GENERATIVE ADOLESCENT MATHEMATICAL LEARNERS:

THE FABRICATION OF KNOWLEDGE

by

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(Under the Direction of Leslie P. Steffe)

ABSTRACT

This dissertation is embedded in a deconstruction of the field of Mathematics Education in order to reconstitute the mathematics student as a generative mathematical learner. The purpose of the dissertation is to understand how generative adolescent mathematical learners (GAMLs) maneuver through their mathematics courses while maintaining such a disposition. As a result, an initial model of constitutive characteristics of the GAML emerged.

A generative disposition is meant to characterize the learner as someone who operates mathematically in ways that reflect an internal sense of authority for knowing and a constructive orientation to the knowledge they come to know. Drawing upon the radical constructivist teaching experiment methodology, I conducted a poststructural qualitative inquiry of students' experiences that may have informed a generative disposition, their orientation toward mathematical knowledge, and the role of schooling in the interactions of these students.

I co-taught the mathematics classes of the studies' three students during April 2005 and interacted with the students as both teacher and classroom researcher. Field notes served as a primary data source, with some classroom episodes and student interviews videotaped in order to aid in the retrospective analysis that followed. Case studies were first developed, and then used to enrich an initial definition of the GAML.

The GAMLs of this study relied on their own thinking to come to know. Confirmation of their teachers' judgments that they were competent thinkers was strongly evident. The three GAMLs demonstrated confidence in what they knew and in their potential to come to know. They considered that mathematics is a human activity and were mathematically interactive among their classroom peers. Further, they enjoyed a high social and academic status among their classroom peers. A disconnection was observed in the three GAMLs between observed classroom behavior and their observed generativity. In particular, they rejected schooling in various ways and engaged in deviant classroom behavior.

INDEX WORDS: Authority, Critical Theory, Critical Postmodern Theory, Deconstruction, Disposition, Equity, Knowledge, Mathematics Education, Mathematics of Children, Personal Epistemology, PostStructuralism, Power/Knowledge, Radical Constructivism, Relational Equity, Social Justice

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"When *I* use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean—neither more nor less."

"The question is," said Alice, "whether you can make words mean so many different things."

"The question is," said Humpty Dumpty, "which is to be master—that's all."

—Lewis Carroll, *Through the Looking Glass*

Before we can be active in any cause we must make it our own, egoistic cause – and that in this sense, quite aside from any material expectations, we are communists in virtue of our egoism, that out of egoism we want to be human beings and not merely individuals.

—Frederick Engels, *Zwischen 18 und 25: Jugendbriefe*

CHAPTER 1

THE PROBLEM AND ITS BACKGROUND

This is a study in social justice. The discourse of equity and social justice has established a foothold in mathematics education, having transcended initial socio-political work of race, gender, and socioeconomic status (Allexsaht-Snider & Hart, 2001; Apple, 1992; Boaler, 1997; 2004; D'Ambrosio, 1990; Gutstein, 2003; Hart, 2003; Martin, 2000; Mellin-Olsen, 1987; Popkewitz, 2004; Reyes & Stanic, 1988; Secada, 1992; Setati, 2005; Skovsmose, 1994; Stanic, 1989; Weissglass, 2000; Zevenbergen, 2001). Sociological constructs such as oppression and resistance (Freire, 1970/2002; Willis, 1977), power and privilege (Foucault, 1976/1990; 1975/1995), and status (Cohen & Lotan, 1997) now find theoretical and practical usefulness in this newly normative language of equity (Martin, 2003; NCTM, 2004). But rather than reiterate decades of documentation demonstrating differential achievement (Fennema, 1984; Fennema & Carpenter, 1981; Matthews, 1984; Yando et al., 1979), resources (Kozol, 1992), treatment (Campbell, 2004), beliefs (Lee, 2003; Scheurich & Young, 1997), and outcomes (Ferguson, 2004; Stevenson & Stigler, 1992; Tate, 1997), the present research is an effort to reconsider the modernist epistemological principles that continue to invisibly guide the contemporary learning of mathematics. I wish to grant learners "a more active epistemological status" (Larochelle & Désautels, 1991, p. 387), one that not only recognizes each learner's construction of mathematical ways of knowing but also engages the educational and research ramifications of such an epistemological status.

The inequities of mathematics education are evident in differential achievement. But more significantly, inequities that are injustices, *iniquities*,[1] are the inadvertent oppressions of the souls of those who are learners in this structure. These iniquities include the oppression of student thinking, autonomy, and authority that occur in the secondary school mathematics setting. Although possibly conceived to be problems of the schooling institution, my focus includes consideration of the ways in which structures of mathematics education may be able to work for the autonomy and authority of children, rather than against it. My goal is to pay close attention, to study the authority and autonomy of the mathematics learner; I want to know more about the ways in which students, near the end of their public school internment, can exhibit these qualities of living. And in knowing more about students with these qualities (and the ways in which students express these qualities), I believe that as a mathematics teacher and mathematics education researcher, I will be more able to effect future students in ways to positively bring forth these qualities, and to engage peers, teachers and researchers, in thinking differently, to conceive of a mathematics education (or a way of studying mathematics education, or a way of preparing mathematics educators) that fosters rather than represses student authority and student autonomy.

To carry out this study, I engage the theory and mechanisms of the postmodern, rejecting universalizing tendencies instead to value power relations, interconnectedness, and discursive construction of knowledge. That I assume this world-view impacts this research project in many ways. I consider the adolescent mathematical learner to operate in such a field, and I am curious to the degree in which this learner perceives these qualities of learning—power relations, interconnectedness, and discursive construction of knowledge—and their relationship to the

[1] Iniquity is used intentionally, as opposed to inequity (Lawler, 2005). I choose iniquity to underscore the gross immorality and injustice associated with a schooling that is self-aware of its systematic oppression.

fabrication of ways of knowing that they may call knowledge. I also am engaged in the same fabrication of knowledge, in particular a fabrication of a knowledge of these knowers. I must not only fabricate, but also reflect on my fabrications and consider how to represent such fabrications of knowledge, fully knowing that the reader again must engage in such activity of knowledge building. This notion of knowledge causes anxiety on multiple levels of this research endeavor; can it be rectified?

This dissertation is postmodern research in mathematics education. I select to take seriously the active learner, in particular high school juniors who demonstrate autonomy and authority in their sense of their own mathematical activity. Coming to some sense of who and what this learner is, and how they are co-constructed within the schooling environment, will inform educational work for social justice. So what is this dissertation about? Social justice? This adolescent mathematical learner? The impossibility of research? The fabrication of knowledge? It is a study of each, AND it is a study on each of the other—a *multiplicity*, an assemblage of a mathematics education rhizome (Deleuze & Guattari, 1987). It is a collection of "heterogeneous actions and entities that somehow function together' (Bogue, 2003, p. 98). This assemblage does not rely on the structuralist AND; instead I invoke a different AND,

> a logic of the AND prior and irreducible to the IS of predictions, which Deleuze finds in David Hume: 'Think *with* AND instead of thinking IS, instead of thinking *for* IS: empiricism has never had another secret.' It is a constructivist logic of unfinished series rather than a calculus of distinct, countable collections; and it is governed by conventions and problematizations, not axioms and fixed rules of inference. (Rajchman, 2001, p. 11)

In effect, this dissertation carries multiple stories, each emerging as the primary thesis depending on who the reader is at the time of her read. But while I intend to work at crafting a dissertation with multiple theses, any paper evokes this same sort of read, that of multiple meanings, invoking lines of flight, never being the same. Whether intentional or not, this dissertation will be a multiplicity. Instead of ignoring this multiplistic quality of the written word, I seek to engage it, and put it to use.

The Study

In this dissertation I will report on my interaction with a small group of adolescent mathematics students and their teachers. These students were identified at the outset of the study to be generative in their mathematical work. This descriptor was chosen prior to engaging the research participants, a notion that captured my visions for the mathematics learner that demonstrated a sense of both autonomy and authority in their mathematical activity; and more importantly, viewed themselves as this mathematical actor and author. By the conclusion of this work, my research will demonstrate both the generative learner and generativity in learners.

The Generative Adolescent Mathematical Learner (GAML)

I select the term *generative* because it is not a common word in other research in the field of mathematics education. When the word is used, it either is not carefully selected, or carries a different meaning than I will emphasize, so I will begin to further clarify my choice of the term. However, because studying what the generative learner may be about is an intention of this research, this initial clarification is tentative.

I use the word generative to carefully carve a niche among the common language of “discovering” and “constructing” that emerged in math education during the 1980’s (and previously in the late 60’s). Considering Boaler’s (2000a) use of the phrase “productive thinking”, I began with what may be at the other end of a spectrum her term suggested. As I worked to make sense of what Boaler intended by the term, and to consider how others may think of it, I considered the productive thinker as one who saw herself as a “producer” of knowledge. It then followed that the other end of such a linear classification would be a "receiver." However, I was dissatisfied with this one-dimensional classification—which I consider to be a disposition toward knowledge, a sense of authority—because it did not leave room to take into account the nature of knowledge; that is I was not able to embrace Vico's (Glasersfeld, 1995, p. 2) view that all knowledge is produced anew. For me, Vico's view opens the conception of the learner and ways of knowing to the ontological question of being and existence—what existed first, the mathematics or the knower? The staging for a sort of binary notion of what reality may be is important here,[2] and will be dealt with explicitly later in the dissertation, in Chapter 3 on Theory. For the purposes of orienting myself early in my research, as well as to orient the reader, I focus now on the use of this ontological question to cross it against the dimension of mathematical learner's role (produce or receive) to emphasize the replication versus the invention of ideas; a nod to the philosophical query, is knowledge discovered or invented? This query I deem to define a binary on the nature of knowledge. I found room within the resulting four planar regions (see Figure 1) to consider the sorts of mathematical learning different orientations toward learning, personal epistemologies, seem to emphasize.[3]

Fellow educational researchers have drawn upon the same languaging I have selected in this model. Larochelle and Désautels[4] use these same words in 1991, "A subject must be conceptualized as a *producer*, and not simply a reproducer of phenomena" (p. 375). Next, Larochelle (2000) states,

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[2] Especially troublesome is to consider what role the observer has in coming to any sort of conclusion on the question. If Vico's statement is taken with seriousness, it is a statement made by an observer, one who could not *know* the truth of such a proclamation.

[3] I present this binaried metaphor for my notions on *generative* cognizant of the dangers such an attempt to name creates and at the same time hides. The metaphor reflects my early attempts to wrestle with other language that worked to name this learner, and continues to serve as a working idea that marks a beginning point to discuss the qualities of such a learner. For now, I allow myself to move forward without troubling the hierarchies this metaphor brings forth.

[4] For accuracy, Larochelle and Désautels are science educators, but participate in and contribute to the field mathematics education.

> Radical constructivism itself approaches an essentially undecidable question as though it were decidable, namely whether we are 'discoverers' (in which case, according to Foerster, we are looking as through a peephole upon an unfolding universe) or whether we are the 'inventors' (in which case we see ourselves as the participants in a conspiracy for which we are continually inventing the customs, rules, and regulations). Radical constructivism does indeed take a position and opts for the latter view. (p. 61)

The intersection of the orientations toward knowledge and learning suggested in these two names, producer and inventor, began my framework toward considering what might be a generative learner.

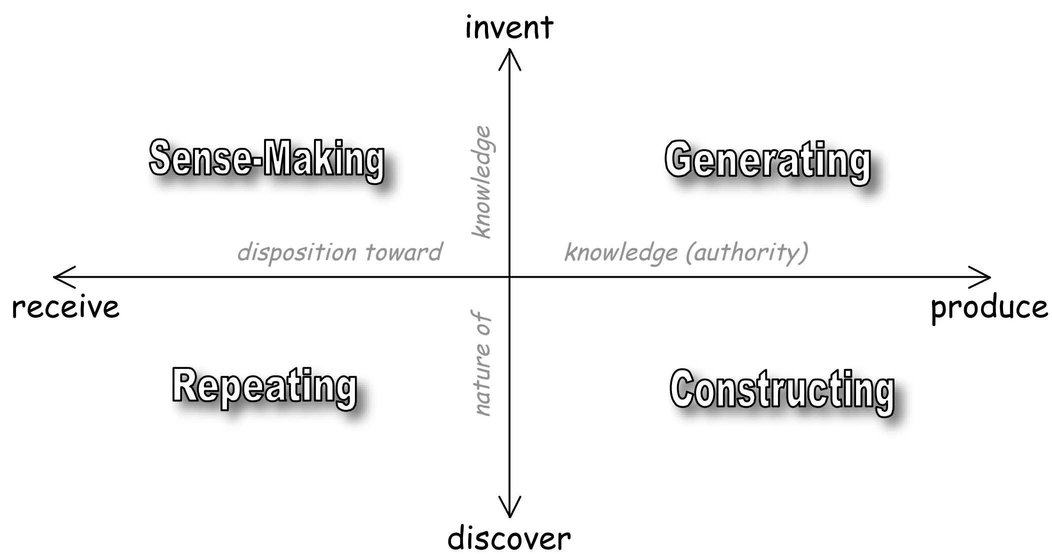

Figure 1: Various ways to theorize mathematical learning.[5]

Next I will say more about how I think of the terms that make up this heuristic. To do so, I refer to a preliminary study I conducted prior to the dissertation in which I interviewed college undergraduate calculus students about their dispositions toward mathematics. I found that the types of responses in this data also helped me to think of each region. I will present that data

[5] The word discover, as do most other words I utilize in this way of organizing, have a long history in mathematics education. Wittrock (1966) notes, "The literature is fraught with conceptual issues, methodological problems, and semantic inconsistencies in the uses of the word discovery" (p. 42). It is my intent to use these terms for the purpose of my model, freed from this historical burden of diverse meanings by my direction to the reader to work only to understand the word within the historical context of my writing of this document.

here, not as direct quotes, but statements similar to those made by the undergraduates as they exemplified each of the regions. The student who saw her mathematical activity as one of *repeating* indicated that, “Knowledge is discovered. It is my task to learn that knowledge so that I can repeat it back for others, on assignments or tests, or in future studies or work. I can tell people about the world, as it exists, after I have experienced it.” The *constructor*[6] reflects a person who sees themselves as much more active in their relationship to coming to know mathematical ideas. “Knowledge is discovered, and I am someone who makes these discoveries. But it all has been known before, well maybe except for some new ideas created by geniuses. I am constructing a way of describing the world that makes sense to me.” This person seemed to be aware of conflicts within the ways they saw themselves as a mathematical learner. I have utilized this word for the purpose of the graphic in spite of the myriad of both carefully- and ill-defined uses of the word. I intentionally selected the word to represent how it is commonly (mis-)used among mathematics educators.

The *sense-maker* may wonder whether or not knowledge is all out there ready to be gathered. However, this person seemed to have the slipperiest or most unsure grasp of their relationship to knowing; they see themselves in a more passive, receiving position as a learner. “Knowledge is invented, so it is my task to make sense of these inventions. Through my work, I can make sense of this knowledge; I make sense of the world I experience.” This position may be characterized by a doubt, unconfidence, or lack of assurance.

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[6] The word “construct” has been carefully identified in math education literature. I have found that the ways in which people use it, however, are not as fine-tuned; there does not seem to be an agreed upon meaning. The use of “construct”, or various derivatives, in the language of math educators is equally as disparate. I use it here with no intention of connecting it to any other definition. I have only intended to create a heuristic for my own thinking, and thus don’t anticipate confusing a reader with the tradition of the word. That said, this use of the word “constructor” is most similar to the radical constructivist’s “trivial constructivist” (Confrey, 1990; Glasersfeld, 1995) where the trivial constructivist is one who—while endorsing the notion that we invent or construct our own reality—at the same time believes in an objective, ontological reality.

Finally is the fourth region, corresponding to the first quadrant of the diagram—the person *generating* mathematical knowledge. This person indicates that, "Knowledge is invented, and I am one of its inventors. My way of understanding is different from others. There isn't one knowledge, even for mathematics—we all have our ways of seeing things. I generate my experiential reality." Steffe, Cobb, and Glasersfeld (1988) bring more clarity to the notion of invention by pointing to the role of the knower's interactions, both with their experiential world and/or among the mental schemes that comprise their knowing. Steffe et al. suggest that while it is likely inventions do occur in the absence of interaction with the knower's experiential reality, these are not prevalent in defining how knowledge of and within one's world is generated. The term invention seems to suggest a lonely voyager, a Robinson Crusoe (Ackermann, 1991b, p. 273). A construction, on the other hand, suggests the production of a novelty as a consequence of interactions. Recall however, it is my intent with this framework is to both name personal epistemologies, most explicitly a *generative* orientation to knowing, as well as to name a knower's ways of thinking about their own processes of learning and knowing. I leave room within the *generative* epistemology for a less detailed perspective on "invention", namely because I posit that it is unlikely a knower will carefully delineate the term in the manner demanded of Steffe et al. in the outlining of his learning theory.

I will also draw upon some uses of the word *generative* in other's educational work to help clarify my initial selection and ideas about applying it here for my research. Wittrock (1974) used the term *generative* to describe a model for considering knowing and learning. This description was a significant introduction to the field, marking an era in which the dominant ideology shifted from positivistic behaviorist models for learning to a postpositivist, cognitivist orientation. This shift left behind a stimulus-response orientation to refocus on the learner as an active agent. "[H]uman learning with understanding is a generative process involving the construction of (*a*) organizational structures for storing and retrieving information, and (*b*) processes for relating new information to the stored information" (p. 182). Wittrock summarized this conception by naming "all learning that involves understanding is discovery learning" (p. 182). The choice of the word *generative* reflects Wittrock's ideal that good teaching stimulates the student to generate meanings, which I take to mean create meaning for themselves. Wittrock's notion corresponds to the positive *x*-axis of Figure 1. The shift espoused by Wittrock emphasized the active nature of the learner, a producer. However, it leaves unquestioned the nature of knowledge. Given the era as well as the avoidance of positioning the subject, i.e. the knower, it is a reasonable conclusion to believe the understanding Wittrock seeks is an understanding of mathematical ideas taken to have an existence prior to the knower. In this way, Wittrock's notion of generativity most likely is reflected in the fourth quadrant by a theme in which I named learning as *constructing*, a productive activity of coming to know what is already assumed to be known by others.[7]

During the same era of educational history, Paulo Freire's seminal writing *Pedagogy of the Oppressed* (1970/2002) gave vision to what a teaching practice for social change may be. Freire was intimately involved in the activity of the teacher and drew no hard separation between the activity of teaching and learning. Freire opposed his theory for learning with a banking model in which the teacher acts as a narrator of "deposits" to be received by the learner, a "receptacle". "The more completely [the teacher] fills the receptacles, the better the teacher she is. The more meekly the receptacles permit themselves to be filled, the better students they are" (p. 72). This banking model, which Freire describes as common to most learning environments, considers the

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[7] For emphasis, I restate that this reflects, in my opinion, a common use of the word among mathematics educators. It is not the constructivists' intended meaning for the word.

learner to be a receiver of what is given to her by the teacher, a notion reflected along the negative x-axis of Figure 1. This idea is of a passive learner, especially when extending the metaphor to think of knowledge as the deposits of this learning. This notion of knowledge takes it as pre-existing, placing the actions of learning to be receiving knowledge, but also replicating what is already known. In essence, any evidence of learning would assume the learner is *repeating*, the third quadrant of Figure 1.

Rejecting the banking model, Freire considered the teacher's role to be to investigate the interactions (the "thought-language") of the student and teacher to learn about the student's manner of perceiving and interacting with what they considered to be reality and their view of the world, places in which their *generative themes* could be found. This generative theme is where the teacher finds the content and potential for education. Freire "termed these themes 'generative' because (however they are comprehended and whatever action they may evoke) they contain the possibility of unfolding into again as many themes, which in their turn call for new tasks to be fulfilled" (p. 102). Although Freire uses the word generative as an adjective on this idea of theme, it is evident he means for the generative theme to be a quality, or a potential, of the knower. Further, this potential has in itself the likelihood for creating new generative themes. Freire recognizes the learner's role in naming a reality and positioning their role in it. He treats the learner as someone engaged in inventing personal meaning, a new knowledge. I believe Freire sees the teacher engaged in the invention of knowledge—knowledge of the generative themes of the student-teacher interaction. It is knowledge emerging in the interaction. But I do not read in Freire's work the troubling of the centrality of knowledge. For now, I place Freire's learners, both the teacher and the student, in Quadrant Two. The teacher, as learner of the student, is there to *make sense* of the learning potentials, the "generative themes," of the student-teacher interaction. These themes are invented, emerging from the interactions of the teacher and student. But the learner is not quite producing them; instead they are becoming aware, and thus receiving an understanding of these themes. Freire does not trouble the idea that the teacher-as-learner is also producing a knowledge of the learner or of this interaction.[8]

Although neither Wittrock nor Freire initially influenced my selection of the term *generative learner*, key ideas are found in each. I think of the learner as creator. And this creativity has in itself a cybernetic structure, one in which the knower as a system has within itself the capacity for replication. The idea of creativity pairs with a notion of authority while the second, the cybernetic structure, speaks to agency, key themes in this research agenda. "Generare" in Latin is derived from "genus" which means "birth." So, my choice of generative is meant to invoke a sense of a learner who perceives themselves to be "giving birth" to new knowledge; both a creative and internal act. Such a perception on one's own learning implies a notion that learning occurs on a continual and personal engagement with a perceived-reality.

As a final comment on the diagram as an introductory notion for what the generative adolescent mathematical learner (GAML) is about, consider the transition of passive to active stance toward perception of self as a learner and in relation to knowledge along the diagram. Moving from West to East clearly reflects a decrease in passivity with regards to learning. And although a South to North move may suggest this, the sometimes contradictory or tenuous ideas evidenced in quadrants 2 and 4 make such an observation less clear. It is more evident that moving toward the Northeast creates the most significant increase in an active stance toward ones own mathematical learning.

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[8] This may be an unfair statement to make about Freire's epistemology in my limited engagement with Freire's work. However, I believe the story, as I've told it, helps to clarify my conception of the four regions through which to contemplate ways to think about learning.

Hopefully an interesting dilemma has emerged in my initial presentation of ideas about achieving a definition of the generative adolescent mathematical learner. To this point, I have only intended to orient the reader toward my thoughts about defining a generative learner; the notion remains a slippery one. As the researcher and author of this project, I perceive all learning to be generative; my statement of theory will address this further. However, at the same time I am willing to call only some high school mathematics students generative. This is first alleviated by a simple clarification, in that my choice of identifying students as GAMLs is by the notion that it is a statement about how they *perceive themselves* as mathematical learners. So not very many high school mathematics students at all fit into this criteria. But, with a bit more thought, the paradox seemingly re-emerges. If I believe all learners are generative, this certainly applies to myself as a learner and knower. So I am creating the disposition that I have assigned to each of these high school mathematics students. That is, it isn't the student who is the GAML; it is *my own idea* about how they see themselves as mathematical learners to which I am associating this label GAML. Does this mean I am not studying adolescents, but rather studying myself? And my interactions with perceptions of things in my environment? So what clues from my interactions with these adolescents make me conclude they do not see themselves as generative? Difficulties and questions such as these lead the rest of this work—to think more carefully about what the GAML may be. This inquiry is one of the intentions of this research project.

What I Did With the GAML

I was curious about generative adolescent mathematics learners, including both a desire to learn more about their ways of acting and interacting in the classroom as well as to possibly gain insight into how this quality of their position toward mathematical learning evolved, or sustained, through a history of public schooling. But further, I was intrigued by this challenge, the dilemma of positing such a view on learning and what it meant for the consideration and study of others as learners. As a classroom teacher, it has been my experience that lower elementary-age children approach activity that I would call mathematical in a playful manner, not as something to be learned or something to do, but as something that they *do* do. If a child in this age bracket is asked to draw a mathematician, they will frequently draw themselves (personal communication, Dorothy Y. White, November, 2004). However, when asked, older students will usually draw an adult, male figure—a more culturally dominant view of what the mathematician is. These drawings hint at a shift in self-concept with regarding to doing and learning mathematics (Garofalo, 1989; Grieb & Easley, 1984; Grouws et al., 1996; Kamii & Housman, 2000; Schoenfeld, 1988; 1989). I will elaborate this shift in self-concept in the review of literature presented in Chapter 2.

At some point in schooling, an expectation shifts from school being an educational playground to school being about the business of preparing for what is to come. I suspect it is acceptable in the American culture to speak about learning mathematics through play with young children, but not for older children. For example, Kamii set forth a project to develop mathematical learning experiences and activities, centered on the engagement in play, which would extend from young children up into the mathematical education of older children. However, at Grade 5 she hit an impasse—not in being unable to develop appropriate learning activity (i.e. mathematical games), but that parents rebuffed her efforts, not allowing for school or learning to be a place for play (personal communication, Constance Kamii, 2002).

Parallel to these observations are the natures of both learning materials and standards for expected learning among the field of mathematics education. Not only are older children discouraged from play, but also the older child’s mathematics is no longer valued. Instead, a particular mathematics that a textbook or test or state standard or professional organization or teacher has deemed must be learned becomes the valued learning content. Although kinesthetic tools such as mathematical manipulatives have made a resurgence in upper grades, the nature of these tools are discovery-oriented, while the counting and game materials available for young learners remain open for play and creativity. No particular mathematical learning goal is pre-assigned to the child's manipulative.

The shift in learning expectations is also evident in professional documents. Steffe (2004) noticed that the NCTM (2000) *Principles and Standards for School Mathematics* had a distinct shift in language from valuing the thinking and creativity of the child to one that expects children to learn an a priori set of mathematical knowledge. This shift also occurs in the mid-elementary years. Apparently it is also our attitude in mathematics education is that once the child attains some age; there is no longer time for play. There is no longer time to take the child's mathematical activity and thinking seriously. Instead, the attitude and actions are guided by the belief "there are things to be learned."

In nine years as a high school mathematics teacher many children I encountered did not seem to display the same playful and creative approach to their mathematical activity as did the elementary age learners I met. And although many of my colleagues, teachers of adolescent learners, expressed a desire to do so, they too did not feel as though they could, or have strategies by which to encourage a playful orientation to doing mathematics. From the adolescent's perspective (and many of their teachers), learning mathematics was a duty—a "job," not something that arose spontaneously from activity. In most cases, students doubted their own reasoning when solving problems. Many sought confirmation from an authority other than themselves—the teacher, the textbook, or the "smart kid" in the class. Yet when I inquired about their own reasoning, in many cases they could provide a strong—and accurate—justification for their solution. But they labored in self-doubt, still seeking confirmation from outside sources. In fact, it is rare even among adult mathematics educators to be satisfied without the external confirmation of their knowing (Cooney, 1985; Thompson, 1985). The radical constructivist position on ways of knowing and operating values this sort of confirmation by another, embracing that it "must play an important part in the stabilization and solidification of our experiential reality" (Glasersfeld, 1995, p. 120). This attention paid to the other for confirmation of ones knowing, what I speak of as authority, opens the doorway to critiquing the binary suggested by viewing oneself or viewing others as a (the) source of confirming ways of knowing. While the former may be rather solipsistic and sociopathic, the latter might be limiting or crippling. Issues and outcomes about my research subjects' locus of authority create some of the interesting findings of this project.

Not all of my former students demonstrated the disaffection from mathematics I describe above. Further, most every student demonstrated that they could be playful and creative in their mathematical activity. However, in most all of these cases, these students would dismiss their learning or problem solving as "not being mathematics". So I have puzzled over these learners who have persevered to maintain their childhood orientations about mathematics and themselves as mathematics-learners. To me, they were mathematically generative, but to themselves, at best some adolescent students seemed to recognize instances in which they felt generative as learners. But they did not perceive themselves to be mathematically generative. What can I learn from those students who perceive themselves to be mathematically generative to inform my teaching? What can the field learn to inform its range of practices—from teaching, to material development and usage, to policy and professional practices? What can the school learn about its purpose, its roles?

These curiosities that I have experienced with children, in conjunction with the theoretical support that suggests a general agreement that “these beliefs are not ‘healthy’ in that they are not conducive to the type of mathematics teaching and learning envisioned in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989)” (Spangler, 1992, p. 19), has led me to pursue this path of more intentional research. In this study, I identified three generative adolescent mathematics learners (GAMLs) and their teachers. I co-taught these students in their classrooms and in private, tutor-like sessions. I also interviewed their teachers, in the context of planning for teaching, to add to and confirm my observations. These students proved to be rather diverse in their mathematical activity, in their relations to the mathematical learning environment, and in their perceptions of themselves as mathematical learners. This diversity allowed me to extend the initial definition of the GAML, as well as better address the questions posed above.

The Place of This Study in Equity Research in Mathematics Education

I began this chapter with the statement, “This is a study on social justice.” In this section I will demonstrate that the adolescent’s mathematical disposition is an important component of equity work in mathematics education, enhancing Becker’s (2002) observation,

> the classroom environment should be a safe one in which students feel comfortable taking risks as they create their own strategies, make conjectures about their discoveries, and debate these strategies and conjectures with other students…. Part of this safety comes from seeing themselves as authorities, rather than relying upon the teacher and text as the sole sources of mathematical truth. (p. 36)

I will extend this argument for careful work to develop authority in children, to show that such work toward equity is also work for social justice. To do so, first I will clarify common meanings for the terms equity and social justice, and situate them in the field of mathematics education. Next I will investigate conflicts that have arisen in such work, conflicts that both unearth the challenges for these sorts of educational goals as well as the inadequacies of certain aims. Third I will explore possible reasons for the persistence of the iniquities that challenge our field. And finally, from this persistence and conflict, I will argue that to address equity and social justice in mathematics education remains equivalent to, and answered by, Dewey's (1902/1964) century-old treatise *The Child and the Curriculum*. With this rich background, I will demonstrate both that there is attention to this topic and suggest new ways of thinking.

Equity and Social Justice

Grand ideas such as equity and social justice have qualities of idealism, as do notions of democracy and freedom. They are the sort of notion Apple and Beane (1995) refer to as a sliding signifier, having no essential meaning but defined by its use within relations of power. Since its foothold was established in the 1989 National Council of Teachers of Mathematics *Standards* document, equity has taken on a normative role in mathematics. Teaching practices are demanded to be equitable; curriculum is written *for all* in order to be marketable; teacher education and mathematics education research programs must incorporate such language to be viable. Equity work has become expected, but the field shares no vision of what this may mean in either theory or practice (Weissglass, 1998). Lacking a shared vision is less so a concern, but of greater significance is that the field uncritically promulgates this very important ideal. Equity as a discourse remains marginalized (personal communication, Laurie Hart, May 11, 2005); it is rare to find theoretical or practical work that directly takes on the challenges of such a vision for mathematics education. The language of equity is either relegated to incoherent or abstruse justifications for research, or as a dependent variable measuring a research outcome.

Equity. Several themes of equity foci have emerged in the field during the past three decades. Weissglass (1998) characterizes these themes through defining five views on equity: (1) Equity as equality; (2) Equity as access; (3) Equity as proportional outcomes; (4) Equity as political change; and (5) Equity as social, psychological, and institutional change. *Equity as equality* is a viewpoint in which the charge is to treat everyone the same. This is a wholly inadequate view given the gross disparities in allocation of resources among citizens and among schools (Kozol, 1992). This view is also obviously inadequate when considering classroom factors such as attempting to take into account the variety of languages spoken in a classroom (Setati, 2005), or another complex issue such as that of the ways status differences play out in the learning environment. Although *Equity as access* is seemingly not a debatable viewpoint, it remains divergently interpreted. It can be manifested in beliefs such as 'Advanced courses are available to all students who do well in a subject' to 'All students must take a college-preparatory sequence' to 'Each school must provide (and create) materials and an environment that enable all students to learn.' This range of responses suggests, "access is too simplistic a concept to be useful" (p. 120). *Equity as proportional outcomes* is reflected in the strong educational checks put in place by the *No Child Left Behind* Act of 2001. One of the key standards of accountability in this act is that states must demonstrate a narrowing of the achievement gaps between subgroups of students. While outcomes may be the definitive measure of equitable educational efforts, this perspective fails to grapple with the complexity of "social and psychological forces involved in teaching and learning and in the challenges of bringing about change" (p. 120). Moreover, this perspective ignores the racist, sexist, and classist structures potentially embedded in the assessment tools intended to 'fairly' measure outcomes.

Advocates of the *Equity means political change* point of view look beyond classroom practices to consider the ways in which schools operate or our economy operates to dictate low achievement for some students, and possibly even among particular students. This position calls for change to the political and economic systems through political action in order to attain equity in education. In the fifth and final view, *Equity means social, psychological, and institutional change*, people's beliefs and biases must be meaningfully addressed in order to address and "eliminate individual and institutional practices and policies that hinder students' ability to learn" (Weissglass, 1998, p. 120). The ways in which racism, sexism, and classism pervade over societal structures *and* our personal ways of acting must be addressed from this perspective in order to fully engage work for equity. I consider this final viewpoint to be a critical sociological position.

Within these five views emerge some conflict over what it may mean to attain equity and possibly even to work for equity. *Equality* and *proportional outcomes* are unlikely to be the same. *Access* to a mathematics discipline as it is, or to a different mathematics? Some argue that changing the mathematical expectations is merely a white bourgeoisie move in order to extend the repression of others in the name of equity (Lubienski, 2000; personal communication, R. Moses, 2007). While Weissglass' second view toward equity considers access to mathematics, the fourth view is more explicitly about access to power and questions the naiveté of a blind march toward simple access. And finally, the potential socio-cultural changes of the fifth view asks us to wonder *Why teach mathematics?* The discourse of equity makes for a valuable target, if not a multiplicity of targets. While most any work with a genuine intention toward equity is likely worthwhile work, if there's such an aversion to a shared meaning, is equity a useful idea, or one that has become hollow? Who admits opposition to work for equity?

For the time, I borrow from Weissglass (1998) a workable definition of equity wherein both student learning and social capacity are included. Further, this definition will retain Apple and Beane's (1995) quality of a sliding signifier, a process not a product, a notion meant to have meaning in discourse rather than beyond or free from humans, and a concept tied to the power relations of its context. Weissglass (1998) names equity to be the ongoing work to increase one's own and society's capacity and commitment to respect individuals as diverse, thinking, and feeling humans, and to ensure necessary resources to assist people in learning, recognizing the effects of prior mistreatment. During the reporting of the processes of this research, I will return to notions of equity in order to extend its definition and maintain its role as a driving force of this research. At the end of this manuscript I will conclude with an effort to sketch what equity might be and mean in and for mathematics education. The multiple meanings and purposes for equity work in mathematics education leads me to prefer to think of my work as a drive for social justice. I draw on the challenges each of these views of equity suggest, but maintain my efforts in the demand for action of a social justice agenda.

Social justice. Gutstein (2003) names teaching for social justice to be the effort to develop within students a "sociopolitical consciousness"—Freire's (1970/2002) *conscientização*. Having opened the working definition of equity up to consider more than the teaching of students to work on one's own and society's capacity, I suggest a broader definition of social justice as well; it is about the drive for "an equitable, compassionate world where difference is understood and valued, and where human dignity, the Earth, our ancestors and future generations are respected" (Arusha, n.d.). Applying this orientation to social justice for education aligns with the efforts of other educationalists, such as those who wish to develop a community-oriented citizenry (Glickman, 1998), moral educators (Noddings, 1984), or multicultural educators (Banks, 1995; Lee, 2003; Ogbu & Simmons, 1998; Sleeter; 1996).

This notion of social justice works nicely alongside the working definition for equity I borrowed from Weissglass as a goal for mathematics education. Social justice as valuing difference and valuing others—lived, living, and yet-to-live—parallels the equity definition, to increase both one's own and society's capacity (by both inward and outward work) and commitment to respect others as diverse, thinking, and feeling humans. The equity I propose is a statement that increases both a learner's own power to act as well as serves to increase that potential in others, and to value each of these amplifications. In particular, I wish to consider a mathematics education in which the learner values difference and the ideas of others. The sort of valuing I have in mind is one in which a learner contemplates the ideas of another, the learner expects and is expected of the other to clarify comments or insights, and mathematics is considered to be a personal understanding of collaborative investigation, where each is responsible to attempt to understand the other's interpretations and justifications; ideas Boaler (2004; 2006a; 2006b; 2008) allude to as *relational equity*.

Conflicting Stories of Equity Research

Attention to equity and social justice has followed an intriguing path in mathematics education. While inequitable outcomes have been documented throughout the history of mathematics education in the United States (for example Fennema, 1996; Schoenfeld, 1988; Secada, 1992), such documentation-oriented work continues. And of course, continues to be important. However, in 1988 Reyes and Stanic challenged the mathematics education research community to go beyond documenting inequities and seek to understand the causes for the differential achievement we know to be occurring. As a result of their meticulous review of literature on disparate achievement in mathematics education, Reyes and Stanic proposed a model to explain differential performance based on group characteristics of race, sex, and socioeconomic status. This model considers factors within schools and classrooms, factors external to schools, and the characteristics of the individuals involved in children's mathematical achievement. In particular, the model draws attention to Societal Influences on Teacher Attitudes, Student Attitudes, and School Mathematics Curriculum. These attitudes interact with Classroom Processes to influence Student Achievement, which itself feeds back into the cycle of interactions. Each link along this cycle suggests a causal connection for differential achievement, most not yet established by research at the time of publication but presented as a guide for future research.

In subsequent research efforts to understand causes for this differential achievement, very interesting contradictions or tensions have arisen—conflict among findings, but also conflicting realms of truth within findings. These tensions speak directly to the difficulty of and difficulty in placing meaning to equity. For example, both policy documents (NCTM, 1989; 2000) and research (Boaler, 1997; Romberg, 1999) suggest the value of problem-solving approaches to mathematical learning, including curriculum design that incorporates open-ended and contextually driven learning experiences. Policy and research states these are desirable and will decrease gaps in levels of attainment. Yet other research designed to investigate these claims, suggests that lower SES students may not benefit from such curricular designs, and may possibly suffer (Lubienski, 2000). In fact, Ball (1995) found that abstract mathematical contexts, rather than contextually-driven problems, often seemed more inclusive, giving more students a sense of common understanding and purpose. Anthony's (2005) findings continued to challenge the focus on or value of contextually-driven problems by noting that a variety of highly successful teachers had quite varying meaning for the term itself as they put it to work in their classrooms. Furthermore, policy intended to increase fairness and justice in education, such as the No Child Left Behind Act (U.S. Congress, 2001), seems to cause practitioners to degrade the learner's experience with a curriculum to the minutia of detail and fact (Noddings, 2007), rather than the open-ended and contextually meaningful approach recommended by the field of mathematics education.

Different tensions have emerged as researchers study the successes of mathematics education seen when cutting along race, gender, or class lines. For example, Martin (2000) and Stinson (2004) found that mathematically successful adolescent black males created (and were aware of) dual and quite separate self-images, one as an African-American male, and the other as a successful student. These young men demonstrated that these multiple selves possessed some degree of social contradiction, and had to actively manage or stand above the negative or conflicting qualities of each.

Apparent challenges also emerge when studying differences between the demographics of mathematics teachers and the student populations. Mathematics teachers are disproportionately white female, while the non-white student proportion continues to grow. Whiteness studies (Avis, 1988; McIntosh, 1992; Sleeter, 1996) challenge the potential for teachers to develop empathetic connections with children of different backgrounds, connections they deem necessary to effectively teach mathematics. Several researchers arrive at similar conclusions when calling for culturally-relevant pedagogical practices (Frankenstein, 1995; Ladson-Billings 1995a; 1995b), that students from different backgrounds will learn mathematics better when it is approached through the context of these student's social, cultural, and economic experiences. Work such as this has sparked a burgeoning area of research that seeks to answer what curricular and pedagogic practices work best for particular students, such as the NCTM series *Changing the Faces of Mathematics: Perspectives on…* (*Gender, African-Americans*, …). This series was designed to communicate good teaching approaches that have been demonstrated to work with these subsets of our student populations. Although well-intentioned and seemingly necessary given teacher demographics, NCTM backhandedly perpetuates the marginalization and essentialization of members of these groups by suggesting certain students possess particular preferences by virtue of their race, gender, or culture (Boaler, 2002). Although such efforts to demark children into particular classifications have egalitarian purposes in mind, they may themselves border on racism (Lawler, 2005).

Persistent Iniquities

Mathematics educators are challenged by the persistence of these iniquities (Lawler, 2005). They continue to appear year after year, decade after decade, in the differential achievement along race, sex, and class lines. That mathematics educators continue to increase the prominence of the equity discourse suggests a sincere desire of members of the field to work to overcome this differential achievement. Yet the pervasiveness of these discords among the work of mathematics educators seems to overwhelm the project, from mathematics educator's identification of potential factors for differential achievement, into the development of solutions, to efforts that measure outcomes intended to be more equitable (Lawler, 2005). Hence, mathematics educators, collectively the field of mathematics education, contribute to the propagation of these injustices through *enforced passivity* (Lawler, 2005).

To clarify this notion of enforced passivity, I pause to explain my use of the word *enforced.* The naïve realist position toward mathematics evidenced in Erdös' naming "the book of mathematics" is a commonly held disposition among mathematicians, mathematics educators, and members of the larger society. This pervasive belief is that mathematics has been written and possesses some sort of existence prior to knowers. The authority awarded this book by the constructing knower allows for, or rather *is* their enforced passivity.

The idea of “the field of mathematics education” must be understood to be a organizing construct of individual knowers, a social system produced and defined by the individuals that make up the system (and by individuals that do not see themselves as part of the system). I do claim that, in certain ways, the self-identified members of this collective operate with some passivity to the inequitable activities characterized to other members of the field, or to invisible structures within which all members seemingly operate. That this is an enforced passivity is a statement about the ways in which individual members define their personal orientation toward what I deem to be constructed ways of knowing, which creates external authority, or lack of autonomy in their own abilities and potentials to act. As an example of this *enforced* nature of a mathematics educator’s passivity, consider the decision many teachers make to “teach” the next page of the textbook, whether or not their students are ready. There is a sort of doomsday message, a guiding “force” that communicates if the teacher doesn’t “cover” the proper mathematics, students won’t be prepared for some sort of test. That a teacher subjects their own know-better and educational values to this seeming truism is this sort of invisible enforcement upon their actions. The notion is akin to a person’s subjugating themselves to the invisible guiding forces of a “destiny.”

First – revered status. This enforced passivity coincides with the revered status attributed to mathematics educators by many members of the American society. Knowing mathematics is attributed to potential for success, and is tightly linked to intelligence within our society. Because mathematics educators are believed to know mathematics through a college or university edict and labeled "highly qualified" (else it is believed they are expelled from the ranks through testing, such as Praxis or CBES), they are stamped with approval and awarded the public's trust in the ability to pass this knowledge (that is, Erdös' book of mathematics) along to children. In being awarded this trust, they are granted, and thus possess this revered status.

As further example of this status of mathematics educators, public advertising campaigns issue dire warnings threatening the dismal future in store for children if they aren't learning mathematics. For example, the National Action Council for Minorities in Engineering, in conjunction with the Ad Council and with support from NCTM, says the purpose of its "Math is Power" campaign is to

> provide information to parents and students about the importance of advanced mathematics courses in high school. The knowledge base of algebra, geometry, trigonometry, precalculus or the equivalent in integrated curricula are crucial gatekeepers for access to a broad range of careers, including engineering, the natural sciences, accounting, investment banking and many others. Students who opt out of academic mathematics as early as eighth grade, essentially forego any future opportunity to pursue a career in such fields. (http://www.figurethis.org/wc/w_grantee_nacme.htm)

Unstated, yet communicated[9] in such rhetoric is that 'no math means no power,' and whether a child 'opts out' or fails out of mathematics, she is doomed to a position in society in which *she* has chosen her relegation to oppression by opting out of mathematics early.

In addition to strong messages in the discourse of education, success in school, and more significantly—potential for future success in school, is measured in large part by standardized tests weighed heavily by scores in mathematics. Strong implications, such as these, about the potential for success in our society and our economy have not only contributed to the powered status of the field and those working within it, but also severely politicize mathematics education (Mellin-Olsen, 1987; Wilson, 2003). Mathematics educators must not only attend to the business

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[9] "Communicated" to the extent that some knowers could attribute meaning.

of developing the child or student in mathematical ways, but also must participate in sociological and political games of power and status.

With the greater power attained by this privileged societal position, mathematics educators, as a collective, have paradoxically also become greater servants to society's demands[10]—simply by the expectations assumed by and attributed to this status. Whether these demands come couched in the technocratic language of human capital theory or as a critique for the unfinished business for schools to address the major problems of a race-, class-, and gender-divided society, these demands put math educators (and as a field, math education) in a position of defense, distracting efforts to respond to and correct weaknesses (McLaren, 1994/1988). The demands engage mathematics educators in externally driven activity such as test-preparation and lawyer-driven paper-pushing at the public school levels, and accreditation documentation and pressure to educate toward a particular ontology[11] in the university, all varieties of reactionary turmoil. The power games associated with these relationships blind educators to the contortions of our actions in light of our democratic goals (Kincheloe & McLaren, 2000; Spring, 1993).

Return to – enforced passivity. The combination of the two elements of our postmodern existence in mathematics education discussed above—powered position and reactionary turmoil—has resulted in a certain passivity in the role mathematics educators play in shaping the goals, practices, and outcomes of our field (Greer & Mukhopadhyay, 2003; Lawler, 2005; Martin, 2003); a passivity marked by reaction rather than proaction. To clarify this passivity, I draw on postmodern efforts to develop epistemologies that seek to blur the strong distinctions

[10] A brief comment on my intent of the use of a word like society: a society does not exist externally to the knower, but the knower acts in relation to it as if it does. When I name "the American society", I do not intend to give it an ontological existence—something independent of a knower's conceptual schemes. With that said, we often operate as naïve realists, treating our conceptual schemes, in this case "American society," as this sort of mind-independent reality.
[11] More specifically, there are pressures to *educate for*, a training mind-set, rather than simply *educate*, an orientation toward growing the mind.

between the cognizing subject and the social realm. These efforts, to me, emerge from the Piagetian and Vygotskian traditions, being not quite satisfied with either because of their inattention to the humanist ontological assumptions that may be read into the theory, those in which the knower (observer) posits a knower-independent, fixed truth. Further, this inattentiveness has caused many readers to consider one perspective of the learner to be of the lone voyager, and the other of the learner as merely a social replicator, the first critique being leveled at Piagetian theory, the second toward Vygotskian. The binary suggested calls for a postmodern approach that values a both/and perspective rather than an either/or. Among postmodern epistemologies are Glasersfeld's (1995) radical constructivism, Bateson's (1972) ecological position, Papert's (Harel & Papert, 1991) constructionism, and Kieren et al's (1995; as well as Davis, 1996) enactivism, and possibly some 'social constructivisms.'

Mellin-Olsen (1987) extends Vygotsky's activity theory to "embod[y] the individual and the society as a unity: the individual acts on her society at the same time she becomes socialized to it" (p. 33). Weissglass (1991) draws on Mellin-Olsen to create a usable definition for Activity: "a learning experience that engages our capacity to take care of life situations" (p. 281). I choose passivity as a sort of antonym for Activity, a disengagement from our capacity for living. *Enforced passivity* is the denial of access to Activity.[12]

The play of power relations (Foucault, 1984/1997) for mathematics educators certainly makes this denial of access to Activity markedly different from the overt hegemonic actions of a common classroom learning environment (Kohl, 1994; Kohn, 1999; Oakes, 1985). When a knower treats "mathematics" as a set of knowledge only obtainable by some intellectual elite,

[12] Weissglass (1991) briefly brings this notion out on p. 291, focusing on the context of learners. My thinking on this idea was sparked here, but I also draw on Foucault's (1984/1997) notions of power relations amid two interactants, that both subjects-in-relation possess power. I disagree with Weissglass' notion that passivity increases as SES decreases. While in a direct way this may be true, but as I attempt to argue here, I believe the inverse relation also holds; enforced passivity is also detrimental to the powered.

that is likely to raise the mathematics educator's status—having assumed that educator has mastered this elite set of knowledge. As a result, the adoration and undiscerning reverence that many knowers attribute to mathematics educators, creating a sort of aristocratic societal position, contributes to mathematics educators' potential to ignore their own complicity in the iniquitable outcomes of mathematics education. We, in the field of mathematics education, are given latitude to justify unequal results through impersonalized and distanced relations, such as the deficiencies attributed to the learner or her family[13], poor curriculum, a lack of time, or under-prepared teachers.[14] When a mathematics educator conceives of a Mathematics as a set of mind-independent knowledges to be imparted to a knower, these impersonal and distanced relations can be very real, and possibly rightfully justified. However, it is this realist treatment of Mathematics that must be altered.

Through this distancing of ourselves to the immediacy of the realities of human interaction, we may be making ourselves sane in the face of the gross inequities we perceive amid our society. However, each of these excuses deflects responsibility from ourselves as mathematics educators. In effect, we are allowed to say, "Don't blame us for the miseducation of children—we weren't provided what we needed to educate them." The quiet acceptances of these standards for a mathematics educator's work, both by ourselves and by the larger society, are examples of the enforced passivity of our field.

Because mathematics education is also engaged in the politics of pressing for change, the brakes of institutional stability and reproduction operate to constrain our facility to act by

[13] Lee, Spencer, and Harpalani (2003) replace this cultural deficiency misconception with a model to integrate cultural socialization and identity development processes into learning as a goal of educational research.
[14] Hill, Rowan, and Ball (2005) argue students could learn more if teachers were better prepared, in particular if their "teacher's mathematical knowledge" improved.

binding us to resource-intensive processes (cf. Crandall et al., 1982).[15] For example, when a school team, engaged in self-study, makes decisions about improving classroom practices, an extensive network of people must be embraced, consulted, informed, and most importantly convinced that such a change is both needed and the correct decision. Next, any sort of change is usually accompanied by rigorous follow-up procedures in order to monitor and test student learning in order to document if the decision was correct or not. I won't attempt to argue that such communication and community investment isn't valuable, but it is also a highly resource-intensive effort. It is not the effort that a German auto-repair shop must go through to switch their machinery or labor practices, nor is it similar to the effort a dental office goes though to put in place the most recent techniques or equipment of the field. The point is that the nature of schools works against change, by making efforts to change highly resource intensive. Because every school site and associated community acts as though it were its own island, unaware of accepted norms and changes in educational processes, that the same conversations and documentation must be replicated in each site. These efforts serve to draw math educator's Activity away from direct effort on our goals for equity, to improve the mathematical education for all students. In effect, the equity work of many mathematics educators is diverted sideways; while working on change in the practices of mathematics teaching and learning, focus and efforts are redirected. Ends become obscured, and we settle for partial and/or ineffective implementation of ideas, or do not engage in the continued learning and change necessary to implement new ideas into practice. Again, this diversion of attention is another form of enforced passivity the powered status of mathematics education invites.

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[15] I am arguing that the status of mathematics education allows us to work unquestioned, unbridled. Yet our status also busies and detracts us with demands for justification—a sort of busy-ness that enforces status maintenance. This sort of paradox I have come to expect in efforts for analytic rationalization.

The Epistemological Conflict

I suggest the problem of equity remains best captured in what Dewey (1902/1964) identified a century ago, that the practices of mathematics education have not found ways to transcend the binaried discourse of teaching the child or teaching the curriculum. Taking the child and the curriculum as binaries, teaching the child is, at the extreme, interpreted as leaving the child be, so as they are to learn what they will, when they show an interest (Benezet, 1935a; 1935b; 1936; Miller, 2002; Neill, 1992). Whereas teaching the curriculum, the adult ways of knowing—Dewey's (1902/1964) "race-expression which is embodied in this thing we call the curriculum" (p. 358), stands at the other end of such a spectrum. Many modern educational philosophers speak directly to this point. Delpit (1988) calls for a more direct approach to teaching the mathematics of the dominant culture, the race-expression embodied in a curriculum. She observes that reform techniques, while egalitarian in theoretical justification, may more greatly mask and thus more strongly replicate the power relations of the society, the dominant culture. Other's unwittingly call for teaching the child; yet wish to teach the child the adult's mathematics (Romberg, 1999). Some may trouble *which* mathematics is to be taught (Frankenstein, 1995; Gerdes, 1997; Powell, 2002; Skovsmose, 1994), but the trouble still allows only for the change among a variety of a priori mathematics. None of these viewpoints are able to get out of the either/or perspective on teaching the child or the curriculum.[16]

This tension between child and curriculum is evident in our professional teaching standards as well. Steffe (2004) documents the contradictory language of NCTM's 2000 *Principles and Standards*, noting the valuation of children's mathematics within the document, yet noting "an unavoidable tension between children's mathematical ways of knowing and communicating and what is regarded as mathematics curriculum for children" (p. 222). Steffe

[16] Skovsmose (1994) challenges this binary in his work.

(2004; Steffe & Weigel, 1996) distinguished three ways of thinking about mathematics and children's ways of operating: mathematics for children, mathematics of children, and children's mathematics. *Mathematics for Children* are an adult's ways of knowing and operating, which are drawn upon in order to hypothesize a zone of potential construction for directing interaction with a child. When being spoken of, this idea is much like Dewey's (1902/1964) "race-expression… embodied in that thing we call Curriculum" (p. 358). However, the Mathematics for Children is likely to remain a mathematics different from Dewey's discipline of Mathematics[17]—a notion referring to knowledges that people attribute external to any one knower. Mathematics for children always remains an individual's conception, an important position to maintain with respect to thinking of a teacher and learner in interaction.

Mathematics of Children refer to the models for children's mathematical knowing and operating that adults (or others) make to explain the observations of a child's mathematical activity. Notice this referent still points to a teacher's ways of knowing. It can be thought of as the tail of an arrow pointing toward Mathematics for Children. Finally, the phrase *children's mathematics* refers to the mathematical concepts and operations that a teacher assumes a child has constructed—the mathematical activity a teacher or some other observer/researcher attributes to the child. The child's 'actual' ways of knowing, that upon which an observer operates, is unknowable by anyone, including the child herself, yet seems apparent through cultural cues; cues that we as observing others are attuned to via influences of our own experiential and cultural domains.

Returning to Dewey's *The Child and the Curriculum* (1902/1964), Dewey emphasizes that some interpret his message to be that schools are to orient themselves to the child as a sort of

[17] I will capitalize names of bodies of knowledge when I wish to point to them as if they existed as disembodied collections of facts—as they are often naïvely treated. My use of lower case will be to point to an individual's way of knowing.

light in the dark skies of society, an unblemished purity that is to be allowed to develop free from culture's tarnish, "to leave the child to his own unguided spontaneity" (p. 357) rather than impart into the child adult ways of knowing. Neither pole reflects Dewey's[18] ideal in his closing passages of the essay. He states there is "no such thing as imposition of truth from without" (p. 357), nor is "such a thing as sheer self-activity possible—because all activity takes place in a medium" (p. 357). In an educative setting, it is naïve to ignore the formulated wealth of mathematical knowledge of the educator. But it is also naïve to believe that these knowledges can be passed to the child. Simply, this wealth of mathematical knowledge may "enable the educator *to determine the environment of the child*, and thus by indirection, to direct" (p. 357). This is the way in which Steffe speaks about mathematics for children, with an important nuance to the meaning intended with the word "environment." For Steffe, the environment is not something there, prior and independent of the perceiver. It is a constructed, externalized world. There explicitly is not a one-to-one match of the educator's and the child's environment. So for Steffe, the goal to determine the environment shifts ever so slightly to emphasize

> An environment that teachers develop by creating what they consider constraints that are likely to guide the student to propitious accommodations. It should never be… an environment based on the assumption that what is obvious to the mathematical initiate [educator] will be obvious to the novice [child] as well. (Glasersfeld, 1990b, p. 23)

Dewey provokes me to think about how I can equitably act as a mathematics educator.[19] I have a particular mathematical knowledge. It is raced, classed, and sexed; constructed in the interaction with a society marked by such principles. It is most certainly powered. But that I understand both that it is not necessarily ever going to be that of the child, and that I inevitably

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[18] "The great resolver of apparent dichotomies" (Stanic, 1990, p. 291)

[19] It is worth noting that my focus in this passage is on "what" it is to be taught. A mathematics teacher must also act intentionally with regards to the learning environment of the child. Glasersfeld (1990b) provides a viable orientation to the "externalization that generates the sphere of experience that we ordinarily call environment" (p. 31)

draw on it in order to direct, even if by indirection, the child, that a consciousness of myself in this mathematical interaction may make possible for the child to assert his present powers, exercise his present capacities, and realize his present attitudes (Dewey, 1902/1964). So the mathematical development of the child—children's mathematics—is never known before it "appears" in interaction, and then only emerges as mathematics of the child. Dewey's concluding observation, "The case is of Child" (p. 357) is then to say; there is no getting around or free from the child. It is she who makes the mathematics she learns. I take this to be my underlying premise for a socially just mathematics education.

Why Thinking about the GAML is Work for Social Justice

So my case is of the child, in particular the high school junior, so as to look at students nearing the completion of their public education. I wish to consider high school juniors who demonstrate that they see themselves as generative mathematical learners, students who *see themselves* as authors of their mathematics, writer's of their world (Freire & Macedo, 1987). Gutstein (2003) argues that beyond students achieving a socio-political consciousness, students must also "have a belief in themselves as people who can make a difference in the world, as ones who are makers of history" (p. 40). This speaks to my notion of who the GAML is, and why this particular disposition is work for social justice.

In what is to follow, I will briefly look at the study of children's mathematical learning, with intent to clarify the goals of this research. I will approach the problem first from an outside in perspective, considering the study of mathematical learning. Next, I will speak from inside my own experiences as a mathematical learner. After setting the research questions, I will re-emphasize the multiple purposes of the paper, and then lay out the organization of the dissertation.

The Authority of Children's Mathematical Learning

As mathematics education researchers, we attempt to know the knowing and the processes of coming to know of others—namely learners of mathematics. The postmodern knowledge-truth-power quagmire is one degree greater than simply problematizing the construction of knowledge. As researchers, we seek to construct knowledge of other's construction of knowledge, within the complex milieu of power relations. Power relations (Foucault, 1980) refer to an attempt to re-capture the interaction, to question the oppressor-oppressed assumption, but instead to notice that relations between any two people have games of power in action, such as subversion. In particular to the study, a mathematical learner and a teacher or researcher interact in ways that might mask, imply, or otherwise provide a distorted view on the knowing of the other (as if the other could be known). The knowledge attributed by the researcher is embedded in the activity of, and relations to, the knower, all of which must be constructed by the researcher. Progressing down this slippery slope of validity and usefulness seems questionable[20], but work in this area continues (e.g. Steffe, 1994; 2001; 2002) and does show signs of value in the mathematical education of children.

But a different, and more rarely asked question is, "What do children learn about the naïve realist's Mathematics as mind-independent knowledge, and themselves in relation to such knowledge, while in a mathematics classroom" (similarly asked by Muis, 2004)? Rather than being a question of student *attitudes*, this focus is on student's epistemological conceptions of mathematics. Does the student perceive herself as inventor or discoverer?[21] In what manners do students acknowledge the roles of others in their learning, and more particularly, their

[20] "Questionable" because it remains arguable that efforts to design a mathematics education based on the mathematics of children will remain valued over a period of time (Lawson, 1991). Presently, such work seems to have established a foothold in the elementary grades (Kamii & Housman, 2000; Carpenter et al., 2000).

[21] I allow myself the use of common words here, because it is highly unlikely a child would communicate notions about themselves using a more carefully defined language of a scholar.

construction of knowing? Rather than taking these questions as curiosities about children and children's mathematical knowing, it must be determined if certain student epistemologies are worth the effort to intentionally develop. The orientation of these questions suggests a preferred outcome, a better goal for mathematics education. This tendency toward assuming antithesis begs for us to address the incivility of the binary, the naming of a 'better.' An alternate approach would be one that seeks to understand the current ways student epistemologies operate, and theorizes what could be.

Why do this work? A view from the outside. If it is confirmed that schooling dissuades children from trusting their own ways of mathematical knowing (what may be labeled "common-sense") and separates this from an official mathematical knowledge, should this be questioned, or accepted as a necessary by-product? Bourdieu (1985) considers this phenomenon "symbolic violence,"[22] in that "students gradually and unconsciously are led to apply the dominant (scientific or pseudoscientific) criteria of evaluation to their own practices of knowledge-constructing, and to think that the production of a symbolic capital is the preserve of a minority of gifted individuals" (Désautels & Larochelle, 1998, p. 124). Secondly, from a learning of mathematics point of view, the curiosity to consider possible associations of student epistemology is just as great. Might a realist epistemology positively correlate to increased or decreased mathematical learning? Would a generative outlook on the self as a learner increase or decrease mathematical learning? And how would we measure such questions? Many would argue that students should learn to see themselves as actors upon their worlds (or a different

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[22] Taken directly from Wikipedia, Oct. 5, 2005: "Bourdieu sees the legitimation of cultural capital as crucial to its effectiveness as a source of power. It is seen as *symbolic violence*, violence that is exercised upon a social agent with his or her complicity. What this means is that people come to experience systems of meaning (culture) as legitimate; there is a process of misunderstanding or misrecognition of what is really going on. So it comes that working class children see it as legitimate that their middle-class peers have more success in the educational system as based on their objective performance."

ontology may say 'the world') and that they should learn mathematics. More careful arguments, in which the nature of knowledge and the notion of "mathematics" are attended to, suggest that students should learn to delve deeper into knowledge games and possess the flexibility to move from the exploration of one game to another in a liberated manner, echoing the Freirian concepts of *conscientização* and *to write the world*. This is a more careful effort because the nature of knowledge and truth and power are brought to the fold.

Not only should it be the goal of pedagogy and curriculum to enhance student construction, rather than "psittaceous repetition" (Larochelle & Désautels, 1991, p. 387),[23] of knowledge, but also to encourage the emergence of a generative epistemology. Further, it is up to educators not only to help our students perform better, but also become competent as social actors. Larochelle and Désautels (1991) capture this idea well:

> "From a strictly individual point of view, the confrontation of one's own knowledge and belief's with a public knowledge… can lead to a depreciation of one's own competencies…. That is why… it is necessary to imagine pedagogical strategies which give students and teachers a more active epistemological status and help them to integrate into their own thinking the reflective, conflictual and unfinished character of any production of knowledge, and thus allow everyone to become familiar with the very serious game of the production of scientific knowledge." (p. 387)

As I've suggested previously, the prevailing relationship to knowledge most often promoted in our schools and mathematics classrooms does not seem to develop this sort of authorship nor agency in relation to mathematical knowledge. I suggest that it seems to shut it down as children proceed from elementary to secondary education. Careful consideration of the epistemological cultivation of the learner is entirely neglected as part of their mathematical education.

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[23] *Psittaceous* is a rather interesting word, meaning of or pertaining to parrots.

Overview of the Study

My study aims to take seriously those adolescent mathematics learners, nearing the completion of high school, that have maintained[24] a vigorous sense of authorship and agency in their production of mathematical knowledge. These are students I consider to demonstrate a generative disposition toward mathematics, a way of acting in the mathematics classroom that suggests they perceive themselves to be the producers *and* inventors of mathematical knowledge, and that they have the authority to evaluate the mathematics of others.

Guiding research questions. In this study, I used a radical constructivist epistemology to create and deconstruct case studies of three high school juniors that demonstrate generative dispositions toward their mathematical activity. Through my own practice of the constructivist teaching experiment methodology and postmodern data analysis, I investigated the subjects' construction of experiences that may have informed this disposition, their orientation toward mathematical knowledge, and the role of schooling in the production of these students. The big question of this study is: How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition? In keeping with both the ontogenetic (radical constructivist) and deconstructive (postmodern) frameworks that will be employed, the exact nature of the smaller questions I answer will emerge through the events of the study, but the following will initially serve as guides:

1. What practices has a generative student created for engaging in mathematical activity?
2. What conceptions of mathematics has a generative student formed?
3. What does a generative student consider to have influenced her disposition?
4. How do generative students perceive the role of themselves in relations with others?
5. What are the relations among generative students and school discourses?

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[24] I have decided to use the word "maintain" rather than "develop", to indicate my belief that this dispositional quality is lost with age, quite possibly schooled out of the child. "Develop," in this context, seems to indicate it is a quality to be taught.

In subsequent chapters, I will clarify the previous discussion of the problem and resulting research questions by developing my theoretical position that guided the maturation of this research inquiry, provided a basis for the analysis of the data, and shaped the results and implications reported in this dissertation.

What's to Come

In this first chapter, I have worked to both introduce the research problem and provide a sense toward my approach to the problem. I pointed toward a way of thinking about the generative adolescent mathematical learner (GAML). Next I worked to clarify how I perceive this dissertation study of the GAML to be work about social justice, by discussing notions of equity and then demonstrating how others see this work as work for social justice, and finally how I am approaching this as work for social justice. I closed the chapter with the research questions that guided my thinking, and then here a summary of the intention to present the study's results.

The work of Chapter 2 is to justify the worthiness of this study, demonstrating the current valuation within the field of mathematics education to create generative learners. To do so, I present a review of the ideas related to the generative mathematical learner that are currently present, or lacking, in mathematics education research. I begin with some in mathematics education who have used the word "generative" when referring to children's thinking. Demonstrating the desire for this type of learner will follow this, and that this learner is evident at younger ages, but no longer seems to be present in the secondary grades. But because this dissertation extends beyond a singular topic, allowing the pursuit of the multiplicities of inquisition necessary for the type of research I conduct—radical constructivist postmodern critical theory—I also review literature that inform other theses that emerged during this research. I explicate the potential for contemporary epistemology to both serve the purposes of such research and set the stage for considering the generative quality of mathematics learners. I will also document that this epistemology is not currently reflected in practice nor valued as a mathematical learning outcome. The conflict of the goal for generative learners and the absence of this epistemology in the field is the cause for the inability to create such learners. To understand the GAML in the context of social justice, I briefly review the contributions of sociology to the field of mathematics education. The chapter closes with an initial foray into postmodern views on knowledge production, giving a glimpse to the paired tensions of the impossibilities of a scientific research, and the fabrication of knowledge.

The dissertation's third chapter explicitly works to point toward the postmodern theory with which I pose questions, interpret data, produce knowledge, and communicate my thinking. While orienting the reader to the macro- and mid-level theories employed in this research, I work to expand on my early attempts to locate my initial conceptions of the generative adolescent mathematical learner (GAML). Save for the second term, each of the words in my naming of my participants will be turned over in this examination. This third chapter opens with a more careful statement of the engagement and implications of epistemological theory. Next, I establish the larger theoretical project within which this work is being done, both to set the stage for the research methodology and to glimpse the larger goal of the dissertation, to deconstruct the fabrication of knowledge in the context of mathematics teaching and learning, and research on mathematics teaching and learning.

Chapter 4 addresses my theory for knowing research that expresses my implementation of my Radical Constructivist epistemology as it mingled with the PostStructural worldview I inhabit. In this chapter I consider the politics of methodology. Next I account for my researcher subjectivity within a critical postmodern frame. This discussion is followed by an overview of the methodological procedures engaged during the study and concludes with an account of the researcher's ethical responsibilities.

Chapter 5 works to characterize directly the methodology applied in this study, and explicitly detail the methods employed for data collection, while marching forward considering the tensions of describing the GAML—the challenge of work for social justice, the violence of the research project, and the fabrication of knowledge. This chapter particularizes the design of and intentions for data collection methods, as well as its theoretical underpinnings. More importantly, this section will capture the evolution of the methods as I became enmeshed in the data and learned about both the subjects and myself as fabricators of knowledge.

Chapter 6 serves to report the results of this research effort. Each of the three research subjects are described via the data collected and analyzed, presented as case studies. Chapter 7 reviews, accumulates, and accounts for what I learned while studying the GAML, both focusing and broadening meaning for such a concept. I return to the driving research questions, but also go beyond them with the both the tangential and emergent questions that arose during the data collection and analysis. Chapter 8 will serve to review the project in its entirety, and bring together findings in order to recontextualize them within the field of mathematics education, and more importantly within the field of research in mathematics education. This concluding chapter reflexively engages what is learned from the results, what these new ideas about the GAML mean for mathematics education's work for social justice. The fabrication of knowledge will be central in this reflection.

While this dissertation reports on research about generative adolescent mathematical learners, it maintains a multiplicity of purposes. It not only reports on the activity of a generative high school mathematics student or the manner in which their experience of schooling occasion the development of this disposition, it also serves to challenge both the researcher and the reader to reconsider the role of knowledge in the powered relations of knowledge production, and the judgment of knowledge production set at play within the science of mathematics education research. This project is my work toward social justice.

CHAPTER 2

RELEVANT LITERATURE

I begin this chapter with a review of generativity in the field of mathematics education. This review establishes that the generative learner is desired within the field, but such characteristics, while present in young children, seem to have faded away in the adolescent student. After demonstrating that this disposition for the mathematical learner is desired in the field, I explore the conflict surrounding its study. This conflict is about the nature of the subject, that fundamental query of man as a thinking subject, a constructed subject, a discursive subject, or possibly other. To explore this unresolved (-able?) trouble, I begin to locate the study of the subject in a discussion of psychological versus sociological epistemological viewpoints. The chapter concludes with an introduction to postmodernism, and especially its contribution to the nature of the fabrication of knowledge and the subject. The postmodern condition may account for the many aspects of the struggle in mathematics educations effort to realize the generative adolescent mathematical learner. It will also allow my epistemological position that names all learners as generative; yet not contradict my act of selecting particular generative students for my research.

Relevant Work in Mathematics Education

Researchers in mathematics education have shown a consistent interest in the mathematical disposition of the learner, valuing autonomy and authority[25] as goals for a mathematical learning environment (cf. Burton, 1999). In the following review of research, I first review work that utilizes the word "generative." Although the use of the word may not exactly be harmonious with my intentions behind engaging its potentials in my study, this review will demonstrate this educational valuation of certain types of dispositions toward mathematics and the learning of mathematics. I continue with research particularly done on younger children, and then adolescents. As I close this section, I begin to narrow the focus on adolescent mathematics students' conceptions of themselves as learners.

The Notion of "Generative" in Mathematics Education

My study is not the first to utilize the adjective "generative" when speaking about the nature of learning, whether it is from a cognitive or affective perspective. In this section I will review significant uses of the term in mathematics education, followed by other efforts to capture qualities of a mathematics education that seem to be related to the maintenance of a generative, or productive, disposition toward mathematical learning.

Wittrock. Wittrock's (1974) paper "A Generative Model of Mathematics Learning," signaled the evolution in the field from behaviorism to cognitivism. The hypothesis that Wittrock, an educational psychologist working from an information processing point of view, put forth in this paper is that learning with understanding is a generative process. This learning theory emphasized the construction of organizational structures for storing and retrieving information, and the construction of processes for relating new information to what is already

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[25] A note for the careful reader: I intend to develop autonomy and authority as ideas within social interaction in Chapter 3, where I attend to my theoretical orientations. I will maintain a consistency among these definitions with the notion that social interaction is a construction of the knower and within the knower's experiential reality.

stored. For educators of the era (cf. Shulman & Keislar, 1966[26]), Wittrock's work was evidence that others were thinking about knowledge production differently, seeing that something can be said about generative thinking—new ways of knowing that result from the cognizing agent's own activity. His work echoed early constructivist's[27] emphasis on the active role of the individual in learning. Like constructivism, this generative model is not a prescription for teaching. Wittrock foresees this confusion stating, "Learning with understanding can occur with discovery treatments or with reception treatments. The important point is what these treatments cause the learner to do" (p. 182). Yet Wittrock does reserve room to comment on teaching, making the ethical claim that respecting and thus seeking to engage the generativity of the learner is the way teachers should introduce, and relate materials to student experiences. Instruction should leave "something important for the learner to do—something to process, to generate" (p. 194).

Although not a mathematics educator, Wittrock did play a role in the development of the field of mathematics education. Many mathematics educators viewed the child as generative (in a generic sense of the word) prior to Wittrock's (1974) pronunciation of "A Generative Model." Hendrix (1936; 1947) explored the nature of transfer of learning, in particular "to what extent, if any, does the way in which one learns a generalization affect the probability of his recognizing a chance to use it?" (1947, p. 197). She found that initializing learning with an authoritative statement of a generalization is not as effective as a method in which the learner is excused once an unverbalized awareness is achieved, demonstrated by revealing possession of the

[26] For accuracy, note that both Shulman and Keislar were educational psychologists. The book cited here is a classic in children's construction of knowledge, laying out issues of learning by discovery, including implications for students, teachers, curriculum developers, and researchers.

[27] In retrospect, there were many "early constructivists", including Piaget, Vygotsky, Dewey, and in particular mathematics educators such as Hendrix (1947) and Davis (1966). The moniker appeared in publication in the mid 1970's (cf. Glasersfeld's 1975 presentation to the Jean Piaget Society [Steffe & Kieren, 1994]; Smock, 1976), and even prior to the 1970's in the work of Piaget and his colleagues (personal communication, Leslie P. Steffe, October, 2007).

generalization through behavior. Furthermore, she hypothesized that efforts to verbalize this generalization may actually decrease the transfer power. Her work pointed in the direction of a learning theory that valued student's unverbalized, sudden awarenesses of patterns, truths, or knowledge. Hendrix' work on teaching and learning can be located as a passageway from the early twentieth century work by leaders such as Montessori, Piaget, and Dewey, who emphasized the child as an active learner, to the modern mathematics (or "new math") movement of the 1960's and early 1970's, which were discovery oriented.

Bruner (1961) worked in the area of "discovery learning"—basically a concept that was set in opposition to learning by being taught. Hawkins (1966) clarified a meaning for discovery learning by stating a *word* can be taught, a name for something if it stands out for naming. "But concepts can be *learned*, which means evolved in the economy of experience" (p. 9). Wittrock (1966) pointed to a dilemma among educational psychologists about discovery learning: "When learning and discovery are measured by one event, discovery cannot be given as a cause for learning" (p. 35). At this stage, the theory had not embraced that all learning is discovery, but instead were mired in a dilemma that involved the effort to teach. Wittrock's (1974) step to develop a generative model for mathematical learning moved past this hurdle.

DiSessa. Andrea DiSessa is a physics educator who collaborated with the Massachusetts Institute of Technology Media Lab's Epistemology and Learning Research group (Harel & Papert, 1991) to design educational computing applications. In conjunction with this design, the group developed an epistemology, entitled Constructionism, along with classroom practices that enacted the computer application in a manner reflective of the epistemological principles. Their work emanated from the conjecture that a computer-based learning environment can open microworlds for the child to create and explore mathematical relationships. The educational software Turtle Logo emerged from this work in the late 70's and early 80's; currently DiSessa (2000) is developing the Boxer Learning environment. In this recent work, he speaks about intuition and generativity as two components of the learner's ways of acting that are to be harnessed in education. "Interests, like almost all components of human intellectual capabilities, are fundamentally generative. They never stay the same, but constantly shift and rebuild themselves according to experiences and contexts" (p. 80).

Muis. Muis (2004) reviewed 33 studies on students' epistemological beliefs about mathematics, what she referred to as personal epistemologies. She defined these personal epistemologies to be beliefs about "the nature of knowledge, justification of knowledge, sources of knowledge, and developmental aspects of knowledge acquisition" (p. 324). As opposed to both Wittrock and DiSessa, Muis emphasized it was her intent to examine the child's *own* sense of their *own* learning, rather than work to develop a learning theory. Because it is "audacious to assume that one knows what is the best epistemological viewpoint" (p. 323), Muis worked to disrupt the common sophisticated versus naïve, and appropriate versus inappropriate ways of thinking about children's disposition. She preferred labels that "should, as much as possible, not convey a value judgment, nor should it have negative connotations" (p. 323). She distinguished beliefs on a continuum of the degree to which they *avail* learning, associated with better learning outcomes. While her effort to avoid judging is noteworthy, the correlation and valuation of personal epistemologies to better learning, as measured by anything—test score, school grade, measure of participation—remains a judgment. Instead of a judgment of the researcher, the judgment is made by a structure imposed by others. Further, that the researcher chooses particular measures of better learning falls right back to a judgment by the researcher.

Moving beyond a critique of her attempt to minimize the hegemony of naming belief dichotomies, the review conducted by Muis (2004) yielded five categories of research: beliefs about mathematics, development of beliefs, effects of beliefs on behavior, domain differences, and changing beliefs. Of these, the first, second, and fifth are most relevant to my research. I next report her findings in these three categories, focusing on grade level and authority, agency, and identity results.

Muis (2004) found that across age levels, students hold non-availing beliefs about mathematics. She includes in these students beliefs that mathematical knowledge is unchanging. The existence of proof, and its use, propagates this notion. Students also believe the goal of problem solving is to find *the* right solution. Students believe that mathematics is passed along to them through some authority, which may be a teacher or textbook author. Muis also found that students believe they are incapable of learning mathematics through logic or reason, while those who are capable of doing mathematics possess an innate ability. Students do not believe they construct mathematical knowledge or can solve problems on their own.

With regards to the development of a personal epistemology, Muis (2004) found that, consistent with literature on development of epistemological beliefs in general, students' beliefs about mathematics become more availing over time. However, there was an example in her review that suggested otherwise. This contradictory example better aligns with other's findings (Garofalo, 1989; Grieb & Easley, 1984; Grouws et al., 1996; Kamii & Housman, 2000; Schoenfeld, 1988; 1989) and the personal observations that led me to this research that young children demonstrate a greater generativity than do high school students. After Muis argued that classroom lessons dominated by recitation and seatwork were common at both elementary and upper levels (see p. 335) and noted that this teaching practice impacts students' personal epistemologies—a second key finding in this category—she argued students' non-availing beliefs demonstrated in elementary schools should also be demonstrated in secondary school. She concluded that, "researchers who have examined the nature of mathematics instruction at this level have found similar results" (p. 335). To account for this curiosity, Muis suggests, "it appears one plausible hypothesis is that formal mathematics education plays a major role in the development of students' beliefs about the nature of mathematical knowledge and learning" (p. 339). Here she speaks of Mathematics education, with a capital M, a particular mathematics taken to exist prior to the learner.

While the effects of beliefs on behavior, Muis' (2004) third category, begin to fall outside the relationship to my research, some interesting findings occurred that help to inform conclusions about teaching that my study may infer. Muis found convincing evidence that students' beliefs influence their engagement in learning and academic achievement—specifically the amount of time they work on a problem, the strategies they use to solve a problem, and their justifications for correctness. The fourth category yielded the conclusion that across ages, personal epistemologies demonstrate domain differences. That is a child, at any age, is likely to show differing epistemologies with respect to learning science versus mathematics. The differences found were such things like the certainty of knowledge (Paulsen & Wells, 1998) and the role of teacher in the generation of knowledge (Stodolsky et al., 1991). In this second case, for example, a study in elementary classrooms indicated student believed they could learn social studies on their own, but the teacher needed to tell them what they needed to know for mathematics.

Finally, Muis (2004) observed that most studies found a positive correlation between changes toward constructivist-oriented classroom instruction and increased availing beliefs.

Constructivist-oriented approaches to learning "embed mathematics in meaningful and authentic contexts and recognize the importance of collaboration and group activity in constructing knowledge' (p. 355). There is "a growing agreement that exploring students' beliefs about the nature of knowledge and learning is an important line of research in education and that such beliefs are an important factor to consider in terms of the impact of beliefs on cognition and motivation" (p. 363). In her conclusion, Muis reported, "epistemological beliefs may be powerful predictors of students' learning methods and achievement" (p. 364).

Muis' (2004) critical review of research confirms that learners' epistemologies are worthy of study. And further, there are types of personal mathematical epistemologies that may be conducive—more *availing*—to more effective learning. Lost in this study is a deconstruction of what mathematics or mathematical learning is. Her conclusion that some personal epistemologies may be more availing to effective mathematical learning does make a statement about 'better' in relation to some thing—a particular mathematics, or a particular way of mathematical thinking—fails the challenge Muis gave herself in choosing the language of 'avail.' The generativity I wish to consider, and wish to attribute to all learners, seeks to transcend this limit to what a mathematics education could be, in a way similar to Muis' challenge.

Analysis. Wittrock (1974) and DiSessa (2000) demonstrate a strong belief that children are generative, producers of knowledge. Muis (2004) reviews a significant expanse of research that considers learner's views of themselves as producers of mathematical knowledge, an availing belief. In this review, it is apparent that the practice of mathematics education falls short of attaining this goal. Muis states, "students commonly hold nonavailing beliefs about mathematics" (p. 364). Muis suggests that research on personal epistemologies would benefit from “examination of the sequential development of beliefs” (p. 364) and “the relationship between students’ beliefs, the impact the environment may have on their beliefs, and how beliefs influence learning and academic achievement” (p. 365). The accumulation of the negative effects of perpetuating one’s own beliefs and a negative impact of the learning environment suggests the conjecture that non-availing beliefs grow the longer students remain in school, and children’s school success correlates to the negative growth of this disposition.

Generativity Observed in Young Children

The next set of studies focuses on the affective domains of young children’s mathematical learning experiences in school. In particular, these researchers considered student autonomy in learning, and the degree to which they saw themselves as autonomous—a way of thinking about seeing themselves as authorities in their own construction of mathematical knowledge.

Yackel and Cobb. Yackel and Cobb (1996) developed tools through which to interpret classroom life, with an aim to account for how students develop specific mathematical beliefs and values, and consequently, how they become intellectually autonomous in mathematics. Data from the mathematical activity of a second grade classroom was used to analyze the sociomathematical norms that develop between the teacher and students. Constructivism, social interactivism, and ethnomethodology were the theoretical perspectives that informed this work. The interactivist perspective proved invaluable by providing the basic assumption that cultural and social processes are integral to mathematical activity. Yackel and Cobb were explicit in stating that their orientation was that coming to know mathematics is coming to learn a social practice. Their work is to study how children come to know a pre-existing Mathematics, external to them. They noted, quoting their prior research, that “From our perspective, the suggestion that students can be left to their own devices to construct mathematical ways of knowing compatible with those of wider society is a contradiction in terms" (p. 474). This comment confirms my note above; in that these authors make clear that the 'mathematics' of this passage is certainly a Mathematics external to the learner, a set of knowledges to be learned.

The construct of reflexivity, drawn from ethnomethodology, aided Yackel and Cobb to clarify how "sociomathematical norms and goals and beliefs about mathematical activity and learning evolve together as a dynamic system" (p. 460). Yackel and Cobb inferred these sociomathematical norms by identifying regularities in patterns of social interaction. The regularities found in their study of the mathematical activity of second grade classrooms included creating meaning for *mathematical difference*, *more sophisticated solutions*, and the development of a *mathematical basis for explanation*. The examples used from the data illustrate that "children can make personal judgments of this kind [that is, those based on experientially real mathematical objects] on the basis of their mathematical beliefs and values that they can participate as increasingly autonomous members of an inquiry mathematics community" (Yackel & Cobb, 1996, pp. 473–474). This development of intellectual and social autonomy is a "major goal in the current educational reform movement, more generally, and in the reform movement in mathematics education, in particular (p. 473), a notion in agreement with Piaget's (1973/1948) treatise *To understand is to invent*.

Yackel and Cobb take care to reject the notion of autonomy as a context-free characteristic of an individual. Instead, autonomy is defined in relation to participation in a community's practice, in this case the mathematics classroom. "Students who are intellectually autonomous in mathematics are aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgments as they participate in these practices (Kamii, 1985)" (p. 473).

Yackel and Cobb (1996) found that teaching with an inquiry approach to mathematics instruction could foster the growth of sociomathematical norms in the classroom of the type that develops autonomy in students. "As we have shown, in the process of negotiating sociomathematical norms, students in these classrooms actively constructed personal beliefs and values that enabled them to be increasingly autonomous in mathematics" (p. 474).

Lampert. While Yackel and Cobb (1996) were particularly interested in student autonomy, Lampert (1990) worked carefully for student authority. Lampert taught a fifth grade class with the intention of making knowing mathematics in the classroom to be more like knowing mathematics in the discipline by deliberately altering the roles of students and teacher in the mathematics classroom. "The issue of intellectual authority is central to this comparison" (p. 32). She sought to examine the sort of intellectually generative activities espoused by groups such as NCTM (1989). Drawing on Lakatos and Polya, Lampert recognized that the activity of doing mathematics is different from what is recorded once it is already done. Mathematics develops as a process of conscious guessing about relationships among quantities and shapes, where validation follows a zigzag path beginning with conjectures and moving through examination of premises through the development of counterexamples. Polya considered courage and modesty to be the qualities essential to the development of mathematical knowledge.

The mathematical context of this study (Lampert, 1990) was to predict the final digit of an exponential statement, along a mathematical learning trajectory of introduction to exponential notation toward developing generalizations about the legitimacy about adding and subtracting exponents on like bases as a shortcut to multiplication and division.

In order to develop the social virtues of courage and modesty in her students, Lampert (1990) had to work deliberately to teach her students “to give up more conventional forms of academic interaction” (p. 58). This teaching involved some telling, some showing, and some doing mathematics alongside students in addition to regular rehearsals—just as someone might instruct dance. Rather than interacting with students in a “conventional ‘knowledge telling’ exchange” (p. 53), she expected students to be the authors of ideas in the discourse structures of the mathematics classroom. She trained students to use phrases such as “I think” and “I want to revise my thinking”. She named student (and her) ideas as hypotheses rather than answers, and refrained from confirming or disconfirming responses. She monitored the potential for students to disengage or distrust their own thinking as a result of status interactions, lack of confidence, etc. She honored all student ideas by recording them to the board with names, but kept each in question by placing question marks alongside them.

Lampert’s (1990) teaching efforts documented that it was possible to teach and learn the mathematical virtues of courage and modesty, virtues that are essential to what she viewed to be knowing mathematics in the discipline rather than coming to know some skewed school mathematics (the focus of Yackel and Cobb’s [1996] research). Her students’ activity suggested that they were “operating with quite a different set of beliefs about what doing mathematics means than those held by other fifth graders in similar school settings” (p. 34). Her students put themselves in the position of authors of ideas and arguments. “In their talk about mathematics, reasoning and argument—not the teacher or the textbook—are the primary source of an idea’s legitimacy” (p. 34). She documents student activity that “engaged almost every member of the class in generative mathematical activity” (p. 46).

As a case study, this work (Lampert, 1990) serves as an existence proof, that a classroom environment is possible that encourages the maintenance of authoritative disposition toward mathematics. While the study documented new ways of mathematical interaction among her students, it remains a problem to define what knowledge they have acquired. Assuming this mathematical interaction and resulting location of mathematical authority is a desirable outcome, what would be necessary to produce it on a larger scale? And what impact would this have on a scale broader than the individual learner, toward social and economic goals? That Lampert has established that students can perceive mathematical authority in themselves further argues for the need to understand this notion of the generative learner. And because Lampert's, and other's work focuses at the elementary grades, students near the end of their public schooling must be considered as well. Before moving onto work done with older students, I consider one more significant study on student disposition, especially in relation to teacher expectations.

Grieb and Easley. Grieb and Easley (1984) published a set of case studies that explored the social mechanism of primary schools in the development of an independent or dependent attitude in creative children. They found that white, middle-class males—a group they termed "pale male math mavericks" (p. 338), or "pm3s" for short—more often than females and minorities, can "survive pressures toward conformity in the early grades with their confidence in mathematical reasoning intact and thus preserve more courage to tackle new types of problems throughout their schooling" (p. 318). It appeared to Grieb and Easley that young white, independent boys have an advantage over girls and minorities of equal creativity because their teachers seem to not expect these pale male math mavericks (pm3s) to conform or adhere to the social norms of arithmetic. Positing a desire for independence and creativity in the child, the authors suggested that all children enter first grade with sufficient creativity and inquisitiveness to pursue mathematics in the same manner as do the pm3s. However, a distinction between groups of children, as identified by the subset of pm3s, is strongly evident by grade 3. It is in the unconscious restrictions of the teacher or school on independent thought and action that stifled a creative and explorative approach to mathematics, and not the opposite. The researchers hypothesized that it is our socio-cultural norms that dictate affordances and constraints in the powered social interaction between teacher and student. If the "authoritarian" (p. 355) role of mathematics teachers were reduced, this would "lead to more pupil thinking and less straining to reproduce procedures" (p. 355). Hence, more students could maintain a productive disposition in their mathematical learning. The study suggested that, because American teachers did not step aside from this authoritarian role, it would be rare to find female or minority adolescents who demonstrate a generative disposition toward mathematical learning.

In the languaging (e.g. "survive pressures toward conformity" [Grieb and Easley, 1984, p. 318]) of their results, it was evident Grieb and Easley presumed that children possess the leanings toward the development of an independent and creative disposition toward mathematical activity, and that it is some sort of social pressures of the schooling experience that tend to disrupt this potential in many young students. This perspective parallels my assumptions about children and their mathematical activity. Children are autonomous, self-regulating, and creative thinkers. At a young age, the child's social interactions that create for them an elementary mathematics environment[28] have not yet fully enacted the oppressive, regulating force of the (constructed) school; the governing of the mind (Popkewitz, 1998) has yet to take

[28] I pause to clarify what I have in mind when I think of the "environment" of a child, or any knower. It is usual to treat what we have categorized as the knower's environment as if it has an existence independent of our own and their own knowing, because the other seems to react to it in a way we think ourselves would react. Glasersfeld (1990b) notes that the radical constructivist can still speak of "environment": "[O]rganism/environment, figure/ground, subject/object, and a host of other dichotomies of the kind are categorizations that cognizing agents impose on their experience and neither of the two mutually dependent terms can ever be less subjective than the other" (p. 33).

hold. In effect, Grieb and Easley believe we can still see a generative disposition in young mathematics learners. And while policy and mathematics educators call for this generative disposition to be maintained, developed, and even enhanced throughout schooling, it seemed to slip away in Grieb and Easley's subjects.

Kamii and Housman. Kamii and Housman (2000) continued and extended the themes of the research found in the collection of studies I have reviewed, that younger children can and do demonstrate a productive disposition toward their own mathematical learning, and that researchers and teachers have ideas about ways in which to design learning environments that elicit such student dispositions. Kamii and Housman worked intentionally to develop autonomy in the mathematical learner. Kamii, a Piagetian scholar, insists that mathematics should be learned through play. The teacher must observe the child's play and identify activities that the child would be most likely to meaningfully and productively engage. Classroom norms, such as those elaborated by Yackel and Cobb (1996), must be developed in conjunction with the students. When such a learning environment is created, Kamii and Housman reported successful student mathematical learning, primarily relying on accepted forms of evaluating mathematical growth such as standardized tests.

The work of the researchers who conducted these four studies demonstrate that the development of a productive disposition toward mathematical learning is valued in the field, even if Grieb and Easley (1984) show that it may not be that way in every classrooms, for every child. Further, they document that such practices are in place, or possible, with young students. This collection of research focuses on the autonomy or independence of the learner, or their sense of their autonomy as mathematical learners and knowers. Lampert (1990) and Kamii and Housman (2000) demonstrate classroom practices that yield this sort of learner. This evidence suggests generativity is valued and possible in the younger years. However, Grieb and Easley point to trends in children's mathematical learning dispositions that suggest struggling students may continue to become further and further removed from their sense of autonomy and authority in learning. Furthermore, even the young pale male mathematics mavericks may be in danger of not maintaining such a disposition, in that it runs counter to what may be valued in classrooms as they continue through school.

Generativity Not Observed in Adolescents

Studies of secondary student's attitudes and dispositions toward mathematics, as well of their sense of themselves as mathematicians, point toward decay of the generative mathematical disposition—that is assuming younger students hadn't already lost such an outlook on their own mathematical learning.

Garofalo. Garofalo (1989) studied high school students' beliefs about themselves as doers of mathematics. He found four beliefs common among students studied. First of these beliefs was that "almost all mathematics problems can be solved by the direct application of the facts, rules, formulas, and procedures shown by the teacher or given in the textbook" (p. 502). This led students to spend their time studying mathematics by memorizing facts and formulas and rote practice of procedures. Students approached mathematical tasks mechanically, attempting to rely on recall. Students did not attempt to make sense of mathematics. A corollary to this belief is that "mathematical thinking consists of being able to learn, remember, and apply facts, rules, formulas, and procedures" (p. 503).

A second belief was that "mathematics textbook exercises can be solved only by the methods presented in the textbook; moreover, such exercises must be solved by the methods presented in the section of the textbook in which they appear" (Garofalo, 1989, p. 503). This detracts from students' willingness to reason through a problem. Students with this belief also viewed mathematics as a highly fragmented set of rules and procedures. A third belief and its corollary were that "only the mathematics to be tested is important and worth knowing" (p. 503) and "formulas are important, but their derivations are not" (p. 503). Clearly, this belief focused students on only studying and caring about knowing the formulas that will occur in evaluation and testing situations.

A fourth and final belief was that "mathematics is created only by very prodigious and creative people; other people just try to learn what is handed down" (Garofalo, 1989, p. 503). Garofalo stated that students with this belief viewed the teacher and the textbook as the authority in, and dispenser of, mathematical knowledge. These students were and "can never be more than copiers or reproducers of other people's mathematics" (p. 504). Garofalo's observation places these learners in the negative portion of the x-axis in my diagram reflecting various ways to consider mathematical learning (see Figure 1). He observed that these adolescent mathematics students saw themselves as receivers or sense-makers of another person's knowledge.

While these beliefs seem to suggest students will not learn much in a mathematics classroom, Garofalo (1989) argued that these students learn quite a bit: "They have learned much about their mathematics teachers, textbooks, tests, and the classroom environment. They have insight about the nature of *classroom mathematics* and have created ways of dealing with it" (p. 504). Garofalo refers here to the same outlook on the Mathematics learned that Yackel and Cobb (1996) desire for the children of their study, a particular Mathematics, not authored by the learner. Garofalo concluded by sketching the image of a mathematics classrooms envisioned by the 1989 NCTM Standards, indicating that such a classroom environment will give students "better opportunities to develop more realistic beliefs about our discipline" (pp. 504–505).

Schoenfeld. Schoenfeld (1988) conducted a case study of a 10th grade Geometry classroom, focusing on both students' mathematical learning as well as what students learned about the mathematics. The analysis of this classroom data, including observation and documentation of teaching practices, and measures of student learning and attitudes, was designed to explore the presence and robustness of four beliefs Schoenfeld established in earlier work, and to seek their possible origin in instruction. Schoenfeld (1985) had previously observed these four beliefs, which are consistent with Garofalo's (1989) later findings:

> *Belief 1:* The processes of formal mathematics (e.g., "proof") have little or nothing to do with discovery or invention.
>
> *Belief 2:* Students who understand the subject matter can solve assigned mathematics problems in five minutes or less.
>
> *Belief 3:* Only geniuses are capable of discovering, creating, or really understanding mathematics. Corollary: Mathematics is studied passively.
>
> *Belief 4:* One succeeds in school by performing the tasks, to the letter, as described by the teacher. Corollary: Learning is an incidental by-product of 'getting the work done.' (p. 151)

Schoenfeld (1988) argued in the present study that, by way of the structures of the state testing system and textbook use, teaching methods were practically prescribed to be geared toward mastery of procedural skill and accuracy rather than learning for meaning. Empirical analysis rather than deductive justification was what constituted proof for the students of his study. Students also learned that proof was as much about form of expression (the particularities of two-column design) as much as it is the substance of the mathematics. These learnings were unintended by-products of their instruction.

Schoenfeld (1988) also found that the Geometry students believed all mathematics problems could be solved in just a few minutes. He demonstrated that short tasks are the only types kids have done all year, observing no students in any observed classes doing “mathematical tasks that could seriously be called problems” (p. 159). And finally, because the “subject matter was presented, explained, and rehearsed” (p. 161) and students practiced until they got it, there was little sense of exploration or opportunity to make sense of mathematics for themselves. As a result, students viewed themselves as passive consumers of others’ mathematics.

Schoenfeld (1988) emphasized that the beliefs that research has documented children hold about mathematics and themselves as mathematical learners is developed from their classroom experience with mathematics. He called for the reexamination of curricular goals, materials, and measurement tools. He argued for the research community to better characterize the knowledge and cognitive processes that comprise thinking mathematically. Further, the research community must devise means to understand the world from the student's point of view, and to find ways to identify the effects of instruction on the development of this worldview. Although Schoenfeld’s research suggests he holds a Platonist or formalist view of mathematics, and a trivial constructivist (Confrey, 1990) view on learning (tending in the negative y direction of Figure 1), these suggestions remain applicable to my research. In part, his message is that what researchers in mathematics education document about the mathematical disposition of adolescents is a by-product of a mathematical educational system steeped in realism. However, it is my contention that a radical constructivist epistemology would more deeply engage these challenges toward a more ethical and equitable mathematics education.

Schoenfeld continued. Schoenfeld (1989) followed up his 1988 study on the development of mathematical understanding in order to extend and situate his previous findings on students’ attributions of success and failure, their comparative perceptions of mathematics with other disciplines, their view of mathematics as a discipline, and their attitude toward mathematics. This study involved 230 grades 10–12, successful mathematics students, in highly regarded schools. These students were highly motivated, and seemingly for good reasons to them: mathematics is interesting, learning mathematics will help them think, and they wish to do well academically. Teaching practices appeared to be respectful of students and a Socratic method of questioning was common. Students believed mathematics could be learned if they worked hard at it. And they did, scoring well on standard performance measures.

However, interesting conflicts emerged in the findings on student beliefs—not only that these positive beliefs and opportunities yielded some undesirable outcomes, but also paradoxical student beliefs about mathematics. Students in this environment believed that learning mathematics was mostly about memorization of facts and procedures, despite their assertions that mathematics can help one learn to think logically and that one can be creative in mathematics. Students believed mathematical problems should be solvable quickly, and that anything taking more than 12 minutes of their effort will turn out to be impossible. Solving problems depended on knowing the rules. Despite student claims that proofs and constructions were closely related, students seemed incapable of drawing on their proof-related knowledge to build constructions on their own. Schoenfeld (1989) concluded that in many situations, the better the student was, the more likely these beliefs were to be held.

Schoenfeld (1989) found the most disturbing aspect of this study to be that “students have come to separate school mathematics—the mathematics they know and experience in their classrooms—from abstract mathematics, the discipline of creativity, problem solving, and discovery, about which they are told but which they have not experienced” (p. 349). I concur that this is a troubling finding, however I consider myself to think about mathematics differently than does Schoenfeld. In his research it appeared students have created a notion of this thing external to themselves, that Schoenfeld named school mathematics. School mathematics are facts and procedures to be memorized and correctly applied to problems. While students believed it aids their creativity and logical thinking, they seemingly have not experienced the doing of something they name mathematics that they would call creative, or logical. For Schoenfeld then to introduce another mathematics—abstract mathematics—seemed to separate these students one step further from a mathematics they are to come to know. This is an ethical problem for teaching, what mathematics is to be taught? Or more difficult, whose mathematics?

Grouws et al. Grouws, Howald, and Colangelo (1996) conducted a study of student conceptions of mathematics. Grouws et al. report that although there are several studies about student attitude toward mathematics, there is little attention being paid to student conceptions of mathematics.

> In reporting data gathered about students perceptions of mathematics in the 1990 National Assessment of Educational Progress (NAEP), Mullis, Dossey, Owen, and Phillips (1991) state that 'national reforms in mathematics education highlight the importance of developing a lasting appreciation and positive attitude toward the use of mathematics to solve problems' (p. 373)…. Across all three grades, 4, 8, and 12, students with more positive perceptions and attitudes had higher mathematics proficiency, with differences among grades suggesting that positive perceptions of mathematics may diminish in high school. (p. 3)

Grouws et al. state that there is a renewed interest the field of mathematics education in philosophical questions such as "What is mathematics?" and "What does it mean to know mathematics?" "Responses to these questions have important implications for the teaching and learning of mathematics" (p. 2). Taking this broader approach to considering the place of mathematics education research, Grouws et al. (1996) studied the conceptions of mathematics held by two groups of high school students, mathematically talented and the more typical student.

Through a review of literature, Grouws et al. (1996) determined two key factors in their research design that would aid the emergence of differences between the groups. First, the researchers decided to use both general and content-specific examples in their questions, because the more concrete the item, the more telling the difference in response patterns. The second of these factors was that a well-defined framework would help them to analyze data on students' conceptions of mathematics in a systematic manner. The framework they devised was drawn from a synthesis "examining existing instruments, reviewing categories of beliefs used by others, and considering the literature linking conceptions and student learning" (p. 12). Four themes were identified: Nature of mathematical knowledge, Nature of mathematical activity, Learning of mathematics, and Usefulness of mathematics. Three dimensions were developed to characterize the first theme: Composition, Structure, and Status of mathematical knowledge. And two dimensions helped clarify the second theme: Doing mathematics, and Validating ideas in mathematics. Each of the resulting seven dimensions was considered on a continuum.

The research team developed a Conceptions of Mathematics Inventory (CMI) survey using some items from previous instruments and writing some of their own. Data was collected from two samples of students. The first were 55 talented high school students enrolled in a summer program. The second sample included 112 high school students enrolled in an algebra or integrated math course.

A few significant differences emerged between the two groups of students. Mathematically talented students tended to view mathematics as a growing system of coherent and interrelated concepts. These students also found that doing mathematics is a sense-making process relying on ones own thinking and reflection to establish validity. On the other hand, the algebra students saw mathematics as a growing field, however it is a discrete system of facts and procedures, requiring more memorization than thinking. Algebra students considered doing mathematics to be implementing known procedures and formulas, and accepting mathematical truths as established by others rather than relying on their own capacity to deduce mathematical knowledge. These differences in the composition, structure, doing, validating, and learning dimensions create an interesting contradiction when considered with the general agreement within the status dimension and the usefulness dimension. “Although both groups see mathematics as a dynamic and useful field, their conceptions of what doing and learning mathematics differs markedly” (p. 31).

This analysis suffers the common validity concern associated with Likert-scale surveys. But particular to the two groups being compared, students in a program for gifted and talented often show particular mathematical strengths, but also often are very school-successful. This school success can translate into trends in survey responses that may be different from their “regular” high school peers. For example, “average” students may have learned through their non-successes in school to carry weaker opinions, and tend toward a “neutral” response rather than a “strongly” agree or disagree. Because many of the questions required a double negative to provide a positive response, more of the “talented” school students may have been better able to negotiate this word play. The researchers conclude, at one point, that “Although there is a significant difference between responses for the two groups for both items, the more concrete the item, the more telling was the difference in the response pattern” (p. 29). Again, it is possible that the “talented” students played the school language game better, and noticed this sort of difference, while for the “regular” students, an adult notion of “general” and “concrete” were equally abstract. These researchers failed to recognize their role as observers and that the concreteness of the instrument is not a quality of the instrument, but a quality imposed by the user. Although the conclusions of this research are worth considering, and help the mathematics education community to think, they also suffer some of the concerns of methods that are not highly reflexive.

Muis' (2004) review of literature demonstrated that student personal mathematic epistemologies are malleable and may be affected by classroom instruction. If the talented group had a constructivist learning environment in their summer project, their conceptions of mathematics may have changed in this short time. Had they been interviewed prior to the summer experience, they may have shown the same results as the regular students.

Another contradictory component of the Grouws et al. (1996) study is the use of the language "discovery" in some of the survey and interview questions (see, for example p. 21). This reference to the generation of new mathematics, rather than the word "invent", implies and suggests a particular ontological standpoint toward mathematics. The researchers own conceptions of mathematics may not only have influenced the responses of the subjects, but also likely skewed their interpretations of the student conceptions of mathematics.

Grouws et al. (1996) found that most students, both mathematically talented and the regular high school students, reported mathematics to be useful in their personal lives both in and out of school, and important for their future plans. This result placed against other data suggesting that many students struggle to pass an Algebra course and may perform very poorly on state and national tests of mathematics creates an alarming situation. Students believe that mathematics is important, yet many receive multiple sources of feedback indicating they are bad at mathematics. What sort of self-image does this develop in children, about themselves as mathematical thinkers and for their potential for success in life?

Díaz-Obando et al. Díaz-Obando, Plasencia-Cruz, & Solano-Alvarado (2003) conducted case studies of two secondary mathematics students in order to consider their beliefs and related issues that emerged from their experiences in mathematics classrooms. The methods were designed to aid the researchers in getting to know the interpretations that the subjects gave to both their mathematical knowledge and actions. Kevin, the first student, was a 15-year old male in a rural public school in Spain. This boy showed a strength in mathematics, but was unsuccessful in school. The second boy, Sam, was 17, attending an urban-marginal school in Costa Rica. The researchers report that he took his studies seriously, having intentions to continue at the university.

Both students were taught by experienced teachers, and both perceived their teacher's role was to enhance students' understanding of mathematics. Teachers were to explain concepts to students. Further, both students reported that their teachers expected the students to do mathematical tasks in particular ways. And both students rejected this notion. Sam believed that the study of mathematics was useful for the future—further study and possible careers. Kevin, however, was less interested in engaging in school mathematics, highly aware of the difference in what his teacher wanted him to do as mathematics in the classroom, versus a mathematics in which his reasoning was valued. Both Sam and Kevin recognized this dichotomy between a school mathematics and the activity of doing mathematics that involved their own sense making; both seemed to prefer making their own meaning. Kevin reacted against the learning only through rules and rote memorization, while Sam seemed to be able to manage both types of learning. However, neither seemed to engage in mathematical tasks for pleasure.

Extending. The Schoenfeld (1988, 1989) research added a layer of interest to this collection by considering the beliefs of secondary mathematics students, and then in particular looking at the beliefs of successful high school mathematics students. It is evident that their beliefs are unlikely to be productive for future mathematics study. This finding is echoed in the work of Boaler and Greeno (2000) in which they learned that successful college mathematics students also display attitudes toward mathematics and their mathematical learning that is seemingly unproductive toward continued mathematical learning.

Grouws et al. (1996) explored dispositions of mathematics students in order to create a framework for analyzing student conceptions of mathematics, and to formulate ideas about how these conceptions relate to student learning. The dimensions used to study student's conceptions of mathematics fell under four categories: the nature of mathematical knowledge, the nature of mathematical activity, learning mathematics, and the usefulness of mathematics. They found that the students who had demonstrated less success seemed to often have a healthier view of mathematics and its purposes. Similar studies have shown the mathematical identities of successful students as non-creative disconnected thinkers to be detrimental to future mathematics learning (Boaler & Greeno, 2000). Clarke, Breed, and Fraser (1992/2004) studied matched pairs of students in a conventional curriculum and an NSF-funded reform curriculum, the Interactive Mathematics Program (IMP) (Fendel & Resek, 1997). They found that students of the reform curriculum also showed a healthier view of mathematics. "Students who have participated in the IMP program appear to be more confident than their peers in conventional classes; to subscribe to a view of mathematics as having arisen to meet the needs of society, rather than as a set of arbitrary rules; to value communication in mathematics learning more highly than students in conventional classes; and to be more likely than their conventionally-taught peers to see a mathematical element in everyday activity" (p. 7). While both Grouws et al. (1996) and Díaz-

Obando et al. (2003) connected lesser success with a healthier[29] view of mathematics, these reform-curriculum students demonstrated equal or greater performance than the students enrolled in the conventional curriculum on the mathematics portion of the SAT in addition to demonstrating this healthier view of mathematics. The Clarke et al. (2004/1992) research demonstrated it was not necessary to perform poorly in mathematics in order to maintain a healthy disposition.

The studies of this section on adolescents consider the affective domain of mathematical learners, yet none focus in particular on the student's disposition toward mathematical activity, as did Lampert in her consideration of authority. Boaler (2000a) and Zevenbergen (2002) conducted research on high school students that focused closely on the development of the student's sense of self in the context of tracked mathematics classrooms. Zevenbergen drew on the theoretical tools offered by Bourdieu, in particular the notion of habitus, "'an acquired system of generative schemes objectively adjusted to the particular conditions in which it is constituted'" (quoting Bourdieu, p. 565). This construct allowed the researcher to consider the "dynamic structure between social reality and the individual" (p. 566). Boaler (2000a) used Lave's (1988) situated cognition to consider student's knowledge production. Both researchers found that the nature of classroom interaction and curriculum expectations had a tremendous influence on the mathematical dispositions students demonstrate.

The Desire for the Generative Adolescent Mathematical Learner

Students as knowledge producers, and students perceiving themselves to be knowledge producers, are valued positions within the field of mathematics education. This message is rather consistent in policy, practice, and results for young learners, but becomes rather convoluted as

[29] "Healthier" in that it is a view reflective of the mathematical disposition the field of mathematics education desires as an outcome of the schooling process (Spangler, 1992).

the learner reaches adolescence. For example, researchers often use the target of 'student's view of themselves as mathematical authors' to be a dependent variable in studies of curriculum, pedagogy or classroom social norms. Both researchers and policy reflects the desire for this sort of learner. But policy, such as the NCTM *Standards*, seems to become contradictory in its call for knowledge inventors as the learner's age increases, instead seeking discoverers or replicators of particular mathematical ways of knowing, failing to question the nature of knowledge and knowing (Steffe, 2004). The languaging of policy documents shift from a focus on mathematics to Mathematics, a shift from focusing on a child's activity and ways of operating, to the mind-independent set of knowledge we call "Mathematics." The expectations for adolescents are placed in the negative y half-plane of Figure 1. Schoenfeld (1988; 1989) and Díaz et al. (2003) demonstrate that adolescents perceive themselves as learning machines designed to regurgitate a particular mathematics and mathematical ways of knowing, that is already known by experts, and that there are outside arbiters who possess the authority to deem knowledge as truth. These ideas of the self as a mathematical learner are presented in the negative x half-plane of Figure 1. These contradictions in policy, and depressing outcomes for student self-concept, come in light of the three decade old paradigm shift in learning theory marked by Wittrock (1974) and embraced by the mathematics community by the 1980's, that posits the student as an active agent in her experiential world.

Contributions of Other Sciences to Mathematics Education

In its early years—the second quartile of the 20th century—the field of mathematics education was dominated by discussion of the purpose of mathematics teaching, efforts toward effective curriculum design, and incorporation of the psychology of learning (National Council of Teachers of Mathematics, 1970, p. 64). As equity research began to grow during the 1970's, research in mathematics education undertook two significant shifts. The first, alluded to above and to be discussed in greater detail below, is a paradigmatic shift to consider the question *What mathematics to teach?* No longer did this question take on the narrow view to ask which particular (Western European) mathematics was to be part of the curriculum, but instead reflects an epistemological shift that entertained the pragmatics of multiple ways of knowing (Belenky, 1986; D'Ambrosio, 1994; Ernest, 1994; 2000). The philosophical work of Lakatos' *Proofs and Refutations* (1976) reoriented the process of knowledge development as taking place through refutation rather than deduction (Ernest; 1998). Mathematical knowledge was now viewed as retaining the possibility of being "challenged by a future counterexample" (Lerman, 2000, p. 22). This fallibilistic orientation to knowledge recognized that the question of "which mathematics" disrupted the assumed a priori nature of the discipline. As a result, new attention was placed on the mathematical knowing of the learner.

The second shift, in a symbiotic relationship with the first, was the influence of sociological theories and methodologies on the study of mathematical learning (Boaler, 2000b). The result of decades of work to justify the field, design effective curriculum, and build upon a psychological focus on learning led to a significant and inexorable open question, *Whose mathematics to teach?*, which lay at the heart of sociological work of the era, the mid 1970's. Sociology, anthropology, and cultural psychology began to have significant effects on the activity of mathematics education by the late 1980's (Lerman, 2000). The sociologist's interest into the effects on the mathematics learning environment, the classroom, of both the external (i.e. society) and internal (i.e. the classroom, a micro-society) cultures[30] led to a great deal of insight into the effectiveness of the mathematics teaching and learning. The tradition of critical theory

[30] Most sociologists I have read take these structures as a unit of study, the "society" and the "classroom," failing to embrace the radical constructivist's orientation to the knower's environment. I use the terms as would a sociologist here.

met this social perspective in mathematics education, and another emphasis emerged; the ineffectiveness and inadequacies of mathematics education captured the critical eye of many researchers.

Influential Sociological Views

As mathematics education, taken as a scientific field, moved beyond curriculum design and a limited focus on the psychology of learning, classroom cultures and the effects of the greater culture on the classroom became focuses of study.[31] Research began to document consistent inequities in schools (Kozol, 1967; 1975; 1992) and classrooms in general (Kohl, 1967; Willis, 1977), as well as in the mathematical learning among various subgroups of children (Becker, 1981; Fennema, 1984; Fennema & Carpenter, 1981; Matthews, 1984). Critical sociologists (e.g. Apple, 1979; Bourdieu, 1977) pointed to the need to learn much more about classroom processes (Reyes & Stanic, 1988). Studies done with underlying theories of oppression and resistance, power and privilege, and the effects of status provided new insights into who was and wasn't learning in the mathematics classroom.

Sociological work in the late 1970's began to be influenced by new theories of power, resistance, and freedom. Previously, power was generally considered to be a product of human agency, a quality with which we are naturally endowed, existing first outside ourselves (Butler, 1995; St. Pierre, 2000a). Humans employed power, possessed it, gave it away and took it back. Power was a bad thing, and philanthropic souls sought to dispense of their power to those who were oppressed (Postman & Weingartner, 1969; St. Pierre, 2000a). Resistance was an act of negation toward this power. Freedom too was thought of as a quality, or possession, of human agency. Postmodern thought about human existence contributed alternate ways to think about

[31] As mentioned above, I will utilize the unproblematized language of the sociologist through this section on *Influential Sociological Views*.

these concepts. Foucault (1976/1990; 1984/1997) wrote about power existing in relations. He suggested that power is always present in human relationships. Power relations are not fixed but are "unbalanced, heterogeneous, unstable, and tense" (Foucault, 1976/1990, p. 93). It is that these power relations are unfixed, mobile, and ever-changing that freedom lies on both sides of a relationship; Foucault rejects an oppressor/oppressed duality. Further, he rejects power as a negative quality. Instead power is productive; it is what produces reality. In the post-enlightenment, Foucault (1975/1995) theorized a new sort of disciplinary[32] power that has emerged and controls the present society. "Discipline blocks relations of power in that it objectifies and fixes people under its gaze and does not allow them to circulate in unpredictable ways (St. Pierre, 2000a, p. 491). Popkewitz (1998) extended this notion of a disciplinary society to consider the governance of the Popkewitz the context of schooling. He saw the invention of psychology as the transformation of Enlightenment beliefs into practical technologies that would assemble people's understanding of their experiential worlds, making possible for the state to govern the soul of the child. So while an intentional effort to bring forth the freedom of individuals, ideals cannot simply be inserted "into discourses of research and reform, they also produce systems of exclusion and inclusion" (p. 560). Critical educationalists draw on these new perspectives of power relations to speak of the political nature of mathematics education (Apple, 1999; Mellin-Olsen, 1987).

A mathematics curriculum, whether imposed at a state or local level, is derived from a selective tradition—particularly European. Political structuring, for the sociologist, is also visible in the structuring of schooling as an institution, contributing to the continuation of the teaching

[32] Read *discipline* here not as a subject of school study, but as training or enforcement to ensure the following of rules.

profession as a classed, gendered, and raced employment.[33] This historical shift to invoke social perspectives into the work of mathematics education, also allowed for a place for the political nature of mathematical learning, as evidenced by those who have co-engaged psychological (cognitive) perspectives along with sociological (cultural). The work of Vygotsky is one source that re-energized such research on learning. For example, recall my earlier connection to Mellin-Olsen's (1987) efforts to extend Vygotsky's activity theory to "embod[y] the individual and the society as a unity: the individual acts on her society at the same time she becomes socialized to it" (p. 33) in the context of thinking about equity. For me, this is a way to engage both the intermental and intramental considerations for learning proposed by Vygotsky (2000/1986). The social view also enriched the view of the individual, in powered relations with others. Instead of monitoring a subject's desire for freedom as a single locus of some great rebellion toward some end goal, resistance is thought of as "generally local, unpredictable, and constant" (St. Pierre, 2000a, p. 492). Bringing these social forces to bear into the immediacy of power relations, interaction and learning, and resistance in the context of disciplinary power makes possible to ways to think about interaction in a classroom and the potential for the mathematical learning of the child. While much can be learned about a child's mathematical ways of knowing in the tradition of Piaget's epistemic subject, the sociologist's perspective challenged the psychological (or epistemological) perspective to better account for environmental impacts on these ways of knowing. While the psychologist treats the environment as a construct of the knower, many sociologists treat the environment as a thing unto itself. These differing views do not expunge either's findings or issues, but challenge each other to more fully develop a "unified" theory.

[33] I believe it would be interesting work to develop a model for interaction between a teacher and student that brings to bear not only mathematical learning, but also a teacher's social class (for example) and the assumptions a child attributes to the teacher based on the child's constructions derived from previous interactions with adults to which they have attributed the same characteristics.

During this era that brought greater influence of sociological work to the education setting, equity studies began to extend beyond documentation of who was and who wasn't learning, into building theory that helped to understand these problematic outcomes. Power and privilege are recurring themes in this work. For example, Apple (1992) demonstrated that "schools are caught in a contradictory situation" (p. 421), where they strive for democratic education and work for a more knowledgeable population, yet operate in an economy where increasing that technical or administrative knowledge available for use is important, not necessarily that a large number of people have it. He recognized that the high status of mathematical knowledge works against equitable practices of teaching mathematics. Skovsmose and Nielsen (1996) refer to this conflation of goals, that teaching a powerful, globalized mathematics develops into a barrier for students to see the relevance of mathematics in relation to their lives and society, as a paradox of general education.

Sociology can go beyond attempting to understand forces inside and outside the classroom that create inequity among students. Sociological theory can also help design classroom interventions, curricular or pedagogical, to promote equitable classrooms.[34] In particular, sociology lends itself to theorizing and impacting central features of the classroom, such as the roles of teachers and students, and the patterns of interaction. For example, Cohen and Lotan (1997) used expectation states theory, an idea of sociology, to derive methods that help to modify inter-student expectations for competence based on academic and peer status.

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[34] Cohen & Lotan (1997) are sociologists that studied classrooms for over two decades in an effort to increase equitable practices and outcomes. Their work assumes that all learners have different and multiple abilities, and reacts to the research base demonstrating the inequitable outcomes of tracking. They use sociological principles, such as increased peer interaction leads to further learning, and status affects learning. Their work led to the development of a structure for pedagogy entitled Complex Instruction (CI). CI has three major components, multiple-ability curriculum designed to develop higher order thinking skills, instructional strategies to train students to use cooperative norms and classroom roles to manage their own group, and awareness and techniques to treat status problems. Boaler's (2004; 2006a; 2006b; 2008) current research on equitable teaching practices emerges from a data drawn from a high school mathematics classroom using Complex Instruction.

These researchers also applied organizational theory, another idea of sociology, to devise methods for the teacher to delegate authority and increase lateral communications, permitting students to use each other as resources and gain meaning from an assigned activity. The resulting pedagogical approach developed by these researchers, Complex Instruction, "enables teachers to teach at a high intellectual level in academically, linguistically, racially, ethnically, as well as socially heterogeneous classrooms" (p. 15). Chapter 10 of Cohen and Lotan's book focuses on what students learn in a middle school mathematics classroom in which complex instruction is used. In this chapter, Cohen et al. (1997) demonstrated a mathematics curriculum compatible with complex instruction's principles and documented student learning utilizing the QUASAR Cognitive Assessment Instrument. Modest improvements were shown in mathematical communication. More significantly, this study showed the challenge for mathematics teachers to enact a multiple-ability curriculum, in part because opening up for the possibility for novel student-generated solutions taxed teacher's mathematical background. While other disciplines consistently showed increased student learning, the single application to a mathematics classroom did not strongly suggest the efforts improved mathematical learning. However, other qualities of student relations and sense of selves showed marked improvement toward more just outcomes.

Complex Instruction is an orientation to pedagogical practices intended to be useful across grade levels and subject matter. The challenges that were documented in the case of its use in a middle school mathematics course sheds some light on its limitations, such as a problematizing the development of an appropriate curriculum, as well as wondering how to act in light of novel, unpredicted student responses. Furthermore, this work failed to push the prevailing practice of mathematics education to be anything beyond getting learners to come to know a mind-independent Mathematics. This work assumes the potential to implement an "intended curriculum" (Ohuche, 1990, p. 316) derived prior to a teacher's coming to know the student's of a classroom. The elaboration of a curriculum is necessarily a post hoc activity (Burton, 1990; Ohuche, 1990); a teacher is unable to determine in advance what a learner may construct, and seems oriented to construct next. Another challenge identified by Cohen and Lotan (1997) was that teacher's were ill prepared to listen and act upon children's "novel" ideas. This, coupled with the implied orientation toward curriculum, indicates a failure to orient their work toward the mathematical development of the child, as opposed to having an intention to deliver a prescribed set of knowledge. This failure to treat mathematical ways of knowing as constructions of a knower is a common problem in most sociological work, one that must be attended to as mathematics educators consider what can be learned and the influences these fields have.

Granted much sociological work has left the a priori treatment of Mathematics unquestioned, Cohen and Lotan (1997) and more recently Boaler (2004; 2006a; 2006b; 2008) have demonstrated that students have achieved as well as students taught in a conventional format as measured by tests of mathematics knowledge. And importantly, these students show healthier views of mathematics and themselves, as well as improved relationships with other students. Not only has sociological theory influenced the building of knowledge about mathematical ways of knowing within mathematics education, it has also contributed to methods for which to teach and interact with mathematical learners. The work of Cohen & Lotan (1997) is evidence of ways in which sociologists have contributed to the field of mathematics education. In particular, this pair of researchers devised a model for instruction with the explicit intent to attend to classroom tasks, teacher practices, and interpersonal relations in ways that increased equitable outcomes.

Sociological Influence Evident in Educational Psychology

From some perspectives, the current nature of research into equitable mathematics teaching practices is at an irreconcilable schism between what psychological and sociological views can answer (Lerman, 1996; Sfard, 1998; Valero, 2002). Many recognize the post-behavioristic learning models of Piaget and Vygotsky as the two most dominant influences on modern learning theory, superficially naming the former psychological (or cognitive) and the latter a more sociological (or cultural) perspective.

Each epistemologist, Piaget and Vygotsky, is tied to modern day learning theories under the varied guises of constructivisms. For Piaget, knowledge is modeled by cognitive structures he called schemes (Piaget, 2001), consisting of three elements: a perceived situation, an associated activity, and the result the activity is thought likely to occur. Such knowledges—schemes—are not pictures, words, or symbolic data structures that correspond to structures in the world. Instead, knowledge is fundamentally action-oriented, a "goal-directed phenomena" (Glasersfeld, 1998, p. 7). Every implementation of knowledge (i.e. a scheme) requires the acting subject to recognize a triggering situation (this recognition referred to as an *assimilation*) because no two situations in a subject's experience are ever quite the same. When a scheme is applied to a new situation and doesn't work, the scheme is modified to fit the environment better, a process named *accommodation*. Together, these activities of scheme recurrence and change are considered *equilibration*. Although equilibration is the most fundamental aspect of development, *reflecting abstraction* is responsible for the bigger leaps that take place, leading to constructive generalizations, operations on operations, knowledge about knowledge. In particular, it is what

Piaget (1983) names "the general constructive process of mathematics: it has served, for example, to construct algebra out of arithmetic, as a set of operations on operations" (p. 125). Reflecting abstraction is a key construct for considering how a learner may move from stage to stage—a theory Piaget is most commonly known for.

Piaget's stage theory provides a model for describing children's thinking at different levels of their cognitive development. As children progress ("advance") through the fixed sequence of sensori-motor, pre-operational, concrete, and finally formal operations, conceptual changes in children, in a manner parallel to theory changes in science, are constructed internally and require deep and fundamental restructuring at each stage. This is how Piaget thought about "spontaneous development", from which a model of learning might be abstracted. However, Piaget's stage theory has not sufficiently accounted for numerous aspects of learning, most significantly the constructive role of contexts and individual preferences in knowledge acquisition (Ackerman, 1991a).

As does Piaget, Vygotsky (2000/1986) strongly emphasized the role of active engagement for learning. However, while Piaget studied development independent of teaching, Vygotsky made teaching the domain of his research. Through careful observations of children's current knowledge, Vygotsky formulated productive ways in which adults could interact to provoke learning in the child. He formulated the notion of the Zone of Proximal Development (ZPD) as the site for learning new knowledge where children can no longer act on their own knowing but can progress with minor influence of more expert knowers. Vygotsky saw learning as an "outside in" phenomena,

> Every function in the child's cultural development appears twice: first, on the social level, and later on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formulation of

> concepts. All the higher functions originate as actual relations between human individuals. (Vygotsky, 1978, p. 57)

Instead of an outside-in process, Piaget insisted on a *genetic* epistemology in which knowledge is not preformed, but a constant, new, and cybernetic construction of the organism. “Knowing reality means constructing systems of transformations that correspond, more or less adequately, to reality” (Piaget, 1970, p. 13 [1st lecture]), what I would label an “inside out” theory for learning, because a newly formed scheme must respond to some interaction for it to become a knowledge structure.

I intend for this simplified orientation to demonstrate how Piaget and Vygotsky differ markedly on this positioning of knowledge. This difference is interwoven with a significant divergence in the purposes of education for the two learning theorists. Piaget states: “Education, for most people, means trying to lead the child to resemble the typical adult of his society… but for me and no one else, education means making creators…. You have to make inventors, innovators—not conformists” (quoted in Bringuier, 1980, p. 132). On the other hand, Vygotsky works from an education-as-socialization perspective, to come to attain, or at least approximate, adult ways of knowing.

However, in spite of these two differences, both theorists allow for the logic of the statement, “knowledge is socially constructed.” This is allowed in Vygotsky’s work by his attribution of knowledge’s existence first on the intermental plane. To make the statement sensible, “socially” might be interpreted to mean that knowledge has a sort of existence amid the relations or interactions of knowers, or that knowledge is constructed in the interaction of learner with knowledge’s existence amid the social (i.e. other people of the) environment. In Piaget’s view, the phrase brings forth the seemingly obvious notion that social interaction is a foundational medium to provoke abstraction. Piaget’s social construction would first be on the level of the learner experiencing a perturbation in their sensory milieu (the "social"), and then through reflective abstraction (Piaget, 2001) a scheme (i.e. knowledge) is constructed. It is worth noting that neither Vygotsky nor Piaget believed that the learner could construct some exact copy of this knowledge. It is the meaning attributed to the word "knowledge" in the phrase "knowledge is socially constructed" that distinguishes the perspectives. After further groundwork in this literature review and some exposition of my relations to theorist's ideas, I will return to this idea of what may be knowledge. It is here in which a fundamental principle of my work resides, both with respect to the study of generative adolescent mathematical learners and in the broader scope of my activity as a mathematics educator.

While constructivist ideas emerging from these two epistemologists pervade the modern-day discourse of learning and teaching mathematics, the premise that knowledge is socially constructed, read from either perspective, has yet to demonstrate few ripples anything greater than a trivial impact on mathematics education. The constructivist viewpoint has had its greatest impact by recognizing that children learn via activity (Larochelle, 2000). But the potential educational implications of this realization, such as increased attention on mathematical play or focus on problem solving, remain trivializations of the learning theory. Children learn in their disengagement as well; it is a matter of learning to place where the active mind of the child resides. This aspect of the incomplete attempts to actualize learning theory fails to recognize that children are always learning. The notion of a "hidden curriculum" (Jackson, 1990/1968) is a well-accepted part of the sociological viewpoint on learning.

Although trivial constructivisms (Confrey, 1990) may be "practiced" in the defining of guidelines for classroom instruction (recall Popkewitz' [1998] governing of the soul), the more radical shift in the enactment of mathematics education reflective of constructivist tenets has yet to emerge. As evidenced by contemporary mathematics teaching practices, a fallibilistic approach to knowledge, that knowledge is to be considered in interactions—including relations between people and the reflective cognitive processes of interacting schemes—has yet to be taken seriously. Contemporary epistemology and ontology questions the a priori status of knowledge, creating a more dynamic, active notion. Yet mathematics is still regarded in education as a static entity, to be filled into the minds of children, a la Freire's (1970/2002) banking model for learning. Our educational theorists (for example, Dewey, 1929) have long ago wrested from Plato the a priori nature of knowledge, yet our field has not yet learned to act on such an ontological repositioning. As noted earlier, Steffe (2004) documents the inconsistency with which the NCTM (2000) *Standards* treat children's ways of knowing as a binary with the Mathematics—a set of knowledge having an a priori existence to the knower—of the *Standards* documents. This different treatment of ontology, and resulting epistemology, demands mathematics education not only leave behind its enlightenment-era visions and agendas in order to teach a particular Mathematics, and but also reorient in order to consider the treatment of knowledge, and especially mathematical knowledge, in the postmodern.

The mathematics education stage is set for the consideration of a very different epistemology, one undergirded by a radical ontology—radical because it rejects the enlightenment thinking, "according to which all human knowledge ought or can approach a more or less 'true' representation of an independently existing, or ontological, reality" (Glasersfeld, 1998, in beginning to explain his radical constructivism as a postmodern epistemology). Mathematics education is ready to consider that:

> The essential lesson we can draw from contemporary epistemology is that the intelligibility of scientific knowledge cannot satisfy itself with aprioristic or empiricist conceptions that reduce this knowledge to empirical and methodological certitudes. Instead, this intelligibility requires a constructivist

> conception that recognizes the inevitability constructed and social character of this knowledge, in both its production and products. (Larochelle & Désautels, 2000, p. 376)

Radical constructivism having taken a postmodern orientation to knowledge, is also a rationalist epistemology (personal communication, Leslie P. Steffe, Nov. 11, 2005); one that has replaced *validity*—some sort of objective representation to an experiencer-independent truth—with a perspective toward internal consistency, or functional fit, utilizing the biological construct *viability* (Glasersfeld, 2001). The tenets of radical constructivism themselves expect each knower to construct a personal meaning for their world. I find a greater (intersubjective) agreement with Glasersfeld's statement that radical constructivism rejects an enlightenment[35] form of thinking, which I conceive to be an orientation that values reason and rationality, and in particular sees as its goal the coming to know of a mind-independent reality. For me, the engagement of radical constructivism as a science could rely on the rationality of internal consistency, or reason, as viability (in place of validity). It might be better said that such a science emphasizes the knower's internal consistency, NOT a consistency internal to or measured against some mind-independent object named Science. The ontological switch suggested by Glasersfeld should return us to a more powerful interpretation of Piaget's socially positioned views on learning—that interaction is a hard-core principle. It is not merely radical constructivism's recognition of the inability to know some ontological reality as a thing-in-itself, but that it takes further the slipperiness of such a statement—making possible the development of an epistemology that has explicitly freed itself from the impossible expectation to demonstrate the existence of such a reality. While Glasersfeld's radical constructivism does not deny reality (personal communication, Leslie P. Steffe, Nov. 11, 2005), as evidenced in constructs such as

[35] For clarification purposes, I use "enlightenment" to refer to the intellectual movement *The Enlightenment* in which reason was the basis of authority.

Glasersfeld's consideration of the affordances as well as the negative functions, i.e. the constraints, of the constructivist's reality, I wish to consider an epistemology that keeps under question a dependence on the ontological assumption of being.[36]

The Postmodern Influence on Knowledge Construction

The postmodern turn, alluded to in the Glasersfeld quote above, refers to the end of the modernist project—a humanist notion that moral, social, and ethical progress was achievable through the incorporation of principles of truth and hierarchy. Instead, progress is an obsolete notion in that constant change reflects the new status quo; "progress" no longer obscures the roles of chance, discontinuity, and the construction of knowledge and its social character, in naming an improved way of knowing. It is of little importance to identify a date when thinking took this turn, but more significant to consider the sort of discourse postmodernism makes possible. Flax (1990) characterized a mindset of the postmodern era when she wrote,

> Postmodern discourses are all deconstructive in that they seek to distance us from and make us skeptical about beliefs concerning truth, knowledge, power, the self, and language that are often taken for granted within and serve as legitimation for contemporary Western culture. (p. 41)

Postmodern discourses are, in many ways, forms of critique, critique that seeks to shuffle assumptions and pry in to that which we most often fail to consider. To debate the existence of the common Western ontological truth (i.e. "reality) is an ineffectual query. Instead, considering what other such potentials might make possible begins to expand this new discourse. As Flax suggested in this passage, such critique opens other doors—truth, knowledge, power, the self, language—leaving questions of *being* or *is* nonplussed in the communiqué of the postmodern.

What follows in this section of Chapter 2 is an intent to broadly sketch ideas that a postmodern framework make possible, following roughly Flax's (1990) characterization above.

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[36] When I write of ontology, I think of it as a way of thinking about being or existence. Thus, it has strong implications for conceptions of reality.

While I have alluded frequently to postmodernism already in the paper, I take a moment now to raise awareness of possibilities I invoke when I point toward a postmodern discourse. By its nature, postmodernism defies definition. By removing the notion of a mind-independent reality, there is not a *thing* called “postmodernism” that waits to be known.

Each knower has a sense to what the idea means for them; and furthermore they can communicate thoughts about the ideas attributed to it. Knowers will even interact as though they understand their uses of the word to mean an identical notion. But embracing the postmodern would remind them that they don’t share the same knowing of this thing—defying an ideal, neatly packaged definition. So to pretend to suggest that here I will say what postmodernism *is*, “is” as if postmodernism existed as some mind-independent notion, would prove I don’t know toward that which I speak. The orientation I offer here will continue to challenge, and hopefully further engage thinking about the generative adolescent mathematical learner as a spiral around the multiplicities present in this dissertation, flowing through the present chapter’s outward orientation toward the GAML, into the next chapter in which I more carefully explicate my theories and what they suggest for considering a definition for the GAML, what I call an inward orientation. In Chapter 3, I will extend the following discussion of postmodernism to more strongly give a flavor to my postmodernist ways of thinking and knowing.

The Step Away from Humanism

The critique embraced by postmodern discourse rejects the humanism characterized by a notion of Enlightenment, in which reason can serve to create an ideal and just world of relations. In the mid-20th century, Adorno and Horkheimer brought forth a more pessimistic concept of enlightenment, noting that while trying to abolish superstition and myth through reason, it ignored its own superstitions. The strive toward certainty of the theories of the Enlightenment worked in a totalitarian manner. Humanism is a similar ethical philosophy, believing that it is within the human capacity to determine what is right through reason. Woodhouse (1980) provided a simple definition for Humanism: "The view that nature and experience, rather than, say, God's will or social custom, ought to be the basis of our religious, moral, social, and political values and ideals." Woodhouse continues, "Humanism is often associated with self-realizationist philosophies and psychologies" (p. 118). St. Pierre (2000a) draws on Flax's (1990) characterizations of postmodern discourses to locate each within a humanist viewpoint: language is transparent; a stable, coherent self exists; reason and science provide an objective foundation for knowledge; reasoned knowledge is true; conflicts between truth, knowledge, and power are resolved through reason; and that freedom is obedience to laws or morals established as a result of reason. Postmodernist practices pry apart these commonly taken-for-granted assumptions.

It is not reason nor rationality that are rejected or cast aside in the postmodern, it is the belief in their potential to know a truth and the exalted status of one rationality over another that are put under examination. The radical constructivist language emphasizing that reasoned knowledge can only be viable, not true, helps to clarify the first of these points. The second serves to disrupt hierarchal ways of knowing by emphasizing that any cognizing agent's rationality is a functional fit for that agent to its experienced world—necessarily a world different from any other person's.

Although the terms *postmodernism* and *poststructuralism* are oft used interchangeably, I choose the former to refer to an era in time/space and the latter to a way of thinking. Henceforth, I will use the latter as I speak more toward my personal orientations, and toward knowledge in particular. Poststructuralism, for me, resounds with Peters' (1996) description: It is a philosophical response to the alleged scientific status of structuralism.[37] Using the methods of deconstruction, one can critique the knowledge claims of humanism. Poststructuralism is similar to structuralism in that both reject the philosophical and theoretical perspectives of empirical positivism, structuralism is embedded within humanism, while poststructuralism makes concerted efforts to remove itself from humanism—attempting "to pass beyond man and humanism" (Derrida, 1978, p. 292).

Another distinction is evident that while structuralism tends to disconnect the individual from social theory, poststructuralism inserts the discursively constituted subject into social theory (Foucault, 1969/1972). Poststructuralists do not search for constraining patterns or structures that provide meaning to human phenomena. Rather, within poststructuralism there is an "incredulity toward metanarratives" (Lyotard, 1979/1984, p. xxiv); emphasis is not on the presence, but the absence of meaning—that which escapes meaning (Derrida, 1974/1997). St. Pierre (2000a) wrote:

> Once we begin to shift our understanding and consider that language is not transparent, that the thing itself always escapes, that absence rather than presence and difference rather than identity produce the world, then the fault line of humanism's structure becomes apparent. At that point, we must begin to use language differently and ask different questions that might produce different possibilities for living. (p. 484)

The poetic license in place here is utilized to continue to work to pry looser a notion of language as a pointer toward a mind-independent knowledge; to recognize that the structures constructed in order to model our experienced reality from what we recognize as presence and identity[38] are terribly incomplete. Incomplete like the frame of a house is a presence that serves as a structure that can help to know the interior living and lived-in

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[37] Briefly, *structuralism* refers to various theories that derive from an assumption that there exists a structural relationship between principle elements of the topic of study.

[38] Identity as in identical to models we already possess.

space, yet the structure says so little about what this living may be. And incomplete in that we are in a way programmed to know/recognize (assimilate) the structure as we can recognize it within what we already know, but could not know it in all the ways that others who may experience the house might know it. These referents to absence and difference explode the notion of knowing beyond the humanist goal to lock into place knowledge, knowledge of a mind-independent reality.

Simply stated, poststructuralism offers a different theoretical perspective as well as new language that allows for reconsideration of concepts found in humanism, such as subject, agency, identify, truth, power, resistance, and knowledge.[39] The very concept of epistemology—a philosophical theory of knowledge—is itself problematic; it is "enmeshed in a [humanist] metaphysics that seeks to rise above the level of human activity" (p. 499).

Deconstruction

It must be emphasized that poststructuralism is much less a rejection of humanism or structuralism, but more a response to, or quite possibly, what each has made possible. While humanism is quite distinctly a knowledge project, poststructuralism functions much more as a thinking project (Butler, 1995). St. Pierre (2000a) draws on Foucault to remind us that:

> while humanism is everywhere, overwhelming in its totality… humanism is not an error and therefore 'we must not conclude that everything that has ever been linked with humanism is to be rejected, but that the humanistic thematics is in itself too supple, too diverse, too inconsistent to serve as an axis for reflection.'" (p. 478)

St. Pierre and Willow (2000) offer a similar caveat that a discussion around poststructuralism is not intended to establish an oppositional binary between poststructuralism and humanism, privileging poststructuralism. They argue, however, that such a practice is difficult to avoid

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[39] I offer more thoughts on each of these concepts later in the text, in particular Chapter 3, as I unfold an orientation to knowledge that underlies the emphasis of this paper—the fabrication of knowledge.

given that "we are always speaking within the language of humanism, our mother tongue, a discourse that spawns structure after structure after structure—binaries, categories, hierarchies, and other grids of regularity that are not only linguistic but also very material" (p. 4). St. Pierre and Willow further clarify, writing:

> Poststructuralism, then, does not assume that humanism is an error that must be replaced—i.e., humanism is evil because it has gotten us into this fix; poststructuralism is good since it will save us. It does not offer an alternative, successor regime of truth, it does not claim to have "gotten it right," nor does it believe that such an emancipatory outcome is possible or even desirable. Rather, it offers critiques and methods for examining the functions and effects of any structure or grid of regularity that we put into place, including those poststructuralism itself might create. (p. 6)

Those who use poststructural analyses find within humanism points of exit from its dominance. Yet the reflective, critical positioning of poststructuralism demands a return to the critique and to critique again. "It is important to understand, however, that poststructuralism cannot escape humanism, since, as a response to humanism, it must always be implicated in the problematic it addresses" (St. Pierre, 2000a, p. 479). This is the obligatory and most challenging demand of the poststructural critique—for its user to return to ask, "what is poststructuralism protecting them from?" (p. 478). This is a double move of both doing and troubling the work, simultaneously.

Language problems. The concept of discourse includes language, complex signs, and practices that order and sustain particular forms of social existence (Leistyna et al., 1996). Foucault (1969/1972) claimed that discourses are "practices that systematically form the objects of which they speak" (p. 49); consequently, "one remains within the dimension of discourse" (p. 76). He identified discourse as both an effect of power and as a point of resistance:

> [W]e must conceive discourse as a series of discontinuous segments whose tactical function is neither uniform nor stable... as a multiplicity of discursive elements that can come into play in various strategies.... We must make allowance for the complex and unstable process whereby discourse can be both an instrument and effect of power, but also a hindrance, a stumbling block, a point of resistance and a starting point for an opposing strategy. Discourse transmits and

> produces power; it reinforces it, but also undermines and exposes it, renders it fragile and makes it possible to thwart it. (Foucault, 1976/1990, pp. 100–101)

Although discourse may structure knowledge, a lack of uniformity and stability make possible resistance, occasioning the development of different discourses. This poststructural attitude toward discourse allows for the understanding of "how knowledge, truth, and subjects are produced in language and cultural practice as well as how they might be reconfigured" (St. Pierre, 2000a, p. 486).

The binary. The natural work of humanism is to account for experience through binaries, existing in a state of one of two mutually exclusive conditions. Poststructuralism provides a means for analyzing these oppositions through Derrida's (1974/1997) deconstruction of language and social practices. Such an analysis recognizes that the first term of the binary, the privileged term, is dependent on the exclusion of the second term. Rather than the first term being more important, this deconstruction demonstrates that primacy of the second, the subordinate. Deconstruction works to reverse, not merely neutralize the violence, the control, the superiority of the binary. The first move in deconstruction is to overthrow this hierarchy

The postmodern deconstruction is an effort to critique the "metaphysical and rhetorical structures which are at work, not in order to reject or discard them, but to reinscribe them in another way" (Spivak, 1974, p. xvii). Spivak characterized deconstruction as work

> to locate the promising marginal text, to disclose the undecidable moment, to pry it loose with the positive lever of the signifier; to reverse the resident hierarchy, only to displace it; to dismantle in order to reconstitute what is always already inscribed. (p. lxxvii)

Deconstruction, as a poststructural tool for thinking differently, "is not about tearing down but about rebuilding... looking at how a structure has been constructed, what holds it together, and what it produces" (St. Pierre, 2000a, p. 482). Deconstruction foregrounds that the world has been constructed through language and cultural practices; consequently, it can be deconstructed and reconstructed again and again (St. Pierre, 2000a).

The Postmodern Subject

Poststructuralism re-characterizes the person as a subject rather than an individual, rejecting the incomplete humanist notion of an essential unified self that fails to embrace the role of social structures on the person. Instead, the subject is at once both subject in and subjected to her experiential world, a subject that is subjugated, but not determined, by social structures and discourses. This is Foucault's (1969/1972) "discursive formation." Lather (1991) calls it a territory "in which structure and agency are not either-or but both-and and, simultaneously, neither-nor" (p. 154).

This reconstitution of the individual as subject seems a threat to the agency of the person, that one is shaped by the world instead of the shaper of a world. Within such a framework is immediately evident a binary, a humanist conceptualization positioning agency as an inherent attribute of humans (St. Pierre, 2000a), an agency in which through the rational intellect, the will of man can be achieved. Poststructuralism re-theorizes the agency of the subject as having both enabling and restricting effects, in particular on the production of knowledge and possibilities of action. St. Pierre explained that the subject exhibits agency in its take up of the available discourses and cultural practices, while at the same time being subjected to those same discourses and practices.[40] Agency is "up for grabs, continually reconfigured and renamed as is the subject itself, [seeming] to lie in the subject's ability to decode and recode its identity within discursive formations and cultural practices" (p. 504). The discursively constituted subject is not a molded subject, but rather "has been opened up to the possibility of continual reconstruction

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[40] I read into this both/and discussion of agency the need to keep in mind both the subject's constructions of discourses and cultural practices as well as their constructions of the ways in which those discourses or practices may have impact on them.

and reconfiguration" (p. 502). The potential to reconstruct oneself is the prospect of this new agency.

Agency theorized in this manner does provide a certain, again newly configured, freedom, to constitute oneself in an unexpected manner, to decode and recode one's identity. Butler (1990) identified this action as "subversive repetition" (p. 32). The agency of the subject makes possible the repetition of oneself in alternate manners because the discourses themselves are open to intervention and resignification. Derrida (1978) argued that discourse is open for the "movement of play" (p. 289), defining play as the "disruption of presence" (p. 292). This movement of play provides a freedom for the discursively constituted subject, a freedom that makes possible subversive repetition. Freedom then may be reinterpreted as play, a generative state because "subversive repetition resignifies the discourse and a resignified discourse allows a different repetition, and so on" (Stinson, 2004, p. 57).

Knowledge Production

Foucault (1975/1995) argued that it is impossible to separate knowledge from power. He writes, "power and knowledge directly imply one another; there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations" (p. 27). So any sort of contemporary epistemology is more useful by focusing "more on the process of the production of scientific knowledge, instead of uniquely on its products" (Larochelle & Désautels, 1991, p. 376). To conclude with knowledge production is insufficient; a postmodern epistemology should examine and theorize the discursive production of knowledges: What made this production possible? What are its effects?

Such an epistemology causes severe problems in the modernist practices of science. If it is logically impossible to compare what is known to a reality independent of the knower that such knowledge is claimed to represent, how can we assess the accuracy of this supposed correspondence or assure the truth of what we believe to be known? Validity of knowing in the postmodern must be reconsidered (Glasersfeld 2001; 1995; Lather, 1993; Scheurich, 1996; 1985) and must be "in radical opposition to traditional points of view" (Larochelle & Désautels, 1991, p. 377). If power and knowledge are inseparable, it cannot be left unasked, whose truth? If there no longer is a sort of direct access to some postulated reality, the validity of knowledge is left as a comparison of an activity of construction to another activity of construction, mediated in this milieu of power relations. Glasersfeld (1985) argued that the knowing subject (the observer, the researcher) is engaged at all levels in the fabrication of the object to be known, the establishment of relations, the construction of conceptual structures designed to model systems of the perceived world:

> From that point of view then, whatever complexity we are facing is of our own making, for it can arise only from the relation between the goals we have chosen and the ways and means we construct for getting there. (p. 8)

Glasersfeld's statement must be taken to understand the postmodern, co-constructed, discursive subject; a concept that will be returned to as I further bring together ideas about the generative adolescent mathematical learner.

Power Relations, Status, Resistance

Next I return to ideas of the subject and of knowledge while readdressing Foucault's conjecture that power and knowledge are inseparable. St. Pierre (2000a) noted within the humanist tradition, power, as is agency, is perceived to be an attribute possessed by all individuals; consequently, it can be deployed, shared, or taken away. She suggested that power in this context is seen as inherently evil or as a negative, oppressive force; therefore, those that are concerned with social justice often “give” power away to avoid domination as they attempt to “empower” others who are less fortunate. A poststructural power folds together this sort of good/evil binary, becoming both an oppressive and liberating force, “found in the effects of liberty as well as in the effects of domination” (St. Pierre, 2000a, p. 491). Foucault (1975/1995) stated:

> We must cease once and for all to describe the effects of power in negative terms: it ‘excludes’, it ‘represses’, it ‘censors’, it ‘abstracts’, it ‘masks’, it ‘conceals’. In fact, power produces; it produces reality; it produces domains of objects and rituals of truth. (p. 194)

Power is reinscribed as a dynamic and productive event that exists in relations, rather than an object to be shared. Instead of treating power in an objective manner, Foucault (1976/1990) spoke of “relations of power” or “power relations” (p. 94). He identified four facets of power relations. Power relations were to be understood as a multiplicity that operate and constitute their own organization; as a process of struggles and confrontations that transform, strengthen, or reverse the relations; as the points of support or resistance of a system; and lastly, as the strategies that design and maintain social structures and discourses. Status, configured as a power relation, is always in flux. Always moving and always movable.

If the relations of power were considered in a push/pull metaphor, the relational powers of either position would be identifiable. However, to yield to the push or succumb to the pull is too an enactment of power, thus the metaphor is incomplete. Foucault (1976/1990) corroborates the error of this metaphor, “Power is everywhere; not because it embraces everything, but because it comes from everywhere” (p. 93). “Where there is power, there is resistance” (p. 95). He believed that power relations concur with a “multiplicity of points of resistance,” arguing that these points

> play the role of adversary, target, support, or handle in power relations. These points of resistance are present everywhere in the power network. Hence there is

> no single locus of great Refusal, no soul of revolt, source of all rebellions, or pure law of the revolutionary. Instead there is a plurality of resistances, each of them a special case: resistances that are possible, necessary, improbable; others that are spontaneous, savage, solitary, concerted, rampant, or violent; still others that are quick to compromise, interested, or sacrificial; by definition they can only exist in the strategic field of power relations. (pp. 95–96)

It is not for Foucault to deny great ruptures, revolutions, but to demonstrate that resistances are also multiple, decentered, and local. The relations, status, resistance, of the subject, and that to which she is subject, cannot be understood as static, or free from power. The postmodern rethinking of concepts such as the subject, agency, and power-knowledge, are implicated in the construction and deconstruction of the mathematical learner, calling for a postmodern epistemology.

A Postmodern Epistemology?

A postmodern science not only has shrugged aside the realist's notion of existence of knowledge-to-be-known, but also works to take seriously the effort of reflexive practice, to deconstruct constructions, to observe and theorize power at play in knowledge construction, and to create multiple ways of knowing the discursively constructed subject. As a reaction to a humanist agenda, poststructuralism seeks to make it uncomfortable. "Humanism is in the air we breathe, the language we speak, the shape of the homes we live in, the relations we are able to have with others…. Humanism is everywhere, overwhelming in its totality" (St. Pierre, 2000a, p. 478). To consider a postmodern epistemology, one must take seriously the reworking of a notion of the subject "ahead of any epistemology" (p. 505). Such a consideration of the subject-epistemology co-construction would necessarily be generative. That is, the generative adolescent mathematical learner (GAML) must be conceived in conjunction with a generative epistemology.

The potential of a poststructural epistemology for the deconstruction and reconstruction of the subject lays forth fertile ground to reconsider the GAML, as it has for poststructural feminists doing other work in education (Britzman, 1994; Collins, 2000; Lather, 2000; Walshaw, 2001, Zevenbergen, 2001). Such an epistemology, would reinscribe the subject, in the case of this dissertation—the generative adolescent mathematical learner. And the nature of knowledge in such a poststructural epistemology would require reexamination of the notion of mathematics. This postmodern epistemology upends sedimented thinking about the adolescent mathematics student—about who they *are*, the point of study in this dissertation. This chapter has placed the problem posed in each word represented by letters of the acronym GAML within the histories of literature current to the fields of mathematics education and educational research. The next chapter will provide the reader with insight into my ways of thinking about each of these ideas G, M, and L; the goals of mathematics education; and the purposes of research.

CHAPTER 3

THEORETICAL FRAMEWORK

In the first two chapters, I established the focus of this dissertation research to be on consideration of the student that may be called a generative adolescent mathematical learner (GAML). In the first chapter, I oriented the reader to the problem, this concept of the GAML, and demonstrated that while this work considers a seemingly singular, namable area of focus—the GAML, the work at hand should more reflectively be considered to be a multiplicity of ideas, each at play when considering the high school mathematics student. It is a dissertation about social justice. It is a dissertation about the impossibility of research and a humanist science. It is a dissertation about the fabrication of knowledge. And it may be more, an assemblage—artifacts of thought and data swirling amidst one another, not in orbit as though hovering tightly to some central notion, but like fireflies viewed from a Georgia porch on a humid, noisy eve. These ideas dance, jerkily, than disappear only to reappear in another location, seemingly new but the same.

The second chapter explored what others may have to tell us about this generative adolescent mathematical learner through their research and theorizing. In Chapter 2, I began to explore more closely each term of this name, wondering what place it has amid current educational research. How do mathematics educators conceive of generative? mathematics? learning, or the learner? In this chapter, I emphasized how the social turn (Lerman, 2000) in mathematics education has made possible both new insights and new questions into the ever present dilemmas posed by a socially just educational effort. Along with the social turn in the mid-level theories, educational research on the whole is now being expanded (maybe exploded) by the burgeoning role of postmodern macro-theory, the embracement and "putting to work" of ontologies and epistemologies that begin to give us ears to hear storylines our humanist science blinds us to.

This next chapter, Chapter 3, is intended to allow for me to examine and explicate my mid-level and macro-level theory, and the ways in which these both forge my notions of the GAML and approach the study of the GAML. To begin, I step away ever so slightly from the field of mathematics education itself and position my work theoretically, beginning with a more nuanced statement of my epistemological and ontological orientations toward mathematical knowledge and learning. I will bend and stretch radical constructivism in ways that seem to better match my personal epistemology. In doing so, I will demonstrate a consistency with this theoretical lens—through which I attempt to consider mathematical learning—to consider myself as a researcher, thus learner, of children doing mathematics. This will lead to the next section of the chapter in which this epistemology is connected to the larger, macro-theory of poststructralism—in particular, a critical poststructuralism. In this portion of the chapter I attempt to situate myself, and hence this study, in a research framework that will allow me to both seek out what I have named and create a name for what I have found. Poststructuralism allows me to not privilege either the signified or the signifier (Walkerdine, 1988). It will also allow my epistemological position that names all learners as generative; yet not contradict my selection of the particular generative students chosen for my research. My research is to broaden and challenge my ways of thinking about the generative adolescent mathematical learner, and through these seeming contradictions I will be stimulated.

Penultimately, I return to my considerations of the generative adolescent mathematical learner with a return to the notions of agency, identity, authority; I will seek to clarify my poststructural subjected subject. This section will locate these concepts in the field of mathematics education, but rely strongly on the epistemology and theoretical framework identified above. I conclude by re-establishing that this study as work for social justice. To not ignore the implicit axiology of the study, I will present my definition for equity and in so doing demonstrate that work for the maintenance, and preferably nurturing, of the generative disposition in children is an important component of a mathematics education for social justice.

Epistemology / Learning Theory

Constructivism, as a theory for learning (or way of knowing) and its implied suggestion for pedagogical strategies, has certainly been adopted as the basic and common tenet for contemporary thinking about how students learn within the mathematics education community. Cognitive and social epistemologies, as well as neuroscience research, suggest and value the learner as an active constructor of her experiential world. Each perspective calls for an engaged, active learner (Kilpatrick, Swafford, Findell, & National Research Council [U.S.] Mathematics Learning Study Committee, 2001). This learner "must now be conceptualized as a *producer*, and not simply as a reproducer of phenomena" (Larochelle & Désautels, 1991, p. 375).

Radical Constructivism

Radical constructivism in particular is a highly matured theory (in a Popperian sense) for a way of knowing. It too posits the learner as an active agent, constructing a world of her own making by means of assimilation and adaptation of perceptions, an experiential reality. Piaget (1937) states "Intelligence organizes the world by organizing itself." But it takes seriously a radical ontological stand, that whether or not a world exists prior to our knowing it, is an unanswerable (and uninteresting) question (Glasersfeld, 1990a). This radical ontological stand is in direct opposition to a naïve realism in which knowledge is treated as a perfect reflection of reality. Such an opposition, *although rarely engaged practically*, "puts into question the largely inherited and dominant representation of the cognitive status of knowledge" (Larochelle & Désautels, 1991, p. 376).

Taking the position that there is not an ontological existence that can be known, "that we only have access to what *we designate as reality* [my italics] by the means of the representations we construct about it" (Larochelle & Désautels, 1991, p. 376), epistemology and in particular knowledge, must be thought anew. My beliefs about knowledge and learning resonate with Ernst von Glasersfeld's (1995) radical constructivism. His is a theory of knowing based on two central principles. First, knowledge is viewed as actively built up by the cognizing subject rather than passively received. In other words, people are self-organizing systems—cognition serves to make sense of one's experiential world. Furthermore, I take as a given that a person's cognitive structures are rational and internally consistent. Although one may experience what she perceives to be inconsistencies or contradictions in another person's cognitive structures, these remain constructions of the observer; "Anything said is said by an observer" (Maturana & Varela, 1980, p. 8). This assumption opens the way for one to regard others as actively engaged in the construction of knowing themselves and their environment. Further, it explodes power relations insisting one's own interpretations must be thought to be no better than the constructions of the other. By imbuing the other to possess a competent mind independent of ones own, a set of experiences as rich as ones own, and the potentials for assimilation and accommodations equal to ones own, differing and equally rational ways of knowing must be created. None of these ways of knowings could be construed of as "better;" each serves the knower with equivalent viability in their experiential reality.

Radical Constructivism's second principle is that the function of cognition is adaptive—it serves the organization of the person's experiential world, not the discovery of ontological reality. In other words, a person's knowing is not an approximation of some pre-existing, singular reality—an approximation that can become "better" by becoming a closer representation of that reality. Instead, a person's knowing serves the organization of her experience, an organization that is ongoing and dynamic. In this organization, a person's knowing can become more viable, i.e., "better," in the sense that it can be modified, in the context of interaction, toward not only working for all previous experiences, but also to fit the new as well. Viability is, essentially, the radical constructivist's notion of truth. It involves how a person's way of knowing fits with "the domain of experience" (Glasersfeld, 1995, p. 14). This assumption encourages valuing each person's experiential world as serving that person. A second order of viability is worth considering as well; it refers to that sort of knowing which one considers to be viable in ones own ways of knowing, but also seems to be so for another person as well. "This bestows a second order of viability to the knowledge and the reasoning we assumed the other to have and act on" (Glasersfeld, 1995, p. 120). Although this second order of viability draws us to believe we, along with others, might be coming to share knowledge, radical constructivism emphasizes both the acknowledgement and acceptance of many different experiential worlds, since no experiential world can be known to be *the* correct one. Thus, knowledge is a human construction, *possessing no existence of its own*, always already in flux.

This theory of knowledge takes as a fundamental assumption that learners are self-organizing systems, not instructable ones (Freire, 1970/2002; Glasersfeld, 1995; Maturana &

Varela, 1980; Steffe, 1996). In other words mathematics students do not passively take in or absorb information from their environment, nor would their understanding mirror another's. Instead, students are actively engaged in building meaning for their experiential worlds through ongoing interaction. This meaning making possesses a stability, or a consistency, with the previous constructions of the learner; accounted for by the radical constructivist's notion of viability. Although the principle and term "construct" appears highly structuralist, emerging from a Piagetian epistemology often framed as "structuralism" (Piaget, 1970), it is not intended to define an existence of a literal "constructed" internal knowledge structures—apply the second principle of radical constructivism, that cognition is adaptive, serving the organization of the person's experiential world. Furthermore, radical constructivism recognizes the injustice[41] of attributing structures to people, recognizing that such a model of the other is merely that—a model, not a template for their being. In these ways, radical constructivism embraces a postmodern epistemology.

However, some instances and instantiations of radical constructivism fail to directly and intentionally challenge the hierarchal, centralized, and distanced ways of knowing (Ackermann, 1991b) that the naïve realist (or modernist scientific) position presupposes. Models, as structures, as presence and identity, are overvalued. The space within the structure, the absence and difference (St. Pierre, 2000a), the space for play, begs to be attended to, toyed with, and used to deconstruct the structure. Papert (1991a, 1991b), a student of Piaget, moves on this trouble in his development of a theory for learning named *constructionism.* The following section will elaborate Papert's constructionism.

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[41] It is an injustice to treat people with the prejudice of assigning ways of knowing to another, and making assumptions about how that person may act—even it is with that person's best interest in mind. Is this something we as constructors of our world can avoid? Possibly not, but for me a worthwhile goal.

Constructionism

Piaget's stage theory provides a model for describing children's thinking at different levels of their cognitive development. As children progress ("advance") through the fixed sequence of sensori-motor, pre-operational, concrete, and formal operations, conceptual changes in children, in a manner parallel to theory changes in science (Lakatos, 1970), are constructed internally and require deep and fundamental restructuring at each stage. Glasersfeld's constructivism engages a significant complement to this stage theory, namely scheme theory. A brief characterization of this way to consider knowing is to think of structures (schemes) that allow for certain actions to lead to a desirable result—this is how we know our experiential world. And intelligence is thought to *be* adaptation, "the ability to maintain a balance between stability and change, closure and openness, continuity and diversity, or, in Piaget's words, between assimilation and accommodation" (Ackermann, 1991b, p. 272). Assimilation involves the interpretation of events perceived from this experiential reality in terms of existing schemes; assimilation can be profitably thought of as recognition. However, when the scheme does not produce the expected or a desirable result, an abstraction might occur, in which the scheme itself changes. Accommodation refers to a change in a scheme to make sense of the environment. In Piaget's work, "he emphasized the importance of rules and invariants as means of interpreting and organizing the world, and he presented abstract and formal thinking as the most powerful ways to handle complex situations" (p. 272). Piaget's structuralist theory of cognitive development and knowledge, his epistemology, emphasized assimilation, possibly in conjunction with his developing stage theory, a more psychological emphasis.

Other theorists focused on differences instead of commonalities. For example, Lave (1988) viewed knowledge as "situated", living and growing in context. Orientations such as hers encouraged researcher's to look at singularities in people's ways of thinking, and to watch how interactions with specific situations evolve over time (Ackermann, 1991b). Consistent with such an approach, Papert among others worked to consider the development of individual learning styles, rather than general stages of development. They questioned the prevalent view that formal thinking is necessarily the most mature form of intellectual development (e.g. Gilligan, 1982; Turkle, 1984). In their work, they demonstrated that different individuals develop different ways of thinking in given situations, but each way is evidence of powerful ways of operating (Ackermann, 1991b). While these findings do not clash with Piaget's scheme theory, they suggest an orientation toward more carefully considering the process of accommodation, rather than assimilation. Here is where Papert (1980) found interest in understanding intelligence (adaptation); rather than attending to the powerful modes of operating made possible by internal organization, he focused on intelligence as being situated, connected, and sensitive to variations in the environment.

Papert (1991a) suggested a more nuanced, but fundamentally deconstructive criticism of Piaget's stage theory, possibly demanding an epistemological paradigm shift—a sort of perestroika.[42] Although Piaget made concerted efforts to build a theory that considered knowledge to be more powerful (i.e. more abstract) when new ways of knowing seemed to make possible a greater realm of possibility, he was confined to making this judgment within his own means of knowing. Even if he allowed himself room to learn alongside the subject of study, Piaget's newly formed knowledge is again a construction of his own mind and could not possibly match that of the child. Again, what is deemed to be more (or not more) abstract is locked within the judgmental capacity and point of view of the observer; the observer's science. Papert and his

[42] Although the term perestroika is most commonly associated with the restructuring of the Russian economy under Gorbachev, Turkle and Papert (1992) use the term more loosely, as a synonym to *restructuring*.

colleagues reinsert and revalue the "concrete" in the dialogue on learning, building upon Piaget's focus on generalization to consider other qualities of intelligence and knowledge development. Their efforts to disrupt epistemologies that may be described as hierarchical-centralized-distanced and create heterarchal-decentralized-personal conceptions of knowledge that "reflects both the political/social and epistemological confrontations in the battle between curriculum-centered, teacher-driven forms of instruction, and student-centered developmental approaches to intellectual growth" (p. 15).

As a part of his emphasis, Papert suggested equal access to even the most basic elements of a child's computation requires accepting the validity of multiple ways of knowing and thinking. From this core assertion, Papert developed a pluralistic epistemology – one that "challenges the hegemony of the abstract, formal, and logical as the privileged canon in scientific thought" (Turkle & Papert, 1992, p. 161). The first challenge against the hegemony of the abstract emerged from feminist scholarship, in which the abstract and rule driven canon of scientific thought is associated with power and elitism and the social identification of science and objectivity as male. Social scientists provide the second challenge by having demonstrated a great deal of powerful and mathematical thinking that works outside the formal school mathematics. There is convergence within these first two challenges to revalue the concrete. The third challenge is presented by the synchronous coming of age of personal computer technology. The computer is often seen as the embodiment of the abstract and the formal. More significantly, however, is that computers provide a context for the development of concrete thinking.

Revaluing the concrete. What is thought to be *concrete* is often understood as tangible, real. Or, instead of focusing on the object but rather the manner of thinking about the object, the more we are able to visualize (or sensorize) an object from a description, the more concrete it is.

For example, describing my pen as a "Papermate ballpoint pen" is more concrete than just the description "pen". In this sense, the more general is equivalent to the more abstract. "Writing utensil" and "communications tool" ascend in levels of abstraction. Simply put, this *standard view* of the concrete follows the logic, "the fewer the number of objects in the world that fall into the description, the more concrete."

From this perspective, it makes sense to want learners to move away from the confining world of the concrete to a level where what they learn can be applied more widely and generally. However, we see that attempts to teach abstractly leave students bored and with brittle, non-useful knowledge (NCTM, 2000). Digging into this idea about what may be concrete reveals serious flaws in the reasoning. For the Southerner, snow may be considered a rather concrete notion—it is the cold, slippery stuff that provides an excellent excuse to stay home from work for 2–3 days. However, for the Eskimo, the word "snow" connotes a large category of many types of snow, each with particular sensory categories. Snow is a vast generalization, an abstract concept for some people. Here, we are reminded of the faulty assumption that each person's experiential reality is identical. Since objects are not given to the senses, they are constructed, it is futile to search for concreteness in the object – it is more reasonable to look within the "person's construction of the object, at the relationship between the person and the object" (Wilensky, 1991, p. 198). Having expanded concreteness to become a property of a person's relationship to an object, concreteness is not a property of snow in and of itself,[43] but of the relationship the knower has with snow. Rather than the standard view of concrete, which is characterized by fewer objects, Wilensky furthers this idea about concreteness by suggesting that the more connections we make between an object and other objects, the more concrete it

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[43] Notice how Constructionism also takes on the postmodern orientation toward the knowing of an object, an object of the experiential world of the knower.

becomes for us. Thus, concreteness refers to the richness of our representations, interactions, and connections with the object—the *quality of our relationship* with the object.

In this new conception of concrete, it can be seen that concreteness is not to be found through an intensive examination of the object, but instead through understanding the modes of interaction a person uses to understand the object. In this way, objects that are not mediated by the senses, objects often considered to be abstract—such as mathematical concepts—can be concrete provided the learner has multiple styles and occurrences of rich engagement with them (Wilensky, 1991). As Minsky (1986) said in *The Society of Mind*:

> The secret of what anything means to us depends on how we've connected it to all the other things we know. That's why it's almost always wrong to seek the "real meaning" of anything. A thing with just one meaning scarcely has any meaning at all. (p. 64)

As a consequence of this view of the concrete, it is evident that rich, interconnected ways of knowing are to be valued. Upon a new awareness, "our nascent understanding of a concept… is often abstract, because we haven't yet constructed the connections that will concretize it" (Wilensky, 1991, p. 201). In this manner, knowledge development for the Constructionist begins with an introduction of an abstract[44] or formal object. A learner's relationship to this object develops through interaction, becoming more intimate and concrete. It can be seen that advancement of knowledge, in its development of a web of interrelations, actually moves from the abstract to the concrete.[45] Wilensky argues, taking a practical stand on people's perceptions of their own knowing, states,

[44] As a point of clarification, while clarifying the Constructionist perspective, I will continue to utilize the language of the constructionist, "abstract" in this instance. In my ideas about ways of knowing, it may be more useful to consider this "nascent understanding" to be initially a more isolated or ambiguous way of knowing, rather than abstract.

[45] I encourage the reader to pause at this sentence. Reflect on the idea of "knowledge;" it is always a construction of a knower, abstracted (in a Piagetian, not Papertian sense) from experience, rather than a thing for the cognizing subject to come to know—suggested in the choice of the word "object." The object of this passage should be conceived to be the knowledge of the constructed object, the scheme.

> The reason we mistakenly believed we were moving from the concrete to the abstract is that the more advanced objects of knowledge (e.g., permutations, probabilities) that children gain in the formal operations stage are not concretized by most adults. Since the concepts/operations are not concretized by most of us, they remain abstract and thus it seems as if the most advanced knowledge we have is abstract." (p. 201)

So long as we perceive more "advanced" objects of knowledge to remain "abstract," we are fooled into believing the process of knowledge development moves from the concrete to the abstract.

The constructionist elucidation of an orientation to knowledge as undergoing a process of concretization, from an isolated or ambiguous initial way of knowing to a more interconnected, personalized knowledge is a valuable heuristic for valuing the need for the knower to interact, and to appreciate the context of and relationships among the interactions. However, the constructionists have not provided a model for how this concretization occurs, in the way the constructivists have developed scheme theory to account for the equilibration of new experiences; coordination of the elements of the manifold.

Overcoming the centralized mindset. In most all aspects of life, ideas of decentralization are becoming a more useful way of thinking rather than applying or enacting a centralized metaphor—including school-based management systems, economic analysis, and brain science. As an example, profitable airlines such as Southwest have abandoned a hub-and-spoke model to adopt a less centrally organized structure, a web-like network of interconnections, in which resources must be more greatly distributed rather than centralized at a small number of hubs. Deleuze and Guattari (1987) suggested the rhizome—a continuously growing root-like plant that puts out lateral adventitious shoots at random intervals—as a metaphor for this decentralized experiencing of living. Decentralized systems typically consist of a multitude of components, each of which interacts in ways that behavior emerges from the system. Making sense of these behaviors can be a challenging task; it often appears quite different than any of the individual components. Resnick (1991) suggested that, although people observe and are likely to participate in decentralized systems, their experiences with such systems are largely passive. It is rare for one to *design* decentralized systems. Without actively engaging, it is unlikely to know some object of the experienced world well. As a result, "people tend to develop a *centralized mindset*" (p. 207). When people observe (sensorize) a phenomenon, they tend to assume—or maybe better said, construct— a centralized control where no centralized structure exists.

Levi-Strauss (1966) introduced the idea of the bricoleur scientist to contrast the centralized methodology of Western science and describe a science of the concrete. The bricoleur scientist does not move abstractly and hierarchically from axiom to theorem to corollary, but instead arranges and rearranges, negotiates and renegotiates with well-known materials. In similar ways, Belenky, Clinchy, Goldberger, and Tarule (1986) identified many adult women making moral decisions by balancing and assessing consequences of actions. Instead of using universal principles, they consider concrete situations. Their work served to disrupt the hierarchical stages of Kohlberg's theory of moral development. These two examples demonstrate practices, intentional or observed, of the decentralized mindset.

Closeness to the object. In the tradition of a humanist science, the essence of objectivity arises in creating a distanced relationship with the object of study. Feminist scholars have argued that this notion parallels the cultural identification of what is male. McClintock (1983), a Nobel Prize winner, demonstrated that her scientific work came through a deep, proximal, conversational relationship with her materials—even as they were neurospora chromosomes (so small that others had been unable to identify them). This evidence of what may be called a "soft" approach to science demonstrates that a “hard” approach is certainly not the only way, and suggests it may not be the most effective or productive.

Another example can be considered as children think about the idea of angle. An observer may attribute to some children a conception of angle as a space between two rays sharing a vertex, while other children’s conception may be of angle as the shape formed by the two rays. Other children bring their bodies into their understanding and seemingly demonstrate a syntonic knowledge of angle as *amount of turn*. One may imagine situations where alone, each way of knowing may run into limitations. However, Turkle and Papert (1992) report that their models of children who appeared to “reason from within” were much less prone to errors produced by a too-simple set of rules. They conclude, “Relational thinking puts you at an advantage: You don’t suffer disaster if the rule isn’t exactly right” (p. 177). This sort of evidence of the power of a variety of sorts of reasoning and concrete ways of knowing lead Turkle and Papert (1992), to conclude Piaget’s idea of formal reasoning is better thought of as a style, not a stage (see p. 22). Abstract thinking is not the only form of matured and powerful reasoning.

Connections to Postmodernism

Papert’s neo-Piagetian epistemology revalues the concrete and emphasizes relationships, interconnectedness, and personal ways of knowing. Additional contemporary epistemologies (such as the radical constructivist’s, and Foucault’s postmodernist orientation to knowledge) press further on the role of interaction while foregrounding the postmodern ontological stand—that knowledge does not exist prior to the meaning maker. Constructivists Larochelle and Désautels (1991) stated:

> The essential lesson that we can draw from contemporary epistemology is that the intelligibility of scientific knowledge cannot satisfy itself with aprioristic or empiricist conceptions that reduce this knowledge to empirical and methodological certitudes. Instead, this intelligibility requires a constructivist

> conception that recognizes the inevitability constructed and social character of this knowledge, in both its productions and products. (p. 376)

In Foucault's (1975/1990) epistemological effort, he co-implicates knowledge with power, a critical insight into the moves of our era's hegemonic[46] relationships. To consider the roles of others in a theory of learning is a must (only possibly superceded by the need to consider our own selves in relation to these others in relation to our construction of a learning theory). Radical constructivism offers a framework for this sort of thinking; Constructionism suggests some deconstructive strategies for theorizing, such as embracing heterarchal[47] structures and decentralization. Maturana's (1980) reminder that "Everything said is said by an observer" called for any effort to describe events to be done at a reflective level, above being immersed in the event. Observers are actors in the reality of their experiences. In the action, or interaction, with another person, they make meaning to the interaction either reflectively or unreflectively. To reflect involves stepping out of the activity in order to re-present it, as a chunk of experienced activity and to look at it as though it were the experienced activity of the moment—with an awareness that it is not.

I suggested earlier that Constructionism had failed to develop a theory on how the concretization of an object occurs. I argue that the radical constructivist notion of *re-presentation* (Glasersfeld, 1995) could be an important notion in creating a viable model for how to take account for the increased quality of a knower's relationship to the object. Glasersfeld characterizes Piaget's re-presentation as always the "replay, or *re*-construction from memory, of a past experience and not a picture of something else, let alone a picture of the real world" (p. 59). Glasersfeld states that he has no way to account for how this occurs; in fact there are no viable models neither of the human memory nor consciousness that this act of re-presentation

[46] I.e. relationships among elements are controlling, ruling, hierarchal, classed…
[47] I.e. relationships among elements are horizontal, power and privilege is distributed, shared…

draws upon, but that in some way both are involved in this re-play of past experiences. The object then is concretized as it is experienced in the moment in conjunction with the other objects of the knower's present experiential world; this refers to the rich, interconnected ways of knowing valued by the constructionist. Yet, the object also is concretized even in its absence, through the conscious recollection, that is re-presentation, of the subject's experiential knowing of the object. When re-represented in other experiences, the object continues a process of concretization. This ability to re-present the object and build these connections called for in concretization must be considered when valuing knowing as a process of moving from an abstract, or more isolated knowing, to this richer, more powerful concrete knowing. This is particularly true for knowing a mathematical object, in which the knowledge-referent may not be externally oriented, but only linked to other internal structures of knowing, to other schemes. Concretization then occurs in the processes of re-presentation.[48]

To be an observer of another in interaction, one necessarily moves to this level of reflection, re-presentation, in a similar manner. The models an observer builds of another knower's ways of knowing are necessarily schemes that may at first be informed by experience of the knower, but then draw upon inferences and conjecture to enrich and create space for further conjecture. The inferences are schemes without any particular experientially derived referent. The ability to reflect upon and concretize these inferences adds a quality of purposeful and experientially derived social interaction, using the social interaction as a tool—for analysis and future action. The opportunity for informed analysis or action creates opportunities for the teacher or researcher.

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[48] It makes sense to attribute some concretization to the enactment of a concept (scheme) in a particular scheme, the constructivist notion re-enactment.

To further refine the role of the observer, radical constructivism classifies two types of observer, the first- and second-order observer (Thompson, 2000). The first-order observer is focused on understanding another's way of knowing, aware that it is different from one's own. The second-order observer considers what they themselves understand about what the other could understand, including for the researcher/teacher the possibly of imagining alternate ways of knowing for the other that might prove more powerful. These distinctions between actor and observer give radical constructivism a mechanism through which to think about ones role as a researcher in interaction, but is not sufficient to create images of the observed's ways of knowing or possible alternative ways of knowing. Models for these ways of knowing must be constructed.

As researchers in mathematics education, Steffe, Glasersfeld, Richard, and Cobb (1983) created models for the possible mathematical knowing of children. These researchers considered two sorts of models, those of a first- and second-order. First-order models are "models the observed subject constructs to order, comprehend, and control his or her experience (i.e. the subject's knowledge)" (p. xvi). The first-order model refers to a (unknowable) way of knowing attributed to the observed—for example, Steffe's *children's mathematics.* Second-order models are "models observers may construct of the subject's knowledge in order to explain their observations (i.e. their experience) of the subject's states and activities" (p. xvi). This is the observer's own construction, a way of knowing being assigned to the other. Steffe's *Mathematics of Children* and *Mathematics for Children* are second order models. The power of second-order models is realized when they are formed through second-order observation, an intentional effort needed by both the researcher and the teacher to act with intentionality in interaction.

There are three significant values to the reflective consideration of observation and model-use from the radical constructivist perspective (Thompson, 2000). For the purposes here, I name the first two.[49] First, it emphasizes that the individual's cognitions are at once, both psychological and social. The observer is challenged to decenter. And second, it brings to the fore that we will tend to see what our theories make possible for us to see. We will interpret an interaction in ways that parallel the theory through which we sort the world.[50]

Ontology / Worldview / Macro-Theory

As a researcher, in my effort to decenter, to be other-oriented in interpreting my subject's perceived realities, to be a second-order observer, there is a challenging trouble that comes with this second value from the radical constructivist perspective. It recognizes that I am drawn to see the world I wish to see. Stirner (1845/1971) wrote, "Man[51] ... cares for each individual, but only because he wants to see his beloved ideal realized everywhere" (p. 83). Postmodernism is a theory that fully embraces the dangers of this draw to make *difference* the *same*. Postmodernism encourages one to critically re-examine observations, to keep conclusions at bay, and to hesitatingly inscribe the world.

Poststructuralism

Epistemologies such as radical constructivism and constructionism reflect a postmodern perspective on knowledge, a view that embraces the inescapable subjectivity of knowing. This

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[49] The third is explicitly about teaching and the interpreters of a constructivist perspective. Thompson (2000) said the constructivist methodologies must be grounded in an epistemology compatible with notions of intervention.
[50] Feyerabend (1975) makes this a point of emphasis in his justification that science must encourage an employment of the principle that "anything goes" (p. 22), encouraging varied manners of doing and interpretations of scientific activity, lest ones own theory—way of seeing—become too tightly bound in the scientific theory for which only counterexample can disprove.
[51] Stirner characterizes "man" not as a person, but "an ideal, a spook" (p. 83). A spook notion, for Stirner, is a concept viewed as prior to the individual, and subsequently reified. He includes among these notions ideas such as truth, right, law, the state, duty, obligation, and love. In each case, people accept these concepts as absolute and then subordinate their own behavior to their reification. Stirner contends that humans allow themselves to become subjects to such "spook notions," and as a result, are possessed.

postmodernity leaves a humanist notion of truth—reinscribed within radical constructivism as viability—perpetually, unmercifully, and always already deferred, never reaching a point of fixed knowledge. Given this position, whether or not an ontological truth exists is an ineffectual question. Questions of *being* or *is* are nonplussed, unaskable, in the communiqué of the postmodern. “Postmodernism is the Western civilization’s best attempt to date to critique its own most fundamental assumptions, particularly those assumptions that constitute reality, subjectivity, research, and knowledge” (Scheurich, 1997, p. 2). Deconstruction is used in the postmodern discourse to make us skeptical of taken-for-granted beliefs concerning truth, knowledge, power, the self, and language (Flax, 1990). This deconstruction is an effort to critique the “metaphysical and rhetorical structures which are at work, not in order to reject or discard them, but to reinscribe them in another way” (Spivak, 1974, p. xvii). Deconstruction is a useful tool to critically re-examine assumptions, and defers our tendency to make other the same.

Postmodernism and poststructuralism both carry the post- prefix to reflect reactions; I prefer to use the former to refer to an era in time/space and the latter to a way of thinking. The postmodern move goes forth from the passing of the humanist project, that through rationality one can establish and/or fulfill an authoritative system of ethics, aesthetics, and knowledge. Poststructuralism, for me refers less to an era of thought, and more to a philosophy or practice of thought. Poststructuralism questions the orientation of a structuralist science, an orientation that seeks to stabilize through naming patterns in observed phenomenon, but is does not denounce nor deny value in doing so. If Mathematics is conceived to be a structuralist science, stabilizing or fixing phenomenon, the poststructuralist burrows in on assumptions attributed to any level of the agenda to stabilize. The knowing of a structuralist science relies on inter-subjective agreements regarding such phenomenon; mathematics thus would be merely the comfort found in repetitious, sedimented, and powered interaction among knowers with regards to unknowable relationships between constructed objects. What might a poststructuralist mathematics entail? Setting forth to fix a definition would of course raise the same issues of the critique of a structuralist Mathematics. On the other hand, such a mathematics may refer to that process of deconstruction of the patterning attributed to our experiential reality.

The question of the possibility of a poststructuralist science might very well be entirely unimportant to the poststructuralist. However, such a science may be worth considering, in part to emphasize the postructuralist deconstruction of a humanist science. If science is some sort of systematic process toward deriving knowledge of a mind-independent reality, the postmodern quality assumed in poststructuralism has already debunked the potential to know such a reality, embracing that all forms of knowing are constructed. Furthermore, these ways of knowing are viable for the knower, serving her in their experiential reality. In this way, science would orient itself to the way a person interacts amid her own experiences of living (or more simply, science is living). If science begins to proclaim to speak for others experiential reality, it cannot do so—the scientist is always an observer, and even the most reflective second-order observer merely can create second-order models of another's way of knowing. In effect, we are back to a conception of science as a person's interactions amid their own experiences of living. Must it be solipsistic? The radical constructivist argues not (Glasersfeld, 1995; Steffe et al., 1983).

Must science be systematic? Maybe it must, to the extent a human being must be systematic.[52] It must be that *systematic* is a name attributed to an observer's experienced interactions with an other, or of her own ways of operating. And other qualities of an effort to name or define science will suffer the same inadequacies, from a humanist knowledge project

[52] What may be left to question is what benefit is served in the delineation of living, knowledge, and science? Especially given that such delineation seems to serve a basis for the establishment of hierarchy, power, and justification for injustice.

perspective. What is left for science then, for the poststructuralist? It is the cycle of knowledge construction and deconstruction that defines keeping what is known at bay, disallowing sedimentation, keeping power/truth/knowledge at play.

My idea of poststructuralism resounds with Peters' (1996) description as a philosophical response to the alleged scientific status of structuralism. Peters concurs that by using the methods of deconstruction, one can critique the knowledge claims of humanism.

The subject. The humanist conception of the subject defines an individual as having an identity prior to the words and actions of knowers (St. Pierre, 2000a). Such an identity would exist separate from the knowing constructed by the knower; the "I" attributed this identity, having an integrity separate and freed from the knower's experiential influences of social practices, change, and time. Such a perspective would be wholly inadequate; it fails to foreground the knower's constructive processes.

Similarly (mis)conceived is the humanist notion of agency, that people can escape oppression by exercising their innate wills. As an example, the concept the American Dream suggests that our society is one of limitless opportunity in which individuals can go as far as their own merit takes them. McNamee and Miller (2004) explain, "According to this ideology, you get out of the system what you put into it. Getting ahead is ostensibly based on individual merit, which is generally viewed as a combination of factors including innate abilities, working hard, having the right attitude, and having high moral character and integrity" (online). They go on to argue that this is not how "the system works" (online) in their debunking the myth of the American meritocracy.

The deconstructive analyses of some poststructural thinkers such as Derrida, Foucault, and Deleuze and Guattari "put the autonomous, present individual of humanism *sous rature*[53] by positing that the subject does *not* [italics added] exist ahead of our outside language but is a dynamic, unstable effect of language/discourse and cultural practice" (p. 502). In St. Pierre's critique of the subject, she may overly emphasize the "does not" quality of the constructed subject, creating an individual with no mind of her own. I take this emphasis within the more common both/and embrace of postmodernists to consider that it is neither one nor the other, but in some way both/and. It is not that the subject no longer exists, or that the identity is a ball of clay, molded by society, *behind* or merely *within* language. In poststructuralism, the subject is characterized by a double move, exhibiting "agency as it constructs itself by taking up available discourses and cultural practices and… at the same time, is subjected, forced into subjectivity by those same discourses and practices" (p. 502). This is a both/and orientation to such binaries as ahead/behind, outside/within. The subject's agency lies in its potential for repeated renewal and reconfiguration, formations that are not determined in advance.

Power/knowledge/truth. Power, in the humanist agenda, refers to a resource that all humans possess allowing them to act in this world. It can be distributed to empower others, and given away to avoid its inherent wickedness (St. Pierre, 2000a). In a postmodern move, Foucault (1976/1990) reinscribes power by recognizing that power comes from everywhere, produced from one moment to the next, in every relation. Foucault rarely used the word "power," but wrote of "power relations," because in human relationships, power is always present. *Relations of power* is a referent "one attributes to a complex strategical situation in a particular society" (p. 93). Further, "relations of power are not in superstructural positions, with merely a role of

[53] *Sous rature* translates as "under erasure," a technique of Derrida to "write a word, cross it out, and then print both word and deletion. (Since the word is inaccurate, it is crossed out. Since it is necessary, it remains legible.)" (Spivak, 1974, p. xiv).

prohibition or accompaniment; they have a directly productive role, wherever they come into play" (p. 94). These power relations are everywhere, most often observed through instances of resistance. "Where there is power, there is resistance, and yet, or rather consequently, this resistance is never in a position of exteriority in relation to power" (p. 95). I take Foucault's continued theorization of power to be what I name a reinscription, in some way a means by which to insert within common notions of power a further textured, nuanced, and challenging ways to think from other directions about what role power may play, and roles people may play with a thing called power. I assume it is not his goal to rewrite or redefine power, but merely to deconstruct, question what may have been left unquestioned or assumed.

Poststructural knowledge is a construction, constituted through discourse and social interaction. "Power and knowledge directly imply one another; there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations" (Foucault, 1975/1995, p. 27). Knowledge is power-laden, and the product of faulty calculations. Yet these errors do not make for a knowledge that can be systematically tracked down (Spivak, 1974); there could be no truth referent when what is taken as truth is always already determined within relations of power. Baudrillard (1981/1994) positioned truth as merely operational, not needing "to be rational, because it no longer measures itself against either an ideal or a negative instance" (p. 2). The *what* of what knowledge really says is merely a *hyperreal*, a "model of a real without origin or originality" (p. 1).[54]

The humanist language with which I am confined to speak/write does not provide me with words to say what knowledge *is*; how can I speak of a knowledge that has nothing like an

[54] Baudrillard's statement makes sense to me when I think of his "origin or originality" to refer to something possessing a mind-independent existence. Such a passage would not make sense for the postmodernist who already would take "origin" or "originality" and place the constructs into the mind of a knower or observer.

existence external to the knower, yet we treat as though it does? I can talk about the ways in which this thing called knowledge *seems* to take on an existence. I can state my models for knowing, through careful description within the play of language. I can state my models for other's knowing. The radical constructivist's second-order knowledge models are exactly these sorts of hyperreality, not unreal, but without a true, knowable reference. A researcher's statements regarding the other, their ways of operating, their ways of knowing, are merely guesses; granted, they are guesses that must be viable and have accomplished a rich second order of viability. Furthermore, they are guesses that live on when their predictive power seems to continue to create viable new knowledges. Yet, in essence, they are guesses, not truths. They are my self trying to create models of the other's ways of knowing while keeping at bay the desire to secure definition and fix mind-independent truth to that unknowable-as-a-thing-in-itself other. These efforts are my self trying to create and secure my unknowable-as-a-thing-in-itself self. Knowing, and similarly knowledge, is always already a construction. The radical constructivist's methodological structures of the first- and second-order observer, and the first- and second-order models for knowing are useful tools for the postmodern researcher to help keep the human, seemingly pre-programmed desire to secure knowing as if it were an isomorphism to a mind-independent reality at bay

As humans, we attribute the assimilations and accommodations of our experienced reality to be our world, to other actors in that world. Thus, I define *knowledge* to be the set of rules one attributes to the governance of one's self and others (living, or otherwise external to that which we call our self) as systems. A "set of rules" can be thought to be equivalent to the constructivist's model, or on a smaller scale the Piagetian scheme; namely a structure initiated by a perceived situation resulting in activity. In a philosophical nod to poststructuralism, I must caution against my desire to hold tightly to this definition, this attempt to fix a notion as though it takes up an existence external to me. Yet, simultaneously I must progress forward, embracing the tools I have access to in order to think further about my objects of study.

In this consideration of knowledge, it is inherently active, bound up in relations (Glasersfeld, 1995; Steffe, 1996; Stirner, 1845/1971). But further, I embrace the subjectivity of knowledge by clearly placing the knower in the definition, and the subjectedness of knowing through my use of the word governance. Although this is a term Popkewitz uses to name a seemingly highly intentional effort to control or regulate others, my intent is to remind myself, as a user of this definition for knowledge, of the constructed nature and tendency toward sedimentation of model building, that one can begin to forget the model is no more than that, a model, desiring further interaction to increase its viability.

A change in knowledge refers to the changes attributed to this set of rules. Thus, *learning* is the perceived *evolution* of the rules attributed to the governance of the system (Davis, 2001; Resnick, 1991). Rules govern the self (system) and the self (system) governs the rules. In this self-referential definition, the knowledge emphasis is in-the-moment, fleeting; it is always already constructed in the interaction, having no need for subsistence. The sedimentation and need to retrieve knowledges are deemphasized and the power of in-the-moment ways of operating is accentuated. Knowledge rules are designed to rebuild ways of knowing, less so to document or conceptualize knowledge as things. Thus, knowledge, as bound to interactions and hence relations, is a power-laden attribution from one onto another, which may be the self. Foucault (1975/1995) wrote, “power and knowledge directly imply one another; there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations” (p. 27). Restating, in order to fully engage the reader to this orientation of power/knowledge: I conceive of knowledge to be the set of rules one attributes to the governance of our self, or others, as a system. And by Foucault's homology, so too is power—that which we attribute to our selves, or others. My postmodern beliefs about knowledge and learning resonate with radical constructivist (Glasersfeld, 1995) principles. Knowledge has no existence; it serves as a marker, a pointer of the observer, to scraps of meaning two in interaction treat as though they share. It serves as an index of constructed power relations. As rules attributed to our own self, knowledge works as Piaget's activity-oriented schemes. Again, I find it more valuable to think about the relations among knowledge that these rules makes possible. This continues the emphasis on interaction, however now this interaction is within and among the ways of knowing of the knower. Knowledge as power now brings forth not only the manners in which interactions shape the governance of the system, but also the way the system constructs a governance of itself. The self-referential nature of this definition of knowledge embraces this cybernetic structure.

While poststructuralism allows me ways to think about dangers such as that of speaking as though structures exist, that knowledge lives free from interaction, that a truth can be known, or that power relations can be overcome, it does not theorize the social drive I feel to improve the experiential world of my fellow humans. Here, my refusal of the status quo, my reaction to what I perceive to be injustice, and my compulsion—and demand—to act, I find to be reflected in critical theory. While the critical theory perspective does not necessarily share the ontological presuppositions of postmodernity or the reflexivity of poststructuralism, there are others who work at the intersection of the two worldviews.

Critical Theory

Poststructural theory encourages me to challenge what I "hold to most tightly" (personal communication, Elizabeth St. Pierre, Fall 2003), to decenter, to think. Critical theory encourages me to act. Social justice, work against oppression, and a struggle for empowerment are key ideas of the critical theorists framework for action. Lather (1991) captured this through defining empowerment as the ability to perform a critical analysis regarding the causes of powerlessness, the ability to identify the structures of oppression, and the ability to act alone or as a group to effect change toward social justice. She maintains empowerment as a process one undertakes for oneself; "it is not something done 'to' or 'for' someone" (p. 4). Lather's statement, made in 1991, has yet to embrace a postmodern orientation toward power and knowledge. Kincheloe and McLaren (2000) summarize the assumptions of critical theory informed by postmodern thought:

1. that all thought is fundamentally mediated by power relations that are social and historically constituted;
2. that facts can never be isolated from the domain of values or removed from ideological inscription;
3. that the relationship between concept and object and between signifier and signified is never stable or fixed and is often mediated by the social relations of capitalist productions and consumption;
4. that language is central to the formation of subjectivity (both conscious and unconscious awareness);
5. that certain groups in any society are privileged over others and, although the reason for this privileging may vary widely, the oppression that characterizes contemporary societies is most forcefully reproduced when subordinates accept their social status as natural, necessary or inevitable;
6. that oppression has many faces, and concern for only one form of oppression at the expense of others (e.g., class oppression versus racism) often eludes the interconnections among them; and finally
7. that mainstream research practices are generally, although most often unwittingly, implicated in the reproduction of systems of class, race and gender oppression. (pp. 139–140)

Kincheloe and McLaren are bound to humanist language, especially when naming oppression. This summary keeps institutional and interpersonal oppression central, yet embraces the unknowability of the postmodern. Kincheloe and McLaren (2000) argued, "A postmodernized critical theory accepts the presence of its own fallibility as well as its contingent relations to progressive social change" (p. 151). In other words, a postmodern critical theory is assumed to be placed under critique, including its principal function—the drive to work for social change. Kincheloe and McLaren also argued, however, that engaging in a postmodern critical theory did not annihilate the concepts of emancipation, empowerment, hope, justice, oppression, praxis, etc. Postmodern theory is not destructive. On the contrary, they suggested that within a postmodern critical frame, these concepts from critical theory are embraced, becoming objects of critique, while demanding from postmodern theory a foundation that precludes the theory from being perceived as nihilistic or inactive. Kincheloe and McLaren concluded their summary, stating:

> To engage in critical postmodern research is to take part in a process of critical world making, guided by the shadowed outline of a dream of a world less conditioned by misery, suffering, and the politics of deceit. It is, in short, a pragmatics of hope in an age of cynical reason. (p. 154)

Kincheloe and McLaren, as did Lather (1991; 1998), brought together postmodernism and critical theory keeping under critique the critical theorist's goals for social justice.

Kincheloe and McLaren's (2000) principles for a postmodern critical theory suggest a centrality of language that may be interpreted to conflict with my constructivism. In particular, the choice of "mediated" in the first and third bullet has a history of discord among epistemological arguments (Lerman 1996; 2000; Steffe & Thompson, 2000) in mathematics education; the discord being about where learning occurs, the intermental or intramental plane. "Mediated" for Kincheloe and McLaren seems to emerge from Vygotsky's (2000/1986) work, which would either be in conflict or subsumed by radical constructivism depending on which perspective, Lerman's or Steffe and Thompson's, is considered. I have criticized this Vygotskian orientation as failing to recognize the constructed nature of knowledge and ways of knowing. However, to more directly speak to how this general notion makes sense for me in Assumption 1 is that *all* (as strong as that may be) thought, no matter motivated by interaction with the environment, or from one's own internal reflections, does engage schemes that have been developed in conjunction or relation to interaction. Each of these interactions has a history, a surrounding set of feelings and experiences and relationships that make all ways of knowing constantly in play amid relations of power, whether the relation be immediate, once-removed, twice-removed…. This commentary is echoed as well in Assumption 2. Assumption 3 introduces an unnecessary duality, for the radical constructivist; the object is the concept. Because the so-called object is always a constructed knowing, it is not worthwhile to treat it as though it took on a mind-independent existence for the purpose of theory building. When it comes to interaction, thinking, collaboration, as knower's we do treat the concept as an object. Yet in the language of interaction with others, we often unquestioningly treat the object-concept as though we shared some intersubjective yet isomorphic way of knowing.

So, I do not criticize Kincheloe and McLaren's (2000) apparent intent of the third assumption, although I would not have contrasted the object and the concept, nor would I have chosen the word mediated—it suggests an external force. I do believe, as do Kincheloe and McLaren, that a knower could never know some particular truth, and further that any approach toward a way of knowing is impacted by that knower's constructions of social relations, such as capitalism's overwhelming and under-understood effects on our ways of acting and interacting. Assumption 4 puts at center the valuation of language for Kincheloe and McLaren's theory. Again, the centrality of language—and possibly the meaning language carries, or has the potential to evoke, free from a knower—may be at odds with a radical constructivist viewpoint. However, I again agree with the direction taken toward the construction of the subject presented in their sketch of a postmodern critical theory. I take this orientation to assume a constructed view of knowledge, and a both/and mindset toward binaried interpretations that could be attributed the sketch. A reader that focused in on the construction of knowledge would recognize the binaried positioning of knowledge that the Piagetian vs. Vygotskian traditions suggest. I think it to be productive to not get mired in this theoretical debate when seeking to act, which is a main reason for me to embrace critical theory. Yet, in one's action, it is valuable to maintain a sense of knowledge as constructed, realizing interactions may operate on an assumption of shared knowing. Furthermore, the person or people with whom I interact may not question an orientation to knowledge that treats it as mind-independent. Maintaining a both/and mindset while working to act can help establish a necessary rapport, and free a group from a sideways orientation (*enforced passivity*) to working on a more critical task at hand, seeking to make change to injustices.

My research effort connects with this postmodern orientation to a critical theory, working toward a social justice in the way Paulo Freire (1970/2002) named to be the goal of education, the realization of *conscientização*—"learning to perceive social, political, and economic contradictions, and to take actions against the oppressive elements of reality" (p. 35). It is the role of schools to construct society, to produce social change. There is no getting around this point; it is what schools do (Dewey, 1937). Dewey called for educators to strive courageously and intelligently in this direction, not to work to maintain a current order of things, or to merely drift along with the present conditions. Critical theory is a social theory oriented toward critiquing and changing society, not a theory merely to organize, understand, or explain. The slipperiest relationship between the macro-theories of poststructuralism and critical theory, and the mid-level, epistemological theories of radical constructivism and constructionism, may be that of the subject, the individual knower, the self. Theories of knowing focus on the subject, the knowing "self."

The Critical-Postmodern-Constructivist Subject

In my research, I do not refuse the self as a knower, but embrace it. It is not poststructuralism's goal to reject the concepts of humanism, but to rethink, and possibly reinscribe them. "Indeed, in the course of their work, scientists need to examine critically their conceptual, as well as material, instruments. In many cases, this examination raises epistemological questions about the problem, including the contribution of the subject and the object in the production of knowledge" (Larochelle & Désautels, 1991, pp. 374–375). The researcher herself is one of these conceptual tools, as is the subject. In engaging in this production of knowledge, both subjects—as research tools—must be critically considered. Radical constructivism takes this position in its positing a model for knowing. Postmodern discourses trouble the knowable self by refusing to place the individual ahead of or outside the social realm. In constructivist language, this positioning is equivalent to saying the self is co-constructed in its interrelations with others. This reflects the double move of poststructuralism, that a subject constructs itself in taking up social knowledge and practices, and also is subjected by these same practices. There can be "no stable referents to the subject, even for the speaking subject, the 'I'" (St. Pierre, 2000a, p. 502). So while not refusing the self, I work carefully to trouble the ways of knowing the other that my observation, perception, and experience constructs. That my subject's and my own subjectivity co-emergence, we influence one another (Davis, 1996).

It is in the awareness of my engagement in critical (self-)reflection, doing, and experiencing, that I define my self. Glasersfeld (1995) modified Descartes' famous pronouncement to fit constructivist theory: "I am aware of thinking, therefore I am" (p. 122). An examination of how we experience our self considers how we recognize both our self as an agent and a construct we distinguish from the rest of our experiential world. To distinguish such a construct requires an observer, a distinction Thompson (2000) made by inserting the reflection on actions one makes that takes one level of awareness beyond merely engaging in the activity, but also being an observer of it. The ethical teacher/researcher works to overcome the draw to replicate one's self; to act as a second-order observer, being aware of one's own activity in interaction, and of the influence this might have on learning.

The subject is a central construct to my ways of thinking about other's use of this notion of knowledge. I consider *mathematical knowledge* to be my way of thinking about the result of an other's production of quantitative, spatial, and symbolic relationships through ongoing interaction. In this way, mathematics (as a collection of mathematical knowledges) is a product of (my) human activity. Although I recognize that people will speak of mathematics as categories of facts, skills, and ways of thinking, I believe mathematics is what a learner reads into (quite literally, actively) their experiential world, not something extracted from their world (Glasersfeld, 1995; Piaget, 1970). In a postmodern move, radical constructivism recognizes that whether or not there is a transcendental knowledge that can be attributed to a learner, we act in ways that do create such simulations (Baudrillard, 1981/1994). Our models of knowledge or ways of knowing are never statements of what *is*, but instead models for considering another's ways of operating as rational. This is an ethical position toward the other.

This exploration of constructivisms, as well as poststructuralism and critical theory, has allowed for me to clarify further two key ideas of the subject of this dissertation, the generative adolescent mathematical learner. I have discussed my personal theory of learning, that which is the perceived evolution of the rules attributed to the governance of a system. And I have clarified what I take to be mathematical, one's own way of thinking about the result of an other's (which could be what one has perceived to be the self) production of quantitative, spatial, and symbolic relationships through ongoing interaction with that other. While each of these positions on "mathematical" and "learner" have strong implications for the concept "generative", implying the active, interactive, inventive, and productive natures of knowledge creation, they have yet to speak about the knower's sense of self; a key qualifier in my definition of the GAML. I return to the postmodern subject to speak about this generative disposition through a poststructural commentary on the humanist concepts of agency, identity, and authority.

Agency, Identity, Authority

The postmodern subject is a co-constructed agent, "an actor who makes something out of what the environment makes of him" (Larochelle & Désautels, 1991, p. 379). Foucault's analysis of power relations challenges us to consider who gets to be this sort of subject in a particular discourse. But he also asks the other part of the question, who is subjected? Again, a more full acknowledgement of the subject respects this double-turn on who she is, or better said how she acts. And in the postmodern, when there can be no stable, knowable subject, what new roles do agency, identity, and authority take?

New Roles for Agency, Identity, and Authority

Poststructuralism is characterized by trying on for size both/and characterizations of notions perceived to be defined along or by the binary of dualistic ways of thinking. Here too—on agency, authority, and identity—is this double move evident. St. Pierre (2000a) wrote that "a subject that exhibits agency as it constructs itself by taking up available discourses and cultural practices and a subject that, at the same time, is subjected, forced into subjectivity by those same discourses and practices" (p. 502). This poststructural subject is not dead, but has been opened up to possibility. "In poststructural theories, the subject is considered a construction, and identity is presumed to be created in the ongoing effects of relations, and in response to society's codes" (p. 503). As a construction, identity is never stable, never "identical to itself" (St. Pierre, 2000a, p. 503, quoting Britzman). "Identity is constructed in the desire to make sense of the world" (p. 504); it is the effort to create meaning.

This in-the-moment consideration for identity is reflected in agency as well. Since the subject must be "constituted again and again implies that it is open to formations that are not fully constrained in advance" (Butler, 1995, p. 135). Thus, the agency of the subject is also continually refigured. "In poststructuralism, meaning can be strategically reinterpreted, reworked, and deferred since there is no referent for the subject" (St. Pierre, 2000a, p. 504). The subject is involved in such games of interaction, power relations. In that we construct our own meaning of ourselves, we refigure ourselves through dis-identification, not-being another. Butler states that agency lies within refiguration, as "subversive repetition" (Butler, 1990, p. 32). Agency theorized in this manner does provide freedom to act. It is "a freedom to constitute oneself in an unexpected manner—to decode and recode one's identity" (Stinson, 2004, p. 57). Derrida (1978) defined play as the "disruption of presence" (p. 292), a theoretical move to open possibility. He goes further to reinterpret freedom as play. Play becomes a generative idea because subversive repetition questions the social world, the interaction. Such a questioning allows for—nay, constructs—a different repetition. And so on. The postmodern agency is found in the freedom of this subversive repetition.

In the postmodern ontology, where what is considered to be knowledge is an ever-changing quality of interaction and construction, invoked through the agency of the subjects amid relations of power, there is no determinacy. In this indeterminacy we can "deflate claims to authority" (Usher & Edwards, 1994, p. 135). The postmodern rejects authority as a notion of the expert knower. Authority is overrun by Foucault's power relations. When asking Bové's (1990) questions in response to what may be a poststructural meaning making: "How does authority function in the discourse? Where is it to be found? What are its social effects? How does it exist?", authority is seen as a tool of power in an era of thought devoted to rationality and reason. In the postmodern, truth has melted away. A truth is bound to a knower, an interaction, a circumstance—always already changed, never fixed. "Truths are illusions about which one has forgotten that this is what they are" (St. Pierre, 2000a, p. 497, quoting Nietzsche). Any notion of authority returns to the root of the word, author—an originator or creator of something. Each knower is then an author, an authority. Further, the authority attributed by a knowing subject, to herself or to others, is also authored by the subject.

Mathematics education researchers Boaler and Greeno (2000) brought together these three ideas, identity, authority, and agency by drawing upon Holland (1998). Boaler and Greeno describe a positional identity, to "refer to the way in which people comprehend and enact their positions in the worlds in which they live" (p. 173). They argue identities develop in and through social practice, i.e. interaction. Boaler and Greeno identify another aspect of identity as "a space of authoring… encapsulated by the notion that 'the world must be answered—authorship is not a choice'" (p. 173). Here again we see authority portrayed as what the subject does. "We are complicit in the production of ourselves" (St. Pierre, 2000a, p. 504). Returning to Boaler and Greene's (2000) research, they identified talented high school mathematics students, i.e. those enrolled in A.P. Calculus, and learned about their sense of identify, agency, and authority in learning mathematics. They found that these students, although highly competent in school-defined mathematical knowledge and ways of knowing, were “received knowers”, a concept developed by Belenky et al. (1986). This received knower considers her knowledge as primarily dependent on and derivative from an authoritative source other than herself. This is not the idea of the generative learner of my study.

The Generative Learner

I return to my discussion around the generative adolescent mathematical learner (GAML) to reposition her within the mid-level and macro-level theories of constructivism and poststructuralism. Although a significant project of this research is to flush out more ideas about this generative learner, the literature and my own ways of thinking have some direction in mind prior to the work. I initially cast aside the name “generative mathematical knower” for the static nature implied by the notion of a “knower”. I conceive of knowledge as the set of rules one attributes to the governance of ones self and others as systems. A change in knowledge refers to the changes attributed to this set of rules. The postmodern ensures such a conception of knowledge to never be static, never fixed, nor is it ever fixable (i.e. knowable). Another’s knowledge is a construction of the knower. To consider, reflect upon, coming to know one’s own knowledge is to have already changed it. Any postmodern conception of knowledge is akin to Baudrillard’s (1981/1994) hyperreal, a real that is “no longer anything but operational” (p. 2). So in the place of “knower” for the purpose of naming the subjects of my research, I chose “learner”. Following on my definition of knowledge, learning is the perceived evolution of the rules attributed to the governance of the system. In this self-referential definition, knowledge has no existence, it is always already socially constructed and thus a power laden attribution from one onto an other—which may be the self. Simply, learning is the evolution of the always changing notion of knowing.

As I built this concept of the GAML, I reviewed my thinking on knowing and learning above. *Mathematical knowledge* refers to my way of thinking about the result of an other's production of quantitative, spatial, and symbolic relationships through ongoing interaction. In this way, mathematics is a product of (my) human activity. Here, I emphasize the parenthetical "my." It is significant to recognize that I attribute mathematics to my own way of organizing my perceived experiences. And so the mathematical knowledge I attribute to the other is necessarily my own.

And finally, the notion of generativity. The author of knowledge where knowledge is a part of a cybernetic system, one that embraces self-referential, self-organizing, and emergent principles (Ackerman, 1995), is truly an author—an originator, a creator. Here, knowledge is authored, or generated. The author is generative. It is this notion of author and authority that remains unexplored in adolescent mathematical learners. Although some research may use this sort of concept for locus of authority as the, or a, dependent variable in a study (for examples, Boaler & Greeno, 2000; Clarke, Breed, & Fraser, 1992/2004; Grouws et al., 1996), I am aware of no work that delves directly into the generative adolescent mathematical learner as the independent variable, or as the focus of an anthropological lens.

With these constructs in mind for learner, mathematics, and generativity, and my views on existence (ontology), knowledge (epistemology), and values/ethics/equity (axiology), I hope I have made it evident that I consider all knowers to be generative mathematical learners. I perceive all humans as being a learning system separate from me—the axiological underpinnings of constructivism. As a part considering knowers as a learning *system*, all knowers are necessarily generative. And I attribute mathematical ways of operating to all these learners. So again, all humans, and all high school age students, are generative mathematical learners. Yet, in this research I seek to distinguish some adolescents as generative and others as less so. While I recognize creating such a demarcation is a false attribution, it is a binary I will employ yet keep at bay, the tools of poststructural critique supportive of this effort. I employ this false demarcation in the initial stages of my research to point toward *adolescents in mathematics classrooms who seem to act as though they recognize their own role in the authorship of mathematics*, or what they may consider to be understanding of mathematics. At the moment, my notion of the generative mathematical learner is independent of the invent/discover binary commonly discussed in mathematics and mathematics education. These generative adolescents view their activity, or play, as mathematical. Non-generative students may view their activity, which I may deem mathematical, as merely play, and not “real mathematics”. The generative adolescent mathematical learner is one I attribute, through my observations and interactions, to possess an identity of mathematical author. They are highly aware of their own agency in the construction, or possibly disruption, of mathematics.

The Generative Learner, Constructivism, Poststructuralism, and Social Justice

Having sketched my initial notions of the generative adolescent mathematical learner within the languaging of constructivism and poststructuralism, I will close this chapter by reconnecting to one of my initial purpose for this dissertation, that is to conduct research for the purpose of social justice. For me, social justice is not an endpoint, but a marker toward which to strive for a belief system about ones self and fellow humans. Difference is valued, not only for the compassion for others, but also because difference provides the environmental resources for one’s own richer constructions of an experiential world. However, society’s institutions—especially school—do not strive to value difference. Do not confuse stated missions or philosophies that may suggest otherwise (Spring, 1993); structures operate with intention, and intention directed at learners is necessarily ordering and regulating.

The GAML's Relation to Social Justice

Stirner (1845/1971) provides a metaphor for the regulating properties of intentionality, *wheels in the head*. He wrote, in *The Ego and His Own: The Case of the Individual Against Authority*, that the dissemination of ideas through schools was fast becoming an important means of domination by the modern state. Modern postmodern critical theorists such as Popkewitz (2004) and Apple (2000) concur. An idea becomes a wheel in the head when the idea owns the individual instead of the individual owning the idea. The generative mathematical learner owns the idea; mathematics is of their own authorship. Mathematics, for them, is not a wheel in the head. This generativity is in resistance to the institutional effort for domination. But more importantly, difference in thinking among generative knowers necessarily emerges in the entropic experience of learning amid generative thinkers.

A second quality of social justice involves ones responsibility to act against injustice. Martin Luther King's statement that "injustice anywhere is a threat to justice everywhere" speaks to a notion that our own best interest is intricately wrapped in and dependent on the best interest of others. The generative mathematical learner recognizes the effect of their activity, that they inscribe the world, that their presence is powerful. Freire (1970/2002) insisted that the goal of education must be toward *conscientização*—"learning to perceive social, political, and economic contradictions, and to take action against the oppressive elements of reality" (p. 35). D'Ambrosio (1990), a peer and mathematical colleague to Freire, wrote of a mathematics education for happiness.

What is Equity?

Inextricable from an educational goal toward social justice, is work for equity. Equity, like social justice, is not best thought of as a state to be achieved, or an ideal toward which to strive. To borrow from poststructural linguists such as Lacan and Derrida, it might be thought of as a signifier, a notion to point toward, yet not attainable. Not an ideal, suggesting a humanist utopia or state of highest knowing. It is forever sliding, a never satisfied state. Consumerism is driven by a similar "sliding signifier" (Apple & Beane, 1995)—desire. No matter the amount of purchases, desire is never satisfied. In this sense, equity is like desire, never satisfiable. Democracy, freedom, justice too, are sliding signifiers.

This conception of equity brings to mind an image of relationships among people, and in particular *just* relationships. Yet, it is not sufficient to think about the embeddedness of equity in relations or interactions unless it is conceived on three planes: the relationship with oneself, the interactions with (the constructed) other(s), and the interactions with (conceived) social structures. For an individual, equity can be realized when that person sees their selves as the author of their world. Not only are they authors, but they are agents in—as opposed to acted upon by—this world. It is key to this notion of agency to recognize the self as a source through which power is exerted upon the perceived reality. Further, it is that one sees others (and by changing perspective) and others see oneself and all other others of their experiential reality as possessing this authorship, that a powerful notion of equity is evident.

Continuing to clarify this point, hand-in-hand with this notion of authority must be the recognition that this authorship attributed to each knower suggests a knowing necessarily different from other's constructions, and thus to consider one's own knowing as *the* "correct" or "true" version is both inequitable and misguided. Such an interpretation leaves other's knowing as less than correct, or untrue. This trouble arises from our taken for granted use of language, reifying truths and knowledge. A more refined manner through which to handle the issue of multiple ways of knowing would address the question how could one know if one's own knowing is actually different from that of an other. Because an individual imputes an experiential reality to another not much unlike one's own reality, they recognize that any model built of another's knowledge is necessarily inadequately informed at the outset, by means of imputing the experienced reality to the knower. Hence, an effort to determine a match or quantify correctness or completeness between the ways of knowing is flawed.

Along with the awareness of the inability to measure degree of truth of one's knowing in conjunction with another must come the humbled recognition that one's own authorship of the world is no "better" than any other person's. Take a very specific instance as an example, the knowing a person one attributes to a life partner. It is true that a new friend will have had much fewer opportunities for interaction. The question is, does one (person A) know their partner (person B) *better* than the friend (person C) knows him or her? To explore this question, I suggest for the moment that "knowing" someone could be aligned with the potential to predict her activity under certain conditions. It is easily agreed the two knowers, A and C, know A's partner, B, differently. It is also agreeable that person A may be better able to predict the activity of person B in most all opportunities for interaction. Yet, for example, person C may be freed from the sedimentation of certain patterned ways of interaction that allow C to better predict the activity of B in some cases. Now, the question might be, how many cases? If the number of cases was left as the measuring tool for how well A and C each knew B, the trouble is this count is unknowable. I would say our egos drive us to say person A. But ultimately, I suggest the mathematics of infinity make the question unanswerable. The measuring tool is problematic when trying to account for "better."[55]

The importance this example illustrates is the unquestioned use of the word *better* when comparing ways of knowing; this is a moment when a term must be used, and then crossed out—*sous rature*. It is not that better is a term to be cast aside; instead it must be deconstructed in the context of measuring knowledge with viability. Each knower's experiential world is assumed to be viable for that knower, stable and free from contradictions that would debilitate the working system, free from non-contradictory "rules" for operating. The model one may make of another's way of knowing in the context of the model of that other's experiential reality may suggest that the other's knowing is less viable in that constructed reality of the other, or even within one's own model of reality. But this judgment is severely constrained; in its best interpretation, it is a judgment defying an assumed access to truth, yet it posits a comparison between what could be considered knowable, one's own way of knowing, and what is unknowable—that knowing of the other.

No equitable resolution, nothing that can respect the viability of another's way of knowing, can come through an attempt to measure or compare ways of knowing between distinct knowers. Instead, an equitable perspective attributes and seeks to recognize the way in which the other's knowing is viable for them. Can "better" be used in a meaningful manner? Can better be

[55] Another context in which we speak of knowing something "better" often used unquestioningly is knowing across time. It might be common to say that I know my partner better now than I did one year ago. How might one evaluate *better*? More able to predict the activity of the other? I believe a similar argument can be made to what I present about person A and person C knowing person B above to deconstruct such an ability to know. Another method of analysis could be considering the sliding scale against which we proclaim to know. I must posit both a model of the experienced reality of the other, my partner, as well as a model for how that other interacts with that experienced reality. I argue that by necessity any quantifiable measure of the growing quality of my knowing my partner's experienced reality is growing at a rate faster than my knowing of my partner's model of their experiential reality. Growth of our interactions with our world trumps the growth of our knowing of these interactions. By that argument, we know a lesser and lesser portion of our experienced reality as time progresses. By extending this reasoning to the knowing of the models we build of our partner, it is clear we know less and less about them the more we get to know them.

used to compare two people's ways of knowing? I suggest it only can be used if one experiential, or a priori world is assumed. When that is the case, the knowing of one may be assumed to make her more viable in that singular context than the knower of the other. But to the extent both experiential realities are distinct, a comparison of better is not feasible. Furthermore, I argue that unquestioned use of the term is dangerous, promotes hierarchy and class defined by knowledge, the *truth regime* (Lather, 1993).

To put the word *better* under erasure, to use it and immediately cross it out, refuse its implications, implores us to do the same with the inferred converse, *no better than*, or equal. It is similarly untenable to conceive of people's ways of knowing to be equal. That is to say, one's perceptions and conceptions of what is, and what is true, are necessarily different from others, yet can have no hierarchal status of superiority. There is no a priori measure by which to gauge such a relationship. It is in the valuing of difference that equity is found, and the devaluation of qualifying or quantifying the difference. Hence, interaction with others attains the characteristic of *equitable* when each person recognizes, respects, and acts/interacts in such a manner that reflects these awarenesses of the differences, yet equalities, of one an other's ways of knowing. Again, hierarchal status relations of perceptions and conceptions would be downplayed, if not refused. Interaction would serve the purpose of coming to know the other, that is, to construct a model for another. Interactions are fellowship, companionship of individuals.

Beyond recognition that the power one exerts over their perceived world, equity must also address interaction with social structures. It must first be noted that social structures—patterns identified in repeated interaction with events perceived to possess similarities—are a construction of the individual. Beyond this recognition that social structures are a construction, others, and possibly the self, participate in these social structures as if they existed, as if they were the norm. Without a critical interaction with these social structures, the structure defines the knower, creates “wheels in the head” (Spring, 1993; Stirner, 1845/1971), even though that structure is a construction of the knower! The uncritical knower succumbs to the perceived structuring forces of these externalized entities, rather than embracing them and using them in their favor.[56] Thus, a deconstructive, or at minimal a critical eye, must be maintained toward assumptions one makes about the patterns they attribute to, and participate in, the world. Equity here demands for people to identify what it is they hold most tightly, and to challenge this desire—that is, to act with a resistance the path of least resistance. Freire’s (1970/2002) *praxis*, reflection and action upon the world in order to reform it, speaks to this demand for thoughtful, critically reflective action.

My Vision of Mathematics Education

Equity in the context of education is about bringing forth these three qualities—the relationship with oneself, the interactions with other(s), and the interactions with social structures—of living/learning in the individual student, in the relations among members of the learning community, and in the ways in which members of the community engage the social structures associated directly with education. Rather than education working to “bring forth” these qualities from the child, I feel it is more respectful, and thus inline with these statements on equity, to recognize them as the current nature of the child, and that the role of education is to work to not diminish these qualities. And while any intention in education or an educative relationship necessarily works to place wheels in the head (Spring, 1993), I feel these wheels are means to keep possibility in play, to challenge the other to think, and to positively invoke power in the teacher-student relation.

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[56] Foucault (1980) reworks power in this manner in his conception of *power relations*.

As is the singular nature of this proposition on the qualities of equity, so to would be any statement on how to work in such ways. With that said, I continue on the role of education, and more particularly mathematics education. I conceive of Education[57] as an organized, intentional effort to order particular learning and knowledge. This Educational structure is inherently counter-designed against goals of equity and social justice. In the instance of naming something to be learned, or an aim for learning, a valuation of what is to be learned is created. Not only does this disrespect the varied ways of knowing of members of the Education system (teachers and learners of all sorts), it points toward a goal of like-mindedness. Any system of Education works against equity even if it's goal is to work for equity.

Yet, we must act. In Education, the interest of the child should define the learning goals for that child. Of course, these goals are defined in the relations and negotiations with the teacher. The goals of education should not be preordained, or constrained by carving our disciplines from particular groups of people's perceived-to-be-common ways of knowing. Mathematics Education should not be about attainment of a body of knowledge, or a particular way of thinking. In fact, there exists no such mathematics, existing externally to any one's experience; and no *one's* mathematics can be deemed what is to be learned. Yet I am willing to call a child's activity that I identify to be mathematical as what the mathematics education of that child is to be composed. When the *child's mathematics* is the curriculum in mathematics education, pedagogy must be thought anew. Listening and learning alongside become central; this is a listening different from "listening for", or "listening to". It is the listening of equitable relations/interactions, a "hermeneutic listening" (Davis, 1997). It is Freire's (1970/2002) development of a "generative theme."

[57] I capitalized Education here to refer to the institution, rather than the practice.

What implication might this initial conception of equity have for the academic study of mathematics education? Simply, it must be the study of child. The generation of models for *children's mathematics* becomes central to the curriculum. Yet that alone is entirely insufficient. An equally great effort must be placed to consider the ways in which educators engage in listening and learning alongside. What actions of the teacher can free her from suppressing the mathematical generativity of the learner? What structures of the school can decrease this oppression? Work must be done to discuss the ways in which we, as mathematics teachers, get in the way of our own efforts for developing children to be authors. We must work to consider how our models of *children's mathematics* both enable and constrain the necessary effort to listen hermeneutically.

Further, mathematics education, including academics, teachers, and learners, must strive to understand the necessarily oppressive roles of the institutions of learning (Skovsmose, 1994; Woodrow, 2003). Beyond the school building, the institution of Education in our culture must be questioned, debated, upset (Frankenstein, 1987; Gutstein, 2003). The current structure of Mathematics Education is discipline-focused, not learner-centered.

The equity I've outlined here emerges from my philosophical beliefs, my worldviews. It is a relational equity (Boaler, 2004; 2006a; 2006b; 2008). It is not an equity about pumping more and more varied children through a sick system. Instead it is an equity that disrupts the system, deconstructing the presuppositions of knowledge, mathematics, and authority to reinsert the potential for the subject's subversive repetition. A science of mathematics education can still exist, however it must find new bearings guiding and forever redefining its course, rather than further tunneling in toward a fixed and stable structure.

To me, equity is letting go; it is release, trust, valuing others. It is rhizomatic, heterarchal, decentralized, personal. It is wrestling with ones desires, confronting ones power to act on the world, and refusing to do so.

With my worldview sketch, and notions significant to this research defined, the next chapter turns toward the manner in which I pursued further study of the generative adolescent mathematical learner.

CHAPTER 4

EPISTEMOLOGY OF RESEARCH

Piaget's interest was in studying "'what people *know how to do*' as opposed to 'what they *think they know how to do*' (Piaget's own words)" (Ackerman, 1991a, p. 370). My interest is in considering the role of one's self as the researcher in this epistemological task; that is, 'what *I think* people know how to do.' Such an orientation needs to be informed in part by 'what I think *they think* they know how to do.' My perspective not only reorients the research to embrace the notion that we are complicit in the construction of knowing, but also is curious to consider the degree to which others—my research subjects—hold this epistemological outlook as well.

I am the sole researcher conducting this study. Insomuch as it will be my responsibility for the shape and findings of the study, I point toward theoretical metanarratives that may tell some of the story of my ways of interacting with the events of my experiences in this research study. Power relations trigger my senses in seemingly all interaction; I exercise power yet abhor the employment of power. Power is my freedom, yet it is this freedom that is my power. I am compelled to remake the world in my image, yet conscious of the stagnation of incentive and extinction of hope such a world would bring. I insist that I am the constructor of my experienced reality, yet recognize some sort of reality, external to my experiencing, constrains my constructions. And redirecting this binary once again, I am the sense-maker[58] of these

[58] I pause to wonder if I may more accurately be, the *generator* of these constraints.

constraints. "[W]e are complicit in the construction of ourselves" (St. Pierre, 2000a, p. 504).[59] An internal/external dualism for considering others or myself in a social world is insufficient. The humanist priority of the individual fails my subjectivity. The sociological mind-independent construct of knowledge fails my constructive orientation. My views align with postmodern thought; my drive for action reflects critical theory. In the field of mathematics education, I share epistemological values with radical constructivism. These theories carry a strong impact on the methodology employed to consider the generative adolescent mathematical learners (GAMLs) of my study. I name the theoretical frameworks that align with my thinking because it is necessarily so that my way of viewing the world defines the methodology I utilize to reflect on these perceptions. Paul and Marfo (2001) identify this connection among personal ontological, epistemological, and axiological beliefs, theoretical frameworks, and the selection of methodological procedure. In the opening of this chapter, I will make an effort to connect the metanarratives of my theoretical framework to the methods of my research.

The methods of this study were initiated by the interaction intentions of a conjecture-driven teaching experiment (Steffe, Thompson, & Glasersfeld, 2000). Additional qualitative methods enrich the data collected during the teaching episodes to more fully address the research problem. Ultimately, I created three cases—initially in order to describe examples of GAMLs, and then for the purpose of examining further the development and deconstruction of a model of the GAML. Each of the cases is not being studied in order to understand, or *know*, that particular subject. Although each interesting in its own right, the intent was not to better understand each particular case—an *intrinsic* study (Stake, 1995; 2000). Rather, this collection of cases has an *instrumental* purpose, to illuminate the theory proposed by this research agenda; the study of the

[59] Stirner (1845/1971): "I myself limit myself" (p. 190). Foucault (1982/1988): We are "much freer than we feel" (p. 10).

case is to gain "insight into the question" (1995, p. 3). The goal for studying these three subjects was to provide insight and new ways of thinking about the concurrence of authority and agency in relation to the construction of knowledge in the high school mathematics student.

In this chapter I will begin with a brief consideration of the politics of methodology, in the relation *between* scientific research and the field of mathematics education, in relation *to* my theoretical framework, and in the relations *with* my subjects. Next I will provide a brief account of my researcher subjectivity within a critical postmodern frame. This discussion is followed by an overview of the methodological procedures engaged during the study, with links to the theoretical frameworks presented previously. I conclude the chapter with an account of the researcher's ethical responsibilities, to the degree I was aware during the study.

The Politics of Methodology

Research is a political act (Eisner, 1988), relating to power and authority, to the social relationships studied within the research, and among those whom the work is to be distributed. If research is an effort to produce knowledge, than that production—that knowledge—is intricately linked to social and historical constraints, and affordances. This knowledge, in its birth, is both constrained socially and historically, but such constraints are always already that of the constructor—the researcher. Yet in its second incarnation, this knowledge belongs to the reader of the research. It is again a whole new knowledge.

Knowledge as a pointer toward truth carries with it the affairs of the social, historical, and perceived realities of the knower. "Discourse and politics, knowledge and power are… part of an indissoluble couplet" (Apple, 1991, p. vii). "Power and knowledge directly imply one another…" (Foucault, 1979/1995, p. 27). So the knowledge production of a research endeavor must be taken seriously, with an eye to keep this knowledge construction at play. "To politicize knowledge production means not to bring politics in where there were none, but to make overt how power permeates the construction and legitimation of knowledges" (Lather, 1991, p. xvii).

The goal for attaining knowledge is a struggle for power. Perceived (constructed) differences in knowledges are a principal component in the different relations, the power relations, among people. The postmodern condition re-presents this mantra time and time again; knowledge is not value-free. "Poststructuralism, [an] example of a truth game, with its claim that the establishment of a truth game necessarily means the repression of other truth games yields an ongoing critique of the dominance of any truth game, whether the game is scientific, Marxist, or any other one" (Scheurich, 1997, p. 35). Hence, a postmodern, or poststructural position does not solve the problem of value-laden or power-laden knowledge. It takes it as a given, and troubles it.

Relations between Research and Mathematics Education

The postmodern recognition that research is political creates a certain power play of acceptance within the research community as a whole for the value of postmodern research, and in particular within mathematics education. When the hegemony of the humanist science is challenged, from within its institutional nature swells resistance. It is a science with instincts to reject what may destroy its authoritative grip. In 2002, the National Research Council (NRC) released *Scientific Research in Education*, arguing that all scientific endeavors share a common set of principles, specialized for the particulars of what is to be studied. The document is written in response to the "evidence-based" policy recently written into federal law that financially supports the bulk of education research programs. The NRC report warned against legislated mandates reifying particular methodological approaches. The authors intend that their inclusive view of science will build a stronger sense of researcher community (Feuer, Towne, &

Shavelson, 2002). The journal *Educational Researcher* published an issue dedicated to this seemingly innocuous document in November 2002 in which several educational researchers argued strongly that this inclusive view of the NRC did not include them, nor would it support the goals it believes it will address (Berliner, 2002; Erickson & Gutierrez, 2002; Pelligrino & Goldman, 2002; St. Pierre, 2002). Instead, the NRC report presented a rather particular, non-encompassing approach to science—one that not only excludes, but also appears to be ignorant of current research theory and practice. The report, in its narrow definition of science as positivism and its methodology as quantitative (St Pierre, 2002), directly excluded the “extreme” (NRC, 2002, p. 20) postmodernism and excluded by omission other theories such as feminist, race, queer, critical, and poststructural; even though it alleged support for scientific diversity. Erickson and Gutierrez (2002) named the NRC presentation of science as that of the “white coat image that appears to the layperson” (p. 22), a far cry from the actual work of scientists, and in particular educational scientists. They declared that a cause-effect science is known simply to be inadequate in education, and more broadly in any scientific field. Furthermore, “real science is not about certainty, but uncertainty” (p. 22). The real danger of this report, to the field of research in education, is the employment of its *disciplinary power* (Foucault, 1979/1995) to claim the valuation of diversity (alongside the brashness of outright rejection of particular views) from within a single epistemological framework (St. Pierre, 2002). “Epistemology is also the issue in the… failure of the committee’s rhetoric of inclusiveness—its rejection of qualitative methodology even though it claims to support it” (p. 26).

In a very similar way, qualitative research in the field of mathematics education maintains only a tenuous grip on acceptance as worthwhile science. Not only has this federal policy shift caused qualitative researchers to move further underground, less likely to receive financial support for their work, be published, or accepted in policy decisions, but has also quite literally removed previously accepted research from readily accessible resources.[60] Many of the theoretical frameworks identified by St. Pierre (2002) are the perspectives that tend to be interwoven with equity and social justice work, theory building in mathematics education, as well as in education writ large. Researchers working with these excluded theories tend to draw heavily on qualitative research methods. Equity work continues to be marginalized in mathematics education (personal communication, Laurie Hart, May, 2005), while equity as justification for research in mathematics education seems to be normalized (Lawler, 2005), a status of ever-present, feckless inconsequentiality.

A very recent exclusionary practice in the name of Science from within the field of mathematics education itself was the self-replicative, tightening of the guard,[61] via its participation in and support of the National Mathematics Advisory Panel's (2008) procedures for making recommendations to the nation on mathematics education. The paper juxtaposes two significant ideas, when placed together make a strong implication on what research is valued. First, the Panel recommends, "Instructional practice should be informed by high-quality research" (p. xiv). Next, the panel defines what type of research is worthy of its own consideration for analysis and influence upon the recommendations of the panel. This research is defined to be "studies that test hypotheses, that meet the highest methodological standards (internal validity), and that have been replicated with diverse samples of students under conditions that warrant generalization (external validity)" (p. 82). They claim for themselves "The Panel took consistent note of the President's emphasis on 'the best available scientific evidence' and set a high bar for

[60] For example, the 2003 decision to remove all research not meeting the federal guidelines from the ERIC database.
[61] My claim is that much "non-scientific" research, with honest concerns for issues of equity and social justice, chip away at the foundations of the theoretical footholds and bridgeheads of mathematics education, and mathematics writ large. I recognize making this claim demands further attention, which must be left for a separate paper.

admitting research results into consideration" (p. xvi). So by defining a particular, white-coated lab technician view of research and claiming it to be the highest of quality, along with recommending mathematics education ("instructional practice") be informed by high quality research, communicates a valuation of a singular, narrow view on research. Even more narrow than the 2002 NRC report. This languaging reflects the power games at play to marginalize certain, powerful, research practices in mathematics education.

A postmodern epistemology, radical constructivism, is also marginalized in mathematics education. Its simple tenet, the call for the active child, has been whole-heartedly taken up within the field, yet the powerful, radical move of its postmodern ontology, is either lost upon or rejected by the field. The resulting trivial constructivism, as the modernist epistemology du jour, yields a convoluted, confusing, and dichotomous learning theory (Steffe & Thompson, 2000) and seems to prescribe a "guess-what-I'm-thinking" (Postman & Weingartner, 1969) pedagogical approach to mathematics teaching. Such an approach follows Freire's (1970/2002) banking model in which what is to be known exists externally to the child and is a matter of the child figuring out what and how she is to think.

The marginalization of equity work and postmodern epistemology in mathematics education is evidence of the politics of methodology. When ways of thinking upset, trouble, or challenge the dominant mindset, power relations, truth games, and the politics of language come to play. Eisner (1988) argued that in the realist mode of thought, propositional language is the vehicle for precise communication. "When terms are made conventional and rules of syntax codified, the possibilities of shared meaning is increased" (p. 16). This desire for precise communication is not only shared by the research community writ large, but a belief that "Mathematics is the ultimate expression of precision in such matters" (p. 16), suggests mathematics education is twice primed for such restriction. Knowledge is defined within this realm of thought—propositional language; certainly this way of thinking about knowledge has been dominant in the field of mathematics education. Those who use language in unique ways are hard to understand (Lather, 1996; St. Pierre, 2000b). The research community concerns itself so greatly about the need to demonstrate the truth of knowledge claims (e.g. Goldin, 2000); “We worry about claims that cannot be tested, and we believe that unless assertions are made in propositional terms, we have no good way to test their truth” (Eisner, 1988, p. 16). The hegemony of these ways of thinking maintains their status and power relations in the field.

To consider a postmodern ontology in mathematics education is to alienate oneself from the field. “The politics of method breeds a sense of community among those who adhere to its principles” (Eisner, 1988, p. 18). Research in mathematics education, like any system or institution, seeks stability, maintenance, repetition. “To maintain control and to ensure ideological immortality we train acolytes to keep the tradition alive (Eisner, 1988, p. 19). A science which values thinking differently (Derrida, 1978; St. Pierre, 2000a) pursues lines of flight, seeking heterarchal, rhizomatic relations (Deleuze & Guattari, 1987), and takes as its method the non-method of “anything goes” (Feyerabend, 1975), is in danger of having no voice in the politics of research in mathematics education.

Relations to Theory – Validity

As a critical poststructural thinker enacting a radical constructivist theory for learning, I am thrust into awareness of the power relations of the research community and the mathematics education community. A benefit of finding my ways of thinking compatible with these macro- and mid-level theories is that I am attuned to the politics of method. While “[e]ach epistemological enactment… is a political enactment" (Scheurich, 1997, p. 49), the methodology emerging from my epistemology is also necessarily political.

It is not to me to have selected a methodological framework within which to work. It is a silly idea that I choose from among an assortment of methodologies, extant knowledges. Instead, it is for me to attempt to communicate the methodology that my ways of seeing the world, my theory, produced. To assume I can select from a variety of methods that which best suit me in the moment or suits "the data" (as though it existed prior to my construction) is a naïve view of method. With an honest attempt to treat the meaning of data, knowledge, and my written language as that which is of my own construction/creation, I will also seek to communicate my ways of thinking about these concepts—the self-referential circle of research—through careful choices of language, citation, and knowledge structures accepted in the realm of educational research to point to my methodologies and rationale for such methods, as well as to attempt to create a sensation of validity in the reader for this poststructural research endeavor on generative adolescent mathematical learners.

Although "Most scholarly work in education pays little, if any, attention to its epistemological assumptions" (Scheurich, 1997, p. 30), I have placed these at the forefront of my work (hence the subtitle, *The Fabrication of Knowledge*). This research takes seriously that it is not a work of Knowledge[62] creation, but work to engage others to think differently. No research could create a statement of what *is* or who these high school mathematics students *are*, let alone why they are generative. These pointed-to notions of our modernist mind-set are merely apparitions, images of fact in place of what is unknowable. Recall St. Pierre's reminder that "'[T]ruths are illusions about which one has forgotten that this is what they are'" (2000a, p. 497, quoting Nietzsche). Nietzsche called the Western knowledge project the will to power

[62] Knowledge here is used in an untroubled, modernist manner, thus capitalized.

(Scheurich, 1997; Spivak, 1974). According to Foucault (1977), Nietzsche argued that a "historical analysis of this rancorous will to knowledge reveals that all knowledge rests upon injustice… and that the instinct for knowledge is malicious" (p. 163). Given that truths are unknowable, those who give false image to know the truth would be gods in such a world. A science in which the pursuit of such knowing is at the fore, then, is a science seeking omnipotence, a project toward reverence and veneration. It is work to establish a demarcation between the knower and the ignoramus. Validity serves as the line separating the two, what Scheurich (1997) called the "line of bifurcation for a two-sided map" (p. 81). He went on to argue, "My contention, then, is that the various kinds of validity, across both conventional and postpositivist paradigms, are a civilizational project, an imperial project" (p. 87).

But what can be made of validity given such a postmodern epistemological stance? The positivist take on validity in qualitative research is a notion derived from a testing perspective, that a test is valid if it measures what it purports to measure. Validity was applied as a truth criterion. Social science asserted that given a proper design and careful treatment of method, research could establish an objective truth, or at least a good approximation of this truth. Validity spoke to the trustworthiness of the research findings (Lincoln & Guba, 1985). Instead of engaging a notion of validity that inscribes a two-sided truth trustworthiness map, I seek to problematize such a dualism as well as its appropriation of those who are othered in the Western knowledge project. I don't seek to reject validity in its design intention: to elicit thinking and to create quality work. I do, however, wish to activate imaginaries in order to unmask and undermine the dualistic regularity that unknowingly shapes our knowledge consumption, to engage multiplicity, and to respect the play of the Other. I seek imaginaries that "reconstruct 'validity' or 'truth' as many sided or multiply perspectival, as shifting and complex" (Scheurich, 1997, p. 88).

Both a postmodern and a radical constructivist epistemology recognize that data do not exist free from the knower. What then to make of the hallowed research precept validity when *data collection* is really *data construction* or *data creation*? Lather (1991) said, "Data might be better conceived as the material for telling a story where the challenge becomes to generate a polyvalent data base that is used to *vivify* interpretation as opposed to 'support' or 'prove'" (p. 10). I seek a "generative methodology" (Lather, 1993, p. 673) in which possibility is at center. Opening up questions rather than answering them is my intent. My method does not seek to convince; it works to provoke thought.

A second overriding concern in my research design, as associated with validity, is to bring forth the subject and the subject's ways of operating in ways that are not merely mirrors of my own knowing. It is my question as a researcher, "how can I engage or interact with 'the otherness of the other without transforming (her or) him into purely one's own (i.e., the Same)'" (Scheurich, 1997, p. 88, quoting Bahktin). Of course, moving beyond mere observation of the subject into dialogue, collaboration, or even teaching is a start toward such a goal. Scheurich presented three possibilities that help move beyond this simple start to avoid saming. He noted that Ellsworth suggests that any non-local prescription of the proper approach is another way of the knower retracting the subject back into a particular truth regime. Ellsworth argued for a local validity of local knowing and local choices. Similarly, White (1991) suggested another posture, or attitude of 'attentive care for the other,' requiring a much stronger injunction to listen to the Other, holding open the space for difference to unfold in its idiosyncrasy. There must be an intentional effort to suspend the habitual assimilation of language, gestures, writing, and behavior of the subject. Good examples of such work can be evidenced in Visweswaren's (1994) *Fictions of Feminist Ethnography* and Stewart's (1996) *A Space on the Side of the Road: Cultural Poetics in an "Other" America*. Yet the actualization of such ideals for research methods is unrealistic. "A civilizational project is so preconceptually embedded in all that we do and think that altering it is disturbingly difficult" (Scheurich, 1997, p. 89). The poststructural realization that we are incorrigibly drawn to engulf the Other into our own does not allow for the fairy-tale naïveté of simplistic corrections to remain unfettered.

Scheurich's (1997) third alternative to methods complicit in saming is evident in Lather's (1993) transgressive validity. In her reconstruction of a poststructural validity, she proposed four specific types of validity: ironic, paralogical, rhizomatic, and voluptuous. Each reflects three particular qualities of Lather's orientation to research: to unsettle truth regimes—her critical focus; to make evident the insufficiency of language, cultivate difference, and embody an explicit tentativeness—her postmodern focus; and to anticipate "a politics that desires both justice and the unknown" (p. 685) through the proliferation of open-ended and context-sensitive criteria—her ethical/political focus. Lather's transgressive validity practice evokes a kind of doubled strategy. She subverts the binary of Same/Other power relations at the same time embraces the play (Derrida, 1978), the possibility, of difference. Yet Lather (1993) posited "the conditions of possibility for validity are also its conditions of impossibility" (p. 687). Instead of fearing the vacuum of Saming the Other, my research methodology emerged as both a concerted effort to invent and invoke counter-practices of authority through reflexivity embedded in both design and method. At the same time I did not shy away from the unavoidable Saming my ways of knowledge production create, both for myself and for the readers of my research. By embracing this imperialistic knowing, I work to subvert its results, disrupt its tendencies, and displace its comfort. Not only did I work "to see what frames my seeing" (Lather, 1993, p. 675), but I must also challenge my reader to do the same. I work toward this challenge through my method, data report, and analysis.

Relations with Subjects

It is in the imperialist potential to Same the Other that I worked carefully, reflexively, to keep the sedimentation of knowing at bay. I did not *know* my research subjects at the conclusion of data collection or analysis. Nor will my reader as a result of my presentation of these generative adolescent mathematical learners. Language "can easily be used to substitute concept for percept, the name of the thing for the thing itself" (Eisner, 1988, p. 17). Eisner goes on to note that this point is also made by Dewey, who in *Art as Experience* (1934) distinguished between seeing and recognizing. Eisner wrote,

> Seeing requires sustained attention to the qualities of an object or situation; it is exploratory in character. Recognition is the act of assigning a label to an object. Once assigned, and classification has occurred, exploration ceases. When in our teaching, our curriculum, and our research methods we emphasize the prompt classification and labeling of objects and events we restrict our consciousness and our likelihood that the qualities of which those objects and events consist will be experienced. Thus, our awareness is always limited by the tools we use. When those tools do not invite further sensory exploration, our consciousness is diminished. (p. 17)

The tools of my research method invited this sort of further exploration. I sought to pose new questions for myself and for my research audience. I hoped to deter knowing. I hoped to make impossible the drive to engineer solutions, to fix perceived inadequacies of my research subjects or other adolescent mathematical learners. To promulgate the knowledge regime would be an injustice to my research subjects, and to my research consumers. Lampert (1990) made an observation which, if taken seriously, can give great pause to our efforts to study children, their knowledge production, and the production of their dispositions to learning: "While they are learning mathematics, these students are also learning, tacitly if not explicitly, to place mathematics appropriately in the lexicon of ways of knowing" (pp. 33–34). If this statement is taken as a given, playing with postmodern definitions of each key term of the statement opens the door for new questions, ideas which may be, at the moment, too hard to consider.[63]

In this section, I discussed the ways in which a methodology unfolds from ones own theory/epistemology, and that such a methodology is the only way that one actually does act within their relations with others. It is the researcher's duty, as a trustworthy data collector and responsible author, to communicate the best she can what this methodology may be. Communication in this research was accomplished by pointing to previously accepted theories and theorists. The effort to build a shared understanding was done through the citation and linking of various thought. To take the stand that the methodology utilized in this research is of my own construction, however, leaves the validity of my work to be more questionable. Why should the findings of this aloof researcher be considered legitimate? I have argued two reasons. First, that as constructors of our ways of knowing, no one's methodology is another's. That is, a researcher can in no way replicate the methodology of another person or of some research tradition. The postmodern is characterized by this loss of absolute frames of reference (Lather, 1993). Second, my research goal is not to create universal truths, data and summaries that can be replicated, or generalizations that will inform future mathematics educators on "what works". It is my goal to challenge the mathematics education—including research—community, and myself, to think differently (St. Pierre, 2000a), to consider the sorts of mathematical thinkers that should be the goal for school education, to maintain a picture of the whole while focused in on one's particular interests. In fact, Lather (1993) positioned a poststructural validity as "an incitement to discourse" (p. 674). In taking these steps, I risked weakening the position of this

[63] A complete address of The Politics of Methodology would include an exploration of my own relations with my self as a knower. To avoid the potential read as self-absorbed, or as a bellybutton gazer, I elect to not and next bring conclusion to this discussion.

research project. I cannot rely on the reader's automatic or blind acceptance of the validity of my research through the repetition of previously accepted standards. My work for validity must return to an effort to simply build a quality documentation and communication of my research endeavor, a smart analysis of data, and the outlining of provoking questions that engage, rather than persuade, the reader. To begin, I took care to see what frames my seeing; I discussed the ways in which I reflected on and managed my own subjectivity within the research process.

Researcher Subjectivity

Subjectivity in qualitative research is about recognizing the researcher's role in the data (Bogdan & Bilker, 1998). The researcher is not only the machine gathering data, but that the data are the constructions of the researcher and that the analysis of the data is the meaning constructed by the researcher. The qualitative tradition has released the notion that an observer should be interchangeable, that the researcher is an unbiased mechanism through which data is gathered, shuffled, sorted, and presented. As an active part of the data (and the whole research process), the researcher must position herself toward both the research (standpoint) and the research community (posture). I have worked toward identifying this standpoint and posture through my careful development of epistemological, theoretical, and methodological views. I conclude this effort through a final phase in which I declare my subjectivity, my biases as well as where I may be particularly poised to enhance illumination.

A researcher's subjectivity is another of these "garment[s] that cannot be removed" (Peshkin, 1988, p. 17). Glesne (1999) connected researcher subjectivity with the rapport built with the subject(s). She explained that the researcher's "capacity and limitation for establishing rapport are affected positively and negatively by your subjectivity" (p. 111). I have been a high school mathematics teacher for nine years. During this time I have worked with numerous student populations, including those from a middle- to upper middle-class western states suburban school, a highly diverse west coast urban magnet school, and two rural-big-city southern schools. I have also worked with these students in a variety of ways beyond just being a classroom mathematics teacher. I have also served as guidance and personal counselor, a swim coach, and an academic coach. Whom I have been influences the relationships I develop.

Rapport

In addition to being this imprecisely defined high school mathematics teacher, I have spent the previous eleven years conducting professional development of high school mathematics teachers. In this position I have met an even more extensive array of teachers, representing most any sort of school imaginable. In my professional learning associated with these positions, I studied formally and informally the role of rapport in relations (Costa, 1994; Garmston & Wellman, 1998; 1999). I value and work to build trust by developing physical and verbal rapport. As an intentional part of my professional relations, this awareness when entering the field for my research was fully engaged. I had collaborated professionally with the teachers of my study for one year prior to approaching them about the study and their willingness to participate. Also during this year of collaboration, I both observed and taught a few math classes in this particular high school. I had begun to develop relationships with a few students, but more importantly had begun to feel at ease in the school environment. This comfort level and familiarity with faculty and staff eased the transition as I became more tightly integrated into the two classrooms that were ultimately involved in my study.

Throughout the study, I sought a relationship with the adolescent subjects to be that of a *teacher-as-student*. I determined this relationship was the interaction I knew best with adolescent children. I felt this interaction would most support the potential for the data I wished to collect for two reasons. First, because it was my most familiar, and second because the way in which I would know these children would hopefully create the greatest potential for sharing experiences with other high school mathematics teachers. In other words, establishing a teacher-as-student relationship not only allowed me to utilize my *ears to hear*, but also opened up the greatest potential for possessing the *words to speak*.

The two teachers of the classrooms in which I worked were also significant sources for both data and data checks. I sought to develop a peer relationship with them. Although they perceived me as possessing some sort of particular expertise on either a curriculum, teaching method, or whatever comes with being a graduate student, I saw them as having a great deal of expertise on teaching mathematics, and in particular on the children of their school and classrooms. As I worked to effect a sort of mutually yet differently-expert status, we were both able to learn a great deal from each other. Not only for data toward my research, but also for teaching high school mathematics.

Glesne (1999) believed that the development of rapport enables the participants to be more willing to talk about personal issues. Rapport is not friendship, in particular in that in rapport one's "need to be liked is overshadowed by the necessity of being accepted and trusted" (p. 96). But the connection to and potential to create rapport are not all that subjectivity is about. Glesne argued, "Reading, reflecting, and talking about subjectivity are valuable, but they are not a substitute for monitoring it in the process of research" (p. 110). Awareness of subjectivity also involves maintaining an awareness of ones emotions. Glesne claimed that awareness of subjectivity contributed not only to trustworthy research, but also to greater understanding of oneself and one's psychological investment in the research (p. 95). She suggested that the researcher, rather than suppressing emotions, should use them "to inquire into your perspectives and interpretations and to shape new questions through re-examining your assumptions" (p. 105). I monitored my subjectivity throughout the data collection by recording both the classroom incidents and private interactions and my reactions to them.

As further evidence of my ability to develop rapport, I remain in contact with the two teachers with whom I collaborated on this project. Although our close, research-based work lasted 5-6 weeks, we have a professional relationship across five years, including over three years since the data collection was completed.

Standpoint

In addition to the role of subjectivity in developing rapport, my outlooks on the goals of mathematics education greatly influenced the manners in which I experienced interactions between and among teachers and students in the classroom. As a mathematics educator who works for social justice, I am often engaged among what I perceive to be injustices in our education system, injustices that negatively affect both children and teachers. It is my tendency to blame others, including teachers or administrators or at times parents. I form this sort of blame to be an ignorance, an inattentiveness, or at times even an outright self-centeredness. However, I balance this sort of blame with a structural awareness. I try to take seriously the impact of the cultures, the histories, the constraints each has constructed of the world within which they operate. This balancing is not a sort of "It's not their fault", but more one of working hard to recognize that people's ways of operating are reasonable and quite sound given the presupposition they want what is best for both others and themselves, and that their own experiential reality guides them toward the decisions they make to be believed to be the best. This is a challenge that pries at my sensitivities toward what I perceive to be unjust; my attempting to rationalize a well intentioned Other calms my indignance. So in a sort of double-reflective move, I remind myself of my initial reaction. This is a statement of the radical constructivist ethics; a sort of decentering ideal that I seek to achieve. All this said, my “blame” is a way of refusing to shrug my shoulders and accept things as they are, but to begin to identify where work needs to be done so that things can be more just (through my eyes).

I am also a critical educator. For me, this encompasses a slightly different focus than being an educator for social justice, fairness. That I am a critical educator means for me that I don’t accept things as they are, but hold tightly and optimistically to ideals, and strive for them. I am critical in the sense that I attempt to focus on improving inequities, rather than ignoring them. Having a critical slant is often interpreted by others as being cynical. Upon reflection, this is a reasonable judgment of comments I make—I expect others to see the world as I do, and can be frustrated when I feel peers are either ignorant of or seem to ignore the issues in mathematics education that I value. Not only does this critical stance impact my interaction with peers in mathematics education, but also significantly impacts the image I have developed for the mathematics classroom, the qualities of a teacher I value, and the qualities of the student I seek to develop. The impact of my subjectivity in relations with my subjects regarding the first two of these qualities has been discussed above. The resulting idealized student plays a significant role in my research.

This ideal for an adolescent mathematics learner, the qualities I am attempting to gather under a name “generative”, developed to a great extent throughout my history as a mathematical learner. Like many young children, I demonstrated a joy for playing with numbers and quantity. As my mother’s first child, each of my insights was spectacular in her eyes, and I suspect her interaction spurned me to continue such inquisitions. One of her favorite stories is of me at a very young age questioning the result of subtracting a larger (whole) number from a smaller, while she worked in the kitchen as I used the bathroom nearby. She claims that I created the equivalent of the negative numbers to allow for this subtraction. Through my own mathematical activity, along with the resulting positive environmental feedback—in this case my mother, at a young age I had begun to find joy and passion in quantitative reasoning. This was reflected early on in my school mathematics being marked by exceptional success. I clearly remember the challenges of counting, by various increments, to 100 that my first grade teacher had us recite to her at her desk. Later in elementary school, I, like other kids, was placed into a math group that was ahead of other students. Eventually, in sixth grade I was working alone on mathematics, completing workbook activities on my own pace.

Upon moving to a new state, due to miscommunication between schools I was placed in the lowest tracked 7th grade math class. I recall a particular moment of the class where for the first time, I became conscious of not enjoying mathematics and furthermore, becoming aware that what was being presented to us by the teacher was unnecessarily tedious and inconsequential. We were being subjected to an expectation to make sense of highly formalized and symbolic statements of the obvious—the associative and the commutative properties for various operations on the integers. Upon expressing my displeasure at home, I was moved to regular 7th grade math at the quarter. This new teacher shortly thereafter moved me again into her pre-algebra course, and I continued stepping through my mathematics education, becoming more and more proficient and less and less enamored. Through my work in high school's top class and a bachelor's degree in mathematics, I found some challenge in learning new ideas and completing procedures accurately. My mathematics education consisted of learning to replicate the teacher's steps and thought processes, something Boaler observed in her research (e.g. Boaler 1997) calling this learning to interpret the *cues* of a teacher or textbook. I believe I came to be able to make some sense of the larger body of mathematics I was being taught through connections I built, or understood my teachers to be pointing out. However, mathematics for me had evolved from an activity in which I sought to answer the questions I posed (on the toilet), to making sense of new ideas that were clearly external to me.

My school experiences paralleled this distancing to the living nature of the school. My loss of experiencing a joyful learning came with an increase in resistance to the authority, manipulation, and coercion of the school environment. As I approached my senior year in high school, I evolved from the studious perfect-attendor to the subversive, rule-manipulating, minimalist. I evolved from participating in school because I enjoyed the interaction and the thinking, to subverting school as a playful intellectual challenge. The danger of pushing up against the boundaries of trouble without crossing became a game for me. Any grade more than a 90% was wasted effort. Any rule-breaking strategy to not attend class was free game. Any move to frustrate authority by playing at the limits of their fiefdom brought thrills. Yet I consistently attended, and participated as a leader, in four music groups and the varsity swim team. I was successful at what I chose to do, or maybe it was that I chose to do what I enjoyed, and by fully attending to my joys I was recognized as being quite successful.

Next, my mathematical learning and my learning about the educational institution turned to that associated with being a teacher (surprisingly?). At the onset of my career, I elected to manage an entirely reoriented view of mathematics education, held by my new colleagues, expected by the NCTM *Standards* (1989), and enacted by a new curriculum being developed through support of the National Science Foundation (NSF) called the Interactive Mathematics Program (IMP) (Fendel & Resek, 1997). As I experienced the mathematics of this curriculum, whether it was working alone, with colleagues, or through interacting with students, I was (re-)

opened to an entire new sense of what math might be. I was reminded that doing math meant solving problems, being creative, guessing and trying, and struggling and thus making one's own sense. I was renewed as a mathematical thinker. Once again I found myself as that child, interested in pursuing ideas (quantitative problems) that were of my own formation.

As I've continued as a teacher (and increasingly as an academic), I have begun to pay much closer attention to the phenomenon of agency, identity, and authority in the mathematics classroom. I've observed students actively and pleasurably involved in significant mathematical activity, arguing, justifying, exploring, and generalizing. Many times I observed this involvement to be followed by a disconnection with what these students considered to be mathematics. My students would say, "That was fun work, but it wasn't real math." Quite clearly, these students, etched in my mind, saw mathematics as NOT as something *they* do and certainly NOT as something they enjoy. They were not the actors in mathematical sense-making or the authority in inscribing mathematics upon their world. As mathematics holds a powered position in our culture, its reinforcement ubiquitous in the classroom, school, and professional community through seemingly innocent means—mathematics as half of HS graduation tests or college entrance exams, NCTM's slogan "Math is Power", and even the rhetoric of "algebra for all" (Lawler, 2005; Martin, 2003)—children suffer, in losing or in winning (locating self as innately inferior or falsely superior), in the meritocracy of cultural capital (Bourdieu, 1977; Nasir & Saxe, 2003) that defines our dehumanizing, postmodern society.

While policy documents call for continuing the sense of engagement and ownership in mathematical learning as necessary through all grades (Allexsaht-Snider & Hart, 2001; and see for example Georgia Department of Education, 2004; Kilpatrick et al., 2001; NCTM, 2000), the same documents work against this goal (Apple, 1992; Martin, 2003; Popkewitz, 2004; Steffe, 2004). I wonder about the possibilities for children to maintain an attitude of play when engaged in mathematical learning, as they grow up, educated in school. Kamii abandoned her intent to extend her work with children learning mathematics through play (Kamii & Housman, 2000) into higher grades because “play is schooled out of children by Grade 5” (personal communication, October, 2002). I suspect some children must still view learning as play even as they near the end of high school. What makes this possible?

Here is where I have arrived, from the young boy doing mathematics with his mother, to the researcher wondering what happened to me~~,~~ and the majority of adolescents in mathematics education, that separated this wonderful field, these mathematical ways of knowing, from our selves. Did my independence emerge from this distancing of school mathematics and my mathematics? Or was it that some sort of independent quality of my being that led toward this split? Maybe it is not an either/or question, but a question to understand how it may be both/and.

Ultimately I seek in adolescent learners this idealized vision of myself as a knower, a constructor of knowledge, an author of my world. I am drawn by a reverence for children in whom I can see images of myself. Stirner (1845/1971) wrote, “Man… cares for each individual, but only because he wants to see his beloved ideal realized everywhere” (p. 83). I ultimately seek in mathematics students the ideal image of myself. My research is guided by my wonder of the ways in which such a student may exist, and the ways in which they are consciously and unconsciously aware of these characteristics of generativity that I ascribe to them. My subjectivity then is both a danger and the driving force behind this research. I have both embraced the constraints through which it caused me to experience the data, and also worked strenuously to step aside and attempt to observe myself observing, collecting, and analyzing the data I created. Such a powerful *and* dangerous prospect!

Posture

Having declared much of what is involved in my standpoint as a researcher, I now consider my posture, toward the research community, and in particular the mathematics education portion of this community. Prior to enrolling as a doctoral student in mathematics education, I was an indirect consumer of research. “Research” is what I was told supported the 1989 NCTM Standards document. This document declared what mathematics was important to know, grades K–12. But more significantly, this document, along with NCTM’s professional (1991) and assessment (1995) standards shaped a vision for mathematics classrooms and the work of mathematics teachers that significantly guided my professional learning. By the mid to late 90’s, research had taken a new meaning—research was the esteemed arbiter of what was the right way to teach mathematics. “What does research suggest?” morphed further into “Does research say this works?” “Research” could be played as a trump card. In a more interesting twist, I learned that there seemed to be research supporting any sort of argument about how mathematics should be taught (cf. Wilson, 2003 on the “math wars”). However, the *anecdote* took on an interesting political role. When the right person spoke, the anecdote about “their child” could be the most persuasive argument on the table.

What really then did research mean if it could yield seemingly opposing conclusions? And what sort of power did it wield? How does this power interact with agents for change? for practicing teachers? and even for children in their classrooms? As a graduate student I have learned how to be a more effective critical consumer of research. But this is a minor result of my schooling. More importantly I have learned much more about the power relations and the people who play among these relations. I saw that the mathematics education researchers, the academics, the professors, are held in a place of esteem. Their intelligence and knowledge is perceived to be fact. They are the creators of knowledge and the providers of truths. Yet to the extent I know some of the people working in the field, I have found that no researcher works as an unbiased, value-free, truth-seeking agent. Each, to one degree or another, seeks to broadcast their own agenda, their own beliefs, their own biases. Further, it is the structure of the institution that researchers must both produce these unavoidably biased truths and promulgate their veracity. Not only is it that quality suffers at the demand for quantity, but more insidiously the institution propagates its existence, bolsters its position, and more resolutely makes its truths the truths of the mathematics education community.

In a postmodern society where this academic institution, this truth regime, unwaveringly and stubbornly holds to an enlightenment-era science, the pressures of two such tectonic plates erupt~~s~~ into severe disconnection between ways of knowing, being, and acting. The classroom mathematics teacher who knows children cannot comprehend the declarations of research-based policies. And educational scientists, engineers, and architects are left blaming teachers when the perfect design does not yield the modeled results. In effect, I have learned, through firmly having one foot in each the research and practice arenas, that a new practice of science must be formed if the research community wishes to speak to the practicing educational community (Wiliam, 2002). It is also the case that the educational community must reinvent itself as a learning profession (Hargreaves, 2003, de Lima, 2001).

It doesn’t surprise me that research speaks many, seemingly incongruent, truths. I doggedly work to deracinate these truths, to place them next to other truths in ways to elicit thought, possibility. The research I value is not that which seeks to declare, but that which seeks to wonder. So my posture toward the research community is a wary one. Bové (1990) provides me with ways to interpret its products, its discourse. I no longer seek to know what something *is* or what is it's *meaning*, but rather: How does a specific discourse, say for example, the discourse of 'constructivism' function? Where is constructivism to be found? How does constructivism get produced and regulated? What are the societal effects of constructivism? How does constructivism exit? This is not a position of distrust; rather it is one of valuing multiplicity. As Foucault (quoted in St. Pierre, 2000a) stated, "I believe too much in truth not to suppose that there are different truths and different ways of speaking the truth" (p. 498). I accept the multiple realities that varying perspectives pose. I value exploring different, nonessentializing questions that trouble the numbing hegemonic discourse of sameness. My posture toward the research community is to produce different science and to produce science differently, to paraphrase St. Pierre (1997).

My research embraces the inextricable link (LeCompte, Preissle, & Tesch, 1993) between theoretical framework and methodology. As a poststructural critical theorist, I am drawn to study oppression and power relations. And rather than study the discourse that constitutes the subject, I prefer to study agency as the "subject's ability to decode and recode it's identity within discursive formations and cultural practices" (St. Pierre, 2000a, p. 504). The tenets of radical constructivism concur with the poststructural notion that "we are complicit in the production of ourselves" (p. 504). As the producer of the knowledge associated with this study, I invoke research methods that acknowledge my centrality to the production of the subjects. In addition to the ongoing work to decenter in the role of teacher-researcher, I will retrospectively analyze this centrality. The data will inform a deconstruction of the role of knowledge in the mathematics classroom. The constructivist teaching experiment and the poststructural deconstruction are natural methods to consider in the design of my research methods. Next I will introduce frameworks for the analysis and data collection and then conclude with an evocation of research ethics.

Methods for Data Collection and Analysis

In this section I present frameworks for my research methods. In this presentation I will deemphasize discussion of the particularities of how I incorporated each of the methods utilized, but instead discuss each method as a technique for research. This will be accomplished through interweaving the theoretical structures I leaned on for collecting the data with a reporting of the pragmatics of actual data collection. Prior to describing in the next chapter exactly what data was collected, how, and when, I make notes on data analysis and the constructivist teaching experiment. This effort is intended to serve as a backdrop to better understand the emergence of methods for data collection and analysis that shaped this study.

Notes on Analysis

The convenience of confining the actual work of data analysis to a bounded, and spill-proof vessel distorts the actuality that it is a person doing the analysis, a person whose mind is not so easily shut on or off, who is not so neatly in the researcher role, than not. The analysis vessel may be better thought of as a Klein bottle—a theoretical mathematical bottle who's inside is it's outside—providing a better metaphor for both my data creation and my analysis of this data. Not only is it nearly impossible to separate my creation of the data from my analysis of it, but to suggest that I have analyzed data only during particular segments of time would be misleading. In her reflection on the research process, St. Pierre (1997) found new and productive ways to think about data. She noted two problems as a result of the shiftiness of presumed boundaries, such as those between data collection and analysis. First is "the notion that data, whatever they are, must be translated into words so that they can be accounted for and interpreted" (p. 179). Once these data, the researcher's constructions, are fixed by words, we believe we are to slice and dice these bits into categories, incorporating various techniques of the modernist knowledge machine. For St. Pierre, there is trouble to this process; in effect we are merely re-languaging with the guise to secure truth. The second problem of data is "the ruthlessly linear nature of the narrative of knowledge production in research methodology" (p. 179). This process, to employ methods that produce data, then categorize and interpret those data, and then to create theory of knowledge, not only ignores the fits and starts of knowledge production, but also suggests that all knowledge produced can be traced back to previously identified data bits. In my work, I did not shy from transgressive data (St. Pierre, 1997) but embraced the desire for validity as a method for data collection. In the spirit of communicating such a validity, I continued with a careful effort to report the methods for creating and analyzing data, although at the loss of embracing the fits and starts by the reader's desire for simple, intelligible, linearity to the reporting of method, data, and analysis.

Certain structures did focus and shape my data collection and analysis. In particular I can identify certain dates and particular research tools I drew upon in order to package this query into reportable investigation. An archeology (Foucault, 1969/1972) of the research question would point back to my experiences as a learner, including those as a boy learning mathematics to a young teacher learning about children who are being mathematical, to a novice academic beginning to theorize the goals of mathematics education, the role of the teacher, and the influence of the school. Instead of pursuing the potentialities of such "lines of flight" (Deleuze & Guattari, 1987), I will attempt to package the data collection and analysis more conveniently.

Once this research was given a go ahead, I was struck with the dilemma of researching the generative adolescent mathematical learner, something I had not yet defined and hoped would emerge from the research. The need to communicate what I sought in students became apparent as *Phase I: Subject Selection* approached. In many ways, my data analysis—data from my lived experiences of the life phases mentioned above—began prior to this first phase of data collection.

Going forth from Phase I, the four phases of my data collection/creation included and were conducted on the following timeline, each to be elaborated later in the next chapter:

Phase I	Subject Selection	February 21–25, 2005
Phase II	Co-Teaching, Observation & Interviews	April 4–26, 2005
Phase III	Member Check	June 27–30, 2005
Phase IV	Analysis	Summer 2005–Spring 2008

During Phase II, I conducted a 4-week co-teaching experience, observation, and interview. This process of co-teaching, observation, and interview emerged from a consideration of the radical constructivist tradition of teaching experiment. It was greatly influenced by my research goals to consider the mathematical learner's disposition, rather than to build a model for a way of mathematical knowing and operating. After an initial period of reflection and data analysis, Phase III was marked by my return to the research site in order to pursue interests and new questions, and to confirm or gather feedback on data collected in Phase II. I chose to distinguish the final phase of my research, further analysis, additional data creation, and writing, to be Phase IV. Rather than a drive to narrow findings and secure knowledge, this final phase is marked by *the intent to raise new questions about generative adolescent mathematical learners and the teaching of high school mathematics*.

My research question, How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition?, begs an answer to an initial question: What is a generative adolescent mathematics learner? As I began this study, I could answer by saying, "I think I know one when I see one." Of course I began with some early sketches of the generative adolescent mathematical learner (e.g. see Figure 1), but these frames needed input from actual high school mathematics students and their teachers. Part of my next steps was to go find out about these children, but I could gain little, if any, through a direct questioning toward my research goals. Grouws et al. (1996) argued that direct interviews about beliefs may not be beneficial, they are odd questions for people and responses tend to not coincide with actions. Further, "most previous work on student beliefs was far too removed from the context of learning" (p. 7). My research is at this intersection of beliefs and learning; it is about student dispositions in relationship with mathematical activity and learning. It only made sense to study these student dispositions in the context of learning and doing mathematics. Before defining mechanisms for selecting my research subjects, I next discuss the constructivist teaching experiment, a method that was designed to learn about children's mathematical ways of knowing and operating in the context of teaching and learning, and collectively doing, mathematics.

Constructivist Teaching Experiment

The constructivist teaching experiment is a conceptual tool, aimed at exploring the mathematical activity of students. It emerged at a time in mathematics education when classical experimental design inhibited efforts to understand student's sense-making activity, and an epistemological paradigm shift occurred in the field (Glasersfeld, 1990a; Noddings, 1990; Steffe, Thompson, & Glasersfeld, 2000). Not only were current research efforts inadequate, mathematics education recognized the postmodern relativism[64] of knowledge construction, both the constructions of the mathematics learner and of the researcher of this learning.

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[64] Relativism makes sense as an issue only when an assumed-to-exist foundational structure appears to be ignored. Cherryholmes (1988) noted, "Relativism is an issue for structuralists because they propose structures that set standards. Relativism is an issue if a foundational structure *exists* that is ignored…. [I]f there is no foundation, there is no structure against which other positions can be 'objectively' judged" (p. 185). Radical constructivism posits no

In a constructivist teaching experiment, the researcher uses teaching as the method to investigate learning (Cobb & Steffe, 1983; Steffe, Thompson, & Glasersfeld, 2000). Through teaching, the researcher is able to understand and explain how the student is operating, reducing the risk that their models are too reliant on theory, "distorted to reflect their own mathematical knowledge" (Cobb & Steffe, 1983, p. 85). As opposed to the clinical interview, the teaching experiment accounts for the experiential content of the teacher-student interaction. Not only does this allow the teacher-researcher to consider the context within which the subject does mathematics, but it also allows the teacher-researcher to embrace her productive relationship to the subject.

In the constructivist view, the teacher-researcher attempts to 'see' both their own and the student's actions from the point of view of the child. This conceptual analysis (Glasersfeld, 1995) of student's mathematical activity is an ontogenetic effort to build a model of a student's mathematical ways of operating. The methodology begins with exploratory teaching, free from hypothesis testing or retrospective analysis. During the course of the teaching experiment, hypotheses for student activity guide the researcher's intentions and activity. Additionally, between and within each teaching episode, the teacher researcher generates and tests smaller-scale hypotheses in order to understand what the student can do, and the ways in which this activity is rational.[65] Through retrospective analysis, the constructivist teaching experiment seeks to design a model for knowing consistent with the subject's ways of operating.

My data collection does not lend itself to a constructivist teaching experiment primarily because I seek to understand not only the GAML themselves and their sense of themselves as

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such a priori foundation, allowing for conjectured structures to be devised as models, yet keeping these models under erasure (Spivak, 1974).

[65] The Radical Constructivist's research agenda is to build models for knowing, models that assume a rational logic of relationships among knowledge and operation schemes; it is a structural project.

learners, but also to consider the context in which they operate as generative mathematical learners, i.e. the classroom. To build models of knowing, rational subjects as well as a superstructure defining the interrelationships of each of these models along with each modeled subjects'' experiencing of this superstructure and the other models within it is well beyond the scope of the project. Further, I did not have the resources to (A) interview high school juniors over the requisite extended period of time for the conjecture-driven series of interviews necessary to build a thorough model for any one GAML, in addition to (B) achieve the need of the research setting for me to participate in the GAMLs' learning environments. As a result, the teacher-as-student relationship I formed in the context of co-teaching the subjects' regular mathematics classroom was the best way to learn about the subjects' mathematical activity, ways of knowing, personal epistemology, and interactions with the other learners of their high school classroom. This teacher-as-student relationship was explicit; that although I enacted a role parallel to their math teacher, my subjects knew that I was also a student of them as mathematical people. I sought to learn what they could teach me about the GAML.

The effort of data collection and analysis I can be better conceived of as an ongoing cycle, rather than having two distinct phases (Shank, 2002)—*data* then *analysis*. Throughout the four phases of my data collection—in between and during—I reflected, coded, organized, and looked for patterns across the data. I formed and tested hypotheses for the subject's mathematical knowing and for their personal epistemologies. Through this ongoing analysis, I developed conjectures from the data, which guided my interaction with the subjects (Cobb & Steffe, 1983; Steffe, Thompson, & Glasersfeld, 2000). Furthermore, as I inscribed[66], reviewed, and discussed

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[66] Here I intend to quite strongly mean "fix" the data, cementing my constructions of activity I attribute to others, through textual encoding. St. Pierre (1997) reminds us, "Yet how can language, which regularly falls apart, secure meaning and truth? How can language provide the evidentiary warrant for the production of knowledge in a

data between and after the teaching episodes, my continuing analysis created new forms of data—not only data generated through my insights, conjectures, and connections of ideas, but also the data of emotions, desire, and new orientations to thinking provoked through the respondents—all forms of transgressive data (Lather, 1993; St. Pierre, 1997). Through this data analysis, I allowed for and reflected upon patterns and themes that emerged. But I also valued the singularities; moments that helped to form interesting stories about each of my subjects, selected in Phase I. After creating these case studies, these mental models of my subjects, through Phases II and III, I returned to the inscription of this data in the fourth and final phase of my data creation/analysis project, a deconstructive effort to consider the position of knowledge in relation to generative learners. In this Phase IV, I turned to refigure the role of knowledge in the cognitive and affective learning of the subjects, and in my construction of the subjects. The intent of this deconstructive effort, paired with the creation of case studies, is to challenge my research audience to think differently.

Data Representation – Case Study

This research considered several cases; more precisely three high school students who have demonstrated some unique ways of thinking, peculiar mathematical activity, and special relationships to mathematics, their peers, and mathematical authority. Case study, by its nature, turns away from the typical to focus on the unique (Shank, 2002). Case study research focuses on the participatory relationship between a single informant and the researcher. "The case study is an exemplar" (Kvale, 1996, p. 273), possibly serving as a vehicle for learning. The case is studied because of its complexity, its peculiarity, and its special interest. The researcher looks at the detail of the case's interactions with its contexts, attempting to understand its activity within

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postmodern world?" (p. 179). Strains in the research effort for knowledge production demand a deconstructive return to the processes of data recording and analysis.

these circumstances. The basic purpose is not to evaluate nor generalize; “the single case has to make its own case” (Shank, 2002, p. 53).

However, the theory employed in this research in congruence with the research question does not attempt to present each case as a known or knowable instance, but instead seeks to consider how each of these subjects had created themselves. This research was *drawn* by a desire for generalization, *compelled* to compare each case and capture themes across cases in order to make bold proclamations about what are defining characteristics for the abstracted generative adolescent mathematics learner. But this draw for generalization is also avoided; “The case itself is important for what it reveals about the phenomenon and what it might represent” (Shank, 2002, p. 54, quoting Merriam). In some ways, the abstract phenomenon of the generative adolescent mathematical learner is the singular case of this study, the desire to model *the* GAML. And this topic is examined through the perspectives of three student subjects, that is, three case samplings. This focus of the fieldwork and its impact on representation is what Shank (2002) refers to as a cumulative case study. A cumulative case study suggests a more generalizable knowing than a single case study could have—in this way it is like survey research.

This research does not seek to create the truths of three singular cases, their mathematical ways of knowing, or their personal epistemologies. Yet this research is also significantly different from a cumulative case study because such a generalizability is expressedly not the goal. Instead of seeking confirmation of similarities or themes between cases, each case here “is deliberately added to make the overall picture richer, deeper, and more complex” (p. 56). An end product will be a richer potential description of the GAML, further insight into how it operates in mathematical activity, how it functions in the school discourses, and what is its concept of itself as a knower and learner?

The impact of this focus on an abstracted phenomenon, the generative adolescent mathematical learner, allowed for the purpose of the results to not focus on the who, what, why of three high school juniors, but instead to use the experiencing of the subjects to report on the singular topic, the GAML. As a researcher, I was freed from the oppressive drive to present my subjects. Instead I considered the peculiarities each expressed in their activity in order to provoke thoughtfulness in the reader about what may be a generative adolescent mathematical learner. As a consequence, the results chapter, Data and Analysis, shied away from being three detailed reports on adolescent mathematical learners, but instead drew on more succinct and narrowed qualities of each student, woven together and built upon one another in order to make the overall picture richer and more complex. Said another way, I did not seek to fully present the being of each subject, but instead will draw out qualities of each to enrich ideas and raise new questions about the GAML. The freedom I sought from presenting my subjects as they *are* reflects the ethics I embrace as a researcher.

Researcher Ethics

Although the National Research Council (2003) was unable to treat ethics in its report *Scientific Research in Education*[67], I draw attention to it because it lived at the center of my work, this effort for social justice. A double bind is presented by the work to do science in the postmodern, the double bind of the co-construction of knowledge. We are both limited and freed by language; I seek to convey meaning through my writing yet embrace the notion that it is the reader who constructs meaning. I am both responsible to my audience but also demand responsibility from my audience. The radical constructivist ethics (Hackenberg & Lawler, 2002; Larochelle, 2000; Steffe, 2000; Thompson, 2000) foregrounds the human need for the other, for interaction, for the

[67] "Research ethics is a complex area that the committee did not have the time nor the expertise to consider fully" (NRC, 2002, p. 153).

cybernetic feedback cycle in order for ontogenetic growth. It is not an ethics that merely says treat others fairly, but one that relies on the other's full capacity to be human in order for one's self to engage others with such an ideal. The art of existence (St. Pierre, 2004) is an entirely circular process; this ethics embraces the co-constructing link in the construction of the self. To intentionally seek to understand—to know—the viewpoint, the experiential world, the worldview, of the other is a move to decenter, to act reflexively. Such a definition of ethics is paralleled in Foucault's (1984/1997) care of the self, implying a learning relationship with others, a care for others.

This definition of ethics was entirely present in my research process in two ways. First on an unconscious level, in the ways a radical constructivist epistemology theorizes the need for others (Glasersfeld, 2008, paragraph 20). But more significantly, during data collection I intentionally worked to decenter, to confer an existence upon the other independent[68] from my own. I worked hard to understand how the other might be reasoning, responding, acting, and behaving in such ways that were entirely reasonable within their ways of knowing. I aimed to posit goals that would make the Other most viable in the relation of these goals to my construction of their experiential reality. My primary tool in order to do this was a "practice of persistent critique" (St. Pierre, 2004, p. 326), asking, How is it that that response or that action makes sense for them? In this effort, I attempted to foreground my subjectivity as I interacted in the moment with my research subjects, and later as I analyzed and reflected upon the data I examined. As I conclude the processes of analysis, I work to deconstruct my relationship to the research question, the subjects of the study, and the situatedness I brought into the effort.

[68] While I choose this manner of characterizing the move to decenter, I recognize human existence is entirely interdependent.

I must also invoke the need to decenter in the representation of this research. The ethical relation between myself as researcher and the academic respondent compels both author and reader to scrutinize the relationship (St. Pierre, 1997). "Such attention is critical since researchers may be encouraged by their colleagues, particularly by respondents in positions of power, to revise methodological practices and to reconstruct texts in ways that do not reflect either their theoretical or ethical positions and, even more importantly, in ways that do not honor their participants" (p. 185). This recognition demands I pay close attention to how I represent the research, and in particular the clarity of this representation. I am limited by language; we treat language as though it carries meaning, yet any such transference is at best messy. Poststructuralists are bound to the words of humanism, yet use them differently. "Posthumanists are… suspicious of language; they tend to use it differently; and their work may not, on first reading, seem so clear" (p. 185). While an emerging body of works troubles this demand for clarity (Britzman, 1995; Lather, 1996; St. Pierre, 2003), it implicates the ethical responsibility of both the researcher and respondent to consider why we write, read, and respond in the ways that we do. An *evaluation* of truth or viability invokes a demand to investigate presuppositions of assumed goals, both our own and those of the communicating other. "This process is about theorizing our own lives, examining the frames with which we read the world, and moving toward an ongoing validity of response" (St. Pierre, 1997, p. 186). A negotiation of sense making is necessary, in which theoretical frameworks and subjectivities are foregrounded, confusion is risked, ways of knowing are reopened, and there is a rejuvenated determination to read harder.

Ultimately, my ethics plays amid the relations of power found in the variety of data sources for this project. The students, teachers, peers, advisors, and colleagues with whom I've interacted have each provided data toward this research. I may be unable to escape Foucault's

(1984/1997) recognition that the Western Culture is infatuated with truth, at the expense of care for the self. So power relations are games of strategy, and our problem is to find the laws, morals, practices—the ethics—that "allow us to play these games of power with as little domination as possible" (p. 298)

CHAPTER 5

METHODOLOGY

This chapter represents an effort to delineate the nonlinear activity of data creation and analysis. To do so as coherently as possible, I follow a Phase-by-Phase script. As I go, the main intention is to detail of the processes of collecting data. While detailing each of these processes, I will connect to meaningful and appropriate theory about each tool. While each of these efforts is seemingly straightforward, I also present some initial data and analysis, to the extent it must be understood in the context of methodology for the entire study.

This study was designed to investigate the primary research question: How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition? The first phase of the data collection was aimed at identifying a small number of research subjects—the generative adolescent mathematical learners of the research question. The second phase involved close mathematical work with and observation of (i.e. co-teaching) the students selected as subjects for the research. Here I worked to create data on the disposition of these students, and how they operated in the mathematics classroom. The third phase primarily was designed to be a member check, and opportunity to follow-up on lingering questions with the research subjects. And Phase IV marks the time during which I worked merely with the data collected, no longer in contact with either the subjects or my co-teachers, in order to theorize amid the possibilities suggested when the initial research question is brought back to the data.

Research Setting – Background to Phase I

In order to launch this study on the generative adolescent mathematics learner, some work was necessary prior to the formalized first phase of data collection. I had to identify a site for collection and sufficiently theorize the GAML in order to identify potential subjects.

Physical setting—the research site

My career in mathematics education involved supporting students, teachers, administrators, and other members of the school community to imagine how mathematics could be taught differently. This work brought me to Prairie High School (PHS) [69] in the winter of 2003. In 2001, the school district had made a concerted effort to realize the *Vision for School Mathematics* proposed by NCTM (2000, p. 3), implementing National Science Foundation (NSF) supported reform curriculum at all grade levels. After a few years of work toward this goal, the district along with the mathematics faculty at PHS decided to increase the formality and structure of their professional development, which led to my work with them.

During my first year of consultation and teaching at PHS, I had observed classrooms and/or co-taught with many of the mathematics teachers. I established relationships with teachers and administrators in the school and at the district. For these and related logistical reasons, this site was well suited to my study. The school was rather typical in structure, a larger high school of approximately 1700 students each year. It operated on a traditional calendar and used a common extended period, "block" schedule. A full one-credit course was completed in one semester. I draw upon the principal's words to further describe the school itself:[70]

> Prairie High School is a comprehensive secondary school engaged in a significant school restructuring initiative. The focus of our change efforts is on converting the facility from a large comprehensive building to one containing five smaller

[69] This pseudonym will be used in place of the high school's real name throughout this research report.
[70] This passage was taken from the school's website on January 5, 2005.

> learning communities through a collaborative process involving community, parents, students, faculty and staff.

The principal goes on to discuss the school's challenges, in particular the student scores on the state high school proficiency assessment.

> Student performance is being monitored through state required testing and assessments linked to student progress with state expectations. Presently, Prairie High School is the first of only two high schools in [deleted] County to use extended block scheduling, 80-minute periods of instruction.

Before the final phase of my data collection was complete, the contracts of this principal and the entire administrative staff were non-renewed. Although I have no direct way of understanding this decision by the superintendent and the school board, my conversations with the high school's administrators and the people I worked with at the district level, coupled with my awareness of the pressures for increased test scores mandated by state and national laws, suggest that the test scores of students in the school demonstrated inadequate progress.

In this introduction to the research site, I have intentionally withheld common treatments on student and teacher demographics, socio-historical data on the school and the community, and common academic measures. While it is undoubtedly the case that each of these qualities interacts with each of my research subjects and may be of interest when considering questions such as, how does the generative adolescent mathematics learner function? get produced? get regulated?, I have chosen to not report these in this iteration of my data presentation. First and foremost, I have not invoked a theoretical framework that would do justice to the subjects—students and teachers—given all the additional analysis that could be done when issues like race, sex, and SES are thoughtfully considered. It is my failing to be able to do this in this context that causes me to push aside and moreover, refuse to report. But I do so secondarily to remind the reader of the potential of the powerful and unconscious draw to biased conclusions by such knowledge. Whether our culture has trained this into us or as humans we are programmed to operate by such a draw toward saming the other, I elect to work with the presentation of my data in ways that push aside whatever presuppositions such information may evoke. These common treatments will not be brought forth as metanarratives, but may emerge as qualities of interactions I participate in with the subjects. I do not intend to ignore the potential socio-historical aspects of my subject's construction. I simply refuse to grant them foreground.

Intellectual setting—the initial conceptualization of the GAML

Having established the research site to be a rather ordinary large public high school, struggling with test scores, I turn to a development of the intellectual setting as the research began. I briefly review the initial conceptualization of the GAML, how it evolved, and how it impacted the selection of research subjects.

The earliest of thoughts about the GAML occurred for me while teaching high school mathematics. I experienced that children could learn Mathematics—the school discipline—by engaging in problem solving activity, sharing ideas with one another, and constructing algorithms, representations, and languaging of their own. My role became one to foster this activity, interaction, and communication, and to label or name the mathematical activity of these children with the conventions of the discipline. In these simple ways, I learned early that children made mathematics for themselves, and that my role as teacher could be much more powerful than merely conceiving of myself as a delivery vehicle for the mathematical Knowledge written in the textbook, district policy, or state standard.

As I grew in the profession, I learned that much theory, research, and even policy communicated the same notion of children (K-12) as active constructors of mathematical knowledge. However, a fracture became apparent when looking closely at the status awarded the nature of mathematical K/knowledge. To speak simply, were these children thought to be inventors of mathematical ways of knowing? Or rather discoverers of a Mathematics that held an existence prior to them, and in fact prior to any human knower? Confrey (1990) distinguished these constructivist viewpoints respectively as *radical* or *trivial*. In my critical bend toward educational goals, I began theorizing the valuation of the autonomous learner and the localization of authority for knowledge. In my opinion, placing oneself as a servant to Knowledge, or worse to an other's knowledge, handicapped a learner's perceived ability to act upon their world.

These key ideas shaped the initial framework I proposed early in the dissertation, the quadrants of Figure 1 suggesting ways of perceiving oneself as a mathematical knower defined by the intersection of two binaries, one's *disposition toward knowledge* and the one's belief in the *nature of knowledge*. First I theorized the poles for one's disposition toward knowledge to be to see oneself as either a *receiver* of knowledge, or as a *producer* of knowledge, noted on the negative and positive x-axis respectively. Second, I demarked one's orientations toward the nature of knowledge on the continuum from thinking of knowledge, and in particular mathematical knowledge, as existing prior to the knower, waiting to be discovered (negative y-axis), to being an invention of the knower (positive y-axis).[71]

From the implications defining the third quadrant, the region bound by the negative x- and negative y-axes, emerged the notion of a mathematical learner who perceived mathematics to exist prior to human knowers, and placed the authority for knowledge external to their self. This pairing defined a mathematical learner who perceived themselves to be a *repeater* of knowledge. Their relationship to mathematics was to be told or shown, and their duty was to replicate.

[71] The naming of binaries may seem to be non-poststructuralist, especially to the extent of my privileging the positive axes. However, I find myself limited to write, and probably even to think, within the structuralist science to which I have experienced as a socio-historically situated knower. I don't deny that these binaries are dangerous. And thus, I don't leave them untroubled; they are not set here as a static notion. They are tools with which I begin to think, understand, and communicate. Yet, as I believe this project in its entirety will demonstrate, these tools must be placed under question, to consider what they may hide, make us blind to see. The binaries, nor the quadrants, create a truth, nor do they carry any meaning unto themselves. Once they are thought, constructed, they must also be deconstructed.

The second quadrant of this model changed the learner's orientation to mathematics as something that is a human invention. Yet, they continued to delegate authority to an external source. I name this type of mathematical learner as a *sense-maker*; they perceive their goal as a learner is to make sense of another's way of knowing—possibly some great mathematician, or more simply the way a teacher, peer, or textbook solves a problem. That other person has been assigned the authority to confirm their own knowing.

The fourth quadrant of this structure characterizing mathematical disposition returned to an a priori status of knowledge. However, the knower's orientation toward their own activity shifts away from a receiver toward a producer of knowledge. They have a sense of themselves as an author rather than reader, and hence a more internalized locus of authority. Although posing languaging difficulties, I elected to name this sort of knower as one who perceives themselves to be constructing knowledge. The Radical Constructivist would bristle at the location of the name, yet I chose to utilize *constructing* to categorize this region because in my professional experiences, this is the orientation toward constructivism that seems to be most common. The sort of personal epistemology suggested in my fourth quadrant—the perception of oneself as constructing knowledge—parallels Confrey's (1990) Trivial Constructivism.

The first quadrant of this sketch is where a mathematical learner who perceives themselves to be the active producer of ways of knowing, and sees knowledge to be the result of this human activity; this knower perceives themselves to be *generating* knowledge. It is my belief that a personal epistemology characterized by this quadrant can aid a knower to most powerfully and effectively read and write the world (Gutstein, 2006) with their mathematical ways of knowing.

Communicating my Conception of the GAML

After formalizing this personal meaning for the generative adolescent mathematical learner, I needed to develop a manner in which to communicate my notions of this learner to others. In particular, I intended to utilize the PHS mathematics teachers to aid in the identification of students displaying indicators of this sort of disposition. To do so, I had to create a survey for these teachers through which they could identify students I might label as GAMLs. I developed an initial draft of this survey and conducted a mini pilot study with mathematics teachers at a high school nearby my home. I asked for them to read the survey and talk me through their interpretations, students they would identify, and why. This feedback that helped develop the descriptors used in the survey instrument later turned out to be some of the first data collected in this study, information that began to reframe how I thought about the theoretical GAML I initially defined above.

In the initial effort to write to mathematics teachers about my ideas of a GAML, I created descriptors of these mathematics students' self-perception, such as:

- A student who perceives herself to be the author of the mathematics she does. The generative learner is a student who sees herself as a key actor in the creation of mathematical understanding. She sees herself as a producer.
- For contrast, the non-generative student would be one who perceives herself in a less active role with regards to her mathematical learning. She sees herself as someone who receives mathematics. She is less likely to label her own mathematical activity as mathematics.

However, the mini pilot study suggested that my theory-laden languaging above was incomplete, or inadequate. Commonly, my mathematics teaching colleagues interpreted my descriptors to indicate children who were gifted mathematically, or high achievers. As a result, I attempted to further clarify my intentions with the statement:

- It is not necessarily the case that the generative learner would be earning good grades, scoring well on standardized tests, or even demonstrating a strong mathematical aptitude. It is also not necessarily the case that a generative learner is the most active child. My naming of a generative learner is much more focused on the way a student perceives himself or herself to be doing mathematics.

When I took this additional care to indicate in the mathematics teacher survey that I did not necessarily mean high achievers, I received confusion and questions in return. “Gifted” or “high-achieving” seemed to be the only categories some teachers had for referring to particular students of their mathematics classroom who had desirable attributes. Correspondingly, I think there was often an assumption that the mathematics students a researcher might seek to study would be successful, good, or otherwise desirable. What I failed to communicate was that the GAMLs I imagined had unique traits of their personal epistemologies—their sense of themselves as learners, in their disposition toward learning, and in their beliefs toward the nature of mathematics.

This difficulty reminded me that my teaching colleagues did not necessarily share the same ideas about learning or about mathematics that I did. Their schemes for a conception of learning, knowing, or mathematics may not have been so intentionally deliberated upon nor as nuanced as mine. I say this to mean that my mathematics teaching colleagues were not recognizing the perspective I had developed and sought to communicate via the survey as something different from their own, certainly not in ways that we could immediately attain a paired manner in which to think about the generative adolescent learner. It is possible that the challenge could have been that they did not see themselves as generative learners. Having a personal epistemology different from my own, and me not having tools to bring this difference into consciousness, may have made the challenge to ask them to identify who they thought might be a GAML unattainable.

To sidestep the roadblock posed by this unintelligibility of uncoordinated epistemologies, I hypothesized a more practical and behavioral delineation of the observable activity of the GAML, based upon both empirical and theoretical students of my career as a mathematics educator and researcher. Instead of focusing on a student's self-perceptions, and then the *not necessarilys* of good grades and active learners, I built up descriptors of classroom behavior I hypothesized for the GAML. By trying to place myself in the mind of my colleagues providing the feedback, I devised the following list of classroom qualities that might be observed in the GAML:

- The generative learner may pursue an idea with greater persistence, less concerned that it may be wrong or lead nowhere.
- The generative learner may be more likely to dispute a conclusion or offer alternative conjectures, even opposing the teacher or a high status student's input.
- The generative learner will reject other's conclusions in favor of their own, or at least until they determine for themselves that the other's conclusion is accurate/ better/preferred.
- A generative learner could possibly be a student who has shut down in their mathematics class because somewhere along the way they learned that this sort of activity was not productive or rewarded in their math classroom. This student may only occasionally show flashes of a generative attitude.

These final additions to aid identification of potential GAMLs completed the survey tool used at my research site. This survey requested the Junior-level mathematics teachers to identify potential generative learners for the purpose of my study (copy in Appendix A). These behavioral additions successfully yielded an initial list of potential GAMLs.

Subject Selection – Data Collection and Analysis in Phase I

The process to select subjects for this research initiated the earliest stages of my data collection and analysis. Here I elaborate on the process to deliver it, the data provided via the surveys, and how this data informed my selection of subjects. I intend to make evident how the Research

Questions were of prominent guidance in this selection of subjects, but that the constraints of the research site and the unknown answer to *How do I know a GAML when I see/hear/experience one?* were significant factors as well. In essence, the process of determining which three PHS Juniors to present in this dissertation reflected a series of selection decisions beginning prior to the start of Phase I of my research—in the site selection—and concluding at the start of Phase IV. Documented next are the procedures to select the three subjects of this research.

Naming 21 (From 200+ Enrolled Juniors to Identifying 21)

In response to the survey (Appendix A), the PHS mathematics teachers identified 21 students. To more easily communicate and account for the data collected during this period, I will label each of the eight teachers with letters *A* through *H*. The initial survey responses included 4 written and 4 verbal. The written responses included three teachers (*A*, *B*, and *D*) who replied within the framework of the survey tool I provided and a fourth (Teacher *H*) who wrote me a handwritten list of names and brief statement of what years he taught them. Included amid the written responses of Teachers *A*, *B*, and *D* were comments that connected to my initial thoughts on the description of the GAML as well as comments that certainly raised once again my concern about how I was describing the students I was interested in. For example, Teacher *D* wrote comments that, in my interpretation most reflected a *desirable* classroom student. Here are the ways he described the three students he recommended:

> *Tchr D*: Very active in class. Shows insight into mathematics. Attendance and class preparation are problems.
>
> *Tchr D*: Very active and interested in class. Does all work needed. Struggles with his knowledge of mathematics.
>
> *Tchr D*: Participates well but could be a bit more vocal (very quiet). Does all work expected and does it well.

In my image of what a teacher would desire in a student, I see qualities such as active, engaged, participates, interested, quick or insightful, completes and completes assigned work. The students identified by Teacher *D* seem to possess each of these characteristics.

Teacher *A* emphasized in his written comments students that extended the mathematics beyond what he expected for most students in the classroom. For example, these two passages from Teacher *A*'s descriptions, referring to two different students, characterize this emphasis:

> *Tchr A*: He is 'generative' in taking his experiential Alice metaphor to the abstract exponents.
>
> *Tchr A*: I sense she will be more open to extension and confident to tackle new problems.

Teacher *A*'s referral to *extension* and *abstract* in the context of these students initially suggested to me he may be interpreting *generative* too strongly as high-level mathematical thinker than I intended for this subject selection. However, Teacher *A*'s comments challenged me to wonder about the relativistic position necessary to call one student's mathematical thinking at a "higher" level than another, or more profound than another's. Maybe from the perspective through which I am thinking about the GAML, "higher" level does have a place, in fact it may be assumed. The "higher" must necessarily be in relation to the knower, but might reflect their intentional continuation of mathematical thought, and thus conjecture upon conjecture yields this sort of higher knowing. As I read the teachers' written responses, beyond the most immediate value of aiding selection of research subject, they continued my thinking about an effort to define the GAML; thus turning out to contribute to the research project through survey feedback in ways beyond what I planned.

Teacher *B* provided the most extensive feedback on each student she recommended. Furthermore her feedback indicated to me a more like mindset on what might be the GAML. I next share some of the passages she wrote when identifying students, not focusing on any one, but to indicate this common mindset and to connect further to what might be characteristics of the GAML. Later, I will return to some of these characteristics when I speak to why I selected her classroom and then again when presenting the subjects I studied in this classroom.

Teacher *B* defined some of her subjects using the "good student" language seen in Teacher *D*, and "mathematically smart" language of Teacher *A*. Each passage below refers to a single student, so the following is drawn from two student descriptions.

> *Tchr B*: His explanations were clear and organized.
>
> *Tchr B*: He will explain his thinking process writing long paragraphs. He works very well in groups. … He makes an effort to make sense of ideas and how they are connected. …wants good grades. he is responsible, turns in his work on or before is due. His papers are usually organized and complete. He uses most of his class time productively.

But when these statements are among others she wrote, I was more convinced that she took to heart my effort to suggest that the GAML was not necessarily the "good student."

> *Tchr B*: …doesn't work hard. … He is absentminded. He comes to class without even a pencil at least twice a week. He wastes a lot of time in class. He turns his work in late most of the times. He is not worried about his grades. He just wants to pass, and have some fun.
>
> *Tchr B*: Sometimes she ignores me. Sometimes she gets 'bored' and starts doing anything else, not related to the work (like singing, talking to her friend).

That Teacher *B* listed these challenging attributes and classroom practices as qualities of the students she was recommending, I took to be indicators that it was not only the "good student" who she considered while thinking of who to suggest for my research.

In similar ways, she also did not maintain a focus on typical notion of "mathematically smart." The next passages she wrote are indicators of this more open attitude toward the mathematical activity she seemed to see in students.

> *Tchr B*: … is a good thinker.
>
> *Tchr B*: He surprised me with a very nice homework once. I asked him to present it to the class. … His explanations were clear and organized. He corrected his mistakes on his own, without any help from me. He knew what he was doing, what went wrong, and why.
>
> *Tchr B*: She 'likes her own way,' even if it doesn't work perfectly.
>
> *Tchr B*: He thinks a lot, he stares at you, writes almost nothing, asks one or two questions, looks at the ceiling or puts his head down, and then he gives his answer.
>
> *Tchr B*: He rarely asks 'why?'

I selected this set of passages to indicate the various types of qualities she referred to when identifying students that I might find useful to study. Although some reflect typical descriptions of mathematical smartness, such as being a good thinker, clear explanations, or just being able to come to solutions in ones mind, other of her comments suggest she valued differing smarts, or did not requires "smarts" as an attribute of the GAML. One student preferred her own way, even if not the best way (not "perfect"). Another student didn't seem to care about an external validation for why things were true.

The responses from Teacher *B* also turned out to provide provocative data to which I returned during the final phases of analysis in order to think further about some of the subjects I ended up studying from her class. Some of this data is hinted at in her responses above, such as the qualities that seem to be difficult classroom attributes—both for a teacher to be accepting of in terms of behavioral expectations in a learning context, but also activities that may not promote mathematical learning. For example, I was challenged to consider why there might be a strong draw toward adherence to social norms (and sociomathematical norms [Yackel & Cobb, 1996]) in the classroom. Is this a structure that a well-functioning group (or system of autonomous thinking agents) takes on? Or is a need placed upon a group so that it works well? The notion of 'well' is maybe of more interest than the chicken-or-egg query posed.

Teacher *B* described one student as "keep[ing] her ideas until somebody else prove a new idea to her." Another student was described:

> He will pursue an idea with greater persistence, he usually likes his own ideas and answers. When he is wrong, he accepts other students' opinions. He listens, thinks, and makes his own conclusions. He won't copy somebody's answer just because he/she always know what he/she is doing. He will if that answer convinces him.

I found that these comments became important for thinking on mathematical authority. These students seemed to place themselves as decision makes about what constituted mathematical correctness, and possibly mathematical knowledge.

I spoke with each of the four teachers who did not reply in writing (Teachers *C*, *E*, *F*, and *G*). Teacher *C* provided suggestions for four research candidates, and *E* offered six suggestions. Teachers *F* and *G* made no suggestions. When I observed the students that Teachers *C* and *E* recommended in their classrooms, I came away with the sense that I didn't communicate successfully with these teachers what my goal for identifying that might be a GAML. However I continued to consider the students they suggested, and also observed closely other students in these teacher's classrooms while attending to the Phase I goals to select my research subjects. I did observe Teacher *F*'s classroom one time. I did not pursue Teacher *G* any further because of hesitation about participating I felt from her in our interactions.

From 21 to 21 Plus Five

None of the written teacher feedback was completed prior to my arrival for Phase I of the data collection. Most of what I did receive was collected from these teachers during the first two days of Phase I, February 24-25. I also began visiting classrooms on these days; Table 1 documents the schedule of classroom observations on which days.

Table 1

Classrooms Observed During Phase I: Subject Selection, Labeled by Teacher

	Taught Jr. Course this Period	Teacher's Classroom Attended				
		Feb 21	Feb 22	Feb 23	Feb 24	Feb 25
Period 1	*B G*	*B*	*B*		*B*	*B*
Period 2	*A C*	*A*	*C*		*C*	*C*
Period 4	*D E F H*	*H*		*F E*	*D*	*H*

By February 24 I had visited the 7 potential classrooms, not including Teacher G because of a reluctance to participate. During these visits, I observed each student candidate as well as teacher, as they would be potential colleagues in the research. Utilizing the same criteria for selection from the teacher survey, I made notes on each of the 21 potential subject's mathematical activity and interactions. Although focused on these 21, I paid attention to other students that satisfied the criteria, and as a result identified five additional students.

The evenings of Phase I consisted of field note review on the subjects and the interactions with teachers, including the verbal and written subject identification they provided via the survey. Based upon factors including how well the teacher seemed to identify with the GAML I described, how interested in conducting the research in collaboration with me—namely co-teaching—the teacher seemed to be, the nature of their classroom climate, and the potentials I saw in the possible subjects in each classroom, I narrowed and ranked my selection for classrooms, as opposed to particular students, that I was interested to pursue for the remainder of the week.

To determine classrooms, first I had to select one teacher at most from each of the three class periods available. I determined Teachers *B*, *C*, and *H* of Periods 1, 2, and 4 to be my primary options. I was drawn to consider participating in all 3 periods of the day, but realized that would be straining on my goals as a researcher—I would not have the time to reflect on and elaborate field notes, nor would I be able to appropriately reflect and plan with three teachers. I decided the time separation between the morning periods and Period 4 would allow some of the time necessary for reflection, so I compared Teachers *B* and *C* with regard to potentials for my research and teaching collaboration, along with the interest I had in subjects of each teacher's classrooms. Several of the subjects of Teacher *B*'s class were very intriguing, and I felt that we could learn together more effectively than I would with Teacher *C*. To conclude Phase I, I selected Teachers *B* and *H*, henceforth known by their pseudonyms Bridget and Larry.[72]

From 26 to Six

Having selected Bridget's Period 1 class and Larry's Period 4, my pool of 26 research candidates was reduced to nine, six in Bridget's classroom and three in Larry's. The process of reducing from nine to six was almost just as easily done. Beyond the broad descriptor of a generative mathematical disposition, I identified students who I suspected would be communicative, willing to attend after-school tutoring twice weekly for one month, and from whom I would learn the most (Kvale, 1996). When indicators of generativity and potential for productive research interaction seemed otherwise equal, I considered diversity of attained levels of school mathematics success and gender diversity. Variety among other factors such as race, SES and family situations emerged in unintentional ways.

My selection decisions were also made on what some candidates did not offer. The first subject dropped was a second language speaker in Bridget's classroom, John (pseudonym, as I will use for each student mentioned hereafter). Although inconsistent in classroom performance and attendance, John seemed to be quite sharp in that he was persistent and productive in solving problems. In my brief interactions, he indicated that some nights he surprised himself how much time he spent on some of his math problems, but that he really enjoyed 'knowing.' I did not

[72] Notice that the teachers have a 2-syllable name, while the students' are monosyllabic.

select him because of his poor record of attendance and, although friendly, he was reserved in the classroom environment.

Bridget pointed out Winn on Tuesday while I was in class observing. She noted that Winn seemed to understand the mathematics well, but Bridget was hesitant on whether I might consider her generative. As the 5th candidate in this classroom, she appeared least interested in thinking about mathematics and most interested in being done with assigned work. I agreed with Bridget's instinct that she may not tell me much about the GAML and did not pursue her as a research subject.

Having eliminated two candidates from Bridget's classroom—leaving me four students in that class alone—I still had three potential candidates in Larry's class. The Phase I Friday was a very snowy day, and only roughly half of the student body attended class. I had not spoken with one of the candidates, Jim, since Monday and he was not in school on this day either. His past attendance history made me decide not to include him in the research. Two students from Larry's class eventually agreed to participate, Fisk and Shea.[73] These two added to my 4 students from Bridget's class: Jack, Jan, Kate, and Kurt. I had identified Shea and Kurt; the others were initially named by their classroom teacher via the survey.

From Six to Three

During Phase II of the study, I co-taught with either Bridget or Larry each of these six students. In addition, each student completed surveys and participated in interviews. While each provided me with rich data sets related to their mathematical activity, both mental and social; insights into their conceptions of mathematics; and the practices in which they engage while doing mathematics; three students stood out with exceptionally dynamic experiences and

[73] Fisk agreed on Thursday, Feb. 24 and Shea on Apr. 4. None of these three Period 4 candidates attended class on Friday, Feb. 25.

personalities that after my first layers of retrospective analysis. Having completed the data collection and several rounds of analysis for all six, I identified three of these high school juniors to present in the dissertation, each demonstrating a generative mathematical disposition. This “personal characteristic” (Shank, 2002, p. 28) strategy was appropriate for the case study methodology of this research.

The first subject I decided to eliminate was Shea; she had very low attendance due to allergies during the Spring of 2005, limiting my data. Kurt turned out to be a very engaging young man, but in my early analysis I found it difficult to find consistent data suggesting useful indicators that told me about him as a GAML, the ways he thought of himself as a mathematical actor, or the ways in which he experienced the school environment. Jan provided a similar difficulty with my data as well. Jan was much more of a difficult classroom student than other subjects (and especially Kurt). By the end of Phase II, Jan and I got along quite well, in fact we conducted a great tutoring session and interview. But because overall I had less data on Jan, I decided not to include her. Although not included in the remainder of the formal data analysis, each of these three subjects did provide me many experiences to think from. These subjects will not appear explicitly in the results, conclusions, or implications of this study, yet memories of my interactions have certainly provided shades and nuances to what I report.

Removing Jan, Kurt, and Shea left me with Fisk, Kate and Jack, three subjects that provided rich data and demonstrated strong orientation toward the GAML that my initial (and emerging) definition suggested. In the next section, I expand on the methods of data collection during Phase II that let to this determination to present these three subjects in this report.

Research Design – Co-Teaching, Observation & Interviews in Phase II

While the constructivist teaching experiment methodology is useful to formulate explanations for student's mathematical activity, methods more often associated with social-cultural research informed the identification and development of the generative learner's disposition. These additional techniques are not intended to be apart from the co-teaching experimental design I enacted, but key components necessary to answer the research questions posed here. In addition to classroom co-teaching, this study incorporated interviews with the student and teacher subjects; videotaped teaching episodes; student-produced video; and collection of archival materials—namely student work. Table 2 provides a summary of the instruments used during Phase II. This research design supported the creation of three case analyses of generative adolescent mathematical learners. The constructivist teaching experiment framework, although not adhered to, provided an organizational structure through which I designed the particular data collection and analysis methods used in this project, allowing for the minimization of the distinction between the two processes that I theorized above.

The Classrooms and Co-Teaching

During Phase II of the research, I collaborated daily in the teaching of Bridget's Period 1 and Larry's Period 4 class. Both of these classes were a college preparatory, untracked, content-integrated math course for PHS Juniors. During the first week of Phase II, students were concluding a unit of study on the operations of exponents and logarithms, for integer and rational exponents on integer and rational bases, structured around a growth metaphor based on the *Alice's Adventures in Wonderland* story by Lewis Carroll. The remainder of the phase involved study of synthetic and analytic geometry in two dimensions, provoked by a line of sight problem amid circular trees planted on the lattice points of a circular orchard.

Table 2

Data Collection/Analysis During Phase II

Phase II. Co-Teaching, Observation & Interviews April 4–26, 2005

	Instrument	Comment	Reference
1.	Subject Survey	Insight into student attitudes toward mathematics and selves as mathematical actors	Appendix B
2.	Subject Mathography	Insight into relationship to mathematics and selves as mathematical actors	Appendix C
3.	Co-Teaching & Observation	Interaction with subject in mathematical activity, and in interactions with other's amid mathematical activity	
4.	Field Notes	Recorded during each class sessions, as teaching/interaction aid and observation records	
5.	Field Note Elaboration	Immediate expansion of field notes, including reflection of subjects' activity	
6.	Co-Planning	Build rapport, ongoing member check of data collected and initial analysis	
7.	Researcher's Journal	Regular, although non-regimented recording of conjectures and ideas about the research questions	
8.	Classroom Video	Twice weekly, showed classroom interaction and what subject's appeared to attend to	
9.	Subject Interview 1	Video or Audiotape	Appendix D
10.	Subject Interview 2	Video or Audiotape	Appendix E
11.	Student Work		
12.	Subject Video Mathography		Appendix F

The class curriculum was a problem-based sequence of mathematical investigation and activity. Typical student activity was initiated by a problem posed by the teacher, the textbook, or a fellow student. Students investigated the problem individually or collaboratively; they were expected to draw upon previous mathematical experiences and their creative mind to consider possible responses or answers to the questions posed. It is important to note that the underlying structure of the curriculum assumed not to expect students to apply mathematical content to word problems, but to utilize questions arising out of mathematical contexts to create mathematical ideas. Through peer and teacher discussion, important mathematical ideas and solutions were to emerge.

The teacher's pedagogical style was as a problem poser, a mediator of the students' ways of knowing with the Mathematical Knowledge to be learned, a manager of student interaction, and an assessor of student thinking. The authors of the textbook used as the primary classroom resource wrote with these curricular and pedagogical frameworks in mind. Bridget and Larry taught using these frameworks in such a way that maintained a high fidelity (Reys, Reys, Lappan, Holliday, & Wasman, 2003) to the authors' intent. In my co-teaching, I attempted to utilize the same pedagogy, working to teach harmoniously with the intentions and goals of Bridget or Larry, in accord with their classroom.

The decision to gather data during Phase II as a co-teacher in the subjects' courses was made based on several factors. First, my relationship with this research site began one year earlier as an external professional development provider. I was hired to help the mathematics department in their implementation of this textbook, and more specifically to develop the pedagogy described above. Part of the agreement with the school and district to participate in my research was the job-embedded peer collaboration and coaching that would naturally result. However, this was not my primary reason for seeking to co-teach; I believed that the mathematical interactions with children that I knew best, the means by which I could best consider their mathematical minds, was to interact with them as I had throughout my teaching career—as the teacher defined by the pedagogical style specified above. This teaching style allowed me to listen carefully to student's mathematical reasoning, through one-on-one interaction or by observing peer interactions in pairs, small and whole-class groups. While doing this with another teacher made the research design simple—I could simply join a class in progress—it also gave me more freedom than a regular teacher would have in that I could attend more closely to my own research subjects. I did not carry Bridget or Larry's "burden" of responsibility for the learning of all classroom members. I could retract from the class into an observer/listener role, or I could approach any of my subjects with the primary concern to listen to them in mathematical activity, attending to understanding their knowing, with a reduced concern to know the mathematical knowing of the other students.[74]

The daily math classes were of 80-minute duration. During the classes, I acted in several ways. Sometimes I was a lead instructor, sometimes the secondary; other times the pair of us worked in harmony with the students. The role of the instructor in this classroom varied—as indicated above. It included such pedagogical practices as large group interaction, with purposes such as initiating mathematical tasks, pausing to discuss what has been learned and what new questions had arisen, to presentation and discussion of solutions. The teacher role also involved interaction with small groups and individuals. These interactions tended to allow for much closer assessment of understanding and ways of thinking for individual students. In this role, the teacher often listened to and observed written representations of student thinking on problems. In addition, the teacher responded to or redirected individual and group questions.

The third main role of the teacher of these mathematics classrooms involved bookkeeping oriented work, such as attendance, making classroom announcements, and collecting work. There was also an ever-present, yet mostly unintended fourth role involved in each of the above activities, a role likely to be perceived in both the conscious and unconscious of the students. Both the classroom teacher and myself were authorities, both disciplinarians and arbiters of knowledge. How each student positioned us is certainly unknown, but provided an opportunity for interrogation of the data, speaking to the researcher's relations with the subject, the subjects' relations with an idea of mathematics, and the subjects' conceptions of their selves amid these relations.

[74] I do not wish to describe a theory for teaching mathematics here. However, I would be remiss not to state that I find much accord between my pedagogical theory and the notions of teaching as listening developed by Duckworth (1996) and Davis (1997). In a few words, by *listening with an intent to learn*, a teacher aids the child in identifying and answering their own questions.

As a temporary co-teacher, I graded no student work. However, I did review most all collected work from all students and commented on it. I both collected and copied all work produced by my students/subjects during this phase. As a teacher-researcher, I recorded data *during* the classroom episodes in two ways. When not acting as a lead teacher and occasionally during group and individual work, I would step to a less noticeable place in the room to record observations, questions, and conjectures to a small reporter's notebook. I also video-recorded the class approximately once per week. Next I elaborate on each of these two methods of classroom data collection, field notes and video. This is followed by discussion of the data collected through interviews and archival records.

Field Notes

While in my classroom researcher's role of co-teacher, I would occasionally withdraw and record to a notebook (my field notes). Though this practice was as a teacher, in this context I had a different intent—to record brief notes, sketches of student work, and other curiosities to be reminders of particular interactions with the subjects.

At the conclusion of each class episode, I immediately sat down at a computer in the department office for 30-90 minutes to elaborate on my field notes taken that class period. In this elaboration, I extended my comments on student activity, my evaluation of their mathematical involvement and understanding, and the ways in which they interacted in the classroom environment. I developed guesses about the subjects' ways of mathematical knowing as well as their ways of seeing themselves as mathematical knowers. I followed up on these guesses with the classroom teacher during our daily peer-debrief. And I continued to refine these hypotheses through the ongoing classroom interaction as well as during occasional formal and informal student interviews. I occasionally wrote in a less directed manner, recording tangential thoughts to the records of my research journal—a separate document.

Most days I also met with Bridget and/or Larry, separately, to review that day's lesson and plan the next. Bridget and I often talked over lunch while Larry and I chatted immediately after school. During these meetings, the time was utilized to conduct member checks of the data I had collected on each student that day. These discussions of the research subjects entailed questions such as, "What did you make of… ?" and "What do you think Subject *A* was thinking when she… ?"

The field notes I kept incorporated a variety of foci. Prior to classroom episodes, they included records of my collaboration with the classroom teacher to design a lesson for the day. Second, during class I recorded notes about the activity of my subjects, as well as about the classes learning as a whole—to inform the lesson planning. I pause briefly in this description of my specific data collection methods to make general comments on field notes. As I began to engage in knowledge construction, the ethnographic practices of anthropological work reminded me to keep in mind a question designed to keep my assumptions at bay, "How do I know?" (Kutsche, 1998). Taking field notes during Phase II was a primary data source, and a way to document, remind, and provoke knowledge that would later help me to respond to my reflexive question, how do I know? Instead of embracing some ethnographers' command to "describe without judging" (Kutsche, 1998, p. 15), I practiced both, finding that my judgments provided another source of data. I allowed myself to record these judgments, often as questions, to allow myself to conjecture and later reflect on such conjectures. Rather than focus on carefully documenting each subject's moments in the classroom (I allowed for the video-recorder to capture a few classroom episodes in this manner), I allowed for my field notes to be a place for thinking. “Field notes are just as much about your impressions and your observations as they are records of ‘who said what’” (Shank, 2002, p. 58). While the video recording can capture the goings on, the words, the actions, it could not capture my thoughts.

Video

I videotaped each classroom approximately twice each week.[75] The purpose for this video was to document the classroom activity, both as a sort of “silent observer” (Harel, 1991, p. 454) aiding as a note-taking tool, and also as an additional viewpoint through which I could re-view my subjects’ classroom activities and interactions

Video-recording, although becoming a normal part of mainstream culture—so much so that many people’s handheld cellular telephones will record short snippets of video, which can then be transferred wirelessly to other devices—is a relatively new data collection method for educational research (Hall, 2000). Visual methods have been used in ethnography since its inception, but primarily as illustrative rather than analytical. Of course, the digitalization potential of the visual medium creates new forms for representation, but also provides new grounds for perception (Ball & Smith, 2001). While videotape creates what some may consider to be a more objective data set—Fisk will raise his hand at the same time mark every time you replay the video—it remains ultimately another subjective source of data (Harel, 1991). As the researcher, I placed the camera and framed what I wanted; I am the decoder of language, inserting potential meanings and ways of knowing.

Video recording offered several particular potentials as a method of data collection (Harel, 1991), including assessment of learning, documentation, comparison, and representation. Video-recording was used in three distinct ways during this project: each of the two classrooms

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[75] Table 3 indicates exactly which dates were videotaped.

involved in the teaching experiment were taped twice weekly for one month, a few[76] subject tutoring/interview sessions were video-taped, and each student created a short video in which they were to capture aspects of their lives that demonstrate the influences on their thinking about the ways they learn and do mathematics. In each of the first two use of video, the recorded images and sounds served as ways to document actual occurrences and timelines of these experiences—a sort of "note-taking tool" (Harel, 1991, p. 454), as well as to provide additional opportunities for assessment of student disposition. As a "silent observer" (p. 452) in the classroom, the video picked up moments of student activity that may otherwise have been compromised when a teacher ambled past, or a researcher was caught noting activity. In the case of the interviews, the video created a "holistic documentation" (p. 450), freeing me to more carefully attend to the subject and take notes during the interview.

The third use of video, in which the subjects created a video mathography, provided a more personal look at what the subject saw of their world from a very unique vantage point. When I left PHS at the end of Phase II, I asked each subject to create this video mathography, in which I hoped to gain one more view of how the subjects perceived themselves as learners. The instructions provided were:

> Reflect on your past experiences in learning and doing mathematics, both in and out of school. With these experiences in mind, consider what has influenced your thinking about the ways you learn and do mathematics. Capture images and video of the aspects of your life that will demonstrate these influences. Do this using 10-15 minutes of video footage.

A copy of this request is available in Appendix F.

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[76] Not every interview was videotaped. Many interviews had to occur in a more spontaneous manner, in that my subjects often failed to attend planned sessions. Because of this, I found that always being prepared to audiotape interviews became a practical solution.

Interview

Phase II of the research also included two interviews per subject. The first interview sought to gather information on decisions made by the subjects while solving mathematical problems (generally), their self-image in relation to mathematical activity, what they consider to be mathematical, and what captured their attention while in math class. The second interview was designed to follow-up on responses in the first interview, the initial surveys, or on particular classroom activity. In cases where a subject wasn't or didn't seem to be likely to attend a scheduled interview, I arranged for the conversation of the interview to occur in a more informal, on-the-spot manner—recorded amid field notes and on video or audiotape.

Kvale (1996) contended that if we "want to know how people understand their world and their life, why not talk with them?" (p. 1). The qualitative research interview conversation opens the doorway to such an experience, whether conceived as an effort to mine for nuggets of knowledge, or as an experience that leads to a telling of the tale of a journey. Interviewing—like other forms of data creation can be artificially separated into two parts, doing the interview and interpreting the interview. In a conventional view, the transcribed text becomes data in a sense very similar to quantitative research. The resulting "aggregates or categories are compared across interviewers, interviewees, times, and places" (Scheurich, 1995, p. 240). The modernist goal for data analysis is to use these "dead, decontextualized monads of meaning [and] the tightly boundaried containers… to construct generalizations which are… used to predict, control, and reform, as in educational practice" (p. 241). In contrast, the postmodernist perspective that guided my work foregrounds the contextual, unstable, and ambiguous relationships between language and meaning, from person to person, situation to situation, time to time. It considers the "severe modernist reduction of the exquisiteness of each lived moment [to border] on a kind of violence" (p. 242). The postmodernist viewpoint recognizes that the interviewer's ways of seeing the world are being overlaid onto the interview. "The constructed overlay is then 'discovered' through 'systematic' analysis and (mis)labeled as the 'valid' meaning of the interview" (p. 244). The danger of the observer seeing what she wanted to see is always present, but further it is that the observer may not be able to see beyond what she is prepared to see.

The radical constructivist research premise encourage the researcher to act in the manner of the second-order observer, a reflective position to remain aware of ones self in the attempt to build models of the other, to decenter. Ultimately, however, for the postmodernist the "interview interaction is fundamentally indeterminate" (Scheurich, 1995, p. 249). The implications of such a standpoint were fundamentally built in to the intentions for my study. As I considered what a generative adolescent mathematical learner may be, how these students exist in schools, and what seems to have interacted with these students in such ways to develop such a disposition, I did not strive to arrive at generalizations, but instead to provide a rich collection of possibilities.

What might be called my initial interviews consisted of written responses; I asked each subject to complete a Survey of Mathematical Attitudes (Appendix B) and a Mathography (Appendix C). The survey was drawn from several resources studying student disposition, primarily from the student questionnaires used in a study by Clarke et al. (1992/2004). The *Mathematics Beliefs* portion of the questionnaire

> examined student perceptions of their mathematical competence, and student beliefs about mathematical activity and the origins of mathematical ideas. Students were asked to report their perceptions of those valued activities, which, in their opinion, assisted their learning of mathematics, in addition to their perceptions of what constituted typical classroom activities in mathematics and their attitudes towards mathematics." (p. 9).

This questionnaire had been adapted from a similar survey employed to measure the student belief outcomes of an innovative pedagogy employing student journals. Every item had been validated through interviews with students.

The *Mathematics World* portion of the questionnaire asked for students to identify the extent to which specific everyday activities were mathematical. Clarke et al. adapted the mathematics world questionnaire for American administration from an instrument employed in a study of community perceptions of mathematical activity. In the questions I drew from this questionnaire, students were asked to indicate whether they thought specific everyday activities were highly mathematical, quite mathematical, slightly mathematical, barely mathematical, or not mathematical.

I also asked my subjects to write a Mathography, a sort of mathematics autobiography. The prompt given to the subjects was:

> People often write about events or experiences from their lives in order to help those who may have had very different experiences view the world from a different vantage point. Reflect on your past experiences in learning and doing mathematics, both in and out of school. With these experiences in mind, write your "mathematics autobiography."

These instructions were clarified further with examples and suggestions, such as,

- Describe a situation when you learned something difficult. Describe a situation when someone helped you learn something difficult.
- Are you good at mathematics? How do you know?

The primary intent was to learn further about the subject's histories and views of themselves as mathematics learners. Other intentions were present, including beginning to gauge the ways in which the subjects would open up to me, to provide additional pre-Phase II data for hypothesis generation, and to begin to establish rapport and a relationship.

In addition to these initial surveys and the classroom interaction of Phase II, I conducted two face-to-face semi-structured traditional question-and-answer interviews with my student subjects. The first interview protocol emerged mostly from the survey mentioned above and student responses to the Mathography (Student Interview 1 protocol available in Appendix D). I developed the protocol for Student Interview 2 (see Appendix E for an example of the protocol used with Fisk) after a few weeks of the co-teaching experience. I prepared some questions in response to particular occurrences involving the subject in the classroom, or would pursue tangents connecting responses to other classroom incidents.

Upon return to the research site in June 2005, I conducted a third and final interview unique to each subject. This interview was designed in part to follow-up on themes that had emerged from initial analysis of the data, as well as to conduct a member check regarding some observations. An example of this protocol can be found in Appendix G. Some of the questions were new while others were intentionally repeated. Some questions were borrowed from other student interviews, usually because they proved productive. And others were created to readdress a curiosity or confirm an observation.

The daily conversations with Bridget and Larry could be construed of as unstructured interviews; these interactions provided excellent opportunity for peer debriefing on what I observed in the subjects during that class period. During the week of Phase III in which I returned to PHS, I observed and interacted with students in the two research classrooms once again. This Phase III of data collection included continued peer debriefing, both informal chats and sit-down unstructured interviews (Kvale, 1996). At the conclusion of Phase III, I also interviewed, in a semi-structured format, the subjects' former teachers. Some of these interviews were face-to-face, while others were conducted through written or email communication (see Appendix H for a sample interview protocol).

Archives

During Phase II of data collection, I collected duplicates of the subjects' classroom work that had been submitted to their teacher for the purpose of a grade. Another form of archival data included my occasional sketches of a student's mathematical scribblings, those initial representations children record while thinking about mathematical ideas. Altogether, these archives of student work provided some insight into how the subject was thinking mathematically—to the extent a written record (representation) of mathematical thinking could. Some of these records were more useful in that I was witness to the subject's generation of the material; others archives were useful because of some extended writing in which student's elaborated the mathematical representations, solutions, and thought process that yielded either.

Transcription

At this point in the process of data collection, the end of April 2005, I had become loaded with data and was ready for some time to decompress and begin to digest what had been collected. During the month between Phase II and III, May 2005, the most significant activity of analysis was to transcribe classroom and interview video and audiotapes. During this time, I also recorded initial observations and conjectures in a journal. Email contact was maintained with the classroom teachers Bridget and Larry, and through their input, I kept up with the activities of my subjects. These email conversations were used to pursue data analysis to some extent, but not significantly. Most of this month-long effort was to prepare for a second round of member checks (the first round occurred during the daily class debrief with each co-teacher) and data triangulation, intended to occur back at the research site in Phase III.

Returning to a few comments on the process of transcription, field notes recorded while co-teaching in Phase II were complemented by digitally recorded visual and auditory data. This included twice-weekly classroom video recording as well as the video or audio recording of two interviews per subject. I transcribed both the audio interview records and much of the classroom video in the pause between Phase II and III, during May 2005. The interview transcription was carefully done, noting each word of both the interviewer and interviewee. The classroom video was also transcribed but in a slightly different manner. Because the audio input of the video records was cast wide among the classroom, it didn't necessarily pick up all conversations or comments of each participant. My video transcriptions focused on capturing episodes of activity by the subject, either written description of individual work, or description and transcription of teacher, small group, and class interaction.

Table 3 provides a daily summary of contacts during Phase II. I provide this table not only to help communicate my interactions with the research subjects, but also to demonstrate the discontinuity the research site provided. For example, although I was on site for three and a half school weeks, 17 scheduled co-teaching episodes with Larry resulted in only 7. He had a family emergency during the second half of April, which severely interrupted his continuity with the class.[77] School was cancelled April 8th due to a campus fire in which a school employee was killed. And class schedules during the third week of June were interrupted for testing. I present this chart to demonstrate the discrepancies between well-laid out research plans and the reality of the research site. Although unsurprising in retrospect, during the research there seemed to be new obstacles at every turn.

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[77] Although I will elaborate the outcomes for the subjects later, Larry did not teach at PHS for the remainder of this school year.

Table 3

Phase II Attendance Summary and Notations of Particular Data Types Collected

	Date in April, 2005																
Name	4	5	6	7	8	11	12	13	14	15	18	19	20	21	22	25	26
Bridget	x^{P}	x^{P}	x	x^{V}			x^{V}_{P}	x^{P}	x^{V}	x^{P}	x^{V}	x^{V}_{P}	^{P}	^{P}	x^{V}	x^{VP}	x^{P}
Larry	x^{P}	^{P}		^{V}		x	x^{V}_{P}	x^{P}		x^{V}_{P}	x^{V}	^{P}			x^{V}	^{P}	^{P}
Kate	x	x	x	x		x	x	x^{I}	x	x	x	x			x	x	x
Jack		x	x	x		x	x	x	x^{I}	x	x	x		^{I} I	x	x	x
Kurt	x	x	x	x		x	x	x	x	x^{I}	x	x			x	x	x
Jan	x	x		x		x	x	x	x	x	x	x			x^{T}_{I}	x	x
Fisk	x	x	x	x		x	x	x		x^{T}_{I}	x			X	x	x	x
Shea	x	x								x					x	x	

Key: x in attendance; class did not meet; ^{V} classroom videotaped; ^{I} student interviewed, audiotaped; ^{T} student tutor session, videotaped; ^{P} teacher interview/planning session, field notes.

Research Design – Member Check during Phase III

During the 4 days of Phase III at PHS, I once again attended Bridget and Larry's same Period 1 and 4 classes, conducted a final interview with each subject and several teachers, did further collection of archival data, and completed paperwork with the research site. The instruments used for collecting this data are summarized in Table 4. The emphasis was not to collect additional data toward developing new hypotheses, but to have an opportunity to look for confirming or contradicting data of initial themes and conjectures. This confirmation was sought not only through further personal observations, but also through input from additional sources, such as a survey of or interview with the subjects' former teachers. I sought to learn two things from them, observations on the subject's perception of themselves as a mathematical learner, and if these teachers experienced mathematical and behavioral characteristics from the subjects similar to my documentation.

Table 4

Data Collection/Analysis During Phase III

Phase III. Member Check			June 27–30, 2005
	Instrument	Comment	Reference
1.	Classroom Observation		
2.	Field Notes		
3.	Field Note Elaboration		
4.	Member Check	Through interviews/discussions with Bridget and Larry	
5.	Researcher's Journal		
6.	Subject Interview 3	Audiotape	Appendix G
7.	Former Teacher Survey		Appendix H

Although I did engage the students again as a co-teacher somewhat, my relationship mostly shifted toward a role as a researcher. My subjects' continued to treat me as a teacher in most ways, such as an aide to help with their mathematical learning, as well as a caring adult. These interactions continued to inform my images of them as mathematical learners, however the interactions I paid closer attention to were the planned interviews (or the informal, spontaneous interviews that occurred when I could catch the subject in the hallway because the planned interview didn't occur).

Phase III included several methods to gather information from the subjects' previous mathematics teachers. I sent each of these former teachers a *Former Teacher Survey* (Appendix H). Surprisingly, this only included a very small number of the PHS teachers. I assumed I may need to collect as many as 12 (six subjects, each with two previous years of mathematics) of these surveys. However, Jan was a new student to PHS, I had dropped Shea by this time, and several students had Bridget or Larry in previous years. As a result, I only needed to survey 4 other teachers for their memories of my subjects. In these Phase III interactions, I sought to clarify, finalize, or complete my intended Phase II purposes for collecting data.

Methods of Data Analysis – Phase IV

Phase IV, summarized in Table 5, is characterized by the analysis taking center stage, with no additional data to be collected in the presence of the subject. Although it should be argued that new data continued to irrupt (St. Pierre, 1997), my effort in writing here is to create a simple pathway to understand my process of analysis—an thus I will ignore the false impressions suggested by a data/analysis duality. After completion of the classroom and interview transcripts, these were replicated and chopped apart, separated and piled for each subject. Then each pile was re-read, coded for future reference, and themes apparent across different data types were identified. I then wrote what I called a storyboard for each of the 5 subjects (not including Shea), including referents to the coded data. After this coding and theming, I stepped back to re-theme referring to frameworks designed by other researchers. The final distinct mode of data analysis was a deconstructive look at the data. Next I review each of these steps in greater detail.

Table 5

Data Collection/Analysis During Phase IV

Phase IV.	Analysis	Summer 2005–Spring 2008
	Instrument	Comment
1.	Interview Transcripts	Audiotape transcribed, word for word
2.	Classroom Video Transcripts	Videotape transcribed, to capture activity
3.	Sorting	By subject—including co-teachers as subjects
4.	Coding & Theming	Location and identification of common and unique elements, and noting of exemplary evidence of these elements
5.	Belenky & Grouws	Themes applied to each subject; contrasted to Belenky et al. model; contrasted to Grouws et al. model
6.	Deconstruction	Implications for further research, on goals for mathematics education, and the notion of mathematics as knowledge

Sorting

My initial methods for data analysis were to create a chronological organization of all data for each subject. This was done to establish a sense of cohesiveness among my experiences with each of the eight research participants (six students, two teachers). I pulled together field notes, research journal entries, and transcripts for each the subjects. I also built archives of all written work completed by the six student subjects. Sorting this work by subject yielded a system that allowed for focus as I began to reflect on each separately. I went back through my field notes and extracted observations, comments, discussions, and reflections on each subject and created new electronic files with these parsed sets of data.

Coding and Theming

Continuing this work of Phase IV, I read through, once again, the data and transcripts I had created in response to experiencing each subject. While rereading, I wrote a summary of the data, to aid my focus. Next, I worked to identify repeated themes, both through another read of raw data, and through utilizing the summary document. I organized these observations into topics. While doing this, I noted evidence that aligned with each topic and subtopic through a dating and coding system, in an effort to document the validity of my findings and to help me return to the datum that supported the theme. I categorized (and recategorized) these themes to develop a storyboard for each of the five student subjects (Shea excluded).

This storyboard then became both a resource for writing the presentation of the subjects, but also for beginning to identify trends, opposites, and singularities among the five student subjects. This effort was less oriented at finding familiarities in the data across subjects, but to find interesting experiences with the subjects, experiences that gave more insight into the notion of the GAML and the way a GAML functions in the mathematics classroom.

Belenky & Grouws' Models

The next step of this data analysis, Step 5, involved re-analyzing each subject using tools developed by Belenky, Clinchy, Goldberger, & Tarule (1986), and Grouws, Howald, & Colangelo (1996). This step was conducted in order to help provide a renewed set of eyes, to challenge me to step outside my framework for seeing, and attempt to use the structures of other researcher's attempts to characterize a mathematical subject's personal epistemology. Belenky et al.'s model proved to challenge my thoughts on the self-perception of each subject's constructive activity, while the Grouws et al. model helped to identify the degree to which each subject oriented themselves to particular conceptions of mathematics. In response to the Grouws et al. framework, I created diagrams of qualities of each subjects' knowing. In this process to revisit the presentation of the subjects I was writing, I drew upon efforts to characterize ways of knowing described in the Literature Review. Boaler and Greeno (2000) used a modification of Belenky, Clinchy, Goldberger, and Tarule's (1986) distinctions among *ways of knowing*, summarized in Table 6, because they were interested in personal epistemologies.

Table 6

Boaler and Greeno's (2000) Ways of Knowing

Received knowing	the individual considers her knowledge as primarily dependent on and derivative from an authoritative source other than herself.
Subjective knowing	the individual considers her knowledge as primarily a result of her affective reactions to information and ideas.
Separate knowing	the individual considers her knowledge as primarily being constructed to comply with rules that establish validity and to be defensible against challenges based on rules for validating knowledge.
Connected knowing	the individual considers her knowledge as primarily being constructed in interaction with other people (either directly, in conversation, or indirectly, through interacting with texts or other representations of others' knowledge and thinking), in a process that depends on understanding others' experiences, perspectives, and reasoning, and incorporates this understanding into the individual's knowing and understanding.

Belenky et al. are careful to observe that these metaphors are not necessarily superordinate stages in an inevitable process of development. While the fourth category *Connected Knowing* sounded most promising for the moral and ethical development of the child, the third reflected an internal authoring to the knowledge, even against some sort of external authority. Connected knowing embraces the notion that other's ways of knowing are likely different from one's own. Although useful as I analyzed my research data, none of these descriptors embraced a radical constructivist viewpoint on the nature of knowledge. The languaging sought to balance a subjective knowing, an inner voice, with some sort of objective knowing, failing to fully embrace the constructed nature of knowledge.

Grouws et al. (1996) research agenda did not embrace a constructivist orientation toward knowledge either. However, this research group did create seven categories to assess a subject's conceptions of Mathematics, namely the 1. Composition, 2. Structure, and 3. Status of Mathematical Knowledge; the 4. Doing and 5. Validating of Ideas of Mathematics; 6. Learning as Constructing and Understanding; and 7. Mathematics as a Useful Endeavor. I found thinking about each subject through these questions to be useful, and in fact often employed the ideas in the report of my findings. During the process of seeking repeating themes within each subject's data set, I (informally) created a visual measure of each subject along these scales. This tool not only helped me to bring together ideas about each subject, it created a useful representation to contrast subjects.

Deconstruction

I consider the final activity in this attending to the data to continue to be one of analysis.[78] I characterize this final step of Phase IV to be generally deconstructive in nature—that is reflexive, determined to open up possibility rather than close off thought, and to provoke challenge to think differently about unquestioned assumptions about mathematics, education, and/or research. Although it is my intention that all phases of data analysis reflected an intentional effort of metacognition and the radical constructivist's second-order analysis, this

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[78] This footnote is intended to communicate that "final activity" is a misnomer. The act of writing continued to be a step of data analysis. In fact, I consider it to continue to be one of data creation as well.

final phase of deconstruction is directed outward instead of only serving as an internal, personal check to maintain credible data analysis. This final deconstructive analysis is captured in Chapter 8, exploring implications for further research, reflections on goals for mathematics education, and interrogations of the notion of mathematics as knowledge.

Deconstruction is the poststructuralist's tool for research, that is, their methodology. Education research, as science, is an intentional, logocentric effort of data creation, analysis, and theorizing. I consider the goal of postmodern educational research to be to make possible for oneself and others to think different. Deconstruction is a critical practice toward this goal, a goal that aims to "'dismantle the metaphysical and rhetorical structures which are at work, not in order to reject or discard them, but to reinscribe them in another way'" (Derrida, quoted in Spivak, 1974, p. lxxv). It is not a destructive critique, but one that rebuilds, valuing possibility. With deconstruction, "knowledge is not closed, and the myth of finitude explodes" (St. Pierre, 2000a, p. 483).

Derrida set about to critique structures through deconstruction by simply remaining alert to the historical sedimentation of the language of the humanist agenda of which we have no choice but to use (Spivak, 1974). "However negative it may sound, deconstruction implies the possibility of rebuilding" (p. xlix). It does not freeze one's possibility to act. Quite the opposite, it demands that one acts, "For we are in a bind… a double bind…. We must do a thing and its opposite, and indeed we desire to do both, and so on indefinitely" (p. lxxviii); a schizophrenia of sorts. Deconstruction is also the ethical imperative to act. A postmodern epistemology recognizes that we construct, and reconstruct the world and ourselves, again and again, and we are "ethically bound to pay attention to how we word the world" (St. Pierre, 2000a, p. 484).

Derrida summarized his deconstructive method as reversal and displacement. Spivak (1974) offered "deconstruction in a nutshell: … to locate the promising marginal text, to disclose the undecidable moment, to pry it loose with the positive lever of the signifier; to reverse the resident hierarchy, only to displace it; to dismantle in order to reconstitute what is already inscribed" (p. lxxvii). Yet this formula offers little, certainly not an improved version of a truth. A deconstruction does not demonstrate what was incorrect or missing.

Although the intention of my analysis activity of Phase IV was not to deconstruct a socio-historically taken-for-granted notion of the mathematical learner (the GAML), I embraced the questioning and critical orientation toward the data suggested by the deconstructive practice. I did intentionally seek to consider varieties of potential meanings or interpretations of the data I had collected. On a scale larger than simply the Phase IV data analysis, this research project is ultimately one of deconstruction, what is possible for high school mathematics education? While practices of deconstruction were invoked throughout the methodology of this exercise, the final work of this project was a deconstruction of the practices of mathematics education, especially when the fabrication of knowledge is embraced in it double meaning—fabrication as construction *and* as concoction.

Although I have spent some time rehashing the postmodern critique of the desire for a hard line between data collection and data analysis, I structured my research design around many conventional methods. In the sections above, I described an adaptation to the constructivist teaching experiment as the methodological framework within which I worked, a framework that relied on many conventional methods. However, understanding that theoretical framework and methodology are "inextricably linked" (LeCompte, Preissle, & Tesch, 1993, p.116), my methods maintained a postmodern quality as well, such as the severe reflexivity that the radical constructivist observer and postmodern deconstruction call for.

Prior to presenting the results of this, what I am willing to label, cumulative case study, I reflect on the tensions between conventional research methods, the radical constructivist epistemology, the postmodern theory, and the restrictions of the logocentric, cultural demand for research representation. The radical, postmodern epistemological stance relieving the drive for a true knowing throws the construction of knowledge into disequilibrium. What is the “known” of such a knowledge? And what can be construed of validity? The language and languaging of modernism is all that I had at my disposal for communicating ideas in need of new words, reconsidering relationships in need of new metaphors, and rupturing presuppositions to which we blindly adhere. I worked in a narrative where meaning was to be fixed, yet I strove to find the play in this knowledge-fixation, to disrupt what is assumed and make possible other realities. Knowing that language deceives, how could I represent not only my own ways of knowing, but the three subjects of my study in a just manner? How could I check my drive to situate, to see merely what I am programmed to see, to make Same the Other? As I worked as a novice researcher amid the ruins (St. Pierre & Willow, 2003), after the crisis of representation and legitimation? Rather than finding despair or a paralysis to act, these ruins made possible for me a new energy about considering what could be. Amid these crumbled and decaying structures rose a freedom to both think and enact new ways of thinking. I embraced this *post-* era and found “possibilities for different worlds, that might, perhaps, not be so cruel to so many people” (p. 1). By engaging my researcher ethics, I drew upon and reconfigured the conventional methods and traditions in ways compatible with the theoretical framework I brought to this study. In this reconfiguration, I engaged my goals to find a more just practice of mathematics education.

CHAPTER 6

DATA REPORTING AND ANALYSIS

This research project was prompted by consideration of the mathematical learning goals, dispositional goals, and citizenship goals for mathematics students in secondary public school, and the achievement contradictions to many of the goals that research repeatedly reports on children in high school math courses. Studies of high school students (Garofalo, 1989; Grouws et al., 1996; Schoenfeld, 1989) did not see mathematics as a construction of their own productive reasoning—a generative disposition. Further, their sense of self as mathematical thinkers was similar to being a replicator of other's ways of knowing. Again, they perceived the value of their schooling to be to duplicate how they were supposed to know pre-existing ideas. And finally, their valuation of the other and toward developing a bond with community is stilted. These attitudes leaned toward recreating authoritarian structures and distancing knowers, rather than developing active, creative, interdependent people. Boaler (2004, 2006a; 2006b; 2008) described such a possible rapport among high school mathematics classmates as *relational equity*.

That few high school students display this generative disposition toward learning, my research was to seek some out and consider who they might be, how they interact in the mathematics classroom setting, and how they have come to maintain such as disposition from a younger age. As a part of such research, two issues became evident, that my work would necessarily be in part to provide a richer definition to this idea of a generative adolescent mathematical learner (GAML), and secondly to understand how the GAML saw themselves as generative—a key dispositional goal evident in the literature for the teaching of mathematics, and as a goal for a socially just education generally. These distinctions in the orientations toward considering the GAML could be conceived as a first-level and second-level consideration of the salient qualities of generativity. The first-level would refer to the generative quality of the mathematical activity of the learner. On this first-level, the characterizations of the GAML are certainly that of the observer, i.e. my characterizations of the mathematical activity, learning, and ways of knowing of the student. Hence, this hypothetical model of the research subject is necessarily of a second-order (Steffe et al., 1983; Thompson, 2000).

The second-*level* for considering the generativity of the learner is marked by a shift in focus; the focus of the observation is on what does the GAML perceives to be the nature of their own mathematical activity, learning, and ways of knowing. The hypothetical model I build for these self-awarenesses of my research subjects continue to be second-order models. Instead of a model emphasizing ways of knowing and interacting, they are hypothetical models for the subject's own self-awareness of their orientation[79] to knowing and interacting.

These two orientations to generativity I wish to emphasize are characterized by the distinctions in a *personal epistemology* and a *mathematical ego*. For the purpose of the following data analysis, a personal epistemology is the subject's self-identity as a mathematical knower and learner. The mathematical ego points to the subject's consciousness of their personal epistemology.

This orienting discussion of the data analysis is intended to initiate the presentation of the three subjects. These high school mathematics students were selected because they showed indicators of activities conjectured to be associated with those of a GAML. The intent of the

[79] Or more explicitly stated, the subject's orientation to the model for their knowing that I have ascribed to them as knowers.

study, that is the research question, was to consider how GAMLs maneuver through their mathematics courses while maintaining such a disposition? I utilized the following orienting questions to design a method for data collection, followed by analysis, in order to more deeply understand this curiosity:

1. What practices has a generative student created for engaging in mathematical activity?
2. What conceptions of mathematics has a generative student formed?
3. What does a generative student consider to have influenced her disposition?
4. How do generative students perceive the role of themselves in relations with others?
5. What are the relations among generative students and school discourses?

Along the way toward answering these questions about the three selected subjects, both of the models that I have been developing for the GAML, that of a personal epistemology and of a mathematical ego, have also been enriched.

This chapter will next present the data collected and its analysis for the three identified subjects in this study, Fisk, Kate, and Jack. After a brief introduction and description of what classroom activity led to their selection, each subject will be analyzed through a presentation of the following organizing principles: Mathematical Activity, Personal Epistemology, Second-Order Viability, and Viability in the Mathematics Classroom. I intend for the category Mathematical activity to be as simple as it appears, and the idea of a personal epistemology was described above. Viability in the mathematics classroom is meant to consider both the subjects' conceptual operations and their mathematical activity, wondering how those ideas as attributed to the GAML allow for a *fit* to the contexts in which the GAML was using them, i.e. the mathematics classroom. This focus is intended to be on the model of the knower that I as the researcher construct. Consideration of the second-order viability of the GAML takes a distinct shift toward the GAML's view of themselves and their ability to thrive in their mathematical learning environment. Of second-order viability, Glasersfeld (1995) says:

> It is obvious that this second-order viability, of which we can say with some justification that it reaches beyond the field of our individual experience into that of others, must play an important part in the stabilization and solidification of our experiential reality. It helps create that intersubjective level on which one is led to believe that concepts, schemes of action, goals, and ultimately feelings and emotions are shared by others and, therefore, more real than anything experienced only by oneself. It is the level on which one feels justified in speaking of 'confirmed facts', of 'society', 'social interaction', and 'common knowledge' (p. 120).

In this section of the data analysis, the orientation will be to understand how the GAML perceived themselves in relations to others, and how they get along among other mathematical learners.

Although the presentation of data analysis for each subject will hold fast to this repeated overarching structure, each subject will unfold in a manner that is unique to my experiencing of and interpretation of the subject. I did not make an explicit effort to carry out each subject's analysis in a parallel at a finer level of detail. In other words, for example the aspects of mathematical activity I emphasize in Fisk were not necessarily attended to in Kate or Jack. The reason for this was to enhance the differing qualities of each subject, rather than gearing the understanding of them toward similarity, or saming.

After summary of key ideas about each subject, I will reconnect to the developing model of the GAML, especially attending to the orienting research questions, but allowing for emergent foci as well. This presentation of the three case studies sets the stage for Chapter 7, which concludes the data analysis with a renewed statement of a general model for the generative adolescent mathematical learner. This model is intended to connect the experiences of the three subjects of the study, to the initial concept of the GAML presented in the opening chapter of this dissertation.

Fisk

As I introduce Fisk through the data recorded in an attempt to document my interactions with him, I will frequently use his spoken words or written responses to provide the reader with a sense of how I am arriving at judgments and conjectures about Fisk's ideas about mathematics, sense of himself as a mathematical knower, and his mathematical relations with others. At other times, I am drawing upon my written observations and reflections; field notes and their extensions. In these cases, I report the analysis that emerged from my rereading, coding, and sorting of these data sources. I do not refer back to the multiplicity of dates that these judgments are based upon for fear of being too pedantic. When significant or clearly defining moments do seem to be of importance, I document the date of the classroom occurrence. I will follow a similar pattern for each subject.

Selecting Fisk

Fisk arrived to his Junior year math class following an atypical approach. His ninth grade class[80] was taught by a sequence of teachers, rotating among predominantly three substitute teachers, and a full-time PHS faculty member. Fisk was next enrolled in an Honors mathematics course as a sophomore. This teacher reported that he showed less effort than necessary to keep up with the class, and shut down near the end resulting in a failing grade. This teacher said he "had a very low opinion of himself as a mathematics learner," reported in the Phase IV "Former Teacher Interview" (Appendix H). Fisk recovered this credit in an after-school program and enrolled in the regular Junior math class with his peers. It was in this class that I met Fisk, through the advice of Larry's response to the "Subject Identification Request" (Appendix A).

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[80] Recall, the mathematics sequence offered at PHS is an integrated, problem-based approach. The traditional high school mathematics topics, including those of an Algebra I, Geometry, and Algebra II course, are included in this sequence.

During Phase I of my research, I observed Fisk in Larry's classroom in order to confirm Larry's recommendation of Fisk as what I might see to be a generative mathematical learner. My field notes of this observation indicated that Fisk was mathematically very active. For example, he volunteered at the front whiteboard to present an idea to the class. What struck me about this presentation was that he seemed willing to share his thinking as it was forming, rather than feeling as though he had to present (and/or defend) an answer, or a method to derive an answer. While presenting, Fisk made connections to previous class work, wondering aloud, "Is it the same formula?" Fisk noticed that the problem he was thinking through was more complex in that a coefficient is now "239,000 rather than 1", and explained the equation he recorded to the board, "$H \cdot 10^x = FH$." Fisk recognized that what he had left to do was solve $10^x = F$, where F was equal to 15, 355.[81] He continued doing some mental work at the board, utilizing his fingers for some sort of calculation. He recorded to the board, in a column,

$10^4 = 10{,}000$
10
$10^5 = 100{,}000$

He next added, off to the side, $10^{4.—} = 15{,}355$. As he recorded this, he elicited from a classmate a possible solution. One of his peers estimated by guessing on a calculator. Fisk recorded the solution offered as,

$10^{4.19} = 15{,}488.1669$

The classroom teacher—Larry—did little to interrupt Fisk's thinking through the problem or the peer interaction. At this point, he challenged his students to *think*, not just copy from the board. Fisk concluded his time leading the class by asking a fellow student who continued to guess and test by using a calculator whether or not she got closer with 2 decimals.

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[81] I did not note the context of the problem in enough detail to understand why he would wish to solve this equality.

This episode was important in my decision to pursue Fisk as a subject for several reasons. He demonstrated that he wasn't burdened by an anxiety of being correct, and seemed willing to think aloud in front of his peers—a dangerous proposition on its own, but possibly even more so when the speaker perceives the environment being one of rightness and wrongness. At this stage, I did not know Fisk's orientation toward mathematics, but that he was willing to take this risk suggested a self-as-author orientation to mathematics. Further, it was apparent that Larry worked to develop a classroom learning culture in which students talked to each other about mathematics, reducing his role as an authority. Fisk showed that he was verbal and not withdrawn; these qualities in conjunction with the signifiers of his potential to be a GAML convinced me to pursue him as a research subject.

Mathematical Activity

Mathematical ability. Phase II of my data collection began with the Subject Survey (Appendix B). In the *mathematical beliefs* portion (questions 1-4) of the questionnaire, Fisk demonstrated an exaggerated confidence in his mathematical ability as well as a very pragmatic usefulness of mathematics. In response to the question, "If I had to give myself a score out of 10 to show, honestly, how good I think I am at math, the score I would give myself would be _____, where 10 means absolutely outstanding and 0 means absolutely hopeless," Fisk scored himself a 10.

Pursuing this quality of an apparent rift between self-perception and classroom success somewhat further, I observed Fisk as a classroom mathematics student to be a very capable mathematical thinker, *on par* with my experiences of many high school juniors. My emphasis in this claim is that Fisk's potential to *reason* mathematically and to *solve mathematical problems* was similar to the majority of junior-age mathematical students I have met in my teaching career, including those in Fisk's current course and school. As I share some examples to clarify my experiences with Fisk, it is important to emphasize that the labels of mathematical reasoning and problem solving I use are what is typically used in schools as a definition for mathematical acuity—qualities such as ability to solve on ones own, speed and accuracy in numerical analyses, and a scientific, rather than intuitive, way of knowing. For example, in my experience as a teacher, typically 3 students in 30 solve a problem through their own exploration about rat population (this problem is discussed in the next protocol). Also, few students generate and demonstrate on their own a logical proof for the equivalence of the two binomials (Fisk's activity presented after the rat problem); however, with coaching on the part of a teacher, most can establish this fact.

To consider Fisk's mathematical activity further, consider a task assigned just as I began Phase II of the research. This was a problem about the growth of a rat population on an otherwise deserted island, a problem intended to involve approximately one week of investigation. Students were asked to do a formal write-up of their exploration and findings on this problem. The problem, as written in Fisk's own words (Archived data: Problem of the Week write-up):

> I have to explain how many rats there will be on January 1st of the next year after the first year. In the first year they're will be 2 rats, 1 male 1 female. These two rats will make offspring under these certain rules.
>
> 1. The number of young produced in every litter is six, and three if those six are females.
> 2. The original female gives birth to six young on every January 1 and produces another litter of six every 40 days thereafter as long as she lives.
> 3. Each female born on the island will produce her first litter 120 days after her birth and then produce a new litter every 40 days thereafter.
> 4. The rats are on an island with no natural enemies and plenty of food, so no rats will die in the first year.

Fisk introduced the situation well, in a complete manner so that the reader knows all that is necessary to solve the problem.

This problem, although reasonably straightforward and easy to compute for the first several months, gets quite complex as time goes on during the first year of rat population growth. In my previous experiences giving this problem to 16-year-old high school students, approximately 10% of the students were able to obtain an accurate solution in the 1-2 weeks allotted to work on the task. Fisk did not solve the problem correctly.[82] Here is a copy of his statement about how he worked, what he determined to be the solution, and his justification.

> Process: What I did to figure out this problem was; I made a chart on the rats birth [not provided]. It was hard because it took a lot of thinking, just because I had to follow certain rules. It wasn't hard figure out how much rats they will be, but it became tougher when I wrote it down. The thing that got me was, once all the new offspring came I couldn't forget about the rats from before, and you still had to add all their offspring in too.
>
> Solution: The solution I got by my chart (don't know if it's right but this is what I got) is 1749 rats. I included the first pair of rats, their offspring and the offspring's, offspring, etc. It was a very tough question because I never did anything like this. It would have been more rats but the offspring couldn't [give birth to] their litter until 120 days after they were born and there are only 365 days. I do believe my answer is right because I found no other out comes when I went back over it.

Not only was Fisk's solution incorrect, but his sense of what was necessary to successfully *communicate* and *represent* his mathematical thinking was *below average* for high school juniors. Similarly aged students typically provided a more carefully documented rationale demonstrating how they arrived at the solution presented. This often looked like a tabular or graphical accounting for and accumulation of the baby rats, usually focusing only on the female babies. Another student in Fisk's class developed a symbolic approach to sum the multiple sums of series of births. Her work resulted in a highly generalizable expression, accurately stating the rat total after one year. Her documentation of her exploration and ability to communicate what she had done to her classmates was typical of a high school juniors' ability to communicate and

[82] I have refrained from including the solution to the problem so that the reader may explore it on their own. I encourage collaboration with others to check on the accuracy of any solution proposed.

represent her thinking. However, her mathematical insight was rather powerful, the first student (or adult) I had seen in my experience with the problem to do such powerful mathematical analysis. By my observation of Fisk's attention and demeanor, I suspect he was able to understand her work and conclusions, but he provided no indication or evidence of connecting her thinking to his own. Again, in his non-evidence, I suspect he was not uniquely talented as a mathematical reasoner or problem solver. Furthermore, the justification he provided for his solution was highly incomplete, and in this way his mathematics was unexceptional, if not poor, as well.

As confident as he proclaimed to be in his abilities to do mathematics, his statement in this problem that "don't know if it's right" suggested something about what he conceived of as being mathematical. Taking both claims simply as truths, that Fisk perceived himself to be exceptionally good mathematically and that he was unsure if his solution was correct forced a consideration of what Fisk might have in mind when asked about being mathematical. It seemed apparent that being correct was less important to Fisk than having mathematical insight. Fisk's was an intuitive knowing, possibly a *subjective way of knowing*, per Boaler and Greeno's (2000) ways of knowing—see Table 6.

During our first interview, as Fisk and I were discussing the example of the problem to factor $x^2 - 12x + 35$ that he had solved differently than his classmates, I asked if he thought his technique would always work. Before replying, he paused and did some mental figuring, moving his fingers. He then made the comment:

F: The technique will always work.

B: You think it will always work?

F: Yeah, and, what was it…

B: I can't quite remember. I have this vague memory, I was sketching it down.

> F: You have $-x$… wait. [writes] $(-x + 5)(-x + 7)$. Then $(x - 7)(x - 5)$.
>
> B: Now most people would do this [pointing to the second], but you wrote it like that [pointing to the first].
>
> F: Yes, I still get x^2. I get minus $7x$. I get minus $5x$. And I do wind up getting 30, I mean plus 30.
>
> B: 35
>
> F: Oh, 35, all right. Same this x^2, $7x$, $5x$, plus 35. That's $x^2 - 12x + 35$. [Then pointing to the other expression] $x^2 - 12x + 35$.

This passage demonstrated that Fisk said confidently that the expressions were equivalent, free from having a formal logical proof in mind. Fisk used a multiplication process reliant on the distributive property to demonstrate their equivalence. But he talked his way through the argument in an informal manner, not attending to appropriate justification of steps that might be the key or most insightful portion of the argument necessary to establish equivalence. But on a greater scale, he allowed this particular example to stand in as an evidentiary proof that his technique would always work. He seemed to rely on his intuitive knowing of this mathematical "truth". I suspect his operations were intuitive and insightful, rather than a re-presentation of conclusions he had already derived, based upon his initial pause. At the very least, the pause early in his response suggested that he had to reconstruct a justification he had previously determined.

One further excerpt from Fisk's solution to the rat problem—taken from the *Self-Evaluation* and the *Self-Assessment* portions of the write-up—continued to demonstrate that his self-confidence was not dependent on achieving correct solutions, but rather a valuing of his ability to think in mathematical ways.

> Evaluation: I do consider it to be educationally worthwhile because it's a logical problem and you can learn something from almost anything. Now I'm ready for more problems like this that deal with logic. Before this I had a decent view on how to approach these problems but now its even better.

> If I could change this question to make it better by changing it to two years instead of one year. That will a lot more thinking than this problem but I believe the problem would be better off that way. …
>
> The question was decent for me not too hard not to easy I was just tiring.
>
> Self Assessment: A Grade I think that I deserve is a maybe a B+. I think I explained myself thoroughly, but I might have gotten the wrong answer. I kind of know I'm a little [below] the real number of rats.

I included these statements to demonstrate and summarize a few of Fisk's self-perceptions. The last sentences indicated he is rather certain his solution is incorrect; yet his work is deserving of a B+. I take this to mean he thinks what he has presented as a mathematical analysis and as a report of this analysis is quite good work, only falling short in that the answer may be—in fact is likely—incorrect. He thought that he explained himself "thoroughly" in this write-up. Again, in my experiences with the abilities of high school juniors to communicate and represent mathematical ways of operating, this aspect of his mathematical work is poor. Thirdly, of most interest was that although Fisk is uncertain of his solution ("kind of know I'm a little [short] of the real number of rats"), he found the problem to be something he learned from and thought it would be better if it were an even harder problem—two years on the island instead of one.

Fisk's self-report of "10 – absolutely outstanding" in response to how good he thought he was in math was *incorrect*. Yet, this evaluation was *my* judgment of him; what was more noteworthy was that the self-report was his own perception of his mathematical ability. The ways in which he engaged in the classroom, and compared himself with his peers likely tells more of the story behind this self-assessment. The notion of the relativism of his perception of self as a mathematical actor, author, or learner will be returned to.

I experienced no interactions with Fisk that particularly demonstrated a mathematical brilliance. I make this judgment relying on my 9 years experience as a high school mathematics teacher. Through my history as a high school teacher, I developed ways of knowing high school mathematics and ways of knowing children doing high school mathematics; what could be construed as my models of the mathematics of children—high school juniors in particular. My experiences with Fisk as a mathematical actor/knower also did not cause me to either deepen or extend my models for or my ways of thinking about the mathematics of high school aged children. Fisk's mathematical activity did not strike me as particularly unique nor powerful, in comparison to other adolescents I have taught in the past.

Classroom engagement. In class on April 12, I noted that, upon arrival, Fisk sat at his desk and left his backpack on. He appeared to be looking at and be "thoughtful about" (classroom video transcript 04/12/05, 5:00) the task Larry had written on the board. Later as the class progressed, I was leading a conversation with a portion of the class on confirming a conjecture someone had made. I said, "Does this hold true for the value 10?" Larry asked Fisk, hoping to engage him, "Why did [Brian] pick 10?" Fisk replied, "Just some random number" (classroom video transcript 04/12/05, 50:00). This response indicated to me that although some behaviors suggested otherwise, Fisk did pay attention to the mathematical activity occurring in the classroom. I say this because Fisk's reply was more or less exactly why I had picked 10—I did not have a special property of 10 in mind for the given problem, simply having selected it to encourage students to substitute and evaluate as a check of the conjecture. His recognition of this, unless a lucky guess, indicated he understood the intent of my mathematical conversation with the group of students.

The classroom episode on the next day, although a somewhat atypical classroom routine, continued to demonstrate his non-stereotypical participation in a mathematics classroom. It also speaks to Fisk's view of himself as a mathematical learner and about his ideas about the nature of mathematics. On April 13, Fisk arrived in class and sat in a desk at the front whiteboard and faced out to the class the entire period (field notes 04/13/05). He kept his backpack on almost this whole class period as well. What was more startling was that this was a test review day in which Larry led the class through problems and topics that would appear on the test while standing at the whiteboard just to Fisk's right. Occasionally Fisk would twist to peer over his right shoulder in order to see what was being written to the board. Most of the time he appeared less engaged, possibly even daydreaming. I pursued Fisk's classroom activity with him in the April 15 interview.

> B: Do you remember Wednesday [April 13] in class?
>
> F: No I do not. … Oh! I was just standing right there, just sitting watching everything go on.

Again, Fisk confirms he was engaged in the classroom activity, "watching everything go on."

> B: Tell me what you were doing that day.
>
> F: Uhh… just taking everything in, since it was basically a review day. Really, if I sit back at the house, reviewing for a test, then I mess up. I never really review for tests ever. I just don't especially [for] math, because there's not really nothing you can review, because if you're paying attention you're going to remember it. You don't really need to review it. Just like, how we did the foil method last year, and I knew I could remember it, I knew there was a quicker way to do it. Then I sat there and named it, and I did it. Cause I always hated doing those boxes [area models]. It takes too long! Sitting there writing it out on your paper, it takes too much space. I just sat back, let [Larry] do his lesson, looked at the board a couple times to see if there's anything new up there, or anything I did and remembered that we went over earlier that maybe I think would have come in handy for the test, but I looked up there, there wasn't really nothing special up there, cause the way I think about it, the way I do it is sit there and break down numbers and count numbers in my head and since the people want to see it, I'll do what I'm thinking in my head on the paper. And those open-ended math questions, they aren't really hard.

It is evident that Fisk's manner of engagement—an unorthodox style of not relying on paper and pencil to record information (knowledge bits) and working out mathematical ideas in his mind—was connected to a particular orientation toward mathematical communication and representation. Fisk stated, "…and since the people want to see it, I'll do what I'm thinking in my head on the paper." His written form of his mathematical thinking was not for himself, either to aid thinking or for future use, it was because other people wanted to see his work.

I will return to this aspect of his classroom engagement after completing a presentation of this snippet of interview protocol. Next, I asked Fisk about a moment during which he did reach into his backpack, and used his calculator during Larry's test review.

> B: I noticed you took out your calculator.… Do you know why you took out the calculator?
>
> F: I think he had asked the class… oh yeah, with the graphs. Yeah, I just wanted to see if I got the same answers as he, and all that then just put it back in… Yeah, it was for a graph because I knew I couldn't think of a graph in my head. Then I put it away, because at that point I was just playing with the calculator. I had stepped into class, what, 10 minutes late, so I already missed the "do now", then it was a review, so we started off a little slow, then it picked up, then toward the end it dropped down again. But even where it picked up I felt like, all right, I know this stuff, I'll be all right, everything will be ok, so it's not that I wasn't paying attention. I was still listening. But it wasn't something, oh I need to jot this down, cause anything that basically is going on in this class right now I basically know.

Fisk continued to show he was engaged in the review session, recognizing that when he was "just playing with the calculator" he put it away. This passage also continues to document Fisk's self-confidence in knowing mathematics: "I know this stuff." What Fisk believed to be an ability to do math in his head and his trust in his intuitive ways of knowing mathematics were what he attributed to his strengths of knowing mathematics. These two episodes of Fisk's unusual classroom activity demonstrate that he worked mentally and that he stayed more engaged than he appeared to be.

The final passage alludes to another aspect of his classroom engagement, that Fisk did much of his mathematical work in his head. He relied little on common, mentally externalized tools of mathematicians—thinking devices such as drawings, symbolic notation, or calculators—only occasionally turning to a graphing calculator or pencil and paper to aid in his thinking. He did not record to paper; in fact, when he did he showed an awareness of a need to conform to conventional usage of mathematical symbols, and he also demonstrated a novice's use of typical algebraic representations. For example, on April 6, students were instructed to explore the effects of the coefficients *a*, *b*, and *c* on the graph of the standard form of the quadratic function: $f(x) = ax^2 + bx + c$. Whether Fisk missed the teacher's re-statement of the standard form, wasn't paying attention, or was thinking on something else, his next question was interesting. He asked me, conjecturing from a warm-up experience with the general linear function $f(x) = ax + b$, if "quadratics would be of the form $abcx^3$?" (field notes, 04/06/06). It was apparent in Fisk's question that he was attempting to develop an ability to understand and communicate with the symbolic language of Algebra.

The next day the activity continued. As I approached a cluster of students that Fisk worked among in order to check on their activity (video transcript, 04/07/08 31:00), Fisk stated "You know I know this stuff." However, the group, including Fisk, had not recorded any findings at this point (field notes, 04/07/08). Fisk picked up a new graphing calculator after noting that the batteries on his were dead. Later in class (video transcript 04/07/08, 45:00), Fisk showed he knew something about the effects of changing the *a* value from positive to negative. I noted in my field note expansion that afternoon that Fisk was able to explore the effects of the coefficients, and appreciate the findings of other students, but that he did not possess standard ways to record his findings. He did not use symbols nor graphs. In fact, he recorded this particular finding about the *a* coefficient using data tables, something like:

4

x	*y*
-2	16
-1	4
0	0
1	4
2	16
3	36

-7

x	*y*
-2	-28
-1	-7
0	0
1	-7
2	-28
3	-63

My observations indicated Fisk understood the need to communicate mathematical ideas through some form of representation. However, he did not demonstrate a strength in using conventional representations, nor did he seem to engage these representations to do mathematical work, doing most work in his mind. Fisk did note that when he needed to build a graph he utilized a graphing calculator because, “I knew I couldn’t think of a graph in my head” (interview transcript, 04/15/05 56:00).

Personal Epistemology

Mathematical ego. Fisk demonstrated an exaggerated confidence in his mathematical ability as well as a very pragmatic usefulness of mathematics in the *mathematical beliefs* portion of the Subject Survey. In response to the question, “If I had to give myself a score out of 10 to show, honestly, how good I think I am at math, the score I would give myself would be _____, where 10 means absolutely outstanding and 0 means absolutely hopeless,” Fisk scored himself a 10.

He went on to indicate, by selecting from 5 statements, that “Mathematics is something I do: ‘Every day as a natural part of living,’ and ‘Mostly in my head.’” By not selecting “Mostly at school,” “With a pencil and paper,” and “With numbers,” it is apparent that Fisk saw himself as a mathematical person, where mathematical thinking was done as a part of living, not as something that was limited to a topic of school study. This embodied orientation to mathematics was again reflected in his responses showing disagreement with the following two statements:

> The ideas of mathematics have very little to do with the real world.
> The ideas of mathematics can only be explained using mathematical language and special words.

Fisk also disagreed that “The ideas of mathematics developed as people needed them in daily life.” It would seem as though Fisk would have agreed with that statement, given that he sees himself as a mathematical actor it would make sense that he thinks of mathematics as something that emerged from people's daily lives. But he didn't; the seeming conflict of ideas might be rectified by conjecturing that Fisk did not hold tightly to a sense of mathematics as a knowledge that takes on an existence external to knowers. If Fisk refuted the initial portion of the phrase, "The ideas of mathematics," from this standpoint, the sentence as a whole made little sense, and thus is disagreeable. It may be that the word "developed" created the disembodied interpretation that justified Fisk's rejection of the statement. I will return to this conjecture later in the data analysis.

There is another interesting conflict evident in these first few sources of data. One teacher reported that Fisk had a low opinion of himself as a mathematical learner—as a sophomore, yet Fisk self-reported an extremely high self-perception of his ability in math. The data gathered to this stage could by no means offer an explanatory reason for this, but it was reasonable to conjecture there were two components at play. One is that Fisk's confidence evolved—that is, it was not static, and second is that a teacher that valued particular ways of being mathematical may view a child's mathematical ability and sense of their own mathematical activity quite differently than the child viewed themselves.

Idiosyncratic ways knowing. I joined Larry as Fisk's co-teacher during Phase II of the data collection. In the classroom, Fisk continued to demonstrate self-confidence as a mathematical learner. His classroom actions suggested a strong belief in his abilities to do and to think mathematically. In fact, this apparent confidence would probably best be characterized as rather egotistical. Although he knew his mathematical knowing wasn't always correct, his source for pride was that he knew he had idiosyncratic ways knowing. In the following passage pulled from our first interview, Fisk recalled a moment in class where his mathematics was represented in a different way than classmates.[83]

> B: Can you think of a time, and maybe not even recently, where you would feel pretty comfortable saying, "This was something I invented?"
>
> F: Mmm [looks up, thoughtfully] Sure, let's say, Tuesday? Somebody asked me if I was dyslexic because I looked at a problem backwards, and instead of using the numbers as negative, I used the variables as negative. And still got the same answer.
>
> B: I remember that.
>
> F: I could say that. In one sense it will work, but if I look at it in a plots and area models it didn't work. But if I did it in the foil method it did work, and I did get the right answer. So I could say that—it's the first thing that popped in my head.
>
> B: And you picked that, I'm guessing, because it was probably pretty different, unique, to how anybody else might do it?
>
> F: I mean, obviously as the people saw it, nobody else saw it that way. That's why [Larry] had to break it down. People realized I got the right answer, but they couldn't figure why I'd do it like that, or look at it like that.

As Fisk described this classroom experience, it was evident in the classroom records of that date (field notes, 04/11/05) that he wasn't certain of the mathematical equivalence of his statement, $(-x + 5)(-x + 7)$, to the rest of the class' result, $(x - 5)(x - 7)$. Fisk stated in the interview that although these two expressions were "the same" (equivalent) in plots (graphs) and area models (an area-based model for multiplication), the other students in the class were using the "foil method." However, Fisk's emphasis was that he had created a solution—that is, a way to solve the given problem: determine roots of $x^2 - 12x + 35$—different from how most everyone else was thinking. Furthermore, this creation of his was interesting enough for the teacher, Larry, to go over it with the whole class.

[83] In transcripts, I will use "B" to indicate something said by me. I will use the first initial of the subject to indicate their comments.

Fisk's prideful orientation to his different thinking was among several facets of his confidence. For instance, due to a high course grade in his first year, he was recommended for and took an Honors section as a sophomore. However, he failed this course. He completed the credits for the sophomore course in an extra-curricular program, and as a junior, he was back enrolled in a standard track course. He was again successful in that he was once again engaged in the mathematical activity of the class, he completed homework, and scored well enough on quizzes to be passing the class. These "successes" were documented through classroom observation and the comments of his teacher, Larry. Further, Fisk himself demonstrated a more positive attitude toward his achievement and ability in the Junior year mathematics class. However, once again Fisk's grades and classroom engagement plummeted near the end of the year, when his regular teacher Larry left and a long-term sub took over the class, as reported by this new teacher. Fisk seemed to be able to be impervious to negative external messages that countered his own positive sense of his abilities. The previous two samples of my interactions with Fisk show examples of this, the first sample reflecting the temporary state of not coming to the same answer as class members and the second being the degree to which teacher judgments did not dissuade his confidence, including poor and even failing grades. Fisk seemed to strongly separate his own ways of knowing and perceptions of his abilities from what school told him about his knowledge and aptitude.

Ontology of mathematics. Fisk responded in the Subject Survey that initiated Phase II, to be in agreement with both of the following two statements:

> The ideas of mathematics were invented by mathematicians.
> The ideas of mathematics were discovered by mathematicians.

This response surprised me. I initially thought people reading these two statements would agree with only one or the other, not both. During our first interview/tutoring session, I followed up on Fisk's response.

> B: Do you remember that survey you filled out for me? One of the questions I wrote on there was, "The ideas of mathematics were invented by mathematicians." And then I wrote after that, "The ideas of mathematics were discovered by mathematicians."
>
> F: No, it wasn't invented by mathematicians because a mathematician was somebody who was good at that position. Somebody else had to realize it at first, therefore they couldn't have been a master at that position.
>
> B: So are you saying somebody before the mathematician?
>
> F: Realized what it was, then somebody else realized they were good at that.

In this passage, Fisk indicated he believed someone *realized* mathematics, and then the mathematician was someone who became good with these mathematical ideas, or activities.

I asked further to learn more about his sense of *realized* in comparison to discovered. Discovered, to Fisk, meant that it was already there, like discovering gold in California. This use of *discovered* seemed to place an object in existence prior to any and all knowers.

> B: How do you think of invented and discovered differently?
>
> F: If something is discovered, that means it's already there. Invented is somebody's idea in their own head, or an innovation on another, something to make it better. Like how you go from an abacus to a calculator. If it's discovered, it was already there. Like, I can't say I discovered sand. But I can say I did invent a sand castle.

Sand, for Fisk, existed prior to any and all knowers. When asked to elaborate on his orientation to invented versus discovered, Fisk created this interesting metaphor using sand; he could not discover sand, but could invent a sand castle. Fisk considered his mathematical activity akin to inventing this sand castle. Maybe he hadn't invented the materials with which he crafted meaning, but it was of his own work to craft that meaning. In this way, his mathematical activity was inventing ways of doing a mathematics. His "sand castle" (mathematics) was not so much a generalized sand castle, but each particular, and unique; hence the potential for invention. Evidenced in a passage above, Fisk noted a classroom example of his invention, when he created the solution $(-x + 5)(-x + 7)$ different from most of his fellow classmates.

Fisk's metaphor to describe *discover* and *invent* discarded the question of the ontological status of mathematics, whether or not it has some existence prior to his knowing. I suspect that he didn't wrestle with his ideas whether mathematics has always been there (like "sand"), in part because it might be a blow to his desire to fulfill his egoism[84] (Stirner 1845/1971) if he were to even momentarily consider that he was merely a discoverer of something already there. It is interesting to consider Fisk's use of the term *realized*. He stated, "No, it [mathematics] wasn't invented by *mathematicians* [emphasis added] because a mathematician was somebody who was good at that position. Somebody else had to realize it at first, therefore they couldn't have been a master at that position." I take this sentence to emphasize *mathematicians* as people other than Fisk himself, and that *he* may be the person who *realized* new ideas. It might be me, as a researcher, or Larry, who are the mathematicians recognizing what Fisk has realized. Taken in this way, it continues to hold that Fisk views himself as an inventor. And it is someone else who comes to recognize or understand, i.e. discover, what Fisk had realized.

Locus of authority. I found it not easy to locate Fisk's sense of mathematical authority. Much of Fisk's activity suggested he located authority internally, as evidenced above in examples such as his convincing himself about the role of the coefficient a via tables, his manner of engaging classmates on solutions and justification to problems, and that he saw himself as someone who realized or invented mathematics. He was dissatisfied with mathematical work

[84] Stirner's egoism can be understood as a view that takes knowers—humans—to be motivated by rational self-interest. Whether this is a conscious state of accepting the engagement of living, or an involuntary actuality of seeking altruistic relations, human's egoism remains.

until things made “sense,” that is made sense for him, in his mind. This was repeatedly evident. For example, as the class work continued on April 22 to convert standard-form quadratics to vertex form, Fisk was wrestling with a problem, thinking aloud, “It don’t make no sense” (4/22/05, 68:00).

He did not ask for the teacher to confirm his accuracy or conclusions, but sought to share his knowing with the teacher to either demonstrate that he was on track or even at times to show off that he understood. If a teacher disagreed, or hinted of an error through their tone or reply, Fisk would not dismiss the teacher, but would think further. So Fisk did not ignore the potential for knowing in others. Fisk certainly paid close attention to what both the teacher and his peers talked about when discussing mathematical problems and solutions, as can be seen, for example, in his self-report on how he listened to the teacher during class the day of the test review and in how he turned to his peers to make sense of the 53.

A somewhat more compelling example of Fisk’s internal locus of mathematical authority but valued input from others was evident in the way he treated the front whiteboard in the room. When things were recorded there, either by the teacher or by other students, he seemed to grant these ideas or solutions a heightened authority, as though sanctioned in some way. Work, solutions, ideas, etc. that were posted to the front board were treated almost like the answer key. Yet Fisk did not quite treat the board as *the* answer key, not responding to it’s contents by erasing his thinking and replacing it with what appeared there. But he certainly knew to pay attention to what was there, and compare his ideas against those. The internal locus of authority I attributed to Fisk did not cause him to operate in a solipsistic manner; he did keep his ideas at play in relation to other inputs of classroom relations.

Usefulness of mathematics. Fisk saw mathematics as a useful tool. He saw mathematics in album cover design and problem solving (stated in Interview 4/15/05, 57:00), in art and sports (Student Survey), and even in figuring out how to use his cell phone! Because he seemed to focus on mathematics as a process of thinking rather than as information, his examples of mathematics focused on ability to do things and ways in which his activity was mathematical as opposed to topics of a school subject with little value in everyday life (Grouws et al., 1996). In addition to being useful and as a part of living, mathematics engaged Fisk's curiosity—not only via the activity of problem solving, but also by engaging with the thinking of his peers. It seemed as though Fisk was engaged by the mental stimulation, and not much by the prodding of others or threat of a grade. As Fisk experienced Larry's classroom, he engaged in challenging mathematical reasoning. In other teacher's classroom, he seemed to disengage. The following excerpt from his Videography, produced at the conclusion of Phase II, demonstrated this:

> Like, math ain't no problem. … My first year I had [Larry] as my teacher. I enjoyed his class. I got an A in his class. Average. And I'm trying to do the same thing now. 10th grade was tough. I had honors. Didn't do so well in that class. I had to take a 5th block class and got my grades up. Got an A in that class. Then I started taking [Larry's] class this year … and just really got into it. … But I just started doing a lot of work. … Got a whole 'nother semester. We'll see how I do.

Notice that Fisk states that he is "really getting into" Larry's class this semester. However, the filming for the Videography occurred after Larry left for the year. Now Fisk has a substitute teacher.

> …and we be sitting here waiting for the answer and all that stuff [he is filming during math class, with a substitute]. We got to slow down the lesson. I mean, I ain't trying to do a whole bunch of work, but I'm just saying I don't want to be sitting here waiting the whole time.

After I left PHS at the conclusion of Phase II near the end of April, a substitute teacher—another member of the PHS faculty, was teaching Fisk's class note. Fisk's comments above demonstrated that he was much less engaged at this stage in the semester, for reasons related to waiting for a teacher-provided solution. In casual conversation during the data collection follow-up of Phase III, Fisk noted that the 5th period course he took to make up the lost sophomore credit was easy. And that he had really stopped working in this current Junior level math course, so as a result wasn't doing well. He wasn't concerned because he intended to simply take another of the 5th Period credit recovery classes. This passage was another indicator of Fisk's ability to thrive in a mathematical and self-perception sense with Larry as his teacher, but otherwise was usually disconnected from the mathematics classroom activity.

Second-Order Viability

Relations with class members. Early in the first week of Phase II, I had not fully developed my role as a teacher in Fisk's classroom. On April 7, Larry had left an agenda on the whiteboard, but after 10 minutes, Larry had not yet arrived to class.[85] Early on, Fisk was chatting with a small group of disruptive girls about being filmed (video transcript, 04/07/05 8:00). The classroom video on this date showed Fisk beginning to write something to paper at 12:00 minutes. This was not work in response to the agenda, but was a response to a mathematical question of a group member. Shortly after this, Fisk's attention returned to the socializing of a nearby group of female students.

I began working with groups, mostly with the intent to focus them on achieving the agenda Larry had left. Between 19:00-24:00, I recorded to transcript some of Fisk's activity evident in the classroom video:

> [Fisk] is helping a pair of the girls as they begin to focus. Fisk continues to drift, not paying attention. He engages in conversation with his neighbor—male. This goes on for a while. A couple of other boys in the room have their attention turned to Fisk.

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[85] Larry never came to class on this date.

On 04/15/05, I observed similar activity in the classroom video. Again, a nearby student for help is asking Fisk.

30:00 Fisk's neighbor asks for calculator help—he does so.

35:00 Fisk helps other neighbor on calculator, but his attention is more on the girl he sits next to.

At approximately 20:00 into this class, I noted during some whole group instruction being orchestrated by Larry: "Fisk is also chatting with others in the room—mathematically." On a later date, April 18, I noted that Fisk appeared to be confident in a solution to a rather challenging problem. Although I encouraged him to share his thinking with his neighbors, he did not. At 40:00, after further encouragement to share with neighbors, I noted that "[Fisk] talks a bit to neighbor, but doesn't impose himself on [her]." And then later in the same period (56:00), "Fisk is chatting with the girl next to him."

Earlier on April 18, Fisk provided a verbal contribution to a whole class discussion in which he generalized a relationship utilizing letters instead of numbers (9:00). I noted that he stated pridefully, "I just gave a formula!" Fisk records his formula to the board and another student joined him to show her idea. While they are up their together—leading the classroom discussion, I noted that Fisk attended closely to her reasoning. He asked aloud, but to no one in particular, "Does every quadratic have to be a perfect square?" (16:00). This episode confirms an self-awareness Fisk had to needing to communicate his ways of knowing mathematically in a standard representation, in this case what he named a formula. Further, the (powerful) question he posed in a pondering sort of way was indicative of the way he engaged in mathematical activity, a generative process of investigation—leading to more questions, rather than a terminating process to secure, or to know.

The quality of Fisk's mathematical interactions continued to be evident on April 22. At 8:00, a male neighbor asked Fisk a mathematics-related question. This time, Fisk seemed unable to provide the requested help. At 15:00, the same neighbor asked another question—seemingly about the calculator. Fisk took the calculator, pressed some buttons while the partner looked on, and then returned it. There was little talk. This series of brief interchanges were indicative of several instances in which Fisk's peers saw him as a resource for help and solutions. Further, Fisk saw himself as someone able to provide answers to his peers. When a question was raised that Fisk may not have been able to answer, for example at 8:00 on April 22, Fisk did not take on an orientation to the peer interaction like, "let's figure it out together." Instead, his responses to peers were continually in the manner of one who saw himself as an answer provider, and an authority—someone who could provide a correct response. Fisk did not seem to be someone who desired to work collaboratively with another to figure something out in small group interactions. However, there were occasions where he did collaborate, and listen attentively to others.

Mathematics, classroom: connection and disconnect. The protocol in the previous section demonstrates ways in which Fisk was often connected, if not immersed, in the activity of the classroom. Although not immediately apparent, he was aware of my teaching intentions as evidenced by the understanding he showed of my selection of testing an idea with the numerical value 10—presented in a protocol several sections earlier. His seemingly casual, uncaring demeanor during the test review camouflaged his engagement in the summary of main ideas Larry brought forth for discussion. Much of this section has so far suggested that Fisk's classroom activity is that he tended to work mathematically in ways disconnected from his classroom peers, or may be off-task mathematically, involved in non-mathematical socialization. While these statements were often the case, they were not the only ways Fisk engaged in class.

There were also moments of real work to understand other's ideas and build upon them in collaborative ways.

On April 11, there was a full class discussion in which students shared their thinking about ways to factor $x^2 - 7x + 10$. Larry had been soliciting volunteers. As one student finished her presentation, having arrived at $(x - 5)(x - 2)$, Larry had stepped outside to speak with a disruptive student. Fisk came to the front of the room on his own accord to present. He showed the class a visual model (Figure 2) that demonstrated a different factorization than the previous presenter showed.

5	$-x$	
$-5x$	x^2	$-x$
10	$-2x$	$+2$

Figure 2: Fisk's model to factor $x^2 - 7x + 10$.

His conclusion was that $x^2 - 7x + 10$ could also be factored as $(5 - x)(2 - x)$. My field notes from that day indicated that I believed he was engaged with the mathematical conversation of the classroom, but also was seeking an alternative to the solution provided. With the classroom disruption, he found an opportunity to share his different thinking with others.

Another classroom episode also gave reason to believe Fisk was highly engaged in thinking through the mathematical reasoning of other's in the classroom. On April 22, Larry was working to support students in understanding how to construct the necessary addend to "complete the square" when shifting quadratics from standard to vertex form (more on the details of the classroom interaction are below). During this whole class discussion, again geared around students sharing their thinking, one student presented a method to solve for this value, but with very little explanation. Fisk volunteered to explain his thinking, and spoke with confidence about an initial step the class had agreed upon. As the difficult step came, he recognized the significance of speaking accurately and in ways that his classmates would understand. I noted at the conclusion of this classroom transcript, “Fisk has consistently shown the utmost respect for all members of the class’ thinking, today and in the past.”

In further analysis, that conjecture was not always the case. The statement should probably be modified to indicate that although friendly and socially respectful to all classmates, Fisk showed a cognitive respect toward only certain member’s of the classroom. These seemed to be people who would volunteer ideas to be considered and understood; ideas that were seeds for Fisk’s mathematical musings. This is significant not only to help describe how he interacted shortly with some of his peers—those who asked him answer-oriented questions, and demonstrated a more engaged attention toward those peers that proffered ideas that engaged his mind. I conjecture further that Fisk would assign ratings to his peers based mostly on this characteristic of their classroom mathematical activity—at least, this activity that Fisk experienced. Further, because he saw himself as someone who was able to make sense of those other’s ways of knowing, and add alternative methods—such as the different solution to the factorization problem—that he saw himself at the highest end of mathematical ability; hence his self-evaluation of a 10 to indicate how good at mathematics he was. This 10 was a comparison to his classroom peers, indicative of his ability to “top” each of those who shared ideas of their own with his own, different method.

Viability

Status. Fisk was a high status student in the classroom (Cohen & Lotan, 1997). This status was less of a social status and more of an academic one. His fellow students thought of him as good at mathematics, at least to the extent they turned to him with questions. I suspect this academic status has developed through Fisk's classroom volunteering to answer teacher questions or to present ideas at the whiteboard; as demonstrated by a rather powerful classroom moment on April 22. On this date, Larry opened the class by challenging the students to convert a quadratic from standard form, $y = ax^2 + bx + c$, to vertex form, $y = a(x - h)^2 + k$. Previously, students had changed the quadratic in the other direction, simply by multiplying and simplifying the vertex form expression. Students also understood the meaning of the h and k in the vertex form to be the coordinates of the vertex, and that the a had some impact on the direction and extent of the parabola's curve. After some individual work time, Larry began a classroom conversation about the problem, focused on developing justification for the class's conclusion that the k value must be equal to -7. Fisk volunteered "Subtract 16 from 9" (classroom video transcript 04/22/05, 36:00), as opposed to most other volunteers stating "16 – 9". Larry accepted Fisk's reply, and asked how he would write this. Larry then records "9 – 16" to the whiteboard. This sort of interaction—placing a student reply on the whiteboard—is a way that a teacher can place academic status upon a student in the mathematics classroom, recognizing them as a sort of expert in their ability to know and do mathematics correctly.

This protocol continued with Larry assigning another problem to the class, and asking individuals to work on it again. The problem was of the same type, simply utilizing different values for the b and c (note: $a = 1$ in both cases). At 40:00, a student put up his response to Larry's new problem, but was unwilling to fully explain how he arrived at the solution. Larry asked Fisk if he would explain the solution. Fisk agreed, and began to do so—moving to the whiteboard. As he talked through the steps, he paused at the point where his comments required him to describe how to determine the k value—the same spot of difficulty for the class in the earlier problem. Fisk said, "I forgot where the 53 came from." Another student suggested some possibilities. Fisk's attention turned to listening to each of them—carefully. As a class, they resolved the problem, with Fisk finishing up what was to be recorded to the whiteboard. This classroom interaction not only documents Fisk's interaction with the ideas of his peers, but also helps to demonstrate the status he had in the classroom. Students paid attention and participated in the conversation he led. Not only did they pay attention, they recognized that something mathematically significant was on the table. This role in the classroom was indicative of a student with a high academic status among peers.

Fisk's own sense of self as someone who could do mathematics was fed by Larry's confidence in him as well as the status he awarded him in the classroom. His peers accepted Fisk as an authority. Fisk seemed to have this belief in himself as well. When it was evident he did not have something figured out, the 53 for example, he was very attuned to listening to other students ideas about the problem. However, when sitting and being asked a more direct question by a neighbor, Fisk seemed to prefer to respond with a correct-answer mentality, either he knew it or did not. As is typical, Fisk's academic status raised his social status in the classroom (Cohen & Lotan, 1997). Fisk engaged with his peers in off-topic conversation frequently, including playful flirting with a female student in the class (noted in classroom video transcripts and field notes on April 15, 18, and 21).

Grading Fisk. The potential result of failing a course did not take on a terrible threat for Fisk; he would make up the credit in PHS' after school credit-recovery program. He saw poor grades as a result of his own decisions not to do this work. He also knew his ability to communicate what he understood as an important part of earning grades in mathematics. This awareness is evident in our interview on April 15, "Some people say show your work" (9:35). He stated something similar later in the interview, "The way I do it is sit there and break down numbers in my head and since the people want to se it, I'll do w hat I'm thinking in my head on the paper" (54:00).

In my experiences with Fisk, he determined strongly for himself when he knew something, and when he did not. From his point of view, he knew most things he was supposed to know in his mathematics class. Recall his attitude about listening during the test review day, and studying for mathematics tests in general: "I never really review for tests ever. I just don't especially [for] math, because there's not really nothing you can review, because if you're paying attention you're going to remember it." If he got an incorrect solution, as in the rat problem, it was not a big issue; accuracy was less what mattered in math class as opposed to understanding. His self-evaluation showed he thought he had done a good piece of mathematical work, independent of an error in the solution: "Self Assessment: A Grade I think that I deserve is a maybe a B+. I think I explained myself thoroughly, but I might have gotten the wrong answer. I kind of know I'm a little [below] the real number of rats."

A course grade was a matter of Fisk's decision to pass or not. When asked about needing to pass a class, he shrugged it off, saying he could always retake the course for credit in one of the school's credit recovery opportunities. Fisk's grades in math classes swung from passing 9th grade with high enough to be placed into an honors 10th grade course. Upon failing this, he passed a credit recovery course and was achieving well in his 11th grade course. For example, his in-class unit test on exponents was 100% accurate, with a few points dropped for slightly incomplete justification on a few of the problems. Yet, shortly after the midway point when a substitute teacher replaced Larry, he stopped with his work and was in danger of failing the course. Fisk himself reported to me in a side conversation that he simply wasn't doing the work.

Since a teacher's grading of a student is so tightly tied to a teacher's knowing of a particular mathematics—a mathematics identified typically by a standards document, district curriculum guideline, or simply the chapters of a textbook, it is hard to say that this grading of Fisk was incorrect. However, the evidence I collected and have presented here strongly suggests that Fisk was mathematical, at least on par with the peers in his classroom.

Summary of claims

Mathematics as invented. Fisk does not show evidence of a firmly held belief that mathematics exists prior to or external to knowers. He acted upon the teacher's intended content of his mathematics course as mathematics he was realizing, or inventing. He was the actor in creating meaning, in response to the problems his teacher posed. Not only was he inventing, but he treated his inventions as new parcels of knowledge. He did not place any emphasis on some realization that he had come to know something that had already been known. In fact he may have seen himself as the person coming up with ideas, and the mathematician as the person who came to understand his ideas.

Typical mathematical achievement. Through my experiencing of Fisk as a teacher and researcher, I did not find anything profound or unique about the cognitive level of Fisk's mathematical activity. By saying this, I intend to imply that he neither showed deficiencies or moments of brilliance in relation to other high school juniors I have met in my experiences teaching. Fisk was able to reason through many challenging problems, such as deriving a generalized method to "complete the square"—converting quadratic equations from standard to vertex form, and demonstrating the equivalence of factored trinomials. He also struggled with more challenging mathematical tasks, such as the rat problem. Fisk did demonstrate some more unique qualities in his mathematical activity however. For example, he did much work mentally, "breaking down numbers in [his] head." I did not see him reasoning with representations to the extent many Junior students do. In some ways, this could be an indicator of a lesser mathematical ability, and be related to why he struggled greatly with the rat problem, one that relied heavily on some sort of accounting for the rats. I would say that one aspect of his mathematical work that was somewhat stronger than many peers was how well he was able to listen carefully to the thinking of colleagues and come to understand their ideas, learn from them, or recognize their flawed understandings. For example, he noted aloud in class on April 22 about a student sharing an idea, "he still doesn't see how…" (47:00).

Fisk measured his own ability to be mathematical against a different norm than was typical in schools. Schools seem to value mathematical knowing as an amount of collected, and replicated facts. This orientation values accuracy. Fisk valued reasoning and understanding; and more specifically, an intuitive way of knowing. This conclusion is more clearly the case when considering that he strongly separated the next stage in his work to be a need to show other people what he knows, either through written or verbal communication. His intuitive knowing posed a challenge for follow-up, sharing it with others. Furthermore, Fisk took pride in the differences he attributed to his ways of doing mathematics. He attributed his differences to coming up with different ways of understanding or solving problems. It was this difference that he attributed to my interest in him as a research subject.

High self-confidence. Third, Fisk demonstrated a confidence in his mathematical ability. This seemed to grow from the discussions above, about how he saw himself as a mathematical thinker. However, this confidence—as most—served as a protection to his *mathematical ego*, to which I use in reference to his consciousness of his own identity as a mathematical learner.[86] It was likely that this mathematical ego was fragile, highly prone to influence from outside sources.

[86] Note, I refer to the idea of a self-identity as a mathematical knower and learner as one's *personal epistemology*.

Fisk demonstrated that he was highly tuned into the feedback he ideas received from peers and from his teachers, Larry and myself. Larry respected Fisk for his mathematical thinking and I expressed an interest in him—to which he attributed to be an interest in his unique ways of mathematical thinking. He felt a safe space in Larry's classroom to share his thinking, even if not on something he knew he understood well. But when not in Larry's class, Fisk shut down. This was noted by both his sophomore mathematics instructor and the teacher that replaced Larry after Phase II of my research. The swings in his perceived mathematical abilities by his teachers spoke to some sort of ego protection. Within such a hypothesis, it makes sense that Fisk attributed his poor grades when not in Larry's class to his decisions to not perform, rather than validate what outside judgments may communicate. And finally, the pride in thinking differently saved him to some degree from comparing his mathematical ability to others, including his fellow students in Larry's classroom who may actually be better mathematical thinkers. The notions of his confidence in his mathematical ability at play with the potential of the fragile mathematical ego made for interesting qualities of Fisk as a generative adolescent mathematical learner.

Context dependence. And finally, as alluded to above, Fisk's personal epistemology seemed very tied to the activity of the classroom teacher and the norms of his learning environment. Because Larry seemed to downplay his own and distribute mathematical authority throughout the classroom—at least through the eyes of Fisk, Fisk found room to stretch his mathematical mind and engage in mathematical activity, freed from a need to protect his ego. Larry demonstrated sincere and consistent belief in Fisk as a mathematical learner.

Informing the Notion of the GAML

As a subject of study to better develop a general model for a generative adolescent mathematical learner, Fisk proved to be a student whose identity as a mathematical learner, his personal epistemology, fell quite squarely into Quadrant 1 of the figure initially proposed to consider various ways to theorize mathematical learning (Figure 1). I perceived Fisk to see himself as a producer of mathematical ways of knowing; he had to come to know for himself, he was his own authority. Fisk also believed that knowledge was something created (or realized) by people, not something humans ran into. His notion of knowledge recognized that someone before him (or another knower) might have realized this knowledge earlier, but ultimately knowledge is the product of the human mind.

I believe that Fisk's mathematical ego resonated with my description of the GAML as well. His self-consciousness of his own identity as a mathematical learner saw mathematics as the results of his own activity. However, it seemed that this defining sense of his mathematical self, his mathematical ego, was fragile. He showed that he did not thrive mathematically in just any learning environment. His relationship with his teacher and the norms of the classroom discourse played an important role to Fisk's mathematical being.

It was also apparent through Fisk that a student might be a generative mathematical learner, but not be terribly profound mathematically in the eyes of other mathematical knowers. Some adults saw Fisk as not particularly able mathematically, including myself. Others saw him more strongly as a student that did not apply himself. However, his peers seemed to generally respect Fisk as someone who was smart mathematically.

Some interesting potential themes emerged from my analysis of Fisk. Of most significance was the role of the learning environment, and especially the role of others in provoking mathematical activity for Fisk, his valuing and working to understand their ways of thinking, and his desire to express his understanding to them. When his mathematics teachers did not demonstrate a similar curiosity and interest in his knowing, he "dropped out" of the learning environment. Although Fisk believed he could do the work, when in some way he was not obtaining the valuable peer and/or teacher rewards, he shut down his efforts to be mathematically engaged. I suspect that when he found he could maintain his social status without the need to engage academically, he allowed himself to drift.

This viability in the mathematics classroom was also sharply marked by a tendency toward deviance from the common norms associated to classroom learning. Fisk sat with his backpack on for long periods of class time. He did not work diligently to record notes, create written responses for assignments, or maintain an organized record of work. He treated the classroom "rules" as what was acceptable to him, his peers, and allowed by his teacher. Although not seeking to push the boundaries of his teacher's tolerance, Fisk was comfortable being himself.

Kate

Kate also had a rather strong personality—without a doubt even more so than Fisk. Kate made the classroom her own in many of the ways Fisk did. And just like Fisk, Kate was a very respectful and courteous student, reflecting the treatment offered by her teacher. I present Kate next, following the same organizing principles used for Fisk. First I will discuss the decision to select Kate as a research subject. I will then go on to analyze Kate's mathematical activity, her personal epistemology, her sense of others and those interactions in relation to her mathematical learning (second-order viability), and my sense of the ways in which she was viable in her mathematics learning environment. After a brief summary of what I learned about Kate, I return to the model of the GAML and my initial research questions in order to continue pushing forward on the development of those ideas.

Selecting Kate

Fisk's classroom teacher, Larry, also suggested I consider Kate—a student he had as a freshman. When I began Phase I of the data collection, Kate was enrolled as a Junior in Bridget's mathematics class. Bridget also recommended that I consider Kate as one of my subjects in my Subject Identification Request (Appendix A), writing that Kate was "Very confident on her capacity/abilities." How this confidence manifested itself proved to be rather interesting. Bridget went on to write that Kate was,

> Outspoken, will explain what she thinks very clearly, trying to convince others…. She seems to keep her ideas until somebody else prove a new idea to her. And she tries to make sense of what she thinks, how is connected to the problem, why it works.

I interpreted these comments to indicate that Kate was a mathematics student that needed to make meaning of a problem or concept for herself. Once she established something to be true, or felt she understood, she was interested to share it with others. This sharing exuded a confidence in knowing, that Kate was willing to explain and even defend. Although she might listen to others' differing ideas, she was not easily swayed—they had to *prove* an idea to her that was different from her own way of knowing. This quality of Kate's second-order viability will be returned to later. For now, the descriptions Bridget offered, in particular this rigidity in the confidence of her knowing, piqued my curiosity about Kate—this seemed to be a student whose confidence in knowing was highly caught up in her need for understanding and own decision that the mathematics, a problem solution, a procedure to solve a problem, or her understanding of a concept, made sense to her. Once it did, she was willing to share this understanding with others.

And although she seemed open to considering other's ideas, her interpretation was not easily swayed.

I attended Bridget's classroom for the first time on Tuesday, Feb. 22, 2005—the second day of Phase I. Bridget's role as teacher on this day provided little room for me to observe the students in mathematical activity. I did note, as a classroom observer, several qualities of Kate's classroom engagement; in summary, Kate's classroom activity confirmed for me that she was potentially interesting for my study. During the period I saw that she was attending closely to a small set of the students in the classroom arguing over the solution to a problem. Later, she asked Bridget about "that log thing" on her calculator. Bridget didn't seem to understand the question. Kate accepted responses from her group members, suggesting to me a dispersed attitude toward mathematical authority. I noted several additional episodes in which Kate approached fellow classmates with questions, to check solutions or get help.

I did have an opportunity to interact with Kate most closely on the final day of this Phase I week, noting that she seemed to move forward on the tasks that she was given in a focused manner. The task she engaged on this day involved working through some numerical examples demonstrating properties of exponents and then, following an inductive logic, generalized the process into conjectures that might hold for all numbers. Kate was given the problem: "A family sets aside $4800 to a savings account. If this account gains 5% each year, what is the value of the account one year later? Two years later?" There were a few more questions about the future value of the account, and then a prompt to create an equation stating the value of the account after any number of years. I noted (field notes, 2/25/08) that Kate's response was "organized very nicely" in which she developed a "multiply and add strategy." This looked something like

\$4800 * 0.05 = \$240	\$4800 + \$240 = \$5040
\$5040 * 0.05 = \$252	\$5040 + \$252 = \$5292
\$5292 * 0.05 = \$264.60	\$5292 + \$264.60 = \$5556.60

Her generalization step involved describing her process of multiplying the previous results by 0.05, then adding this to that previous result. Interestingly, her "vocalization of the general steps was good, but her symbolization was nonstandard and inadequate, not capturing all operations" (field notes, 02/25/08).

She progressed smoothly through this task; I was struck by the confidence with which she worked. I recorded to my field notes that she and the three other students she sat with seemed to indicate that the generalization part of the activity was less significant to them. This might have been because the hard work for them was to have worked out the numerical examples; thus, through that work the process to generalize was already apparent to them. I also noted that Kate's notation, although sensible, was not exactly a standard or most simplified manner to represent the exponential expression. That Kate worked diligently indicated an interest and an orientation that she could do this. That her results—especially evident vocally—were mathematically sound yet nonstandard in written form, suggested that she did not hesitate from generating her own mathematical ideas and determining a way to represent them. In other words, she did not appear to be limited to using previously taught convention, but more importantly didn't find this lack of teacher direction to limit her exploration.

Mathematical Activity

Confidence in knowing. Kate's confidence in her potential for mathematical thinking and knowing, as well as in her mathematical conclusions were difficult to identify. As a part of the class's work on her teacher's task, to create a generalized statement relating b and c in the standard quadratic equation ($ax^2 + bx + c = 0$), I recorded the collection of classroom ideas to the front board. I followed this with one particular suggestion on how to generalize the students' ideas relating *b* and *c*. Kate volunteered a different and mathematically accurate statement of this generality, which I recorded to be mathematically correct (classroom video transcript, 04/12/05 75:00). Her confidence in knowing was apparent in the nature of volunteering in a risky environment—to the whole class—but also indicative of the sort of "I know for myself," anti-authority positioning that Kate seemed to exhibit. This social posturing may have helped Kate maintain a cool-factor with classmates, maintaining a status based on her academic abilities.

In another classroom episode, students were sharing final results of their investigations of the rat problem (mentioned in Fisk's data analysis). Again acting as teacher, I gathered the variety of student responses to the question posed in the Rat problem and recorded them to the board, including the number Kate and her collaborators arrived at—314. That there were 8 different solutions recorded did not seem to shake her confidence in her own solution (unusual among the students of this classroom), nor did it seem to capture her curiosity to confirm her result. I went on to say, wondering aloud, "What if I said none of these solutions were correct?" Again, there was little response from Kate. Next I stated that one of the solutions on the board matched what Bridget and I obtained after we worked on the problem together, and then invited the students who had prepared to present portions of their investigation to the class. One of the student's presentations developed a solution larger than 314. Still Kate was unperturbed; until one of her peers with whom she worked on this problem seemed to express a doubt in their solution. At this point, I noted a significant change in Kate's body language, suggesting that she was considering for the first time that her solution might have been incorrect. Kate took notice of challenges to her knowing that emerged among the classroom learning environment. Again, Kate's knowing showed off a high self-confidence, yet one that was susceptible to peer input for confirmation and real confidence.

There were several indicators that made it quite apparent that Kate was not as certain in her mathematical knowing and understanding that she seemed to project in the cocky or inattentive demeanor frequently demonstrated in the classroom. During the videotaped classroom episode described above on April 7, 2005, Kate frequently checked in with the focus of the whole class conversations. In my impression, if she felt confident in her understanding of the topic at that moment, she would turn her attention back away from these discussions (field notes, 04/12/05). As another example of her dependence on confirmation from the social realm of the classroom, while she worked to understand the process being shown to her by her group mate, she would check her work with that group member. Kate's brashness in the classroom was not all due to her view of herself as a generative learner, I conjecture it also had to do with an attitude toward school.

Kate paid a sort of underground attention to the teacher, savvy enough to pick up cues on what was important to know, including mathematics as well as knowledge bits regarding course grades—her previous teacher noted this in particular. She watched to ensure her solutions were correct. And she listened for assignments that might be collected for a grade. For example, on April 7, a group summary was to be submitted during class of the task being investigated. On that date, I recorded to my field notes that her classroom activity maintained a stability between her own self-confidence in knowing and being correct, with a maintenance of social status—reminding me of Cool Pose Theory (Majors & Billson, 1992). This theory posed that a subject's posturing counters stress caused by social oppression, rejection, and racism, providing a sense of control, strength, confidence, and stability that helps to deal with negative messages. In the language of this theory, “cool pose” is a stylized, institutionalized posture of camouflage that mystifies the White man but poses no threat to him. I was reminded here of the theory, designed for social analysis of black males, because there seemed to be a connection to shunning the external, oppressive power associated with (mathematical) learning, maybe the teacher or maybe the school. Again, her classroom stance indicated a need for social feedback on her ways of mathematical thinking, yet reflected an attitude about the social environment of the classroom, or school, that was slightly combative, or maybe more simply ego-defensive or –protective.

As another example of her socially dependent self-confidence, on April 11, 2005, I instructed students to begin a warm-up activity. Kate turned to a neighbor to instead check on a solution to one of the assigned homework problems. After this check, she announced “I was right.” That she checked with the neighbor indicated an uncertainty, but the pronouncement of self-confirmation served to re-establish a self-assuredness, as well as demonstrate to her peers that she was right all along.

Graded work. At the midterm of the semester, April 18, 2005, Kate was earning a C+ in the class—a 78.7%. She ended the course with a high B. Kate’s course grade consisted of 5 categories, including homework, class work, problems of the week, unit portfolios, unit tests, and exams. The first three categories best reflect typical daily class work, understanding, and consistency as a student. Kate’s grades in these categories were centered tightly around 90% (at midterm—88, 88, 92 to be precise). Kate’s unit portfolios tended to be of exceptional quality; in fact she had a midterm mark of 101. Kate’s test categories were both much lower, in the mid-C range. Bridget’s grading system used a weighting method that seemingly made these test scores impact significantly on the overall course grade.

In a cursory manner, it appeared as though when a graded assessment provided Kate greater time flexibility and greater leeway on demonstrating what she knew in her own way, she was better able to perform. Bridget used a typical testing environment in the classroom, where students worked individually, in a timed setting, in which the accuracy of solutions was most valued. The tests were Kate's lowest scores. The portfolios, on the other hand, were Kate's highest scores. The portfolio was another graded assessment, but one in which the student had more say in the selection of the work to be included, with the intent to polish the work done during the unit of study in such a way to show best what you learned and understood. Kate earned high marks on this assessment for two primary reasons. First, she assembled a very tidy and well-presented collection of her work, obtaining some extra credit. More significantly, the cover letters she wrote and the tasks she included demonstrated a competent awareness of the mathematics topics studied in a unit. For example, Kate wrote:

> Equation of a circle (center in (0, 0) and center (a, b)) was the third main idea we discussed in the "Orchard Hideout." For the center in 0, 0, the equation is $x^2 + y^2 = r^2$. Then, as we worked, we saw that for the center a, b is $r(x - a)^2 + (y - b)^2 = r^2$ [note – the initial r is mathematically incorrect]. Lastly, the standard form for the equation of a circle is $x^2 + y^2 + cx + dy + f = 0$.

This was one paragraph of a 4-page unit overview that Kate wrote. Her writing on other topics was similar. A similarly stated, rather complete summary of topics was presented. However, little demonstration of deeper understanding, such as why equations/formulas were as they were, or application to problems, was provided. The passage above was the depth to which she developed and justified the mathematical ideas she presented. In other words, she did little more than state learned facts. This did earn her high marks on the portfolio assessment. Again, the strong scores in the portfolio seemed to reflect a good ability to reflect and pull together main ideas of the unit of study, and to present them well.

The scores on Kate's daily coursework and homework were most aligned with her final grade in the class, and seemed to land between slightly lower test scores and very high portfolio scores. As an example of this work, in response to the following prompt:

a. Find an algebraic expression for a quadratic function whose graph has its vertex at (3, 4).
b. Make a partial table of values for your function and use the table to sketch a graph of the function.
c. Does your graph seem to confirm that (3, 4) is the vertex for your function?

Kate responded $y = (x - 3)^2 + 4$; created an x, y table with x-entries -3, -2, … 3 and corresponding y pairs; sketched by hand a graph with domain (-3, 3) and range (0, 40) with points associated with the function accurately placed. Her reply to question *c* was that, "I am positive because it is the lowest point on the graph."

This assignment once again demonstrated a nicely presented and complete response to questions asked. The mathematical representations in Kate's response matched an accurate conventional answer, however, there were indicators that she did not fully appreciate the conceptual importance of the vertex being the minimum of the function. Kate did not extend either the table or the graph beyond a fairly standard set of integral domain values, -3, … 3, to show that indeed the graph of the function did increase after the minimum (3, 4).

Kate scored a 75% on her in-class test for the exponents unit. This test explicitly asked students to explain their answers. Instructions, and three sample problems from the first portion of the test were:

> Seven equations are shown here. Some may be true and some may be false. You have to decide. Do part *a* and *b* for each of the equations.
>
> *a.* State whether the equation is true.
>
> *b.* Explain your answer.
>
> - If you think the equation is true, say why. If possible, state and explain a general principle that the equation illustrates.
> - If you think the equation is false, change the right side of the equation to make it true, and explain why the new equation is true.

1. $10^5 \cdot 10^{12} = 10^{17}$
2. $\frac{3^8}{3^2} = 3^4$
3. $\sqrt{10^{16}} = 10^4$

Kate's three responses were as follows:

1. True – Law of exponents says same Base just add the exponents & keep the Base.
2. False – 8 - 2 = 6 not 4. Law of exponents says to subtract not divide.
3. False – The answer equals $10^{8.}$

Bridget recorded a checkmark to the right of each of these replies, suggesting she awarded credit. Bridget also wrote "Why?" next to the third. Bridget's scoring indicated she required a rather shallow justification when asking students to "Explain your answer." I would consider a more robust indicator of a student's knowing of a problem like 1. To include an indication that the student understood 10^5 represented the product of five 10's, and that $10^5 \cdot 10^{12}$ represented the product of seventeen 10's, hence a solution that could be recorded 10^{17}. Kate's justification, and Bridget's acceptance of it, was rule-oriented, and less so emerging from mathematical reasoning.

The written mathematical work of Kate's that I assembled, including the portfolio, class work, and test examples shown here, suggest that Kate recorded her mathematics in a neat, and usually conventional mathematical format. However, the depth of her understanding—which I considered evident in the quality of her justification of her responses to mathematical problems and in the explanation of her thinking when opportunities were present, is uncertain. This element of mathematical activity—justification—was valued in some ways in Bridget's classroom. Students were asked to explain. Yet Bridget may have communicated a valuation of shallow explanations, through feedback such as full credit on tests. Kate had not developed a personal value for communicating her reasoning—especially this justification. She apparently was challenged to do so, or did not have a depth of mathematical knowing that allowed her to accomplish this explanation.

Personal Epistemology

Self-concept. When asked to give herself a score from 0 to 10 on "how good I think I am at math," Kate rated herself a 7. She wrote in her Mathography, completed at the beginning of Phase II, that, "I personally don't think I am the best in math. I know this about myself because one minute I can be doing the greatest on a problem and then you change something and I do horribly."

To provide additional context for this score and self-critique, I return to the grade feedback Kate received from Bridget during the course, and my evaluation of her class work. At midterm, Kate was earning a C+ in the class—a 78.7%. Kate's course grade consisted of 5 categories, the first three representing typical measures of daily class work, understanding, and consistency as a student. Kate's grades in these categories were centered tightly around 90%. Kate's fourth category, unit portfolios, included very high scores—an average of 101. Kate's test categories were both much lower, in the mid-C range. Bridget's grading system used a weighting method that seemingly made these test scores impact significantly on the overall course grade, hence the C+ overall grade.

This summary of grade feedback and reflection on Kate's submitted mathematical work was also intended to compare with Kate's self-evaluation of her mathematical ability in the context of her mathematics classroom. Kate evaluated her own ability at mathematics as a 7 on a 0–10 scale. She received grade feedback from her teacher that confirmed that assessment in certain ways—test scores that averaged approximately 74%, nearly 7 out of 10. However, Kate also received more positive grade feedback on other coursework, including the equivalent of 9 out of 10 on daily work, and 10 out of 10 on her portfolios.

Kate directly addressed her self-concept of her mathematical ability in her Mathography. As documented above, she wrote, “I personally don’t think I am the best in math. I know this about myself because one minute I can be doing the greatest on a problem and then you change something, and I do horribly.” This statement seems in line with her rating of a 7 in 10. She provided more insight into her self-perception in the Mathography. She stated:

> When I have learned math alone outside of the class, I felt good because all the attention was focused on me and we weren’t going to proceed until I was positive on the work, but I also felt slow because I was alone not knowing anything. In class, I have other students who also do not know the work so I didn’t feel bad, but when I was alone and I didn’t know it I felt stupid.

This passage indicated Kate’s image of how good she was in mathematics was quite dependent on her sense of what other people “know.” Kate compared herself to others, and when no one else was available, she might catch her self-confidence and revise it to be a more self-doubting perspective.

Usefulness of mathematics. Kate replied to the prompt, “Mathematics is something I do” by selecting almost all responses available, except “Mostly in my head.” She did select:

> Every day as a natural part of living.
> Mostly at school.
> With pencil and paper.
> With numbers.

This response indicated that she might see mathematics in her daily routines of life; suggested most clearly by the first response. However, I would suspect she might have also selected “Mostly in my head” had that been the case, an option she ignored. It might be the “mostly” set her off. The next three responses from the survey, “Mostly at school”, “With pencil and paper”, and “With numbers,” all were indicative of a classroom orientation to where mathematics occurs.

Kate returned to this question in her Videography at the conclusion of Phase II as well. As she filmed herself walking to a bus stop to get home from school, she spoke about how she always found herself doing mathematics. She stated,

> I'm walking home, well I'm walking to the bus stop. And this involves math because how many steps I'm taking to get to the bus stop, which is on 7th street, from the high school… the distance from the high school to the bus stop… see how long it would take me…. Different stuff like that which involves math. And then, when I get to the bus stop, I'm taking the bus to my house, and I could work out, like, how much fuel the bus burns if I really wanted to do it. And plus I have to pay to get on the bus, so that's another part of it.

As she is filming her trip home, Kate is commenting on activities she is engaged in or could be—right at that moment—that are mathematical, or as she said, "doing math." She continued in the Videography:

> Another thing I could think about when I'm walking home is how fast the car is going. You know, when you cross the street you think: I wonder if he's going to hit me? I wonder how fast he's going? Is he going to slow down. That's another part of math that you don't know that you're involved in math, but it's still there….
>
> So simple things I think of, even though I'm not thinking that I'm doing math, I'm still using it in a way.

Throughout this video, Kate identified her orientation to mathematics' usefulness or application to life. She saw that it is always present in some way, that people are always using it.

However, on the survey provided prior to Phase II, Kate circled "Disagree" disputing the notion that "The ideas of mathematics have very little to do with the real world." This response contradicted both her previous comments on the survey, and what she discussed in her video. Continuing to examine Kate's survey, she responded that "Playing a sport" was Slightly Mathematical; Kate was a softball player, so *she* played a sport. She also marked both of the final two options in the survey, "Using a calculator to work out interest paid on a housing loan over 20 years" and "Cooking a meal using a recipe" as Highly Mathematical. She responded that each of the following items was Barely Mathematical:

> Traveling to school or work.
> Planning a family's 2 week holiday.
> Upkeeping a domestic vegetable garden.
> Playing a musical instrument.
> Chopping down a large tree.
> Buying clothing at a sale.
> Painting the house.

At the moment she completed this survey, I conjecture that she did not see a great range of life activities—given the possible selections—to involve much mathematics or mathematical activity. The interest computation sounds of typical school language about what is mathematics, as might the cooking situation. Kate may have a stronger personal connection to each of these two ideas, as well as "Playing a sport," influencing her selection of these responses as more mathematical.

The most strongly evident contradiction in her notions about the usefulness of mathematics appeared between her response to the survey question, calling "Traveling to school or work" Barely Mathematical, and in her Videography—produced 6 weeks later--calling her walk from school to the city bus stop an example of how there is mathematics behind much, if not all, lived activity: "That's another part of math that you don't know that you're involved in math, but it's still there…." I conjecture that the initial survey responses were her first reaction to questions such as these. But during the next 6 weeks, her attitudes changed as she thought more on the idea.

To follow up on the inconsistencies I noticed within Kate's survey responses, I probed further into her thoughts about the usefulness of mathematics during our interview:

> B: Do you think you learn mathematics outside of class?
>
> K: yeah, mm-hmm

B: For example…

K: Like driving, [people] all use mathematics. People say we don't use mathematics, and I'm like, yes you do! Like, when I'm coming up the steps, I'm like, 'Hmm, lot's of steps. I wonder how many steps I'm walking up? I'm counting steps. Or I'm coming to school, I count houses, or how long does it take to get from my house to school if we go this way, if we stop at the light, or like walk down the street. I wonder how long… Every time I walk home, cause I walk home a lot.

I think, [since] I get like different times, I wonder what pace or speed I was walking at. Just like different things you think about like simple stuff you do, like how you write your name sometimes, like I'm going to write it big, like form here to here. Now I want to have my letters, you think about different stuff. So math is an every day thing.

In this response to my question about *learning* math outside of the classroom, Kate strongly expressed her belief that people *do* math as an every day thing. She sees much of her own regular activity as mathematical. It seems reasonable to conclude that Kate sees mathematics as "an everyday thing." Her answers about the degree to which something is mathematical should be understood within this context—that all is mathematical, just more or less.

Regarding the apparent inconsistencies in Kate's replies about the usefulness of mathematics, some consistency emerged. She repeatedly spoke in both the interview and her Videography about people and herself *using* mathematics daily. And although not every aspect of life might be highly mathematical, there was some small amount of mathematics in most anything people are doing—even when writing their name. Kate's responses strongly suggest that she saw mathematics as a useful endeavor (Grouws et al., 1996). However, Kate might not believe one *learns* mathematics in everyday living; she does not address the idea that people (or herself) may construct or generate a mathematics. This notion is important to understand further when thinking about Kate as a Generative Adolescent Mathematical Learner, one whose self-epistemology is that of a person who generates mathematics. To understand Kate's orientation to mathematics further, I next turned to data that spoke to her thoughts on the relationship of mathematical knowledge to its knowers.

Ontology of mathematics. In her initial survey, Kate responded that she *disagreed* that "The ideas of mathematics were invented by mathematicians" and *agreed* that "The ideas of mathematics were discovered by mathematicians." Further, she *agreed* that "The ideas of mathematics have always been true and will always be true" and *disagreed* that "The ideas of mathematics developed as people needed them in real life." The collective of these responses provided a rather consistent message that Kate believed that whatever it was she considered to be "the ideas of mathematics" had an eternal existence, being there prior to people's use or awareness of them. That she disagreed that ideas of mathematics were invented by mathematicians but rather discovered, supports this interpretation of Kate's orientation to mathematical knowledge.

I followed up on Kate's survey responses in our first interview. In her initial reaction to my posing of the question, she rethought through the questions and her initial responses. It was apparent that she gave a renewed effort to think about the idea.

B: Do you recall this survey? There are two responses I want to consider at the same time. I asked, "The ideas of mathematics were invented by mathematicians." And you said disagree. And I asked, "Were they discovered by mathematicians." And you wrote agree. Do you remember responding to that?

K I must not have been focusing.

B Oh! Would you change your answers then?

K Yes, I mean… wait, I have to have had a reason for not putting… it's the same question. Invented is one thing, but discovered is like they found it, invented is they found it so, I don't know…

Kate's initial reaction indicated she would be thinking through the question again as she spoke to me during the interview. It was evident she sorted through what each word might mean. Next I asked her what she saw as the difference between *invented* and *discovered*?

> K Invented means you sat there and like worked it out, [for example, someone might think of something and decide] I'm going to name this mathematics. Discovered is somebody invented... like you invented it, but I found it, so I'm going to claim it. That's what I thought. So, like I find that some … like I don't know why I put that.

As Kate re-considered the terms *invented* and *discovered*, she developed meanings for each word that kept them related to ideas or knowledge, as opposed to a thing—such as a printing press. Although she said, "Discovered is somebody invented…" it is my interpretation that Kate meant someone first must invent an idea—"like you invented it"—and then another person might happen upon this invention, in which case she would say that person discovered this invention.

Continuing the interview protocol regarding the ideas of mathematics being discovered or invented:

> B Well, you don't have to tell me why you put that at the time. What do you think right now?

> K Right now… I still think like they may have, they just, they found it. But they didn't sit down and write 2 + 2 = 4. Like maybe a mathematician came along the way and said, oh I'm going to call that math. But I don't think he sat there and did it. So like [someone] taught it to him and taught it to them. That person who may have been a caveman knew what they were doing—they were not a mathematician.

Kate created a sort of story about how it might have been that someone—maybe a caveman—"discovered" mathematics. This person, this mathematical discoverer, although *not* a mathematician, understood and "knew what they were doing." Her third sentence established a role for the mathematician in her story. It seems as though the mathematician may not have been the initial discoverer, but the person who was taught the mathematical idea by this initial discoverer. The mathematician would then have decided to name the discovery mathematics, and possibly put mathematical words or symbols to it, like 2 + 2 = 4.

That Kate placed a distinction between the original inventor and then the realization of the idea by the mathematician fashioned room for all people to be creators of mathematical ideas.

B oh. So what if we changed the word to people. Was mathematics invented by people? Or was it discovered by people?

K I would agree with both. Cause like people…

B you would agree for both?

K yeah, cause when it's invented by somebody, it is somebody came and still found it, so I would still agree to both cause people did, you don't know specifically who. Mathematicians, they're a group of people. You may be a mathematician, I'm not a mathematician. When we say people, it's like ok, go ahead, you can put both those together.

Kate's final comments suggested again that it is people who do invent mathematics, and since mathematicians are people, they have every possibility to be those inventors as well. And as the previous protocol indicated, it might be that the mathematicians discover someone else's invention, and name it mathematics. Although Kate's comments have not explicitly addressed the notion of the existence of mathematics prior to the inventor, the storyline evident in the various sources of data suggest that she sees new mathematical ideas being invented by people, not that the ideas have existed forever prior to these inventors.

I predicted a much stronger statement from Kate indicating a belief that all mathematics was to be found, whether by a person or by a mathematician. Recall that Kate's initial response indicated an agreement with the statement, "The ideas of mathematics have always been true and will always be true." It is difficult to say what Kate's exact thoughts were on the ontology of mathematical ideas; there appeared to be some conflict in possible meanings. I do believe that she does place a strong role in the invention of such ideas by any human, not only mathematicians.

Self-confidence in knowing. While in class, Kate not only asked others for help when uncertain, but she also kept her awareness open to the larger classroom learning environment for counterexamples or conflicts that threatened her confidence in knowing. However, this openness was concealed; her actions of attentiveness were guarded. Recall Kate's hesitancy in accepting the potential that her count of rats in the Rat problem may be off. She told me during the interview, that as she worked with some friends, they were aware that other students had a different solution. She said, "And we all argued for a while, based on they [the other students] had this answer. I'm like, no—do it this way." I sought to inquire further into Kate's self-confidence in knowing during the April 13 interview:

> B: You were just describing when there are pieces of a problem you aren't certain about, you would talk to other people and see how they're thinking. How about when you decide you're certain something is correct, what do you do in class then?
>
> K: I want to show it, like I'll be the first to raise my hand like I want to do it, 'cause I know it's right. Like I know it can't be wrong.

Kate's response demonstrated she liked to know that she was correct and had a confidence about her knowing—this certainty that something is correct that I prefaced my question with. She would present her ideas to others, and had a feeling she couldn't be wrong. However, she was open—with a qualifier—to such a possibility.

> K: I mean, if it's wrong I'm like, Dang! How did I do it wrong? I go back and check. Maybe you get the whole process right but you put a wrong number in here, or a wrong number in there, so it's like you did it right, you just did the wrong number.

This qualifier was that she probably was correct, just made some simple numerical or arithmetic mistake. Again, her comments on the whole indicated a disposition leaning toward confidence rather than self-doubt. She would be surprised if her solutions were off; her initial assumption was that she could have made a small error, rather than there being an error in her ways of understanding the problem. “I still feel positive, ‘cause I know I can do it—I just slipped up with a number.”

I asked Kate if it would be OK to learn she had made a mistake or if she would prefer not to know. She replied, “I’d rather find out. Because if I make one mistake and I don’t know, I keep making the mistake.” Although the response alone doesn’t seem all that unusual, I asked to learn about how guarded she was about her own knowing. I asked this question after observing the class day where she really seemed to reject the possibility of her solution to the Rat problem to be in error. Although Kate is open to others’ solutions and ideas—in fact, she relies heavily on them—she is not quick to modify or reconsider her own knowing. Her confidence in knowing comes with a stubbornness as well.

Second-Order Viability

Self-concept of classroom activity. In reviewing these classroom behaviors during instruction, Kate seemed to follow more of her own agenda rather than allowing for the teacher to direct the focus and deem what was important for her to pay attention to. This emerges when she wrote in her Mathography about classroom work:

> In most of my Math classes, I have noticed that group work seems to occur a lot. This can be both good and bad. Knowing myself, I can focus working alone or in a group, but in a group I may have a friend and still complete the work, but I’m not as focused as I would have been if I worked alone.

Rather than focusing on Kate’s self-reflection on her focus during individual vs. group work, notice that she takes a responsibility for the intensity of her attention to the mathematical learning. This is seen in another context, later in her Mathography, when she wrote about her attitude toward mathematics. “My attitude about Math affects my learning a lot. If I don’t like the form of Math we are learning I shut down and don’t learn anything.” Again, her comments on her engagement place herself at the center of decision making about her activity. Kate’s outbursts, inattentiveness, and reflections on her lapses of attention suggest an egocentric focus, something akin to an attitude is that “the classroom is there for *her* learning.” Her written reflections indicated that she regarded the organized activity of the classroom as important to her opportunities to learn mathematics. Yet, often her outward behaviors suggested an attitude that the classroom activity was a waste of time. This conjecture is consistent with the previous teacher’s summary of Kate as cocky, and her current teacher’s statement that “sometimes she ignores me.” Kate’s observed “attitude” during mathematics class may well be a characteristic of a generative mathematics student in the context of how mathematics is taught in the schools. When she needed a break, was interested in something else, or confident of her knowing, she acted in such a way that ignored her classmate’s needs for feedback and discussion regarding the topic of instruction. These lapses of attention and resulting behavior formulate an egocentric position of oneself in the social setting of the mathematics classroom.

Relations with class members. Kate first wrote about the role peers play for her in class and with regards to mathematical learning in her Mathography. She wrote,

> In most of my Math classes, I have noticed that group work seems to occur a lot. This can be both good and bad. Knowing myself I can focus working alone or in a group, but in a group I may have a friend and I still complete the work, but I’m not as focused as I would have been if I worked alone….
>
> Mainly I enjoy group work because you have people there if you need help and if you feel like working alone you can simply keep quiet and work single handedly.

Kate notes that much of her mathematics classroom activity has involved sitting in small groups with other students. She indicated she found good and bad in such an arrangement, stating that there is a social pleasure of working with friends, but the tradeoff may be less focus during the work time. She continued noting the benefits:

> But a good thing about group work is that you have other people with different methods to solving problems that may help you improve your skills, or no one can understand the problem and you're back to square one….
>
> I feel that when I am completing long mathematical problems is when you should work in groups because you can see each person's way just in case you mess up. On short problems, I feel better working alone because I focus more and work faster.

Kate recognized that other people might contribute different ideas about solving or methods to solve particular problems. This indicated Kate does not see there to be only one way for mathematics to be done.

The summary of data in the previous section on Kate's interactions during teaching indicated that Kate kept an ear tuned into the classroom activity, seeking confirmation of the conclusions she came to hold or other's methods to solve a problem she was shaky at or wished for a more efficient solution. It was apparent that Kate engaged in the mathematical tasks brought forth in the classroom, and that she wished to reason through them in a way she was satisfied with her understanding. This understanding, although personally developed and held tightly to, relied heavily on confirmation from others.

The way that Kate checked with others was confirmed in our first interview on April 13, 2005. She responded to my inquiry regarding what she did when not certain of a solution or part of a process to solve a problem, "I ask people in the group, or basically [Wendy]. [Jan], she knows her stuff, so we ask her now." Earlier in the interview she stated, "If there's somebody around, like if my mom is home, I'll ask her to help me. Or if I'm with a friend or somebody who had math, show me how to get this and from there I'll be OK—let me try and get the next part."

Viability

Classroom behavior. Kate was a very interesting student in the classroom. In the survey of previous teachers, her sophomore teacher indicated that she was “very confident and at times rather cocky about her abilities. As a result she did not work as hard as she should have and handed in work that was not up to her abilities.” He went on to describe this attitude serving Kate well because, “She knew what was involved in getting the grade that she wanted out of class so she just did the work needed in the end and received a good grade.” However, “Her impression of herself made it difficult to succeed at the level that she should have because she thought that doing a lot of … the work was not important to her.” Kate exuded an inappropriate self-perception of her ability, inappropriate possibly in concordance to what she achieved as a course grade and her teacher’s perception, but also likely in terms of classroom activity—for a teacher to indicate a student was cocky suggested some inappropriate behavior during teaching.

Most days her junior mathematics class began with some sort of homework review; either small group discussions, student presentations, or teacher summary. On April 7, 2005, in my role as teacher I began class with a summary activity of the homework with invitations for students to share their investigations from the night before. The topic involved a review of factoring quadratic equations written as trinomials. While a fellow student presented, Kate’s attention was turned to a student sitting next to her (classroom video transcript, 04/07/05 9:00). This other student was showing Kate what she knew about a method to factor these sorts of problems; both ignored the presenting student. Kate worked some on her own, occasionally checking in with the other student. It appeared as though Kate paid little attention to both the instruction of the presenting student and the teacher, in fact her back was to the focal point of the room most of the period (15:40). However, Kate did listen attentively to the instruction of the peer sitting next to her.

As the class transitioned to the next phases of work, Kate slapped a “high five” with another student in the room as she returned to her seat after retrieving a Kleenex (classroom video transcript, 04/07/05 24:00). The rather blatant disengagement with the teacher’s instructional agenda evident on this morning continued a trend of data that suggested Kate operated on her own agenda, reflective of the cocky-confidence of a student who knew what she needed to (and wanted to) learn.

The classroom activity following the discussion of homework—and the high-five—on this day built upon the review and practice of factoring from the previous evening’s homework. Kate and her group of four students collaborated on this new task—Kate often checking in with the peer that showed her the technique earlier (classroom video transcript 04/07/05, 30:00). Again, students were solicited to share ideas in a whole class discussion after approximately 15 minutes of work. Kate occasionally peered to the front, apparently to check in on the work of the presenter.

In a role of teacher, I asked each group to prepare a summary response, for collection, concerning the activity. Kate’s actions indicate she paid attention at this moment, immediately joining, if not leading, her small group to begin the task-to-be-collected. This teacher instruction provoked another round of group work followed by whole class discussion of questions. Again, during the whole class time, Kate was not paying attention, and was increasingly disruptive. In fact, the student presenting asked Kate to be quiet (classroom video transcript, 04/07/05 46:00). At this time, Kate’s attention turned to the presenter as did her body. After, she returned to work among her small group. Kate maintained some focus on productive mathematical activity for much of the remaining class time on this date, drifting off occasionally as the end of the period arrived.

Inattentiveness to the speaker during whole class discussions, presentations, or teacher instruction occurred frequently for Kate. Certainly, this inattentiveness was not always indicative of disconnection from mathematical activity. In the example above, much of Kate's inattentiveness was due to listening to another student's ideas and making sense of them for herself. However, this was not always the case. On the date discussed above (April 7), at a moment when sirens were heard outside, Kate turned to off-task conversation with a neighbor. On other days, Kate's off-task behavior during classroom instruction was even more aggressive. For example, on several days during class, she sang during class work time; often this singing involved group members (e.g. field notes, 04/14/05). On April 12th, prior to reviewing for a Midterm exam, Bridget and I designed the lesson to begin with some warm-up problems that reviewed recent work and helped prepare students for the Midterm. While the teacher was going over these problems, Kate said, practically shouting, "Let's work on this damn midterm!" (field notes, 04/12/05).

Reconsidering mathematical activity: learning. I return to the analysis of data regarding Kate's mathematical activity and her confidence in knowing to continue to consider her relations with peers in the context of *doing* and *learning* mathematics. Recall that Kate seemed to see herself doing mathematics throughout aspects of life, but learning mathematics may have been only been attributed to the classroom. Using the interview context, I sought to understand further the ways in which Kate distinguished between doing and learning, and especially the role of herself and classmates in her *learning* mathematics.

> B: Tell me about what mathematics you are studying right now that you feel like you're just learning. What's new?

> K: The whole graphing thing and all that. I know how to graph a simple plot on a graph; I'm fine with that *x*-axis and *y*-axis, I'm good with that. But when it comes to, ok, put it in the calculator, or like now when you say find two *x*-intercepts, that's something [new]…. Every time we move on, I just learned what we just did.

Kate is referring to the context of seeking *x*-intercepts of a quadratic function both by tracing a graph on the graphing calculator and by determining them through factoring. Her response indicated there were new techniques and procedures she is learning, and that the class moved forward quickly with these ideas.

I asked this question to draw up in her memory what she was currently learning so that she might have a context to think about the role she and others play in her mathematical development.

> B: I asked you that previous question because I wanted to put your mind into the things that you're learning right now. Tell me about the role that you play in learning this new mathematics, like these new things.
>
> K: I think like when there's one person in class, not that it's me, but when one person in class that gets it, sometimes it's easier to understand from your peers than from your teacher. Sometimes I feel like you can sit there and they won't get frustrated with you 'cause they were frustrated the same way learning. That's why the teachers, even though they don't want to show it, they get aggravated and I feel like I'm a burden. Like I feel if I learn something that's benefiting somebody else, I could help. They could understand me better than they can understand the teacher.

Clearly Kate valued talking with other students in her mathematics classroom, whether to learn from them or help them learn. This data suggests that Kate has a communal orientation to *learning* mathematics; however, *knowing* mathematics was done by each learner. Although what Kate has communicated does not get at where mathematical knowledge may lie—such as is it generated/constructed? or is it deposited to the mind, as money to a bank (Freire, 1970/2002)?—it is clear that she believes she and other students come to know mathematical ideas through their own thinking or through interactions with—help from—others.

> B: When somebody else in class does get it like that, what job do you have to do then?
>
> K: If I don't get it, I'm going to try to learn from whoever I get to first—teacher or student. But I feel like, ok—if she gets it I could get it. If she gets that easily, why can't I get that? I want to see what she did to figure this out, just like that, without having no one to help.

Kate further expressed a strong belief in her own potential to learn mathematically. She indicated that if another student seems to be able to understand, that she herself must be able to also. And further, Kate expected of herself to be able to figure things out without the help of another—just as that peer did for herself. Again, Kate demonstrated an unwavering confidence in her ability to learn mathematics. This confidence carried though her belief in her own ability to learn math with understanding, to do mathematics accurately, and to see and use mathematics in her natural part of living. Even though my questions, in the interview, Mathography, and Videography, prodded her to consider learning mathematics outside the math classroom, her language when speaking about those experiences repeatedly indicated that she *did* mathematics in her daily, usual experiences of the world. It was only in the context of the mathematics class where she spoke specifically about *learning* mathematics. Kate's viability as a mathematical learner was tightly associated with, in fact limited to, the mathematics classroom.

Self-satisfaction. Kate described later in the interview about enjoying being able to know she knows how to do mathematics: "Like the functions: When we were doing them well, I was like really into this, I know how to do this, I want to show everybody how to do it. When I know how to do stuff, I feel very positive about myself and I get drawn to it." Again, her pleasure in knowing was tied to being able to show others what she understood. I wondered if this pleasure was connected to being able to *help* others or if it was related to a more egocentric desire to *show off* to others. Kate also stated, "I feel good because I showed them and they didn't get it at first." This comment indicated that Kate's enjoyment came from perceiving herself as more able than her peers. Kate's enjoyment of expressing her mathematical knowing created an opportunity to *fit* into the social network of the mathematics classroom. She saw her knowing as an ability to maintain viability, a drive to survive among the knowing others.

Considering that Kate's ego may be involved in her enjoyment of mathematical activity, I looked into additional data. In further excerpts of the interview protocol above, Kate noted:

> K: When I understand stuff, that really draws me to the work. I really want to do this; I want to focus on it.
>
> B: Does it ever catch you attention when you don't understand something?
>
> K: I tend to draw away. It depends. When we do certain stuff, like when we did graphing, … I paid attention because of the test today…. But some stuff, like say the quadratic functions, for me first—if she [Bridget] didn't teach it to me, the first day I'd be like (sigh). The second day, I looked at it like oh I'm enjoying this; I like doing it…. After a while I start to fall into it.

Kate's continued commentary supported my conjecture that she very much enjoyed a feeling of confidence associated with knowing. It also suggested that her experience of the mathematics classroom is associated with coming to learn new ideas that are presented.

Prior to the interview, she also wrote in her Mathography about liking mathematics. She "only enjoyed four years of this challenging subject." After recognizing that *her* focus levels were what caused her to advance more greatly, she noticed that "when the teacher really enjoys the subject himself or herself, I find myself appreciating the subject even more." She also noted that she had the same teacher in grades 5, 7, and 8. She wrote that he gave her first D in Math—during 5th grade. At that point, she "always wanted to try harder so that I could prove to him I would do good and accomplish and understand whichever Math we were learning." Kate recognized that her own attitude about Math affected her learning. "If I don't like the form of Math we are learning I shut down and don't learn anything. If I am enjoying it, I am alive and waiting anxiously to complete more problems."

Kate highly associated her mathematical learning with engaging in the math classroom. More than anything thing, this evidence suggested that when she considered *learning mathematics*, it was tightly associated to topics being studied in her mathematics course. Established earlier, while working in math class, she valued working with peers. Not only for the social nature of the activity, but also because she sometimes feels she can learn well, if not better, from them. More significantly, she enjoys being able to show others what she knows, and hopefully teach them as well. This opportunity to *show* seemed tied to Kate's ego; she was very confident in her ability to know, learn, and do the mathematics studied in the math class. She was somewhat guarded of this knowing; what I perceived to be somewhat of an ego-protection mechanism. She was always monitoring classroom activity and discussion, whether it is teacher-led or other students thoughts. She rejected most ideas until they were strong enough to create a crack in her own knowing, challenge her own ability to do something, or refute a conclusion she derived. When these moments occurred, she expressed some frustration, yet also returned to the problem to think through it herself or concurred with a peer in order to correct her misunderstanding.

Summary of claims

Self-confidence. Kate came across as a confident if not cocky mathematics student. She put forth an image of someone who knew what she was doing—for example saying to a group mate after checking a homework solution, "I was right." Her previous and current teacher identified this quality. This confidence manifested itself in several ways. For one, it contributed to Kate's status in the classroom. She was perceived to be a good student by her peers, probably in part because she acted as though she knew mathematics well. This academic respect afforded her a powered role in the classroom. She enjoyed *showing* other students how to do problems, or helping them understand an idea. This status and ability to show others that she knew seemed to contribute to her ego in which she identified herself as a competent mathematical knower and learner. However, she scored herself a 7/10 on a self-assessment of her mathematical ability. She further described her abilities as "I'm not best at math" in her initial survey. This slight contradiction of outward posing and inward self-doubt indicated some degree of fragility associated with her own sense of self. Much of her classroom behavior, such as seemingly ignoring contradictions to her knowing and being slow to be open to new learning indicates an ego-protective mechanism. Kate did compare herself to others with regards to her ability to learn mathematics. She assumed that if they could do it, she could as well. Again, this orientation speaks to a cockiness she held in her own abilities.

Self as authority. The data strongly indicated that Kate positioned herself as an authority for knowing. Answers to problems, or discrepancy in what she understood had to emerge from her own reasoning for her to accept it. Bridget initially wrote that Kate sometimes ignored her. Kate also seemed reluctant to consider the possibility of error in her solutions. It was quite evident on the other hand, that Kate relied tremendously on her peers—including the teacher—for feedback that might support or refute her own ways of knowing. In this way, others played an important role in Kate's mathematical knowing. Kate's disposition toward knowledge leaned away from being a received toward that of being a producer. This corresponds to the right side, the positive *x*-axis, on the initial model of ways to theorize mathematical learning that I proposed in Figure 1.

Humans as inventors. Kate also saw people as the inventor of mathematical ideas, whether or not those people were Mathematicians. A Mathematician was someone other than herself; but by what defining characteristics was left unlearned. I suspect that Kate conceived of a Mathematician to be someone who self-identified their job or regular activity as the learning of mathematics. This learning might be coming to recognize what others had invented (i.e. they were mathematical discoverers), or the mathematician may be an inventor themselves. Although Kate never directly implicated herself, I believe Kate would say she is discovering mathematics that other people realized before her, in most cases. However, there may be the occasional moment where Kate might invent something new herself. Key to this question about the emergence of mathematical ideas was that it was done by people. This orientation to the nature of mathematical knowledge positioned Kate in the positive y-axis of Figure 1. The data collected during the study confirmed that Kate tended toward the first quadrant of my initial model to theorize various ways of mathematical learning, suggesting she was a good representative of a student I would identify as generative.

High status. Kate's orientations to the nature of knowledge and her disposition toward knowledge, her sense of authority, manifested themselves in the classroom in several ways. In particular, Kate maneuvered with these tools to create a position of high status in the classroom. To consider this claim, first recall that Kate valued other knowers in the class as resources from which to learn and through which to test ideas. However, equally, or possibly more importantly, class members were there for her to show off her own knowing. Kate's status in the classroom was important to her, she wished to maintain her academic standing among students, being perceived as someone who often understood well the topics or problems being studied. She leveraged this peer respect into a heightened social status as well. Most of Kate's classroom social interactions were limited to a small subset of the more socially powerful students. It may be that her guardedness about her own ability in mathematics was connected to maintaining this access to the higher-status peer network.

Mathematics as a part of living. Kate identified mathematics as something that was part of living; people are doing mathematics all the time. She included her own counting of stairs and finding the best way home, along with her playing of sports and signing her name, as mathematical activity. I believe that because she felt herself to be so present in her work to do and understand what she identified as mathematical ideas and problems that she readily saw what she identified to be mathematics in all she experienced. The interesting quality of this orientation to mathematics is that she seemed to separate *learning* mathematics to being bounded by the classroom environment. She did not readily identify herself has developing or learning mathematics while experiencing the world. In this manner, she did not self-identify has mathematically generative.

Borderline classroom demeanor. A concluding observation about Kate's mathematics classroom activity was that she was well versed in what social-norms were necessary to adopt in order to achieve good grades in her mathematics class. Further, she valued achieving these grades. It was apparent that she didn't directly associate high grades with the quality of her knowing, although she did not outright refute this either. Kate elected to take part in the social norms necessary to obtain these grades, such as being regular with completing homework and turning in major projects. She also demonstrated a concern for doing well on course exams. However, while Kate acquiesced to the norms required in the classroom to earn her grade, she was often quite abrasive with regards to other norms, especially toward maintaining a degree of civility toward the teacher. Kate did interact one-on-one very well with adults, so this abrasiveness likely had some degree of association with what was accepted in her peer group. But it also certainly had much to do with the protection she kept up of her own mathematical knowing; she was both loathe to be challenged, but possessed a strong desire to listen for incomplete knowings of mathematics that she had constructed. She subversively listened to the teacher for input to what she knew and was correct in understanding.

Informing the Notion of the GAML

As did Fisk, the data collected during the study of Kate supported the idea that she was a generative adolescent mathematical learner. And, as with Fisk, my inquiry was designed to better conceive of what the GAML may be—to enrich a model of the GAML as a learner and to understand better how the GAML exists and interacts amid its experiential reality, in particular among peers in the mathematics classroom setting.

Kate demonstrated that herself, as a GAML, was a key figure in coming to understand and to know. Mathematical learning was done by her, not to her. Her personal epistemology was very confident, and resolute, in order to understand mathematical ideas. Fisk was similar in that he believed highly in his mathematical abilities. However, each of their self-confidence showed signs of fragility. Kate expressed self-doubt in a survey. On the other hand, Fisk expressed a very high self-evaluation, yet received less consistent high marks from a teacher. Kate seemed to protect the fragility of her mathematical ego somewhat more aggressively than Fisk. When Fisk would shut down when given negative or unsupportive feedback, Kate seemed to buckle down harder in order to make sense.

Kate and Fisk’s knowing were both heavily dependent on the opportunities for interaction with others. Interestingly, both learners placed a seemingly equality of value of feedback from peers and from the teacher. It is evident that both of these GAMLs dispersed mathematical authority, rather than locate it squarely in a teacher, textbook, or other expert.

In both students' case, external evaluation—namely that of their classroom teachers and myself—did not label these students as star mathematical performers, but they also did not indicate mathematical development below what was common for their peers.

However, both students did possess qualities that did not fit well into the typical norms of a high school mathematics classroom. Neither student's behavior meshed well with the codified rules of the typical classroom. Each expressed their own demand of space and time and other's attention in ways that tended toward an egocentrism; they arrived to class as they wished, they paid attention to ideas, problems, conversations, their own musings, as they wished. Kate would sing during class, shout across the room, or sit with her back directly to the teacher. Fisk kept his backpack on throughout class, spoke with classmates on disconnected topics, and clearly expressed his own role in deciding when to pay attention, study, learn, and when not to. It seemed significant to both students that their teachers had patience for this pressing of the boundaries on classroom decorum. Both GAMLs were allowed to be mavericks.

Jack

If it were that Kate were somewhat abrasive, hard, aggressive; Jack was yin to Kate's yang. As was Kate, Jack was well liked by many people. Yet it was his nature to be much more open and approachable. His peers, by no means an exaggeration to say everyone in the school, called him "Fridge." Fridge, like his namesake—the mid 1980's football player William "the Refrigerator" Perry—was a large young man; happy, warm, and welcoming. I next present this young man utilizing the organization found in Fisk and Kate, beginning with comments on how he was selected, then continuing with data analysis following the pattern: mathematical activity, personal epistemology, second-order viability, and viability in the mathematical learning environment. After a summary of key findings, this data presentation and analysis for Jack will conclude with a reflection over the key points in order to consider how what was learned from Jack impacted the development of a model for the GAML.

Selecting Jack

Jack was not identified as a potential subject by any of the PHS mathematics teachers, unlike Fisk and Kate. However, I noticed Jack while observing Bridget's class on the second day during Phase I. I recorded to my field notes (02/23/05) that he had done some good mathematical work in class on this day. In particular, during a warm-up activity meant to be orientated toward test preparation, he shared with the class an idea about how to select from among 4 multiple-choice options a line-of-best-fit to a picture of data, see Figure 3. Jack's method was to insert a single value for *x* into each of the four options for the linear function and see which output, that is corresponding *y*-value, best fell into the picture of the data. I interpreted his idea as a creative method to solve the problem posed, rather then an adherence to a learned procedure or algorithm. On that day, I also presented him with an extension question to the idea he offered the class. When I checked in with him later, he seemed to have resolved the task I offered.[87]

Jenny studied the effect of light on plant growth. She graphed a scatterplot to represent her data.

Which of the following **best** represents the equation for the line of best fit for the data shown?

A. $y = -0.4x + 5$
B. $y = 0.4x + 5$
C. $y = -4x + 5$
D. $y = 4x + 5$

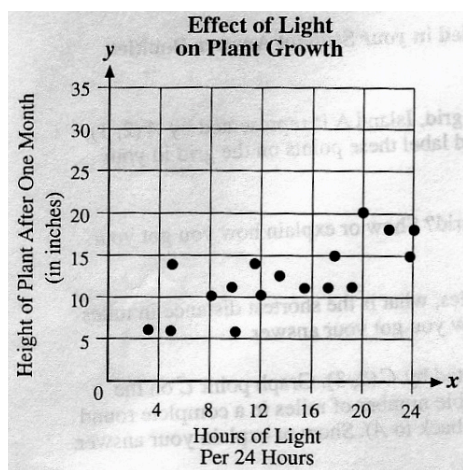

Figure 3. A problem to determine a best-fit line, studied in the context of test preparation.

[87] My notes on the particularities of Jack's classroom activity are somewhat incomplete; recall I was focused more on observing students that were suggested by PHS teachers.

Bridget arranged her classroom so that students sat in small groups of 3-4 members. Later in class on this same date, February 23, when given a mathematical activity to work on with colleagues, Jack moved to a different group to do his work. As his work with the young lady he sat with drew to an end, he elected to gather an overhead acetate sheet and pens in order to prepare to present his solution to the class. The task given to the groups was on the second day of introduction to the notation for the concept of logarithms. Previously, students estimated solutions to exponential equations. The logarithmic notation was shown as a way to represent the solution. The textbook used stated the example, "$\log_{10}162$ means 'the power to which I should raise 10 to get 162'" (Fendel et al., 1997, p. 414). After learning the notation, students estimated the solution to problems like, "Between what two whole numbers does the value of $\log_{10}162$ lie? Explain your answer" (p. 414). With this introduction to logarithms, emphasizing the equivalence to a corresponding exponential function, the task Jack worked on was as follows (abbreviated for emphasis):

1. For the function $y = \log_3 x$, choose values for x for which you can easily compute the value of y, and plot the resulting points. Choose enough points to allow you to sketch the entire graph.

…

4. How does the graph of a logarithm function compare to the graph of the corresponding exponential function?

Jack presented his thinking to the class, sharing a picture of the graph he developed with his fellow student. I recorded to my notes that his overhead acetate showed a graph of the function $y = \log_3 x$, with the x and y axes switched. Jack also discussed his response to question 4, regarding the relationship between his graph and the associated exponential function. Although I didn't note his exact wording, I recorded that he seemed to have a "gut-level sense of understanding how $y = \log_3 x$ and $y = 3^x$ related" (field notes, 02/23/05).

In class that day, I spoke to Jack about the switch of the *x*- and *y*-axes in his graph. I wrote in my field notes (02/23/05) that he seemed to not immediately understand what I pointed to, but later on he made sense of my comment, as he had done with my challenge to the warm-up problem regarding his test for the best line-of-fit. I noted that the way in which he demonstrated this knowing to me, in both cases, indicated he understood what I had suggested he consider. Although I hadn't been paying close attention to Jack since no PHS teacher had suggested him, these two mathematical experiences suggested to me a potential that he engaged in mathematical thought for himself, created meaning, had an intuitive way of knowing, and the confidence to share his ideas with others. I began to consider him a possible candidate on this date.

The next day of class, I decided to invite Jack after experiencing another mathematical moment with him. Again during a warm-up activity, he observed a pattern in a series of fractions connecting the denominator to the sum of the series. His statement of this pattern did not adhere to conventional mathematical language, but I took note that he was willing to volunteer what he "saw". I perceived his activity to be highly mathematical; that he readily expressed it made possible access for me as a researcher. Furthermore, I believed much of the mathematical ways of knowing and learning I perceived during these two days of observation were indicative of a generative learner.

Because I invited Jack, I also asked Bridget to write about her impressions of Jack, utilizing the same format completed when she advised me of other potential subjects (Subject Identification Request). Bridget wrote,[88]

> Jack doesn't work hard. But when he works, he believes he will "get it." He will find the solution; somehow, he is very confident.
>
> His papers never show all his work. They are not very organized and he doesn't like to write a lot. But he is able to connect ideas in his own way, and

[88] The lines are numbered for future reference.

> he can explain what he is doing very well. He just won't do a detailed written explanation.
>
> He thinks a lot, he stares at you, writes almost nothing, asks one or two questions, looks at the ceiling or puts his head down, and then he gives his answer. He will pursue an idea with greater persistence, he usually likes his own ideas and answers. When he is wrong, he accepts other students' opinions. He listens, thinks and makes his own conclusions.
>
> He won't copy somebody's answer just because he/she always know what he/she is doing. He will if that answer convinces him.
>
> His questions are "how do you do this?" or "what is this for?" He rarely asks "why?" And his answers to "why?" are "because it works" or "because I knew it had to be 49."
>
> He is absentminded. He comes to class without even a pencil at least twice a week. He wastes a lot of time in class. He turns his work in late most of the times. He is not worried about his grades. He just wants to pass, and have some fun.
>
> He can work in groups. He loses his concentration easily, because of not-related-to-math reasons, like conversations, games, anybody at the door, etc.

This information about Jack's mathematical and classroom activity helped to confirm for me that he seemed to be oriented to learning as a generative learner; in particular he matched the initial model's ideas of about one's own sense of a productive role in learning. Bridget described Jack's reasoning as central to his coming to accept his own understanding of a mathematical problem: line 11, "He listens, thinks and makes his own conclusions." Bridget also suggested that Jack was unwilling to accept another's solution just for seeming to be smart, but "will if that answer convinces him" (line 13), a strong argument about Jack's internal locus of authority. Jack apparently was quite confident in his ways of knowing, as well in his ability to come to know: "He believes he will 'get it'" (line 1). Many other aspects of who Jack was as a learner were interesting, and I will build upon these in the following sections, once Phase II of my data collection and analysis began.

Mathematical Activity

Written records of mathematical activity. Jack and I spoke during class and in our interview about his work on the rat problem, the problem to determine a rat population after one year on an island first identified in the case of Fisk. On April 5, Bridget and I prompted students to share what they had learned so far about the problem. Most work was done in a whole group setting, but individual work and small clusters of collaborative activity was encouraged as well. The time was simply meant to be a work session, part of an ongoing expectation that students work on this problem for approximately one week before it was due. I recorded in my field notes (04/05/08) that Jack appeared to have come unprepared, not having spent any significant time exploring the problem. However, he "followed discussions well." During class work, I saw that he had an idea about the problem that seemed potentially productive, and worth sharing with others. When I asked him to share this with the class, he responded by asking me, "Is it right?" His reaction could be an indicator of many factors—lack of confidence, externalized locus of authority, a simple surprised reaction to realizing the teacher had heard his thoughts. This response certainly suggested that he valued, in some way, the solution to this problem more so than the thinking involved.

When the finished assignment came due, Jack submitted his write-up of the problem late. It was both incomplete and inadequate, in terms of the level of justification expected of the high school juniors in this math class. Bridget scored his work, available in Figure 4, a 40%, awarding 5/20 points on the first section (Problem Statement), 20/50 on the Process, 5/20 Solution, and 0 points on each of the final two sections. Although a failing grade and a terribly incomplete thinking through of the problem, Jack's mathematical response is worth a brief note. The problem was quite involved; here it is restated in Fisk's words:

> I have to explain how many rats there will be on January 1st of the next year after the first year. In the first year they're will be 2 rats, 1 male 1 female. These two rats will make offspring under these certain rules.
> 1. The number of young produced in every litter is six, and three if those six are females.
> 2. The original female gives birth to six young on every January 1 and produces another litter of six every 40 days thereafter as long as she lives.
> 3. Each female born on the island will produce her first litter 120 days after her birth and then produce a new litter every 40 days thereafter.
> 4. The rats are on an island with no natural enemies and plenty of food, so no rats will die in the first year.

The problem involved accounting for the accumulation of rats. This was a challenging problem for two particular mathematical and organizational reasons, students must keep track of how many rats were eligible to give birth on each of 10 birthing dates during the given time period, and they must design a way to count total rats. In my experience with students working on this problem, it was the ability to appreciate the complexity of these two aspects and the ability to develop a recording system that helped track the rat population. In a way, there seemed to be a need for students to develop figurative material through which to work; the problem is too cognitively taxing for many to remain in the activity of mental operations.

Jack's response did allow for some inference of what he did do with the problem. He recognized that several generations would be born, and each newly born rat would produce children. Jack has identified several generations: the first being 3 male and 3 female born to the Mother (3m and 3f); the second 6, 6, and 6 born to Kid 1, Kid 2, and Kid 3, of there are 3 male and 3 female; a third generation, labeled 6 6 6; a fourth of 18's; a fifth of 54's, and a sixth and final generation of 162's. Jack final solution was to count the number of 162's, $9 \cdot 18 = 162$. It is unclear why Jack selected 6 generations; there are ten 40-day cycles in one year, when including January 1 as cycle 1. There are only four 120-day cycles for newly born rats to mature to give birth.

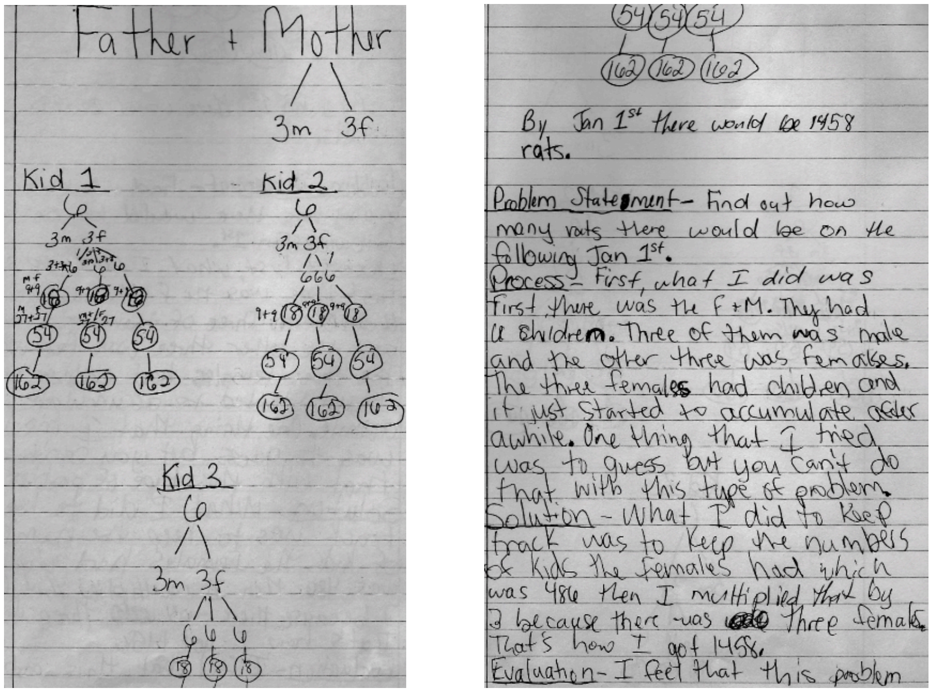

Figure 4: Jack's response to the rat problem.

One possible explanation is that Jack took away from the classroom conversation that his solution needed to be larger than the $9 \cdot 18 = 162$ that he may have originally obtained by watching the 120-day new rat birth cycles, possibly having some sense that other rats kept having babies every 40 days. However, he certainly did not reconcile that some rats would be giving birth every 40 days—even after having their first litter. This is strongly evident in that his organization showed no indication that he recognized the original parents would have another set of 3 male 3 female babies. In the first interview, Jack responded to my inquiry:

> B: Remember the rat problem? I know you worked on it to some extent; tell me about your decisions when you stopped with that problem, or when you decided you were done.

> J: Well, you said the number I had was 500 somethin'. Then you said it was more than a thousand, then I said I wasn't finished, so I had to keep working at it. And I felt like I knew I had the answer so I stopped.

It seemed that Jack suspended his initial thinking, which I told him would result in a solution just greater than 500, when I suggested that the actual solution was much higher. Rather than holding to what made sense to him, he let go of his own knowing on this task.

Not too much about Jack's mathematical ability, or potential, should be deduced from this single example of work. What can be stated is this is a problem he did not complete, neither from a perspective of working through the challenge of the mathematics, as well as the write-up of his findings. Even the last sentence on the paper, "I feel that this problem", was left unfinished. I suspect Jack simply knew he needed to turn something in, it was already late, and rushed to complete something. The Mathography Jack submitted for me was similar: I asked several times for him to complete the response, but he never did until I sat him aside at a computer during my Phase III visit. And as was his rat problem write-up, this response was rather brief—in comparison to that of the other research subjects. Bridget wrote in lines 3-6 of the Subject Identification that Jack didn't like to write, and that his explanations do not show all of his work. These experiences with Jack on the rat problem and Bridget's initial reflection on his activity seem to confirm that he will readily engage in mathematical activity, but may do little of this outside the mathematics classroom.

A particular aspect of Jack's work was apparent during my classroom observations of Phase I, which was the intuitive knowing he seemed to rely upon. Bridget confirmed this observation in lines 15-16 of her Subject Identification, indicating Jack was satisfied to justify his thinking with brief replies such as "Because I just know", or "Because it has to be…."

Following up on the evidence of this intuitive knowing regarding the relationship between exponential and logarithmic functions evident in Phase I data, he repeatedly showed a solid “knowing” of this relationship in other contexts. In the cover letter for his portfolio for the properties of exponents unit (Archived data: Jack’s Exponents Portfolio), Jack wrote to Bridget:

> The central theme of this unit was exponential functions, logarithms, and scientific notation. Exponential function is a function involving a variable or unknown quantity as an exponent or part of an exponent…. Another function is logarithms and is the power to which a fixed number or base (usually 10) must be raised in order to produce a given number. If the base is 10, the logarithm of 1,000 is 3.

Although some of the language Jack used in defining and giving an example of the logarithm operation sounded of adult or mathematician language (i.e. “fixed), there is enough language of his own apparent that on the whole, he has made meaning for himself of this mathematical operation.

Continuing to look at his ways of understanding logarithms and their relationship to exponents, I considered two problems on the In-Class unit exam. First, having been asked to “State whether the equation is true” and “Explain your answer”, Jack’s response to the mathematical sentence: $\log_2 8 = 3$ was, “True – because $2^3 = 8$” (Archived Data: Jack’s Exponent Tests). On the Take-Home portion of the test, Jack was instructed:

1. Sketch the graph of the function $y = 1.5^x$ from $x = -3$ to $x = 3$. Label at least five points and their coordinates.
2. Explain and show how to use your graph to estimate these values.
    a. $1.5^{-0.5}$
    b. $\log_{1.5} 2$

His response, provided in Figure 5 (lighter markings were Bridget’s feedback), suggested less understanding of the inverse relationship of the exponent and logarithm operations than previous work suggested. Although points are reasonable—relative to one another—Jack’s record of the five coordinates poorly match the function. Based upon my classroom observations of his mathematical activity, I suspect Jack utilized the graphing calculator to graph the function, and then copied it to his test paper. He may have estimated from this graph, the calculator window, or possibly traced on the calculator in order to record the values he did, namely: “(-3, 0); (2, 0); (0, 1); (2, 2); (2.5, 3).” The integral solutions he recorded suggested that he did not have an appreciation for the significance of the decimal values providing a more accurate representation of the exponent operation.

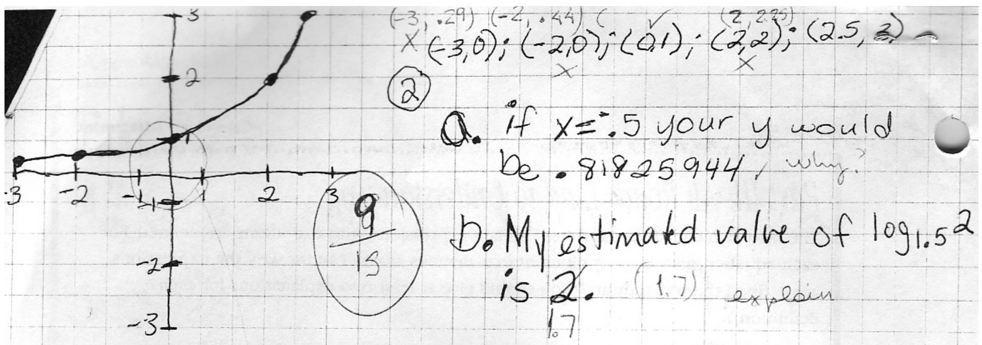

Figure 5: Jack’s response to logarithm question on Take-Home test.

The number of decimal places carried through in Jack’s response to question 2a further indicated that he used the Trace feature of his graphing calculator. When computing directly $1.5^{-0.5}$, the calculator Jack used would respond .8164965809. If a student utilized the trace feature, the calculator would still report the correctly rounded mathematical computation of 1.5^{x}. However, the value for x couldn’t have been traced to exactly –0.5 and thus the corresponding y-value would not have been an accurate result either. Jack could be justified in his processes had he been thinking that he “used his graph to estimate these values.”

Jack’s response to the question, “use the graph to estimate… $\log_{1.5}2$” was 2. Because Jack provided no explanation of his thinking, I considered some possible rationalizations for this response. Had Jack utilized the reasoning evident in the In-Class response confirming that $\log_2 8 = 3$ because $2^3 = 8$, he may have thought “1.5 to the *what power* equals 2?” Using Jack’s description of the logarithm in his portfolio cover letter, “the logarithm is the power to which a fixed number or base (usually 10) must be raised in order to produce a given number" (Archived data: Exponents portfolio), he may have asked equivalently, "what power must 1.5 be raised in order to produce 2?" Had Jack guessed and checked, he likely would have followed a path of reasoning (my words in italics) similar to below, utilizing the calculating possibility provided by the calculator:

$1.5^1 = 1.5$	*I knew that. The solution must be bigger than 1. Try replacing the 1 with 2.*
$1.5^2 = 2.25$	*Too big. Try something smaller than 2. Maybe 1.8?*
$1.5^{1.8} = 2.0747$	*Still a little too big. Try 1.7.*
$1.5^{1.7} = 1.9923$	

Of course, this reasoning could continue to refine the estimate. However, since Jack's solution was 2, i.e. he proposed $1.5^2 = 2$, I don't believe he checked the accuracy of his reply by computation. This conjecture remained consistent with the error in the coordinates provided in response to question 1. Further, because all of his previous responses to this question seemed to come from the graph, I suspect that he solved this problem in a similar manner.

Had he solved using the graph, it would be uncertain to deduce what he knew about the inverse relationship between exponent and logarithm. If Jack knew that estimating the value of $\log_{1.5}2$ was equivalent to determining what x value would make $1.5^x = 2$ in the graph of $y = 1.5^x$, he might have looked to his graph and read across the vertical axis at the height $y = 2$ to determine that $x = 2$. In this way, his reasoning may have been sound, and reflected some connection between the operations. It was unfortunate that the point was identical numbers, (2, 2). As such, Jack might also have read up from the horizontal axis at $x = 2$ and determined the matching coordinate $y = 2$, thinking this number solved the problem for him.

Reconsidering Jack's response to the previous problem to interpret $\log_2 8$, that he recognized that $2^3 = 8$, could have been an act of guessing and decoding rather than one of knowing. These small, whole numbers are well-known quantities in the mind of an adolescent, and much can be maneuvered readily in operative thought. The understanding of the relationships of the exponential and logarithmic graphs may have been a sort of first level awareness that *x*'s and *y*'s are "swapped", but that when asked to use that understanding—especially in the context of more challenging numbers ($\log_{1.5}2$)—Jack was unable.

Interaction. When given several options to chose from, Jack completed the Subject Survey prompt, "Mathematics is something I do" by selecting *only* the option, "mostly at school." He also wrote in his Mathography that, "I really don't learn math outside of the classroom." This appeared to be a direct reply to a prompt in the Mathography instructions: "Consider experiences where you were doing and/or learning mathematics outside of your mathematics class. What mathematics were you doing and/or learning? What was different about your learning compared to math class? What was similar?"

I followed up on this school-focused perception of his mathematical activity in our first Interview (04/14/05) by asking, "Do you learn mathematics outside of classroom?" After a pause, Jack asked, "Like going to other math classes?" My question seemed discordant; how might he do the classroom work of mathematics anywhere else? I replied,

> B: Are you aware of any times that there was something you would say that you did that was you doing something where you learned some mathematics? Or are you just willing to say, 'it seems like I probably would.'
>
> J: yeah, seems like I probably would.

I found throughout the interview that Jack wasn't very vocal about how he was thinking, nor did he do much to discuss his thoughts. I did not doubt his willingness to participate; his answers were not short and his body language was not indicative of wishing to wrap up and be done. I found that I might have led him to answer in this case, rather than give an answer of his own. I worked again to get back to the question by asking about some of his home activities, but was not able to draw out further expression about doing or learning mathematics outside the classroom context.

Returning to the survey, where Jack had just emphasized that he believed mathematics was only done in school, he was next asked, "When I am doing mathematics at school, I am likely to be". This is a selection of his responses:

a.	Talking	Often
b.	Writing numbers	Sometimes
c.	Writing words	Seldom
d.	Drawing diagrams	Often
e.	Working on my own	Seldom
f.	Working with a friend	Often
g.	Working with a group	Often

Jack characterized much of his classroom mathematical activity to involve talking and working with friends and groups. He saw himself as *Seldom* working on his own. This interactive nature of Jack's classroom activity was repeatedly evident in the classroom video of his work.

When most classrooms move to a mode of whole class discussion, usually this does not actually involve the *whole* class discussing. It typically means that the teacher is talking and 2-5 students are replying, usually to the teacher. Bridget's classroom operated in this manner, with Jack often being one of these verbal participants. During class on April 7, the students were reviewing factored polynomials, quadratics in particular. The emphasis was on the motivation to be able to quickly identify the x-intercepts of a quadratic function when in factored form. Initially, students worked in small gatherings of 2-4 students. During this work, Jack was a part of a productive group. I described his engagement as, "partial,… working in his head, but not needing to record. He seemed aware that [Brad] was recording for the group" (classroom field notes, 04/07/05). As Bridget moved the focus from small group investigation to debriefing in the large group, students were asked to volunteer solutions and methods. Not all students had finished the activity, but this was intentional in Bridget and my planning. I did have to draw people's attention to the presenters (Classroom video transcript, 04/07/05 29:00). Shortly after, Jack volunteered a solution to a question that I posed (31:00). Class continued weaving in between small group and large group work, as students refreshed their memory of processes to factor quadratics. During another presentation, Jack asked one of the student presenters a mathematical question (48:00), indicative of intent listening as well as mathematical understanding. And again, during a later episode, Jack stated to the presenter, "You got to combine like terms" (59:00).

Very similar interaction was evident on April 12 as well, although on this day Jack was a bit more pesky in his contributions; he seemed to be participating to keep himself entertained, rather than out of mathematical engagement. Jack shouted out an answer (Classroom video transcript, 04/12/05 8:00) after a couple of volunteers spoke (8:00). A short while later Bridget asked the class if they heard a small-voiced volunteer speak. Jack jumped in berating his classmates for talking and thus, not hearing the presenter.

The conversation extended from the initial question to identify which of two quadratics was a perfect square, to justify the response given (Classroom video transcript, 04/12/05 11:30). Bridget asked, "What makes you think this is a perfect square?" Jack replied, "It is perfect." This lack of justification was not a cause of concern to Jack, he saw that as his proclamation of how he knew.

The next significant amount of time on this date involved students reviewing mathematical topics for a midterm test the next day. During this time, Jack repeatedly called for my attention, and then later Bridget's; Jack solicited my attention at 38:00, 51:00 (three questions, each seemingly to keep me there), 60:00, 63:00, and then Bridget's at 67:00. As this class came to a close, Bridget brought forth a final problem for the whole class (Classroom video transcript, 04/12/05 73:00). I noted that almost immediately, Jack provided an answer. Bridget asked Jack how he knew, to which he replied, “Because I’m smart like that.” Bridget’s intent was to present some example quadratic functions, in standard form, and ask students the coordinates of the vertex—prompting a need for a format to more simply locate those coordinates, as was done with the process of factoring in order to locate the x-intercepts. Bridget offered one more example (75:00), to which again Jack offered a solution and Bridget followed with, “How do you know?” This time, Jack said more than “because I’m smart” but started using more mathematical reasoning.

Jack seemed to keep a finger on the pulse of mathematical activity in the room, he interacted with classroom presenters and with students across the room that vocalized questions. He also interacted much with the teachers as they led class discussion. Although sometimes verging on play, usually this tendency was short lived and Jack returned to mathematical activity.

Oral mathematical activity. Bridget noted in lines 3-6 of her response to the Subject Identification that Jack didn’t like to write much, but that she felt like he usually knew what he was doing. Bridget stated (line 1) that Jack didn’t work hard. A previous teacher described him as lazy: “He can do work, but most of the time he was being lazy” (Previous Teacher Survey). In fact, Jack even called himself lazy in his Mathography, saying that, “at times I can be lazy and not fully apply myself.” This derogatory way of seeing Jack, both by his teachers and by himself, raised questions about who Jack was, and a sort of which-came first: was he lazy, and observed to be by school teachers, or was he named to be lazy, and then lived up to such expectations—taking on the expectation for himself. Much research has established that students meet teacher expectations (cf. Good, 1987), so it is difficult to not assume that may be the case here. But there may also be another factor involved, and that is that Jack has a strong preference for an oral manner for interaction, and thus does not thrive in a logo-centric social environment, such as the school and the mathematics classroom. That both teachers, as well as myself, observed Jack to be highly mathematical, supported this conjecture. Jack's classroom mathematical activity evident in the previous section demonstrated a pattern of oral engagement in mathematical activity.

As further confirmation that he may have a preference for oral communication, Jack demonstrated corroborating evidence through the nature of his inability to articulate his reasoning. This inability was evident in the contexts of both providing verbal and written solutions to mathematical problems as well as in the context of the interview—during which I explored some of his views on life in a context more broad than the mathematics classroom. The examples of Jack's justification on his exponent tests above showed that when asked to explain in writing, he provided little more than a statement of conclusion. He did not provide more intermittent, causal or deductive logic to justify what he determined to be a solution, or to be true.

Prior to my observations during Phase II, Jack completed another extended problem, like the rat problem, in which deductive logic was explored. The assignment began with instruction about drawing conclusions from true statements, with the following two examples from Lewis Carroll's work (Fendel et al., 1997, pp. 382-383):

> Example 1
> a. John is in the house.
> b. Everyone in the house is ill.
>
> If you know that statements a. and b. are both true, then you can deduce that John must be ill. So "John is ill" is a valid conclusion.
>
> Example 2
> a. Some geraniums are red.
> b. All these flowers are red.
>
> In this case, knowing that statements a. and b. are both true does not tell you whether any or all of "these flowers" are geraniums. They might be other kinds of red flowers. So there isn't anything new you can definitely deduce from the two statements in Example 2.

Having been provided these examples, and example explanations, students are set with 6 problems, and asked to create two “sets of statements” (p. 384) of their own. Jack’s responses (Classroom Artifact: Logic problem), duplicated next, demonstrated more about the nature of his deductive logic as well as his ability to express his reasoning:

1. a. No medicine is nice.
   b. Senna is a medicine.
   Senna is not nice.

3. a. Some pigs are wild.
   b. All pigs are fat.
   Some pigs are fat and wild.

Jack answers to each of these two problems – the third line in each instance—was indicative that he understood the intention of the problems. It also showed that he could reason in ways that seem to appear rational, deductive. His second reply might not have been what a teacher might predict, but it was logically sound; since all pigs are fat, those that are wild are *both* wild *and* fat. However, again in this work Jack did not make any attempt to explain his reasoning, although explicitly asked in the instructions.

The next problems got more difficult; problem 4 utilized opposites.

4. a. Prejudiced persons are untrustworthy.
   b. Some unprejudiced persons are disliked.
   Prejudiced persons are untrustworthy and disliked.

My analysis of the problem as given was that all prejudiced people were untrustworthy, a description for people who may or may not be prejudiced. The second statement provided the information that some, but not all unprejudiced people were disliked. The two statements made no comment about the trustworthiness of unprejudiced people, nor about how well liked prejudiced people might be. They also left room for some untrustworthy people to be liked and for some to be disliked. Jack’s conclusion that “prejudiced persons are untrustworthy and disliked” might have been true for *some* prejudiced persons, but not necessarily all. Jack's response was not a direct contradiction, but also could be understood as falling into a trap of bias from experience or opinion outside the two given statements in the problem—something that the problem author may have intended. Also, again Jack made no effort to explain.

I provide one more of Jack's replies.

5. a. Babies are illogical.
   b. Nobody who is despised can manage a crocodile.
   c. Illogical persons are despised.
   Illogical people is despised and cannot manage a crocodile.

Briefly, *a*. and *c*. together suggested that babies were despised; then taken with *b*. could lead to the conclusion that babies could not manage a crocodile. Jack's conclusion was valid, but did not draw upon all three statements.

Bridget scored Jack's work on this task as an 80%. In my experiences with high school sophomores and this problem, it challenged many of them. In his ability to use logic, this reply indicated to me that Jack was rather similar to many high school age students as a mathematical reasoner. But that he made no effort to explain once again could be attributed to the laziness described by both him and his teachers. This lack of explanation may also be partly described by a difficulty Jack has in communicating his reasoning, or possibly even connecting his expressive self with the thinking self that does this work.

Even Jack himself expressed surprise at his seemingly miraculous, unrationalized ways of knowing. The following protocol from the first interview (04/14/05) demonstrated some interesting comments by Jack.

B: What sorts of things [in math class] catch your attention? Do they catch your attention because they're harder or because it takes more work for you to figure out?

J: It takes a lot of work for me to figure it out.

B: Are there other things that catch your attention, like are there times where you know you've been daydreaming and then you'll tune back in and…

J: There's plenty of times when I been daydreamin', then like sometimes I daydream and I come right back in and I answer a question and then I'm right and I'm like, how was I right?...

Soon as I come back, she may ask a question and I might know the answer and I'll just answer it.

In the final two paragraphs of this protocol, Jack indicated that he had answered questions correctly, without he himself really understanding how he could have been right. Not that he offered a random guess, but he seemed to indicate he couldn't access the reasoning that may have yielded his response.

In summary of Jack's mathematical activity, he confirmed Bridget's assessment that he didn't do much school mathematics outside school. Further, he didn't perceive himself to learn mathematics outside of the mathematics classroom. Most of Jack's mathematical activity was oral. He did record solutions when working on problems in class and for assignments and tests, but this written representation served little more than record keeping of conclusions he reached. He did not demonstrate much explanatory discussion or logic. Furthermore, the opportunities to create figurative material upon which to do further mathematical activity was also absent, or not of productive use. The most potential within his generated representations may have been the graph created in the Take-home test on exponents. Less productive were the sketches for the rat problem. And Jack showed no evidence of utilizing written representations to imagine the sets of items under consideration in the logic problems—a problem type in which Venn-like diagrams might have been helpful. Lines 7-11 of Bridget's initial description of Jack as a mathematics student addressed Jack's classroom mathematical activity:

> He thinks a lot, he stares at you, writes almost nothing, asks one or two questions, looks at the ceiling or puts his head down, and then he gives his answer. He will pursue an idea with greater persistence, he usually likes his own ideas and

> answers. When he is wrong, he accepts other students' opinions. He listens, thinks and makes his own conclusions.

Jack seemed to be operative, but not strong at creating figurative material with which to do mathematical work. And finally, Bridget's comments showed once again that Jack did mathematics in conjunction with others, listening and contributing.

Personal Epistemology

Ontology of mathematics. On the survey Jack completed just prior to our Phase II classroom interactions, Jack answered the prompt, "The ideas of mathematics: Have always been true and will always be true" by circling "Disagree." This statement reflected a position indicative of the GAML I though I experienced from Jack during Phase I; Jack had an orientation to mathematics that seemed to not grant it an eternal existence.

He replied to the next four phrases in the following way, "The ideas of mathematics:"

b. Were invented by mathematicians	Agree
c. Were discovered by mathematicians	Agree
d. Developed as people needed them in daily life	Agree
e. Have very little to do with the real world	Disagree

Jack circled "Agree" for both invented and discovered, as did Fisk, which again surprised me. I inquired during our first interview:

> B: There was one question I asked on [the survey] where I wrote, "The ideas of mathematics were invented by mathematicians. Do you agree or disagree?"
>
> J: Agree
>
> B: And then I wrote after that, "The ideas of mathematics were *discovered* [said with emphasis] by mathematicians."
>
> J: I think I said disagree.
>
> B: What you just said to me was "agree" with invented, but "disagree" with discovered. Why are you thinking that way?

J: Because invented is something that somebody created and discovered is finding something that somebody had already done for many years that we never knew about.

Jack's amended reply, and the reasoning he shared—by defining the terms—more strongly confirmed that Jack positions mathematics as a set of growing knowledge, not principles or axioms that have existed prior to any knower. Furthermore, Jack placed mathematics as a human endeavor, "something that somebody created." He positioned the idea of discover similar to Kate, in that what is discovered is still a discovery of some previous person's invention.

Next I sought to explore further whether Jack considered the invention of mathematics to be only for the realm of mathematicians, or for people more generally.

B: OK. So you're willing to say that mathematics was invented by mathematicians?

J: Yeah.

B: Do you ever invent mathematics?

J: I probably don't know personally, but I probably do without even knowing it. Everybody, probably in some way, like invents math and don't even know.

So, this second sentence demonstrated that Jack imagined that it could really be anyone who invented some new mathematics. Of note is how Jack emphasized how that inventor might not know they actually created something mathematical—himself for instance.

Usefulness of mathematics. Evident within this survey was an interesting divergence in some of the ways Jack saw himself as doing and learning mathematics. Jack completed the second question of the survey by *only* circling option B., which completed the sentence starter "Mathematics is something I do…" in this way: "mostly at school." The full survey question was,

2. *Circle one or more*
   Mathematics is something I do
   A. Everyday as a natural part of living
   B. Mostly at school
   C. With a pencil and paper
   D. Mostly in my head
   E. With numbers

It was reasonable to assume that Jack replied by selecting only one answer, the one he agreed to most strongly, given that most multiple selection questions presented in a schooling context are designed to have one correct answer, i.e. a multiple choice test. With this in mind, I didn't readily throw out the possibility that Jack may have agreed with other statements as well. But since he did circle "Mostly at school," I inferred that he did not see himself as mathematically active—doing, learning, or inventing—outside of his mathematics classroom. In fact, as presented in the previous section, Jack indicated in our first interview that he did not see himself *learning* mathematics outside of the classroom. Yet he recognized that he might actually—in fact probably—invent mathematics in the course of living. I didn't notice him consider this conflict further.

That he disagreed that "Mathematics have very little to do with the real world" continued to confirm a sort of unresolved dichotomy in Jack's orientation to the usefulness of mathematics. I concluded that he believed mathematics was both done and invented by people, in their everyday living. And I suspect, if pressed, he would agree to the same for himself. But when the question was approached not connected to who invents mathematics, Jack's mindset oriented him to consider "mathematics" as a class in school. This mindset toward mathematics involved learning facts and ideas that were disjoint from his daily living. School mathematics was distinct from experiences of living outside the classroom walls.

Locus of authority. Jack's orientation toward the invented quality of mathematics suggested that he would see himself as a mathematical author, and hence an authority for his knowing. However, previously presented interactions may have provided contradictory evidence. For example, on April 5 when I asked him to share an idea with the class, he immediate response was to ask me, “Is it right?” Such a question could be interpreted as possibly a concern toward sharing aloud to his classmates, a fear of being wrong. His question might also have marked surprise to the possibility that what he offered in private to me was surprisingly possibly correct. Considering Jack’s frequent verbal participation in classroom discussion, the second possibility seemed more likely, and consistent with other data.

That Jack saw that mathematics emerged for people as either an invented or discovered phenomenon suggested it would be likely he viewed himself as the arbiter for determining the viability of the mathematical ideas he invented or discovered. This conjecture is confirmed in observations of his classroom activity that has previously been noted, in particular Bridget’s comment on Jack’s manner of reasoning, the private decision making that involved no paper or pencil, simply his inward reflection.

Second-Order Viability

Jack more than any other of the subjects seemed to take notice, and possibly engaged in class differently, as a result of my expressed interest in him as a research subject. He wrote in a Personal Growth reflection at the end of a portfolio (Archived data: Exponents portfolio), “I have been doing so well that this student-teacher [me] wants to do a report on me because he likes the was I solve problems and the way I am thinking about problems.” He also wrote in his Mathography—which wasn’t completed until June, the beginning of Phase II—that, “You [again, me] influenced me to do math when you were here for a week, but I still don’t like math.”

More evidence of my influence on his classroom activity was present in some of Jack’s classroom behaviors the first week of Phase II of my classroom teaching. During this time, I observed Jack to be often engaged in class, willing to volunteer ideas and listen carefully to others. He seemed to want to check in with me frequently; this was especially evident during factoring activities on April 12 (Classroom video transcript). As time elapsed during Phase II, this early manic-ness of Jack's interactions toned down somewhat. Yet, his general demeanor and presence in the classroom did not change significantly.

Bridget reported no differences in how Jack acted, learned, or seemed to work on mathematics during Phase II as opposed to other times during the semester-long course. Along with the feedback she provided about Jack's mathematical activity and sense of self, I am confident that the disposition that emerged from the data collected was a reasonable representation for Jack as a mathematical learner.

Worth noting was that Bridget's openness to Jack as a learner—possibly as a different learner, but more certainly her simple belief in him—impacted Jack's belief in himself, and thus affected his classroom activity. Jack wrote about this impact in two places. First, in the reflection portion of his Exponents portfolio (Archived data, undated), "Since I been in the math class my teacher has confidence in me that I know what I am doing and can help others." And similarly, in a portfolio from the Quadratics unit (Archived data, 05/25/05), "My teacher also has a lot of confidence in me when I am doing math because she know I can do math. She like the way I do math that she wants me to share my ideas with other students in the class."

Relations with class members. Jack wrote his Mathography very much a series of direct and brief replies to the prompts suggested. The first one asked him to…

> Consider all the mathematics classes you have taken over the years. What did you do in those classes that helped you to learn? What happened in some of those classes that influenced or affected your learning? What happened in some classes that made it hard for you to learn?

He replied,

> I really did not learn anything because the classes that I had was full of bad kids and I was not able to learn. They were bad and the teacher could not control them, I was not able to understand also because I had teachers that were foreign and really couldn't speak English.

Jack reported in his Mathography that in previous years he didn't learn much when he was in a class of 'bad' kids and the teacher couldn't control them. He continued his response, apparently now focused on his Junior year math class:

> I had some good experiences working in groups because I learned some things faster and also I helped my fellow group members. It's good to work alone when you need to be alone and work by yourself. It's best to work with others when you have a hard problem and need some assistance.

It was evident Jack felt he learned when working with others, but that he also needed to work alone at times.

During my classroom data collection, Jack appeared to be respectful of all his classmates, tuning into any student's contributions to classroom discussions. He turned over their ideas, considered them if the idea contradicted his, and at times responded with his own ideas. For example, a group mate presented a sketch to the class to argue his idea that there were 13 lattice points inside or on the boundary of a circle of radius 3 units (field notes, 04/26/05). After this student returned to his desk, a member of Jack's group, Jack asked him why he did not count an additional lattice point directly north of the westernmost point. It was evident that Jack considered seriously his classmate's ideas—which were quite different from his own—and asked for his reasoning behind not including a particular point to be inside the circle.

On April 19, the class was summarizing and reviewing the quadratics unit. Bridget asked students to volunteer to record responses to the previous nights homework, in which they were instructed to summarize topics, terms, and techniques developed in the unit. Although Jack came to class with nothing—not even a bag—he did contribute several ideas to the board (field notes, 04/19/05). During this time, he engaged in mathematical conversations with other people recording to the board. As he returned to his seat, he remained attentive to the people and activity in the room. Jack saw himself as a member of a group of learners, caring to learn himself and help the learning of his classmates.

Self-concept of classroom activity. Jack seemed to detach from class work occasionally. As noted previously, much of his mathematical work was done in his head. Often times what appeared to be detachment from the classroom would be unforgettably concluded with a surprisingly correct or insightful connection to a topic being discussion or question asked. I asked him during the first interview about his perception of his activity in the classroom, in particular what prompted his beaks from intense mathematical work.

> B: At one point I noticed during the test, on your calculator screen you had a quadratic function graphed and it looked like you were tracing it. But I'd say for like 3 minutes you weren't doing much. When you're working in class, sometimes do you just take a break?
>
> J: Yeah, sometimes you just have to do it…. It's gonna be times when you gonna need a break, but sometimes you gotta keep on working. And that's the part where I had to do something about Alice [referring to a metaphor in the exponents unit], I had to take a break 'cause there was wearin' my mind [best guess on transcription] but everything else was cool though.

Although Jack often appeared disconnected from the classroom, much data discounted a lack of engagement, such as providing correct answers to teacher questions and appropriate inquiries related to the task at hand. He also engaged immediately in whole group interaction. While he perceived himself to be engaged in the mathematics of the classroom, he also recognized that at times he did disconnect. Interestingly he attributed the disconnection to needing a break from the difficult thinking as opposed to wanting to chat with friends.

As another example of the intensity of his classroom work, on April 19 students were working, optionally in collaboration with one another, on the task to convert the quadratic

$-16t^2 + 92t + 160$ to vertex form. I had earlier suggested to Jack that a first step that would greatly simplify the process would be to factor out a (–16). Jack continued to work, mostly by himself, using the calculator and little else, often appearing as though he were doing little. Once again, he confirmed the internal/operative nature of his mathematical activity; he exclaimed, “Man, this is hurting my brain.”

As I prepared for the first interview, I was intentional to follow-up on the distracted nature of the appearance of Jack’s classroom participation. I wanted to get at what might engage him in class, what might catch his attention. I was also curious to understand the degree to which he was aware of his distraction and reconnection to the classroom. During the interview, I asked specifically about what caught his attention.

B: What sorts of things catch your attention in math, in the class?

J: Like, basically like word problems and the algebra. Word problems get me.

B: Tell me a little more about that; you said they get you.

J: Like word problems, like in the math IMP, like they take easy algebra and word ’em harder into word problems, so instead of just put the algebra it’d be much easier if they didn’t have the word problem. So that’s why the word problem is a little bit hard.

B: Do they catch your attention because they’re harder or because it takes more work for you to figure out?

J: It takes a lot of work for me to figure it out.

To this point, I learned that Jack seemed to find the challenge of figuring out the difficult word problems to “catch his attention,” or engage his mind during math class. Again, the prompt seemed open to socially oriented responses, yet Jack again focused on mathematics. I still wanted to pursue the manner in which he seemed to work.

> B: Are there times when you've been daydreaming and then you'll tune back in?
>
> J: There's plenty of times when I been daydreaming. Then like sometimes I daydream and I come right back in and I answer a question and then I'm right and I'm like, how was I right?

Jack seemed to be aware that although he disconnected from what is going on, he somehow was not so disconnected because he provided solutions to questions asked. This quality surprised him. In fact, he talked more about this surprise as the interview continued.

> B: Do you think your mind was working while you were daydreaming?
>
> J: Maybe, probably was.

Jack had not come to a rationalization about how these sudden knowing came to be, he surprised himself with how he knew and answered questions correctly.

During the interview, I also brought up a classroom moment when he may have had one of the spontaneous knowings he described in the previous protocol.

> B: I noticed two days ago there was a 'Do Now' up, and the question was which of these is a perfect square. I noticed that during the 5 minutes of getting started, you and [Sam] were mostly talking…. As soon as [Bridget] asked the class you responded with an answer. Was that an example of how you were just saying, 'I listen when the teacher asks a question, sometimes I can just answer right away.'
>
> J: That was I reading the problem. I read the problem, looked at it, analyzed it some, and started talking to [Sam]—but it wasn't about the problem. I just started talking. And then when she asked me, I knew that that was the answer.

The situation of this protocol was different from one of spontaneous knowing, although from my perspective as a classroom observer, seemed to yield similar classroom behavior. In this case, Jack had thought about and come to a conclusion about the problem posed.

However, another interesting twist was raised in his languaging the final sentence. Jack might have intended to mean that because he had already thought about the question, he knew that he had the answer. However, he may also have meant that his interpretation of the teacher's asking him to share his solution was that he knew that his answer must be correct. This would indicate he deemed a sort of omnipotent status upon the teacher. I don't find this possibility to be far-reaching, given that Jack apparently was willing to attribute some of his knowing to a sudden and miraculous awareness.

Mathematical ego. Earlier in presenting Jack, I provided teacher and self-assessment about Jack being "lazy", or as Bridget stated, "He doesn't work hard." In his Mathography, Jack wrote of himself, "I can be good in math if I really put my mind to it, but at times I can be lazy and not fully apply myself. I know because there are some problems I can do real fast but others I just don't feel like doing." In the Subject Survey, the first question was score yourself on a 0 to 10 scale to report "how good I think I am at math." Jack recorded a 7. My interpretation of a 7 would suggest he perceived himself to be not great, but not so bad. A score of 5, which might be thought of as so-so, carries the connotation of a failing score, so I initially assumed Jack's self report to be something similar to a self-evaluation of "so-so." Based upon other interactions, and the next response on the Survey in which he reported math to be something done at school, I suspected this was a self-evaluation of how he does in math class, not a perception of himself as a mathematical person. Other data provided deeper insight into this sense of himself, as a mathematical do-er, creator, inventor.

As a hint to this, he wrote he could *do* math when he put his mind to it. He also indicated that his teacher, Bridget, believed in him as a mathematical thinker. On April 11th, I recorded to my field note's portions of an interaction with Jack that reflected many classroom episodes in which he was engaged more on his own as a learner. Students of the class were working on a task to graph a quadratic function, given in vertex form—although students were not yet aware of the value of this form. They were asked to find the vertex for the graph, and explain how they could be certain that they had the exact vertex. Jack was making good progress on the task, utilizing his graphing calculator. I wrote,

> On one problem he [Jack] indicated to me was that the graph was too high for a vertex. I suspected he was thinking about *x*-intercepts.

I suspected this confusion because the first two problems students looked at were $y = (x-2)^2$ and $y = (x+4)^2$, in which for both cases the vertex was the same as the *x*-intercept (see Figure 6). The third problem—the one that prompted Jack's question—was the function $y = (x-3)^2 + 2$. This function did not have any *x*-intercepts, hence Jack's initial response that it "was too high for a vertex."

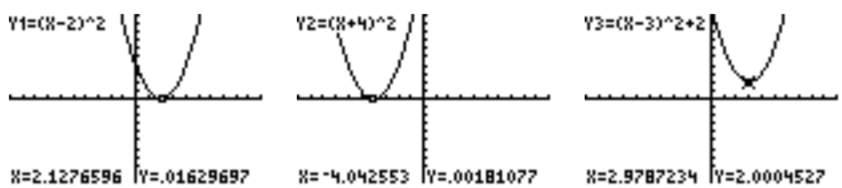

Figure 6: Possible calculator screen images of student work on the given quadratic functions.

> My field notes continued,
>
> I reminded him that no matter where a parabola was graphed, it had a bottom-most [or top-most] point. He recognized this quickly and returned to his work. When he got to question #3, he seemed stuck.

Question #3 asked students to, "Explain how you could find the vertices for functions like those in questions 1 and 2 without graphing."

> I encouraged him to notice the results he had found for the vertex, and to look back at the numbers in the equation written in the book. He looked some. Then, using his calculator, I pointed out that by zooming in on the graph of $y = 3(x+1)^2 + 16$, the vertex appeared closer to (-1, 16) than the estimate he had recorded. The notion of zooming on the calculator seemed like a strategy he hadn't considered, but seemed to make sense to him. He said, 'So where did I get my answer from?' I indicated that his was just a rougher estimate, and he seemed comfortable with that. I say comfortable because Jack will respond not just to satisfy the teacher, but understanding seems to be central to him—what I mean is that he's not one to nod to satisfy the teacher, or if he does, it is apparent to me when he is still thinking about something that was said vs. when he understands.

> It's in his body language. Shortly after, he said he saw the answer to question #3. I didn't check with him if it was correct, but I did see that he had decided for himself he had found a solution.

The description of Jack's activity that I recorded on this day indicated that he was engaged in mathematical exploration. The point of emphasis was to demonstrate an example of when he seemed to decide that he understood, or knew something. I identified a distinction between when he understood what I had to say, and when he resolved the mathematical question he was working on. In this manner, his location of mathematical authority leaned inward. In this, and some further examples, it was evident that he solicits input from others, but did not assume this input to be truths—he must own the understanding for himself. This protocol confirms a previous notion that Jack seemed to locate mathematical authority within himself.

I wrote in my field notes at the conclusion of this period, one that I perceived to be a day in which Jack felt like he learned something, and that he made sense of it, that, "I seem to recall Jack saying in class, right near the end, that he was good at math."

Viability

Status. Jack has a moderate to good social and academic status in the classroom. That everyone knew his name, or nickname "Fridge", was a start. In many high school classrooms, not all students' names are known by their peers. It was evident in the classroom video transcripts that Jack spoke regularly with a wider-than-typical portion of his classroom peers. Jack was a consistent member of his small, collaborative group. He contributed and they looked to him for contributions. During the first 2 weeks of my teaching during Phase II, Jack sat with two boys. Although I noted that there was not much interaction (field notes, 04/06/05), they did work well together; there was a sort of equality in the playing field. One student listened more to the others, but insisted things made sense. A second tended to act as a recorder—as it was clearly not a strength that Jack contributed. I asked Jack a question to check his understanding and the group's conclusions. As Jack spoke, this second student nodded, showing that he agreed with Jack's mathematical reasoning.

He listened as intently to his peers as his teacher for mathematical ideas and argument. Bridget reported, "When he is wrong, he accepts other students' opinions" (Subject Identification Survey). I took this to mean that she perceived Jack to be open to being convinced by his peers, regarding their mathematical opinions as important. He also shared materials, including a borrowed eraser during the mid-term test on Apr 13. This care for the welfare of his fellow students extended in many other circumstances, getting paper or Kleenex for others whenever it was that he needed some for himself. On April 7, Jack showed a female classmate something about the use of the calculator and on April 12.

One status play seemed to occur during my teaching and observations of Phase II. On April 11, one week into our work together, Jack asked for me to show him how to multiply through $(x-2)^2$ (field notes, 04/11/05). I noticed this gathered attention from several nearby students so I responded more publicly than I usually would have. I later recorded in my field notes that as a showed and carefully recorded steps of this procedure, Jack was confirming what he already knew. He replied to my help with a comment indicating not that he already knew how to answer the question, but that he asked because asking questions like this was what students were supposed to do. Recall that the previous I believed that Jack's classroom activity appeared slightly hyper because he was aware that I was interested in his mathematical activity. In this episode, I suspect his internal awareness that he was being watched continued—and thus sought out the attention. However, in the context of status with his peers, in this school it was not the coolest of things to be friendly with or reliant upon the teacher. And as such, I conclude that his comment regarding the asking of questions was made to slightly offset negative impact on his posture with classmates.

The classroom was a very social place for Jack. Although not outgoing in an eccentric manner, Jack's interactive, social, friendly, and helpful nature was well received by his peers. Additionally, people respected him mathematically. It was not that his peers perceived him to be gifted or unique in any way; but no one seemed to think of him as a dummy, dismissing his ideas.

Classroom behavior. Jack was fortunate to have Bridget as his classroom teacher. It was clear that she observed Jack to be doing a lot of mathematical activity, even if he recorded little to paper and other behaviors suggested otherwise. She had described his work in the Student Identification Survey as, "He thinks a lot, he stares at you, writes almost nothing, asks one or two questions, looks at the ceiling or puts his head down, and then he gives his answer." During our class debrief on April 15, Bridget and I discussed an episode in class where Jack noted it was very hard to convert a quadratic in standard form "backwards" to one written in vertex form. I recorded to my field notes, as she was describing her observations of Jack's work and her interactions with him,

> Bridget showed me that Jack would write $x^2 - 6x + 4$. Then he noticed in [a previous problem] and saw that the term inside the parentheses was always half the *b* coefficient. He then wrote $(x-3)^2 - 5$. [She told me] All the work is done in his head, writing nothing on paper. He stated, 'You need half of this, then you need next something to get the perfect square.' She asked him, 'How do you know?' He stated, 'I don't know, I'm asking you!'
>
> Bridget followed with a comment [to me] indicating that no matter what she said, he seemed to always lean back in his chair, and decide for himself whether or not he was correct. She characterized that much of his work was this sitting back and thinking. 'And he won't just trust the teacher. He sits back and thinks about it.'

Bridget recognized Jack's mathematical activity, even when there was little he seemed to be generating in terms of written work, or if he looked like he might not be engaged in work at all.

Jack also had a side of him that was not what might be considered to be an ideal student in general. Recall Bridget's characterizations of his activity in the Student Identification Survey:

> He is absentminded. He comes to class without even a pencil at least twice a week. He wastes a lot of time in class. He turns his work in late most of the times. He is not worried about his grades. He just wants to pass, and have some fun.

He usually got up to wander around the room 1 to 2 times per day. Given that the classes were 80 minutes long, this is not an unusual need for a person. But for a teacher, certain elements of Jack's behaviors could easily get under one's skin. For example, he usually came with little or no materials: pencils, books, or completed homework. These walks were often to collect a pencil from someone, or to move to work with a new group. Because of his size, there was no missing his movement. However, a little more unnerving was his distracted and distracting conversations with peers, as he would become disengaged or not begin class promptly. Bridget regularly used an activity at the start of the bell called a "Do Now." This was meant as a classroom management tool to get students started immediately, and also as review, as foreshadow of the day's lesson, or as test practice. Typically, Jack did not arrive to class, and sit quietly in his desk to work on this individual task. April 12 was a typical start to class for Jack. I recorded to my field notes that during the first five minutes of class he was chuckling with a neighboring student. As Bridget begins a whole class debrief of the Do Now problem, the classroom video on this date showed Jack shouting out a reply (8:00).

The data collection I relied on Jack for was also very difficult to get him to complete. I received several permission forms significantly later than I requested of him. As previously noted, I had to ask him to complete the Mathography—hopefully to be submitted by the end of the week following my invitation to participate in the study during Phase I—during his class time during Phase III. The Subject Survey, also given during Phase I, was completed in class on the second day of Phase II. His Videography was incomplete; in fact the tape he submitted was of friends introducing themselves. And he missed several scheduled interviews.

Grading Jack. As mentioned above, Jack rarely came to class with homework. Although daily completion of homework did not directly affect his course grade, he also failed to turn in the larger, outside of class assignments as well. However, he eventually did turn in missing work, certainly in part as a result of Bridget's prodding. Most all of his work, including the work completed outside of class and tests he took in class, was scored between a mid-C and a mid-B (Artifact: Progress Report). Jack's write-up of the rat problem, shared earlier, had the only score outside this range—a 50%, due in parts for inadequate explanation, incomplete investigation and solution, incomplete write-up, and lateness in submission.

Jack did attend class regularly. And when in class, he was usually on task—even when seemingly daydreaming, as described previously. Jack was not a paper-oriented, or written-communication oriented person. He would do enough of this, however, to ensure he passed the course. Although most of his assignments asked for explanations of solutions or conclusions, this was usually an ignored part of his replies. The scoring system Bridget utilized gave Jack room to perform in class, be engaged as he was, and submit written work and tests and pass the class, while having fun. This seemed to match Jack's own goals for his mathematics class.

Summary of Claims

Oral communicator, operative reasoner. Jack's mathematical activity was most strongly distinguished by the oral nature of his work. His mathematics was almost singularly operative; Jack reasoned with constructions of his mind, he did not seem to create or work productively from representations. His knowing was also intuitive. Jack seemed to either have little conscious access to the rationalized or reasoned qualities of his mathematical knowing. Or, he did not draw upon these potentials to satisfy his marked confidence in knowing. Jack's mathematical activities were also strongly interactive. He listened exceptionally well to others for ideas, confirmation of or contradiction to his own ideas, and out of sheer interest.

Constructed knowing. Jack did not see mathematics as an a priori body of truths for which his purpose was to come to know. The learning of mathematics happened as he engaged in thinking and creating understanding. The ideas of mathematics were invented, and then frequently rediscovered; invented *and* discovered by people, including both mathematicians and himself. Jack had a dichotomous orientation to where he located mathematics; he identified it both as a regular part of living, yet quite sharply saw it as what he did in his math classroom. Jack was himself the arbiter of mathematical truth. That he perceived himself as such was not apparent in the data.

Interactive. Jack valued working with others not only to solve hard problems, but also to be able to test his own knowing. And reflexively, he valued being able to draw upon others to cause him to reconsider his own conclusions. He saw his mathematical activity as thinking hard, and needed to occasionally rest his mind. Jack was certainly confident his ability as a thinker, and his ability to solve problems as given—provided he was not too lazy to complete the task. His self-evaluation of his performance in mathematics reflected that he did not always do all he could to demonstrate all he was able to do.

Unconventional. Jack was not an ideal student; it would be easy to wish to get Jack to stay organized, keep track of papers, write down his homework assignments, turn things in on time, and not talk off topic or move around in the classroom at inappropriate times. Missing and incomplete work—mostly for being left unjustified—kept Jack's class grades down lower than he might have achieved. His classroom behavior also did not appear to be that of an ideal student. He often appeared disengaged or uncaring about the topics being studied. However, this perception was repeatedly proven to be an incorrect conclusion; Jack did pay close attention to the mathematical activity of the classroom most every day. He had a tremendous respect for the ideas of his classmates, evident by his constant awareness of other's ideas and thinking. The egalitarian quality of his perceptions of his classmates' mathematical power was reflected back to him in the appreciation and respect shown him as a mathematical thinker. Jack possessed both a good academic and social status in the classroom. He was well liked, by both his peers and the adults that knew him.

Informing the Notion of the GAML

My interactions with Jack did not locate him toward either of the invent/discover poles that help to define an orientation toward the nature of knowledge. Because he believed that someone invented mathematical ideas and then other people came along and might (re-)discover them, my thoughts are to lean Jack toward the positive y-axis. None of the three subjects made a clear demarcation about toward the nature of knowledge along these poles. I conjecture that the nomenclature I initially chose may be inadequate to distinguish potential variations in orientation to knowledge.

Jack most certainly had a pull toward the positive x-axis in his disposition toward knowledge, his locus of authority. It was strongly evident that Jack's confidence in knowing was always put to test with the other mathematical minds in his classroom. And again, some of his actions provided indicators that he may give potential authority to others, such as the teacher especially, he still used the other's feedback as material for further thinking, rather than submit to the differing feedback as accepted truths. Jack's leanings toward mathematical knowledge and locus of authority confirmed the initial selection of not only a student that was a generative adolescent mathematical learner, but also saw himself as such.

Jack was not mathematically talented in a way that distinguished his performance from that of classmates. This predicted notion has not been challenged by evidence from any of the subjects in this investigation. What has been interesting is that mathematics as a highly rational way of knowing—in that proof is central to what constitutes truth and fact—was not valued by Jack. He didn't rely on the rationalization of error, or of different conclusions, in order to be swayed. He reacted to people's differing ideas. He would turn each over for himself to determine if it contradicted his knowing. Further, he did not demonstrate a valuation of the need to justify his mathematical conclusions or ideas. He was quite satisfied with an intuitive orientation, a "because I know" validity.

Jack's classroom activity as well as his behavior more general as a student did not meet the traditional description or expectations of a model student. Jack seemed to flourish because his teacher recognized him as mathematical, and granted a great flexibility to invite him to participate in the classroom community.

While both Fisk and Kate seemed to demonstrate a fragility in their sense of their own mathematical selves, Jack seemed to demonstrate no such fragility. If anything, he demonstrated a much stronger disconnect between needing to be correct, or valid in his reasoning. If anything, he pursued knowing because he wanted to know, not as a protection for his self-concept as someone who was smart. This is quite a distinction from Fisk and Kate.

But very similar to both Fisk and Kate was the role others played in Jack's ways of knowing. It was very important to him to listen to how others thought about things. Jack's purpose leaned toward seeing how other's thought, as it seemed Fisk did. Kate seemed to need this interaction to confirm her own ways of thinking and solution to problems.

CHAPTER 7

SUMMARY AND CONCLUSION

This chapter is meant to summarize the findings of the research project, and to present an updated and abstracted model of the generative adolescent mathematical learner.

Enriched Understanding of the Theoretical GAML

One component in achieving the goals of this research was to enrich my working definition of the Generative Adolescent Mathematical Learner (GAML). In previous chapters I outlined my theories on qualities of knowing and learning, and on the nature of knowledge, which provided some initial meaning for the GAML situated within theoretical framework I reflect during this research. I cannot conceive of knowing apart from a constructing person. For me, this means any body of knowledge deemed mathematical (i.e. "mathematics") must be a constructed—or a fabricated—thing. The truth of such a knowledge is not prior to the knower, it is wrapped up in the knower and their existence in relation to others. So for me, mathematics is not discovered, it is generated. The process of learning, what we attribute to be the evolution of the rules governing our own ways of knowing the world, is something that all people are doing, at all times. We are constantly re-working our knowledge.

I take care to restate these notions of mathematics as a fabricated knowledge, and learning as inherently generative, in order to restate my operating principle that all adolescents would be generative adolescent mathematical learners. But this is certainly how *I see* them. This research was guided by a curiosity in high school students who may *see themselves* as generative mathematical learners. Initially in this research, I set forth to characterize the GAML as a student who perceived herself to be mathematically generative.[89] At the outset, this meant little more than showing a disposition that seemed to match my own perception of how all people exist in relation to ways knowing and learning, and in particular knowledge. Through the study, I hoped to sharpen my ideas about a sort of definition for the GAML. It was not my goal to lock into place *the* definition of such a person. Such a goal would not be in accordance with my postmodern orientation on *being*. Instead, I sought to explore the ways in which the GAML—a person who saw themselves as generators of mathematics—functioned in the school environment. To do so, I taught mathematics to my research subjects. Through this process of teaching, I investigated my subjects' construction of experiences that may have informed this disposition, their orientation toward mathematical knowledge, and the role of schooling in the production of these students.

The focus question of this study was: How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition? I devised the following additional questions to help orient my data collection toward further understanding the primary query:

1. What practices has a generative student created for engaging in mathematical activity?
2. What conceptions of mathematics has a generative student formed?
3. What does a generative student consider to have influenced her disposition?
4. How do generative students perceive the role of themselves in relations with others?
5. What are the relations among generative students and school discourses?

With these as guides, I now return to the summaries of my experiences with each of the GAMLs in my study to explore facets that emerged through my analysis of the data. These facets expand

[89] Recall Figure 1, in which I use the ideas *receiving* versus *producing* knowledge, and *discovering* versus *inventing* mathematics, to aid a description of tendencies in which a person might see themselves as a learner: repeater, constructor, sense-maker, or generator.

my thinking about my notions of the GAML. In the remainder of this chapter, and the dissertation, I intend to interrogate the analysis that has been presented so far. I will seek to identify both trends and interesting oddities, “singularities” (Ackermann, 1991b). I will present analysis that may further a definition, but also help to think about the self-aware GAML as a goal for mathematics education. I conclude the study with a return to the overall direction associated with this research program, to consider what the impact of considering the notion of generative adolescent mathematical learners may have upon a just mathematics education.

The GAML in the Mathematics Learning Environment

The three subjects of my study worked in some common ways in the mathematics classroom. All seemed to need some private time, whether to investigate for oneself, or to make sense of another’s ideas, to make mathematical meaning for themselves. All clearly valued the interaction with others in the room, in order to gage the accuracy of their own thinking, or to see how someone else may be thinking about something. Jack, and to some degree Fisk, demonstrated a strong ability to think with others about a mathematical problem. In this manner, they were not so focused on determining correctness, but it was more of an exploratory activity. If the intent of the interaction was to be more conclusive, Jack and Fisk saw the interaction as not the final step – they still needed to make sense for themselves.

Making sense for oneself was a common thread for all of my subjects; all saw logical thought as what validated mathematical ideas as opposed to an outside authority (Grouws et al., 1996). Because each subject believed that all people might have a different way of understanding or knowing, it simply made sense to them to be responsible to know for oneself. Few expressed a feeling or pressure to know someone else’s way of knowing, such as needing to replicate a teacher’s method or “the right way to do something.” To varying degrees, each subject did look to the teacher or to peers as having a sort of veto power over their own conclusions. If someone else presented a different answer, the GAML had to reason through that to make sense of the other answer or method, and then possibly have to reevaluate their own. Fisk took pride in thinking differently, yet this different thinking did not always arrive at the same conclusion, or follow a logic of conventional mathematics. To some extent, this was not too problematic for Fisk. Jack's teacher, Bridget, strongly pointed out that he did not defer to the teacher; he always had to decide for himself.

While each of my subjects did value mathematical interaction, each had varied relationships with the teacher, both as a mathematical authority (discussed above) and as a contributor to self-esteem. Having someone interested in them, or showing a confidence in their ability (as Jack indicated his teacher did this year) brought out much more mathematical activity. This suggested to me, that while the GAML may appear to be individualistic and highly self-confident, the role of others was valuable for them. Others, and I suspect in particular someone with an authority status of some sort, like a teacher, supplied a needed confirmation. Jack was almost needy—the type of student who would call the teacher over again and again. Fisk was not so outwardly needy, but thrived when teacher interaction assumed him to be smart and well intentioned, rather than inept or misguided. Kate needed someone who could both be invisible, but guide her and confirm her ways of thinking. The social confirmation and locus of authority I spoke about in the previous sentences characterized each subject, in an order that I considered to be most independent to least (although I consider each of the subjects to tend toward independence in relations to classmates).

It is clear that each GAML experienced confirmation in two ways, and that both were very important. First was a mathematical confirmation from within themselves, that they strongly had to confirm their own knowing. Second was an external confirmation that came from the teacher. This confirmation had les to do with communicating a mathematical correctness, but instead was to confirm that the GAML was a capable and competent mathematical learner. Later as I discuss school grading, it was interesting to note that the degree to which each subject focused on grades was indirectly proportional to the subject's need for confirmation of their knowing in this teacher interaction.

The GAML and Characterizing a Mathematics Knowledge

Related to conclusions about the social interaction with the classroom teacher were conclusions about where the GAML placed the authority for knowing mathematics. Each subject leaned strongly toward their own logical thought as being the source for validating mathematical ideas. However, each subject suggested different and interesting ways to consider the ontological existence of mathematics, and also expressed differing degrees of confidence in the ideas shared on the topic. Most tended toward a reply that something about mathematics, like numbers, have always existed. Jack was the only subject that consistently indicated that he had a highly people-centric notion of mathematical knowledge. He indicated that he and probably everyone are always inventing mathematics in some way. While Jack seemed to not even pause on the notion that mathematics didn't have an a priori existence, other subjects did—and most wrestled somewhat with this idea. For example, Kate emphasized that someone had to originally work out some idea, i.e. "invent", and then others might reinvent, or discover later. However, neither subject presented this position as unquestioningly as Jack. Fisk, interestingly, was the most certain of an a priori existence to mathematics. Yet even his comments suggest that mathematics is created by people from some sort of raw material, like a sand castle is made from sand.

What stood out was that each GAML placed an important role for people in relations to mathematical knowledge. Each replied that it was people who created mathematics. This stood in opposition to Erdös' belief that god invented mathematics, and that it was for man to discover her book of truths (Schechter, 1998). This highly people-oriented perspective on mathematical knowledge is an additional interesting notion these subjects suggested about the GAML. Not only did they see the role of people creating knowledge, they consistently reported that each person had different ways of thinking about or knowing things. This orientation was almost a badge of honor, a place of being an individual thinker amidst others. Fisk and Jack especially demonstrated a heightened awareness and pride in their unique ways of thinking and knowing. I believe each subject demonstrated Belenky et al.'s (1986) "Connected Knowing" in which the individual considered her knowledge directly or indirectly in interaction with people, in a process that depended on understanding others' experiences, perspectives, and reasoning.

That mathematical ways of knowing were so people-centric to the GAMLs, it was not surprising each subject also communicated consistently about mathematics as being a part of daily activity. None of the subjects isolated mathematical activity to the classroom or school, although most recognized, to some degree, a distinction between their mathematical activity in and out of school. Each subject demonstrated an awareness of engaging in mathematical activity outside the classroom—beyond simplistic answers such as calculating change or balancing a checkbook. Fisk gave specific examples of using mathematics, including simply to use his cell phone! His, and other's, examples indicated that they treated mathematics as mathematical activity, rather than as a set of static entities, things to be known. Kate spoke in her videography about how when her mind might be otherwise at rest, she was engaged in mathematical activity, some of it unconscious and some more in her awareness. For example, she spoke about being compelled to walk the shortest distance. She discussed counting steps and stairs; and wondering how far away something may be. She noted each of these ideas was related to rate of change—showing an experiential understanding of mathematical topics she studied. None of the subjects treated mathematical knowledge as a collection of isolated pieces; facts, formulas, and algorithms (Grouws et al., 1996). Jack had the greatest split in his identification of mathematics, as what was studied in the classroom, and the possibility of mathematics being a part of daily activity. He did keep a distinction in mind, but it seemed to be a practical one—a way to separate school life from his experiences out of the classroom.

GAML's Perception of Self as a Mathematical Learner

Each subject saw themselves as operating at times as an author of mathematics, and at times as a reader. They might need to "read" another person's way of thinking about an idea, and make sense of this. Other times, they are authored their own mathematical ways of knowing. No GAML spoke of the process of creating meaning for the other person's idea as a process of authoring. This would be an indicator, to me as the researcher, that the subject positioned themselves in the radical ontological stand suggested in my definition of the GAML, as one who understood because they were the inventor (paraphrasing Piaget [1973]). Although not surprised that I did not come across a student who possessed this orientation about their own learning—that even in trying to understand another, they were creating a new knowing—I continue to wonder if any adolescent may have such a self-concept of their relation to knowledge or learning; a consideration of the second-orderedness to their knowing of the knowing of others.

The subjects of the study possessed a confidence in their ability to do mathematics. This notion tied together with what they considered to be mathematical—numerical, spatial, and logical thinking activity, as opposed to memorization of facts—and the nature of knowing, determining for oneself. The internal locus for the nature of knowing and the nature of activity likely allowed for a high degree of confidence, in that it didn't overly value the mathematical confirmation of some external authority, whether than be a teacher, peer, or text. Several subjects went a bit further and indicated they enjoyed the challenge presented by mathematics. Kate discussed zoning out unless something really challenged her, and that she often found this challenge in figuring out mathematical problems. Jack expressed a similar enjoyment of the challenge, even stating the need to rest his mind sometimes, because it was working so hard.

The GAMLs of my study treated mathematical ways of knowing as internally determined. This point of view also appeared to be related to a sort of ego safety net. While Jack seemed to have a belief that he could do seemingly anything he set his mind to, other subjects had a more fragile confidence. Fisk almost didn't want to hear he was incorrect. Kate certainly didn't, nor was she thrilled about beginning problems anew or beginning new problems. Kate didn't like the uncertainty associated with not knowing. Kate also seemed to believe that her social status was related to the mathematical ability she was perceived to have. This put a high price on knowing/not knowing for her, creating a sort of dangerous gamble. Should she engage in exploratory mathematical activity, that which involved uncertainty? If she demonstrated understanding, her social feedback was positive. But such exploration risked not understanding or appearing smart, possibly losing status.

The Social Relations of the GAML

All of the GAMLs had a good social status among peers. None were outcasts or loners, none were thought of as stupid. In fact, all were academically respected to some degree, if not for getting good grades, at least for being able to do mathematics well (maybe just lazy, e.g. Jack). Each GAML also demonstrated positive relationships with their peers. Other students in the room treated them respectfully and considered seriously the contributions they made to the mathematical interaction of the classroom. The GAML valued the ideas of other students, as well as the teacher's ideas, when doing mathematics. None expressed a frustration about discussion. Kate paid close attention to conversations, yet in an almost secretive manner, keeping one ear tuned to it while engaging in what she thought to be status-building activity. Jack most certainly paid attention to the mathematical conversations of the room. When whole group discussions began, he was always ready to contribute, having considered Bridget's question or worked on the given task.

With regards to the GAMLs' perception of themselves in relation to other's knowledge building, they seemed to believe others learned from them. Each did demonstrate a modesty about this, except Fisk. He was the only to say that maybe he was an author more than others, who spent more time as readers. Recall, Fisk took some pride in being identified as someone who thinks differently from others. This seemed to be one of the few moments any of the GAMLs set themselves apart, made their ways of knowing, distinct from their peers.

The GAML and School Structures

All three subjects rejected school structures in differing ways. Common to each was that this rejection was about the way it influenced their concept of themselves as learners and as capable thinkers. This rejection was not about refusal to participate or accept the importance to earn passing grades. It was a rejection to subsume their selves, their identities, to the perceived norm for being a subjugated knower. None of the subjects of this study were singularly motivated by achieving good grades. Kate certainly paid the most attention to meeting the teacher's Some high school students, especially by the time they are juniors, are driven by a "just tell me what I need to do to get a good grade" mentality. The subjects of this study all demonstrated a desire to understand the mathematics. Kate believed this would translate to earning a good grade. Jack seemed to just find a way to squeak by with grades. He would complete just enough work that, coupled with the consistent oral demonstration of understanding, earned him passing grades. Fisk seemed to reject any concerns of grades. He indicated during Phase III of the research that he didn't expect to pass due to the long-term sub and him not getting along, but that he didn't care because he could get the credit in an easy, extra-curricular program.

Kate was the most antagonistic toward external authority. She demonstrated some defiance in her classroom actions, namely not engaging in mathematical tasks when asked by the teacher, and talking (even singing) during class work. When I asked her about this, she seemed somewhat unaware of her activity, and expressed that she knew it was wrong and should improve upon it. Jack, although not antagonistic, seemed to disregard some of these same classroom norms as well. His attitude seemed to be much more of a "you can't really make me unhappy" perspective, as opposed to an egoist demand for individuality. Fisk treated classmates almost on an equal level as the teacher when it came to influence of mathematical ways of knowing and proper classroom behavior. During mathematical discussion, he would attend equally to other students as the teacher. He was one of the few students in the classroom that would interact directly with other students during large group discussion. Jack participated during large group discussion; Kate listened more and interacted less.

Fisk did not engage in the defined student role in the social game of school. He did not acquiesce to institutional standards for operating, including socializing often during class, not completing class work by due dates, or taking out his work. I observed entire 80-minute periods where he did not remove his backpack. One day, after arriving late to class, he sat at the front of the room facing the class with his backpack on for almost the entire period. He opened it at one point to take out his calculator. Even more interestingly, the teacher was leading a midterm test review session on this day. Occasionally Fisk would turn to the board during the whole class discussion. I asked him about this later. He stated he understood everything they were talking about, checking the board occasionally to make sure. He took out his calculator to make sure he was getting the same answer as others during the review. Although he was not playing school in the expected way, he was engaged and having some sort of success in his activity. Other subjects acted in similar ways in the classroom, finding nonstandard ways to participate in school. The two teachers with whom I co-taught, Bridget and Larry, were more open to this classroom behavior than many of my subject's previous teachers at PHS.

Each of the GAMLs in this study had quirks in their role as a student, in their relationship to the school structures. Each also demonstrated interesting qualities in the perception of themselves as mathematical learners. Through my teaching of these generative adolescent mathematical learners, data collection and analysis, and inscription of ideas about these learners—experiencing Richardson and St. Pierre's "writing as a method of inquiry" (2005)—I have greater understanding of the initial concept of the GAML. More importantly, I have an improved sense on how they may have maintained their relationship to mathematic learning through schooling, and what these insights might tell us about the practices of mathematics education and mathematics education research. Through my discussion of my experiences of these subjects, I hoped to contribute and energize a database about the adolescent mathematical learner, intending to vivify (Lather, 1991) and trouble mathematics education's goals for the productive learner, for a learner with an agency and identity as a mathematical author and authority.

The Abstracted GAML

Having returned to the initial series of research questions that were designed to further consider the GAML's activity as a mathematical learner, a mathematically social being, and a high school student, I will conclude the data analysis with an effort to create an updated model of the GAML, abstracted from my experiences with the three research subjects. Initially, I proposed a framework to help orient the reader to what I considered to be a generative disposition toward mathematical learning. I will recreate that here, with minor modifications based upon the conversations I had with the students of the study. I will also review the initial description of the GAML that I offered to the teachers at PHS when asking for their thoughts to help identify candidates for the research.

This section will conclude with the recreation of a model for the GAML, emergent from the experiences of this research. I will orient the reporting of this abstracted model around the principals used in analyzing each subject, namely: Mathematical Activity, Personal Epistemology, Second-Order Viability, and Viability in the Mathematics Classroom. I will conclude with a discussion of the GAMLs' mathematical ego—the subjects' consciousness of their own identity as a mathematical learner—what I claimed to be a key indicator that separated the second-level GAML from the first-level, to which I attribute all learners.

Ways to Theorize Mathematical Learning

I initially constructed this model with the intent for it to be useful in communicating with others about what it might mean to be a generative mathematical learner. I identified two scales upon which the *nature of knowledge* and *disposition toward knowledge* were meant to be considered for oneself as a knower. During the course of the research, the model was both used in conversations to communicate the idea of what generative might mean, but also to formulate and design the data collection activity. It is worth noting that many of the words of Figure 1—discover, invent, produce, receive, repeat, make sense, construct, generate—carried different meanings for many people. By no means could I match the personalized meanings for all readers, but through descriptions evident in my text above, or conversations during the early Phases of my research, I found it to be a useful talking point.

It is worth a brief note that the word "construct" carries similar, yet significantly distinct meanings among mathematics educators. I chose to use it in its more common convention, although a radical constructivist would object. The realm of generate, the first quadrant of Figure 1, is likely a better match to their orientation. I found correlating usage of my word choice in the work of two French-Canadian constructivists, Larochelle and Désautels. "A subject must be conceptualized as a *producer*, and not simply a reproducer of phenomena" (1991, p. 375). Also,

> Radical constructivism itself approaches an essentially undecidable question as though it were decidable, namely whether we are 'discoverers' (in which case, according to Foerster, we are looking as through a peephole upon an unfolding universe) or whether we are the 'inventors' (in which case we see ourselves as the participants in a conspiracy for which we are continually inventing the customs, rules, and regulations). Radical constructivism does indeed take a position and opts for the latter view. (Larochelle, 2000, p. 61)

The intersection of the orientations toward knowledge and learning suggested in these two names, producer and inventor, began my framework toward considering what might be a generative learner.

Initial edits. However, through my work I asked each of my subjects their notions on whether or not mathematics was an invented or discovered idea. I found that each wrestled with the idea, and the answers provided spoke to a more practical status of knowledge, and didn't wrestle with a metaphysical statement about the existence of mathematical knowledge. Given these responses, I determined that the ideas *invent* and *discover* carried meanings that were more dependent upon a person's thoughts about the nature of knowledge than they were insightful into what they perceived to be the nature of knowledge. For this reason, I modified the figure to reclassify the poles of the y-axis to be *a priori* or *a posteriori*. The terms "a priori" and "a posteriori" refer primarily to how or on what basis a proposition might be known. A proposition is knowable a priori if it is knowable independently of experience. A proposition is knowable a posteriori if it is knowable on the basis of experience. Thus, the a priori/a posteriori distinction is epistemological.

In my usage, both are meant to be strongly tied to experience, in which a priori refers to mathematical knowledge having an existence independent of experience, where a posteriori mathematical knowledge is dependent on experience.[90] A posteriori replaces *invent* appropriately in that the intention of the word invent was that the knowledge grew from the constructive activity of the mathematical knower. And a priori replaces *discover* for a similar reason, discover was intended to indicate the result of the activity of the mathematical knower was to come to know. As a result, the minor modification to the original figure is recreated here, in Figure 7.

Further edits? At this stage in my development of this 4-quadrant descriptor, I reviewed for what use is it to me? Initially it was a place I carved out some differences I saw in people's way of viewing mathematical learning. I refined it to share with others. Once again I am at a stage where it is intended to be most meaningful to me. I have claimed that I assume all learners to be generative, this is the Radical Constructivist learning theory that I embrace. But, not all people, and in the case of this research agenda—high school matheamtics students, see themselves as generative learners. Ultimately, this research was about which students do have such a disposition, a consciousness of their own--and equivalent to the generative prientation—

[90] Each word carries significant meaning within philosophy, of which I am rather unfamiliar except to be able to recognize their changing meanings over time. I fear that the selection of these words runs into the same trouble as invent and discover, that each person's own ontological, and possibly metaphysical, stance overcomes the notion that the words can have a somewhat shared interpretation free of what a person brings to interpreting them in the context of the diagram.

epistemology, Which students have a mathematical ego so that they would place themselves in Quadrant 1 of this figure?

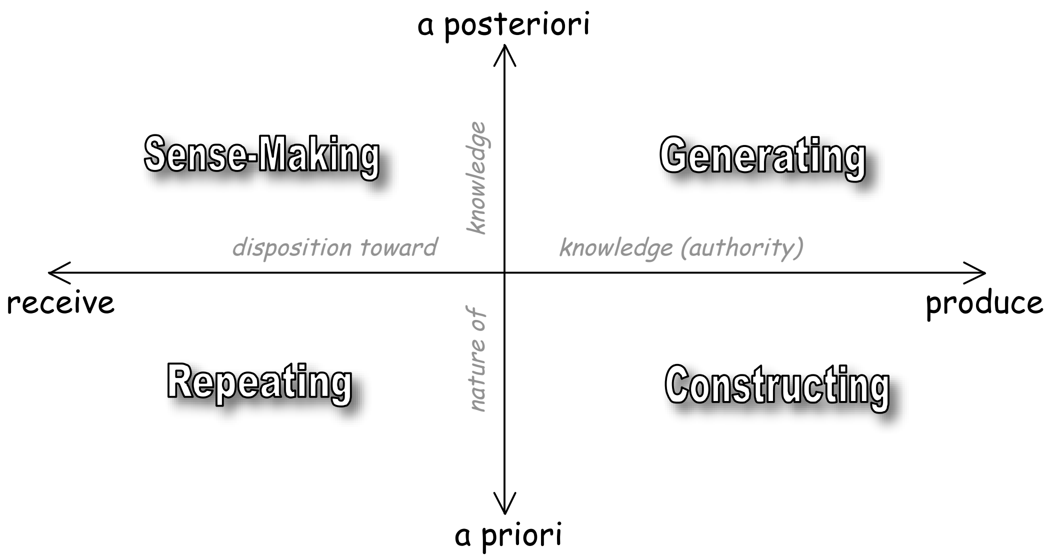

Figure 7: Various ways to theorize mathematical learning – revised.

For summative purposes, it is worth returning to the initial design, that the two axes reflected the two basic tenants of Radical Constructivism, the first being a statement on the activity of learning and the second on the nature what comes to make up that learning—the *x*- and *y*-axes, respectively.

1. Knowledge is built up by the actively cognizing subject, rather than the passive recipient.
2. Knowing serves the purpose of organizing one's experiential world, not coming to know an ontological reality.

However, the changes made to the *Ways to Theorize Learning* model were to orient its use toward evaluation of mathematics students' self-perception of their disposition toward knowledge creation and the nature of knowledge itself. In this way, the *x*-axis is a self-perception of the degree to which their own activity matters, and the *y*-axis is their self-perception toward the relativism of the constructed knowledge.

At this stage in this larger research project, I no longer know the usefulness of this model. Maybe it could be useful limited to its original intent, reformulated to first exist in a manner that acknowledges all learners are generative—a true representation of the Radical Constructivist learning theory. And then, in a second-level version, the model might evolve to better communicate how a learner may view themselves in orientation to their own learning activity and notion of knowledge.

As a result of this study, the model of the GAML has grown richer than what a 2-dimensional figure can communicate. Although it might be possible to develop the figure on an *n*-dimensional scale, that binaried structure may limit possibility too severely, and simplistically. Thus, I intend to drop further pursuit of the figure. Instead, next I will enrich the idea of the GAML by devloping descriptive passages of qualities having emerged from this research.

A Model of the Generative Adolescent Mathematical Learner.

Prior to beginning this research, I worked to develop a manner through which to describe the type of student I sought, high school juniors who saw themselves as generative mathematical learners. I wrote, “By generative, I mean a student who perceives him or herself to be the author of the mathematics he/she does. The generative learner is a student who sees him or herself as a key actor in the creation of mathematical understanding.” Through some verbal interactions with mathematics teachers and a brief pilot study, I realized that asking teachers to consider how a child perceived herself as a learner was not something the teachers regularly thought about—although most could. I found more success with more teachers when I focused more on potential activity of a learner that the teacher perceived to be generative. I asked teachers to look for these qualities by describing

- The generative learner may pursue an idea with greater persistence, less concerned that it may be wrong or lead nowhere.
- The generative learner may be more likely to dispute a conclusion or offer alternative conjectures, even opposing the teacher or a high status student's input.
- The generative learner will reject other's conclusions in favor of their own, or at least until they determine for themselves that the other's conclusion is accurate/better/preferred.
- I think a generative learner could possibly be a student who has shut down in their mathematics class because somewhere along the way they learned that this sort of activity was not productive or rewarded in their math classroom. This student may only occasionally show flashes of a generative attitude.

I also noted that the generative learner may or may not be earning good grades, scoring well on tests, or particularly active.[91] Not only was I able to collect feedback from practicing teachers on how they might interpret my requests to identify students they thought might be generative, these initial descriptions helped to inform my thinking on the initial hypothetical model of the GAML. Once I had the experience with the three subjects of my study, I revised this early model, abstracting from my data analysis a richer view of the GAML.

Mathematical activity. The mathematical activity of the GAML is marked by the atypical ways in which it uses traditional representations, relies heavily on operative reasoning, values intuitive reasoning over deductive logic, and greatly draws upon interaction for developing and shaping ways of knowing. Although much of this activity showed to be atypical from the conventional classroom activity associated with a high school mathematics classroom, there remained an appreciation for compliance with the emergent socio-mathematical (Yackel & Cobb, 1996) norms of the math classroom. The GAML is freed to think in their own best modes of operating, but recognizes and values the need to communicate how they are thinking about various problems or tasks. The classroom teacher influences the development of socio-

[91] The descriptors can be reviewed in the Subject Identification Request, Appendix A.

mathematical norms, and in this way, the GAML shows a valuation for conventional notations, vocabulary, and representations.

Possibly because they do not limit themselves to thinking within the context of these norms, they do not reason well with them. The GAML may likely be less-able to work with externalized representations of mathematical ideas, such as reasoning with symbols or pictures; The GAML may be seem to mostly be dependent on operative thought. This orientation to reasoning also seems to heighten the oral nature of their mathematical activity. And while mathematics may be defined as a particular discipline, carving itself away from the more general activity of numerical, spatial, and logical reasoning, by distinguishing itself by the valuation of deductive, reasoned logic, the GAML demonstrates a propensity for inductive reasoning as well as intuitive ways of knowing; and accepting such conclusions as valid.

Personal epistemology. The GAML perceives mathematics to be ideas, practices, formulas, solutions, etc. that is generated by knowing minds; it is a human endeavor.[92] And such, mathematics is a part of the daily activity of people. The GAML may see a distinction between what is learned in the classroom—School Mathematics—and this mathematical activity of living, but likely only to the extent that there is a particular Mathematics to be *learned* in school.

Knowing mathematics is a result of one's own thinking through ideas and coming to conclusions that they believe to be valid. In this way, the GAML perceives herself as the authority for knowing. Yet, the GAML finds great need to interact with others about this knowing, to test the waters. The GAML is highly interested in listening to other's ways of thinking, for the value of confirming or contradicting one's own thinking or conclusions, as well as continuing to learn and be challenged by new ideas. Each other person seems to carry a

[92] In 1970, Harold Jacobs published a mathematics textbook "for those who think they don't like the subject" of this same name—*Mathematics: A Human Endeavor*.

reasonable sense of authority as well, in such a way that it is respected well enough to cause the GAML to reevaluate their own knowing. The authority attributed other's is certainly not of a quality that would cause the GAML to abandon their own conclusions, just to review it. Maintaining such a strong positioning of the self as knower opens the GAML's ego up for damage, it can be a precarious position. The GAML's sense of mathematical self is related to the larger self, one's identity as a knower and as a peer. The social status issues that are bound up in classroom behavior, perceived intellect, alignment to school norms, etc. All these relations impact the GAML's degree to which it is protective of its mathematical ego. The GAML is drawn to be able to confirm it's ways of knowing by sharing their thinking, but to do so in an environment that does not feel safe is likely to shut down the GAML's mathematical activity.

Second-order viability. The second-order viability of the GAML takes a distinct shift toward the GAML's view of itself and it's ability to thrive in its mathematical learning environment. Glasersfeld (1995) describes second-order viability:

> It helps create that intersubjective level on which one is led to believe that concepts, schemes of action, goals, and ultimately feelings and emotions are shared by others and, therefore, more real than anything experienced only by oneself. It is the level on which one feels justified in speaking of 'confirmed facts', of 'society', 'social interaction', and 'common knowledge.' (p. 120)

The consideration of the second-order viability of the GAML firstly allows for the consideration of how the GAML locates authority and the role that other's play in the ways in which they feel confirmed in their knowing. But further, this second-order opens thought to consider the GAML's self-concept of it's classroom activity as well as their own mathematical activity, quite possible their mathematical ego.

As mentioned previously, the GAML draws heavily on interaction with others to confirm their own knowing, to feel justified to speak of confirmed facts, as Glasersfeld (1995) described. The GAML certainly recognizes this aspect of their interaction with others. They value and respect the differing perspectives, ideas, and methods that people other than themselves bring into discussions. While each member of the mathematical community seemed to impact the knowing and the viability of the GAML's knowing, ultimately determining one's ways of knowing lay within the GAML. This is not to say that the GAML might outright reject the ideas of others, but that the modifications interaction with others may cause remains a decision of the GAML.

This valuation of the ideas of others, and the seeking to be understood and to understand, seems to contribute to a shared appreciation and mutual respect between the GAML and her classmates. There is an understanding and expectation for difference; it is valued as material for further mathematical activity.

A question more difficult to consider than the identifying the GAML is to attempt to understand their own perception of themselves as learners. What is their own consciousness of their personal epistemology? How can you notice their mathematical ego? Much of the data that yields the model above comes from self-description of mathematical activity, often through interview follow-up to confirm inferences from observed classroom activity. What stands out most strongly is that the GAML perceives their learning to be the result of their effort, their thinking, and the sense making that they do. The GAML is a very confident learner, believing that if they decide, they can come to know and do what they conceive to be mathematics.

Viability in the mathematics classroom. Viability in the mathematics classroom is meant to consider both the subjects' conceptual operations and their mathematical activity, wondering how those ideas as attributed to the GAML allow for a *fit* to the contexts in which the GAML was using them, i.e. the mathematics classroom. As a mathematics classroom learner, the GAML is not likely to be highly adaptable to any context. Their mathematical activity is highly dependent upon a classroom setting that values exploratory mathematics, interaction and discussion, and diverse and divergent ways of thinking. The topic of mathematics cannot be reductionist in nature, nor can the goal for thinking be training-oriented. The GAML is not meant to know or think in a way equivalent to another. The GAMLs teacher must show a belief in the GAML as a productive mathematical reasoner, believing that they are capable. The teacher must distribute authority.

With these necessary conditions for the GAML to flourish, it is still the case that the GAML will not conform to the neat and tidy expectations of the knowledge receptacle. The GAML will set her own classroom agenda, may demonstrate little care for grades, other than to pass a required course, and not create documents of activity or knowing that may be otherwise expected of a "good" student. However, the GAML values the classroom environment, and holds in high regard the others of the class. For the teacher, the GAML may demonstrate deviant behavior, for her peers, she is not unusual—except that the GAML is likely to be viewed as smart.

In summary, I believe the GAML's potential to endure as a viable mathematical learner is influenced strongly by two things, the strong identity of the self as a mathematical authority and enough classroom (including influences of the course content, pedagogy, and teacher beliefs) opportunities to express this authority and remain a viable student in the school and mathematics classroom. It seems that a strong orientation to mathematics as human activity, and the confidence to productively engage in this activity correspond to this internal locus of mathematical authority. And, as classroom learners, that they may be mavericks (Grieb & Easley, 1984) to some degree must be embraced.

CHAPTER 8

THE FABRICATION OF KNOWLEDGE[93]

I begin this chapter with a brief summary of the study and its conclusions, revisiting the research questions that guided the study. This summary is followed by a general discussion, providing the limitations of the study, concerns about a goal for generative adolescent mathematical learners, and recommendations for future research. I conclude the chapter by outlining implications of the study for the field of mathematics education, namely what it might mean if the field took seriously ideas suggested by the postmodern view on the truths of knowing, the fabrication of knowledge.

The Study

This study is my work for social justice in education; I consider it to be equity work. The modern institution of education is founded on knowledge-as-information; it is discipline-centered and culturally reproductive. Such an institution could only further cement the inequalities found in a society. This research project begins to wonder what if the learner's relation to knowledge and perception of self as a knower could reconstruct unjust school structures? Mathematics education purports to strive for equity. It also purports to desire the curious, self-directed, productive learner. One who constructs knowledge, and who feels empowered by their potential to read and write the world; a contributing author of mathematics and a responsible authority for their knowing.

[93] "So it is the process of the fabrication of knowledge with its boldness, detours, contradictions and negotiations, that is masked" (Larochelle & Désautels, 2000, p. 386)

Resetting the Framework

This study grew for me while my career in professional development in mathematics education matured. I observed such contradiction between goals stated for mathematics education and so many of its practices, including documentation stating what children needed to know when, and the prevalence of Freire's (1970/2002) banking model for teaching in high school classrooms. Yet goals of mathematics education called for thinkers, problem solvers, and communicators. Furthermore, in the wake of decades of unequal financing, unequal support structures, and unequal outcomes, a national push was underway to heighten awareness and reduce these inequalities. I was in a position to observe both the increased societal demand for better schooling, and school's reactions to the demand by withdrawal from promising work toward an even more tightly focused machinery for social replication.

As a high school mathematics teacher, I had the opportunity to work with educators who intentionally sought to both continue the development of children's problem solving and achieve more equitable successes for our students. Yet even with these opportunities, I didn't experience many adolescents who perceived themselves as generative mathematical learners.

Consequently, I wondered how could mathematics teachers work in order to develop a child's disposition that maintained their path toward a confidence and competence as readers and writers of their world? What structures confound this effort, and are—simply put—in the way? My graduate studies served as an informal archaeology (Foucault, 1972), weaving through the various discourses all seeming to be a part of the histories of the teaching of mathematics. Upon churning up stances in response to questions such as What is mathematics? and Why teach mathematics?, I arrived at a curiosity about the nature of knowledge itself, uninterrogated in the queries above. These questions, and in particular the fabrication of knowledge, motivated the purpose of this study.

The purpose of this study was to create case studies of three high school juniors that demonstrate generative dispositions toward their mathematical activity. Through my own radical constructivist's methodology involving co-teaching, observation, and interview, I conducted a poststructural qualitative inquiry of the subjects' construction of experiences that may have informed this generative disposition, their orientation toward mathematical knowledge, and the role of schooling in the production of these students. The big question of this study was: How do generative adolescent mathematical learners maneuver through their mathematics courses while maintaining such a disposition? In keeping with both the ontogenetic and deconstructive frameworks that were employed, the initial sub-questions oriented my design and informed my analysis, but also evolved during the study:

1. What practices has a generative student created for engaging in mathematical activity?
2. What conceptions of mathematics has a generative student formed?
3. What does a generative student consider to have influenced her disposition?
4. How do generative students perceive the role of themselves in relations with others?
5. What are the relations among generative students and school discourses?

The emphasis of this study was not to identify particular sociocultural influences on the subjects' success in school and academics, nor to exact specific qualities of the subjects' being that allowed for them to maintain a generative disposition toward mathematical learning. The methods of this study emerged to consider how the generative adolescent mathematical learner may have maintained such a disposition while sociocultural influences, in particular those of school, seemingly work against such an outcome. The intent of this inquiry was to better understand the conception of a generative adolescent mathematical learner, and as a result consider educational goals for generative learners, which I perceive all learners to be.

This study drew upon traditions of radical constructivism and critical postmodern theory in order to peer behind the backdrop and off into the wings of a knowledge-centric stage. The ontological stance of radical constructivism moved the question of truth out of the way when considering knowing and learning. Postmodernism, and in particular poststructural theory, encouraged the circumvention of the desire to name, to fix an origin, rejecting a cause-effect linearity. I was able to focus too on the detection of "the incidence of interruptions" (Foucault, 1972). Through deconstructive techniques, postmodern qualitative research strives to consider the effects, limits and possibilities of the structure of some "object of knowledge," in this case the learner in mathematics education. And critical postmodern theory (Kincheloe & McLaren, 2000; Lather, 1991) kept the social and historically constituted relations of power and privilege at the fore, allowing me to connect this effort to my passion to contribute to an education for social justice.

Conclusions

I considered the three subjects of this study in the way I knew best to learn about children, through teaching them mathematics. I co-taught these juniors' mathematics course as if I were a regular faculty member in the school, during which time I had the opportunity to interact as a teacher both in and out of the classroom, as an occasional tutor, and as a classroom observer. The subjects' current and previous teachers told me much about their mathematical and school activity, as did the subjects themselves, through interviews and surveys. I was in contact with the subjects over the course of four months, one month of which I engaged daily as their teacher, in an attempt to answer the research questions posed above. The following paragraphs summarize my findings in response to that inquiry. The summary I present is not intended to be a statement of truths about the generative adolescent mathematical learner, but rather a gathering of awarenesses that the subjects brought forth in me, impactful possibly because they appeared in many of the subjects, or possibly due to the assertion of a singularity.

The generative adolescent mathematical learners of this study suggest that the GAML sees mathematics as activity, rather than as a static entity. Mathematical knowing was discussed in relation to problems and metaphors for ideas, as opposed to named and fixed notions. The subjects did not suggest that they were engaged in coming to know a fixed body of knowledge, but rather that they were engaged in the study and practice of a dynamic system of knowing and thinking.

While mathematics was located in activity, the GAMLs understood ideas that they attributed to be mathematics by needing time to make sense for themselves. Subjects expressed a need for individual time for this to occur. However, they also needed interaction with others, either to spark ideas, to check understanding, or to expand one's own current understanding. Furthermore, the GAMLs recognized that each person's knowing was different, and valued this possibility. Sometimes the value was in having a variety of ways to approach problems, either individually or when working with others. Sometimes the value was expressed in the potential this gave for improving upon one's own understanding.

The nature of mathematical activity and knowing was highly social for the subjects of my study. This coincided with each of my subjects having a good status in the classroom. They interacted with peers well, and each had respect from others for their mathematical abilities. Although they varied in their ego-driven sense of themselves in relations to peers competence, each demonstrated that they valued others ideas. The ways in which they attributed intelligence or mathematical ability to others seemed to encompass more about how hard people worked, or the speed to which they may figure something out; not to their peers capacity for or capability to know or do. The research subjects indicated that they could learn better from peers because they were better able to relate. I have heard many students over the years tell me this, but often times it was in the context of being better able to explain a static mathematical concept, as opposed to the relational and social emphasis I observed in the GAML.

While mathematics had an important social role, the GAML maintained a strong notion that it was for them to determine a truth to the knowing. The disagreement of another served as a catalyst for further inquiry and drive to make meaning, as opposed to proof provided by an external authority. This extended to the relationship with the teacher as well. The GAMLs demonstrated a need for validation from the teacher, but validation of themselves as people and as thinkers, as opposed to a validation for accuracy of knowing. I observed this to be a much more relational drive for companionship and security from an adult figure, rather than a need for confirmation from an authority outside of oneself.

Independent of the ontological status of mathematical knowledge, each GAML placed people in a primary role in the creation of mathematics or mathematical ways of knowing. In conjunction with the aforementioned activity-orientation toward mathematics, the GAML saw mathematics in all activity. Mathematics was a part of what humans did. This echoes with Belenky et al.'s (1986) Connected Way of Knowing, in which the individual considers her knowledge as primarily being constructed in interaction with other people.

Although none of the subjects conceived of themselves as constructors of knowledge in the radical constructivist manner,[94] all saw themselves as authors of mathematics at times, and readers at others. I've said that the GAMLs expressed the need to make sense for themselves;

[94] That is to say, none of my subjects took the double turn to foreground that not only did they construct mathematical knowledge for themselves, but that they constructed a knowing of their fellow students as knowers of another mathematical knowledge. That the subjects of my study unquestioningly used the notion that they read others ways of knowing suggested that they hadn't deconstructed their use of language, so I don't know how they might reply if this idea was pursued further.

each equated this to authoring. But they spoke of needing to read another's way of thinking, or their response or answer, in order to think further. The GAMLs location of themselves as mathematical knowers coincided with a high confidence in their mathematical abilities. From my perspective, their judgments of themselves were valid. None of the subjects of this study stood out in their mathematical ways of thinking or learning as particularly strong or unique in relation to other high school juniors I have met, but all were very capable, creative, thoughtful, and engaged. Not all were as competent with their thinking, as determined by my understanding of the discipline of Mathematics. In particular, some of the GAMLs demonstrated weaknesses in the ability to communicate their knowing of an idea. This is related to another struggle I observed, and that was an underdeveloped sense of the conventions of mathematical recording and communication. Fisk struggled with this, but was flexible and highly in tune with his classmates and did find an ability to relate mathematically with them. He demonstrated an awareness that there was a convention and he did pay attention to come to learn to use it. Jack, on the other hand, struggled with convention but seemed to care somewhat less about this. He, like Fisk, was successful in understanding other's mathematical ideas. And, Jack did manage to record much mathematical work—by the end of a unit of study—in conventional ways. So convention seemed to be learned along the way, but less valued.

This relationship to convention and externally derived expectations played out in another way for these GAMLs. Most demonstrated moderate to moderately severe dysfunction in relation to the classroom or school structure in one way or another. The two teachers I worked with in the study were more open and flexible with students, but other teachers of my subjects reported a variety of trouble with these students. The GAMLs of my study could have been regarded as lazy, uninterested in doing well in school, talkative, too confident in their ability, disrespectful of rules and classroom norms, and even aggressively defiant of authority. That these GAMLs were able to find some degree of success in the classrooms context I studied them in recalled the work by Grieb and Easley (1984) which identified the "pale male math maverick" (pm3): young boys who demonstrated strong mathematical abilities or insights and whose teachers allowed them greater space for some dysfunctional classroom activity. Their teachers reported a belief that these boys were succeeding fine and did not need to be held to the social norms of arithmetic. The researchers concluded that the dysfunctional roles taken on by the pm3 provided them with an advantage for future learning, increased opportunity to develop habits of independent thinking in mathematics.

Limitations

An expressed goal of this research program was to create new questions, both for the reader and for the researcher. With such a goal in mind, limitations to this study immediately occur when having to come to a place to conclude; many questions are left unanswered, and even unexplored. Before pausing to collect what may be implied by the current state of the inquiry, I will outline some of the questions that have arisen from within the study, including some of the issues pressing for further research. First I will analyze the design of the research to address points of limitation, or perceived limitation, in its structure. Next, I will bring forth concerns about both the GAML and about Mathematics Education that arose through this work. And finally, I will suggest avenues for further research.

Limitations of the Design

The first limitation of this research is the scope at which I studied the subjects. I paid attention most closely to each subject as an independent entity, choosing to not carefully study the actors and structures with which they interacted. For example, although I worked closely with their classroom teachers Larry and Bridget as co-teachers, I did not engage in an effort to collect data about Larry or Bridget per se. Each was interviewed and observed, and were often a part of my field notes or journal entries about each subject; each contributed to my analysis of each subject. But neither was studied in and of themselves. This study did not identify the classroom learning environment, the pedagogical techniques nor teaching beliefs, or resources available in the school or classroom in which the subjects participated. This served to make the study manageable and to encourage focus on the learner's experiences. But aspects of a more complete, richer story are not told.

A second limitation to the study is to attempt to understand to what degree the findings I reported are tied to the subjects I selected. Although this was very much a purposive sampling, the working definition of the GAML (and the associated means used to identify subjects) was certainly less evolved than it is now. In other words, this was a study designed to learn more about students who exhibited the characteristics of the disposition initially described as a GAML. Conclusions that suggested the GAMLs of my study located authority in themselves were not surprising. Yet that I selected students with this disposition created the opportunity to understand more about such a positioning of themselves to mathematical knowledge and learning, as well as the associated activity and ways to maintain such a disposition within a social-cultural milieu that tends to quash these attitudes in other students. Further, new awarenesses were raised, such as the possibilities for what might be dysfunctional behaviors in the social-cultural context.

This third limitation is a complex issue, going right to the heart of epistemology and the potentials for science. It may be irresolvable, but certainly can be attended to in scientific research via thoughtful methodology—either through intentional design or analysis. This third limitation is the question of whether or not I have found my idealized self in three high school students. Stirner (1845/1971) suggested that the egoist that we all are makes for no other way of being. I prefer the radical constructivist outlook that although it may be our tendency, we are constantly experiencing unpredicted feedback from those others of our experiential world, and in that way we know that they are unique and separate from us. Thus, as a researcher I took seriously the challenge to seek out those surprises and pay especially close attention to them, not washing them away into sameness, but working to theorize a subject different from myself.

The postmodern qualitative inquiry position embraces the notion that this sort of limitation, a researcher's reflexivity, exists in all data collection and analysis (Pillow, 2003; Scheurich, 1997). Reflexivity should be conceived as a limitation when not identified and attended to. To address this limitation further, I do find much of my own disposition in these students. Enough so that at times I have worried that this uncovering threatens the validity of the research. On a scale of learning broader than just mathematical, I too need to make sense for myself, but value others ideas and input greatly. My preferred manner of work is with others, enjoying the camaraderie of learning together, of discussion and debate of ideas, of striving for and arriving at common goals together. I have a confidence in my knowing, and to whatever degree it is a blind confidence, externally judged for its competence, I proceed with my ways of knowing. Yet, I do value confirmation from peers. Are my activities dysfunctional? With regards to attaining certain successes, they certainly are. But from within and to the degree I am aware of the constraints and affordances of my disposition, I am quite happy, as I believe each of the subjects of my study were. Maybe of most importance was that this was not a study to justify the desire, or argue for the valuation of the GAML in mathematics education. It was a study that located a sort of fissure in what policy called for—the productive learner, and what policy structured—a replication of a particular way of knowing, what I have termed school Mathematics.

Concerns for the work of Mathematics Education

I argued early in this study that the field of mathematics education points toward a goal to create the GAML as a result of education. NCTM (2000) values students who are problem solvers, and calls for students who are actively engaged in learning. Constructivist theories are currently the prominent discourse in mathematics education. Taken across its various forms, this learning theory values the active learner, the child who makes sense of the world in mathematical activity, who constructs understanding. Among others, Becker (2002) and Grouws et al. (1996), argue for the development of students who view themselves as authorities of their knowledge or ways of knowing. Boaler and Greeno (2000) characterize and value productive thinking as creative and connected. They claim such a mathematical disposition is necessary for future success in mathematical study. Freire (1970/2002) and Gutstein (2003) go further with a notion of authority to describe the student who not only has learned to read her world, but also understands her place in (and responsibility to) write it.

In this study, I posited a sort of learner that matched these desires, and learned more about the way they operate in the classroom, how they view mathematics, and how they view themselves as learners. Having learned what this study suggests about the GAML, it is appropriate to return to the assumed position that this is the type of student desired in mathematics education.

If we ultimately desire a generative learner, to what degree does that endanger a discipline[95] of Mathematics? As mathematical ways of knowing are generated in the mathematics classroom, what mathematics will become valued? The teacher can exert her will, can present a mathematics identified as the socio-culturally defined and approved knowledge, but to what extent will the GAMLs knowledge be a match? And to what extent may a group of GAMLs accept or reject this dominant knowledge? On a classroom scale, there is a distinct game of power in motion in the discourse of knowledge. An expressed valuation of the students as generative knowers may tip a precarious balance that maintains the status of a specific way of knowing toward a different accepted knowledge. Or differently, a hierarchal knowledge may be replaced with a more heterarchal, decentralized mindset with regards to ways of knowing. But will this knowledge be returned to a very novice, inexact science, ignorant of the phylogeny of intellectual development? Or, would it build upon, meld with, or somehow interconnect with current ways of knowing Mathematics?[96]

The social activity and valuation of understanding may also serve to disrupt the discipline of Mathematics. Even more so, this may deteriorate the hierarchal structures of the expert/novice paradigm within which schooling currently operates. If the learner establishes herself, and her peer network, as authority, the school is no longer a resource for knowledge, but a place in which knowledge is created. The role of the expert must shift; or possibly the notion of expert would be reconfigured into an entirely different principle.

[95] In this sentence, I intend to mean the body of knowledge we treat as the mathematical truths, what Dewey (1902/1964) referred to as the curriculum. But to keep meaning at play, and to foreshadow my closing comments, also consider the disciplining potential for such a relation to knowledge (Foucault, 1975/1995).

[96] Of interest, and surprising coincidence, is the manner in which this same question arises in the flattening of knowledge-structures made possible by the Internet and knowledge-sources such as Wikipedia (http://www.wikipedia.org) and Connexions (http://cnx.org). Each of these is a heterarchal, decentralized resource in which peer-review is the validator of knowledge.

A heightened valuation of the social activity is likely to markedly change social relations as well. If status issues reflected by the dominant culture are replicated, the school and in particular the mathematics classroom would serve to propagate the oppressive structures of the dominant society. Or possibly, in some settings a new oppression would emerge. Possibly a new pedagogy could develop that monitors and influences that role of status, in ways that maintain value to all voices. Boaler's (2004; 2006a; 2006b; 2008) work suggests a sort of relational equity that such an environment may produce. She defines relational equity as equitable relations in classrooms, "relations that include students treating each other with respect and considering different viewpoints fairly" (2008, p. 2). This focus on student relations is intended to think of schooling not only (or possibly, minimally) for the purpose of test scores and related school-centric outcomes, but also as extending into student lives in and beyond school.

Other than the hazard to current structures of mathematics education on the modern school, the mathematical egoism[97] of the adolescent learner expressed within the GAML must be addressed. While this egoism may be unavoidable in the 17-year old child, maybe mathematics learning can be designed more fully aware of how this quality operates in the learning environment. Is egoism a powerful way of operating? Or is a more humbled attitude, maybe one that would be named mathematical altruism, toward knowing and knowledge a desirable trait while students maintain a perception of themselves as authors of their knowledge? I suspect that the educational effort would seek to foreground the quality of the GAML that recognized each person has a different way of thinking. The egoism/altruism binary may settle into a both/and rhythm, rather than the either/or prevalent in schooling currently. In some way, the notion of how to value the relativism of knowing could become an object of study in itself.

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[97] Taken to mean that self-interest is the conscious motive of mathematical learning activity.

Recommendations for Future Research

The concerns identified above raise a bevy of interesting questions, some of which need theorizing and meting out in order to revitalize policy. Design for structure, practice, and outcomes of mathematics education should be reconfigured. Research in the field can inform these developing ideas, both within educational theory and in its practices.

Particular research questions have emerged for me through the course of this study, the first of which emerged from the noted limitations. The discourses of typical sources for authority in the classroom were given little attention in this study. How does a teacher's relationship to mathematical knowledge function in the interaction with the GAML, especially with respect to their mathematical learning and disposition? This study also gave almost no attention to the demographics associated with the school setting and the research participants. I believe Collins' (1991) Afrocentric feminist epistemology could further the development of descriptors for the activity of the GAML. She writes, "The significance of an Afrocentric feminist epistemology may lie in how such an epistemology enriches our understanding of how subordinate groups create knowledge that fosters resistance" (p. 207). Her work embraces four assumptions about knowing, especially the notion of knowing-in-relations: 1. Concrete experience as a criterion of meaning; 2. The use of dialogue in assessing knowledge claims; 3. The ethic of caring; and 4. The ethic of personal accountability. These assumptions are harmonious with findings in this research, and the theorizing of the GAML I suggest asserts a sort of subordination to a powered knowledge structure, in particular Mathematics.

A second potential line of research is to trace the prevalence of this disposition across the years of schooling. Utilizing this more nuanced characteristic of the desired mathematical learner, research should theorize the decay of this quality of the learner. Better understanding where it occurs is one step of such an inquiry project. Work should also be done to understand the function of discourse in the classroom that influences the child's disposition, and how this discourse is evolving over the K-12 educational cycle. In addition to understanding the classroom discourse, attention must also be placed upon the policy discourse, which is the discourse shaping the activity of the mathematics educator.

A third research orientation should become more pronounced in the field, to develop Mathematics of Children (Steffe, 2004; Steffe & Weigel, 1996) as an important knowledge base for Mathematics Education, and as an expansion to a discipline of Mathematics. There are currently research projects doing this work. However, few understand neither its use or value to the field of Mathematics Education, nor its implications for the activity of teacher development. Although not research per se, work must be done to establish the role of this knowledge in the culture and practices of Mathematics Teacher Education.

Implications

In the concern for mathematics education section above, I raised several points that question the desire for the GAML as a goal for mathematics education. Such a goal seems to place at risk the status of the discipline of Mathematics, as well as the status of the expert knower of this discipline. I also suggested that the role of the mathematics teacher would change, focused less on the provider of knowledge and arbitrator of truth, to a moderator of perspectives and monitor of status. In what follows, I will return to the undercurrent of this research effort to bring forth the postmodern characteristic of knowledge in a deconstructive effort to not only better understand the GAML, but to encourage thinking differently about mathematics, mathematics education, and research in mathematics education. I conclude with a return to the purpose of this paper, to engage the reader in thinking about mathematics education, equity, and social justice.

The Fabrication of Knowledge

The adolescent mathematical learners of this study were taken to be generative, that is constructors of knowledge and truths.[98] These students are learners who fabricate knowledge, where fabrication is taken to mean build, design, construct. Although the field of mathematics education seemingly has embraced the constructivist notions of the active learner and the constructing mind, it is most certainly a "softer" (Larochelle & Bednarz, 2000, p. 3) constructivism enacted in schools. The modernist truth agenda remains in place in schools and other educational structures. While student's points of view may be increasingly valued in policy documents and elicited in the classroom, such elicitation only serves to determine what is "wrong" about a student's point of view. Wrong, used in this manner, to mean from the perspective of a pre-existing knowledge, a truth-regime, something that is to be taught. In this soft version of constructivism, the fabrication of knowledge takes on a different meaning, it is a concoction, an invention, a forgery. In essence, the soft constructivism creates a perspective toward the learner as to be one who constructs untruths, who fabricates lies. The political and social ramifications for a generative view of learners, and the related constructed view of knowledge, has yet to be enacted in the mathematics classroom, nor taken seriously when conceiving of the activity of or goals for mathematics education.

Treating children as fabricators of knowledge, as little liars, may in fact be a greater injustice to the learner than teaching with the intent to deposit knowledge into the account of the knower, paraphrasing Freire's banking model for teaching and learning. In the present model for teaching our adolescent fabricators, we engage them in activity, engendering them with a momentary belief that we are truly interested in what they are thinking about their world. And then we tell them how it is, how it should be, how they should have figured, how they should

[98] As are all knowers in my personal theory of knowing and learning.

think. We not only continue to act in accordance with a belief that language may somehow transmit knowledge, of course an illusory notion (Glasersfeld, 1998), but we seem to enforce the modernist knowledge-as-truth agenda onto the adolescent learner. When unquestioningly engaged in this epistemology of soft constructivism, we treat the learning activity as a process of discovery, holding tight to a knowledge that is to be discovered, listening *for* (Davis, 1997) cues to hear in the child our own ways of knowing this knowledge. The pedagogical practices of the teacher devolve to a guess-what-I'm-thinking state; the pressure of time and the testing of this pre-existing knowledge drive the maddening process of an education that began with a hopeful premise—that children make meaning through active engagement with their experiential world, that children are knowledge constructors, fabricators.

If the radical epistemology of constructivism is embraced and the fabrication of knowledge is recognized not as a construction of untruths but as other truths, a different mathematics education must be conceived. Such a mathematics education would mature from this postmodern epistemology of radical constructivism, and its concordant poststructural concept of power/knowledge (Foucault 1980; 1975/1995).

What might be a Mathematics Education?

Most immediately to be addressed is a new conscription for the question, *What is mathematics?* The field[99] must undertake an archaeology of the discipline, to unearth its multiple histories to further understand the way mathematics operates in the structures and functions of our society. Much of this work is currently in place. However, this work must be reconstituted through the postmodern lens to better understand how mathematics, as a discipline, a taken-as-truth structure, was and is politically, socially, and economically implicated in our current

[99] For the purpose of this section, I treat the field as practicing educators, policy makers, researchers, and theory builders.

knowledges, ways of operating, and in particular notion of mathematics education. Once mathematical knowing is reconceived, wrested from Erdös' "book", and returned to the activity of the fabricator, can an education that is mathematical be conceived.

A second question in this new field would be, *What is mathematics education?* How would it function? What would be its goals? I suspect that in the fashioning of new goals, a discipline of mathematics that reflects the accumulation, or histories of experiences, of mankind would have minimal status, possibly equivalent to any other knower in a collection of learners. I don't think it could disappear if a teacher, an adult knower, were present. For that knower would bring with them these histories of knowledge, and through her interaction with other knowers they would persist. However, I don't necessarily think a form of mathematics education that ignores the socio-historical knowledges of mathematics is a reasonable solution. To some degree such a mathematics education continues along an either/or trajectory of knowledge, a binaried way of thinking of possibility. A both/and direction is most valuable. The discipline of Mathematics would be embraced for what it is (that is, perceived to be), including its histories and power structures. But in addition, the mathematical activity, the fabrications of knowledge, which emerges in the classroom are taken with equal interest and quality and value. Student-student and student-teacher interaction would be characterized by *learning alongside*.[100] Knowledge emerges amid the social relations between people, ways of knowing are valued in their socio-cultural terrain. Learners position themselves as authors; the awareness of and responsibility for their fabrications are part of a pedagogy. An expressed desire for this mathematics education is that children learn to act justly. Herein lies a new conception of mathematics education, one that I believe could be equitable. It is my work for such an education

[100] Davis' (1997) "hermeneutic listening."

that I equate to my work for social justice. This point will be rearticulated in the next and final section of the dissertation.

First, however, to return to the academic side of mathematics education: What implications for mathematics education research? What does this mean for a field in which its science was/is conceived to be knowledge creation? What role for research? What does it mean to build theory? Does mathematics education continue to have a place as a science? I think that a renewed field of Mathematics Education would emerge. Firstly, this field would continue to document, and thus empower, *Mathematics of Children* as a valid knowledge. Such a contribution would validate children's ways of knowing, aid teachers in their work to listen with and learn alongside their students, and also broaden the base for and of adult mathematics. Second, Mathematics Education research must reassert itself in the study, design, and theorizing of a mathematics pedagogy. Even currently this is a most underserved area of study (Boaler, 2008). The larger system of Mathematics Education must take seriously the task of teaching teachers how to act in ways that are very different from the their current perceptions of the activity of a teacher. To give serious study to pedagogy may be as difficult a shift as it has proven to be to achieve the radical move of constructivism, in which knowledge is conceived to represent a true, or nearing true, representation of some independently existing reality, to a relationship of viability between knowledge and one's experiencing of the world. The current practices in Mathematics Education repeatedly, and seemingly endlessly, focus on designing curriculum, phylogenetically ordering topics, and then prodding and poking our learners—children and teachers alike—to see if they understand what we demand they do, in the way that we do. Mathematics Education, in current practice, fails the ethical demands to recognize a second-order of knowing.

Mathematics Education, as a field, currently operates in a state of treading water. It spends its resources responding to external questioning and criticism rather than progressing forward. It is locked in a battle of proof and refutation, rather than a creative endeavor to grow; it is reactive rather than proactive. It is a shell of its potential self, desperately clinging to a status as a worthy field. Mathematics Education *is* in a crisis, but not the one it thinks it is in. While reacting to society's demand for equal results, the field both bemoans the misunderstandings of its goals AND responds to ignorant solutions to the challenge. The historical cycle of social commentary is re-approaching an era that questioned the value of a mathematics education, one of such particularities. Mathematics Education must reassert itself by resetting an agenda of its own. This was accomplished two decades ago by the National Council of Teachers of Mathematics (1989) through the gathering of then current understanding about mathematics, teaching, and learning, and setting a vision for the work of the profession. A similar move is needed today.[101]

Social Justice in Mathematics Education

When knowledge is understood as a fabrication, as a construction, as a viable set of rules useful in the moment toward accomplishing a goal one has set for oneself (Glasersfeld, 1998), a new and more powerful meaning for equity can be valued. No longer is equity constrained to an equal opportunity, equal treatment, equal outcome (Fennema, 1996) simplicity, but socio-cultural, political, and power factors (Weissglass, 1998) can be understood as well. When learners position themselves as authors, and when teachers position learners as authors, the awareness of and responsibility for the fabrication of knowledge must become a part of the pedagogy. Mathematical knowledge

[101] Similar moves have occurred in recent years, such as the *Adding It Up* (Kilpatrick et al., 2001) report. But none have engaged the field to move forward as did the NCTM series of standards documents (1989, 1991, 1995). An American gathering may be hopeful toward this goal, the "Math is More" group (http://www.mathismore.net/).

becomes more tightly bound to the student-student and student-teacher relations. Not only does a learner come to know through this interaction, there is a *need* for this interaction to be equitable. Not a need of the teacher, but a need of the individual; all ideas are necessarily heterarchal being that each person has a different and worthwhile way of knowing, and the other is needed to provide feedback to the viability of one's own knowing. A spontaneous outgrowth of this mathematics education is that children would learn to act and interact equitably.

Boaler has identified a relational equity (Boaler, 2004; 2006a, 2006b; 2008), a useful concept when, as above, one's knowledge is understood to possess a meaning in relation to one's goals, and goals are necessarily in relation to others. Boaler's relational equity emerges in classrooms that promote student respect for each other's differences, and listen to others who have different opinions, perspectives, or experiences in order to engage as equals. When relational equity was observed in mathematics classrooms, the following strands of student activity took place (2008, p. 8):

1. Respect for other people's ideas, leading to positive intellectual relations.
2. Commitment to the learning of others.
3. Learned methods of communication and support.

The development of relational equity, a new orientation for equity work in mathematics education, provides a more integrated backdrop for work for social justice. I have defined social justice in this paper to be not only the development within students of a "sociopolitical consciousness" (Freire, 1970/2002), but also about the drive for an equitable, compassionate world where difference is understood and valued, and where human dignity is respected. I suspect that when children learn to listen to one another and value difference in the mathematics classroom, they will become more fully aware of conditions of their culture or society that are inequitable. A mathematics classroom can seed this sociopolitical consciousness simply through teaching children new ways to conceive of knowledge, and teaching them to interact equitably. But a further effort must be made to teach them how to *act* upon this sprouting rage toward injustice.

Further understanding of the generative adolescent mathematical learner can begin to inform a new agenda for mathematics education. In fact, it is necessary to recognize the functions and dysfunctions associated with maneuvering through their mathematics course while maintaining a sense of oneself as an author and authority with relation to mathematical knowledge. These insights provide data upon which to reconsider the structures of education, including curricular and pedagogical practices as well as epistemological beliefs. When Mathematics Education is able to adjust its practices in ways that bring forth the positive qualities of the GAML, a rejuvenated effort for equity and social justice can be realized—one that isn't fighting upstream against the duplicative structures of the knowledge-régime, but one that flows unfettered from the learner's mathematical interaction, i.e. the fabrication of knowledge itself.

AFTERWARD

The essential political problem for the intellectual is not to criticise the ideological contents supposedly linked to science, or to ensure that his own scientific practice is accompanied by a correct ideology, but that of ascertaining the possibility of constituting a new politics of truth. The problem is not changing people's consciousnesses—or what's in their heads—but the political, economic, institutional régime of the production of truth.

—Michel Foucault (1977/1980, p. 133), *Truth and Power*

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APPENDIX A

SUBJECT IDENTIFICATION REQUEST

Brian R. Lawler
The University of Georgia
Department of Mathematics Education
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Athens, GA 30602–7124
(706) 542–4194
blawler@coe.uga.edu

February 21, 2005

Ms. <*PHS Junior Math Teacher*>,

I will be conducting my dissertation research at Prairie HS, selecting students at the end of February and gathering data in April. My study will be focused on 3 juniors at Prairie, but will rely on a great deal of input from their classroom teachers. I want to explain a little more about my study, and then ask if you can help in my selection of students.

The purpose of my research is to learn about high school students who demonstrate a productive attitude toward mathematical learning. As a result of many schooling factors, it is often the case that students dislike mathematics, and/or perceive mathematics as a set of facts and skills to be learned. Mathematics teachers have been working to expand student's perceptions of mathematics so that they see it as a tool for making sense of and acting in the world. Along with being able to use mathematics in these ways, teachers wish for students to see themselves as being willing and confident to tackle new problems.

In my research, I will identify and study three high school juniors who demonstrate a productive attitude toward mathematical learning. I hope to learn about the ways they do mathematics, perceive their mathematical activity, and interpret the forces that may have shaped their attitudes toward mathematics. To do this, I will observe and teach, alongside their regular teacher, in their math classroom for one month. During this period, I will also conduct twice-weekly one-hour tutoring/interview sessions. All tutoring sessions and some classroom episodes will be videotaped. Finally, the three students will be provided equipment and asked to create a 10-minute video showing what they believe to be the influences on their doing and learning mathematics.

My request of you is to help in identifying the students I will study. Prior to my weeklong visit in February, I would like for the junior teachers to identify students who may fit my selection criteria for the study. This will help me to focus my February observations on a few students, and make my final selections. Please read my description of a "generative mathematical learner" on the back of this letter, then complete the form in which you identify 1-4 of your students who best meet this description. Please take a few days to consider which students you would identify, but don't delay too long in your reply.

Please return your reply to Nancy's mailbox using the envelope provided. Your participation is optional and I appreciate the time you take in completing this information. Call or write if I can answer any questions.

Thank you very much,

Brian R. Lawler

<PHS Junior Math Teacher>,

Again, thank you for opening your classroom and being willing to help with my doctoral research project "Generative Adolescent Mathematical Learners." For this project I will select three juniors from your high school and interact with them in order to learn about the ways in which they perceive themselves to be engaged in mathematical activity. I am looking to select students with some particular qualities, which I incorporate into the name "generative."

By generative, I mean a student who perceives him or herself to be the author of the mathematics he/she does. The generative learner is a student who sees him or herself as a key actor in the creation of mathematical understanding. They see themselves as a producer.

For contrast, the non-generative student would be one who perceives him or herself in a less active role with regards to their mathematical learning. They see themselves as someone who receives mathematics. They are less likely to label their own mathematical activity as mathematics.

> I would like to emphasize that it is not necessarily the case that the generative learner would be earning good grades, scoring well on standardized tests, or even demonstrating a strong mathematical aptitude. It is also the case that a generative learner is not necessarily the most active child. *My naming of a generative learner is much more focused on the way a student perceives himself or herself to be doing mathematics.*

I ask that you aid my selection process by helping to identify 1–4 juniors in your class(es) who you would identify to be the *most* generative learners. Please use the included form and return to Nancy within one week using the enclosed envelope. For the sake of helping you think, I've listed a few qualities that you may observe in a generative student in the classroom:

- The generative learner may pursue an idea with greater persistence, less concerned that it may be wrong or lead nowhere.
- The generative learner may be more likely to dispute a conclusion or offer alternative conjectures, even opposing the teacher or a high status student's input.
- The generative learner will reject other's conclusions in favor of their own, or at least until they determine for themselves that the other's conclusion is accurate/better/preferred.
- I think a generative learner could possibly be a student who has shut down in their mathematics class because somewhere along the way they learned that this sort of activity was not productive or rewarded in their math classroom. This student may only occasionally show flashes of a generative attitude.

Thank you very much for your input. Please don't hesitate to call or email if I can clarify this request. I look forward to talking more about the students you've identified, and what this notion of a generative learner might mean. It is a concept of a student I am very curious about, but don't have much handle on who they may be, or how I as a teacher may influence this kind of learning.

Brian

Generative Adolescent Mathematical Learners
Teacher: *<PHS Junior Math Teacher>*

1.	Student Name	Class Title	Period
Please make a few comments in the space below about your identification of this student.			

2.	Student Name	Class Title	Period
Please make a few comments in the space below about your identification of this student.			

Generative Adolescent Mathematical Learners
Teacher: <*PHS Junior Math Teacher*>

3.	Student Name	Class Title	Period
Please make a few comments in the space below about your identification of this student.			

4.	Student Name	Class Title	Period
Please make a few comments in the space below about your identification of this student.			

APPENDIX B

SUBJECT SURVEY

NAME: ____ DATE: _________

This questionnaire includes items of different types. Answer each item by choosing the alternatives which best show what you think. Read the instructions for each item carefully.

1. If I had to give myself a score out of 10 to show, honestly, how good I think I am at math, the score I would give myself would be

   Write your score in the box;
   10 means absolutely outstanding;
   0 means absolutely hopeless.

2. *Circle one or more*

Mathematics is something I do

a. Every day as a natural part of living

b. Mostly at school

c. With a pencil and paper

d. Mostly in my head

e. With numbers

3. *Circle "Agree" or "Disagree" depending on whether you think the statement is true or not.*

The ideas of mathematics

a.	Have always been true and will always be true	Agree	Disagree
b.	Were invented by mathematicians	Agree	Disagree
c.	Were discovered by mathematicians	Agree	Disagree
d.	Developed as people needed them in daily life	Agree	Disagree
e.	Have very little to do with the real world	Agree	Disagree
f.	Are most clearly explained using numbers	Agree	Disagree
g.	Can only be explained using mathematical language and special words	Agree	Disagree
h.	Can be explained in everyday words that anyone could understand	Agree	Disagree

4. *Circle the word which best describes how often the particular thing is true for you.*

When I am doing mathematics at school, I am likely to be

a.	Talking	Always	Often	Sometimes	Seldom	Never
b.	Writing numbers	Always	Often	Sometimes	Seldom	Never
c.	Writing words	Always	Often	Sometimes	Seldom	Never
d.	Drawing diagrams	Always	Often	Sometimes	Seldom	Never
e.	Working on my own	Always	Often	Sometimes	Seldom	Never
f.	Working with a friend	Always	Often	Sometimes	Seldom	Never
g.	Working with a group	Always	Often	Sometimes	Seldom	Never
h.	Listening to the teacher	Always	Often	Sometimes	Seldom	Never
i.	Listening to other students	Always	Often	Sometimes	Seldom	Never
j.	Copying from the board	Always	Often	Sometimes	Seldom	Never
k.	Working from a textbook	Always	Often	Sometimes	Seldom	Never
l.	Working from a worksheet	Always	Often	Sometimes	Seldom	Never

5. *Decide whether you believe the activity to be mathematical or not, then decide to what extent the activity is mathematical or not mathematical.*

Show your responses by circling either HM (Highly Mathematical), QM (Quite Mathematical), SM (Slightly Mathematical), BM (Barely Mathematical), or NM (Not Mathematical).

Circle one choice for each item.

a.	Playing a sport	HM	QM	SM	BM	NM
b.	Traveling to school or work	HM	QM	SM	BM	NM
c.	Using a calculator to work out interest paid on a housing loan over 20 years	HM	QM	SM	BM	NM
d.	Cooking a meal using a recipe	HM	QM	SM	BM	NM
e.	Planning a family's 2 week holiday	HM	QM	SM	BM	NM
f.	Upkeeping a domestic vegetable garden	HM	QM	SM	BM	NM
g.	Playing a musical instrument	HM	QM	SM	BM	NM
h.	Chopping down a large tree	HM	QM	SM	BM	NM
i.	Buying clothing at a sale	HM	QM	SM	BM	NM
j.	Painting the house	HM	QM	SM	BM	NM

APPENDIX C

SUBJECT MATHOGRAPHY

Mathematics Autobiography

People often write about events or experiences from their lives in order to help those who may have had very different experiences view the world from a different vantage point. Reflect on your past experiences in learning and doing mathematics, both in and out of school. With these experiences in mind, write *your* "mathematics autobiography."

Here are some things to think about as you write. Please respond to each.

- Consider all the mathematics classes you have taken over the years. What did you do in those classes that helped you to learn? What happened in some of those classes that influenced or affected your learning? What happened in some classes that made it hard for you to learn?
- Compare experiences you have had working in a group with experiences you have had working alone. When is it best for you to work with others? When is it best for you to work alone?
- Consider experiences where you were doing and/or learning mathematics outside of your mathematics class. What mathematics were you doing and/or learning? What was different about your learning compared to math class? What was similar?
- Describe a situation when you learned something difficult. Describe a situation when someone helped you learn something difficult.
- Are you good at mathematics? How do you know?
- Do you like or dislike mathematics? Why? What do you like about doing math? What do you not like?
- Who or what influenced (either positively or negatively) your feelings about mathematics?
- How do you think your attitude about math affects your learning of math?

<*subjectname*>,

Thank you again for participating in my research effort. I realize your contributions have been your own generous giving of time and energy toward helping me to understand better the ways in which high school students conceive of themselves as authors of mathematics.

You are welcome to hand write or type this response. If possible, please send me your work through email. If I can answer questions about this task, please don't hesitate to call or email me.

Sincerely,

Brian R. Lawler
(706) 338–0578
blawler@uga.edu

APPENDIX D

SUBJECT INTERVIEW 1 PROTOCOL

1. I'm curious about you're thinking when working on math tasks, especially your decisions about being done, or correct. For example, the Rat problem. Tell me about your decisions when you know you have solved a problem.

2. Describe the mathematics you're learning now.
   Tell me about the role you play in learning this mathematics.
   Do you recall the survey where you responded to the questions "The ideas of mathematics were invented by mathematicians." And "The ideas of mathematics were discovered by mathematicians." What do you think? What if I use the word "people" instead of mathematicians.

3. Do you learn mathematics outside of class?

4. What sorts of things catch your attention in math class?

APPENDIX E

SUBJECT INTERVIEW 2 PROTOCOL

1. When you see a new problem in your math class, how confident are you that you'll be able to solve it? Rate your confidence on a scale 1–10. Comment.

    a. What role do peers have on your learning?
    b. What role does the teacher have? Has it been different for past teachers?
    c. The role of other aspects of school? Family? Other parts of life?
    d. So when you are learning, there are a variety of people involved in your learning. [list examples mentioned] When it comes to learning new mathematical ideas, which person plays the most significant role?
    e. What kind of things can you learn/do on your own?

2. What aspects of school encourage you to learn? What discourages you?

3. What happens in between the teacher saying it and you knowing it?

    a. What happens in between you reading a problem—like the Distance Formula activity in Orchard Hideout—and you knowing the Distance Formula?
    b. Are there other ways you come to know math?
    c. Which makes you know best?
    d. Which way happens most in your classrooms?

4. What has influenced the ways you learn and do mathematics?

    a. The last question I asked was "What has influenced the ways you learn and do mathematics?" This one is slightly different: What has influenced your thinking about the ways you learn and do mathematics?

5. Do you remember learning math when you were young? Describe. Tell me about that. What do you remember about learning?

    a. What is the same and what is different now about how you are learning mathematics? Has your role or your actions changed?

6. What do you do when your teacher says something you don't think makes sense?

7. How do you perceive yourself as a math learner—a reader or an author? Tell me more about your response.

APPENDIX F

SUBJECT VIDEO MATHOGRAPHY

Video Mathography

Reflect on your past experiences in learning and doing mathematics, both in and out of school. With these experiences in mind, consider what has influenced *your thinking about* the ways you learn and do mathematics. Capture images and video of the aspects of your life that will demonstrate these influences. Do this using 10 – 15 minutes of video footage.

<*subjectname*>,

This is the part of my research I am very curious about how things will turn out. Please don't put a great deal of pressure on yourself to make a perfect videotape—no need to take & retake shots, or to try to edit your work. Please do your best to respond to the task using video. I hope this is more fun than it is trouble for you.

Pick the camera up from *Ms. Edwards* on your scheduled morning. The battery will last for approximately one hour, so if you are careful to turn the camera off when not recording, you can probably do without the power cord. However, you are welcome to take it if you wish. Keep the camera until the end of the next school day. Please return the camera to *Ms. Edwards* by that time so the next person can use it. I encourage you to take the camera home and to other places you visit outside of the school building. However, if you are not comfortable being responsible for the camera, you may leave it with *Ms. Edwards* at the end of the first school day.

As always, please don't hesitate to contact me if you have questions or concerns. Best of luck, I look forward to your video!

Sincerely,

Brian R. Lawler
(706) 338–0578
blawler@uga.edu

Brian R. Lawler
The University of Georgia
Department of Mathematics Education
105 Aderhold Hall
Athens, GA 30602–7124
(706) 542–4194
blawler@uga.edu

April 27, 2005

To Whom It May Concern:

Please allow <*subjectname*> to videotape small segments of this school environment. <*subjectname*> is collaborating with me on my research on adolescent mathematics learners. As part of the project, she is creating a 10–15 minute video of what has influenced her thinking about the ways she learns and does mathematics.

This research has been approved and is supported by superintendent Dr. <*superintendent*>, principal <*principal*>, and mathematics chair *Nancy Atwater*. Please feel free to contact me with any questions or concerns regarding the research.

Thank you for your consideration.

Sincerely,

Brian R. Lawler
cell........(706) 338–0578
email.....blawler@uga.edu

APPENDIX G

SUBJECT INTERVIEW 3 PROTOCOL

1. When you see a new problem in your math class, how confident are you that you'll be able to solve it? Have you solved problems that the teacher hasn't first shown you how to solve?

2. What role do your peers have on your learning?

3. You wrote in your mathography that the other people around you can be both good and bad, and that one of the bad things can be that sometimes you get too many ideas. Say more about that.

4. Tell me a bit about comparing your teacher this year with teachers you've had in the past, in terms of that original question: when you see a new problem in your math class, how confident are you that you'll be able to solve it?

5. What other aspects of school affect your confidence level in your math class?

6. What kind of things can you learn and do on your own? What aspects of school encourage or discourage you to learn?

7. What happens in between the teacher saying something and you knowing it?

8. Describe an example of learning mathematics when you were young. Is there anything you see the same between when you were learning math then and when you're learning math now? When you're learning math… or anything different?

9. What has influenced the ways you learn and do mathematics? How you perceive yourself when you're doing mathematics? What role do you have? And what do you think has influenced that?

APPENDIX H

FORMER TEACHER INTERVIEW PROTOCOL

<*teachername*>,

Thank you for being willing to help with my doctoral research project "Generative Adolescent Mathematical Learners." For this project I selected six juniors from your high school and have interacted with them in order to learn about the ways in which they perceive themselves to be engaged in mathematical activity. I selected students with particular qualities, which I incorporate into the name "generative."

By generative, I mean a student who perceives him or herself to be the author of the mathematics he/she does. The generative learner is a student who sees him or herself as a key actor in the creation of mathematical understanding. They see themselves as a producer.

For contrast, the non-generative student would be one who perceives him or herself in a less active role with regards to their mathematical learning. They see themselves as someone who receives mathematics. They are less likely to label their own mathematical activity as mathematics.

I would like to emphasize that it is not necessarily the case that the generative learner would be earning good grades, scoring well on standardized tests, or even demonstrating a strong mathematical aptitude. It is also the case that a generative learner is not necessarily the most active child. My naming of a generative learner is much more focused on the way students perceive themselves to be doing mathematics.

You taught <*subjectname A*> during their sophomore year. Please tell me a little bit about them as learners. You can type into this document then email it back to me, or just write your reply in an email. Or you could print the document and write in it. Also, I am happy to meet with you briefly and audiotape your reply. I will be available Wednesday except 1:00–3:00 and Thursday morning.

In particular, I would like you to respond to these questions:

1. Describe what you recall about this student's *perception of themselves* as a mathematician and/or as mathematics learners.
2. Describe ways this helped them succeed as a student in your class.
3. Describe ways this hindered their success as a student in your class.

Thanks again *teachername*. Enjoy your summer.

Brian Lawler

(706) 338–0578

blawler@uga.edu

Student Name	Class Title	Grade (if you recall)
<subjectname>		
Describe what you recall about this student's *perception of themselves* as a mathematician and/or as mathematics learners.		
Describe ways this helped them succeed as a student in your class.		
Describe ways this hindered their success as a student in your class.		