# GENDER DIFFERENCES IN MATHEMATICS STRATEGIES USED BY THIRD AND FIFTH GRADE CHILDREN

By

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ABSTRACT

This study was designed to determine whether gender differences in strategy use found among first and third grade children continue into the fifth grade. Seventy-eight third and fifth grade children from two suburban public elementary schools participated in this study. Children solved number facts, word problems, extension problems, and non-routine problems individually in the spring. Strategy use was assessed based on observation and the children's reports. Third grade girls were more likely than third grade boys to use strategies utilizing manipulatives across all problem categories. Fifth grade boys were more likely than fifth grade girls to use invented strategies on word problems and across all problem categories. No gender differences were found in the children's use of retrieval or standard algorithms. No gender differences were found in the total number of correct responses for any problem category.

INDEX WORDS: Gender differences, Math strategies, Mathematics.

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# INTRODUCTION

Gender differences in mathematics have received much attention in the literature. It is a common notion that mathematics is a male domain and that females are at a disadvantage to males. Some evidence exists to support this assumption. Significant gender differences favoring males have been found in performance on complex mathematical tasks (Seegers & Boekaerts, 1996). Lummis and Stevenson (1990) found reliable gender differences among third and fifth graders favoring boys in such tasks as the solving of word problems while Marshall (1984) found that the gender differences in word problem solving ability in favor of boys first appeared in the sixth grade. By the end of high school, gender differences in mathematics achievement on standardized tests generally favor males (Carpenter,  Lindquist, Mathews, & Silver, 1983; Swafford, 1980).

Although we see some gender differences diminishing, such as the gender differences in mathematics course enrollment and overall mathematics performance (Carpenter, Lindquist, Matthews, Silver, 1983), gender differences have recently emerged at the fourth grade level with boys outperforming girls in a number of problem categories including items in the geometry, spatial sense, and measurement categories (Ansell & Doerr, 2000). Gender differences are seen again in the mathematics achievement of 17 year old students with males outperforming females (Carpenter et. at., 1983). Females suffer the most in the area of standardized testing such as the SAT-M in which, Johnson (1993) finds, females score anywhere from three to sixty-six points lower than males do but have equal or better grades.

Despite the perception that girls are poorer mathematicians, girls are not always outperformed by their male classmates.  Lummis and Stevenson (1990) found no gender differences when examining the counting skills, conceptual knowledge, or the simple arithmetic skills of kindergarten children from the US, Taiwan, and Japan. Females also tend to be superior to boys in computational, algorithmic activities while males tend to be superior in arithmetic reasoning, application and problem solving (Marshall, 1984; Varmeer, Boekaerts, & Seegers, 2000).

The source of gender differences in standardized mathematics achievement tests is unclear. A number of explanations have been offered some of which range from motivation (Schunk, 1989), socialization (Beal, 1994), spatial processing (Geary, 1996), speed of processing (Royer, Tronsky, Chan, Jackson, Marchant,1999), and early strategy differences (Carr & Jessup, 1997; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Although the current study focuses on gender differences in mathematics strategy use, other explanations of gender differences in mathematics achievement are presented to give the reader a general background.

## Theories Explaining Gender Differences in Mathematics

### *Motivation*

Researchers consistently find that girls have lower self-efficacy in mathematics than boys do (Pajares & Miller, 1994; Seegers & Boekaerts, 1996). According to Bandura and Cervone's (1986) self-efficacy theory, self-efficacy judgments, accurate as well as inaccurate, help determine which activity children are likely to undertake and how much effort they will exert in the face of obstacles. Bandura and Cervone further state that the higher self-efficacy, the less likely one is to give up when faced with difficulties. Also, the more positive the self-efficacy, the harder the individual will work to accomplish a task. This suggests that children with high self-efficacy in mathematics continue to do mathematics, which in turn increases their confidence as well as performance in mathematics. Children with low self-efficacy, on the other hand, avoid mathematics, are more likely to give up, less likely to ask for help, and eventually place less value on mathematics.

*Socialization*

The role of parents cannot be overlooked in the development of children's beliefs. Parents' beliefs and expectations have been related to child's performance on cognitive tasks (McGillicuddy-DeLisi,1985) as well as self-perceptions of ability and achievement expectancies (Jacobs, 1991). In the area of mathematics, parents' beliefs play a particularly important role in which sex differences in attitudes are greater than performance differences (Chipman, Brush, & Wilson, 1985). Parents of girls were found to hold more stereotyped beliefs, favoring males, than parents of boys. Mothers, in particular, have lower mathematics ability beliefs for daughters and higher mathematics ability beliefs for sons. Research also shows that mother's beliefs have a strong influence on the ability beliefs of their children (Jacobs, 1991).

More subtle gender differences by which parents send messages to their children about mathematics have also been found. When compared to parents of boys, parents of girls are less likely to buy mathematics-related games and toys (Astin, 1974) and are more likely to report that mathematics is not as important as other subjects (Parsons, et. al., 1982). As a result, girls get a message that it is OK not to do well in mathematics because mathematics is a male subject and is not important for girls.

*Spatial Processing*

It has been argued that boys' advantage in mathematics is a result of an advantage in visual-spatial skills (Maccoby & Jacklin, 1974). While a number of studies indicated that boys outperformed girls in visual-spatial ability (Connor & Serbin, 1985; Voyer, Voyer, & Bryden, 1995), other studies found no such gender differences (Linn & Peterson, 1985; Manger & Eikeland, 1998).

When looking at a specific spatial ability, however, a number of studies indicated strong and consistent gender differences favoring boys in performance on mental rotation tasks (Linn & Paterson, 1985; Voyer, Voyer, & Bryden, 1995). Another study indicated that performance on mental rotation tasks is related to mathematics performance (Casey, Nutall, et. al., 1995). Using the Vanderberg Mental Rotation Test (Vanderberg & Kuse, 1978), Casey, Nuttall, and colleagues (1995) found large gender differences favoring boys across diverse samples. These studies are in line with the "Bent Twig" theory proposed by Sherman (1987). She maintains that overall, boys tend to have more interest in spatial tasks than girls which further increase the boys' exposure to spatial activities. As a result, boys are more likely to participate in spatial activities when they are available to them and will spend more time on these activities than girls.

*Speed of Processing*

Royer and his colleagues (1999) maintain that males are more likely than females to rapidly and automatically retrieve correct answers to addition, subtraction, and multiplication problems. Furthermore, they believe that math fact retrieval at an early age contributes to enhanced performance on speeded standardized mathematics tests such as the SAT-M.

Speed of processing is significantly related to mathematics test achievement (Royer et al., 1999). Royer and his colleagues divided first through eighth grade children into three categories according to the children's response times to math facts. For each grade level, they formed a fast, an average, and a slow group. For example, in the third grade, Royer and his colleagues selected the four fastest males to form the fast group, four average males to form the average group, and the four slowest males to form the slow group. The same was done for females with the same number of girls in the fast, average, and slow groups. All children responded verbally to addition number facts; 23 out of 25 second grade and all third through eighth grade children were also administered subtraction number facts. Multiplication number facts were administered to third through eighth grade students only. The accuracy of children's responses was recorded along with the accurate as well as inaccurate response times. Royer and his colleagues found that, beginning around the fourth grade, a consistent pattern emerged with faster males having an advantage over faster females in the speed and accuracy of math-fact retrieval. The opposite was true, however, at the low end of the curve with boys tending to be less accurate and slower than the slow girls (Royer et al., 1999).

*Early Strategy Differences*

Other researchers believe that the gender differences in mathematics achievement may be influenced by gender differences in mathematics strategies (Carr & Jessup, 1997; Fennema, et al., 1998). Gender differences in strategy use emerge in first grade and have been documented in second and third grade children (Fennema et al., 1998). Carr and Jessup (1997) found that first grade girls are more likely to use manipulatives while first grade boys are more likely to use retrieval. Fennema and her colleagues also found gender differences in strategy use among first through third graders (Fennema et al., 1998). First through third grade girls were more likely to use manipulatives while third grade boys were more likely to use retrieval and invented strategies. No overall gender differences in correct responses were found in the research by Carr and Jessup (1997) and Fennema and colleagues. However, Fennema and colleagues (1998) found that boys outperformed girls in the solving of extension problems. No research has been done to examine whether gender differences in strategy use continue in the later elementary years.

If early gender differences in strategy use continue into and beyond the later elementary years, they may affect performance on complex mathematics tasks. We know that gender differences exist in the strategies girls and boys use to solve complex mathematics tasks (Seegers & Boekaerts, 1996) and we also know that boys and girls vary in performance on mathematics standardized tests (Johnson, 1993). We do not know whether the early gender differences in strategy use are early precursors to later emerging gender differences in strategy use causing gender differences in mathematics performance. In order to test this possibility, gender differences in strategy use need to be traced to higher grades. The study at hand seeks to determine whether the gender differences in mathematics strategies found among first (Carr & Jessup, 1997) and third graders (Fennema et al., 1998) continue into the fifth grade.

The next section will discuss the research on the development of mathematics strategies and the connection between strategy use and conceptual knowledge. Before we can explore gender differences in strategy use and their connection to mathematics performance, we must understand the typical development of mathematics strategy use in elementary children. Following this will be a section on the contributors to strategy development. Metacognition and conceptual knowledge, and how they are related to strategy development, will be discussed.

## The Development of Strategy Use

Mathematics strategy is defined as any method used to solve mathematics problems. By observing how strategies develop, we can better understand the processes behind strategy use. To give the reader a general background on the development of strategies, the typical development of mathematics strategies will be discussed in this section.

### *Addition*

Although children learn to use different strategies at different times, a general pattern of strategy development exists. Around four years of age, children begin to use overt strategies to count (Siegler & Jenkins, 1989). At this point they tend to use fingers, counters, or other external objects to represent numbers and operations with the numbers. To add, children start by putting out fingers, or counters, to represent each number in the problem and then count all of the fingers or counters; this is called the *counting-all* strategy. Children then advance to the *counting-on-max* strategy; they start counting from the smaller addend and count on the larger addend. For example, to count 3 + 6, the child counts: “3, 4, 5, 6, 7, 8, 9 - the answer is 9.” This is a transitional strategy in that some children use the *counting-on-max* strategy and some do not use it at all. As soon as children learn that it is easier to count from the larger addend, they rely on the *counting-on-min* strategy. Using the *counting-on-min* strategy, they start counting on from the larger addend. As an example, to solve 3 + 6, the child will now count: “6, 7, 8, 9, the answer is 9”(Ashcraft, 1982, 1987; Kaye, Post, Hall, & Dineen, 1986; Svenson, 1975). Both, the *counting-on-max* and *counting-on-min,* strategies usually develop in the first grade.

Around the age of seven, children become increasingly able to make mental representations which allow them to mentally count without using external objects. During the first grade, some children also start using *retrieval,* that is, they are able to recall the answer to a problem from their memory without needing to do any computations (Carr & Jessup, 1997). By the time children enter second grade, they rarely use the *counting-all* strategy; rather they may use the *count-on-max* or *count-on-min* strategy(Carpenter & Moser, 1984; Siegler, 1987). They are also more likely to count verbally rather than with the help of counters or other manipulatives such as fingers (Carpenter & Moser, 1984; Siegler, 1987).

In later elementary years, children begin to rely more on retrieval and invented strategies to solve addition and subtraction problems (Carpenter & Moser, 1984). Invented strategies are those strategies that are created by children by manipulating numbers. They are not algorithms taught step by step by the teacher. For example, to solve 18 + 23 using an invented strategy, a child might proceed through the following steps: “10 + 20 = 30, then 8 + 30 = 38, 38 + 2 = 40 and now add the 1, the answer is 41.” By the sixth grade, children appear to be using retrieval to solve the majority of basic math fact addition problems (Ashcraft & Fierman, 1982; Geary, Widaman, Little, & Cormier, 1987; Kaye, et al., 1986).

*Subtraction*

When children are about five years old, they start subtracting by using manipulatives, such as fingers or other counters. Manipulatives allow children to physically represent the numbers and keep track of the subtraction process. When children first begin to learn subtraction, at about 5 and 6 years old, they initially use *counting down* and later develop the *counting up* strategy. When counting down, a child counts down from the minuend the number of times of the value of the subtrahend. For example, to solve 11 - 6, the child will count: "10, 9, 8, 7, 6, 5 the answer is 5." Many children hold up their fingers and then count backward putting their fingers down one by one until they arrive at the answer. When using the *counting up* strategy, the child starts with the subtrahend and counts up one by one until the minuend is reached. Therefore, the child counts: " 7, 8, 9, 10, 11, the answer is 5," to solve the '11 - 6' problem.

Although some children are able to successfully use retrieval as early as first grade (Carr & Jessup, 1997), most children do not begin using more complex strategies for subtraction, such as invented strategies, until in the later elementary years (Carpenter & Moser, 1984)**.** Ilg and Ames (1951) found that by third grade, children know most, if not all, of the basic subtraction facts "by heart." It is also around this time that most teachers introduce standard algorithms and children become increasingly dependent on using them to add, subtract, and multiply. Subtracting using columns is a common standard algorithm taught to children. While most children write down the columns to keep track of the subtraction process, some become able to mentally represent the columns without having to write them down (Fuson & Kwon, 1992b). This is referred to as the *columnar retrieval* strategy. When children solve problems using this strategy, they go through the same process as they would using pen and paper, except they do all of the computations in their head.

*Multiplication*

The development of multiplication strategies mirrors the development of addition and subtraction skills. Children first begin multiplying by modeling using their fingers or counters. The earliest strategy to develop is *direct counting* by which children model with counters one by one as they count (Kouba, 1989; Mulligan & Mitchelmore, 1997). Most children then proceed to one of two common strategies when first learning to solve simple multiplication problems. Using the *repeated addition* strategy, the child represents the first number the number of times that is indicated by the second number and then adds those numbers together. To solve 2 x 3, the child will count: "2 + 2 is 4, 4 + 2 is 6, the answer is 6,"(Geary, 1994) . *Counting by n* is another strategy used by children when first learning to multiply. This strategy is dependent on being able to count by 2s, 3s, and so on. Using this strategy, the child counts the sequence of the numbers. For example, to solve the above problem, the child counts: "2, 4, 6, the answer is 6." As children count, they hold up their fingers for each number until they are holding up the number of times indicated by the multiplier (Geary, 1994). This is called direct modeling; as children become better able to represent numbers mentally, they stop using manipulatives.

Finally, children begin using retrieval and invented strategies some time in the later elementary years (Geary, 1994) . To use invented strategies, children break down the multiplication problem into several manageable tasks. For example to multiply 13 x 4, the child counts: "10 x 4 = 40, 3 x 4 = 12, 40 + 12 = 52, the answer is 52." As the standard algorithm is introduced by teachers, many children increasingly multiply using columns. At this point in time, some children begin using columnar retrieval to multiply; they solve the problem by retrieving columnwise products (Geary, 1994). To solve 13 x 4, the child counts: " 4 x 3 = 12, note the 12, 4 x 10 = 40, now 40 + 12 = 52, the answer is 52."

*Division*

Children's ability to divide depends on their ability to add and multiply. The first strategy usually used by children to divide involves a form of repeated addition. The child starts with the divisor and adds the value of the divisor until the dividend is reached (Ilg & Ames, 1951). So, to solve 15 / 3, the child counts: " 3 + 3 = 6, 6 + 3 = 9, 9 + 3 = 12, 12 + 3 = 15, the answer is 5."  In the second strategy children use to divide, they rely on their knowledge of multiplication (Ilg & Ames, 1951). Using this strategy, the children solve a problem by multiplying the divisor by *n* to obtain the dividend. For example, to solve 15 / 3, a child uses his or her knowledge that 5 x 3 is 15 to arrive at the answer, which is 5 in this case.  If the child has not mastered his or her times table yet, the child will count: "3 x 2 = 6, 3 x 3 = 9, 3 x 4 = 12, and 3 x 5 = 15, the answer is 5."

<u>Variability in Strategy Use</u>

Great variability exists in the use of mathematics strategies. Variability in children's strategy use is evident as early as kindergarten with some children already using counters, some needing minimal instruction to count with counters and others requiring considerable instruction in the use of counters to count all (Baroody, 1987). Young children use a variety of strategies with an individual child often using up to five or more different approaches to solving problems (Baroody, 1984; Carpenter & Moser, 1982). The use of increasingly mature strategies is not characterized by simply substituting one strategy for another (Ashcraft, 1982). Rather, the development of strategies can be seen as a mix of existing strategies combined with the construction of new ones and abandonment of old ones (Siegler & Jenkins, 1989).

According to Siegler's Adaptive Strategy-Choice Model (1996), children generate a variety of strategies to solve a particular problem. Depending on the nature of the task and the child's goals, certain strategies are selected and become used more frequently than other strategies. With practice and maturation, the child gradually begins to use the newly acquired strategies more frequently causing the strategies to become easier to use. Eventually, the child abandons simpler, less effective strategies for more complex, but effective strategies. Therefore, according to Siegler (1996), the development of strategies does not proceed a step-like fashion. Rather, it can be viewed as a series of overlapping waves with the distribution of the waves changing over time.

Although multiple strategies are available to children of all ages, the frequency with which strategies are used changes with age (Bjorklund, 2000). While older children rely on verbal counting strategies more than on counting on their fingers or using counters, they occasionally use simpler strategies, such as counting on their fingers, as a back up (Siegler,1987). Siegler also found that although younger children can use retrieval on simple problems; they use counters for more complex problems.

Children also vary in how abruptly they shift from one strategy to another. Children who use a variety of strategies are more gradual in shifting than children who use fewer strategies (Alibali, 1999). Furthermore, when children are given instructions to use a particular strategy, they appear to shift abruptly to start using the instructed strategy as their dominant strategy (Alibali, 1999).

Contributors to Strategy Development

A number of factors come into play in the development of mathematics strategies. In this section, conceptual knowledge and metacognition and how they are related to strategy development will be discussed. A positive relationship was found between conceptual knowledge and strategy use (Rittle-Johnson & Siegler, 1998; Cauley, 1988). It was also found that children’s metacognitive knowledge plays a role in how fast they begin to use newly discovered strategies (Crowley, Shrager, & Siegler, 1997). Furthermore, monitoring themselves as they work on problems, seems to lead to a gain in children’s conceptual knowledge and strategy development in mathematics (Chi, Bassok, Lewis, Reimann, & Glasser, 1989; Renkl, 1997).

We know that conceptual knowledge and strategy use are positively related (Rittle-Johnson & Siegler, 1998; Cauley, 1988). Most kindergarten children, for example, understand that each addend must be represented once and only once and that the order of addends is irrelevant, before they invent and start using the min strategy (Siegler & Crowley, 1994). Results from a longitudinal study following children from first grade to the beginning of fourth grade also support a relationship between conceptual understanding and strategy use. Children having conceptual understanding of multidigit addition and subtraction were more able to invent and adopt strategies than their classmates who did not show the same conceptual understanding of multidigit addition and subtraction (Hiebert & Wearne, 1996). Accordingly, children with conceptual knowledge seem to be more likely to develop and use invented strategies than children lacking conceptual understanding (Hiebert & Wearne, 1996). Thus, conceptual understanding plays a major factor in how children do mathematics.

Another contributor to the development of strategies is metacognition. Children who monitor and explain things to themselves as they read and solve problems, are more likely to acquire conceptual knowledge and develop new and better strategies than children who do not monitor themselves (Renkl, 1997). Also, children who monitor their strategy use and reflect more on their mathematics become better able to use and understand complex strategies (Carr & Jessup, 1997). Thus, the children develop better problem-solving skills. In contrast, children who believe that mathematics is a rote application of procedures might be less likely to reflect on their strategy use and mathematics knowledge (Carr & Jessup, 1997). They are less likely to develop invented strategies reflecting conceptual knowledge.

Also, children's metacognitive knowledge plays a role in how fast they begin to use newly discovered strategies. Children show variability in how much insight they have into the strategies they discover (Crowley, Shrager, & Siegler, 1997). Children who show the greatest level of explicit insight, or metacognitive awareness of the strategy, at the moment they discover the strategy, generalize the strategy faster and more completely than children not showing such insight. Children who discover a strategy without being able to provide a reasonable explanation for how they discovered the strategy, are less likely to generalize the strategy to other problems. As a result, they continue to use less efficient strategies. Thus, metacognitive knowledge seems to accelerate the generalization process (Crowley et al., 1997).

The above literature indicates that strategies develop in a consistent manner but that variability exists in children's strategy use. Conceptual understanding and metacognition are major contributors to mathematics strategy development but are not currently thought to be the source of gender differences in strategy use. The next section will discuss the research on gender differences in strategy use.

<u>Gender Differences in Strategy Use</u>

While some studies have indicated gender differences in the mathematics strategy use of children in the kindergarten and early elementary years, others have not. No gender differences were found in mathematics strategies used to solve simple addition and simple subtraction problems among Chinese and American kindergarten children (Lummis & Stevenson, 1990).

Although no gender differences were found in strategy use by kindergarten children, gender differences in mathematics strategy use began to emerge in first grade with American girls being more likely than American boys to count on their fingers when solving simple addition and simple subtraction problems (Geary, Fan, & Bow-Thomas, 1992). Similar results indicating that girls and boys use different strategies to solve mathematics problems were found in a number of studies (Carr & Jessup, 1997; Carr, Jessup, & Fuller, 1999; Fennema et al, 1998). After interviewing first graders about strategies used in addition and subtraction problems, Carr and Jessup (1997) concluded that, although no gender differences were found in total correct responses, girls were more likely to count on their fingers while boys were more likely to use retrieval. Although it is not clear why these gender differences exist, it was found in a subsequent study that after controlling for strategy use, first grade girls were less capable than first grade boys in their retrieval of arithmetic problems from memory (Carr & Davis, 2001). This suggests that some differences in strategy use are, to some extent, based on differences in retrieval skill. While these findings indicate that boys' tendency to use retrieval and girls' tendency not to use retrieval is based on their differences in ability to retrieve facts from memory, these findings should be approached with caution. Because gender differences in strategy use have only recently emerged, they might be of contextual, rather than inherent nature, perhaps due to practice effect (Carr & Davis, 2001). For example, Geary, Fan, and Bow-Thomas (1992) found gender differences in the strategy use of American first grade children but not Chinese first grade children suggesting that these differences do not exist in all children, rather, they reflect differences in instruction.

Fennema and her colleagues (1998) found similar gender differences in mathematics strategies among first, second, and third graders. As early as first grade, girls were more likely to use strategies involving counters or counting on their fingers, while boys tended to use invented strategies (Fennema et al.,1998). Gender differences in strategy use continued into the third grade with girls using significantly more standard algorithms than boys. No gender differences were found in the number of correct solutions to number facts, word problems, or non-routine problems, those involving multiple steps and requiring interpretation and analysis. However, third grade boys solved significantly more extension problems, those requiring flexible knowledge of place value and multidigit operations, than third grade girls did.

After separating the students in the second grade into either *invented algorithms group,* which consisted of children who used invented strategies by the fall of second grade, or *standard algorithms group,* made up of students moving directly from using counting strategies to using standard algorithms, two divergent patterns of learning multidigit procedures emerged. The *invented algorithms group* seemed to have developed a conceptual understanding, knowledge of place value and multidigit operations, which was evident in their ability to operate flexibly with large numbers. On the other hand, the *standard algorithms group* seemed to have started using standard algorithms before they showed the conceptual understanding required to develop invented strategies. No gender differences in correct responses were found within either of the groups. However, third grade girls in the *invented algorithms group* solved significantly more extension problems than third grade girls in the *standard algorithms group*. These findings suggest that using invented strategies in the first and second grades seems to be beneficial for solving extension problems successfully in the third grade.

Regarding gender differences in older children, similar gender differences in strategy use were found in certain areas of mathematics among high school students. Gallagher and DeLisi (1994) found that high achieving adolescent girls were more likely to rely on conventional strategies while high achieving adolescent boys tended to use unconventional strategies when solving items on the SAT-M. As in the other studies, no overall differences in the number of items answered correctly were found.

However, the females outperformed the males in the solving of conventional problems, those with a well defined method of solution, while the males outperformed the females in the solving of unconventional problems, those requiring the use of an atypical solution or an unfamiliar algorithm, or those problems that could be solved faster using some type of estimation or insight.

## CURRENT STUDY

The current study sought to determine whether the gender differences in strategy use of first graders (Carr & Jessup, 1997) and third graders (Fennema et al., 1998) continue into the fifth grade. It was hypothesized that the gender differences in strategy use seen in first grade and third grade, with the emerging gender differences among third graders in the ability to solve extension problems, will continue into the fifth grade. It was hypothesized that third and fifth grade boys will be more likely than third and fifth grade girls to use retrieval and invented strategies while third and fifth grade girls will be more likely than third and fifth grade boys to use manipulatives and standard algorithms.

It was also hypothesized that the gender differences in strategy use, of both third and fifth grade children, will be accompanied by gender differences in performance on complex word problems, specifically, extension problems. Boys in the third and fifth grade were hypothesized to outperform girls in the third and fifth grade on extension problems, respectively.

# METHOD

## Participants

Seventy-eight students from two elementary schools serving a suburban middle to upper class population in north-east Georgia participated in the study. One of the schools was involved in ability grouping. For that school, beginning in the fifth grade, all children were separated into classrooms based on a their mathematics ability. The children's mathematics ability was determined at the beginning of the school year by administering a test which covered all areas of the mathematics curriculum which reflected the McMillan and McCraw textbooks. About 35 to 40 percent of children were assigned to the intermediate group, the rest were either in the highly focused or exceptional groups. As the year progressed, students were moved between the three ability groups as needed based on their progress in mathematics. Twenty-two fifth graders, 14 boys and 8 girls from the intermediate group were used for the current study. The rest of the children came from traditional classrooms not involved in ability grouping.

All 78 students agreed to participate with the informed consent of their parents and teachers. The third grade sample consisted of 21 girls, with the mean age of 9 years and two months (SD = .45) and 21 boys with the mean age of 9 years and 4 months (SD = .50). Thirty-three White, 5 African American, and 2 Hispanic third grade children participated. The fifth grade sample consisted of 17 girls, with the mean age of 10 and 9 months (SD = .25), and 20 boys with the mean age of 11 and 1 month (SD = .45). Thirty-four fifth grade children were White and two were African American.

## Procedure and Materials

Starting in January, the children were interviewed individually in a quiet place outside of the classroom. Each interview took approximately 30 to 45 minutes and was videotaped to capture both verbal as well as non-verbal responses. Participants were instructed that the researcher wanted to know how children solve problems and that after each solution the interviewer would ask the children how they arrived at the answer. The interviewer also told the children that although it is very important to do their best, the results would not be shown to their teacher.

Children were allowed the time necessary to solve each problem. A pencil with an eraser, paper, and counters were present for all problems. Each child was given 5 number facts, 5 word problems, 5 non-routine problems, and 5 extension problems. The problems, similar to those used by Fennema and colleagues (1998), were carefully constructed by the author and checked by a team of teachers from one of the schools involved in the current study to ensure for the appropriate difficulty level. See Appendix A for third grade problems and Appendix B for fifth grade problems. All names in all problems were gender neutral, for example Sam or Alex, to minimize any possible gender bias. All children at each grade level received the same problems. The problems were randomly presented. All problems were written on a half of sheet of paper; they were first read to the children and then placed on their desk as the children were solving the problems. Problems were reread at children's request; no help in solving the problems was provided.

The children's responses to each question and observation of behavior were used to code the strategies. All addition and subtraction strategies were categorized as guessing, counting on fingers or verbal counting, invented strategies, standard algorithms, or retrieval (Carr & Jessup, 1997). They were further coded as correct or incorrect. All problem categories were categorized using the same criterion.

Multiplication and division problems were coded as strategies utilizing manipulatives, guessing, standard algorithm, invented strategy, or retrieval. All strategies involving the use of fingers or counters were categorized as strategies utilizing manipulatives. All problems were further categorized as correct or incorrect. Because the current study only had one multiplication number fact and one division number fact, all five number fact problems were collapsed across before analysis was calculated.

For all types and categories of problems, when a child provided an answer without verbally counting or using manipulatives, the interviewer asked the child how he or she arrived at the answer. If the child said that he or she "just knew it," and the amount of time to solve the problem was short, the strategy was coded as retrieval. If the child took a long time to solve the problem and than stated that he or she "just knew it," the interviewer further inquired what number the child started counting from. Based on the child's answer, the strategy was coded either as retrieval or verbal counting. If the child quickly provided an answer without doing any obvious calculations, the strategy was also coded as retrieval.

To be coded as a standard algorithm, the interviewer looked for the use of columns. If the child solved the problem by using columns on a paper, the strategy was coded as standard algorithm. If the child said he or she counted in his or her head in the same way he or she would have done using columns on a paper, the strategy was also coded as a standard algorithm. To be coded as an invented strategy, the interviewer looked for the breaking down of a problem into more manageable parts. For example, the child might report that to add 38 + 26 he or she solved "30 + 20 is 50, then 8 more makes 58, add 6 and the answer is 64"(Fuson, Wearne, Hiebert, Human, Murray, Olivier, Carpenter, & Fennema, 1997).

Children mostly used a single strategy to solve number facts. Because children sometimes tried several strategies before arriving at an answer for the word problems, extension problems, and non-routine problems, the last strategy used to solve the problem was identified as the strategy used for that particular problem. New categories of strategies were added as necessary to ensure that all strategies were coded appropriately.

# RESULTS

Analyses were run separately on third grade children and fifth grade children because different problems were used to assess strategy use. The first step in the analysis was to determine whether the gender differences in strategy use found in the prior studies were replicated (Carr & Jessup, 1997; Fennema et al., 1998; Carr & Davis, 2001). To determine whether boys were more likely than girls to retrieve from memory, gender differences in attempted and correct use of retrieval, invented strategies, strategies using manipulatives, and standard algorithms by girls and boys at each grade level were examined. At each grade level, strategy use within the four different categories of problems were analyzed separately to determine whether gender differences in strategy use were affected by problem type. The overall strategy use on all problems at each grade level and how it related to gender was then explored. The means and standard deviations for boys' and girls' attempted and correct use of each strategy on each problem category are listed in Table 1. It was also determined whether gender differences at each grade level were present in the total number of correct responses for each problem category.

*Retrieval*

To determine whether boys were more likely than girls to attempt retrieval, two ANOVAS, one for third grade and one for fifth grade, were run with attempted retrieval on number facts as the dependent variable and gender as the independent variable. No gender effect was found for third grade children, $F(1,39) = .39$, $p > .05$, or fifth grade children, $F(1,35) = .60$, $p > .05$. Two ANOVAS, one for third grade and one for fifth grade, were also performed with correct retrieval on number facts as the dependent variable and gender as the independent variable. No significant effect for gender was indicated for third grade students, $F(1,39) = .11$, $p > .05$, or for fifth grade students, $F(1,35) = .60$, $p > .05$.

Although retrieval might have been used by the children on word problems, extension problems, or non-routine problems, no additional analysis on retrieval were done because none of the children used retrieval to solve the problems in these categories unless it was used as a part of a standard algorithm or invented strategy.

*Invented strategies*

To determine whether boys were more likely than girls to use invented strategies to solve number facts, two ANOVAS, one for third grade and one for fifth grade, were performed with the attempted use of invented strategies on number facts as the dependent variable and gender as the independent variable. No significant gender differences for third grade, $F(1, 39) = .62$, $p > .05$, or fifth grade, $F(1,35) = 3.62$, $p > .05$, were found. The relationship of gender and correct use of invented strategies on number facts was explored next. Two ANOVAS, one for third grade and one for fifth grade, with the correct use of invented strategies on number facts as the dependent variable and gender as the independent variable indicated no significant gender differences for third or fifth grade children, $F(1,39) = .46$, $p > .05$, $F(1,35) = 3.03$, $p > .05$, respectively.

The use of invented strategies on word problems was examined next. Two ANOVAS, one for third grade and one for fifth grade, with the attempted use of invented strategies on word problems as the dependent variable and gender as the independent variable indicated a significant gender effect in fifth grade, with boys being more likely than girls to attempt invented strategies on word problems, $F(1,35) = 4.67$ $p = .04$. No gender effect was indicated for third grade children, $F(1,39) = 1.01$, $p > .05$.

The data for gender differences in the correct use of invented strategies on word problems were explored to determine whether gender differences in the correct use of invented strategies on word problems were mirrored in gender differences in attempted use of invented strategies on word problems. To do this, two ANOVAS, one for third grade children and one for fifth grade children, were calculated with the correct use of invented strategies on word problems as the dependent variable and gender as the independent variable. As a result, no gender effect for third grade students was indicated $F(1,39) = 1.01$, $p > .05$. However, a significant gender main effect was indicated for fifth grade children, $F(1,35) = 5.36$, $p = .03$, with boys being more likely than girls to correctly use invented strategies when solving word problems.

In order to determine whether boys were more likely than girls to attempt invented strategies on extension problems, two ANOVAS, one for third grade and one for fifth grade, with the attempted use of invented strategies on extension problems as a dependent variable and gender as the independent variable were performed. No gender effect was indicated for third grade, $F(1, 39) = 2.42$, $p > .05$, or fifth grade, $F(1,35) = 1.59$, $p > .05$. To determine whether boys were more likely than girls to correctly use invented strategies when solving extension problems, two ANOVAS, one for third grade and one for fifth grade, were calculated with the correct use of invented strategies on extension problems as the dependent variable and gender as the independent variable. No gender differences were found for third grade students, $F(1,39) = 2.09$, $p > .05$, or fifth grade students, $F(1,35) = .85$, $p > .05$.

After observing the frequencies of the attempted use of invented strategies on extension problems, with $M = .30(SD = .95)$ for third grade and $M = .08(SD = .36)$ for fifth grade children, a floor effect was noted which might have accounted for the lack of significant results as were indicated by previous studies (Fennema et al., 1998). Similar floor effect was noted after observing the frequencies of correct uses of invented strategies to solve extension problems, with $M = .20(SD = .68)$ for third grade and $M = .03(SD = .16)$ for fifth grade.

To determine whether gender differences were present in the use of invented strategies on non-routine problems, two ANOVAS, one for third grade and one for fifth grade, with the use of attempted invented strategies on non-routine problems as a dependent variable and gender as the independent variable were calculated. No gender differences in the use of attempted invented strategies on non-routine problems were found for third grade, $F(1, 39) = .30$, $p > .05$, or fifth grade, $F(1, 35) = 1.21$, $p > .05$. Further analysis of correct use of invented strategies on non-routine problems was calculated using two ANOVAS, one for third grade and one for fifth grade, with the correct uses of invented strategies on non-routine problems as the dependent variable and gender as the independent variable. No significant main effect was indicated for third or fifth grade children, $F(1,39) = .30$, $p > .05$, and $F(1,35) = .49$, $p > .05$, respectively.

Again, the distribution of scores for the attempted use of invented strategies on non-routine problems indicated a floor effect which might have prevented the detection of any significant main effect with M = .07(SD = .26) and M = .20(SD = .64) for third and fifth grade children respectively. Furthermore, a floor effect was noted for the correct uses of invented strategies on non-routine problems for both third and fifth grade children, M = .07( SD = .26) and M = .11(SD = .39) respectively.

*Strategies Utilizing Manipulatives*

To determine whether girls were more likely than boys to use manipulatives to solve number facts, two ANOVAS, one for third grade and one for fifth grade, with attempted strategies utilizing manipulatives on number facts as the dependent variable and gender as the independent variable were calculated. No gender effect for third grade children, $F(1,39) = 2.01$, $p > .05$ or fifth grade children, $F(1,35) = 1.84$, $p > .05$, was indicated. It was next determined whether any gender differences existed in the correct use of strategies utilizing manipulatives on number facts. Two ANOVAS, one for third grade and one for fifth grade, with the correct use of manipulatives on number facts as the dependent variable and gender as the independent variable revealed no main effect for third grade, $F(1,39) = .43$, $p > .05$, or fifth grade, $F(1,35) = .18$, $p > .05$.

After careful observation of first the frequencies of attempted strategies utilizing manipulatives and then correctly used strategies utilizing manipulatives on number facts, a floor effect was noted for third grade, with $M = .8(SD = 1.02)$ and $M = .7(SD = .85)$ for attempted and correctly used manipulatives respectively. A similar floor effect was noted for fifth grade for both attempted and correctly used manipulatives, $M = .14(SD = .42)$. Thus, the failure to detect a gender effect may be attributed to the floor effect.

To determine whether girls were more likely than boys to use strategies utilizing manipulatives when solving word problems, two ANOVAS, one for third grade and one for fifth grade, were performed with attempted strategies utilizing manipulatives as the dependent variable and gender as the independent variable. No gender differences were indicated for either third, $F(1,39) = 3.22$, $p > .05$ or fifth grade children, $F(1, 35) = .02$, $p > .05$. Two ANOVAS, one for third grade and one for fifth grade, with correct use of strategies utilizing manipulatives on word problems as the dependent variable and gender as the independent variable indicated no gender effect for third, $F(1,39) = 1.35$, $p > .05$, or fifth grade children, $F(1, 35) = .2$, $p > .05$.

Upon observing the frequencies of attempted and correct uses of strategies utilizing manipulatives on word problems, a floor effect was noted for fifth grade children with $M = .11(SD = .39)$ and $M = .08(SD = .28)$ respectively, which might have had a role in the failure to detect a significant gender effect. Although a floor effect was not observed for the attempted uses of manipulatives on word problems by third grade children, the frequency distribution of correct uses of manipulatives yielded a floor effect, $M = .5(SD = .75)$.

Gender differences in the use of strategies utilizing manipulatives when solving extension problems were explored next. Two ANOVAS, one for third grade and one for fifth grade, with children's attempted use of strategies utilizing manipulatives on extension problems as the dependent variable and gender as the independent variable, indicated no main effect at either grade level, $F(1,39) = .83$, $p > .05$ and $F(1,35) = .03$, $p > .05$, for third grade students and fifth grade students respectively. Furthermore, two ANOVAS, one for third grade and one for fifth grade, with correct uses of strategies utilizing manipulatives on extension problems as the dependent variable and gender as the independent variable indicated no main effect for third grade, $F(1,39) = 3.34$, $p = .08$, or fifth grade, $F(1,35) = .29$, $p > .05$.

After a careful observation of the frequencies of attempted strategies utilizing manipulatives, a floor effect was noted which might have contributed to the failure to detect any significant main effect, $M = .70(SD = 1.09)$ and $M = .43(SD = .65)$, for third grade and fifth grade students respectively. Also, a floor effect was noted for the distribution of correct uses of strategies utilizing manipulatives on extension problems for third grade children as well as fifth grade children, $M = .20(SD=.51)$ and $M = .35(SD = .59)$, respectively.

Next, two ANOVAS, one for third grade and one for fifth grade, were run with the attempted strategies utilizing manipulatives on non-routine problems as the dependent variable and gender as the independent variable. Main effects for third grade children, $F(1,39) = 1.61$, $p > .05$, and fifth grade children, $F(1,35) = .02$, $p > .05$, were not indicated. Two ANOVAS, one for third grade and one for fifth grade, with the correct use of manipulatives on non-routine problems as the dependent variable and gender as the independent variable also indicated no gender main effect for third or fifth grade children, $F(1,39) = 1.37$, $p > .05$ and $F(1,35) = .03$, $p > .05$, respectively.

A floor effect which might have lessened the chances of a significant result being detected was observed for the distribution of attempted strategies utilizing manipulaives on non-routine problems for fifth grade only, $M = .06(SD = .95)$. A similar floor effect was noted for distributions of correct uses of manipulatives on non-routine problems for third, $M = .50(SD = 1)$ and fifth $M = .20(SD = .58)$ grade.

*Standard algorithms*

To determine whether girls were more likely than boys to use standard algorithms on number facts, two ANOVAS, one for third grade and one for fifth grade, with the total attempted standard algorithms on number facts as the dependent variable and gender as the independent variable were calculated. No gender differences were indicated for third, $F(1, 39) = .00$, $p > .05$ or fifth grade, $F(1, 35) = .78$, $p > .05$. Next, two ANOVAS, one for third and one for fifth grade, were run with the correct uses of standard algorithms on number facts as the dependent variable and gender as the independent variable. No gender differences were indicated for third grade children, $F(1,39)=.22$, $p>.05$. Also, no gender differences were found for fifth grade children, $F(1,35)=.17, p>.05$.

In order to determine whether girls were more likely than boys to solve word problems using standard algorithms, two ANOVAS, one for third grade and one for fifth grade, with the attempted use of standard algorithms on word problems as the dependent variable and gender as the independent variable were calculated. No gender effect was found for third, $F(1,39) = .38$, $p > .05$, or fifth, $F(1,35) = .41$, $p > .05$, grade children. Two ANOVAS, one for third grade and one for fifth grade, with the correct uses of standard algorithms on word problems as the dependent variable and gender as the independent variable were calculated next. No gender differences were indicated for either third, $F(1, 39) = .15$, $p > .05$, or fifth grade students, $F(1,35) = 1.40$, $p > .05$.

To see whether girls were more likely than boys to solve extension problems using standard algorithms, two ANOVAS, one for third grade and one for fifth grade, with the children's attempted use of standard algorithms on extension problems as the dependent variable and gender as the independent variable were calculated. No gender effect was indicated for third or fifth grade students, $F(1,39) = .89$, $p > .05$, and $F(1,35) = .45$, $p > .05$, respectively. Two ANOVAS, one for third grade and one for fifth grade, with the correct uses of standard algorithms as the dependent variable and gender as the independent variable yielded similar results. No gender effect was detected for third or fifth grade children, $F(1,39) = 1.28$, $p > .05$ and $F(1,35) = .36$, $p > .05$, respectively.

To determine whether any gender differences existed in the children's use of standard algorithms when solving non-routine problems, two ANOVAS, one for third grade and one for fifth grade, with the attempted uses of standard algorithms on non-routine problems as the dependent variable and gender as the independent variable were run. No gender effect was indicated for either third or fifth grade children, $F(1,39) = .27$, $p > .05$ and $F(1,35) = .85$, $p > .05$, respectively. Next, two ANOVAS, one for third grade and one for fifth grade, with the correct uses of standard algorithms on non-routine problems indicated no gender effect for third grade $F(1,39) = .40$, $p > .05$ or fifth grade $F(1,35) = 1.65$, $p > .05$.

A careful observation of the distribution of frequencies of attempted standard algorithms on non-routine problems revealed a floor effect for third grade children, M = 1.30(SD = 1.30), which might have accounted for our failure to detect a significant gender effect for third grade children's use of standard algorithms on non-routine problems. A floor effect was also noted for the distribution of correct uses of standard algorithms on non-routine problems for third as well as fifth grade children, M = .39(SD = .59) and M = .90(SD = .99), respectively.

*Overall strategy use*

The small number of problems in each category might have accounted for the failure to detect gender differences in strategy use. In order to determine whether gender effects would emerge across a larger number of problems, all problem categories were collapsed and analyses were run for each strategy.

To determine whether boys were more likely than girls to use invented strategies overall, two ANOVAS, one for third grade and one for fifth grade, with the attempted use of invented strategies as the dependent variable and gender as the independent variable were calculated. No gender effect was indicated for third grade children, $F(1, 39) = 1.01$, $p > .05$; however, a significant effect was indicated for fifth grade children with boys being more likely than girls to attempt invented strategies overall, $F(1,35) = 4.67$, $p = .04$. Additionally, two ANOVAS, one for third grade and one for fifth grade, with the correct uses of invented strategies as the dependent variable and gender as the independent variable indicated a similar significant gender effect for fifth grade children with boys correctly using more invented strategies than girls, $F(1,35) = 5.17$, $p = .03$. Again, no gender effect was found for third grade children indicating that girls were as likely as boys to attempt and correctly solve problems using invented strategies. A careful observation of the distribution of frequencies of correct uses of invented strategies indicated a floor effect for third grade children, $M = .80(SD = 2.00)$, which may have accounted for the failure to detect a significant gender effect.

To see whether girls were overall more likely than boys to use strategies utilizing manipulatives, two ANOVAS, one for third grade and one for fifth grade, with the attempted use of strategies using manipulatives on all problems as the dependent variable and gender as the independent variable were calculated. No significant gender effect was indicated for either third or fifth grade children, $F(1,39) = 2.94$, $p > .05$ and $F(1,35) = .64$, $p > .05$ respectively. Two ANOVAS, one for third grade and one for fifth grade, with correct uses of strategies utilizing manipulatives as the dependent variable and gender as the independent variable indicated a significant gender effect for third grade, $F(1,39) = 4.94$, $p = .03$, with girls correctly using more strategies utilizing manipulatives than boys. No such interaction was found for fifth grade children, $F(1,35) = .04$, $p > .05$.

After separately observing the frequencies of attempted and correctly used strategies utilizing manipulatives, it was noted that a floor effect was present for the attempted and correct use of manipulatives for fifth grade children, $M = 1.40(SD = 1.52)$

and M = .80(SD = 1.08). Thus, the failure to detect a significant gender main effect at the fifth grade level might have been a result of the minute number of children using these strategies.

The use of standard algorithms across all problems was then explored. To determine whether girls were more likely than boys to use standard algorithms overall, two ANOVAS, one for third grade and one for fifth grade, with attempted use of standard algorithms as the dependent variable and gender as the independent variable were calculated. No gender effect was indicated for third or fifth grade children, $F(1,39) = 0$, $p > .05$ and $F(1,35) = 1.48$, $p > .05$ respectively. The correct use of standard algorithms across all problems was then explored. Two ANOVAS, one for third grade and one for fifth grade, with correct use of standard algorithms across all problems as the dependent variable and gender as the independent variable indicated no gender effect for third or fifth grade, $F(1,39) = .13$, $p > ,05$ and $F(1,35) = 1.64$, $p > .05$ respectively, indicating that both boys and girls were using standard algorithms at each of the grade levels.

No additional analyses were calculated on retrieval because, unless used in combination with other strategies, the children retrieved answers from memory to basic number fact problems only. No significant gender differences in the number of correct responses to number facts, word problems, extension problems, or non-routine problems were found.

# DISCUSSION

The results of this study replicate earlier research indicating that boys are more likely than girls to use invented strategies when solving mathematics problems. However, the gender differences found in the current study do not reflect those found by other researchers (Fennema et al.,1998). Specifically, Fennema and colleagues found gender differences in the use of invented strategies in the third grade. The current study found gender differences in the use of invented strategies among fifth grade children only. In particular, boys were found to attempt and correctly use invented strategies on word problems more often than girls. Gender differences in the use of invented strategies were not evident in any other problem category, however, when collapsed across all problem categories, gender differences emerged with boys attempting and correctly using invented strategies more often than girls. It should be noted, however, that although the boys used invented strategies more than girls did, both genders rarely used them, See Table 3.

The gender differences in the use of invented strategies should be interpreted with caution. An examination of the means shows that neither girls nor boys used invented strategies frequently. Boys attempted invented strategies more than girls, but they only did so 1.55 times in the solving of the twenty problems while girls attempted them .24 times. One reason for the failure to find much use of invented strategies is that, the children in the current study came from traditional classrooms as opposed to the children studied by Fennema and colleagues (1998) whose teachers were participating in a three year program “designed to help teachers understand their students’ intuitive mathematical ideas and to understand how those ideas could form the basis for the development of more formal ideas”(Carpenter, Fennema, & Franke, 1996; Fennema, et al, 1998). Students in these classrooms were given ample time and were encouraged to invent ways to solve problems and alternative strategies were discussed with them. The children in the current study were likely taught to use standard algorithms with no emphasis being placed on the invention of other strategies.

We have reason to believe that strategy use and conceptual understanding are closely related (Rittle-Johnson & Siegler, 1998;Cauley, 1988). This could mean that children using invented strategies have an enhanced conceptual knowledge required to invent and use invented strategies (Seegers & Boekaerts, 1996, Fennama et al., 1998). In the current study, the children rarely used invented strategies and relied mostly on standard algorithms. Most of the children were not able to correctly solve the extension or non-routine problems. This finding is in line with Fennema and colleagues' (1998) finding. In their study, children who were more likely to invent and use invented strategies performed better on extension problems than the children who were not likely to invent and use invented strategies, regardless of their gender.

One can argue whether the gender differences in approaches to doing mathematics found among first grade children (Carr & Jessup, 1997) and third grade children (Fennama et al., 1998) continue into the fifth grade where standard algorithms are the focus. For the most part, no gender differences were found in the current study. Gender differences that might have emerged under other contexts did not emerge in this data perhaps because both boys and girls were oriented to use the algorithms.

The hypothesis that third and fifth grade boys will be more likely than third and fifth grade girls, respectively, to use retrieval was not supported. Thus, the results of prior studies (Carr & Jessup, 1997; Carr & Davis, 2001) indicating that boys are more likely than girls to attempt and correctly use retrieval were not replicated. The gender differences in retrieval were found among first grade children before standard algorithms were introduced, which is usually in the second grade (Carr & Jessup, 1997; Carr & Davis, 2001). It may be, that as a result, children become over reliant on standard algorithms and less likely to use retrieval. Additionally, retrieval is not as effective as standard algorithms when solving more complex problems, such as many of those used in the current study, specifically, the extension and non-routine problems which consisted of multiple steps.

When looking at gender differences in the use of strategies utilizing manipulatives, the current study yields similar results to those of prior studies (Carr & Jessup, 1997; Fennema et al., 1998; Carr & Davis, 2001) which indicated that third grade girls were more likely than third grade boys to use manipulatives to solve mathematics problems. In the current study, this is only the case with the correct use, not attempted use, of strategies utilizing manipulatives when the data was collapsed across all problem categories. No gender differences in strategies utilizing manipulatives were found when looking at the problem categories separately. The failure to detect gender effects for the attempted and correct use of strategies utilizing manipulative within each of the problem categories might have been due to the low number of children attempting and correctly using strategies utilizing manipulatives, see Table 3. As children advance in their mathematics skills and the difficulty of problems increases, they gradually move away from using manipulatives.

The final hypothesis that girls at each grade level will be more likely than boys at each grade level to solve problems using standard algorithms as indicated by prior research (Fennama et al., 1998) was not supported. No gender differences for third or fifth grade children were indicated in the current study for the overall attempted and correct use of standard algorithm as well as the attempted and correct use of standard algorithms on the separate problem categories. The current study extended research indicating that gender differences in the use of invented strategies may continue into fifth grade. Because children in the current study relied heavily on standard algorithms, we were not able to detect gender differences that may be present in the strategy use of children whose curricular act ivies allow the children to invent alternative strategies. As more curricula view students as problems solvers and encourage students to freely solve mathematics problems, gender differences may emerge. When examining strategy use of children, it would be of special interest to include a variety of classrooms with diverse curricula which do not emphasize the use of standard algorithms. It would also be interesting to examine whether the gender differences in the use of invented strategies continue into middle school and perhaps beyond. If so, these gender differences in strategy use may be the precursors to the gender differences in strategy use found among young women and men in the solving of items on the SAT-M(Gallagher & DeLisi, 1994). If this is the case, it would be beneficial for children to invent strategies and the curricula should place emphasis on alternative strategies as opposed to instructing children to rely solely on standard algorithms. Further research is also needed to examine the relationship between invented strategies, conceptual understanding, and mathematics performance.

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# APPENDIX A

Third grade problems.

Number facts

5 + 11,

7 - 9

11 x 2

8 x 3

9 / 3

Word Problems

- 248 boys and 168 girls came to a picnic. How many children came to the picnic?
- 301children were playing on a playground. 121 left; how many children were playing at the playground?
- The cafeteria bough 510 apples for a snack; 331 children ate apples. How many apples were left after snack?
- The store had 102 pens; 92 were blue ink, 6 were green ink, and the rest were red ink. How many red ink pens were at the store?
- There were 189 pelicans and 199 eagles at the ZOO. How many birds were at the ZOO?

Extension problems

- Pat had $4. Pat spent $1 and 86 cents for a toy. How much money did Pat have left?
- Sam has $398. How much more would Sam have to save to have $500?
- Alex received $15 for allowance and $55 for B-day. Alex spent $6 and 15 cents. How much does Alex have?
- Karri went to the Zoo with $56. Ticket cost $3 and 50 cents. Karri bought tow drinks for $1 and 50 cents each and a souvenier for $17. How did Karri have left.
- Tyler had $201 and spent $ 111 and 56 cents. How much did Tyler have left?

Non routine problems

- 49 children are taking a mini-bus to the ZOO. They will have to sit either 2 or 3 to a seat. The bus has 17 seats. How many children will have to sit 3 to a seat and how many will sit 2 to a seat?

- Jamie had 5 bags with 4 candies in each bag. Jamie also had 3 bags with 6 candies in each. How many bags could Jamie make with 2 candies in each bag?

- 5 pizzas with 8 slices each were divided among 17 children. How many children had 2 slices and how many had 3?

- Terry bought 16 green marbles, 48 blue ones, 56 yellow and 272 red ones. Terry went outside and lost 6 greens , 18 blue ones, and 71 red ones. How many marbles did Terry have at the end of the day?

- The pet owner bought 9 yellow parrots, 12 blue, ones, and 9 white ones. The pet owner only had 6 cages to put all the parrots in. How many birds were in each cage after being equally divided?

# APPENDIX B

Fifth grade problems.

Number facts

16 x 2

48/8

31 - 12

15 + 18

3 x 12

Word problems

- Sam had 589 Legos. Kerri gave Sam 101 Legos. How many Legos did Sam have?

- Terry had 801 marbles; Terry lost 299 marbles. How many marbles did Terry have?

- The bug collection consisted of 192 ants, some butterflies, and 106 lady bugs. Together, the collection consisted of 302 bugs. How many butterflies were there?

- The cafeteria bought 461 apples and split them all in two halves. 222 children each ate half of an apple. How many halves of apples were left?

- The store had 102 pens. 92 were sold and a new shipment brought 16 new pens to the store. How many pens were at the store?

Extension problems

- Terry invested $250; every year Terry adds $25. How much money will Terry have in 7 years?

- Pat is saving money to buy a bike that costs $150. Pat starts with $35 and gets $10 a week for allowance. How long will it take Pat to save money for the bike?

- Kerri bought 5 bags of candy for $2.13 each. Kerri started with $15. How much does Kerri have left?

- Sam went shopping and bought 5 pens at $1.80 each, 3 notebooks at $2.10 each, a book-back for $86 and a pair of sneaker for $34. How much money did Sam spent?

- Grandmother lives 250 miles away. A gallon of gas costs $1.23. Car has a 10 gallon tank will 25 miles per gallon. How much money will it cost to go see Grandmother and come back home?

Non routine problems

- Jamie had 15 bags with 8 candies in each bag. Jamie also had 6 bags with 11 candies in each. How many bags with 6 candies in each can Jamie make?
- 43 children are taking a bus to the ZOO. They will either sit 2 or 3 to a seat. The bus has 19 seats. How many will sit 2 to a seat and how many will sit 3 to a seat?
- 5 pizzas with 8 slices each were divided among 17 children. How many children had 2 slices and how many had 3?
- Kerri bought 4 bags with 23 cadies in each and 11 bags with 19 candies in each. Kerri's class consists of 16 students . If Kerri divided all candy among all the students and gave what is left to the teacher. How many candies will each student have?
- The pet owner bought 18 yellow parrots, 24 blue ones, and 38 colored ones. The pet owner only has 7 cages to put all the parrots in. How many birds per cage after they are evenly divided?

Table 1. Means and standard deviations for each strategy used by grade and gender. Range is from 0 (never attempted or correctly used the strategy) to 5 ( strategy attempted or correctly used on all five problems within a problem category).

| | 3rd grade | | 5th grade | |
|---|---|---|---|---|
| | Girls | Boys | Girls | Boys |
| Number Facts | | | | |
| Correct Retrieval | 1.62 (1.12) | 1.5 (1.15) | .88 (.7) | .7 (.73) |
| Attempted Retrieval | 1.71 (1.06) | 1.5 (1.15) | .88 (.7) | .7 (.73) |
| Correct Invented Strategies | .24 (.7) | .4 (.82) | .12 (.33) | .7 (1.34) |
| Attempted Invented Strategies | .24 (.7) | .45 (1) | .12 (.33) | .75( 1.33) |
| Correct Manipulatives | .76 (.94) | .55 (.75) | .24 (.56) | 0 (.22) |
| Attempted Manipulatives | 1 (1.2) | .6 (.82) | .24 (.56) | 0 (.22) |
| Correct Standard Algorithms | 1.1 (.89) | 1.25 (1.2) | 3.47 (1.12) | 3.3 (1.38) |
| Attempted Standard Algorithms | 1.28 (.9) | 1.3 (1.17) | 3.7 (.92) | 3.35 (1.42) |
| Word Problems | | | | |
| Correct Invented Strategies | 0 (.3) | .3 (1.13) | 0 (0) | .25 (.44) |
| Attempted Invented Strategies | 0 (.21) | .3 (1.13) | 0 (0) | .3 (.57) |
| Correct Manipulatives | .62 (.86) | .35 (.59) | 0 (.24) | .1 (.31) |
| Attempted Manipulatives | 1.14 (1.61) | .55 (.60) | .12 (.49) | .1 (.31) |
| Correct Standard Algorithms | 2.86 (1.82) | 2.65 (1.53) | 3.7 (.92) | 3.3 (1.13) |
| Attempted Standard Algorithms | 3.67 (1.65) | 3.95 (1.28) | 4.65 (.79) | 4.5 (.61) |
| Extension Problems | | | | |
| Correct Invented Strategies | 0 (.22) | .35 (.93) | 0 (0) | 0 (.22) |
| Attempted Invented Strategies | 0 (.22) | .5 (1.32) | 0 (0) | .15 (.49) |
| Correct Manipulatives | .33 (.66) | 0 (.22) | .29 (.47) | .4 (.68) |
| Attempted Manipulatives | .8 (1.12) | .5 (1.05) | .41 (.62) | .45 (.69) |
| Correct Standard Algorithms | 1.38 (1.2) | .95 (1.23) | 2.18 (1.55) | 1.9 (1.23) |
| Attempted Standard Algorithms | 3.9 (1.26) | 3.45 (.18) | 3.88 (1.11) | 3.65 (.99) |

| | | | | |
|---|---|---|---|---|
| Non - routine Problems | | | | |
| Correct Invented Strategies | 0 (.3) | 0 (.22) | 0 (.24) | .15 (.49) |
| Attempted Invented Strategies | 0 (.3) | 0 (.22) | .12 (.33) | .35 (.81) |
| Correct Manipulatives | .71 (1.1) | .35 (.88) | .24 (.75) | .2 (.41) |
| Attempted Manipulatives | 2.9 (1.87) | 2.2 (1.67) | .65 (1.06) | .6 (.88) |
| Correct Standard Algorithms | .33 (.58) | .45 (.6) | 1.1 (.93) | .7 (1.03) |
| Attempted Standard Algorithms | 1.24 (1.26) | 1.45 (1.36) | 3.11 (1.4) | 2.7 (1.34) |

Table 2. Means and standard deviations for total correct responses by problem category, grade, and gender. Range is from 0 (no correct responses within a problem category) to 5 (all 5 problems correct within a problem category).

| | 3rd grade | | 5th grade | |
|---|---|---|---|---|
| | Girls | Boys | Girls | Boys |
| Number facts | 3.71(1.01) | 3.70(1.03) | 4.71(.59) | 4.75(.44) |
| Word Problems | 3.57(1.36) | 3.40(1.47) | 3.94(.90) | 3.75(1.07) |
| Extension Problems | 1.81(1.33) | 1.45(1.36) | 2.82(1.47) | 2.65(1.53) |
| Non-routine Problems | 1.14(1.01) | .85(1.14) | 1.41(1.18) | 1.1(1.12) |

Table 3. Means and standard deviations for attempted and correctly used strategy, by grade and gender. Range is from 0 (strategy never attempted or correctly used) to 20 (all 20 problems attempted or correctly solved using the strategy) for each strategy except retrieval. Range for retrieval is 0 (not attempted or correctly used) to 5(attempted or correctly used to solve all number facts).

| | 3rd grade | | 5th grade | |
|---|---|---|---|---|
| | Girls | Boys | Girls | Boys |
| Correct Retrieval | 1.62(1.12) | 1.50(1.15) | .88(.70) | .70(.73) |
| Attempted Retrieval | 1.71(1.06) | 1.50(1.15) | .88(.70) | .70(.73) |
| Correct Invented Strategy | .43(.87) | 1.10(2.71) | .17(.53) | 1.15(1.70) |
| Attempted Invented Strategy | .43(.87) | 1.30(3.37) | .24(.66) | 1.55(2.50) |
| Correct Manipulatives | 2.43(1.86) | 1.30(1.34) | .82(1.07) | .75(1.12) |
| Attempted Manipulatives | 6.05(4.63) | 4.00(2.71) | 1.63(1.54) | 1.21(1.51) |
| Correct Standard Algorithms | 5.66(3.32) | 5.30(3.29) | 10.47(3.04) | 9.20(2.98) |
| Attempted Standard Algorithms | 10.10(3.71) | 10.15(4.12) | 15.35(3.22) | 14.20(2.55) |