HIGH SCHOOL MATHEMATICS TEACHERS' USE OF TECHNOLOGY IN THEIR LEARNING AND TEACHING: THREE CASES

by

HYEONMI LEE

(Under the Direction of John Olive)

ABSTRACT

This study was designed to investigate how mathematics teachers used technology in their teaching, what and how they learned in their teacher education program in terms of using technology for teaching mathematics, and whether there were links or gaps between their learning and teaching in terms of using technology. The purpose of this study was to find useful ideas that may enrich technology-integrated courses in teacher education programs. Instrumental genesis theory and three metaphors: Zone of Proximal Development, Zone of Promoted Action, and Zone of Free Movement influenced the theoretical orientations of this study working as instruments for collecting and analyzing data. Goos' (2005) categories of teachers' working modes with technology were also used for this study. Three high school mathematics teachers who were graduates from the mathematics education department at the University of Georgia, where two technology integration courses were required, participated in this study. Using qualitative research methods, the main data sources include formal and informal interviews, and field notes and video recordings from observations. Inductive data analysis method for class observations data and content analysis methods for text data including transcribed interview data were used to find patterns. Comparisons between participant teachers' learning and their teaching were made to determine what learning experiences in their teacher education program fostered their actual use of technology in meaningful ways. Also, comparisons between participant teachers in terms of their ways of using technology were made to discuss how different factors such as their working environments and their professional development opportunities affected their ways of using technology. Findings revealed that there were links between their learning and teaching in terms of using technology for teaching mathematics. Direct links were identified where the same mathematical topics at the same level were encountered in both their learning and teaching. Indirect links were found concerning their disposition to act toward using technology. However, their foci on using technology varied depending on their beliefs about students and students' mathematics. Positive and/or productive meanings created during teachers' use of technology also varied based on their foci.

INDEX WORDS: mathematics education, teacher education, technology integration, secondary school mathematics, qualitative research, case study

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DEDICATION

I dedicate this work to my family, my teachers, and friends for their unconditional love and constant support.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

CHAPTER 1

INTRODUCTION

Mathematics Teachers and Teacher Education

There are many important questions in the field of mathematics education. Among those concerning the preparation of mathematics teachers, I consider the following questions: Are school mathematics teachers well-qualified? What makes them qualified? Do teacher education programs adequately perform their function to produce qualified mathematics teachers? These questions are not only issues for teachers, teacher educators or professionals in the field of education, but also for the wider society. In fact, it is common to come across articles about these issue on the cover of a news magazine or to find TV programs in which these topics are discussed. For example, in the recent report of the TIME magazine (Wallis, 2008), the reporter Claudia Wallis started the article by asking the following questions:

- "How do great teachers come by their craft?
- What qualities and capacities do they possess?
- Can these abilities be measured? Can they be taught?
- How should excellent teaching be rewarded so that the best teachers remain in a p rofession known for low pay, low status and soul-crushing bureaucracy?" (p. 28).

She reported that American public schools were struggling to attract and retain high-quality teachers, and it was urgent to prepare an alternative plan to recruit great teachers considering the retirement of teachers in the baby-boom generation. The article focused on the matter of recruiting great teachers raising the question of whether or not teachers are well-qualified and reporting recent results from the national comparison tests. The article also revealed the public's great concern about teachers, and teacher education. Indeed, there has been a controversy on the issue of teachers' qualifications throughout the whole society for a long time. Since the third international mathematics and science study (TIMSS)[1] team (1997) reported their findings, the controversy became more intense than ever. Furthermore, accelerated social change and the advent of highly advanced technology seemed to fuel the controversy since school change is too slow to act in concert with changes in other areas (Sorto, 2006).

In response to these social concerns and social changes, many mathematics educators and researchers have launched studies on mathematics teachers from many angles including their beliefs and knowledge and their learning and teaching. Based on their findings, they have designed teacher education programs, provided the opportunities for teachers' professional development through workshops, and suggested reform. However, as it is, there are discrepancies between ideal and reality as well as between theory and practice. For instance, Cohen (1990) revealed in his study that there were great discrepancies between the intention suggested by a new framework for educational reform and the ways it was implemented by a teacher in her teaching. According to him, although the teacher went through workshops for the new framework, her learning was superficial and not enough to reach the intended goals. He found that the teacher's beliefs about mathematics and teaching mathematics were not

[1] In the recent report of the 2007 trend in TIMSS, both U.S. fourth- and eighth-grade students performed better compared to 1995. In the average score of mathematics, 11 points for fourth graders and 16 points for eighth graders were increased respectively. 10 percent of U.S. fourth-graders and 6 percent of U.S. eighth-graders scored at or above the advanced international benchmark in mathematics. The Asian countries: Chinese Taipei, Korea, Singapore, Hong Kong SAR, and Japan and some countries in Europe: England, Hungary, and Russian Federation outperformed U.S. students, on average, overall mathematics scale. (Retrieved from http://nces.ed.gov/timss/results07.asp)

compatible with the view from the new framework, and therefore, her beliefs worked as an obstacle to correctly interpreting the intention of the new framework. Cohen's findings revealed how important teachers' beliefs are in learning to teach and interpret educational theories and ideas given by educators or policy makers. The findings clarified the necessity that teacher education programs should play a role to challenge teachers' beliefs and make them aware of their possible misconception. Teacher education programs, therefore, are not just for helping teachers learn to teach; they also need to help teachers change their fixed ideas, develop their reflective thinking from different views, and consider themselves as autonomous beings with authority (Cooney & Shealy, 1997). The findings in Cohen's study implied that more research would be needed to bridge the gap between the theory and the practice.

Further according to the national review on teacher education research in United States by Wilson et al. (2001), research on teachers and teacher education was limited compared to other topics in the education field. Among the 300 research articles on teachers and teacher education they reviewed in their study, they found that only 58 of them met their criteria for inclusion in their summary. None of the reviewed research focused on the issue of pre-service teachers' learning of technology use in teacher education programs. This implied that more research on that issue needed to be conducted. Technology became essential in teaching and learning (NCTM, 2000; Garofalo et al., 2000). However, we know little about how pre-service mathematics teachers' learning about technology integration impacts on their actual use in their teaching. What do pre-service teachers learn about technology integration to teach mathematics in their teacher education program? How does their learning about using technology for teaching impact on their actual use of technology in their teaching after graduation? Are there any gaps or links between their learning and teaching regarding the use of technology for teaching? If there are some, what are they? These questions need to be answered to help pre-service teachers better prepare for their future teaching. The intent of this study is to address these questions.

What is Technology?

Some terms used in educational field are often elusive (Wilson, Cooney, & Stinson, 2005). "Good teaching" and "technology" are the examples. The term technology and, therefore, teaching with technology may mean different things to different people. The word technology as it is known comes from the Greek t*echnología* (τεχνολογία) — *téchnē* (τέχνη), 'art' or 'craft' *and* -logía *(-λογία),* the study of something. So technology can be interpreted as the branch of knowledge of *téchnē* (τέχνη), 'art or skill'[2]. However, it commonly means material objects that are the results of using scientific and systematic methods and organizations. All in all for the term technology, some may think of hardware such as computers, graphing calculators, overhead projectors, SMART Board, document camera[3], and the like. Others may consider the whole of scientific knowledge, methods, or skills that are used to do something as technology. In this regard, software such as GSP, Fathom, and Excel Spreadsheet links the two opinions since its role is to enable the user to express ideas through the use of hardware. For this study, the meaning of technology is limited to the use of hardware and software, and does not include methods or ideas, no matter how systematic or scientific are those methods or ideas.

Technology in teaching and learning

The matter of technology use in classrooms has become a very important component in teacher education. In 1991, the National Council of Teachers of Mathematics (NCTM, 1991)

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[2] For the etymology of technology, retrieved from http://podictionary.com/?p=68

[3] Document cameras look similar to opaque projectors but they can do more. They can capture and display images in real time. They are also able to magnify the images and project actual three-dimensional objects as well as transparencies.

suggested, in their Professional Standards for Teaching Mathematics, that mathematics teachers should "help students learn to use calculator, computers, and other technological devices as tools for mathematical discourse" (p.52). Since then, there has been a general consensus in educational circles for teachers and students to make effective use of technology. The abilities required for living in the future include the ability to readily acquire new knowledge, to solve new problems, and to engage in creative and critical thinking. Therefore, technology became essential in teaching and learning as new knowledge (about the technology) to be acquired, and as tools to solve problems and develop creativity and critical thinking. In response to the increasing significance of technology use in schools, the Panel on Educational Technology (PET) was organized under the President's Committee of Advisors on Science and Technology (PCAST) in April 1995. The mission of the PET was to "provide independent advice to the President on matters related to the application of various technologies (and in particular, interactive computer- and network-based technology) to K-12 education in the United States" (p. 5). The PET reviewed the research literature and submitted professional opinions from all levels of the educational system, and from professional and industry organizations involved in various ways with the application of technology to education. In 1997, the PET reported the recommendations related to aspects of using technology for educational purposes. In particular, the PET summarized specific recommendations related to the aspects of the use of technology within America's elementary and secondary schools as follows:

1. Focus on learning with technology, not about technology.
2. Emphasize content and pedagogy, and not just hardware.
3. Give special attention to professional development.
4. Engage in realistic budgeting.
5. Ensure equitable, universal access.
6. Initiate a major program of experimental research. (p. 7)

The Panel found that the ready availability of computer systems at school did not guarantee teachers' actual use of the systems, and moreover, when teachers did use the systems, they were often used for either teaching students about the technologies or for drill and practice sessions for acquiring separated basic facts or skills. The panel contended that:

> The more ambitious and promising pedagogic applications of computers call for considerably more skill from the teacher, who must select appropriate software, effectively integrate technology into the curriculum, and devise ways of assessing student work based on potentially complex individual and group projects. Not surprisingly, most teachers reported that computers make their job more difficult. Despite the daunting challenge of using computers and networks appropriately within an educational context, however, teachers commonly report that they have not received adequate preparation in the effective ue of computers within the classroom. (p. 47)

Therefore, it was natural that the role of colleges of education was seen as important in the panel's report. However, they found that "Overall, teacher education programs do not prepare graduates to use technology as a teaching tool" (p. 29).

In 2000, the NCTM made even stronger recommendations for the inclusion of technology in the teaching of mathematics. The organization added a *technology principle* as one of the six principles in the NCTM's *Principles and Standards for School Mathematics*. The technology principle states that " technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhance students' learning" (p. 16). The NCTM's recommendations, together with the PET's (1997) recommendations for teacher preparation, has brought about an increase in the number of technology-integration courses in teacher education programs. In addition, much research on the effects of technology on learning and teachers' beliefs about the use of technology has been conducted (Booers-van Oosterum, 1990; Dunham & Dick, 1994; Goldenberg & Guoco, 1998; Groves, 1994; Hoyles, Noss, & Kent, 2004; Kaput, 1992, 2001; Lagrange, 1999; Olive, 2000; Hegeuds & Kaput, 2004; Strout, 2005). However, little attention has been given to describing what technology-integration courses in teacher education programs should look like, and how graduates from such programs, actually use technology in their teaching.

Olive and Leatham (2000) pointed out that little attention had been given to the technological preparation of pre-service mathematics teachers despite a great deal of interest and research on technology integration into mathematics classrooms. Olive and Leatham found that rich opportunities for pre-service teachers to use technological tools for learning and doing mathematics in their courses were not enough to ensure their actual use of technology in their future teaching although the school that they would work for would be equipped with the same kinds of technological tools that they used before in the courses. Olive, therefore, applied his findings to his technology integration curriculum course with concrete examples that pre-service teachers could directly apply. However, it was left unanswered whether his modifying the course would make any difference. Lee (2005) extended Olive and Leatham's study by providing pre-service teachers in her technology integration methods course with opportunities to work with eighth grade students. Lee expected that the opportunities might provide some sort of perturbations to the pre-service teachers that would drive their learning and the opportunities would foster their actual use of technology in their future teaching. However, her study does not yet answer what her treatments enable us to predict about the pre-service teachers' future teaching in terms of using technology. Questions concerning the links (if any) between how teachers use technology in their teaching and what they learned from technology-integration courses in their teacher education programs, are still left unanswered. (Goos, 2005; McCoy, 1999). My goal in this study is to address some of these unanswered questions.

Researcher and Research Questions

My interest in teacher education originated in my personal experiences with two different teacher education programs: as a pre-service teacher and as an experienced mentor teacher in South Korea, and as a graduate student at UGA in the U.S. The teacher education programs in South Korea are all university-based systems, but there is no alternative institution for teacher certifications. Mathematics teacher education programs in South Korea consist of courses for advanced mathematics, courses for general education theories including educational psychology, history, philosophy, and measurement, and a month field experience in school in the last year of the programs. There were not specific subject-oriented education courses or mathematics courses for teaching in school, which combine a specific subject and education theories together. After early 2000's, courses in which subjects and pedagogy are considered together appeared but only a few courses in each program. To sum up, South Korean mathematics education majors are similar to U.S. counterparts who majored in mathematics with some credits from education courses for the teacher certification and a longer period of fieldwork. There have not been technology-integrated courses yet in South Korean teacher education programs, and the most recent national curriculum and standards suggests to use calculator and computers but "carefully" not to hamper the acquisition of mental math for basic computations. In conclusion, South Korean teacher education programs and the national curriculum and standards focus on what to teach but not how to teach. How to apply educational theories into teaching is left to individual teachers.

In contrast, the mathematics teacher education program at UGA provides different courses and different opportunities together with what to teach and how to teach. It focuses more on practical aspects of how to teach and emphasizes pre-service teachers reflecting on their own teaching of mathematics. Technology-integration courses are required as mandatory in the program. As an international graduate student at the University of Georgia, I was fortunate to have the opportunity to assist Dr. Olive in his technology integrated curriculum course for secondary school mathematics (EMAT 3500). This course sparked my interest since I value the potential that technology has and I consider teachers as key agents to lead educational change. In many other countries like South Korea, again, one cannot find such a course in any teacher education program.

To me, who experienced a different teacher education under the abovementioned conditions, the EMAT3500 course in the mathematics and science education department in the University of Georgia is something wonderful. In this course, computers are fundamental tools that are used on a daily base; several educational software tools: GSP, Fathom, Excel Spreadsheet and TI-Nspire are used; Lab experiments are conducted and analyzed mathematically, which shows connections between mathematics and everyday life or physics and does applicable aspects of mathematics accordingly. The contents covered in the course are algebra topics: the numbers and operations, expressions, functions [especially trigonometric functions], and conic sections. Pre-service teachers in this course are required to find standards that match contents. Several articles are discussed to challenge pre-service teachers' beliefs of mathematics and teaching mathematics. I believe that the course meets much of the needs from research findings and social concerns such as preparing pre-service teachers to use technology for teaching mathematics and challenging pre-service teachers' beliefs about mathematics and teaching mathematics. While I have assisted and taught this course for several semesters, I was anxious about how well teachers who completed this course are doing in their teaching with technology. This study was intended to satisfy my curiosity. Through this study, I sought to find some connections between what they learned in their teacher education programs and how they actually teach in school. Specifically, I investigated the following questions.

1. What are the patterns that teachers, who have taken technology-integration courses including the EMAT 3500 course, share in their teaching, and where do the patterns originate?
2. What links are there between what they learned from technology-integration courses and how they are teaching in their classroom after graduation?
3. In what ways do teachers use technology, and what factors of technology-integration courses are related to teachers' use of technology in their teaching?

Basically, I aim to look at the specific impacts of the courses on real practices, and I expect that findings will provide ideas to further develop those courses so that teacher education programs can enrich pre-service teachers' learning in practical ways.

CHAPTER 2

THEORETICAL FRAMEWORK

I aim to observe technology-involved didactic situations, in which technologies are used as mediators among teachers, students, and mathematical concepts. My focus is on how technologies are used in classroom tasks. Didactic situations become more complicated when technologies are included. Teachers need to select types of technology and ways of using the selected ones for teaching. When planning lessons or working with technology, other situational factors (such as the availability of technological resources) affect teachers' choices. I'm interested in seeing how teachers deal with the physical, social and psychological aspects of their teaching with respect to their knowledge of using technology for teaching. I, therefore, consider that socio-cultural constructivism would provide the best lens through which to study teachers' teaching with technology. In order to clarify this choice of a theoretical perspective, I discuss how different epistemological stances can suggest different models for didactic situations, examine how the component of technology fits within different models for didactic situations, and finally provide the model for my study.

The Models Grounded on Different Epistemological Stances

In this section, I discuss metaphors for didactic situations based on two different perspectives: objectivism as a conventional perspective and socio-cultural constructivism. Objectivism takes the traditional approach to answer the questions: what is knowledge and how is it acquired? Objectivism holds that knowledge exists independently as an absolute truth, regardless of the operation of any consciousness. People, therefore, acquire knowledge by discovery (Crotty, 2003). This perspective supports the fact that teachers can relay what they already discovered to their students. In contrast, constructivist perspectives contend that knowledge is neither what is discovered nor what one can deliver to others. Rather, knowledge is acquired by individual's constructing through certain processes. Socio-cultural constructivists, rooted in the work of Vygotsky (1986), consider that knowledge is a product of individuals' social interaction based on their cultural contexts. It looks at the interconnection among students, teacher, and their social and cultural environments when teaching and learning comes to be considered. Since different perspectives suggest different aspects of knowledge acquisition, they imply different models for didactic situations with different components. In the following paragraph, I compare such models based on the above epistemological stances.

A comparison of two models by Steinbring (2005) and Zbiek et al. (2007) illustrates how conventional perspectives (objectivism) and socio-cultural perspectives suggest different metaphors for classroom didactic situations. In the following figure, Steinbring's didactic triangle is on the left and Zbiek et al's didactic rectangle is on the right.

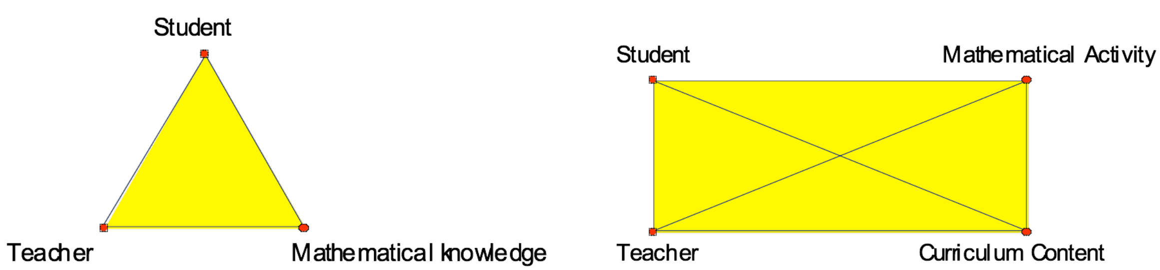


Figure 2.1. Traditional perspective vs. socio-cultural constructivism in didactic situations

I consider there is a clear distinction between the above two models based on their different epistemological stances. The model by Steinbring provides the metaphor for traditional didactic situations with three components: student, teacher, and mathematical knowledge. In that model

(See left side of Figure 2.1), mathematical knowledge is fixed as one component as if it exists out there as an object that students need to wrestle with in order to attain it. On the other hand, Zbiek et al. identified their epistemological stance with the socio-cultural perspective and they provided the model based on their perspective. The model on the right in Figure 2.1 shows this. The major difference is that mathematical knowledge is not shown as a component of the model, but is, rather, constructed by the student as a result of the interactions of the components of the model. In addition to the interaction between the three components of student, teacher, and curriculum content that are present in a traditional model, the socio-cultural perspective emphasizes mathematical activity as a fourth component within their model as an essential medium of interaction. However, these two models are not adequate in explaining the didactic situations with technology. In the following section I consider how the introduction of technology changes the didactic situation and examine how the component of technology fits within different models for didactic situations.

Technology in Didactic Situations

Since technologies entered into the teaching and learning contexts, the didactic situations have been changed. For examples of such change, Kaput (1992) noted that by “off-loading” routine computations, technologies could help students learn efficiently, in that technologies compact and enrich experiences. He also pointed out that technologies replaced paper with computer screens and provided more vivid examples through the Internet for the same tasks in didactic situations so that teachers could save their time to demonstrate procedures, and plan their lessons. Pea (1987) contended that cognitive technologies[4] help "transcend the limitations of the learners' mind in thinking and learning" (p. 91).

Technology as an Instrument in the Didactic Situation

Hoyles and Noss (2003) viewed technology use in the classroom using the notion of *instrumental genesis,* which means "the process of an artifact becoming part of an instrument in the hands of a user" (Drijvers, Kieran, & Mariotti, 2009, p. ??). The instrumental approach as a theoretical framework is grounded in instrumentation theory that suggests the distinction between artifacts and instruments (Rabardel, 2002). Artifacts are objects that are used as tools (e.g. a hammer, a piano, or a Dynamic Geometry software tool). An instrument (as a psychological construct) is more than an artifact. In addition to the artifact, "the instrument also involves the techniques and mental schemes that the user develops and applies while using the artifact." (Drijvers et al., 2009, p. ??) That is to say, to speak of an instrument, a meaningful relationship between the user and artifact is essential. Instrumental genesis is the process through which an artifact becomes part of an instrument through being used by a user in a meaningful way. That is, before a user establishes a meaningful relationship with an artifact through his or her mental schemes and techniques, the artifact is not yet an instrument (Drijvers et al., 2009, Kieran et al., 2007). Following Hoyles and Noss (2003), Drijvers et al. (2009) articulated a bilateral relationship between technologies (as artifacts) and users:

> [W]hile the student's knowledge guides the way the tool is used and in a sense shapes the tool (this is called instrumentalization), the affordances and constraints of the tool influence the student's problem solving strategies and the

[4] Cognitive technologies refer to technologies that reflect "learners' thought processes or strategic choices while engaged in mathematical activity" (Zbiek et al., 2007). They, therefore, can help users to go beyond limitations of the human mind in imagining and thinking about complex objects and to perceive properties of such objects.

> corresponding emergent conceptions (this is called instrumentation). The dual nature of instrumentation and instrumentalization within instrumental genesis comes down to the student's thinking being shaped by the artifact, but also shaping the artifact. (p. ??)

Whatever the artifacts are, the ways to use them are determined by the users through the process of *instrumental genesis,* or they are left as just artifacts. The users (teachers and students), therefore, are the main factors that determine the roles of technology. Instrumentation theory provides researchers with a focus on the meanings created by users when using technological tools. From this viewpoint of instrumentation theory, the intervention of technologies in didactic situations made learning and teaching contexts more complex for educators to understand and describe. Hence the 2-D models suggested by Steinbring (2005) and Zbiek et al. (2007) are no longer appropriate as metaphors for the interactions of elements in technology-involved didactic situations (the focus of this study).

Three-Dimensional Models for Technology-involved Didactic Situations

Olive and Makar (2009) applied the Piagetian theory of the assimilation, accommodation, and perturbation principles to teaching and learning with technology. According to these authors, technologies can be assimilated into the existing learning and teaching contexts, without redefining classrooms, by making electronic worksheets and structured lessons on the computer screen. Alternatively, the teaching and learning context can undergo an accommodation when teachers overcome perturbations that originate from use of the technologies in profound ways. In the process of this accommodation, new learning is possible. Olive and Makar (2009) suggested a transformation of Steinbring's didactic triangle into a didactic tetrahedron as a metaphor "for the transforming effects of technology when it is accommodated by the didactical situation rather than assimilated into it. It literally adds a new dimension to the didactical situation." (p. 136)

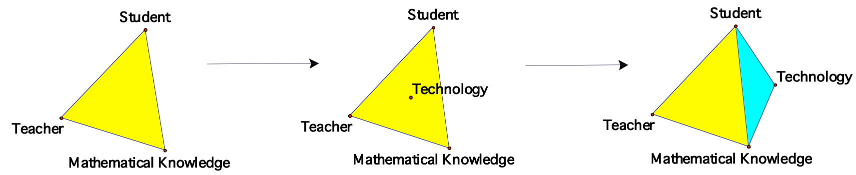


Figure 2.2: Transforming the Didactic Triangle into the Didactic Tetrahedron (from Olive & Makar, 2009)

Zbiek et al. (2007) also suggested that when technologies are involved in teaching and learning, the didactic situations become more complex and thus need higher dimension models, 3-D models than those without technologies to better describe them. They extended their didactic rectangle to form a didactic pyramid with Tool (Technology) as the vertex of the pyramid (see Figure 2.3).

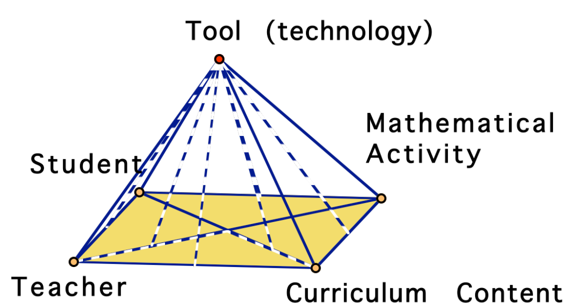


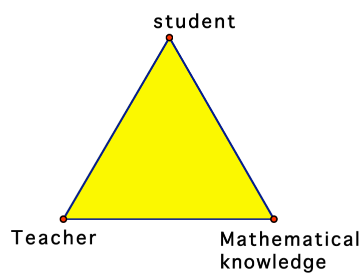
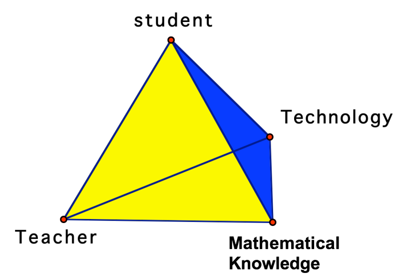
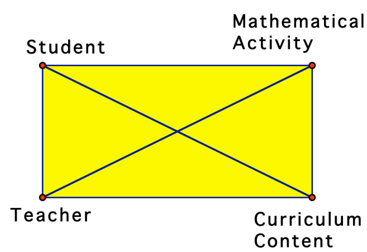
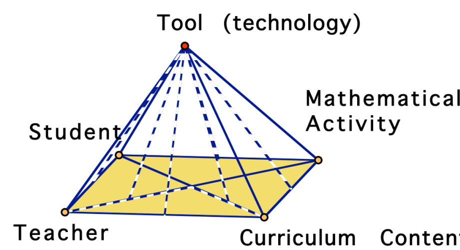
Figure 2.3: Didactic Pyramid (from Zbiek et al., 2007)

Technology as a tool is important in the socio-cultural view as a semiotic mediator in developing knowledge. Falcade et al. (2007) contended that "artifacts of any kind are central elements in the Vygotskian theoretical frame: products of human activity, they play a fundamental role in cognitive development" (p. 320). However, Vygotsky pointed out the differences and commonalities between signs and tools as artifacts. While signs are internally oriented, tools are externally oriented, that is to say, they are contrasted in terms of the ways that they orient human behavior. Both have commonalities in terms of their functioning as mediators.

All in all, I summarize four different models based on different epistemological stances in the following table.

Table 2.1

Models in didactic situations

Perspective	Without technology	With technology
Traditional Objectivist		
Socio-cultural Constructivist		

As shown above, teaching and learning include many factors and different people can have different definitions of teaching and learning. It might be natural that different people take different approaches to researching teaching and learning based on what aspects of teaching or learning they want to investigate. Scientists use telescopes when they explore stars in the sky, but they use microscopes when they examine minute particles. As such, I suggest that students' learning can be largely considered in two aspects: intra-personal and inter-personal. Learning has extremely private aspects and I consider these as intra-personal aspects. For example, the topic of what is going on in one's brain when one is studying or of what knowledge processing looks like in one's brain is intra-personal. However, in the case of classroom teaching context, we need to zoom out to see a broader scene by considering what teachers provide to the class, how students interact with teachers or materials, and how teachers' interpretation about students' responses affect their following teaching. In this case, we need to see not only students but also out side of students to see what influences their learning and also what influence teachers' teaching. Cobb (2007) suggested two criteria to select an appropriate theoretical perspective for research studies in the field of mathematics education. One is how one conceptualizes individuals and the other is what perspective is useful. I consider that the socio-cultural perspective is useful to study teaching and learning in technology-involved complicated situations and establish the model for the study.

The Model for This Study

In this study, my aim is to 1) investigate what factors from technology integration courses foster beginning teachers' instrumental genesis, 2) to look at how teachers make decisions for selecting types of technology and ways of using technology to teach mathematics, and 3) to see what affects their decisions. In order to construct a theoretical model to also address aims 2) and 3) above, I turned to research by Goos.

Goos (2005) contended that teachers' ways to use technologies were affected by many factors. For example, they were influenced by time and opportunities in their pre-service education or professional development, skills and previous experience with technologies, beliefs about mathematics and teaching mathematics, curriculum or requirements, and availability of technologies. Goos considered technologies as cultural tools that re-organize students' cognitive processes and transform classroom practice. In a part of the longitudinal study that spans the transition from pre-service to beginning teaching of secondary school mathematics, Goos (2005) focused on the case of one participant while she investigated how both pre-service teachers' and beginning teachers' working modes with technology change over time and across different classroom settings.

In order to describe the changes in teachers working modes with technology, Goos (2005) suggested four metaphors: *master*, *servant*, *partner* and *extension of self*. The followings are the meanings for each category.

- *Master*: Teachers mainly use technologies due to external pressures from education systems, but their knowledge and competence limit the range of operating with the technologies.
- *Servant*: Teachers use technologies for a fast and reliable replacement of pencil and paper, but do not change the nature of classroom activities.
- *Partner*: Teachers develop an affinity for technologies, and therefore, it is possible that students can have new types of tasks or new ways to approach the existing tasks so that the students can construct powerful knowledge.
- *Extension of Self*: Teachers use technologies for course planning and everyday practices in the classroom. Technologies become part of their pedagogical and mathematical repertoire.

Goos (2005) also used a modified Vygotsky's Zone of Proximal Development (ZPD) applied to novice teachers' development and two other zones from Valsiner (1987): Zone of Promoted Action (ZPA) and Zone of Free Movement (ZFM), which were developed as extensions of Vygotsky's ZPD (1986) to describe what factors influenced the participant's working modes. Vygotsky's ZPD originally refers to the symbolic space between a learner's actual developmental level and the level of potential development that can be reached under adult guidance, or in collaboration with more capable peers. In the teacher education context Goos (2005) explained Vygotsky's Zone of Proximal Development as "a symbolic space where the novice teachers' emerging skills are developing under the guidance of more experienced people" (p.37). Goos (2005) modified Vygotsky's ZPD to include teachers' beliefs, knowledge, and skills in working with technology.

Valsiner (1987) extended the concept of Vygotsky's ZPD and suggested his theoretical framework that includes three zones: ZPD, ZPA and ZFM. According to Linda Galligan (2008) in discussing Valsiner's ZFM and ZPA:

> While acknowledging students' freedom of action and thought, the Zone of Free Movement represents a cognitive structure of the relationship between the person and the environment, seen in terms of constraints that limit the freedom of these actions and thoughts. This environment is socially constructed by others (teachers, administrators, the curriculum writers) and the cultural meaning system they bring to the environment, but the ZFM's themselves can either be set up by these 'others', the students themselves or through joint action, but are ultimately internalized. While the ZFM suggests which teaching or student actions are possible, the Zone of Promoted Action (ZPA) represents the efforts of a teacher, or others to promote particular skills or approaches. For example, a nursing department promotes students to go to numeracy classes. However the ZPA is not binding; thus students may not wish to actively participate in this course. (p. 3)

Goos (2005) applied the ZFM and the ZPA as well as her modifed ZPD to describe teachers' working mode over time. She explained that the ZFM represents environmental constraints that limit teachers' actions and thought, and the ZPA represents what teachers learn in their teacher education or how their supervising teachers, educators, or their colleagues influence them to develop their teaching skills or approach.

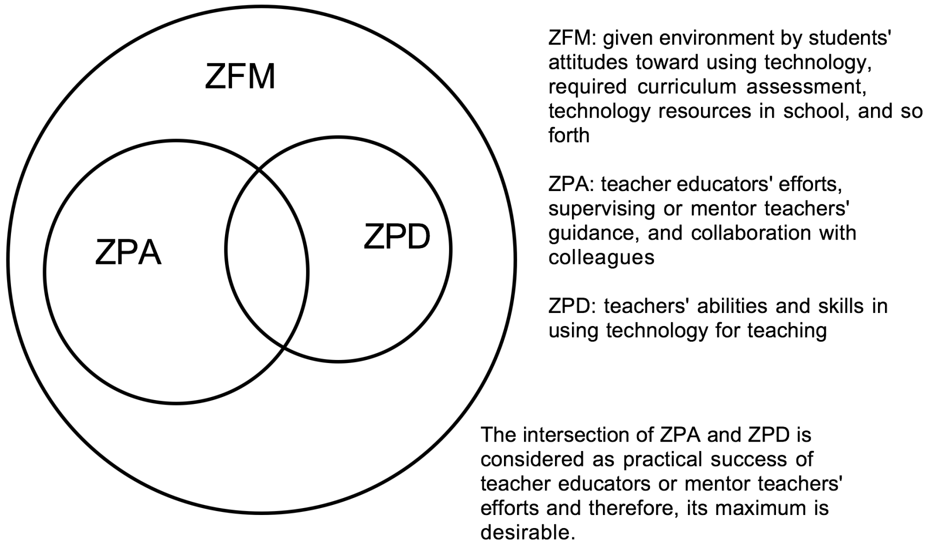


Figure 2.4: Three metaphors of ZPD, ZPA, and ZFM in working with technology

Goos (2005) described her participant teacher's dynamic working modes over time based on these theories. For example, she followed the teacher from his student teaching to his beginning teaching in school. She explained, for both his student teaching and beginning teaching, what the teacher experienced in his teacher education program and professional development opportunities that he attended (using the concept of ZPA); how the teacher's working mode affected by environmental constraints (using ZFM); What the teacher was capable of doing with technology (using ZPD). She also categorized different types of working modes using her four metaphors to describe the role of technologies used in teaching mathematics. According to Goos, the participant teacher in her study used technology as *servant* in his student teaching, but he could go beyond *servant* and *partner* modes toward the *extension of self* mode in his beginning teaching.

In my study I used Valsiner's concepts: the ZPA and the ZFM and the concept ZPD modified by Goos (2005). I simplify the factors that can influence a teacher's teaching in order to make my study doable. I selected participants who were in technology-rich environments as teachers in their second or third year of teaching. I, therefore, considered that I needed to investigate the participant teachers' teaching along multiple perspectives:

- ZPD: teachers' beliefs, knowledge, and skills in working with technology
- ZPA: teacher education, professional development, and teaching experience with colleagues
- ZFM: access to hardware, software and laboratories, access to teaching materials, support from colleagues, curriculum and assessment requirements, and students' attitudes and abilities (Goos, 2005).

In my study I consider ZFM as providing both affordances and constraints for the teachers, especially with respect to the access to hardware, software and laboratories. Where teachers had ready access to these resources, the ZFM acted as an affordance rather than a constraint.

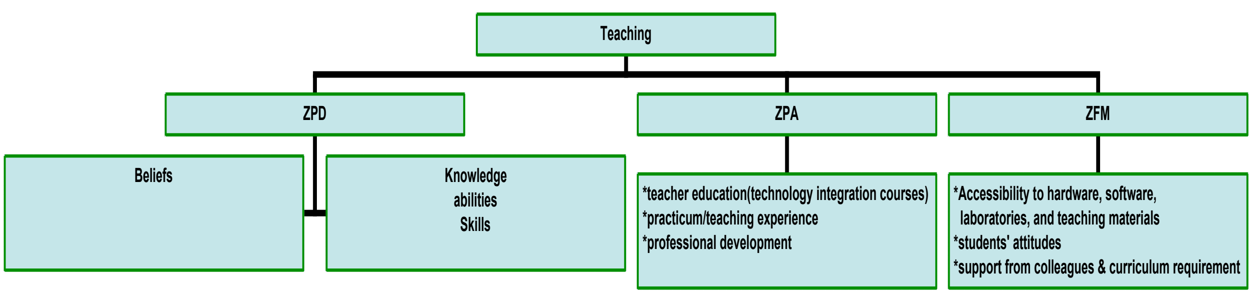


Figure 2.5. Dimensions of factors that influence a beginning teacher's teaching

I focused on the impact of the technology integration courses among elements of the university ZPA of each teacher in his or her teaching. That is, I compared the teaching differences against the experiences those courses provided among all variables. The following model (Figure 2.6) encapsulates the two main aspects of this study: Teachers' experiences in mathematics education courses and teachers' teaching in their classrooms, as well as the possible relationships between these two aspects. I have used a modification of Zbiek et al's didactic pyramid for teachers as learners in technology-integration courses (left side of figure 2.6). In the right side of the figure, I have incorporated the three zone theories as vertices of a tetrahedron, with the teacher as the fourth vertex and teaching emerging from the interactions among all four vertices.

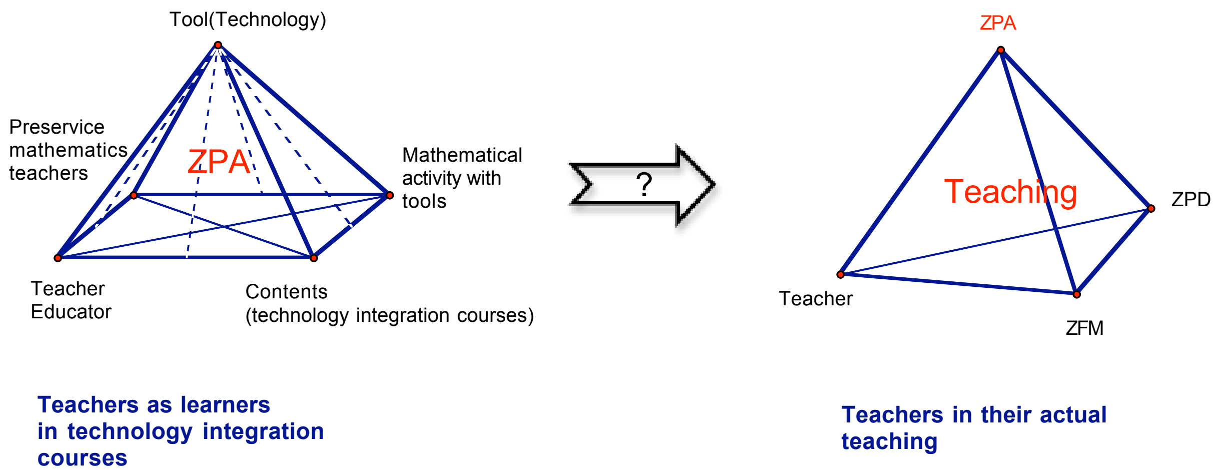


Figure 2.6: A model for the focus of this study

The focus of my study is represented in figure 2.6 by the double arrowed box containing the question mark. Thus, the study is focused on the ways in which the experiences represented by the pyramid on the left related to the observed and perceived aspects of the teaching situation represented by the tetrahedron on the right.

CHAPTER 3

METHODOLOGY

Case Study

The case study, one of the ways of doing social science research, is considered "a research approach, situated between concrete data taking techniques and methodologic paradigms" (Lamneck, 2005). The case study has become one of the most popular research strategies in the education field, especially educational evaluation. Nonetheless, it has a short history (Stake, 1995). Tellis (1997) explained this by contending that the case study is an ideal methodology for a holistic, in-depth investigation, and therefore it is ideal for research in education. Case studies are often confused with qualitative research since they are often used in qualitative research. However, any mix of quantitative and qualitative data can be used in a case study. According to Hays, "case studies are used to provide information for decision making or to discover casual links in settings where cause-effect relationships are complicated and not readily known, such as school reform or a particular government policy" (Hays, 2004). Since the focus of my study is on understanding the impact of what teachers learned from the technology integration courses on how they are teaching in the classroom, the case study method fits my study. For the purpose of illuminating links between these two experiences, I employ a *qualitative* case study method. Patton (2002) contended that a case study is expected to comprehend the complexity of a unique case; however, that single case can be layered with smaller cases (Patton, 2002). He also suggested that cases in a case study are units of analysis; thus, in my study, the teaching of teachers who went through technology integration courses is the single case study of the top layer, and participant teachers, as sub layered-cases, are the units of analysis. The links between layers are expected to inform the shared patterns among teachers.

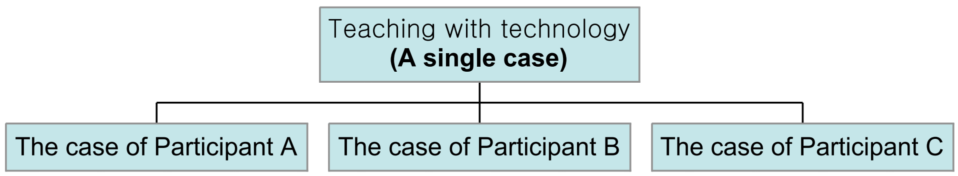


Figure 3.1: The structure of the study

Study Settings and Data

Participants Selection

Since I wanted to investigate teachers who satisfied specific conditions, I used purposeful sampling (Patton, 2002). The criteria to select participants are the following:

1. The participant was a practicing teacher with at least two years of teaching experience and a graduate of the mathematics education program at UGA. In this study, I examined how teachers who went through courses (including the EMAT3500 course) in which technology was integrated into teaching mathematics were actually using technology. For this reason the participant had to be a graduate of the program.

2. The participant was using technology in their teaching and working in school. Again, the research questions required this condition. I assume the target teachers learned how to use various technologies for teaching mathematics in a technologically rich environment. Thus, I was also interested in seeing what technology they selected to use for teaching specific topics and why they did so.

3. The participant's school was located in the Clarke County School District. This

condition was necessary to comply with human subjects research approval.

Once the criteria were set, I sent out the letter (See Appendix for the letter) to recruit participant teachers. Finally three teachers: Alvin, Theodore, and Simon[5] met the criteria. They expressed their willingness to take part in this study showing their interests.

As graduates from the University of Georgia who majored in mathematics education, they all completed the EMAT 3500 course in which I have worked over the last two years. This allowed me to have knowledge of what they did in the EMAT 3500 course, for instance, assignments, essays that they submitted, computer outputs that they constructed, their projects, test scores and so forth as main data sources. The study basically took place in their school classrooms in Athens, Georgia. Since one main data source is from the EMAT 3500 course, it would be reasonable to provide the course description of EMAT 3500 in the following section.

Description of EMAT 3500[6]

The EMAT 3500 course is entitled "Exploring Concepts (with Technology) in Secondary School Mathematics", and it is a required course for mathematics education majors. The course prerequisites are Differential Calculus, Integral Calculus, Linear Algebra, and one of the following: Introduction to Higher Mathematics, Multivariable Calculus, or Elementary Differential Equations. The goals of EMAT 3500 are basically to provide the opportunities to become familiar with curriculum standards, concepts of secondary school mathematics and to use application software to solve mathematical problems and investigate mathematical concepts. It also aims to help pre-service teachers think reflectively about becoming mathematics teachers by providing opportunities of reading literature, writing reflections, and discussing pedagogy. All

[5] All names of participants are pseudonyms. They all were white male teachers.
[6] For the details, visit http://math.coe.uga.edu/olive/emat3500s07/Syllabus_emat_3500.html

assignments and projects are closely tailored to meet the course goals. Course topics are 1) numbers and operations, 2) expressions and equations, and 3) functions, before the midterm exam, and 4) statistics and probability, and 5) mathematical modeling with real data following the midterm exam. Students work in small groups (3 or 4 students) to construct instructional units that incorporate technologies in meaningful ways as their final product of the course. Excel Spreadsheet, Geometers' Sketch Pad (GSP), and Fathom are the main programs that the students use; they also use TI-83+ or 84 calculators and real-time data collection tools such as the Calculator Based Ranger (CBR) and Calculator-Based Laboratory (CBL) probes. As the course title indicates, the course emphasizes mathematical exploration with technology. While the course instructor helps pre-service teachers explore mathematical concepts with technology, the instructor focuses on students' instrumentalization in that he emphasizes students reshaping the technological tools through their mathematical reasoning activities. For instance, students create GSP sketches to investigate transformations of functions dynamically. In this way, the geometric construction tool becomes a tool for dynamically representing geometric changes in function graphs related to changes in the numerical parameters of the function equation[7].

Data Collection

Yin (1994) recommended six primary sources of data for case study research. The six primary sources are documentation, archival records, interviews, direct observation, participant observation, and physical artifacts. It is not necessary to include all the sources in a case study, but using multiple sources of data not only increases the reliability of the study but also ensures the construct validity. Tellis (1997) added, "The rationale for using multiple sources of data is

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[7] See the Java Sketchpad example at
http://jwilson.coe.uga.edu/olive/EMAT4680.2000/Javagsp/Dynamic_Quadratic.html

the triangulation of evidence" (p.10). Considering the complexity of relationships among components in didactic situations observed in my study, I used multiple data of documents, archival records, interviews, direct observation, and so forth. The following table shows the specific sources that I used.

Table 3.1

Data Source

Source	**Data Item**
Documents	• Lesson plans • Handouts for students • Computer output, etc.
Archival records	• Teachers' GPA and taken courses • Their assignments and essays • Computer outputs from their EMAT3500 • Questionnaires
Interviews	• Interviews at the end of the semester
Direct observation	• Classroom observations
Other	• E-mails • Informal conversations

The process of data collection

First of all, I started collecting some documents and archival records that are related to participants' course work by accessing the database that Dr. Olive established for the EMAT3500 course during 2004 fall semester and 2005 fall semester. I was very familiar with all the assignments, topics of their essays and the course itself thanks to my teaching experiences in the course. I could examine participants' beliefs about mathematics, technology, and teaching mathematics from their essays and their knowledge by reviewing their assignments and work when they were taking the course.

In the second step, I provided questionnaires (see Appendix) to them to understand their backgrounds, for instance, their reflections on their teaching, school life as high school mathematics teachers, opinions about students' learning, climates in school, what they learned while they were teaching, ideas about mathematics, teaching mathematics, and using technology, how often and what types of technology they used in their past teaching and so forth.

From September to December in 2008, I observed teaching and video-recorded some of that. In broad strokes, classroom observations were weekly based.

Formal interviews followed the classroom observation during January in 2009. All interviews were recorded and transcribed. While I interviewed the teachers, I could clarify what they did in their teaching and deepen my understanding about their beliefs, which were unlikely to be revealed in their actions. I also collected e-mails and informal conversations.

Validity and Reliability as Trustworthiness

Yin (1989) proposed that validity and reliability are critical criteria in determining the value of a qualitative study. Reliability means the extent to which one can believe in the research findings for the collected data while validity means whether researchers are observing what they want to observe and measuring what they want to measure. In order to support the validity and the reliability of my study, I applied several strategies to my study: data source triangulation, participants' feedback, and external audit. According to Stake (1995), triangulation is the protocol that is used to ensure accuracy of researchers' interpretations and conclusions. "Triangulation can occur with data, researchers, theories, or methodologies" (Tellis, 1997 p40. 2). Denzin (1984) identified four different types of triangulation such as data source triangulation (multiple data sources), investigator triangulation (multiple investigators), theory triangulation (multiple theoretical perspectives, and methodological triangulation (multiple methodologies). Among them, I used data source triangulation by using multiple sources of data. Second, I used member checks by asking my participants to review my interpretations before and after my observing their classrooms. Their feedback provided the opportunities to correct my interpretations of observed phenomena, or serve as evidence to support my interpretation. Last, I consulted my colleagues throughout the process of data collection or analysis. Their feedback increased the validity of my study as external audits. I believe that all these strategies increase the trustworthiness of my study.

Participants and Settings

Alvin

Alvin was teaching in a public 9-12 high school located in a mid-city in Georgia. The school year was divided into two 18-week terms, and the school used a 4 by 4 block system in which students took four courses each semester and each class period was one and a half hours long. The student body of the school was racially diverse with 1480 enrollments, and the majority of students were from lower level class families. The graduation rate of the school was about 50%. The school had the priority in terms of receiving government support, so the school was technologically very rich. For instance, every classroom in the school had a SMART Board system. A SMART Board system hooked up to teachers' computers supported teachers' and students' presentations with more accurate expressions and audio-visual effects. In that school, the SMART Board already replaced the role of the existing white board and the white board was used as an auxiliary or bulletin board to announce homework, school news and the like. The school also purchased the Schoolview software system that allowed teachers to view each student's computer screen on the teacher's computer and also allowed the teacher to project any student's computer screen on the SMART Board. The following figure 3.2 shows Alvin's classroom that I observed.

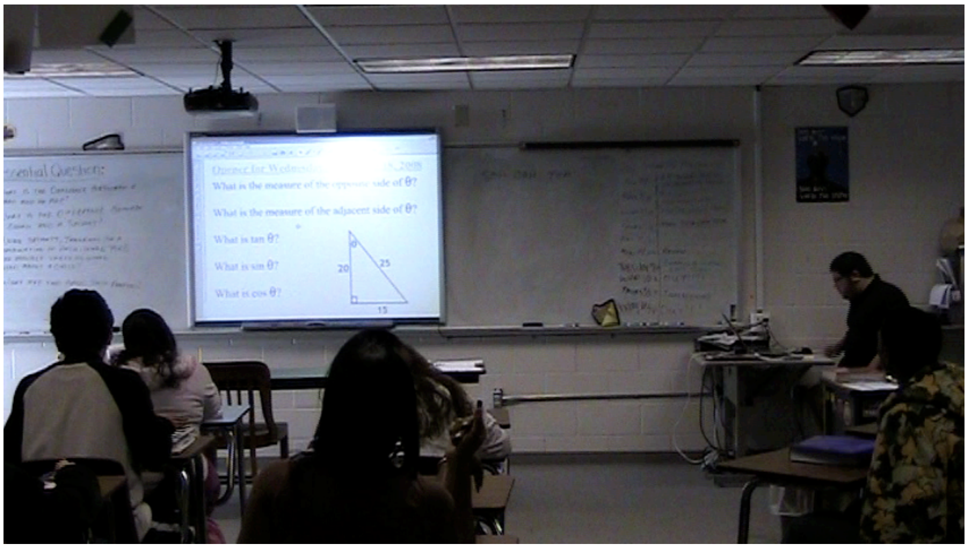

Figure 3.2: Alvin's classroom

Specifically I targeted his Geometry class for observation. In his geometry class, he had a total of 27 students. Among them, 17 were African-American; 8 were Hispanic; 2 were Caucasian. His Geometry class met in the third block that had three sub-blocks: the first sub-block from 12:00 to 12:45, the second from 12:45 to 1:15 for lunch, and the last from 1:15 to 2:00. Alvin expressed that his Geometry students were "bad". He clarified the meaning "bad" included their behaviors or attitudes as well as their performances.

Alvin just started his third year of teaching when I observed. He was teaching 30 hours per week and coaching a women's softball team. He had teaching experience with Algebra II for three consecutive semesters, Geometry for two consecutive semesters, and Algebra III for two semesters. Algebra II and Geometry seemed the most familiar subjects to him, so I came to observe his Geometry class since he did not teach Algebra II in fall 2008. Alvin expressed his special concern about managing a classroom several times, took workshops for classroom management, and had an ESOL certification. This explained the facts that the school was diverse in its student population and students' attitudes were tough for teachers to control. He was in a master's degree program for mathematics education and talked to the researcher that observing others' classes was his best way to learn how to teach in terms of integrating technology and managing classrooms.

The adjectives that described teaching characteristics that he pursued and also as his own were 'down to earth', 'organized', 'methodic', and 'thorough'. He believed that different students needed to develop different types of mathematical abilities based on their aptitudes and therefore, different types of teaching styles and different types of tests. His belief was reflected in his reflection on the article of instrumental and relational understanding by Skemp (1987). He wrote:

> As with every person, I had a particular apptitude[sic] for something. It happened to be math. […] What is left to reason are the "benefits" of relational instruction to those who have no apptitude [sic] or interest toward math. How does it help someone to monopolize their time trying to teach them something that just is not important to them? They will probably never see its usefulness. One may say that we should learn for learning's sake and not aim toward good grades (as Skemp submits would be a downfall of instrumental learning.) However, this is surely unrealistic. It is not possible for all of us to understand everything.

He preferred the traditional teacher-led lecture style in his teaching, and this was constantly witnessed by classroom observations. The following table shows the dates of observations and topics that were covered in each day.

Table 2.2

Observation dates for Alvin

Observation Date	**Lesson Content**	**Used material**
Sep-03-2008	Types of quadrilaterals and their properties	SMART Board system, Guide note as a handout for student
Sep-17-2008	Congruent polygons	SMART Board system, Guide note as a handout for student
Oct-01-2008	Circumference and Area of circles	SMART Board system, Guide note as a handout for student
Oct-08-2008	Area and perimeter of plane geometries (Reviewing topics for the following test)	SMART Board system,
Oct-22-2008	Surface area and volumes of solid geometries (Individual practice for the following test)	SMART Board system, The Schoolview software, Laptops for the teacher and all students, graphing calculators
Nov-13-2008	Inscribed angles and central angles in a circle	SMART Board system, Guide note as a handout for student
Nov-19-2008	Basic trig functions	SMART Board system, Guide note as a handout for student
Dec-02-2008	Reviewing geometric terms and notation for EOCT Individual practice	SMART Board system, The Schoolview software, Laptops for all students, graphing calculators
Dec-04-2008	Reviewing congruence and similarity for EOCT Individual practice	SMART Board, The Schoolview software, Laptops for all students, graphing calculators

For the first four days in the table, I did take field notes but could not video-tape his classroom since the consent forms for video-recording were still being collected. For the rest of the days, I video-recorded his teaching while taking field notes.

Theodore

Theodore was teaching in the same school as Alvin. Thus, Theodore had the same working environment. However, he was teaching an ESOL class for Algebra I. This seemed to make a difference between their working conditions. The number of students in his class was 12 (five girls and seven boys). One was from Ethiopia and the others were from Peru and Mexico. The following figure 3.3 shows his small classroom.

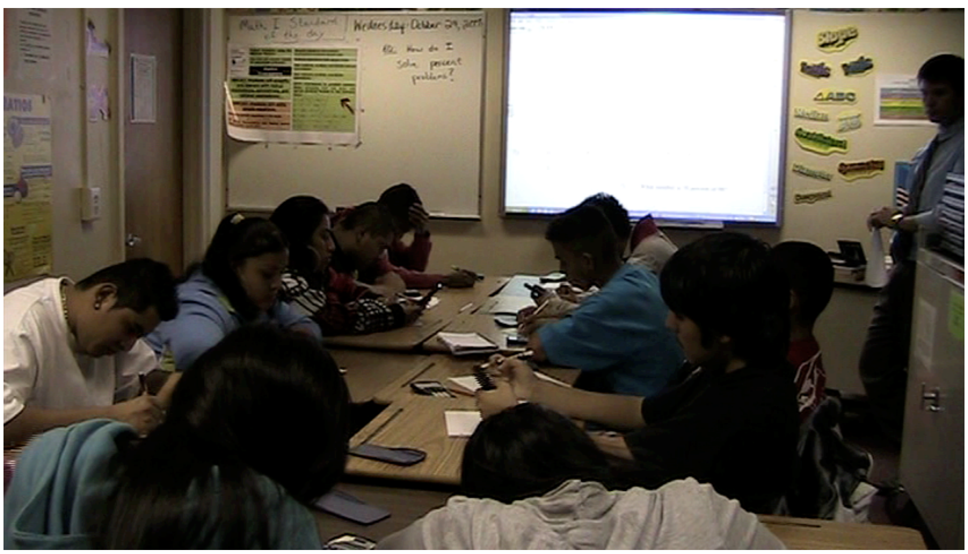

Figure 3.3: Theodore's classroom

His Algebra I ESOL class also met in the third block. Theodore described his class as comfortable, enjoyable, and easy to control. This was a contrastive opinion with what Alvin expressed about his class.

Theodore started his fourth year of teaching when I met him in school on the third day of September in 2008. He was teaching 22.5 hours per week and coaching a baseball team. He had teaching experience with Algebra I for six consecutive semesters, Geometry for one semester, and Algebra III for one semester. He was teaching Algebra I and Math I. The school operated its co-teaching system, so he was co-teaching Math I but not Algebra I. Thus, we decided on the Algebra I class to observe his teaching. The adjectives that described teaching characteristics that he pursued and also as his own were ‘mild mannered’, ‘even keeled’, and ‘approachable’.

He expressed special concern about getting students engaged in the class. This was in accordance with his teaching style which was student centered. In the interview, he was saying:

> “I prefer the student centered for the main reason that when your classroom is student centered and they’re learning in a student centered classroom, I believe it’s easier for a teacher to determine whether or not they’re learning the material. If I’m, and my first year was definitely the opposite of student centered. It was write on the board, take notes, and then when they leave you have no idea whether or not what you were teaching was actually getting through, whether they learned any of it or not. But when it’s student centered and they’re having discussions, they’re answering questions, they’re asking questions, and you can kind of see them working, it kind of gives you more insight into what they’re thinking. And so you kind of – you can see better see, I think, whether they’re actually learning or not.”

His ideas were fairly well represented throughout my observations in his classroom. His class was like a small community where each student was an integral part in exchange of their knowledge. In his class, it seemed easy for all students to ask questions, to answer the questions, and to express ideas. The following table shows the dates of observations and topics that were covered in each day.

Table 3.3

Observation dates for Theodore

Observation Date	**Lesson Content**	**Used material**
Sep-03-2008	Laws of exponents	SMART Board system, graphing calculators
Sep-17-2008	Properties of radicals	SMART Board system, laptops for students, graphing calculators
Oct-01-2008	Product of binomials	SMART Board system, Laptops for students
Oct-08-2008	Product of binomials Factoring	SMART Board system, Algebra tiles
Oct-22-2008	Individual practice for the following test (online problem bank)	SMART Board system, Senteo[8], laptops for students
Oct-29-2008	Percent and proportion	SMART Board system, graphing calculators
Nov-05-2008	Application of percent	SMART Board system, laptops and handouts for student
Nov-12-2008	Proportional equations (Pair work)	SMART Board system, Laptops for all students, graphing calculators
Nov-19-2008	Radical expressions and	SMART Board, Laptops for

[8] Senteo interactive response system is an assessment tool that can be purchased with the SMART Board. The system includes a computer in which Senteo software is installed, a Senteo receiver, and Senteos, which are hand-held clicker devices. Students use the Senteo clickers to enter their responses for given questions. Then, the Senteo receiver collects all responses and transfers them into the computer. The installed software organizes the received data to produce summative and formative assessment results. Teachers can import questions in xml format and export the result to an Excel spreadsheet. (See the example of the result in figure 4.25 on page 79).

	proportional equations (Individual practice)	all students, graphing calculators
Dec-03-2008	EOCT practice (Individual practice)	SMART Board system, Online resource, laptops, Senteo, graphing calculators
Dec-04-2008	EOCT practice (Individual practice)	SMART Board system, Online resource, laptops, Senteo, graphing calculators

For the first five days in the table, I did take field notes but could not video-tape his classroom since the consent forms for video-recording were still being collected. For the rest of the days, I did video-record his teaching while taking field notes.

Simon

Simon was teaching in a Catholic 9-12 high school located in a mid-city in Georgia. The school year was also divided into two 18-week terms. All students had five classes a day and the first class started at 8:15 am and the last class ended at 3:30 pm. Each class period was fifty-minutes long. However, the class schedule was not on a weekly basis; it was on a four-day cycle, thus Monday's class repeated on the following Friday. The majority of students in that school were white with 120 enrollments and they were from middle class families. The graduation rate of the school was about 95%. The school was not technologically rich especially compared to the school where Alvin and Theodore taught. The school provided a laptop for each teacher, but he did not have a projector in his class. Laptops for students were not available in his classroom. The students were supposed to have their own graphing calculators. As such, technological environments were sharply contrasted between two schools. However, Simon had special interests in integrating technology into his teaching. He personally purchased a DLP projector, Elmo document camera, and graphing calculator computer software and installed them in his classroom. The following figure 3.4 shows Simon's classroom that I observed.

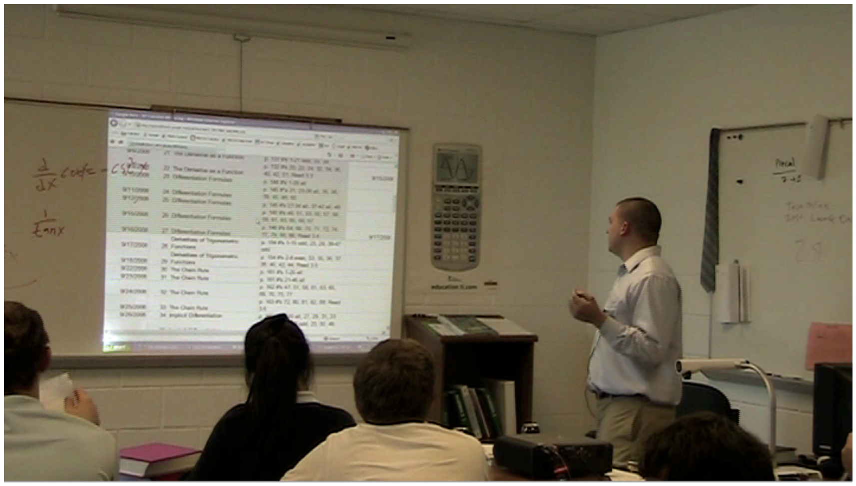

Figure 3.4: Simon's classroom

I observed all of the subjects that he taught. I could not choose just one of his courses since his class schedule was not fixed weekly, while the other school and I had weekly based schedules, but I focused more on his Calculus classes. He had a total of 8 students (four boys and four girls) in his Calculus classes. They all were white except one who was Asian American. Simon expressed that his class was big since he had had four or five students in his other calculus class.

Simon was starting his third year of teaching when I first met him in August 29, 2008. He was teaching 25 hours per week. He had teaching experience with all of Algebra II, Pre-Caclulus, and AP Calculus for four consecutive semesters. Simon expressed his special concern about students' performance in the AP Calculus test. He was a member of the AP Calculus learning community that met monthly to discuss issues that were relevant to members' classroom teaching. He presented on how to use Winplot in teaching Calculus at the Georgia Mathematics Conference during October in 2008. This, together with his personal purchases of devices for teaching, well reflected his enthusiasm toward learning about educational technologies and sharing the ideas with others. The adjectives that described teaching characteristics that he pursued and also as his own were ‘enthusiastic’, ‘inspiring’, and ‘adventurous’. I often witnessed that his expedition delved into the world of mathematics. In his essay, he wrote:

> I hope to be a teacher that is passionate about mathematics. I hope that my passion will be apparent and that some of my passion will be transferred to my students. I hope that I am liked by my students and never seem too busy to be approached for help. I wish for my students never to be held back by my own deficiencies, but at the same time I hope I don’t know all the answers. Knowing all the answers is no fun, only in discovering solutions lies pleasure.

The following table shows the observation date and lesson contents for his teaching.

Table 3.4

Observation dates for Simon

Observation Date	**Lesson Content**	**Used material**
Aug-29-2008	Algebra I: Addition and subtraction of signed numbers	Document camera system[9], graphing calculators, GSP
Sep-05-2008	Algebra II: Correlation and best fitting lines Calculus: Derivative functions	Document camera system, graphing calculators, GSP, Excel Spreadsheet
Sep-18-2008	Calculus: Derivatives of trig functions	Document camera system, graphing calculators, GSP, graphing calculator software
Sep-24-2008	Calculus: Derivatives of trig functions (Problems reviewed)	Document camera system, graphing calculators, Virtual-TI
Oct-03-2008	Algebra II: Translation of functions	Document camera system, graphing calculators, GSP, Excel Spreadsheet

[9] This represents the system that consists of a document camera, a DLP projector and a computer.

Oct-30-2008	Calculus: Application on optimization problems	Document camera system, graphing calculators, graphing calculator software
Nov-20-2008	Calculus: Application on sketching function graphs	Document camera system, graphing calculators, graphing calculator software
Feb-6-2009	Calculus: Definite integral of a function	Document camera system, graphing calculators, Winplot

For all the dates, I took brief field notes but I started video-recording from my second visit.

Data analysis

I applied inductive data analysis to analyze the data from classroom observations. It began with specific observations and moved on to build general patterns. Patton (2002) suggested, “Categories or dimensions of analysis emerge from observations as the researcher comes to understand patterns that exist in the phenomenon being investigated” (p. 56). For the transcribed interview data and documents, I used content analysis. According to Patton, it often refers to searching texts for repeated words or themes, and it is appropriate to analyze text data rather than field notes from observations. Patterns or themes mean the core meaning found from content analysis. I also used case-by-case comparison analysis technique to find shared patterns across the cases of different participants. Finally, I formulated findings and conclusions based on the above three analyses.

CHAPTER 4

THE INDIVIDUAL CASES

In this study, I aimed to investigate how mathematics teachers use technology in their teaching. Were there links or gaps between their past learning in technology-integrated teacher education courses and their actual teaching in their classroom in terms of using technology? If so, what caused the links and gaps? I used these questions as a map for the study, connecting research questions to literature review, data collection, and analysis.

In this chapter, I provide brief descriptions for what I observed in the participant teachers' teaching of Mathematics and, at the same time, contrast these observations with what they learned in their technology integration courses. For each of the participants, I discuss types of technologies that they used, ways of using them, and their pedagogical intention of using technology on an observation day basis.

Alvin

During the semester when the data were collected, Alvin was teaching Algebra I and Geometry. I chose to observe his Geometry class because he was teaching Algebra I for the first time. He used OnCourse[10] software to plan lessons electronically. According to Alvin, when planning lessons, his primary focus was on how he could gain his students' attention and pique their interest from the beginning of each class. In addition, he was concerned with what material he needed to cover, essential questions that would drive the lesson, standards that were to be addressed, and how he could bring closure to the class in a meaningful way (Interview, 9-25,

[10]For further information about the OnCourse system, visit http://www.oncoursesystems.com/

2008). At the beginning of every lesson, he wrote the essential questions that would drive the lesson on a dry-erase board for students to solve while he was walking around the classroom to check homework.

Observation day 1 (9/3)

Mathematical Topic

The topic of the lesson was types of quadrilaterals and their properties. Types of quadrilaterals were introduced with their definitions. Properties of quadrilaterals are used to compare one type of quadrilateral to the others. Among quadrilaterals, similarities and differences were discussed to categorize the quadrilaterals. The question of "what makes it special?" was constantly asked.

Tools and Materials

For the class materials, Alvin gave each student a guide note on which they had a graphic organizer in the form of a Venn Diagram (Figure 4.1) and opened a prepared file that was similar to the guide note on the SMART Board screen for his lecture. The guide notes included more information than was given on the screen, including students' practice problems. Alvin drew types of quadrilaterals on the screen and gave the definitions of them. The SMART Board system supported Alvin's drawing with accurate and colorful representations. He was capable of using many other functions of the system: highlighting, transforming, animating, and hiding or showing figures or words. After students learned types of quadrilaterals, volunteers were called upon and asked to come over to the SMART Board screen and move the words to the proper places in the Venn Diagram. The students liked the activity of clicking the words and moving them to the corresponding places on the screen. The following figure shows both the guide notes for students and those on the SMART Board screen during the activity.

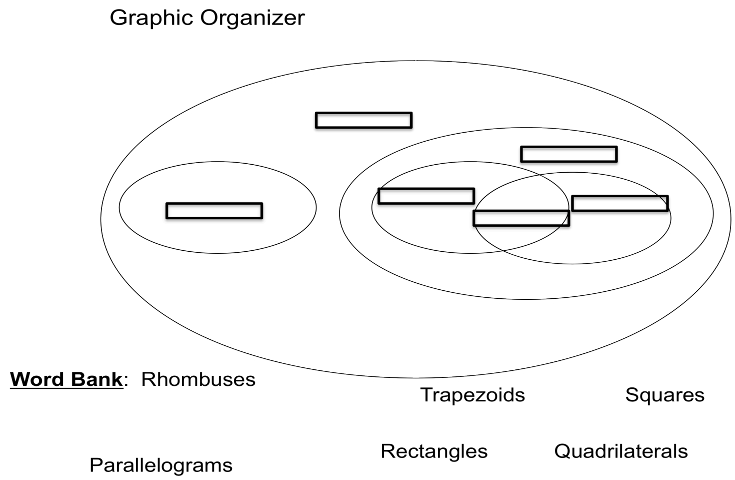

Figure 4.1: A sample of guide notes (graphic organizer).

Activity

While Alvin was lecturing, his students were supposed to take notes on their guide notes. The class was a teacher-centered lecturing style. Alvin gave definitions of quadrilaterals with their figures and introduced their properties. Teaching by telling was predominant throughout the class. For instance regarding the property of the same length of two diagonal lines in a rectangle, the students learned the SAS and HL triangle congruence theorems in the previous lesson and practiced on using the theorems to solve problems like the given in figure 4.2; this problem tells that rectangles are quadrilaterals with equal diagonals. However, the class did not reconsider the previous lesson to build the new concepts in quadrilaterals upon what students already learned.

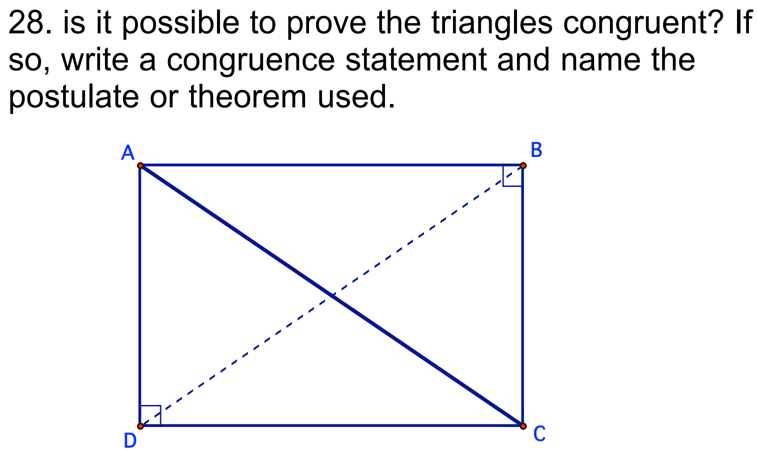

Figure 4.2: An example problem excerpted from students' workbook.

Ways in Which Technology Was Used

The technological tools that Alvin used for the lesson included the SMART Board system and the laptop to which it was connected. When he used them, his focus was on creating a better presentation that would make his demonstration easier and make students excited about the lesson. I, therefore, considered that he used the SMART Board technology in a meaningful way. However, his attention was given to 'pedagogically effective' ways of using visual supports rather than 'mathematical exploration'. In particular, when the class was classifying quadrilaterals on the SMART Board screen, one student suggested an interesting claim that could be worth discussing. When Alvin asked, "What are rhombuses?" she answered, "Rhombuses are small parallelograms." He asked what she meant by "small" and she answered, "one pair is longer than the other pair in parallelograms but they are not longer in rhombuses." Alvin simply made a rhombus bigger by dragging a corner of the rhombus next to a parallelogram and asked if the new rhombus was still smaller than the parallelogram. He pointed out her expression was neither mathematically precise nor appropriate to explain rhombuses. He did not use her limited conception to explore further the properties that distinguish squares from rhombuses and rectangles from parallelograms but just used a visual representation changing the size of a rhombus.

Observation day 2 (9/11)

Mathematical Topic

The topic for the lesson was congruent polygons. However, at the beginning of the class, Alvin's essential questions were problems in Probability and Statistics. For instance, the questions concerned finding the range of the given data, calculating the average for the given data, and answering the probability for a certain case. There was no connection between the essential questions and the lesson for the day.

Tools and Materials

Tools and materials included the SMART Board system and handouts containing practice problems for students.

Activity

Naming polygons, discussing the meaning of "identical," and finding corresponding congruent polygons were the main activities of the class. Again, the teacher led the class for the entire class period, and the activities were focused on learning facts rather than exploring or understanding concepts.

Ways in Which Technology Was Used

For the activity, Alvin used the SMART Board system in a meaningful way. The SMART Board system allowed him to flip, rotate, translate, or highlight text and figures. In order to show the corresponding parts of two congruent figures, Alvin selected a figure and then rotated and translated it. His demonstration was very visual, providing exact representations that students could use to check whether two figures would satisfy the definition of "being congruent." In using technology, he focused on creating better presentations and getting students' attention. In the interview, he said:

> For my students, I used a lot of technology for my presentations and, actually, it helps to be able to – because you have digital SMART Boards and digital presentations, I can go through and write over a presentation, quickly erase it, quickly adjust it, add to it just on the fly much more so than with chalk and dry-erase boards, and I can always save it. … a lot of the way I use technology is to make things simpler but not necessarily you know, it's more exciting, maybe more colorful (Interview, 1-21, 2009).

<u>Observation day 3 (10/1)</u>

Mathematical Topic

The topic for the lesson was the circumference and area of circles.

Tools and Materials

The SMART Board system and handouts that included practice problems for students were used.

Activity

Alvin asked his students to say loudly the formulas of circumference and area of circles. However, he was only going through the motions of asking. As soon as he asked, he usually answered himself. It seemed that following his plan was very important to him, and he believed that he knew what and how students knew. Students were supposed to memorize the formulas and use them to solve individually the given problems after the teacher's demonstration. There was no conceptual teaching or learning. Rather, students were drilled in finding the circumferences and areas of circles by using the formulas.

Ways in Which Technology Was Used

For the activity, Alvin used the SMART Board system in exactly the same way as on day two. In using technology, his focus, again, stayed on creating better presentations and getting students' attention.

Observation day 4 (10/8)

Mathematical Topic

The topic for the lesson was the area and perimeter of plane geometries. A test on the topic was scheduled for the next class. By reviewing problems in the topic, the class was preparing for this test.

Tools and Materials

The SMART Board system and dry-erase boards for individual groups were used for the "Jeopardy" game during the entire class period.

Activity

The SMART Board system allowed Alvin to create a Jeopardy game for the class. Students were grouped into teams of five. The teacher hid the prepared questions on the SMART Board screen. Questions were given scores according to their difficulty. In terms of student engagement, the class was very successful. However, many questions were given in a multiple-choice style and students were interested in acquiring scores rather than learning the mathematics behind the activity.

Ways in Which Technology Was Used

For this activity, the SMART Board system delivered the goods. It supported all of the functions of hiding and showing by touching the screen. It also provided an "applause" sound effect whenever answers were correct. It surely helped to excite students' interest in the activity.

In the personal conversation at the end of the class, Alvin told me that his intention of using technology was given to integrating the work of doing mathematics with an entertaining activity. He was satisfied with the lesson saying that his using the SMART Board system helped students to collaborate with others in their group and the lesson was successful in terms of engaging students in the activity and keeping their attention. (Personal conversation, 10-8, 2008)

Observation day 5 (10/22)

Mathematical Topic

The topic for the lesson was surface areas and volumes of solid geometries. A test was scheduled for the next class, so students reviewed and practiced problems to prepare for it.

Tools and Materials

Alvin used the SMART Board system to present problems and the Schoolview technology to monitor students' work. Students used laptops to solve work problems given on a website.

Activity

Students' individual practice followed the teacher's demonstration of using formulas to solve the problems. By using Schoolview technology, Alvin could take a different approach to provide students with individual practice opportunities via online resources while he was assessing students' learning.

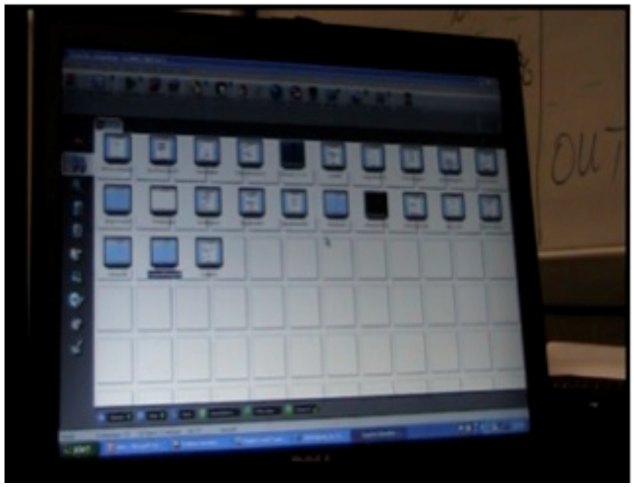

Figure 4.3: Alvin monitoring students' work. Each square shows a student's computer screen.

Ways in Which Technology Was Used

The Schoolview technology allowed Alvin to monitor students' work on their practice problems (See figure 4.3). If his students attempted to surf the Internet, which was not involved in the work at hand, then he could control their Internet access. He could also zoom in on each student's computer screen to see whether they were struggling and what they knew and could do. This allowed him to make informed decisions about his future teaching (personal conversation, 10-22, 2008). This technology allowed Alvin to control the classroom and observe all students' work at once.

<u>Observation day 6 (11/13)</u>

Mathematical Topic

The topic for the lesson was inscribed angles and their corresponding central angles.

Tools and Materials

The SMART Board system was used, along with guide notes that included practice problems for students.

Activity

The lesson began with Alvin defining inscribed angles and the central angles of circles. He also explained that the inscribed angle theorem states that an angle x inscribed in a circle is half of the central angle y that subtends the same arc on the circle, regardless of the position of its apex on the circle. Alvin used the SMART Board screen to show the written statement and a figure example for a central angle and its corresponding inscribed angle as in figure 4.4.

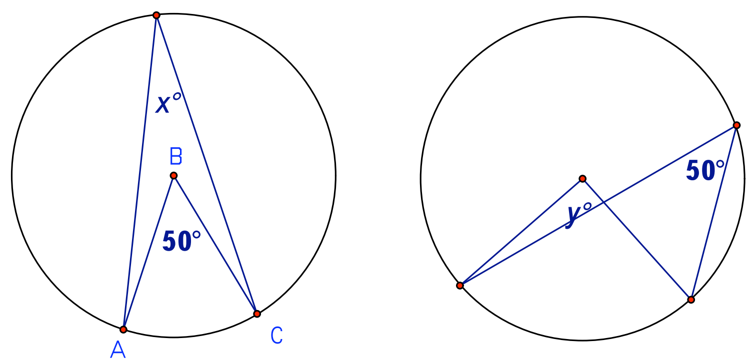

Figure 4.4: Finding x and y by using the inscribed angle theorem.

After his lecture, he demonstrated using the inscribed angle theorem to solve problems like the above. The remaining class time was given to students for their individual practice with similar problems that Alvin demonstrated. The didactic situation was typical of a teacher-led lecturing style.

Ways in Which Technology Was Used

As part of his pedagogical repertoire, Alvin used the SMART Board system to present his lecture. However, the figures on the SMART Board screen were fixed, and he did not go further than teaching by showing and talking about the fixed figures. Although the figures and written statements on the screen were precise and beautiful, they could also have been drawn and written on the chalkboard. Therefore, SMART Board was used as a simple replacement of the chalkboard in traditional classrooms. Alvin did not attempt to incorporate technology to explore mathematical concepts. In teaching this lesson, he focused on letting students know mathematical facts and how to use them to solve problems.

However, he said in the interview, “GSP is one thing I would definitely love to involve. The rest of it [other types of technologies], I don’t necessarily think I’m interested in doing too much with. But GSP, I would love to see my students in a computer lab using GSP, and basically every geometry concept that we go through, that they do some GSP exploration. I see that software as that valuable.” This contrasted sharply with his actual teaching although he could have used GSP on his SMART Board. The way he taught with technology in this lesson also contrasted with his major experience about using technology in the courses in his teacher education program. His learning about GSP was focused on exploring mathematical concepts by using it dynamically. By using dragging and tracing functions of GSP, he learned how to explore the invariant rules associated with mathematical concepts at hand. The following figure shows the example that Alvin created for one of the course assignments.

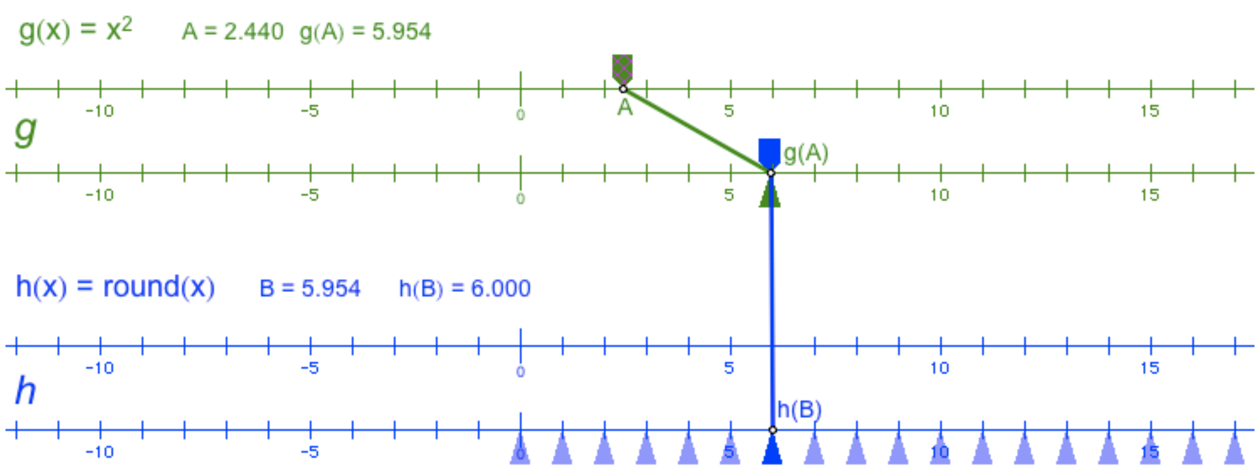

Figure 4.5: Alvin's work in his EMAT 3500 course

In figure 4.5, he had two functions: a quadratic and a round function. What he did was to construct the composition of the two functions by merging the input point of h(x) with the output point of g(x) and examined the range of the composed function by tracing the triangle attached to point h(B) and dragging point A. As point A was moved, the output point, g(A) moved according to the function rule, in this case as the square of the distance of point A from the origin. The output point h(B) also moved according to the function rule for h(x), in this case jumping from integer to integer values on the number line. The trace illustrates the discrete aspect of the range of this composed function. This can be a different way of representing functions as well as a way of exploring function behaviors and properties.

Based on how he was capable of using GSP, I considered that Alvin could show the invariant rule that the inscribed angle is always half of the central angle by dragging the apex point on the circumference of the circle, if he had used GSP to create figure 4.4 rather than the static figures he produced using the SMART Board.

<u>Observation day 7 (11/18)</u>

Mathematical Topic

The topic for the lesson was the three basic trigonometric ratios: sine, cosine, and tangent.

Tools and Materials

The SMART Board system and guide notes were the materials for the lesson. Students did not have individual practice time, but they did have guide notes for copying down what Alvin was writing on the SMART Board screen.

Activity

At the beginning of class, Alvin checked students' homework as usual, which took about 10 minutes. Before the class, Alvin prepared some problems about missing measures that could be solved by using the definitions of trigonometric ratios. He posted these on the SMART Board screen. Figure 4.6 shows these opening questions for students while Alvin walked around the classroom checking students' homework.

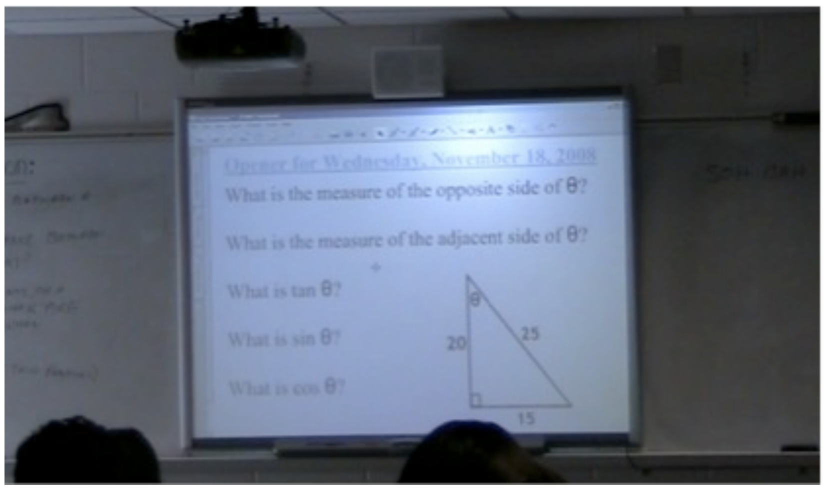

Figure 4.6: Opening questions.

After reviewing the opening questions, similar problems were given one by one. Although Alvin asked the students questions such as "What do we know? And what do we need?", he solved all the problems himself. Alvin covered 7 similar problems in the class. Figure 4.7 shows one of them.

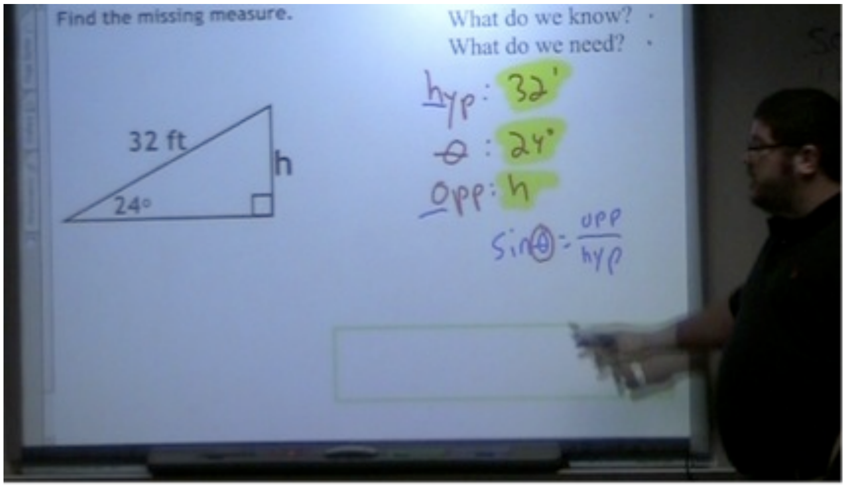

Figure 4.7: Alvin's demonstration.

Alvin used the whole class time to work on finding the missing measures of right triangles. Specifically, the class consisted of 10 minutes for checking students' homework, 7 minutes for reviewing the opening questions, and 30 minutes for Alvin's demonstration. Shortly after Alvin's lecture, students left the classroom for lunch.

Ways in Which Technology Was Used

Alvin used the SMART Board system in a meaningful way in terms of delivering his lecture. He used the functions of hiding, showing and highlighting by touching the screen that the SMART Board system supports. Colors and precise figures on the screen enhanced his lecture. However, he did not yet use the available technological tools to investigate mathematical concepts.

In terms of Alvin's past learning experience, there was a clear gap between his learning and his teaching, in that he was focusing much more on procedural teaching and learning. In a course assignment, he had a chance to write his reflection on using dynagraphs (dynamic function graphs on GSP -- see figure 4.5). After playing with dynagraphs to examine the properties of functions, he wrote:

> Introducing something new [dynagraphs] could often act to peek [sic] one's interest and engage their attention. In doing so, general concepts and ideas can be enhanced by the use of these types of learning tools. […] Dynagraphs provide a colorful, clear representation of what happens to a value after it is taken through multiple functions. Dynagraphs are an excellent tool that I can use in my own classroom. They are fun to build and engage students in the learning process. Requiring direct interaction between students and their learning tools, what results is a more invested effort into understanding the topic at hand. Such ideas as functions and their compositions can be more clearly enhanced. In the end, a more diverse classroom is achieved and the learning curve improves.

This reflection shows one example in which his learning focused on exploring mathematical concepts by using technology and he valued such ways of using technology as teaching and learning tools in his future classroom.

Observation days 8 and 9 (12/2 and 12/4)

Mathematical Topic

The classes during this whole week were dedicated to preparing for the end of course test (EOCT). The Geometry EOCT provided by the Georgia DOE consists of six domains, as follows:

- Domain I: logic and reasoning
- Domain II: points, lines, planes, and angles
- Domain III: congruence and similarity
- Domain IV: polygons and circles
- Domain V: perimeter, area, and volume
- Domain VI: coordinate, transformational, and three-dimensional geometry

The topic for day 8 was points, lines, planes, and angles in Domain II; for day 9 it was congruence and similarity in Domain III.

Tools and Materials

The SMART Board system for Alvin's monitoring and laptops for individual students were used for both days.

Activity

Alvin started the classes by reviewing contents in each domain of Geometry (See figure 4.8).

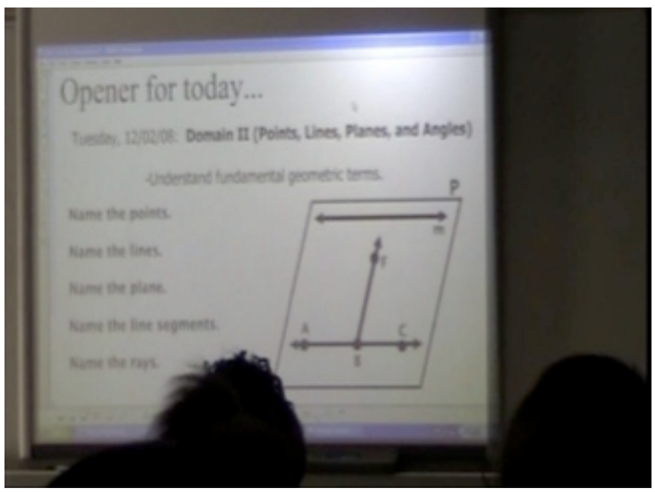

Figure 4.8: An example of how Alvin started the EOCT preparation.

After reviewing theorems and formulas, Alvin distributed laptops to students and they logged on to the school website to visit the USATestprep website, which the school paid to use. USATestprep, Inc.[11] provides practice questions and content descriptions determined by the Georgia Performance Standards. Students visited the website and found the corresponding domain to practice. Alvin monitored students' work and walked around to help students individually. This occupied the entire class time.

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[11] For more information, visit the website, www.usatestprep.com.

Ways in Which Technology Was Used

Alvin used the Schoolview technology to monitor students' work. The software was installed in all of the school's computers and allowed teachers to view all students' screens at once. Without it, a teacher's view was limited to line-of-sight. The Schoolview technology provided Alvin with a kind of microworld of the classroom in a panoramic view. He used it like he did on day 5. The following figure shows how he could zoom in on individual students' work and interact with students inside the microworld. The system supports the functions of sending messages, chatting, and making announcements by both voice and written messages. Figure 4.9 shows these functions.

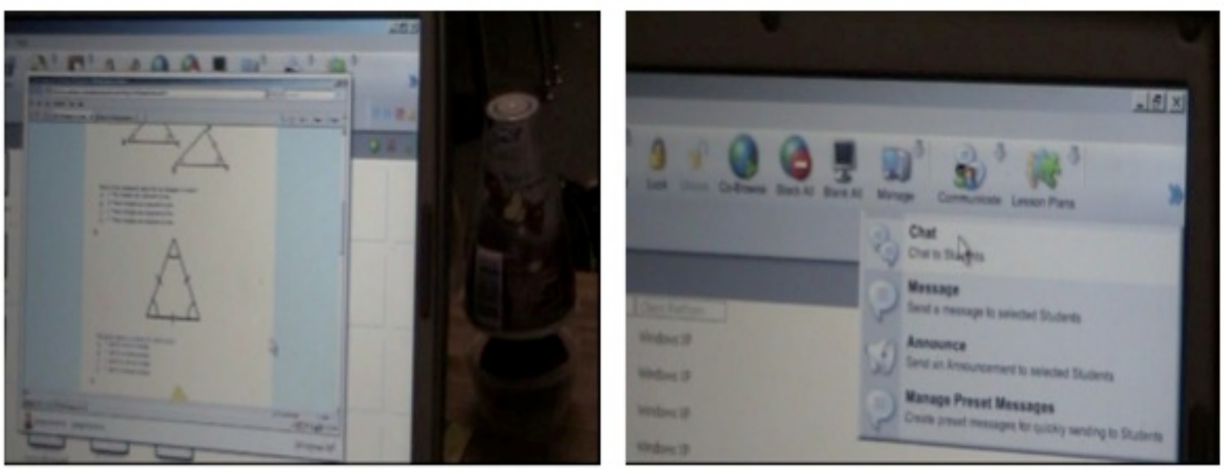

Figure 4.9: Alvin's use of the Schoolview technology.

Alvin showed a pedagogically meaningful way of using the Schoolview technology. Yet he was not observed to use technology to explore mathematical concepts.

A consistent pattern that appears to emerge from these observations is Alvin's use of technology to enhance the pedagogical aspects of his teaching. However, I did not observe him using technology for students' mathematical exploration (i.e. to enhance the learning aspects of his teaching). The overall themes or patterns resulting from my analysis of these observations of Alvin's classes will be presented in detail in the next chapter.

Theodore

Theodore was teaching Math I and Algebra I during the semester when the data were collected. I observed his Algebra I class, since Math I was being taught for the first time, and therefore he was busy learning it himself and preparing to teach it. Like Alvin, Theodore used the OnCourse software for lesson planning. He said that he preferred a student-centered class, and indeed he usually provided students with a large amount of time for their individual practice. Compared to Alvin, his demonstrations were brief. His typical way of opening class was checking students' homework and reviewing the homework problems.

<u>Observation day 1 (9/11)</u>

Mathematical Topic

The topic for the lesson was the laws of exponents. Specifically, the class focused on applying exponent properties involving products with the two laws: $a^n a^m = a^{n+m}$ *and* $\left(a^n\right)^m = a^{nm}$.

Tools and Materials

Theodore used the SMART Board system, and students used laptops and graphing calculators.

Activity

Theodore started the class by checking students' homework. He checked whether students completed the homework, but not how they did it. A review of the homework problems followed. He wrote some of the homework problems on the SMART Board and demonstrated how to solve them step-by-step. Then, much time was given to students for their individual practice on problems similar to those that were reviewed. Each student visited the textbook publisher's website[12] and found the e-workbook in chapter 8, Exponents and Exponential Functions in Algebra I. While they practiced individually, Theodore walked around the classroom and helped students who were struggling or had questions.

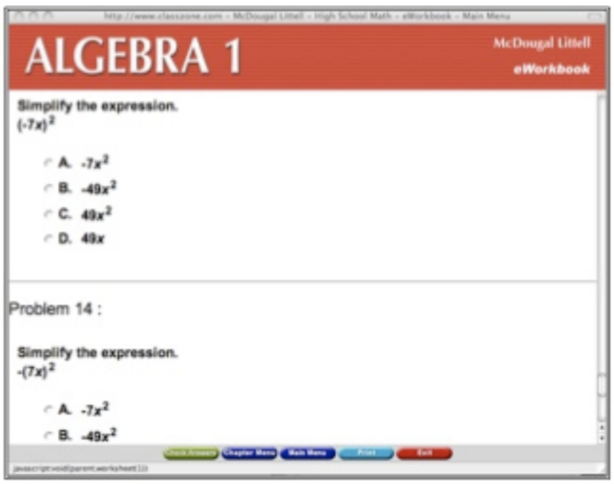

Figure 4.10: The e-workbook that Theodore's students were using.

Figure 4.10 shows where many of the students were struggling. Their answers were incorrect for the problems with negative signs inside or outside of parentheses. For instance, many students thought that the coefficients $-(3a^2)^5$ *and* $(-3a^5)^2$ were both negative. At the end of the lesson, the teacher assigned another set of homework problems on the same topic.

Ways in Which Technology Was Used

Theodore used the SMART Board just like a dry-erase board, and did not use any of its functions other than writing and erasing. Thus, there was no difference between using a dry-erase board and using a SMART Board for him. This revealed that the SMART Board technology was

[12] www.mcdougallittel.com. This website has now moved to www.holtmcdougal.com.

not an instrument for him[13]. Computers and the Internet were instruments for his students, however. It seemed that his students were familiar with working with computers and using online resources. It took little time for them to access the Internet and find the e-workbook, and none of them needed to ask how to enter their answers and see the results. However, by incorporating technologies into his teaching, Theodore took a different approach in terms of students' working mode for practice and types of classroom resources. His primary concern in using technology was engaging students in the class and imparting to them a sense of responsibility for their learning. About the benefits of technology use in teaching and learning, he said, "I've – just teaching for these few years technology has really helped out a lot with the students that have struggled with math in the past and maybe don't like it as much as some other students that normally would just go to sleep or just do something else. They're actually starting to engage in class a little bit more" (Interview, 1-23, 2009).

<u>Observation day 2 (9/17)</u>

Mathematical Topic

The topic for the lesson was properties of radicals.

Tools and Materials

The SMART Board system, online resources, and laptops for students' practice were used.

Activity

At the beginning of the class, Theodore introduced the following properties of the radicals: $\sqrt{a}\sqrt{b} = \sqrt{ab}$ *and* $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$. He asked students to use calculators and substitute *a* and

[13] As I discussed earlier in chapter two, instrumentation theory suggests that instruments are distinguished from artifacts. Artifacts become instruments only when they are used in meaningful ways.

b with any positive numbers to see whether or not the two equations are always true. Students used graphing calculators, inputting different values for *a* and *b,* and none of the students reported a case in which the equations were not satisfied. The class concluded that the equations are always true for any positive numbers *a* and *b*, based on their finite number of tests of these properties using their calculators.

This lesson might encourage the students to develop misconceptions about mathematical proofs, as well as leading them to give too much authority to calculators, resulting in high dependency on the technology. About two weeks before, the class had covered the topic of properties of exponents and the students had already learned the meaning of square roots (see Observation day 1 for the details). The students' learning in the previous lessons provides a sort of foundation on which they could build new concepts of properties of radicals. In several of his mathematics courses in his teacher education program, Theodore was exposed to developing proofs from previously learned concepts. For example, he could have demonstrated a proof of the above properties of radicals by squaring both sides separately and using the laws of exponents, assuming the associative and commutative properties of multiplication hold for radicals as well as for rational numbers. The left side of the equation ends up as $\left(\sqrt{a}\sqrt{b}\right)^2 = \left(\sqrt{a}\right)^2\left(\sqrt{b}\right)^2 = ab$ and the right side as $\left(\sqrt{a}\sqrt{b}\right)^2 = ab$. This approach would allow students to prove the property mathematically and give the authority to their own mathematical ability in doing mathematics using their logical thinking rather than depending on calculators.

After their "proving" activity (from Theodore's perspective) students had time to practice simplifying radicals. Theodore assigned homework problems in the textbook at the end of the class. Overall, his teaching focused on students' gaining their procedural fluency in dealing with radical expressions and solving related problems. He was leading the class in a traditional lecturing fashion.

In contrast with what he showed in his teaching, he, as a pre-service teacher, took different perspectives on teaching and learning mathematics. For instance, Theodore wrote the following in his reflection on how constructivist teaching and learning environments foster productive learning, which was given as a course assignment:

> [In traditional classrooms,] the students are told what to think, and are misled into thinking that there is only one way to solve many problems. By adopting these new and progressive teaching styles [constructivist styles], students will see that mathematics is not a discipline with strict rules and procedures that one must follow, but a discipline that gives the student freedom to explore and construct their own ideas.

Ways in Which Technology Was Used

To Theodore, the SMART Board system was no different from a dry-erase board; he only used it to write and erase. The SMART Board system was not an instrument for him yet. However, he did provide students with new materials, in the form of online resources and laptops, for their practice. In this regard, I considered that his teaching moved away from traditional didactics, while he lectured the lesson.

<u>Observation day 3 (10/1)</u>

Mathematical Topic

The topic for the lesson was multiplying monomials with polynomials and binomials with binomials.

Tools and Materials

The SMART Board system, online resources, and laptops for students' practice were the main materials for the lesson.

Activity

Students' individual practice was the class activity during the whole class period. Theodore monitored students' work on his computer screen. The following figure shows a part of the e-workbook that students practiced.

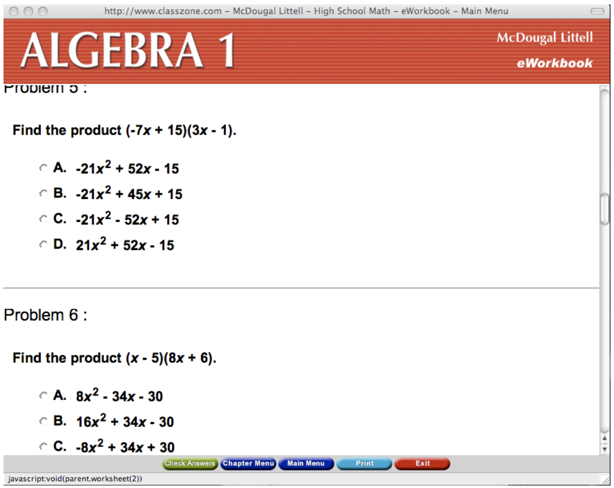

Figure 4.11: E-workbook for multiplying polynomials.

Theodore could find students who needed his help by observing students' work, and communicate with them either through online messages or by talking to them directly. He could select those problems with which many of the students struggled.

Ways in Which Technology Was Used

To observe students' computer screens, Theodore used SMART Sync classroom management software. The software let him view the screens on his computer as full sized or thumbnail images. According to him, the software allowed him to assess students' learning and give them his support directly. It also enabled students who were shy to ask for help by messaging him privately. The software provided more active interactions between him and his students. His use of technology focused on pedagogical purposes: assessing, interacting with students, saving time in preparing handouts, and so forth. He was therefore engaged in using the technology in a pedagogically meaningful way, and the software became an instrument for him.

Observation day 4 (10/8)

Mathematical Topic

The topics for the lesson were the products of two binomials and factoring quadratic equations.

Tools and Materials

The tools and materials were Algebra Tiles[14] for hands-on activities involving expanding and factoring quadratic equations, the SMART Board system, online resources, and laptops for students' practice.

Activity

Theodore demonstrated how to model simple quadratic equations with Algebra Tiles.

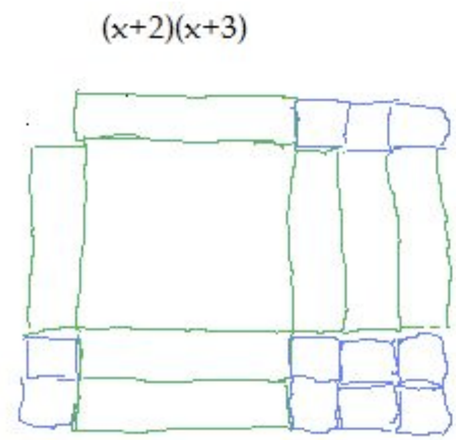

Figure 4.12: Theodore's illustration on the SMART Board of how to model the product (x+2)(x+3) with Algebra Tiles.

[14] Algebra Tiles are plastic tiles in rectangular and square shapes of different colors. Small squares are used to represent known units, while the non-square rectangles are used to represent the unknown value, *x,* and the large square is used to represent x^2. They are designed specifically for modeling the product of two binomials.

Theodore drew rectangles or squares as in figure 4.12 on the SMART Board. Non-square rectangles represented x; small squares represented 1; a big square represented x^2. After drawing these by hand, he counted the number of big squares, the number of rectangles, and the number of small squares to determine the coefficients of the three terms of the equation. However, his drawing was unclear in terms of counting the different shapes to determine the coefficients of the binomial expansion. In the figure that he drew, the top row and the first column stood for the two given binomials that were multiplied and should not be counted. Many students included the top row and the first column in their determination of the coefficients when solving problems in the workbook. When Theodore recognized that his students were producing wrong answers by miscounting, he responded simply by telling them that they should not count those tiles since they were parts of the givens in the problems. After students had practiced these activities, Theodore introduced factoring quadratic equations as the reverse process of expanding the products of two binomials. He also demonstrated how to use Algebra Tiles to factor an equation. The following figure shows an idealized version of his hand-drawn demonstration.

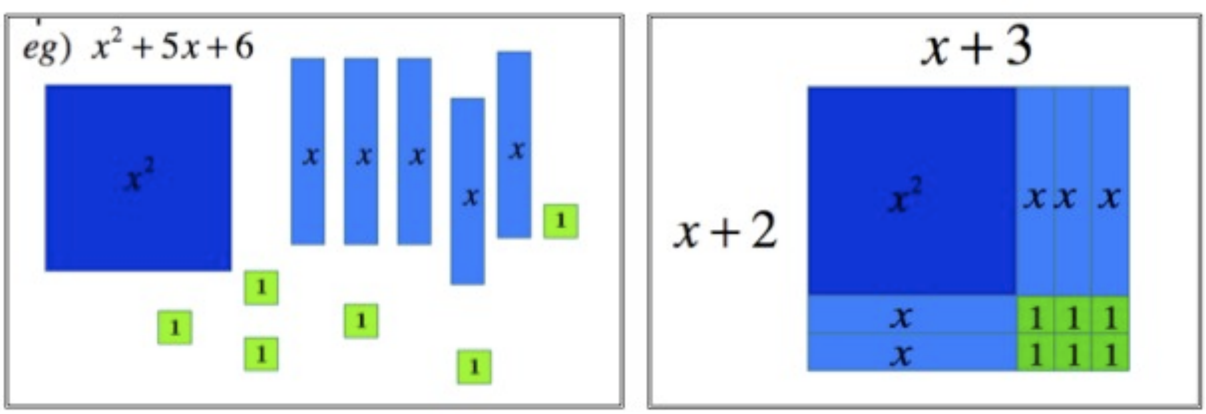

Figure 4.13: Factoring demonstration with Algebra Tiles.

After the factoring demonstration, students worked individually with the physical Algebra Tiles to solve factoring problems in the e-workbook. The lesson covered quadratic equations with positive coefficients. Factoring quadratic equations with signed coefficients was left for the next class. Theodore finished the lesson by assigning homework problems.

Ways in Which Technology Was Used

Algebra tiles worked well in terms of letting students engage in the learning activity and helping them understand the concepts of expanding and factoring quadratic equations. However, Theodore did not use the SMART Board for any purpose other than writing and erasing. He could have used the SMART Board's drawing tool for accurate and colorful representations of figures. At the end of the class, he explained this lack of use of the SMART Board functionality by saying that drawing by hand was easier and quicker for him. The SMART Board system was not yet an instrument for him, since it was used as a simple replacement of a dry erase board. On the other hand, students' working mode was not a passive and traditional style. In this regard, the didactic situation was different from a traditional lecture.

<u>Observation day 5 (10/22)</u>

Mathematical Topic

Mathematical topics covered quadratic equations and functions from chapter 10 of quadratic equations and functions.

Tools and Materials

Theodore used the SMART Board technology, including the Senteo interactive response system, to monitor students' work and collect their answers. Students used laptops, Senteos, and graphing calculators.

Activity

Students practiced to prepare for the chapter test they would have in the next class. Theodore explained how to set up the Senteo device so that he could collect students' answers for the problems. He prepared the test problems as a Microsoft word file and imported it into the SMART Board system. In the last 10 minutes of the class, he exported the test results to the SMART Board system and opened it on the screen. He and his students could see the mean score and review the problems, especially those which many of them had answered incorrectly.

Ways in Which Technology Was Used

The SMART technology was used in a pedagogically meaningful way, although it was not the way that affects students' mathematical understanding. In particular, it helped the teacher assess students' learning, and therefore modify and refine his teaching of mathematical concepts before being tested. The Senteo interactive response system of the SMART technology changed Theodore's working mode of assessing students' learning, and enabled him to use the statistics of the students' test results to focus on students' problem areas. He evaluated the technology, saying that it allowed him to save time preparing test papers, grading tests, and analyzing the test results. In this lesson, he was using the SMART technology in a pedagogically productive way. I considered this technology to be user-friendly, as it can easily become a pedagogical instrument for many teachers.

<u>Observation day 6 (10/29)</u>

Mathematical Topic

Percent and proportion was the lesson topic.

Tools and Materials

Theodore used the SMART Board system, and graphing calculators were given to students.

Activity

The class started by reviewing homework problems from the previous lesson that covered the topic of proportion. Percentages, therefore, were the main topic for this lesson. Theodore demonstrated solving word problems that involved percentages. Students' individual practice followed his demonstration. At the end of the lesson, students took a five-minute quiz that helped assess their learning about percent and proportion concepts.

Ways in Which Technology Was Used

Theodore used the SMART Board system as a simple replacement of the dry-erase board. He gave a traditional lecture for the lesson. Graded test papers, the textbook, and graphing calculators were given to students. No generative use of technology was observed in this lesson.

<u>Observation day 7 (11/5)</u>

Mathematical Topic

Application of the concept of percentages was the topic for the lesson.

Tools and Materials

The materials for this lesson included the SMART Board system, online resources, and graphing calculators and laptops for students.

Activity

Theodore planned to relate the concept of percentages that his students were learning to real life. He used the topic of the 56th United States presidential election, held on November 4, 2008, to the class regarding the percent problem. He introduced the history of the 1948 presidential election poll, stressing the importance of sampling methods to predict election results accurately. Specifically, he used the following two tables and two problems.

1948 Presidential Election Poll Prediction				
	Candidates			
Pollster	Dewey	Truman	Thurmond	Wallace
Crossley	49.9%	44.8%	1.6%	3.3%
Gallup	49.5%	44.5%	2.0%	4.0%
Roper	52.2%	37.1%	5.2%	4.3%

Percentage of Votes Counted in 1948 Election			
Candidates			
Dewey	Truman	Thurmond	Wallace
45.0%	49.4%	2.4%	2.4%

1. According to the poll predictions, who would win the presidential election in 1948? Did t he poll accurately predict who won the election?
2. The total number of votes counted in the 1948 presidential election was 48,836,579. Use t he percentages in Table 2 to determine how many votes each candidate received.

Figure 4.14: The handout given to students for this lesson

After the lecture, Theodore asked students to learn about the ongoing presidential election surveys regarding gender, race, education, and so forth on the Internet. Students tried to represent the survey results with the percent concept (See figure 4.15).

Figure 4.15: Learning about the 2008 presidential election survey through the Internet.

Ways in Which Technology Was Used

In this lesson, students' use of the Internet brought them a learning opportunity using current events. They read about the presidential elections through the Internet. This learning included social study, the history of American elections, and mathematics. The teacher regarded this as an interdisciplinary learning opportunity. Within mathematics, they had a chance to discuss not only the concepts of percentages and fractions, but also statistical concepts, including sampling methods. Although the students did not have in-depth discussions about this information, Theodore achieved a community in the classroom where each person had an integral part in the exchange of knowledge. There was no room for power and arrogance. In this way, he promoted students' self-confidence. This revealed an aspect of the innovation that modern technologies can bring forth in education. On the other hand, Theodore's use of the SMART Board system did not make any difference in using technology to explore mathematical concepts per se.

<u>Observation day 8 (11/12)</u>

Mathematical Topic

The topic for the lesson was proportional equations.

Tools and Materials

The tools and materials used in this lesson included individual dry-erase boards called lapboards, markers, and the SMART Board.

Activity

After reviewing homework problems, students worked in pairs to solve proportional equations in their textbooks. Students used lapboards and markers while discussing each step in solving the problems. Many of them also used graphing calculators for arithmetic calculations.

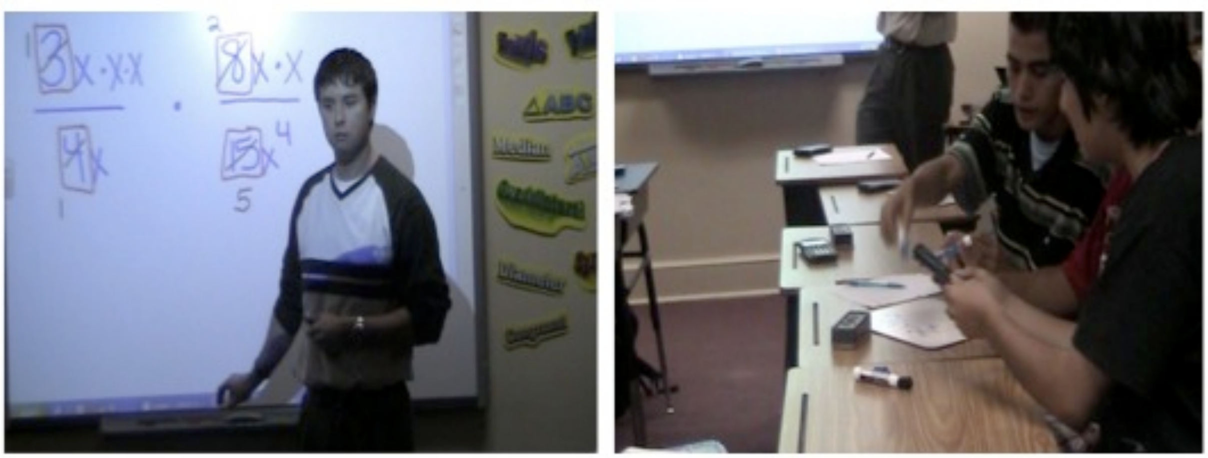

Figure 4.16: Reviewing homework problems and students' pair work.

Theodore walked around the classroom to observe students working. In general, students' practice occupied a lot of time in his class. At the end of the class, he announced that students would have a test next time on what they had practiced.

Ways in Which Technology Was Used

This lesson did not include a significant use of technology. In Theodore's demonstrations, he simply wrote and erased procedures on the SMART Board, which could just as easily have been done on the dry-erase board. Students' use of individual lapboards helped them share their ideas, but this could also have been done with pencil and paper.

<u>Observation day 9 (11/19)</u>

Mathematical Topic

The topic for the lesson was radical expressions and proportional equations.

Tools and Materials

The class used the SMART Board system, online resources, and laptops for students' practice.

Activity

The lesson proceeded in the same way as in the previous observation. Theodore reviewed some of the homework problems, with students' practice following. In this lesson, the class used e-workbook for their practice. The following figure shows the website that the class often visited to use e-workbook.

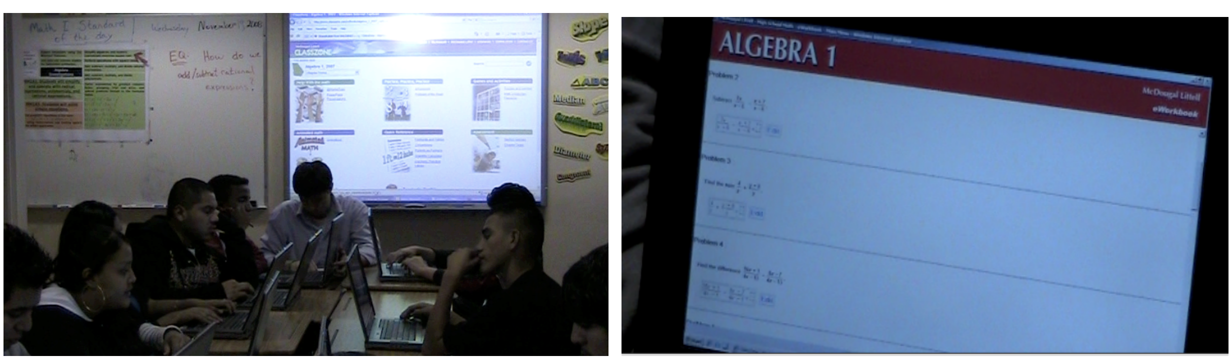

Figure 4.17: E-workbook on the Classzone.com website.

At the end of the class, students turned in the paper on which they showed their work with the e-workbook.

Ways in Which Technology Was Used

Using the e-workbook from classzone.com was an important part of Theodore's pedagogical repertoire. In many lessons, he used the e-workbook together with the textbook. He used the e-workbook so frequently because he often needed more problems due to the large amount of practice time he allowed his students in class. The e-workbook allowed him to save his time in preparing the practice problems. He believed that doing by learning was a better way than listening to teachers, and this also provided the students with opportunities to work with their peers (personal conversation, 11-19-2008).

<u>Observation day 10 and 11 (12/3 and 12/4)</u>

Mathematical Topic

The topic for the lesson was a review of the entire semester. To prepare for the end of the course test (EOCT), Theodore decided to review what the class covered during the semester for five days. He believed one of the best ways of reviewing would be actually taking the EOCT from several years ago.

Tools and Materials

Theodore used the SMART Board technology, including the Senteo interactive response system, to monitor students' work and collect their answers for analysis and review. Students used laptops, Senteos, and graphing calculators. Of a total of 85 questions for the actual test, the class worked on the first 45 questions on Dec. 3 and 40 questions on Dec. 4. The following figure shows the Georgia Department of Education website (www.doe.k12.ga.us) where the test questions were loaded, along with the released test questions that students used for practice.

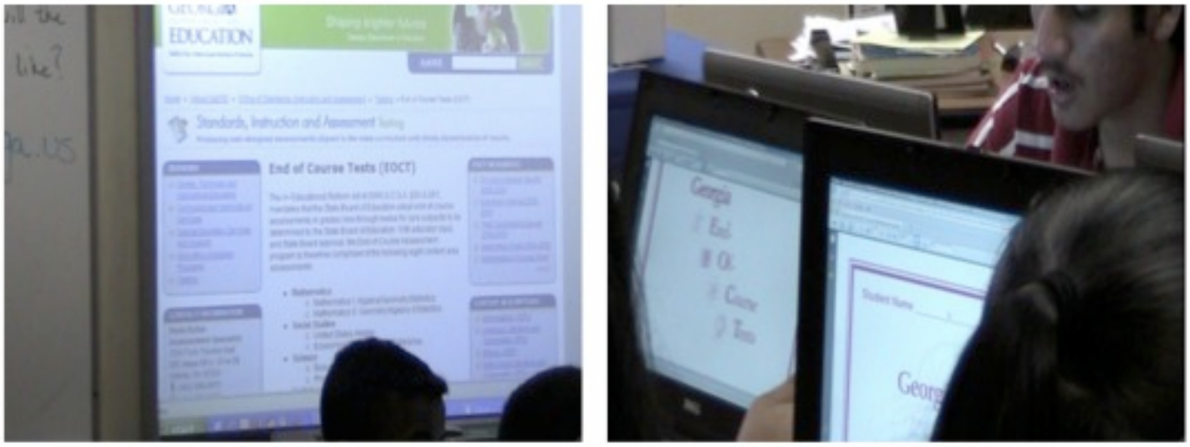

Figure 4.18: The Georgia Department of Education website and the released test questions on it

Activity

The class took a test of released EOCT test questions. As figure 4.18 shows, first, Theodore asked students to visit the Georgia Department of Education website to find the released test questions. Then, Theodore explained how the students should set up their Senteos to allow him to monitor the test and collect the results.

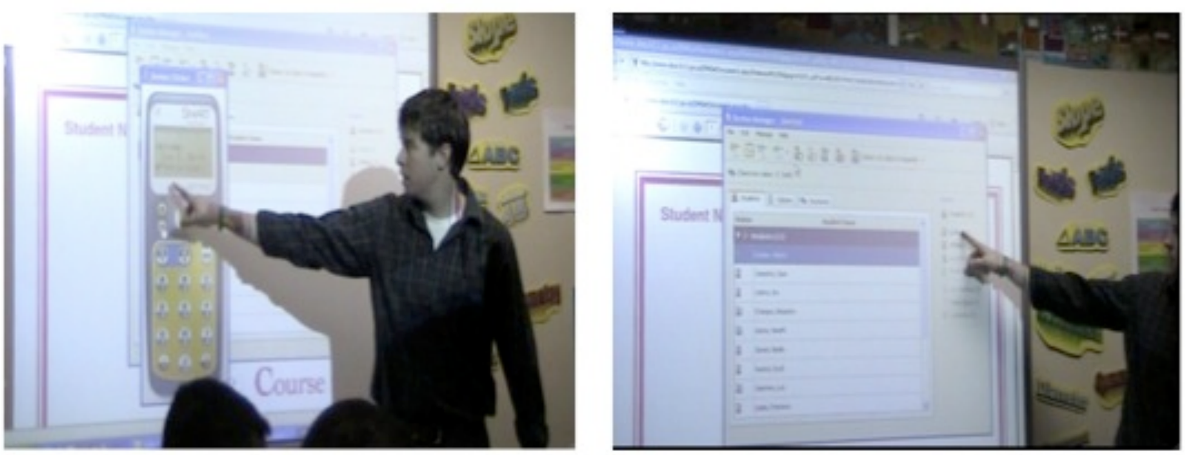

Figure 4.19: Theodore explaining how to set up Senteos and checking setup status.

Figure 4.19 shows the class setting up the Senteo interactive response system in the classroom. Students were supposed to find the correct class and teacher name to enter the system. Figure 4.20 shows a student' entering her answer.

Figure 4.20: A student entering answers into her Senteo hand held device.

Theodore also explained how to enter answers into the Senteos and asked students to turn in papers that would show their work at the end of the class. Graphing calculators were handed out, but students used them only for arithmetic calculations. While students worked, Theodore monitored their progress. He was able to keep track of the number of students who answered each question and the elapsed time, as shown in Figure 4.21. When the assigned time was up, he opened the test results on the SMART Board screen for the students. The following figures show his monitoring, the result of the test, and one of problems that many of students answered incorrectly. Figure 4.22 shows Theodore indicating one problem that only one student answered correctly. Each question was represented as a bar, on which incorrect and correct answers are shown in red and green, respectively.

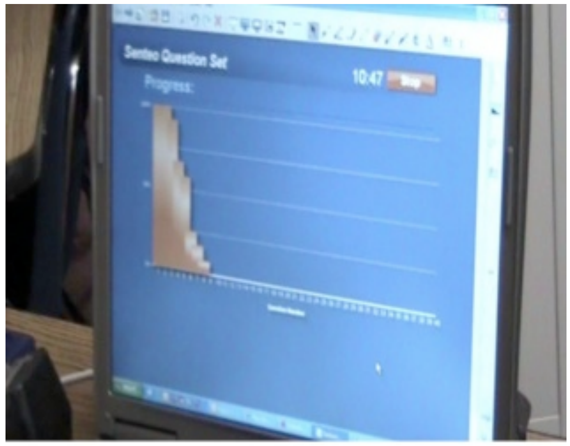

Figure 4.21: Theodore's monitoring of the progress of the test.

Figure 4.22: Test results.

In Figure 4.22, the horizontal axis represents question numbers and vertical shows the response rate as a percentage. As the figure shows, many of students did not complete the test in the time allotted. The white finger pointer in the figure indicates that, among the problems that all of the students answered, problem 13 had the lowest percentage of correct answers. Theodore pulled out the problem on the screen as in figure 4.23.

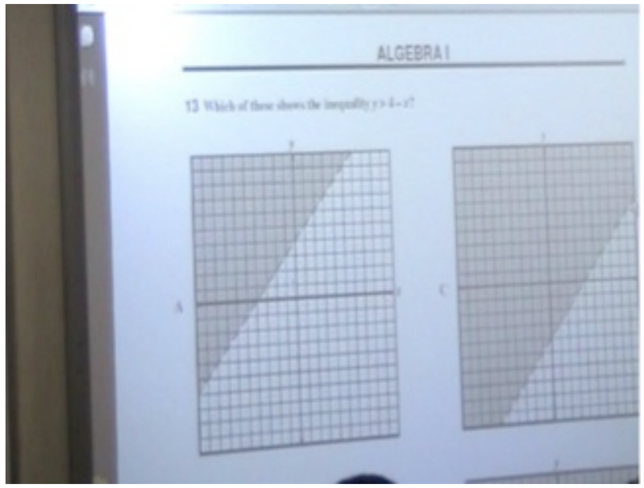

Figure 4.23: The question with the lowest percentage of correct answers.

Over two days, the class completed both section 1 and section 2 of the test. The class decided what would be reviewed during the next week. Theodore announced the class would go over the test problems for three days. Students were assigned individual review as homework.

Ways in Which Technology Was Used

Theodore used the SMART Board system productively and efficiently in terms of assessing students' learning. The Senteo response system allowed him to export the test results into an Excel spreadsheet. In the Excel file, he could see which questions had lower rates of correct answers and what answers were their choices for those questions. This told him immediately what he needed to go back over. Without such help, teachers need much more time to assess students' understanding based on tests. In his case, what Theodore needed was to sort out problems that had lower rates of correct answers, so that he could then see what their misconceptions were. Theodore also realized that students needed more time to complete the test. He set up the test time as a 40-minute period. He told the class that he would emphasize time management in the next test for section 2. Figure 4.24 and Figure 4.25 show Theodore exporting the test results and the exported test results in an Excel file.

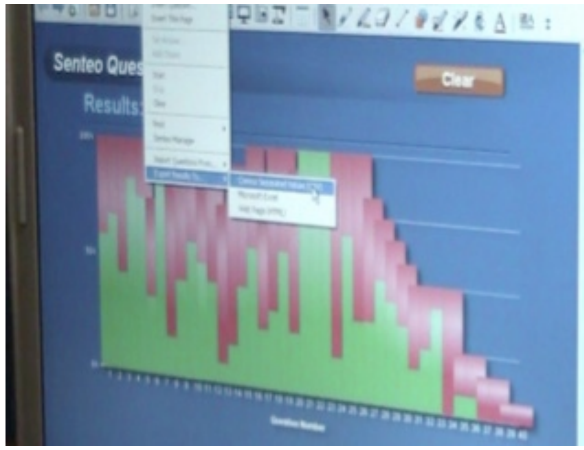

Figure 4.24: Theodore exporting the test result into an Excel file.

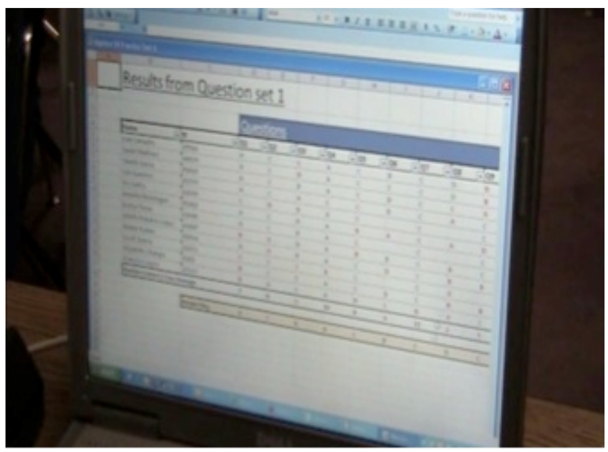

Figure 4.25: The test results for the first section.

Theodore used the SMART technology in ways that generated pedagogically significant meaning. Although his use of technology was not directly related to exploring mathematical concepts, his way of using the SMART technology was pedagogically meaningful in that his way of assessing students' learning took a major step forward.

A consistent pattern that appears to emerge from these observations is Theodore's use of technology to enhance the pedagogical aspects of his teaching. However, like Alvin's case, I did not observe him using technology for students' mathematical exploration (i.e. to enhance the learning aspects of his teaching). The overall themes or patterns resulting from my analysis of these observations of Theodore's classes will be presented in detail in the next chapter together with those of other teachers' classes.

Simon

Simon was teaching Algebra I, Algebra II, and AP Calculus during the fall semester of 2008. He was in his third year of teaching in the school and had previously taught Algebra II, Pre-Calculus, and AP Calculus. Algebra I was an honors class, and therefore was new to him. The class schedule at his school was not week-based, so I was able to observe all three classes that he was teaching. Simon used a personal website linked to the school website to communicate with students, who visited the website often to check on homework assignments and the course schedule, including tests. Simon did not write his lesson plans down, but he did spend a lot of time on lesson planning -- 15 hours per week on average. By contrast, Alvin and Theodore spent 5 and 7.5 hours respectively (answers to question 4 on the survey questionnaire). According to Simon, most of his lesson planning time was spent on testing software that was to be used in the class, proving theorems and mathematical properties ahead of time, and solving example problems for demonstration. In response to the question about his primary concerns regarding lessons planning, he said:

> When I do lesson planning, I mainly consider the problems that I want students to be able to solve at the end of the lesson. Then I work backwards. I try to determine what techniques, ideas, etc. are necessary for students to understand in order to solve the selected problems. Then I decide the best manner to present those concepts in class; hopefully they can be introduced with a demonstration, example, or connection to previous work. Demonstrations on the computer or calculator seem to work best. The students love learning "calculator tricks." I like to break up the class period into sections to keep the students from getting bored. Usually a lesson is broken into homework discussion, demonstration and presentation of new material, and individual or group work.

Like Alvin and Theodore, he generally started class by checking and reviewing homework problems.

Observation day 1 (8/29)

Mathematical Topic

His Algebra I honors class covered the topic of addition and subtraction of signed numbers. Specifically, the class discussed number systems, additive inverses, and absolute values of real numbers. However, Simon did not relate the additive inverse concept to the additive identity. For that, he simply said, “Zero tells us where everything goes.”

Tools and Materials

Simon set up his own document camera system in his classroom that included a document camera, a digital light processing (DLP) projector, and his laptop. He used this system every day. He also used the Geometers’ Sketchpad (GSP) software in this lesson.

Activity

Simon used a teacher-led lecturing style. He checked students’ homework problems by walking around the classroom. The assigned homework problems were about adding signed numbers. The class discussed some of them based on students’ requests, and the discussion was continued in the lesson. Simon explained the concept of additive inverse and absolute value. He defined additive inverses as numbers that add to zero and absolute values as the distance between a number and the origin on the number line. He explained that absolute values could not be negative by their very definition. Following are reviewed problems and examples used in explaining definitions of terms and associated concepts.

Reviewed problems asked by students:

1.What is the right order to solve $21+(-8)+(-7)$?

The order does not matter.

$21+\{(-8)+(-7)\}=\{21+(-8)\};\ 21+(-15)=13+(-7)$.

The associative property of addition was discussed.

Simon stressed that the final sign in adding signed numbers should always follow the sign of the larger absolute value of the two numbers. Simon confirmed by asking his students if they preferred adding signed numbers to subtracting. For instance, they converted $7-2\ \textit{to}\ 7+(-2)\ \textit{or}\ 7-(-2)\ \textit{to}\ 7+(+2)$. He said that this was something he did not know until he started teaching in school (personal conversation, 8-29-2008).

2. $-6\frac{4}{5}+\frac{3}{5}=-\frac{34}{5}+\frac{3}{5}=-\frac{31}{5}=-6\frac{1}{5}$.

Simon emphasized converting mixed numbers into improper fractions in order to add them. When I asked him at the end of class whether he had considered alternative ways to solve the problem, for instance, decomposing the mixed number into negative six and negative four-fifths, which would require students to think about the role of the negative sign in front of mixed numbers and have the sense of combining like-terms with respect to one-fifth, Simon said, "I don't know. It's just the way I have done it. You know, the way I was taught and I practiced." I considered that this might imply his lack of proficiency and flexibility with calculation strategies with fractions.

<u>Notes from his lecture:</u>

Topic 1. Number system:

Definitions according to Simon:

- Counting numbers: natural numbers used in counting items
- Whole numbers: counting numbers with zero

- Integers: whole numbers and their opposites
- Real numbers: all numbers on the number line

Simon asked his students, "What is the opposite of zero?" One of his students answered “Infinity.” He did not ask why the student thought so, or whether any other students agreed; he simply ignored the unexpected answer.

However, based on his ability that he showed in his course work in his teacher education program, I believed that he was capable of explaining what he meant by “opposite” regarding the additive inverse concept. For example, in his exploration of a composition of two functions, he had the rational function, $\frac{1}{x^3}$ that has the value infinity when x is at zero.

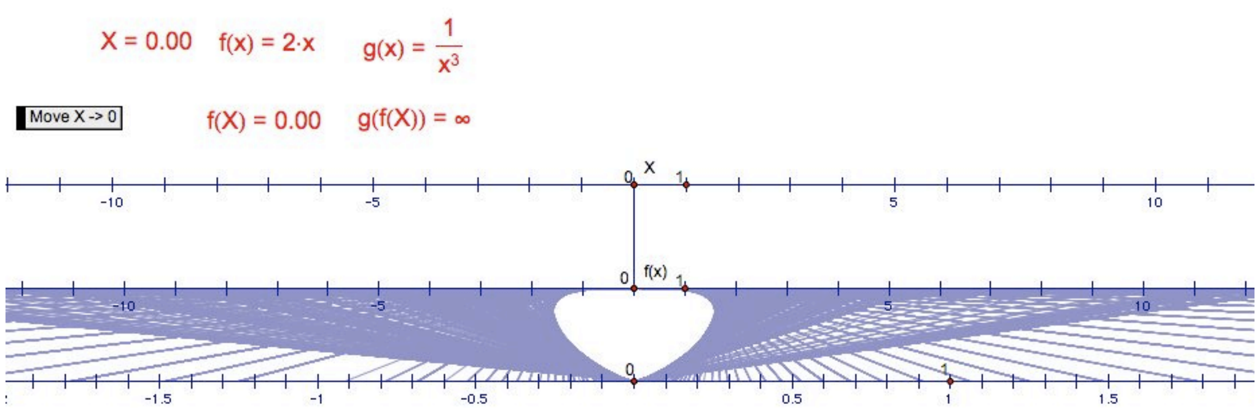

Figure 4.26: Simon’s exploration of a composition of two functions in his course work

Although his exploration of the composition of two functions was not directly connected to the concept of the multiplicative inverse of x, he explored how zero and infinity were related to the reciprocal relationship between x and y.

In his review of number systems, Simon did not mention rational numbers before moving to real numbers from integers. When I asked him why, he said that discussing fractions was not necessary in this lesson, since the focus of this lesson was on signed numbers and a discussion of fractions would have taken too much time away from that topic. He also indicated that fractions were taught and learned in earlier grades in elementary or in middle school. Regardless of whether students established strong enough knowledge background with fractions, according to him, it was not necessary to teach the topic of fractions in high school. This attitude towards fractions in high school would imply that students are unlikely to have the chance to re-visit the learning of fractions after graduating middle school.

Topic 2. What is the additive inverse of 7? That is, $7+(\ )=0$.

He explained how to find additive inverses strictly following the definition. However, he did not discuss what role zero plays in adding signed numbers. The role of zero as the additive identity is important for extending students' number sense from counting numbers, to rational, to real numbers. In fact, he was exposed to this idea in courses in his undergraduate program. For example, as one of mathematics content courses, he took the Advanced Abstract Algebra course in which groups and fields are important objects of study. He could have learned that fields provide a useful generalization of many number systems, and the identity properties for the given two operations is important for a set with those operations to be considered a field. Another example of the importance of the identity property is found in his EMAT 3500 course. The following figure shows what he played with to explore both additive and multiplicative identities in the course. Called "mystery machine", this dynamic number line has three points: A, B, and A+B. In figure 4.27, the figure shows that B must be positive since A+B is right side of A and the difference between A and A+B tells the absolute value of B. In order to find the position of zero, one needs to find certain arrangements of the three points by dragging the points.

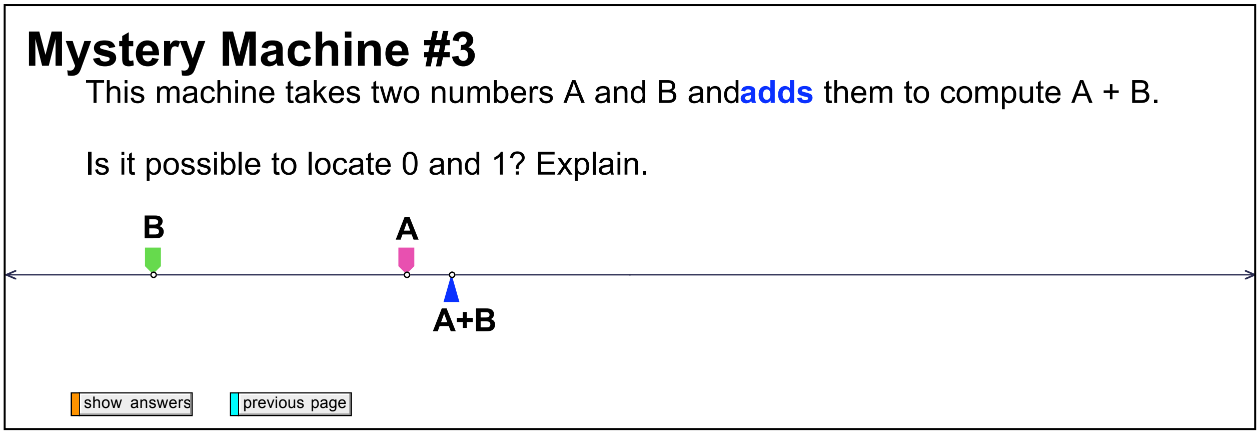

Figure 4.27: Simon's playing with Mystery machine

By dragging these points, he could find the position of zero but not one. For this activity, he wrote the following in his reflection:

> To find zero, consider what it means to add zero to a number. Any number plus zero is your original number. Thus by fixing A at any point, and moving B, I can find a point such that A+B = A. When A+B = A, B must be zero.
>
> It is not possible to find one with the given data. We are given no scale on our number line, so it is impossible to determine any more points on the number line. Two known points are required for a uniform scale, and we only know one.

Simon did not use the term, identity but he certainly used the identity property to find the position of zero. Besides this activity, Simon had experiences to play with dynamic number lines for more topics in numbers such as finding the position of 1/8 given the position of ½ on the number line by using two moveable points and the point representing their product. Another activity was, given three points on the number line, to find the hidden relation among them.

Topic 3. Absolute value of n: $|n| \geq 0$ since $|n| = d(n,0)$, $|4| = d(4,0) = 4$ & $|-4| = d(-4,0) = 4$.

What's inside if $|\ \ | = 17$?

In general, his teaching was abstract, symbolic, and conceptual compared to other teachers. He believed this would help his students think in depth and understand what the abstracted symbols and representations meant. Students' individual practice followed his lecturing.

Ways in Which Technology Was Used

Simon pulled out a number line on his laptop screen by using GSP and his laptop screen was projected onto the dry erase white board through the DLP projector. He also used the document camera system to project the textbook, handouts, and notes onto the white board to make sure where the students were and what were the given problems for them. In this lesson, he used tools to enhance his writing and drawing. His tool use was pedagogically meaningful, but not mathematically meaningful. He used GSP simply to draw a number line. However, he could have used GSP to help students' become aware of their misconceptions or explore the mathematical concepts, in ways similar to activities Simon had experienced in EMAT 3500. In the personal conversation, he said to me that he was teaching this course for the first time and did not have enough time to think about incorporating technology for teaching. I considered that his limited use of GSP was mainly due to this being his first time teaching the class.

<u>Observation day 2 (9/5)</u>

Mathematical Topic

On this day, I observed two classes. The topics of his Algebra II class were writing equations of lines (homework review) and correlation and best fitting lines. Finding derivatives was the topic in his calculus class.

Tools and Materials

For his Algebra II class, Simon used his document camera system and graphing calculator. He gave his students handouts that guided the lesson. For his Calculus class, he used the document camera system, graphing calculators and GSP.

Activity —Algebra II

The class started by reviewing homework problems that were about finding equations of lines. Simon uploaded the workbook on his website for students. Students could download and print it for their homework. For homework problems, students practiced finding equations of lines for several types of conditions: slope and y-intercept, slope and a point, two points, a point and a parallel line, and a point and a perpendicular line. Figure 4.28 shows the case that the given condition is two points. Simon projected the workbook onto the white board and wrote coordinates or equations on it. Although he was using the document camera system, his way of using it was similar to Alvin's.

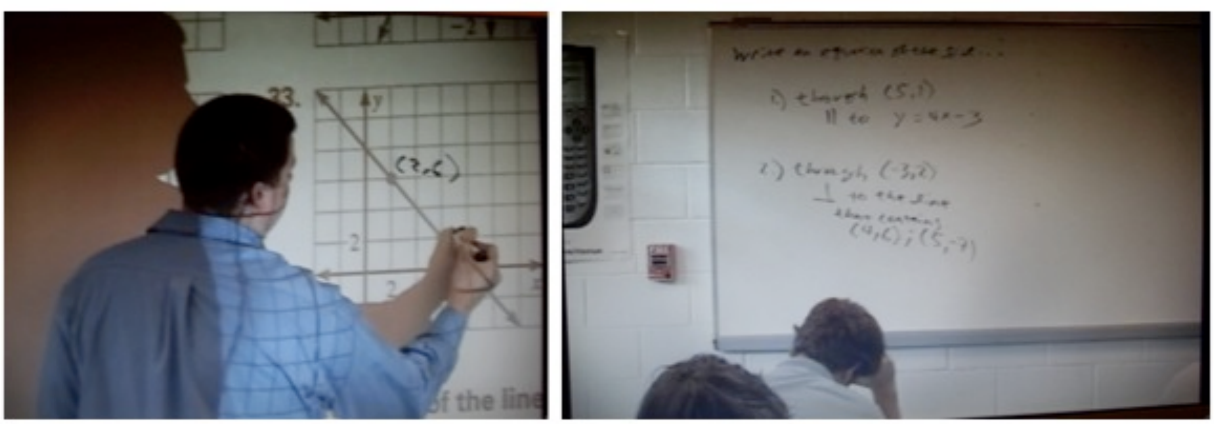

Figure 4.28: Simon reviewing homework problems and students' practice.

Half of the class time was spent on reviewing. Simon provided more practice questions, shown in figure 4.28. For the main lesson, handouts were given to students. These included lesson goals, as in the left picture of figure 4.29.

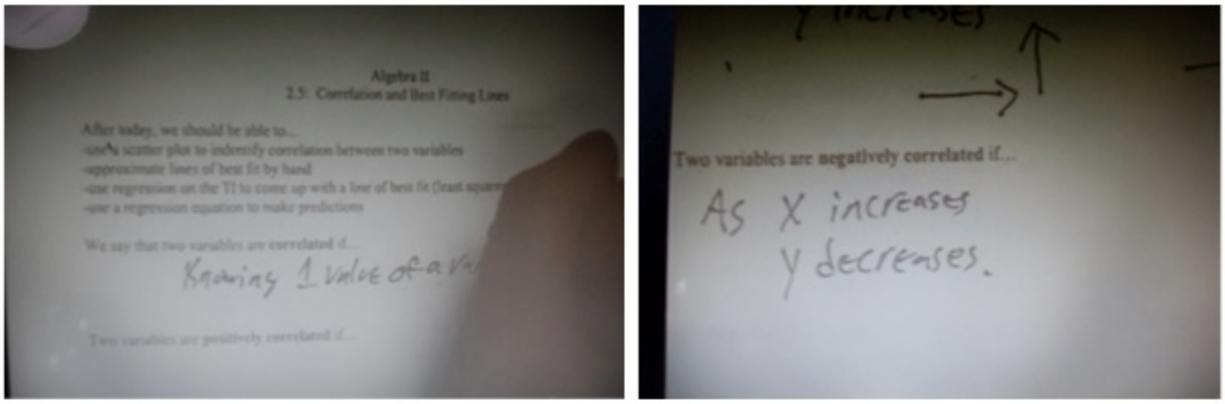

Figure 4.29: Handouts for guiding the lesson.

The lesson goals was stated as follows:

After today we should be able to…

- use a scatter plot to identify correlation between two variables.

- approximate lines of best fit by hand.

- use regression on the TI to come up with a line of best fit.

- use regression equation to make predictions.

The two pictures, above, show how the class went. Simon projected the handout and talked and wrote at the same time for each question. The questions were not multiple choice, but required the students to explain concepts. In the right picture in figure 4.29, the question was “Two variables (x and y) are negatively correlated if…” The students were supposed to complete the sentence, and Simon commented after they answered. Topics and activities were all covered in goal statements during the lesson, although Simon was not sure whether individual students met the goals. At the end, he visited his website to assign homework problems and let students know the topic for the next class.

Activity—AP Calculus

The topic was finding derivative functions by using the definition via difference quotients, that is, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Simon demonstrated how to use the definition to find the derivative function of $f(x) = x^2 - 6x + 8$. An exploration using GSP followed his algebraic solution, shown in figure 4.30.

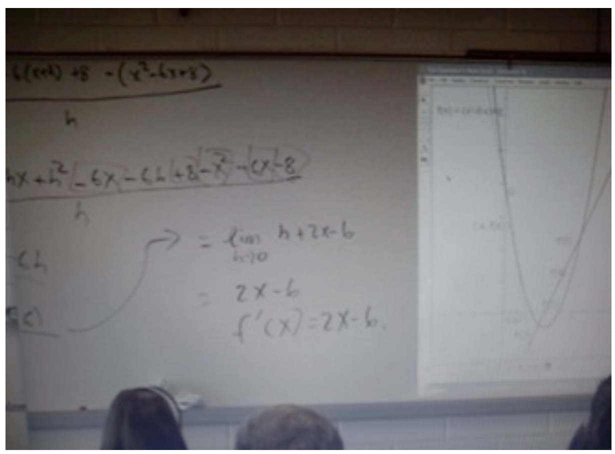

Figure 4.30: Finding derivative functions and exploring the concept of derivatives using GSP.

After demonstrating with several polynomial functions, Simon projected GSP onto the white board to explore the concept further. For instance, he plotted a quadratic function first, selected some input values of the function, and drew tangent lines at those points. GSP allowed him to measure the slopes of the tangent lines. He plotted points with pairs of inputs and the corresponding slopes on the plane to show students that the pattern of plotted points would be linear. Students asked Simon for more examples, and he showed the cases of a cubic function and a linear function. Since students could not use GSP individually, Simon also used a TI graphing calculator to explore the concept for their personal practice.

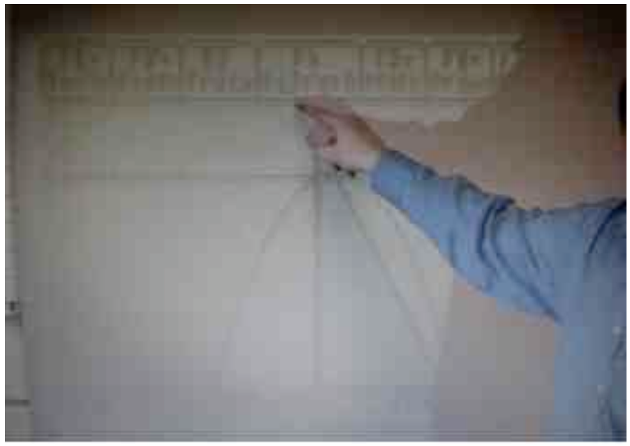

Figure 4.31: Using a TI-84 graphing calculator to explore the concept of derivatives.

Figure 4.31 shows a quadratic equation and its derivative projected from the screen of Simon's graphing calculator onto the white board. The vertex of the quadratic was a point on the x-axis (perfect square form). Simon is pointing at the derivative, asking students what the intersection point of the x-axis and the derivative function meant, and why this intersection point passed through the vertex point of the parabola. Many students correctly answered that it meant the slope [of the tangent line at the vertex point of the quadratic function] was zero when x was equal to the x-intercept of the derivative, and the vertex of the parabola is the intersection point. They also answered that the parabola changed from increasing to decreasing at the intersection point. Although students were making arithmetic errors in their algebraic procedures for finding derivatives, their understanding was conceptual and strong.

Ways in Which Technology Was Used

Simon used a teacher-led lecturing style. However, his way of using technologies engaged the class in active discussions with "what if" questions. While Simon was manipulating the GSP software or a TI-84 graphing calculator, students asked him to drag certain points or lines, change some of the given conditions, and so forth. In this way, they could use technologies indirectly through Simon. His use of the document camera system provided efficient presentations, and at the same time allowed him to explore mathematical concepts together with students. The presentation with his document camera system was inferior to that of the SMART Board system in terms of the quality of visual supports, but it also went beyond the limits of traditional chalkboard a didactic situations. For instance, in traditional teaching without a tool like a document camera, it is not possible for teachers to show how they are manipulating the graphing calculator to all students at the same time. All the students had their own graphing calculators, and they followed Simon's demonstration to find the correlation between two variables and the regression line.

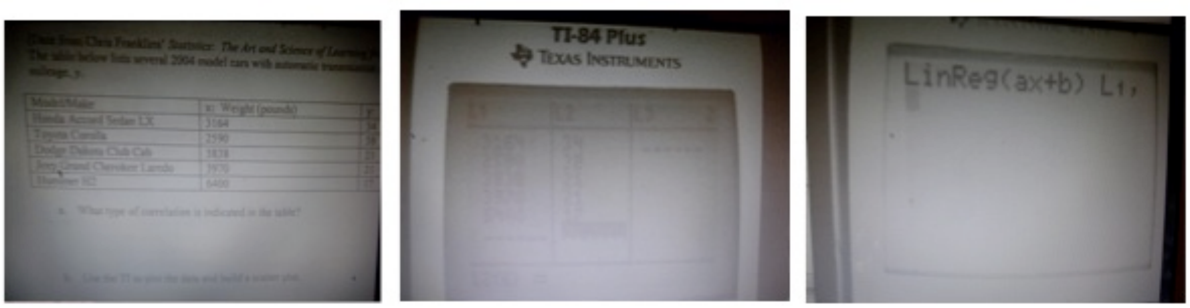

Figure 4.32: Understanding bivariate data using graphing calculators.

Figure 4.32 shows some parts of the process of finding the correlation coefficient between the weight of vehicles and their mileages per gallon. Three pictures show the process of dealing with the given data, projected onto the white board in the classroom. Students could predict mileage for a certain vehicle and compare it to its actual mileage. Students enjoyed working with the data about vehicles. This may mean that Simon was successful in connecting the lesson to real life, and made mathematics meaningful by incorporating technology in analyzing and interpreting data. Certainly, his graphing calculator was a highly effective instrument for him. Another example of his creating mathematical meaning was in his using GSP to teach the concept of derivatives in his AP Calculus class. His use of GSP allowed the students to consider the given functions and their derivatives at the same time by measuring slopes of tangent lines at some input variables and plotting pairs of the input variables and their corresponding slopes as (x, y).

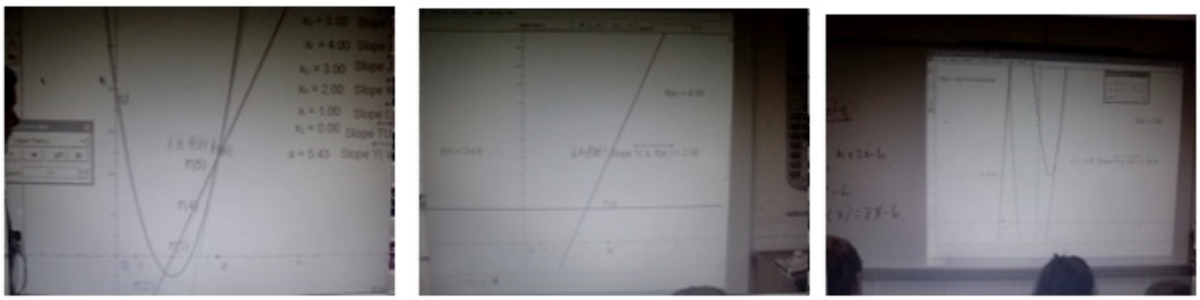

Figure 4.33: Exploring given functions and their derivatives.

Simon demonstrated for a quadratic function and its derivative. His students asked for the case of a linear function. Figure 4.33 shows this including a cubic function. Compared to the other two participant teachers, Simon focused more on mathematical exploration in using technology. To this end, I considered the types of meaning that he created by using technology were both pedagogical and mathematical**.**

<u>Observation day 3 (9/18)</u>

Mathematical Topic

The topic for the lesson was derivatives of trigonometric functions, including the squeezing theorem[15] or the squeeze principal and its application.

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[15] This is a theorem regarding the limit of a function in Calculus. It states that $\lim_{x\to a} g(x) = L$ if

Tools and Materials

In this lesson, Simon used his document camera system, a graphing calculator, the GSP software, and the PC version of the graphing calculator software. His students also used graphing calculators.

Activity

First, Simon showed how to obtain the derivative of a sine function by using GSP.

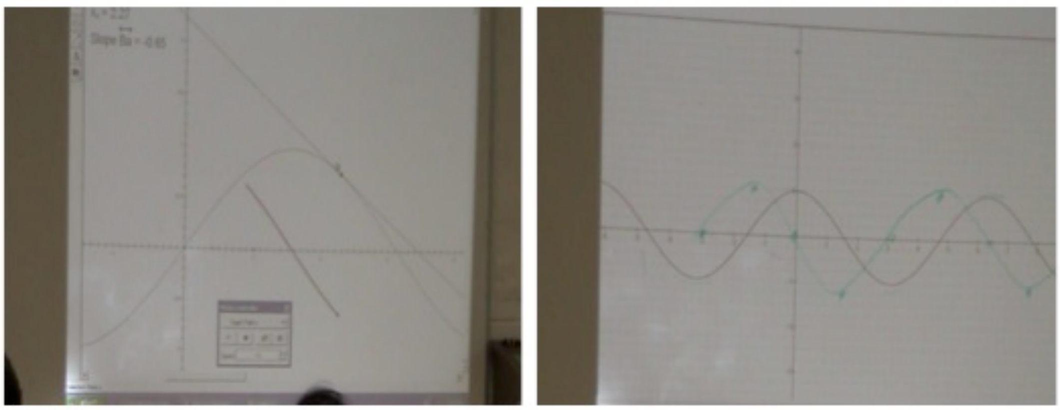

Figure 4.34: The derivatives of sin(x) and cos(x).

He found the tangent line of a movable point at x on the sine curve and measured its slope. The derivative function was drawn by tracing the point that has as its x-coordinate the x-coordinate of the movable point, and as its y-coordinate, the slope of the tangent line at x. The students were already familiar with using GSP in this way. However, this stimulated their curiosity to check it algebraically. Several students shouted, “It’s cosine.” Simon responded, “Really? You think it is? Let’s see if it really is cosine.” He demonstrated using the definition via difference quotients again to find the derivative. For the derivative of cosine, he used the graphing calculator software

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$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$ where $f(x) \le g(x) \le h(x)$ for all x near a, but $x \neq a$.

as in the right picture of figure 4.34. First, he plotted the function of cos(x) and marked where the slopes of tangent lines were zero, 1 or -1. He drew the curve connecting the marked points, as in the picture. Students answered that it was sin(x). Simon also plotted sin(x) to reveal students' misconception. They realized it was –sin(x) but not sin(x). Students' practice followed their investigation of derivatives of sin(x) and cos(x). The class explored the squeezing theorem as well.

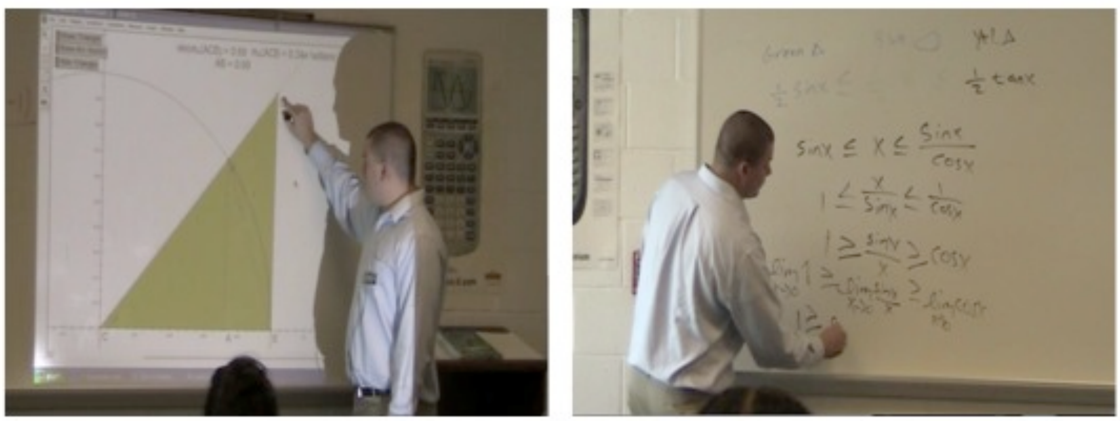

Figure 4.35: The squeezing theorem.

The left picture of figure 4.35 shows what Simon constructed to visualize the squeezing theorem concept. In the picture, he constructed a unit circle and a movable ray that started from the origin. By constructing the vertical line to the x-axis from the intersection point of the ray and the unit circle, he constructed the smaller right triangle. It was obvious to students that the height of the constructed right triangle is shorter than the length of the arc determined by the angle between the ray and the x-axis. He also constructed the line tangent to the unit circle at the intersection of x-axis and the unit circle. By finding the intersection point of the tangent line and the ray, he found a bigger right triangle. The height of the bigger right triangle was longer than the length of the arc. This inequality was written on the white board as shown in the right picture in figure 4.35. In that picture, x represented the angle between the ray and the x-axis. By moving the intersection point of the ray and the unit circle around the circumference towards the x-axis (angle x → 0), he demonstrated the limiting inequality among the height of the small right triangle, the height of the bigger one, and the length of the arc between the two right triangles. After a visual representation of the theorem, Simon took the algebraic approach in the right picture of figure 4.35. The investigation concluded with acquiring the properties of $\lim_{x \to 0} \frac{\sin x}{x} = 1$ *and* $\lim_{x \to 0} \frac{\tan x}{x} = 1$. The application problems were given for students' practice.

In his EMAT 3500 course, Simon constructed the unit circle to explore trigonometric functions and learned about the origin of the term, tangent. He learned neither the squeezing theorem nor did he use GSP to explore the concept of limiting trigonometric functions. I consider that he went beyond copying what he did in the technology-integrated courses in using GSP to explore mathematical concepts and this example illustrates his instrumentalization regarding what he learned in EMAT 3500.

Simon also introduced a calculator trick. He believed his students loved learning "calculator tricks" (Personal conversation, 11-3-2008). He explained how to find the angle addition properties for trigonometric functions using a TI-89 graphing calculator.

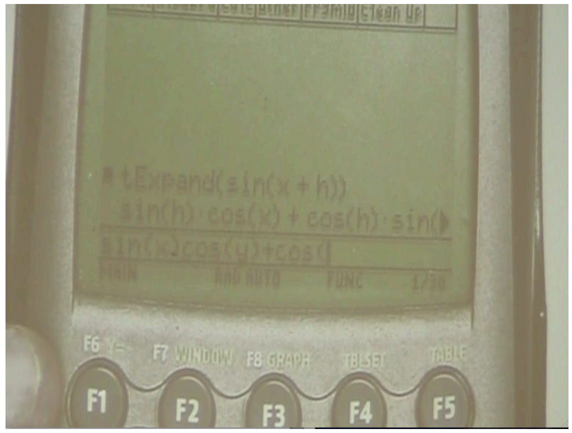

Figure 4.36: Introducing calculator tricks to find formulas using the Document Camera.

As shown in figure 4.36, Simon selected the menus: Algebra-trig-tExpand in this order and typed (sin(x+h)) into the calculator and pressed the enter button. The calculator then showed the answer, sin(h)cos(x)+cos(h)sin(x). For the next problem, he chose the menu item "tCollect" under the Trig menu to show the opposite process, producing sin(x+h) in the calculator display. In this way, students enjoyed learning 'calculator tricks.'

Ways in Which Technology Was Used

Simon focused on exploring mathematical concepts by using technology, as shown in the above figures. He used a graphing calculator in the Algebra II class to help students understand and interpret data mathematically. In contrast, although Alvin and Theodore distributed calculators to students, their ways of using calculators in their classrooms were limited to arithmetic calculations. Simon used GSP in the Calculus class to explore the concepts of derivatives and the squeezing theorem. It helped students to visualize cognitively demanding concepts. In this regard, Simon's use of technology illustrated the process of instrumental genesis, as he created pedagogically and mathematically significant meaning through such use.

Observation day 4 (9/24)

Mathematical Topic

The class involved a problem solving session to prepare for a test. Simon uploaded the workbook for students' practice on his website. The workbook included problems involving finding limits of functions and finding derivatives. Students were supposed to work on those problems in the workbook before the problem solving session. They reviewed the problems based upon students' requests.

Tools and Materials

In the problem solving session, Simon used the document camera system, Virtual-TI[16], and his website. Students used graphing calculators while Simon used Virtual-TI.

Activity

Simon downloaded the workbook linked on his website to start the problem solving session. Based on his students' questions, he demonstrated how to solve problems and explained related concepts. Examples of problems that were reviewed follow:

1. Find the limit, $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta + \dfrac{\sin\theta}{\cos\theta}} =$
2. Find the derivative of $y = v(a\cos v + b\cot v)$.
3. For $x(t) = 8\sin t$, find the position, the velocity, and the acceleration when $t = \dfrac{2\pi}{3}$.

[16] This is a feature- rich graphing calculator emulator for Microsoft Windows written in C++. It is also called VTI.

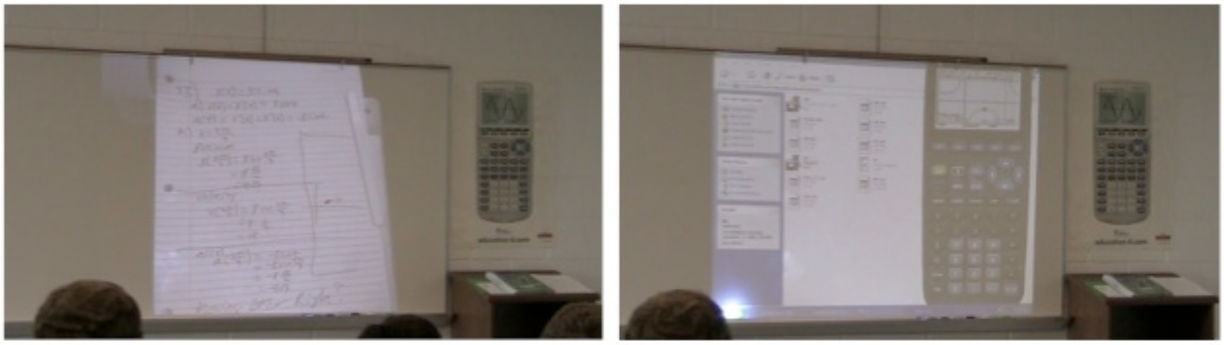

Figure 4.37: Problem solving.

As figure 4.37 shows, Simon preferred to solve problems on paper. Simon explained that the problems in Calculus generally required long procedures, and therefore required more space than the white board offered. He also used Virtual-TI to show how to use a graphing calculator to solve problems. According to him, Virtual-TI had some advantages. When he used a graphing calculator, his hand hid some keys on the calculator when he pressed a certain key, and students often had to ask what key was selected. Virtual-TI solved this problem as it used a cursor to select keys, allowing students to see clearly what he was doing. In this lesson, problem-solving strategies were discussed and shared. Related concepts to solve problems were also discussed and reviewed.

Ways in Which Technology Was Used

At the beginning and end of the class, Simon usually visited his website to check on or announce the homework or class schedule. He also used his website to upload guide notes and homework problems for students. The left picture of figure 4.38 shows him downloading linked handouts to see assigned problems for the problem solving session. In right picture, taken at the end of class, he is showing the assigned homework problems on his website.

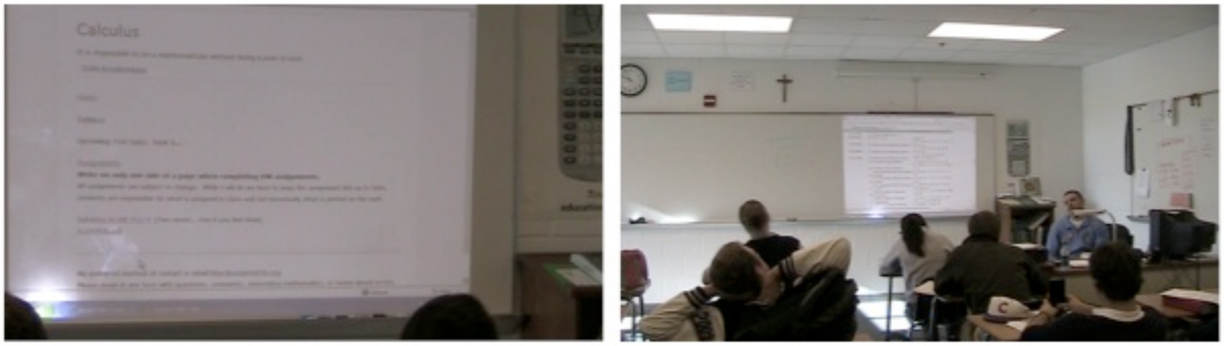

Figure 4.38: Simon's ways of using his personal website.

Due to the nature of the problem solving session, the class solved problems mainly using algebraic procedures. Thus, tools were used for demonstrating solving strategies, rather than exploring mathematical concepts. The use of graphing calculators helped students to develop ideas by seeing graphical representations and checking whether their answers were correct. Both ways provided them with chances to think about concepts, not just to do basic calculations. In this regard, I considered that technology was used in this lesson to create mathematical as well as pedagogical meaning.

<u>Observation day 5 (10/3)</u>

Mathematical Topic

The topic for the observed lesson was transformation of functions in Algebra II. Simon conducted the class as a problem-solving session for a test to be given in their next class.

Tools and Materials

In the problem-solving session, Simon used the document camera system, Excel spreadsheets, and graph papers. The same graph papers and handouts that included problems for practice were given to students.

Activity

This class went similarly to the previous observation of Simon's Calculus class (observation day 4). He demonstrated how to solve problems, discussing strategies and related concepts. To prepare for the test in their next class, students needed a strong understanding of how transformation affects the original function in terms of graphical and symbolic representations. This is also what the problems in the given handout covered. Figure 4.39 shows how Simon used graph paper placed on the handout.

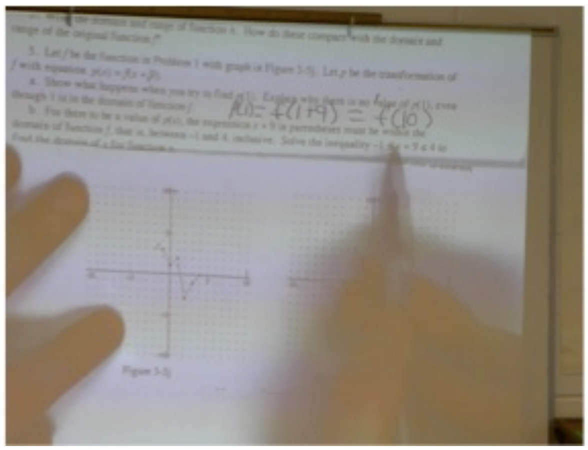

Figure 4.39: Using graph paper to draw a function transformation.

Among the different types of transformations, translation and dilation were the main target concepts. Simon also used an Excel spreadsheet to compare transformed functions to the original function. While Simon demonstrated, students copied what he was doing onto their handouts and graph paper (see left side of figure 4.40). This took the entire class time.

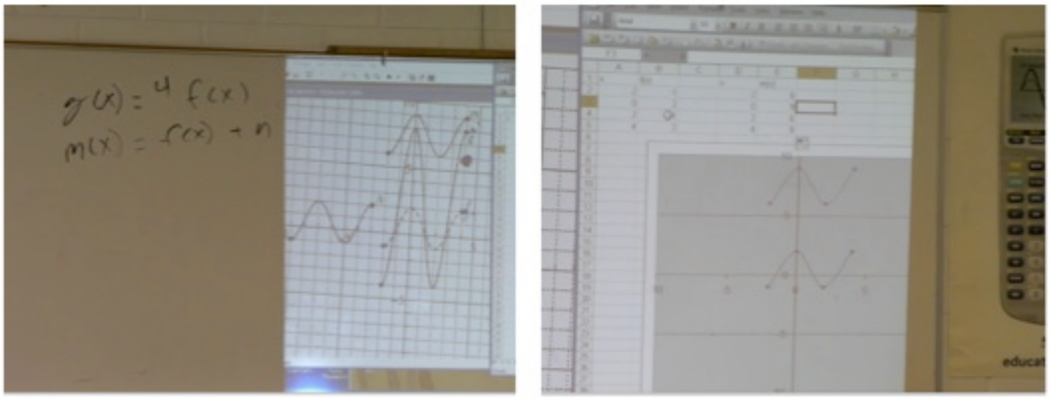

Figure 4.40: Using Excel to understand transformation of functions.

Ways in Which Technology Was Used

Simon's use of the document camera system was the same as in the previous observations. However, his use of the Excel spreadsheet was new. This showed how transformations affect the original function in their equations and graphs. Thus, he used the Excel spreadsheet to explore the concept of transformation. As the right picture in figure 4.40 shows, Simon typed conditions for the equation of the transformation function and filled out the whole column by dragging down to the bottom cell of the table. Then, the linked table showed the transformed function. Although use of technology was limited to the teacher, the students could also use it indirectly by asking the teacher to do anything that they wanted. I considered his using technology was successful, at least, in that it grabbed students' attention and excited their interests. This was confirmed by listening to students in their responses like saying “cool” (Video, 9-24-2008).

Observation day 6 (10/30)

Mathematical Topic

The topic in Simon's AP Calculus class was applications on optimization problems in Differential Calculus. In this lesson, the class had another problem-solving session for a test. Terminologies, including sharp corners and extrema and concepts of “differentiable” and “continuous,” were reviewed before the problem-solving activity led by Simon. In this problem-solving session, the class spent much time on reviewing concepts rather than solving problems.

Tools and Materials

For the problem-solving session, Simon used the document camera system, GSP, and a graphing calculator.

Activity

The class activity consisted of Simon lecturing and demonstrating, and students listening and copying throughout the entire class time. Thus, the class was basically a typical lecture, although students frequently asked questions to check their understanding. The class spent more than half of the class time on reviewing concepts and learning strategies for solving the problems.

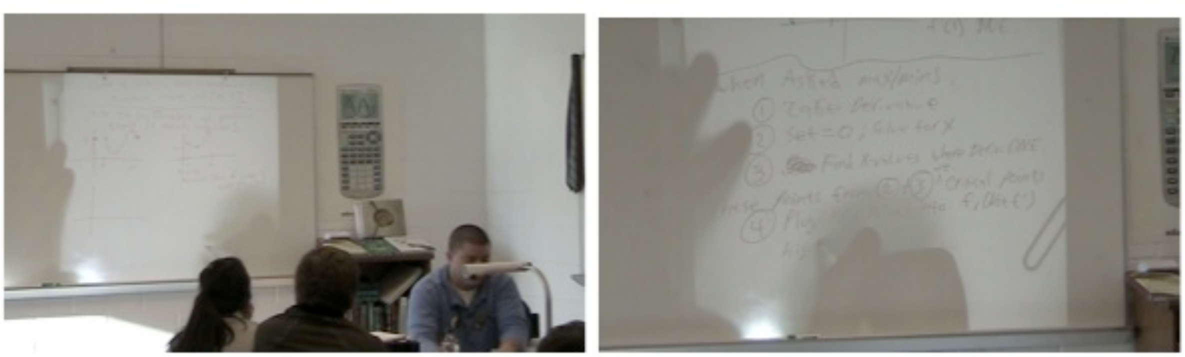

Figure 4.41: Reviewing concepts and learning problem-solving strategies.

Simon was explaining what “differentiable” and “continuous” meant with function graphs and suggesting a step-by-step strategy to come up with the question that asked for maximums or minimums. Figure 4.41 shows these two in order. The remaining 20 minutes were used in solving problems, applying what the students had reviewed earlier in the class. Figure 4.42 shows an example of how the class went. For the problem that asked for the absolute maximum and minimum of $f(x) = x^3 - 3x^2 + 1,\ -\frac{1}{2} \le x \le 4$, the left picture shows they solved the problem by using step-by-step algebraic solving procedures, and the center picture shows Simon pointing to the maximum after he plotted the function by setting the window on the graphing calculator with the given interval.

Figure 4.42: Algebraic solving procedures by hand and with the aid of two technologies

He also used GSP to plot functions in the same manner as he did with a graphing calculator and explore the slopes of tangent lines, especially at extrema (the right-hand picture in figure 4.42). Simon said that he was using both GSP and a graphing calculator in his demonstration since GSP was not available to his students but all of them had graphing calculators. They solved two more problems in the same way before the time was up.

Ways in Which Technology Was Used

Among tools that Simon used, the document camera system was used to make his presentation easier and better. For that purpose, it worked very well for him, providing him with pedagogical help. It also helped Simon teach ways of using a graphing calculator to solve problems and explore concepts on GSP together with students. In fact, without the document camera system, his use of the graphing calculator, GSP, and other software could not be connected to students' learning. On the other hand, GSP and the graphing calculator were not only useful for finding solutions, but also for exploring mathematical concepts. Simon used technological tools in pedagogically and mathematically meaningful ways.

<u>Observation day 7 (11/20)</u>

Mathematical Topic

The lesson topic of his AP Calculus class was "limits at infinity: horizontal asymptotes." However, the topics also included using derivatives to sketch function graphs. Since the same topic was continued from the previous lessons, Simon and his students reviewed what they had worked on and applied those concepts to solving problems.

Tools and Materials

Simon used the document camera system, GSP, the graphing calculator software, and a graphing calculator in the lesson.

Activity

At the beginning of the lesson, Simon discussed the difference between vertical asymptote and horizontal asymptote and summarized how to discover types of horizontal asymptotes by comparing degrees of two functions in the numerator and the denominator of

$\frac{f(x)}{g(x)}$. In figure 4.43, Simon indicates that if $\lim_{x \to \#} f(x) = \pm\infty$ there is a vertical asymptote of $f(x)$ when x= #, and if $\lim_{x \to \pm\infty} f(x) = \#$ then there is a horizontal asymptote of $f(x)$ at $x = \#$. The figure also shows three cases: i) no horizontal asymptote, ii) horizontal asymptote of y=0, and iii) horizontal asymptote of y=the ratio of leading coefficients of $f(x)$ *and* $g(x)$.

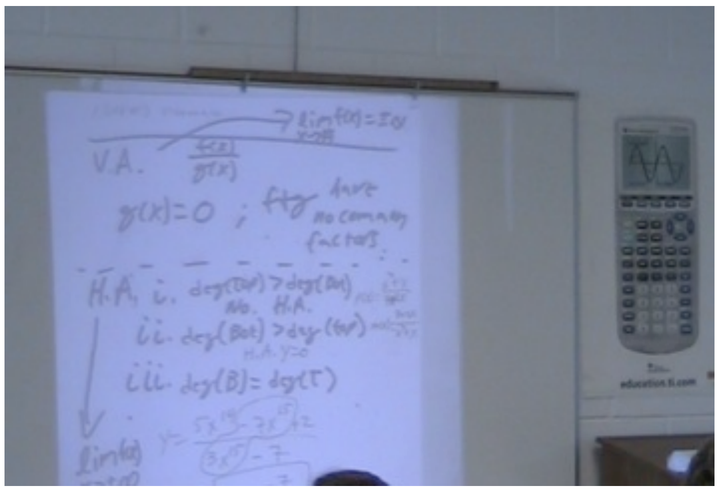

Figure 4.43: A summary of types of horizontal asymptotes.

Later, Simon added the case of slant asymptotes, which were included in his category of deg(Bottom) < deg(Top).

In the follow-up interview with Simon after this lesson, I suggested that the model of deg(Bottom) – deg(Top) would give students a clearer-cut distinction for determining the type of horizontal asymptote rather than the model that he used. For the polynomial functions of Bottom and Top in Simon's model, the following shows what he wrote as in figure 4.43.

deg(Top) >deg(Bottom): No horizontal asymptote

deg(Bottom) > deg(Top): Horizontal, y=0

deg(Top) = deg(Bottom): y=the ratio of two leading coefficients of Bottom and Top

On the other hand, the model suggested by me was:

deg(Top) – deg(Bottom) > 1: No horizontal asymptote

deg(Top) – deg(Bottom) = 1: Slant asymptote, y=linear

deg(Top) – deg(Bottom) = 0: y=the ratio of two leading coefficients of Bottom and Top

deg(Top) – deg(Bottom) < 0: y=0

He liked the model of deg(Top) – deg(Bottom) since this model distinguishes the case of no horizontal asymptote from the case of slant asymptotes. He told me that the new model included all the different types of horizontal asymptotes and gave the information to students in an organized manner. However, he said that the textbook the class used dealt with the case of slant asymptotes separately from the other cases implying this influenced his model.

After reviewing types of asymptotes, the class used asymptotes and derivatives to sketch function graphs. In this lesson, they could sketch only three function graphs, since sketching function graphs was time-consuming work when it is done by hand. One of the functions was $y = x\sqrt{5-x}$, and figure 4.44 shows them sketching the function graph.

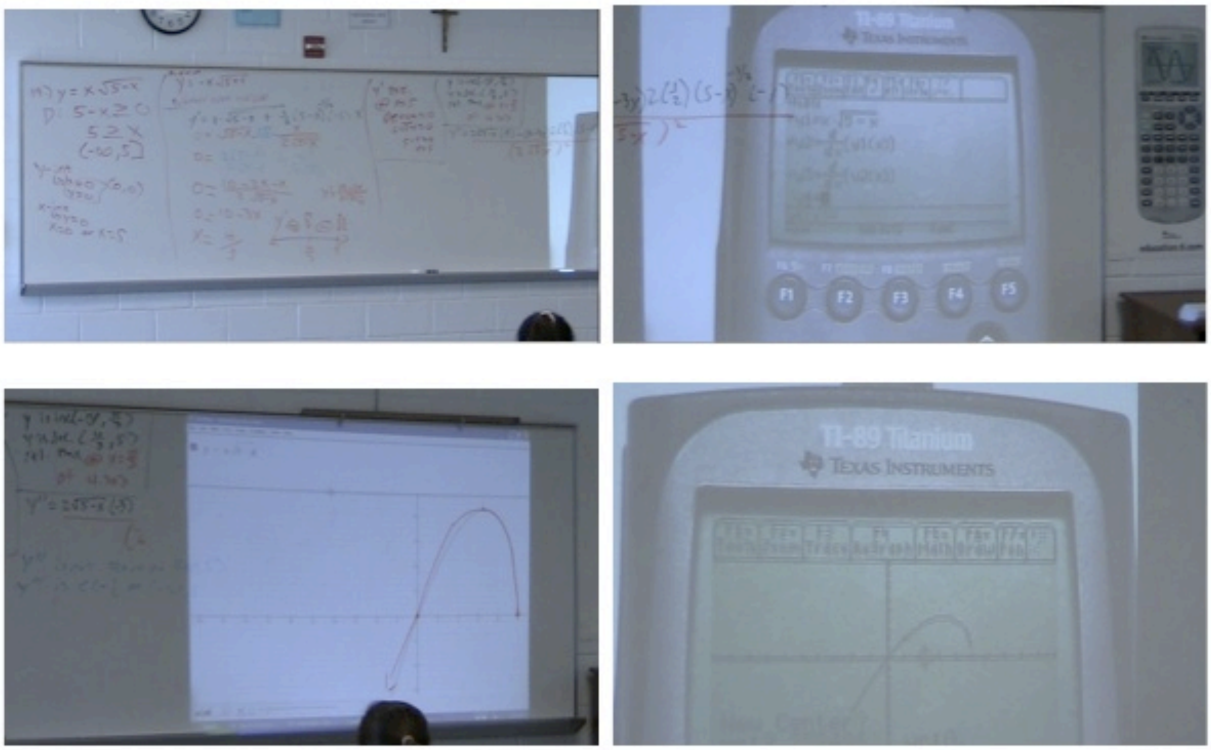

Figure 4.44: Sketching a function graph.

The left top picture in figure 4.44 shows algebraic solving procedures. Simon followed all the steps of finding the domain and the range and x and y intercepts, checking asymptotes, using the first derivative to find increasing or decreasing intervals, and using the second derivative to find the concave or convex of the graph. Whereas by hand this algebraic solving process required a great deal of time, using graphing calculators was very helpful in terms of saving time and reducing errors in calculations. In the right top picture, Simon has typed the original function and commanded the first and the second derivative. By asking to solve for x when the functions are zero, Simon could find all x values for extrema and inspection points. The class, then, compared the results gained by using calculators to the results of solving by hand. The left bottom picture shows Simon's sketching by hand based on the resulting information and the plotted function graph by typing the function equation into the graphing calculator software.

The students were making “wow” sounds as this picture was taken. Simon also provided graphing calculators for students’ individual use. The last picture shows this. They covered two more function graphs in the class. At the end, Simon assigned homework problems that were similar to what they had done in class.

Ways in Which Technology Was Used

Simon represented the same concept in multiple ways using different technologies. While using multiple representations, students were invited to explore concepts in different ways. The students especially enjoyed comparing sketched function graphs by hand to those plotted by using the graphing calculator software. Complicated algebraic solving processes often cause people to become lost in the middle of the solving process, which happened to Simon when he demonstrated sketching the function $y = \frac{x^2}{x^3 + 3}$ by hand. He often checked whether or not he was on the right track thinking back what he had done. We discussed this situation in the follow-up interview. In the interview, he explained:

> We were doing graphing but we were supposed to, you know, find min’s and maxes and inflection points, vertical asymptotes, horizontal asymptotes, discontinuities of, you know, point discontinuities and do all that. And I think we probably only did two, maybe three problems in that entire class period but those problems were – they were big problems but there were little problems within it.

However, because the use of graphing calculators reduced the complexity of the algebraic solving processes, this use provided Simon and his students with a big picture of the long problem-solving process, with an emphasis on the important concepts. Therefore, this technology helped them focus more on mathematical concepts per se.

<u>Observation day 8 (2/6, 2009)</u>

Mathematical Topic

This observation was in spring semester 2009. The lesson topic for the second part of Simon's AP Calculus course was finding volumes of revolutions in Integral Calculus.

Tools and Materials

Simon used the document camera system, Winplot, the graphing calculator software, and a graphing calculator.

Activity

The class was a teacher led lecture. At the beginning, the class was given a simple question that asked for the volume of the revolution that is obtained by revolving the area bounded by the function, $y = x^2$ and $y = 4$ around $y = 4$. Simon sketched the solid of the revolution on the white board and wrote the equation, "volume $= \pi \int_{-2}^{2} (4 - x^2)^2 \, dx$" from the sketch. Once they established the equation, finding the volume was easy for the students. If the area to be revolved was bounded by two more complicated functions (e.g. sin x and e^x), then the students struggled to draw the revolved solid. Therefore, establishing equations and finding the volumes became harder for them. Simon said that mental imagination for such solids of revolution or hand drawing of them on paper was hard, and suggested that Winplot could do the cognitively demanding work, allowing students to focus more on the concepts rather than calculating.

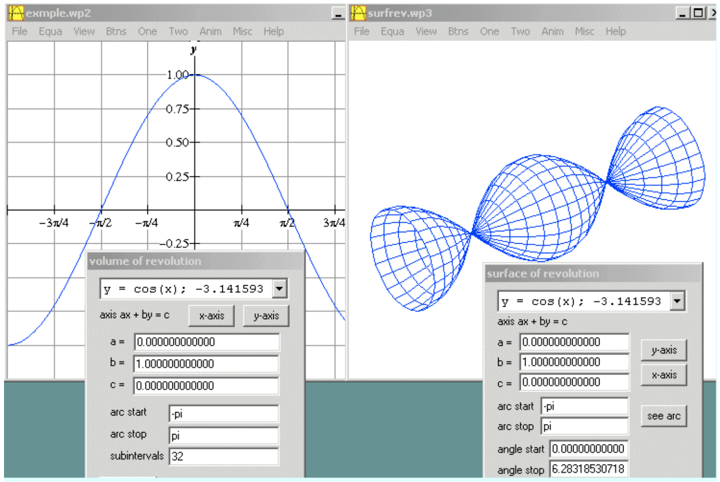

Figure 4.45: An example[17] of using Winplot for the solids of revolution.

Figure 4.45 shows Simon using Winplot to learn about applications involving the Definite Integral. In this figure, the original function is $y = \cos x$, and the solid is obtained by revolving the function around the x-axis. By using Winplot, Simon could help students establish integral equations and find the volumes of the solids formed. For several more problems, Simon showed the solids of revolution by using Winplot, established the integral equations himself, and entered them into his graphing calculator to find volumes of the solids. He left some problems for students to solve on their own.

Ways in Which Technology Was Used

In conversation with me, Simon often mentioned "Winplot is a great tool for teaching Integral Calculus." In the class, he suggested his students use the software that could be downloaded for free online. His students enjoyed seeing solids of revolution drawn on Winplot. In this regard, his use of Winplot was successful in engaging the class in studying applications of the Definite Integral concepts. He used the graphing calculator software to draw the two given

[17] This is not an example from Simon's class. Video -recording was not available on that day.
The image was retrieved from http://faculty.matcmadison.edu/alehnen/winptut/winpltut.htm#3DGraphsn

functions in a two-dimensional plane and see intersection points to find intervals for the integral. His way of using the software and the graphing calculator allowed him to represent mathematical concepts in different ways and helped his students understand the concepts better. This implies that his use of technological tools was meaningful both pedagogically and mathematically.

A consistent pattern that appears to emerge from these observations is that Simon focused more on using technology to explore mathematical concepts with students, although his ways of using his personal website and representing mathematical concepts by using software also enhanced pedagogical aspects of his teaching. Along with the cases of Alvin and Theodore, the overall themes or patterns resulting from my analysis of these observations of Simon's classes will be presented in detail in the next chapter.

So far, I have described what technology the participant teachers used in their teaching and in the ways in which they did so. I also provided some examples from their course work to compare their teaching to their learning. In the next chapter, I will discuss patterns that each participant teacher showed in using technology for teaching mathematics. Similarities and differences among these patterns will follow the discussion. Finally, I will summarize what patterns the three teachers shared.

CHAPTER 5

DISCUSSION OF THE CASES

In this chapter, I present the patterns that participant teachers evinced in their teaching mathematics in terms of ways of using technological tools. First I discuss each participant's use of technological tools individually; then I discuss the similarities or differences between them. The research questions of this study include 1) how teachers used technology in their teaching and what were the patterns that each participant teachers had in using technology, and 2) whether or not there were links and/or gaps between their learning and teaching experiences in using technology and what (if any) were these links and/or gaps. Before answering the questions, I discuss micro theories used as instruments for this study. Then, several sections are given for each participant to answer question 1 and finally, the last section is devoted to answering question 2.

Micro Theories as Instruments for Analysis

I revisited the theories from the theoretical framework in chapter two and used them as instruments for analyzing data and for answering the research questions. Specifically, I used Vygotsky's Zone of Proximal Development (ZPD) theory together with Valsiner's theories of Zone of Promoted Action (ZPA) and Zone of Free Movement (ZFM). These theories provided me with a way of explaining complicated teaching situations and allowed me to examine the dynamic relationship among major factors that affect teachers' decision-making in classroom teaching situations. For instance, Goos (2005) described teachers' working modes with technology by using the Venn diagrams showing the relationships among ZPD, ZPA, and ZFM (see figure 2.4). In a similar way, I use the diagrams to describe teachers' working modes with technology as well as changes in their working modes with technology over time. Figure 5.1 below shows an example of Goos' ways of using the Venn diagram to illustrate change in ZPA over time. In his past teaching, this hypothetical teacher's ZPA was small, but he expanded it by attending workshops and professional development opportunities. Teachers can also expand their ZFM by purchasing computer software or technological tools, encouraging students to develop a productive disposition to use technology in learning mathematics, or encouraging colleagues to see the positive effects of technology on teaching and learning mathematics.

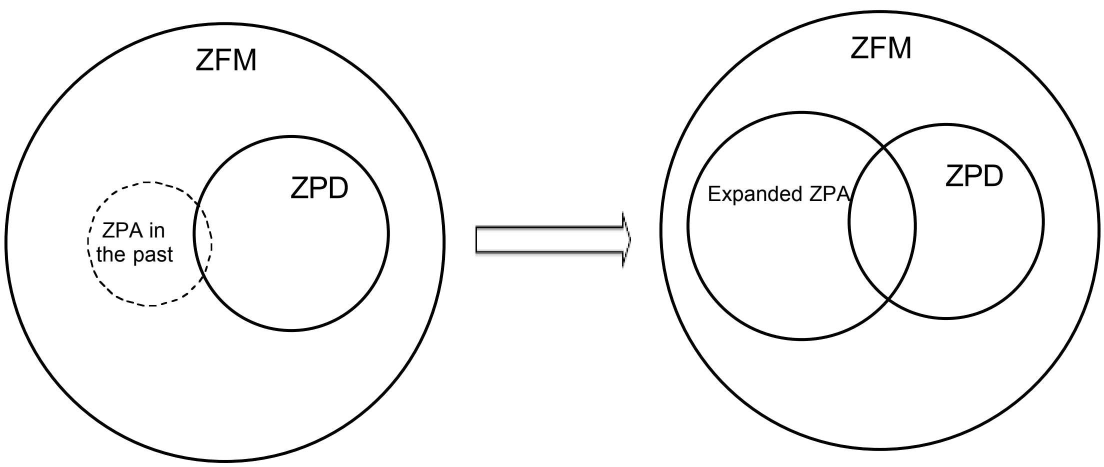

Figure 5.1: The change in a teacher's working mode with technology over time.

I also used *Instrumentation theory* as an analysis tool for identifying *instrumental genesis* for each teacher. As mentioned in chapter two, *Instrumentation theory* draws a distinction between instruments and artifacts. Artifacts are considered tools that can be used in any way whereas instruments are artifacts with the additional quality of "being used meaningfully." *Instrumentation theory* identifies a bilateral relationship between user and tool, which is characterized by both instrumentalization and instrumentation. Instrumentation explains how the affordances or constraints of technologies affect users' thinking; instrumentalization, on the other hand, explains how users shape tools to help them solve particular problems. Figure 5.2 illustrates this bilateral relationship.

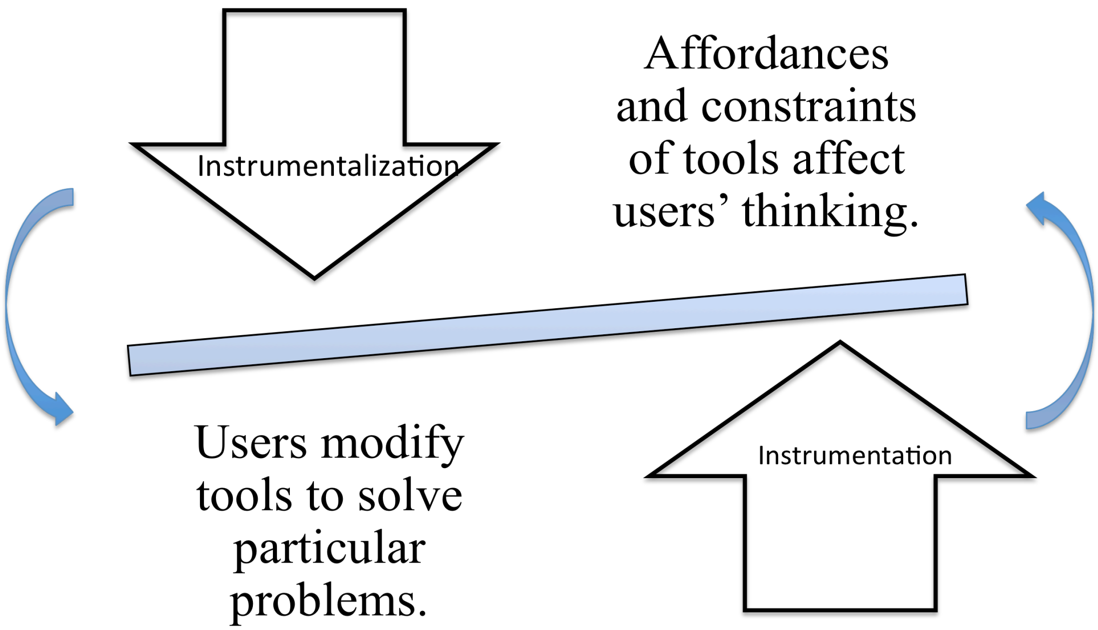

Figure 5.2: The bilateral nature of instrumentation theory

The counter-clockwise process from instrumentation to instrumentalization works as the driving force for the advance of technologies. Through instrumentation theory, I was able to become aware of teachers' meaningful use of technology while observing them using technology in their teaching of mathematics. For instance, if a teacher uses technology for exploring mathematical concepts, then the technology is used in a mathematically meaningful way. On the other hand, if the teachers' focus is given to providing better presentations with more accurate and illustrative representations, then technology is used in a pedagogically meaningful way. In addition, the participant teachers used technology for word processing and record keeping. These types of uses improved their work efficiency but were not seen as pedagogically or mathematically related. These types of uses are not only related to the teaching area in school but also almost all areas in any workplace. I labeled such uses of technology as ‘global’. To better describe teachers’ meaningful uses, I categorize positive and productive uses of technology into three groups: global, pedagogical (from teaching-related but not subject-specific work), and mathematical (from both teaching-related and mathematics-related work). In the discussion of the individual cases I use the phrase “create meaning” with respect to teacher’s use of technology in a meaningful way. That is the way the teacher used the technology added value to the activity, whether it was global, pedagogical or mathematical activity. For example, Simon explained the squeezing theorem by using GSP to provide students with a visual representation of it. GSP allowed the students to see functions and their derivatives at the same time, and this helped the students understand the concept of derivative in depth. In this way Simon’s use of technology helped “create mathematical meaning.”

In Chapter 2 I referred to the four categories for teachers’ working modes with technology from Goos’ (2005) study: *master, servant, partner,* and *extension of self*. I had intended to use these categories to explain the reason behind teachers’ decisions to use certain tools to teach certain topics. However, I did not find her categories helpful in explaining differences among the participant teachers’ working modes with technology in my study because the three teachers were mainly using technology as a partner or extension of self. I, therefore, decided to create three new levels of working with technology based on instrumentation theory. The three levels of use are as follows:

Level 1: Technologies are simply replacements of tools used in traditional classroom environments. Thus, using technologies does not change the nature of teaching and learning in traditional classrooms. At this level, using technology stays at the level of artifacts and does not reach the instrument level. For instance, although it has innovative functions, the SMART Board can be used just like chalkboards or whiteboards.

Level 2: Teachers provide different teaching and learning activities that are not possible without using technologies. At this level, teachers use technology as instruments while teaching. However, the meaningful use of technologies by the teacher stays at the level of general pedagogy, which means that teachers intend to use technology to provide better presentations, hold students' attention and so forth. Teachers do not intend to use technology for mathematical exploration.

Level 3: Teachers use technology in mathematically meaningful ways, contributing to their *instrumental genesis* with respect to their use of technology. At this level, technology is used in more than simply a pedagogical fashion, and is involved in students' explorations of mathematical concepts. At this level, students can learn different mathematics than in mathematics classrooms without technology or in which technology use remains at levels 1 or 2.

The participant teachers' use of technology in teaching mathematics was analyzed using the three metaphorical instruments of ZPD, ZPA and ZFM, instrumentation theory, and the three levels of categorized working modes as illustrated in figure 5.3.

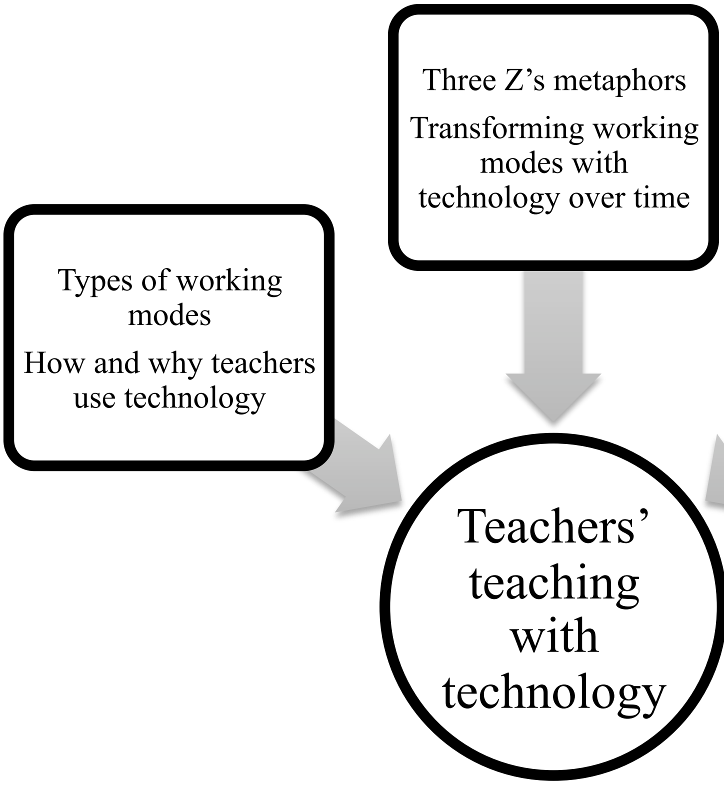

Figure 5.3: Three Components of Analysis

Teacher tasks include various domains: presenting materials, assessing students' learning, lesson planning, recording and keeping recorded grades and attendance, and so forth. The tasks in which teachers use technologies are closely related to different meanings and different levels that are described above. In order to articulate in what specific domain the teachers used technology, I limit the domains to those that I could observe in their teaching mathematics. The domains are as follows.

- Teaching
    - Presenting mathematical facts and/or procedures for solving problems
    - <u>Exploring mathematical concepts per se</u>
- Assessing
- Lesson planning
- Recording and keeping attendance and grades
- Communicating with students to announce homework or to update and refresh course content

As mentioned earlier, I wanted to see what meanings were created when the participant teachers used technologies. Among the above domains, I underlined the category of exploring mathematical facts per se to distinguish the subject-related meaning, which is mathematical meaning, from pedagogical meaning shared by all subjects. Except for the underlined item, I consider meanings created in the domains of teaching, assessing and lesson planning as pedagogical. The last meaning can be created in all areas, since the tasks of recording and communicating with others are required in all areas in and outside of school. For instance, if a teacher used a technological tool to explore mathematical concepts to deepen students' conceptual understanding and this was successful, I consider that the teacher created mathematical meaning through his *instrumental genesis* for the technology. If a teacher used his website simply to better communicate or interact with his students and it showed productive results, I consider that the teacher created global meaning while managing the website. Using all these theories, I will discuss patterns that were found in each participant teacher's use of technology in the following sections.

Alvin

The prevailing activities in Alvin's classroom were lecturing and demonstrating. He described these activities as his preferred teaching style: "I always prefer to – almost always prefer to teach in a traditional manner, teacher-led, because of my inability to keep students on task and out of trouble" (Interview, 1-21-2008). In fact, the students in his Geometry class were unmotivated underachievers. I often observed that his well-organized and down-to-earth lectures were falling on deaf ears. His primary concern, therefore, was to teach his students the minimum knowledge required to pass the test. Thus, he focused more on procedural fluency rather than on conceptual understanding in teaching mathematics. According to Alvin, his expectations for his students' willingness to learn became significantly lower over time. He said, "I had extremely high expectation of students with content, and now it's a lot -- my expectation of their ability is not lower, but my expectation of the reality of their willingness is lower, significantly lower" (Interview, 1-21-2008). In his reflection on instrumental and relational understanding, written during his EMAT 3500 course, he wrote:

> What is left to reason are the "benefits" of relational instruction to those who have no apptitude [sic] or interest toward math. How does it help someone to monopolize their time trying to teach them something that just is not important to them? They will probably never see its usefulness. One may say that we should learn for learning's sake and not aim toward good grades (as Skemp submits would be a downfall of instrumental learning.) However, this is surely unrealistic. It is not possible for all of us to understand everything.

His lowered expectations and beliefs about mathematics affected the choices he made about using technology. Basically, he used technology to have his students listen to him and engage in the class, as well as to save his time in doing other tasks like lesson planning.

Types of technologies and their meanings for Alvin

Alvin's favorite technology was the SMART Board (Interview, 1-21-2008). The SMART Board was connected to his computer at all times to form the SMART Board system. He used the system to present mathematical facts and to demonstrate how to use the facts to solve problems. He usually taught by telling what the definitions and theorems meant and writing procedures or drawing associated figures to solve problems on the SMART Board. The board worked as a template for each day's lesson. The SMART Board provided high quality visual supports and interactivity. Alvin was very familiar with the SMART Board's functions and used them to enhance his presentations. By touching the screen, he could select different colors, drag, rotate, enlarge or shrink figures, and hide or show text and figures. These manipulations would be difficult or impossible in a traditional classroom with only a chalkboard.

He did not have to erase anything since the SMART Board provided unlimited space. Rather, he could save what he wrote and draw and print it out or send it to students electronically. His students also enjoyed the visual aspect of the SMART Board when they got the chance to present their answers. Based on Alvin's use of the SMART Board system, I considered that he created positive pedagogical meanings in that he got students engaged and created effective and precise presentations.

Alvin also used the OnCourse software for lesson planning and recording and keeping students' attendance and grades. Although this was not directly related to his teaching, the software helped him save time and made all his tasks easier and more streamlined. OnCourse became an instrument for him, providing both pedagogical and general significance.

The Schoolview software was another instrument Alvin used. It helped him monitor and assess students' understanding with its panoptic view. With the students' laptops connected to the Internet and the Schoolview software installed in Alvin's laptop, Alvin could select and enlarge any one of the students' computer screens and communicate with the student either online or offline. He could also access statistics about students' correct or incorrect answers, which told him what he needed to go back over. In this way, the Schoolview technology offered him a new, efficient way to assess students' learning and understanding. This meant it made a difference in education in his classroom.

All in all, Alvin used technologies as instruments, not merely artifacts. He imbued them all with pedagogical meanings, and some with global meanings. However, he did not create mathematical meaning; he did not use technologies to explore mathematical concepts. I never observed him proving or verifying mathematical theorems or properties. Thus, in terms of the levels described earlier in this chapter, he stayed at level 2.

Mathematical ability stressed in teaching with technology

Kilpatrick et al. (2001) suggested five strands of mathematical proficiency: conceptual understanding, procedural proficiency, strategic competence, adaptive reasoning, and productive disposition. Among these five strands, Alvin focused most on procedural fluency.

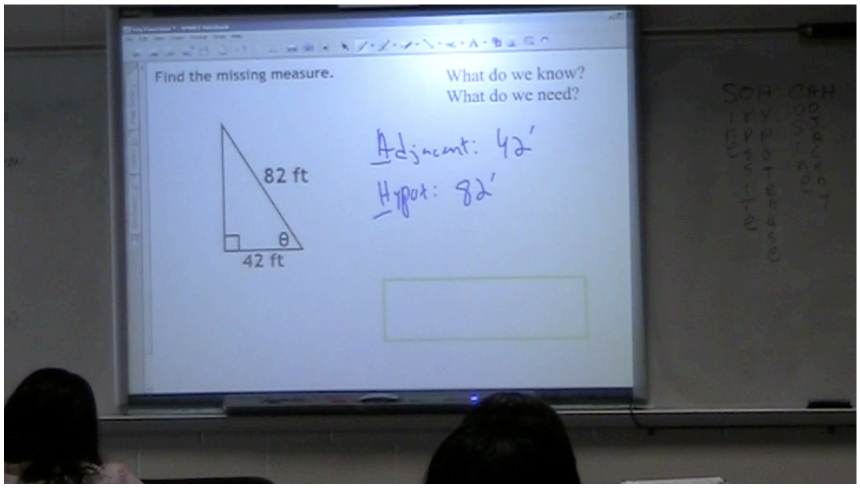

Figure 5.4: Alvin mostly stressed recalling facts in his teaching.

As figure 5.4 shows, in his demonstration, Alvin usually asked his students what they knew and what they needed to solve problems. By emphasizing remembering and knowing facts, his teaching did not provide the students with the chance to think “why?” He had difficulty stimulating students’ mathematical curiosity. Alvin attributed his difficulty mainly to students’ poor mathematical backgrounds and attitudes (Personal conversation, 11-18-2008). Instead of emphasizing mathematical curiosity, he attempted to develop students’ productive disposition toward mathematics by introducing fun activities that focused on procedural fluency in order for his students to be successful on tests. His use of technology was also geared toward these goals.

Identified links or gaps between learning and teaching

In what I was able to observe, there was no identifiable link between what Alvin learned in his teacher preparation and his teaching in his classroom with regard to using technology to teach mathematics. Although he said in the interview that he used GSP and Virtual-TI to investigate mathematics, I did not see any evidence of this. As he pointed out, the SMART technology, which was his favorite, was not introduced in his teacher education program. Rather, his major experiences with using technology in his teacher education program were about educational software – for instance, GSP and Fathom – and about how to use them to explore mathematical concepts. Thus, his learning was more about creating mathematical meaning to deepen conceptual understanding. In contrast, in his actual teaching in school he focused on creating pedagogical meaning through the use of technology.

As a student, the technology Alvin experienced in high school consisted solely of a graphing calculator. However, the graphing calculator was used by his teacher (through a display device), not by the students in his class. Compared to his colleagues, he considered that he was well trained in using technology and recognizing its significance in some of the mathematics education and statistics courses at the University of Georgia (Interview, 1-21-2008). I could see that teachers in his school often asked him about technology and how to use it in their classrooms. According to Alvin, some teachers still simply emulated the ways in which they themselves had been taught. He clearly recognized the benefits of using technology and the importance of using technology for better teaching and learning. As discussed earlier in chapter four, technologies had already become a large part of his teaching repertoire. In this regard, there were indirect and unobservable links between his learning experiences and his practice. At the very least, his learning experiences in his teacher education program provided him with new perspectives and helped him step out of the long-standing customs of mathematics teaching, with respect to his pedagogical use of technology.

Theodore

The prevailing activity in Theodore's classroom was students' individual practice, which followed his lectures and demonstrations. He spent little time lecturing, instead opting to let students learn by doing things themselves. Like Alvin, Theodore identified a preferred teaching style. In the interview (1-23-2008), he said that he preferred a student centered class because it was easier for him to determine whether his students were actually learning. The students in his Algebra I, ESOL class were underachievers, like those in Alvin's class. However, they were different from Alvin's students in that they listened to the teacher, constantly asked for help, and collaborated with each other more. The small class size of only 12 students was one reason for this. Another reason was that they were all English-language learners. Theodore's students helped each other if one of them did not understand what he meant, and peers' explanations often worked better than the teacher's, especially to them as second language users. Theodore took advantage of this by providing them with plenty of time for collaboration. His mild manner and approachability also supported their active working with others. These teaching strategies were also evident in his ways of using technology. Basically, he used technology to provide his students with practice materials. However, his use of technology to assess students' learning was outstanding, and he saved a lot of time preparing handouts, test questions, and lesson planning by using technology.

Types of technologies and their meanings to Theodore

The SMART technology was Theodore's favorite like Alvin. The SMART technology[18] included many subcategories, one of which was the SMART Board system. Although he used

[18] Among the SMART technologies in Theodore's school, I observed the SMART Board system, the SMART Interactive response system, the SMART Notebook collaborative learning software, and the SMART Sinc classroom

the SMART Board system in his everyday teaching, the SMART Board was not an instrument for him yet. As described in chapter 4, he used it just like a blackboard or whiteboard, and did not take advantage of its high quality of visual support or its interactive aspects in a meaningful way. He usually taught by telling what definitions and theorems meant and demonstrating them by writing procedures to solve problems on the SMART Board. Thus, I considered that he stayed at level 1 in terms of using the SMART Board.

However, Theodore's use of the SMART Interactive response system to assess students' learning was significantly meaningful. Together with the SMART Sinc classroom management software, he interacted with and monitored his students while the students practiced doing problems. The technology allowed Theodore to import the results and generate statistics to understand the overall performance. He also collected students' work on paper to learn about their misconceptions. These techniques greatly assisted him in preparing remedies for students' misunderstandings in the next class. The SMART Interactive response system was a wonderful instrument for him, with special pedagogical meaning -- for instance, knowing what students know and what they do not know, and consequently getting through to the students.

Like Alvin, Theodore also used the OnCourse software for lesson planning and recording and keeping students' attendance and grades. Again, although it was not directly related to his teaching, the software helped him save his time and made all his tasks simpler. The OnCourse software was a helpful instrument to Theodore, having both pedagogical and general significance for him.

The SMART Sinc classroom management software and the Schoolview software were additional instruments for Theodore which helped him monitor and assess students'

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management software.

understanding in a panoramic view. The only materials required were students' laptops connected to the Internet. Theodore could select and enlarge one of students' computer screens and communicate with the student online or offline. This let him know what he needed to go back over. This technology offered him a new way to assess students' learning and their understanding.

All in all, with the exception of the SMART Board, many technologies that Theodore used were instruments, and they all had pedagogical meanings and some global meaning in his hands. However, he did not create mathematical meaning from using technologies. I did not observe him proving or verifying mathematical theorems and properties or exploring mathematical concepts in his classroom. Thus, in terms of the levels described earlier in this chapter, he stayed at level 1 or 2.

Mathematical ability stressed in teaching with technology

Among the five strands of mathematical proficiency -- conceptual understanding, procedural proficiency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001) -- the most stressed strands in Theodore's teaching and use of technology were procedural fluency and productive disposition. For instance, with regard to productive disposition, he connected the lesson for the topic of percentages to the presidential election that was going on at the time. He let the students use the concept to solve problems while discussing current events. This helped them see the practical side of mathematics, as well as the fun aspects of learning mathematics. Although it was not observed, he told me that he liked to visit websites like the National Library of Virtual Manipulatives (NLVM) to allow students to do mathematics while playing. For the strand of procedural fluency, his way of using a calculator or using problems in e-workbook are good examples. He saved a good deal of time for students' practice with problems in the e-workbook, as mentioned in chapter 4. For problem solving, it seemed that all that students needed was to know about mathematical facts and/or how to use the facts. Like Alvin, Theodore did not provide the students with the chance to think "why?". This was confirmed when he said, "For the most part, the students are pretty good at – they're not that great at asking 'why' questions but they're usually good at asking, 'Slow down,' if they need me to slow down." (Interview, 1-23, 2008). This shows that his focus on procedural fluency and productive disposition was mainly due to his beliefs about students. The following shows his beliefs about mathematical proofs and students.

> I think proof is one of those topics that we've judged to be, you know, that we've judged as making students uncomfortable. They don't enjoy it. They hate it. So I think it's just become the trend over time to lessen the importance of it. ...[Even if I do prove] the students don't know the differences because they've never really shown proof or taught proof or what proof is. (Interview, 1-23, 2008)

Like Alvin, instead of focusing on the exploration of mathematical concepts, Theodore focused on developing students' productive disposition toward mathematics and procedural fluency so that his students could do well on tests. But in Theodore's case he provided students with more opportunities for individual work and peer collaborations to encourage ownership of their own mathematics. Accordingly, his use of technology was geared toward fostering their productive disposition and procedural fluency through students' independent and collaborative work.

Identified links or gaps between learning and teaching

Although I could not observe Theodore using GSP to explore mathematical concepts, he told me that he used GSP for exploring mathematical concepts in his Math I class. For instance, when his Math I class covered the topic of centers of triangles, he provided a group task of finding centers, given three different towns that form a triangle. Through different constructions, the students were required to find the different centers of the triangle: inscribe center, circumcenter, orthocenter, and so forth. He also asked which center was the best place to put a cell phone tower. He told he spent three days on that topic. First, he started the lesson providing a hands-on activity with rulers and protractors. In the next lesson, he did GSP simulation. In the interview (1-23-2008), he told me that he copied this activity from one of his mathematics education courses. He said that he chose the activity since he found it fit with what he was to teach. I identify this example as a direct link between his learning and his actual teaching. In the interview, I asked him how often he used GSP. He answered that he was using GSP frequently in his Math I class, which I did not observe. He said that topics in the geometry chapter of Math I required him to use it more, and he had other ideas for that chapter like his ideas for the centers of triangles from what he learned in technology integration courses. However, he said that he did not have ideas for GSP activities for his Algebra I class.

In his Algebra I class, Theodore used Algebra Tiles to teach the product of two binomials and factoring quadratics (See page 67). According to him, he also learned about using Algebra Tiles to teach those topics in one of his mathematics education courses. He could remember the activity that fit the lessons that he was to teach, and so he implemented the activity into his lessons. Although Algebra Tiles are usually considered manipulatives (non-electronic technology) rather than (electronic) technology, this did reveal that he was using what he learned in the teacher education program in his teaching.

Much like Alvin, the SMART technology was Theodore's favorite technology, and it was not introduced in his teacher education program. His major experiences using technology in his teacher education program were with educational software – for instance, GSP and Fathom – and concerned how to use them to explore mathematical concepts. Thus, his learning was more about creating mathematical meaning to deepen conceptual understanding. In contrast, he focused on pedagogical purpose in his actual teaching in school. Although I found that he was using technology for mathematical purposes, the pedagogical one prevailed.

As a learner, a graphing calculator was the only form of technology that Theodore experienced in high school mathematics classes. In the interview (1-23-2008), he compared himself to his colleagues in terms of incorporating technology into teaching:

> As far as a learner, most of my teacher education was done at the University of Georgia and I think they did a very good job as far as starting to get the teacher used to using technology. I used a lot of technology in my undergraduate courses, so I became familiar with a lot of different pieces of technology and how to use that technology in the classroom, which I think teachers coming from a lot of other places may not have the same knowledge coming out of the teacher education program as I did, if they're coming from a different place other than the University of Georgia.

He also mentioned that he was required to read a lot of teacher education articles in his undergraduate and graduate courses. These helped him develop reform-based ideas and made him able to adapt to new things. As discussed in chapter 4, technologies had already become a big part of his teaching repertoire. Together with Alvin, he was taking the lead among teachers in his school in terms of technology integration for teaching and learning.

To this end, there were both direct and indirect links between Theodore's learning experiences and his practice. His learning experiences in his teacher education program helped him make a difference in teaching mathematics.

Simon

The prevailing activities in Simon's classroom were exploring and verifying concepts. Although Simon usually led the activities, students actively engaged in the class by answering and asking questions. I found that these types of activities were associated with his enthusiasm for mathematics:

> I like Math a lot because it's like a puzzle basically. I really love puzzles. So, you know, Math is all about asking questions and then finding truth and so one big game usually and it's fun because you can get lost but it's also frustrating because, you know, you can – that's my recurring nightmare is whatever problem I'm working on, you know, I wake up in the middle of the night thinking that I've solved it and then can't even remember what it was when I wake up. But I don't know, it's – I think the more Math I do the more I see it around me, too, which is interesting.

His teaching style was very structured, and using technology was a big part of that structure. He wrote or verbally stated lesson goals after homework review and discussed how the current lesson would be related to the previous or the next lesson (See figure 4.29). In contrast, Alvin and Theodore did not set up lesson goals, and when they did they did not do so clearly. Simon usually broke a lesson into homework discussion, demonstration and presentation of new materials, and individual or group work. According to him, connections between lessons and the best manner of presenting mathematical concepts were his primary concerns when he did his lesson planning. His students were good at asking "why," and in fact he goaded them on to ask why (See day 3). His enthusiasm was contagious. Like the other participants, his enthusiastic and inspiring character was well reflected in his teaching and his students' learning. His primary concern in teaching was to let students make sense of what they learned and develop the habit of doing so. Thus he focused more on conceptual understanding than procedural fluency in teaching mathematics. He also said that he felt more obligated to prove theorems in Calculus than in the algebra classes. Like the other two teachers, Simon's focus was also associated with his ways of using technology. He mainly used technology to explore mathematical concepts or verify mathematical theorems or properties. He also used his website to communicate better with students. In his classroom, nothing was unexpected or unclear in terms of course schedule, tests, homework, and the like.

Types of technologies and their meanings to Simon

In the interview, Simon said that the GSP software was his favorite technology. To explore mathematical concepts with his students, he needed to project his laptop screen onto the board. However, his school did not provide him with a projector or other technological resources except a laptop. He personally purchased all the technological tools he needed: a document camera, a DLP projector, TI-84 and TI-89 graphing calculators, and some computer software. These were the types of technologies that he used.

In terms of the quality of visual support, Simon's document camera system was not nearly as good as the SMART Boards that Alvin and Theodore had. However, it was more widely applicable. He did not have to be in front of the board to write or draw. He could do so sitting in a chair, projecting his paper onto the board using his document camera. He also used it to project handouts, the textbook, and calculators onto boards, which the SMART Board system could not do. It was especially essential to Simon for teaching how to use calculators to explore mathematical concepts and calculator tricks. Simon was very capable of shifting the document camera system back and forth between his computer screen and other materials. Technology failure was not observed at all. Clearly, what he was doing was not possible in traditional classroom without such technologies. By using technologies, he could provide better lessons with multiple representations. I consider that this tool was used in meaningful ways, and he created a new didactic situation.

Simon also used the graphing calculators and the Virtual-TI software to represent concepts in a different way or to explore concepts in depth. Simon often used the calculators to show graphical representations of symbolic expressions. According to him, his students liked learning calculator tricks, and he was constantly researching and sharing the tricks with colleagues. I believe that this led him to become a calculator expert. This also allowed him to help students use formulas without just memorizing them, since they needed to think about the principles behind the formulas (See figure 4.36), and this, therefore, could help them focus more on learning about concepts. His use of calculators and Virtual-TI, therefore, directly and indirectly worked to create mathematically valuable meanings.

Simon's favorite software, GSP, was another instrument for him. It helped him explore and verify mathematical concepts with students for their better understanding. For instance, he explained the squeezing theorem by using GSP to provide students with a visual representation of it, rather than just writing the symbolic expressions and telling them about it. GSP allowed the students to see functions and their derivatives at the same time, and this helped the students understand the concept of derivative in depth. Although Simon led all the GSP activities, students actively engaged in those activities, suggesting what they wanted Simon to change in manipulating GSP.

Excel was another piece of software Simon used for mathematical exploration in his Algebra II course for the topic of function transformations. Since Excel provided the link between a data table and its graph, he could use it to show how modifying a function equation in the table affects its graph (See figure 4.40). This linkage helped his students generalize the function equations for translation, reflection, dilation, and their combinations. For instance, students could tell that $g(x) = af(x-b)+c$ meant that the function g is acquired by horizontally shifting the function f by b, then stretching it by factor a, and then vertically shifting by c. The students enjoyed asking their teacher to edit the function equation to see the resulting graphs.

Graphing calculator software is generally used to plot exact function graphs. However, Simon often used it to project an xy-plane onto the board and draw function graphs on it by hand. He often used it to compare graphs that he sketched by hand to the exact functions plotted by the software. Students enjoyed seeing every moment of the functions plotted by the software over what their teacher drew. This software gave accurate function graphs with colors. Simon used this software to represent equations in graphical ways, whereas GSP was mainly for exploring mathematical concepts in both algebraic and geometric ways at the same time. Thus, the graphing calculator software had less meaning in terms of mathematical exploration than did GSP.

Winplot was another piece of software that Simon used in his Integral Calculus class. Often, imagining figures of revolved solids is difficult even for teachers, since solids of revolution were 3-D figures acquired by revolving two-dimensional regions about given axes. Simon used Winplot to deal with this cognitively demanding work. Regarding the Winplot software, he said,

> Winplot is also really good because anything – I can spin it in front of them and that gives them a much better feel for what it looks like, to be able to see if from all sides. But I can actually print it and give it to them, you know, in a Word document if I wanted to which is good because then they have a more accurate drawing in their notes, or I can put it online or anything, it's very nice to have something that's – if it goes into Word, you know, you can put it on anything. (Interview, 2-6-2009)

Winplot and the graphing calculator software were basically used for the same purpose. They helped the students understand concepts better with accurate representations, but were not directly related to exploring concepts per se.

Simon also used Calculator Based Laboratory (CBL) toolsets that he found in the science materials room in his school. According to him, he did not know that it was available until he talked to a science teacher about the mathematical modeling of real world phenomena. When their discussion turned to Newton's Law of cooling, which gives an exponential function model, the science teacher informed him of the toolset. Simon decided to conduct the experiment in a way of demonstrating it for the students. The class discussed mathematical meaning of the transferred data by their teacher after the experiment. This was another example of using technological tools for mathematical exploration.

In general, the technologies that Simon used were all instruments and they had three levels of meaning -- mathematical, pedagogical, and global -- in his hands. Proof and verification of mathematical theorems and properties were constantly observed in his classroom. Thus, in terms of the levels described earlier in this chapter, he was at level 3.

Mathematical ability stressed in teaching with technology

Among the five strands of mathematical proficiency (Kilpatrick et al., 2001), the most stressed strands in Simon's teaching and use of technology were conceptual understanding and productive disposition. Many examples of his emphasis on conceptual understanding have already been discussed. Often, he started showing GSP simulations to stimulate students' curiosity before actually working on a new concept. For instance, he showed the cosine function as the derivative of sine by tracing the point whose x and y coordinates were x and the slope of the tangent line at x, respectively. Students could tell that it was cosine but not why. So the class started investigating why it was cosine. Although he was concerned about students' achievement and their success at tests, he did not focus on procedural fluency. Even in his lesson planning, his primary concern was the best way to investigate mathematical concepts, and consideration of how he would connect one concept to another as quoted in the previous chapter.

As discussed earlier, Simon challenged his students to think “why?” and showed them the practical side of mathematics by conducting the experiment of Newton’s Law of cooling and interpreting the result mathematically. I considered that this was the way in which Simon sought to help his students develop their productive disposition toward mathematics.

Identified links or gaps between learning and teaching mathematics with technology

Simon’s teaching revealed significant connections between his learning and his actual teaching with technology. First, his way of using GSP was very similar to that taught in his EMAT 3500 course. According to him, Simon took the course twice, registering once and auditing once. This could be why he was very proficient with GSP. He also introduced the origin of the trigonometric term “tangent” by pointing out that the vertical side of the right triangle used to construct the tangent function was “tangent” to the unit circle. He said that he had learned this from the course instructor. Unlike the other teachers, he not only used what he learned in his teaching, but also created his own techniques. For instance, he applied his knowledge to create the GSP file for teaching the squeezing theorem. He went beyond the level of simply repeating what he had learned.

The experiment using a CBL toolset was the second identified link between Simon’s learning and teaching. As mentioned above, he copied the same experiment that he had done in the EMAT 3500 course. According to Simon, the document camera system played an important role in demonstrating the experiment. Although it was not a full experiment in which all students actually collected data, the system showed all the details of how to use the tools and the transferred data.

Simon’s way of using his website was another example for links. Some of the mathematics educators whose courses Simon had taken used their personal websites to announce assignments, course schedule, course materials, and course-associated information. Simon used his website exactly in this way. By visiting his website, I could also get an idea about what he would do in class before I observed it. He could communicate with his students both in and outside of school by managing the website, whereas Alvin and Theodore lacked this capability. Simon was making a difference in interacting with students outside of school by using the website. Although I could not determine whether managing the course website affected students' achievement, it was clear that Simon was creating a different classroom atmosphere, providing a sense of belonging to his students in their learning community.

As a learner, Simon went through the same teacher education program as Alvin and Theodore in learning to use technology for teaching. However, he was also a member of the Partnership for Reform in Science and Mathematics (PRISM[19]), a state-wide initiative of the Board of Regents in Partnership with the Georgia Department of Education. I believed that this had contributed to his development as a teacher. He worked and met regularly with high school Calculus teachers in the learning community of PRISM. In 2008, he attended a Georgia Mathematics Conference as a primary presenter and introduced an application of Winplot. This showed how actively he was working on becoming part of the learning community.

Like the other two teachers, Simon clearly recognized the benefits of using technology and the importance of using technology for better teaching and learning. As discussed in chapter four, technologies had become a big part of his teaching repertoire. Simon's case showed the largest overlap between learning experiences and actual practice among the three participant teachers.

[19] For more information, visit www.coe.uga.edu/prism/About_PRISM.htm

Summary

Alvin, Theodore, and Simon shared very similar experiences in the mathematics teacher education program at the University of Georgia. They all started teaching right after graduation, and when I observed their teaching, they were all in the same master's program in mathematics education. In spite of taking the same courses in their teacher education programs, their foci and intentions regarding using technology for teaching mathematics were not the same. To see what made the differences and the similarities, I consider these three teachers in terms of their professional growth in using technology. First, I use the three metaphors: ZPD, ZPA, and ZFM.

I discuss Alvin and Theodore together since they worked in the same school, sharing the similar growth in their profession with the same school recourses and culture.

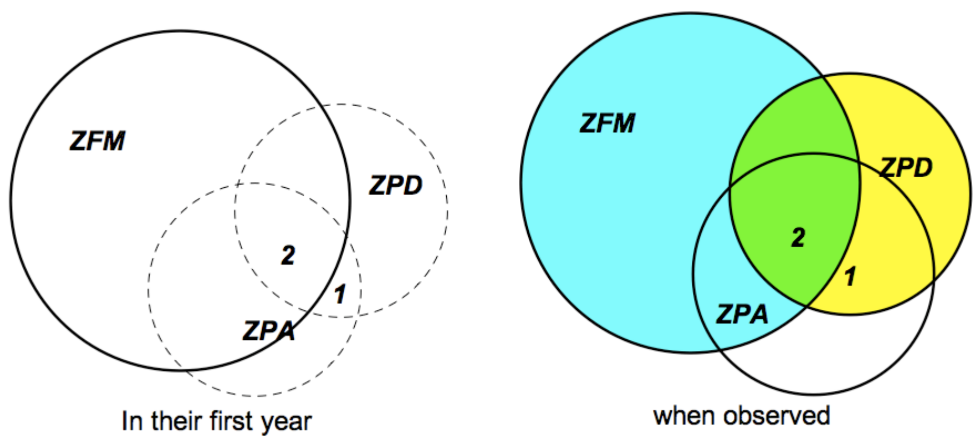

Figure 5.5: The relationship between the three zones in the cases of Alvin and Theodore

In figure 5.5, comparing the left picture to the right one, one can see how their working with technology changed over time. The two teachers' ZPA and ZPD expanded somewhat, whereas their ZFM did not change. They were continuously learning in the master's program in mathematics education and they also participated in several workshops. This was the reason for their growing ZPA and ZPD. However, their use of technology (especially the SMART Board) constrained their ways of teaching and promoted a lecture method, thus indicating instrumentation of their teaching by the technology; as such it could limit their ZPA.

In the right picture, the yellow region outside of ZFM represents where teachers and/or teacher educators could not manage to use technology for teaching due to environmental constraints -- for instance, poor technological resources and students' negative responses to technology. Region 1 includes their use of technology for exploring mathematical concepts, which was not observed but they were capable of doing so. On the other hand, region 2 represents their actual use of technology for pedagogical or general purposes in their classroom, which I could observe. I concluded that the factors of students and assessed curriculum were the main reasons for region 1. Alvin said that he was overburdened with covering assessed curriculum, so he focused on preparing his students for the graduation test or the end-of-course test. He also said that these tests would not contain proof questions, so he did not feel obligated to teach proofs. Alvin and Theodore believed that students hated proofs, and they did not want to lose their students. They seldom used technology to explore mathematical concepts, since proving and mathematical exploration require students' deep thinking. Region 1 represents this kind of constraint. However, their major experience with using technology in their teacher education program was using technology for exploring mathematical concepts, and they each mentioned the possible use of technology for mathematical investigation. I considered that, if some of the constraints on their teaching were lifted, they had the potential to use technology for such purposes.

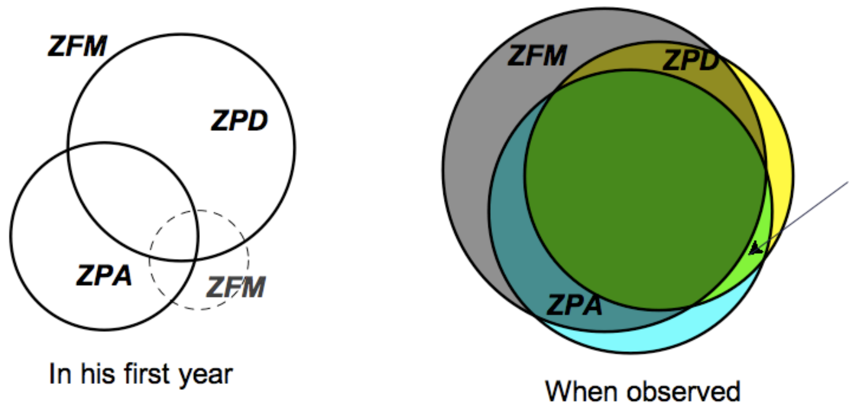

Figure 5.6: The relationship between three zones in the case of Simon.

In figure 5.6, two pictures show how Simon's working with technology changed over time. As mentioned earlier, he was not given any technological resources by his school except a laptop. The region of ZFM in the left picture describes this. All he could do at the beginning was create his personal website and communicate with students through the website. By purchasing technological tools, he expanded the ZFM in his workplace.

Compared to the two other participant teachers, Simon revealed more knowledge about technology integration for teaching concepts and types of technology. Although all three participants shared learning experiences from the same teacher education program, Simon had a different attitude. Examples include taking the EMAT 3500 course twice and his professional development with conferences and in the learning community with PRISM. All these tell how he could significantly expand his ZPA and ZPD. As discussed earlier, his ways of using technology and the types of technology he used were similar to those of the mathematics educators in his teacher education program. This is shown in figure 5.6. Thus, there was a considerable overlap between his ZPD and ZPA.

The light green part indicated by the arrow includes full lab experiments, students' individual use of computers or other tools, and a computer lab, which was not possible for him to access due to circumstances beyond his control. However, that part belongs to his ZPA and ZPD, since he experienced lab experiments, personal investigation of concepts with GSP, and the like in courses in his teacher education program. Simon's ZFM gave him more constraints than Alvin and Theodore in terms of materials and resources. However, the student factor was not a drawback for Simon. Rather, Simon's students welcomed his use of technology, whether it was for mathematical exploration or just for presentation.

In terms of how the participants created meaning while they used technology for teaching and which technological tools they preferred, I summarize this in figure 5.7.

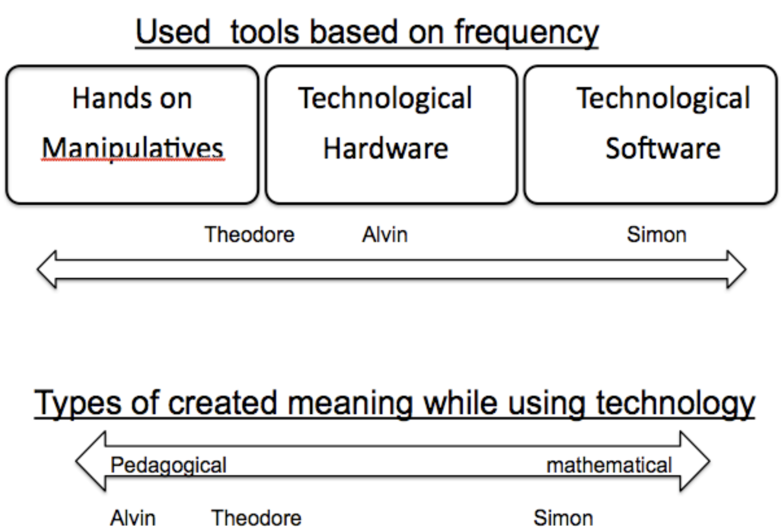

Figure 5.7: Differences between teachers in using technology.

I consider there to be three main factors that caused differences among the participants. First, their beliefs about students and mathematics lay behind their different approaches. For instance, developing students' positive disposition toward mathematics and learning mathematics was their common concerns. However, they took different approaches to doing this. Alvin focused on providing fun activities and better presentations with visual supports; Theodore developed students' learning community in the class, providing chances to collaborate with peers; Simon challenged students with "why" questions, stimulating their mathematical curiosity. Second, external pressures influenced their ways of using technology. In the interview and informal conversations, Alvin and Theodore constantly mentioned heavy burdens on them for covering assessed curriculum and students' test results. Simon also felt these pressures, but not so much as Alvin and Theodore did. Third, students' attitudes and abilities worked as affordances for Simon whereas they worked as constraints for Alvin and Theodore in terms of the different levels of working modes with technology.

In spite of their different working conditions, the three shared several things as follows:

- First, they all recognized the significance of using technology for teaching mathematics in school, and they were seeing the benefits from using it. I considered that they developed a productive disposition toward incorporating technology into teaching. This implied their potential to adapt to new technology in the future.
- Second, technology became a big part of their everyday lessons. Whatever the purpose of using it, the lessons that they taught incorporated technology. I never observed them teaching in a purely "chalk and talk" style.
- Third, they were all taking the lead in integrating technology into teaching in their schools. This was indirectly confirmed by their informal conversations with their colleagues during my visits and in interviews, although I could not confirm this by observing others' teaching.

- Fourth, they were creating positive meanings while using technology, regardless of the types of meanings. Technology was meaningful to Alvin, especially, in presenting his lectures, whereas it was meaningful to Theodore in assessing students' learning and their understanding and in using online resources. Simon usually created mathematical meanings while using technology. It was a very important instrument for him for exploring mathematical concepts, which was the core of his teaching.

In this chapter, I briefly reviewed the theories that worked as instruments for this study. I summarized how each participant teacher used technology and what their patterns were in using it. I also considered the three participant teachers together to make comparisons between them. I summarized the differences and similarities between them at the end. In the next chapter, I go back to the beginning and reconsider the research questions. By doing so, I elicit implications of this study and for the direction of future research.

CHAPTER 6

CONCLUSION

This chapter provides the overview of the study, the research questions and findings with respect to the model for this study. Implications and concluding remarks follow the overview.

Overview of the Study

Simply speaking, the purpose of my study was to see whether the graduates from the mathematics education department at the University of Georgia would demonstrate meaningful use of technology in their teaching of mathematics, and the answer was yes. Specifically, I aimed to see what factors of their learning experiences with respect to using technology for teaching mathematics in their teacher education program influenced their actual teaching. Three graduates who were in their second or third year of teaching in high school participated in this study. Their teaching was observed more than eight times per teacher. Observations were recorded either by using video or field notes. To increase the trustworthiness of study findings, I used methodological triangulation, which involves using multiple data resources such as interviews, observation, questionnaires, and documents (Denzin, 1978). Inductive data analysis, content analysis, and case-by-case comparison methods were used to analyze the data.

Research questions and findings

The research questions for this study were:

1. What patterns do teachers who have taken technology-integrated courses, including the EMAT3500 course, share in their teaching, and where did these patterns originate?

2. What links or gaps are there between what they learned from technology-integrated courses and how they are teaching in their classroom?

3. In what ways do the teachers use technology, and what factors of such courses are strongly related to teachers' use of technology in their teaching?

Returning to the model for this study in chapter two, I traced the participants' ZPA and ZPD; I sought to understand how those two factors affected their actual teaching with technology within their ZFM; I investigated how the teachers interacted with their ZFM. I discussed the answers for the above three questions at length with evidence in chapter 4 and 5. In particular, I focused on the answers to how each participant teacher used technology, what patterns they showed in using technology, and what were the differences and similarities between them in using technology. In this section, I zoom in on the arrow-shaped box between their learning and teaching and describe it with identified gaps and links (See figure 6.1 below).

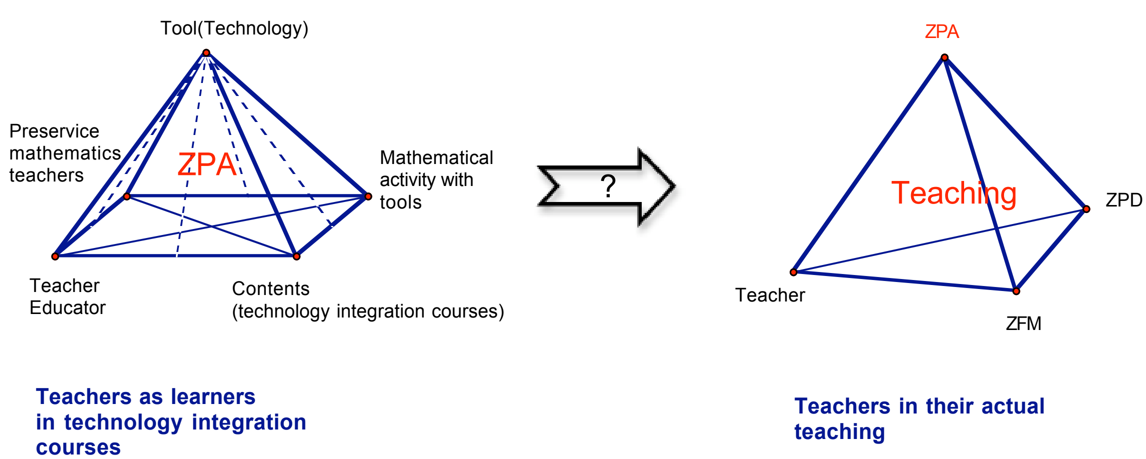

Figure 4 Model for this study

Gaps

Identified gaps between Simon's and Theodore's learning and teaching were found in their ways of using technology. Evidence, from the reflections and interviews with Alvin and Theodore revealed that their beliefs about students and mathematics attributed for students were the main factors that contributed to the gaps. In particular, Alvin told me that he believed that students with different aptitudes needed different mathematics. Such a belief affected his ways of using technology. Many of his students were unmotivated underachievers and the dropout rate of the school was about 50%. Thus, his primary concern was to help his students stay in class and be successful on tests. He, therefore, stressed procedural knowledge rather than conceptual knowledge and used technology to help his students become engaged in lessons. In this regard, I consider Alvin's belief about his students was the main factor that limited his ZFM. This also affected his ZPD since he focused on learning about fun activities or managing classes rather than learning through mathematical exploration using technology (*e.g.* his participation in a workshop for classroom management--as indicated in the background survey questionnaire, learning about the SMART Board applications for fun activities like Jeopardy Game on observation day 4). Theodore shared overall aspects of Alvin's use of technology except that he focused on fostering students' collaboration providing his students with hands-on manipulatives and graphing calculators. I asked Theodore regarding his changes in beliefs about students or teaching. He answered:

> You know, when you first get out of school and, you know, what you've been hearing in your undergraduate courses the whole time is, you know, "We're gonna go out there and we're gonna change every student, we're gonna turn things around." And so you start off thinking, you know, "Okay, every student, they are gonna make a complete 180. You're gonna…," but then you start to realize that there's some students that have already decided they're not gonna learn. (Interview, 1-23-2008)

In contrast, Simon revealed a narrower gap than Alvin and Theodore. Simon's primary concern in using technology was for mathematical explorations as discussed earlier. Figure 5.5 and 5.6 showed the difference between Simon and the others. Limited technological resources in his school were the main factors that limited his ZFM and contributed to the gap between his learning and teaching. In contrast, abilities and attitudes of Simon's students contributed to expand his ZFM, whereas those of Alvin's and Theodore's students limited their ZFMs.

Regarding the gap between Simon's learning and his teaching with technology, he usually used GSP or other technologies including his Newton's Law of cooling experiment in demonstration modes. His school was not equipped with technological tools and laptops for students' use. His students, therefore, used those technologies indirectly by asking Simon to try certain things. However, he used technologies as a primary user in his learning in technology-integration courses in his teacher education program. This was the salient difference between his learning to use technology and his teaching using technology.

Concerning overall links, I largely divide them into two: direct links and indirect links.

Direct links

Alvin did not show direct links between his learning and actual teaching in using technology. Theodore and Simon each showed some links.

Theodore used GSP in teaching centers of triangles. His lesson for the centers of triangles was not created, but copied from a mathematics education course. I considered that the way he used GSP to teach centers of triangles and the way he had learned it were identical, although he did add a hands-on activity with rulers and protractors to the lesson. Although Algebra Tiles are not technological tools, his use of Algebra Tiles, which he had learned to use in a mathematics education course, showed another strong link between his learning and his teaching. According to Theodore, he remembered what he had learned about GSP activities for exploring centers of triangles and using Algebra Tiles. He utilized his learning since he found that it fit with what he was to teach.

Simon's teaching revealed many links. First, his use of GSP in his Calculus class overlapped, in many ways, with that in his EMAT 3500 course. For instance, he constructed tangent lines and traced a point to teach the concept of derivatives, and used a unit circle with measures to teach trigonometric functions. Another link can be found in his Newton's Law of Cooling experiment. He basically conducted this experiment in the same way as he did in the EMAT3500 course. Some instructors in his teacher education programs used their websites to communicate with students, providing materials and information. He adopted this as an efficient way of managing his classes and communicating with his students. I consider his ways of using his personal website to be the third link between his learning and his teaching. Figure 6.1 summarizes direct links found in this study.

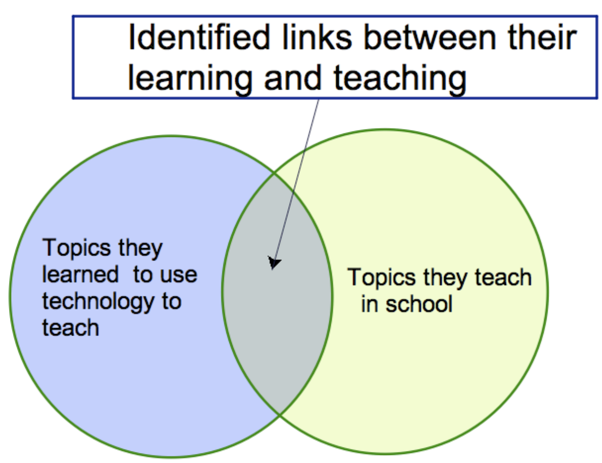

Figure 6.2: Identified direct links between participants' learning and teaching.

Although Simon showed that he applied what he learned to create new activities, he also borrowed some activities from mathematics education courses and used them. I consider that he internalized his learning with GSP so he could apply his learning to his teaching.

Indirect links

Indirect links were found when participants discussed their beliefs about technology and using technology. They all said that they were seeing many benefits from using technology in teaching, and were taking the lead in teaching with technology in their schools. I consider that their productive dispositions toward using technology for teaching belong to their ZPA, overlapped with their ZPD. As mentioned earlier, Theodore said that mathematics education articles were often assigned to him in his mathematics education courses. This led him to be able to adopt new perspectives in teaching mathematics. He also said, “I think they [mathematics educators] did a very good job as far as starting to get the teacher used to using technology.” I think these new perspectives could help the teachers use technology in meaningful ways and these indirect links presented valuable potential for use of technology.

Implications

The findings have several implications. First, in the example of Simon, he learned from his professors the idea of utilizing a personal website for keeping materials, sharing them with others, and communicating with students. This shows that teacher educators need to build their online community for their courses, since teacher educators are the role models that pre-service teachers look to in terms of using technological resources. Considering that the next generation will live more and more online, this is important. The recent increase in online courses is part and parcel of this development.

The identified direct links between what teachers learned and how they were actually using technology imply that when teacher educators teach how to use technology for teaching mathematics, they should consider using more core concepts in school mathematics that directly relate to the mathematics that pre-service teachers will be asked to teach. Alvin's and Theodore's knowledge about GSP was not productively used, as indicated by their limited activities with GSP. The only cases in which Theodore used GSP for mathematical exploration were when he was able to copy the same activities that he had learned in his courses to teach the same concepts. In the interview, he actually stated that he used the activity *because* he found that what he had learned was identical to what he was to teach. Although Simon showed that his knowledge of using GSP was internalized and therefore could be used to create new activities, he also copied the same activity or idea in the cases of the lab experiment and the origin of the term "tangent."

The lack of direct links of Alvin and Theodore supports Leatham and Olive's statement, "Learning technology for teaching does not guarantee their actual use" (Leatham and Olive, 2000). However, the direct links that I observed in Simon's teaching could suggest that by taking the EMAT 3500 course twice he was able to implement more of what he learned. One implication from Simon's experience is that pre-service teachers need to experience using technology for teaching mathematics in more courses in the teacher education program. We cannot simply expect Simon-like teachers. If more teacher educators were to use technology in their courses, a similar effect to Simon's repeated course-taking could be achieved.

Professional development opportunities and workshops for inservice teachers are important with regard to using technology for teaching. Such opportunities allow teachers to stay aware of what is going on in mathematics education and update or reinforce their knowledge about using technology. As far as technology integration is concerned, I believe that we can expect better results from inservice teacher training than teaching pre-service teachers. It seemed that inservice teachers are better ready to learn about using technology to teach mathematics and see the necessity. As Alvin said in his interview,

> [...]Recently, because of limited time and because of all the use of my SMART Board […] I use the SMART Board technology on my SMART Board as opposed to GSP, but I've definitely used old GSP files and would definitely recommend -- if I was talking to an undergrad right now who was taking EMAT 3500, I would definitely say to them to take the time to really create good files with GSP and Fathom, and take it seriously. And file it away in an organized manner so that you can retrieve it later, and understand that it really is not just a homework assignment for a college class, but that you're preparing for your future teaching. And I think, at the time, at least for myself and probably several others, we didn't really take it that seriously 'cause we never really -- you know, you never really see it that way as an undergrad (Interview, 1-21-2008).

When Simon was sitting in the EMAT3500 course for the second time, he was teaching in school. He did not explicitly talk about the reason for auditing the class a second time, but it seems to have been similar to what Alvin was talking about (taking the assignments seriously) and the necessity he found for using technology in the classes he was teaching.

The above has also implication for redesigning teacher preparation programs. The participants' readiness to learn increased when they were in their own classrooms faced with the reality of actually teaching. Considering this, a 5-year teacher education program that culminates with a full year of supervised teaching internship during which teachers have the support of both university professors and mentor teachers in their schools could be more productive. Such a model could optimize teachers' ZPA within the constraints of their ZFMs.

Limitations of the study

There is a lack of variability among the three participants in terms of backgrounds and teaching situations. At the beginning, seven teachers were considered as participants, but four teachers withdrew during the process. The four teachers who withdrew were teaching in different educational locales (urban, suburban and rural), included one female teacher and all had only a bachelor's degree.

The participant teachers were all male. Although I sent the recruitment letter (see Appendix F) to more female teachers than male (about 70% female), only one teacher among seven responses was female. As the recruitment letter emphasized the purpose of the study was to look at teachers' use of technology, there could be a gender issue in terms of either using technology or participating in a research study.

All participant teachers were in their master's program in mathematics education when their teaching was observed for this study. Thus, this study provides a narrow view limited to those who were in the master's program in mathematics education.

This study was conducted in a mid-sized city in Georgia. Thus, it is possible that cultural and environmental factors in that region could affect the study. This study might find different results from teachers outside of the city or the state. However, as a case study, the goal was not to generalize findings to other situations but rather to build explanatory models that other researchers or teacher educators could use to help explain their experiences of gaps or links in learning and teaching with technology.

Concluding remarks and future directions

One cannot stress enough the significance that the teacher has as a factor in education. Just like our past society needed good teachers to educate young generations, today's world also fully realizes the value of good teachers in school. However, the definitions of 'good teachers' are vague. A certain aspect about the definition is the fact that it depends on the time period in students' lives and the culture of the society. Considering education is a future oriented enterprise, good teachers should be capable of helping their children better prepare for their future life. In this regard, today's good teachers should help students deal with the rapidly changing world with highly advanced technology. In fact, technology integration in teaching has already become another component that teacher education programs should cover and the number of technology integration courses has been increasing accordingly. What I wanted to see from this study was how teachers' learning experiences in technology-integrated courses affect their actual teaching. The model that I developed based on the three zones helped capture and simplify the complexity of teaching situations. Findings in this study revealed a mixed but positive correlation between learning and teaching. What teachers bring to the classroom is a function of their ZPD and ZPA given their ZFM. In order to make sure the positive relationship is maintained and/or reinforced, further research efforts are required by teacher educators and researchers in order to optimize both teachers' ZPD and ZPA given their ZFM. To this end, I suggest the following future research as an extension of this study:

- First, researchers need to examine the same research questions for more teachers with different learning experiences in different teacher education programs to further refine or modify my explanatory models.
- Second, could this model be applied to teacher educators in teacher preparation programs to improve their teaching?
- Third, the gender issue in using technology for teaching is an interesting topic although it does not stem directly from the results of this study. Researchers need to investigate

whether gender influences teachers' using technology for teaching; if so, then how? Further, researchers should explore whether there are differences between genders in terms of dispositions and beliefs toward technology and using technology.

- Finally, I wonder how using technology for exploring mathematical concepts influences students' disposition toward mathematics and learning mathematics. I am especially interested in those students who are unmotivated and underachieving. Alvin and Theodore considered "students" as the reason why they did not prove or verify mathematical theorems and properties. They did not feel that their students would enjoy exploring mathematical concepts. Examining this topic will either support their beliefs or reveal their misunderstanding about their students. Thus I recommend including a focus on students' learning and affect within future research designs that look at links between teaching and learning with technology.

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APPENDIX A

Background Survey Questionnaire

Thank you so much for agreeing to participate in my study. I hope you will enjoy working with me. The following questions are very important for me to understand you as a mathematics teacher and your teaching. Please take your time to answer them thoughtfully.

Your teaching in School

1, How many years have you been teaching? Where (school) and when (year)?

School name	**From**	**To**

Total number of years:

2. What grades and what subjects have you taught? Find the corresponding cells by semester and subject, and write "grade level/number of classes" taught for that cell (e.g. 9/3 means grade 9, 3 classes for that subject). Leave blank for non-applicable cells.

Teaching Semester	**Alg. I**	**Alg. II**	**Geo-metry**	**Trig**	**Pre-Calc**	**AP Calc**	**AP Stat**	**Other**
Fa. 2006								
Sp. 2007								
Fa. 2007								
Sp. 2008								

3. Please list any extra school activities in addition to teaching (eg. coaching a baseball team in school, administrative work for school, or anything for school or students):

4. Normally, when do you start work in school, and when do you finish?

From: _______________ in the morning to:__________________ in the afternoon or evening.

How much time do you normally spend on school-work per week? List the activity and average amount of time spent on this activity during the week.

Activity	**Time spent per week**
Teaching classes	

Professional development or work

5. Have you attended workshops or conferences related to mathematics education? If yes, what are they and how many did you attend? Were they mandatory or voluntary?

Title of Conferences or workshops	**Period**	**Voluntary or mandatory**

6. If you have experience in #5, did the conferences or workshops that you attend meet your expectation?

Check one of the following.

- Very satisfied
- Satisfied
- Neutral
- Dissatisfied
- Very dissatisfied

7. If you want to learn from professional development opportunities, what kind of help will directly work for you to teach math for students?

That is to say, what do you want to learn for your teaching from the opportunities?

8. Do you have other experiences to develop your teaching career (degree program or non-degree program)?

Technology to you

** You can consider computer software, hardware, calculators, computerized tools, physical artifacts for hands on activities for class activities or things like that.*

9. Do you have any technologies (software, hardware, or physical artifacts for teaching) that you purchased personally for use in your classroom?

If yes, what are they?

10. What kind of technology have you been using in your mathematics teaching in class and how often?

	Technology type	How often it has been used
G		
E		
O		
M		
T		
R		

Y		
A	Technology type	How often it has been used
L		
G		
B		
R		
A		
	Technology type	How often it has been used

**You can correct the subject and add other subject if the given is not appropriate to your experience.*

From this question, I want to know whether you have specific subjects that you prefer to use technology to other subjects. Plus, what technologies are usually used in what subjects. I really appreciate your cooperation!

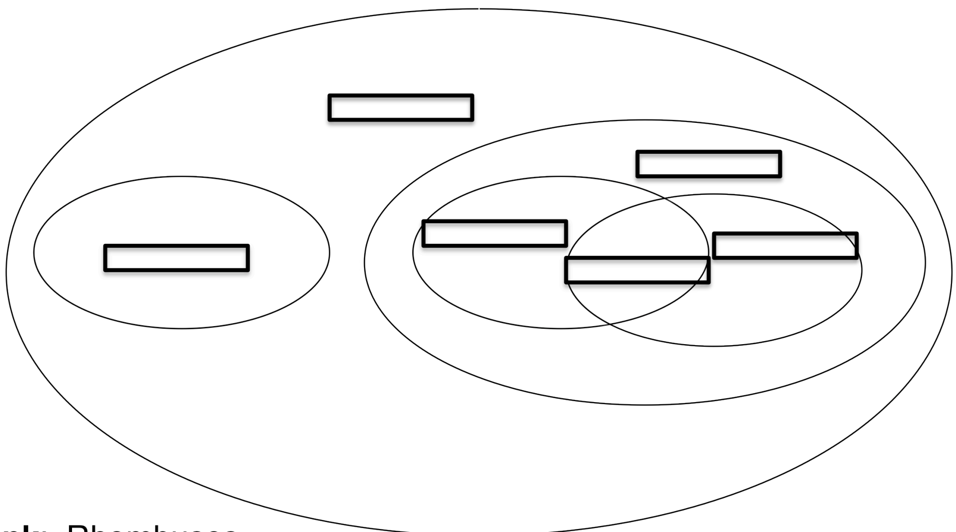
APPENDIX B

Interview Protocol

1. Why don’t we start with reflecting on your teaching? Did the lesson go the way you intended?
2. Can you compare what happened to what you had planned?
3. Why did you choose them (physical artifacts, technologies, or other) for the class activities?
4. In terms of teaching the concepts (at hand), how do you think about your choice? Was it the best choice? Or are there alternative ways to teach them, which were not available to you? (Depending on teaching situations, more questions including mathematical questions, the way to use technologies or activities would be framed.)
5. What did you intend for students to learn from those activities? Do you think students’ Performance reached your expectation? Could you evaluate your teaching and students’ learning?

APPENDIX C

Sample Handout from Alvin's class (09-03-2008)



Word Bank: Rhombuses

Trapezoids Squares

Parallelograms Rectangles Quadrilaterals

Properties:

1. Opposite sides are parallel →
2. All sides are congruent →
3. All angles are right angles→
4. Opposites angles are congruent→

5. Diagonals bisect each other→
6. Diagonals are perpendicular→
7. Opposite sides are congruent→
8. Diagonals are congruent→
9. Diagonals bisect angles→
10. Consecutive angles are supplementary→

<u>What do I know about a Parallelogram?</u>

- A ________ of a parallelogram divides the parallelogram into two congruent triangles.
- Opposite sides of a parallelogram are ________.
- Opposite angles of a parallelogram are ________.
- Consecutive angles of a parallelogram are ________.
- The diagonals of a parallelogram are ________ each other.
- A rhombus is a ________.
- A rectangle is a ________.
- The diagonals of a rhombus are ________.
- The ________ of a rectangle are congruent.
- The diagonals of a kite are ________.
- A ________ is a rectangle.
- A square is a ________.

- The diagonals of a square are ________ and are the ________ bisectors of each other.

What conditions of a Parallelogram do I know?

- If two pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
- If one pair of adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.
- If the diagonals of a parallelogram bisect the angles of the parallelogram, then the parallelogram is a rhombus.
- If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

APPENDIX D

Sample Handout from Theodore's class

Lesson NAME ________________________ DATE____________

11.3 Math and History Application

For use with page 662

History Public opinion polls are a means of gathering information. They generally gather opinions on different topics including government and politics. Polls are often used by the media to monitor opinions of voters about political candidates.

One of the first national polls was the Gallup Poll, which was developed by a man named George Gallup. The first Gallup Poll was reported in the *Washington Post* on October 20, 1935, about people's opinions of Roosevelt's New Deal programs. Polls had been conducted before this, but this poll gained national attention because it was printed on a full front page of the *Post*. Other public opinion pollsters include Roper and Crossley, who also conducted political polls.

Early polls were conducted by people going door to door collecting information. Beginning in the 1950s, telephone polling became the standard way to conduct a poll. Now people are polled by the telephone or by participating in the Internet poll.

Math Early polls had begun to predict presidential elections with great accuracy. However, the 1948 election poll was considered a disaster because it wrongly predicted the election results and the incorrect results were even printed in newspapers. Many people lost confidence in polls after this election, but the public confidence has returned and public opinion polls are still used today.

Table 1

1948 Presidential Election Poll Prediction				
Candidates				
Pollster	**Dewey**	**Truman**	**Thurmond**	**Wallace**
Crossley	49.9%	44.8%	1.6%	3.3%
Gallup	49.5%	44.5%	2.0%	4.0%
Roper	52.2%	37.1%	5.2%	4.3%

Table 2

Percentage of Votes Counted in 1948 Election			
Candidates			
Dewey	**Truman**	**Thurmond**	**Wallace**
45.0%	49.4%	2.4%	2.4%

1. According to the poll predictions, who would win the presidential election in 1948? Did the poll accurately predict who won the election**?**

2. The total number of votes counted in the 1948 presidential election was 48,836,579. Use the percentages in Table 2 to determine how many votes each candidate received.

APPENDIX E

Sample Handout from Simon's class

Algebra II
2.5: Correlation and Best Fitting Lines

- After today, we should be able to…
- Use a scatter plot to indentify correlation between two variables
- Approximate lines of best fit by hand
- Use regression on the TI to come up with a line of best fit (least squares regression line)
- Use a regression equation to make predictions

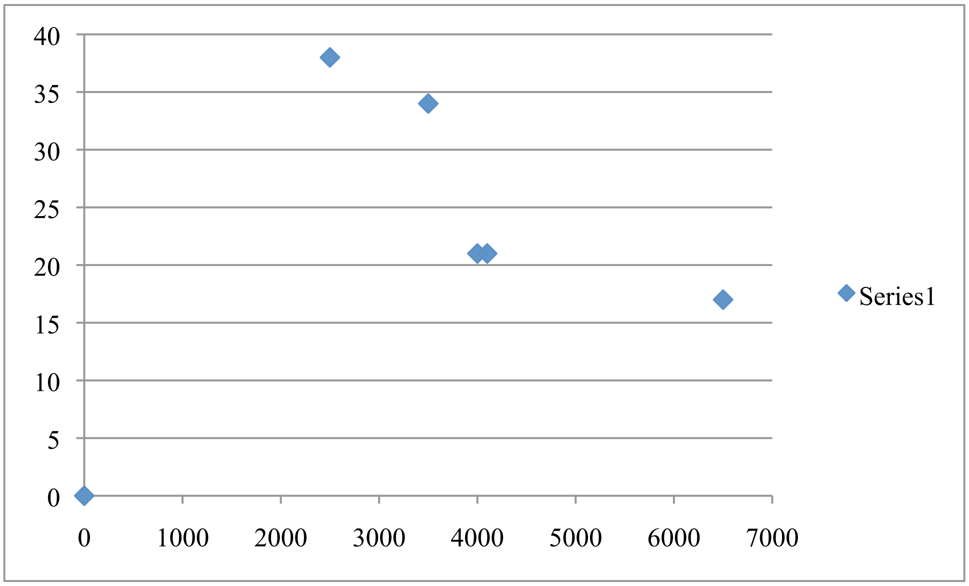
We say that two variables are **correlated** if…

Two variables are **positively correlated** if…

Two variables are **negatively correlated** if…

Two variables have relatively **no correlation** if…

Approximating a line of best fit:



The **strength of correlation** can be thought of graphically as…

Weak Negative Correlation:

Strong Negative Correlation:

Weak Positive Correlation:

Strong Positive Correlation:

Using the TI to determine an equation for the least-squares regression line:

(Data from Chris Frankins' Statistics: *The Art and Science of Learning from Data*)
The table below lists several 2004 model cars with automatic transmission and their weight, x, and gas mileage, y.

Model/Make	X: Weight (pounds)	Y: Mileage (miles per gallon)
Honda Accord Sedan LX	3164	34
Toyota Corolla	2590	38
Dodge Dakota Club Cab	3838	21
Jeep Grand Cherokee Laredo	3970	21
Hummer H2	6400	17

a. What type of correlation is indicated in the table?

b. Use the TI to plot the data and build a scatter plot.

c. Use the TI to determine the regression equation. Write the equation and graph the line of best fit on the scatter plot.

d. A particular Ford Mustang weights 3350 pounds. Use the regression equation to predict the gas mileage for the car.

e. If the actual gas mileage for the Ford Mustang is 20mpg, is the prediction in part d an underestimate or an overestimate? What factors might cause the predicted and actual mileage to differ?

Steps for creating a scatter plot on the TI

1. Clear everything from your "y="

2. From the home screen, press [STAT], select edit.

3. Enter x-values into list 1 and y-values into list 2.

4. From the home screen, press [2nd], [y=] to get to the "stat plot' menu

5. Select plot 1.

    a. Turn it on
    b. Be sure the scatter plot is selected
    c. Make sure the x-list is L1 and the y-list is L2
    d. Choose your marker
6. Press [Zoom], select option 9 "zoom stat"

Steps for linear regression on the TI:
1. Plot the data first
2. From the home screen, press [stat], [→], and select "linreg(ax+b)"
3. Once you have "linreg" on the home screen, input: L1, L2, Y1
    a. To get L1, press [2nd], [1]
    b. To get the comma, press the button above the 7
    c. To get L2, press [2nd], [2]
    d. To get Y1, press [vars], [→], [Enter], [Enter]
4. Press [Enter] to run the regression.
5. Press [Graph] to see the line. You should be on zoom stat.