Statistical Model for the Diffusion of Innovation and Its Applications

by

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(Under the direction of A.N. Vidyashankar)

Abstract

A well-known statistical model - the Bass new product diffusion model is introduced to describe the diffusion of an innovation. The basic assumption of the model is that the timing of a consumer's initial purchase is related to the number of previous buyers. We also introduce two methods of estimating the model's parameters: the ordinary least square (OLS) method and nonlinear least square (NLS) method. We then apply this model to several consumer products data. We obtain the statistically significant estimates for the model's parameters, and good predictions of the sales peak and the timing of the peak. We also perform a long range forecast for the sales of ATM cash card.

Index words: Diffusion of Innovation, Bass Model, Coefficient of Innovation, Coefficient of Imitation, New Product, Ordinary Least Square (OLS), Maximum Likelihood Estimation (MLE), Nonlinear Least Square (NLS)

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Chapter 1

Introduction

The diffusion of an innovation traditionally has been defined as the process by which that innovation is communicated through certain channels over time among the members of a social system [3]. As such, the diffusion process consists of four key elements: innovation, communication channels, time, and the social system.

The field of research on the diffusion of innovations took off after formation of the diffusion paradigm by Ryan and Gross [3]. Figure 1.1 could show us the number of diffusion research publications in developing versus developed countries between 1940's and 1980's: the former represent about 30 percent of all diffusion publications, while about 70 percent have their setting in developed countries. In each succeeding two-year period, the number of diffusion publications has increased considerably, until the late 1970s when the data on the number of publications are only approximate due to the lag in obtaining those publications.

One fundamental marketing concept for managing resource commitments to a new product is the product life cycle (PLC). The PLC hypothesizes that sales of a new product, over time in a target market, go through the stages of launch, growth, maturity, and decline [1]. Several descriptive, normative, behavioral, managerial, and analytical models and frameworks have been proposed to depict, explain, forecast, and manage the life cycle of a new product [2].

As a theory of communications, diffusion theory's main focus is on communication channels, which are the means by which information about an innovation is

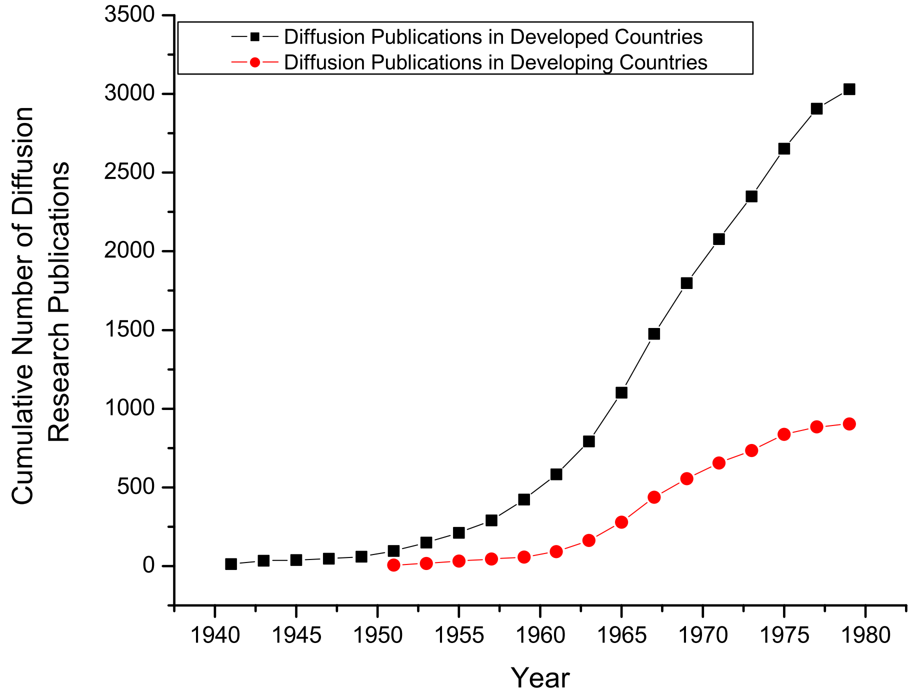

Figure 1.1: Cumulative number of diffusion research publications by year.

transmitted to or within the social system. These means consist of both the mass media and interpersonal communications. Members of a social system have different propensities for relying on mass media or interpersonal channels when seeking information about an innovation. Interpersonal communications, including nonverbal observations, are important influences in determining the speed and shape of the diffusion process in a social system.

The main impetus underlying these contributions to the diffusion of innovation is a new product growth model suggested by Bass in 1969 [4]. The Bass model and its revised forms have been used for forecasting innovation diffusion in retail service, industrial technology, agricultural, educational, pharmaceutical, and consumer durable goods markets [5]. Representative companies that have used the model include Eastman Kodak, RCA, IBM, Sears, and AT&T [6].

The emergence of diffusion modeling literature in marketing could be categorized as the following five subareas:

(1) Basic Diffusion Models. Definition of innovators/imitators and the formulation of relationship between innovators and imitators over time. etc.

(2) Parameter Estimation Considerations. Ordinary least square estimation procedure, maximum likelihood and nonlinear least square estimation procedure, Bayesian and feedback estimation (time-varying parameter) procedure, etc.

(3) Flexible Diffusion Models. Systematic (or random) variation in parameters over time, flexible diffusion patterns in terms of timing and magnitude of peak of adoption curve, etc.

(4) Refinements and Extensions. Dynamic diffusion models (market saturation changes over time), multi-generation models (timing and adoption of different generations of an innovation), space/time diffusion models (diffusion of an innovation occurs simultaneously in space and time), etc.

(5) Use of Diffusion Models. Forecasting, descriptive, normative (derivation of optimal pricing, advertising, etc), etc.

In the product innovation context, diffusion models focus on the development of a life cycle curve and serve the purpose of forecasting first-purchase sales of innovations. That is, in the first-purchase diffusion models one assumes that, in the product planning horizon being considered, there are no repeat buyers and purchase volume per buyer is one unit. The number of adopters defines the unit sales for the product. The best-known first-purchase diffusion models of new product diffusion in marketing are those of Fourt and Woodlock [7], Mansfield [8] and Bass [4]. These models attempted to describe the penetration and saturation aspects of diffusion process. In this thesis work, we will also restrict ourself to this basic first-purchased diffusion model. The development of this model over the last three decades can be reviewed in book by Mahajan, etal. [9].

The outline of this thesis is as follows: Chapter 2 reviews the theoretical background about the basic Bass diffusion model and the methods of estimating the model's parameters. Chapter 3 demonstrates the applications of the Bass diffusion model in several different areas. Chapter 4 presents the remarks and conclusions.

Chapter 2

The Basic New Product Diffusion Model - The Bass Model

2.1 Introduction

It has been documented that the natural growth of many phenomena can be depicted by a S-shaped pattern [10, 11]. Examples include phenomena as diverse as the future populations of cars and computers, the life expectancy of creative geniuses, the frequency of economic booms and buts, the number of fatal car accidents, the incidence of major nuclear accidents, and the number of deaths from AIDS.

The analytical and empirical evidence for the existence of the S-shaped pattern to represent the first-purchased growth of a new durable product in marketing was first presented by Bass [4]. Unlike other growth studies in physical or social sciences that do not concern themselves with the underlying processes that generate the S-shaped regularity, the Bass model relies on the diffusion theory to mimic the S-shaped growth patterns of new durable products [12].

The Bass growth model is best reflected by growth patterns similar to that shown in Figure 2.1. Sales grow to a peak and then level off at some magnitude lower than the peak. The stabilization effect is accounted for by the relative growth of the replacement purchasing component of sales and the decline of the initial purchase component. We shall be concerned here only with the timing of initial purchase. In addition, the theoretical framework presented here provides a rationale for long-range forecasting which is easier to guess than other models.

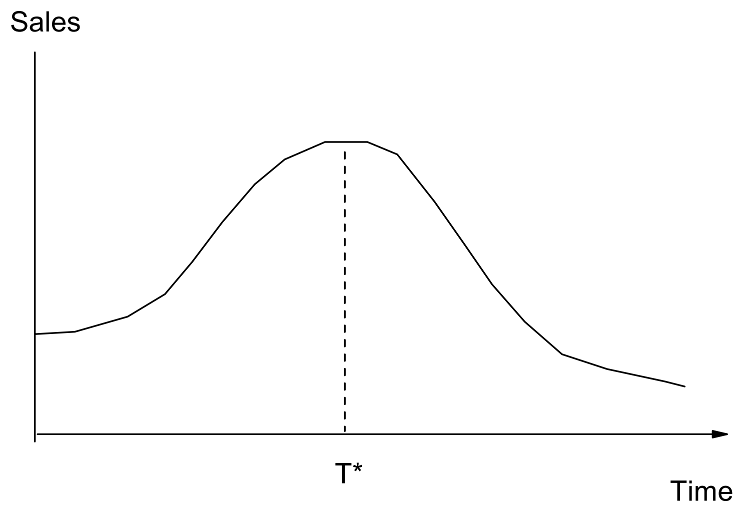

Figure 2.1: Growth Pattern of a New Product. Sales grow to a peak and then level off at some magnitude lower than the peak.

2.2 The Bass Diffusion Model

2.2.1 Adoption and Diffusion

In the discussion which follows an attempt will be made to outline the major ideas of the theory as the *timing* of adoption.

Some individuals decide to adopt an innovation independently of the decisions of other individuals in a social system. These individuals are called *innovators*. Apart from innovators, adopters are influenced in the timing of adoption by the pressure of the social system, the pressure increasing for later adopters with the number of previous adopters. They are defined as *imitators*. The Bass model assumes that potential adopters of an innovation are influenced by two means of communication - mass media (external influence) and interpersonal (internal influence). Interpersonal communications, including nonverbal observations, are important influences in determining the speed and the shape of the S-shaped pattern of the diffusion process in a social system.

In applying the theory to the timing of initial purchase of a new product, Bass formulate the following precise and basic assumption: *The probability that an initial purchase will be made at T given that no purchase has yet been made is a linear function of the number of previous buyers.* In the section which follows, the basic assumption of the theory will be formulated in terms of a continuous model and a density function of time to initial purchase. We shall therefore refer to a linear probability element as a likelihood.

2.2.2 The Bass Diffusion Model

Two assumptions characterize the model:
(1) Over a period of interest ("life of the product") there will be m initial purchases of the product. Since we are dealing with infrequently purchased products, the unit sales of the product will coincide with the number of initial purchases during that part of the time interval for which replacement sales are excluded.
(2) The likelihood of purchase at time T given that no purchase has yet been made is

$$P(T) = \frac{f(T)}{1 - F(T)} \tag{2.1}$$

where *f(t)* is the likelihood of purchase at T. $F(T)$ is the distribution function with $F(0)$=0. Eq.(2.1) is also called *Hazard Rate*[20]. Its derivation can be seen in Appendix A.

According to Bass's basic assumption, we have

$$P(T) = p + \frac{q}{m} Y(T) = p + qF(T) \tag{2.2}$$

where we set $F(T) = Y(T)/m$, p and q are two coefficients. Also

$$F(T) = \int_0^T f(t)\,dt. \tag{2.3}$$

Since m is the total number purchasing during the period for which the density function is constructed.

$$Y(T) = \int_0^T S(t)\,dt = m \int_0^T f(t)\,dt = mF(T) \tag{2.4}$$

is the total number purchasing in the $(0, T)$ interval. Sales at T is

$$S(T) = mf(T) = P(T)[m - Y(T)] = [p + q \int_0^T S(t)/m\,dt][m - \int_0^T S(t)\,dt]. \tag{2.5}$$

Expanding above equation, we get

$$\begin{aligned} S(T) = \frac{dY(T)}{dT} &= pm + (q-p)Y(T) - \frac{q}{m} Y(T)^2 \\ &= p[m - Y(T)] + \frac{q}{m} Y(T)[m - Y(T)]. \end{aligned} \tag{2.6}$$

Since $\frac{dF}{dT} = f(T) = [p + qF(T)][1 - F(T)]$, solving this non-linear differential equation we will get

$$F(T) = \frac{q - pe^{-(T+C)(p+q)}}{q(1 + e^{-(T+C)(p+q)})}. \tag{2.7}$$

We now use condition $F(0) = 0$ to get C, and then we will obtain the following useful formulas

$$f(T) = \frac{(p+q)^2 e^{-(p+q)T}}{p(\frac{qe^{-(p+q)T}}{p} + 1)^2} \tag{2.8}$$

$$F(T) = \frac{1 - e^{-(p+q)T}}{\frac{qe^{-(p+q)T}}{p} + 1} \tag{2.9}$$

and

$$S(T) = \frac{m(p+q)^2 e^{-(p+q)T}}{p(\frac{qe^{-(p+q)T}}{p} + 1)^2}. \tag{2.10}$$

In order to find the time at which the sales reaches its peak, we differentiate $S(T)$ and will get

$$\begin{aligned} T^* &= \frac{ln(\frac{q}{p})}{p+q} \\ S(T^*) &= \frac{m(p+q)^2}{4q} \\ Y(T^*) &= \frac{m(q-p)}{2q}. \end{aligned} \tag{2.11}$$

The expected time to purchase can also be derived (see Appendix B). It is

$$E(T) = \frac{ln(\frac{p+q}{p})}{q}. \tag{2.12}$$

So, once we know those three parameters: m, p and q, we will obtain the useful information about the diffusion procedure of a specified new product.

2.2.3 Notes on the Bass Model

The behavioral rationale of the Bass model are summarized:

(1) Initial purchases of the product are made by both "innovators" and "imitators". Innovators are not influenced in the timing of their initial purchase by the people who have already bought the product, while imitators are influenced by the number

of previous buyers.

(2) m is also called the ultimate market potential; p is called the coefficient of innovation; q is called the coefficient of imitation; Y(T) and S(T) are the cumulative and noncumulative number of adopters (sales) at time T. If q=0, it reduces to the exponential distribution of Fourt and Woodlock's [7]; If p=0, it reduces to the logistic distribution of Mansfield's [8];

(3) Figure 2.2 shows the analytical structure underlying the Bass model. As depicted, the noncumulative adopter distribution peaks at time T^*, which is the point of inflection of the S-shaped cumulative adoption curve.

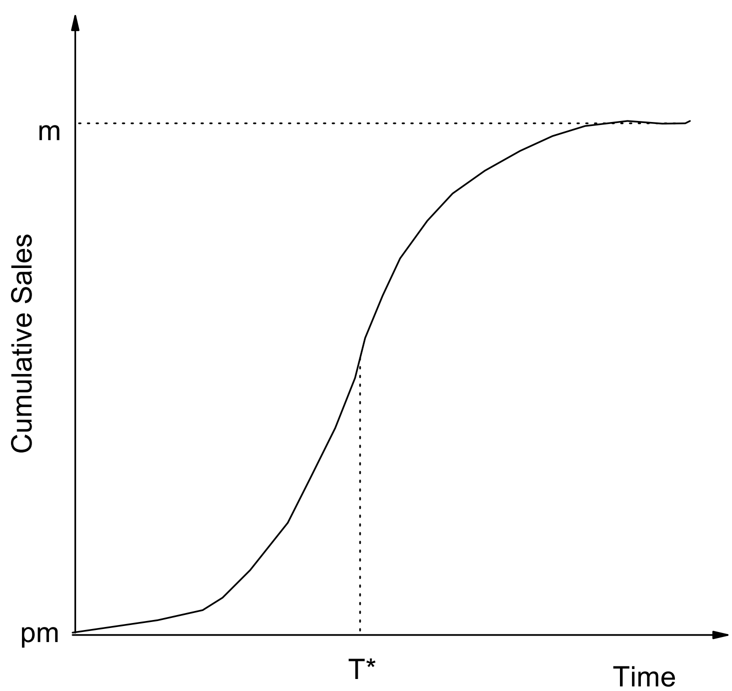

Figure 2.2: Analytical Structure of the Bass Model. The noncumulative adopter distribution peaks at time T^*, which is the point of inflection of the S-shaped cumulative adoption curve.

2.3 Parameters Estimation for The Bass Model

2.3.1 Estimation Procedures

The use of the Bass model for forecasting the diffusion of an innovation requires the estimation of three parameters: p, q and m, and a minimum of three time periods are required to estimate these parameters. The estimates of these parameters are sensitive to the number of data points used to estimate them. Several estimation procedures were proposed. For example, a meta-analyzing the results of 15 such diffusion studies reported average values of 0.03 and 0.38 for p and q, respectively (see [5] and references therein).

One of the first procedures suggested to estimate the diffusion parameters is the ordinary least squares (OLS) procedure proposed by Bass [4]. It takes the discrete or regression analog of Bass model (i.e., Eq.(2.5))

$$S(T) = pm + (q-p)Y(T-1) - \frac{q}{m}Y(T-1)^2 + \epsilon(T) \tag{2.13}$$

where $E[\epsilon(T)] = 0$ and $Var[\epsilon(T)] = \sigma^2$ and $\epsilon(T_i)$ is independent of $\epsilon(T_j)$ for $i \neq j$.

The OLS procedure is applicable to many diffusion models. Its main advantage is that it is easy to implement. However, it has some shortcomings [13]. First, in the presence of few time-series data points and multicollinearity between variables ($Y(T-1)$ and $Y(T-1)^2$), one may obtain parameter estimates which are unstable or possess wrong signs. Second, the procedure does not directly provide standard errors for the estimated parameters since nonlinear relationships exist among p, q and m. Third, there is a time-interval bias because discrete time series data are used for estimating a continuous model (i.e. the solution of the differential equation specification of the Bass model).

The Maximum Likelihood Estimation (MLE)[13], and the Nonlinear Least Square (NLS)[14] procedures are two other estimation procedures. These procedures specifically eliminate the time-interval bias and provide the standard errors for the parameter estimates.

The approach of MLE also possesses some shortcomings [14]. For example, the MLE method is applicable in a direct way only to those diffusion models for which $F(T)$ in Eq.(2.1) can be expressed as an explicit function of time. NLS estimation procedure can provide for a better fit and lower forecast errors for durable product categories and the existence of a downward bias in MLE standard error estimates, it generally become the standard in diffusion research [15, 16]. It is also noted that the more the data points used, the better the NLS procedure.

There are also other estimation procedures. For example, the algebraic estimation (AE) procedure [17] and a numerical technique proposed by Scitovski and Meler [18], etc. In the following section, we will present the principle of NLS estimation technique, and use it in the applications of Bass model for next chapter.

2.3.2 Nonlinear Least Square (NLS) Estimation

This approach is designed to overcome some of the shortcomings of the maximum likelihood approach [14, 19].

Approach 1 (NLS1)

Eq.(2.8) is the cumulative distribution function (c.d.f.) for eventual adopters:

$$F(T) = \frac{1 - e^{-(p+q)T}}{\frac{qe^{-(p+q)T}}{p} + 1}. \tag{2.14}$$

Therefore the parameter estimates $\hat{p}$, $\hat{q}$ and $\hat{m}$ can be obtained by using the following expression for the number of adopters $S(T_i)$ in the ith time interval (T_{i-1}, T_i):

$$S(T_i) = m[F(T_i) - F(T_{i-1})] + \epsilon_i^1 \tag{2.15}$$

where ϵ_i^1 is an additive error term. Based on Eq.(2.14), the parameters p, q and m and their asymptotic standard errors can be directly estimated by using nonlinear least squares procedures.

Approach 2 (NLS2)

Using the nonlinear least squares procedure, an alternative formulation to estimate the parameters of the Bass model is based on following principle. Since the probability that an individual who has not purchased the product up to period T_{i-1} will purchase the product in the ith time interval (T_{i-1}, T_i) is $(F(T_i) - F(T_{i-1}))/(1 - F(T_{i-1}))$, the number of individuals $S(T_i)$ adopting the product in the ith time interval is

$$S(T_i) = [m - Y(T_{i-1})][\frac{F(T_i) - F(T_{i-1})}{1 - F(T_{i-1})}] + \epsilon_i^2 \tag{2.16}$$

where $Y(T_{i-1})$ is the cumulative number of adopters up to time T_{i-1}, ϵ_i^2 is the error term, and the cumulative distribution function $F(T)$ is given by Eq.(2.13).

Approach 3 (NLS3)

Another possibility to estimate the parameters via nonlinear least squares is to use Eq.(2.3), which gives the cumulative number of adopters at time T. That is,

$$Y(T_i) = mF(T_i) + \epsilon_i^3 \tag{2.17}$$

where ϵ_i^3 is the error term. Since $Y(T_i)$=$S(T_1)$+$S(T_2)$+...+$S(T_i)$, the errors ϵ_i^3 are likely to be heteroscedastic (i.e. error variance increasing with i). It is expected that the estimation of Eq.(2.14) (NLS1) will provide more accurate estimates than the estimation of Eq.(2.16) (NLS3).

It should point out that since the nonlinear least squares algorithms employ various search routines to estimate the parameters, parameter estimates may sometime be very slow to converge or may not converge. Also, the nonlinear least squares procedure is applicable directly for only those diffusion models in which $F(T)$ can be expressed as an explicit function of time. A new numerical method for the parameter estimation could read the paper shown in [18]. This method can be used regardless of whether the analytical solution of the differential equation describing the model is known or not.

Chapter 3

Applications of The Bass Diffusion Model

3.1 IBM Mainframe Computer Products

The first application of the basic Bass diffusion model presented here is about the IBM mainframe computers. We illustrate this model's applications for four generations of IBM mainframe computers: first generation (vacuum tubes), second generation (transistors), 360 family (integrated circuits), and 370 family (silicon chips).

The data can be obtained in Phister[21] and [22] (for more information about IBM in these decades and later and an extensive history of the U.S. computer industry, see [23]). We will see that each of the four generations can capture the growth pattern of the new products.

Table 3.1, 3.2, 3.3 and 3.4 contain the estimations of Bass diffusion model's three parameters and the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$ for the first generation, second generation, 360 family and 370 family IBM mainframe products, from the respective OLS, NLS1, NLS2 and NLS3 estimation procedures. We can see that all 4 estimation procedures can provide for good estimates of the parameters from the magnitude of the p-values for those 4 generation computer products.

Now, we present the comparisons of the observed values with the predicted values (and 95% $C.I.$) by OLS, NLS1, NLS2 and NLS3 four procedures for those 4 generation products. Figure 3.1, 3.2, 3.3 and 3.4 respectively show the observed values and the values predicted by those four estimation procedures for those 4 generations

Table 3.1: Parameter Estimates for the 1st Generation IBM Mainframe Computer. Numbers in parentheses are estimated standard errors (asymptotic approximations). Since p, q and m are nonlinear functions of the OLS parameters, standard error estimates are unavailable for OLS. Pr>F is the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$.

method	**p**	**q**	**m**	**Pr>F**
OLS	0.0391	0.5565	15799	< 0.0001
NLS1	0.0152(0.00116)	0.6579(0.0180)	15682(292)	< 0.0001
NLS2	0.0164(0.00139)	0.6270(0.0174)	15858(58)	< 0.0001
NLS3	0.0152(0.000882)	0.6339(0.0136)	15861(44)	< 0.0001

Table 3.2: Parameter Estimates for the 2nd Generation IBM Mainframe Computer. Numbers in parentheses are estimated standard errors (asymptotic approximations). Since p, q and m are nonlinear functions of the OLS parameters, standard error estimates are unavailable for OLS. Pr>F is the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$.

method	**p**	**q**	**m**	**Pr>F**
OLS	0.0314	0.4552	88246	< 0.0001
NLS1	0.0081(0.00161)	0.6080(0.0360)	84500(3610)	< 0.0001
NLS2	0.0119(0.00274)	0.5107(0.0379)	88474(1111)	< 0.0001
NLS3	0.0094(0.00159)	0.5356(0.0313)	87994(911)	< 0.0001

Table 3.3: Parameter Estimates for the 360 Family IBM Mainframe Computer. Numbers in parentheses are estimated standard errors (asymptotic approximations). Since p, q and m are nonlinear functions of the OLS parameters, standard error estimates are unavailable for OLS. Pr>F is the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$.

method	**p**	**q**	**m**	**Pr>F**
OLS	0.0411	0.4391	133858	0.0003
NLS1	0.0208(0.00444)	0.5138(0.0496)	134293(8109)	< 0.0001
NLS2	0.0220(0.00480)	0.4835(0.0482)	135348(3045)	< 0.0001
NLS3	0.0183(0.00258)	0.5168(0.0362)	133445(2261)	< 0.0001

Table 3.4: Parameter Estimates for the 370 Family IBM Mainframe Computer. Numbers in parentheses are estimated standard errors (asymptotic approximations). Since p, q and m are nonlinear functions of the OLS parameters, standard error estimates are unavailable for OLS. Pr>F is the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$.

method	**p**	**q**	**m**	**Pr>F**
OLS	0.0353	0.4856	79765	0.0068
NLS1	0.0215(0.00479)	0.4354(0.0982)	92303(17610)	< 0.0001
NLS2	0.0213(0.00495)	0.4517(0.1033)	88640(15954)	< 0.0001
NLS3	0.0193(0.00224)	0.5204(0.0698)	79369(8878)	< 0.0001

of IBM mainframe computers. We can see that for each product those four estimation procedures describe the general trend of the time path of growth very well. In addition, each estimation procedure can provide a very good fit with respect to both the magnitude and the timing of the peaks for all of the 4 generation products and hence the parameter estimates seem reasonable for the model.

3.2 Electronic Banking Product - ATM Cash Card

Electronic banking, also known as electronic fund transfer (EFT), uses computer and electronic technology as a substitute for checks and other paper transactions. EFTs are initiated through devices such as cards or codes that one use to gain access one's account. Many financial institutions use an Automated Teller machine (ATM) card and a personal identification number (PIN) for this purpose. The federal Electronic Fund Transfer Act (EFT Act) covers some consumer transaction.

So, to most people, electronic banking means 24-hour access to cash through an automated teller machine (ATM) or paychecks deposited directly into checking or savings account. You generally insert an ATM card and enter your personal identification number (PIN). Some ATM's impose a surcharge, or usage fee, on consumers who are not members of their institution or on transactions at remote locations.

The Survey of Consumer Finance (SCF) (http://www.federalreserve.gov) is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of U.S. families. The survey also gathers information on the use of financial institutions. The links to the surveys provide summary results of the surveys, codebooks and related documentation, and the publicly available data. These surveys are the most direct precursors of the SCF.

In this section, we use information from this web site to extract the data which include ATM cash card sales in 1989, 1992, 1995 and 1998. We then use OLS and NLS methods to estimate the parameters of the Bass diffusion model. We also use this model to do a long-range forecasting based on the model's parameters we estimate from those limit data.

Table 3.5 contains the estimation of Bass diffusion model's three parameters and the model's p-value from the OLS, NLS1, NLS2 and NLS3 procedures, respectively. The NLS estimation procedure could provide for a better estimate of the parameters from the magnitude of p-values. However, since fewer data points are available, larger standard errors are observed.

Table 3.6 show us the series of estimates of ATM cash card sales predicted by basic Bass diffusion model using above estimated parameters. It could demonstrate the slowing down of growth rates as sales near the peaks. In focusing on the theoretical issues the Bass diffusion model may serve to aid management in avoiding some absurd forecasts. However, since fewer data points are available, we observed larger standard errors and hence these predictions can not present lower and upper error limits of prediction.

Table 3.5: Parameter Estimates for ATM Cash Card Sales. Numbers in parentheses are estimated standard errors (asymptotic approximations). Since p, q and m are nonlinear functions of the OLS parameters, standard error estimates are unavailable for OLS. Pr>F is the model's p-value for the null hypothesis H_0: $p = 0$, $q = 0$, $m = 0$. Note that since fewer data points are available, larger standard errors are observed.

method	**p**	**q**	**m**	**Pr>F**
OLS	0.0505	0.2088	208602	0.2322
NLS1	0.0415(0.0705)	0.1980(0.1826)	235270(419716)	0.0367
NLS2	0.0448(0.063)	0.2068(0.1716)	216733(322114)	0.0355
NLS3	0.0364(0.0634)	0.1883(0.1487)	267980(483139)	0.0076

Table 3.6: Forecast of ATM Cash Card Sales (2001 - 2016). Numbers in the Year 1989, 1992, 1995 and 1998 are the actual sales. Due to fewer observations we have not presented the standard errors associated with the prediction.

Year	**Sales (NLS1)**	**Sales (NLS2)**	**Sales (NLS3)**
1989	10740	10740	10740
1992	11557	11557	11557
1995	14059	14059	14059
1998	14829	14829	14829
2001	**16484**	**16284**	**16802**
2004	**16972**	**16584**	**17578**
2007	**16984**	**16365**	**17941**
2010	**16518**	**15654**	**17858**
2013	**15625**	**14534**	**17334**
2016	**14397**	**13125**	**16422**

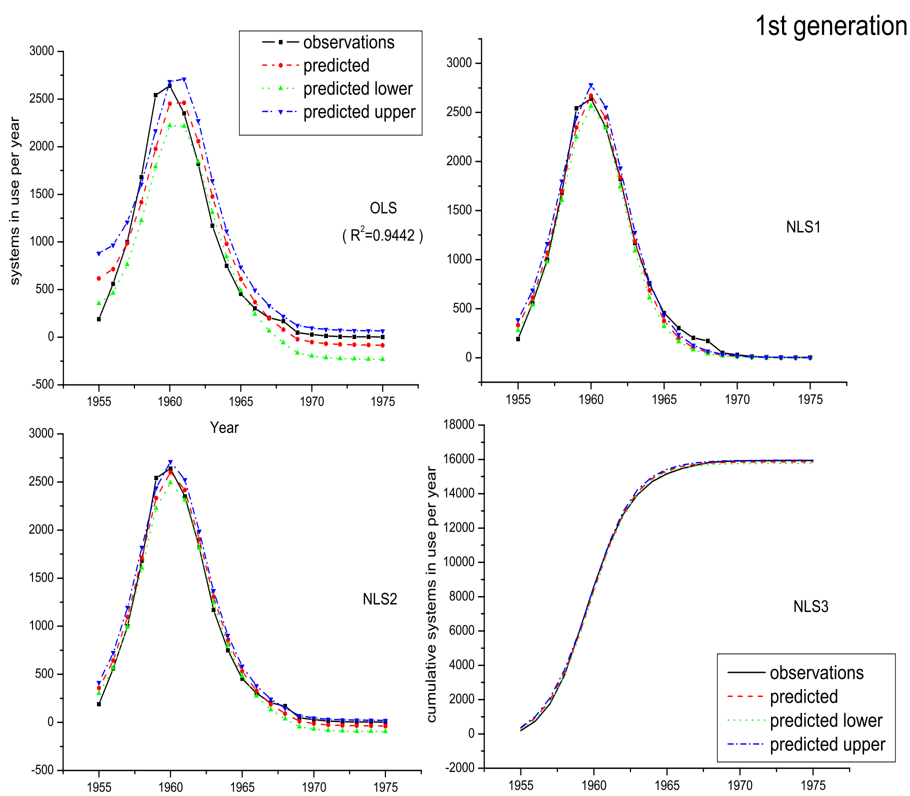

Figure 3.1: 1st generation IBM mainframe computer in use per year: observed vs. predicted by OLS, NLS1, NLS2 and NLS3 estimation procedures. NLS3 give us the cumulative systems in use.

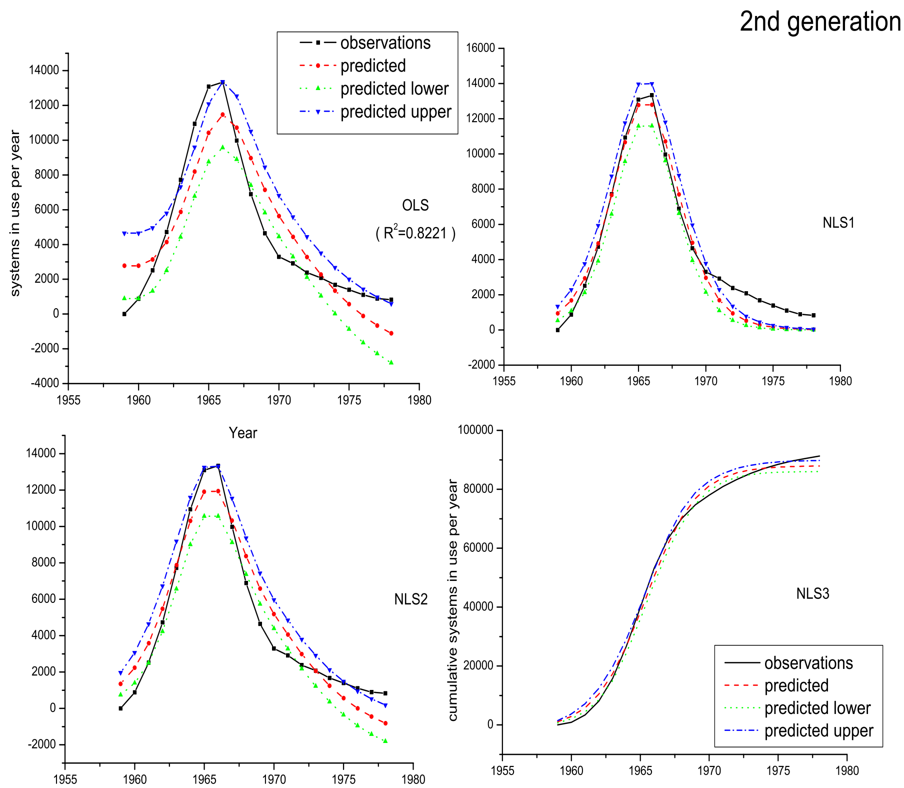

Figure 3.2: 2nd generation IBM mainframe computer in use per year: observed vs. predicted by OLS, NLS1, NLS2 and NLS3 estimation procedures. NLS3 give us the cumulative systems in use.

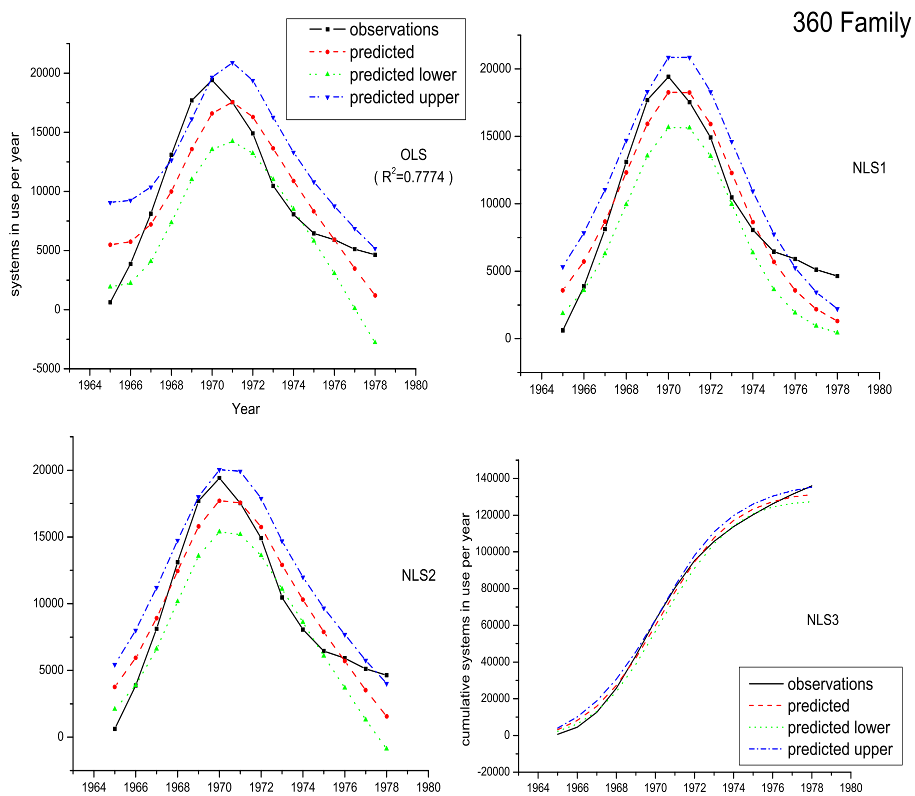

Figure 3.3: 360 family IBM mainframe computer in use per year: observed vs. predicted by OLS, NLS1, NLS2 and NLS3 estimation procedures. NLS3 give us the cumulative systems in use.

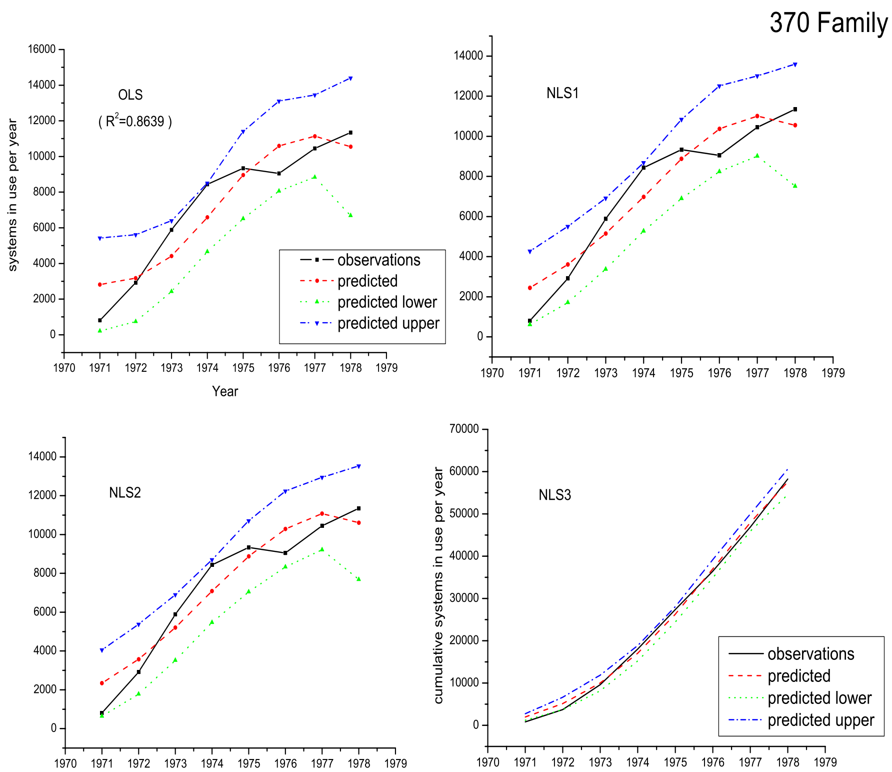

Figure 3.4: 370 family IBM mainframe computer in use per year: observed vs. predicted by OLS, NLS1, NLS2 and NLS3 estimation procedures. NLS3 give us the cumulative systems in use.

Chapter 4

Conclusions

The Bass growth model for the diffusion of innovation in this work is based on an assumption that the probability of purchase at any time is related linearly to the number of previous buyers. There is a behavioral rationale for this assumption.

Data for several products to which it has been applied are in good agreement with the model. Parameter estimates derived from OLS and NLS estimation procedures provide good descriptions of the growth of sales. This model is also useful in providing a rationale for long-range forecasting.

Research on new product diffusion models which are based on or extended from the basic Bass diffusion model has resulted in a body of literature consisting of several hundred articles, books, cases and software tools [5]. In order to make these models effective and realistic, there still exist lots of research possibilities. For example, when should a monopolist introduce a product if both positive and negative word of mouth affect diffusion process? How should product be advertised over time? How does an industry set a price of a new product class over time? etc.

The Bass diffusion model and its extensions have many managerial applications, such as new product planning and decision making. They can describe the rate of diffusion, to provide a better understanding of the drivers of adoption, to predict the future penetration trajectory, to provide inputs for investment, pricing, advertising, and product development decisions (normative use), etc.

In conclusion, the Bass-type statistical models for the diffusion of innovation could contribute to an deep understanding of the process of new product adoption. We expect further work could be done in this field.

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Appendix A

Derivation of Eq.(2.1)

A basic quantity, fundamental in survival analysis, is the hazard function. It is also known as the conditional failure rate or simply as the hazard rate. The hazard rate is defined by

$$\tilde{P}(x) = \lim_{\Delta x \to 0} \frac{P[x \leq X < x + \Delta x | X \geq x]}{\Delta x}. \tag{A.1}$$

According to the property of conditional probability, we have

$$P[x \leq X < x + \Delta x | X \geq x] = \frac{P(x \leq X, X < x + \Delta x)}{P(X \geq x)}. \tag{A.2}$$

If *F(X)* denotes the distribution function, then (A.2) can be written as

$$\frac{P(x \leq X, X < x + \Delta x)}{P(X \geq x)} = \frac{F(x + \Delta x) - F(x)}{1 - F(x)}. \tag{A.3}$$

Therefore, we get

$$\begin{aligned} \tilde{P}(x) &= \frac{\lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}}{1 - F(x)} \\ &= \frac{\frac{dF}{dx}}{1 - F(x)} \\ &= \frac{f(x)}{1 - F(x)}. \end{aligned} \tag{A.4}$$

(A.4) is just the Eq.(2.1).

Appendix B

Derivation of Eq.(2.12)

The expected time to purchase E(T) is

$$\begin{aligned} E(T) &= \int_0^\infty P(T>t)\,dt \\ &= \int_0^\infty Tf(T)\,dT \\ &= \frac{1}{p}\int_0^\infty \frac{xe^{-x}}{(1+Be^{-x})^2}\,dx \end{aligned} \tag{B.1}$$

where x=$(p$+$q)T$, B=q/p. Let $Y = e^{-x}$, then $x = -ln(Y)$, $dx = -\frac{1}{Y}dY$. The integration in Eq.(B.1) becomes

$$I = \int_1^0 \frac{lnY}{(1+BY)^2}\,dY. \tag{B.2}$$

Let $dv = \frac{dY}{(1+BY)^2}$, then we get $v = -\frac{1}{b(1+BY)} + c$. Eq.(B.2) becomes

$$\begin{aligned} I &= \int_1^0 \frac{lnY}{(1+BY)^2}\,dY \\ &= \int_1^0 \frac{Y lnY}{1+BY}\,dY - \int_1^0 \frac{ln(1+BY)}{B}\,dY \\ &= \frac{ln(1+B)}{B}. \end{aligned} \tag{B.3}$$

Therefore, we get

$$\begin{aligned} E(T) &= \frac{1}{p}\frac{ln(1+B)}{B} \\ &= \frac{ln(\frac{p+q}{p})}{q}. \end{aligned} \tag{B.4}$$