about the ways in which Tess and Clark were implementing tasks in terms of facilitation and hindrance, which further helped me develop their stories and develop themes. The tables permitted me to create a picture of how the cycles were affecting the teachers' use of higher-order thinking strategies in their classrooms across cycles and within mathematical units. (See Appendix E for examples.)

In addition to these frequency counts, I used the tables to record notes on the substantive categories. I completed an overall summary of each cycle and more detailed summaries for each meeting. These summaries included all substantive categories and how they were related to the way in which the cycles could be affecting the teachers' use of higher-order thinking strategies in their practice. After considering the substantive categories across all cycles, I determined which categories affected the teachers' instruction and created descriptions of the categories with examples from the cycles.

Following the organizational and substantive categorization of the data and frequency counts, I looked at the data by focusing on the units of analysis: the reflective teaching cycles. I recomposed the transcriptions and created documents that included a planning meeting transcription, observation notes, and a reflection meeting transcription for each cycle. I read the documents together to get a sense of the events that transpired in a cycle and wrote narratives to tell the story of each individual cycle. These narratives allowed me to find confirming and disconfirming evidence of themes created from initial analyses of the data (Erickson, 1986). I was able to determine general themes for the teachers' selection and implementation of tasks as well as for the reflective teaching cycles' influence on those processes.

The analysis allowed me to create stories for Clark and Tess that described the way in which they thought about mathematics, pedagogy, and higher-order thinking. It also helped me determine how the reflective teaching cycles influenced the teachers' selection and implementation of tasks that had the potential to develop higher-order thinking. Finally, this analysis showed the ways the teachers considered their students while selecting and implementing tasks.

CHAPTER 4

RESULTS

The purpose of this study was to examine the influence of reflective teaching cycles on the teachers' selection and implementation of mathematics tasks that had the potential to facilitate higher-order thinking. In this chapter, I present the data about influences on teachers' decisions. First, I describe themes that influenced the teachers as they selected and implemented mathematics tasks. These themes were coded as follows: (1) pacing and assessment, (2) engagement of students, (3) teaching experiences, and (4) doing mathematics. Next, I present the teachers' stories, which provide further detail into each teacher's practice as well as how each theme influenced the individual teacher. The final section describes the different ways that the reflective teaching cycles influenced the teachers' selection and implementation of tasks. I provide episodes from planning and reflection meeting transcripts as examples of each theme and of the reflective teaching cycles' influence.

Description of Themes

The teachers chose and implemented tasks for various reasons. During the reflective teaching cycles I noticed that Clark and Tess attended to specific aspects of teaching and learning mathematics as they made these decisions. Based on my readings of the transcripts, I identified four themes that captured these influences. A general description of each theme is presented below.

(1) *Pacing and assessment*: Evidence for this theme included instances in which teachers chose tasks or talked about what mathematical topics needed to be covered and the amount of

time they would spend on specific mathematical topics. These instances included times when teachers altered the district-pacing guide, created or modified unit assessments, or selected additional tasks to cover mathematical topics that would be on the quarterly or end of year assessments.

(2) *Engagement of students*: Evidence for this theme included instances in which teachers selected or implemented tasks based on how they thought they could successfully engage students in mathematics during class and help them learn the material; that is, students needed to be confident, have fun, have problems base in real-life contexts, or needed direct instruction. This theme included instances in which teachers talked about what they thought students could or would do with respect to mathematical tasks, which included conversations where teachers chose tasks based on students' mathematical ability or motivation level. The theme also included conversations in which teachers selected or implemented tasks based on classroom management and discipline concerns.

(3) *Teaching experiences*: This theme was signaled by instances in which teachers talked about events in their current classrooms and how those events informed the selection and implementation of tasks. Moreover, the theme included instances in which teachers talked about previous teaching experiences, such as in a different school, in previous years, or with Project ISMAC, and how those events informed the selection and implementation of tasks

(4) *Doing mathematics*: Evidence of this theme included instances in which teachers talked about their views on what it meant to do mathematics, what understanding mathematics meant, or how they assessed mathematical learning. Episodes in this theme incorporated times when the teachers talked about mathematical concepts, state standards for mathematics content,

instructional materials, or their beliefs about mathematics and mathematics teaching and learning.

Story of Clark

Clark organized his fifth-period class so that students could work individually, in pairs, or in groups of 3 or 4 students. Each student had a separate desk that could be moved when needed. Usually, Clark began each period with students sitting alone and facing the front of the room. He would then have them move together, if needed, during the course of the lesson. The fifth block met during fifth period on A days and third period on B days. There were 26 boys and 2 girls in the class. Eight students were categorized as special needs, but generally there was no special educator present for this period. Sometimes a special educator came to aid Clark during this period, although that happened in a nonroutine manner.

Clark's typical lesson began with a "warm-up." The warm-up was a problem or set of problems that engaged students in mathematical thinking. The goals of the warm-up changed from day to day and included reviewing previous concepts, providing students multiple-choice test item practice, previewing concepts, or preparing for the lesson by reading excerpts from the textbook or task. After the warm-up, Clark began the day's main task or tasks. The task started with a teacher-directed whole class discussion. This opening or "launch" reviewed or introduced the context for the problem or the mathematical ideas needed to engage in the problem. Following the launch, there was a "work session" in which students explored problem(s) in the investigation individually, in pairs, or in groups. Clark would periodically stop the class to make sure students were staying on task, to make sure they were understanding the problem, or to bring the class together to discuss solutions. Near the end of the class period, Clark brought the students together to summarize the lesson. This summary did not always happen because sometimes students worked until the end of the period.

Pacing and Assessment

Clark considered the mathematics that was important for students to know as he selected and implemented tasks. He believed that certain mathematical topics were important or could be difficult for students and thus altered the pacing that he, Betty, and Tess had agreed upon to extend time spent on these topics. For example, Clark believed that his students needed to develop the ability to write mathematical formulas or equations from written descriptions. During Cycle 3 the teachers planned lessons for Investigation 3 in the Variables and Patterns unit. The context of the first two problems was an amusement park, Wild World, and students were asked to describe different patterns of change by using tables, graphs, equations, and written descriptions. Although in the planning meeting the teachers had decided to complete the first two problems as written in the textbook over the next 3 days, Clark altered his plan to focus on writing equations. On the first day, Clark had students formulate rules based on verbal descriptions. On the second day, students only completed a table for the first problem, even though the directions included describing patterns of change using words, writing an equation, sketching a graph, and analyzing how the patterns of change appear in the two representations. In the reflection meeting, Clark talked about his reasoning behind this choice as well as the repercussions of this selection and implementation of the tasks.

1 Eileen: So, was your goal in that lesson to just talk about that table?

2 Clark: Yes.

3 Eileen: Okay.

Clark: That was my total goal that whole day for that group because I knew that that group had had a hard time for the last four weeks.

Eileen: Okay.

Clark: Um, dealing with anything, I mean simplest problem. They couldn't get through it, and I knew they were gonna have a problem when you asked them to, "Okay, um, you came up with this rule. I want you to apply it." They were going to have issues with it because they've never been made to actually sit down and apply what you tell, what they tell you.

Eileen: Right.

Clark: And so, I knew it was going to take the full class period for them, and um, and it actually helped today. It really, it really helped us today because when I went to introduce [Problem] 3.2 to them, um, we got to talking about going to this place and this Wild World, and you know, what's gonna happen here, what are our variables? And they were able to tell me before I showed them the charts. So then I asked them to turn the book to those pages and say, "Okay, now this is your opportunity to show what you learned from yesterday and this mini talk we just had." I put the timer up. I gave them five minutes, and I gave them all calculators. Because it wasn't my intent to worry about the actual calculations. It was the procedure that I wanted to make sure that they understood. And I gave them all calculators, and I said, "Okay, I'm gonna set it for five minutes. Now what, this is what I want from you: I want you to fill in the chart. I want you to fill in those two lines there. And I want you to graph them. And that's five minutes.

Now, is that enough time?" "Oh yeah, we can do this. We can do this. We got it. We got this." "Okay, go!" And I backed up. And lo and behold, they had it.

Clark thought that his students needed time to complete a task for themselves since they had not had regular opportunities to do that (lines 7–11). He used his knowledge of students to predict how long the problem would take them (line 13), which is why he decided to select only a portion of the task for them to complete. This decision positively affected his students' engagement in the second problem because it built their confidence and gave them the ability to do the problem on their own (lines 27–29). On the third day, Clark decided to have the students work on this investigation independently so they could illustrate what they had learned the day before (lines 18–21). By choosing to do the tasks in this way Clark was able to talk about determining variables, creating tables, and writing equations to enhance students' understanding of the concepts.

Another example in which Clark wanted to change the agreed-upon pacing because of the difficulty of the topic occurred during the fifth cycle. In the planning meeting Tess reminded Clark that they had scheduled a quiz for the next day (Wednesday) to assess the first investigation. Classes were shortened to 30-minute blocks because of a school function. Hence, Clark wanted to cut down or delay the assessment in order to begin the second investigation, in which students would be applying their new knowledge of integers to develop algorithms for addition and subtraction, a topic on which students typically struggled.

Clark: To solve these five simple problems? If they can do that, okay. We have done what we needed to do for that part of it, and let's move on into two, Investigation 2. Instead of wasting a lot of time. Because we're gonna spend a lot of time getting the kids to understand, um, again, the subtraction, the multiplication and division, that kind of stuff. That's where most of your time is going to be spent. Distributive property using negative numbers, um, associative property using negative numbers. Those are things where you're gonna have to spend the majority of your time.

Tess: But, we, we, it's harder there if we don't have a solid understanding of what is a negative number. Because we really start from scratch. These are the kids that have never had it before. So, I see growing confidence and understanding.

Clark: I see what you're saying. But what I'm saying is instead of having a large test—.

Tess: Oh. But 30 minutes. I'm saying that whole period just—.

Clark: I'm saying, let's test the three skills [using the horizontal and vertical number lines and modeling integer operations]. Let's test the three things we wanted them to get out of there. We want to be able to order from least to greatest, greatest to least. We need for them to be able to go to a number line and be able to show, ah, ah, ah, the opposite, or—.

Clark wanted to select quiz items to check his students' understanding of ordering rational numbers and using the two models for integer operations (lines 16–20). In lines 1–6 Clark defended this action by pointing out the difficulty of the next topics (i.e. operations and properties of rational numbers) and the time it would take to adequately address them. Tess insisted that the assessment was important because students needed a strong foundation to be successful on the next concepts (lines 9–10). Clark agreed to give the quiz with the understanding that the assessment would be brief enough for them to complete in a short amount of time. This decision allowed Clark the freedom to adjust the pacing to implement the next investigation over more class periods and to focus on the concepts he knew from experience would be difficult and time consuming.

Sometimes, in order to maintain the pacing, Clark would use his warm-ups or ELT to provide students more practice with certain mathematical concepts. During the sixth cycle, Clark and Tess discussed if and when they should implement the fifth problem in Investigation 2. Investigation 2 addressed the addition and subtraction of positive and negative rational numbers— a topic Clark had mentioned in the fifth cycle would be difficult for students. Problem 5 dealt with coordinate graphing. The textbook proposed placing this concept here to allow students time to “use positive and negative numbers to graph in four quadrants and to model and answer questions about applied settings” (Lappan et al. , 2006a, p. 2). Tess wanted to give the students another day to practice addition and subtraction, whereas Clark wanted to move on to Problem 5. Even though Clark also thought that some of his students could use more practice adding and subtracting rational numbers, he was willing to provide additional exercises during warm-ups and ELT.

Clark: But, I do, I feel like we move a little, move further because it is the coordinate plane, and they’ve been introduced to one side of it already. And you just basically, and you been introduced to the vertical number line and horizontal number line.

Tess: Right.

Clark: So, by just, you know, getting to that point where we’re showing them the coordinate plane and how to locate coordinates on that coordinate plane.

It's fine. I mean, I think it's great. You can, you can, ah, have a mini-lesson on the adding and subtraction end just to review, and you do that every day.

Tess: Yeah.

Clark: Yep. But for right now, I think we do need to go ahead and step up to the next step, the coordinate plane.

In lines 1–7 Clark explained that by working with coordinate graphing, the students could have time to apply what they had learned about the horizontal and vertical number lines. Clark also considered this problem to be a "break" from "heavy math" for his students. He thought that they deserved to do something different since they had been working so hard, and especially since the next problem introduced multiplication of integers, which would be difficult for the students.

Clark thought about pacing and assessment as he selected and implemented tasks by deciding what concepts would be targeted because of their importance or difficulty. He altered the agreed-upon pacing, modified unit assessments, and focused on concepts that were important for the understanding of future concepts or were challenging for his students. In order to stay faithful to the agreed-upon pacing, he was also willing to review concepts during warm-ups or ELT when students needed more exposure to those concepts. Clark was also continually assessing how he could engage his students in mathematics and higher-order thinking.

Engagement of Students

Clark engaged his students in higher-order thinking while solving problems but thought that certain groups of students would be able to do this independently, whereas others would need more guidance. Clark thought that students who had discipline problems, low motivation, or low achievement to require additional assistance in order to successfully engage in mathematics, and thus differentiated his instruction. Students needing close attention, in his view, included special education students, students who had low CRCT scores, or students who habitually were the recipients of disciplinary action. Henceforth, I refer to these students as "low-level."

Clark understood higher-order thinking as a focus on conceptual rather than procedural content. In general, he wanted his students to do the thinking and for them to tell him how to do problems rather than relying on him to provide answers. He tried to lead class discussions in which students explained their thinking, justified their strategies, and accessed and extended their prior knowledge. For example, during the third cycle Clark described an event in his class in which he pushed students to explain and justify their solutions.

Clark: And boy they were telling me some things. I'm like, "Okay! So … what happens if you're the owner of this company and you're standing in the back, and you got a clicker. And you just saw 42 people come in, and you walk up to the person taking your money, and he hands you 400 dollars. What are you thinking? Did he give you the right money? And if so, how do you know?"

Eileen: That's great. That's a great question.

Clark: "What?! 400 dollars!" One of the kids, "No! No! He cheat me out of 70!" So [Clark said,] "How do you know?" [Student responded,] "Well, I went to count my money, and I asked him, 'How much you charge?'" I said, "What do you mean?" Well, he said, "They got the group rate. That's 70 dollars. He cheated me out of it." [Clark asked,] "How do you know? How do you know he cheat you out of 70 dollars?" [Student responded,] "He's

> gotta charge them 50 bucks, and 42 times 10 is four hundred." I said, "Hold on. How do you know it's 10 dollars a person?" "In order to get the group rate," a kid told me, he raised his hand, "this is the only way you can get the group rate is you gotta pay 50 dollars up front, and then you gotta pay 10 dollars per person. That's 42 people. That's 420 dollars. My 50 bucks, that's 470. If he only gave me 400 dollars, that means he's cheating me out of 70." I said, "What if he didn't give you the group rate? What if he didn't give them the group rate? How you know he gave the group rate?" [Student responded,] "Well, that's the minimum he cheated me out of then." I said, "Oh, okay! So, if he didn't give the group rate, what else happened?" [Student responded,] "Well, then he cheated me out of more because it's 21 dollars a person. If you put it back where they're in charge, they're gonna come quick."

Clark asked his students to explain and justify their thinking by asking questions such as "How do you know?" and "What do you mean?" He pushed his students to use higher-order thinking to solve the problem, rather than just give him the correct answer, by asking his students to explain their thinking (lines 5–6, 9, 10–11, 12–13, 15) (Boaler, 2002; Lobato et al., 2005). He also facilitated higher-order thinking by asking his students to revise their thinking when presented with a new situation (lines 20–23) (Stein et al., 2000; Wenglinsky, 2000).

Another way Clark engaged his students in higher-order thinking was to facilitate class discussions that pushed them to come up with their own strategies for solving problems (Lobato et al., 2005). In the fifth cycle, the teachers were discussing how to help their students understand how to combine mixed numbers. Clark planned to have the students come up with their own strategies first so they could share their thinking with each other before he discussed the problem. He described how he would do this activity during our meeting. In his words,

> Yeah, I'm gonna let them do it first before I talk to them about it. Just let them go through it and discuss it amongst themselves and on the board. Because I'm gonna put these things on the Smart Board. One at a time. Each slide. One on a slide. And then how I'm going to do it is, I'm going to have the kid go up and let everybody work it and then somebody go up and explain it to us. What did you do? How did you do it? Talk us through it, you know, and that type of thing. So that we all get some kind of—.

Clark wanted his students to discuss the concept with each other and then to present it while he asked probing questions. In this way, Clark would give the students the responsibility for determining strategies for solving the problem.

However, Clark frequently differentiated his strategies for students he considered to be low-level. He regularly talked about the additional guidance these students needed in order to successfully engage in mathematical problem solving. In the first cycle as the teachers planned the fourth task in Investigation 1, Clark intended to show his students a *t*-table they would need to create. He was not confident that they would independently and efficiently create a table and did not want to waste time on a cosmetic aspect of the problem. Clark talked about letting the gifted and advanced students in the fourth and sixth block do the problem on their own by giving them the launch and allowing them to work in groups or individually. However, he planned to "guide" the other two classes.

Clark: But what I would do, in, in, in my case, it's like, say, with my third block, guide it. What I mean is, I would go in I would have them I would work with them with the first two points.

Tess: Yes. That's how far I would go.

Clark: Okay. I would give, "Okay guys, how did we do this?" And let them tell me how to do it, and I question them on it and make sure they understand it.

"Okay now, if we've done the first two together, I want you to continue on and do the rest."

The first goal of the task was to have students create a table using data on a graph, a task they had done several times already in this unit. Clark planned to graph the first two points so he could make sure they understood what they were supposed to do. This practice was in contrast to the teacher resources, which suggested talking about the situation, looking at the graph together to identify the variables, reading the directions, and allowing the students to work individually or in pairs. Clark did not follow that advice, because he thought that the some students needed to be shown what to do in order to successfully engage in the task.

He additionally spoke of using alternative pedagogical strategies, such as flexible grouping, so that he could more closely monitor and assist low-level students in regular classes. Clark thought that certain students could not be trusted to work independently or were not capable of performing the necessary mathematics. Part of the issue was the discipline problems Clark experienced with some of his students. Certain students disrupted class and prevented others from learning. Clark thought that he needed to separate these students so that he could better control their behavior and provide them additional guidance. In his words,

> And hopefully that everybody's listening, but a lot of times in these larger classrooms, uh, and I can see exactly what you're saying, when you get to that point where you're trying to scaffold them to get it, you done lost some kids along the way because they, they probably know it. They won't say that they know it, but they're getting bored. Because they're sitting there waiting on Joe over here, and his group of clowns is clowning, to get it, and you're right there with them. Those guys is clowning, trying to get them to pull it out. So, this person's lost now. So, I'm, you know, I had to rethink about that last night and … wondering if maybe … I need to have two types of lessons in that class. And maybe separate those kids based on their skills, and say, "Okay. We'll have a common open Getting Ready." And we'll put them to work. "This group, you do this." And you put them, you let them go at it. And then I go to the other group and work with them more closely.

Clark claimed that some students in his classes who understood the material were getting lost because he had to scaffold some students in order for them to understand. According to Henningsen and Stein (1997),

> Scaffolding occurs when a student cannot work through a task on his or her own, and a teacher or more capable peer provides assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task. (p. 527)

As Clark provided additional assistance to some students, others disrupted the class and preventing students from learning and him from teaching. He believed that if he could separate the class according to skill-level, he would be able to differentiate his instruction and work with students who needed additional scaffolding or closer attention.

Some students who needed closer attention were those who seemed to be unmotivated. During the fourth reflection meeting, Clark talked about how his fifth block “doesn’t want to do anything” and wanted “to clown.” Most of the students in the class were failing, and Clark needed to come up with new strategies to improve the situation. One method that Clark spoke of on several occasions was “flexible grouping.” That strategy separated classes into two or three groups for differentiated instruction. Some groups would be given tasks and work with limited direction, while others would be “coached.” Clark would get higher ability students started on a task and spend his time working with low-level students. By doing this grouping, Clark could provide more guidance to low-level students and give the necessary attention to students that were not working or had difficulty understanding tasks. Clark thought that if he could separate students, he could explain the mathematics to them.

Clark: You know, let them try that while these [are] working. And once I introduce them just like, follow the script, okay, now, you go ahead and do the rest of these. I’ll come back to you in a few minutes. Let me check on the other students while you’re working. And then I go over and check on the students. Just like I used to do it in elementary school. I had three groups in that room, and there was just one of me. But I was able to go work the group, and I had special ed, gifted, and regular ed, all in one class.

Tess: That makes sense.

Clark: And it worked. It works.

Clark wanted to be able to have the option of focusing on a smaller group of students while other students who could work independently or understood the concepts would work separately (lines 3–4). He had used this strategy successfully as an elementary school teacher (lines 5 & 9), and believed that “it worked.”

Clark tried to engage his students in higher-order thinking and worked to differentiate instruction to include all students. He thought that even if students struggled, if he provided the appropriate accommodations or instruction, they would be able to successfully engage in mathematical problem solving. He believed that all of his students would eventually succeed and that they all should be expected to use higher-order thinking, but they needed to know that they were capable and sometimes needed more guidance to get there. Clark tried to gain his students’ trust early so he would be able to challenge them and push them to think more deeply about mathematical concepts with less guidance from him later. He felt that he should help build students’ confidence and trust by spending time helping them experience early success so that they would be more likely to engage in more difficult problems later. During Cycle 3, Clark said, “I’ll start with something easy and then start to increase as you go along, so you building up their confidence as they do it.” He thought that he would be able to push his students harder in the future if they gained confidence in their ability to do mathematics. They would be more likely to struggle if they knew they could find the answer. Clark described the effect this practice was having on his students and how he would proceed. In his words,

> Um and they're at that point now that they, they, they felt that success. They know that they can do it, and just right after class was ended [I] was talking to the kids in the hall and coming in the room; they were so proud of themselves. They know that they can do it. Now I can step it up a little bit more and constantly build more into them to get them to give me more. And each time reminding them that, "Hey, you did this yesterday. We just adding a little bit to it."

In this way, Clark reminded students of their previous achievements to encourage them to challenge themselves.

Clark regularly shared these types of teaching experiences during the cycles to show how different strategies were working with his low-level students. He also talked about being optimistic that he would eventually reach all students and felt that he was able to chose and implement tasks that was helping his do this. Clark's experiences in the classroom, both this year and in previous years, were a major influence on his practice.

Teaching Experiences

Clark's current and previous teaching experiences influenced his selection and implementation of tasks. He was influenced by events that happened with his current students and reflected on students' thinking while planning for lessons. He was also influenced by his prior teaching experiences, including those with Project ISMAC. He thought about successful strategies he had used in the past and considered how he could employ them with his present students. Clark thought that his students were learning with tasks and pedagogical strategies used by Project ISMAC facilitators and wanted to continue using these ideas in the current year.

Current Teaching Experiences. Clark selected and implemented tasks based on student comments, questions, and explanations of mathematical thinking either in class or on written assessments. He considered how his students engaged with new concepts and assessed students' prior knowledge to build new knowledge. He decided on tasks or pedagogical strategies during lessons in response to students' remarks. He assessed his students' understandings of mathematical concepts during classroom interactions in order to make decisions on tasks he would use and how he would implement them. Clark sometimes thought he needed to extend students' thinking and would thus push students to think more deeply about the current concept or introduce advanced topics. Alternatively, he was not willing to move on in the pacing before he felt his students were ready. Therefore he would sometimes change his lesson plans to review old concepts or remediate new ones.

Since I had already been observing the teachers' classes through Project ISMAC, I was able to bring up pedagogical issues with the teachers beginning in our first planning meeting. In that meeting, we discussed strategies for engaging students who finished a task ahead of other students. The tasks the teachers were discussing were part of Investigation 1 of the Variables and Patterns unit. This investigation had students making tables, graphs, and verbal descriptions to represent relationships between variables and "interpret and create data representations and begin to think about the strengths and weaknesses of each type of representation" (Lappan et al., 2006b, p. 3). During one problem in which students created a graph from a table, Clark talked about what he did when a student finished the task before the rest of the class.

Clark: What I do is I have that happen a lot, too. But what I do is, I have a set of questions that I would give those kids.

Tess: Mm hm.

Clark: Like if those, um, the kid told me he was finished. I gave him a couple of questions to answer from the graph. "Make a prediction for me, sir. What would happen in 30 minutes? What would happen in five minutes? What would happen in the next 10, 15 minutes?”

Tess: Okay.

Clark: Um, “Can you make that prediction? Based on what you have here.” And have him go and do it. And then come up with what happens to the graph if we change it from 10 minutes to the scaling from 10 minutes to 5 minutes?

Clark asked his students to think critically about the task by making predictions (lines 4–7), conjecturing alternatives and drawing conclusions (lines 9–12) (Facione, 2009). This focus on critical thinking allowed Clark to facilitate higher-order thinking as he responded to the student who completed the task.

Clark reacted to student thinking during instruction by asking questions that pressed students to explain their thinking and justify their solution strategies. During the third cycle, students worked on a task that asked them to consider the admission price for an amusement park. The task provided two options for entering the park: regular admission for $21 per person or a special group rate of $50 plus $10 per person. After the students described the relationship between the number of customers and the total admission cost, Clark facilitated a discussion about their conclusions. Clark began the discussion by asking his students to explain and justify their thinking and did not simply accept numerical answers. He pushed his students to explain *how* they came up with their solutions. He also extended the problem to get students to consider other alternatives, thus altering the task based on students’ comments. This alteration facilitated higher-order thinking by requiring students to defend their answers, explain their conjectures, justify their answers, and extend their thinking (Facione, 2009).

Clark built on students' developing understanding of the seventh-grade content to help them create new ideas. Questioning techniques played a central role in his ability to facilitate higher-order thinking by building on students' knowledge. As I facilitated the cycles, I regularly asked him to reflect on the types of questions he asked students, consider the effects of questions on students' learning, and to predict student difficulties and misconceptions about the upcoming material in order to compose questions that could be used as the taught. I believed this strategy would help him facilitate higher-order thinking because these premeditated questions would more likely emphasize conceptual understanding rather than procedural knowledge (Lobato et al., 2005). During the fifth cycle, Clark described an encounter with a student in which he used questions to remind the student of previous tasks to help him access knowledge to solve the current problem. The previous tasks involved developing algorithms for integer addition and subtraction using chip and number line models. The current problem had students applying the new algorithms.

Clark: I found that, and yeah, I need to have some questions when I have the kids doing this because what they were doing, they was, you know, it's hard for them to try to bridge things together. Bring it together. So you have to guide them into bringing it together. And one of the questions in the third block was and, you, what was the kid's name? Smith – Smith said, "Mr. Welling, what's going on here?" I said, "You tell me." I said, uh, he said, "How do I do this?" I said, "Do you remember what happened in the groups when we were adding integers in Investigation 2.1?" [He said,] "Yeah." I said, "What do you see here?" [He said,] "I see integers." [Clark said,] "What do you see?" He said, "Problems with two integers, two of the same signs and problems with two different signs." I said, "Okay, so what are you supposed to do? You drew the conclusion. What are you supposed to do here? What do you know?"

Eileen: So, I wonder if asking him, (sigh) that seems like so general. If I was a kid, I'd be like, what?

Clark: I was trying to get him to understand that he—. Okay, when I look at these problems, if I got two. If I'm adding integers and they both are negative—.

Eileen: What answer should I get?

Clark: What's gonna happen here? He said, "I know the answer's gonna be negative, but what type of—." [Clark said,] "Okay, you said it was negative. You're right in that aspect. But what do I really do here? What do I do?" [He said,] "I don't know." I said, "Okay. What strategies do you know?"

Eileen: Okay. So then you get to the strategies. "What strategies do you know?"

Clark: And so I made him tell me some strategies. He said, "Well, there's three that we learned in class."

Eileen: Which one do you like? Yeah.

Clark: "Which one do you want to use?" So then he says, "Well, I'm gonna try the chip method." [Clark said], "Okay, explain to me the chip method, and while you're doing it, do it on paper, and explain it to me. Talk me through it. I don't know."

Clark initially prompted the student in lines 9–13 to recall his experience with the previous investigation. Then Clark helped the student access his knowledge of that investigation (lines

22–24) and finally to apply the knowledge in order to solve the current problem (line 27–30). This episode illustrates how Clark used questioning to facilitate higher-order thinking by helping a student access and build on prior knowledge to solve problems (Stein et al., 2000). However, Clark not only used his current students' experiences in class to help select and implement tasks. He also considered students' elementary school experiences through his own knowledge and experiences teaching mathematics to elementary school students.

Prior Teaching Experience. Clark's experience as a former elementary school teacher allowed him to build on mathematical concepts students had been exposed to in Grades K–5. Clark used his knowledge of the elementary school curriculum to help his students access prior knowledge when learning new concepts. He helped them use their prior knowledge to engage in tasks and use their own strategies as they grappled with new material.

For example, the concept of fact families came up as a useful tool in learning several seventh-grade concepts. A *fact family* is "a set of facts, each of which relates the same three numbers through addition or subtraction" (http://www.math-glossary.info/definition/1078-fact_family) or multiplication and division. Fact families help students in elementary school understand the commutative properties of addition and multiplication. They also help students see connections between addition and subtraction and between multiplication and division. During the reflection meeting in the second cycle, we discussed Tess's warm-up in which the students were asked to solve four one-step equations. During my observations, I noticed that several students had difficulty with the problem $\frac{x}{12} = 4$. The two students who had been called up to the board to show their work found $x = 3$. Tess shared that this was a mistake she had encountered in all of her classes. Clark suggested that she could have built on students' prior knowledge of fact families to help them understand how to solve the equation.

Clark: Well, you know, and another way I would have looked at it, I would have told the kids, "Okay, guys, for those that can't conceptually see this, look at this problem. How many ways, what's my families that I can write from this? I could say, um, something times, um, well, $12 * 4 = x$. Can I say that?"

Eileen: Is that something that they've done before? So you're saying—.

Clark: They've written fact families in elementary school.

Eileen: For multiplication and division?

Clark: For multiplication and division. And that's what that is.

Eileen: Oh.

Clark: They wrote fact families.

Eileen: That's good to know. I didn't know that they had already done that.

Clark: Mm hm. Because I taught that. That's why I know. They wrote fact families. They did it in third grade and they did it in fourth.

Eileen: But not with variables. It was like with a box or something like that?

Clark: No, it was with a box. But you could, you know, you could say, okay, write me a fact family using these. Using what you have here.

Tess: *Fact* stands for factor?

Clark: Mm mm. Fact families. In other words like, you know, $12 \div 4$ is 3. $12 \div 3$ is 4.

Tess: Well, that's what they did.

Clark: Twelve. No.

Tess: They quickly went to a family and then they—.

Clark: But they didn’t realize—.

Eileen: But they did the reciprocal. Instead of, they were going to a family, but just the wrong family.

Clark: Wrong family. They should have said, “$x \div 12$ equals to four.” What number divided by 12 would give me—.

Eileen: And that’s what she wound up talking to them about, but I was just saying that it was interesting to me. And actually, one of the girls wrote, instead of writing four divided into 12, wrote 12 divided into four is equal to three. But when she said it, she said “Four divided by 12.” So she wrote 12 divided into four, but said out loud, “Four divided by 12.F So I thought that was also pretty interesting.

Tess: And that was the person most competent to go to the board and she messed up. Yikes.

Clark: But even if they had said, ah, “$x \div 12$ is equal to 4,” then you also have 4 * 12 is equal to, like, you know. So it made them—. They had to write down all the fact families that belonged to that.

Clark suggested using fact families to help students understand how to solve for the variable (lines 1–5 & 16–17). By suggesting this strategy, he was recommending that Tess build on the students’ prior experiences with fact families. Tess viewed students’ prior knowledge as problematic because they immediately thought of the wrong family to solve the equation (lines 25–28). Nevertheless, Clark thought that implementing the task by building on students’ prior knowledge could aid in the development of students’ understanding of solving equations by writing down all they knew about the fact family 4, 12, 48 (lines 37–39).

Another example of Clark building on students' prior knowledge occurred as students learned to add, subtract, multiply, and divide positive and negative rational numbers. He noticed that students struggled with computations with fractions, mixed numbers, and decimals even though they had begun having experiences with fractions as early as the first grade. During the planning meeting for Cycle 6, Clark recalled one particular problem students struggled on: 12.6 – (3/4). When a student suggested writing 12.6 as a fraction Clark asked the student how he would do that, but the student did not know how to answer. Clark stopped his lesson and reviewed concepts from sixth grade and elementary school in order to help students extend their knowledge and be able to solve problems with negative numbers, a new seventh-grade concept.

Clark: Yeah. What I did with my kids though, I went to my cabinet. I got these little round manipulatives that I had when I was teaching elementary school, and I brought them out, because there are only nine of those kids in that room.

Eileen: Right.

Clark: And we sat around, together, made a little circle, and I started playing with them, and I said, "Okay, um, if I separate a whole into 10 pieces, which one of these represent that?" I had them to go back and find it.

Eileen: Right.

Clark: "Okay, now, if I took one out of that, how would I write the one that I took out of it?" And they tell me, "One-tenth."

Eileen: Okay.

Clark: Okay. "Now, if I was writing that one-tenth as a decimal, how would I write it?" [Student responded,] "Decimal point, one."

Eileen: Right.

Clark: Oh, right, now we [are] getting somewhere. Now we [are] getting somewhere. “Okay, now, I’m going to take two pieces out of this 10. Tell me … what does this represent out of this picture of 10 that I took away?” [Student responded,] “Oh, two-tenths.” I said, “Oh, it is?” [Student responded], “Yeah.” I said, “Okay. Um, is there another way I could say that?” [Student responded,] “Yeah, we could say one-fifth.” [Clark said], “So how do you know that’s one-fifth?” “Well,” he said, “that, that’s ah, that’s ah, that’s ah, that’s ah.” I said, “What is it called?” I kept—. I made him tell me. And he finally came up with the answer. So I said, “Okay now, how can I write this as a decimal?”

Eileen: Well, but, I’m confused now because how is that helping address the issue of a problem like 12.6 – (3/4)?

Clark: I knew, no, I wanted—. First, I wanted them now because I knew they were having a problem converting this decimal into a fraction.

Eileen: So they were having trouble doing this $\left[12.6 = 12\frac{6}{10}\right]$? Or they were having trouble doing this $\left[12\frac{6}{10} = \frac{126}{10}\right]$?

Clark: Yeah! No ma’am. They were having a problem understanding

Eileen: Writing 12 and six-tenths.

Clark: Yeah. They were having a problem that they didn’t understand that that’s $\left[12\frac{6}{10}\right]$ what that [12.6] really was. They did not understand.

In lines 1–8 Clark talked about stopping his lesson to review fraction and decimal concepts. He used his familiarity with students' elementary school experiences to implement the task address prior content using elementary school pedagogy and manipulatives. He reviewed some basic concepts get students thinking about fractions and equivalent fractions, which he wanted to lead to a conversation about changing 12.6 to a mixed number (lines 26–35). Clark felt the need to review this concept because it "bothered" him they did not know how to change decimals to fractions. He tapped into his own experience as an elementary school teacher to "trigger their memory" about their own elementary school experiences.

In addition to Clark's elementary school teaching experience, he frequently referred to experiences within Project ISMAC as influencing his current practice.

Experience in Project ISMAC. Project ISMAC helped Clark change his pedagogy with his seventh-grade students the previous year and was continuing to do so in the current school year. He saw how his students benefitted from chosen tasks and pedagogical moves used by Project ISMAC facilitators and wanted to emulate them. As he executed the same techniques, he experienced success with his students in terms of their engagement in and understanding of mathematics. Thus, Clark chose tasks that he felt would help him implement these same strategies.

During implementation, Clark talked about having students justify and explain their thinking. He said that he was teaching for understanding rather than for procedural knowledge, a theme on which Project ISMAC focused. Part of Clark's interpretation of conceptual understanding was that students should not simply learn rules or procedures, but should be able to apply what they learned to solve problems. In our sixth reflection meeting, he said, "You know, I want them to understand the rule. I don't want them to just know it. I want them to understand it." Clark commented on how higher-order thinking strategies, such as promoting thinking and pressing students to use cognitive and metacognitive strategies (Peterson, 1988), were working with his students. He observed changes in his classroom in student success, participation, learning, and understanding. Even in the first reflection meeting, Clark commented,

> To be honest with you learning from what you all have been teaching me to do this is actually working for them. Because here it is, you take this low level group of students who, um, the average score in there was a 780 on the CRCT. Um, and here they are, they're using the vocabulary because that's what we're talking vocabulary. We're talking a lot all day long and pushing that issue and we're forcing them to do it because they have to answer using that vocabulary. We're not giving them the answer; they have to do it themselves. And I think the best thing that really helped me was when you told me to use that clock. Put a timer on it, give them an opportunity to think about it and do it in pairs where they come up with the answers. They discuss it with each other, and then they're sharing it and then they share it with the rest of the class. It works.

In this episode, Clark explained how he held students accountable for communicating using mathematics (lines 3–6 & 9–10) and thinking for themselves (lines 7 & 9). He was encouraged that even his low-level students were able to engage in mathematical thinking in this way and credited this ability to what he had learned in Project ISMAC (lines 1–2 & lines 7–8).

There were several occasions in which Clark talked about how Project ISMAC showed him that he needed to change his instruction. During the second reflection meeting, he described how he was helping students learn mathematics by promoting thinking and not explaining procedures. This year, Clark wanted his students to come up with ideas on their own. He thought that this approach would lead students to internalize and understand the mathematical concepts on which the tasks focused. He even compared his instruction this year with his experiences from several years before when he taught the eighth grade. He thought that he had made mistakes by focusing on rules rather than conceptual understanding. In his words,

> Because I was trying to get my kids to come up with that [understanding] on their own. Because I found that, and this is where I made my mistakes last year. If you tell them to do stuff, they won't think for themselves. They look for you to guide them through it and you want them to discover that this is the way that you do it and the reason why. Because if they do it they'll remember it.

Clark also mentioned this issue again in the third reflection meeting when he said, "once I stopped doing that, the kids got better. They did. They really got better."

The examples above illustrate how experiences in Project ISMAC the previous year had affected Clark's understanding of teaching and learning mathematics. However, Dorothy White and the doctoral students involved in Project ISMAC continued to influence Clark's teaching during the current school year. White regularly visited College, attended planning meetings, and observed classes. While in classrooms, she frequently interjected and wound up demonstrating pedagogical strategies. During the third cycle, White attended Clark's class and noticed students were not engaged in the problem. She challenged students to think about the problem through her questioning. Clark was excited about how his students reacted and wanted to try this out the challenge with his students himself.

Clark: And what she did is she challenged the kids and then she told them, "Okay, don't say a word. 30 seconds. I want you to think about it for 30 seconds. Just don't say anything." And the kids were quiet for that 30 seconds. And they were actually thinking about it. And I think that if, and here's where my problem is. I think that I haven't done that enough. That maybe if I did that, that would actually help our kids.

Eileen: Well, what is the difference about what she was asking them to think about? Like, you know, why do you think they were thinking about what she was talking about?

Clark: Because what she did was, she made it relevant to what was interesting to them. What we're doing textbook wise seems to be boring to them.

Eileen: But it was the warm-up that Tess provided. It was that candy warm-up.

Clark: Yeah, it was your warm-up. It was the candy warm-up. What she did is she challenged. Some of those kids in that room are very smart. And they shut down when it's something that seems too easy for them. And they start to act out. Whereas some kids in that classroom, it's hard for them.

Tess: Yeah.

Clark: So instead of us, instead of me gearing it down, what I need to do is, is tier it up into both areas. Ao that, okay, the kids that got this can think about it this way while this other group can think about it the other way.

Tess: And they could also be writing a sentence too or something like that.

Clark: Mm hm. And that what she was basically doing. She was tiering it up each time. She would go up some and have some down here. Okay, you think about this. And they were thinking. And I mean, they were *really* thinking about it. And then when they, the smarter kids started getting the problems, started having problems with this one problem, then the kids who were having a hard time and the [inaudible] wanted to do the same stuff.

> So that's what she did. So when she did that, it made the other kids who were the lower level want to reach up and grab some of it too. Because she's, they saw some of the other kids doing it. It's the old competition thing. So, maybe, I need to just, you know, beef it up a little bit.

Clark witnessed his students actively working and thinking about mathematics. He felt that if he could facilitate class as White did, that he could help his students engage in mathematics in the same way (lines 5–6). White challenged students while differentiating her questioning to include different ability levels (lines 25–31). Clark concluded that he would better serve his students if he raised the level of the task rather than reducing it (lines 18–20).

Clark appreciated the university team's interventions in his classroom. Clark thought that the conversations during the cycle meetings as well as his experiences the previous year with the Project ISMAC team were helping him focus his attention on his practice rather than blaming problems on his students. He thought that these experiences were "making [him] a better teacher" and helping him "make things better for the kids." This attitude made Clark open to suggestions and willing to try new strategies, both important components to implementing change in his practice (Silver et al., 2007). Nevertheless, his views on what it meant to do mathematics and his understanding of mathematical concepts sometimes affected how he selected and implemented tasks in opposing ways.

Doing Mathematics

Clark selected and implemented tasks depending on his understanding of what it meant to do and understand mathematics. He believed that doing mathematics included explaining, justifying, and interpreting solutions and ideas. He believed teaching mathematics for understanding involved helping students learn how to think independently, make conjectures, draw conclusions, and communicate their thinking. Clark wanted his students to understand concepts rather than rules. However, in some cases, and with some mathematical concepts, Clark focused on procedural knowledge rather than conceptual understanding.

Clark believed teaching mathematics involved helping students learn how to think independently and to communicate their thinking. He wanted his students to develop their own strategies and gain conceptual understanding. In Cycle 2, Clark continued to talk about using questioning to steer students in the right direction rather than explaining procedures or rules. In the planning meeting, he said, “Let them go ahead and do it their way like she [Eileen] said and then come back afterwards and make them rethink it by questioning, not by telling.” Clark wanted to make sure that he did not provide the students with rules to follow because he wanted them to conceptually understand mathematics. During Cycle 5, when Clark talked about using the number line model for integer addition, he wanted students to understand $|-12| > |8|$ in order to see that when you added these two integers, the answer would be negative.

Clark: They [students] were kind of confused on it. Until you could show it on a number line and if you’re talking about the distance from zero, yes, it is bigger because of its absolute value. We didn’t get into that right away because my concern was…looking at the pattern. I wanted them to understand the pattern, not understand some rule that Mr. Welling was telling them. And as we got to understanding those two squares, those two groups, and we go down into the lesson, then the absolute value will come up. I just wanted them to look at it and see from what—.

Eileen: Right, because you never wound up, in the class that I observed, you never actually wound up using that term.

11 Clark: That’s right. That’s right.

Instead of providing the students with the definition of absolute value, which he thought would be providing the students with a rule (lines 3–6), he worked to help the students observe patterns in the problems to draw their own conclusions.

It was important to Clark that his students not rely on rules or procedures. He thought that mathematics consisted of concepts that students needed to explore to understand. During the fifth reflection meeting, Clark brought up how other teachers in the school focused on rules and how that could be a disadvantage for seventh-grade students when they moved to eighth grade.

1 Clark: That’s right. That’s right. Exactly. And this is what’s happening with our
2 kids right now because (sigh), and I’m not trying to bring this one up, but
3 that’s what’s happening over there [Grade 8]. The way we taught I, we
4 didn’t teach rules so our kids. They [the eighth-grade teachers] say, “They
5 don’t know the equation of a line.” No, they don’t know the name of the
6 equation or they don’t know the rule…
7 Eileen: They’re so wed to rules over there. It’s ridiculous.
8 Clark: They know how to *obtain* that equation or recognize it when they see it
9 because we didn’t teach the rule. We taught the concept.
10 Eileen: Right.
11 Clark: And that’s what we’re doing here and it helps because math is not rules.
12 Tess: Right.
13 Clark: Math is concepts to get where you want to be.

Clark made a distinction between rules and concepts and believed that he and Tess were focusing on concepts (lines 3–4, 8–9, 11). When students brought up rules they heard from other sources, Clark was adamant about steering them away from a dependence on rules to engage them in conceptual learning.

Clark: Today it came up again about this negative times a negative is a positive. It came up again in my classroom, and I said, “Listen. … Do you remember every rule that every teacher’s told you since you’ve been in elementary school?” No. So, we don’t teach rules. So—.

Tess: I just say, “We’re getting to that next week.”

Clark: So, we’re, we, I just tell them, “We don’t teach rules.” I said, “I want to know the concept. You going to tell me the concept. That’s all I want to know. And if you can tell me the concept, then you can do that problem. And I don’t want you to memorize rules because rules change.” I said, “When your mama tell you not to go outside and you go outside, she tear your hinny up, don’t she?” I said, “That one day, but then the next day she tell you to go outside, and you go outside and she might say, ‘That’s okay baby.’ Rules change. But concepts don’t.”

Clark did not want his students trying to use the rule for multiplication of positive and negative numbers. He thought that they would misuse the rule and try to generalize it to other situations. Therefore it was important to Clark that his students understand that they were responsible for talking about concepts since that’s what would help them do mathematics (lines 6–8). He did not want them to memorize rules since rules did not work in all situations (lines 9–13), especially as students extended the number system to include rational numbers.

Clark was able to focus on conceptual understanding when he felt comfortable with or had a strong grasp of a mathematical topic. In such cases, his instruction tended to facilitate higher-order thinking. When Clark and Tess reflected on the implementation of the end of unit task for Variables and Patterns, they discussed students' learning opportunities. Specifically they wanted students to make the connection between patterns of change and mathematical equations or formulas. The frameworks task they were using had students represent the pattern of change between a stack of cups and the stack's height. The students built different stacks, recorded their data in a table, created a graph, and described the change in words and with a formula. During the fourth planning meeting, I encouraged Tess to spend more time on the task to allow students to explore the connection between the pattern and the formula. In the class I observed, Tess provided students with the slope-intercept form of a line and told them to use it for the formula without allowing them to come up with an equation on their own. Clark and I tried to convince Tess that implementing the task in a more open and conceptually based way would help students make connections between representations. Clark implemented the task by connecting it to previous problems and by helping students determine the formula on their own. In his words,

> But see, even in the book when you—in [Problem] 3.1 I believe it was—when you had the go-cart, and it was 20 dollars initial fee, and it was 5 dollars per hour, and, you remember that? And you see, I built on, I used that, you know, when we got to that equation to build on to what we were doing. So that's why my kids were able to look at it and say, "Oh yea, yea, yea! I remember this! It's the same thing." So, and I kept asking them, "Well, how do you know? Can you explain this to me? How do you know that?" [Inaudible] "So it's a half an inch each time. How do you know that? Well, if you put another cup in there and measure it again, you going up half an inch each time. Okay. So what happens when I put it for 20 cups? You going to stack 20 cups up and do it?" No. "Well, what are you going to do? I could add 20 cups. A half, 20 times, or I can do, 20 times one half. Okay. That will work. But now, what if somebody told you, you had *n* cups. In other words they told you, you had the letter *n*."

Clark used his students' prior knowledge from a similar problem to help them understand how to come up with a formula for this situation (lines 1–5). He used questioning to help students develop the formula on their own (lines 6–12). This questioning facilitated higher-order thinking by asking the students to use inductive reasoning to come up with a general formula for the pattern (Facione, Facione, & Giancarlo, 2000).

Clark's desire to build conceptual understanding was sometimes challenged by how he understood mathematical learning and concepts. More specifically, Clark thought that student understanding could be assessed by what they said or could mimic rather than what they could explain, justify, or analyze. Thus, he assessed students' understanding through their ability to mimic his words or actions. For example, in the second planning meeting, Clark talked about making students use the correct mathematical terminology when describing patterns of change. In his words,

> Is that, you're talking about a constant rate of change? If it's a constant rate of change, then what do we have? We have a straight line. So, it's linear. Am I correct? Yes. And making them use the same words so they understand exactly what it means.

Clark thought that the correct use of vocabulary translated into mathematical understanding, and when students used correct terminology but were unable to successfully apply a concept he did not understand. This thinking extended to students' ability to mimic procedures. During the second reflection meeting, he noted that students were able to tell him how to create a scale for a graph, but then consistently created scales using irregular intervals.

> Clark: But the funny thing was, every time, we was talking about it, they could tell you, "Oh yeah, we got to change the scale to be going by fives or tens or twos." They could tell you those things.
>
> Eileen: But then you'd go and look at their paper and—.
>
> Clark: And they wouldn't do it.

During this same cycle, Clark talked about how he could assess his students' understanding of the interquartile range by having them describe what steps they would need to take to compute it. We discussed a warm-up Clark had used to review descriptive statistics, and I asked Clark about his implementation strategy.

Eileen: So, when you were going through the answers [to the warm-up], they [students] seem to have a pretty easy time with the range question, and they struggled a little bit more with the interquartile range question and what I noticed was that the question that you kept asking them was, “What do you do?”, “What do you do?”, “What do you do?” And so, I was—. What made you decide to phrase question that way do you think?

Clark: Because I wanted them to tell me the steps.

Eileen: Okay.

Clark: I wanted them to talk me through it, so I had an understanding of where they were going wrong. What happened to get this wrong answer?

Eileen: Okay.

Clark: Um, if they could talk through steps, then I know, okay, this is where he missed it.

Eileen: Right. And so when they continue to have difficulty describing the steps, what did that tell you about their knowledge?

Clark: That told me that they didn’t really know. They didn’t really know the interquartile range. How do you get the quartiles?

Eileen: Okay, so they didn’t know the definition of the interquartile range? Or they didn’t know how to find it?

Clark: They knew the definition. They didn’t know how to find it.

Clark focused his question on the procedure for finding the interquartile range. He wanted students to tell him the steps they would need to perform so that he could assess their understanding of the concept. He thought if they could not provide the steps, they did not "really know the interquartile range" (lines 9–16). I challenged Clark about his assessment technique (lines 18–19) but Clark assumed the students knew the definition. So if they couldn't find the interquartile range, they didn't understand the concept. During my observation, I had not heard anyone give a precise definition of *interquartile range*. Because of this lack, it was hard for me to tell what the students understood without asking them what they knew about the concept. During our discussion, Clark shared what his students were telling him to do: "You had kids saying, 'I'm subtracting the maximum from the minimum.' That kind of stuff." Because they were able to give him this information, he was confused when they gave him the range instead. In his words,

> And it's [a] funny thing, because they can tell you how to get there, but they, where they have the problem was, "How do I take and get the interquartile range?" Which meant I subtract the lower quartile from the upper quartile. They kept trying to give me the range.

Clark was "shocked" at this performance because the students were able to recite a procedure but were still unsuccessful.

The ability for students to mimic his words or actions was directly affected by how Clark himself understood the mathematical content in the task. The second unit of instruction, Accentuate the Negative, had among its goals to "develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers" and to "write and use related fact families for addition/subtraction and multiplication/division to solve simple equations with missing facts" (Lappan et al., 2006a, p. 2). Furthermore, students were to develop and use two different models for representing addition and subtraction of integers: a number line model and a chip model. When the teachers began the series of tasks that had students working with the chip model to represent addition and subtraction, Clark introduced integer addition with the chip model by having the students explore how to use the chips. However, when he began to talk about integer subtraction, he changed his instructional strategies because he thought that subtraction of integers would be difficult, especially how to apply the chip model to represent this operation. During the fourth cycle, Clark described how he introduced integer subtraction in a previous class.

> They really liked that. They liked getting up to that Smart Board and showing it [adding integers using the chip model]. So we did that and then I said, "Okay, guys, here's something else. We talked about addition, but in one of your problems it said subtract a negative number, or subtraction. How do I represent the chip method in subtraction?" One of my kids said, "You just draw the negative. If it said 'negative 3,' you just draw three negative chips." [Clark responded,] "No, sir. That's not what we do. You can do it that way, but at some point you're gonna get a little confused."

Clark have his students to explore how the representation could be used to model the operation because he thought they would eventually get confused (lines 6–7). Instead of letting students work with the model and determine their own strategies for using chips, Clark provided students with his own method for subtracting integers.

Clark: That's right. So I told them. I said, "Look." I gave an example. I said, "If I have five minus two." I did this. I did two things. First I started off with five minus two and I said, "You know, … whenever I'm doing subtraction, I'm thinking any number after the subtraction sign is a pair, what we call a zero pair, a set of zero pairs. A negative and a positive. So, let's illustrate that, and let's see." Started out with five pluses. Kids told me there was five pluses. And I said, "Because we got two over here, we're gonna set up two sets of pairs—one positive, one negative." And I said, "Now, what does subtraction tell me?"

Tess: That's zero.

Clark: Hold on. Hold on.

Tess: Okay.

Clark: But what does the subtraction tell me? Subtract the two what? The two positives. Now, I've gotten rid of my two positives. What do I—?

Tess: No.

Clark: Hold on. Hear me out.

Tess: But what you had up there wasn't two. It was zero. That's what I was saying. Before you subtract—.

Clark: Hear me out Ms. Freeman. Hear me out.

Tess: Are you showing them? Okay.

Clark: So, now, I've gotten rid of my two positives. So what do I do next? I create my pairs. I'm getting rid of my pairs. And what's my answer? I got three positives left.

By giving students his method, he expected them to do the same steps every time in order to get the correct answer. These steps are illustrated in Table 3 with *p* representing +1 and *n* representing $^{-}1$.

Table 3 *Clark's Explanation of Integer Subtraction Using the Chip Method*

Clark's explanation (line numbers)	**Chip representation**
Start out with five positives (6)	*p p p p p*
Set up two sets of zero pairs (7)	*p p p p p* *n n* *p p*
Get rid of two positives (14)	*p p p p p* *n n* ~~*p p*~~
Get rid of my pairs (22)	*p p p* ~~*p p*~~ ~~*n n*~~

The mathematical goals of the investigation were to help students develop and use the chip model to represent integer addition and subtraction. Students were to learn how to flexibly rename integers using different combinations of positive and negative chips, which would only be necessary only in certain situations. This renaming was needed when the subtrahend could not be taken away from the minuend. Thus, by renaming integers, one could represent subtraction by combining opposites to make zero; that is, use zero pairs. For example,

$$^{-}4 - 2 = (^{-}6 + 2) - 2 = {}^{-}6 + (2 - 2) = {}^{-}6 + 0 = {}^{-}6.$$

Clark's implementation hindered higher-order thinking by leading his students to explicit steps to perform without relating them to conceptual understanding of the model (Stein et al., 2000). Clark defended his method and talked about how it enabled his students to successfully subtract integers. He thought the concept and model were confusing, so his students needed to be given a method or procedure they could use for every problem. His students were able to mimic this procedure successfully, and Clark believed that this mimicry reflected their understanding of the concept. In his words,

> And to be honest with you guys, like I said, in my classes—. Today I started this in my classes and my kids, they like it, they understand, they're able to get up to the Smart Board and explain it to me, and it's working for them because they know, that, "Okay, I can do it this way. I can see what I need to take off and what do I need to pair up."

Part of the reason for Clark's decision to implement the subtraction of integers using a chip model in this way had to do with his understanding of the concept and use of the model. In the beginning of the Cycle 5, I brought up Clark's method to ask him what his goals were for teaching the concept in this way.

Eileen: But does it [Clark's method] get at what you want them to walk away with from [Problem] 1.4? And if you, and if what you want them to walk away with in 1.4 is building an intuitive understanding about these integers, I don't know if this helps. This'll help later in section 2.2 when they're starting to have to develop algorithms for how to subtract, but I don't know if it helps here.

Clark: Hmm. … Never thought about it like that. But most kids gonna get confused when they see this part here because they're gonna say, "Well, I got a negative eight here and if I take all these away then I have nothing left." At which point we're talking about zero pairs. That's why it to me it seemed better to come here to explain what zero pairs are and how it would help.

As I connected the purpose of the current task to future tasks, I challenged Clark's ideas about the use of the representation (line 7). He still believed his students would struggle, which was why his method would be useful (lines 7–12). Furthermore, Clark said that he provided his students with the method so they could do the computations accurately and use the information to explore patterns and make conjectures. In his words,

> But I'm just looking at it … some consistency because there are gonna be some cases where you gotta use the zero pairs. And what I was trying to do was eliminate the guess work and saying, "Okay, if I did this, this way every time, and it does work every time, then I can see my patterns and then I can generalize what is really going on here."

Clark's interpretation and understanding of the model and mathematical goals of the task influenced his implementation. In the fifth planning meeting, Clark talked about his experience the previous year with Dorothy White that led him to develop the method for this year.

Clark: Yeah, because see remember last year when we taught this we kept talking about doing the opposite? Remember that? And I remember, and I guess I'm so stuck on the, the pairs, the zero pairs because I learned that from Dr. White and it's, it's like, "This is easy. We can do it this way!" You know?

And that was the biggest difference. Because the kids were confused last year because we [the teachers] kept saying, “Adding the opposite. Adding the opposite.” It was always adding the opposite.

Tess: Yeah, I didn’t do that. I remember you sent me a link on that and that doesn’t make sense in my mind. I’m not smart enough to get that. But I see it on the Web.

Clark: Yeah. There’s a lot. Add the opposite. Add the opposite.

Tess: There’s a lot of stuff. The inverse.

Clark: Mm hm. Inverse, and that kind of a thing. And that got the kids confused. And I remember that specifically last year because I remember Dr. White coming to me and saying, “Let me show you another way to do this.” And she said, “Okay, think about the zero pairs.” And that’s how she did it down there in the room. And she said, “When you look at a problem,” and she put a problem, I believe it was something 3 – 1 or 2 – 1, and she asked you, “What does this really say? It says subtract a positive one. So subtract one positive number.” So we’re looking at the chips set. It’s saying take a positive number from there. Take a positive sign from it. So if I was to put—.

Tess: Take two positives and you take one away.

Clark: Mm hm. Exactly. Or if I’m looking at zero pairs and it says take away a positive, that means okay, now if I got 2 – 1 and I got the zero pairs there I’ll take one positive away. All right, I got a negative, and I still got two

> positives over here. I take them off, and I got one left. And that's, that stuck with me because, man, this is an easier way to teach it.

White's description of zero pairs affected Clark's understanding of the model. He interpreted White's explanation to mean that he should always make zero pairs for subtraction problems. This interpretation changed the problem from subtraction to adding the opposite. Thus, as his own mathematical knowledge developed, his worked to match his instruction with his emerging understandings.

Clark's instruction for writing equations was different than his instruction for integer operations. In the former, he focused on conceptual understanding and on connecting and extending new and prior knowledge to make predictions and create new ideas. In the latter, he focused on providing a procedure for students to follow in order to avoid confusion. It seemed that this difference was connected to Clark's mathematical understandings.

Summary

Clark selected tasks based on his understanding of the mathematical needs of his students. When the pacing called for him to spend time on a concept, he either followed or changed the pacing to make sure that he was serving his students. In particular, he wanted to make sure that they understood the material before he moved on, or that they would have more time with important or difficult concepts. He also selected tasks to engage his students in mathematics. He wanted to build their confidence and be able to address previous concepts in order for students to work on current seventh-grade mathematics.

To implement tasks, Clark considered how he could engage his students, reflected on his teaching experiences, and thought about what it meant to do mathematics. Clark tried to mimic ideas he had seen in Project ISMAC. For example, he started to challenge his students while introducing tasks and through his questioning based on how well his students responded to challenges from Dorothy White. He also focused a great deal on his questioning techniques. He thought about what had worked with his elementary students, but also tried to react to his students' contributions during class.

Story of Tess

Tess's room had approximately 25 individual student desks and one large table in the back of the room. The desks were placed in rows facing the front of the room. The rows were created so that pairs of desks were connected and Tess encouraged students to work individually or in pairs during the lesson. Tess's fourth block collaborative class met at the same time every day. This class included 28 students, of which 16 were boys. The special educator present during the period, Ms. Lane, was there to serve three students. However, she walked around to help any student as needed, and periodically commented on the lesson. In the middle of the period this class had a bathroom break, which lasted about 10 minutes. The seventh-grade teachers had decided to institute this break during fourth period to cut down on hallway traffic throughout the day. The seventh grade had been experiencing discipline problems and it was thought that this decision would help teachers better manage the hallway and reduce the number of students leaving class during instruction.

Tess's typical lesson was much like Clark's. Lessons began with a warm-up, followed by a launch and a work session. Tess's work sessions were frequently teacher-directed class discussions interspersed with individual or pair work done by students. In the CMP2 textbook design, teachers were supposed to pull the class together and help students "explicitly describe the mathematics of the Problem, ideas, patterns, relationships, and strategies they found and used" (Connected Mathematics Project, 2009) in a summary at the end of the period. Tess planned to present summaries but usually did not because she frequently allowed students to work until the end of the class.

Pacing and Assessment

Tess worked to abide by the district-pacing guide or the pacing she, Clark, and Betty agreed upon. She was especially careful about addressing concepts that would appear on quarterly state exams during class and implemented lessons with this as her goal. Tess felt that her students needed to be exposed to every concept before they were tested, even if that meant she could only dedicate a short amount of time to a concept before an assessment.

During Cycle 3, Clark informed Tess during the Tuesday planning meeting that the quarterly assessment would be given the following Monday. Tess had planned to give a mid-unit assessment on Friday, but after hearing of the quarterly assessment wanted to alter this plan in order to cover concepts that would appear on the test.

Tess: No, no, no, but that definitely depends. Friday I was going to give a test but instead I'll do distributive property probably.

Clark: No, that's right. And I've been doing that a little bit and I've been doing it a lot in ELT.

Tess: Me, too.

Clark: But we haven't done it in the classroom as much because we've been getting through this. I've been putting it in my warm-ups. But you know, we do what we can.

Eileen: Yeah.

Tess: Oh, what's the other thing that needs to get done here?

11 Clark: And we just build on it for each year. But Benchmark is Monday. I got told
12 yesterday afternoon.
13 Eileen: That's crazy.
14 Clark: In the leadership meeting they told us. So, you know what the deal is.
15 Tess: Okay, so, I'm sorry, so what else are we missing then? Those questions,
16 analyze, subtract. What does "add and subtract linear expressions," mean?
17 Does it mean, so if I have—.
18 Eileen: Sounds like combining like terms.
19 Tess: $a+3+4a+2$ then I'm just—.
20 Clark: Like terms. And understand this is a Benchmark [quarterly assessment].
21 We're going to see more of this and they'll get more questions. Just like it
22 did last year when we did the data and we had missed certain things and
23 they go along and got better.
24 Tess: So, I hit that a little bit. Okay. It's going to be distributive.

In lines 1–2 Tess talked about covering the distributive property instead of administering a unit quiz. She was also worried about what else would be on the test (lines 10 & 15) and wanted to make sure that she went over these concepts with her students, even if she just "hit that a little bit" (line 24). Tess reiterated this intention in the reflection meeting when Betty came in to clarify the pacing and find out where the teachers were. Tess said, "I'm just saying I want to get the properties in before Benchmark."

Tess frequently felt pressed for time during her lessons. As a result, Tess sometimes provided students with answers or procedures in order for them to complete a task. During the sixth cycle, Tess described her reasoning behind her implementation decisions before a unit quiz.

She planned a quiz over adding, subtracting, multiplying, and dividing positive and negative rational numbers for the second half of class. In the first half of class, Tess had students work through groups of four or five problems subtracting integers. The focus on the lesson was on subtracting negative numbers and the students struggled. During my observation I noticed some students trying to use the rule that two negatives make a positive. Others did not see the difference between $5 - 3$ and $5 - {}^{-}3$. Tess showed students how to do the problems using the number line only since she didn't have enough time to talk about using the chip model. In the 30 minutes before the students had their bathroom break, they had worked on 17 problems. During the reflection meeting we discussed how much time she was giving the students to think about the problems and if she could have postponed the assessment.

Tess: I did [feel pressed for time]. And then the stupid bathroom break, and everything else there.

Eileen: I kind of felt that. Um, so I wonder if doing less examples—.

Tess: Maybe.

Eileen: Would that have been able to give them more time and then gotten it across anyway?

Tess: We took so long. There was so much misunderstanding and I didn't feel like we could leave that before giving a quiz, so I felt just stuck.

Eileen: Right. Are you flexible enough where you could think, like, okay, so I don't think they're ready. I'm going to give them the quiz on Monday morning instead. You know, would that have been an option? Or

Tess: Not at this point because I've already not done the graph and…No, I mean, that's why I'm saying, I wish I had

Clark: You could bring the graph into warm-up.

Eileen: More time.

Clark: You could bring the graph into warm-up. You can do the graph in the warm-up and through a week time, and do pieces instead of time and then pull it all together.

Tess: But we're very tight on this. What we have yet to do (inaudible–Eileen talking).

Tess did not know what to do because she felt pressed for time, but did not want her students to have misunderstandings about the operations (lines 1–2 and 7–8). She did not feel that she could skip the quiz because she had already altered the pacing and skipped a problem (lines 9–13). If she did not do the scheduled assessment, this would cause her to fall further behind. This was a problem since they were already "very tight on this" (line 19), meaning that the teachers didn't have much time to cover all of the concepts that would appear on the next quarterly assessment. When Clark urged Tess to be more flexible and to not let the pacing get in the way of her "having to teach," she explained that the assessment provided her with useful information about her students' understanding of the material since she could not informally assess such a large number of students during class.

In the class I had observed for this lesson, Tess had two problems involving fractions on the board for students to complete while they had their bathroom break: $2+\left(-\frac{1}{4}\right)$ and $\left(-\frac{1}{4}\right)+\left(-\frac{3}{2}\right)$. When the students returned, Tess quickly showed them how perform the operations using the number line model. Tess explained why she felt showing the students how to do the problems without discussing them first was appropriate.

Tess: We had done that before. There was a problem in CMP2 where it was one and a quarter. So I had done that last week, not a long, long time ago. I just wanted to have it up there on the board so they could see that we were dividing into four parts.

Eileen: What um, how do you think, how do you think it would have changed the information you got from the assessment if you had not done that? Like, what if you had just not gone over that?

Tess: I don’t know.

Eileen: Do you think anyone would have been successful on the problem?

Tess: Oh yeah.

Eileen: Yeah?

Tess: Definitely. Because in my other classes particularly, we went through that problem. They did it. They wrote it, they counted, they gave me, you know, examples and stuff. Some of them immediately went into, like, what he [Clark] was saying before, um, decimals and they changed it to .25, 0.25, whatever, blah, blah, blah, and then tried to make other kinds of addition and subtraction sentences others went. Somebody made 7/4. Somebody make1 3/4. So the other classes we did–without the bathroom break–we had time to expand on that.

Eileen: I hate that bathroom break.

Tess: And without such, so long on the warm-up.

Eileen: Right.

Tess: We had time to do that properly. So, in my mind it wouldn't have been fair to not put anything up there.

Eileen: Right, because everybody else had—.

Tess: Comparably. Not that that necessarily helped anybody except for the idea that I can represent this. I have *a* way.

Tess illustrated how to perform the operations and left the answer on the board so the students would have a way to solve the problem. She had covered this earlier but thought that too much time had passed for students to remember (lines 1–4). In lines 23–24 she explained that she was being fair to this class since the other classes had been able to discuss different methods for solving the problems (lines 12–19).

In Cycle 6, Tess chose to shorten an assessment in order to do integer addition and subtraction computations in class that the students would feel good about.

Tess: So, um, and my assessment's going to be short like that too, but I think, um, I think the other thing dynamics in my class is that…for the first time I'm seeing some sense of competency with some kids that haven't been competent all year.

Clark: Mm hm. Mm hm. Yep. Have you noticed that?

Tess: And they're excited about it.

Tess wanted to continue with the current topic because she saw her students' excitement as a way to increase their confidence. She did not experience this excitement and confidence frequently in her room and said, "That's why I don't want to give up quite yet on this; they want to feel competent at learning that kind of thing, and um, they don't always get a whole lot of chances for that."

Tess felt that she had to teach her students everything they would see on the quarterly assessments. At the same time, she wanted her students to have fun in class and to build their confidence. As a result, she sometimes spent more time on topics than was allotted for in the pacing guides and then had to teach several topics right before assessments. Tess considered how her students would react to problems when she selected tasks and planned for instruction and focused on building students' confidence because she felt that was one way in which could engage them in mathematics learning.

Engagement of Students

Tess engaged her students by selecting and implementing tasks that would build their confidence, which she believed was a necessary ingredient for their ability to figure out mathematical problems. In order to facilitate confidence, Tess selected tasks that would be fun for her students and implemented tasks in a way that would limit the amount of struggle or frustration they would encounter. Moreover, Tess strove to help her students learn how to apply their knowledge to solve problems and worked to connect mathematics to the world in a meaningful way. She believed by putting mathematics in a context, her students would better understand mathematical concepts.

However, Tess believed her students needed close monitoring in order to stay focused, especially if a task was initially difficult for them. In her experience, students did not take the initiative to think about problems, but instead waited for her to help them. In the third reflection meeting, I asked the teachers to think about how they could challenge their students' thinking without actually telling them the answer.

Tess: I don’t have that kind of intention going on in my classroom. I don’t. I could—. My problem, why I jump in sometimes too quickly, is because if there’s too much down time—.

Clark: Yep.

Tess: There’s not enough focus that they’re working on it. Some of them are sitting there like this. And others are starting to think about what’s happening at four-thirty.

Tess felt that she needed to provide additional guidance for her students. She limited the amount of time they worked individually or with other students because if students struggled too much, they would disengage from the learning process and their minds would start to wander (lines 6–7).

Tess saw her students as having difficulty with concepts and felt that students would not be able to focus or take the initiative to solve problems. Therefore, her instruction frequently became teacher-centered meaning she did most of the talking, thinking, and explaining. In Cycle 6, Tess talked about how she thought she needed to be more explicit in her instruction because her students were struggling. In this case, she knew her students were continuing to have difficulties subtracting negative numbers. She had them look at patterns, utilize the models, and try to develop algorithms but they still were not consistently working through problems with success. In her words,

> But that’s because I feel it, my kids have been confused. I’ve done the group things, looked for patterns, and then it’s been very confusing. It’s been better for me to try to be more elemental about this number, and then they really have understood subtracting a negative from looking at using the strategy. Not so much from the patterns or anything like that. Patterns are about rules really, with these things.

Because of these issues, Tess wanted to be “more elemental” and have them start using the “strategy,” which meant she wanted them to start using rules with the number line model.

Earlier in this same meeting, Tess and Clark talked about implementing a problem in which students would compute several integer addition and subtraction expressions. The launch would give students two real life situations to talk about which model, number line or chip, would be more useful to employ while solving. At this point, the students had a lot of experience using both models, and this problem was to help move them to more abstract understandings of the operations.

Eileen: Right. And that’s really the real point of the launch is for them to start thinking about, you know, “Which one’s [method is] better to use? Which one can I use here or there?” And then—.

Tess: And then to do this one and me just quickly show you could do positive eight plus two using the chips or you could do it on the number line. It would already be up there. Now, on this side of the paper do this whole group whichever you want, and with the (inaudible–Clark talking).

Clark: So, why should we have to tell them you could do it on a number line or with the chips?

Eileen: Because that’s in the launch.

Clark: But remember—.

Tess: It says, “Use chip models or number line.”

Clark: Right, but remember back in Investigation 1 where we explored the number lines, vertical and horizontal number lines, and the chip set and how we solved our problems using those? Okay. In this particular investigation you’re going to use those methods to solve problems in the two groups. Uh, you know.

Eileen: And to try to find patterns so that we can develop an algorithm. I mean, I guess, I guess.

Tess: It's where the mental resources are.

Eileen: Right.

Tess: If they spend a long time trying to figure out what they're supposed to do on the paper instead of using them to solve all those problems and see what they're doing, then I've lost the focus of what the point is. That's the thing.

Tess thought that she would need to not only go through the launch, but also evaluate one of the expressions with her students with both models (line 4–7). In lines 8–17, Clark challenged this decision based on the students' prior experience in Investigation 1. Tess defended her idea in lines 22–24 by expressing her concern that students would spend too much time trying to figure out which method to use and would get off track. The task required the students to think for themselves, something Tess was not confident they would do, and she did not want them to struggle.

Confidence. Because Tess had doubts about the students' ability and motivation to do mathematics, she decided to try to engage her students in mathematical thinking by building their confidence. One way Tess tried to build students' confidence was to select tasks that would be fun for her students. For the lesson in Cycle 2, Tess selected a warm-up from the previous year because she "had a good time" with the task. The activity involved students creating sentences for graphs and the students did "all kinds of different things. It was very fun." In Cycle 3 as she reviewed assessment materials for the first instructional unit she said, "These were kind of good. Did you see those when you looked through there? Or these could be warm-ups. They could be kind of fun." Later in the sixth cycle during the second instructional unit, she talked about activities that would “build that sense of math is fun and I can be good at this.” Tess wanted her students to enjoy mathematics and to build their confidence so they would engage in the material.

During Cycle 2, Tess described a warm-up she had selected that had her students practicing solving one-step equations and evaluating expressions. She thought would help build students’ confidence and assumed that it would be fun for them. However, during implementation she was unable to provide enough time for students to grapple with the problems. As it turned out, students took more time to complete the problems and made some mistakes so Tess had to rush through at the end in order to start her lesson.

Tess: Um, so that’s going to be a lot. And I thought it would be easy and fun for them to do independently while I was doing the role, and it wasn’t so fun. But it wasn’t not easy. I mean they weren’t afraid to dig in there, so.

Eileen: Um, do you think they had enough time? She [Tess] had 4 problems. One was $3x + 6 = 12$. The other one was $x/12 = 4$. The other one was $x + 1 = 10$. And then the last one was $6m(4) = 48$. So those were the four problems that were up on the board, um, and, you wound up taking about ten minutes on that [$x/12 = 4$]? So, and at the end you wound up having to just, you know, you couldn’t have them talk about the problems.

Tess: One, yes, and people had made some mistakes. I wanted to rebuild confidence.

Tess thought the problems would be easy and fun (line 1) so they would build confidence. However, since she spend so long on one of the problems, $x/12 = 4$, she was not able to allow students to work it out themselves (lines 7–9). Tess was concerned that two of her "better" students had calculated $x = 3$ and wound up spending a great deal of time reminding the students that the equals sign meant that both sides of the equation were balanced. She drew a scale on the board and showed the students how if $x = 3$, the sides would not be equal, and then talked about how they should know the answer needed to be larger than 12 since 12/12 = 1. After this discussion, which lasted most of the ten-minutes allotted for warm-up, Tess had to begin the next part of the lesson and only had enough time to tell students the answers to the last two problems. She thought that she needed to spend so much time on the one problem and at least give the students the answers to the remaining ones so that she could "rebuild confidence" (lines 10–11) by helping them get the correct answer.

In Cycle 5, while the teachers discussed an upcoming assessment, Tess expressed concern over the students' understanding of the material. She wanted to make sure that the problems she chose for the quiz increased their confidence because currently she saw "growing confidence and understanding" and wanted that to continue. In Cycle 6, Tess was excited because some of her lower-achieving students understood how to use the chip model for integer addition and subtraction better than the students who normally grasped concepts.

Tess: Some of the kids who've not got anything are, they'll get this.
Clark: Are really starting to come together.
Tess: And like some of my smarter kids that are, you know, naturally thinking at a higher level are getting confused.
Eileen: Right.
Clark: Something wrong. Something wrong.
Tess: So then I've got like [Student A], you know, I got it, and he can work those chips. And then I've got [Student B], so smart, and she's like, ah. And I

gave her some extra steps to take home to help her try to clarify. Um, she's just over thinking some stuff.

Eileen: Yeah.

Tess: Um, so, and she wants those rules, and he's, he likes those chips, but he can, he can work those chips. 20, 25's. Maybe not fractions, in all cases, and that's why I don't want to go there yet.

Eileen: Right.

Tess: I want to build that sense of math is fun and I can be good at this. And I know that this is hard because I've never learned it before. You know, this is a 7th-grade thing.

Eileen: Yeah.

Tess: And I'm good at this. I'm getting this.

Eileen: Yeah.

Tess: And I need that in my class.

Tess saw students doing well with integer operations, and she wanted to continue talking about it before moving on to using rational numbers, which was where students started to struggle again (lines 13–14). She wanted to continue to show students that math could be fun and they could succeed (lines 16–18) and build their confidence, which was something she thought she needed in her class (line 22).

Context. Another way Tess worked to build student confidence and help students engage in mathematics was to embed problems in real-world contexts. She chose tasks to connect to her students and to make them enjoyable. Tess valued making connections with "real-life" and wanted to make mathematical concepts concrete. She worked hard to put everything into contexts to deepen student understanding and focused on connections between representations within mathematics to facilitate higher-order thinking (Jones, 2004; Ladson-Billings, 1994). Tess would do much of this work after school hours or at home. Her commitment to her students' learning and desire to engage them in mathematics was one of Tess's biggest strengths as a teacher.

During the second cycle, the students were going to work through a problem in which they would have to decide on the independent and dependent variables in a situation. The textbook encouraged teachers to allow students to make their own decision on which variable should go on which axis, knowing that the situation was challenging enough that many students would most likely choose the incorrect variable for the *x*-axis. This choice would lead to rich discussions about variables and how choosing variables in certain ways made it easier or more difficult to make sense of data.

Eileen: So how are you going to present it though? So, because I think the launch in this case is going to be important, because if the idea is, what I'm hearing from you guys is that, it's, you know, it is again practicing making a graph from a table again, doing that, that's the same thing, but the deeper issue is, having them be able to … decide on the variables.

Tess: I was thinking about maybe tonight, looking for some very clear examples of independent and dependent variables, graphs. And talking about other experiments that had been done or something, like three or something. Just to show how, you know, how that made sense.

Eileen: So in your launch you'll reiterate the independent and dependent.

Tess: Yeah.

Eileen: Mm hm.

Tess: And it would be nice to have an example of when you, you know, if I put up there an example of, well what would happen if you did put the dependent variable on the *y*- on the *x*-axis? Um, you know, what would it look like? When would it be wrong? You know, like make a clear example of when it goes awry.

Tess planned on opening the task by giving the students some clear examples of independent and dependent variables (lines 6–9) and possibly even illustrating a situation where the independent and dependent variables were on the incorrect axes (lines 13–17). In this way she said she could “get them in the mode” of thinking about the problem. While connecting mathematics to the world and other disciplines in a meaningful way facilitated higher-order thinking, this particular situation could have hindered students’ higher-order thinking by removing the problematic aspect of the task (Stein et al., 2000). That is, since the problems’ intent was to have students grapple over the definition of variables, if Tess launched the problem by bringing this issue up and showing the students what could go wrong, she could possibly steal their “aha” moment.

Another example of how Tess focused on real-life contexts to introduce tasks occurred during the first instructional unit. Students were learning how to represent verbal phrases using algebraic symbols as well as simplifying algebraic expressions. Tess thought that this two-step process was challenging for students, especially the idea that a variable x was equivalent to $1 * x$. Tess created a real-life scenario to help clarify this idea for her students and explained her approach in the third planning meeting. In her words,

> Then I go into this one where you have to go to the vet. So, I’ve got three cats, two dogs, and a bird. So then we could build an equation, who’s cheap, where’s cheaper to go? And

> again, saying, “Well, somebody has one less dog then minus *d*. Three *d*’s minus d is two *d*’s. Again, trying to make it concrete the idea of variables and using variables to solve problems like which clinic to go to, and then using equations, like you said, as abbreviations of longer mathematical sentences, like more English type sentences.

By embedding the concept in this context, Tess thought that she could engage her students in learning the meaning of variable coefficients and help them remember the mathematical conventions.

When Tess was unable to find suitable contexts in the textbook, she either looked in alternate resources or created her own tasks. During the second instructional unit, the teachers addressed operations on integers, and students had difficulty understanding how to subtract negative numbers. Tess found different contexts for this concept and talked about the ways to connect the concept to real life phenomena.

Tess: So, negative. Sandbags are negative. They pull the balloon down. The balloon starts at zero, let’s say. And helium is positives. It sends the balloon up. So, if I put more helium, if I add, if I put more helium in, the balloon goes up. If I add more sandbags, more negatives, it goes down. If I take away a negative, if I take off a sandbag, what does the balloon do?

Eileen: It’s gonna go up.

Tess: It goes up. That’s a very strong intuitive image to build for the kids. I saw it, because I mentioned it yesterday in all my classes. And today when we had a similar thing like that, I reviewed that before we went into the investigation. The sandbags, I mean, they all said, “Oh, subtract a negative, it’s going up.”

Eileen: That’s interesting. I never heard of that.

Tess: It makes sense. Last year I did this little thing with if you take away a bad thing you feel happy. Like, if you take away a bee sting, you feel positive. So, taking away bad things makes you positive. But this was way better. I could see the gears moving.

Both the sandbag and happy/sad examples provided students a way to remember what would happen when they had to subtract a negative number. Tess liked that these examples provided students with strong images (line 7) because the students were then able to use the ideas to successfully perform computations (lines 10–11 & lines 15–16).

Tess worked to help her students develop understanding of mathematical concepts by giving them additional guidance, providing procedures, building their confidence, and relating mathematics to real-life contexts. She thought that her methods were appropriate based on her assessment of students' abilities and the types of activities that would motivate them to learn.

Teaching Experiences

Tess's teaching experiences over the past 3 years influenced how she selected and implemented tasks. She had experienced difficulties with classroom management and discipline as a middle school teacher and as a result was hesitant to attempt new pedagogical strategies. She experienced difficulties when she tried new techniques, which reinforced her trepidation. Therefore, she used strategies that had worked for her in the past or said that she wanted to see something working before she attempted it herself. However, when she experienced success using a particular task or pedagogical strategy, she was willing to repeat it.

Tess's classroom management difficulties caused her to question the effectiveness of student-centered learning. For example, she was hesitant to use group work in her classroom to allow students to grapple with problems. She thought that her students, especially the boys, would not work well together and that she would have trouble managing groups. Even when Clark spoke about his own personal success with this strategy, she was not convinced.

Tess: Well, that's kind of like, the personalities not necessarily group (inaudible–announcement). They're not always good group personalities. So the kids that I have, like in my group work, that know the answers. And I'm thinking like [Student A] and [Student B]. The other boys resent that. So, they would be in a group, but they're the only ones that—.

Clark: But they'll have more of an appreciation for them though. If they're in a group with them, and they don't understand something, and they have to ask these guys, and these guys are helping them, they'll start to build a more appreciation—.

Tess: Maybe.

Clark: Of those kids because I had that last year.

Tess: Okay.

Clark: And it worked. You know. The kids started to appreciate these kids more because of what they could do, and then they started learning from them.

Tess: Okay.

Clark: And it's a good way to mimic good behaviors too.

Eileen: But it's also, I mean, it is also a challenge to manage, you know, 24 students in a room that are in groups of four. So you have—.

Clark: True.

Tess: They're not good behaviors. These are boys.

Clark: And that's a problem. Yeah.

22 Tess: That's why the kids don't like them. They're boys and they're arrogant.

23 Clark: Yeah.

24 Eileen: Maybe they could be in a group with each other.

25 Clark: Don't do that.

26 Tess: They'd be fighting in a second.

In Tess's experience, the personalities of her students would hinder their ability to successfully work together. She thought the more competent boys in the class were resented (lines 2–4) and if she put them together, they would fight (line 23–25). Clark said that he had a similar experience the previous year, and he experienced success with groups because students appreciated learning from each other (lines 6–13). Nevertheless, Tess needed someone to demonstrate that groups could be used with her students before she would try the strategy. In the fourth reflection meeting she specifically asked for someone to demonstrate how groups could work with her students. In her words,

> Maybe this is something that you [Eileen] or Dr. White or somebody could show us; them [groups] effectively working. My classes of 27, with more than half of the people who failed everything, working effectively in groups. Because it's obviously not something that I believe or have seen germinated effectively.

Tess thought that in addition to personalities and behavior, her students' ability level affected their capacity to respond to particular pedagogical strategies such as group work. In the seventh reflection meeting Clark talked about utilizing flexible grouping with his students. She said,

> I'd like to see what *these* kids with a kind of fragmented skill bag that they bring, or whatever. I'd like to see it [flexible grouping] work because I just—. I mean, I'm game, but there are so many times whenever I do anything that nobody has enough knowledge to move forward.

Tess thought that her students' lack of skills would hinder them from being able to engage in mathematical problems independently. In her experience, her students did not have enough knowledge to begin working on problems.

One strategy Tess used to persuade students to participate in class was to encourage them to try anything and worry about correctness later. She asked students to do or write anything, even if it was not mathematically correct, so she could see them busily working. During the third reflection meeting she explained,

> I've been explicit that I don't care if it's the right answer. I just want to see something written down, and then test it. Maybe you can find out if it's the right answer or not. Otherwise, eventually, we'll solve it, so you know, it's worth working on it. And then I go around and it gives me a chance to talk to some kids about, "What are you thinking here?"

In her experience, this practice of asking students to stay busy by writing something down for her to check gave Tess time to talk to students, assess their thinking, and give individual help to students. It did, however, hindered higher-order thinking because by accepting unclear or incorrect student explanations, Tess was not holding her students to high standards (Smith et al., 2000).

Tess thought that she needed to choose tasks that would clearly explain mathematical concepts. For example, when the teachers reviewed a task in the second planning where students would have to determine independent and dependent variables, Tess chose a warm-up that would provide students with "some very clear examples of independent and dependent variables" and graphs to show students ahead of time how to choose the variables. She asked students to determine the variables in several situations such as "Does the age of a mouse effect how long it takes to run a maze?" She asked them to fill in the blanks of the statement "______ depends on ______" to further help them identify which variable was independent and which was dependent. She did this activity because she thought it would help students access their prior knowledge.

To implement tasks, Tess thought that she needed to clearly explain concepts and provide procedures. If she did not, her students would wait for her to give them the answers, get off task, or waste time with incorrect thinking. Tess thought that if she promoted independent thinking by, for example, using a timer to encourage students to work alone, many students would still wait for her to explain how to do the problem. In the first reflection meeting, Tess talked about how during the lesson her students were either not working on the task or did not know how to begin. She wished she had used the timer but at the same time was not sure that the timer would have persuaded her students to complete the task. She thought that many students would still have "waited for it to be explained on the board." While students waited, they sometimes became disruptive, and she struggled to keep them engaged in the task.

Tess also thought that her students would have problems understanding task instructions. In the fourth reflection meeting, Tess said,

> Being able to keep several instructions in their mind at the same time and follow through seems to be a weakness for many of them. And then when they can't do it and you have more than two tables that can't do it then they just start talking, especially when they're more than two people.

Once the students realized they could not do the problem, they would "just start talking" especially if they were in groups. If she introduced a problem and had the students work alone or with each other, she was worried that if they did start to work, they would go down the wrong path and waste time. In the first planning meeting she talked about not wanting students "to get started and work away" because she would then "come over there and it's like they've made a circle." However, when Tess provided students with procedures or additional guidance, she witnessed how they could then complete problems. For example, Tess provided one particular student with a method for remembering how to determine when integer products would be positive or negative. Once he had this method, he was able to solve problems, whereas before he had experienced problems.

Tess attempted to facilitate higher-order thinking in her classroom. For example, she tried to push students to explain and justify their thinking (Facione, 2009), and explore concepts (Lobato et al., 2005). But sometimes her efforts would fail and she would be discouraged.

Tess: Because I mean I just flip flop. One day I'll be trying to, you know, build an environment where there is more kind of exploration, and then the next day I'm like, oh, you know, they failed miserably. So I'm going to tell them every step of the way and—.

Eileen: Well it's not really an environment for explanation, it's an environment, it's just a way for us to be explicit and so—.

Tess: No, I know what you're saying. Still. One lends itself, you know, to the degree that we um … are, sometimes it just takes so long to really clearly state everything, you end up telling them what to do kind of thing.

Eileen: Yeah.

Tess: It's being careful not to do that.

Tess tried to create an environment in which students explored mathematical concepts independently. When her students were unsuccessful, she thought about telling students how to do the task (lines 1–4), which would hinder higher-order thinking (Stein et al., 2000). But Tess said she wanted to avoid this approach (lines 9–11). Other times, Tess's efforts were successful and she witnessed how her students could think independently and communicate their reasoning. For example, in the sixth planning meeting, Tess said,

> I really did let kids go up and explain. And so sometimes, somebody who could do it well would be up here and somebody would say, "I still don't get it." And then I'd say, "Try

> to ask them. What method do they use?" And then they might say, "Chip." And I know she uses number line or just the algorithm, but she showed them [the chip method]. And that was just, it was good. It was a very good class.

This particular student was able to use either model to represent computations and explain her reasoning. This was encouraging to Tess and she thought it was a "very good class."

Although many of Tess's experiences with pedagogical strategies reinforced ideas that students needed to be told what to do or how to think, she continued to work towards implementing strategies that facilitated higher-order thinking by encouraging independent thinking (Raudenbush et al., 1993) and mathematical communication (Jones, 2004). She intended to help students develop higher-order thinking, but her discipline problems, lack of time, and struggles with student-centered teaching sometimes hindered her ability to do that development.

Doing Mathematics

There were two ways in which Tess understood what it meant to do and understand mathematics that influenced the way she selected and implemented tasks. First, Tess's understanding of how mathematical concepts were situated in the standards, the pacing of the course, and the textbook she used affected how she chose tasks and taught lessons. Second, Tess thought that in order for students to build conceptual understanding of mathematics, they needed to be able to link it to the real-world. She thought she needed to create connections to help students do mathematics. Therefore, when students had to work within purely mathematical contexts, Tess thought that they needed rules or procedures in order to come up with correct answers.

Standards, pacing, and textbook. Tess's interpretation of the meaning of mathematical concepts and content standards influenced the way in which she chose and implemented tasks. She considered the learning goals for students, and either focused her instruction on rules or concepts depending on her understanding of mathematics and the meaning of what it meant to do mathematics.

In the fourth planning meeting, Tess reflected on her implementation of the first unit's final task. In this task, students were developing tables, graphs, and formulas to represent the height of a stack of cups and the number of cups in the stack. During her lesson, Tess did not allow her students to determine an equation, but rather gave them one to use without discussing how it could be used to represent the pattern. During our meeting, she talked about why she had made that decision.

Tess: I feel like [in] Variables and Patterns, the emphasis was on the relationship between two variables and showing that essentially in tables and graphs.

Eileen: But when you wrote down the formula, I felt like it was not getting at the relationship. You were like, "A formula is gonna have *y* equals something *x* plus or minus something else."

Tess: Initially.

Eileen: But that's totally decontextualized

Tess: Right.

Eileen: And it's not focusing on the relationship. And then you went to the one times *c* but it wasn't clear to me that any of the students understood that what you were talking about was that that one was your constant rate of change. Always changing by one. So why is it 1 * *c*? Why isn't it plus one?

Tess: Because I don't feel like that's been part of our standard yet. I feel like that's Moving Straight Ahead. Maybe I'm wrong, but I don't feel like it's part of the understand, the Variables and Patterns start of, I thought that that's more introduced—the whole formula aspect in—.

Clark: But, see it's actually talking about the rate.

Tess: Moving Straight Ahead.

Clark: But we're talking about the rate.

Tess: *y* equals *kx* and all of that comes into that next one.

Clark: But, Tess, on this one, it's talking about the rate. Rate is part of this, in the aspect, it's starting to change when we get to—.

Tess: The rate of change. Constant change.

Clark: That's what—.

Tess: And showing it in a table, and showing it in a graph. But I don't know that, I didn't understand that formulas were a big—.

Clark: Yeah, that was a big deal because you're supposed to be able to write formulas.

Tess: [Reads from the content standards] "Analyze graphs and tables to determine the relationships between varying quantities. Collected as occurs as a result of the relationship between varying quantities. Organize data into tables. Tables and graphs, tables and graphs. Explain how changes in one variable affects another. Use tables and graphs, written descriptions. Result of relationships." My feeling is that the formulas are more emphasized in Moving Straight Ahead.

Tess gave her students a formula to use without focusing on the formula's relationship to the situation (lines 4–9). Her understanding of the pacing, content standards, and textbooks was that she should stay focused on tables and graphs rather than formulas (lines 13–18). Not only did the content standards not explicitly mention formulas (lines 29–34), but also there was a future textbook unit, Moving Straight Ahead, that covered writing linear equations in more detail (lines 34–35). Because writing equations was the focus of this later unit and she had not formally taught the concept, she decided to provide the formula and move on.

As Tess and Clark continued to talk about the content standards, they found the algebra standards explicitly called for students to "translate verbal phrases to algebraic expressions" and "given a problem, define a variable, write an equation, solve the equation, and interpret the solution" (Georgia Department of Education [GADoE], 2005, p. 11). The task itself called for students "to represent the relationship between the number of cups in a stack and the height of the stack using a table, a coordinate graph, a formula and a written description" (Grade 7 Mathematics Frameworks, Unit 2: Patterns and Relationships, p. 20). Regardless, Tess said that she did not want to have her students engage in this part of the activity because "there's not really a lot of investigation prior to that," and Tess would not spend any more time on the task even though some of her students hadn't competed it.

Tess: Well, I'm not going to spend any more time on the task.

Eileen: Some of your kids didn't have it completed.

Tess: By the end of the summary they had most of the thing. I mean most of them had it, certainly some. The majority had everything answered and copied down, even if they didn't create it all themselves. My intention was to finish that day because we were starting [Problem] 1.1 today. So there's maybe a couple kids that didn't, but—.

Tess thought that since the majority of the students had at least copied down all the representations they needed to represent this pattern of change, even if they hadn't figured it out on their own, she should move on to the next topic (lines 3–6). She had scheduled the task for just 1 day and since most of her students had completed most of the problem, or at least copied the answers down, she had satisfied her goals for the task. Tess's understanding of the mathematical goals of the task, the unit, and the content standards contributed to her implementation of the task and her decision to limit her students' exploration into one topic. In her mind, students needed to focus on tables and graphs and formulas would come later, while in actuality this task and unit were building a conceptual foundation for writing formulas.

Tess selected tasks based on her understanding of the state's content standards. For example, in the first cycle planning meeting, the teachers discussed if they should go over a particular problem in the investigation with their students. The problem asked students if they should connect the data points on a graph. This had been the topic of a problem in the last investigation and was the first problem now. Tess decided to do the problem with the students, but said, "It's not, that's not like a very deep connection to our standards or anything." Because of this view, Tess would spend a short amount of time in a whole-group discussion about it rather than giving students a lot of time to think about it on their own.

Context. Tess wanted her students to develop conceptual understandings of mathematical concepts. She wanted them to be able to explain and justify their thinking as well as to make generalizations and conjectures as they explored concepts. For example, during the second unit when students learned about addition and subtraction of integers, Tess wanted them to develop number sense by using their intuition about number size and quantity and to make generalizations about patterns they saw. She did not want the students to memorize rules. In the fifth reflection meeting Tess talked about what it meant for students to understand the concepts.

> The most exciting language this year has been coming from kids who have really to me felt that, and have been able to say "It can't go on that side of zero because there's not enough to move it there," and like, who are really looking at that kind of, and that works with chips or—. It's basically you can't go in the black, or you can't get in the positive because there's too much negative. Like that kind of language to me is the most important because they're actually feeling the numbers. And then what's happening with the operations. So, I've heard more of that this year.

She thought that her students this year understood the concept, and particularly the chip model, because they were "feeling the numbers" and understanding what was "happening with the operations" rather than simply applying rules or procedures. The chip model was an example of a purely mathematical context that helped students gain conceptual understanding. But in general Tess thought that purely mathematical contexts would not make sense to students or they would not connect with them. Tess thought that such contexts, such as mathematical patterns, were not powerful or legitimate ways to present or think about mathematics.

Tess's understanding of mathematical patterns was that they were less important and powerful than real-life connections. During the sixth reflection meeting as we planned for Tess's class, she talked about shifting her focus away from patterns and providing her students more practice in adding and subtracting integers.

Eileen: Do you think a good thing to bring up also would be, you know and also think about, you know, the patterns that we saw? Because you did talk about patterns. So, you know, when you answer a question, and I would love for us to start trying to get this in more because it helps a lot on standardized tests, but, you know, what do you think the answer should be? So before I even do the problem, I should think about, you know, is the answer going to be positive or negative? So then when I do the problem, and I get an answer—.

Tess: Right, but you know what? I don’t see that as a pattern thing so much as an understanding the number line.

Eileen: Okay, okay. So—.

Tess: So, but they—.

Eileen: Let me look at these numbers and try to decide. If I’m doing 7 subtract 9, I should know ahead of time what kind of answer I’m going to get. I know it’s going to be negative. So when I do it, I have a sense of, what my answers going to be.

Tess: Right.

Clark: You know—.

Tess: But that’s because I feel it, my kids have been confused. I’ve done the group things, looked for patterns, and then it’s been very confusing. It’s been better for me to try to be more elemental about this number, and then they really have understood subtracting a negative from looking at using the strategy. Not so much from the patterns or anything like that. Patterns are about rules really, with these things.

When I suggested reminding the students of the patterns they had investigated previously to develop ideas about when computations would result in positive or negative answers (lines 1–8), Tess talked about how she felt about these patterns. The class had already talked about patterns, and her students were still confused, so it was time for her to move on (lines 19–24). In fact, “patterns are about rules, really, with these things,” meaning that patterns were used to develop algorithms and now the students needed to be able to solve problems and understand methods; specifically the number line model (lines 9–10). In the seventh reflection meeting she again referred to patterns as rules and thought that when students noticed patterns, it represented "an artificial display, so it's not so deep and, you know, organically natural that that's what they should be noticing." Therefore, when Tess thought about using contexts to do mathematics, she mostly considered real-life connections.

Tess used real-life connections to build conceptual understanding, but she needed to create connections when they were not explicit in the instructional resources. If real-life connections were not available or if Tess thought those she had were unsuitable for her students, she focused on teaching the students rules or procedures for determining correct answers. Moreover, Tess thought that using rules and procedures was appropriate if real-life connections were provided ahead of time. For example, during the third planning meeting, Tess and Clark discussed how to help students understand how to write formulas from written descriptions. They thought it was a hard concept for students because they had to first determine variables and then translate written descriptions into symbolic notation. To help students in this process, Tess created problems that would be more concrete and related to students' experiences. In this first problem, Tess wanted to help her students identify operations in written descriptions.

Tess: This is what we were talking about, using the language, this is the EQ [essential question]. So, they have to say, "Well, for every dollar you get 10 pieces of candy." Somebody's going to say something like that. Or 10 pieces of candy per dollar. So getting that "per" related to multiplication, or whatever. And then from that developing, well, I can say, "Oooh, now, it's really bad. Candy is 10 times." And then this point, though, they're coming up with variables. They're talking through, but we end up with an equation that then, if [the principal] wants to give me 500 dollars, then, I mean 10, I can just say 500 times 10. We've got 5000 pieces of candy, whereas I'm not gonna extend my table like that.

Clark: That's right.

Eileen: Right.

Tess: So, then—.

Clark: That's similar to what's on page 56 in the book. Problem number five, six, and seven. And that's what I put. I put one up for my warm-up.

Tess: Okay. Same idea then.

Clark: Mm hm.

Eileen: Well, yeah, it's the same idea, but this one is—.

Clark: Yeah, more concrete.

Eileen: Table to formula and the other one was formula to table.

Tess: And the other thing is, wanting them to say, "A dollar a candy." I'm trying to really make it concrete.

The purpose of this example was to get students to clearly understand that "per" would translate to multiplication (lines 2–5). This problem that Tess created was the same idea as the textbook problems, but was more concrete for the students (lines 14–19 & lines 21–22). In another example Tess worked to help her students translate multiple operations and variable coefficients.

Tess: So I said, "Okay, how much does it cost to go to the movies?" So we get a couple, you know, we agree that okay, children are $6 and adults are $8, something like that, right? So then everybody has to write an equation that would work for the cost of their family to go to the movies. So then, what I'm hoping [for] is six times three and eight times two, or whatever, plus. And so then we're beginning to look at multiple operations. Except I've got three adults, you know, and so then we went around the room and there was a fair amount of interest because, "Oh, you've got three adults in your family." So then looking at it: Cost equals six times the adults, I mean the children, plus eight times the number of adults. Then you can say, "Well, what if one adult can't come?" Then you can subtract, um, that's not gonna work at this point.

Eileen: Well, you can subtract \$8.

Tess: Can subtract 1*a*.

Eileen: Yeah.

Tess: 8*a* minus 1*a*. I wouldn't with this one. I did it with the next one. But you can then discuss a little bit of like terms in one and make sure that everybody knows that you don't need to have a one.

In this case, Tess wanted to relate to students' experiences by having them think about going to the movies with their families, which made the problem more interesting (lines 3–4 & lines 6–8). After creating the expressions, Tess wanted to help students figure out what it meant to subtract (lines 10–14) and to understand the meaning of the term 1*a*. However, this example did not achieve this second goal because it was mathematically incorrect. If a = the number of adults going to the movie, and one adult did not come, the equation would change to $8(a - 1)$ or $8a - 8$ rather than $8a - 1a$ (lines 13–14). The expression $8a - 1a$ would translate to \$8 for each adult minus $1 for each adult, or $7 for each adult, whereas Tess meant to have $8 for each adult when one adult was missing. Tess did not see this mistake and thought that this example would help everyone understand what the 1a represented (lines 16–18). This error showed how Tess was sometimes unsuccessful in her attempts to connect mathematical concepts to real-life contexts in meaningful ways.

In other situations, Tess was not comfortable with a representation in the instructional resources, or she did not think it related to students' lives. This was the case for the multiplication and division of rational numbers. It was important to Tess that her students walk away knowing how to apply rules, especially because she did not see a way to embed multiplication and division in real-life contexts that would mean something to the students. There are many ways to teach negative numbers, but historically teachers have struggled with understanding how to relate negative numbers to students and talk about operations with negative numbers (Arcavi & Bruckheimer, 1981; Hefendehl-Hebeker, 1991). During the sixth planning meeting, Tess talked about how she understood multiplication and division of integers and what she wanted her students to know.

Tess: It's still—. I still do multiplication and division of integers in a more rule, or much more algorithm based—.

Clark: I understand what you're saying.

Tess: I don't with addition and subtraction. But, and I was looking around last night, like on Dr. Math and stuff, and they would—. And you can go on Dr. Math and say, what's a real-life example, blah, blah, blah? And so they give lots of real-life type examples of addition and subtraction like we've been teaching. Concept based. But when you get to dividing a negative and multiplying a negative, and I was looking, negative times a negative, there aren't just, it's not as naturally conceptually linked. It is more of an algorithmic or a rule-based procedure.

Eileen: Well, yeah.

Tess: We can do multiplication, still based on repeated addition and we can build that connection, but with division, it's—. I mean I'm not going to not have my kids understand the rules at the end of the day. At the end of the day.

Tess thought that multiplication and division were based on rules (lines 1–2). She concluded that in part through her inability to find a real life example (lines 4–11) that her students would connect with to help them understand the concepts. She was able to connect multiplication with repeated addition, but did not think to use the analogous connection of repeated subtraction for division. As a result she knew that she wanted her students to know the rules (lines 13–15).

The CMP2 textbook used a number line/motion model for students to understand multiplying and dividing negative numbers, but to Tess it was a contrived example. In the seventh planning meeting we spent a good bit of the time discussing the model and working through the problems in the investigation. Tess thought that it was difficult and knew she needed to spend extra time puzzling over it before she implemented the lesson. Before my observation, I co-taught a lesson with Tess to help her understand how to use the model during instruction. By the time Tess implemented the lesson alone, she was able to use the model to help her students connect integer division with a context. However, she thought that the model was not integral to student learning, and did not want students to focus on the model as they solved division problems. Instead, she wanted to illustrate how the same model that "worked very well with multiplication" could be used with division, and how division was "really grounded in the same things that could happen." Tess thought the model was a way to convince students that integer division had some connection to real-life so they would not think it was a purely mathematical concept. Thus, she did not feel the need to have the students really understand the representation; rather, she just needed to show it. In that way she could reinforce the rules while illustrating how the students could still, in her words, "do it in a concrete way, and it would still work."

Rules seemed to always have a place in Tess's practice. Rules could give students the ability to solve problems as in the example above or rules could reinforce concepts. Several times Tess talked about her special education assistant, Ms. Lane, and how Ms. Lane's contribution to the classroom affected student understanding. In the fifth reflection meeting, Tess talked about Ms. Lane's strategy for determining whether the solution to an integer addition expression would be positive or negative. Tess had used the textbook's approach of employing the fact that the number with the larger absolute value could be used to predict answers. Ms. Lane had a different approach.

Tess: But that's where I had to use the absolute value because, you, because Ms. Lane also she has the same thing, "Big boy gets the sign," and they [students] like that, and that seems fine with me. That doesn't seem like it is a silly rule.

Eileen: No.

Tess: So, big boy is 12, but it's absolute value of big boy because obviously eight is more than negative 12.

Clark: That's right.

When Ms. Lane suggested this approach in class, students liked it, so Tess thought that it was fine to use (lines 3–4). It was not a “silly rule,” because it could be directly related to absolute value (lines 6–7), which she had already talked about.

In the sixth planning meeting, after Clark commented that Ms. Lane “knows that math,” Tess discussed Ms. Lane’s contribution to the class. Because the class was collaborative, Tess wanted to include Ms. Lane in the lessons. When she saw her working with students, she regularly would ask Ms. Lane to share what she was doing with the class. In one episode, Tess shared how Ms. Lane had a worksheet with rules for multiplying and dividing positive and negative numbers, such as *keep change* or *keep keep change*, where *keep* = sign or operation stays the same and *change* = take the opposite sign or inverse operation. Tess did not mind these rules because Ms. Lane had been “successful over the years” and she shared the rules after the class had completed the investigation.

Tess thought that rules could help students understand mathematics and thus saw them as useful tools for her practice. However, if students tried to use procedures or rules before Tess had talked about concepts in class, she did not approve. During the sixth reflection meeting, I asked Tess about something I had heard one of her students say during my observation. The problem on the board was $^{-}5 – 11$ and a student said, “But I thought two negatives make a positive.” Tess talked about how this was true of the expression $5 – {}^{-}11$ and talked to the students about “turn turn,” a method with the number line. This method helped the students understand when you moved left or right on the number line depending on whether you were adding (turn right) or subtracting (turn left) or if you had a positive (turn right) or negative (turn left) number. When I asked Tess why she thought her student got these two number sentences confused, she talked about her disappointment in the students' desire to use procedures like "two negatives make a positive."

Tess: I think it's because, I think it's that urge to have a rule, kind of thing. Because I think if you see negative minus 5, negative 5, and if you're going down and you're going down then it's not confusing. In this case you start at negative 5. It works for me easier on the number line, in this case, and then you know you're gonna turn turn or change change. You wouldn't change change there. You wouldn't turn turn there 'cause there's just one. But I think if they're just looking for signs. That's the problem. They're looking for signs and looking for rules. It's a problem.

Clark: Yeah, it's like they've been taught nothing but rules, rules, rules and you know

Eileen: So you think they're just trying to hold onto that again.

Clark: Mm hm.

Tess: Yes.

Eileen: So, they're not thinking conceptually about it.

Tess: I think that, because I never said two negatives give a positive. Ever. But somebody else might have said that even, like, maybe Ms. Lane might have said that. I don't know. But another student might have said that.

Clark: But do they had somebody else for ELT. Who did they have for ELT?

Tess: I don't know, who knows.

Eileen: Because we know [a non-math ELT teacher] already said that in terms of multiplication.

Clark: That's so. That's what I'm worried about. They may be trying to remember.

Tess: Okay, but I never. That's not even the way I think about it. So…

Eileen: Because then Ms. Freeman got up here and said, "You know, if I'm subtracting, I'm facing this way and then when I'm a negative number I have to turn around again." So, she was like actually physically showing them what she meant by that, too.

In lines 1–9 Tess describes the rules students were attempting to use and how this was a problem. She did not know where students were hearing about these rules (lines 15–17) but knew that it was negatively affecting their understanding of the mathematics. To help counteract this affect, Tess focused on her own use of the number line during her instruction (lines 25–28). In this way, Tess tried to help students think about why and how the rules they were using worked and to thus facilitate higher-order thinking.

As Tess chose and implemented tasks, she asked students questions that encouraged them to make generalizations and establishing conceptual knowledge of algorithms. However, her understanding of what it meant to do mathematics affected how she was able to do this and caused her intentions and actions to contradict each other. In particular, while she spoke of students gaining conceptual understanding of mathematical concepts, she simultaneously focused on rules and procedures to insure that students would be able to successfully find correct answers to computational problems for particular topics.

Summary

Tess wanted to make sure that her students would be successful on the quarterly assessments and end of year test. To do this, she chose tasks in order to cover all concepts during class and would alter her pacing when needed. Tess frequently felt pressed for time, which also affected how long she could spend on tasks and concepts. Tess tried to engage her students by choosing tasks that would build their confidence and relate to their experiences.

During implementation, she closely monitored and guided her students in order to make sure they were working. Tess thought that this type of instruction would help students stay focused on mathematics. Tess based this view on her prior teaching experiences, which included struggles with discipline and classroom management. Tess wanted to facilitate higher-order thinking and to implement student-centered instruction, but she had negative experiences with both her ability to facilitate class and in her students' ability to engage in activities, which caused her to shy away from these methods. Even so, Tess was able to facilitate higher-order thinking in particular ways, like connecting mathematics to the real-world (Jones, 2004; Ladson-Billings, 1994). She worked to select and implement tasks in ways that highlighted connections. In many cases these helped build student understanding, but in others her own mathematical understanding and views on what constituted legitimate mathematics may have hindered student understanding.

Influence of Reflective Teaching Cycles

The previous two sections described influences on teachers' selection and implementation of tasks. In this section, I look at how the reflective teaching cycles influenced how teachers chose and implemented tasks that had the potential to develop higher-order thinking. Based on my analysis of the data, I found the reflective teaching cycles influenced the teachers in three important ways, *collaboration, building teachers' knowledge of mathematics,* and *reflecting on pedagogy.*

I begin this section with descriptions of the influential components of the reflective teaching cycles. The influential components manifested in different ways depending on the mathematical content. I present examples spanning several cycles that focused on a particular mathematical topic. These examples illustrate how the components were represented through the interactions during the meetings.

Influential Components of the Reflective Teaching Cycles

The *collaborative nature of the cycles* influenced teachers' ability to realize changes in their practice. The planning and reflection meetings allowed us to collaborate about mathematics, pedagogy, and higher-order thinking. Our conversations helped Clark and Tess critically examine their practices, challenge each other's thinking, and provide each other support. Clark and Tess were able to provide critical feedback to each other on their mathematical thinking, selection of tasks, and implementation of tasks. They viewed each other's practice from their own perspective and offered alternatives for instruction, discipline, and motivation. My role as the facilitator also contributed to the type of collaboration that occurred during meetings through my questioning, which focused the teachers on mathematics, pedagogy, and higher-order thinking. Evidence for this theme was embedded within the two other themes because the teachers collaborated as they discussed mathematics and pedagogy.

We also spent time *building teachers' knowledge about mathematics* Clark and Tess were expected to teach. We talked about mathematical concepts, mathematical goals of tasks, content standards, and learning goals for students. These discussions allowed the teachers to critically examine chosen tasks and think through the implementation of tasks. We clarified mathematical concepts, representations, or language the teachers' encountered in the text or in other mathematical resources and discussed misconceptions or misunderstandings. The teachers also helped clarify mathematical concepts or goals to each other and would sometimes have to work through disagreements. Evidence of this theme included instances in which the teachers talked about tasks or worked through tasks they used or would use during instruction. Other evidence included episodes in which the teachers talked about general mathematical concepts or specific mathematical goals.

Finally, we *reflected on pedagogical strategies* the teachers used and discussed possible new strategies they could try. As we talked about pedagogical strategies, the teachers thought about how they could facilitate higher-order thinking, gave each other suggestions, and considered ideas that I presented. During the meetings, Clark or Tess sometimes identified instances in each other's teaching they thought hindered higher-order thinking, provided critical feedback, and challenged each other's thinking. We considered how particular strategies affected student learning and thought about ways to facilitate higher-order thinking. Evidence for this theme included episodes in which teachers talked about pedagogical strategies, including introducing tasks and using questioning. Other episodes where we discussed strategies I had observed in their instruction also indicated this theme.

The cycles focused on planning for and reflecting on mathematical lessons in two instructional units. We discussed the first instructional unit in Cycles 1–4 and the second instructional unit in Cycles 4–7. Below I present two examples, one from each instructional unit, that illustrate the three ways in which the reflective teaching cycle influenced the teachers' selection and implementation of tasks. This presentation highlights specific mathematical concepts that dominated the discussions and how we wound up discussing the same concepts over several cycles. We discussed the first mathematical topic, V*ariables*, through Cycles 1–3 and the second topic, *Subtracting Integers*, through Cycles 4–6. These examples will also show

how the influences of collaboration, building teachers' knowledge of mathematics, and reflections on pedagogical strategies surfaced through conversations about mathematics teaching and learning.

Variables

In Cycles 1–3, the teachers planned lessons in the first unit of instruction that focused on variables and patterns of change. The way students understood the definitions of independent and dependent variables and identified variables in different problems was an important component of the lessons. In these conversations the teachers collaborated by examining their practice and reflecting on students' thinking. They collaborated by supporting each other to figure out the best way to help students gain a deeper understanding of variables. We built teachers' knowledge about mathematics by discussing the definitions of independent and dependent variable. This allowed us to reflect on pedagogical strategies and determine why students were engaging in tasks in particular ways. Specifically, we considered how the definitions of independent and dependent variable could be hindering flexibility in student thinking and tried to figure out the best way to support student learning and promote understanding.

During the first cycle, the teachers' planned for a task in which students were given data in the form of a graph and were asked to interpret the graph, to create a table based on the graph, and to compare patterns of change in both representations. During the planning meeting, Clark and Tess talked about how defining variables was necessary for doing the problem, but how this was not the main mathematical goal for the task. This was the fourth task in the investigation and the students had been introduced to the notions of variable, dependent variable, and independent variable in the first task. The students were expected to use their emerging understanding of these concepts to create tables and graphs and analyze information. Thus, they did not spend a great deal of time thinking about how their students understood the concepts of independent and dependent variable or how they would discuss this with students during class. Instead, Clark and Tess talked about how they would implement the task to make sure the students could tell them what they needed to consider as they created a table or a graph; that is, variables, scales, and labels.

During my observations of this lesson, I noticed that Clark and Tess asked questions that led to one-word responses from students. For example, Clark wanted students to use the word *variable*, and when they did not he asked questions such as “Independent what?” and “Once we find our independent variable, we then find our what?” In a similar way, Tess asked questions that prompted students to guess her thinking or to insert a word into an unfinished sentence, such as “They change so we call them?” and “I’m thinking of one other thing I like to see.”. The students answered these questions, but because of the nature of the questions, it was not clear to me if they understood the definition of independent variable or dependent variable or what they knew about variables in general. The teachers were also not being precise with their language as they talked about the variables. In the previous task, the variables were time and total distance traveled. In this task, the variables were time and distance from a fixed point.

During the reflection meeting, I asked Tess to consider what she knew about her students’ thinking at this point to help her determine future lessons. She thought that her students were still confused about how to identify and define variables. She had seen a “glimmer of understanding there,” but it was “not too deep.” Therefore, even though to me Tess’s questions had not given much information about students’ thinking, she knew where she wanted to go and what her students still needed to work on; that is, she still had “more to do in terms of looking at the relationship between two variables.”

When I spoke with Clark during the reflection meeting, I used my observations to ask him questions about how he was using questioning and mathematical language during his teaching. We examined his practice and reflected on his pedagogy, specifically his questioning. Clark described his intentions.

> What I was asking them was, ah, "How did you come up with that answer? Um, what, what, how do you know that this is actually the coordinate that you were looking for? What is it called? Ah, which one is my, my independent variable?" I was using the vocabulary that we're asking the kids to mimic, or to memorize to know—. Um, so what I wanted them to do is I'm asking you the question, and I'm going to ask it to you using the vocabulary that we're requiring you to know. And I want you to answer me with that same vocabulary that I'm presenting to you, um, so they know that they can talk mathematical language, you know? They need to understand that things are different as you move up in grades. You're presented with new vocabulary, and you're expected to use it. And I was trying to show them that I use the same thing, and I wanted them to use it with me.

It was important to Clark that students were able to use the appropriate vocabulary words in class. His intention was to let students know that they needed to communicate mathematically by using appropriate mathematical terminology. Pushing students to communicate and reason with mathematics would facilitate higher-order thinking (Jones, 2004; Ladson-Billings, 1994), but based on my observations, this was not what the questions accomplished.

Therefore, I challenged Clark to think more deeply about how he phrased questions. I knew that he wanted to engage his students by building their confidence, and had already experienced some success with this. He had also talked about pushing his students harder once they were more confidence. I reminded him of this and he reflected on how he could alter his questioning techniques to align with this intention.

Eileen: Like you'll say, like you'll say something like, you know—instead of just saying, "What are the variables?" You'll say, "Okay, what's this? This is the … something variable?" And they'll say, "Independent." So you know you're giving—.

Clark: Mm hm.

Eileen: So you know I would just encourage you to maybe—.

Clark: Back up a little.

Eileen: Back up. Because they are having that at least that class you know is having success. So now asking you know a little bit more general questions.

Clark: Gotcha.

Eileen: Because I think that would raise, raise the [cognitive] demand; their demand, you know, a little bit more.

Clark: So, what and, and, so like tomorrow. Um, my warm-up is going to actually be that exact graph sitting there, and my question is going to be, “What do you need to build a table from this graph?”

Eileen: Mm hm.

Clark: And I want them to be able to write it out. To be able to tell me those four items that you would actually need. I’m going to take it off the board so that they don’t see it and I want them to be able to tell me.

Clark had hindered higher-order thinking by asking closed and direct questions that focused on correct answers (lines 1–3 & lines 8–11) (Doyle, 1988; Henningsen & Stein, 1997; Romagnano, 1994). We reflected on his pedagogy to consider how he could better facilitate higher-order thinking by asking open questions that would emphasize meaning and conceptual understanding (Stein et al., 2000). Clark talked about how he could alter his questioning in the next lesson that he thought would reflect my suggestions (lines 20–26). This examining and critiquing of Clark's practice promoted collaboration and allowed me to offered feedback and suggestions on pedagogy.

During the second cycle, the teachers planned for a task in which the students were again going to identify variables, create graphs and tables, and analyze information. The main goal of the task was to understand and identify independent and dependent variables. The task was going to help students think more flexibly about variables and required students to think deeply about what it meant to be an independent or dependent variable. Tess was concerned the difficulty of the task and her students' ability to understand the situation. The independent variable was the amount of money charged for a bicycle tour and the dependent variable was the number of customers who would pay that price. Clark and Tess thought this problem would be difficult because of students' previous experiences with independent and dependent variables (i.e., "money's always been the dependent variable"). The problem required a "sophisticated understanding" in order for the students to effectively and correctly graph the data. According to the teachers' guide, the students should be encouraged to define the variables however they chose, even if it was incorrect. This approach would require students to use higher-order thinking because they would have to evaluate their thinking, justify their decisions, and reflect on the reasonableness of their answers (Facione, 2009; Facione et al., 2000). Clark and Tess decided that they would follow the guides' suggestions. Tess even said that she hoped for an example where students reversed the variables in order to have a good conversation during class. As the teachers' thought about the task, they collaborated by supporting each other to determine how best to introduce the task and by building knowledge about the task and about students' thinking. In particular, they thought about why it made sense for the independent variable to be the amount of money charged and determined that the clearest way to see this was through the initial point;

that is, it only make sense with the correct definitions for the variables. One reason for this decision was that the definitions of independent and dependent variable were not clear.

Up until this point, the teachers and I had not talked about the definitions of independent and dependent variable. I was under the impression that the teachers were using the textbook's definitions, however, as we discussed the concepts further, I realized that our own definitions were creating problems with how *we* understood the independent and dependent variables. We talked about the independent variable being "independent of manipulation" and the dependent variable "depending on" the independent variable. This made the problem tricky because if we used this language, one could be convinced that the money depended on the number of customers or that the number of customers depended on the money.

This conversation illustrated how students would struggle, so the teachers began to prepare themselves by considering pedagogical strategies they could use to introduce the task. Tess's solution was to provide a more detailed opening and launch to prepare the students for the thinking that would be involved. Tess was going to select tasks that she thought would help her students engage in the problem. She planned to provide her students with clear examples of independent and dependent variables and even show them situations where the two could be confused in order to limit the amount of struggle or frustration they might feel while working through the task. By giving the students clear examples and showing them what could go wrong, she would hinder higher-order thinking by removing the problematic aspect of the task (Stein et al., 2000).

Alternatively, Clark planned to help students access their knowledge from the previous tasks and allow the students to explore the situation without too much introduction. In his words,

> My whole goal is, I'll have my warm-up, and I'll talk about what we did today, which would be the day before. And then I would ask them to complete [Problem] 2.2 and I will

> give them a time limit so that I will have enough time to go over and talk about it and see what everybody got.

This would facilitate higher-order thinking because he would be asking students to apply their knowledge to solve the problem (Raudenbush et al., 1993) and to communicate their thinking (Jones, 2004).

During teaching, Tess used her warm-up to get her students ready to solve the problem by talking about several situations and asking the students to identify the independent and dependent variables. For example, she asked the students to think about the amount of fertilizer used and the growth of a plant. She asked them if the growth depended on the amount of fertilizer or if the amount of fertilizer depended on the growth. After asking this, Tess did not give the students much time to consider the answer, and asked the students to try to fit the situation into a fill-in-the-blank sentence. After the warm-up, Tess facilitated a whole-class discussion that led the students to the correct graph and definition of variables. She drew the graph on the board and had her students sketch a graph based on hers. Throughout the period, Tess's questions and comments hindered higher-order thinking by being focused on correct answers and requiring one-word answers (Henningsen & Stein 1997; Stein et al., 2000). Therefore, I was still unable to assess her students' understanding of how to determine and define variables.

In Clark's class, I observed him introducing the context, allowing the students to define the variables, and engaging them in a conversation about the choices they made concerning the independent and dependent variables. His questioning techniques during this class were more open and focused on eliciting explanation and justification of ideas from his students. For example, when Clark initiated the class discussion, he asked the students which variable would be on the x-axis or be the independent variable, and then asked the students why. He further asked the students to reflect on the reasonableness of their answers by considering how the data made sense when represented in the different ways. Clark's questioning reflected our conversation in the first cycle because he worked to use questions that would facilitate higher-order thinking. He focused on conceptual content by pushing his students to consider how they should analyze the situation and why (Lobato et al., 2005). Nevertheless, based on the answers to the questions and the students' talk during class discussions, I was still unclear about what their understanding of independent and dependent variable was.

Therefore, in our reflection meeting, I wanted to help the teachers further build their knowledge of variable so that we could better understand how to support student thinking. I did this by focusing the teachers' attention on student thinking and talked about an interaction I had with one of Tess' students. As the students were working independently during the bathroom break, I had gone over to check on this one student's progress. I asked her how she was determining the variables, and she talked about how the independent variable *determined* the value of the dependent variable. Tess reflected on this definition and how it was different than her own. She began to realize that she needed to think more deeply about the mathematics in the task beforehand in order to be prepared for different contributions from students and to be able to better explain the concepts. In her words,

> The very first problem today was, um, the number of students that go on the trip depended on the price. And so, it just seemed more clear. It was like, no. Because your price, your price can't depend on the number of students because your price depends on renting the bike, purchasing the first aid kits, having the tires. That's what your price [is]. So in this case … if I had thought that ahead I could have been more clear.

Tess thought that if she had been able to think about the situation ahead of time and described the problem in this way, her students would have been clearer on the independent and dependent variables. As she reflected on her pedagogy and how she could introduce the problem context, she remained committed to clearly explaining the concepts to her students so that they would engage in the task.

Later I asked Clark and Tess to consider what pedagogical strategies they could use in order to help students learn how to read graphs, and specifically identify the variables and the nature of the covariation. The teachers continued to reflect on their pedagogy and we collaborated as I challenged the teachers' thinking.

Clark: And I start out at the beginning when I'm opening. [I] ask them, "What are the four things that we need to make a table or a graph?" And we go in order. First thing is my, and they tell you, "Variables. I need to know the variables."

Tess: Right.

Clark: "Second thing I need to know; which one is independent? Which one is dependent?" And that's the same thing when we analyzing these graphs here. We need to know, okay, what are the variables? Which one is dependent? Which one is independent?

Eileen: But what I'm hearing the students start doing as the problem goes on, is that, I don't want to say they're becoming sloppy in their language, but that's kind of what it is. So if it's, like for example, when we're doing the bicycle, um, examples, the first distance was total distance.

Clark: Right.

Eileen: For that first situation. In the second, it was distance from a fixed point. And what I noticed in those two classes was that they were not being specific with their language. Total distance or distance from Lewee. I think was the name of the town or whatever.

Clark: Right.

Eileen: And so, it was, so when the distance dipped, some of the kids got additionally confused because if they're just thinking, distance, in terms of total, so it's like, what's confusing them? Is it the variables or is it the fact that they're not thinking that this is distance from a specific point? And so as Tess was saying, you know, talking it out in the sentences and making them be really precise.

Tess: And, the necessity for the sentences are that both variables are named and that the directionality, or whatever, I said that you use the words increase, decrease as appropriate. So, if you have your independent variable and it's increasing, it's always increasing, right? Then what's happening to your dependent variable? That's your sentence and that's also you're title. It has to show the relationship. It has to bring in the independent and dependent together. Title has to have both, um, variables, in it and talk about how they react. How they're related.

Clark knew that he wanted scaffold his students' thinking by giving them a way to access their prior knowledge (lines 1–9). I challenged the teachers to examine student thinking by talking about my observations. In particular, I had noticed the students were being "sloppy" with their language, which had caused some confusion over the variables (lines 10–19). We used this event as a way to build knowledge about students thinking, and specifically about what caused the confusion (lines 20–23). We reflected on pedagogy by brainstorming ideas for instruction that could alleviate the problems. Tess made two suggestions (lines 26–33), both of which would facilitate higher-order thinking by encouraging students to interpret the problem (Lobato et al., 2005) and communicate using mathematics (Jones, 2004). Then, during the third cycle, we revisited the definitions of independent and dependent variables.

Before the third planning meeting, as I reviewed my notes and listened to the conversations from the previous cycle, I was struck by the way the teachers had been talking about independent and dependent variables. In addition to this, as I continued to attend classes, I witnessed an ongoing confusion on the part of the students about how to identify and describe variables in different problems. Therefore, before planning for the upcoming task, we discussed the teachers' interpretations of independent and dependent variable, which included the independent variable standing on its own or being more dominant. Then we looked in the textbook.

Eileen: Well, because that's the thing—. Because we kept on saying, "Its something that can stand on its own." But, then we had that struggle with, "Well, when you're talking about number of customers and money, neither of them really can stand on their own." You know? So, we started having difficulties. So, I was just wondering—what are they [independent and dependent variable]? Is there an index? What do they say in the book?

Tess: They say the quantity being measured or counted whose value is determined by choice.

Eileen: Okay. So, that's a little bit different than standing alone, right? So, it's like, what do you get to choose? So, for the money and number of customers—in one case, I was choosing the number of customers and then telling them how much they had to pay. And then the second example, I was choosing how much I was going to charge, and then I was figuring out how much, how many people I would get. So, it's like—.

Tess: Choose is a funky word to me.

Clark: [Reading from the textbook.] "One of two variables in a relationship. Its value determines the value of the other variable called the dependent variable. If you organize a bike tour, for example, the number people who register to go, the independent variable, determines the cost for renting the bikes, the dependent variable."

Eileen: So, they say in the … it says, "It's value *determines* the value of the other variable." And that's what that little girl said the other day. That's interesting. So, I don't know. I was just wondering, like, being a little bit more flexible with our definitions. Saying it a couple different ways. I wonder how that would affect the students' flexibility with the understanding of the concept.

Tess: It's complicated.

Eileen: It is complicated.

Tess: Yeah, I mean, it's sure nicer when you have a very clear definition.

Eileen: Yep.

Tess: Like *mean* or something.

Clark: But in this case, you don't.

Tess: We don't.

Clark: We don't have a clear definition because the independent variable can mean many different things.

Eileen: And like you said, it's total context dependent.

Tess initially gave a definition of independent variable from memory (lines 7–8). When I tried to use her definition, she said it was "funky" (lines 9–15), so Clark read the textbook's definition. This definition used the idea that the independent variable determined the value of dependent variable. Clark and Tess thought that this was complicated and was not a clear definition (lines 27–33) because the independent variable depended on the context of the problem. Even through the teachers did not think that this definition was clear, this conversation gave them another way to think about and talk about variables and how they could support students' learning.

The conversations during the cycles pushed the teachers to think about their understanding of independent and dependent variable. The teachers were able to identify the variables in tasks for themselves, but had difficulty describing their thinking to the students. Based on my classroom observations, I concluded that their descriptions were not supporting student learning because their language was ambiguous and sometimes confusing. The cycles helped the teachers become aware of how they needed to be more precise with their language and communication. I prompted them to assess student thinking to understand why and how students were getting confused. Once Clark and Tess saw that their language was a problem, I prompted them to think more deeply about their own mathematical knowledge and understanding of the concepts.

As the episodes above show, we spent a good bit of time grappling with mathematical concepts to *build knowledge of mathematics*, specifically about what it meant to be an independent or dependent variable. Moreover, we considered what students were thinking and their understanding. By *collaborating* to build this knowledge and reflect on student thinking, we examined the teachers' practices. Through this examination, we were able to *reflect on pedagogical strategies* and think of ways the teachers could facilitate higher-order thinking, such as helping students communicate and reason with mathematics (Jones, 2004; Ladson-Billings, 1994), explain and justify their thinking (Facione, 2009), access and apply their prior knowledge (Raudenbush et al., 1993), interpret problems (Lobato et al., 2005), and using questioning techniques to focus students on conceptual content and meaning (Lobato et al., 2005). But the fact that the teachers were not comfortable with the textbook definition and continued to struggle with facilitating tasks requiring the definition of variables, illustrated to me that the teachers needed more time to think about the mathematical concepts in order to fully internalize a change in how they were thinking about variables.

Positive and Negative Rational Numbers

In Cycles 4 – 7, the teachers planned lessons for the second unit of instruction that focused on positive and negative rational numbers. Using the chip model for integer subtraction was an interesting topic of conversation because the teachers collaborated by critiquing each other's pedagogical decisions and challenging each other's mathematical knowledge. In these cycles, the teachers debated the mathematical goals for tasks and the best way to introduce students to the model and facilitate its use. This was different than the first three cycles where the teachers collaborated by examining their practice and working together to establish definitions and pedagogical strategies that could support student learning.

Clark first brought up using the chip model for representing integer subtraction in the fourth reflection meeting. He told us about how he had introduced the topic by telling students they needed to perform a particular procedure to avoid inevitable confusion. Tess challenged the mathematical accuracy of the method that Clark described, and the teachers debated its meaning. The problem discussed was 5 – 2. As shown in Clark's story, he told the students to create two zero pairs, take away two positive chips from those pairs, and complete the problem by finding and removing all remaining zero pairs. Tess thought that Clark changed the problem from 5 – 2 to $5 + {}^{-}2$. In her mind, instead of starting with a minuend and subtrahend, he started with the minuend, 5, added zero, $5 + ({}^{-}2 + {}^{+}2)$, then removed ${}^{+}2$ to end up with $5 + {}^{-}2$. The issues for Tess were that Clark changed the operation from subtraction to addition and that he took away ${}^{+}2$ from the zero pair rather than the minuend. Clark saw these actions as equivalent.

Clark: It's saying subtract two positives and that's going to leave you with two negatives. And then you're going to pair them off.

Tess: But you're talking the two positives from here [the zero pairs].

Clark: No, we're not. We're taking them from this group [the five positive chips].

Tess: Mm mm (no).

Clark did not see the difference because in both cases he was taking two positives away (lines 1–5), and did not see how he was changing the problem. This episode began a long conversation about Clark's method in which the teachers continued to challenge each other and defend their positions. Clark advocated his method because it worked every time and gave students something they could use for every integer subtraction problem. His students understood how to employ the method and were successfully using it to compute subtraction problems. Furthermore, for certain computations, Tess seemed to use Clark's method. For example, when Tess described how the textbook described how to use the chip model for the problem $5 - {}^{-}2$, she started with 5, added two zero pairs, and subtracted two negative chips from the zero pairs. Clark saw this as the same principle as he was using in his method. I tried to explain the difference between the problems.

Eileen: You're saying—you're saying one, two, three, four, five. In this group of five, I have no negatives. So, what I can do is, I can look at my zero pairs because that doesn't change my expression at all. Now, suddenly, I have my two negatives, and so I can subtract them out of the problem. And I'm left with seven.

Tess: Yes.

Eileen: Okay? That's what Tess just said. If I want to do that same kind of thing with 5 – 2, all right? One, two, three, four, five. I don't need to add a zero pair. I don't have to think. I don't have to because I already have five positives. So, I can subtract two of those positives away.

Clark: True.

The difference was whether or not the students would *need* to use zero pairs to determine the answer. If the subtrahend could be taken away from the minuend, the zero pairs were unnecessary. Clark's method added on an additional step even if it did, as in Clark's words, "get to the point." He thought that even if his method required students to perform an additional step, students could find the correct answer every time by using the method and would not have to worry about "changing signs and all this other stuff." In this case, it seemed that finding the correct answer was important. This method made it easier for his students to see because it was "consistent," and was "just as good" as how Tess described the computations.

I tried to get Clark to think about how the decision to teach this procedure would affect his students' conceptual understanding of the model and operation since they would no longer have to *decide* when zero pairs were needed and why. I challenged Clark to think about how he would handle a situation where a student thought about the problem as Tess did, namely when he or she wanted to take away two positive chips from the five positive chips without creating zero pairs.

Clark: It would be fine, but my thing is you don't have every kid that can do that. You don't have every kid. Yeah, they can look at it and say, "5 – 2 = 3."

Tess: Yeah, five take away two.

Clark: But, but if we're talking about integers and we want them to show us a representation, we want them to be able to actually put it there. Show me a representation of it.

Clark thought that not all of his students would be able to think in this way, and thus wanted to provide a way for everyone to represent the problems. Even though he said he understood that his method required and additional step, it was important for students use it because the additional step would help them understand how to complete the computations. This was an interesting choice because when he talked about engaging his students in mathematical learning, he tried to differentiate instruction based on which student needed additional guidance. However, for this concept, he taught his method to everyone.

Tess expressed concerns that Clark would confuse students by providing them with this method because she planned to follow the textbook and have them decide when zero pairs were needed. In particular, she thought that the students who had Clark for ELT and her for regular mathematics class would have trouble. She asked Clark to not do this method, but he said, "My thing is I'm going to keep doing it this way because it's working. And my kids are understanding it." This created some tension because the teachers disagreed and neither was budging. At this point, I tried to alleviate the tension by asking them to reflect on pedagogy by considering what they could do if students expressed confusion over the two methods. I pointed out that Clark needed to be aware that some students would notice that his extra step was not needed and he should be prepared for questions. He said that he would allow students to use a different method if his confused them, but he would tell them, "If you can do it and come up with the answer, great! But I want you to be able to understand that sometimes it going to be easier for you to do it that way and sometimes it's not." He believed that his method was appropriate and possibly better for students to use. His evidence was based on the fact that his students were able to successfully mimic the procedure and get the correct answers, which was consistent with his understanding of what it meant to do and understand mathematics. In his words,

> And to be honest with you guys, like I said. In my classes—today I started this in my classes and my kids. They like it; they understand; they're able to get up to the Smart Board and explain it to me. And it's working for them. Because they know that, "Okay, I can do it this way. I can see what I need to take off and what do I need to pair up."

For Clark's method, students would have to know to add zero pairs when they saw the subtraction operation. But with addition, they would have to remember to just add the appropriate number of chips. The students would not be considering *why* zero pairs were needed with certain problems, so his method could hinder higher-order thinking because students were performing the same steps each time without considering how it was connected to the conceptual meaning of subtraction (Stein et al., 2000). Tess thought that this was a rule that could cause confusion. In addition, students could misuse the procedure. She said,

> But for me it would be so confusing to know when to put two there and when to put zero there. When my equations look like this, I would be thinking, "Am I supposed to put two there or zero?" How do I know that?"

In addition, the rules for addition and subtraction were not supposed to be discussed until the next investigation. Tess was concerned about following the pacing and wanted to make sure the current investigation built an intuitive understanding of operations and models, as stated in the teacher's guide. She explained to Clark,

> But you're not supposed to be learning how to subtract. You're supposed to be learning how to do that on a chipboard. You're using this equation to model it on a chipboard. It's not, to me it's not the same as if you saw that you could solve it without a chip board.

As we critically examined Clark's method and thought about what it meant mathematically, at no point did we talk about how this method was connected to the important mathematical principle that subtraction is equivalent to adding the opposite. If this connection would have been discussed, we could have talked about developing students' conceptual understanding of subtraction thorough its connection to addition. We reflected on Clark's pedagogy to consider how his choice to implement this strategy affected student learning, but we did not consider how this was an alternative way to build an intuitive understanding of negative numbers and operations with negative numbers. Even so, his method did not correspond with how the textbook introduced and used the chip model.

After Clark left the meeting, Tess confided in me some concerns; that her students would be hearing this approach and get confused. I encouraged her to talk to him about what he was doing in his classroom. In this way, they could continue to collaborate by having critical conversations to learn more about each other's mathematical understandings and pedagogical strategies. Having open lines of communication would help them determine how to handle situations when students expressed confusion over hearing different things in both classes. I told her,

> I'm saying you guys don't have to do the same exact thing in your classroom. But knowing that that's what he's doing and what he's saying will prepare you for if a student says, "Well, Mr. Welling told me to do this." [You can] say, "Well, it's actually the same thing." You know? You know, you could show them [how the methods are similar].

The fourth reflection meeting described above was collaborative because the teachers challenged and critiqued their practice and mathematical ideas. The meeting helped us build knowledge about mathematics; namely, the chip model and the mathematical goals of the investigations. We reflected on pedagogical strategies, Clark's method, to understand how it facilitated or hindered higher-order thinking. Clark thought his method was easier for students to do and a good way for them to understand how to subtract negative numbers using chips. He made this assessment based on prior teaching experiences from last year when his students struggled with understanding the model and integer subtraction. Clark thought that even if his method was procedural, it was justified because it gave the students a reliable tool to make computations and find correct answers they could use to draw conclusions.

In the fifth planning meeting, I challenged Clark to reconsider this stance. I wanted him to think about the mathematical goals of the task again and to reflect on his pedagogy to see if the two matched.

Eileen: I thought the goal of [Problem] 1.4 was to start to have the students to build an intuitive understanding about what it means to use these chip models and what it means to subtract and add positive and negative integers. So, if you immediately go to this [Clark's method], it's to me is more procedural, more like a rule, as opposed to building an intuitive understanding that I want to take away more than what I have. What do I need to do? I need to add a zero pair.

Clark: Hmm.

Eileen: I mean, you know, this is a fine strategy. And you're going to get the right answer. And it's going to work every time with subtraction. But does it get at what you want them to walk away with from 1.4? And if you—and if what you want them to walk away with in 1.4 is building an intuitive understanding about these integers, I don't know if this helps. This'll help later in section 2.2 when they're starting to have to develop algorithms for how to subtract, but I don't know if it helps here.

Clark: Hmm. Never thought about it like that. But most kids [are] going to get confused when they see this part here because they're going to say, "Well, I got a negative eight here and if I take all these away then I have nothing left." At which point we're talking about zero pairs. That's why it to me it seemed better to come here to explain what zero pairs are and how it would help.

Eileen: Well, you do explain zero pairs. Because, like, let's say you have something like, um—I mean. I mean. (Sigh) All right, ask that question again. Because to me, it's like, if I have ${}^{-}7 – {}^{-}8$, I look at my group of negative seven things. Can I take away eight? No.

Clark: So how do I do it?

The task was a stepping-stone towards creating algorithms for subtraction, which would be talked about in the next investigation. Clark's method proceduralized the concept (lines 1–15), especially if its connection to addition was not made explicit. As Clark considered my ideas, he realized that he had never thought about it that way (line 16) but was concerned whether his students would be able to understand the textbook's approach (lines 16–20). This concern, a product of the previous years' struggles, was the reason he presented the problems using his method.

As we worked through some problems and talked about the goals, Tess joined the conversation to offer support.

Clark: Uh … it makes sense … for me … but my thing is will it make sense to my kids?

Eileen: Oh, yeah, I mean, I don't know.

Clark: That's the thing.

Tess: My kids understand it the way the book has it and the way I've been teaching it. I've got some pretty low kids. Because it seems like it's more directly what it's saying. Like, an operation is an operation. You're adding or subtracting. So you start with your five and then you do what you have to do to it. So, you don't do another step first. You either see "Can I take away three?" Yeah. I could just take away three or, "If I, I can't take away seven and in that case I'm going to have to add two zero pairs or," yeah. Because I can take away five and then two positives and then I'd end up with $^{-}2$. But I don't see why I would add seven zero pairs and then … Does that make sense? Are you following me?

Clark: I'm hearing you all and I'm understanding what you're saying. I understand exactly what you're saying.

Tess: Okay. So I don't need to write it.

Clark: But I'm just looking at it—some consistency. Because there are gonna be some cases where you gotta use the zero pairs. And what I was trying to do was eliminate the guess work and saying, "Okay, if I did this, this way every time—and it does work every time—then I can see my patterns and then I can generalize what is really going on here."

Tess supported Clark by telling him that even her "low kids" could do it (lines 5–6). She pointed out that thinking about subtraction using the textbook's strategy was related to the operation of subtraction because the students would be thinking about *subtracting* from the minuend rather than *adding* zero pairs to be removed (lines 6–13). In lines 18–22 Clark reiterated his intent in teaching the method. He thought he would facilitate higher-order thinking by giving his students tools to consistently compute subtraction problems so they could use their answers to make conjectures and draw conclusions (Facione, 2009; Facione, Facione, & Giancarlo, 2000).

This meeting displayed how we used collaboration by examining, critiquing, and supporting pedagogy and mathematical thinking while we discussed the subtraction integers using the chip model. We continued to build knowledge about the mathematical concept and goals of the tasks by comparing Clark's method to other ways of using chips and zero pairs. We built knowledge about the textbook by discussing how this investigation was connected to the next investigation, and the purpose of the learning trajectory in the textbook. Finally, we reflected on the pedagogical strategy of presenting a procedure and discussed how this method could hindered higher-order thinking if the understanding of the procedure shifted from the meaning and connection to mathematical principles to the correctness of the computations (Stein et al., 2000).

Clark's method resurfaced during the sixth planning meeting when he described an event in his classroom where one of his students successfully used his method to compute the problem, ${}^{-}5 - {}^{-}7$. As the student applied the method, he came up with the generalization that subtracting a negative was equivalent to adding a positive. Initially, the student computed the answer to be $^{-}12$ using the number line. Clark encouraged the student to try the problem again using the chip model. This was a pedagogical strategy we had talked about in previous meetings that facilitated higher-order thinking. Specifically, we had discussed how the teachers could model metacognitive strategies, such as self-correction and self-questioning (Facione, 2009), to help students use higher-order thinking. As the student did use the chip model to compute the expression, Clark had him describe his steps.

Clark: So he took away seven negatives. I said, “Okay, so now what?” He said, “Now I got to take these pairs and pair them together.” And he said, “Oh, I got a positive two.” I said, “So what happened with the number line and the chips? They don’t coincide, so something’s wrong. What’s going on?” I’m trying to make him look at it.

Eileen: Yeah. Yeah.

Clark: And he said, “Um … I’m adding.”

Eileen: Because subtracting a negative’s the same as adding.

Clark: “Oh,” he said, “I’m adding here.” And I said, “Huh?”

Eileen: How did he know it was adding?

Clark: Because he said, “The only way this could be a positive two, I had to add. Something had to be positive somewhere.”

Eileen: Yeah, so he really—.

Clark: And so he looked at it and he said, “Well, this had to be positive now, so that’s adding.”

Eileen: That’s interesting.

The student calculated the answer using the chip model and saw that it was not the same as his previous answer. He decided the chip method was correct without considering why his answers were inconsistent. Clark asked the student to think about what had happened (lines 3–5) in order to facilitate higher-order thinking by having him evaluate why the statements were contradictory (Facione, 2009). The student instead made a generalization about subtracting a negative number (lines 7–15), which is what Clark wanted his students to do; that is, use the method to draw conclusions about computations. Thus, this event most likely reinforced Clark's belief that his method was beneficial for his students; that is, his student successfully calculate a subtraction problem, and used his answer to make a generalization about subtraction.

This event showed that our conversations during the last two cycles about the chip model and subtraction may not have been internalized by the teachers. Clark did not challenge the student when he employed Clark's method. But neither Tess nor I challenged Clark on the use of his method during class. This could be evidence that our knowledge was still developing and that we needed to have more conversations and experiences with the model and concepts before being consistent in how we wanted to use the model; that is, using the textbook's method or Clark's method.

Our conversations about the subtraction of integers using the chip model were *collaborative* in the sense that we examined instructional strategies and classroom events and supported teachers in figuring out how to address student thinking in ways that would facilitate higher-order thinking. However, the teachers also challenged and critiqued each other's pedagogical decisions and mathematical understandings. In these discussions, I helped Clark and Tess *build their knowledge about mathematics* and the textbook by engaging them in conversations about the chip model, how it could be used to facilitate student learning, and the mathematical goals of the investigations in the unit. However, based on our inability to connect Clark's method with an important mathematics principle (i.e., subtraction is equivalent to adding the opposite) and to reflect on pedagogical decisions to use the method illustrated that additional work with this mathematical concept would be necessary in order for the teachers to fully understand the repercussions of their instructional choices.

Summary

The reflective teaching cycles helped Clark and Tess become more reflective in their practice and by doing so, influenced the way they selected and implemented tasks. The *collaborative nature* of the reflective teaching cycles influenced the teachers by providing them with a venue to examine, critique, and support each other's practice as well as to examine and reflect upon their own practice. By working together, the teachers were able to see aspects of the other's practice that was not visible to the other teacher. This helped them see how certain decisions facilitated or hindered their students' opportunity to engage in higher-order thinking.

The cycles helped Clark and Tess *build knowledge about the mathematics* they were expected to teach and how their textbooks presented this material. The teachers would ask questions about the meaning of certain concepts or representations. We would collectively try to understand these mathematical ideas and the mathematical goals of the textbook to see how they supported student learning. Once the teachers knew more about tasks and the mathematical goals of the tasks, they made decisions about what tasks they would implement and how they would do so.

Finally, the cycles helped Clark and Tess *reflect on their pedagogical strategies* and to begin to understand how their pedagogical strategies may be affecting higher-order thinking. With this knowledge, they were then able to decide if they wanted to make changes in their instruction. They thought about how they could facilitate higher-order thinking, if they already were, and how this may be happening. Specifically, they were able to reflect on whether their instruction actually accomplished their stated learning goals for their students concerning higher-order thinking.

CHAPTER 5

DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

> I'm learning a lot. I appreciate it because I'm learning a lot. And you know what? It's making me a better teacher … And it's actually helping me because what I do…I have a reflection that I write into it. And that helps me, so it makes me think about what I'm doing wrong and what I can do to make things better for the kids. And so I can go and type that stuff in: how I can change the lesson and the way I presented it.
>
> Clark, Reflection Meeting, Cycle 3

> Me too. And also, it keeps me flying right. It's always easy for my attention to go in different areas and think about the problem behaviors or, you know. This really helps me stay focused on what's the most important thing is like, what I can do better.
>
> Tess, Reflection Meeting, Cycle 3

As evident from the comments above, Clark and Tess thought that the reflective teaching cycles were helping them become better teachers. The cycles gave them a space to engage in conversations about teaching and learning mathematics. In order to figure out how to teach their students, the teachers thought about issues they faced as teachers, such as discipline, testing, student engagement, understanding mathematics and their textbooks, and what it meant to learn mathematics. Their main concern was how they could teach their students *better* and what *they* could do better to help their students learn mathematics.

Discussion

This study took place within the larger context of the two-year research and professional development project, Project ISMAC. Over the course of Project ISMAC, Clark and Tess developed into a *high-functioning teacher team* because they engaged "in collective dialogue about student engagement and achievement, the effects of practice on student performance, and how to provide an appropriate level of challenge and support to every student" (Gajda & Koliba, 2008).

When Project ISMAC started, the teachers were focused on procedures and rules. Their students had not performed well on the CRCT the previous year, and they were having problems facilitating student-centered lessons in which their students explored tasks and actively took part in developing their own understandings of the content. However, from the beginning of Project ISMAC, Clark and Tess committed to regular meeting times that were structured and focused on the specific goals of improving student achievement and changing their pedagogy. They were dedicated, motivated, and conscientious teachers who were interested in learning about mathematical content and how to teach mathematics to help their students be successful.

During this study, we focused on facilitating higher-order thinking. As we worked together through the reflective teaching cycles, Clark and Tess began to think more deeply about how to support students' development of conceptual understanding and to be able to identify when they were or were not facilitating higher-order thinking. They still believed that students needed to develop procedural knowledge, especially for content that was confusing or did not lend itself to real-life connections. However, they tried to encourage conceptual understanding through selecting and implementing tasks that had the potential to facilitate higher-order thinking by using the textbook and framework tasks that were of high demand. They worked on introducing tasks by building on students' prior knowledge (Stein et al., 2000) and connecting the mathematics to the world and the students' experiences in a meaningful way (Jones, 2004; Ladson-Billings, 1994). They tried to use questioning techniques to encourage students to evaluate, justify, and explain their thinking (Facione, 2009) and to apply, connect, and extend new and prior knowledge to make predications and conjectures (Lewis & Smith, 1993; Thompson & Zeuli, 1999). However, there were times that Clark and Tess hindered higher-order thinking through their task choice and implementation.

Since the teachers were using the CMP2 curriculum, which was designed using higher-order tasks, the issues of higher-order thinking came up when the teachers supplemented the text. When Clark and Tess felt that they needed to provide the students with additional problems for particular concepts, they chose to use other mathematical texts that they were familiar with or they created their own problems, which were frequently lower demand and would not facilitate higher-order thinking. Generally these tasks focused on explicit procedures or steps to perform without connections to conceptual meaning (Stein et al., 2000).

For implementation, Clark and Tess talked about facilitating higher-order thinking strategies, but during my observations and as they described their actions in the classroom, I noticed that they were not always successful at doing so. For Clark, sometimes his understanding of the concept, as with the chip model, or unfamiliarity with the types of questions that could facilitate higher-order thinking, as with his fill-in-the-blank questions, hindered his ability to teach as he intended. Tess regularly reviewed the textbook and teacher's guides, which helped her understand the textbooks' suggestions for questioning and introducing tasks, and she talked about following these suggestions. She generally planned to implement lessons based on the textbook's teacher guides, and saw the validity in this (i.e. to build their number sense). However, during her instruction, she sometimes altered her plans and reduced the cognitive demand of tasks, which hindered higher-order thinking. This was because of her teaching experiences, including time and classroom management, and her beliefs about students' ability and motivation to engage in challenging mathematics.

The previous chapter described what influenced the teachers' selection and implementation of tasks and specifically the reflective teaching cycles' influence. In this discussion, I will illustrate how a series of reflective teaching cycles influenced the way the teachers *chose* and *implemented* tasks that had the potential to facilitate higher-order thinking. I will also discuss how the teachers considered their students while selecting and implementing mathematical tasks.

Reflective Teaching Cycles

The reflective teaching cycles influenced how Clark and Tess selected and implemented tasks by providing them a space to learn about facilitating higher-order thinking and changing their practice. The cycles provided the teachers time to think about mathematics and pedagogy, helped them learn more about their textbook, and to consider what it meant to do mathematics. The engagement in the cycles went beyond the teachers' participation in Project ISMAC and provided them with different learning opportunities.

Project ISMAC did not have the structure inherent in the cycles. The cycles had structure within their individual components as well as across the series. Although Project ISMAC's team met with the teachers and helped them plan on a weekly basis, we did not regularly reflect on what happened after the teachers implemented their lesson. In contrast to this, the regular planning, teaching, and reflecting components of the cycles worked together and provided time for the teachers to reflect on each lesson that we planned together as well as on other events that transpired during the week. We were able to both plan *and* reflect in each cycle. Also, the planning and reflection meetings overlapped, as the teachers regularly engaged in both of these activities each time we met. This was unlike Project ISMAC where the meetings were clearly focused on planning.

Project ISMAC's team was in the school on a weekly basis and we tried to attend all grade-level common planning meetings as well as several classes. However, because of administrative duties, testing, and other school-wide responsibilities, we were not always able to meet with the teachers or engage in the activities we had scheduled. On the other hand, the teachers and I always met for our planning and reflection meetings and were always able to concentrate on their practice. Moreover, I was always able to attend their classes to observe how they were implementing the task we had discussed. But there was more to the observations than this.

Project ISMAC's team worked with the teachers in their classrooms by co-teaching, doing demonstration lessons, and working with students during class. During the cycle's observations, I tried to *not* intervene with Clark and Tess's instruction since my intent was to understand how the teachers were selecting and implementing tasks. However, since I played a dual role of researcher and Project ISMAC team member, the students were used to having me be a part of the class discussions. Thus, to balance my two roles, I wound up helping students during the observations while they worked individually or in groups. This helped me formulate questions for planning and reflection meetings built on specific examples of student's thinking, which allowed us to have rich discussions when we met about their practice and its influence on student learning.

Below I discuss how the influential components of the cycles helped me answer my research questions about how the series of reflective teaching cycles influenced the way the teachers chose and implemented tasks.

Collaboration. Clark and Tess's established relationship contributed to the reflective teaching cycles' impact on the teachers. They began working at College the same year in the same grade level, and this was their third year working together. They regularly expressed how much they valued each other's input and support. They were comfortable talking about what they wanted to change about their own practice, their strengths and weaknesses, and what they did or did not know about mathematics or CMP2. This type of relationship was cited by Kitchen and his colleagues (2007) as characterizing highly effective schools that serve the poor; that is, faculty who collaborate, supports each other, and focuses on building relationships. By building close and trusting relationships, teachers in Kitchen's study could focus on their teaching rather than on students with behavioral issues. They chose to make time to collaborate so they could talk about what was expected, how to help students, and what needed to be done or done differently. They used this time to figure out the curriculum, mathematics content, and lesson plans in a safe and encouraging environment. Kitchen and his colleagues (2007) felt that collaboration was the "one magic bullet" (p. 128) to affect students' learning and achievement. They characterized collaboration as work with colleagues to modify and write curriculum, share teaching ideas, and discuss students' strengths and weaknesses. They knew that there was little doubt that the collaboration the teachers engaged in was among the primary reasons why certain schools were highly effective. In a similar way, Clark and Tess used their collaboration to become more effective teachers.

The cycles provided the teachers with a chance to have regular conversations in which they could critique, examine, and support each other's practice. This collaboration allowed them to reconsider how they selected and used tasks to support students' higher-order thinking. There was value in the teachers' collaborating because they were able to talk about teaching issues, which is an important first step in changing practice. Issues must first be recognized and identified before they can be changed. In particular, the teachers were able to normalize their problems of practice (Little & Horn, 2010) and offer each other support. This support allowed Clark and Tess to focus on their instruction, as evident in their quotes that opened this chapter.

Pappano (2007) concluded that collaboration needed to focus on specific goals, such as higher-order thinking, and needed to have teachers reflecting on what and how they want their students to learn. Clark and Tess participated in many forms of collaboration such as offering advice, suggesting approaches to tasks or concerns, generally helping one another with daily classroom work, and engaging in conversations that prompted improvements in overall instruction and changes classroom practices. They regularly engaged in these practices during the cycles as they talked about discipline strategies, instructional techniques, and created assessments together. The nature of their relationship and desire to learn about teaching and learning allowed them to critically think about their own practice and challenge each other to explain and defend pedagogical choices. This represented a commitment to change that was essential for their individual growth as teachers (Briscoe & Peters, 1997).

The nature of the teachers' collaboration changed over time and across instructional units. During the first instructional unit, the teachers spent more time supporting each other and examining each other's practice in nonthreatening ways. When we began to discuss the second unit, and specifically the subtraction of integers, the collaboration became more critical and challenging. At times, I became uncomfortable with the conversation and tried to alleviate tension in the meetings. Although I was uncomfortable when the teachers began to argue, as I reflect on these meetings, I am glad that the conversations became critical. The teachers' needed to experience discomfort in order to challenge their own knowledge and thinking. Therefore, facilitators need to learn how to deal with discomfort and welcome these types of exchanges. One way to do this is to focus the teachers on how to settle their disagreements by considering "why the issue related to the disagreement matters and how it might be addressed in their own classroom" (Crespo & Featherstone, 2006, p. 101).

Building knowledge of mathematics. The cycles provided the teachers with regular times to think about mathematics teaching and learning. The cycles also gave them time to think about the mathematics they were expected to teach and to work to understand how the textbook could support student learning. This opportunity to share ideas of their practices helped them build knowledge of content and pedagogy, which was another necessary factor in their individual growth (Briscoe & Peters, 1997). Clark and Tess showed growth as teachers when they were able to plan assessments based on mathematical goals and student learning and when they understood the need to spend more time working through tasks to gain a better understanding of the content before they implemented them in class. Clark showed growth by beginning to work through tasks and read the teacher's guide before meetings. By doing this he was better able to engage in conversations about the mathematics in the tasks, which helped both of them concentrate on students' conceptual understanding of the material and advance students' conceptual understanding (Superfine, 2006).

The teachers' mathematical knowledge developed throughout the series of cycles as is evident from their continued attempts to choose and implement tasks that were of high cognitive demand and maintain the level of tasks. According to Stein and Kaufmann (2010), teachers are able to implement high-quality lessons when they prepare lessons by taking into account the big mathematical ideas within the textbook. *High-quality lessons* are ones in which the cognitive demand of the task is high and teachers maintain that demand. But, if teachers do not understand the mathematics in tasks, they may "fail to recognize the mathematical integrity of the task, thereby altering it in ways that (unintentionally) change (and often reduce) the level of cognitive demand of the task" (Stein & Kaufmann, 2010, pg. 671). This could have been why Clark proceduralized the integer subtraction task. Furthermore, if teachers do not understand the mathematics in the task, they may not be able to "appreciate mathematical insight to be gained from students' devising various routes through the problem space" (Stein & Kaufmann, 2010, pg. 671). This could have been why Tess gave her students the slope-intercept form equation of a line to use to represent a linear pattern of change instead of giving them the opportunity to explore the concept on their own. But in general, I found that the teachers' were developing their understanding of mathematics, and by doing so were becoming better able to focus on advancing their students' conceptual understanding and supporting their students' thinking in order to facilitate higher-order thinking.

Reflecting on pedagogy. Briscoe and Peters (1997) found that collaborative meetings were a valuable opportunity for teachers to reflect on what pedagogical strategies worked or did not work, and helped teachers continue to try problem-centered activities. Initial change in teachers' practices occurred because they had "the opportunity to plan together, to share ideas, and to discuss with one another alternative ways of teaching" (p. 57). Clark and Tess were changing their practice through their reflection of their pedagogy and their participation in the reflective teaching cycles.

As Clark and Tess reflected on their teaching, they thought about the types of tasks they wanted to use and the strategies they could employ to encourage students to use higher-order thinking. Even if the teachers needed several cycles to think about particular tasks or to try pedagogical strategies, the cycles encouraged them to think about changes and consider alternatives to traditional instruction. If Clark and Tess did not have the opportunity to engage in this type of talk, it is possible they would not have reflected on such issues. Therefore, it is important that teacher educators provide sustained and ongoing professional development that incorporates regular reflection.

Sustained professional development takes into account the fact that it takes a long time for teachers to internalize changes in their practice and to be able to apply those changes in their classroom (Arbaugh & Brown, 2005). Teachers need to know what it is that they want to change or should change, and to be aware of what they are doing before they are able to make changes. For example, Clark talked about Project ISMAC showing him how what he was doing last year was wrong and what he wanted to change; that is, guiding students and telling them what to do. Furthermore, Clark changed how he used questioning to hold students accountable for higher-order thinking over the course of the cycles. During the first instructional unit, Clark asked questions such as, "Independent what?" During the second unit, he asked his students to justify and explain their thinking, which facilitated higher-order thinking. For Tess, she did not understand *fact families* or how to use them in instruction until the last cycle. We had talked about it in Cycle 2 and Cycle 4, but it took her until Cycle 7 to internalize the concept and apply it in her instruction. The fact that many issues we discussed about pedagogy and mathematics reoccurred throughout the cycles illustrated the need for continued conversations.

With the knowledge of how their pedagogy could be impacting higher-order thinking, Clark and Tess were able to decide if they wanted to make changes in their instruction. They thought about how they could facilitate higher-order thinking, if they already were, and how this may be happening. During the reflection meetings, I would ask Clark about the times that his actions did not correspond with his intentions to facilitate higher-order thinking. He reflected upon what happened and why. He thought about what he could do differently in the future and listened to my suggestions on how to alter his practice. Clark's prior teaching experiences contributed to his reflection and willingness to act because of past success he had in Project ISMAC. In contrast, sometimes when we reflected on Tess's instruction, she would give reasons for why things happened the way they did that were not related to her instruction; that is, she ran out of time, the bathroom break was too disruptive, or she had to help students who started to struggle. Her reality in the classroom may have limited her ability to reflect on her practice and when she tried to implement higher-order thinking, she had a hard time seeing past the students who would disengage to realize there were others that were engaged and enjoyed the challenge. The cycles gave Tess an opportunity for her to talk to Clark and I about upcoming tasks. She had a chance to hear what Clark was going to do or did, and to hear my suggestions for what tasks to use and how to implement them. This helped her start to focus on her practice rather than the external factors.

Generally speaking, Clark was more comfortable talking about what he had already done in class rather than what he was going to do because he typically had not read ahead or looked at the teacher's guide. Clark tried to employ strategies that we discussed during the cycles and considered alternatives to his current instruction. Tess was more comfortable talking about the task she was going to teach and how she planned to implement that task rather than what she had done and why. Further research is needed to understand how and why Clark and Tess reacted to the cycles in these ways and their beliefs about the purpose of the cycles.

Facilitator. Another major influence of the reflective teaching cycles was my presence as a facilitator in the meetings. Research has shown the importance of having a facilitator present in teacher learning groups (Arbaugh, 2003; Arbaugh & Brown, 2005; Crespo, 2006), and during common planning time (Brown, Arbaugh, Allen, & Koe, 2000). Brown and colleagues (2000) found that the presence of a facilitator, or "expert other," during common planning time increased the quality of teach talk during planning meetings. Facilitators are able to promote and sustain discussions in study groups by pushing teachers to elaborate on their ideas, encouraging them to comment on other's ideas, and challenge teachers to reflect on their mathematical knowledge and teaching (Arbaugh & Brown, 2005; Crespo, 2006). Prompts and questions that engage teachers in conversations play a significant role in promoting collaboration among teachers by helping them actively examine and critique the ideas of others (Crespo, 2006). I also found that my role as a facilitator in the planning and reflection meetings during the cycles influenced how the teachers talked about their selection and implementation of tasks.

My facilitation played a role in the teachers' ability to consider student thinking and learning while selecting and implementing tasks. For example, when the teachers planned for an assessment during the second cycle, they based their conversation on pacing and what they needed to cover. Later in the fifth cycle, the teachers planned an assessment based on mathematics and student thinking. In the planning meeting, they worked through a disagreement about when to give a quiz by discussing the mathematical goals for the assessment and basing their arguments on student learning; that is, spending more time on future concepts versus building a strong foundation. In the later cycle, the teachers' based their decisions on their understanding of mathematics and the mathematical needs of their students.

Arbaugh and Brown (2005) found that their facilitation in a teacher study group changed over time. In the beginning of the professional development experience, Arbaugh, as the facilitator of the group, acted as an "expert other" by "asking probing questions and challenging the teachers to reflect verbally on their knowledge and teaching" (p. 507). However, the researchers found that Arbaugh's guidance as the group's facilitator became less evident as the year progressed "as the teachers became more comfortable with guiding their own learning" (p. 507). Similarly, I found that Clark and Tess became more comfortable asking each other the types of questions I used during our meetings. These questions intended to focus their attention on how and why they selected particular tasks, how they planned to introduce tasks and use questioning to facilitate higher-order thinking, and to reflect on the influence of their actions on student learning. However, since the series of cycles was for a short duration of time, I found that my facilitation did not diminish over the series of cycles, and I continued to focus our discussions on our goals and to ask the majority of the questions. An extended period of time with a facilitator in reflective teaching cycles would have given the teachers the time to learn how to ask themselves different questions about their practice. They could have guided their own learning by reflecting on what pedagogical strategies helped facilitate higher-order thinking. They could have used the cycles without facilitation and continued to learn how to respond to students' thinking and become more comfortable with conversations in the classroom being driven by the students.

Since the cycles occurred outside of the regular school day, sustaining them would have been difficult. If we had been able to meet during their common planning period, this limitation could have been avoided. This was not possible because the math coach, Betty, ran the planning meetings and could not allow us to conduct our cycles at those times. One of the reasons for this was that Betty was required to carry out particular administrative duties during common planning. She had been hired in part because of the school's poor performance on the CRCT, so a large component of her job concerned monitoring student performance and helping teachers develop strategies to improve test scores. When she had the teachers plan lessons, I found that she frequently lowered the level of the conversation to focus on surface features of the tasks or how the teachers could incorporate instructional tools, such as graphic organizers, into their lessons. Similar to Lamberg and Amador (2009) I found that Betty had a difficult time allowing the teachers to engage in discussion while making sure that she had accomplished a particular goal for the meeting. She would regularly shift the conversation away from the teachers working through the tasks, discussing mathematical content, or reflecting on events that transpired in their classrooms, to setting the pacing guide or talking about what students needed additional interventions to help them pass the test. This derailed conversations that Clark and Tess attempted to have that would have contributed to their learning.

It is important that mathematics coaches who are responsible for facilitating common planning time learn how to ask the types of questions that promote teacher learning. My role as a facilitator in the reflective teaching cycles could serve as a model for coaches. I provided the teachers with broader views on issues of teaching and learning and helped support teacher learning of mathematical content. I helped teachers engage in discussions that advanced their insight into mathematics and pedagogy. These contributions in our planning and reflection meetings were the types of involvement that a mathematics coach charged with affecting student achievement should give.

Consideration of Students

This study also sought to answer the questions, “In what ways do mathematics teachers consider students while *choosing* mathematical tasks that have the potential to develop higher-order thinking?” and “In what ways do mathematics teachers consider students while *implementing* mathematical tasks that have the potential to develop higher-order thinking?” Clark and Tess considered their students by attending to how their students were going to engage in mathematics. When Clark considered how to engage his students, he thought about helping them feel confident doing mathematics. But, he thought that he could do this early in the year so he could push them harder later. He thought that some students needed to be challenged and that others needed closer monitoring and additional guidance in order to engage and remain engaged in classroom activities. He believed he could accomplish this through differentiating instruction and considered how he could do this through his task choices and implementation strategies.

Tess strongly believed in the power of real-life contexts and making meaningful connections to students' lives. These connections were crucial to student' engagement in tasks and she spent a good bit of time considering what contexts she could use for the mathematical concepts she was teaching. Tess also thought that she should engage her students by choosing tasks that would build students' confidence, were fun, and did not cause frustration. She wanted to avoid too much struggle during implementation because, in her experience, when students started to struggle, they disengaged. She thought that her students needed more guidance in order to engage in tasks and to remain engaged in problem solving. Therefore, even though Tess wanted to facilitate higher-order thinking in her practice, her views on how to best engage her students sometimes caused her to not be able to follow through on her intentions. In particular, Tess sometimes chose tasks that would lead students to correct answers (Doyle, 1988; Henningsen & Stein, 1997; Romagnano, 1994). She avoided or altered difficult tasks because she worried about classroom management and students' ability or motivation to engage in tasks. She implemented tasks in whole group discussions that focused attention on the communication of her mathematics rather than supporting the development of students' mathematics (Lobato et al., 2005).

Other considerations that Clark and Tess had for their students were how they could help their students gain conceptual understanding of mathematical content, and how they could help their students be successful on standardized tests. Clark considered how to accomplish these goals by reflecting on and reacting to student thinking, which allowed him to differentiate his instruction as he saw appropriate. But his understanding of what constituted a rule or procedure affected his practice. He talked about the value in supporting students' conceptual understanding and not relying solely on rules, procedures, or algorithms. However, Clark provided his students with a procedure for using the chip model with integer subtraction and did not see this as problematic because the procedure was one that he came up with himself based on his understanding of the concept and model. The procedure was not a standard mathematical rule, such as two negatives make a positive, and he distinguished these two types of procedures. He thought that when he taught *his* rules or procedures, he was helping to facilitate higher-order thinking by helping the students comprehend a difficult mathematical situation and make a connection between the model and the operation (Lobato et al., 2005).

Tess wanted her students to gain conceptual understanding, but her interpretation of the standards and the mathematical goals of the textbook influenced what she thought her students should do during lessons. If a concept was addressed in detail in a later unit or standard, and was needed for the current task, Tess covered the concept procedurally to provide her students with the means to finding the correct answer. As she considered her students, she thought about what guidance she would give during class and how she could help them access the mathematics. She also became more elemental about concepts if her students did not initially understand them. It was important that her students knew about and could apply rules and algorithms so they could be successful on assessments.

The teachers wanted students to succeed, learn how to think, and do the best they could. The fact that Clark and Tess engaged in the cycles as a team helped them normalize their problems of practice (Little and Horn, 2010), which led to their ability to work through challenges, find solutions, and focus on their practice. The reflective teaching cycles were a needed space in which the teachers could take the time to think about their practice, their students, and how to best teach them. Through the cycles, Clark and Tess began to focus on what they could do rather than on discipline problems. They continued to support each other in determining strategies to engage students in learning, which sometimes required thinking about discipline actions such as point deduction for non-participation. However, they focused on their practice and said that the cycles were helping them do this. Kitchen and his colleagues (2007) similarly found that teachers at highly effective schools serving the poor made conscious decisions to focus on structures they could control, such as ones they had created to support students' mathematics learning, rather than on "students circumstances that were beyond their control" (pg. 164).

Implications

There is a growing body of research on the practice of facilitating teacher groups and on how facilitators can prepare of this practice (Crespo & Featherstone, 2006). Since "engaging teachers in professional conversations of teaching is not without serious challenges" (Crespo & Featherstone, 2006, p. 99), more research needs to be done to help teacher educators learn how to facilitate teacher groups and what helps support teacher learning. Lord (1994) stated that teachers needed to learn how to engage in a "critical colleagueship," which he defines as "an alternative professional stance where teacher move beyond sharing and supporting one another through the change process to confronting practice – the teachers' own and that of his or her colleagues" (as cited in Crespo & Featherstone, 2006, p. 100). Further research on reflective teaching cycles should focus on how the facilitator can help encourage critical colleagueship. A measure of this colleagueship could be the level and type of discomfort that occurs during conversations and how well the facilitator encourages the teachers to report failures and confusions as well as successes in their practice (Crespo & Featherstone, 2006). Clark and Tess were able to engage in critical colleagueship because of their participation in the reflective teaching cycles, as evident from the change in their collaboration from support and examination to critique and challenge.

This study prompted other questions about the effect of a facilitator, like myself, on reflective teaching cycles. How could teachers engage in reflective teaching cycles on their own? Would they experience the same collaboration, knowledge development, and reflection? What questions did I ask the teachers or what actions did I take that prompted the teachers to reflect and want to implement change? Little and Horn (2007) found that the leadership or facilitation in collaborative meetings was important in "sustaining the group's attention to problems of practice" (pg.90). The facilitator was able to pose questions, prompt teachers to give accounts of their classroom practice, and preserve a focus on mathematic teaching and learning. How to keep the focus of meetings on teaching and learning, helping teachers see how the issues in the classroom are worth examining, and encouraging reflection are questions that should be answered. This research could help teacher educators and professional developers determine the most effective way to use reflective teaching cycles and the types of facilitation that would be most successful at promoting generative change in teachers' instruction.

Assessing the influence of professional development should be based on its impact on student learning (Smith, 2001). However, professional development needs to be sustained in order for generative change to take place (Garet, Porter, Desimone, Briman, & Yoon, 2001; Wenglinsky, 2000). The series of reflective teaching cycles that I conducted with Clark and Tess only occurred over an 8-week period. This time period was too short to consider how the reflective teaching cycles affected student learning. But, it is interesting to note that before Project ISMAC, College Middle School had not made AYP in over five years, and after working with the Grade 7 teachers for two years, the seventh-grade students were the only ones in the school that passed the CRCT in the spring. There were factors that could have contributed to this improvement including the teachers becoming more familiar with the curriculum, the teachers' district-facilitated professional learning activities, or the school's test initiatives such as after-school and Saturday tutoring. I believe that the teachers' ability to participate in ongoing collaborative planning through the reflective teaching cycles during this time was a factor in the achievement success of their students. This improvement supports the findings of Saunders and his colleagues (2009) that teacher teams that were given time to collaborate in structured meetings increased students' achievement. This could also be further evidence that when teams "engage in high-quality dialogue, decision making, action, and evaluation around a shared purpose" (Gajda & Koliba, 2008, p. 145) they can improve student achievement. So while I believe that the reflective teaching cycles played a role in the students' increased performance, more research is needed to measure this effect.

An alternative way to assess the cycles' effectiveness would be to "look for changes in what teachers know, how they think about teaching and learning, and what they do in their classrooms" (Smith, 2001, pp. 51–52). In particular, since the goal of the reflective teaching cycles was to influence the way in which the teachers chose and implemented tasks that had the potential to facilitate higher-order thinking, the effectiveness of the cycles could be measured by determining how the cycles affected the teachers' knowledge of higher-order thinking, mathematics, and pedagogical strategies.

I worked with Clark and Tess for a year before data collection began and felt that I had a good understanding of their knowledge and beliefs about mathematics teaching and learning. Thus, I did not collect data on the teachers' knowledge and beliefs before we started the series of reflective teaching cycles. Nevertheless, researchers interested in using a series of reflective teaching cycles with teachers to influence their practice should collect data before the cycles begin to establish a baseline measure of the teachers' prior experiences; mathematical knowledge, including what they think it means to do mathematics; and pedagogical strategies, including what they think is important with respect to engaging students in mathematics. This information should be used to during planning and reflection meetings. The facilitator should cater the cycles to the teachers by building on their prior knowledge and experiences. During the cycles, facilitators interested in influencing teachers' selection and implementation of tasks can continually evaluate the affect of the cycles by looking for changes in teachers' collaboration, mathematical conversations, reflections, and openness to new ideas. After the series of cycles has concluded, the facilitator can assess the extent to which the cycles affected the teachers by reexamining teachers' knowledge and beliefs.

Final Thoughts

My research contributes to the literature on professional development in urban schools by helping to understand how reflective teaching cycles can influence the ways in which teachers select and implement tasks that promote higher-order thinking. It extends our understanding of using reflective teaching cycles as a context for professional development for mathematics teachers and begins to fill a gap in our knowledge about using reflective teaching cycles with middle school mathematics teachers through different instructional units. This research highlights important outcomes for middle school teachers who participate in reflective teaching cycles, particularly in regard to collaboration, building mathematics knowledge, and reflecting on pedagogy. It is important for us to understand these outcomes since instruction focused on higher-order thinking is not typically used in urban schools and teachers need support in implementing standards-based curriculum. As we begin to better understand the factors that influence teachers' selection and implementation of tasks, we are able to create and provide worthwhile professional development experiences for teachers. If we believe that facilitating higher-order thinking in mathematical classrooms can improve student achievement, then we need to continue to learn more about how teachers select and implement tasks that can engage students in this type of thinking. Further, we need to better understand what influences teachers' instructional decisions so that we can better support them as they learn how to choose tasks and put them into practice.

Facilitating higher-order thinking in mathematics classrooms is a worthy and necessary objective. Understanding how teachers choose tasks and implement higher-order thinking, and implications of these decisions for professional development, is essential for attending to the needs of all mathematics teachers. The more we know, the more our students will benefit.

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Appendix A
Interview Protocol–Reflective Teaching Cycle
Higher-Order Thinking in Middle School Mathematics Classrooms

Planning Questions

1. What opportunities to learn mathematics are afforded by the task?
2. Is the level of the task appropriate for the students at this time? How do you know?
3. What prior knowledge and experience would students need in order to engage in the task successfully? Do your students possess this knowledge? How do you know?
4. How can you make the expectations of the task clear enough to create an environment in which the students engage in high-level thinking?
5. How would you expect students to go about solving the task?
6. How can you use scaffolding and modeling high-level thinking during the lesson?
7. What would be a sufficient amount of time to give to the students so that they can grapple with the demanding aspects of the task and have time to expand their thinking and reasoning?

Task introduction or set up: Focus on the messages being conveyed both implicitly and explicitly to the students.

1. What are the students being asked to do?
2. How are the students being asked to do this?
3. What resources do the students have at their disposal? How will they support the students in their engagement with the task?

Reflection Questions

Implementation: How deeply did the students grappling with significant mathematical ideas?

1. Did the students deal with mathematical meaning as they worked?
2. Was the students' talk grounded in mathematical reasoning and evidence? Did the students actually produce mathematical explanations and justifications? How did you encourage this?
3. Was the students' talk focused on memorized procedures and symbols that are disconnected from underlying ideas? How did you encourage this?
4. Were multiple-solution strategies used? How did you encourage this?

Summary of lesson:

1. What were the main mathematical ideas in the lesson?
2. What evidence was there that the students learned those ideas?
3. What did you do to facilitate or inhibit students' learning of those ideas?
4. How did you reduce or maintain the level of thinking in the classroom?
5. How did you scaffold or model high-level thinking?

Adapted from (Stein, Smith, Henningsen, & Silver, 2000, p. 36)

6. What was the nature of the questions asked by the students? By you?
7. What factors appeared to support students' engagement in mathematical activity?
8. What factors seemed to hinder such engagement?
9. What should be the mathematical target of instruction in the next lesson?
10. What knowledge did the students demonstrate that would serve as a foundation for constructing new knowledge?
11. What task would accomplish the learning goal?

Adapted from (Smith, 2001, pp. 9-10)

Appendix B
Observation Tool & Descriptors
Higher-Order Thinking in Middle School Mathematics Classrooms

Observation Tool

Teacher:	Time Lesson Begins/Ends:
Date of Observation:	Duration of Lesson:

1. Describe briefly the main topic and/or purpose of the lesson based on the planning meeting.

2. Describe the main activities that occurred during the class period and the amount of time devoted to each activity.

   Example: Opening problem–5 minutes; Review homework–10 minutes; Instruction by teacher–15 minutes; Group work–10 minutes; Summary by teacher–5 minutes; Students work individually on homework–10 minutes

Activity	**Time**

Which of the following **best describes the primary emphasis** of the lesson? (See descriptions below adapted from Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344–350.)

__ Memorization / __ Procedures without connections / __ Procedures with connections / __Doing mathematics

Memorization	**Procedures without connections**
• The lesson involves either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory. • Tasks cannot be solved using procedures because a procedure does not exist or because the time frame in wcih the task is being completed is too short to use a procedure. • Tasks and quesitons are not ambiguous. The tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • The tasks have no connection to the concepts or mearning that underlie the facts, rules, formulas, or definitiosn being learned or reproduced.	• Tasks are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task. • Tasks require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. This is reflected in teacher questions and answers. • The class is focused on producing correct answers instead of on developing mathematical understanding. • No explanations are required or explanations focus solely on describing the procedure that was used.
Procedures with connections	**Doing mathematics**
• The teacher and task focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • The task suggests pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with repsect to underlying concepts. • Tasks and student ideas are usually represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. The teacher focuses on making connections among multiple representations to help develop meaning. • The task requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.	• The task requires complex and nonalgorithmic thinking–a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, the instructions, or a worked-out example. • The task requires students to explore and understand the nature of mathematical concepts, processes, or relationships. • The teacher and task demand self-monitoring or self-regulation of the students' own cognitive processes. • The teacher requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • The teacher requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • The task requires considerable cognitive effort and may involve some level of anxiety for the students beceause of the unpredicatble nature of the solution process requred.

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Classroom Events

1. The lesson provided opportunities for students to make conjectures about mathematical ideas. **Examples:**
2. The lesson fostered the development of conceptual understanding. **Examples:**
3. Connections within mathematics were explored in the lesson. **Examples:**
4. Connections between mathematics and students’ daily lives were apparent in the lesson. **Examples:**

5. Students explained their responses or solution strategies. **Examples:**
6. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class. **Examples:**
7. The teacher encouraged students to reflect on the reasonableness of their responses. **Examples:**
8. The teacher encouraged students to actively engage in thought-provoking activity that often involved the critical assessment of procedures. **Examples:**

Lesson Flow Recording Sheet

Take notes describing the activities of the teacher and students occurring during the class period. Provide a time stamp in the "Time" column to correspond with the events (record a time stamp at least every 4 minutes).

Line	Time	Event
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Observation Tool Descriptors

1. **The lesson provided opportunities for students to make conjectures about mathematical ideas.**

There are three types of conjectures: (1) guessing about how to solve a problem based on experience solving problems with similar solution strategies *(making connections between a new problem and problems previously seen)*; (2) guessing about the truthfulness of a particular statement and subsequently planning and conducting an investigation to determine whether the statement is true or false *(investigating validity of their own or someone else's guess)*; and (3) creating a generalization by reasoning from specific cases of a particular event, testing in specific cases, and logically reasoning the generalization to be acceptable for all cases of the event *(looking for patterns and making generalizations)*.

2. **The lesson fostered the development of conceptual understanding.**

Conceptual understanding occurs when students are given the opportunity to create meaning for the mathematical symbols and procedures that they use. They link procedural and conceptual knowledge through the problems with which they engage and teacher questioning. Conceptual knowledge is when one understands the relationships and among general patterns and principles of mathematics (Raudenbush et al., 1993), or understands connections within mathematics and between mathematics and personal experience, the world, and other disciplines (Jones, 2004). Procedural knowledge is when one understands mathematical symbols, symbolic manipulation, rules, algorithms, and step-by-step procedures without understanding the meaning of the symbols or actions.

3. **Connections within mathematics were explored in the lesson.**

Students have the opportunity to think about relationships and connections among mathematical topics. The lesson goes beyond examining fragmented pieces of information to having students look for an discuss relationships among mathematical ideas, express understanding of mathematical topics, or explain solution strategies for relatively complex problems in which two or more mathematical ideas are integrated.

The tasks and solutions are represented in multiple ways (i.e. visual diagrams, manipulatives, symbols, graphs, tables, verbally) and the connections among these representations are used to help develop meaning.

4. **Connections between mathematics and students' daily lives were apparent in the lesson.**

The teacher explicitly makes real life connections between the mathematics under study and the students' daily lives. The teacher elaborates on these connections in ways that underscores the importance of the topic and/or generates interest in the topic. While doing mathematics, students are required to access relevant knowledge and experiences and make appropriate use of them in working through the task. The task builds on students' prior knowledge and they have the appropriate prior knowledge to engage with the task.

5. **Students explained their responses or solution strategies.**

The teacher encourages students to elaborate on their answers or solution strategies by justifying their approach, explaining their thinking, or supporting their results. There is a sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback. The teacher encourages discussion and justification of mathematical ideas (Boaler, 2002) and encourages students to communicate and reason with mathematics (Jones, 2004).

6. **The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.**

The teacher uses student responses during instruction to pose questions that simulate further discussion, illustrate a point, or relate the response to other aspects of the lesson. For example, the teacher can use questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

The students have the opportunity to serve as the mathematical authority in the classroom. For example, the teacher did not provide an answer to the question or tell the students if they were correct or not. The students themselves engage in conversation about responses and as a class came to a decision on the task.

7. **The teacher encouraged students to reflect on the reasonableness of their responses.**

Students sometimes do not check the reasonableness of their answers. Evaluating the reasonableness of a solution involves the connections between procedural and conceptual knowledge. Thus, unreasonable responses could be a result of the lack of connections between symbols and their meaning. Students may rely on rules or procedures to obtain correct answers and not have the conceptual knowledge to help them evaluate reasonableness of the answer. Teachers should encourage their students to reflect on the reasonableness of their answers and facilitate discussion emphasizing conceptual understanding.

The cognitive demand of a task is maintained when students are provided with means of monitoring their own progress by actively thinking about how their decisions and actions will affect their progression through the task. Teachers can do this by encouraging students to use metacognitive strategies such as self-questioning.

One type of self-questioning that is useful in this regard is reflection questions. These questions prompt students to *reflect* on their understanding and feelings during the solution process. Examples of these types of questions include the following: “What am I doing?”; “Does it make sense?”; “What difficulties/feelings do I face in solving the task?”; “How can I verify the solution?”; “Can I use another approach for solving the task?”

The cognitive demand of a task is reduced when the teacher shifts focus to the procedural aspects of the task or on correctness of the answer rather than meaning and understanding. Teachers need to provide encouragement for students to make conceptual connections and press students for explanation and meaning.

8. The teacher encouraged students to actively engage in thought-provoking activity that often involved the critical assessment of procedures.

Teachers encourage students to actively work on tasks as well as actively think about how their decisions and actions could clarify future decisions and actions in their investigation.

If the problematic aspects of the task become routinized (e.g. students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform or the teacher takes over the thinking and reasoning and tells students how to do the problem), then the students are not actively engage in thought-provoking activity that often involved the critical assessment of procedures. Teachers need to make sure that the feedback, modeling, or examples that are given are not too directive so that they still leave the complex thinking for the students.

Teachers should encourage students to reflect on what they are doing and to clarify for themselves how to go about the investigation. The teacher should answer questions that clarify the task and encourage students to try out their own ideas. Teachers should refrain from affirming students' strategies or answers and from providing students with the next steps or actions to take. However, some students may need affirmation to continue with the investigation. As long as the students have had time to reflect and critically examine their ideas first, this may encourage students to continue and struggle more with the mathematics.

The cognitive demand of a task is maintained when students are provided with means of monitoring their own progress by actively thinking about how their decisions and actions will affect their progression through the task. For example, students may be encouraged to use metacognitive strategies such as self-questioning.

One type of self-questioning that is useful in this regard is strategic questions. These questions prompt students to consider which strategies are appropriate for solving the given problem/task and for what reasons. In addressing strategic questions, students describe *what* (e.g., What strategy/tactic/ principle can be used in order to solve the problem/task?"), *why* (e.g., "Why is this strategy/tactic/principle most appropriate for solving the problem/task?"), and *how* (e.g., "How can I organize the information to solve the problem/task"; and "How can the suggested plan be carried out?").

Appendix C
Observation Scales
Higher-Order Thinking in the Middle School Mathematics Classroom

	Higher-Order Thinking Skills	**Cognitive Demand - Tasks**		**Cognitive Demand - Implementation**
Researcher		*Procedures With Connections*	*Doing Mathematics*	*Maintaining High Cognitive Demand*
(Raudenbush et al., 1993)	Understand the relationships among general patterns and principles of mathematics *(#3)*	Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.	Require students to explore and understand the nature of mathematical concepts, processes, or relationships.	Teacher draws frequent conceptual connections.
	Apply knowledge to solve problems, think critically *(#8)*, analyze theories and conduct inquiry *(#1 & #7)*		Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.	Tasks build on students' prior knowledge. *(#6)*
(Peterson, 1988; Zohar & Dori, 2003)	Use good judgment, active mental construction, and abstract thinking *(#7)*		Require students to analyze the task and actively examine tasks constraints that may limit possible solution strategies and solutions.	
	Gain content mastery and conceptual understanding *(#2 & #7)*	Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.		Teacher draws frequent conceptual connections.
(Boaler, 2002)	Encourage discussion and justification of mathematical ideas *(#5)*			

(Jones, 2004; Ladson-Billings, 1994)	Communicate and reason with mathematics *(#5)*			Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback.
	Connect mathematics to the world and other disciplines in a meaningful way *(#4)*			
(Peterson, 1988)	Use cognitive and metacognitive strategies *(#7)*, such as comprehension, problem-solving, self-questioning *(#8)*, hypothesis generation *(#1)*, and study skills		Demand self-monitoring or self-regulation of one's own cognitive processes.	Students are provided with means of monitoring their own progress.

Appendix D
Higher-Order Thinking Codes
Higher-Order Thinking in Middle School Mathematics Classrooms

Facilitating Higher-Order Thinking

1. Gain content mastery and conceptual understanding.
    a. Ask questions that emphasize meaning and conceptual understanding.
    b. Teacher draws frequent conceptual connections. Understand the relationships among general patterns and principles of mathematics (Abstract thinking).
    c. Focus on conceptual content instead of procedural content; learning how to determine when, how, and what to tell and why; conceptual content includes ideas, images, meaning, why procedures work, comprehension of mathematical situations, and connections among ideas (Lobato, Clarke, & Ellis, 2005).
        i. Justification of ideas.
        ii. Teacher's actions promote thinking and do not explain procedures (Lobato, Clarke, & Ellis, 2005).
        iii. Describing a new concept
        iv. Summarizing student work
        v. Presenting counterexamples or providing information to allow students to supply their own
        vi. Presenting new representations
    d. Evaluation (judging credibility, comparing strengths and weaknesses of alternate interpretations, judging if two statements are contradictory, judging if evidence supports conclusions) (Critical thinking–Facione, 2009; Facione et al., 2000)

2. Communicate and reason with mathematics (Jones, 2004).
    a. Push students to communicate and reason with mathematics, specifically with mathematical language and ideas.

3. Use cognitive and metacognitive strategies.
    a. Comprehension
    b. Problem-solving (i.e. generalize and form hypotheses (Lewis & Smith, 1993)
    c. Self-regulation (self-examination, self-correction, self-questioning)
        i. Students are provided with means of monitoring their own progress.
        ii. Reflect upon the reasonableness of their answers.
        iii. Differentiation strategies
    d. Use good judgment and active mental construction.
    e. Study skills

4. Connect mathematics to the world and other disciplines in a meaningful way.

5. Apply knowledge to solve problems (Raudenbush et al., 1993).
    a. Apply concepts to solve unfamiliar or unique problems (Wenglinsky, 2000).
    b. Connect and extent new and prior knowledge to make predictions and conjectures, create new ideas, test conjectures, and decide what to believe.
        i. Inference (querying evidence, conjecturing alternatives, drawing conclusions) (Critical thinking–Facione, 2009; Facione et al., 2000)
    c. Tasks build on students' prior knowledge and student thinking.
    d. Focus attention on "the development of students' mathematics rather than on the communication of the teacher's mathematics" (Lobato, Clarke, & Ellis, 2005, p. 109).

6. Students given a sufficient amount of time to explore mathematics (not too much, not too little).

7. Teacher modeling
    a. High-level performance
    b. Remind students of previous ways of thinking, i.e. representations, definitions, already established rules, vocabulary (Lobato, Clarke, & Ellis, 2005)

8. Think critically (Facione, 2009; Facione et al., 2000)
    a. Interpretation (categorization, decoding significant, clarifying meaning)
    b. Analysis (examining ideas, detecting arguments, analyzing arguments)
        i. Interpretation and analysis (Lewis & Smith, 1993).
    c. Explanation (describing methods and results, justifying procedures, presenting full and well-reasoned arguments, proposing and defending causal and conceptual explanations with good reason) **(F1)**
        i. Use deductive (argue from general law to specific) and inductive (argue from specific observation to general) reasoning

Hindering Higher-Order Thinking

1. Ask for correct answers.
    a. Focusing on correct answers and procedures (Doyle, 1988; Henningsen & Stein, 1997; Romagnano, 1994)
    b. Shift emphasis from meaning, concepts, or understanding to the correctness of completeness of the answer. (Stein, Smith, Henningsen, & Silver, 2000)
    c. Ask students questions with one answer (closed & direct questioning). **(H4)**

2. Use questions, comments or give directions that lead to explicit procedures or steps to perform.
    a. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher takes over the thinking and reasoning and tells students how to do the problem). (Stein, Smith, Henningsen, & Silver, 2000)
    b. Break task down into smaller subtasks (Smith, 2000)
    c. Adapting tasks or teaching suggestions to be consistent with personal notions of effective teaching and learning (Arbaugh, Lannin, Jones, Park-Rogers, 2006; Clarke, 1997; Lloyd & Wilson, 1998; Remillard, 1999).

3. Use questions, comments, or give directions that are not specific or lead the students to guess what the teacher is thinking.

4. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are giving the impressions that their work will not count toward a grade). (Stein, Smith, Henningsen, & Silver, 2000)

5. Inappropriateness of task or amount of time for task. (Stein, Smith, Henningsen, & Silver, 2000)
    a. Students do not engage in high-level cognitive activities due to lack of interested, motivation, or prior knowledge needed to perform.
    b. Task expectations are not clear enough to put students in right cognitive space.
    c. Not enough time to wrestle with demanding aspects of task.
    d. Too much time allows students to drift into off-task behavior.

Appendix E
Frequency Tables
Facilitation and Hindrance of Higher-Order Thinking

Facilitation of higher-order thinking during task implementation introduction episodes

		Total	Clark	Tess
F2	***Gain content mastery and conceptual understanding.***			
a	Ask questions that emphasize meaning and conceptual understanding.	1		1
b	Conceptual connections and relationships among general patterns and principles of mathematics.	2		2
c	Focus on conceptual rather than procedural content. Learn how to determine when, how, and what to tell and why. Justification of ideas.	7	5	3
d	Promote thinking and do not explain procedures.	5	3	3
e	Evaluation.	2	1	1
F3	***Push students to communicate and reason with mathematics.***	7	7	1
F4	***Help students use cognitive and metacognitive strategies.***	1	1	0
a	Comprehension			
b	Problem solving			
c	Self-regulation	15	10	6
d	Good judgment and active mental construction			
e	Study skills			
F6	***Connect mathematics to the world, other disciplines, and itself in meaningful ways.***	15	4	11
F7	***Apply knowledge to solve problems.***	4	2	2
a	Apply concepts to solve unfamiliar or unique problems.	1	1	1
b	Connect and extend new and prior knowledge to make predictions and conjectures, create new ideas, test conjectures, and decide what to believe.	7	4	4
c	Build on students' prior knowledge and student thinking.	12	9	4
d	Focus on developing students' mathematics rather than communicating teachers' mathematics.	2	1	1
F8	***Give students a sufficient amount of time to explore mathematics.***	2	2	
F9	***Model high-level performance.***			
b	Remind students of previous ways of thinking, i.e. representations, definitions, already established rules, vocabulary.	6	1	5

F10	***Think critically.***	1	1	0
a	Interpretation	4	1	3
b	Analysis			
e	Explanation (deductive and inductive reasoning)	13	6	7
		107	59	55

Hindrance of higher-order thinking during task implementation introduction episodes

		Total	Clark	Tess
H1	***Ask for correct answers.***			
a	Focus on correct answers and procedures.	6	2	4
b	Shift emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.	2	2	0
c	Ask the students questions with one answer (closed and direct questioning).	1	1	
H2	***Use questions, comments, or give direction that lead to explicit procedures or steps to perform.***	3	0	3
a	Problematic aspects of task become routinized. Teacher takes over thinking and tells students how to do the problem.	22	12	10
b	Break task down into smaller subtasks.	13	7	7
c	Adapt tasks or teaching suggestions to be consistent with personal notions of effective teaching and learning.	7	4	3
H3	***Use questions that are not specific or lead the students to guess what the teacher is thinking.***	1		1
H5	***Do not hold students accountable to high-level products or processes.***	2		2
H6	***Inappropriateness of task or amount of time for task.***			
a	Lack of interest, motivation, or prior knowledge needed to perform.			
b	Expectations are not clear enough to put students in right cognitive space.			
c	Not enough time	1		1
d	Too much time			
		58	28	31