ESTIMATING GROWTH USING THE CENSORED-INFLATED STRUCTURAL EQUATION MODEL WITH FLOOR EFFECTS DATA

by

TABITHA MCKINLEY

(Under the Direction of Under the Direction of Deborah Bandalos)

ABSTRACT

A "floor effect" arises when performance is as bad as possible in all conditions. It is also known as the lowest possible measure of an individual's performance or achievement. The floor may serve as a proxy for how low an observation could actually go if not for the lower bound which restricts the performance to a certain point. The floor effect can also be an artifact of an interaction between the assessment and examinees, which causes individuals to reach the bottom of their capacity to answer test items correctly. Much work has been conducted on factors affecting student achievement and achievement growth trajectories, however, there is very little regarding how to model the growth trajectories of a floor effect. The censored-inflated model is unique and seemingly appropriate to be employed with these types of data since two growth models are simultaneously estimated. The first is a continuous growth model, and the second a model specific for floor effects.

A simulation study was conducted to test model fit under various conditions. The study provides promising evidence that the censored-inflated model is best used for these highly specific data.

INDEX WORDS: floor effect, growth modeling, censored-inflated model, structural equation modeling

ESTIMATING GROWTH USING THE CENSORED-INFLATED STRUCTURAL

EQUATION MODEL WITH FLOOR EFFECTS DATA

by

TABITHA MCKINLEY

BS, HOWARD UNIVERSITY, 2000

MEd, GEORGE WASHINGTON UNIVERSITY, 2007

MA, UNIVERSITY OF GEORGIA, 2007

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial

Fulfillment of the Requirements for the Degree

DOCTOR OF EDUCATIONAL PSYCHOLOGY

ATHENS, GEORGIA

2010

ESTIMATING GROWTH USING THE CENSORED-INFLATED STRUCTURAL

EQUATION MODEL WITH FLOOR EFFECTS DATA

by

TABITHA MCKINLEY

Major Professor: Deborah Bandalos

Committee: Allan Cohen
Stacey Neuharth-Pritchett
Eric Houck

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
December 2010

DEDICATION

This paper is in thanksgiving to God, who gave me the idea to pursue a degree in a field I had never heard of. Thanks to my parents, Sid and Ivy Nickolas, my siblings, Nelson, Nicholas, and Joanna, and my in-laws, Leon and Robenia McKinley for always doing whatever it took to help keep me going.

Thank you to my maternal grandmother Hazel Cosper “Nana” who could not have been admitted to UGA because of the segregated south in 1938. You have supported and encouraged me every step of the way.

Aunt Teresa, Uncle Bernard, and my research partner Rhea who I have spent countless hours crying to. I love you so much for just giving me a place to vent, and understanding what I was talking about!

To my sorority sisters of Alpha Kappa Alpha Sorority, Incorporated, thanks for always encouraging me to keep going. Monica, my “Front”, I will miss our gripe session about our papers and how “No one understands me but you.”

Thank you to my committee members for giving of your time. I have had personal relationships with each and every one of you and can attribute my personal successes to specific times in my life where you have had an influence.

To my advisor Dr. Deborah Bandalos “Bandy”, you have been an academic parent, mentor and friend to me. Thank you for never letting me fail. Thank you for disagreeing with me as your “child” when you knew better than I. Thank you for never letting my anxiety to complete this degree cause me to be mediocre.

Finally, thank you to my family. To my wonderful, loving and supportive husband, Malik Sr, thank you so much for putting up with me and this whole process. It has not been an easy road, but I would not rather travel it with anyone else. This finished product would not be possible without you in my corner! To my adorable and amazing kids Malik Jr. and Maria Kathleen who have been on this journey with me since it all began in 2006, thank you. I appreciate your sacrificing the many Saturdays when Mommy had to work on this paper and couldn't go fun places. It will all pay off. We have worked hard together and now we can play hard together!!

I love you all more than words can say. This degree is for all of us!

Love,

Tabitha (Malik Sr., Malik Jr., Katie, Mom, Dad, Nelson, Nick, JoJo, MIL, FIL, Nana, Aunt Teresa, Uncle B, Rhee, Mon) Marie Bellamy-McKinley, PhD.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

CHAPTER 1

INTRODUCTION

On June 10, 2008, the United States Department of Education (through Secretary of Education Spellings) granted permission to a total of 17 states to begin employing the use of growth modeling for the purposes of assessing Adequate Yearly Progress (AYP) in the school systems. A panel of experts in the fields of academia, State and District Practitioners, and Education Organizations was composed to review petitions. The techniques used in these growth models vary in approaches, but most often those used by state departments of education have been within the family of Hierarchical Linear Models (HLM). The Value-Added (VA) model developed first in Tennessee, has been most popular and widely adopted by other states. Though the family of HLM models (specifically the value added model), of data assessment are widely accepted and frequently utilized, the hierarchical linear modeling (HLM) family of models does not contain the kind of flexibility, measurement extension possibility, or control for measurement errors found with the Structural Equation Modeling (SEM framework). Regarding flexibility, Structural Equation Modeling is seen as a flexible and powerful analysis tool. For example, in the SEM framework of growth curve modeling, the assumptions for errors stemming from homoscedasticity and independence can be relaxed and analysis based on heteroscedastic and autocorrelated errors can be completed. Another strength of the SEM framework is the ability to disaggregate the effects into direct, indirect and specific indirect as well as calculate standard errors for each of these components simultaneously across all levels (Bollen, 1987).

For growth modeling using the SEM approach, parameters for the slope and intercept are estimated through the use of latent factors. In most computer programs, the maximum likelihood approach to parameter estimation is employed based on the fact that the ML method provides unbiased, consistent and efficient estimators of parameters. This method also assumes that the population is multivariate normal and that the sample used for the purposes of analysis is a direct reflection of that population. Due to the fact that standard errors can be very high in populations where students' ability levels may be below the mean, it is most effective to use Structural Equation Modeling to analyze the data for the purposes of the study.

For the purposes of this study, the population under investigation with regard to growth modeling is those who display what is referred to as "floor effects". According to Cohen (1995), a "floor effect" arises when performance is as bad as possible in all conditions. It is also known as the lowest possible measure of an individual's performance or achievement. The floor may serve as a proxy for how low an observation could actually go if not for the lower bound which restricts the performance to a certain point. In the case of this discussion, this term is used to describe those individuals whose resulting test scores indicate that they have reached the bottom of their capability to answer test items correctly. Most often this phenomenon is observed in special needs student populations when for reasons often unknown to teachers, school administrators and policy makers, students display an inability to adequately respond to assessment items.

Much work has been conducted on factors affecting student achievement and achievement growth trajectories. However, there is very little research regarding how to model the growth trajectories of students who are consistently low achievers. Essentially, students exhibiting a floor effect display different growth trajectories from those in the mainstream population. With this in mind, combining the two groups when estimating growth model parameters will likely result in biased estimates for both of the groups. Methods of analysis that allow the floor effects to be modeled specifically are necessary to prevent such biases.

The method of interest for the purpose of this study is the censored-inflated model (Muthen, 2003). The censored-inflated model is unique and seemingly appropriate to be employed for floor effects data analysis because in this method, two models are simultaneously estimated. The first is based on the normal observations within the sample, while the second is a model that is specific for the individuals that display floor effects. Though there is little research that has been conducted using the censored-inflated model, it is hypothesized that this model will provide more accurate parameter estimates for data with floor effects than will traditional growth curve models. Currently, floor effects data are often treated as missing observations, resulting in a loss of power due to loss of observations. In instances where these observations are simply included for omnibus analysis, the parameter estimates are negatively biased based on the extreme low values of the floor observations.

CHAPTER 2

SPECIAL EDUCATION, GROWTH MODELING, AND FLOOR EFFECTS

Why Special Student Populations?

Based on the fact that federal law mandates specific structures must be in place for students with learning disabilities, it is imperative that public school systems throughout the country not only take the appropriate steps to ensure an adequate and equitable learning environment for students with special needs, but find appropriate methods for measuring their progress. Since Individualized Education Plans (IEPs) are legally binding educational contracts, these students cannot be ignored in classrooms and therefore, the way in which they learn (as measured through standardized assessments) is a matter which requires immediate attention. The presence of students with special needs in schools and their performance on standardized tests can ultimately affect measures of Adequate Yearly Progress (AYP) as mandated by the No Child Left Behind (NCLB) legislation.

According to the federal legislation, 95% of the total population of a state school system must be included for the purposes of analysis in order to determine whether criteria for AYP were met. With the new permission granted by the Secretary of Education, growth models are being piloted in school systems around the country in order to display schools' meeting criteria for AYP. As a result, it is necessary that educational researchers continue to engage in studies which increase the knowledge base regarding how to best model the growth patterns for this population of students. Currently, the states that have been granted permission to use growth models are: Alaska, Arizona, Arkansas, Delaware, District of Columbia, Florida, Hawaii, Iowa, Minnesota, Missouri, Nevada, New Hampshire, New Mexico, North Carolina, Ohio, Pennsylvania and Tennessee, and the guidelines for state-level implementation of growth models are as follows:

1. Ensure that all students are proficient by 2014 and set annual state goals to ensure that the achievement gap is closing for all groups of students;
2. Set expectations for annual achievement based upon meeting grade-level proficiency and not upon student background or school characteristics;
3. Hold schools accountable for student achievement in reading/language arts and mathematics;
4. Ensure that all students in tested grades are included in the assessment and accountability system, hold schools and districts accountable for the performance of each student subgroup and include all schools and districts;
5. Include assessments, in each of grades 3 through 8 and high school, in both reading/language arts and mathematics that have been operational for more than one year and have received approval through the NCLB standards and assessment review process for the 2005-06 school year. The assessment system must also produce comparable results from grade to grade and year to year;
6. Track student progress as part of the state data system; and
7. Include student participation rates and student achievement as separate academic indicators in the state accountability system.

(http://www.ed.gov/news/pressreleases/2008/06/06102008.html)

In the case of students who display floor effects, it can be assumed that the growth trajectories are of a non-traditional linear fashion, which may mean that the growth displayed is stagnant or flat at lower bounds observations. This means that the growth models currently being employed under the assumption of a normally distributed population are inefficient to be used when students within the group display floor effects. The modeling of such trajectories requires additional investigation into alternative methods for assessing growth other than the traditional linear models.

Special Education in America

The history of special education programs in the United States dates back as early as the 1960s when many of the states in the country did have programs for special student populations in place. However, the problem was that there were an almost identical number of states that did not possess such programs, and many students with disabilities were left without necessary services within an educational setting. In some states students with disabilities were not allowed to attend public school education. In 1965, the U.S. Congress passed the Elementary and Secondary Education Act (ESEA), and part of the funding of the first ESEA was given to states for special education services. This initial federal funding was in response to reports that found up to two-thirds of the nation's disabled children were not receiving "appropriate education."

Two of the most significant court cases with respect to special education are the *Pennsylvania Association of Retarded Children (PARC) vs. Commonwealth of Pennsylvania and the Mills vs. Board of Education*. In 1971, a federal district court found that every mentally challenged child in Pennsylvania had a right to a public education and in the 1972 *Mills vs. Board of Education of the District of Columbia* case, the U.S. District Court found that a school district cannot exclude any exceptional children from public education, even if the school district has insufficient funds to provide such services. Based on the outcomes of these cases, the U.S. Congress amended the Elementary and Secondary Education Act (ESEA) in 1974 to require states to provide a "free and appropriate education to all children." The following year, the U.S. Congress passed the Education for All Handicapped Children's Act (EHA) of 1975, which provided additional federal funding for special education.

This “Public Law 94-142, the Education for All Handicapped Children Act" was a law that for the first time gave opportunities to individuals, ages 3 to 21, for a free and appropriate education regardless of disabilities, and included procedural safeguards to protect the rights of students and their parents. It also included the opportunity to receive an education in the least restrictive environment, utilizing an individualized educational program for each child as necessary, and provided for parental involvement in educational decisions related to their children with disabilities, and fair, accurate, and unbiased evaluations. This law also required that educators provide "due process procedural safeguards". As a result of *Mills vs. Board of Education*, laws that "clearly outlined due process procedures for labeling, placement, and exclusion" were created, and "Procedural safeguards to include right to appeal, right to access records, and written notice of all stages of the process" were also required (Chinn & Gollnick, 2006).

This legislation has been amended through the years and is currently known as the Individuals with Disabilities Education Improvement Act (IDEIA 2004). According to this regulation, children ages three through nine who experience “developmental delays” or “health impairments” which impede or interfere with the learning process(es) of a student are included for the purposes of public (or if necessary private) school education. This legislation also mandates that each state “adopt specific criteria” which determine whether a child has special needs. IDEIA 2004 regulations require that teachers, specialists, school psychologists, or other psychologists diagnose the disability that would elicit services under the legislation. Since each state is at liberty to identify special needs students according to individual standards, there is no actual formalized definition of what conditions are considered as special needs, only suggestions and inclusions (such as Tourette’s syndrome). Additionally, there is no pre-specified age or grade level at which a learning disability is to be determined by school officials, but such assessments are recommended to be completed on an “as needed” basis.

Due to the fact that the terminology of special education is considered to be ambiguous or state specific, it is necessary to operationalize the term for the purposes of this paper. In the instances where the term “special needs” or “special education” is used, the term will refer to students who have been identified by the state of origin as learning disabled based on cognitive deficits in processing, or those with behavioral or emotional disabilities which impede learning, and as a result have been provided with an Individualized Education Plan for classroom accommodations and instructional modifications. It is important to note, that in most instances, though highly controversial, Limited English Proficient (LEP), students who are also known as English Speakers of Other Languages (ESOL) are not usually included in the IDEIA 2004 legislation, and therefore in this paper will not be identified as special needs. The special needs population under investigation often possesses patterns of growth that are not linear in nature, or are linear with growth trajectories containing a slope of zero. In these cases, it is more appropriate to seek methods of growth modeling which best suit this type of growth.

Linear Growth Modeling

According to the National Center for Education Statistics (NCES; 2002), the definition of a longitudinal study is one in which the same respondents are surveyed repeatedly over time. Longitudinal studies are multilevel or hierarchical in nature, based on the fact that time points at which the data were collected are nested within the individuals being used as the sample. For any type of statistical modeling, the term longitudinal refers to observations made over a specified period of time, which are then combined, often in a composite form, in order to observe differences among time points within the same dataset. Longitudinal data may provide useful information regarding specific trends in the data as well as give opportunities to directly pinpoint where in time changes occur. Work done by Bryk and Raudenbush (1987), Willett and Sayer (1994), and Muthén and Khoo (1998) has provided strong evidence for the hypothesis that growth modeling can be used as an important method when analyzing patterns of change-especially as it relates to the field of education. The same researchers also agree that in order to gain a more accurate measure of a respondent's change or growth over time, it is necessary to observe more than one "snapshot" picture of the individual, and employ a more holistic approach, which takes into account the time factor and allows for multiple observations within a specified period.

Longitudinal data are inherently nested, and multilevel by nature (Byrk & Raudenbush, 1987, 1992). Although cross-sectional analyses can be useful in answering some research questions, when a research question requires the observation and analysis of growth over time, the cross-sectional approach is not fitting because there are not multiple time points collected on each individual for the purposes of analysis. Cross-sectional analysis provides a snapshot of performance which may not be useful if interest is in change over time. Given these reasons, it is not surprising that a large portion of educational research conducted in, for or about public schools is conducted within a longitudinal framework.

For most statistical modeling of cross-sectional data, analysis is conducted based on the assumptions that: the data are normally distributed, there is independence of observations, homogeneity of the sample, and linearity of growth. However, for many longitudinal studies there may be violations of these assumptions. Work conducted by Chiu and Khoo (2005) discussed a number of problems in the data that may lead to violations of the assumptions of the dataset and may impact the outcome of the study.

One problem the authors consider is the violation of the independence assumption. Methods such as ordinary least squares are based on the assumption of independence between observations. In a longitudinal study, the sample is the same across time points, and observations made from subsequent years may be the direct effect of some variable from a previous time point. Additionally, there may be cohort effects within the data which have formed over time (such as groups of students who have traveled together from classroom to classroom for 3 years) that impact the respondents' answers and are unaccounted for by any cross-sectional analysis.

According to Curran (2003), the way to rid the model of problems linked to time-dependent data is to include the measures of time as fixed values in the analysis. He goes on to state that by doing this, it is then possible to disaggregate the level-1 (occasion) and level-2 (person) covariance structures in a single "partitioned covariance matrix S" which will in turn be used as the unit of analysis when employing estimation procedures in a growth model using the structural equation modeling (SEM) framework.

A problem with multilevel modeling within a longitudinal study perhaps not encountered in cross-sectional studies is the fact that there can be a lack of stability of the effects of explanatory variables across time. Because the same sample is used over time, a condition known as group heterogeneity may occur where the effects of the explanatory variable(s) may have different effects on an individual level that emerge over time (Goodman, Ravlin & Schminke, 1987). The same effects may change over time within the entire group which is called nonstationarity (Dabbs & Ruback, 1987; Goodman et al., 1987). This is a violation of normal statistical assumptions that the explanatory variable has the same effect on the group of respondents across time.

Additionally, in a longitudinal study, the assumption of linearity in the growth may be violated, especially in special student populations where floor and ceiling effects may be observed. According to Cohen (1995), a "floor effect" arises when performance is as bad as possible in all conditions. Inversely, in the case of "ceiling effects", these instances arise when performance is as good as possible in all conditions. In such situations, growth cannot be measured linearly. In addition, students at the floor or ceiling are unaffected by explanatory variables due to a lack of variability.

When confronted with the changing face and demographics of students within the public school system, it is imperative to be able to employ methods of analysis which not only are able to measure the nested effects of variables within the dataset, but to be able to accurately estimate individual student growth trajectories, which may not follow traditional patterns of growth. With regard to longitudinal data, regular regression techniques cannot be applied as tools for analysis due to the violation of the assumption of independence within the observations. As a result, growth curve or mixed-effects models are employed and widely accepted for use based on the consideration of the joint probability density function for the repeated measures (Fitzmaurice, Laird, & Ware, 2004; Meredith & Tisak, 1990).

A method of analysis often preferred for use with longitudinal data is Structural Equation Modeling. Within the family of modeling techniques known as Structural Equation Models is the latent growth model (LGM).Latent growth model is a type of structural equation model that is estimated using a mean structure. The analysis in an LGM usually requires three components which are :1) a continuous dependent variable measured on at least three different occasions, 2) scores that have the same units across time and are not standardized, and 3) data that are all time structured (Kline, 2005).Additionally, typical LGM estimation allows models to be fitted to raw data which may contain incomplete or unbalanced observations (Jennrich & Schluchter, 1986, McArdle & Hamagami, 1992, Mehta & West, 2000, Blozis, 2004). Latent growth models are considered to be extremely flexible due to their ability to handle missing or unbalanced data which can occur when individuals within a specific sample are observed at different time points (an event that can occur often within longitudinal studies).

In a latent growth model which is based on linear change, the equation commonly is denoted:

$$\eta_{0i} = \alpha_0 + b_{0i} \text{ and } \eta_{1i} = \alpha_1 + b_{1i} \quad (1)$$

Where i=individual, η_{0i} = expected response when x_{ti}=0, η_{1i} =expected change rate for the individual, and x_{ti}=the individual's characteristic (such as age, grade or reading level) at time t, α_0=population value when the individual characteristic x_{ti}=0, α_1 = change rate.

The random effects b_{0i} and b_{1i}indicate the individual's deviations from the average coefficients. In the SEM framework of growth curve modeling, the assumptions for errors stemming from homoscedasticity and independence can be relaxed and analysis based on heteroscedastic and autocorrelated errors can be completed.

The set of random coefficients is assumed to be normally distributed as

$$\begin{pmatrix} \eta_{0i} \\ \eta_{1i} \end{pmatrix} \sim N\left[\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}, \begin{pmatrix} \phi_{00} & \\ \phi_{10} & \phi_{11} \end{pmatrix}\right] \quad (2)$$

where the variances of the random intercept η_{0i} and slope $\eta_{1i}$are denoted by ϕ_{00} and ϕ_{11}, which indicate how much individuals differ on the change characteristics, while covariances represented by ϕ_{10}display the linear relationship between the response level and the change rate.

Latent growth models have proven to be popular in various statistical analyses due to their "ease of application and interpretation" (Blozis, 2007). These models are much like the standard unrestricted factor analysis model in which the mean vector and covariance matrix of the manifest variables have imposed structures. In this model, the columns of the factor matrix define the general shape of the response trajectories over a specified period of time and can be pre-determined or some parameters can be left free for the purposes of estimation. With regard to leaving some parameters to be estimated, there may be some preference for this approach in that it allows for some flexibility in defining the mean curve when it cannot be specified in advance (Blozis, 2004). This is one method of obtaining a nonlinear growth trajectory since when estimating the random coefficients one may discover that these portions of the model can become nonlinear.

The estimation of these models yields estimates of the fixed effects and of the variances of the random effects as well as the corresponding covariate effects at the second level. Also, in this model there is the possibility of estimating variances and covariances which relate to the time-specific errors at the first level of analysis (Blozis, 2007).

When using a latent growth model based on linear change, the underlying assumption is that the changes in a behavior occur in a linear fashion and at a particular rate. Furthermore, the assumption is that responses across individuals are dependent on a common form of change although they may vary in their dependence on this commonality (Meredith & Tisak, 1984, 1990). In the case where a latent growth model contains a linear change value, the individual's response can be described by a model that includes an intercept and a linear time effect which can be unique to the individual (Blozis, 2008). These individual-specific effects are called random coefficients and are assumed to be random deviates which vary about a corresponding population value.

Though there is a component of individual-specific effects that are included in the linear change models, there is also the assumption that the change in the individuals has a common component. Because of this fact the linear model may not be appropriate for use with floor effects data. In the case of floor effects which are most often observed in special needs student populations; there are major differences about the change that takes place or the way in which it occurs when compared to the change of the normal counterpart.

Nonlinear Growth Models

Nonlinear methods for modeling data are appropriate for usage with longitudinal data when it can be assumed that the response pattern of individuals is not linear or contain non-linear portions. When combining linear data with nonlinear (or floor effects) data, nonlinear growth trajectories are often discovered, as can be observed in Table 1. The table below is a comparison between a normal and floor effects dataset when using a regular linear regression model The table used was based on a simulated (from the Monte Carlo function in MPlus (Muthen, 2003) sample of N=250 at T=4 time points. The two groups used for comparison were comprised of the same number of observations, however with the floor effects data there were 14% of extreme lower bounds observations included for the purpose of analysis.

Table 1: Comparison of the linear regression outcomes for normal and floor effects data

Obtained Statistics	Normal Data	Floor Effects Data
R	0.50	0.37
R Square	0.25	0.14
Adjusted R Square	0.24	0.13
Sum of Squares Residual	2250.00	6635.03
p-value	0.00	0.00

The results displayed in the table indicate that in the case of the floor effects dataset, the adjusted R^2 value is only *0.13* versus the *0.24* observed in the normal dataset. Of special importance to note is the sum of squares for the residuals between the two models. In the regression model used for the normal dataset, the sum of squares residual was 2250.00, while for the floor effects data the value obtained was 6635.03. This basic comparison provides evidence for the assertion that traditional methods for data analysis are not appropriate when dealing with floor effects datasets.

In cases where the data are so non-normal that the growth trajectories would be nonlinear, it is suitable to investigate alternative models for analysis. Models that should be explored could include: latent basis curve, quadratic, exponential, and multiphase models. Models for zero-inflated data, including the censored-inflated model, are then discussed.

Latent Curve Models

Latent curve models are models that allow individuals to observe how outcomes change over time based on time-variant and invariant features of independent variables (Kaplan, 2009). These models are effective not only for handling linear forms of change, but also some nonlinear forms including polynomial functions. Although polynomial functions are appropriate for modeling curvilinear patterns, many longitudinal data patterns in the area of education tend toward an asymptote, and polynomial models do not fit such patterns well. Since these types of models often do not provide an adequate fit for the data or match the theory that underlies the manifest behaviors (Francis, et al, 1996), polynomial models are often abandoned for the latent basis curve models.

Latent Basis Curve Models

With latent basis curve models, change trajectories are estimated from the data rather than fixing the parameters to some predetermined values that model a particular shape. By using the data to attempt to find the optimal shape, change is determined through the use of the vectors $A_0[t]$, and $A_1[t]$ for the intercept and slope coefficients, respectively, where the first and last latent basis coefficients for $A_1[t]$ are fixed to 0 and 1, respectively, for the purposes of model identification and to establish the interpretation of the intercept when $A_1[t] = 0$. This model may be preferred to quadratic or polynomial models due to the fact that it is able to capture all of the *nonlinear* aspects of intraindividual change with the use of just a single vector ($A_1[t]$) and a single intraindividual differences variable.

Latent basis curve models require that individuals are measured according to the same time points, which somewhat limit their flexibility. Additionally, in this form of modeling, the responses of an individual are assumed to be a linear combination of sets of basis functions which are common to all members of the population as well as a set of weights that are unique to the individual composed of both the intercept and time effect (Blozis, 2008).

Quadratic Growth Models

A quadratic growth model allows for a specific form of nonlinear change in an individual's response over time. The model, which usually includes linear change as well as acceleration rates, is specified as

$$Y_{ti}=\eta_{0i} + \eta_{1i}x_{ti} + \eta_{2i}x^2_{ti} +\varepsilon_{ti} \qquad (3)$$

where η_{0i} =expected response when time=1, η_{1i} = change rate for the individual when x_{ti}=0, η_{2i} =acceleration rate when x_{ti}=0, ε_{ti} = error term.

Between individuals, the random intercept, linear and quadratic time effects are assumed to be normally distributed as:

$$\begin{pmatrix} \eta_{0i} \\ \eta_{1i} \\ \eta_{2i} \end{pmatrix} \sim N \left(\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \right) \qquad (4)$$

where the variances of the random coefficients ϕ_{11} , ϕ_{22} , and ϕ_{33}, measure the degree of individual differences in each change feature, and covariances represent the linear relationships among them. The covariance between the intercept and linear time effect is denoted by ϕ_{21}, and

$\phi_{31}$and ϕ_{32} are the covariances between the quadratic intercept and time effects. Quadratic models may be appropriate to employ as a standard way to introduce complexity in intraindividual change which displays interindividual differences. In these models, there are three aspects of intraindividual change included which are the intercept ($A_0[t]$), the linear change ($A_1[t]$), and quadratic change ($A_2[t]=A_2[t]^2$). The assumption for this model is that at each time point, growth takes place at a fixed rate beginning with a linear baseline.

Rindskopf (2003) states that one advantages of utilizing a quadratic model for modeling nonnormality within a dataset is that it results in a model that is most parsimonious. He makes this assertion based on the fact that the quadratic model requires a single curve and fewer parameters for estimation than a model such as the latent basis. He also suggests that in the case of nonnormal outcomes, a nonlinear transformation be made to essentially “normalize” the data in order to retain a simpler model.

However, in the case of floor effects data, the group containing such observations may actually be obliterated (due to the treatment of these observations as missing if the actual obtained values for the observations is zero) and excluded from analysis. In the case of such data, there are differences between those with and without floor effects in the slopes and intercepts that may vary greatly from the normal distribution. Thus performing a nonlinear transformation to the data may suppress these group differences (Bauer & Curran, 2003 pg. 389). Bauer and Curran (2003) state that while parsimony may be the goal of most researchers when faced with model selection, the ultimate goal should be to find a model that proves to be the best fit based on the reality of the data.

Exponential Growth Models

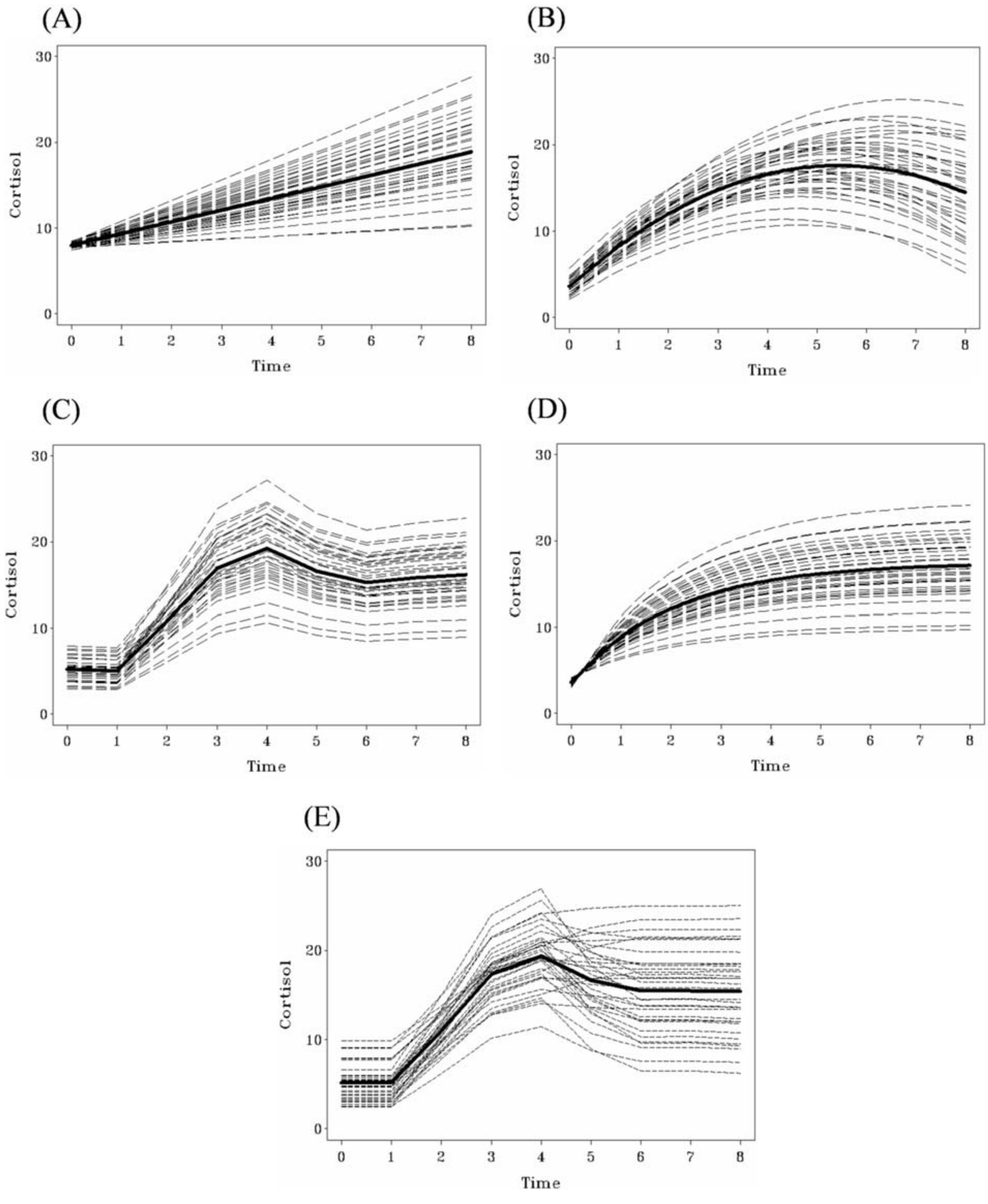
In the exponential growth model, certain portions of an individual's change vector are estimated from the raw data (Ram & Grimm, 2007). The two vectors used to estimate the change are A0[t]= the latent intercept, and $A_1[t]=e^{[-\alpha(t)]}$ where the value of α is estimated from the data. The interindividual differences in intraindividual change are modeled through the use of random variables. In this model, the latent intercept describes the individual's capacity to reach the limit of outcome variable "Y".

In this model of growth it is assumed that Y will be reached simultaneously among all individuals in the sample. If there are any deviations from the exponential form of the model, those observations are simply treated as error. In the case of floor effects, there would be large decelerations from the outcome variable Y. Those lower boundary values would be treated as error when estimating model parameters and biases would be the result.

Multiphase Models

The multiphase model, also called "spline regression" (Ram & Grimm, 2007 page 311), is one in which two or more regression lines are connected to model multiple processes that may influence the intraindividual change over time. The multiphase models are created through a fixed number of time points that display the number of times at which certain aspects of growth are "turned on" and "off". The model may include three phases: 1) A baseline phase, 2) a production or growth phase, and 3) a dissipation or decline phase. This model allows for the disaggregation of data and growth according to specific phases in order to adequately determine when growth occurs. In this model, the time points may be fixed to predetermined values in order to show that the change occurs at these specific times. With regard to growth curve modeling, the multiphase model is preferred for data which display multiple growth phases occurring over time (Browne & Cudeck, 1993). This model is often employed in biological studies where certain hormones or chemical markers are expected to “turn on” at various points in time. In these instances the model allows researchers to pinpoint actual growth that has taken place at the various phases within the period of time being observed.

Figure 1 is an example of graphs (Ram & Grimm, 2007) displaying individual growth trajectories showing the differences in modeling using (A) linear, (B) quadratic, (C) latent basis, (D) exponential, and (E) multiphase growth curve models (in each of these models, the bold line is the predicted average, while the dashed lines are predicted individual trajectories).



Images included with permission from Ram and Grimm (2007) Using simple and complex growth models to articulate developmental change: Matching theory to method.

In the case where the data may contain extremely low values, the types of nonlinear models discussed above may not be appropriate to employ for data analysis. In the aforementioned methods of data analysis, there is an assumption of normality within the dataset, and the extreme nature of the floor effects is not truly considered. Regarding floor effects, these data are denoted as "censored from below" meaning that the measured variable has some fraction of observations at the minimum.

Models for Zero-Inflated Data and Floor Effects

The Poisson distribution is a discrete probability distribution which expresses the probability of a number of events occurring in a fixed period of time if they happen within a known average rate and independently of the time since the last event. This type of distribution focuses on certain random variables N that count the number of discrete occurrences that take place during a time-interval of given length. If the expected interval of these occurrences is λ then the probability that there are exactly k occurrences (k=0, 1, 2, …) is

$$f(k;\lambda) = \Pr\,(Y = k) = e^{-\lambda i}\lambda^{y}/k! \qquad (5)$$

where e is the base of the natural logarithm, k is the number of occurrences of an event, k! is the factorial of k, λ is a positive real number equal to the expected number of occurrences that occur during the given time interval.

In the literature pertaining to zero-inflated datasets the term is used when referencing datasets which contain a preponderance of zeros. This phenomenon most often occurs when dealing with counts data which appear as a probability mass that clusters at zero (Tooze, Grunwald, & Jones, 2002). McCullagh and Nelder (1989) and Martin et al. (2005) described zero-inflated data as a unique case of overdispersion of zeros in which the variance is greater than it should be based on the shape and central tendency of a distribution.

As with floor effects, ordinary least squares (OLS) methods are not appropriate for analyzing these data. According to Min and Agresti (2004) zero-inflated data cause a linear model to display lack of fit due to the disproportionate amount of zeros. This type of dataset can arise from various reasons such as 1) true zeros which are a result of an absolute absence of a quantity, or 2) sampled zeros which are a result of errors in sampling. Due to the large amount of these zeros in the dataset, the observations cannot simply be deleted from the data which would results in a loss of power. In addition, it is often recommended that with nonnormal data, transformations be made to the extreme low observations. However, in the case of zero values the transformation would be useless since the natural logarithm of zero is not defined (Zhou &Tu, 1999). Although it is possible to add small numbers (such as 0.001) to each zero observation in order to take the log of that number in an attempt to create a nonlinear transformation of the data, the resulting parameter estimates and residuals will be biased and nonnormally distributed.

The Hurdle Model

In order to attenuate the effects of such data, two types of counts models can often be employed. The Hurdle model was developed separately by Mullahy (1986) and King (1989). The two-part distribution for the random component is given by the transition stage g_1 probability mass function:

$$\Pr(y_i = 0) = p \tag{6}$$

which models whether the response crosses the hurdle of zero. Assuming a Poisson distribution the random component for the event stage (which is the stage where the change or growth occurs) distribution is

$$\Pr(y_i = k) = 1\text{-}p/1\text{-}e^{-\lambda i}\ e^{-\lambda i}\lambda_i^k/k!\ ,\ k = 0,1,2\ \ldots \qquad (7)$$

The generalized linear models as a function of covariates are then

$$\log p_i/1\text{-}p_i = x_{1i}B_1 \text{ and } \log(\lambda_i) = x_{2i}B_2 \qquad (8)$$

The maximum likelihood estimates are then derived using this notation (Min, 2003)

$$\ell_1(\beta) = \sum [\log P_1(y_i{=}0;\beta_1, x_{1i})] + \sum [\log(1\text{-}P_i(y_i{=}0;\beta_1 x_{1i}))] = \sum x_{1i}, \beta_1 \text{-} \sum \log(1+e^{x1i\beta1}) \qquad (9)$$

$$\ell_2(\beta) = \sum [y_i x_{2i}\beta_2\text{-}e^{x2i}\beta_2\text{-}\log(1\text{-}e^{-ex2i\beta2})]\text{-}\sum \log(y!) \qquad (10)$$

Due to the fact that the two models are functionally independent (since L1 is for the censored portion and L2 continuous portion), the likelihood functions can be maximized separately (Min & Agresti, 2004; Min, 2003; Cameron & Trivedi, 1998; Mullahy, 1986) and the total log likelihood estimate is

$$\ell(\beta_1,\beta_2) = \ell(\beta_1) + \ell(\beta_2) \qquad (11)$$

The estimation for the negative binomial Hurdle models is usually performed through specialized maximum likelihood methods such as the Newton-Raphson algorithm.

<u>Zero-Inflated Poisson (ZIP) Model</u>

The zero-inflated Poisson (ZIP) model is one that can be used when zeros in a dataset are generated by chance and systematic factors (Min & Agresti, 2004). In this model the transition stage addresses the zero-inflated portion of the model while the event-stage addresses the continuous portion of the data (Jang, 2005).

In the ZIP model the equation for the event stage is

$$\Pr(y_i = k) = p/1\text{-}p\ \ e^{-\lambda i}\lambda_i^k/k!\ ,\ k = 0,1,2\ \ldots \qquad (12)$$

However, for the transition stage the model varies greatly from the Hurdle model in that instead of $\Pr(y_i = 0) = p$, there is an inclusion of the probability of a nonzero which is multiplied by an exponentiated Poisson mean. As opposed to modeling an occurrence of zeros at the transition stage, the ZIP model takes into account the probability that the actual counts have a Poisson distribution which allows two sources of zeros (which are zeros due to error and those which are true observations) into the analysis.

The equation for the event state is

$$\log(\lambda_i) = x_{2i}B_2 \tag{13}$$

and the equation for the transition stage is

$$\log p_i/1\text{-}p_i = x_{1i}B_1 \tag{14}$$

Unlike the Hurdle model, the ZIP likelihood cannot be maximized separately for event and transition stages (because of violations of independence), and has been derived as (Lambert, 1992).

$$L=\sum \log (e^{Gi\gamma} +\text{-}e^{Bi\beta} + \sum(y_iB_i\beta\text{-}e^{Bi\beta})\text{-}\sum \log (1+e^{Gi\gamma}) - \sum \log (y!) \tag{15}$$

Where βB is the vector of coefficients and matrix scores of the means and variances for the event stage and γG is the vector of coefficients and matrix scores for the transition stage, where iterations are based on the EM or Newton-Raphson algorithms (Min, 2003; Lambert, 1992).

Though sufficient for dealing for data with a preponderance of zeros, the count data analysis methods previously mentioned are not appropriate for floor effects for two reasons. The count data differ from the continuous observations used in the case of floor effects (particularly in an educational setting). In addition the preponderance of zeros in the datasets used for analyses with the aforementioned methods would be highly unlikely in educational data based on communication with officials from the following public school systems: New Jersey, Delaware, Florida, Hawaii, and Nevada that the typical dataset contains 0.01 to 15% zeros. If there was a standardized testing dataset which included greater than 40% zeros, analysis would likely not continue as there would be an assumption of a problem with the data, the assessment used, or both.

Another method commonly referenced for nonlinear continuous data with binomial outcomes is logistic regression (Cohen et al, 2003). This method is discussed here in relationship to the censored-inflated model based on the fact that the censored (from below) and normal parts of the data could also be considered binomial as Y=0 for censored values and Y=1 for noncensored.

With binomial data, the form of the growth trajectory is more of an S-shaped curve than a linear function and therefore a nonlinear function should be imposed to allow the predicted probability for an individual $\hat{p}_i$ to have a nonlinear relationship with the actual predictor variable X. This nonlinear relationship is based on "odds", which is the probability that an individual will score 0 versus that of an individual scoring a 1.

In order to model this nonlinear relationship, the logistic function is denoted as

$$\hat{p}_i=1/1+e^{-(B_1X_i+B_0)}=e^{(B_1X_i+B_0)}/1+e^{(B_1X_i+B_0)} \quad (16)$$

Where $(B_1X_i+B_0)$ is the predicted score for a linear ordinary least squares regression which is transformed to the odds (nonlinear) function, and $\hat{p}_i$ is the odds ratio for the predictor. Though logistic regression is often used as a preferred method for binomial data with censored values, it is not sufficient when attempting to model growth in a population in which censored and normal samples are combined. The resulting parameter values would be biased toward the lower bounds of the data.

One method for dealing with floor effects in analysis that may commonly take place in current growth modeling methods would be the option of deleting observations displaying extremely low values through a method such as listwise deletion. In this method of exclusion of observations, if a specific case contains a floor effect, the entire case would be deleted. Though this may appear to be a simple and possibly effective way to rid the dataset of floor effects, the clear disadvantage of this method is the fact that the result is a loss of data and shrinkage of the entire sample size. Additionally, in the case of floor effects as it relates to special education students, these individual observations cannot be eliminated from the dataset because at least 95% of the students in the school system must be included in analysis. If there is a loss of data due to deletion, there could be a shrinkage that results in less than 95% of students being included in analysis, which is a violation of federal legislation.

Another method sometimes employed as a way of dealing with floor effects would be to treat the low values as missing not at random (MNAR) based on the hypothesis that the low values or zeros in the dataset are due to the fact that the floor effect exists and data are therefore not missing at random. In those cases, one may opt to estimate the values of the zeros by using traditional EM algorithms or multiple imputation methods. Those imputed values may be biased due to the fact that the estimation methods used for these missing observations are based on the assumption that data are either missing completely at random (MCAR) or missing at random (MAR).

Two-Part Model

A study frequently referenced that serves as the precursor for the censored models of data analysis is that of Olsen and Schafer (2001). The researchers conducted studies with substance abuse data using a model they referred to as a “two-part random-effects model for semicontinuous longitudinal data”. In this model they employ the estimation of both a linear and logistic model. The two part model includes random coefficients into both portions of their model.

For the logistic part of the model, Y_{ij} is the semicontinuous response for person i=1,…m at time j=1…n_j which is recoded into two variables:

U_{ij}= 1 if Y_{ij} =0 , and 0 if $Y_{ij} \neq 0$

and

V_{ij}= g(Y_{ij}) if $Y_{ij} \neq 0$ and irrelevant if Y_{ij}=0,

where g is a simple increasing function (log) that will make V_{ij} approximately Gaussian. In this design the responses are modeled by a pair of correlated random-effects models. One is for the logit probability of U_{ij}=1 and one for the mean conditional response $E(V_{ij}|U_{ij}=1)$. The logit for part one of the logistic model is:

$$\eta_i=X_i\beta_i+Z_ic_i \qquad (17)$$

Where $\pi_{ij}=P(U_{ij}=1)$, η is a vector with elements $\eta_{ij}=\log(\pi_{ij}/1-\pi_{ij})$, j=1,…,$n_i$ and $X_i(n_i \text{ x } q_c)$ and $Z_j(n_i \text{ x } p_c)$ are matrices of covariates that pertain to the fixed and random effects. Time measures may be included in X_i and possibly Z_j, allowing slope and intercept to vary per subject. In this model, the β and c parameters are estimated.

In the linear part of the model, the equation is:

$$V_i=X^*_i\gamma + Z^*_id_i+\varepsilon_{i*} \qquad (18)$$

Where V_i = the vector of length n_i* (the number of time points included in the analysis) which contains all relevant values for V_{ij} for subject i with values that correspond to Uij=1. In this model, the residuals of the error term are assumed to be distributed as N(0, σ^2 1) and

X_i*(n_i* x q_d) and Z_i*(n_i* x p_d) are matrices of covariates, and the γ parameter is for the random event. In this portion of the model, the random coefficients from the two parts are assumed to be "jointly normal and possibly correlated" (page 732).

$$bi=\begin{pmatrix} c_i \\ d_i \end{pmatrix} \sim N\left(0, \Psi = \begin{pmatrix} \Psi_{cc}\Psi_{cd} \\ \Psi_{dc}\Psi_{dd} \end{pmatrix}\right) \qquad (19)$$

In this model when an individual's score is n_i*=0, they have no impact on the estimation of parameters: γ, d, σ^2, (conditional distributions based on priors obtained through Bayesian estimation),Ψ_{dd}, andΨ_{cd}. Additionally, if Ψ_{cd} =0, the two parts of the model separate which makes the Uij and Vij values independent. If the separation does take place, the assumption of independence of the values indicates that occurrence or non-occurrence at one occasion has no influence on occurrence/non-occurrence at the next time point.

Model for Ceiling Effects

The semicontinuous variables studied by Olson and Schafer (2001) identify zero values in the dataset as real zeros. This is different from a zero that is truncated or left-censored, because the zeros are "valid self-representing data values, not proxies for negative or missing values" (page 730 Wang et al, 2008). This theoretical basis is not consistent with that underlying floor effects in that the zeros in the datasets are known to be proxies of some real value that cannot be obtained due to the floor. Based on the fact that the lowest possible scores have been obtained by the individual, it is hypothesized that the low value is actually a proxy for the true score an individual could have earned if not "censored" (Wang et al, 2008).

A search of the existing literature revealed few studies that directly addressed the estimation of growth models for data containing observations at the floor. However, given that ceiling effects introduce essentially the same issues as floor effects, the literature on ceiling effects was also considered relevant. There are important similarities in the effects of floor and ceiling effects on parameter estimates and model fit. For example, ignoring either floor or ceiling effects within the data and analyzing the data without paying special attention to these phenomena can result in using the wrong model for analysis and problems with biased parameter estimates (Wang et al, 2008). In 2008, Wang et al, conducted a study which employed the Tobit model as a method for analyzing the data that displayed various levels of ceiling effects with categorical data. The Tobit model is a categorical model with nonnegative constraints, and was decided upon based on the fact that it is a semiparametric censored regression model. According to Chay and Powell (2001), a regression model is considered censored if the observations based on the dependent variable cut off outside of a "certain range within multiple endpoints of the range" (page 29). Whenever the data are censored, the effect of the regressors on the dependent variable is underestimated when OLS is used. To attenuate for the effects of such inconsistency in estimation, the Tobit model used by Wang et al. (2008) and is based on the equation:

$$Y = \begin{cases} a & \text{if } x'\beta + \varepsilon < a, \\ b & \text{if } x'\beta + \varepsilon > b, \\ X'\beta + \varepsilon & \text{otherwise} \end{cases} \qquad (20)$$

Where y=the observed dependent variable, x=a vector of observed explanatory variables, β= a vector of unknown regression coefficients, to be estimated, ε= an error term, and a and b= censoring endpoints.

For the study conducted by Wang et al. (2008) datasets were simulated based on a sample size of *N=200*, over *T=5* distinct time points where ceiling effects (also referenced as "right-censored data") were varied between 10-40% of the sample, and repeated 500 times. The data were initially analyzed using a latent growth curve model. In the results both the mean vector and covariance matrix of the slope and intercept were underestimated. Because of the fact that the Maximum Likelihood Estimator (MLE) is based on fitting the sample mean and covariance matrix to the model, when these parameters were biased, the final results of the model were also biased with bias increasing proportionally to the amount of ceiling effect present in the data.

When using the Tobit model to analyze ceiling effects, Wang et al.(2008) included only Bayesian methods to obtain estimates of parameters. The team used WinBUGS to specify prior distributions for the parameters and continued the analysis by fitting a Bayesian Tobit growth curve model to the ceiling data that were generated. According to the results obtained by the team, the Bayesian Tobit growth model recovered all of the parameters in the model with more accuracy than a traditional growth model.

Wang et al. (2003) created data to fit a linear model. They then imposed ceiling thresholds at different points (13, 14, 15). Their hypothesis was that, with a higher ceiling, the linear model would fit more poorly and a quadratic model would be needed to fit the data. They found that the data without ceiling thresholds fit the linear model 95.4% of the time, but this percentage decreased as varied thresholds of ceiling effects (extreme high observations) were introduced into the simulation (Ceiling Threshold=15, linear model fit =34.2%: Ceiling Threshold=14, linear model fit=2.2%: Ceiling Threshold=13, linear model fit=0%). However, the results of the fitted Tobit model displayed identical proportions of correct model selection (based on what was previously known about population parameters from Bayesian estimation, called “Expected Recovery”) across ceiling thresholds as can be observed in Table 2.

Table 2: Comparison of Latent Growth and Tobit Model for Ceiling Effects Data

	% Correct	Selection	
Ceiling Threshold	Latent Growth	Tobit	Expected Recovery Percentage
0	94.50%	25%	25%
15	34.20%	25%	25%
14	2.20%	50%	50%
13	0.00%	75%	75%

The results of the study are promising for investigating the impact of ceiling effects in longitudinal categorical data. The research team also implied that many of the same theoretical bases which exist for ceiling data are identical to those in floor effects research, and that future studies are needed in order to discover ways to adequately select models and recover necessary parameters.

Censored-Inflated Model

For the purposes of model estimation with a unique population such as those that display floor effects, it is imperative to be able to estimate two related growth models in order to attenuate the biases that may be present when estimating a single homogeneous growth model. Further investigation of probable models to employ in order to attempt to obtain accurate and unbiased model parameters when floor effects are present in longitudinal data, the structural equation model considered was the censored-inflated model. The censored-inflated model developed by Muthen (2003) is unique in that the "Censored" option allows researchers to determine which outcome variables will be censored as well as whether the censoring will take place from above (as in a ceiling effect) or below (as in a floor effect) the data. In this model, the censoring limit is determined by the data and, the residual variances of the outcome variables are estimated and not allowed to vary across time and the residuals are not correlated by default. In addition, the censored-inflated model was created with the assumption that the data used in analysis were collected at the same time points and measured at equidistant occasions, so that time is not allowed to vary across participants.

In the censored-inflated model, the similarity with the Olson and Schafer (2001) two-part model is that *two different* growth models are estimated simultaneously. The first model is created based on the continuous part of the outcome for individuals from the sample who can assume values of the censoring point and above; while the second model models the inflation part as the probability of an individual being able to assume any value other than the censoring point.

Additionally, in this model parameterization for the continuous part of the model is based on the intercepts for all outcome variables being fixed at zero and means and variances, and covariances of the growth factors are estimated. In the censored portion of the model the intercepts of the outcome variable are held equal across all time points. The intercept for the growth factors is fixed at zero. The mean of the slope growth factor and variance of the intercept is estimated, but the variance of the slope is fixed to zero. The latter specification implies that all covariances involving the slope for the censored part of the model are also fixed to zero. The censored-inflated model is estimated as a censored regression model, using the same mathematical functions as the logistic portion of Olson and Schafer's two-part model (2001), and uses the robust least squares estimator. The robust least squares estimator (RLSE, also called the M-estimator) is often employed in instances when the data is heteroskedastic and it is hypothesized that least squares estimators (LSE) will not sufficiently obtain unbiased estimates due to the violation of the assumption of homoskedasticity. In addition, it is thought that the RLSE provides greater power than traditional LSE for these types of data (Stromberg, 2004).

The robust least squares estimator (RLSE) was introduced by Huber (1964) and is based on the linear equation

$$yi = \alpha + \beta 1xi1 + \beta 2xi2 + \cdot \cdot \cdot + \beta kxik + \varepsilon i$$
$$= x'i\beta + \varepsilon i \tag{21}$$

For the ith of n observations and the fitted model is

$$yi = a + b1xi1 + b2xi2 + \cdot \cdot \cdot + bkxik + ei$$
$$= x'ib + ei \tag{22}$$

The general M-estimator minimizes the objective function

$$\sum_{i=0}^{n} p(ei) = \sum_{i=0}^{n} p(yix'ib) \tag{23}$$

where the function ρ gives the contribution of each residual to the objective function. It is used when ρ is the following:

- $\rho(e) \geq 0$
- $\rho(0) = 0$

The RLSE is not a popular method of obtaining estimates in regression analyses due to the fact that it is more computationally demanding to obtain these estimates and many statistical software packages do not include these estimates (Stromberg, 2004).

In order to illustrate the usefulness of the censored-inflated model, a simple experiment was conducted using *N=1000* observations over time points *T=4* (Variable names Y11-Y14, with *n=250* students per time) that compared the parameters of a traditional linear growth and quadratic models with those from the censored-inflated model with 24% of the observations censored. The results of the comparison are displayed in the table below (Table 3).

Table 3: Comparison of Parameter Estimates for Linear Growth, Quadratic and Censored-Inflated Model (continuous portion)

Parameter	Linear	Quadratic	Censored-Inflated
ResidualY11	2.29*(0.29)*	2.27 *(0.78)*	1.44 *(0.14)*
ResidualY12	5.99*(0.39)*	5.91 *(0.39)*	1.52 *(0.10)*
ResidualY13	8.95*(0.12)*	8.98 *(0.52)*	1.64 *(0.12)*
Residual Y14	11.31*(0.75)*	8.88 *(1.56)*	1.23 *(0.21)*
Mean Slope	1.13 *(0.04)*	0.85 *(0.12)*	1.52 *(0.03)*
Variance Intercept	1.09 *(0.26)*	1.84 *(0.77)*	1.64 *(0.12)*
Variance Slope	0.36 *(0.10)*	1.76 *(1.23)*	0.33 *(0.04)*
Chi-square	21.55	*NA*	*NA*

The main difference that can be observed is in the residuals for all four time points. In the linear model, the residuals range in size, increasing over time until the final value of 11.31 is obtained at the fourth time point (Y14). The residual values for the same four time points in the censored-inflated model are relatively smaller than those of the comparison with the largest estimated value of 1.64. This can serve as evidence for support of the assertion that the censored-inflated model does model growth more accurately (including observations at the floor) and can explain the growth more efficiently than a traditional linear growth model.

CHAPTER 3

METHODOLOGY

Practical Model Investigation

In order to determine parameters appropriate for an analysis model, preliminary investigation of the generic framework for the censored-inflated model (Muthen, 2003) was conducted. The results of the investigation are in Table 4 below. The information within the body of the table is based on varying sample sizes (N=1000, N=5000 and N=10,000). At each simulation the number of integration points was manually changed from less than five to greater than eight in order to determine the minimum and maximum numbers needed for model convergence. The steps of manually manipulating parameters was continued with minimum and maximum percentage of observations at the "floor" of the dataset in order to also discover the optimum values for; number of integration points, residuals and standard errors (Table 4) .

Table 4: Summary results from Mplus Censored-Inflated Simulations

Sample Size	Integration Points	Floor Minimum	Floor Maximum	Residual Y11 (Error)	ResidualY12 (Error)	ResidualY13 (Error)	ResidualY14 (Error)
800	5	-1	-1.5	1.43(0.11)	1.54 (0.08)	1.64 (0.09)	1.18 (0.12)
900	6	-1	-1.99	1.60 (-0.13)	1.58 (0.11	1.67 (0.12)	1.36 (0.19)
1000	7	-1	-1.99	1.68 (0.14	1.63 (0.10)	1.62 (0.11	1.34 (0.18)
1500	7	-1	-2.49	1.43 (0.11)	1.54 (0.08)	1.64 (0.09)	1.18 (0.12)
2000	8	-1	-2.5	1.51 (0.17)	1.56 (0.11)	1.66 (0.13)	1.32 (0.19)
8000	8+	-1	-2.5	1.42 (0.05)	1.53 (0.03)	1.62 (0.04)	1.23 (0.06)
10000+	8+	-1	-2.5	1.43 (0.04)	1.53 (0.03)	1.62 (0.04)	1.23 (0.05)

As can be observed, the censored-inflated model works optimally when employed with large sample sizes. Smaller sample sizes require fewer integration points for model convergence, but error messages in output indicate that these models produce parameter estimates that are less stable. In addition, the percentage of values at the floor of the dataset can be varied between 10 and 35% (corresponding to floor minimum and maximum of -1 and -2.5) with higher convergence rates in larger sample sizes. There are distinct restrictions on how much variation (in percentage of floor effect) can take place in a small sample which is indicated by lack of model convergence with larger floor maxima. Finally, with larger samples more integration can be used. This is important because use of more integration points should result in more accurate estimates.

Further investigation of the censored-inflated model with real and simulated data (based on real data parameters) was conducted. The data were taken from a dataset provided by a state school system which employs growth modeling for assessing longitudinal student achievement. The sample size was N=1,000 and included observations collected over three time points denoted as variables named y11-y13 and contained a floor effect of less than 1%. The first investigation was conducted in three steps by first employing a linear, then quadratic, and finally the censored-inflated models to the data for analysis. The result of the residual variances (with error in parentheses) is located in table 5 below.

Table 5: Residual Variances of the Linear and Censored-Inflated Models when used with real data at N=1000 and T=3 time points

Model	Time 1	Time 2	Time 3
Linear	5217.84	3877.59	3166.75
	(-14.02)	*(-19.76)*	*(-9.89)*
Censored-Inflated	3749.02	3295.02	2686.94
	(-222.78)	*(-168.21)*	*(-242.69)*

** Standard error estimates not given by computer software used for the quadratic model*

Table 5 displays the residual variances are smallest in the censored-inflated model with the data in its unaltered form, containing less that 1% of a floor effect. Of interest was the result of analysis based on data that contained a greater amount of observations at the lower bound of the dataset. The parameter estimates obtained from the censored-inflated analysis on the data in Table 5 were used to create a similar dataset through Monte Carlo simulation in Mplus (Appendix A) using a linear model. These simulated data were used in subsequent censored-inflated model analyses in which varying levels of floor effects (5% and 10%) were inserted into the datasets. The residual variances resulting from analyses of data with no data at the floor, and with 5% and 10% of data at the floor are shown in Table 6. The values from the real dataset are shown in the first row.

Table 6: Residual Variances of Continuous Portion of the Censored-Inflated Model at T=3 time points

Data Type	Time 1	Time 2	Time 3
Real Data	3749.01	3295.02	2686.94
	(222.78)	*(168.21)*	*(242.69)*
C-I with 5% floor effect	237.29	296.61	289.14
	(50.49)	*(26.58)*	*(75.35)*
Linear with 5% floor	80595.34	48838.67	1146.04
	(3991.15)	*(2388.42)*	*(60.36)*
C-I with 10% floor effect	253.26	297.02	281.65
	(39.96)	*(22.41)*	*(94.74)*
Linear with 10% floor	581.29	1343.44	1040.97
	(64.85)	*(127.39)*	*(214.73)*

Table 6 shows the results of what takes place when floor effects are introduced into the data, residual variances obtained from a censored-inflated model are smaller than those obtained from a linear model of the same data. In addition, standard errors decrease when employing the censored-inflated model with these data. However, note that when floor effects are not present, use of the censored-inflated model results in inflated estimated of residual variances (first row of Table 6). These basic analyses provided evidence for the hypothesis that the censored-inflated model is best when analyzing data that contain noticeable amounts of floor effects and should be further investigated using larger datasets.

Simulation Study

A simulation study was conducted to study the performance of the censored-inflated model across a variety of data conditions thought to be representative of those encountered in state or district assessment data. Parameters for the simulation study were derived using an actual raw dataset from a state assessment data source. The data for analysis as well as the model under investigation will be implemented via Mplus 5.0 (Muthen&Muthen, 2005). Mplus was selected because it currently is the only statistical software that can generate floor effects through the MONTECARLO command, while simultaneously employing the censored-inflated model for data analysis.

For the simulation study, there were three consecutive time points at which observations were analyzed, each representing a standard reading test score for a student grades 3,4,and 5. The decision to use three time points was made based on characteristics of state test datasets, for which three time points are typically the maximum that is available.

The simulation focused on: specific sample sizes (1,000, 5,000 and 10,000 examinees), floor effect (thresholds of -1.0 (5%), -2.0 (10%), -3.0 (15%), and -4.0 (20%)), and the number of integration points needed for optimal model convergence at each level of floor effect (5, 6, and 7). There will be four conditions of floor effect, three trials of integration points, and three sample sizes, resulting in 4x3x3=36 cells. One hundred replications were conducted per cell.

After running the censored-inflated model and obtaining parameter estimates, the resulting data were read into SPSS version 15 to calculate the bias between the empirical and estimated values.

In order to analyze the data using the censored-inflated model, it was first necessary to determine which variables within the dataset are censored and which values accurately reflect the censoring limit of the data. After population parameters were specified, the model was indicated. Once analysis was terminated and the model converged, the information of interest was: means of the slopes and intercepts for the continuous portion of the model, mean of the slope for the censored-inflated portion of the model, variances of the slope and intercept for the continuous part of the model, variance of the intercept for the censored-inflated part of the model, and the residual variances. In addition, a special look was given to the standard error bias resulting from this model, since these are supposed to be lower when using the censored-inflated model. Finally, all fit indexes were discussed using descriptive statistics.

The simulation study was used to evaluate the ability of the censored-inflated model to reduce the residual variances when empirically used with floor effects data analyses. The research questions are: 1) What effect do sample size, percentage of floor effect, and number of integration points have on bias in the parameter estimates and standard errors; and does a reduction in the number of integration points at each level of floor effect produce parameter effects identical to those replications in which a larger number of integration points is used? 2) Do parameter estimates obtained through the censored-inflated model display less bias than those obtained through traditional growth modeling?

CHAPTER 4

RESULTS

Censored-Inflated Model

The censored-inflated model was used in a two-dimensional fashion. The first step of analysis was to create the data using the Mplus DATAGEN command. This was executed a total of thirty-six times, with each simulation producing a total of N=100 datasets. Within the datasets the amount of floor effect (from one to twenty percent [-1.0, -2.0,-3.0, and -4.0 in command codes]), the sample size (from one thousand to ten thousand), and the number of integration points (5, 6, and 7) were varied. After the data were generated, the Monte Carlo command was used to analyze the simulated data using the censored-inflated model. The code for this analysis is listed below:

```
DATA: FILE =C:\Documents and Settings\TMCKINLEY.dat;
      type = montecarlo;
 VARIABLE: NAMES ARE Y11-Y13;
     CENSORED ARE Y11-Y13 (bi);
   ANALYSIS: INTEGRATION = 7;
   MODEL: i s | y11@0 y12@1 y13@2;
   ii si | y11#1@0 y12#1@1 y13#1@2;
   [y11-y13@0];
   y11 * 289.42;
   y12 * 301.32;
```

```
y13 * 290.64;
[y11-y13@ -1.0] (1);
[i*297.234 s* -0.672 ii@0 si * -0.262];
i*283.109; s*300.697; i with s*-176.777;
[ii @ 0];
[si * -0.262];
Si @ 0.000;
ii * 0.047
i with ii *-.155;
s with ii * .075;
Ii with si @ 0;
s with si @0;
i with si @0;
OUTPUT: TECH1  TECH9
SAVEDATA: results are C:\Documents and Settings\TMCKINLEY.res;
```

All of the parameter estimates obtained through the censored-inflated model were saved to results ("res") files to be used for secondary analysis. Appendix A contains the output files that are a culmination of all parameter and standard error estimates used by the analysis.

<u>Findings from Secondary Analysis</u>

When compared with the empirical parameter estimates, and standard errors, the bias in those obtained from the censored-inflated model varied according to the different conditions: sample size, floor, number of integration points, and interactions between the three. A full factorial Analysis of Variance model was completed on values for each parameter and standard error bias, taking into account the three conditions and their interactions in order to discover which had the greatest impact on bias. Due to the large total sample size (N=3600) each parameter and standard error bias appeared to be significant when using only the *p value* as an indicator of statistical significance. Because of this, the partial eta squared statistic was included as an indicator of power. An effect size of 0.14 or greater based on Cohen's medium effect size criteria (Cohen, 1973), was interpreted as meaningful using this estimator.

Parameter Bias

The following tables are of the results of parameter bias based on the obtained values from the censored-inflated compared with the empirical values. The percentage of bias within the parameter was calculated using the following formula:

$$\text{(Estimated value- Empirical value)/Empirical Value} * 100 \quad (24)$$

The empirical means for each parameter are listed above the table and the *p*- and partial eta squared values are included.

Table 7: Nu (intercept for the censored-inflated part of the model) Parameter Bias

Floor values of 1%, 5%, 10% and 20%

Nu (population values): -1.00, -2.00, -3.00, and -4.00)

Model	***p value***	**Partial η^2**	
Sample Size	0.00	0.40	*
Integration Points	0.00	0.02	
Floor	0.00	0.32	*
Sample Size * Integration Points	0.07	0.00	
Sample Size * Floor	0.00	0.57	*
Integration Points * Floor	0.00	0.01	
Sample Size * Integration Points * Floor	0.90	0.00	

** indicates an eta squared value greater than the .14 cutoff*

The information in Table 7 indicates the results of the Nu or floor parameter estimate (the intercept for the censored-inflated portion of the model). This was the floor condition which was fixed four times at -1.0, -2.0, -3.0 and -4.0 to represent approximately 1%, 5%, 10% and 20% of a floor effect within the simulated data.

When comparing the estimated Nu or floor parameter to the empirical value effect sizes larger than the .14 criterion value were obtained for two of the manipulated conditions (sample size, number of integration points) individually, and the interactions between them. At each level of floor effect and sample size, η^2 values of greater than 0.14 were detected indicating that all sample sizes and percentage of floor effect have significant power on this parameter. There was no effect detected when observing the influence of the number of integration points used for model analysis.

Table 8: Theta 1 (residual variance of Y11) Parameter Bias

Theta 1 (population value: 289.42)

Model	***p value***	**Partial η^2**	
Sample Size	0.15	0.01	
Integration Points	0.00	0.14	*
Floor	0.00	0.01	
Sample Size * Integration Points	0.63	0.00	
Sample Size * Floor	0.01	0.00	
Integration Points * Floor	0.00	0.01	
Sample Size * Integration Points * Floor	0.93	0.00	

** indicates an eta squared value greater than the .14 cutoff*

With regard to the Theta 1 or residual variance for variable Y11 parameter, or the residual variance of Y11 there was an effect detected in the number of integration points used in analysis. As the number of integration points used for analysis increased from five to seven, the bias in this parameter decreased. All of the other conditions failed to produce an effect on this parameter estimate, as can be observed in Table 8.

Table 9: Theta 2 (residual variance of Y12) Parameter Bias

Theta 2 (population value: 301.32)

Model	***p value***	**Partial η²**
Sample Size	0.91	0.00
Integration Points	0.01	0.00
Floor	0.58	0.00
Sample Size * Integration Points	0.28	0.00
Sample Size * Floor	0.56	0.00
Integration Points * Floor	0.52	0.00
Sample Size * Integration Points * Floor	0.58	0.00

Table 9 displays the obtained results when calculating bias between the estimated and empirical mean of the Y12 residual variance. This table shows the resulting lack of effect of any of the 3 single conditions or interaction models on the parameter bias.

Similarly to the Theta 1 or residual variance of Y11, the residual variance of Y13 Table 10 shows little effect of any conditions with the exception of the number of integration points used for analysis (with large amounts of bias seen at five and six integration points, but none at seven). Sample size, percentage of floor effect and interactions with all conditions had little to no effect (when using the η^2 value as an indicator of significance) on this parameter.

Table 10: Theta 3 (residual variance of Y13) Parameter Bias

Theta 3 (population value= 290.64)

Model	***p value***	**Partial η²**	
Sample Size	0.01	0.00	
Integration Points	0.00	0.20	*
Floor	0.00	0.01	
Sample Size * Integration Points	0.57	0.00	
Sample Size * Floor	0.09	0.00	
Integration Points * Floor	0.00	0.01	
Sample Size * Integration Points * Floor	0.98	0.00	

** indicates an eta squared value greater than the .14 cutoff*

Table 11: Alpha 1 (continuous intercept) Parameter Bias

Alpha 1 (population value: 297.23)

Model	*p value*	Partial η^2
Sample Size	0.05	0.00
Integration Points	0.80	0.00
Floor	0.91	0.00
Sample Size * Integration Points	0.17	0.00
Sample Size * Floor	0.97	0.00
Integration Points * Floor	0.75	0.00
Sample Size * Integration Points * Floor	0.75	0.00

Table 11 shows the results of the Alpha 1 or the intercept from the continuous portion of the censored-inflated model. The empirical and estimated parameter comparison indicated that none of the manipulated conditions had an influence on this value. Table 11 is of special importance because this table contains some of the pre-specified information of interest mentioned in Chapter 3 of the paper. The intercept for the continuous portion of the censored-inflated model was being investigated to see how this parameter behaved based on the varying conditions. According to the results, this parameter is not influenced in any way by the sample sizes, percentage of floor effect or number of integration points used to analyze the data when the censored-inflated model is used on the continuous data.

Table 12: Alpha 2 (continuous slope) Parameter Bias

Alpha 2 (Population value: -0.67)

Model	*p value*	Partial η^2	
Sample Size	0.00	0.86	*
Integration Points	0.00	0.86	*
Floor	0.00	0.90	*
Sample Size * Integration Points	0.00	0.92	*
Sample Size * Floor	0.00	0.95	*
Integration Points * Floor	0.00	0.95	*
Sample Size * Integration Points * Floor	0.00	0.97	*

** indicates an eta squared value greater than the .14 cutoff*

Table 12 displays the results from the analysis of the bias within the Alpha 2, or slope for the continuous portion of the censored-inflated model. The table indicates that all manipulated conditions produced large effect sizes, with the largest observed in the interaction between all three conditions. The Alpha 2 parameter was another of interest as specified in Chapter 3. This table displays an omnibus effect on the mean of the continuous slope, which originates from all three conditions. The information obtained here serves as evidence that the slope of the continuous portion of the data is highly influenced by the sample size, percentage of floor effect present in the dataset, number of integration points used to analyze the data as well as the interactions between all three. For this parameter, all effect sizes were large when sample size, number of integration points and floor were varied. When observing these effects, the influence was linear, meaning that as sample size, number of integration points and percentage of floor effect increased, the effect increased as well.

Table 13: Alpha 3 (censored-inflated slope) Parameter Bias

Alpha 3 (population value: -0.26)

Model	***p value***	**Partial η^2**
Sample Size	0.33	0.00
Integration Points	0.54	0.00
Floor	0.33	0.00
Sample Size * Integration Points	0.04	0.00
Sample Size * Floor	0.20	0.00
Integration Points * Floor	0.68	0.00
Sample Size * Integration Points * Floor	0.98	0.00

The results of the bias between the empirical and estimated mean of the Alpha 3, or slope for the censored-inflated part of the model display that, like the Alpha 1 parameter, none of the conditions had effects on this value. The mean of the slope from the censored-inflated portion of the model was also of interest as specified in the previous chapter, as it was important to know how this parameter would behave when introduced to the varying conditions. The results indicate that the mean of the slope for the censored-inflated data is not influenced by the floor, sample size of number of integration points used to analyze the data.

Table 14: Psi 1 (variance of the continuous intercept) Parameter Bias

Psi 1 Bias (population value: 283.11)

Model	***p value***	**Partial η^2**	
Sample Size	0.84	0.00	
Integration Points	0.37	0.15	*
Floor	0.33	0.01	
Sample Size * Integration Points	0.11	0.00	
Sample Size * Floor	0.12	0.00	
Integration Points * Floor	0.63	0.01	
Sample Size * Integration Points * Floor	0.79	0.00	

** indicates an eta squared value greater than the .14 cutoff*

The Psi 1 or variance of the continuous intercept, parameter shown in Table 14 is the bias in the variance of the intercept for the continuous part of the censored-inflated model. The variance of the intercept from the continuous portion of the model is another parameter of interest because nothing was known about the behavior of this parameter under various conditions. With this parameter, only the number of integration points used to analyze the data had a substantial effect on the parameter bias. Again, with this parameter, the smaller numbers of integration points than those suggested in Muthen's example (seven) produced larger amounts of bias.

Table 15: Cov 1 (covariance of the continuous slope and intercept) Parameter Bias

Cov 1 (population value: -177.77)

Model	*p value*	Partial η^2	
Sample Size	0.45	0.00	
Integration Points	0.00	0.14	*
Floor	0.00	0.02	
Sample Size * Integration Points	0.59	0.00	
Sample Size * Floor	0.02	0.01	
Integration Points * Floor	0.00	0.01	
Sample Size * Integration Points * Floor	0.59	0.00	

** indicates an eta squared value greater than the .14 cutoff*

The information obtained regarding the Psi 2 or the covariance of the continuous slope and intercept parameter, displayed that under all conditions only the number of integration points had an effect on this parameter estimate. The amount of bias decreased as the number of integration points increased. Sample size and percentage of floor did not matter when using the censored-inflated model on the data.

Table 16: Psi 2 (variance of the continuous slope) Parameter Bias

Psi 2 (population value: 0.69)

Model	*p value*	Partial η^2	
Sample Size	0.46	0.00	
Integration Points	0.00	0.20	*
Floor	0.00	0.03	
Sample Size * Integration Points	0.65	0.00	
Sample Size * Floor	0.00	0.01	
Integration Points * Floor	0.00	0.01	
Sample Size * Integration Points * Floor	0.78	0.00	

** indicates an eta squared value greater than the .14 cutoff*

In comparison with the Psi 2 parameter, the Psi 3 or variance of the continuous slope displays much of the same pattern. Like the covariance of the slope and intercept for the continuous portion of the data, this bias on this parameter is primarily impacted by the actual number of integration points used to analyze the data, with a decrease in bias as the number increased.

Because the two covariance parameters shown in Tables 17 and 18 were not designated as particular parameters of interest, they will be discussed together as opposed to individually. For these two parameters, no design factor reached the .14 criterion level. Finally, the variance of the censored-inflated intercept bias displayed an effect size greater than the .14 criterion related to the percentage of floor effect; bias decreased as the percentage of floor effect increased from five to seven.

Table 17: Cov 2 (covariance of the continuous intercept with censored-inflated intercept) Parameter Bias

Cov 2 (parameter mean: -0.15)

Model	***p value***	**Partial η^2**
Sample Size	0.02	0.00
Integration Points	0.03	0.00
Floor	0.59	0.00
Sample Size * Integration Points	0.07	0.00
Sample Size * Floor	0.17	0.00
Integration Points * Floor	0.30	0.00
Sample Size * Integration Points * Floor	0.27	0.00

Table 18: Cov 3 (covariance of the continuous slope with the censored-inflated intercept) Parameter Bias

Cov 3 (parameter mean: 0.07)

Model	***p value***	**Partial η^2**
Sample Size	0.05	0.00
Integration Points	0.12	0.00
Floor	0.45	0.00
Sample Size * Integration Points	0.19	0.00
Sample Size * Floor	0.01	0.00
Integration Points * Floor	0.26	0.00
Sample Size * Integration Points * Floor	0.55	0.00

Table 19: Psi 3 (variance of the censored-inflated intercept) Parameter Bias

Psi 3 (parameter mean: 0.04)

Model	***p value***	**Partial η^2**

Sample Size	0.00	0.03	
Integration Points	0.00	0.05	
Floor	0.00	0.20	*
Sample Size * Integration Points	0.00	0.00	
Sample Size * Floor	0.00	0.02	
Integration Points * Floor	0.00	0.02	
Sample Size * Integration Points * Floor	0.05	0.01	

** indicates an eta squared value greater than the .14 cutoff*

For the variance of the censored-inflated intercept, there was only one condition that influenced this parameter, and that was the percentage of observations displaying a floor effect. As the floor effect increased, the variance of the censored-inflated intercept increased.

Standard Error Bias

The standard errors were all of concern in this study, because the first research question specifically addresses how sample size and percentage of floor effect impact these. The tables below (20-32) contain data regarding the standard error bias of all aforementioned parameters, again using the partial η^2 value of 0.14 or greater as an indicator of statistical significance. Standard error bias was calculated as

$$(\text{Estimated Std Dev- Empirical Std Dev})/ \text{Empirical Std Dev} *100 \qquad (25)$$

Below is a series of tables and graphs that display the individual effects on standard error bias according to the manipulated variables of percentage of floor effect, sample size, and number of integration points used for analysis.

The first test done was a multiple analysis of variance (MANOVA) test to see if there were any significant effects on the standard error biases of the manipulated conditions. For the omnibus MANOVA, all conditions and their interactions exceeded values of .14 for eta-squared. The follow-up univariate values are shown in Table 20 below.

Table 20: Parameter Standard Error Bias

Source	Dependent Variable	F	Sig.	Partial Eta Squared
N	SEbiasnu	1.66	.19	.00
	SEbiastheta1	2784.05	.00	.61*
	SEbiastheta2	372.95	.00	.17*
	SEbiastheta3	1059.57	.00	.37*
	SEbiasalpha1	3624.96	.00	.67*
	SEbiasalpha2	516.24	.00	.23*
	SEbiasalpha3	2018.56	.00	.53*
	SEbiaspsi1	1397.90	.00	.44*
	SEbiascov1	1023.22	.00	.36*
	SEbiaspsi2	66.56	.00	.04
	SEbiascov2	15.43	.00	.01
	SEbiascov3	107.87	.00	.06
	SEbiaspsi3	2.74	.07	.00
INT	SEbiasnu	.05	.95	.00
	SEbiastheta1	11189.68	.00	.86*
	SEbiastheta2	798.71	.00	.31*
	SEbiastheta3	21945.30	.00	.93*
	SEbiasalpha1	9847.29	.00	.85*
	SEbiasalpha2	1363.08	.00	.43*
	SEbiasalpha3	342.07	.00	.16*
	SEbiaspsi1	16872.27	.00	.90*
	SEbiascov1	10320.47	.00	.85*
	SEbiaspsi2	17495.85	.00	.91*
	SEbiascov2	16.06	.00	.01
	SEbiascov3	475.58	.00	.21*
	SEbiaspsi3	.08	.93	.00

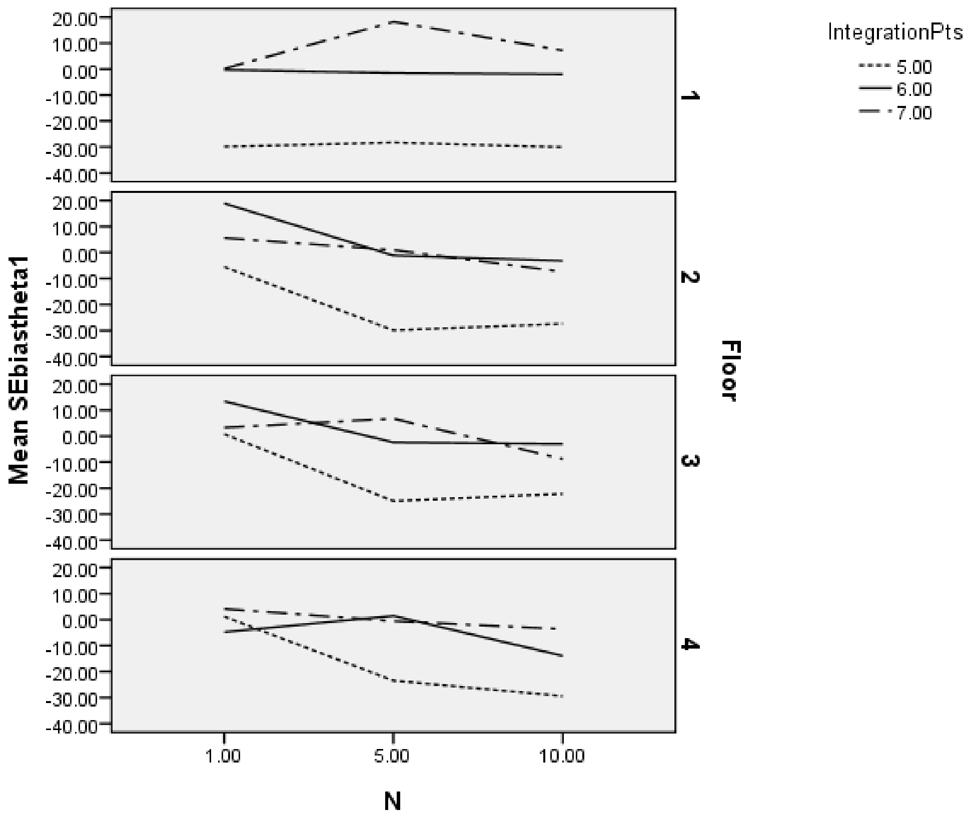
Floor	SEbiasnu	1.60	.19	.00
	SEbiastheta1	145.45	.00	.11
	SEbiastheta2	324.37	.00	.21*
	SEbiastheta3	92.35	.00	.07
	SEbiasalpha1	365.53	.00	.24*
	SEbiasalpha2	2426.58	.00	.67*
	SEbiasalpha3	126.16	.00	.10
	SEbiaspsi1	465.72	.00	.28*
	SEbiascov1	401.11	.00	.25*
	SEbiaspsi2	51.99	.00	.04
	SEbiascov2	90.32	.00	.07
	SEbiascov3	315.07	.00	.21*
	SEbiaspsi3	1.16	.32	.00
N * INT	SEbiasnu	1.53	.19	.00
	SEbiastheta1	665.17	.00	.43*
	SEbiastheta2	237.52	.00	.21*
	SEbiastheta3	471.24	.00	.35*
	SEbiasalpha1	6084.57	.00	.87*
	SEbiasalpha2	3164.24	.00	.78*
	SEbiasalpha3	907.61	.00	.51*
	SEbiaspsi1	930.53	.00	.51*
	SEbiascov1	713.90	.00	.45*
	SEbiaspsi2	1464.88	.00	.62*
	SEbiascov2	11.12	.00	.01
	SEbiascov3	173.75	.00	.16*
	SEbiaspsi3	.94	.44	.00
N * Floor	SEbiasnu	1.61	.14	.00
	SEbiastheta1	564.35	.00	.48*
	SEbiastheta2	264.57	.00	.31*
	SEbiastheta3	86.79	.00	.13
	SEbiasalpha1	3703.33	.00	.86*
	SEbiasalpha2	3237.51	.00	.85*
	SEbiasalpha3	537.79	.00	.48*
	SEbiaspsi1	417.29	.00	.43*
	SEbiascov1	168.06	.00	.22*
	SEbiaspsi2	115.52	.00	.16*
	SEbiascov2	112.71	.00	.16*
	SEbiascov3	182.71	.00	.24*
	SEbiaspsi3	1.37	.22	.00

INT * Floor	SEbiasnu	1.30	.25	.00
	SEbiastheta1	608.24	.00	.51*
	SEbiastheta2	365.91	.00	.38*
	SEbiastheta3	312.61	.00	.35*
	SEbiasalpha1	1364.66	.00	.70*
	SEbiasalpha2	2468.55	.00	.81*
	SEbiasalpha3	594.97	.00	.50*
	SEbiaspsi1	283.57	.00	.32*
	SEbiascov1	134.92	.00	.19*
	SEbiaspsi2	179.94	.00	.23*
	SEbiascov2	136.42	.00	.19*
	SEbiascov3	431.90	.00	.42*
	SEbiaspsi3	1.49	.17	.00
N * INT * Floor	SEbiasnu	1.09	.36	.00
	SEbiastheta1	139.74	.00	.32*
	SEbiastheta2	267.63	.00	.47*
	SEbiastheta3	100.28	.00	.25*
	SEbiasalpha1	1555.96	.00	.84*
	SEbiasalpha2	1577.70	.00	.84*
	SEbiasalpha3	422.57	.00	.59*
	SEbiaspsi1	231.21	.00	.44*
	SEbiascov1	270.76	.00	.48*
	SEbiaspsi2	388.52	.00	.57*
	SEbiascov2	91.57	.00	.24*
	SEbiascov3	288.70	.00	.49*
	SEbiaspsi3	.81	.64	.00

**indicates an eta squared value greater than the .14 cutoff*

As can be seen from the table, all parameter standard errors with the exceptions of the censored-inflated thresholds (Nu) and the variance of the censored-inflated intercept (Psi3) were affected by the manipulated conditions at levels greater than the .14 criterion. Figures 1 through 11 display the results of the individual standard error bias by manipulated conditions tests.

Figure 2: Graph of manipulated variable interaction on Standard Error bias for Theta 1 (residual variance of Y11)



For the standard error of the residual variance of Y11, there was substantial negative bias with five integration points for most levels of floor and sample size. At this number of integration points, standard error bias increased unexpectedly with sample size. With six or seven integration points, bias was close to zero under most conditions, and generally decreased with larger sample size and larger floor values. With seven integration points, however, there was a tendency for bias to increase slightly at the middle sample size of 5000.

Figure 3: Graph of manipulated variable interaction on standard error bias for Theta 2 (residual variance for Y12)

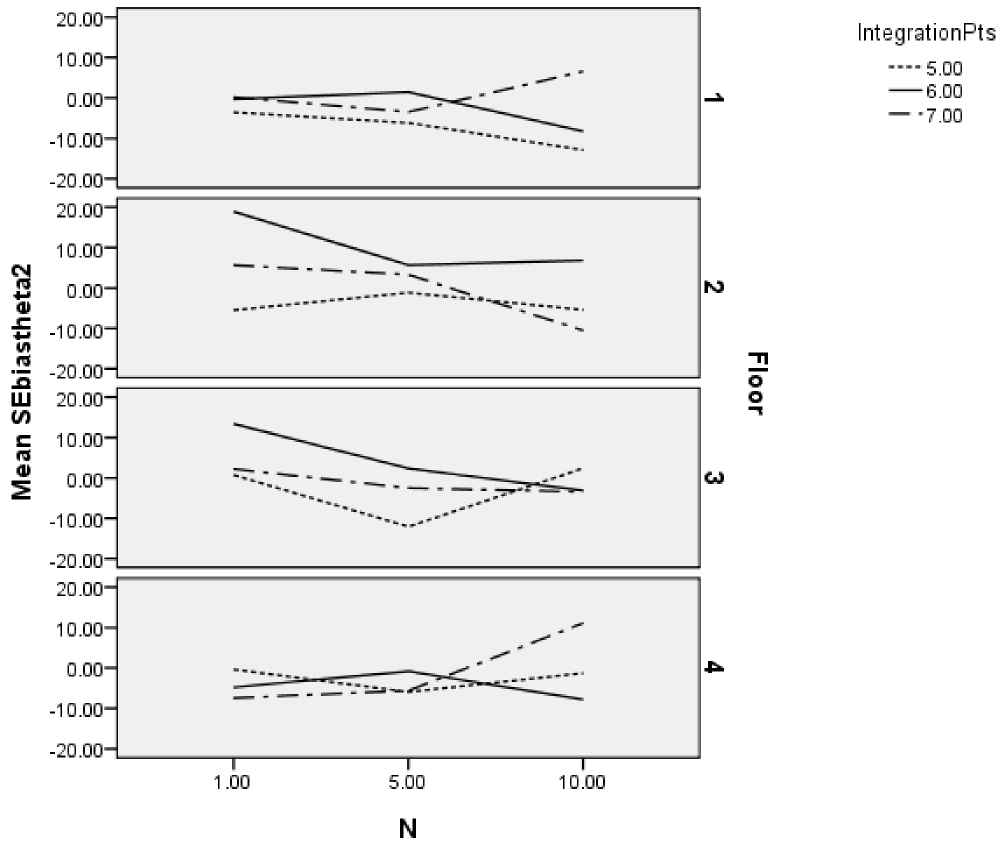
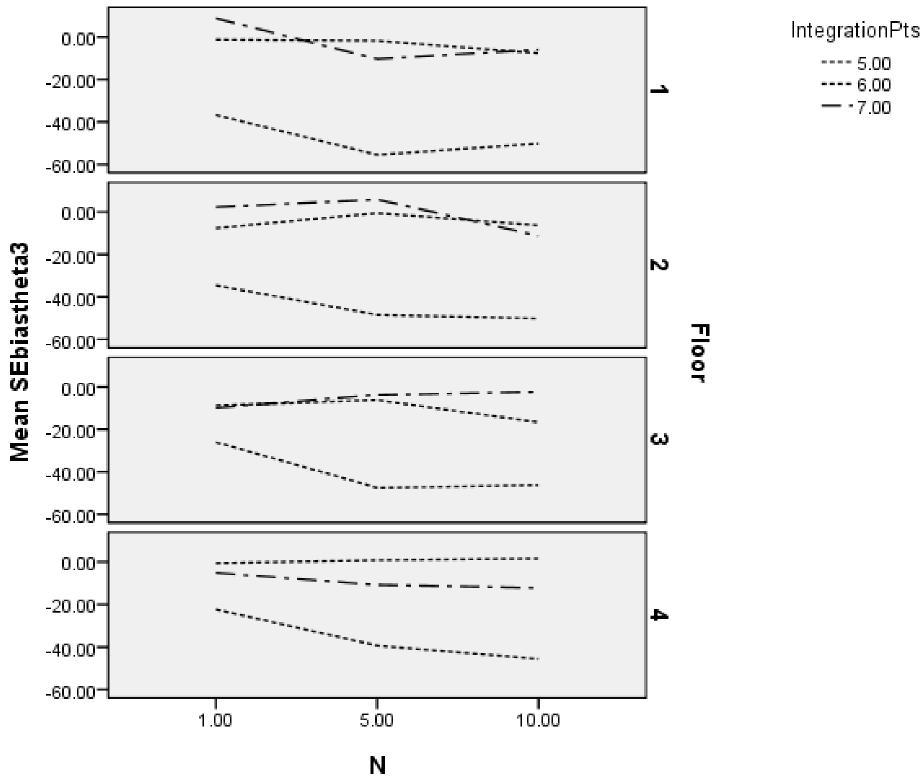


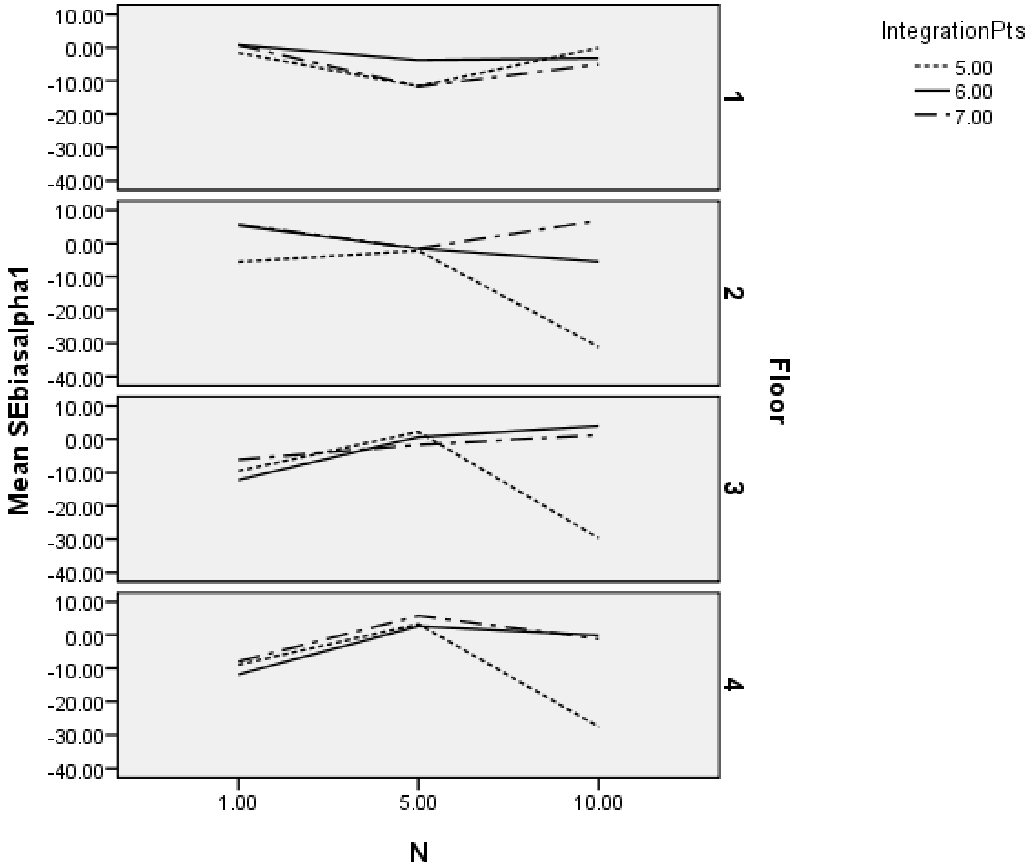
Figure three shows the bias in the standard error of the Y12 residual variance. Overall, standard error bias for this parameter was minimal under all conditions. The use of five integration points resulted in the most standard error bias for this parameter. However, with seven integration points at floor condition of one and four, positive bias of up to 10 was found. With six integration points, bias generally decreased with sample size, as would be expected.

Figure 4: Graph of manipulated variable interaction on standard error bias for Theta 3 (residual variance for Y13)



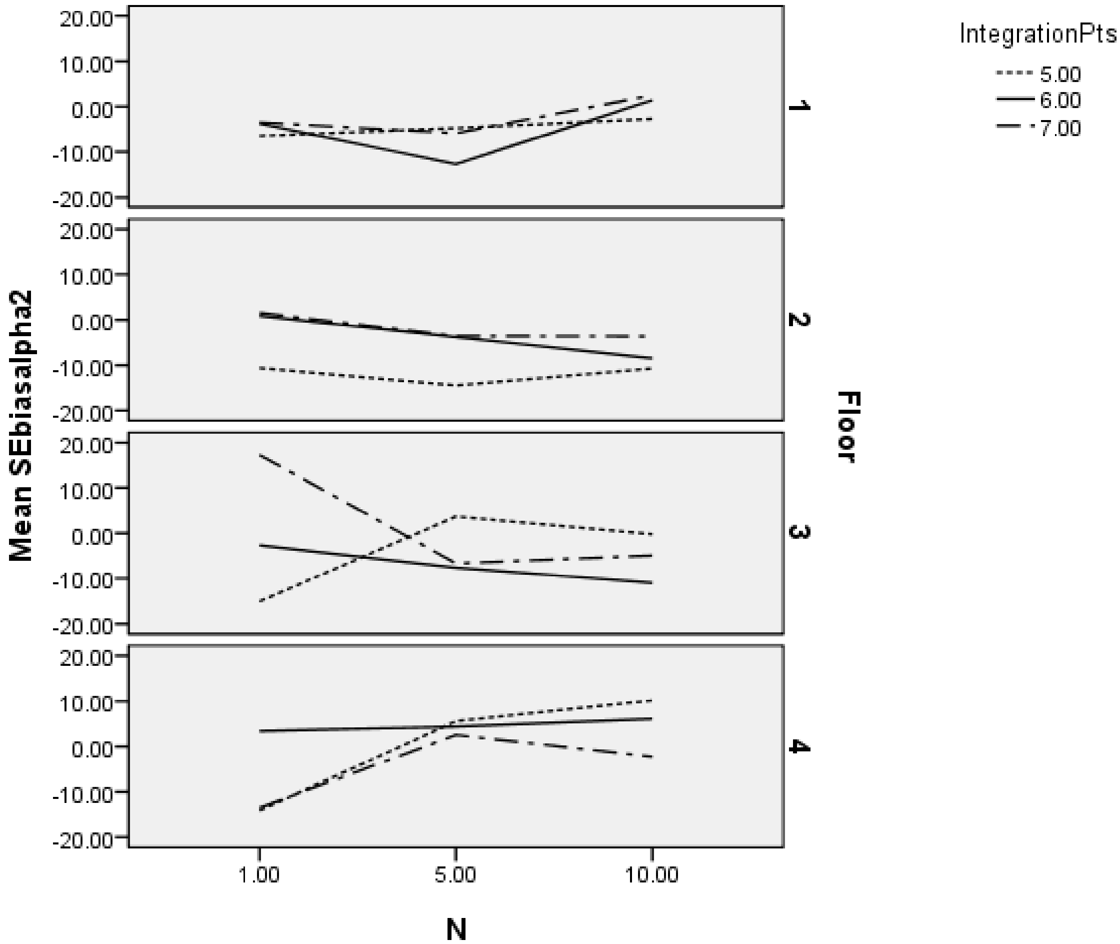
In figure four, the graphs displays that negative bias in the standard error of the residual variance of the Y13 parameter is quite severe with five integration points. Standard error bias also increases with sample size for this parameter. With six or seven integration points, bias is very slight, hovering around zero.

Figure 5: Graph of manipulated variable interaction on standard error bias for Alpha 1 (Continuous intercept)



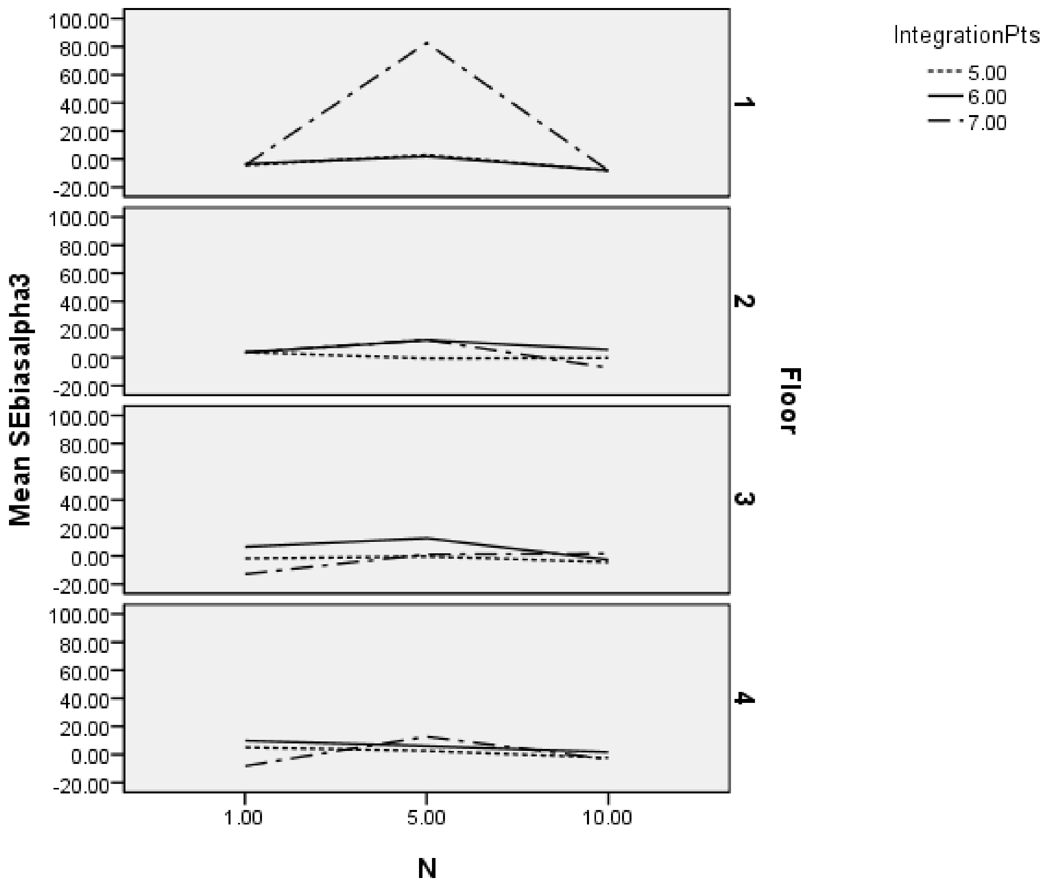
For the mean bias of the standard error of the continuous intercept, bias was negligible under most combinations of conditions, but substantial negative bias was seen for conditions with five integration points at floors of two, three, and four and sample sizes of 10,000.

Figure 6: Graph of manipulated variable interaction on standard error bias for Alpha 2 (Continuous slope)



The mean bias of the standard error for the slope of the continuous part of the model, was minimal at floors of one or two, not exceeding 10% for any condition at this floor. At a floor of three or four, however, negative bias of up to 20% was noted for conditions combining five integration points and small sample sizes. Positive bias of up to 20% was evident with seven integration points, a floor of three, and a sample size of 5000. Conversely, a combination of seven integration points with a floor of four resulted in negative bias of about 15% at a sample size of 5000.

Figure 7: Graph of manipulated variable interaction on standard error bias for Alpha 3 (Censored inflated slope)



The bias for the slope of the censored inflated portion of the model, was greatest at five x integration points and a sample size of 5000. Other integration points displayed minimal bias values across floors and sample sizes.

Figure 8: Graph of manipulated variable interaction on standard error bias for Psi 1 (Variance of the continuous intercept)

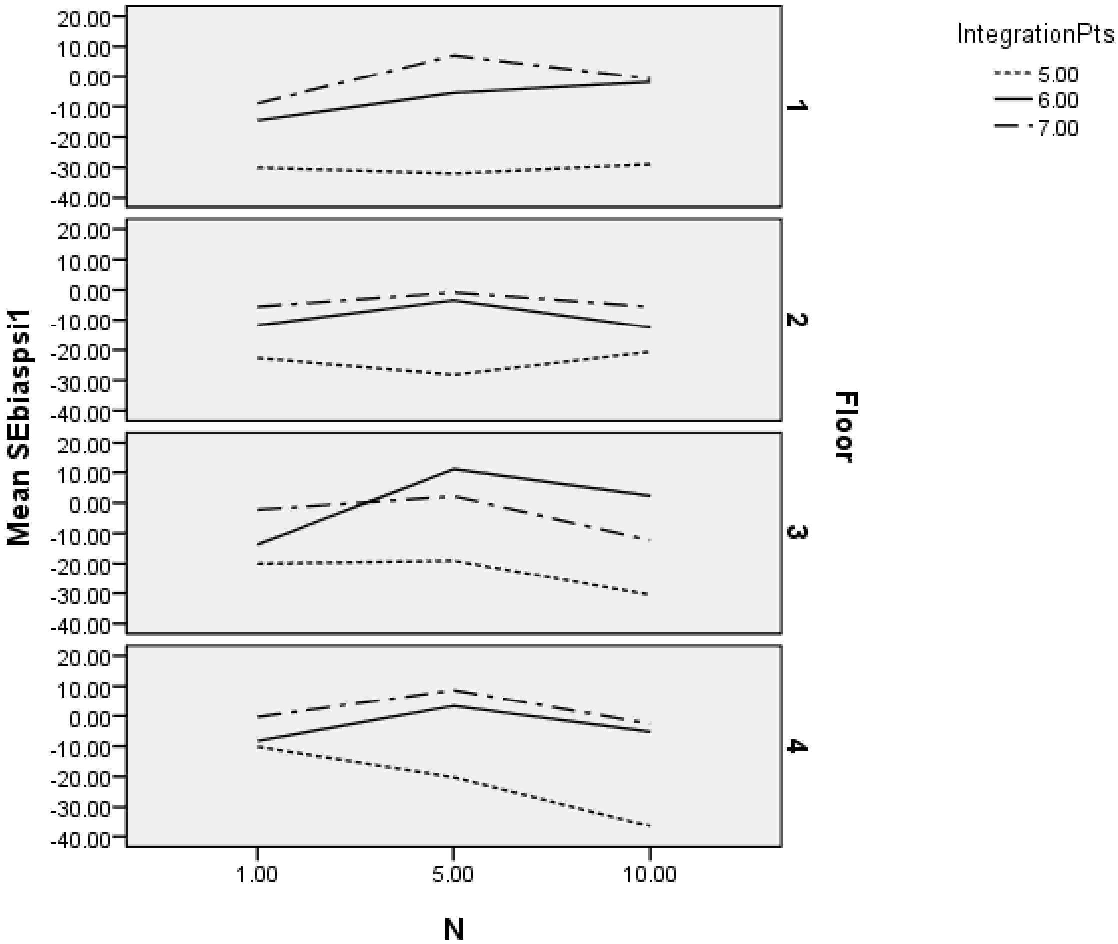
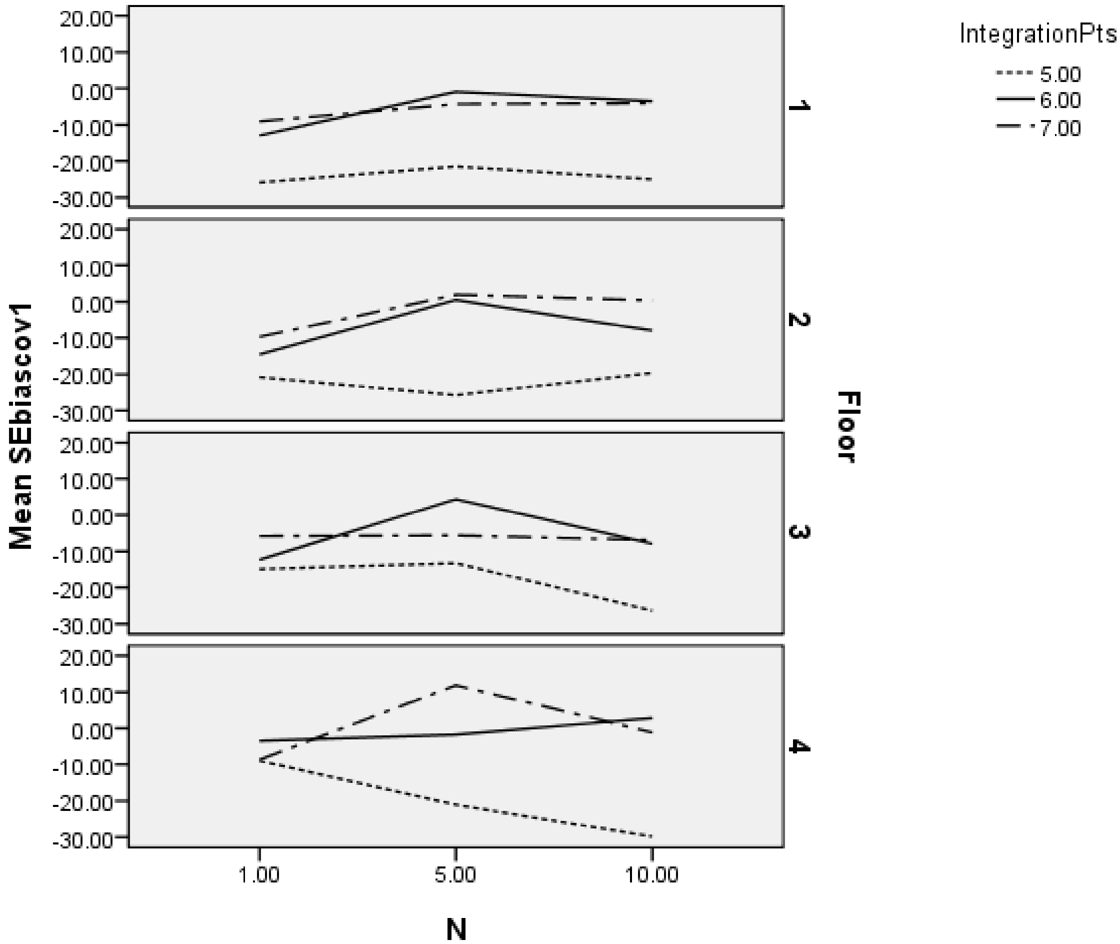


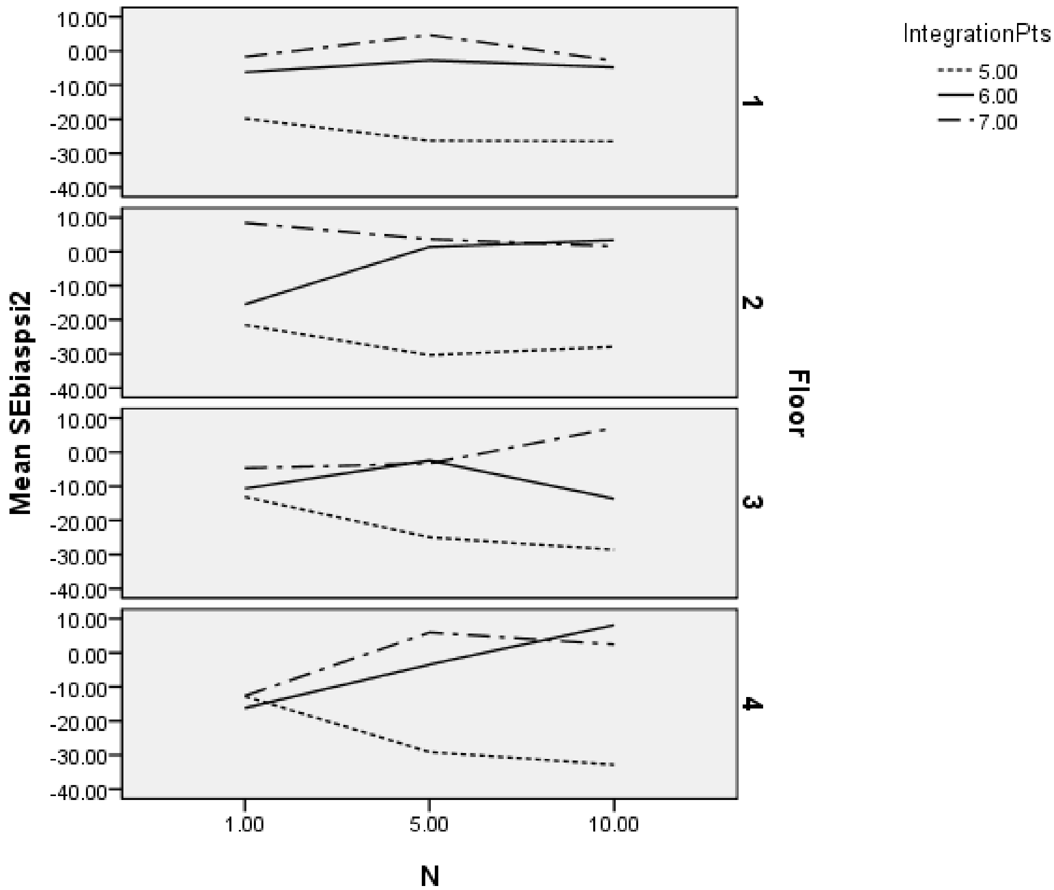
Figure eight shows that mean bias in the variance of the continuous intercept is most severe with five integration points across all sample sizes. Though the six and seven integration point lines display variations in biases as the floors and sample sizes increase, the seven integration point line displays a mean bias of approximately zero across all sample sizes at the second floor.

Figure 9: Graph of manipulated variable interaction on standard error bias for Cov 1 (Covariance of the continuous slope and intercept)



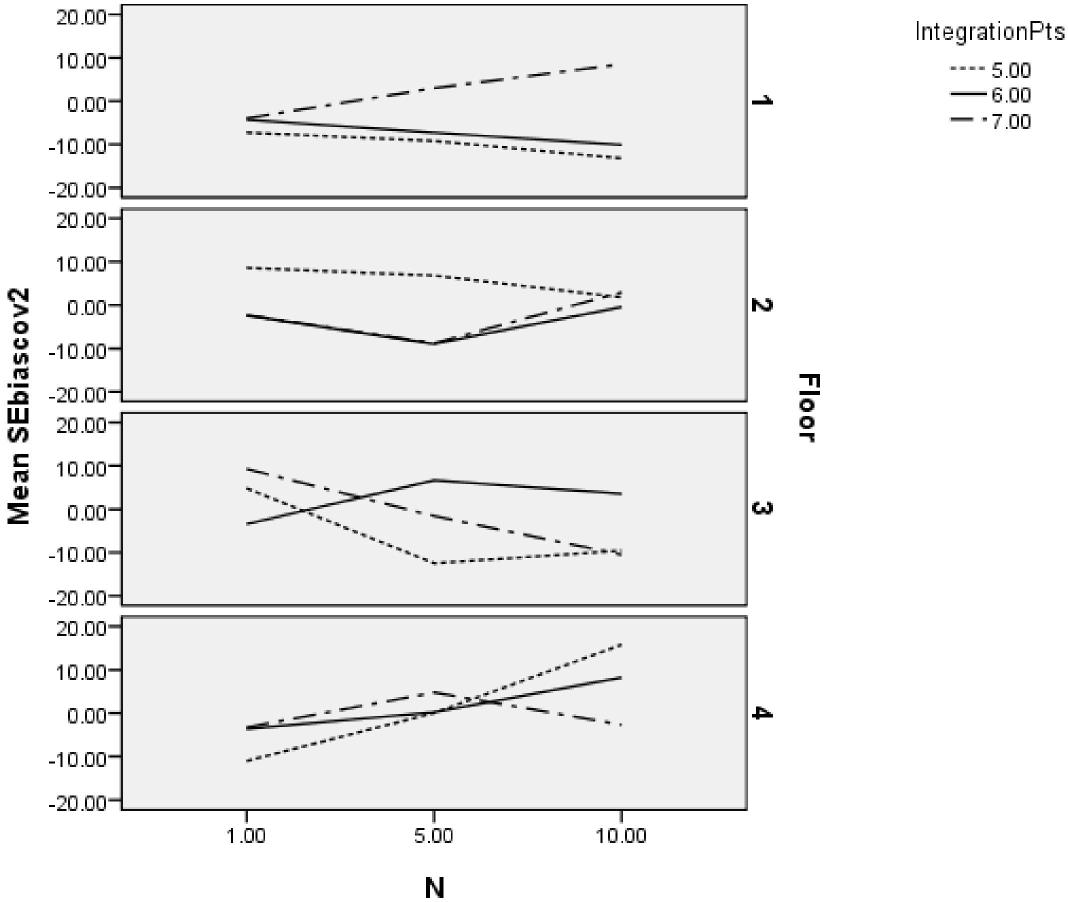
The mean bias in the continuous slope/intercept covariance, is greatest in the five integration point line across floors and sample sizes with a linear effect displayed at the fourth floor according to increases in sample size. The six integration point line displays a decrease in bias as floors increase from one to four, with the mean bias value of approximately zero across all sample sizes at floor four. The seven integration point line shows the least amount of bias at floor three with a mean bias value of approximately zero across all sample sizes.

Figure 10: Graph of manipulated variable interaction on standard error bias for Psi 2 (Variance of the continuous slope)



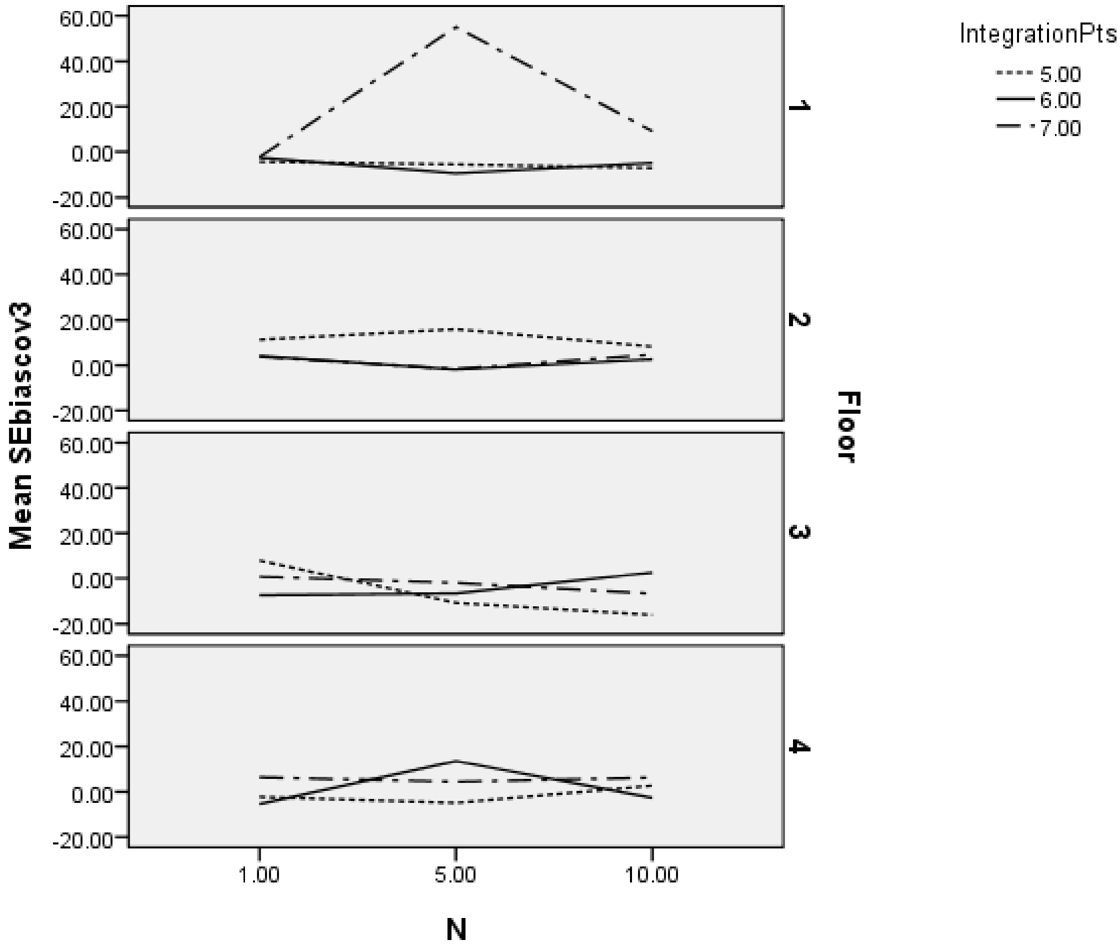
The mean bias for the continuous slope variance in all four floors is greatest for the five integration point line regardless of sample size. The six integration point line is constant at zero in floor one, approaches zero with sample size of 10000 at floor two, fluctuates with sample size at floor three, and linearly increases with increases in sample size at floor four. The seven point line shows as increase in mean bias as the floor increases from one to four, with greatest bias displayed at floors three and four and sample sizes of 10000 and 5000 respectively.

Figure 11: Graph of manipulated variable interaction on standard error bias for Cov 2 (Covariance of the censored inflated and continuous intercepts)



These graphs display that the standard error bias of the covariance of the censored-inflated and continuous intercepts was minimal at all conditions. The greatest amount of bias can be observed in the seven integration point line at floor one and five integration point line at floor five with increases in sample size.

Figure 12: Graph of manipulated variable interaction on standard error bias for Cov 3 (Covariance of the censored inflated intercept and continuous slope)



For the mean bias of the covariance of the continuous slope and censored-inflated intercept, most integration points displayed minimal values across sample sizes. The exception was the seven integration point line at the first floor and a sample size of 5000, where the greatest bias was detected-approximately 50.

A Comparison of Linear versus Censored-Inflated Models

The second research question of this study was one that inquired about the differences in parameter estimate bias that might be discovered when comparing the linear and censored-inflated models on identical datasets. It was hypothesized that the parameter bias obtained through the use of the censored-inflated model would be less than that from the results of the linear model. In order to test this hypothesis, the datasets created using the Mplus Monte Carlo command were analyzed in the same software using a linear model. The code for which is below:

```
type=montecarlo;
VARIABLE: NAMES ARE Y11-Y13;
USEVARIABLES ARE ALL;
MODEL: i s | y11@0 y12@1 y13@2;
```

The saved results files were then exported to SPSS 15.0 in the same way as for the censored-inflated model. Once exported into SPSS, descriptive statistics were obtained and displayed below in tables 21 and 22. Table 21 shows the results for the censored-inflated and 22 is for the linear model. Though the censored-inflated had more initial parameters, in order to compare the parameter estimates in identical parameters, the datasets were matched according to the parameters that they had in common.

Table 21: Descriptive Statistics for the Parameters of the Censored-Inflated Model N=3600

Parameter	Minimum	Maximum	Mean	Standard Deviation
Nu	-95.10	47.87	1.6727	17.77436
Theta1	-53.96	44.71	-3.3896	8.98658
Theta 2	-22.65	19.27	.1838	4.24444
Theta 3	-55.05	53.15	-5.2381	12.14028
Alpha 1	-.77	.91	.0020	.17872
Alpha 2	-344.36	383.60	-.9329	71.43957
Alpha 3	-217.38	181.69	.5824	30.97517
Psi 1	-35.29	53.00	3.5124	9.45795
Psi 2	-42.09	55.33	4.0422	10.69029
Psi 3	-21.72	30.75	2.8033	6.61130
Psi 4	-9547.75	9736.12	24.7724	1210.89295
Psi 5	-13897.69	19693.91	30.9730	2136.09263
Psi 6	-99.88	5369.69	281.4844	536.42696

Table 22: Descriptive Statistics for the Parameter Estimates of the Linear Model N=3600

Parameter	Minimum	Maximum	Mean	Standard Deviation
Nu	-2295580.90	-58256.72	-817397.1241	578490.75461
Lambda 1	164.83	7611.06	2311.5934	1830.28881
Lambda 2	66.66	7265.17	1863.5683	1593.81193
Lambda 3	-52.54	1.58	-9.6932	11.18793
Theta 1	-100.91	-93.15	-98.2062	1.64125
Theta 2	-644.82	1056.23	3.6009	136.10842
Theta 3	-786.23	270.36	-152.4121	81.63061
Alpha	-340.41	585.86	-12.7344	73.96391
Beta	-85.50	350.90	26.5958	111.14861
Psi 1	-92.24	176.07	-28.1145	64.36247
Psi 2	-749.77	-122.20	-281.6817	158.80413
Psi 3	-89.00	3.07	-60.1278	24.94084

When looking at the two tables, it appears that the parameter estimate biases are substantially larger in the linear model than in the censored-inflated. However, to confirm this suspicion a dependent samples t-test was conducted to further investigate the significance at each parameter bias according to group membership.

Once the censored-inflated and linear datasets were combined, they were dummy-coded "1" for censored-inflated and "2" for linear and the parameter estimates that were common to the two analyses were compared. Common parameter estimates across the two models were the intercepts for the continuous variables (Nu; note that only one value is estimated as values for the three time points are constrained to be equal), the residual variances at each of the three time points (theta1-3), the intercepts and variances of the continuous slope and intercept, and the covariance of the continuous slope and intercept. The results from the descriptive statistics and dependent samples t-test are located in tables 23 thru 24.

Table 23: Descriptive Statistics for the Bias Parameters of the Censored-Inflated Model N=3600

Parameter	Minimum	Maximum	Mean	Std. Deviation
Nu Bias	-636.92	-104.90	-355.93	119.70
Theta1 Bias	-53.96	44.71	-3.39	8.95
Theta2 Bias	-22.65	19.27	.18	4.22
Theta 3 Bias	-55.05	53.15	-5.16	12.14
Alpha 1 Bias	-.77	.91	.01	.19
Alpha 2 Bias	-344.36	383.60	-.56	71.09
Psi 1 Bias	-35.29	53.00	3.51	9.44
Psi 2 Bias	-322.40	-233.16	-274.84	11.23

Table 24: Descriptive Statistics for the Parameters of the Linear Model N=3600

Parameter	Minimum	Maximum	Mean	Std. Deviation
Nu Bias	58056.72	2295380.90	843281.92	590947.02
Theta1Bias	-100.93	-92.96	-98.09	1.717
Theta 2 Bias	-623.30	1010.56	-.21	130.37812
Theta 3 Bias	-811.44	283.97	-154.04	84.48
Alpha Bias	-335.08	570.64	-15.23	72.16
Beta Bias	-199537.05	-6511.85	-56370.31	48502.11
Psi 1 Bias	-92.24	176.07	-27.71	63.51
Psi 2 Bias	-749.77	-122.20	-282.63	156.68

There are apparent differences between the descriptive statistics obtained using the censored-inflated and linear models for analysis. To further investigate the differences between the means of these two types of analyses, a dependent samples t-test was performed.

Table 25: Results of the Dependent Samples t-test N=7200

Parameter	T	*p*-value	
Nu Bias	60.23	.00	*
Theta1 Bias	-90.09	.00	*
Theta2 Bias	-0.02	.98	
Theta3 Bias	-70.48	.00	*
Alpha Bias	-12.52	.00	*
Beta Bias	-53.88	.00	*
Psi1Bias	-21.40	.00	*
Psi2 Bias	-212.82	.00	*

**indicates significance at p<0.05*

When comparing the parameters that could be matched between the censored-inflated and linear models, all comparisons except the Theta 2 or the residual variance for variable Y12, displayed statistically significant results. This simple test provides support for the research question that asks if there is a difference in the parameter estimate bias obtained through using the censored-inflated instead of linear modeling for floor effects data. The results display that there is in fact a statistically significant difference between seven of the eight parameter biases obtained when using the two methods of data analysis.

In order to further investigate the differences in means produced by the two methods of data analysis, a means comparison test was done in order to closely examine those produced by each method. Group one designated those observations analyzed with linear modeling, and group two was censored-inflated.

Table 26: Results of the Mean Comparisons N=7200

Group		NuBias	Theta1Bias	Theta2Bias	Theta3Bias	AlphaBias	BetaBias	Psi1Bias	Psi2Bias
1.00	Mean	843281.93	-98.09	-.21	-154.04	-15.23	-56370.31	-27.71	-282.63
2.00	Mean	-355.93	-3.39	.18	-5.16	.01	-.56	3.51	-274.84

The t-test in addition with means comparison analyses shows that the censored-inflated model consistently produced parameter estimates with less bias than those resulting from linear modeling.

CHAPTER 5

CONCLUSION

<u>Discussion</u>

In Chapter three, the research questions were: 1) What effect do sample size, percentage of floor effect, and number of integration points have on parameter estimate bias and standard error bias? 2) Do parameter estimates obtained through the censored-inflated model display less bias than those obtained through traditional growth modeling?

Effects of Manipulated Variables on Standard Errors

Through data generation using the Mplus Monte Carlo method, analysis with the censored-inflated and linear models the questions were answered. For the question regarding the effect of sample size and floor effect on parameter estimate bias and standard errors bias, tables 27 and 28 provide checklists visual representations of the way in which the parameter estimates and standard errors were impacted by the manipulated factors.

Table 27: Results of Censored-Inflated Model Parameter Estimates by Manipulated Factors

Parameter Estimate	Sample Size (N)	Floor (F)	Number of Integration Points (INT)	N*F	N*INT	F*INT	N*F*INT
Nu (floor)	✓	NA	✓	✓	✓	✓	✓
Theta 1 (varianceof Y11)	NA	NA	✓	NA	NA	NA	NA
Theta 2 (variance of Y12)	NA	NA	NA	NA	NA	NA	NA
Theta 3 (variance of Y13)	NA	NA	✓	NA	NA	NA	NA
Alpha 1 (mean of cont. intercept)	NA	NA	NA	NA	NA	NA	NA
Alpha 2 (mean of cont. slope)	✓	✓	✓	✓	✓	✓	✓
Alpha 3 (mean of C-I slope)	NA	NA	NA	NA	NA	NA	NA
Psi 1 (variance of cont. intercept)	NA	NA	✓	NA	NA	NA	NA
Cov 1 (covariance of cont. intercept and slope)	NA	NA	✓	NA	NA	NA	NA
Psi 2 (variance of cont. slope)	NA	NA	✓	NA	NA	NA	NA
Cov 2 (covariance of CI and cont. intercepts)	NA	NA	NA	NA	NA	NA	NA
Cov 3 (covariance of cont. slope with C-I intercept)	NA	NA	NA	NA	NA	NA	NA
Psi 3 (variance of CI intercept)	NA	✓	NA	NA	NA	NA	NA

✓ *indicates a η^2 value of 0.14 or greater*

When observing the Nu or floor parameter the effects of the sample size, number of integration points and the interactions between all manipulated variables were detected at a greater than .14 cutoff. These effects were linear in nature meaning that as sample size and number of integration points increased, the bias decreased. The increase in the number of integration points had a decrease on the effect of the bias for the residual variance of the Y11, Y13, mean of the continuous slope, the variances of the continuous slope intercept, and the covariance of these two parameters. When increased, all of the manipulated conditions decreased the bias of the continuous slope. Finally, for the variance of the censored-inflated intercept, only the actual percentage of observations at the floor had an effect on this parameter. As the floor increased, the bias decreased. This served as evidence that the censored-inflated model is best to be used specifically with floor effects data in that a datasets containing smaller floors (between one and five percent) may not need such a highly specialized model for analysis.

Table 28: Results of Censored-Inflated Model Standard Errors by Manipulated Factors

Parameter Estimate	Sample Size (N)	Floor (F)	Number of Integration Points (INT)	N*F	N*INT	F*INT	N*F*INT
Theta 1 (variance of Y11)	✓	NA	✓	✓	✓	✓	✓
Theta 2 (variance of Y12)	✓	✓	✓	✓	✓	✓	✓
Theta 3 (variance of Y13)	✓	NA	✓	✓	✓	✓	✓
Alpha 1 (mean of cont. intercept)	✓	✓	✓	✓	✓	✓	✓
Alpha 2 (mean of cont. slope)	✓	✓	✓	✓	✓	✓	✓
Alpha 3 (mean of C-I slope)	✓	NA	✓	✓	✓	✓	✓
Psi 1 (variance of cont. intercept)	✓	✓	✓	✓	✓	✓	✓
Cov 1 (covariance of cont. intercept and slope)	✓	✓	✓	✓	✓	✓	✓
Psi 2 (variance of cont. slope)	NA	NA	✓	✓	✓	✓	✓
Cov 2 (covariance of CI and cont. intercepts)	NA	NA	✓	✓	✓	✓	✓
Cov 3 (covariance of cont. slope with C-I intercept)	NA	✓	✓	✓	✓	✓	✓
Psi 3 (variance of CI intercept)	NA	NA	NA	NA	NA	NA	NA

✓ *indicates a η^2 value of 0.14 or greater*

The effect of the sample size on standard errors was detected for all parameters except the covariances of the continuous slope with the continuous intercept and with the censored-inflated intercept, and the variance of the censored-inflated intercept. The effect of the percentage of floor effect on standard error was large for all standard errors bias except that for

Theta 1 (the residual variance for Y11), the covariance of the censored-inflated and continuous intercepts, and the variance of the censored-inflated intercept.

The number of integration points used in analysis had an effect on all standard errors except those for the covariance of the censored-inflated intercept with the continuous intercept and the variance of the censored-inflated intercept.

The interaction between the sample size and percentage of floor effect was significant on all except the Theta 3 (residual variance for Y13), and variance of the censored-inflated intercept.

The interaction between the sample size and number of integration points used in analysis displayed effects in a small number of standard error biases. Those that it did not affect were the covariance of the censored-inflated intercept with the continuous intercept and slope, and the variance of the censored-inflated intercept.

The interaction between the percentage of floor effect and number of integration points used for analysis was greater than .14 in all except the variance of the censored-inflated intercept.

Finally, the interaction between all three manipulated conditions: sample size, number of integration points used for analysis and the percentage of floor effect has an effect size of greater that .14 on all standard errors except for the variance of the censored-inflated intercept.

Censored-Inflated versus Linear Model Bias

The parameter estimate bias was smaller for the censored-inflated model as compared to the continuous linear model for all parameters common to the two models. This provides evidence for the assertion that if a floor effect of any level is present within a dataset, it is imperative to treat those data in a different manner than linear modeling allows for.

Effect of Number of Integration Points on Parameter Estimates

Based on the fact that so little was known about the censored-inflated model, it was important to try to explore as many aspects as possible in order to gain more insight on its workings. The recommended number of integration points provided by Muthen's example was 7 (Muthen, 2005). However, through preliminary experimentation as referenced in Chapter 3, it was determined that between five and seven would allow the model to converge without error or warning messages. With that information, the analyses were conducted at each of four levels of floor effect, varying the sample size and using between five and seven integration points for analysis.

The results section of this paper provides great detail about how the number of integration points, the sample size, percentage of observations at the floor and the interactions between the three effect the bias of the standard errors, however there is still room for experimentation with all of the parameters used here as well as others that can be inserted into the data by a future researcher.

Practical Use

As shown by the results obtained through analyses, the censored-inflated model consistently produced smaller bias in parameter estimates and standard errors than linear growth modeling. This supports the assertion that the censored-inflated model would best be employed in situations where there are examinees who display floor effects, when attempting to truly capture the essence of their growth over time.

Since this method assumes that data are nonnormal, there is no treatment required of the data before analysis can take place.

However, caution should be used with this method when large floor effects are discovered as this could indicate a problem with the assessment as opposed to the examinees. In such instances if at all possible, the measure should be examined for construct relevance to make sure that it is not an incorrect fit for the examinees. This examination should include making sure that the ability level of the examinees matches that being assessed by the measure, and ensuring that the material being assessed by the measure has adequately been taught to the examinees beforehand.

Caveats

The censored-inflated model is a very promising model to be used specifically with data that contain greater than 1% of a floor effect. The results of the study have displayed evidence that floor effects data are unique in the make-up of the observations that reside in the lower bounds and should not be treated as normally distributed data with linear effects by using linear growth modeling analyses.

The censored-inflated model however is a highly specialized and complicated model. There are many components of this model to maintain in order to ensure a successful analysis. In implementing Monte Carlo data to be used for further studies, there is a high level of precision required in order to obtain successful results. Background information obtained from some real dataset to be used as a model, such as all of the starting values for the various means, intercepts and slopes, and percentage of the floor within the dataset must be known *a priori* or the model will fail.

In addition, the multiple parameters obtained through the model, may be more than necessary for such a small group of observations. If there is a small floor effect of one or five percent, this model may be a bit overbearing, in that favorable results could likely be obtained through more conventional growth modeling techniques.

Preliminary research results displayed in Chapter 3 showed that due to the complex nature of this model, it cannot be expected to produce successful results with small datasets (smaller than N=1000). It also showed that there are very specific parameters of the percentage of the floor effect that can be present in the data (those tested were between 1 and 20%), and the number of integration points that can be used for the model can converge varies between five and seven. Fewer integration points than five resulted in a failed model, and greater than seven resulted in a plateau effect. The same was observed when and a sample size of more than 10,000 was used in the analyses.

Implications for Future Research

Future studies may focus specifically on further investigation of the differences between the empirical and estimated parameters between the censored-inflated and linear models. This study only included three time points for analysis. A future study may include more time periods to observe more of a long term change in the floor effects dataset, and compare that volume of information between the censored-inflated and linear models.

Also from the preliminary research, there was a point at which the sample size of greater than 10,000 actually did not result in any differences in parameters; however there could be fortification of this area. A future study could include more observations (perhaps at the same number of time points), and integration points to determine whether there is a plateau effect similar to the one observed and discussed in Chapter 3.

Because so little is known about this model, there are many possibilities for research in the future. For instance, though there is some information known about the sample sizes and numbers of integration points that can be accommodated, there is still much to investigate as far as the actual percentage of floor effect that can be used with this model. From the preliminary experimentation conducted it appears that the smallest percentage of floor effect that can be accommodated is one percent, and for the purpose of this study it was capped at 20, however, the true bounds of this variable are unknown. Depending on the purpose for using a floor effects model, a greater percentage of floor effect may be desired, and little to no information is available regarding the behavior of this model with a larger floor effect.

It is hopeful that a greater amount of floor effect could in essence provide results comparable to the zero-inflated (ZIP) model that can be used for the over dispersion of lower bounds data.

REFERENCES

Bauer, D.J. & Curran, P.J. (2003). Distributional Assumptions of Growth mixture Models: Implications for Overextraction of Latent Trajectory Classes. *Psychological Methods*, 8 (3) 338-363.

Bauer, D.J. & Curran, P.J. (2003). Overextraction of Latent Trajectory Classes: Much Ado About Nothing? Reply to Rindskopf (2003), Muthen (2003), and Cudeck and Henly (2003). *Psychological Methods*, 8(3) 384-393.

Blozis, S.A (2007). A Newton procedure for conditionally linear mixed-effects models. *Behavior Research Methods*, 39(4) 695-708.

Blozis, S.A. (2008). On Fitting Nonlinear Latent Curve Models to Multiple Variables Measured Longitudinally. *Structural Equation Modeling*, 14(2), 179-201.

Bollen, K.A. (1989). *Structural Equations with latent variables*, New York: Wiley.

Browne, M.W. & Cudeck, R. (1993). Alternative ways of assessing model fit. In K.A. Bollen and J.S. Long (Eds.), *Testing structural equation models* (pp 136-162). Beverly Hills, CA: Sage.

Byrk, A.S. & Raudenbush, S.W. (1987). Application of hierarchical linear models to assess change. *Psychological Bulletin*, 101, 147-158.

Byrk,A.S. & Raudenbush, S.W. (1992). *Hierarchical Linear Models: Applications and data analysis methods*. Thousand Oaks: Sage.

Chay, K.Y. & Powell, J.L. (2001). Semiparametric Censored Regression Models. *Journal of Economic Perspectives*, 15(4) 29-42.

Chiu, M.M, & Khoo, L. (2005). A New Method for Analyzing Sequential Processes: Dynamic Multilevel Analysis. *Small Group Research*, 36, 600-631.

Cohen, J. (1973). Eta-squared and partial eta-squared fixed factor ANOVA designs. Educational and Psychological Measurement, 33, 107-112.

Cohen, P.R. (1995). *Empirical Methods for Artificial Intelligence* (pp 87-92). Cambridge, MA: The MIT Press.

Cudeck, R. & Henly, S.J. (2003). A Realistic Perspective on Pattern Representation in Growth Data: Comment on Bauer and Curran. *Psychological Methods*, 8(3), 378-383.

Curran, P. (2003). Have Multilevel Models Been Structural Equation Models All Along? *Multivariate Behavioral Research*. 38 (4), 529-569.

Fitzmaurice, G.M., Laird, N.M., & Ware, J.H. (2004). *Applied Longitudinal Analysis*. New York: John Wiley.

Fox, J, (2002). Robust Regression. An Appendix to an R and S-Plus Companion to Applied Regression.

Huber, P. J. (1964). Robust Estimation of a Location Parameter. *Annals of Mathematical Statistics* 35, 73-101.

Hui,L. & Powers, D.A. (2007). Growth Curve Models for Zero-Inflated Count Data: An Application to Smoking Behavior. *Structural Equation Modeling*, 14(2), 247-279.

MacCullum, R.C., Kim, C, Malarkey, W., &Kielcolt-Glaser, J. (1997). Studying multivariate change using multilevel models and latent curve models. *Multivariate Behavioral Research*, 32, 215-253.

McArdle,J.J. (2004). Latent growth curve analysis using structural equation modeling techniques. In D.M. Teti (Ed.), *The Handbook of Research Methods in Developmental Psychology* (pp. 340-466). New York: Blackwell Publishers.

Mehta, P.D., & West S.G. (2000). Putting the individual back into individual growth curves. *Psychological Methods*, 5, 23-43.

Meredith, W. & Tisak, J. (1990). Latent curve analysis. *Psychometrika*, 55, 107-122.

Miller, J.M. & Miller, M.D. (2009). No Zero Left Behind: Comparing the Fit for Zero-Inflated Models as a Function of Skew and Proportion of Zeros. (University of Florida Department of Psychology Working Paper 1-28). Retrieved June 5 from

Muthen, B. (1989). TOBIT factor analysis. *British Journal of Mathematical and Statistical Psychology*, 42, 241-250.

Muthen, B.O., & Curran P.J. (1997). General longitudinal modeling of individual differences in experimental designs: A latent variable framework for analysis and power estimation. *Psychological Methods*, 2, 371-402.

Muthen, B., & Khoo, S.T. (1998). Longitudinal studies of achievement growth using latent variable modeling. *Learning and Individual Differences*, 10, 73-102.

Muthen, B. (2003). Statistical and substantive checking in growth mixture modeling: Comment on Bauer and Curran (2003). *Psychological Methods*, 8 369-377.

Muthen, L.K. & Muthen B.O. (2006).*Mplus user's guide*(4th ed). Los Angeles, CA: Muthen & Muthen.

Olsen, M.K., & Schafer, J.L. (2001). A two-part random-effects model for semicontinuous longitudinal data, *Journal of American Statistical Association*, 96, 730-745.

Ram, N. & Grimm, K. (2007). Using simple and complex growth models to articulate developmental change: Matching theory to method. *International Journal of Behavioral Development*, 31(4) 303-316.

Grimm, K. J., & Ram, N. (in press). Non-linear growth models in Mplus and SAS. *Structural Equation Modeling: A Multidisciplinary Journal.*

Responsiveness to Intervention and Learning Disabilities (2005). Report prepared by the National Joint Committee on Learning Disabilities. Washington, D.C.

Rindskopf, D. (2003). Mixture or homogeneous? Comment on Bauer and Curran (2003). *Psychological Methods*, 8 364-368.

Seltzer, M. Choi, K. & Thum, Y.W. (2003). Examining Relationships Between Where Students Start and how Rapidly they Progress: Using New Developments in Growth Modeling to Gain Insight into the Distribution of Achievement Within Schools. *Educational Evaluation and Policy Analysis*, 25 (3) 263-286.

Smith-Bailey, D (2004). Who is learning disabled? Report published by the American Psychological Association Public Policy Office. Washington, D.C.

Stromberg, A. J. (2004). "Why write statistical software? The case of robust statistical methods". *Journal of Statistical Software*.

Tobin, J. (1958). Estimation of relationships for limited dependent variables, *Econometria*, 26(1), 24-36.

Wang, L., Zhang, Z., McArdle, J. J., & Salthouse, T. A. (2008). Investigating ceiling effects in longitudinal data analysis. *Multivariate Behavioral Research*, *43*(3), 476–496.

Willett, J. (1988). Questions and answers in the measurement of change. In E.Z. Rothkopf (Ed.) Review of research in education, 15 (pp. 345-422). Washington, D.C., American Educational Research Association.

Willett, J.B., & Sayer, A.G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin*. 116, 363-381.

APPENDICES

Appendix Table A: Mplus Data Generation Code

```
MONTECARLO:
    NAMES = y11-y13;
    NOBSERVATIONS = 10000;
    NREPS = 100;
    SEED = 1471;
    GENERATE = y11-y13 (cbi 0);
    Censored are y11-y13 (bi);
    REPSAVE = ALL;
    SAVE = C:\Documents and Settings\TMCKINLEY\Desktop\datanew*.dat
    MODEL POPULATION:
    i s | y11@0 y12@1 y13@2;
    ii si | y11#1@0 y12#1@1 y13#1@2;
    [y11-y13@0];
    y11 * 289.42;
    y12 * 301.32;
    y13 * 290.64;
    [y11#1 - y13#1 @ -1.0](1);
    [i*297.234 s* -0.672 ii@0 si * -0.262];
    i*283.109; s*300.697; i with s*-176.777;
    [ii @ 0];
    [si * -0.262];
    Si @ 0.000;
    ii @ 0.047;
    i with ii *-.155;
    s with ii * .075;
    Ii with si @ 0;
    s with si @0;
    i with si @0;
```

Appendix Table B: Mplus Censored-Inflated Model Code

```
MONTECARLO:
   NAMES = y11-y13;
   NOBSERVATIONS = 1000;
   NREPS = 100;
   SEED = 2343;
   GENERATE = y11-y13 (cbi 0);
   Censored are y11-y13 (bi);
   REPSAVE = ALL;
   SAVE = E:\Dissertation 1\data1i5n1*.DAT;
   MODEL POPULATION:
   i s | y11@0 y12@1 y13@2;
   ii si | y11#1@1 y12#1@2 y13#1@3;
   [y11-y13@0];
   y11 * 289.42;
   y12 * 301.32;
   y13 * 290.64;
   [y11#1 - y13#1 * -1.0];
   [i*297.234 s* -0.672 ii@0 si * -0.262];
   i*283.109; s*300.697; i with s*-176.777;
   [ii @ 0];
   [si * -0.262];
   Si @ 0.000;
   ii @ 0.047
   i with ii *-.155;
   s with ii * .075;
   Ii with si @ 0;
   s with si @0;
   i with si @0;
```

Appendix Table C: Mplus Linear Model Code

```
DATA:FILE IS C:\Documents and Settings\TMCKINLEY\Desktop\datanewlist.dat;
type=montecarlo;
VARIABLE: NAMES ARE Y11-Y13;
USEVARIABLES ARE ALL;
MODEL: i s | y11@0 y12@1 y13@2;
SAVE:  results are E:\Dissertation 5\linear1i7n10.res;
```

Appendix Table D: SPSS Parameter Estimate Bias Syntax

```
COMPUTE nubias1=(Nu-(-1))/-1*100.
EXECUTE.
COMPUTE Theta1bias=(Theta1-289.420)/289.42*100.
EXECUTE.
compute Theta2bias=(Theta2- 301.32)/ 301.32*100.
EXECUTE .
compute Theta3bias=(Theta3-290.64)/290.64*100.
EXECUTE.
compute Alpha1bias=(Alpha1-297.234)/297.234*100.
EXECUTE.
compute Alpha2bias=(Alpha1- -0.672)/-0.672*100.
EXECUTE.
compute Alpha3bias=(Alpha3- -0.262)/-0.292*100.
EXECUTE.
compute Psi1bias=(Psi1-283.109)/283.109*100.
EXECUTE.
compute Psi2bias=(Psi2--177.77)/-177.77*100.
EXECUTE.
compute Psi3bias=(Psi3-300.697)/300.697*100.
EXECUTE.
compute Cov1bias=(Cov1--.155)/-.155*100.
EXECUTE.
compute Cov2bias=(Cov2-0.075)/0.075*100.
EXECUTE.
compute Cov3bias=(Cov3-0.047)/0.047*100.
EXECUTE.
compute Stderror1bias=(Stderror1-0)/0*100.
EXECUTE.
compute Stderror2bias=(Stderror2-40.78)/40.78*100.
EXECUTE.
compute Stderror3bias=(Stderror3-21.25)/21.25*100.
EXECUTE.
compute Stderror4bias=(Stderror4-50.91)/50.91*100.
EXECUTE.
compute Stderror5bias=(Stderror5-0.8424)/0.8424*100.
EXECUTE.
compute Stderror6bias=(Stderror6-0.7740)/0.7740*100.
EXECUTE.
compute Stderror7bias=(Stderror7-0.0621)/0.0621*100.
EXECUTE.
compute Stderror8bias=(Stderror8-41.79)/41.79*100.
EXECUTE.
compute Stderror9bias=(Stderror9-27.90)/27.90*100.
EXECUTE.
compute Stderror10bias=(Stderror10-0.0853)/0.0853*100.
EXECUTE.
compute Stderror11bias=(Stderror11-1.603)/1.603*100.
EXECUTE.
compute Stderror12bias=(Stderror12-1.36)/1.36*100.
EXECUTE.
compute Stderror13bias=(Stderror13-0)/0*100.
EXECUTE.
```