TEACHERS MAKING SENSE OF A

MATHEMATICAL PROFESSIONAL DEVELOPMENT EXPERIENCE

by

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(Under the Direction of Paola Sztajn and Chandra Orrill)

ABSTRACT

The purpose of this study was to understand how teachers make sense of their professional development experience for their own learning, their students' learning, and their teaching. Three teachers were observed and interviewed during a professional development course where the goal of the course was for the teachers to develop their mathematical content knowledge. The mathematics instruction of the course was similar to how these teachers are expected to teach in their classrooms with a course emphasis on using technology to explore mathematics. The participants' experiences were broken into their making sense of the mathematics, technology, and problem solving, and their making sense was observed as assimilation (content was not problematic) or perturbation (content was problematic) and how they dealt with each. Each of the participants' experiences is presented as case studies followed by a cross-case analysis.

INDEX WORDS: Mathematics education, Professional development, Educational reform, Constructivism, Assimilation, Perturbation, Accommodation, Case study, Cross case analysis.

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CHAPTER 1

INTRODUCTION

Jasmine, a kindergarten teacher, decided to take an InterMath course hoping it would allow her to review the mathematics content that she would be tested over on the Praxis II teaching exam. She often expressed that she was uncomfortable answering in front of the entire class because of her self-perceived low level of mathematics ability. On the last night of class, Jasmine caught me in the hall, away from the rest of the class, and told me that she was more confident in her mathematics abilities. She also said that she realized that they were all there to learn from each other and that there was not always one correct way to solve a problem. I was very proud that Jasmine had come so far in such a short time.

Rationale

I have been teaching/observing mathematical professional development courses for three years. While working with teachers in these courses, I have noticed trends in participant comments and engagement in activities. The participants' observable interactions with the mathematics, each other, and the instructor (often myself) were of interest to me as I wondered how these experiences "made sense" to them. Because of this curiosity and the continuing need to understand how teachers learn new ways of teaching and what their roles are in facilitating learning experiences that are in line with current curriculum standards (Putnam & Borko, 2000), I have studied teachers attending a professional development course in mathematics. The overarching research question that guided my work was: how do teachers “make sense” of their professional development experiences?

In the last fifteen years, mathematics educators have seen the release of *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics [NCTM], 1991) and *Principles and Standards for School Mathematics* (NCTM, 2000). These documents propose a reformed mathematics curriculum not only focused on when certain concepts should be taught, but also on how they should be taught. Teachers are encouraged to move away from instruction where students obtain knowledge through the transmission of information and mastery of basic skills and computational procedures before exploring “higher order” problems (Goldsmith & Schifter, 1994). While moving away from teaching strategies that are more teacher-centered, teachers are encouraged to move toward standards-based teaching practices that are more student-centered (NCTM, 1991; 2000).

Standards-based instruction[1] allows students to actively participate in the mathematics classroom by using technology to explore mathematics, solve problems, communicate mathematical ideas, connect topics within mathematics and to other subjects, and move between multiple representations (NCTM, 2000). These ideas promote teaching mathematics for understanding rather than memorization.

The national standards have influenced the development of some state standards, including the new Georgia Performance Standards. Georgia’s teachers are now expected to teach mathematics content while incorporating process standards, which include similar ideas to those set forth by NCTM such as technology use and communicating

[1] Standards-based instruction refers to the teaching practices proposed by NCTM.

mathematics. “The performance standards isolate and identify the skills needed to use the knowledge and skills to problem-solve, reason, communicate, and make connections with other information” (Georgia Department of Education [GADOE], n.d.). More specifically, the Georgia Performance Standards include a variation of the following process standards for all grades, k –12:

- Using the appropriate technology, students will solve problems that arise in mathematics and in other contexts.
- Students will investigate, develop, and evaluate mathematical arguments.
- Students will use the language of mathematics to express ideas precisely.
- Students will understand how mathematical ideas interconnect and build on one another and will apply mathematics in other content areas.
- Students will create and use pictures, manipulatives, models, and symbols to organize, record, and communicate mathematical ideas. (GADOE, n.d.)

In these standards, the emphasis on student learning requires a different style of instruction (Lappan & Briars, 1995). For many teachers, this new teaching style is a “substantial departure from [their] current practice” (Borko & Putnam, 1995). For example, research tells us that teachers are not comfortable emphasizing problem solving in their classrooms, partially due to their lack of experience in doing so, their tendency to use teaching practices they experienced as students, and their lack of deep mathematical knowledge (Ball, Lubienski, & Mewborn, 2001). Based on the classroom activities and learning environments that teachers are now expected to foster in their classrooms, Cohen and Ball (1990) questioned, “how can teachers teach a mathematics that they never learned, in ways that they never experienced?” (p. 352). Due to this dilemma, professional developers of mathematics teachers have recommended that professional development courses should be taught in a way that is similar to the way in which teachers are expected to teach (Schifter, 1998). In particular, professional development courses that focus on teaching mathematics content should engage teachers in using technology, problem solving, communicating mathematics, and making connections.

The professional development used as the context for this study is called InterMath. In InterMath, teachers are exposed to standards-based teaching strategies through their engagement as learners of mathematical content. This includes engaging teachers in mathematical problem solving using technology. While experiencing these standards-based teaching strategies as learners in InterMath courses, teachers may come to question their fundamental beliefs about mathematics: what mathematics is, what it means to know mathematics, how students learn mathematics, and what teacher roles are in the mathematics classroom (Wilson, Hannafin, & Ohme, 1998). The approach used in InterMath focuses on developing mathematical power including understanding, using, and appreciating mathematics, with the main interest being in empowering teachers through the use of technology in mathematics exploration, open-ended problem solving, mathematics interpretation, development of understanding, and mathematical communication (Wilson et al., 1998).

Calls for teaching mathematics for understanding have been around for many decades (e.g., Brownell, 1947; Polya, 1962). With some of these writings dating back almost sixty years, it is still my experience that teaching strategies focused on understanding are often new to teachers when they experience them in an InterMath professional development setting. Therefore, InterMath courses are designed to teach mathematical content while supporting teacher learning in a context where standards-based teaching strategies are being implemented.

In observing that these teaching strategies are new to some participants of the professional development, it is also my experience that teachers may attempt to understand the teaching that is going on rather than solely focusing on learning the mathematics content. This process of understanding was the focus of my research, as this study sought to explore the following question:

- How do teachers make sense of their experiences in a mathematics content professional development course?

Model of Making Sense

An initial hypothesis of this study was that there existed a hierarchy that the participants went through while trying to make sense of their professional development experiences. In InterMath, the participants were placed in the role of the student because the major goal of the course was for them to learn mathematical content. In the proposed hierarchy, participants were expected to consider what and how their students learn and how they teach only after attempting to learn the content for themselves. This has been evident in previous courses where participants actively discussed implementation, or plans of implementation, of different teaching strategies in their classrooms based on their experiences in the InterMath courses.

Figure 1 presents a model showing this hierarchy. In this model, participants first try to make sense of their experience for their own learning, considering whether the teaching style used in InterMath is beneficial to them as learners (i.e., can I learn from the teaching approach used in InterMath?). Once the participants make sense of the experience for their own learning, they can then think about how this experience makes sense for their students' learning (i.e., if I can (or cannot) learn this way, can my students learn this way?). Once participants consider their student learning, they can then try to make sense of the experience for their teaching (i.e., if I think my students can learn this way, can I teach this way?).

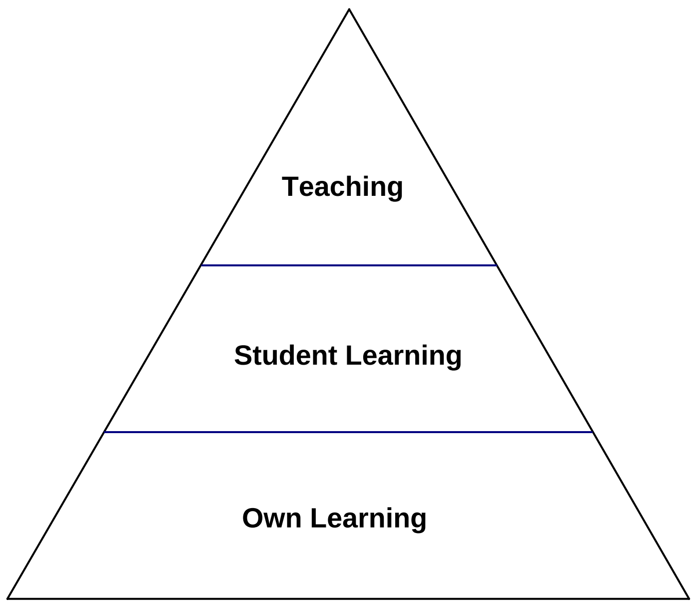

Figure 1. Hypothesized hierarchy of making sense. This model shows the process that participants may go through when trying to make sense of the professional development for their own learning, then for their students' learning, and finally for their teaching.

As a concrete example for understanding the proposed hierarchy, we can consider the InterMath use of technology to explore mathematics. According to the model, the teacher must first see using technology as beneficial for her learning of mathematics before she can think about how it would benefit her students. She may find that exploring the mathematics using technology actually allowed her to make connections between what were once disjoint mathematical ideas. She may then decide that her students may also benefit from using technology when investigating certain topics. Once she sees the value in using technology to enhance student learning, then she may attempt to incorporate different technologies in her teaching.

The hypothesized hierarchy indicates that teachers do not think about implementing new teaching ideas without going through the process of thinking about benefits to their own learning and benefits to their students' learning first. The research presented here tests this model. Thus, my more focused research questions can be stated as:

1. How do teachers make sense of their experiences for their own learning, for their students' learning, and for their teaching in a mathematics content professional development course?
2. What model best represents how teachers make sense of their professional development experiences?

CHAPTER 2

CONCEPTUAL FRAMEWORK/LITERATURE REVIEW

Making Sense

How teachers *make sense* of their learning experiences is the investigation focus for the study presented here. Simon (1995) noted that thoughts and assumptions are modified as one attempts to "make sense" of classroom experiences. Cobb (2000) further asserted that, "teachers reorganize their beliefs and instructional practices as they attempt to make sense of classroom events and incidents. Hence, teachers' learning, as it occurs in a social context, can become a direct focus of investigation" (p. 312).

Based on the Mathematics Teacher Development Project, which involved pre- and in-service elementary teachers in a professional development that focused on establishing standards-based teaching practices, Tzur, Simon, Heinz, and Kinzel (2001) reported on how teachers made sense of the project by looking at differences/similarities between teachers' thinking and practice. The study considered teaching as a reflection-interaction cycle through which knowledge was always changing based on teachers' ideas about mathematics, mathematical activity, and teaching-learning processes of mathematical content. For the researchers, each observation of the teachers led to an account of practice that represented their, "commitment to comprehend how the teacher organizes her or his experiential reality with respect to teaching mathematics. [The researchers] assume that everything the teacher does makes sense from her or his perspective" (p. 234).

Tzur et al. (2001) report on the case of Nevil, a fifth grade mathematics teacher. The researchers found that Nevil understood the mathematics that he taught, set mathematical goals that focused on understanding, and was aware of students' failure to understand the mathematics; however, Nevil was unable to question his fundamental assumptions about mathematics (e.g., mathematics exists in an objective reality). The researchers found it was never clear if different views of mathematics were ever addressed in the professional development Nevil experienced, and he may not have been aware of other possible perspectives about mathematics that he could/should think about.

Tzur et al. (2001) contributed to the research on how teachers make sense of professional development by making connections between the mathematical assumptions held by the participant, the goals of the professional development, and observed teaching episodes. They concluded their report by inviting others to consider "how teachers make sense of what happens in their classrooms and what they encounter in teacher development situations" (p. 250).

In her doctoral dissertation, Nipper (2004) examined teachers' sense-making processes in professional development situations. She focused on the context of teaching practices and looked at how teachers made sense of their professional development experience (a) *from their practice* based on their prior experiences; (b) *for their practice* based on their intended experiences for teaching; and (c) *in their practice* based on their post-professional development teaching experiences. Nipper looked at the differences in how teachers constructed meaning from their professional development during, immediately after, and throughout the next school year and what this meant for their teaching practices. She found that the three teachers involved in her study showed evidence of change in their "knowledge and beliefs about mathematics, mathematics teaching, and mathematics learning" (p. 105). She noted that all of the teachers seemed to make sense of their professional development experiences in different ways. Nonetheless, she concluded that the teachers' understandings progressed from personal learning to teaching practices, such as in the case of Sue, one of the teachers in the study, who progressed from problem solving as a goal to achieve to problem solving as a means for teaching.

Nipper (2004) contributed to the literature by examining connections made by the professional development participants during, immediately after, and throughout the following school year as they connected the ideas of the professional development to their teaching practices. A major shortcoming of her research was that the professional development only lasted for one week. Therefore, the participants did not have a lot of time to really think about the ideas of the professional development while attending it.

As a professional developer, one cannot pour beliefs about teaching, student learning, or mathematics into the heads of the teachers—just like the teacher cannot pour mathematics into the heads of the students. Cobb, Wood, and Yackel (1990) discovered that they were negligent in assuming that they could "transform the teachers into [people] who thought just like [the researchers/professional developers] did" (p. 145). The fact is that professional developers can only provide the teachers with experiences for them to make sense of for themselves. Therefore, it is important to continue searching for an understanding of how teachers make sense of those experiences.

Constructivist Learning Theory

Researchers in mathematics education have produced a wide body of evidence that supports the view of learning as "the process of an individual mind making meaning from the materials of its experience" (Knoblauch & Brannon, 1984, as cited in Lappan &

Briars, 1995, p. 133). This view of learning is referred to as constructivism, a theory positing that learners build knowledge based on experiences while relying "on their peers, tutors, teachers, and themselves for feedback" (Lappan & Briars, 1995, p. 133). Richardson (2003) defined constructivism as a:

> learning theory that suggests that human knowledge is constructed within the minds of individuals and within social communities. The theory states that individuals create their own new understandings based on the interactions of what they know and believe with the phenomena or ideas with which they come into contact. (p. 403–404)

Piaget's work on children's learning laid the groundwork for constructivism. In this work, Piaget studied how living organisms organize and integrate "their experiences and activities into a system" (Penrose, 1979, p. 18). When a child is thinking and learning, he is "absorbing his experiences" and "integrating them into his internal mental or cognitive structure" (Penrose, 1979, p. 18). In studying this organizing, absorbing, and integrating, it is student thinking that is being studied. Therefore, constructivists value student thinking (Lappan & Briars, 1995).

Constructivism is multifaceted. It is considered an epistemology, philosophy, and learning theory. Constructivism is not considered a theory of teaching; however, it has implications for teaching because it requires that teachers focus on what students think and what students can do with the material presented to them (Noddings, 1990). Even if one does not accept constructivist premises, one can embrace the pedagogical methods that come out of constructivist ideas (Goldin, 1990; Noddings, 1990). In the mathematics-teaching arena, constructivists argue that children must be given opportunities to think as they construct their individual mathematical understandings (Baroody & Ginsburg, 1990). More specifically, Goldin (1990) suggested that children be given opportunities involving guided discovery, meaningful application, and problem solving instead of imitation and rote learning. Cobb, Wood, and Yackel (1990) called this a form of "teaching compatible with constructivism" rather than "constructivist teaching" (p. 146) because, as Simon (1995) warned, constructivism "does not tell us how to teach mathematics" (p. 114).

Teaching practices proposed by the NCTM (1991, 2000) are in line with the forms of teaching that are compatible with constructivism as teachers are encouraged to allow students to engage in problem solving, reasoning, and proof while communicating the mathematics, making connections between mathematics and other subjects, and using multiple representations of the mathematics (NCTM, 2000). Again, Lappan and Briars (1995) said that constructivists "*value students' thinking*" (p. 133, italics in original) and the teaching strategies presented by the NCTM foster students doing just that – thinking. Thus, the idea of standards-based teaching is broadened to include the teaching that is compatible with constructivism.

Professional Development Supporting Standards-Based Teaching

One way to address teachers' lack of experience with standards-based teaching is for teachers to attend professional development initiatives that present these teaching ideas while supporting teachers' thinking about student learning and mathematics (Smith, 2001). Smith stated that such professional development requires a "great deal of learning on the part of the teachers" (p. 3). What is actually being called for is a major paradigm shift by the teachers in their classrooms as they are encouraged to move from a classroom promoting memorization, repetition, and correct answers to a classroom promoting communication, inquiry, and investigation. Therefore, teachers participating in professional development need to create new understandings based on their interactions, beliefs, and ideas.

Professional development that aims to help teachers learn about student thinking and mathematics may be facilitated by highlighting the former, the latter, or both. With this in mind, one can consider Cohen and Ball's (1999) work focusing on "interactions among teachers and students around educational material" (p. 2) to think about professional development (see Figure 2). This instructional triangle shows the interactions that occur in the mathematics classroom between the teacher, students, and mathematics. We can think of the professional development of teachers using a similar triangle with the professional developer outside of this triangle, encouraging participating teachers to think about student learning (or students) and mathematics.

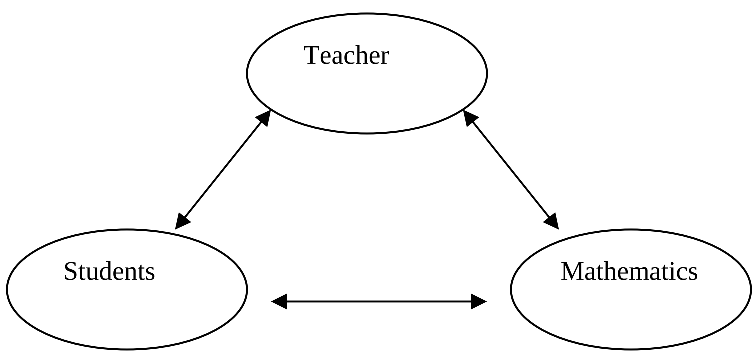

Figure 2. Instructional triangle. This triangle is based on interactions between teachers, students, and mathematical content based on Cohen and Ball (1999).

In the following section, three approaches to professional development are discussed: (a) those that encourage teachers to think about mathematics *and* student thinking; (b) those with main focus on student thinking; and (c) those with the main focus on mathematics. For each of these three approaches, the professional development goals are discussed in the context of the instructional triangle and then two examples are given. The professional development examples discussed here are not meant to be an exhaustive list. Rather, the discussion of these examples aims at providing examples.

Professional Development Focused on Mathematics and Student Thinking

The professional development efforts in this category have the dual purposes of building mathematical content knowledge of teachers while thinking about student thinking. In representing these types of professional developments with the instructional triangle, we have major interactions between the teacher and student thinking (changed from “students” in the original triangle) and between the teacher and the mathematics (see Figure 3). When the activity of the professional development is focused on student thinking, the teachers can also ponder the mathematics through the lens of student thinking. When the activity of the professional development is focused on mathematics, the teachers can also ponder student thinking through the lens of mathematics.

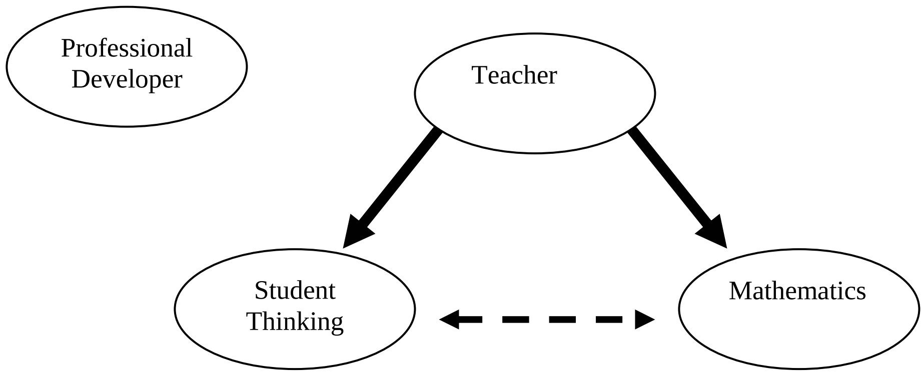

Figure 3. Mathematics and student thinking interactions triangle. This professional development interactions triangle is based on the goal of professional development being for teachers to focus on mathematics *and* student thinking. The dashed arrow between students and mathematics denotes teachers thinking about mathematics through student thinking *and* thinking about student thinking through mathematics.

One project that aimed at improving teachers' knowledge of mathematics and of student learning was the "New Jersey Project" (Maher & Alston, 1990). The professional developers claimed that:

> by *doing* mathematics, (teachers) have an opportunity to become more *aware* of their own mathematical thinking as they work to build greater understanding of some mathematical ideas. As they engage in their own problem solving, opportunities naturally evolve in which they become more aware of their personal approaches and begin to consider the *implications* of this experience for the learning of their *students*. (p. 151, italics in original)

In this professional development, there were two stated goals for the teachers. First, the teachers were to develop their own mathematical content knowledge through problem solving. The teachers explored mathematical situations while seated around a table containing different manipulatives such as chips and pattern blocks. Teachers chose a manipulative to construct a mathematical situation and justify their solutions before being asked to share with their peers. The professional development encouraged teachers to move from their roles as problem solvers to reflecting on their own problem-solving behavior and then to consider student learning. Student learning was explored as a separate activity where the teachers' task was specifically to pay attention to children's thinking. The teachers watched videotapes of students doing mathematics, followed by each teacher interviewing a student and building a model of the student's thinking about the mathematics observed in the video.

Findings of the New Jersey Project involved major changes in teaching practices as teachers moved from direct instruction to encouraging students to find more than one way to approach and solve problems, understand other students' solutions, and accepting responsibility for finding and correcting their own errors. Therefore, this professional development focusing on mathematical content development and student understanding, resulted in the teachers implementing teaching strategies in their classrooms that may contribute to students getting a deeper understanding of the mathematics that they are learning.

Similar to the New Jersey Project, the Developing Mathematical Ideas (DMI) (Cohen, 2004) professional development project also emphasized teachers learning mathematical content and student development of mathematical ideas. DMI seminars allowed teachers to "engage in the study of the meanings and complexities of the mathematics of the elementary curriculum" (p. 11), including, but not limited to, number, operations, and geometry. DMI's content addressed complex mathematical concepts that both teachers and students struggle with if they are not afforded opportunities to explore them.

In addition to addressing complex mathematical concepts, DMI allowed teachers to examine elementary school students' mathematical thinking related to the concepts addressed in the curriculum. The teachers engaged in careful analyses of children's mathematical thinking by reading written cases or viewing video cases and then discussing them as a group. Further, teachers read and discussed essays connecting the mathematical content to research on the development of these understandings in children. This allowed the teachers to "build a larger picture of what might be generalizable" from what they have been studying as a link to the broader research community (p. 12). DMI findings included that all of the teachers who participated came to believe that children can and do generate mathematical thoughts. For those teachers interviewed and observed, their teaching practices moved toward increased focus on student reasoning (Education Development Center, 2000).

Professional Development Opportunities Focused on Student Thinking

Some professional development projects have the major goal of providing opportunities for teachers to examine student learning and thinking. While these professional development environments allow teachers to also think about mathematics, teacher learning of mathematics is a secondary goal. In representing these types of professional developments with the instructional triangle, we have the major interaction between the teacher and student thinking (see Figure 4). Even though building mathematical content knowledge is not the major intended goal, teachers can still ponder mathematics through the lens of student thinking.

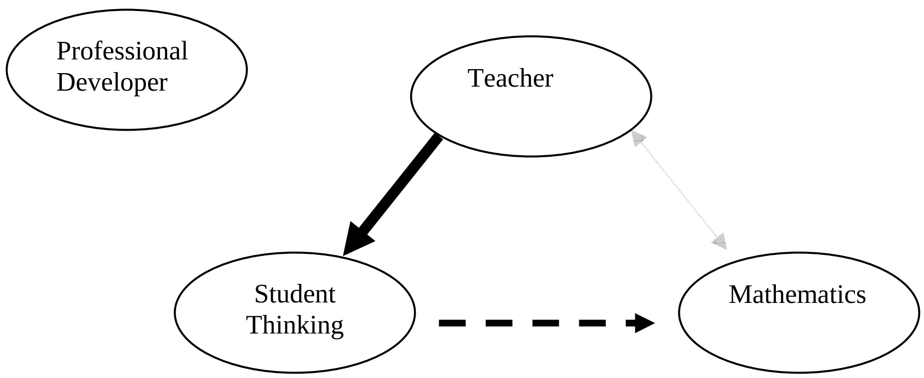

Figure 4. Student thinking interactions triangle. This professional development interactions triangle is based on the goal of professional development being student thinking. The thick arrow of teachers engaging in student thinking denotes the main goal. The secondary goal is denoted by the dashed arrow of teachers engaging in the mathematics through their students' thinking.

Cognitively Guided Instruction (CGI) (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996) was a well-known example of a professional development based on students' thinking. When CGI began, the professional developers believed that there was a relationship between teachers' study of children's thinking and change in teaching practices that could lead to greater student achievement in mathematics. They developed an intervention that was based on helping "teachers build relationships between an explicit research-based model of children's thinking and their own students' thinking by encouraging reflection on how the model can be interpreted in light of their own students and classrooms" (p. 405). Teachers who attended CGI workshops watched videos of students solving problems, examined student work, and engaged in discussions of what and how the students were thinking about the mathematics. Although mathematics was involved as the context in which the students' thinking was intertwined, development of the teachers' mathematical content knowledge was not the primary goal of the professional development. Rather, the focus was on developing teachers' knowledge of the research findings on students' mathematical thinking. Some findings from CGI included that teachers who participated were more likely to agree with problem solving being the focus of instruction and spent significantly more time on problem solving and less time on teaching number facts than did the control teachers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

The Purdue Problem-Centered Mathematics Project (Cobb, Wood, & Yackel, 1990) was another professional development effort designed to allow teachers to think about student learning. This project conducted research that attempted to "coordinate a constructivist view of leaning mathematics with the practice of teaching for the purpose of analyzing children's mathematical leaning within the setting of the classroom" (p. 125). The project team's original purpose was to conduct research on a single teacher based on the belief that classrooms were learning environments for teachers. This teacher watched videos of interviews done with her second-grade students and then discussed her understanding of her students' thinking with the research team. The teacher seemed to be memorizing a list of technical terms she could use with the research team that seemed to have no relevance to her teaching.

Due to the interactions with this one teacher, teacher development became a primary focus of the Purdue Project along with the researchers original focus on children's learning. Participating second-grade teachers went to a summer institute conducted by the research team/professional developers, were visited in their classrooms during the school year by the research team/professional developers, met in small groups once a week to discuss classroom experiences, and participated in after-school work sessions. Teachers watched videos of students working through mathematics problems and were given opportunities to discuss their observations. Findings of the Purdue Project included the teacher participants questioning their assumptions about student learning and developing their pedagogical knowledge and beliefs (Cobb, Wood, & Yackel, 1990)

Professional Developments Focused on Mathematics

Other professional development projects have the primary goal of engaging teachers in mathematics with the intent of strengthening their mathematical content knowledge. While these professional development environments allow teachers to also consider student thinking, this is a secondary goal. In representing these types of professional developments with the instructional triangle, we have the major interaction between the teacher and the mathematics (see Figure 5). Even though engaging in student thinking is not the major intended goal, teachers can still ponder student thinking through the lens of mathematics.

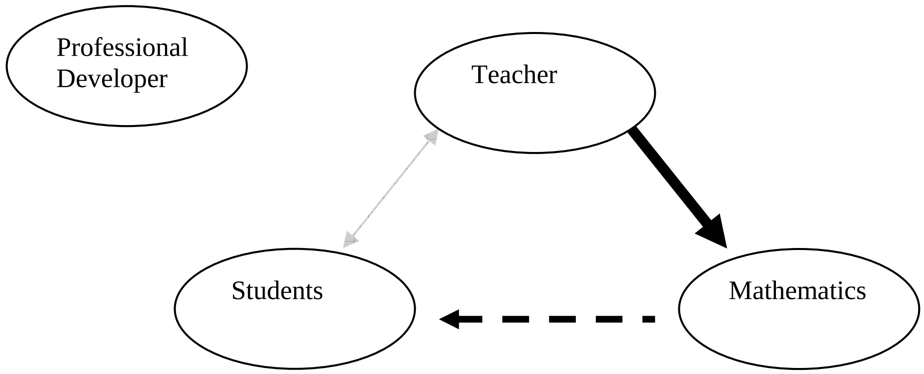

Figure 5. Mathematics interactions triangle. This professional development interactions triangle is based on the goal of professional development being mathematics. The thick arrow of teachers engaging in mathematics denotes the main goal. The secondary goal is denoted by the dashed arrow of teachers engaging in the student thinking through their mathematics.

One example of a current professional development that focuses on teacher learning of mathematical content is the Math in the Middle Institute Partnership (Lewis, Heaton, McGowan, & Jacobson, 2003). The main stated objective is enhancing teachers' mathematical knowledge. Other goals of the program include teachers conducting action research about their teaching practices, developing leadership skills, and applying acquired knowledge and skills to their classrooms. The professional development comes in the form of a 36-hour graduate program that can be completed by teachers in 25 months. Some of the graduate courses included in this program have titles such as Mathematics as a Second Language, Experimentation, Conjecture and Proof, and Using Mathematics to Understand our World, as well as courses in calculus, number theory, discrete mathematics, and statistics. At the time that this was written, the researchers had not yet reported any findings.

Another example of a professional development that focuses on mathematical content knowledge is InterMath. Wilson et al. (1998) explained that InterMath supported teachers in expanding their mathematical content knowledge while using technology to explore the mathematics through open-ended problem solving. As previously stated, the main goal of InterMath is for teachers to build on their mathematical foundation. In my experience, teachers consider student thinking during an InterMath course but the professional developers do not target this as a learning outcome of the project. The InterMath professional development is the context of the study presented here.

Because InterMath courses are mathematical content courses, there is an important similarity between InterMath and the courses participants teach: the main goal of both is for learners to develop mathematical knowledge. Therefore, the pedagogical approaches used by the facilitator in an InterMath course may provide instructional ideas for the participants to use in their own classrooms.

In a typical InterMath class, the participants and facilitator begin by discussing mathematical concepts. They collaborate to solve a mathematical investigation, and then each participant moves to a personal workstation and investigates another mathematical problem alone or with a peer. Each InterMath participant gets to set his own learning focus, select which problem(s) to investigate, and choose which, if any, technologies to use for the problem-solving process. There are over 500 problems to select from on the InterMath website (http://intermath.coe.uga.edu), which are divided into the strands of Number Sense, Algebra, Geometry, and Data Analysis.

Using Constructivist Ideas to Study Professional Development

All of the professional development efforts discussed here are consistent with what Smith (2001) called for in professional development of mathematics teachers. Smith described learning as a dichotomy where learning was either transformative or additive. Learning that is transformative in nature involves “sweeping changes” occurring in deeply held beliefs, knowledge, and habits of practice (p. 3). Learning in an additive sense involves new information and skills simply being added to what is already known and understood by the learners. Smith called for professional development that promotes transformative learning for the teachers.

Smith (2001) called for the professional development of mathematics teachers that creates disequilibrium in teachers’ existing patterns of thought. For example, mathematics teachers should be challenged to think about their beliefs about mathematics, who can learn mathematics, and how they learn mathematics. By reflecting on these beliefs in the context of new experiences, teachers may see the limitations of current practices and begin to build new ones. However, this state of disequilibrium could also “serve as a rationale for rejecting new ideas” so the professional developer must proceed with caution (p. 44).

These ideas used by Smith (2001) are very similar to the concepts constructivists use to discuss learning. In using constructivist terminology, assimilation is similar to Smith’s additive process of learning, perturbation is the same as disequilibrium, and accommodation is the transformative process of learning. Teachers as learners in a professional development setting, “construct knowledge through the assimilation and accommodation of new ideas with what they already know and believe" (Nipper, 2004, p. 6). Therefore, it is appropriate to look at teacher learning using these constructs.

The constructs of assimilation, perturbation, and accommodation will be used in this study to continue the discussion of how teachers make sense of their professional development experience. "Making sense" is about participants making connections from their experiences in the professional development to their own learning, their students' learning, and their teaching. Therefore, making sense involves the individual thinking about his own learning even if he is not necessarily learning new content.

Definitions of Constructs

Assimilation is "the process whereby changing elements in the environment become incorporated into the structure of the organism" (Nash, 1970, in von Glasersfeld, 1995, p. 62). Assimilation allows the learner to take new information and fit it into his existing schemes, which are "mental categories that influence the ways in which a person sees the world and interprets personal experiences" (Penrose, 1979, p. 19). When assimilating, the learner is able to force new information into existing categories (Penrose, 1979). However, new information can only be assimilated if it is somewhat familiar to the learner (Baroody & Ginsburg, 1990), that is, if it fits with what the learner already knows. If assimilation occurs as a participant is trying to make sense of the professional development experience, the content or the pedagogy matches what the participant perceives to already know or implement, which means that the participant does not find it to be problematic.

If a learner is not able to assimilate new information, a perturbation arises. A *perturbation* is a mental agitation or its cause (Stein, 1988). A perturbation occurs within a learner when she cannot fit new information into existing schemes through assimilation, which may cause disappointment or surprise to the learner (von Glasersfeld, 1995). In trying to make sense of the professional development, the participant will find something that is problematic because it does not match what the participant already knows or does.

When a learner experiences a perturbation, the learner will attempt to find equilibration, that is, eliminate or resolve the perturbation (von Glasersfeld, 1995). One way that a learner may eliminate a perturbation is through the process of *accommodation*. In accommodation, the learner is unable to assimilate information into existing schemes, experiences perturbation, and reorganizes her thinking in such a manner that the perturbation is reconciled. This reconciliation may occur after a long period of time and changes the way the learner thinks about an idea. Accommodation of new knowledge is often considered as "real learning." An accommodation in making sense conveys that the participant has reconciled a perturbation that involved making connections to their own learning or their classroom.

Slavin (2003) provided an example of these constructs in the context of a young infant. The young infant enjoys banging small objects. When the young infant is given a new object that is familiar in the sense that it may also be banged, the young infant may bang the new object. The child assimilated this new piece of information (the new object) into existing schemes. However, if the new object given to the infant is an egg and the infant bangs it on the table based on existing schemes, the egg will surely break causing the child to possibly modify her existing scheme of banging small objects to accommodate the idea that some small objects should not be banged.

In this study, besides assimilation, perturbation, and accommodation, I am also considering another construct in my making sense framework that is only briefly mentioned in the constructivist literature. I refer to this final construct as “shutting down.” Smith (2001) warned that the state of perturbation can “stimulate new learning” but can also serve as a “rationale for rejecting new ideas” (p. 44). Baroody and Ginsburg (1990) claimed that any information that was “incomprehensible” to the learner would cause the learner to “quickly lose interest” and “tune it out” (p. 56). Loucks-Horsley, Love, Stiles, Mundry, & Hewson (2003) claimed that when perturbation arises, learners often reject the new information. Therefore, when a learner is faced with a perturbation, she does not always accommodate the new information. She can simply reject the idea and *shut down*. Shutting down may be a conscious or an unconscious decision. Regardless of the decision level, the learner finds the information to be too far removed from her existing schemes or not worth thinking further about.

Figure 6 shows how these ideas are related to each other when one is trying to make sense of any content. In the diagram, one can see that new content can be assimilated, where it is placed into existing thinking patterns, or it can cause perturbation, where it does not fit into existing thinking patterns causing the learner to undergo another process. Once the learner has been perturbed, the information can be accommodated, where thinking schemes are reorganized to accommodate this new information, or it can cause the learner to shut down. A learner can only assimilate or become perturbed by content that she is trying to make sense of, so content can be disregarded altogether as no connections are made or attempted. Also, a perturbation may not be reconciled immediately. Therefore, participant may remain in a state of perturbation for an extended period before content is accommodated or the participant shuts down.

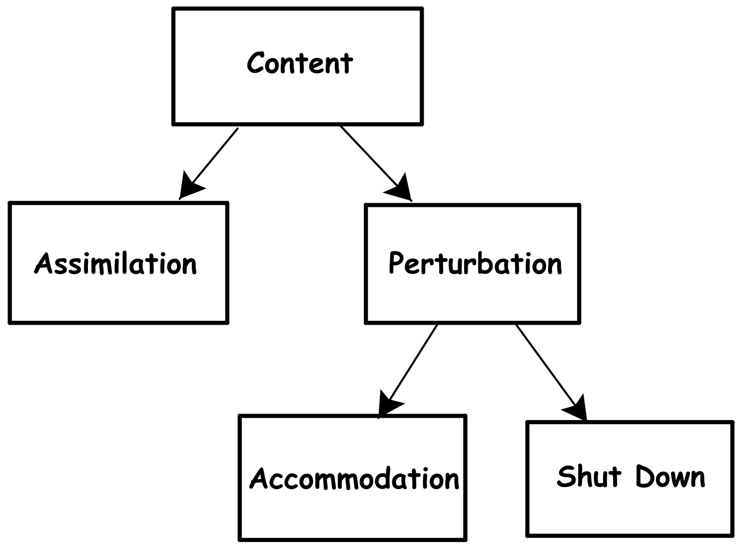

Figure 6. Making sense of content model. This model shows the processes that a learner can undergo. The learner may assimilate the content or experience a perturbation. Once the learner has experienced a perturbation, the learner may accommodate or shut down.

When Piaget used the ideas of assimilation and accommodation, he did not want to merely describe what he was observing about children's thinking – he wanted to understand it (Penrose, 1979). The same is true for this study; in observing adult participants, the goal is to understand these adults. Even though Piaget used these constructs to talk about children's learning, these processes explain how anyone makes sense of their experiences. Therefore, the concepts developed by Piaget to initially explain children's knowledge development, continue to support our understanding through adult life when one is faced with new information (Hill, 1997). Adults have more schemes than children and those schemes have been refined and broadened. Nonetheless, adults continue to resolve the perturbations they experience (Penrose, 1979).

Because it is often impossible to know what another person is thinking or how the person is developing knowledge, one must be able to observe actions or utterances and conclude that the process is going on internally. The observer determines that the learner is going through assimilation, accommodation, or shutting down based on those actions and utterances. Therefore, it is important to define the actions or utterances that allow the researcher to make assertions about them.

Observations of Assimilations and Accommodations

Many other studies that use a constructivist learning framework have used intense observation of students' actions to determine whether assimilation or accommodation is taking place. For example, Olive and Steffe (2002) studied children's construction of numbers and fractional schemes by observing children investigating problems involving fractions. During these observations (and the many hours of analysis of video tapes that followed the original face-to-face episode), Olive and Steffe attempted to name the fractional schemes that the children constructed. They determined whether a child assimilated information or the child became perturbed by the operations the child used. The two episodes that follow show one instance in which they determined that a child was assimilating information and one in which they decided the child was perturbed by a piece of information.

In the first example, Laura attempted to combine 81 cookies with 30 cookies by counting on from 81 by ones. She made a counting error as she arrived at the number 110 instead of 111, but "experienced no perturbation in arriving at 110 when adding 30 onto

81” (Olive & Steffe, 2002, p. 417). Laura was not at all bothered by the sum as she assimilated the operation of counting on by ones instead of attempting another counting scheme. In another episode, Joe became perturbed by the problem that he was working on involving doubling a fraction. Joe was to use a software program to create a stick that was 4/7 the length of a given stick. He solved this task easily and was then asked to create a stick that was twice as long as the 4/7 stick he just created. Joe appeared to count to eight to himself and then created a stick that is divided into 8 parts only to erase it when asked how long that stick was. He said that the stick was “eight” in response to the question about its length. When probed further, he completed his answer with “sevenths.” He said, “no,” moved his stick around, and then said, “I don’t know.” When he was told that his answer was correct, he asked how it could be EIGHT sevenths with emphasis on the eight. In this episode, Olive and Steffe claimed that Joe was perturbed because he was reflecting on his actions and trying to make sense of the problem that he was solving.

In his doctoral dissertation, Tzur (1995) did similar research and looked at teacher-learner interactions and children’s fraction learning. Like Olive and Steffe, Tzur also determined when students assimilated new tasks into existing schemes and when they were perturbed. For example, Jordan was given 7/10 of a “pizza” and was asked to re-construct the whole pizza. The first thing that Jordan did was to partition the pizza into 10 equal parts, which was an assimilation of previous tasks where he partitioned the object into equal parts based on the denominator of the fraction. Jordan then realized that this way of acting would not lead toward the desired solution so he stopped to think. Tzur (1995) called this act of realizing that the task would not work like others had and thinking about it as a perturbation for Jordan. Jordan then "modified the activity of the scheme to neutralize the perturbation and solve the task" (p. 313).

Although the work mentioned above was done with young learners, Hill (1997) looked at the constructs of assimilation and perturbation with adult learners. She observed an undergraduate mathematics classroom and looked for instances of assimilation and perturbation in the students' thinking. For her study, she hypothesized that "students who encounter difficulty with new approaches to familiar areas of work would be drawn to assimilate new problems to familiar schemata instead of modifying existing schemata to accommodate the new approaches" (p. 26). She focused her observations on students who were having difficulty applying new mathematical techniques to problems that they had previously learned to solve in a different manner. For example, a student experienced difficulty in differentiating an equation using a method of converting the equation to its parametric equivalent before differentiating. The student had differentiated many equations prior to this without converting to the parametric equivalent first. His difficulty with this new method allowed Hill to observe how he made sense of the new technique.

In Hill's observations of this student, she noticed that he seemed to be unconvinced of the necessity to use this new method of converting to the parametric equivalent of the equation before differentiating. The student reverted to his familiar method of differentiating only to find problems that were not experienced using the new method. It was only after Hill had a discussion with the student about this problem that the student came to an understanding of why the new approach was necessary. Hill determined that the student was surprised that the problem "provided an opportunity to learn more mathematics" (p. 28). The student had attempted to assimilate solving the problem using a familiar technique that caused more problems than the new method had. Hill considered that this student only assimilated this new technique because he was focused on finding the solution to the particular problem rather than learning the new technique for solving the problem. After their discussion, the student seemed to accommodate this new technique into his differentiating schemes.

Hill recognized that this student would have simply assimilated this new technique if it had not been for the scaffolding that she provided in her discussion with the student. She claimed that this was in part due to assimilation being much easier to achieve than modifying schemes in order to accommodate. She saw other instances where students accommodated by "linking" new methods to previously learned methods of solving problems. She also noted the hypothesized assimilation as some students showed no difficulty working through new methods and others simply reverted back to familiar methods.

Simon, Tzur, Heinz, & Kinzel, (2000) used the constructs of assimilation, accommodation, and perturbation to study teachers in professional development. They investigated, "what sense the teachers make of the (professional development) experience, what they consider important, and what they take to be problematic" (p. 598). In one instance, Simon led the teachers in a discussion about children's development of number by stating that everyone was born without the concept of number embedded in their brains. Simon asked, "how does somebody who doesn't see number in the world come to see number in the world, come to see the world in terms of number?" (p. 598). In their discussions, teachers considered how a child could come to understand a particular quantity related to its number name, which was not what Simon had in mind. Simon was thinking of the larger idea of number in general rather than how a child learns to understand the quantity that belongs to the word “two.” The teachers were unable to grasp the question and, therefore, did not find children’s development of number problematic. The researchers concluded that the teachers assimilated these ideas about how children develop the concept of number.

Examples of cases in the literature in which researchers examine instances of a participant shutting down are not as common as descriptions of assimilation and accommodation processes. In most research studies from a constructivist perspective, researchers are interested in the paths that lead learners to perturbations and accommodation. This is often considered to be the path of “real learning.” Steffe (personal communication, 3/28/06) suggested that the idea of “shutting down” is not discussed much in the literature because researchers using the constructs of assimilation and accommodation to study learning are mostly interested in the accommodation process. It is not that these researchers do not see their participants shut down; most of the time, they are just not interested in studying it.

There may be specific reasons why teachers shut down during a professional development with the most obvious being that a teacher may not see relevance for what is being done in the professional development to the teacher’s own classroom. Tzur (1995) provided one such example from his experience in teaching a methods course for elementary teachers. Three participants in the class were working on a measurement task where they were to create two different sized sticks and measure the longer stick using units of the shorter stick. One participant supported the activity, the second was neutral at first but was then swayed to support the activity, while the third participant rejected the activity seeing no point in “re-invent(ing) the ruler.” The participants carried on with the activity and discussed with the entire class whether or not they should “re-invent the ruler.” The third participant seemed perturbed by the activity because she was not allowed to dismiss it as the group was involved in “heated discussions” about the activity. Her ultimate rejection of the activity may be considered a shut down because even after the “heated discussions,” the participant still did not see any value in the lesson.

Refined Research Questions

Goldsmith and Schifter (1994) commented that there are a large number of professional development projects that focus on exploring different models for intervention in teaching professional development. They suggested that more needs to be understood about the psychological mechanisms of teacher change. This study attempts to do just that. It uses the constructs of assimilation, perturbation, accommodation, and shutting down to understand how the teachers make sense of professional development experiences developed to help teachers learn mathematical content. This leads to refined research questions for this study:

- What components of the InterMath professional development experience (mathematics, technology, problem solving) tend to cause assimilation or perturbation in the participants?
- What problematic components of the InterMath professional development experience tend to cause accommodation or shutting down in the participants?
- How can these experiences be combined to create a model for how the teachers make sense of their professional development experiences?

- How do these experiences allow the teachers to make sense of their professional development experience in terms of their own learning, their students' learning, and their teaching?

CHAPTER 3

METHODOLOGY

Bogden and Biklen (1992) offer five characteristics of qualitative research: the natural setting is the direct source of data; it is descriptive; the concern is process, not outcomes; analysis is inductive; and “meaning” is of essential concern. The research study reported here aims at understanding how the teachers made sense of their experiences in professional development settings. Because of the interest in the setting, the experiences of the teachers, and the process of making sense, I used qualitative methods to collect and analyze data. More specifically, the research presented here is a participant observation study in which “the researcher enters the world of the people he or she plans to study, gets to know, be known, and trusted by them, and systematically keeps a detailed written record of what is heard and observed” (Bogden & Biklen, 1992, p. 2). As is consistent with the name participant observation, I acted as a participant in the research setting, and I observed the participants who were being researched.

I acted as a full participant in this research project by taking on the role of the professional developer for an InterMath Number Sense course in fall 2005, where I studied three of the participants in the course. In my experiences teaching the different InterMath courses, each course has been filled with interesting people who bring to class different experiences in their backgrounds, educations, and mathematical and technological knowledge. The participants also have different goals for their own learning, which causes many of them to attend to different components of the professional development. In typical classes I previously taught, some participants viewed InterMath as a course about technology, others viewed it as a mathematics course, while others viewed it as a course on how to implement technology into their mathematics lessons. Due to such differences in prior experiences and expectations, participants have different experiences in the course. Their experiences are what I studied.

Participant Observation

As previously stated, I was a full participant in this study. When conducting a participant observation, the participant observer has the dual role of being an ordinary participant and a participant observer (Spradley, 1980). Peshkin (1988) refers to these "two" people as the "*human* participant observer," who is the person in everyday life, and the "*research* participant observer," who is the person doing the research (p. 270, emphasis in original). Thus, as a participant observer, I had two purposes for my interactions with participants in the InterMath course, and I engaged in activities as if I were both an ordinary participant *and* an observer of the actions of others.

My role as human participant observer came out in my role of instructing the InterMath course. I had taught four other InterMath courses prior to this, so I was already familiar with the situation as an ordinary participant and was comfortable in this role. Due to this familiarity, and because of the research design in which I had chosen to better understand my research participants' understandings of our shared experiences, I had the highest level of involvement in a participant observation (Spradley, 1980) as I instructed the course as a complete participant. The role of research participant observer was a new role for me during this study, as I had never attempted research on participants in previous courses I taught in this capacity. I had conducted research in previous courses but never as a participant observer.

For this study, it was important for me to be the instructor of the InterMath course because it was important that the InterMath instructor provide an experience for the learners that was consistent with my views and goals for InterMath —that is, in a manner that I considered consistent with the framework for the project (Simon, 2000). For example, a researcher studying discourse would only want to study a class where discourse was implemented in the manner intended by the researcher. In my research, I knew how I wanted the teaching strategies to be implemented in the InterMath course, so it made sense for me to take on the dual role of teacher and researcher. Because of my dual role, I was in a privileged position as a researcher: being the teacher in the course, I was able to ask questions that helped me construct models of the participants' thinking. As the teacher, I could further probe participants' thinking when necessary, benefiting both the teaching and the research in this situation.

Lubienski (2000a; 2000b) and Orrill (1999) conducted research using similar methodologies, as both took on a dual role of instructor and researcher in educational settings. In Lubienski's work, it was important for her to be the teacher as well as the researcher due to her coming from a similar lower socio-economic class to that of her students. In order to "guard against potential problems" (Lubienski, 2000a, p. 381), she had colleagues conduct interviews with students to gain information that students may have been unwilling to tell her as the teacher.

Orrill (1999) engaged in a professional development effort as the facilitator while acting also as the researcher. She expressed a "conscious effort…to balance being an insider and being an outsider" (p. 74). In her role as the researcher, she wanted to collect data for her research. In her role as the facilitator, she wanted to provide a high-quality professional development experience for the teachers. Orrill stated that she made conscious decisions to move more into the role of researcher when the two roles were in conflict. In order to alleviate some issues of validity and trustworthiness with her playing dual roles in the study, she had other educators observe her participants in order to provide an outside view of the participants' practices. She then compared their observations to her own.

The methods Lubienski and Orrill used in order to ensure that their studies were valid and trustworthy drove my own methods, as I took similar precautions in order to ensure validity and trustworthiness of my data collection and analysis. In order for something to be valid, it must be "based on fact or evidence" (Wiersma, 1995). In order to make my study more valid overall, I collected various forms of data and attempted to triangulate across data sources and across time. I had an outside observer attend three of the thirteen classes in order to provide a potentially different interpretation to my own. I also videotaped a specific portion of each class so that I could continue in my role as instructor during the lesson and could watch it later with the eyes of the researcher. These data collections will be discussed in the next section in greater detail.

Context

In the fall of 2005, I taught an InterMath Number Sense course in a computer lab at a middle school in a suburb of a major southern United States urban area. The course consisted of 13 classes that met one night per week spanning 16 weeks (we did not meet weeks that included holidays). There were eight participants in the class with varied teaching backgrounds – 1 kindergarten teacher, 2 elementary school teachers, 3 middle school mathematics teachers, 1 middle school special education teacher, and 1 high school mathematics teacher.

The Number Concepts course basically involved sets of numbers. This content included such topics as whole numbers, integers, rational and irrational numbers, prime and composite numbers, even and odd numbers, triangular, square, and star numbers, and abundant, deficient, and perfect numbers. The problems used in the course are referred to as investigations due to the investigative nature in which the participants attempt to solve the problems. The sets of numbers are investigated in problems specifically addressing the set (e.g., an investigation involves finding the first six perfect numbers) and in problems where the main idea includes a concept such as least common multiples, greatest common factors, or divisibility rules (e.g., an investigation may involve using divisibility rules to simplify a fraction leading to a discussion of prime and composite numbers).

Participant Selection

I selected my research participants on the first night of the course. Seven of the eight expected participants came to the first class meeting. Three of those seven met my criteria to be a participant in my study. I wanted to focus my study on teachers who were the original target audience for the InterMath professional development courses, that is, I was interested in middle grades (4-9) mathematics teachers. In previous offerings of the course, like in this particular offering, participants had come from a variety of grade levels and included teachers from other content areas. Although these participants had been successful in InterMath, they were not the original target audience of the course. Therefore, they were not considered for my participant pool.

Prior to our first class, I had chosen to work with three teachers. In my previous research with InterMath teachers, I had found that more than one teacher provided me with some overlap in findings, yet also provided unique, distinct cases. I was not comfortable choosing two participants for fear that one participant may not complete the course. In selecting three participants, I thought it was likely that at least two of them would complete the InterMath course during this offering (which would mean a not unusual loss of 30% in the course of the professional development).

At the first InterMath meeting, I discussed my research with the three teachers who met my criteria. I told them that I was interested in their experience in the course and there would be no right or wrong answers. I explained that the extra work that they would be expected to do in order to participate in my research involved completing three interviews with me that would last 30 to 60 minutes. All three middle grades mathematics teachers agreed to be research participants on the first night of class and all three completed the course. I report about them here using pseudonyms.

Data Collection

Because I had the dual role of instructor and researcher in the InterMath course, I divided the collected data into two categories: instructor data collection and researcher data collection (Lampert, 2001). As the instructor, there were data that were collected as normal duties of any instructor. These data included participant work, reflections in their journals, and classroom discussions. These sets of data were collected for all InterMath participants, including the three research participants. There were also data that were not collected as part of the duties of everyday teaching but were needed for research such as formal interviews and videotaping of class sessions. Table 1 divides the data collected into these two categories.

Table 1

Data Collection in Two Categories

Instructor Data Collection	**Researcher Data Collection**
▪ Participant work (pre- and post-exams and weekly assignments) ▪ Reflections in journals ▪ Notes on discussions and conversations	▪ Formal interviews (audio taped and transcribed) ▪ Videotaped sessions ▪ Outside observer report

Participant Work

All InterMath participants[2] took a pre- and a post-exam designed to measure any mathematical content knowledge growth. Both exams were exactly the same and were given on the first and the last night of class. The exams consisted of two Balanced Assessment questions (Concord Consortium, 2006) (see Appendix A for the exam). The InterMath participants also had seven assignments of completing a "write-up" where they explained their thought processes as they investigated/solved a mathematics problem.

Reflections in Journals

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[2] "InterMath participants" or simply "participants" refer to everyone taking the InterMath course including my research participants and those who were not research participants. When specifically addressing only my "research participants," they are referred to as just that.

During the course, the InterMath participants were asked to send a weekly journal entry to me via email. This has been common practice for all InterMath courses that I have taught. During the first several weeks of this particular course, the participants were not given any direction for the journal entry except to write about anything having to do with the class or what they were thinking about. Teachers could write about their thoughts of mathematics, technology, instruction, InterMath in general, issues in their own classrooms, or whatever was important to them at the moment they were writing the entry.

In this particular course, only two InterMath participants (neither were research participants and both had previously taken an InterMath course) regularly emailed journal entries to me. Due to the lack of participation in this activity, mid-way through the course I began to email writing prompts to the class and asked them to respond. Because the entire class still did not participate, I made the decision to start each class by giving the participants 15 – 20 minutes to respond to my questions via email. From then on, all participants responded to all of the questions.

Notes on Discussions and Conversations

As the instructor, I was able to communicate with the participants in the course during whole group discussions, during class time, as well as during individual work time We were able to have informal conversations before, during, and after class, in the classroom, in the hall, and in the parking lot. After each class, I wrote in my journal about these episodes.

Formal Interviews

Research participants were asked to take part in three formal individual interviews – one at the beginning of the course for me to obtain baseline data about their views of mathematics and learning for themselves and for their students; one interview in the middle of the course and one at the end to find possible differences between the original interview answers and the teachers' later answers as they progressed through the InterMath course. The interviews were conducted over the telephone due to the schedules of all involved and were audio taped and transcribed verbatim. The participants were given the option of doing the interview in class after the whole group discussion (during the time set aside in the course for participants to work on their individual assignments). All participants decided that they would rather work on their assignments during class time and preferred telephone interviews scheduled at their convenience. (See Appendix B for Interview Protocols.)

Videotaped Sessions

Because I held the dual role of instructor and researcher, I videotaped the whole class discussion in each class. This generally included the first half of each class (approximately 2 hours). As the instructor this allowed me to continue facilitating the mathematics lesson without having to worry about missing a discussion that was important for my research.

The computer lab was set up with three horizontal rows of computers facing the front of the room where the board and computer-projection were located. All of the participants sat in the first row and the camera was located on one end of that row. So the camera was very close to some participants and far from others. There did not seem to be any other option in the location of the camera due to the room configuration and the heights of the computers. Two research participants usually sat at the computers nearest the camera while the third moved throughout the course. Therefore, two of the research participants were often captured on camera discussing the mathematics or helping each other with the technology while the third was not captured as often.

Outside Observer Report

I arranged for an observer to attend the class three times (approximately 1/4 of the class sessions). The observer was the project manager of InterMath. She had worked on the project for over one year and knew what I was trying to accomplish as I taught the InterMath course. She was instructed on her role in this research and what she would be observing. She was to look for instances where the participants talked about their own learning (what they were learning, if they were learning, how they were learning, etc.), their students' learning (connecting their own learning experiences to those of their students, talking about how their students learn or how their students could learn, examples of instances where students learned, etc.), and their teaching (how they have taught lessons, how they could teach lessons, adaptations that could be made for their own classrooms, etc.). The observer created an observation sheet that she used while observing to list the three components mentioned (see Appendix C for Outside Observer Report). After each observed session, the observer and I discussed her observations.

Data Analysis

It is expected in qualitative research that data collection and analysis be conducted simultaneously as data analysis drives further data collection, which was the case in this study. As I collected and analyzed data, I applied the constant comparative method

(Merriam, 1988) where I developed categories and tentative hypotheses about how the research participants were making sense of this professional development experience. I noted what other data needed to be collected to better understand what I was observing and hearing. Most of the added data collection came as added interview questions, where I essentially asked the research participants to complete seemingly incomplete thoughts or to clarify ideas for my understanding. Again, this allowed me to generate hypotheses that would lead to my building models of participants' making sense of their experiences.

The analysis for each research participant was organized according to the three InterMath components of mathematics, technology, and problem solving for their learning, for the their students' learning, and then for their teaching. Since my goal was to understand these teachers and not to judge their teaching and learning, in this report I present the data from the point of view of each research participant with as many of their own words as possible. Once the data have been presented, I provide my interpretation of how each participant made sense of each component of the course using the constructivist framework that includes assimilation, perturbation, accommodation, or shut down.

Observations of Assimilations and Accommodations

When analyzing my data, I looked for instances of assimilations, perturbations, accommodations, and shut downs by the research participants. I had to determine when participants assimilated or were perturbed by the content of the course, although these constructs are "subjective and (may) depend on unobservable states" in the participants (von Glasersfeld, 1995, p. 66). If I noted that the content resulted in the participant experiencing a perturbation, I then sought to determine if that led to the participant accommodating the information or shutting down.

In my analysis, assimilations are understood as the lack of visible reactions of discomfort. The evidence of this comes from the research participants claiming or seeming to be comfortable with their experiences. I also considered assimilations to happen when a participant contradicted himself or herself or the InterMath rationale in making claims of coherence. For example, a contradiction exists when a participant says that she employs problem solving in her classroom, similarly to that is done in InterMath. However, the participant also says that students must be taught the basic skills and then practice prior to solving a problem, which is not how I view the problem-solving component of InterMath. Because the participant saw these two ideas as coherent, despite my understanding that they contradict each other, I considered that the participant assimilated the idea of what it means to do problem solving in the classroom.

If assimilation and perturbation are used as a dichotomy, then a perturbation is a visible reaction to discomfort. In the cases of perturbations, participants may have mentioned being uncomfortable not knowing the mathematics or how to use the technology. However, I only claim that a participant experienced a perturbation when a particular challenge was unexpected. When participants viewed content as problematic and this was unexpected, the participant experienced a perturbation. When content is not problematic or problems are expected, the participant experienced assimilation.

Once an instance was declared a perturbation, the second task was to determine whether it led to an accommodation or to the participant shutting down. If the participant openly discussed how they had been influenced by something that happened in the InterMath class or seemed to change how they were thinking about the content, then the instance was considered an accommodation. Shutting down was an easy construct to observe because the participants would stop participating in the class discussion and would no longer direct attention to others, including me as the instructor. Participants would also tell me that they shut down or “tuned out.”

All instances of assimilation, accommodation, and shutting down were triangulated by the data across sources and time, where possible. Wiersma (1995) discussed triangulation as “qualitative cross-validation” where multiple data sources show the same findings. Triangulation also allows for the research to be trustworthy as the researcher presents multiple data saying the same things.

I then created a model of each participant’s experience in the InterMath course for their own learning, for their students’ learning, and for their teaching based on what they attended to in the course and if they assimilated, accommodated, or shut down. With these models in hand, a cross-case analysis was done in order to search for patterns and themes that cut across the individual experiences (Patton, 2002). Specifically, I looked at participants’ different (or similar) perspectives on the central issues of their own learning, their students’ learning, and their teaching across the mathematics, technology, and problem solving components of the InterMath course.

Limitations

The first limitation of any research study involves the impossibility of disconnecting your human self with your research self. Peshkin (1988) claimed, “Our personal proclivities do more than incline us to investigate certain problems. They lead us to take sides” (p. 269). Peshkin also said that it is not possible for our personal subjectivities to not show up in our work, which would certainly be true of any research, including participant observation. Some of my own subjectivities that will show up in my work include: teachers are life-long learners and learn from each other and from their students; teachers take professional development courses because they want to improve their teaching for the better of the students; and the teaching methods that I implement in InterMath are the teaching methods that teachers should be implementing in their classrooms. All of my assumptions listed here are part of the research design of this study because without these assumptions, I have no research questions.

The second limitation of this study is that I only used self-reported data about teaching practices from the participants. The study would have been richer if I had observed the participants' classrooms, but that was not the purpose of this study. Instead of relying on them telling me what problem solving looks like in their classrooms, I could have observed it for myself. I was also not interested in how (or if) the participants took any ideas from InterMath back into their classrooms. In this study, I was only interested in how they were experiencing the InterMath course and what they were thinking about in doing so. How (or if) the participants implemented teaching strategies from InterMath would be a nice follow-up study to this one.

CHAPTER 4

PARTICIPANT JUDY

This chapter and the next two provide brief descriptions of each research participant, often using their own words to capture their interactions and learning from the InterMath course. These chapters also present my analysis of each participant's sense making, using the concepts of assimilation, accommodation, perturbation and shut down. Specifically, the analysis is arranged according to the InterMath components of the mathematics content, the technology, and the problem solving approach used in the InterMath course. Within each InterMath component, I look at the participant making sense for personal learning, for student learning, and for teaching using my research questions. Finally, an overall model for each research participant is presented as my interpretation of how each made sense of the professional development in terms of their own learning, their students' learning, and their teaching.

Description

Judy was in her late 40's and was the daughter and niece of several teachers. When she was younger, Judy he did not want to be a teacher because she did not want to "fall into" the profession just because her mom and dad were teachers. Therefore, she did not pursue teaching as a career until 1990. She did some substitute teaching and then was hired as a full-time teacher in the fall of 1994. She had taught middle school since then on 2-, 3-, 4-, and 5-teacher teams. She had taught language arts, science, and mathematics that consisted of inclusion, gifted, and general education students. Most of Judy's teaching had been done at Monument Middle School (MMS) (pseudonym) except for a 4-year stint in Texas when her husband accepted a calling as a minister there. Judy was teaching sixth grade mathematics at MMS to general education and gifted students at the time this research was done.

Judy took the InterMath course in order to get teaching ideas for her classroom, as she wanted to become a "better teacher" (Interview 1 Page 5; Interview 2 Page 1). Judy attended twelve of the thirteen InterMath classes in the Number Sense course. She was very active in the course, answering and asking questions, explaining her own solutions or helping others explain theirs, and helping nearby participants with the mathematics. She was confident with her mathematics abilities (Interview 1 Page 5), but she often solicited help from others while using the technology.

InterMath Mathematics

For her Learning

Mathematics as mostly review. Judy claimed to already be familiar with a lot of the mathematical content in the InterMath course – she called 85% of the content review (Interview 3 Page 13). She also said,

> You've refreshed my memory on a few things that I knew but that the dust had grown fairly thick on it and so it was not at the forefront of my brain anymore so that has been refreshed for me and brought back to the forefront. (Interview 2 Page 17)

Since the course was Number Sense, many of these topics were also included in the mathematics curriculum that Judy taught to her sixth grade students. Judy claimed that InterMath focused more on problem solving and that, "it's just a matter of applying what you've already learned" (Interview 2 Page 17), again indicating that Judy believed she knew a lot of the mathematical content already.

Understanding the mathematics. A regular occurrence in the InterMath course was for the participants to explain their solutions. In class 9 (11/08/06), we discussed alternative algorithms for multiplication of numbers including the Russian Peasant Method and Lattice Multiplication. Judy seemed to understand why the Lattice Multiplication worked based on the common algorithm that she uses with her students. We worked through the multiplication of 12 x 13 using the Russian Peasant Method and I asked the participants to think about why this algorithm worked and Judy said, “oh, we want to know why it works” (Video – Class 9). Judy seemed content that the algorithm worked without knowing why it worked. She and the other InterMath participants attempted to explain why the algorithm worked and found an explanation on the Internet, but Judy claimed to still not understand why it worked after reading the explanation and listening to others. Judy finally said that she could do the algorithm but could not explain it. I asked participants to think about how their students would feel in a similar situation where they were taught how to do something with no understanding of how or why it worked. Judy said her students would want to know how it worked and that it would be confusing for them if they did not understand this. After discussing another algorithm that they collectively decided they did not understand, Judy turned to another InterMath participant and said that a gifted student would understand the algorithm (Observer Notes – Class 9).

Mathematics as new content. In her second interview, Judy was asked what new content she was learning in the InterMath course. She specifically responded “history of math,” “formulas for finding the sequences and the series,” and “subscript one” in reference to variables with subscripts (Interview 2 Pages 17-18). In her third interview, she added “understanding patterns” and “star numbers” as new content (Interview 3 Pages 12-13).

Judy did not respond to all of the new mathematics content in the same manner. In class 7 (10/25/06), we explored a problem called *Theater Seating* where there were 25 seats in the first row of the theater, 27 seats in the second row, 29 seats in the third row, and so on. The problem asked that if the pattern continued, how many seats would be in the theater if there were 15 rows in all? In investigating this problem, the InterMath participants found the number of seats in each row and then added the series of numbers together by adding the first to the last, the second to the second to last, etc., where this method is commonly credited to the famous mathematician, Gauss. When exploring summing in this manner, we derived the formula for adding a series of numbers, $S_n = n\left(\frac{a_1 + a_n}{2}\right)$. We derived this formula based on familiar mathematics as Judy was familiar with adding the series of numbers this way. However, she claimed to have never seen this method of adding “in the context of a formula” or formulas that were comprised of variables with subscripts. She was not bothered by this and attributed her not knowing the formula or the concept of variables with subscripts to her graduating “high school… a long time ago and Algebra II was just as far as you went” (Interview 2 Page 18).

Although variables with subscripts as new content were not problematic for Judy, other new content was. Following her interview statement about not learning about variables with subscripts because she graduated so long ago, she also claimed the Greek alphabet to be something else that she missed out on. Occasionally, a letter of the Greek alphabet (Rho, Sigma, or Delta) would enter our group discussion as part of a formula that another InterMath participant would say or simply ask about how a specific Greek letter related to mathematics.

For example, during the discussion of *Theater Seating* (see Appendix D for entire problem), the InterMath participants came up with an equation ($y = 2x + 23$) to find the number of seats in a specific row in the theater (y) based on the row number (x). An InterMath participant brought in the idea of functions and the slope-intercept form of a linear equation. Another InterMath participant asked about the Greek letter used to define slope. After several attempts at addressing her inquiry of the Greek alphabet and still not understanding what she was getting at, I asked if she was talking about “Delta y over Delta x.” Several other InterMath participants then said they remembered that. In the video of this episode, Judy seemed to be paying attention and nothing seemed problematic for her (Video – Class 7). However, in her second interview, which took place the day after this class session, she said, “I don’t know Delta, Sigma, or Rho. No. I know what the letter is but I don’t know anything about its connection to math” (Interview 2 Page 19). She then said she had thought about taking the secondary mathematics PRAXIS II exam but when she looked at a practice test, she decided, “there’s no way I could do that. There are too many Sigmas in there …I don’t know anything about Sigma. Isn’t that funny?” (Page 19). She added, “I guess I’ll spend the end of my career in elementary school” because she desperately did not want to take the teaching examination due to the Greek letters that she had seen on the practice test. She would rather settle for teaching elementary school where the examination was not required.

Later, in class 11 (11/08/05), while we were picking an investigation to explore as a group, I clicked on several different problems and asked if they were interested in exploring any of them. One problem involved a summation of fractions that was shown with a Sigma in the problem. When I asked if they wanted to solve that problem as a group, Judy quickly and loudly said, “no” (Video – Class 11).

Mathematical comfort zone. Judy said that she was never put outside of her comfort zone in relation to her mathematical knowledge. She explained that as the instructor, I was “so supportive” (Interview 3 Page 14) that she did not worry if she did not know certain aspects of the mathematical content of the course. When we worked through an investigation, I (in the role of the instructor) would often say, “I don’t know. Let’s look at this together.” Judy claimed that this made her comfortable because she thought, “well, she’s working on her doctoral thesis and if she doesn’t know, then poor little me …who barely has a master’s degree…” (Interview 3 Page 14). Her thought trailed off into laughter, but it appeared that Judy thought that it was okay for her to not know the mathematics if I claimed that I did not know it.

My interpretation. Judy assimilated most of the mathematics content for her own learning by calling 85% of it review. Even when some of the mathematical content was considered new to her, she was able to assimilate most of it partly due to the supportive InterMath classroom environment and partly due to her mathematics background.

Not understanding why an algorithm worked was not problematic for Judy. It was only after I encouraged the InterMath participants to explain why the Russian Peasant method worked that Judy said, “Oh, we want to know *why* it works.” If I had not asked them to explain why the algorithm worked, I do not think that Judy would have pursued thinking about that. This was evident earlier in the course when we discussed the divisibility rule for 11, as we never came up with reasons why it worked but Judy said that her students would love it. What was briefly problematic for her was that if she were to show her students these alternative algorithms, she claimed that her students would want to know why the algorithms worked and she would not be able to explain them. Nonetheless, Judy moved back into the mode of not being troubled by this by commenting that a gifted student would be able to understand even if she did not. Due to the lack of observable discomfort with understanding the alternative algorithms, I argue that Judy assimilated this mathematics.

However, some mathematical content was problematic for her. Judy had previously experienced a perturbation by the presence of the Greek alphabet in mathematics when she was thinking about taking her teaching examination. She was so perturbed by the Greek alphabet showing up on the teaching examination that she decided that she would rather teach elementary school than take that test. Therefore, prior to the InterMath course, she had already shut down in relation to understanding the use of Greek letters in mathematics. She was slightly perturbed by this same issue again in the InterMath course even though she stated that she was never placed outside of her comfort zone in our course. Consistent with her prior reactions to the Greek alphabet in mathematics, Judy continued to shut down by simply claiming that she thought it was funny that she did not know anything about the letters and by not wanting to investigate any problems involving the letters.

For her Students' Learning

Judy hardly ever discussed her students in conjunction with the InterMath course unless specifically asked. When asked about the mathematics that her students learn, Judy responded that their mathematics was basic operations and skills, which was different from the mathematics that she was learning in the InterMath course because she already possessed the skills needed to investigate the problems. Because of this, she seemed unable to connect the mathematics of InterMath to her students' learning, which allows me to claim that there was nothing for her to assimilate or be perturbed by for her students' learning of mathematics.

For her Teaching

Understanding the mathematics. Judy did not discuss InterMath in relation to her teaching of mathematics often. However, in Judy's first journal entry, she wrote that she had "already shared the divisibility rule for 11 with [her] students; we had a lot of fun with it" (Judy's journal – 9/22/05). As with the alternative algorithms for multiplication, the participants in the course never came up with an explanation of how the divisibility rule for 11 worked.

Understanding algorithms was mentioned again when Judy discussed what her students needed to know in order to perform well on assessments. Judy said,

> the kids enjoy hands-on and yet we need to prepare them … for a benchmark that is not hands-on so they need to be prepared to be able to *do* the algorithm and know that it works without knowing *why* it works. (Interview 1 Page 8)

She questioned how her students could be asked to "*show how you got the answer* to the problem on a multiple choice test" (Interview 1 Page 8).

My interpretation. Judy was not perturbed about not understanding why the alternative algorithms worked for her teaching. It was not problematic for her that she did not understand the divisibility rule for eleven, but she shared it with her students anyway. Again, Judy was not perturbed by her own or her students' inabilities to explain algorithms or rules perhaps because students would not be asked for such explanations on standardized tests. Therefore, Judy seemed to assimilate the notion that being able to explain mathematics was important or at least worthwhile, which allowed her to teach without explanations.

Judy's Making Sense of the Mathematics

It can be argued that Judy learned mathematics in the InterMath course as her post-test score showed improvement from her pre-test score. She scored a 17/21 (81%) on the pre-test and a 21/21 (100%) on the post-test 15 weeks later. I claim that Judy assimilated most of the mathematical content because it was rarely problematic for her. She saw most of it as review and considered the main premise behind InterMath was applying mathematical skills she already possessed to exploring problems. The mathematical content that was new to her was also not problematic, partially due to Judy's lack of perceived need to understand the mathematics. The only topic Judy did not assimilate was the use of Greek Letters, in which case she shut down.

Considering her teaching, Judy showed her students some of the mathematical algorithms performed in our class. But they seemed to be presented more for an awe factor, and there was no expectation that the students would develop an understanding of the mathematics involved. Other than this, Judy seemed to make no other mathematical connections to her teaching, probably due to the difference she perceived between the nature of the mathematics that she teaches and her students learn (basic skills) and the mathematics of the InterMath course (application of basic skills). Therefore, she assimilated the mathematics of InterMath for her teaching.

InterMath Technology

For her Learning

Computer issues. Judy lacked basic computer literacy as she talked about the computers in her room as the "2 white Dells" and the "black one," "Dell XP," or the "one black Dell XP." She asked if I knew "those computers" (Interview 1 Page 7 and Interview 2 Page 4). She also hesitated when she said, "I'm just always afraid that I'm going to click on a button and lose it all" (Interview 2 Page 3).

Judy was very fluent in using the computer for specific tasks. Twice during the semester MMS had problems with their grade book program where all input data disappeared. Several teachers came into the computer lab where our class met and re-entered all of their grades in order to meet the school's deadlines. Judy often helped those teachers with this task. She was also fluent with email and word processing, but not so much with using the Internet. She had a "dial-up" Internet connection at her home so "surfing and looking for sites…is so unbelievably slow" (Interview 2 Page 3) making it an undesirable activity. Therefore, she seldom used the Internet unless she was at school and claimed to have very little time while at school to use it.

Spreadsheets. Spreadsheets were commonly used in the InterMath classes to investigate problems, which was an unfamiliar use of computers for Judy. In the first several weeks, I often suggested using spreadsheets to solve the problems because I was trying to introduce the idea of using spreadsheets for problem solving to the class. During the second class, I suggested using a spreadsheet to explore a problem and Judy said, “a spreadsheet. Wow” (Video – Class 2), indicating that she may not have thought about using a spreadsheet to solve this particular problem on her own. As I demonstrated setting up the spreadsheet with input from the participants, Judy visibly sat up in her chair, leaned forward, and watched where the spreadsheet was being projected (video – Class 2).

Several weeks later, when I interviewed Judy for the second time, I asked her what she was learning. She talked about how she had “picked up on some good websites [and that] trying to navigate FrontPage[3] has been good” (Interview 2 Page 2). She further claimed to not be completely comfortable with FrontPage yet, which led her to talk about her lack of comfort using spreadsheets.

> Every time you get on that spreadsheet, I never do click on. It’s just sort of scary to me and so I never do click on it and I would like to get more comfortable with it. So every time you get on it, I’m learning a little bit. I think mostly I’m afraid of the language [like] the asterisks instead of the multiplication symbol (Interview 2 Page 2).

After talking with Judy, I noted that she watched me work with the spreadsheet program but did not try much on her own. Judy commented that she was afraid that she would miss something if she tried to create a spreadsheet at her workstation while we discussed the problem (my notes – 11/1).

In Judy’s second interview, she requested that I demonstrate spreadsheet set-up more slowly in the context of investigating a problem so that she could follow along with

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[3] FrontPage was the web editing software used in this InterMath course for their on-line portfolios.

her own computer. In response to her request, I structured the following class (class 8) in a way that mimicked my normal organization, but focused on mathematical content we had previously discussed so that Judy could be more comfortable focusing on the technology instead of the mathematics. During this class, I heard Judy say, “I love this” multiple times and that she really liked using the spreadsheet for that particular problem. She asked many questions about the logistics of setting up the spreadsheet and another participant offered Judy additional support with the spreadsheet as she investigated the problem (video – Class 8).

Three classes later (Class 11), another participant suggested using a spreadsheet to investigate a problem. Judy did not appear to be paying attention to the discussion about the mathematics of the problem or the spreadsheet used to investigate it as she often looked at her neighbor’s computer and talked to him. She seldom looked at the projection of the spreadsheet on the wall and did not answer many questions about the mathematics involved in the problem, which was out of character for her. When talking to her neighbor, she clearly said something about the CRCT and something else about a vertex – neither of which related to the problem we were discussing. Clearly, Judy was not engaged in this spreadsheet-based investigation (video – Class 11).

When the course was over, Judy said that she was still not comfortable enough using spreadsheets to implement them in her classroom and that she “just needed to practice a little bit” (Interview 3 Page 1). Judy never put any spreadsheets in her write-ups (Participant Products) and aside from her asking me during the interview to slowly go over creating a spreadsheet in the context of a problem, she never suggested that we solve a problem using a spreadsheet during our class meetings (Videos). Near the end of the course, she wrote in her journal, “while we are on the computer throughout our InterMath class time, I don't think the computers are indispensable” (Judy’s journal – 11/15/05).

My interpretation. Judy was not necessarily computer literate and it was very problematic for her to use computers in ways that she was not accustomed. Using a spreadsheet to investigate mathematics was new to Judy when she entered the InterMath course and is a specific example of her perturbation with learning technology for herself. She seemed engaged in learning how to use a spreadsheet as evidenced by her sitting up in her seat when I demonstrated spreadsheets and then in her request that I slowly walk her through the process of setting up a spreadsheet. Later in the course, she seemed to lose interest in spreadsheets. This could indicate that Judy shut down in terms of learning and using spreadsheets to explore the mathematics. However, at the end of the course, she claimed to still not be comfortable using spreadsheets and noted that she needed more practice. Thus, overall, learning technology perturbed Judy, and I would argue that Judy left the course still lingering in a state of perturbation about using spreadsheets for herself.

For her Students’ Learning

As with the mathematics, Judy never discussed the technology component of InterMath in relation to her students’ learning. When asked about her ideal classroom, Judy said, “I believe that students truly need computers instead of this one black and one white one that I have here in the classroom” (Interview 2 Page 6). She seemed to think that computers had a place in her classroom and that they were important for her students’ learning; however, she never discussed how technology could be implemented in her classroom. Therefore, Judy did not seem to make any real connections between the technology component of InterMath to her students' learning, leaving nothing for her to assimilate or be perturbed by.

For her Teaching

Computers in general. The technology component of InterMath seemed to be a concern for Judy from the beginning in relation to her teaching. In her first interview, she asked if "this is going to be mainly a computer program. Are we going to be on the computer? Are we going to be encouraging our students to be on the computer an awful lot?" (Interview 1 Page 5). This interview was two days after our first class meeting where the participants took a survey[4] that had 72% of the questions asking specifically about technology, investigated a problem using a spreadsheet, and created their home pages for their on-line portfolios using web-editing software. I assured her that we would be using technology in every class but that there was no course requirement for her to implement the technology in her classroom during the semester. She seemed relieved and then asked if she could get on her "soapbox for a minute" (Interview 1 Page 6).

Judy proceeded to explain that she did not have much access to computer technology in her school because they only have two computer labs for 1400 students. She needed to reserve the lab 3-4 weeks in advance without knowing whether she and her students would be on track for completing a computer lab activity by the time they actually got to go into the lab. Judy said that if she took any of her classes to the computer lab, then she wanted to take all of her classes to the computer lab. She wanted to instruct the same activities to all of her classes on the same day so that no single class

[4] This survey data was collected for InterMath evaluation and is only mentioned here as a specific instance where Judy was introduced to the technology component of InterMath. The survey data was not analyzed for this study.

would get ahead or behind of the others according to her pacing guide. Therefore, Judy found it very difficult to instruct a lesson to all of her classes using the computer lab.

In her classroom, Judy had three computers but only one that worked, which was the computer that she used for inputting grades and word processing for herself. She said that she would not plan any computer activities until all 3 computers in her room were working (all were working by the end of the InterMath course) because she had 27 students and it would be too difficult to have them equitably work on only one computer. Judy mentioned that she had access to a laptop cart with 20 laptops for her students to use, but that there would still not be enough computers for her students and some would have to share. She also was not sure about using the laptop cart because there was no “little bubble in the ceiling,” so she did not know if the computers were “connected or not” (Interview 1 Page 7). In the midst of her soapbox speech, Judy admitted that, “if my desire to do it was great enough, I would work at it even harder” (Interview 1 Page 7).

Co-teaching technology. Once the InterMath course had ended, I interviewed Judy for the third, and final, time. At the end of the interview, I asked if she would ever be interested in me helping her plan and teach a lesson in the computer lab that used the technology that we had been using in InterMath. She said, “that would be so neat” (Interview 3 Page 18). When I started explaining how I had done this with other teachers, she interrupted me and started talking about how I could take her gifted students to the lab and asked if I would mind taking the other sixth grade teachers’ gifted students to the lab on the same day. She was very excited about this prospect and said, “It’s easier to do with the kids now than ever. They have their own login[5]." I saw Judy several times the following spring semester and reminded her that I was still willing to help her with this activity. She was always appreciative and said that she would get back with me on that. She never did.

My interpretation. Judy talked very little about how she could use technology in her teaching and a lot about how she could not. She presented a laundry list of reasons why she could not implement technology in her classroom. She seemed to be too constrained in using technology in her teaching, which was especially evident in her comment about her desire to use technology not being great enough for her to work on it. Therefore, the technology component, in general, was a source of perturbation for Judy with regards to her teaching.

In order to help ease Judy out of her perturbed state, I offered to co-teach a lesson using technology with her. She never discussed this with me except for when I continued to offer. Even then, she seemed more concerned with the logistics rather than thinking about what the students could learn from this kind of lesson. By the end of the course, she was unable to reconcile her perturbations about teaching with technology, as she never discussed how she was thinking about implementing technology, in general, in her teaching or spreadsheets, specifically. I claim that she did not shut down here because she claimed to still need more practice in order to become more comfortable with the technologies so that she could use them with her students.

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[5] Students needed their ID cards earlier in the semester, but now had an ID number that allowed them to access the student desktop on the computer without the card. This had been a constraint for Judy earlier in the year since she could never guarantee that all of her students would be able to access the computer once they were in the lab if they had forgotten or lost their cards.

Judy's Making Sense of the Technology

The technology component of InterMath was a great source of discomfort for Judy from the very beginning when she asked if she would be expected to encourage her students to use computers. Many of her comments were about her teaching, but the issues she was raising involving her teaching with technology really seemed to impact her ability to learn, for herself, the technologies used in the InterMath course. She experienced a lot of perturbation for her own learning of the technology – she was scared of the technology in some instances and was unable to repeat procedures from week to week. Thus, she left the course still in a state of perturbation about the technology component of InterMath for her own learning.

Judy was also unable to make connections to using technology for her students' learning. Although she thought her students needed to use technology, she never discussed *how* her students needed to do so. At the end of the course, she still said that she needed to try to implement technology in her classroom, but that she needed more practice with it herself first. Therefore, I argue that Judy left the course in a state of perturbation with regard to using technology in her classroom.

InterMath Problem Solving

For her Learning

Investigation-based approach. Judy claimed to be "very comfortable" with the problem-solving approach taken in the InterMath course (Interview 2 Page 5). As stated previously, Judy thought that InterMath basically allowed her to apply the mathematics that she already knew in the problem-solving component of the course (Interview 2 Page 17). When asked to define problem solving, Judy wrote,

> Problem solving is applying already known skills in a new situation. All the skills needed for the situation are already in place. Those skills might have to be arranged in a different format in order to be useful. Knowing which step (skill) will be needed first and which step will need to follow is crucial to problem solving. (Journal – 11/29)

One could attribute Judy's comfort with the problem-solving approach used in InterMath to her claims of already knowing so much of the mathematical content covered in the course. However, Judy attributed her comfort with the problem-solving approach to the regular problem solving that she does with her students where she claims to use an approach similar to that used in the InterMath course (Interview 2 Page 5). She found nothing hard about using this approach (Interview 2 Page 5) and liked the summary of the mathematics at the end of each problem as a review of all the different mathematics that was part of solving the problem (my notes – 10/25).

My interpretation. For Judy, problem solving is something you do after you know the basics. In InterMath, she already knew the basics and could solve the problems presented. Thus, Judy found nothing hard about the problem-solving approach taken in InterMath meaning that it was not problematic for her. This may have been because she claimed to use a similar problem-solving approach with her students or because she already knew the mathematical content. Regardless of the reason, Judy assimilated the problem-solving component of InterMath for her own learning.

For her Students' Learning

Similar to the other InterMath components, Judy rarely discussed anything about student learning in relation to the InterMath course, which was also true for the problem-solving component. She did mention that her students often lacked basic skills and according to her definition of problem solving, if her students lacked in basic skills, then it was difficult for her to engage them in problem solving. Due to the differences in skill levels between Judy and her students, I argue that Judy did not make any connections between the problem-solving component of InterMath to her students' learning, leaving nothing for her to assimilate or be perturbed by.

For her Teaching

Problem solving in general. Judy reported that she engaged her students in a lot of problem solving like that done in InterMath. She also claimed that students needed to have a foundation of the basic skills before they could engage in the problem-solving activities in her class. She clarified this need for problem solving to only occur after students had the fundamental skills when she gave the example of her students not knowing how to convert a fraction to a decimal as

> what I'm facing when I say that my kids don't have those skills. So a problem-solving approach where that might pop up, where they would have to apply those things… then I've just cheated them out of success on the problem since they don't have the skill to do it with. (Interview 3 Page 15)

Aside from the basic skill requirement, Judy found a similarity in her teaching of problem solving and the InterMath approach. She discussed incorporating cooperative groups in her classroom (Interview 2 Page 6). She said that she appreciated having someone to "bounce ideas off of" in our InterMath course and she claimed, "that's the way that I work with my students. There is always someone that might have a different take on it and be able to see something that I didn't see before" (Interview 2 Page 5).

My teaching philosophy and her teachable moment. In the last interview, I asked Judy what she thought my teaching philosophy was. She called my approach to teaching "very honest" as I taught as if it was "okay for the teacher to learn with the students" (Interview 3 Page 7). She said that this came through, as I would say to the class, "I don't know. Let's work on it together" (Interview 3 Page 7). When I asked Judy if she was able to be that honest in her teaching, she said, "I try to be honest. I hate to admit when I don't know. But I do." Later in the same interview, Judy told me about a "teachable moment" that she took advantage of in one of her classes where she admitted to her students that did not know the answer to a question.

In one of Judy's classes, a student asked Judy a "what if" question about a problem that he was working on. In the interview, Judy said, "it was a totally new question for me. I had no idea" so she decided that this was a good time to allow herself to learn with her students (Interview 3 Page 10). She asked the student several questions in front of the whole class that led the other students to also engage in the problem even though they were not all operating at the same skill level. In the end, she decided that all of the students "learned a lot during that 18 minute period" (Interview 3 Page 11). She said that she thought, "Thank you, God, this is great" and went home to tell her husband, "I love my job" (Interview 3 Page 12).

My interpretation. Judy claimed that the problem solving in InterMath was very similar to how she conducted problem solving in her own classroom; however, her idea of developing basic skills prior to engaging in problem solving did not seem to match the ideas presented in InterMath. For most of her teaching, Judy assimilated the problem-solving component of InterMath. She did not incorporate problem solving with her students unless they already possessed the basic skills required for them to be successful with the problem. Perhaps she thought that all of the InterMath participants possessed the basic skills needed to solve all of the InterMath problems of our course. Judy's perceived similarities in her own teaching of problem solving and the InterMath course allowed her to assimilate most of the problem-solving component.

Near the end of the semester, Judy had a teachable moment in her classroom and was able to allow her students to explore a problem. Even though Judy never seemed perturbed by the problem-solving approach used in InterMath, I claim that in this case she experienced a perturbation because she seemed to have accommodated this teaching moment into her teaching repertoire by allowing her students to explore the mathematics in a collaborative effort even though they did not all possess the same levels of basic skills. Therefore, Judy seemed to accommodate the problem-solving approach used in InterMath for her teaching, at least in this episode.

Judy's Making Sense of the Problem Solving

I argue that Judy assimilated most of the problem-solving aspects of the InterMath course because she never showed any discomfort nor did she ever talk about the problem solving in our course being problematic for her – as a learner or as a teacher. One major reason for this may have been Judy's confidence that she was teaching her students in a similar fashion, although these similarities were not made clear to me. Like the other InterMath components, she made no connections between the problem-solving component of InterMath to her students' learning.

There was only one instance that implied that Judy did not assimilate everything about the problem-solving approach used in InterMath even though she never indicated that anything was problematic. The one episode involving Judy's teachable moment indicated that she experienced a perturbation about my honest teaching approach and may have accommodated this perturbation in this particular teaching episode.

Summary

Throughout my observations of and interactions with Judy, I found that she assimilated most of the components in the InterMath course. She considered herself to be familiar with many of the ideas in the course, particularly in relation to the mathematics and problem-solving strategies. Still, Judy did experience perturbations in those areas for her own learning with using the Greek alphabet in mathematics and for her teaching with using the investigative-based approach used in InterMath problem solving. Judy shut down in the first case and seemed to experience accommodation in the second.

Judy's most enduring perturbation came from using technology. She had a difficult time thinking about teaching with technology due to her constraints ranging from learning the technology to her limited access to technology. She was unable to reconcile these perturbations during the InterMath course as she left still perturbed by the idea of using spreadsheets for her own learning and technology, in general, in her teaching based on her comments that she needed more practice.

The following table summarizes Judy's making sense of the professional development for her learning, for her students' learning, and for her teaching in terms of assimilation, perturbation, accommodation, and shut down (see Table 2). Any accommodations or shut downs resulted from experiences of perturbations. A component is only considered a perturbation in the table if Judy did not accommodate or shut down; otherwise, it is listed as the latter.

Table 2

Summary of Judy's making sense. This table shows Judy's making sense of the professional development for her learning, for her students' learning, and for her teaching in terms of assimilation, perturbation, accommodation, and shut down.

	For her Learning	***For her Students' Learning***	***For her Teaching***
Assimilation	Mathematics – all but one topic Problem Solving		Mathematics Problem Solving – most
Perturbation	Technology		Technology
Accommodation			Problem Solving – teachable moment
Shut Down	Mathematics – Greek alphabet		

In Judy's table, one can see that the column, *for her students' learning*, is blank, which is due to Judy not seeming to make any connections from the InterMath course to her students' learning. Mathematics was assimilated for her learning and for her teaching, except for the Greek alphabet, which is the only mathematics listed elsewhere. Since Judy assimilated most of the problem-solving component for her teaching, it is listed as assimilation in the table and only that one instance of the teachable moment is listed as an accommodation.

Judy's Model

From the beginning, I was trying to create a model of how each participant made sense of the professional development for their own learning, for their students' learning, and for their teaching. Judy's model for the professional development did not match my hypothesized model that teachers first make sense of the professional development for their own learning and then move on to making sense of it for their student's learning, and finally make sense of the professional development for their teaching.

Judy rarely talked about how students learn unless asked specifically. She mainly discussed how she learned and how she attempted to teach in a similar fashion to how she liked to learn. She liked cooperative learning for herself, so she tried to implement cooperative learning in her teaching She was comfortable with the problem-solving approach in InterMath for her own learning because she claimed to teach a lot of problem solving in her classes. She had many constraints in using technology in her classroom and had great difficulties in learning the technologies used in InterMath for herself. She did not always understand how or why algorithms worked but taught them to her students without providing any explanations. Even the episode of Judy's teachable moment seemed to move directly from her learning in the InterMath course to her teaching, bypassing her thinking about her students' learning.

Judy never seemed to connect her own learning to her student's learning. She also never seemed to make any connections about students learning to her teaching. Since Judy never became comfortable with the technologies of the InterMath course, she said that she would not use them in her classroom until she was comfortable enough with them herself. Therefore, student learning with computers was completely out of the question until she learned the technology better herself. If Judy did make any connections between student learning and her learning or teaching, she did not discuss them in the InterMath course or in her interactions with me.

Again, she rarely focused on her students' needs in her classroom. Due to this bouncing back and forth from her thinking about her own learning and her teaching, my organization of Judy's model of her making sense of the professional development also moves back and forth between her learning and her teaching practices while her students' learning seemed to be a disjoint idea (see Figure7).

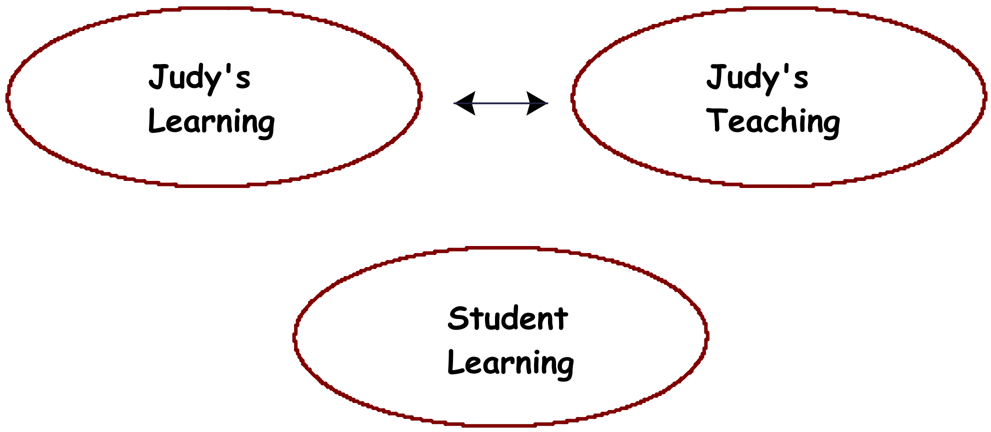

Figure 7. Judy's model. This model shows Judy's experience in thinking about her learning, her students' learning, and her teaching.

CHAPTER 5

PARTICIPANT TOM

Description

Tom went to college intending to get a music degree so that he could teach music. Upon graduation, he decided against teaching music and found himself working as a substitute teacher while getting his master's degree in middle grades education. He spent four years teaching social studies at an alternative school and was starting his sixth year of teaching mathematics at Monument Middle School (MMS) at the beginning of this study. He had an education specialist degree in school administration but had never applied for an administrative position because he said that he would rather focus on becoming a good classroom teacher. At the time of this study, Tom and Judy (the research participant previously presented) were both teaching at MMS and they collaborated often. Like Judy, Tom taught general education and gifted students in sixth grade.

When Tom signed up for the InterMath Course, he was familiar with the philosophy of InterMath and the amount of work that would be involved in taking the course because he had taken a similar course with the mathematics education professor who designed and initiated the InterMath project. Tom was taking the InterMath course hoping to get ideas for teaching specific content that he taught in sixth grade. He was comfortable with both the mathematics and the technology as evidenced by a consistent sense of confidence displayed when he answered questions about mathematics and technology. He also often helped Judy with her technological difficulties.

InterMath Mathematics

For his Learning

Comparing mathematics to music. Music appealed to Tom at a very early age and to him, mathematics was similar to music. He said that in mathematics

> you would sort of be given a problem-solving tool and then you would go away by yourself and practice using that tool to solve similar problems until you mastered using that tool. The other thing that you needed was the ability to recognize what problems required which tool. There are two skills going on in mathematics. (Interview 1 Page 4)

Music was similar as he compared a problem-solving tool in mathematics (algorithm, rule, or procedure) to an etude in music as something "you practiced and studied and… it would challenge you… and you would sort of practice until you got it and then you would go back and learn and study another" (Interview 1 Page 4). Without being asked, Tom again addressed these similarities in his second interview as he said,

> An etude teacher is a certain kind of musical phrase that you practice a whole bunch of ways in this supposedly musical piece but it's not really music because you are practicing something. And a math class is very similar. You go to class and learn an algorithm and then you take a bunch of problems and you solve it so that by the next class you can learn the next algorithm. (Interview 2, page 17)

When probed further about etudes, Tom said that the musical piece would be "somewhat musical," but that it was "not a piece composed for enjoyment. It's a piece designed for musicians to practice something on" (Interview 2, page 17). Therefore, Tom seemed to think that school mathematics is to be practiced before it can be applied for enjoyment.

Mathematics as review. From the beginning, Tom made it clear that he did not expect to “learn” much mathematics in the InterMath course when he said, “I don’t really actively learn math anymore” (Interview 1 Page 4). He also made it clear that he did not perceive the mathematics of the InterMath course as being new material to him as he said, “it has been a review” (Interview 2 Page 2). Later in his journal, he wrote, “I'd say that these concepts we already know. We are just applying them to interesting problems. Math classes in my past required much more practice, especially out-of-class as I was learning the content” (Journal – 11/10). Still at the end of the course, he reinforced the idea of the content being review as he compared the InterMath content to middle grades mathematical content when he stated, “you’re not really teaching us how to differentiate equations and maybe other stuff that we hadn’t done before. They’re all basically concepts that students are going to use in middle school” (Interview 3 Page 18). He claimed to be familiar with a lot of the content that we covered such as greatest common factor, sets of numbers, and ratios and proportions as he said, “I had heard of all of that. I had studied that in school” (Interview 3 Page 19).

Enjoyment of abstract mathematics. Tom really liked mathematics puzzles and the abstractness of the subject as he said, “I like math because as you get deeper and deeper into math, it becomes very abstract…. So I like math in terms of puzzles and challenges, having to think. And I think of it as being good for the brain” (Interview 2 Page 16). Tom claimed to “like working problems with pencil and paper” (Interview 2 Page 17), which he referred to as “really abstract” tasks (Interview 1 Page 7). He said that he could enjoy paper and pencil mathematical tasks because "of course, I've gone through puberty and maturity, which my students haven't. And my brain is full now. It's done developing." (Interview 2 Page 17). Therefore, Tom seemed to think that he was able to think about mathematics on an abstract level where his students could not due to their brain development.

My interpretation. Tom considered most of the mathematics in the InterMath course to be review. In his comparison of practicing basic skills in mathematics to practicing etudes in music, it seemed that Tom considered each to be building blocks for applying them to a real mathematical or musical piece. Since Tom already had the basic skills, he was able to apply that mathematics to the problem solving in our course. Tom never seemed to think that the mathematics of the course was problematic for his learning. Of course, he came into InterMath not expecting to learn any mathematics. Therefore, I would argue that he assimilated the mathematics component of the course for his own learning.

For his Students' Learning

Concrete vs. abstract. When asked specifically how he believes students learn, Tom said that students should have a "visual or some kinesthetic experience" as a "fun or involved experience that you transfer to the abstract" (Interview 1 Page 7). For example, he talked about one task in which his students were learning about area as "they pushed all the little squares together and made a rectangle and understood that length times width equals area" (Interview 1 Page 7). Once the students had experienced the mathematics in a concrete manner with the little squares, then they could move on to the abstract formula.

When talking about the sixth graders he teaches, Tom observed, “students at this level…are not ready to fully engage in the abstract…. Some of the gifted students enjoy math as a very abstract concept but the others…what they really like is the applied math” (Interview 2 Page 16), but “by and large, they are not at that level to think abstractly” (Interview 3 Page 20). Tom further explained that when a student is trying to attend to a problem that has been written on the board, “it's too abstract. They will need something in their hands in front of them” (Journal – 11/10). He later restated this as, “paper and pencil tasks are very abstract really no matter what it is because everything that (his students) are writing down is something that they have to visualize” (Interview 3 Page 20).

Tom seemed to think that because his students could not think abstractly, they could not appreciate or be interested in mathematics in its abstract form. For Tom, his students could only appreciate mathematics if they could see that the content existed in the real world and “especially if they can interact with it” (Interview 2 Page 18). When saying this, he referred back to a discussion of limits in our InterMath class, where he said,

> if I say (to my students), “there are these things called limits, which means approaching a number… closer, closer, closer, but would never, never get there.” That wouldn’t be interesting to them…. Now, if I said, “walk halfway to that wall and stop. Now walk halfway to that wall and stop. Now keep doing it and keep doing it.’ That would interest them. (Interview 2 Page 18)

My interpretation. Tom seemed to think that his students needed to see real world connections and engage in the mathematics concretely before they could engage abstractly if they were going to be interested in the mathematics enough to try to learn it. Tom was able to engage in the mathematics of the InterMath course, but he liked abstract mathematics and, according to him, was mature enough to engage abstractly. Tom never mentioned anything about InterMath being concrete and he did not seem to make any connection between the mathematics of InterMath and his students' learning because it was so different. Since no observable connection was made, there was no assimilation or perturbation experience.

For his Teaching

Concrete vs. Abstract. Tom said from the very beginning of the course that he wanted to help his students make concrete, real world connections. He discussed wanting to take his students outside and measuring physical objects in order to study measures of central tendencies as a way of providing a concrete, real world example (Interview 3 Page 20). However, he wanted to get better at this as he said, "I would like to get very proficient in explaining (mathematics) and making real world connections with (mathematics) for students" (Interview 1 Page 4). Since he believed that many of his students could not engage in abstract concepts without having a concrete experience, Tom visualized himself as a teacher who would provide his students with those.

Engaging his students. Tom saw a direct link between his students' engagement in the class and his performance as the teacher. "The more engaged the students will be, the more confident I am and the more exuberant I am because I know the kids are going to be involved in it" (Interview 1, page 16). Tom found it important to use activities that would engage his students in the mathematics whether it was a real-world activity or a story that he could tell to get them interested. He hoped to "start off with a story to get them engaged and then say ‘here’s the problem.’ Hopefully, I transfer their interest in what I’m telling them to the written problem” (Interview 2, page 14). He claimed that he wanted to be a better “storyteller” so his students “all stop and listen [to] something that engrosses them and catches them before I present” (Interview 1, page 6).

Tom discussed a task from the Georgia Performance Framework (GADOE, n.d.) that he did with his students about basketball claiming that “it was fun” (Interview 2 Page 4). This was an example where the mathematics was “embedded in something that is meaningful to students. When it is real. When it is wrapped up in a story or … when it is linked to something else that kind of hooks the kids” (Interview 1 Page 7). Tom did not want his classroom to be a place where “students are practicing problems, but are applying something that they know in an activity that has engaged them.” (Interview 2 Page 8).

Work in progress. After ten years of teaching, Tom claimed to still be “a work in progress as a teacher” (Interview 2 Page 8). He elaborated,

> I’m not quite the teacher that I need to be. I’m still at the point in my career where I have to focus on, more than I would like to, what I’m teaching in the lesson. I’m trying to get better at knowing the students and their strengths and weaknesses and knowing what they know and what they need to know. I still sort of think that I have to focus too much on myself and what I’m doing. When I have that down… hopefully as I get a little more experienced, I’d like to think that I will be able to focus more on the students and really pay attention to whether they are actually receiving the information. So that what I’m doing will be second nature so that I can really pay attention to whether they are getting it or not. I can think

about what they know and what they don't know more than I do. (Interview 2 Page 8)

Tom later discussed how important it was for him to know his students better and their abilities. He said that because he did not know more about them, he was taking "a little bit more caution" when giving his students activities (Interview 3, page 16).

My interpretation. Because he believed his students needed to experience mathematics concretely, Tom envisioned himself facilitating those concrete experiences in his classroom. He wanted to engage his students more in the mathematics by giving them real-world connections or stories to hook them. It was unclear if Tom was thinking about any part of the mathematics of InterMath for his teaching. He seemed to have all of these ideas about his teaching prior to the InterMath course. Tom had also said that he took the InterMath course because he wanted to get teaching ideas for the specific content that he teaches. Since Tom perceived the mathematics of InterMath to be so similar to the mathematics that he teaches, he could have gotten specific teaching ideas based on the problems that we explored that addressed his middle grades content. However, Tom never mentioned any connection of this sort. Tom did not seem to make any connections from the mathematics component of InterMath to his teaching except for the idea that the content was similar. Because there were no connections, there were no assimilations or perturbations.

Tom's Making Sense of Mathematics

If Tom's tests scores were to reflect his mathematical learning, it could be argued that he did not learn any mathematics in the InterMath course. His posttest score actually showed a decline in his mathematical abilities as he scored a 19/21 (90%) on his pretest and only a 10/21 (48%) on his posttest. One reason for the decreased score came from him getting the answers correct and showing his work on the pretest but not showing his work on the posttest, when the question stated that work must be shown for credit. Another major mistake on his post-test came from him finding the correct ratio of 8:1 on one of the problems, but then using a ratio of 4:1 to answer the rest of the question. He got this particular problem completely correct on the pre-test but due to this error, Tom had several points deducted from his posttest score. There was only one question that Tom incorrectly answered on the pretest that he got correct on the posttest. The second question that he had missed on the pretest was in two parts and both were also answered incorrectly on the posttest.

In Tom's comparison of mathematics to music, he seemed to think that algorithms were meant to be practiced and later applied to problem-solving activities. He seemed to compare the mathematics in our course to real music and that he no longer needed to practice algorithms – just apply them. Thus, the mathematics content in the InterMath course never seemed problematic to Tom for his own learning. He claimed that he did not actively learn mathematics anymore because, in general, the mathematics that he studied presently was mainly a review of topics that he had studied once before. It seemed that he only considered that he could actively learn mathematics if it was new content to him. Therefore, I would argue that Tom assimilated the mathematics component of the InterMath course for his own learning.

There could be several reasons why Tom seemed engaged in the mathematics in the InterMath course. Tom's engagement could have been related to his perception that the course was a review of mathematics. Maybe it was the result of Tom perceiving the

InterMath problems as being presented as interesting puzzles. Perhaps it was the real world context in which some of the problems were presented. Regardless of the reason(s), Tom was always engaged in the mathematics just as he wanted his students to be engaged in the mathematics that he taught them. He had experienced this for his own learning, claimed it was important for his students' learning, but he had not yet figured it out for his teaching.

Tom rarely talked about student learning regarding mathematics except when asked specifically. He discussed how he enjoyed the abstractness of mathematics in paper and pencil tasks but that his students could not engage in the mathematics at this same level. When talking about what he liked about mathematics and what his students needed in order to learn mathematics, he said that they were very different due to his personal maturity. Perhaps, Tom viewed the mathematics of InterMath as too abstract for his students especially since he taught his students the skills before they could be applied in problem solving. Because no connections were made from the InterMath mathematics to his students' learning or his teaching, Tom experienced no assimilation or perturbation.

Tom had many views of mathematics. He thought about mathematics as partially concrete in nature because his students can use concrete manipulatives to visualize the concept or see a real world application. At the other extreme, he thought about mathematics as abstract in nature but that only the mature student could engage at this level after mastering the concrete aspects. Further, Tom asserted that learning mathematics required practice and mastery of a set of skills before they can be applied in problem-solving situations. He asserted that because he had already learned all the mathematics that he needed that he could engage in the mathematics of the InterMath course. His students would not be able to engage in this manner without the concrete images, the real-world applications, or the practice of basic skills beforehand, which was not the message that I was trying to convey in the InterMath course.

InterMath Technology

For his Learning

Spreadsheets. When asked why he was taking InterMath, Tom said that one reason was to "get better versed in using Excel[6]" (Interview 1 page 2) and that he was "looking forward to the practice (he would) get with Excel" (Journal – 8/31). Tom had experience with creating spreadsheets, but wanted to get better at it. With this goal in mind, he always seemed to pay attention when we created a spreadsheet during class and he often helped others create spreadsheets at their individual workstations (Videos).

Early in the course, spreadsheets were difficult for him. During the third class meeting, Tom was investigating *Splitting Fractions into Two* where the goal was to find two unit fractions that summed to 2/5, 2/7, and 2/11 (see Appendix D for entire problem). He worked for a very long time using a trial-and-error method to no avail. It appeared that his issue was not with the mathematics, but rather his organization of his trials as he scribbled sums all over his paper. Due to the lack of organization of his trials, Tom often tried a sum a second and third time not realizing that he had already tried it and that it did not work. Tom asked me for help and since we were at the beginning of the course, I suggested that Tom try using a spreadsheet. So, with his input, I helped him create a spreadsheet to find these unit fractions still using trial-and-error, but now also noticing patterns in the unit fractions giving the desired sums (My notes – 9/13). In the end, Tom

[6] Excel was the spreadsheet program used throughout the course.

claimed not to completely understand how the spreadsheet worked, so he wrote up the problem without the spreadsheet. Instead of inserting the spreadsheet in his write-up, he discussed how he made an organized table of trials to find the solution with my help (Participant Product). Later he said, “that problem was very difficult. It was pretty impressive the way that you were able to use Excel to find the solution” (Interview 2 Page 5).

Using spreadsheets seemed to get easier for Tom as the course progressed. Later in the course, he still asked for help when creating spreadsheets on his own (Video – Class 8; Class 9), but, in class 10, he suggested creating a spreadsheet as a solution strategy (Video). In his third interview, Tom said that he had picked his write-ups so that he could practice using Excel (Interview 3 Page 6) and that he really learned how to use Excel just by doing it (Interview 3 Page 7). Tom’s use of spreadsheets in his write-ups was more obvious in his later write-ups as his fourth, fifth, and seventh write-ups all clearly involved using a spreadsheet to solve the investigations as the spreadsheets were embedded in his write-ups (Participant Products). However, at the end of the course, he claimed not to be completely comfortable with this technology as he said “the technology, I hope will come” (Interview 3 Page 10).

My interpretation. Tom had prior experience with spreadsheets and was hoping to get better versed at using them, which allowed him to be open to the experience of learning them for himself. Tom came to the course wanting to learn spreadsheets and he learned them. Because he came to the course *expecting* to learn how to use spreadsheets, learning them cannot be problematic in the way that I am talking about perturbations in this study. In this study, perturbations are the result of an *unexpected* problem. Even if

Tom's difficulties with the technology were unexpected to him, they were not unexpected by me for him. Therefore, I argue that Tom assimilated spreadsheets for his learning even though he still expressed discomfort with them at the end of the course.

For his Students' Learning

Tom did not seem to make any connections between the technology component of InterMath and student learning. Upon entering the course, he seemed to think that his students should have experiences using technology in his classroom since technology would always be accessible to them. Tom did not mention any other uses of technology that would benefit his students' learning other than specific software that he used from time to time to review mathematics concepts. Since Tom made no connections here, there were no experiences of assimilation or perturbation here.

For his Teaching

Access to technology. Tom said that he had "very good access to computer-based technology" at MMS (Interview 1 Page 3). He had four computers in his room and access to computer labs. When asked about his ideal classroom, Tom indicated that technology would be a part of it,

> because the kids that I teach now are always going to have technology… technology is always going to be at their disposal in probably whatever job or career that they go into. So ideally, technology would be a part of the classroom. (Interview 2 Page 7)

Spreadsheets and mean, median, and mode. Tom wanted to learn Excel in the InterMath course. After just one class, Tom said, "I'd love to teach class using Excel with the frequency you did the other night" (Journal – 8/31). Tom specifically wanted to be able to incorporate spreadsheets into his classroom to teach mean, median, and mode.

Prior to the InterMath course, Tom had "co-taught" a lesson on mean, median, and mode using Excel during summer school (Interview 1 Page 3). Tom had solicited help with this lesson from David, the "technology person that summer," who was also a mathematics teacher. Tom wrote the lesson[7] and implemented the data collection as the students "found their mean, median, and mode and the range of their performance. They compared it to the class" (Interview 2 Page 4).

In the computer lab, David taught the students how to input their data in a spreadsheet to analyze it. "Off the top of his head," David walked the kids "step by step through Excel." Tom said the lesson "was just great and I always wanted to use Excel to teach mean, median, and mode" (Interview 2 Page 3). The whole lesson took several days, but "the kids were engrossed in everything they did. It was not a problem and they got it" (Interview 2 Page 4).

Tom said that the lesson was fun and that by watching David, he "got the sense that there were a lot of really neat things that you can do with Excel" (Interview 2 Page 4). During this lesson, Tom decided that he wanted to become proficient in that technology as well. At the end of our course, he claimed,

> I would be apt and much more comfortable to use Excel now to do mean, median, mode, average, and analysis of data. Put some data in there and let the kids generate the different charts… pie charts, bar graphs, and stuff like that. (Interview 3 Page 10)

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[7] Tom's lesson involved his students "shooting baskets" in the gymnasium. The students got in groups and took several turns shooting 5 baskets.

However, when asked if he had done anything with his students this school year involving mean, median, and mode, he responded, “that’s not really in our curriculum anymore” (Interview 3 Page 3). He said that he had not thought of any ways that he could use technology to teach any of his remaining content although he was “not opposed to it” (Interview 3 Page 3).

Technology planning takes time. Tom was impressed by the ease in which David was able to navigate through Excel himself and how he was able to teach the students how to use the program with little effort. Tom claimed that David did not have “to sit down the night before and go step by step through how he was going to explain it to the students” (Interview 2 Page 4). Tom said, “it takes more work to arrange instruction for technology when you’re not familiar with it yourself” (Interview 1 Page 3). At the end of the semester, Tom was still saying, “technology takes a while to plan for, for me anyways” (Interview 3 page 10). He was uncertain about using technology in his classroom and was still not completely comfortable with the technologies used in the InterMath course for himself.

My interpretation. One reason that Tom gave for taking the InterMath course was because he wanted to practice using spreadsheets with the intention of possibly incorporating them into his teaching. Prior to the InterMath course, Tom had co-taught the lesson on measures of central tendency. It was never clear where Tom had gotten the idea to use spreadsheets for teaching this lesson, but it was prior to our course. Tom never mentioned any other ways in which he could use spreadsheets, or any other technology, in his teaching even though he claimed that he had good access to technology in his school and that students should have experiences using technology.

Tom knew that implementing technology would require more time for planning those lessons because he was not as fluent as he needed to be to use the technology in his classroom with the ease he desired. He was still hoping that he would get to the point where the technology was second nature and he could implement it without having to plan for it as much. Not learning the spreadsheets well enough to incorporate them in his teaching was a source of perturbation as he expected to learn spreadsheets in InterMath and wanted to learn them well enough to teach with them. By the end of the course, he had not gained the comfort level that he was hoping for, which was unexpected and caused him to leave the course still in a state of perturbation about using technology, in general, and spreadsheets, specifically, in his teaching.

Tom's Making Sense of Technology

Tom essentially spent the first half of the course learning how to use spreadsheets. He had previously been exposed to spreadsheets and liked the idea of learning how to use them, which was a reason he gave for taking the InterMath course. He wanted to learn how to use spreadsheets and he did. Because he expected to learn spreadsheets, any problems arising in his learning of spreadsheets was not viewed as a perturbation for this study. Therefore, Tom assimilated using spreadsheets for his learning because the problems he had in trying to learn the spreadsheets were not the kind of perturbations that I was looking for in this study as I expected Tom to have difficulties in learning how to use spreadsheets even if he did not.

Prior to the InterMath course, Tom had a specific idea for teaching with technology and he seemed to have the goal of teaching this lesson by himself with ease. The use of technology in teaching this particular concept seemed very important to Tom as he discussed it in each of his three interviews. I claimed that Tom assimilated the spreadsheet sub-component for his own learning, but he experienced a perturbation for his teaching as he never got comfortable enough with using the program himself to incorporate it in his classroom. Tom also talked about how he had no ideas for incorporating technology into his teaching of future content. Therefore, Tom seemed to leave the course in a state of perturbation about using spreadsheets and technology, in general, in his teaching.

InterMath Problem Solving

For his Learning

Context for review. When Tom was asked how he would explain InterMath to a colleague, he said,

> you will sort of get a refresher class in all different math concepts but they will come up as result of the math class. You won't get direct instruction on what the commutative property is, but it will likely come up in a problem some place. (Interview 3 Page 12)

For Tom, the problem-solving component of InterMath was the context for reviewing the mathematical content that he already knew. When talking more about the commutative property, he said that I, as the instructor, did not have to "worry about doing" it with the InterMath participants because I was able to "assume" that most of them knew the commutative property and if they did not, "no harm, no foul." Regardless of their knowledge of this particular property, Tom said that I refreshed their memories "by talking about it collectively" with the group as it came up in the context of the problem (Interview 3 Page 18). Tom also claimed that "by using a problem-solving approach, you will address the skills that we (need) in each problem" (Interview 3 Page 18).

My interpretation. Tom considered problem solving as the context in which the mathematics of InterMath was reviewed, which again implied that Tom did not view the mathematics of InterMath as new content. The course simply provided a "refresher" when the mathematics was not easily remembered. Tom did not seem to find the problem-solving component of InterMath to be difficult, which allows me to claim that he assimilated this component for his learning.

For his Students' Learning

Basic skills a must. For Tom's students, problem solving involved "students practicing problems (and) applying something that they know in an activity that has engaged them" (Interview 2 Page 8). Tom said his students need the "requisite skills" in order to be successful at problem solving (Interview 2 Page 7) and that "students need to be more successful than not to be motivated to continue" (Interview 3 page 20). Therefore, his students needed skills first before engaging in problem solving so that they could be successful in his classroom.

My interpretation. Tom did not seem to make any connections from the problem-solving component to his students' learning because he viewed problem solving as an engagement in applying already acquired basis skills. Since his students did not have the basic skills, he could not allow them to engage in problem solving. If he tried to do problem solving with his students, they may not feel the success that he wanted them to experience due to their lack of skills. Due to the lack of connections, Tom did not experience any assimilation or perturbation here.

For his Teaching

Student success. When discussing student success in relation to student learning, Tom quickly moved into discussing student success in relation to his teaching. Tom was willing to accept the blame if his students gave up on a problem “because I have not put that bridge there for them to get to it” (Interview 2 Page 6) or “because I haven’t prepared them for it” (Interview 2 Page 11). In both of these statements, he discussed the necessity for him to prepare his students by giving them the requisite knowledge necessary to answer the problem-solving activities that he gave them.

In class, Tom talked about giving his students a task from the GPS framework (Observer Notes – Class 5). He had issues with the task because his students could not be successful as he said that his

> students are frustrated because they are trying to work a problem they really can’t. And I’m frustrated because it’s a performance task that the kids are interested in, but if I took it away from them and taught them the skill that they needed and I give it back to them in a couple of days, the interest in it will be gone. (Interview 3 Page 14)

He decided that he could remedy this by identifying “the skills with which the students need to solve the problems” (Interview 2 Page 7) and teaching his students how to solve the problem the day before he gave them the actual performance task (Interview 3 Page 14). In doing this, his students would be “able to solve the problems successfully and practice the requisite skills that are part of the curriculum that they are studying” (Interview 2 Page 7). This would also allow his students’ “interest level [to be] there because they couldn't solve the problem and you showed them how to do it" (Interview 3 Page 15).

Thorny problems. Tom liked the idea of giving his students "thorny" problems (Interview 2 Page 1; Interview 3 Page 16) similar to those of InterMath, with "a lot of things going on in them" and "multiple ways to solve them" (Interview 2 Page 1-2; 9). In giving his students these thorny problems, he wanted his students to be "suitably challenged by a problem but not overwhelmed by it" (Interview 2 Page 11).

Changed teaching practices. In Tom's final interview, he said, "my teaching has changed. You provided a model that sort of caused my instruction to change a little bit" (Interview 3 Page 10). He really liked the problem-solving approach where "you would put a problem on the board and you would really get us all interested in thinking about it. I have since tried to do that with my students" (Interview 3 Page 9). Another change in Tom's teaching practices, according to him, involved the kinds of problems that he was using for his students' warm up activities. "My warm ups are no longer '3/8 equals what as a decimal?' They are more convoluted. They are bigger problems than that. And I attribute that to InterMath" (Interview 3 Page 11).

My interpretation. Tom claimed that his students needed the basic skills before engaging in problem solving, and it was his job to provide the direct instruction of basic skills so that his students could be successful. Therefore, Tom seemed to prescribe the steps that students must take in order to solve a word problem instead of allowing his students to explore. Tom made little connection from my teaching problem solving in InterMath to his teaching problem solving in his classroom other than the possible use of "thorny problems." This was mainly due to the difference in skill abilities of our students

– Tom seemed to think that my students already possessed the necessary skills where his did not. Of course, in our InterMath course, it was not always the case that every participant had every skill used to solve every problem in every class, but Tom did not attend to this. He seemed to assume that because they were all teachers, then they must all have the necessary skills. Therefore, Tom assimilated most of the problem-solving component for his teaching.

However, Tom attributed some changes in his teaching to InterMath, which may indicate that he was perturbed by the problem-solving component of InterMath for his teaching. He never discussed anything being problematic or appeared uncomfortable with the problem solving in our course, but he had to reflect about his teaching while engaging in the problem solving of InterMath for him to make changes. It was unclear what these "bigger, more convoluted" problems looked like in Tom's classroom. It was also unclear of what I was doing to get the participants interested in the problems that Tom was trying to emulate in his classroom. Regardless, he was attributing some change in his teaching practice to InterMath. I claim that Tom experienced a perturbation and accommodated it in regard to the problem-solving component of InterMath for his teaching. This would have gone totally unnoticed if Tom had not reported these changes in his final interview.

Tom's Making Sense of Problem Solving

For Tom, InterMath was a review of mathematics that he already knew and problem solving was simply the context used for the review, which allowed him to assimilate this component for his learning. He liked the InterMath approach to problem solving for himself, but he already had all the requisite basic skills, which enabled him to engage in the problem solving without getting frustrated the way that his students might.

Tom was unable to make any connections between the problem solving of InterMath to his students' learning because he claimed that all of the InterMath participants already possessed the basic skills necessary to engage in the problem solving of InterMath. This was different from what his students could do in his classroom because they had to be taught the basic skills first. Because of the lack of connections between the problem-solving component of InterMath to his students' learning, there was no assimilation or perturbation experienced.

Again, Tom wanted to engage his students in problem solving but not until he had taught them the skills needed to do so. It would seem that Tom assimilated the problem-solving approach of InterMath for his teaching as well. If he had not mentioned in his final interview that his teaching practices had changed due to InterMath, I would have claimed complete assimilation. However, since Tom claimed to change his teaching practices, he had to experience a perturbation with the problem-solving component of InterMath for his teaching that he was able to accommodate. Not understanding the exact nature of the changes in his teaching is unimportant because he attributed this change to InterMath.

Summary

Throughout my observations of and interactions with Tom, I found that Tom assimilated all of the components in the InterMath course for his own learning. He was familiar with the mathematics allowing him to engage in the problem-solving component without having to learn any new mathematics. He expected to learn about spreadsheets, but not any new mathematics. Because the participants in the InterMath course were developmentally different from his students, their abilities, skills, and engagement with the mathematics was different than what he expected of his students, causing him to make no connections to student learning and few connections to his teaching.

Tom often discussed his teaching, in general, and the connections that he made from InterMath to his teaching were the only instances in which Tom experienced perturbation. He remained in a state of perturbation about using technology in his classroom and seemed to accommodate a perturbation in regard to his teaching that related to the problem-solving component of InterMath.

In the following table, I attempt to summarize Tom's making sense of the professional development for his learning, for his students' learning, and for his teaching in terms of assimilation, perturbation, accommodation, and shut down (see Table 3). In Tom's table, you can see that the column, *for his students' learning*, is blank, which is due to Tom not seeming to make any connections from the InterMath course to his students' learning. Tom assimilated all components for his learning and the idea that problem solving can only be done if students have the necessary skills to solve the problems. He accommodated some ideas for problem solving in his teaching and left the course in a state of perturbation about using spreadsheets in his teaching.

Table 3

Summary of Tom's making sense. This table shows Tom's making sense of the professional development for his learning, for his students' learning, and for his teaching in terms of assimilation, perturbation, accommodation, and shut down.

	For his Learning	***For his Students' Learning***	***For his Teaching***
Assimilation	Mathematics Technology – spreadsheets Problem Solving		Problem Solving – need of skills
Perturbation			Technology – spreadsheets
Accommodation			Problem Solving – change in teaching practices
Shut Down			

Tom's Model

From the beginning, I was trying to create a model of how each participant made sense of the professional development for their own learning, for their students' learning, and for their teaching. Tom's model for the professional development did not match my hypothesized model that teachers first make sense of the professional development for their own learning and then move on to making sense of it for their student's learning, and finally make sense of the professional development for their teaching.

Tom rarely talked about his students' learning unless asked specifically. When he did respond to a question about student learning, he always turned it back to himself and his teaching. Because his learning was so different than his students' in that his students would actively learn the mathematics that he was teaching them since it was new material to them, he was unable to make any connections between his learning of the mathematical content through problem solving to his students' learning or to his teaching. Tom also did not make any connections between the technology and problem solving components of InterMath to his students' learning.

Tom came to the InterMath course thinking that his students would benefit from using technology, especially Excel. He also knew that he needed to learn Excel well enough to incorporate it in his teaching. He had already made these connections between his students benefiting from using technology to his learning to his teaching. Since Tom came to the InterMath with the specific goal of learning the technology so he could teach with it, he really was only thinking about his teaching. This was also evident in his statements about him being a work in progress as a teacher and that he was trying to get better at teaching by hoping to know his students better once he no longer had to focus on himself in the classroom.

At some point in the course, Tom experienced an accommodation for teaching problem solving. Not knowing more about this experience, I do not know if he connected the problem solving for his own learning or from his students' learning to his teaching, which is another disconnect.

Due to Tom's lack of connections made during the InterMath course for his learning, his students' learning, and his teaching, the model that I have created for Tom's making sense in the InterMath course is very simple as there are no arrows drawn denoting connections (see Figure 8). If Tom did make any connections between his learning, his students' learning, and his teaching, he did not discuss them with me.

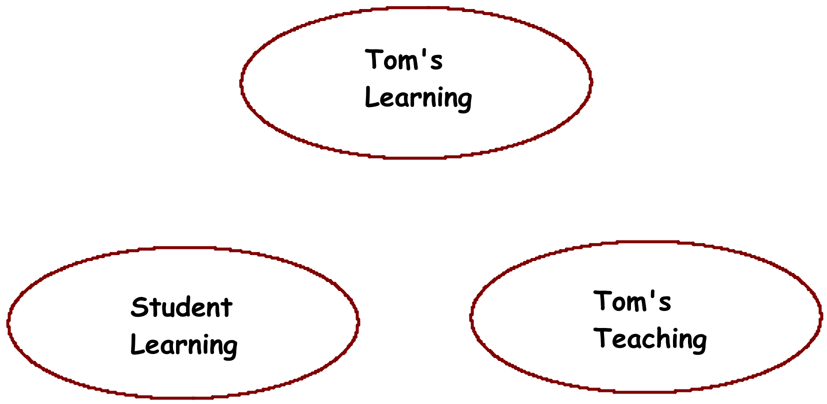


Figure 8. Tom's model. This model shows Tom's making sense of thinking about his learning, his students' learning, and his teaching.

I would argue that Tom's model of making sense could have been made even simpler since he was mainly focused on his teaching rather than his learning or his students' learning (see Figure 9). Tom's focus on his learning of the spreadsheets was actually a focus on his teaching because he wanted to use the spreadsheets in his teaching. Therefore, he was only focused on learning the spreadsheets so that he could incorporate them in his teaching, which was a connection between his learning and his teaching made before the InterMath course.

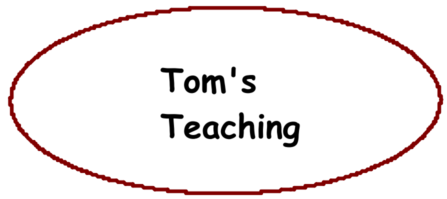


Figure 9. Tom’s simpler model. This model shows Tom’s making sense of the professional development, which is only about his teaching.

CHAPTER 6

PARTICIPANT SUSAN

Description

Susan graduated from college with an early childhood degree, which she reported did not require much mathematics. She remained in college, added middle grades to her certification, and then taught middle school for eleven years before starting her master's degree in middle school education. She alternated between teaching 5th grade and middle school for several years. She eventually quit teaching middle school mathematics because she did not have the newly required mathematics concentration. She had the concentration for language arts, social studies, and reading, but enjoyed teaching math, so she decided to teach fifth grade since that level does not require the concentration to teach mathematics. Susan was in her twentieth year of teaching and had taught at least one mathematics class for more than half of those years. At the time of the InterMath course, she was teaching all subjects, except for science, and an extra mathematics class at Flat Rock Elementary.

Susan was often quiet during our InterMath class but appeared to be engaged. She often wrote in her notebook and offered solutions and explanations (Video – Class 1; 2; 4; 5; 6; 7; 9; 10; 12). She appeared confident in her mathematics abilities and scored a 19/21 (90%) on her pretest. She also seemed confident in using the technology. At the halfway point, she said that she was happy that she had taken the course because after a particularly bad day of school, the InterMath class "was actually the better part of the day" (Interview 2 Page 12). She claimed that the class was "calming and soothing" and she "enjoyed it" (Interview 2 Page 12).

InterMath Mathematics

For her Learning

Content to just "do" most mathematics. As a learner of mathematics, Susan was content to just "do it [the way the teacher showed them] because this works." She "accepted that and went with it" (Interview 1 Page 2). She claimed that the mathematics instruction that she got when she was younger "was more 'just do it because this is the way you do it'…. It was just do it because you want to get that good grade. You want to move on" (Interview 1 Page 4). She "never thought about why…. Just do it because it is less stress than to stop and ask questions" (Interview 1 Page 4). Susan was content to mimic procedures "taught" to her by her teacher.

When asked how she would approach a university level mathematics course today, she said, "if all I needed to do was get that credit, then I probably wouldn't [try to understand it]" (Interview 1 Page 8). Susan attempted to clarify this when she laughingly said,

> if I were taking it just because I had to have it and it had nothing to do with my everyday… if I wasn't teaching math or anything like that, I would probably just [do the mathematics] because I'm a little basically lazy. (Interview 1 Page 8)

Susan also noted that she needed to understand some mathematics – the mathematics that she teaches her students. She realized that there were kids out there who were like her in that they just wanted "to get through [the mathematics] and move on to something that they like better. I guess it depends if you see the validity in it and how it's going to relate to whatever you're doing next" (Interview 1 Page 8). She realized that a lot of her students were not content to just do the mathematics, so she needed to really understand the mathematics that she teaches so she could help her students understand it. Of mathematics content, she said,

> if it is something that you need to get across to the children and make sure that they understand, then that's a different story. If you're just doing it because you need five hours[8] or whatever, then you're just going to get your five hours and move on. But if it is something that you're going to use to actually help your situation day to day, then I think that's different because how can you explain it to them if you haven't bothered to ask any questions and figure it out yourself? (Interview 1 Page 8-9)

Intimidation versus comfort. Susan claimed to be intimidated by a lot of the other InterMath participants mainly due to her perceptions of their backgrounds in mathematics, "especially the high school [teachers, who] have a lot more knowledge or experience with things" (Interview 2 Page 5). Susan "hadn't taught a lot of that stuff in these lower grades" (Interview 3 Page 4) and said,

> I'm a little intimidated because I never had any college math at all. I [tested out of] the math 101 and went from there. So, I don't have the background that the teachers have and I haven't been forced to remember all that stuff from high school because I haven't taught those higher levels. So, I guess I feel a little bit intimidated in that regard." (Interview 3 Page 13).

She said that she was never uncomfortable in the class, but that "the [teachers of the] higher levels were just so comfortable and throwing out [mathematical ideas] and I

[8] "Five hours" refers to the professional learning units (PLU) given for the InterMath course. Teachers in Georgia have to acquire ten PLU over each five-year period in order to keep their teaching certificates.

was like 'oh, what are they talking about?'" (Interview 3 Page 13). In staying in her comfort zone, she said,

> things kind of got above my head a time or two…. But I was interested to hear what they had to say and then if it just got too far blown, I kind of tuned into something that was more relevant to my situation. (Interview 3 Page 13)

She laughed when she said that she sometimes thought, "oh, I don't know, this is too much…how would I even start to do that? Skip that and go to another one" (Interview 2 Page 5). And then in a specific class, we discussed slope and Susan said, "'whew, too many years since slope.' I think I kind of tuned out at that point and started looking at [a problem I could teach my students]" (Interview 2 Page 5).

Even though she was familiar with some of the mathematics of the course, Susan also found that a lot of the mathematics was new to her. She knew that "square numbers make a square, but Fibonacci… series… a lot of the terms and the patterning and the methods of doing things were new" (Interview 3 Page 5). Occasionally, Susan tried to push through the mathematics that she was uncertain about. "Sometimes it can be a little intimidating but then when you get it, it kind of lights up your whole night. It's like 'oh, I got that'" (Interview 2 Page 7). Overall, she said the mathematics content of the InterMath course was "good" and "appropriate," and that it would be

> hard to find something that's going to work for the range that we have [in our InterMath course] and make everybody feel comfortable, but feeling comfortable is not what it's all about either. It's about learning what can you do and kind of stretching yourself. And you do feel good when you get it or you feel like you've

> contributed to the process. If everything was too easy we would all be wasting our time. (Interview 2 Page 10)

Getting the answer – quick! Susan was also content to hurry through a problem, get the answer, and move on to the next problem. Of the mathematics instruction she experienced as a learner, she was used to following steps, so she "was more used to 'alright, get the answer, move'" (Interview 2 Page 4).

Sometime during the first half of the course, Susan investigated a problem and then said,

> I thought it was kind of cool… 'oh wow, I actually know this. Hurry up, I know the answer.' Then after a few minutes [of whole group discussion], it was 'ok, you don't know it all after all.' You know you felt good at first but then you still got a lot out of it at the end. (Interview 2 Page 2)

Susan laughed when she said she realized that she "did not know it all after all." Before, she "was thinking, 'well, we have to solve these problems like a race… hurry up, whatever'" (Interview 3 Page 10). By the end of the course, she claimed to have "become a lot better at getting the most out of the problem" (Interview 3 Page 10). She was spending more time on each problem trying to get as much out of it as she could and she seemed to attribute this to InterMath.

When asked what she was learning in the InterMath course, Susan laughingly said,

> to be patient. I guess I've learned… before it's like, you have to hurry up and do all these problems and make sure that you just get the right answer and not so much delving into the whys and how you got there. So, I guess, kind of accepting

> every answer and presenting it the way you always [do]… 'Well, what about that?' 'What do you think about that?' 'What does that do?' All those wonderful questions that you ask to just kind of get you to pay more attention to the particulars of the details of each problem and what's going on instead of hurry up and get the answer and go to the next one. (Interview 2 Page 1)

My interpretation. Susan came to InterMath knowing that she needed to understand the mathematics that she teaches to her fifth graders because many of her students were not content to learn the mathematics without understanding. By taking this InterMath course, she may have expected to learn mathematics that she teaches better. When talking about the different mathematics courses that she could take with the goal being to learn mathematics, she talked about how the content of the course was important for her engagement. InterMath seemed to provide mathematical content that Susan needed to learn so that she could facilitate student understanding in her classroom. Therefore, Susan expected to learn some mathematics in more depth. This allowed her to assimilate the mathematics content that was similar to the content she teaches.

However, not all of the InterMath content was in her fifth grade curriculum. She had stated that she needed to see relevance in the mathematics content for her everyday life in order to try to understand it implying that she may need to see relevance for her *teaching* in order to try to understand it. There was content covered in InterMath that she did not teach, but she engaged in the mathematics anyway. Because Susan only expected to want to learn content that she taught, she may have been perturbed by the presence of content that she did not teach. Susan did not deal with all of this in the same way. Sometimes she pushed through the mathematics allowing her to accommodate this perturbation of learning content that she did not teach. Other times, like during the discussion of slope, she tuned out and found something more relevant to her teaching. Since she tuned out the conversation on slope, she shut down in that perturbation. Therefore, Susan assimilated, accommodated, and shut down in terms of the mathematical content for her own learning.

Another source of perturbation for Susan was the time that we spent discussing problems. Susan was accustomed to working through problems as quickly as possible so that she could move through them and be finished with the assignment. She realized early in the course that InterMath was not a race and that there was more to learn in each problem than just arriving at the correct answer. Susan was able to accommodate this pace perturbation as she learned patience in the course and how to get more out of the problems.

For her Students' Learning

Students' need for "whys." As stated previously, Susan realized prior to the InterMath course that her students were not content to "do [the math] this way because this works" the way she had (Interview 1 Page 2). She attributed this realization to her son, who had

> to understand all the whys, and wherefores, and why nots, and all that before he can make any sense of it and put his best effort into math. I know he's not the only child like that so I just want to get all the ideas I can to try to make my kids understand what they're doing and make them want to do well and know that they can do well. (Interview 1 Page 2)

Her students “need explanations and they need to be able to manipulate and try to get the concrete definitely before they can envision anything more” (Interview 1 Page 6). When talking specifically about teaching elapsed time, she said, “if they don’t have that clock in their hands, moving those little hands around those minutes, they can’t figure it out. It doesn’t make any sense [to them]” (Interview 1 Page 5). More generally, she said that her fifth graders needed anything that “they can manipulate physically… touch, that kinesthetic thing and that kind of helps them to experiment because they are so into the concrete. They need to have something to work with instead of trying to visualize in their minds” (Interview 3 Page 7). Susan knew that her students required more of her than just rules and procedures. Her students needed to know why the mathematics worked and to manipulate concrete objects while attempting to understand the mathematics.

Her students’ shoes. Susan often discussed how she thought her students would react when put in similar situations to those she experienced in the InterMath course. When she discussed being intimidated followed by feelings of success, she said that her students may also need experiences similar to those that she got in InterMath to help them get over their own intimidation (Interview 2 Page 7). After exploring the InterMath problem *Apples and Oranges* (see Appendix D for entire problem) for 75 minutes and not solving it, Susan said that she understood why her students would quit (Video – class 5). When she claimed that the mathematics got over her head, she talked about her fifth graders getting stuck and how she wanted to be like them as “this is too much trouble. I don’t want to bother with this. Let’s go on to something else” (Interview 2 Page 10).

When talking about how she had to push through the mathematics in the InterMath course but that she was able to get there and feel good about herself, she said,

> so maybe the kids need that. If I could lead them to that first…especially if it's like a word problem where they don't know what to do. I could find those questions to guide them through to get that down. Maybe they would be a little more willing to stick with it or try a little harder. (Interview 2 Page 7)

She also reported her experiences in the InterMath course to her students. She told them about the InterMath test where "there were 4 questions and there were 2 on there, 'oh ok, I think I got these. But these other 2, I don't know.' They were looking at me like 'ahah, now she sees what it's like'" (Interview 3 Page 13). About InterMath, Susan said that it was good to be "learning from each other and just getting a different feel and also, being good as the student instead of just the teacher and you can kind of see both sides" (Interview 3 Page 4).

My interpretation. Susan definitely thought about her students while taking the InterMath course. First of all, she knew that her students needed to understand the mathematics, so that was a priority for her and her main reason for taking the course. But by also being placed in the role of the student, she often compared her situations in the course to those of her students and seemed to understand how her students may feel when placed in similar situations. She claimed to understand why her students would want to give up when trying to solve a problem that they did not completely understand or find relevant or had worked on for a long time with no answer, how they may feel intimidated by their classmates' mathematical abilities, and how they may feel success when she is able to guide them through a problem that they would have otherwise given up on. Of course, she experienced all of these situations in the InterMath course as a student.

Because of Susan's learning experiences in the InterMath course as a student, she was able to think about her students' learning experiences. She did not seem to experience *new* perturbations in terms of her students' learning; however, she was able to relate the perturbations that she experienced for her own learning to her students' learning. By putting herself in her students' shoes, she accommodated these perturbations for her students' learning.

For her Teaching

Teaching the "whys." Susan knew that she could not "just tell them" (Interview 1 Page 5) – her students needed to know why the mathematics worked making it her responsibility as the teacher to teach for student understanding. She needed "to try to help them see how all the things fit together" even though she said, "it was hard to think that way since I wasn't really geared into it…it was very taxing to try to stop and think 'well, how can I make this make sense to them?'" (Interview 1 Page 4).

Susan knew that some of her students were content to just do the mathematics as she had been, but she wanted those students to understand "that you're not just learning this for tomorrow's quiz. There's a reason for this and you're going to build upon this" (Interview 2 Page 11). Since many of her students wanted to understand the mathematics, she wanted to teach in a way that would help them understand. She also wanted to instill this value of understanding in her students who were not concerned with understanding the mathematics.

My interpretation. Again, Susan came to the course wanting to learn the mathematics that she teaches better so that she could teach her students for understanding. She claimed that it was difficult for her to think about teaching her students this way because she was not accustomed to it for her own learning. Susan was probably perturbed by this at one time but had accommodated because she was now trying to teach her students for understanding.

Susan's Making Sense of the Mathematics

Prior to the InterMath course, Susan realized that her students needed to understand the mathematics, so in order for her to teach her students mathematical understanding, she needed to understand the mathematics herself. Due to this need for mathematical understanding for her students' learning and her teaching, Susan seemed to used this as a reason for taking the InterMath course in order to develop her own understanding of mathematics even though she did not explicitly say so. Therefore, Susan's thinking about the mathematics seemed to move from her thinking about student learning first, to thinking about her teaching, and then to thinking about her own learning.

Susan was satisfied to only understand the mathematics that she needed to teach to her fifth graders and was not interested in understanding mathematics that she was just getting "credit" for. Susan saw no need to understand mathematics that did not directly relate to her life somehow, and at this point, the only connection of mathematics to her life seemed to be the mathematics that she taught. Therefore, in the InterMath course, Susan tried to understand the mathematical content that she found relevant to her teaching, which was the mathematical content that she taught her fifth graders. She was able to assimilate this mathematics, as she came to the course expecting to learn that content in more depth.

However, she also experienced mathematical perturbations in the course even though she said that she was never "uncomfortable." She talked about how she was intimidated at times and that the mathematics was over her head. Sometimes, she tried to push through the mathematics even though she wanted to quit like she had seen her students do. In these instances where she pushed through the mathematics, she may have accommodated the perturbations of not knowing the answers right away. At other times, she shut down when she experienced mathematics that she did not find relevant to her teaching as she turned her focus to problems that were relevant to her teaching. The mathematics component of InterMath allowed Susan to experience assimilation, accommodation, and shutting down in terms of her own learning.

Susan's learning experiences as a student allowed her to think about her students' learning experiences. She did not seem to experience *new* perturbations in terms of her students' learning or her teaching; however, she was able to relate the perturbations that she experienced for her own learning to her students' learning and then for her teaching. By putting herself in her students' shoes, she accommodated these perturbations for her students' learning. When thinking about her students needing similar experiences to those that she had gotten in the InterMath course, she said that she needed to provide those experiences in her classroom. Therefore, she accommodated these perturbations for her teaching as well. In this instance, she was able to use her learning experiences as the student to think about her students' learning and then to think about her teaching.

InterMath Technology

For her Learning

Using computers to do math. After the first night of class, Susan said that she had gotten the impression that we would be using the computers a lot based on the InterMath survey (Interview 1 Page 2). She claimed to be "not very good with computers" but that she was “so excited to be able to tie [technology and mathematics] together a little bit” (Interview 1 Page 2), which she claimed to not know how to do since she usually used computers for word processing and presentations for language arts (Interview 1 Page 3).

We often created spreadsheets that allowed us to explore the mathematics in our classes. Susan usually followed along creating the same spreadsheet at her individual workstation (My notes). In class 7, Susan suggested that the group create a spreadsheet to solve a problem that we were discussing (Video). She said that in that particular class, she really focused on learning the basics of the spreadsheet program because she thought her students would like it (Interview 2 Page 2). She seemed to be learning how to use spreadsheets in mathematics, but she only mentioned using a spreadsheet to investigate the mathematics in one of her write-ups (Participant Products).

My interpretation. The technology component of InterMath was unexpected by Susan; however, she did not display any feelings of discomfort about this. She was excited about learning to use computers to investigate mathematics, which she was previously inexperienced with. Therefore, Susan did not experience a perturbation here, which implies that Susan assimilated the technology component for her own learning.

For her Students’ Learning

InterMath component. When Susan said that she did not know that InterMath would involve a technology component, she added, “I know that the kids really love [technology] so I’m excited to be able to tie those two together a little bit” (Interview 1 Page 2). After creating several spreadsheets in the InterMath course, she saw the relevance for her students to also use spreadsheets as she said that her

> kids would enjoy that instead of having to work everything out. They hate all that constant computation and I think, sometimes, if they had a tool like that and they knew they could use it, then maybe they would spend more time on the thought process instead of 'I'm not going to do this problem because I would have to do all that multiplying' or something. I think maybe they would stick with it a little bit longer if they knew they could use tools like that. (Interview 2 Page 2).

My interpretation. Susan seemed to immediately see relevance in the technology component of InterMath for her students, because they would enjoy using it. During the course, she talked about her students having access to a computation tool, which would allow them to spend more time thinking about the mathematics rather than getting bogged down by the computations. Where her students could have easily used a calculator for these computations, Susan referred to the spreadsheet program that we used in InterMath as this kind of tool. Susan assimilated the technology component for her own learning, which seemed to be based on the idea that her students would enjoy using the technology. Therefore, Susan assimilated the technology component for her students' learning also because it was never problematic for her when thinking about her students.

For her Teaching

Using computers to do math. At the beginning of the course, Susan was unsure of how to combine mathematics and technology (Interview 1 Page 2; 3). She basically used the computers in her room for word processing and presentations for language arts (Interview 1 Page 3), and only used them in mathematics for "fun" as "little math games" for her students to drill and practice (Interview 2 Page 3). She knew that computers could specifically be used for "graphing… but didn't remember how" (Interview 2 Page 4).

In our course, we used computers for many activities. Specifically for mathematics, we regularly investigated problems using spreadsheets, we explored a problem using geometry software, and we often looked up mathematical topics (e.g., star numbers) and mathematicians (e.g., Hypatia) on the Internet to see what we could find. Susan claimed to have not thought about "doing the research on [the computer]…. I hadn't really thought about using [computers] for that purpose in math" (Interview 3 Page 9). Susan said,

> With all of the things that are on-line for you to use, like the dictionary… I always knew they were there but I never took the time… really spent enough time looking at it or seeing what I could do. (Interview 2 Page 3)

By the end of the course, she decided that one way she could use computers in her mathematics classroom was "kind of like in language when we don't know a word, we just look in the dictionary real quick but you could just as easily send someone over to the computer to find the answer" (Interview 3 Page 9).

Susan said she learned technology in the InterMath course because she "was at nowhere to start with" (Interview 3 Page 7). She said, "I'm a lot more willing to try different things [with the computers]…and to let the kids try different things than I was before [InterMath]" (Interview 3 Page 7). By the end of the course, she had not tried anything with spreadsheets with her students similar to how we used them in our class because she "was always afraid of that because it was something that you have to take time to figure out and then do" (Interview 3 Page 3).

My interpretation. Susan was unsure of how to incorporate technology into mathematics, but she got several ideas during the InterMath course including using the computers for research purposes, such as using the Internet to look up topics and mathematical vocabulary. She never got to the point where she was comfortable using spreadsheets in a manner similar to how we used them in our InterMath course as she claimed to be “afraid” of the time that it would take to plan for those experiences. She had experienced a perturbation about the technology component of InterMath and had accommodated it for her teaching as she now had ideas of how to involve technology in her mathematics instruction. However, incorporating spreadsheets in her teaching still seemed to be problematic for her. She never indicated that she would or would not continue trying to incorporate spreadsheets in her teaching. Therefore, she left the course still in a state of perturbation about using spreadsheets in her classroom.

Susan’s Making Sense of the Technology

Susan was unaware of the technology component of InterMath, but she did not experience any visible discomfort. She claimed that her students loved technology and that she was excited about learning how to tie together technology and mathematics. Therefore, the technology component did not seem problematic for Susan for her own learning or for her students’ learning so I claim that she assimilated this.

For her teaching, Susan was unsure of how to incorporate the technology in her mathematics lessons. Because of this uncertainty, I claimed that Susan experienced a perturbation about the technology component for her teaching. She got ideas of how to incorporate the technology, which implied that she accommodated this perturbation. She also experienced a perturbation in using spreadsheets with her students similar to the ways in which we used them in our course. Incorporating spreadsheets in her teaching would require time for her learning and time for planning. Susan did not accommodate this perturbation of using spreadsheets in her teaching and she did not seem to shut down either. Therefore, Susan left the InterMath course in a state of perturbation about using spreadsheets in her teaching.

InterMath Problem Solving

For her Learning

InterMath approach. The problem-solving approach taken in InterMath required “training” on Susan’s part. The approach was different than what she was accustomed to because the goal was different. She said that in InterMath, the “object is not to just do these problems and not have any homework tonight” but to get “a better understanding of what was going on and how all the different math aspects work together” (Interview 2 Page 5). She also said that the approach “opens your mind and brings back a lot of things that maybe you weren’t thinking about” (Interview 3 Page 4) and it “ has been a good thing for me to see” (Interview 2 Page 2).

Even though the problem-solving approach was new to her, Susan “enjoyed how it pulled everything together” (Interview 3 Page 3) and how “you can get the most out of each of us… keep asking those questions and make us think and put it out there but at the same time it was very low stress situations” (Interview 3 Page 6). Susan said that these low stress situations

> didn’t make me feel uncomfortable at all. It was very easy going. You’re so accepting of whatever it is that we say and “let’s look at it this way.” If we do give you something that’s out in left field, you just kind of bring us back around. So it was new to me, but it didn’t make me feel uncomfortable. (Interview 2 Page 4)

Susan also liked the "debriefing" at the end where we discussed all of the mathematics that we had used to solve the problem. She said, "I didn't even realize what all we had talked about and just going back and mentioning it was a good way to end" (Interview 2 Page 5).

My interpretation. The problem-solving goal was different than what Susan was accustomed to. Like the mathematics, she was not used to the goal being *understanding*. Susan experienced a perturbation because it was different, but she seemed to accommodate it as she said that she liked that the goal was understanding. Because the problem-solving goal was different, the problem-solving approach being different was not problematic for her as she liked the approach that was used to get at this understanding. She claimed that the problem solving never made her feel uncomfortable because it was so "easy going." Due to her lack of discomfort, she assimilated the problem-solving approach for her learning.

For her Students' Learning

Problem-solving training. Susan said that this approach was different than what her students were used to as they normally worked through problems more quickly with the focus on getting the correct answer. Because of this, her students

> would have to kind of be trained in that as I was. This is different, we're not doing what we've been doing…. I think they would have to be trained in it and kind of led what to expect. Maybe walk them through step-by step, but in the end, it would benefit them more because there would be a better understanding of what was going on and how all the different math aspects work together. (Interview 2 Page 5)

My interpretation. Having to train her students using the InterMath approach to problem solving did not seem to be problematic for Susan. She seemed to think that it was something that could be done and that her students would benefit from using this in the classroom. Since this was not problematic for Susan, she assimilated the problem-solving component of InterMath for her students learning.

For her Teaching

Pace. Susan talked about slowing down and working through the problems instead of speeding through them like she was used to doing. The same was true for her students – they did not need to be rushed because rushing them did not allow time for understanding. “I think we are trying really hard to reach the students and to get them to know the things that they need to know, but it is really hard at the pace [we]’re going at” (Interview 3 page 5). For her teaching, she said that she needed to “kind of focus myself so that [my students] will do the same thing. I think if I slow down and they see that I am putting more importance on all the little steps and what we can draw out of it, then they will take it more seriously, too” (Interview 3 page10).

In her classroom, Susan

> tried to slow the pace a little bit, especially the word problems because it was kind of frustrating to me that there’s the word problem in the book and two or three of the truly math kids are on it and everybody else kind of has no clue and so then I would more or less explain to them how we got this. But I’m trying to give it a little bit more time and I’ve got a long way to go. But to try to lead more of them to see what to do or how to guide them into it more, I guess, instead of just “ok. This is the way it is. Don’t you see it? (Interview 2 Page 6)

When asked what she was learning in InterMath, she said, "just to slow down and give them more time" (Interview 3 Page 3). So, she was trying to slow down with her students to give them time to understand the mathematics without telling them everything that they need to know. She was hoping that by slowing down that her students would take problem solving more seriously (Interview 3 Page 10).

InterMath teaching ideas. A lot of the InterMath investigations had extensions that required more thought once the original problem had been solved. In our course, we often talked about extensions that we could add to the problems that did not have them. For example, when working through a problem that involved thinking about positive rational numbers, we would then talk about how the problem would change if we used negative rational numbers instead or integers as opposed to whole numbers.

Susan liked extensions for her students because "you've got those kids that are ready to move on that are real quick in math and they don't want to just sit there. They need to be challenged to think further" (Interview 2 Page 11). She discussed using the idea of the extension with her students who needed another task to keep them busy while others were still working on the original task. She said that these extensions could even "go past the curriculum… especially for those who are ready to go on just to expose them [to more mathematics] if nothing else" (Interview 2 Page 11). She also talked about using the problems posted on the InterMath website. "Those [problems] are great…. I would like to try some of those with my students" (Interview 3 Page 11).

Another teaching idea that Susan got from InterMath involved cooperative learning. She already did this in her classroom, but it was essentially a student helping or getting help from the students sitting on either side in their "horizontal rows" (Interview 1

Page 6). In our course, the InterMath participants tended to work together and to "feed off each other" (Interview 2 Page 5). Susan wanted to

> teach [her students] how to work together… don't just give the answer or whatever but teach them how to actually discuss the problem and arrive at the solutions… and for them to be able to explain how they got that. (Interview 2 Page 5)

Susan wanted to teach her students to engage in the mathematical problem solving the way that she had been able to engage in our course.

My interpretation. Susan originally experienced a perturbation in the InterMath course about the amount of time that we spent on any one problem. She was able to accommodate this as she was able to realize that the goal for problem solving was different so it made sense for the approach to be different even if the approach meant possibly spending an hour discussing one problem. Susan was able to link this accommodation to her students' learning as she realized that they need more time to think about the mathematics, which she linked to her teaching – she needed to give her students more time to think about the mathematics. Once she accommodated her original perturbation about the pace at which we solved problems, she was able to link it to her teaching without it being problematic. Therefore, she seemed to assimilate the pace of the problem solving for her teaching.

Aside from thinking about giving her students more time to think about the mathematics, Susan got other problem solving ideas that she could incorporate in her teaching. She liked the idea of the extension that could be given to her "quicker" students, who may be ready to move on before the rest of the class. With the extension, those students could be challenged to think further about the mathematics. She liked the idea of teaching her students to discuss the mathematics the way that the InterMath participants had in our course. She also mentioned possibly trying some of the InterMath problems with her students. These seemed to be no discomfort for Susan when thinking about problem solving and her teaching. Because she wanted to provide her students with more opportunities to understand the mathematics, none of this was problematic for her. Therefore, she assimilated the problem-solving component for her teaching.

Susan's Making Sense of Problem Solving

The goal of problem solving in InterMath was different from what Susan was accustomed to, which caused her to experience a perturbation that she was able to accommodate, as she liked that the goal was understanding. Since the goal was different, it was not problematic for her that the approach taken in InterMath was different. Therefore, she was able to assimilate the problem-solving approach used in InterMath once she was able to accommodate the problem-solving goal. Also due to this accommodation, Susan was able to connect the problem solving that she was learning to her students. Because she had accommodated this for herself, she was able to assimilate it for her students, as it was not problematic for her.

Susan got teaching ideas from InterMath about problem solving. She liked the extensions, the summaries or debriefings, the problems, and the collaborative effort that they put into solving the problems. These ideas never seemed problematic for her, which may have been due to her desire to provide her students with more opportunities for understanding the mathematics. Because of this, she was able to assimilate the problem-solving component for her teaching.

Summary

Susan experienced assimilations and perturbations with respect to the mathematics, technology, and problem-solving components. There was mathematics that she assimilated, mathematics that she accommodated, and mathematics that caused her to shut down. Once Susan was able to accommodate the goal of problem solving, she was able to assimilate other aspects of problem solving. She initially assimilated the technology component of InterMath for her own learning, but then left in a state of perturbation about using spreadsheets in her teaching.

Student understanding was at the forefront of Susan's goals for taking the course and Susan often considered her students' learning during the course. Aside from the idea of her students needing to learn mathematics with understanding rather than rote, she often put herself in their shoes. She often discussed how her students would feel in similar situations and how it was good for her to have this InterMath experience as a student, but to be able to think about it also as a teacher.

The following table attempts to summarize Susan's making sense of the professional development for her learning, for her students' learning, and for her teaching in terms of assimilation, perturbation, accommodation, and shut down (see Table 4). In Susan's table, you can see that she assimilated, accommodated, and shut down in relation to the mathematics for her learning. She accommodated a majority of the components for her learning, her students' learning, and her teaching. She also left the course in a state of perturbation about using spreadsheets in her teaching.

Table 4

Summary of Susan's making sense. This table shows Susan's making sense of the professional development for her learning, for her students' learning, and for her teaching in terms of assimilation, perturbation, accommodation, and shut down.

	For her Learning	***For her Students' Learning***	***For her Teaching***
Assimilation	Mathematics - that she teaches Technology – spreadsheets Problem Solving – approach	Technology – spreadsheets Problem Solving	Problem Solving
Perturbation			Technology – spreadsheets
Accommodation	Mathematics – that she does not teach Mathematics – pace Problem Solving – goal	Mathematics – experiences	Mathematics – experiences Technology – teaching ideas
Shut Down	Mathematics – that she does not teach		

Susan's Model

From the beginning, I was trying to create a model of how each participant made sense of the professional development for their own learning, for their students' learning, and for their teaching. Susan's model for the professional development did not match my hypothesized model that teachers first make sense of the professional development for their own learning and then move on to making sense of the professional development for their student's learning, and finally make sense of the professional development for their teaching.

Prior to the InterMath course, Susan had decided that her students needed experiences in trying to understand the mathematics. She had also decided that she needed to teach in a way that allowed her students to understand the mathematics she was teaching them. She had already made the connection about her students' learning and her teaching – her students needed to know the "whys" of mathematics so she needed to teach them the "whys." Since Susan had never cared to learn the "whys" for her own learning, she realized that she needed to try to understand the mathematics better herself so that she could help her students understand it. Therefore, in terms of understanding mathematics, Susan first thought about her students' learning, then about her teaching, and finally about her own learning as she realized that she needed to learn the mathematics better so that she could teach her students to understand it better themselves.

Even though she had already made the connection between her students' learning and her teaching of the mathematics, she still often thought about how these components impacted each other. When learning about the technology component of InterMath, Susan immediately related that to something that her students would love and she got teaching ideas about how to incorporate technology into her teaching. When thinking about how slowly we investigated each problem in InterMath, Susan related this to her students needing more time to synthesize the mathematics and attempted to slow down in her teaching.

Susan seemed to only think about her learning as it related to her students' learning as influenced by her teaching. Therefore, Susan's learning was directly impacted by her students needs and the content that she teaches causing her model to look like that in Figure 10.

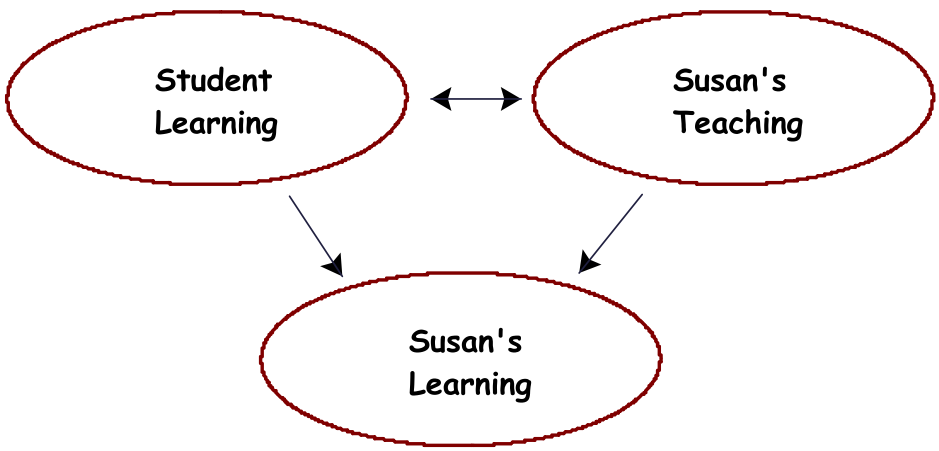


Figure 10. Susan's model. This model shows Susan's making sense of thinking about her learning, her students' learning, and her teaching.

CHAPTER 7

CROSS-CASE ANALYSIS

In this chapter, I examine the experiences of the InterMath participants using a cross-case analysis, which allows me to search for patterns and themes that cut across individual experiences (Patton, 2002). Specifically, I look at the research participants' similar and different perspectives on issues related to their own learning, their students' learning, and their teaching across the mathematics, technology, and problem solving components of the InterMath professional development course. The research questions guiding this study organize the presentation of the cross-case analysis. Thus, in this chapter, answers to the questions raised in this research study are presented.

Assimilation and Perturbation

What components of the InterMath professional development experience tended to cause assimilation or perturbation in participants?

To answer this question, the components of the InterMath course were combined based on whether each participant responded to the component with assimilation or perturbation. Table 5 shows the InterMath components that the participants assimilated and Table 6 shows the InterMath components that caused a state of perturbation. Some of the InterMath components fit in their totality within one cell, but others had to be broken into sub-components in order to be appropriately placed in the tables. For example, Susan assimilated some aspects of the mathematics content for her own learning but experienced perturbation when faced with other aspects of the mathematics content for her own learning. Therefore, the mathematics component was broken into sub-components in terms of the more specific mathematics that caused Susan to assimilate the ideas or experience perturbation.

Assimilation

Table 5

InterMath components assimilated by each participant.

	For Their Learning	***For Their Students' Learning***	***For Their Teaching***
Judy	Mathematics – all but one topic Problem Solving		Mathematics Problem Solving – most
Tom	Mathematics Technology Problem Solving		Problem Solving – most
Susan	Mathematics – content she teaches Technology Problem Solving – approach	Technology Problem Solving	Problem Solving

Assimilation of mathematics. The participants in this study seemed to assimilate most of the mathematics content of the course for their own learning, which may have been due, in part to, the apparent similarity between the topics covered in the InterMath course and the mathematics they teach. This was a Number Sense course involving working with whole numbers, integers, fractions, and other sets of numbers—important topics in the middle grades mathematics curriculum. Judy and Susan found specific mathematical content that was problematic and therefore they were unable to assimilate the entire mathematics of the course. Judy was the only participant who assimilated the mathematics other than just for her own learning. Specifically, she introduced rules and procedures to her students that we discussed in our InterMath course, but was unable to explain why or how they worked. Judy did not find her inability to understand the rules to be problematic for herself or for her teaching.

Assimilation of technology. Tom and Susan appeared to be very computer savvy from the beginning of the course. Neither of them seemed bothered by this component of the InterMath course for their own learning. Tom came to the course wanting to learn more about using spreadsheet technology, and Susan, from the beginning, liked the idea of incorporating technology in mathematics. Judy was the only research participant who did not assimilate any of the technology component of InterMath even though she was fluent with several computer applications. When first introduced to the goal for our class, Susan claimed to see merit in her students using technology in mathematics. She extended the assimilation of technology to her students' learning, making her the only research participant to assimilate technology for her students' learning.

Assimilation of problem solving. The participants seemed to assimilate much of the problem-solving component of the course. They did not find the problem-solving approach used in the InterMath course to be problematic for their own learning because they claimed to have already developed the basic skills necessary to explore the mathematics in the manner presented in the course. In discussing their teaching of problem solving, the problem-solving approach used in InterMath was often claimed by the participants to be similar to the approaches used in their own classrooms. This seemed to allow the participants to assimilate the problem-solving component for their teaching as well. Susan, again, was the only participant who discussed connecting problem solving in InterMath to her students' learning.

Perturbation

In using assimilation and perturbation as a dichotomy for participants' reactions to thinking about the content of the InterMath course, if the participant did not assimilate the component, then they must have been perturbed by it. Of course, this dichotomous assumption only applies when the participants were able to make connections from the components of the InterMath course to their learning, their students' learning, or their teaching. If no connection was made, then the participants did not assimilate or become perturbed. Table 6 contains a summary of the InterMath components that caused the participants to experience perturbation.

Table 6

InterMath components causing a state of perturbation.

	For Their Learning	***For Their Students' Learning***	***For Their Teaching***
Judy	Mathematics – Greek alphabet Technology		Problem Solving – teachable moment Technology
Tom			Problem Solving – teaching practices Technology
Susan	Mathematics Problem Solving – goal	Mathematics	Mathematics Technology

Perturbation of mathematics. Judy and Susan found specific mathematics content that was problematic for them, which always involved mathematics that they did not teach. The pace in which we covered the mathematics in our InterMath course also perturbed Susan. She was accustomed to working quickly through several mathematics problems with the goal of getting the correct answer. In our course, we spent a lot of time on each problem and focused more on the thought processes than the final solution. Susan was also the only one who experienced perturbations about the mathematics in connection to student learning and teaching. Tom was not perturbed by the mathematics component at all.

Perturbation of technology. Again, Susan and Tom seemed to be very comfortable with the technology component of InterMath especially for their own learning; however, Judy had difficulties throughout the course with learning to use the technologies herself. None of the participants experienced perturbations about the technology component for their students, and all of the participants experienced perturbations in connecting the technology component of InterMath to their teaching.

Perturbation of problem solving. Susan was the only participant who experienced a perturbation about the problem solving for her own learning. She was initially perturbed by the goal of problem solving in our course. As previously mentioned, the participants seemed to assimilate most of the problem-solving component of InterMath for their teaching. However, Tom and Judy claimed to change their methods of teaching problem solving during the InterMath course, which seemed to imply that they experienced some sort of perturbation.

Discussion

It was surprising to observe how much the research participants in this study assimilated the components of InterMath. This assimilation seemed to be mostly due to their claims of familiarity with the mathematical content of the course and with the use of computers. One of the goals of this InterMath course was for participants to experience a teaching style that was different from what they know, so there was an assumption that the teaching style used in the InterMath course would be different from what the research participants were accustomed to as learners and as teachers. However, the participants of this study claimed to be familiar with the teaching style I used in general and even compared their own teaching style to mine, using claims of similarity.

According to Hill (1997), it is easier to think about things in terms of what one already knows than to try to see differences and understand these differences. Therefore, it is easier to assimilate in learning by comparing new information to what we already know instead of examining how the new information differs from what we know. This statement seems to imply that one should not be surprised by the assimilation of content that may or may not be familiar to the learner.

Although the participants were familiar with a lot of the mathematics, using computers themselves, and the problem-solving approach used in InterMath, they were unfamiliar with using technology in their mathematics teaching. In fact, Tom was the only participant who discussed using technology as a teaching tool for mathematics. Ideas were presented in the InterMath course involving teaching with technology that were new to the participants, which became a source of perturbation for the participants because they were unable to assimilate. These teaching ideas seemed to cause a state of perturbation because the participants had very little to compare them to in their personal existing schemes.

In looking at the assimilation and perturbation tables, it is intriguing that only Susan made connections to her students' learning. Judy and Tom never talked about student learning unless specifically asked and then often turned the discussion back to their own learning or teaching.

Accommodation and Shutting Down

What problematic components of the InterMath professional development experience tended to cause accommodation or shutting down in the participants?

In order to answer the second question, the InterMath components organized in Table 6 that caused the participants to experience a perturbation were then sorted into three tables illustrating how the participants dealt with these components once they were perturbed. The perturbations were reconciled during the course in two ways: accommodation (see Table 7) and shutting down (see Table 8). The last table represents the perturbations that the participants were unable to reconcile during the course (see Table 9).

Accommodation and Shutting Down

Table 7

InterMath components accommodated by the end of the course.

	For Their Learning	***For Their Students' Learning***	***For Their Teaching***
Judy			Problem Solving – teachable moment
Tom			Problem Solving – change in teaching practices
Susan	Mathematics – that she does not teach Mathematics – pace Problem Solving - goal	Mathematics	Mathematics Technology – teaching ideas

Table 8

InterMath components that caused the participants to shut down during the course.

	For Their Learning	***For Their Students' Learning***	***For Their Teaching***
Judy	Mathematics – Greek alphabet		
Tom			
Susan	Mathematics – that she does not teach		

Accommodation and shutting down of mathematics. Judy and Susan shut down when faced with specific mathematical content that was different from what they teach. Judy specifically shut down when the Greek alphabet appeared in the mathematics of our class. Susan shut down during various discussions as she quit paying attention once the mathematics got over her head and turned her focus to other problems that were similar to the content that she teaches. Therefore, when the mathematics extended beyond mathematics that the participants teach, they occasionally shut down because they seemingly could not see relevance to the mathematics that they teach.

Susan was the only participant of the three perturbed by some of the mathematical content of the InterMath course for her own learning, her students' learning and her teaching. She eventually accommodated some of the content that she does not teach as she continued to explore the problems that were not relevant to her teaching while shutting down during other explorations. She also accommodated the pace of the mathematical explorations and her mathematical experiences, in general. Susan seemed to be making sense of her mathematical experiences in the course as listening to other participants discuss the mathematics, answering questions even if unsure of the correctness of the answer, and spending more time on a problem even after the answer had been found. During the course, she discussed how the mathematical experiences that she had in the InterMath course would also benefit her students, which implied that Susan was able to accommodate the mathematics component for her students' learning as well as for her teaching.

Accommodation of technology. Susan was the only participant who was able to reconcile any of her perturbations with the technology. She was originally perturbed by using technology in her teaching in general, which she was able to accommodate in that she did get teaching ideas that involved using technology in her classroom. Unlike Tom, Susan did not come into the course thinking about teaching with technology because she was unaware of the technology component. Unlike Judy, Susan was not concerned about not being able to incorporate technology in her teaching due to her access to it.

Accommodation of problem solving. All three participants were able to reconcile their perturbations caused by the problem-solving component of InterMath. Susan was able to accommodate the goal of the problem solving as exploring the mathematics in depth rather than working as many problems as quickly as possible, which was her original problem-solving goal. Tom and Judy were able to accommodate parts of the problem-solving component for their teaching. It was previously stated that Tom and Judy assimilated the problem-solving component of InterMath for their teaching; however, there must have been a perturbation that they were able to accommodate, as they must have thought about their teaching with regards to the problem-solving approach used in InterMath. Judy discussed a "teachable moment" in her classroom where she told her students that she did not know the answer to a particular student-asked question, which allowed her students to discuss the mathematics and explore it together without her feeding them the information. Tom claimed to have changed his teaching practices based on the InterMath problem-solving approach. Therefore, there must have been a perturbation that they were able to accommodate during the course.

Leaving in a State of Perturbation

Table 9

InterMath components leaving participants in a state of perturbation

	For Their Learning	***For Their Students' Learning***	***For Their Teaching***
Judy	Technology		Technology
Tom			Technology
Susan			Technology – spreadsheets

Remaining technology perturbations. Technology was the only component of InterMath that caused irreconcilable perturbations for all the participants. Tom and Susan did not have difficulty learning or using the technology for themselves but were unable to reconcile their perturbations about using the spreadsheet technology in their teaching. Judy was unable to learn the technology for herself, which caused her to leave the course in a state of perturbation about the technology component of InterMath for herself. Judy also left the course in a state of perturbation about using technology in her teaching. In the semester-long InterMath course, the participants did not have time to reconcile the perturbation generated by the technology component of the course through accommodation or shutting down.

Discussion

It was only in their last interviews that Tom and Judy mentioned a change in their teaching styles that they each somewhat attributed to the InterMath course. Nothing they mentioned prior to this would lead one to think that they were reflecting on the teaching they experienced in the InterMath course or on their own teaching. However, before the course ended, Judy had allowed her students to explore mathematics in a way that was similar to how it was done in the InterMath course and Tom had given his students less straightforward word problems. These were the only accommodations that either participant experienced in the course.

Both Judy and Susan shut down as a result of a perturbation that occurred during the InterMath course. Both of these shut down experiences were connected to mathematics that the participants did not find relevant to their teaching. Therefore, connections to one's own instruction seem to be important for participants when it comes to assimilating or accommodating mathematics content.

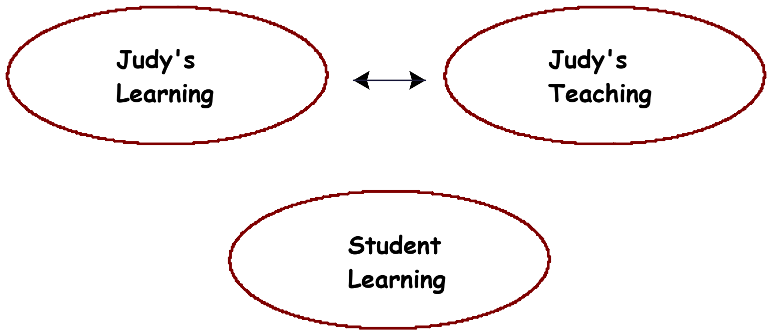
All of the participants left the course in a state of perturbation about the technology component of the course. Judy was never able to reconcile her perturbations about learning the technology for herself while Tom and Susan had no problems with the technology themselves as they assimilated and accommodated the technology component for their own learning. However, all of the participants left the course in a state of perturbation about using technology in their teaching. Based on my original hypothesis, it was not surprising that Judy was unable to move into thinking more about the technology for her teaching given that she did not understand the technology for herself. Tom and Susan seemed to try to accommodate their perturbations that involved teaching with technology, but were unable to reconcile these during the course.

From the three participants, Susan was the only one who accommodated or shut down for her own learning, for her students' learning, and for her teaching. She was able to accommodate the mathematics, the technology, and the problem-solving goal for her own learning and then the mathematics and technology for her teaching. Susan seemed to approach the course differently than Tom and Judy as she often claimed to not be as good at the mathematics as the others and her goals for the course were different. Tom and Judy wanted teaching ideas; Susan wanted to better understand the mathematics that she teaches so that she could help her students understand mathematics. Susan was also the only participant who came to the course thinking about student learning and made connections to her students' learning.

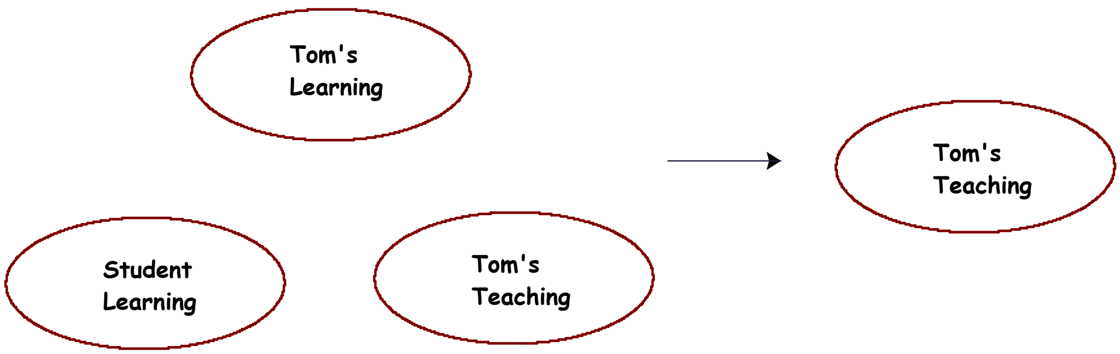
Participant Models

How can these experiences be combined to create a model for how the teachers made sense of their professional development experiences?

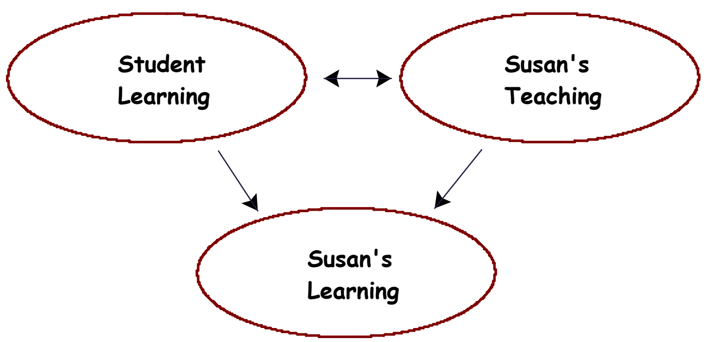
It was initially hypothesized that the participants would first make sense of the InterMath components for their own learning, then make sense of it for their students' learning, and finally, for their teaching. This was not the case for the participants in this study. In the process of analyzing the data from participants, I created a different model for each participant, which were previously presented for each participant (see figure 11).



Judy's model about her learning, her students' learning, and her teaching.



Tom's model about his learning, his students' learning, and his teaching, which is really only about his teaching.



Susan's model about her learning, her students' learning, and her teaching.

Figure 11. Participant models. All of the participant models showing them making sense for their own learning, their students' learning, and their teaching.

All three participants came into the InterMath course saying that they wanted teaching ideas and all of the participants' models depicted connections between their making sense of their experiences with InterMath and their teaching. Thus, making sense of the experience for one's own teaching was an important component in the models for all three participants. The models for Tom and Judy, although simplistic, represent their goals for the InterMath course, which paralleled what they made sense of in the course. Tom's model evolved into him only thinking about his teaching, whereas Judy's model showed her making connections between her learning and her teaching without thinking about her students' learning.

An initial assumption of this study was that teachers thought about their students' learning before thinking about their teaching. In other words, teachers sought instructional methods that they thought would help their students learn and understand the mathematics. This assumption was based on the idea that teachers already thought about how they could help their students make sense of the mathematics and then implemented teaching strategies that would facilitate that learning. This assumption did not prove to be valid. Judy and Tom did not seem to consider their students' learning when thinking about their teaching – they used teaching methods that made sense to them as the teachers and as the learners.

From the three models of making sense presented here, Susan's model was the only one that depicted connections with student learning. She was the only one who made sense of the InterMath components for herself, her students' learning, and her teaching. When considering Susan's goal for attending the course and its relation to teaching for student understanding, her interest in students' learning can be understood. Susan wanted to better understand the mathematics herself so that she could teach her students to understand the mathematics. Therefore, student learning was at the forefront of her goals for the course and she often attempted to make sense of her experiences in the InterMath course in relation to her students' learning.

Perhaps also due to Susan's goals for the course, she had different experiences in the course. She experienced perturbations involving the mathematics of the course that she accommodated and others that caused her to shut down. She was originally perturbed by the problem-solving goal of the course, which she was able to accommodate thus allowing her to assimilate other aspects of the problem-solving component of the course for her own learning, for her students' learning, and for her teaching. Again, it may have been due to Susan coming into the course with the goal of understanding the mathematics for herself and her students that allowed her to become perturbed more often than her colleagues allowing her to have a more connected experience.

Making Sense

How did these experiences allow the teachers to make sense of their professional development experience in terms of their own learning, their students' learning, and their teaching?

This study was broken into several components. First, the participants' experiences in the InterMath course were studied as how they made sense of the professional development for their own learning, for their students' learning, and for their teaching. The InterMath professional development was also divided into the components of mathematics, problem solving, and technology. Finally, my observations of the participants in the course were divided into assimilation, perturbation, accommodation, or shutting down. Based on my analysis of these experiences, I make the following assertions about the three participants in this study and how they made sense of their InterMath experience.

1. The goals of the participants shaped how they made sense of their experiences in the InterMath course.

The participants in this study had different goals for attending the course, which seemed to shape how they made sense of the InterMath course. The participants were able to focus on the aspects of the course that allowed them to fulfill their goals. Consequently, participants learned what they came to InterMath to learn. Even though the participants assimilated so much of the content, each experienced a perturbation in relation to their goals. Tom and Judy wanted teaching ideas from the course and both were able to accommodate a perturbation they experienced about problem solving for their teaching as they both implemented new ideas in their teaching before the InterMath course ended.

Susan wanted to better understand the mathematics that she teaches so that she could help her students better understand the mathematics. Susan struggled with some of the mathematics of the course where she would think more deeply about the content and other times she would divert her attention to mathematics that she found to be more relevant to her classroom. Susan also experienced a perturbation in relation to her initial goal for the course as she wanted to better understand the mathematical content that she teaches. In the course, she was able to accommodate some of the mathematical content that she perceived to not be relevant to her teaching.

Learning using this lens of assimilation, perturbation, and accommodation is often considered for children. However, it is still appropriate to use this lens with adults even though there are major differences in children learning and adult learning. The main difference that is addressed here is that adults often have specific goals for learning where children do not. Because adults often have goals for their learning, the learning is more self-directed (Merriam, 1993). This first assertion is in-line with this idea of self-directed learning because each participant was able to fulfill their goals for the course even though their goals were different.

2. The mathematics that the InterMath participant teaches influences the mathematics of which the participant chooses to make sense.

Susan and Judy shut down when the mathematics in the InterMath course was not relevant to the mathematics that they teach. At times, Susan continued to explore mathematics that she claimed was not applicable to her teaching, but at other times, she claimed to independently explore other problems that were relevant to her teaching while the rest of the class discussed the other mathematics. Tom claimed that the mathematics of the Number Sense course was very similar to the mathematics that he teaches in middle school. Therefore, the participants were able to make connections between the mathematics content of the course to the mathematics content that they teach, which seemed to allow them to see relevance in engaging in the mathematics.

This finding is consistent with prior research conducted around InterMath courses. In the two pilot InterMath courses, it was found that participants saw the website housing the InterMath resources as a tool to be used in their own classrooms, which is not the purpose of the website. In viewing the site as a tool to be used with middle school students, many participants in the pilot courses completed problems that they felt that their students could also complete. This led to many participants not challenging their mathematical abilities (Orrill, 2006). The same was true for participants in this study as the participants engaged more often in the mathematics that was considered similar to the mathematics that they teach and shut down when they saw no relevance to their teaching.

3. Participants do not always consider student learning in making sense of InterMath for their own learning or for their teaching.

Susan was the only research participant who thought about her students as she thought about her teaching. She wanted to help her students understand the mathematics because she knew that simply telling them how to do the mathematics was no longer good enough. In order to help her students understand the mathematics, Susan knew that she, too, needed to understand the mathematics, which meant that she needed to ask questions and fill in any gaps in her knowledge. The other participants never mentioned their students or student learning unless specifically asked indicating that they made no connections between their experiences in the course to their students.

Part of my original hypothesis included teachers thinking about student learning as a fundamental step between their own learning and their teaching. In the literature review previously presented, professional development efforts that had the main focus on student understanding, mathematics, or both were discussed. The InterMath course had the main focus on developing teachers’ mathematical content knowledge, which I had assumed would allow the participants to consider student learning as they explored the mathematics for themselves. However, in this study, it was not found that the participants engaged in thinking about student learning, except for Susan. Therefore, student learning may need to be an explicit goal of the professional development in order for participants to engage in this.

4. The teaching style used in the InterMath mathematics content course provided structure for teachers to make sense of their own teaching of mathematics.

At first, Tom and Judy were able to match components of my teaching style with their own in that they were unable to see many differences but a lot of similarities, excluding the technology component. They had claims of teaching problem solving similar to how it was addressed in our course as implementing cooperative groups and questioning students in their exploration of the mathematics. However, by the end of the course, both discussed changes in their teaching practices that they attributed to InterMath. Even though it was not clear during the course that either of these participants was thinking about my teaching as being different from what they did in their classrooms, they both seemed to reflect on their own teaching in relation to my teaching style and implemented changes.

It was previously stated that teachers should experience the teaching of mathematics in the way that they are expected to teach (Schifter, 1998) and that InterMath is taught in this manner. All of the participants came to the course with the goal of getting teaching ideas for their own classrooms and all of the teachers got teaching ideas. It was surprising that the teachers implemented some ideas that they had gotten from InterMath in their classrooms before the course was over especially since it was never made clear that these particular participants were even reflecting on the teaching of the InterMath course in relation to their own teaching. Therefore, InterMath provided an environment for the participants to reflect on their teaching with respect to the teaching style of InterMath even though it was unknown to me as the instructor during the course.

5. Participants' perceived availability of technology in their schools and classrooms relates to the way they make sense of the use of technology in professional development situations.

Judy perceived that she had limited access to computers in her school and room. She listed many constraints as to why it was so difficult to implement technology in her teaching: the computers in her room were not all working; the computer lab was too hard to book; she had to plan a separate activity for students who could not use the computers; etc. Judy also said that her desire to implement technology in her teaching was not great enough for her to really try at it. In contrast, Tom and Susan claimed to have great access to technology in their schools. An interesting aspect of this assertion is that Tom and Judy taught at the same school and had the same access to computers in their classrooms and computer labs. Therefore, it was the participants' "perceived" access to technology that seemed to affect how they attended to the technology in the InterMath course.

The InterMath research team recognized that technology access in schools often prohibited the use of technology similar to how it was used in InterMath (Orrill, 2006). However, this assertion is different because two of the participants had access to the same technology, but had very different views of using technology in their classrooms due to their perceived access, which seemed to cause them to have different experiences in the course. Judy's perception of having little access to technology seemed to provide an obstacle that she was unwilling to fully undertake.

6. The teachers in this study reported a lack of willingness to incorporate technology in their teaching unless they felt completely comfortable using the technology for themselves.

Where Tom and Susan expressed high levels of comfort in using the different technologies for themselves, neither of them became comfortable enough in using the technologies to implement them in their teaching. Judy fell into this same category, as she definitely did not get comfortable enough with the technology to incorporate it in her teaching. Judy's experience was different from Tom and Susan with respect to the technology because Judy never got comfortable with the technology herself causing her to leave the course still in a state of perturbation with regards to the technology component for herself and for her teaching.

Prior InterMath research reported similar findings in that participants indicated that they were not comfortable with the implementation of technology-enhanced problem solving in their classrooms when they finished the course. Earlier InterMath participants asserted that they needed more practice themselves with the technologies before they could implement InterMath in their classrooms (Orrill, 2006). This previous finding implies that participants considered InterMath as teaching mathematics as explorations with technology where the technology was an integral part and that InterMath was intended to be an all or nothing teaching idea. The participants in the study presented here simply did not get comfortable enough with the technology to incorporate it in their classrooms, but were comfortable with other components of InterMath that they could and did implement in their classrooms.

7. Learning to use technology takes more time than was allotted in the InterMath course.

As expressed above, two of the three participants seemed comfortable with the technology for their own learning, but never became comfortable enough to incorporate the technologies in their teaching while the third participant never got comfortable enough with the technology for her own learning. The participants received credit for 50 seat hours where most of that time involved using technology in some capacity. Since all of the participants left the course in a state of perturbation about using technology, there must not have been enough time in the course for them to accommodate these perturbations or to shut down. If the participants had had more time, they may have been able to reconcile this.

Although this assertion is related to the previous finding, it can stand alone. The previous assertion is more about the participants' comfort levels with the technology themselves that allowed them to implement the technology in their classrooms. This assertion is about the time factor that is involved in learning technology in general. It could be argued that if the participants had more time, then they may have become more comfortable in using the technology for themselves, which could lead to the use of the technologies in their classrooms. However, there is not enough information provided here to come to that conclusion. If the course continued into a second semester, perhaps they would have had enough time.

CHAPTER 8

CONCLUSIONS

As a teacher of secondary mathematics and a participant in professional development, I often wondered what I was expected to "take away" from the professional development. When I became a professional developer, I started to wonder about what the attendees of my classes took away from the experience. In the last few years, I have taught several InterMath courses designed to enhance teachers' mathematical content knowledge through the use of technology to solve mathematical problems. Aside from gaining a deeper understanding of mathematics, problem solving, and technology, I also expected teachers in my courses to gain insights about the implementation of several different teaching strategies for their classrooms such as use of communication, connections and representations.

The study presented here focused on understanding how three participants in an InterMath Number Sense course made sense of their experiences. I originally hypothesized that participants had to make sense of their experiences for their own learning (i.e., am I learning?) before making sense of their experiences for their students learning (i.e., I can learn this way. Can my students learn this way?); only after making sense for their learning and for their students' learning, would participants make sense of their experiences for their teaching (i.e., I can learn this way and my students can learn this way. Can I teach this way?).

Professional development projects designed for mathematics teachers may focus on different learning goals. Some focus on teachers' mathematical content knowledge, other focus primarily on teachers' knowledge of student thinking, while others focus on both. My original hypothesis implied that teachers would make sense of their experience for their student thinking and for their teaching even when they engaged in a professional development such as InterMath, which has as its primary goal the development of teachers' mathematical content knowledge.

Conceptual Framework

To study how the participants made sense of the InterMath professional development, I used the constructivist concepts of assimilation, perturbation, and accommodation to develop a model of how teachers make sense of their experiences (see Figure 6). Assimilation allows the person to fit new content into existing schemes, but it can only take place if the learner is somewhat familiar with the content (Baroody & Ginsburg, 1990). If new content cannot be assimilated, then the person experiences a perturbation, i.e., an agitation or its cause (Stein, 1988). In perturbations, participants cannot fit the new content into existing schemes, and once the learner experiences a perturbation, he will attempt to find equilibration in order to eliminate or resolve the perturbation (von Glasersfeld, 1995).

A perturbation may lead to accommodation when existing schemes and thinking patterns are reorganized to fit new content. This reconciliation may not be immediate and may only occur after a long period of time as the learner changes the way s/he thinks about an idea. A perturbation may also lead to shut down. In addition to these three constructs, my model includes a fourth, *shut down*, because, in my observation, it is an important facet of how teachers make sense of their professional development experiences. Once a participant experiences a perturbation, he may "tune out" to discussions that are irrelevant or incomprehensible to him.

Based on these constructs, my research questions were stated as:

- What components of the InterMath professional development experience (mathematics, technology, problem solving) tend to cause assimilation or perturbation in the participants?
- What problematic components of the InterMath professional development experience tend to cause accommodation or shutting down in the participants?
- How can these experiences be combined to create a model for how the teachers make sense of their professional development experiences?
- How do these experiences allow the teachers to make sense of their professional development experience in terms of their own learning, their students' learning, and their teaching?

Data Collection and Analysis

Because InterMath was designed primarily for middle grades teachers, I invited the three middle grades teachers who came to class the first night to participate in this study. Their participation was voluntary. Two of the teachers, Tom and Judy, taught sixth grade mathematics at the same middle school; the third teacher, Susan, taught fifth grade.

I had a dual role in this study as I was the instructor of the course and the researcher. Data collection was divided into two categories: that collected as instructor, which included assignments and informal conversations, and that collected as researcher, such as formal interviews and videotaped sessions. The data analysis process used a constant comparison method. Data from multiple sources and across time allowed me to triangulate information.

In the analysis of the data, I determined whether the participant assimilated or became perturbed by the content. For example, if a participant showed no signs of discomfort when attending to specific content, I considered that he was assimilating the ideas. On the other hand, when visible discomfort was observed I considered that the participant had been perturbed by the ideas. For each participant, I determined if a perturbation led to accommodation, shut down, or whether the person left the course in a state of perturbation. The data was analyzed across the three cases to find similarities and differences. Seven assertions were the result of the cross-case analysis.

Findings

The research presented here attempted to take a closer look at how teachers made connections from the InterMath professional development to their classrooms, which assumed that the teachers would be open to learning the mathematical content while considering the teaching methods used in the professional development as plausible for use in their own classrooms. Consistent with Nipper's (2004) findings, this study also found that teachers made sense of their experiences in different ways. Based on my analysis, I made the following assertions about the three participants in this study and how they made sense of their InterMath experience.

1. The goals of the participants shaped how they made sense of their experiences in the InterMath course.
2. The mathematics that the InterMath participant teaches influences the mathematics of which the participant chooses to make sense.
3. Participants do not always consider student learning in making sense of InterMath for their own learning or for their teaching.

4. The teaching style used in the InterMath mathematics content course provided structure for teachers to make sense of their teaching of mathematics.

5. Participants’ perceived availability of technology in their schools and classrooms relates to the way they make sense of the use of technology in professional development situations.

6. The teachers in this study reported a lack of willingness to incorporate technology in their teaching unless they feel completely comfortable using the technology for themselves.

7. Learning to use technology takes more time than was allotted in the InterMath course.

It was originally hypothesized that the participants in the professional development would progress through a hierarchy in which they would first make sense of their experience for their own learning, then for their students' learning, and then for their teaching. This hypothesized hierarchy was based on my previous experiences of teaching InterMath courses where the participants seemed to progress through this kind of hierarchy. The participants here did not follow any one, clear path in making sense of their professional development.

For the technology component of InterMath, participants moved from their own learning to their teaching without considering student learning. Two of the participants seemed to do well with the technology during the course for themselves but claimed at the end to not want to use the technology with their students until they were more comfortable with it. The third participant never seemed to learn the technology well enough herself to even think about using the technology in her teaching. So, in this instance, the hierarchy (excluding student learning) seemed to hold because the participants wanted to be comfortable with the technology themselves before implementing it in their teaching.

Nipper (2004) found that participants in her study about teachers making sense of their professional development experience tended to move from their own learning to their teaching. It was interesting that only the technology component of InterMath course seemed to fit the original hypothesized model (without student learning) across the participants. The technology component seemed to be more about acquiring skills to use the technologies, where the other components seemed to be more about understanding. Similarly, prior to the course, two of the three participants claimed to already have developed the mathematical skills necessary to explore the problems of InterMath. Therefore, it may be that the hypothesized triangle may fit when the learning simply involves skills and when student learning is removed.

All of the participants came into the course thinking about their teaching and having the goal of getting new ideas for teaching and all of the participants thought about their teaching during the professional development, without necessarily considering their own learning or student learning. Two participants did not consider student learning at all and rarely discussed their own learning. The third participant also thought about her own learning and student learning. This participant came into the course already thinking about student learning and had a goal that involved student learning. She also thought about the mathematical content as she had another goal of learning the content that she teaches better for the sake of her students and her teaching.

She seemed to think about student learning first, then the mathematical content for her own learning, and then her teaching, which was different from what I proposed.

Schwab (1973) acknowledged four phases of concern that teachers, in general, experience when thinking about the classroom. Teachers first concern themselves with the classroom environment, then their teaching, then the effect of their teaching on student learning, and then finally the content being taught. Similar to Schwab's work, the participants in this professional development thought about their teaching before they considered their students, if they considered their students at all. Therefore, it seems that there must be an explicit goal of the professional development for the participants to consider student learning because they may not necessarily think about their students otherwise.

Applying Schwab's (1973) work to professional development initiatives that use new teaching ideas for the classroom, it is important to know how open the teachers are to learning these new ways of teaching. Consistent with this statement, the participants of this study were open to learning new ways of teaching as they all indicated this as a goal for taking the course. Thus, participants' goals determined what they were open to learn and all of the participants got teaching ideas for their classrooms from the InterMath professional development course. Similarly, two of the participants were open to the idea of using technology in the classroom, while the third was not. This third participant resisted the idea of using technology in her classroom during the entire InterMath course and insisted on her perception that there was no appropriate technology available at her school.

Overall, in this study, there was not one set of hierarchical steps that all participants followed with all of the InterMath components. Therefore, the ways in which teachers make sense of their professional development experience seemed to be influenced by their initial goals for the professional development (e.g., their personal goals of getting teaching ideas influenced what they made sense of for their teaching), how they thought about their teaching (e.g., the mathematics that they teach influenced what mathematics in which they would engage), how they thought about student learning (e.g., only one participant considered student learning and it influenced how she made sense of the professional development), and their perceived capabilities and constraints in the classroom (e.g., one participant's perception of the lack of technology available to her influenced how she engaged with the technology component of InterMath).

Implications for the InterMath Instructor

I have taught several InterMath courses and continue to conduct professional development workshops based around the ideas of InterMath while using the resources of the InterMath website. In my experiences, I now know that not every participant will reflect on her/his own learning, student learning, and teaching in relation to the InterMath course due to their goals. As the InterMath instructor of this course, there were several things that I learned and will apply to future courses.

I will purposefully push the participants in future InterMath courses to think about their students so that they can better develop their ideas about student learning. I was very surprised that two of the research participants did not discuss their students in the context of their experiences in our class. Because I think that student learning should always be at the forefront when we make decisions involving our classrooms, I was actually disturbed by the finding that this is not always the case to all teachers. To me, the classroom is not about the teacher; it is about the students; as an instructor of professional development courses, I will strive to include this idea into my discussion with other teachers

I will push participants in my courses to think more deeply about the mathematics that they do and do not teach. It seems to make sense that a teacher would be more concerned about the mathematics that she teaches; however, I think that it is important to understand the mathematics that one does not teach as well. No one can ever understand everything that is mathematical, but one should expand their mathematical horizons beyond what they teach.

I will push the participants to think differently about the mathematics and problem solving of our course and its relationship to student learning and teaching. I was able to do this only on some level as Tom and Judy claimed to change their teaching based on methods that I used in the course. In discussing the problem solving of the course, they both claimed that students must develop basic skills before they can engage in problem solving that requires the use of those skills. It is unfortunate that they did not see that not all of the InterMath participants in our course had developed the basic skills necessary for solving all of the problems that we explored. They seemed to assume that all of the InterMath participants had developed the basic skills needed to explore the problems and missed that component in my teaching where basic skills were discussed/taught to some of the participants.

Even though I have listed several implications for my instructing of future InterMath courses, I still feel that this InterMath course was a success. The main goal of InterMath is to engage participants in mathematics, which all students in the course did.

One of my main goals for the professional development involves the participants thinking about how they teach and the participants did this as well. Therefore, the goals of the professional development and of the professional developer were fulfilled to an extent.

Implications for Further Research

There are many implications for future research. The research presented here merely begins to explore the connection between teachers learning for themselves, for their students, and for their teaching. This research also begins to explore teacher learning through the lens of assimilation and perturbation. Other research is needed to explore these broad ideas and other specific findings from this study. Future research questions may include:

- What connections exist between teachers learning for themselves, for their students, and for their teaching? How do participant goals impact the connections made? How do participants' feelings about their abilities as teachers and their self-efficacy impact the connections made?
- Once they left InterMath, were the participants able to accommodate perturbations or did they simply shut down? Would a second InterMath course allow the participants to reconcile some of these perturbations?
- How does student learning fit into teachers' beliefs about teaching? What does it mean for a teacher to teach without thinking about student learning?
- How long does it take to learn to use technology? Would a second InterMath course allow the participants to learn it better for themselves and then for their teaching? How many technologies are reasonable to learn in a 50-seat hour course?

There are many directions that future research could take and they are all very important in helping us understand the significance of professional development in the teachers’ classrooms. Once we understand how the teachers make sense of their professional development experiences, we may then attempt to help the teachers overcome their perceived constraints that inhibit them from attempting new teaching strategies in the classroom.

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Appendix A: Exam

At The Supermarket

1. The grocery manager at *Central Supermarket* needs to set the selling price for canned corn. Two competing stores are selling the same kind of corn in cans of the same size. At *Gourmet Grocery*, the price is **7 for $3**. At *Shop and Run*, the price is **5 for $2**.

a. Which of the competing stores (*Shop and Run* or *Gourmet Grocery*) has the lower price for corn? Show how you get your answer.

b. The manager at *Central Supermarket* wants to set her store's price **in between** the prices at the other two stores. She wishes to do this by filling in the two blanks on this shelf label **with whole numbers**. Fill in a pair of numbers that she could use. Underneath, show how you know that your answer is correct.

CORN
Whole Kernel Niblets
8 oz. can
______ for $______

2. The bakery chef at *Central Supermarket* is collecting the ingredients to bake some brownies. His cookbook contains this ingredient list:

INGREDIENTS FOR BROWNIES

1/2 cup butter
1.5 ounces unsweetened chocolate
1 cup sugar
2 eggs
1 teaspoon vanilla
3/4 cup all-purpose flour
1/3 cup chopped nuts

Use a **6-inch by 9-inch** baking pan.

However, the chef needs to use a commercial-size baking pan, **18 inches by 2 feet**. Figure out how much of each ingredient would be needed to bake brownies in this pan. (The brownies should still have the same thickness as in the original recipe.)

Larger, Smaller, In-between

1. **a.** Which is larger, 0.009 or 0.0013?

b. Write a decimal whose value is between 0.009 and 0.0013.

2. **a.** Which is smaller, $\frac{4}{7}$ or $\frac{6}{11}$?

b. Write a fraction whose value is between $\frac{4}{7}$ and $\frac{6}{11}$.

3. **a.** Which is larger, 2^{300} or 3^{200}?

b. Write an exponential expression which is between the given values.

Appendix B: Interview Protocol

First Interview to be conducted at the beginning of the course

- What is mathematics?
- How do you think about mathematics for your own learning?
- How do you think about mathematics for your students' learning?
- How do you think about mathematics for your teaching?
- Describe a typical lesson in your mathematics class.
- Based on what you have seen and know to be true, how do you think students learn?
- Picture in your mind a math classroom where learning is taking place – get a mental image of that room in your mind:

- What is the teacher doing?
- What are the students doing?
    - How do you know that learning is taking place?

- What access do you have to computer-based technology?
- How do you use technology in your classroom?
- Why are you taking InterMath?
- What do you hope to learn?

Second/Third Interview to be conducted mid-way through the course & at the end

- What is mathematics?
- How do you think about mathematics for your own learning?
- How do you think about mathematics for your students' learning?
- How do you think about mathematics for your teaching?
- What have you learned so far?
- What have you been focusing on so far?
- How comfortable are you with the technologies you've been using in InterMath?
- Have you been able to try out any of the technologies in your own classroom?

- If so, which ones? How did you use them? How did that go?
- If not, do you intend to – why or why not?

- How comfortable were you with investigation-based approaches when you began this course?
- How comfortable are you with investigation-based approaches now?
    - Is there anything you find hard about them?
- What do you think of investigation-based approaches for use in your own classroom?

Third Interview to be conducted at the end of the course (in addition to above questions)

- Based on what you have seen and know to be true, how do students learn?
- Picture in your mind a math classroom where learning is taking place – get a mental image of that room in your mind:

- What is the teacher doing?
- What are the students doing?
    - How do you know that learning is taking place?

Appendix C: Observer Record

Name: ______________________________ Date:____________________

Class Topic: _______________________________

Questions to Consider	**Observer's Comments**
How/What does the participant see InterMath affecting his/her learning?	
How/What does the participant see InterMath affecting his/her student's learning?	
How/What does the participant see InterMath affecting his/her teaching?	

Appendix D: InterMath Problems

APPLES AND ORANGES

(found at: http://intermath.coe.uga.edu/newInterMath/nmcncept/integers/a126.htm)

They say you cannot add apples and oranges. However, when you subtract one fruit from another you get an interesting result, as shown in the puzzle below. Each letter represents a different digit from 0 to 8. The digit 9 does not appear anywhere. Can you break the code?

ORANGE
– APPLE
 MELON

SPLITTING FRACTIONS IN "TWO"

(found at: http://intermath.coe.uga.edu/newInterMath/nmcncept/fractns/r04.htm)

2/5 can be written as the sum of two unique unit fractions. For example, 2/5 = 1/3 + 1/15. Try to find two unique unit fractions whose sum is 2/7. What about 2/11? 2/13? Is there a pattern?

THEATER SEATING

(found at: http://intermath.coe.uga.edu/newInterMath/algebra/patterns/a15.htm)

The Mathematics Theater has twenty-five seats in the first row, twenty-seven seats in the second row, twenty-nine seats in the third row, and so on. How many seats are in the theater if there are fifteen rows in all?