OBESITY AND COUNSELING

by

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(Under the Direction of Michael E. Wetzstein and Susana Ferreira)

ABSTRACT

A theoretical model is developed addressing habit formation and time-inconsistent preferences in consumption of unhealthy foods. In particular, the effects of counseling in altering the consumption satiation point and lowering individual discount rates are investigated. The model highlights the importance of health counseling and offers support to initiatives such as low-cost obesity screening and counseling.

INDEX WORDS: Obesity, Counseling, Habit formation, Time-inconsistent preferences
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CHAPTER 1
INTRODUCTION

With obesity rates nearly doubling from the 1980s to 2008, yielding epidemic proportions globally, it is one of the most widely pressing health problems (WHO, 2012). Empirical analysis suggests that an increase in caloric intake, rather than a change in caloric expenditure, is responsible for much of the obesity trend (Cutler, et al., 2003). Public policies directed at discouraging caloric consumption include taxes on sugar-sweetened beverages (SSBs), educational programs, and product labeling such as nutrition information panel. Given the magnitude and importance of obesity to health outcomes, substantial literature has emerged investigating the effectiveness of these policies (e.g., Coestier et al., 2005; Finkelstein et al., 2012, Lin et al., 2011; Mokdad et al., 2001; Zhen et al., 2011). However, mechanisms for curbing obesity, involving behavioral counseling or therapy, have been largely ignored in economic analysis. Behavioral counseling fundamentally differs from other price or labeling based policies in that it is a mechanism directly aimed at an individual’s underlying perception and attitudes toward food consumption. Such behavioral counseling is a foundation of the U.S. Affordable Care Act, which substantially reduces the cost of preventive services for obese patients. However, there is limited or no empirical work in economics investigating the effectiveness of behavioral counseling. One exception is an article by Kan and Tsai (2004), which tests the relationship between obesity and risk knowledge. On the theoretical side, modeling addictive consumption has expanded substantially since the seminal work on rational

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1 Gortmaker et al. (2009) defined the SSBs category to include all sodas, fruit drinks, sport drinks, low-calories, and other beverages that contain added caloric sweeteners (sweet tea, rice drinks, bean beverages, sugar cane beverages, horchata, and non-alcoholic wines and malt beverages).
addiction by Becker and Murphy (1988). Based on this expanded work, a new direction is developing models that address external intervention, which may alter individual’s consumption preference and time preference. As a first attempt, a theoretical model is developed that highlights the role of behavioral counseling within the framework of rational addiction.

Three unique contributions result from the theoretical model. First, two plausible explanations are proposed for the failure of taxes on SSBs and information programs aiming to discouraging caloric consumption. One explanation is the presence of habit formation or addiction in certain food consumption (Benton, 2010; Carroll et al., 2011; Davis and Carter, 2009; Gearhardt et al., 2009; Khare and Inman, 2006; Richards et al., 2007; Zhen et al., 2011). Another explanation is individual time-inconsistent preference. Time-inconsistent preference causes failure of realization of weight loss plans (Bleichrodt and Gafni, 1996; Cawley and Ruhn, 2011; Scharff, 2009). Second, the potential role of counseling is formalized in mitigating the mentioned two problems. This effort expands the literature modeling addictive consumption to incorporate counseling as an external intervention, which endogenizes individual consumption preferences and time preferences. Third, results from the model provide support to initiatives such as low-cost obesity screening and counseling and are consistent with new requirements for intensive behavioral intervention for obesity by the U.S. Preventive Service Task Force (USPSTF).

In this paper, the investigation of behavioral counseling is grounded in the health belief theory. It is motivated by the theory that individuals’ personal beliefs on the consequences of obesity and the probability of developing conditions caused by obesity may comprise their

---

2 Health belief theory is derived from an established body of psychological and behavioral theory, and is a commonly used theory in health education and health promotion. The underlying paradigm is that personal beliefs influence health behavior. Individual beliefs, such as individual perceived seriousness of and susceptibility to the condition, benefits from, and barriers to change in behavior along with cues to action and self-efficacy are the main determinants of health behavior change (Glantz et al., 2008).
commitment to health behavior changes. Furthermore, individuals’ perceptions as to whether a new behavior is preferred to existing behavior also affects whether the behavioral change will actually occur. Counseling addresses individuals’ unrealistic perception of their own weight, confusion regarding causes and consequences of obesity, and biased expectations of weight loss. It emphasizes that even a small weight loss can significantly reduce diabetes risks\(^3\). The effectiveness of obesity counseling foreseen in health belief theory is supported in numerous field studies (e.g., Post et al., 2011, Powell-Wiley et al., 2012).

The theoretical model based on health belief theory assumes counseling obese patients will result in weight reduction through two major channels: it alters their consumption preferences and time preferences. Assuming an obese patient is still forward looking but exposed to imperfect information, counseling helps the patient establish an unbiased perception and healthier consumption pattern. In the model, an individual’s consumption and time preference are endogenous to counseling. Counseling is a repeated process through time and triggers changes in consumption preferences via changes to individuals’ underlying utility function. However, the effects of counseling sessions can be temporary, gradually decaying back to zero resulting in individuals relapsing to their original preference until the next secession occurs.\(^4\) Counseling also affects individual time preferences via lowering time discount factors. Similar to a preference change, an altered time-discount factor also relapses after a counseling secession. Individuals choose the optimal time to join and quit counseling by maximizing the present value of utility through time.

\(^3\) For example, a 5% weight loss reduces type 2 diabetes risk by 60% (Clifton, 2008; Wadden, 2011).

\(^4\) This assumption is consistent with high rate of patient relapse as sustained weight loss is especially problematic—up to 95% of patients regain their weight within five years (Freedman, 2008).
CHAPTER 2
LITERATURE REVIEW

Since Becker and Murphy (1988), the rational theoretical approach has become the standard to developing addiction models in economics. But, the assumption of perfect foresight has also received critiques. Later extensions of rational addiction models are generally motivated by relaxing the fully rational assumption allowing partial awareness of the potential harm associated with consuming an addictive substance. These extensions include uncertainty and regret (Orphanides and Zervos, 1995), bounded rationality (Suranovic et al., 1999), endogenous time preference (Orphanides and Zervos, 1998) and quasi-hyperbolic discounting (Gruber and Köszegi, 2001).

Modeling food consumption within this framework is a recent trend. Different perspectives on the causes of obesity lead to a variety of models. The theoretical work addressing obesity can be divided into three areas. The first is within the framework of household production models. This assumes individuals maximize utility subject to their ability to produce commodities for personal use, their budget constraint, and time constraint. Mancino and Kinsey (2004) incorporate the visceral influences from Loewenstein (1996) into the Becker (1965) household production model to depict how individual’s food choices are affected by time delays and situational factors. An individual’s utility is assumed to be a function of food consumption, composite non-food consumption, leisure time, and the individual’s perceived health status and the visceral factors. The model allows individual’s knowledge about health to play a role on how they perceive their own health from a change in bodyweight. An ideal weight is given and
deviation from it causes disutility. Their research indicates that although knowledge about the 
importance of eating well should increase consumer’s intention to follow a healthier diet, the 
intentions can be thwarted by visceral factors such as hunger, a hectic schedule, and where 
consumers choose to obtain food.

The works of Philipson and Posner (1999) and Lakdawalla and Philipson (2009) develop 
a similar but simpler dynamic framework addressing the role of technological change in the rise 
of obesity. They assume individuals derive utility from bodyweight directly, and bodyweight 
rises with consumption but falls with individual’s home or market production activities. 
Deviation from an ideal bodyweight also causes disutility. They conclude that approximately 
40% of weight gain is due to the expansion of food supply, and 60% to the reduction in the 
physical requirements.

The second area is categorized within the rational addiction framework. Dockner and 
Feichtinger (1993) expand the addiction capital in Becker and Murphy (1988) model from one to 
two: eating capital (or addictive capital) and weight capital. The eating capital presents the 
addictive force causing current consumption to rise with past consumption and the weight capital 
presents the satiating force driving current consumption to decline as habits accumulate. By 
introducing weight as another consumption capital, their model is able to explain the cyclical 
consumption that the Becker and Murphy model fails to demonstrate.

In Levy (2002), eating is neither addictive or a formed habit, individuals take into 
consideration the probability of dying associated with deviation from physiologically optimal 
weight when maximizing expected utility over their life time. The model demonstrates a steady 
state of overweightness given no physiological, psychological, environmental, and sociocultural 
effects. When the socio-cultural norms are incorporated, the steady state for overweight
individuals is lower, and stationary weight of lean individuals is greater than otherwise. Levy
(2003) distinguishes the consumption of health food from junk food and highlights the effects of
age on individual’s health condition and the role of the relative price of junk-food.

Dragone (2009) generalizes the Levy (2002) model to allow for possible presence of
habits in food consumption. The Dragone (2009) model deviates from the traditional method of
assuming that past consumption has effects on the marginal utility of current consumption,
instead it assumes changes in food consumption are costly. Dragone (2009) shifts the emphasis
from studying the role of the level of consumption to changes in consumption. The results
indicate that, with consumption habits, following a monotonic path can be too costly if it requires
too rapid changes in the amount of food consumption. Thus, a slower convergence to steady
state bodyweight is required and it is associated with fluctuations above and below the steady
state body weight.

Yaniv (2002) aims at explaining the failure to adhere to a low-fat dietary regimen.
Individuals decide on whether or not to adhere to the prescribed diet. They maximize the present
value of their expected lifetime utility stream from the consumption of high-fat and low-fat
products, taking into account the adverse effect of high-fat consumption. Slightly different from
Levy (2002, 2003), adverse effects of high-fat consumption are modeled through the probability
of experiencing a heart attack. In Yanic (2009), a rational choice model is developed to examine
the effect on obesity of a tax on junk-food and a subsidy to cooking ingredients. Results indicate
for non-weight conscious individuals a fat tax will unambiguously reduce obesity, whereas a thin
subsidy may increase obesity. For a weight conscious individual, particularly one who is
physically active, even a fat tax may increase obesity.
A recent model within the rational framework is developed in Dragone and Savorelli (2012). They continue modeling habit as changing consumption and incorporate the social ideal bodyweight. Their model accounts for the social pressure individuals suffer when bodyweight does not conform to an ideal social weight.

All the work within the rational framework (considering habit or not) view weight gain as the outcome of rational choices that reflect willingness to tradeoff some future health for the present pleasures of less restrained eating and lower physical activity. The individual consumption preference and discount rate are fixed through the life-time maximization. This type of model does not consider the substantial evidences of bounded rationality and time-inconsistent preferences of obesity patients from psychology and behavioral economics (Borghans et al., 2006; Courtemanche et al., 2011; Komlos et al., 2004; Ikeda et al., 2010; Zhang et al., 2008).

The third area raises problems with full rationality and provides an alternative. Suranovic and Goldfarb (2005) develop a behavioral model of food consumption choices that illuminates observed patterns of dieting. The key assumption is excessive computational costs constrain decisions to be made only for the current consumption periods. Unlike the property of adjacent complementarity displayed in the Becker and Murphy model,5 individuals do consider the future health effects of current consumption. However, they do not consider how this consumption will affect tomorrow’s consumption decisions or how future decisions will affect current choice. Consumption habit is incorporated by assuming a habit level of consumption and sudden reductions induce negative effects.

5 “Adjacent complementarity” indicate that due to reinforcement, the quantities of the addictive good consumed in different time periods are complements; as a result, current consumption of an addictive good is inversely related to not only the current price of the good, but also to all past and future prices. It is a critical hypothesis derived from the Becker and Murphy (1988) model of rational addiction.
Based on the previous literature, an economic approach is developed for understanding obesity with emphasis on obesity counseling. Individuals are still assumed to be rational and forward-looking, provided evidence by Gruber & Köszegi (2001). However, full rationality is limited by a time-inconsistent discount rate. Time preference is endogenous to obesity counseling by assuming that counseling triggers a change in individuals’ discount rates. The model focus on the effect counseling has on the change in individual behavioral, so the utility framework developed in Dragone and Savorelli (2012), which focus on the change of consumption, is applied.
CHAPTER 3

THE MODEL

The objective is to model obesity counseling as a potential mechanism for influencing established consumption habits and the preference discount rate. Individuals’ consumption patterns are modeled under habit-formation and inconsistent discount rates. A dynamic model is developed to solve for the optimal starting time of counseling, duration of counseling, and to compare different patterns of counseling. Following Dragone and Savorelli (2012), the individual’s pre-counseling instantaneous utility of is then stated as

\[ U(t) = c(t)[c^F - (c(t)/2)] - 1/2[w(t) - w^H]^2 - (\beta/2)[w(t) - w^G]^2. \]

The first term, \( c(t)[c^F - (c(t)/2)] \) is the utility from calories through food consumption \( c(t) \geq 0 \) at time \( t \) and \( c^F > 0 \) is an individual’s satiation point, above which the marginal utility from food is negative. The maximum utility occurs when \( c \) is equal to \( c^F \). Consumption below or above the satiation point \( c^F \) yields a lower utility level.

The second term in (1), \( 1/2[w(t) - w^H]^2 \), represents the utility from individual bodyweight. The variable \( w(t) \) is bodyweight at time \( t \), and \( w^H \) represents a healthy bodyweight. According to WHO guidelines (2000, 2004), when an individual’s BMI is between 18.5 and 25, a person is considered to have a normal weight. Deviations from the healthy bodyweight have a negative impact on utility. An overweight person may experience some physical inconveniences such as back pain, while an underweight person may have trouble regulating body temperature. The last term, \( (\beta/2)[w(t) - w^G]^2 \), refers to the effect of a socially desirable bodyweight \( w^G \). As in Dragone
and Savorelli (2012), \( w^G \) is assumed to be exogenously determined. The parameter \( \beta > 0 \) measures the intensity of the cost due to a difference between an agent’s bodyweight and the socially desirable bodyweight.

This Dragone and Savorelli (2012) non-counseling utility function (1) may be considered in a dynamic framework with counseling starting at time \( s_0 \) and ending at time \( s_1 \). Counseling has value in terms of alerting individuals to adjusting their satiation point \( c^F \) and their socially desirable bodyweight \( w^G \) to maximize their utility stream. This satiation point and their socially desirable bodyweight may maximize their utility at time \( s_0 \), but might be too high for maximizing their lifetime utility stream. As they continue through time to consume around the levels of \( c^F \) and \( w^G \), their utility declines as they realize the long-term consequences of this inflated satiation point and socially desirable bodyweight. At some point this decline in utility may trigger a change in the satiation point, \( c^F \), and their socially desirable bodyweight, \( w^G \). Such a trigger is termed counseling, where it could result from a physical health checkup, a psychological session, and/or external peer or internal pressure. An example are individuals who know their consumption of high calorie junk foods are detrimental to their long-term health, but currently for instantaneous utility are willing to consume them. But through time their continued consumption of junk foods starts to be a concern and at some point they decide to adjust downward their consumption. Thus, within the interval \( s_0 \) and \( s_1 \), \( \partial U / \partial t < 0 \).

Non-learner

For modeling the optimal timing for receiving counseling, first consider the case when obese patients are naïve and do not alter their satiation point, \( c^F \), their socially desirable bodyweight \( w^G \), or time preferences after counseling (non-learners). Some individuals may regardless of counseling not change their preferences, but with counseling, their utility may
return to the utility level experienced at $s_0$. Individuals may under internal or external (friends, family, and coworkers) pressure get counseling to relieve this pressure, but then just disregard it. The relieved pressure then restores their utility to that experienced level at $s_0$.

Consider the case of only one counseling session (cycle) with an individual’s discount rate $\rho$. The present value of one cycle consisting of a utility stream starting at time $s_0$ and ending at time $s_1$ is then

$$\gamma(s_0, s_1, 1) = \int_{s_0}^{s_1} U(t)e^{-\rho(t-s_0)} dt.$$

where $\gamma(s_0, s_1, 1)$ represents the stream of utility starting at $s_0$ and ending at $s_1$ with one counseling session. The optimal terminal time, $s_1^*$, is then determined by the condition $U(s_1^*) = 0$. With no subsequent cycles there is no transversality condition on terminal time, so utility is driven to zero. Individuals will not consider counseling until they do not receive any positive utility from continuing their current level of consumption. If the utility never reaches zero, then there is no terminal time and it is an infinite horizon problem. This is the scenario employed by previous research, most recently by Dragone and Savorelli (2012), which then concentrates on determining the optimal trajectory of calories, $c(t)$, through time. Figure 1 illustrates this optimal time, $s_1^*$, when utility does dissipate to zero. The cumulative utility increases at a decreasing rate, given utility per unit of time decreases through time. With only one counseling session, the optimal time for counseling is when the utility per unit of time is zero, at $s_1^*$.

Equation (2) may be extended by assuming an individual has the objective of maximizing a stream of utility over an infinity life, $\gamma(s_0, s_1, \infty)$ rather than the one cycle (2) along with a constant cost $C$ of a counseling session. This cost of counseling is in terms of both explicit cost such as paying for a doctor’s appointment and implicit cost involving the disutility of experiencing a counseling session. This stream of utility involves multiple counseling sessions.
with associated length between each session, $s_I$. This assumes individuals will not just receive one counseling session throughout their lives but will incur multiple sessions. Examples are individuals having annual physicals or attending WeightWatchers on a regular basis. Without loss of generality, assume utility will start at age zero, $s_0 = 0$ (birth). The present value of the utility stream is then

$$
\gamma(0, s, \infty) = \gamma(0, s, 1) - e^{-\rho s} C + e^{-\rho s} \gamma(0, s, 1) - e^{-\rho s} C + e^{-\rho s} \gamma(0, s, 1) + ... \\
= [\gamma(0, s, 1) - e^{-\rho s} C][1 + e^{-\rho s} + e^{-\rho s} + ...] \\
= \frac{1}{1-e^{-\rho s}} [\gamma(0, s, 1) - e^{-\rho s} C].
$$

This is the present value of a perpetual utility annuity of the amount $\gamma(0, s, 1) - e^{-\rho s} C$ received every $s_I$ years. The optimal duration between checkup, $s_I^*$, is then determined by

$$
\frac{\partial \gamma(0, s, \infty)}{\partial s_I} = -\rho e^{-\rho s_I} [\gamma(0, s, 1) - e^{-\rho s} C] + \frac{e^{-\rho s}}{1-e^{-\rho s}} [U(s_I) + \rho C]) = 0.
$$

Rearranging terms:

$$
U(s_I) = \frac{\rho}{1-e^{-\rho s_I}} [\gamma(0, s, 1) - C].
$$

Considering the opportunity cost of lost utility by not having counseling results in (4) where the individual does not reduce utility down to zero when considering counseling. The greater future utility (feeling better after counseling), the more impatient the individual will be to receive counseling, but this impatience is mitigated by the cost of counseling, $C$. Figure 1 illustrates this transversality condition; yielding counseling at $s_I^*$. As illustrated in the figure, consideration of utility from the second and subsequent sessions leads to an earlier time for counseling. This assumes the cost of counseling is less than the associated future utility. For maximizing (3), the
individual should seek counseling when the utility of not having counseling equals the flow which could be realized by an immediate counseling session.

\[
U(t) = \frac{\rho}{1 - e^{-\rho s_1}} \left( \int_0^{s_1} U(t) e^{-\rho t} dt - C \right)
\]

\[
= \frac{\rho}{1 - e^{-\rho s_1}} \Psi(s_1).
\]

**Figure 1.** Optimal time for counseling

Substituting (2) into (4) and rearranging terms yields

\[
U(s_1) = \frac{\rho}{1 - e^{-\rho s_1}} \left( \int_0^{s_1} U(t) e^{-\rho t} dt - C \right)
\]
where \( \Psi(s_1) \) is the net present value of accumulated utility at the time of counseling. The net present value, \( \Psi(s_1) \), received is a perpetual annuity received every \( s_1 \) years. The recovery factor, \( \frac{\rho}{1 - e^{-\rho s_1}} \), converts this annuity into a constant flow of utility. The denominator yields the present value of the annuity and the numerator converts this present value into a continuous flow.

Figure 2 illustrates a non-learner’s optimal time path for \( U(t) \). Utility depreciates with the length of time since the last counseling. At the time thresholds \( s_1, 2s_1, \ldots \), the non-learner’s utility has declined to the point of triggering a counseling. With the counseling, utility is then restored to its initial level \( U(0) \).

\[
U(t) = \frac{\rho}{1 - e^{-\rho s_1}} \left( U(0) e^{-\rho t} dt - C \right)
\]

**Figure 2.** A non-learner’s optimal time path

**Case One**

As a special case of a non-learner, consider a non-learner that does not experience a decline in utility as a checkup is postponed. The non-learner derives no benefit from a checkup in terms of relief after the checkup. In this special case, utility is constant through time, so
\[
\frac{\rho}{1-e^{-\rho s}} \left( U \frac{1-e^{-\rho s}}{\rho} - C \right) = U - \frac{\rho}{1-e^{-\rho s}} C.
\]

Which implies \( C = 0 \). The individual will never get a counsel. The optimal counsel is zero. In Figure 2, this corresponds to a constant \( U(0) \) across time with no checkups.

**Case Two**

For this second special case, consider a non-learner with a zero discount rate, \( \rho = 0 \), but does not retain a constant utility as assumed in case one. Such individuals value current utility and future utility the same. They are not impatient, so they have no time discounting of preferences. Considering (5) and employing the L’Hospital’s Rule

\[
\lim_{\rho \to 0} \frac{\rho}{1-e^{-\rho s}} = \frac{1}{s_i},
\]

Thus, (5) becomes

\[
U(s_i) = \frac{1}{s_i} \Psi(s_i).
\]

The optimal counseling is where the utility at counseling equals the average undiscounted utility over time. Figure 1 provides an illustration of this result. The cumulative undiscounted utility over time is a strictly concave function starting at some initial time. The zero discount solution is \( s_i^0 \), which maximizes the average undiscounted revenue over time.

Figure 1 illustrates an individual will get counseling sooner as the discount rate increases. However, in general this relation is indeterminate. From (5)

\[
(6) \quad \frac{\partial}{\partial \rho} \left( \frac{\rho}{1-e^{-\rho s}} \Psi(s_i) \right) = \psi(s_i) \frac{\partial}{\partial \rho} \left( \frac{\rho}{1-e^{-\rho s}} \right) + \frac{\rho}{1-e^{-\rho s}} \frac{\partial \Psi(s_i)}{\partial \rho}.
\]
The first term on the right-hand-side is positive (negative only when ρs₁ is less than 0.1) but the second term is negative, given \( \frac{\partial \Psi(s_1)}{\partial \rho} < 0 \). Thus, the sign of (6) is indeterminate. A rising discount rate, ρ, increases the recovery factor, \( \rho / (1 - e^{\rho s_1}) \), which increases the cost of delaying counseling. This delay is mitigated by the decline in the present value of future utility as the discount rate increases, \( \frac{\partial v(s_1)}{\partial \rho} < 0 \). For relatively low counseling costs, the second term will not be sufficient to completely offset a positive response. Thus, a rise in the discount rate will result in an unambiguously earlier counseling. This is an interesting result. Normally we would expect individuals with a higher discount rate would delay counseling since people value future utility less.

**Learner**

Consider the case where individuals are learners, and thus after just one counseling session permanently modify their satiation point, \( c^F \), along with their socially desirable bodyweight, \( w^G \). The utility function (1) developed by Dragone and Savorelli (2012) can be modified for considering such a permanent learner

\[
R(t) = c(t)[c^{FL} - (c(t)/2)] - 1/2[w(t) - w^H]^2 - (\beta/2)[w(t) - w^{GL}]^2.
\]

where \( c^{FL} \) and \( w^{GL} \) represent the new satiation point and desirable bodyweight, respectively, after one counseling session. Assuming counseling will cause a decrease of the satiation point, \( c^F \), and the individual’s perception of socially desirable bodyweight \( w^G \), then \( c^{FL} < c^F \) and \( w^{GL} < w^G \). Again denoting \( s_t \) as the starting time of the first and only counseling period, the individual’s stream of utility is then

\[
\gamma_1(s_0, s_1, \infty) = \gamma(s_0, s_1, 1) - e^{-\rho(s_1 - s_0)} C + \int_{s_1}^{\infty} e^{-\rho(t - s_0)} R(t) dt.
\]
The optimal time, $s^L_1$, occurs when $U(s_1) = R(s_1) - e^{\rho(s_1 - s_0)} \int_{s_1}^{\infty} e^{-\rho(t-s_0)} \partial R(t)/\partial s_1 dt - \rho C$. The first and only counseling session should be postponed until the utility at $s^L_1$ equals the utility that could be obtained from a session, $R(s_1)$, minus the change in utility flow after counseling, $e^{\rho(s_1 - s_0)} \int_{s_1}^{\infty} e^{-\rho(t-s_0)} \partial R(t)/\partial s_1 dt$, and the incremental counseling cost, $\rho C$. The higher the value of the utility flow after counseling, the sooner will be the checkup. In contrast, the higher the counseling cost, the longer will be the delay in receiving counseling. As individuals delay their start of the counseling session, their utility upon entering counseling may increase. They realize an increase in the value of counseling after having delayed it, $\partial R(t)/\partial s_k > 0$.

Only if the aggregate utility for the time interval between $s_1$ and $\infty$ is independent of when counseling starts will $\partial R(t)/\partial s_k = 0$. In this case, the optimal time, $s^L_1$, occurs when $U(s_1) = R(s_1) - \rho C$. Figure 3 illustrates this learner’s optimal utility time path. At the time threshold $s^L_1$, the learner’s utility has declined to the point of triggering a counseling session. With the checkup, the learner’s utility, $R(s^L_1)$, is higher than the initial level $U(0)$. The learners do modify their utility after counseling, so they do not revert back to the initial activities associated with $U(0)$.
Figure 3. A learner’s optimal time path

*Learner with Relapse*

For many dynamic preferences, individuals may at first adjust their preference following counseling, but then through time revert back to some prior preference behavior. Individuals may at first follow the advice and adjust their diet, but over time they return to their pre-counseling diet. This behavior is consistent with the evidence that rebound weight gain is common (Swinburn et al., 2007; Blackburn, 2006; Fleck et al., 2008). Thus, repeated sessions may be required for mitigating relapses toward prior preference behavior.

The time path for repeated counseling sessions can be modeled by defining the binary control variable $\theta_k$ specifying the $k^{th}$ counseling session

$$\theta_k = \begin{cases} 1 & \text{counseling} \\ 0 & \text{otherwise} \end{cases} \text{ with } \theta_0 = 0 \text{ and } k = 1, 2, \ldots \infty.$$ 

Theoretically, an individual can have an infinite number of counseling sessions as $k$ approaches infinity.

Counseling can change individuals’ preferences by altering their satiation point, $c^F$, (reduce their desired level of caloric intact), by lowing their socially desirable bodyweight, $w^G$. 

$$R(s_1) = U(s_1) + \rho C$$
(realize the social benefits in losing weight), and by changes in the discount rate, \( \rho \), (made aware of the long-term costs of current consumption habits). Considering these potential effects of counseling, after counseling an individual’s utility is

\[
R(s_k, u_k, k) = c(t)[e^{-\tau(t-u_k)}c^F - (c(t)/2)] - 1/2[w(t) - w^G]^2 - (\beta / 2)[w(t) - e^{-\tau(t-u_k)}w^G]^2.
\]

Where \( s_k \) and \( u_k \) denote the beginning and ending of counseling session \( k, k = 1, 2, \ldots \infty \).

Counseling triggers a decrease of the satiation point, \( c^F \), and the individual’s perception of socially desirable bodyweight \( w^G \). This is modeled by the exponential adjustment factors \( e^{r(t-u_k)} \) and \( e^{s(t-u_k)} \), which decrease to one as \( t \) increases to \( u_k \). At the time of the counseling session, individuals adjust their satiation points and socially desirable bodyweights downward, but subsequent to the session, these target levels increase exponentially to their previous non-counseling levels. Through time these adjustments dissipate, so, at time \( u_k \) the individual’s target levels of satiation and socially desirable bodyweight are restored to their pre-counseling levels.

The discount parameters \( r > 0 \) and \( s > 0 \) measure the intensity of this reversion.

Possible changes in time preference may be realized through different discount rates.

Before counseling, utility is discounted by the factor \( e^{-\rho t} \), during counseling, the discount factor is \( e^{-\tau t} \), and after counseling it becomes \( e^{-\nu t} \). So, \( \rho, \tau, \) and \( \nu \) denote three different discount rates for the three different stages: pre, during, and post counseling. It is assumed \( \tau < \nu < \rho \) considering that the effect of counseling in lowering the discount rate may still remain after the effective counseling period.

Thus, an individual’s generalized stream of utility is

\[
\gamma(s_0, s_1, s_2, \ldots, u_1, u_2, \ldots, \infty) = \sum_{k=1}^{\infty} (1 - \theta_{k-1}) \int_{s_{k-1}}^{s_k} e^{-\rho(t-u_k)}U(t)dt
\]

\[
+ \theta_k \left[ -e^{-\tau(t-s_k)}C + \int_{s_k}^{u_k} e^{-\tau(t-u_k)}R(t)dt + \int_{u_k}^{u_{k+1}} e^{-\nu(t-u_k)}U(t)dt \right].
\]
This utility stream is maximized subject to the change in bodyweight influenced by calories consumed and burned as specified by Dragone and Savorelli (2012).

The generalized set up of (9) can be simplified to the maximization problem in Dragone and Savorelli (2012) by assuming a single constant discount rate without counseling. Specifically, if for all $k$, $\theta_k = 0$, and $\tau = \nu = \rho$, (9) reduces to $U(0, \infty) = \int_0^\infty e^{-\rho t} U(t) dt$, which corresponds to equation (3) in Dragone and Savorelli (2012).

The conditions for maximizing (9) determine the optimal levels of the control variables, $c(t)$, and the optimal starting and ending point of counseling, $s_k$ and $u_k$. Dragone and Savorelli (2012) provide the conditions for determining the optimal time path of $c(t)$.

Optimal Counseling Starting and Ending Conditions

The optimal starting and ending points for counseling sessions can be determined by treating the control variables as parameters, which transforms the optimal control problem (9) into a calculus of variations problem (Kamien and Schwartz, 1992).

\[
\begin{align*}
\partial t / \partial s_k &= \theta_{k-1} [e^{-\tau(s_k - s_{k-1})} U(s_k) + \int_{s_{k-1}}^{s_k} e^{-\tau(t-s_k)} \partial U(t) / \partial s_k dt] \\
&\quad + (1 - \theta_{k-1}) [e^{-\tau(s_k - s_{k-1})} U(s_k) + \int_{s_{k-1}}^{s_k} e^{-\tau(t-s_k)} \partial U(t) / \partial s_k dt] \\
&\quad + \theta_k [e^{-\tau(s_k - s_0)} \tau C - e^{-\tau(s_k - s_0)} R(s_k) + \int_{s_k}^{s_{k+1}} e^{-\tau(t-s_k)} \partial R(t) / \partial s_k dt] = 0, \quad k = 1, \ldots,
\end{align*}
\]

\[
\begin{align*}
\partial t / \partial u_k &= \theta_k [e^{-\tau(u_k - s_k)} R(u_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_k)} \partial R(t) / \partial u_k dt] \\
&\quad - e^{-\tau(u_k - s_0)} U(u_k) + \int_{u_k}^{u_{k+1}} e^{-\tau(t-s_k)} \partial U(t) / \partial u_k dt = 0, \quad k = 1, \ldots.
\end{align*}
\]

where

\[
\partial R(t) / \partial u_k = -\tau c(t) e^{r(t-u_k)} c^F - sfw^G e^{s(t-u_k)} [w(t) - e^{s(t-u_k)} w^G] < 0, \quad k = 1, \ldots
\]
The condition $\partial R(t)/\partial u_k < 0$ follows from assuming $[w(t) - e^{s(t-u_k)}w^G] > 0$, which assumes individuals at $t$ are above their desirable bodyweight, $w(t) > e^{s(t-u_k)}w^G$.

For interpretation, the second and fourth terms in (10),
\[
\int_{u_{k-1}}^{s_k} e^{-\nu(t-s_0)} \partial U(t)/\partial s_k \, dt \quad \text{and} \quad \int_{s_{k-1}}^{s_k} e^{-\rho(t-s_0)} \partial U(t)/\partial s_k \, dt
\]
are considered zero. This implies the aggregate utility derived prior to the start of counseling is not affected by when counseling starts. In contrast, the second term in (11),
\[
\int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial u_k \, dt,
\]
is not considered zero, given $R(s_k, u_k, k)$ in (8) is a function of $u_k$, $\partial R(t)/\partial u_k < 0$. The utility following counseling is a function of its ending point $u_k$. In terms of when counseling starts, $s_k$, influencing the utility stream of counseling, $R(s_k, u_k)$, and when counseling ends, $u_k$, influencing the utility stream of post-counseling, $U(u_k, s_{k+1})$, these influences may not be zero.

Specifically, generally the last terms in (10) and (11),
\[
\int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial s_k \, dt \quad \text{and} \quad \int_{u_k}^{s_{k+1}} e^{-\nu(t-s_0)} \partial U(t)/\partial u_k \, dt
\]
may not be zero. Again as individuals delay their start of counseling sessions, their utility upon entering counseling may increase. They realize an increase in the value of counseling after having delayed it, $\partial R(t)/\partial s_k > 0$. Only if the aggregate utility for a fixed time interval between $s_k$ and $u_k$ is independent of when counseling starts will the integral be zero. Similarly, the marginal utility over the course of post-counseling may change depending on when the counseling period ends, $u_k$.

The interpretation of (10) and (11) can be developed given the values of the binary control variables for counseling, $\theta_k$, $k = 1, 2, \ldots$. If $\theta_k = 0$, $k = 0, 1, 2, \ldots$, there are no counseling sessions and (9) reduces to the Dragone and Savorelli (2012) problem.

One Counseling Session
Considering only one counseling session, \( \theta_1 = 1 \) and \( \theta_k = 0 \), for \( k = 2, \ldots \), then (10) and (11) reduce to

(12a) \( \partial \gamma / \partial s_k = e^{-\rho(s_k - s_0)} U(s_k) + \int_{s_k}^{s_{k+1}} e^{-\rho(s_k - s_0)} \partial C - e^{-\tau(s_k - s_0)} R(s_k) + \int_{s_k}^{s_{k+1}} e^{-\tau(t-s_0)} \partial R(t)/\partial s_k dt = 0, \ k=1 \),

(12b) \( \partial \gamma / \partial s_k = e^{-\rho(s_k - s_0)} U(s_k) = 0, \ k=2, \)

(12c) \( \partial \gamma / \partial s_k = e^{-\rho(s_k - s_0)} U(s_k) = 0, \ k=3, \ldots, \)

(13a) \( \partial \gamma / \partial u_k = e^{-\tau(u_k - u_0)} R(u_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial u_k dt - e^{-\tau(u_k - u_0)} U(u_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial U(t)/\partial u_k dt = 0, \ k=1, \)

(13b) \( \partial \gamma / \partial u_k = 0, \ k=2, \ldots. \)

The condition for the one counseling in (12a) is determined where \( e^{-\rho(s_k - s_0)} U(s_k) \) represents the gain in utility for postponing counseling one period and the remaining terms, \( e^{-\tau(s_k - s_0)} \tau C - e^{-\tau(s_k - s_0)} R(s_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial s_k dt \), represent the cost of this postponement. This cost is in terms of lost utility from the postponement, \( e^{-\tau(s_k - s_0)} R(s_k) \) minus the incremental counseling cost, \( e^{-\tau(s_k - s_0)} \tau C \) and the aggregate counseling period benefits \( \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial s_k dt \). The term \( e^{-\tau(s_k - s_0)} R(s_k) \) is the loss in utility from one period postponement of counseling and \( \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial s_k dt \) is the aggregate utility gain from postponement. At the optimal counseling time, \( s_1 \), the marginal benefits are just equal to the marginal costs of instigating a counsel. From (12b and c), when \( k = 2, U(s_k) = 0 \). With no subsequent counseling there is no transversality condition on terminal time, so utility is driven to zero. As with the non-learner optimal condition (2), individuals will not consider counseling until they do not receive any positive utility from continuing their current level of consumption.
If the utility never reaches zero, then there is no terminal time and it is an infinite horizon problem.

Similar to (12), the one counseling optimal condition (13a) states the marginal benefits from an additional counseling period, $e^{-\tau(u_k - s_0)}R(u_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial u_k \, dt$, are just equal to the marginal costs of the post counseling period, $e^{-\nu(u_k - s_0)}U(u_k) - \int_{u_k}^{s_{k+1}} e^{-\nu(t-s_0)} \partial U(t)/\partial u_k \, dt$. The marginal benefits are the added utility from postponing post-counseling one period, $e^{-\tau(u_k - s_0)}R(u_k)$, minus the aggregate period costs, $\int_{s_k}^{u_k} e^{-\tau(t-s_0)} \partial R(t)/\partial u_k \, dt$. The benefits are then balanced with the net costs. The cost of delaying post-counseling one period, $e^{-\nu(u_k - s_0)}U(u_k)$, minus any aggregate period benefits of postponement.

The optimal conditions for a one-counseling session lead to the following proposition.

**Proposition 1:** Positive counseling cost creates a hurdle rate for counseling sessions.

Based on these optimal conditions (10) through (14), the implications are established. With a positive counseling cost, $C$, (10) indicates a wedge between pre-counseling and counseling utility. The counseling must overcome this cost before it will occur. A hurdle rate exists, which is the minimum acceptable rate of return (utility) that must be achieved before individuals will undertake counseling. The higher the counseling cost, the larger is the hurdle rate. A lower counseling discount rate, $\tau$, relative to the pre-counseling rate, $\rho$, will mitigate this hurdle rate. If counseling costs are high, for a counseling session to occur the session must trigger a substantial change in individuals’ time preference, however, if costs are low, a smaller change in time preference is required. The implication for health policy is given the difficulty in changing
individuals’ time preferences, lowering the counseling costs maybe an effective way to increase participation. This is consistent with the idea of the Affordable Care Act. Of relevance, Type 2 diabetes screening for adults with high blood pressure, diet counseling for adults at higher risk for chronic disease and obesity screening, and counseling for all adults are included in the preventive services. Also, obesity screening and counseling for children are covered. If going into a clinic for an obesity check-up is free, we should expect the participation rate to be high.

In general Proposition 1 provides a theoretical foundation for the classical RAND experimental results of rejecting the null hypothesis that health spending does not respond to the out-of-pocket price (Aron-Dine et al., 2013). Specifically, a higher out-of-pocket price will delay health care purchases. In terms of the optimal transition from counseling to post-counseling (11), there is no associated hurdle rate.

Continuous Counseling

The condition for continuous counseling, represented as $\theta_k = 1, k = 1, 2, \ldots$, yields the optimal conditions (12a) and

$$
(14) \quad \frac{\partial \gamma}{\partial s_k} = e^{-\tau(t_k-s_0)}U(s_k) + e^{-\tau(t_k-s_0)}\tau C - e^{-\tau(t_k-s_0)}R(s_k) \\
+ \int_{t_k}^{s_k} e^{-\tau(t-s_0)} \frac{\partial R(t)}{\partial s} dt = 0, \quad k=2, \ldots
$$

The second condition (14) differs from (12b) with subsequent counseling yields a transversality condition. This second condition is analogous with the non-learner problem (3) where individuals will not just receive one counseling session throughout their lives but will incur multiple sessions. If the pre-counseling discount rate, $\rho$, is the same as the post discount rate, $\upsilon$, then the two conditions in (12a) and (14) collapse into one. In terms of
the ending for the counseling time, for the case of continuous counseling, the first condition (13a) holds for all \( k, k = 1, \ldots \).

Interpretation of these continuous-counseling optimal conditions (12a), (14), and (13a) can be summarized by the following propositions.

**Proposition 2:** Continuous counseling shortens relapse duration between counseling sessions.

The optimal conditions for continuous counseling are stated in (12a), (14), and (13a) for \( k = 1, \ldots \). These conditions may be compared with the discontinuous scenario (12) and (13). Specifically, for \( k = 2 \), with discontinuous counseling there is no transversality condition (12b). This is in contrast to \( k = 2 \) with continuous counseling (14) with transversality condition
\[
e^{-\tau(s_k - s_0)}\tau C - e^{-\tau(s_k - s_0)}R(s_k) + \int_{s_k}^{u_k} e^{-\tau(t-s_0)}\partial R(t)/\partial s_k \, dt.
\]
Considering this opportunity cost of increased utility from counseling, individuals will seek counseling earlier. Future counseling sessions will then reduce the relapse duration between counseling.

The importance of continuing counseling leading to reduced relapse time between counseling is consistent with intensive behavioral intervention for obesity recommended by the U.S. Preventive Service Task Force (USPSTF). In order to promote sustained weight loss for obese adults, the USPSTF recommend high-intensity counseling, which is defined as two or more person-to-person sessions per month for at least the first three months of treatment for a total of six counseling sessions per calendar year. Scheduling follow-up contacts to provide ongoing assistance is highlighted in the counseling framework designed for effective intensive behavioral intervention (U.S. Preventive Service Task Force, 2003).

The comparative statics influence of discount rates on the starting and ending of continuous counseling periods may be investigated by assuming aggregate utility for a fixed time
interval is independent of when counseling starts and ends. This results in the integrals in the optimal conditions (12a), (14), and (13a) drop out.

**Proposition 3:** As the wedge between the pre-counseling discount rate \( \rho \) and post-counseling rate \( \tau \) widens, individuals will seek their first counseling earlier, and as \( \tau \) declines they will reduce the time interval between subsequent counseling.

**Proof:** Setting \( s_0 = 0 \), (12a), (14), and (13a) for, \( k = 1, \ldots \), reduce to

(15a) \[ e^{-\rho} U(s_k) + e^{-\tau} \tau C - e^{-\tau} R(s_k) = 0, k = 1, \]

(15b) \[ e^{-\rho} U(s_k) + e^{-\tau} \tau C - e^{-\tau} R(s_k) = 0, k = 2, \ldots, \]

(15c) \[ e^{-\tau} R(u_k) - e^{-\rho} U(u_k) = 0, k = 1, \ldots. \]

The comparative statics conditions derived in the appendix are then

(16a) \[ \frac{\partial s_k}{\partial \rho} = \frac{e^{-\rho} u(s_k)}{e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k}}, \quad < 0, k = 1, \frac{\partial s_k}{\partial \rho} = 0, k = 2, \ldots, \]

(16b) \[ \frac{\partial s_k}{\partial \tau} = \frac{e^{-\tau} C + e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k}}{e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k}}, \quad > 0, k = 1, \ldots, \]

(16c) \[ \frac{\partial s_k}{\partial v} = 0, k = 1, \frac{\partial s_k}{\partial v} = \frac{e^{-\rho} u(s_k)}{e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k}}, \quad < 0, k = 2, \ldots, \]

(16d) \[ \frac{\partial u_k}{\partial \rho} = 0, k = 1, \ldots, \]

(16e) \[ \frac{\partial u_k}{\partial \tau} = \frac{e^{-\tau} R(u_k)}{e^{-\rho} \frac{\partial R(u_k)}{\partial u_k} - e^{-\tau} \frac{\partial U(u_k)}{\partial u_k}}, \quad < 0, k = 1, \ldots, \]

(16f) \[ \frac{\partial u_k}{\partial v} = \frac{-e^{-\rho} U(u_k)}{e^{-\rho} \frac{\partial U(u_k)}{\partial u_k} - e^{-\tau} \frac{\partial U(u_k)}{\partial u_k}}, \quad > 0, k = 1, \ldots, \]

The signs of the comparative statics are determined by the effects the starting and ending times have on utility. Recall that individuals realizing the long-term consequences of an inflated satiation point and socially desirable bodyweight yields \( \partial U(s_k)/\partial s_k < 0 \), and if individuals realize an increase in the value of counseling after having delayed it, then \( \partial R(s_k)/\partial s_k > 0 \). This
yields the resulting signs in (16 a, b, and c). The signs for (16 e and f) are determined by first recalling from (8) $\partial R(u_k)/\partial u_k < 0$. Similar to $\partial R(s_k)/\partial s_k > 0$, $\partial U(u_k)/\partial u_k > 0$ assume individuals realize an increase in the value of returning to their pre-counseling preferences after having delayed it, $\partial U(u_k)/\partial u_k > 0$.

From (16a), an increase in the pre-counseling discount rate, $\rho$, reduces the time for seeking the first counseling session and from (16b), a decrease in the counseling period discount rate, $\tau$, reduces both the first counseling and all subsequent counseling time for seeking counseling. This inverse influence the two discount rates have on the timing of counseling leads to Proposition 3.

**Corollary 1.** As the wedge between the discount rates $\tau$ and $\upsilon$ widens, individuals will prolong the counseling period.

**Proof:** From (16e), a decrease in the counseling period discount rate, $\tau$, will delay its end and from (16f), an increase in the post-counseling discount rate, $\upsilon$, will also delay the counseling period. This inverse influence the two discount rates have on the counseling period ending leads to the corollary.

As indicated in Proposition 3 and its corollary, the greater the effect counseling has on lowering the counseling period discount rate, $\tau$, relative to the pre- and post-counseling discount rates, $\rho$, and $\upsilon$, respectively, the longer will be the counseling period. Programs and policies that influence this wedge between the discount rates can have a marked influence on individuals’ diets.
CHAPTER 4

CONCLUSION

Obesity counseling as an external intervention affecting individual consumption and time preference is theoretically modeled. Both cases where individuals do not modify their preferences (non-learns) and modify their preferences (learner) from counseling sessions are considered. Three major propositions are derived from the theoretical model. Based on these propositions a lower market price of counseling will provide incentives for increased participation in counseling, which supports efforts to provide affordable preventive services such as type 2 diabetes screening, obesity screening and counseling to the public. The propositions also indicate the importance of continuous counseling when taking obesity relapses into consideration. Continuous counseling leads to reduced relapse time between counseling and reinforces weight loss. This finding supports high-intensity counseling recommended by USPSTF. Finally, the propositions highlight the role of preference discount rate modification. The lower the discount rate after counseling, the shorter the time interval between counseling Thus, results indicate that counseling should also aim at lowering the time discount rates for obesity patients. It is suggested that self-control strategies in dieting and exercising should be incorporated in obesity counseling sessions.
REFERENCES


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Appendix: Comparative Statics Solution

Differentiating (15) with respect to the discount rates \( \rho, \tau, \) and \( \upsilon \) yield

\[
-e^{-\rho} U(s_k) + e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \rho} - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \tau} = 0, \quad k = 1,
\]

\[
e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \rho} + C \tau e^{-\tau} + e^{-\tau} R(s_k) - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \tau} = 0, \quad k = 1,
\]

\[
e^{-\rho} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} - e^{-\upsilon} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} = 0, \quad k = 1,
\]

\[
e^{-\upsilon} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} - e^{-\upsilon} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} = 0, \quad k = 2, \ldots,
\]

\[
e^{-\upsilon} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} + C e^{-\tau} - \tau C e^{-\tau} + e^{-\tau} R(s_k) - e^{-\tau} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \tau} = 0, \quad k = 2, \ldots,
\]

\[
-e^{-\upsilon} U(s_k) + e^{-\upsilon} \frac{\partial U(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} - e^{-\upsilon} \frac{\partial R(s_k)}{\partial s_k} \frac{\partial s_k}{\partial \upsilon} = 0, \quad k = 2, \ldots,
\]

\[
e^{-\upsilon} \frac{\partial R(u_k)}{\partial u_k} \frac{u_k}{\partial \rho} - e^{-\upsilon} \frac{\partial U(u_k)}{\partial u_k} \frac{u_k}{\partial \rho} = 0, \quad k = 1, \ldots,
\]

\[
-e^{-\upsilon} R(u_k) + e^{-\upsilon} \frac{\partial R(u_k)}{\partial u_k} \frac{u_k}{\partial \upsilon} - e^{-\upsilon} \frac{\partial U(u_k)}{\partial u_k} \frac{u_k}{\partial \upsilon} = 0, \quad k = 1, \ldots,
\]

\[
e^{-\upsilon} \frac{\partial R(u_k)}{\partial u_k} \frac{u_k}{\partial \upsilon} + e^{-\upsilon} U(u_k) - e^{-\upsilon} \frac{\partial U(u_k)}{\partial u_k} \frac{u_k}{\partial \upsilon} = 0, \quad k = 1, \ldots,
\]

Solving for the partials of the starting and ending periods of counseling respect to the discount rates yields (16).