BIAS AND PRECISION OF EIGHT MULTIVARIATE MEASURES OF ASSOCIATION FOR A FIXED-EFFECTS ANALYSIS OF VARIANCE MODEL

by

Soyoung Kim

(Under the Direction of Stephen Olejnik)

ABSTRACT

A number of multivariate effect-size measures for MANOVA contexts have been proposed in the statistics literature. These measures however overestimate the strength of relationship between independent variable and dependent variable. A procedure by Tatsuoka (1973) and a procedure by Serlin (1982) have been suggested to adjust for the bias. The purposes of proposed study are to investigate the sampling distribution of selected eight measures of strength of association and to evaluate the two adjustment procedures using a computer simulation method. The results, when there are no true effects, indicate that eight effect- size measures are highly biased with small sample size and large number of variables. When two groups are compared, Serlin adjustment provides a better adjustment than Tatsuoka adjustment. When three or more groups are compared, Serlin adjustment for SGI, SEI, and CNI can provide an appropriate adjustment.

INDEX WORDS: Multivariate analysis of variance (MANOVA), Multivariate effect-size indices, Tatsuoka adjustment, Serlin adjustment.

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SOYOUNG KIM

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SOYOUNG KIM

Approved:

Major Professor: Stephen Olejnik

Committee: Deborah Bandalos
Joe Wisenbaker

Electronic Version Approved:
Maureen Grasso
Dean of the Graduate school
The University of Georgia
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LIST OF TERMS

Term	Meaning
Λ	Wilks test statistic in MANOVA
V	Hotelling-Lawley test statistic in MANOVA
U	Bartett-Pillai test statistic in MANOVA
Θ	Roy's test statistic in MANOVA
V'	$[V(df_e - p - 1)]/df_e$, where df_e is the degree of freedom for error
r	min(p,q)
b	max(p,q)
p	number of variables
q	degree of freedom for hypothesis (the number of groups – 1)
N	total sample size
Tr( )	trace of matrix ( )
ρ^2_{can}	squared canonical correlation
K	number of groups
η^2	univariate eta squared
ε^2	univariate epsilon squared
ω^2	univariate omega squared
$SS_{Between}$	sum of squares for between groups
SS_{Within}	sum of squares for within groups (error)
SS_{Total}	total sum of squares

r^2	measure of effect size in regression context (squared Pearson correlation coefficient)
R^2	measure of effect size in regression context (squared multiple correlation)
SS_{reg}	sum of squares for regression
$\Sigma(y_i\text{-}Y)^2$	total sum of squares, where Y is the mean of y_i
E	sum of squares and cross products for error (SSCP matrix)
H	sum of squares and cross products for hypothesis (SSCP matrix)
T	total sum of squares and cross products (SSCP matrix)
I	Identity matrix
λ_j	jth characteristic root (eigenvalue) of $\mathbf{H}^* \mathbf{E}^{-1}$
$\vert\text{-}\vert$	determinant of - matrix
WI	Wilks index of effect size in MANOVA context
HI	Hsu index of effect size in MANOVA context
SI	Stevens index of effect size in MANOVA context
SGI	Shaffer-Gillo index of effect size in MANOVA context
SEI	Serlin index of effect size in MANOVA context
TSI	Tasuoka-Sachdeva index of effect size in MANOVA
HRI	Hotelling-Rozeboom index of effect size in MANOVA
CNI	Cramer-Nicewander index of effect size in MANOVA
TA	Tatsuoka Adjustment procedure
SA	Serlin Adjustment procedure
Σ	common variance covariance matrix

η^2_m	effect size parameter (multivariate eta squared)
α_{jk}	$\mu_{jk} - \mu_{j.}$, where μ_{jk} = population mean for variable j and population k,
	μ_j = grand mean for variable j
ζ^2	effect size parameter for SGI (multivariate zeta squared)
ξ^2	effect size parameter for SEI (multivariate xi squared)
τ^2	effect size parameter for CNI (multivariate tau squared)
ω^2_{multi}	multivariate omega squared
$\boldsymbol{\mu}_j$	vector of population mean for the group on variable j

CHAPTER 1

INTRODUCTION

Statistical hypothesis test and effect size

Statistical hypothesis tests have been criticized for many years (Carver, 1978; Fan, 2001; Kirk, 1996; Thompson, 1996). Kirk (1996) identified three major criticism of statistical significance testing. The first criticism is that "null hypothesis significant testing and scientific inference address different questions" (1996, p.747). In other words, when researchers use the significance test, they want to know the probability of null hypothesis given obtained set of data. But what the probability of the hypothesis test reports is the probability of obtaining these data if the null hypothesis is true. The second criticism is that "null hypothesis is always false, a decision to reject it simply indicates that the research design had adequate power a true state of affairs" (p.747). The problem with statistical significance testing that it relies too heavily on sample size. The third criticism is that statistical significance testing " turns a continuum of uncertainty into a dichotomous reject-do-not-reject decision" (p.748), and this dichotomous decision may "lead to the anomalous situation in which two researchers obtain identical treatment effects but draw different conclusions" (p.748).

For many years, researchers have been strongly encouraged to assess and report effect-size estimates as a supplement to statistical hypothesis tests (Kirk, 1996; Thompson, 1996; Wilkinson & TSFI, 1999). Today at least 23 journals require authors to report some measure of effect-size when they present quantitative research findings.

A magnitude of effect-size means that "how much of the dependent variable can be controlled, predicted, or explained by the independent variable (s)" (Snyder and Lawson, 1993, p.335). Besides, the magnitude of the effect-size can clarify whether the statistically significant result has any practical significance. According to Kirk (1996), there are three categories in effect sizes; strength of association measures (r^2, R, R^2, Hays's ω^2, Kelly's ε^2, Tatsuoka's ω^2_{multi} ,etc), standardized mean difference measures(Cohen's *d, f, g* , Hedges's *g*, etc), and other measures (Cohen's U_1, U_2, U_3, Relative risk, Risk difference, etc). Maxwell and Delaney (1990) classified magnitude-effect-size into measures of effect-size and measures of association strength. In the measures of effect-size category, there are mean difference indices, estimated effect parameter indices, and standardized differences between means. In the measure of association category, there are η^2, partial η^2, Hays's ω^2, Kelly's ε^2, R^2, Ezekiel's adjusted R^2, the Lord formula, etc. According to Snyder and Lawson (1993), Hays's ω^2, Kelly's ε^2, Ezekiel's adjusted R^2, and the Lord formula are the corrected effect-size measures for biased estimators (e.g., η^2 or R^2).

Multivariate measures of strength of association

Many researchers are using multivariate statistical techniques due to increased availability of comprehensive computer programs (Bray & Maxwell; Onwuegbuzie & Daniel, 2003). When multiple outcome measures are compared in a multivariate analysis of variance (MANOVA), a measure of strength of association can be used to for measuring the effect size. It was not until in the early 1970s that the use of multivariate effect-size index was discussed at least in the behavioral sciences (Huberty, 2002).

When multiple outcome measures are compared in a multivariate analysis of variance (MANOVA), several effect-size indices have been suggested. Table 1.1 presents several popular indices of effect-size for the MANOVA context.

Table 1.1

Multivariate strength of association indices

Wilks Index (1932)	$\eta^2_{mult\text{-}WI} = 1 - \Lambda$
Hsu Index (1940)	$\eta^2_{mult\text{-}HI} = \dfrac{V}{1 + V}$
Stevens Index (1972)	$\eta^2_{mult\text{-}SI} = \dfrac{V'}{1 + V'}$
Shaffer-Gillo Index (1974)	$\eta^2_{mult\text{-}SGI} = \dfrac{Tr(HE^{-1})}{r + Tr(HE^{-1})} = \dfrac{V}{r + V}$
Serlin Index (1982)	$\eta^2_{mult\text{-}SEI} = \dfrac{SS_{Between}}{SS_{Total}} = \dfrac{U}{r}$
Tatsuoka(1970)-Sachdeva (1973) Index	$\omega^2_{mult} = \dfrac{(N\text{-}K) - (N\text{-}1)\ \Lambda}{(N\text{-}K) + \Lambda}$
Hotelling (1936)-Rozeboom (1965) Index	$R^2_{mult\text{-}HRI} = 1 - \Pi^r_{j=1} (1\text{-} \rho^2_j) = 1 - \Lambda = \eta^2_{mult\text{-}WI}$
Cramer-Nicewander Index (1979)	$R^2_{mult\text{-}CNI} = 1 - \dfrac{\lvert \mathbf{S}_{error} \rvert^{1/p}}{\lvert \mathbf{S}_{total} \rvert^{1/p}} = 1 - [\ \Pi^r_{j=1} (1\text{-} \rho^2_j)\ ]^{1/p} = 1 - (\Lambda)^{1/p}$
	$R^2_{mult\text{-}CNI1} = \dfrac{Tr(\mathbf{S}^{-1}_{total}\mathbf{S}_{reg})}{Tr(\mathbf{S}^{-1}_{total}\mathbf{S}_{total})} = \dfrac{\sum^r_{j=1} \rho^2}{p}$

where Λ is Wilks test statistic in MANOVA, V is Hotelling-Lawley test statistic in MANOVA, V′ is $[V(df_e - p - 1)]/df_e$, where df_e is the degree of freedom for error, r is min(p,q), where p is the number of variables and q is the degree of freedom for hypothesis, U is Bartett-Pillai test statistic in MANOVA, N is overall sample size, Tr( ) is trace of matrix ( ), ρ^2 is the squared canonical correlation, and K is the number of groups

Smith (1972) also presented a generalization of the univariate eta squared in the multivariate context. However, his formula, "based on stepdown procedures" (Huberty, 1983, p.709), "do not yield values that are invariant under alternative orderings" (Smith, 1972, p.371). Therefore, Smith (1972) index is not considered in this study. These indices (presented in Table 1.1) can be categorized into three classifications based on how they were developed: 1) generalization of the univariate eta squared (Hsu, 1940; Serlin, 1982; Shaffer-Gillo, 1974; Stevens, 1972; Wilks, 1932), 2) generalization of the univariate omega squared (Sachdeva, 1973; Tatuoka, 1970), and 3) as a function of the squared canonical correlation (Cramer-Nicewander, 1979; Hotelling, 1936; Rozeboom, 1965). SPSS (2002, version 11.0) reports Shaffer-Gillo index, Serlin index, and Cramer-Nicewander index under requested in the General Linear Model – Multivariate program. But, SAS (SAS Institute INC, version 8, 1999-2001) does not report any of these indices.

Adjustment procedures in MANOVA

Tatsuoka (1973) found that Tatsuoka index, ω^2_{mult} (TSI), is positively biased when the number of variables is large and the sample size is small. To reduce the bias in TSI, he developed an adjustment formula. He maintained that this adjustment would be sufficient for most MANOVA contexts and could be used with Wilks index and Hsu index as well as the Tatsuoka index. Serlin (1982) indicated that the Serlin index (SEI) is a biased estimator, and proposed another adjustment analogous to Ezekiel's (1930) adjustment for squared multiple correlation coefficient. Bray and Maxwell (1985) have recommended Serlin's adjustment while Huberty (1994) favors Tatsuoka's adjustment. Table 1.2 presents these adjustments for multivariate measures of association.

Table 1.2

Adjustment of multivariate measure of strength of association

Tatsuoka Adjustment (1973)	$(\omega^2_{mult})_{adj} = \omega^2_{mult} - \frac{p^2 + q^2}{3N}(1 - \omega^2_{mult})$
Serlin Adjustment (1982)	$(\eta^2_{mult\text{-}SEI})_{adj} = 1 - \frac{N - 1}{N - b - 1}(1 - \eta^2_{mult\text{-}SEI})$

where p is the number of variable, q is the number of group minus one, and b is max(p, q), and where N is the total sample size

Statement of problem and purpose of study

Although the multivariate effect size measures are known to be biased, among statisticians, applied researchers are generally unaware of this problem. For example, SPSS (2002, version 11.0) reports Shaffer-Gillo index (SGI), SEI, and Cramer-Nicewander index ($R^2_{mult\text{-}CNI}$ = CNI) when effect-size is requested in the General Linear Model – Multivariate program, but provides no indication that the estimates are biased. In review of a convenience sample of 14 multivariate textbooks published since 1985 only 10 textbooks discussed multivariate effect-size measures and only four commented on bias. Table 1.3 provides a list of book titles, publication dates, effect-size measures discussed, and type of adjustment suggested.

Table 1.3

Analysis of multivariate textbooks

Author	Title	Year	Effect Size	Adjustment
Bary, J. H. Maxwell, S. E.	Multivariate Analysis of Variance	1985	$\eta^2_{mult\text{-}S}$	Serlin Adjustment

Diekhoff, G.	Statistics for the Social and Behavioral science: Univariate, Bivariate, Multivariate	1992	η^2_{mult-W}	None
Edwards, L. K.	Applied Analysis of Variance in Behavioral Science	1993	None	None
Flury, B.	A First Course in Multivariate Statistics	2002	None	None
Huberty, C. J.	Applied Discriminant Analysis	1994	η^2_{mult-W}, $\eta^2_{mult-SG}$, η^2_{mult-S}, ω^2_{mult}, $R^2_{mult-CN1}$	Tatsuoka Adjustment
Jobson, J. D.	Applied Multivariate Data Analysis (volume п: categorical and multivariate methods)	1992	η^2_{mult-W}, ω^2_{mult}	Tatsuoka Adjustment
Marcoulides, G.A. Hershberger, S.L .	Multivariate Statistical Methods-A first course	1997	None	None
Rencher, A. C.	Method of Multivariate analysis (second edition)	2002	η^2_{mult-W}	None
Sharma, S.	Applied Multivariate Techniques	1996	η^2_{mult-W}	None
Srivastava, M. S.	Method of Multivariate Statistics	2002	None	None
Steven, J.	Applied Multivariate Statistics for the Social Science (3rd edition)	1996	η^2_{mult-W}	None

Tabachnick, B.G. Fidell, L. S.	Using Multivariate Statistics (second edition)	1989	η^2_{mult-W}	None
Tatsuoka, M. M.	Multivariate Analysis	1988	ω^2_{mult}	Tatsuoka Adjustment
Timm, N. H.	Applied Multivariate Analysis	2002	None	None

In addition to bias, the precision with which these statistics estimate measure of association has been given little attention. Furthermore, the adequacy of the two procedures for adjusting for bias has not been examined. The purposes of the present study are 1) to examine the degree of bias and precision in eight of multivariate measures of association and 2) to evaluate the effectiveness of the Tatsuoka and Serlin procedures for adjusting the eight effect-size measures.

Method

To address the purpose of this study a computer simulation method is used using SAS/IML (SAS Institute INC, version 8, 1999-2001). The factors considered in this study are the number of compared groups (k=2, 3, and 5), sample size (n=10 and 50), the number of variables (p=3, 5, and 10), and population effect size (η^2_{mult}=0, .1, .3, and .5). While the effect-size measures can be used in more complex designs, the present study only considers the one factor multivariate analysis of variance context and when all MANOVA assumptions are met.

Significance

The reporting of an effect-size measure is currently required by several prominent education journals. For this requirement to be useful the effect-size measure reported should be unbiased and estimated with precision. Multivariate effect-size measures suggested in many textbooks and those currently reported on computer output provide biased estimates population differences. Many researchers are unaware of this bias and are unaware of procedures that are available to adjust these effect-size measures. The present study provides estimates of the magnitude of the bias and compares two adjustment procedures to reduce the bias. The results of this study should be of interest to authors of multivariate related textbooks, to methodologists interested in the distributional properties of the multivariate effect-size measures, and to applied researchers using MANOVA and are interested in an unbiased estimate of the effect size.

The next chapter reviews the development of both univariate and multivariate effect-size measures. In addition, studies that have examined the multivariate effect-size measures are discussed. In chapter 3, the sampling conditions, the generating populations, and the generating samples are described. In the results chapter, the degree of the bias of unadjusted eight effect-size measures, the degree of the bias of adjusted eight effect-size measures using the Tatsuoka adjustment and the Serlin adjustment, and the precision of eight unadjusted/adjusted effect-size measures are presented. Finally, chapter 5 summarizes the results and discusses the implications of the findings.

CHAPTER 2

LITERATURE REVIEW

The purposes of this study are: 1) to examine the degree of bias and precision in eight multivariate measures of association and 2) to evaluate the effectiveness of the Tatsuoka and Serlin procedures for adjusting the eight effect-size measures. This chapter describes topics related to research purposes: 1) multivariate analysis of variance, 2) the measures of association in the univariate and the multivariate contexts, 3) two adjustment procedures of bias in MANOVA, and 4) previous investigation on multivariate effect-size measures.

The literature reviewed on the related studies presented in this chapter were identified by searching ERIC (Educational Resource Information Center), PsycINFO (Psychology Information), GALILEO (Georgia Library Learning Online), and references from previous research. Key word used in the search are “effect size”, “measures of strength of association”, “MANOVA”, “multivariate measures of strength of association”, and “adjustment procedure of bias in MANOVA”.

Multivariate Analysis of Variance

MANOVA is an analysis of variance (ANOVA) model that is suitable for the analysis of data with more than one dependent variable. When there is more than one dependent variable, MANOVA is recommended because this procedure can control experimentwise error rate that is inflated in the univariate analyses, if each dependent variable is considered separately. Besides, it makes researchers can take into consideration the correlations among dependent variables. Huberty (1983) notes that there is some natural scalar-matrix correspondence between ANOVA and MANOVA. An ANOVA between group sum of squares, $SS_{Between}$, generalizes to, a hypothesis SSCP (sum of squares and cross products) matrix **H**. Similarly, within sum of squares generalizes to **E**, and a total sum of squares (SS_{total}) to **T** (Huberty, 1983).

The hypothesis tested using MANOVA is that the population mean vectors, or centroids of k populations are equal to each other (where k is equal to number of populations). To test the null hypothesis composite scores are created by an optimally weighted linear combination of dependent variables. When a set of weights (raw discriminant function coefficients e.g., a_1, a_2, …a_p) is multiplied by their respective dependent variables (Y_1, Y_2, … Y_p), it yields the weighted linear combination of dependent variables ($lj = a_1Y_1 + a_2Y_2 + \ldots a_pY_p$) (Hasse & Ellis, 1987). "These linear combinations of dependent variables are called Linear Discriminant Functions " (Huberty, 1994, p. 206). The number of Linear Discriminant Functions (LDFs) is determined by either the number of dependent variables (p) or the degree of freedom for the hypothesis (q), whichever is smaller. In addition, the number of LDFs to consider may be determined in one of three ways; statistical tests, proportion of variance, and LDF plots (Huberty, 1994). Each LDF is associated with eigenvalue (λ_i), where "an eigenvalue is a measure of concentration of shared variance between a MANOVA effect and a Linear Discriminant Function" (Hasse & Ellis, 1987, p. 408).

There are four test criteria in MANOVA. They are Wilks' Λ, Bartlett-Pillai's U, Hotelling-Lawley's V, and Roy's Θ. They can be computed as a different function of eigenvalues (λ_i), where λ_j is the jth characteristic root (eigenvalue) of $\mathbf{H}^* \mathbf{E}^{-1}$; Wilks' $\Lambda = \prod 1/(1 + \lambda_i)$, Bartlett-Pillai's $U = \sum \lambda_i/(1 + \lambda_i)$, Hotelling-Lawley's $V = \sum \lambda_i$, and Roy's $\Theta = \lambda_1/(1 + \lambda_1)$ (where λ_1 is the largest eigenvalue). Instead of using the four test criteria, a F-test approximation, which is transformed from Wilks' Λ, Bartlett-Pillai's U, Hotelling-Lawley's V, and Roy's Θ to F, is used for the test statistic in MANOVA. If the F-test approximation test is significant, the follow- up test (e.g., contrast analysis and discriminant analysis) can be conducted.

In addition, as Keselman et al (1998) noted, data conditions should be considered because all ANOVA-type statistics require that data conform to distributional assumptions in order to provide valid tests of statistical hypotheses. The assumptions in MANOVA are:

1. The observations on the p dependent variables follow a multivariate normal distribution in each population.
2. The population covariance matrices for the p dependent variables in each population are equal.
3. The observations are independent. (Stevens, 1992, p.245).

Measure of Strength of Association

Univariate context

Pearson (1905) proposed η, correlation ratio, to describe a nonlinear relationship between the grouping variable and the dependent variable. It reflects the relationship between the grouping variable and the dependent variable within a sample. Later, Fisher (1925) described the squared correlation ratio (η^2) as a measure of strength of association in the ANOVA context. The notation η^2 was defined as:

$$\eta^2 = \frac{SS_{Between}}{SS_{Total}} = 1 - \frac{SS_{Within}}{SS_{Total}}$$

where,

$SS_{Between}$ = sum of squares for between groups,

SS_{Within} = sum of squares for within group (error),

SS_{Total} = sum of squares for total variation.

However, it is a positively biased estimator, that is, it over estimates the relationship between the grouping variable and the dependent variable. Kelly (1935) suggested an adjustment of the eta squared (η^2), ε^2. The notation ε^2 was defined as:

$$\varepsilon^2 = 1 - \frac{(N-1)\ SS_{Within}}{(N-K)\ SS_{Total}}$$

where,

SS_{Within} = sum of squares for within group (error),

SS_{Total} = sum of squares for total variation,

N = number of total sample size,

K = number of groups.

In 1963, Hays proposed another estimator of strength of association in the ANOVA context, ω^2, to reduce the estimation bias associated with the eta squared (η^2). Epsilon squared (ε^2) and omega squared (ω^2) were proposed for inferential purposes, they estimate strength of association within the population (Richardson, 1996; Huberty, 2002). The notation ω^2 defined as:

$$\omega^2 = \frac{SS_{Between} - (K-1)\ MS_{Within}}{SS_{Total} + MS_{Within}}$$

Where,

$SS_{Between}$ = sum of squares for between groups (hypothesis),

SS_{Total} = sum of squares for total variation,

MS_{Within} = mean squares for within group (error),

K = number of groups.

These measures of associations (η^2, ε^2, and ω^2) represent the proportion of variance in the dependent variable that is explained by the grouping variable (Richardson, 1996; Olejnik and Algina, 2000). Carrol and Nordholm (1975) and Keselman (1975) studied empirical comparisons among η^2, ε^2, and ω^2 using computer simulation method.

Carrol and Nordholm (1975) evaluated sampling distributions of ε^2 and ω^2 using computer simulation study within the context of one-way ANOVA. In the study, they considered equal and unequal sample sizes (total sample sizes yielded 15, 30, and 90) and three levels of variance conditions (homogeneous variances, slight heterogeneity, and marked heterogeneity) when three groups were compared. The results indicated that 1) ω^2 was slightly biased and ε^2 was not biased when equal sample size and homogeneous variances were considered; 2) both ω^2 and ε^2 underestimated independent-dependent variable relationship when homogeneous variances and unequal sample were considered; 3) both ω^2 and ε^2 substantially underestimated independent-dependent variable relationship when the relationship between heterogeneous variance and unequal sample size was positive; 4) both ω^2 and ε^2 substantially overestimated independent-dependent variable relationship when the relationship between heterogeneous variance and unequal sample size was negative. With regard to precision, both ω^2 and ε^2 had “large standard deviations when small samples were used” (Carrol and Nordholm, 1975, p.549). However, the standard deviations of ω^2 were consistently lower than those of ε^2.

Keselman (1975) compared the sampling distributions of η^2, ε^2, and ω^2. He considered normal and non-normal distributions, three levels of population effect sizes, and two levels of variability of population. He found that ω^2 was the least unbiased estimator among them and the standard deviation of η^2 was smaller than those of ε^2 and ω^2.

“Edgeworth (1892) used the expression coefficient of correlation for the symbol ρ (parameter and statistic were not then commonly differenciated)” (Huberty, 2002, p.229). Pearson began to “popularize the correlation coefficient, r, around 1896” (Huberty, 2002, p.229). Currently, the notation ρ is considered as a parameter and the notation r is considered as a statistic. In other words, the squared rho (ρ^2) represents the proportion of variance in the dependent variable that is explained by “its regression on the independent variable within the population” (Richardson, 1996, p.16). On the other hand, the squared Pearson correlation coefficient (R^2) represents the magnitude of strength of association within a sample. The notation R^2 defined as:

$$R^2 = \frac{SS_{reg}}{\Sigma(y_i - Y)^2} = \frac{SS_{reg}}{SS_{total}}$$

where,

SS_{reg} = sum squares for regression (hypothesis),

$\Sigma(y_i - Y)^2$ = sum squares for total, where Y is the mean of y_i.

"In 1914, Pearson proposed the expression coefficient of multiple correlation when he used the symbol R" (Huberty, 2002, p.233). In multiple regression, the notation R^2 (squared multiple correlation) is used as a measure of the strength of association between a dependent variable and a linear composite of independent variables within a sample. The squared multiple correlation (R^2) is a positively biased estimator. Ezekiel (1930) proposed an adjustment to get an unbiased strength of association of parameter. The adjustment derived as a function of sample size and number of independent variables. The Ezekiel's adjustment is defined as:

$$R^2_E = 1 - \frac{N - 1}{N - P - 1}(1 - R^2)$$

where,

N = sample size,

P = number of independent variables,

R^2 = squared multiple correlation.

Multivariate Context

Several multivariate strength of association indices were derived by a generalization of the univariate correlation ratio (eta squared and omega squared) and a function of canonical correlation, which reflects a relationship between a linear composite of dependent variables and a grouping variable (Huberty, 1994).

Wilks index

"Multivariate generalization of η^2 have been proposed by Wilks (1932) and Hsu (1940)" (Huberty, 1972). The Wilks' multivariate generalization of the eta squared index can be derived simply as shown below.

$$\eta^2 = \frac{SS_{Between}}{SS_{Total}} = 1 - \frac{SS_{Within}}{SS_{Total}}$$

from the relationship among the SS's,

$$1 - \eta^2 = \frac{SS_{Within}}{SS_{Between} + SS_{Within}}$$

this generalizes to

$$1 - \eta^2_{mult} = \frac{|\mathbf{E}|}{|\mathbf{H} + \mathbf{E}|} = \Lambda$$

where, $|\mathbf{E}|$ = determinant of error sum of squares and cross products (SSCP) matrix,

$|\mathbf{H} + \mathbf{E}| = |\mathbf{T}|$ = determinant of total SSCP matrix.

Thus, $\eta^2_{mult}(WI) = 1 - \Lambda$

Alternatively, $\eta^2_{mult}(WI) = 1 - \prod 1/(1 + \lambda_j)$, where λ_j is the jth characteristic root (eigenvalue) of $\mathbf{H}^* \mathbf{E}^{-1}$.

The lamda (Λ) is one of the multivariate test criteria and is actually a "product of two matrices, $\mathbf{H}^* \mathbf{E}^{-1}$" (Huberty, 1994). When λ_j is the jth characteristic root (eigenvalue) c), Λ can be expressed by $\Lambda = \Pi^r_{j=1} 1/1+\lambda_j$. Wilks' index ($\eta^2_{mult} = 1 - \Lambda$) can be given by $|\mathbf{H}|/|\mathbf{T}|$, a ratio of the determinants of the hypothesis SSCP matrix and the total SSCP matrix. When the between group variation is large relative to the total variation, then Λ will be close to zero, and hence $1 - \Lambda$ will be close to 1. On the other hand, when the between groups variation is small relative to the total variation, then Λ will be close to 1, and 1 – Λ will be close to zero.

Hsu index

Hsu proposed a multivariate generalization of eta squared by suggesting that V equals $\eta^2/1-\eta^2$ (Huberty, 1972,; Stevens, 1972). The Hsu's index is defined as:

$$\eta^2_{mult}(HI) = \frac{V}{1+V} = \frac{\Sigma \lambda_j}{1+\Sigma \lambda_j}$$

where, V = $\Sigma \lambda_j$, where λ_j is the j^{th} eigenvalue of the **H* E**$^{-1}$ matrix (Hotelling – Lawley trace Criterion).

According to Stevens (1972), the difference between 1 – Λ and V/(1+V) is small. To prove why the difference between 1 – Λ and V/(1+V) is small, he showed that Λ and 1/(1+V) differ by little because V/(1+V) equals 1 - 1/(1+V). The Λ can be expressed using V as a function of λ_j (only when the number of dependent variables are more than one). He presented that as shown below:

Two groups : $\Lambda = \frac{1}{1+\lambda_1} = \frac{1}{1+V}$

Thus for two groups, since there is just one eigenvalue, Λ and 1/(1+V) are equal.

Three groups : $\Lambda = \frac{1}{(1+\lambda_1)(1+\lambda_2)} = \frac{1}{1+V+\lambda_1\lambda_2}$

Four groups : $\Lambda = \frac{1}{(1+\lambda_1)(1+\lambda_2)(1+\lambda_3)} = \frac{1}{1+V+\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_3}$

Because the third eigenvalue of **H* E**$^{-1}$ usually less than .05 and remaining eigenvalues are still smaller, for the K groups case (assuming more dependent variables than groups), the sum of all products involving all different pairs of eigenvalues, plus the sum of all products involving all different triples of eigenvalues, plus the sum of all products involving all different quadruples of eigenvalues, . . . , plus product of all nonzero q eigenvalues will be negligible (Steven, 1972). Therefore, there is little difference between $1 - \Lambda$ and $V/(1+V)$.

Stevens Index

In 1972, Stevens proposed a modification of the Hsu index. According to Stevens (1972), "Ghosh (1963) suggested that a modification of the global measure involving V might be better and showed that $E(V) = df_e \Sigma \lambda_j (df_e\text{-}p\text{-}1)$" (p.375), where df_e is the degrees of freedom for the **E** matrix, p is the number of dependent variables, and the λ_j are the population eigenvalues of $\mathbf{H}^* \mathbf{E}^{-1}$. An unbiased estimate of the population sum of roots for V is given by $V(df_e - p - 1)/ df_e$. The Stevens index is defined as:

$$\eta^2_{mult}(SI) = \frac{V'}{1 + V'}$$

Where,

$$V' = V(df_e - p - 1)/ df_e.$$

Shaffer and Gillo index

Shaffer and Gillo (1974) proposed an alternative multivariate generalization of the univariate correlation ratio (η). They argued that in the univariate context, W + B = T, where W = sum of squares for within groups (error), B = sum of squares for between groups, and T = sum of squares for total variation thus, univariate correlation ratio can be computed as 1 – W/T or B/T. However, in the multivariate context, when $|\mathbf{E}|$, $|\mathbf{H}|$, and $|\mathbf{T}|$ are taken as the multivariate generalization of univariate W, B, and T, then $|\mathbf{E}| + |\mathbf{H}| \neq |\mathbf{T}|$. Therefore, "the two definitions of the correlation ratio do not produce the same measure, using these multivariate definitions, and Wilks himself regarded both $1 - |\mathbf{E}|/ |\mathbf{T}|$ and $|\mathbf{H}|/ |\mathbf{T}|$ as different possible multivariate generalization" (Shaffer and Gillo, 1974, p.523). In contrast to the Wilks index, the Shaffer and Gillo index is based on the additive decomposition $Tr(\mathbf{TE}^{-1}) = Tr(\mathbf{EE}^{-1}) + Tr(\mathbf{HE}^{-1}) = p + Tr(\mathbf{HE}^{-1})$, where p is the number of dependent variables and Tr( ) is the trace of the matrix product named in the parentheses. They insisted that their index is a more suitable multivariate generalization of the univariate correlation ratio. The Shaffer and Gillo index is defined as:

$$\eta^2_{mult}(SGI) = 1 - \frac{Tr(\mathbf{EE}^{-1})}{Tr(\mathbf{TE}^{-1})} = \frac{Tr(\mathbf{HE}^{-1})}{Tr(\mathbf{TE}^{-1})}$$

where ,

$Tr(\mathbf{EE}^{-1})$ = trace of matric product of $\mathbf{EE}^{-1}$,
$Tr(\mathbf{HE}^{-1})$ = trace of matric product of $\mathbf{HE}^{-1}$,
$Tr(\mathbf{TE}^{-1})$ = trace of matric product of $\mathbf{TE}^{-1}$.

It is equivalently expressed as:

$$\eta^2_{mult}(SGI) = \frac{Tr(\mathbf{HE}^{-1})}{r + Tr(\mathbf{HE}^{-1})} = \frac{V}{r + V} = \frac{\Sigma \lambda_j}{r + \Sigma \lambda_j}$$

where,

r = min(p,q), where p is the number of variables and q is the number of group minus one,
V = the Hotelling – Lawley trace statistic = $Tr(\mathbf{HE}^{-1})$.

> "It can be regarded as a weighted average of the estimated correlation ratios for each of the discriminant functions, with each weight equal to the total sum of squares for that discriminant function after the functions have been standardized so that each has the same within groups sum of squares" (Shaffer and Gillo, 1974, p.523).

Tatsuoka index and Sachdeva index

Tatsuoka (1970) proposed ω^2_{mult} as a multivariate analogue to the univariate Hays' ω^2. It is obtained by "replacing each sum of squares by the determinant of the corresponding SSCP matrix, with one exception: $SS_{Between}$ is replaced by $|\mathbf{T}| - |\mathbf{E}|$ rather than $|\mathbf{H}|$" (Huberty, 1972).

Hays' univariate index is defined as:

$$\omega^2 = \frac{SS_{Between} - (K-1)\, MS_{Within}}{SS_{Total} + MS_{Within}}$$

Tatsuoka (1973) examined four expressions for a multivariate omega squared effect size measures to prove that the most plausible estimator of ω^2 is the ω^2_{mult} presented in 1970 based on the conditions. It is that $1 - \Lambda$ converge to ω^2 when $N \rightarrow \infty$ and p (the number of dependent variables) increases. The result indicated that ω^2_{mult} is the most plausible estimator of ω^2 among four expressions (Tatsuoka, 1973). Tatsuoka's multivariate index is defined as:

$$\omega^2_{mult} = \frac{|\mathbf{T}| - |\mathbf{E}| - (K-1)\, |\mathbf{E}|/(N-K)}{|\mathbf{T}| + |\mathbf{E}|/(N-K)}$$

where,

N = sample size,

K = the number of groups,

$|\mathbf{T}|$ = determinant of total SSCP matrix,

$|\mathbf{E}|$ = determinant of error SSCP matrix.

Since $|\mathbf{E}|/|\mathbf{T}| = \Lambda$, an equivalent expression using Λ is:

$$\omega^2_{mult} = \frac{1 - \Lambda - (K-1)\, \Lambda/(N-K)}{1 + \Lambda/(N-K)}$$

$$= \frac{(N-K) - (N-1)\, \Lambda}{(N-K) + \Lambda}$$

Sachdeva (1973) also arrived, independently, at the same index as Tatsuoka. According to

Sachdeva (1972),

$$\omega^2 = \frac{SS_{Between} - (K-1)\, MS_{Within}}{SS_{Total} + MS_{Within}}$$

$$= \frac{SS_{Between} - (K-1)/(N-K)\, SS_{Within}}{SS_{Total} + 1/(N-K)\, SS_{Within}}$$

the multivariate extension of Hays ω^2 is obtained by replacing each sum of squares by the determinant of the corresponding matrix of sums of squares and sums of cross products,

$$\omega^2_{mult} = \frac{|\mathbf{H}| - (K-1)\, |\mathbf{E}|/(N-K)}{|\mathbf{T}| + |\mathbf{E}|/(N-K)}$$

where,

$|\mathbf{H}|$ = determinant of hypothesis SSCP matrix.

It was simplified to the expression using Λ (Sachdeva, 1973).

$$\omega^2_{mult} = 1 - \frac{N\Lambda}{\Lambda + (N-K)}$$

Sachdeva proposed another formula using "The ω^2_{mult} as defined above expression can also be estimated by the F-ratio using the fact (Rao, 1965) that" (Sachdeva, 1973, p.629)

$$F = \frac{(1 - \Lambda^{1/s})u}{\Lambda^{1/s}}$$

where,

$s = \sqrt{[p^2(df_h)^2 - 4] / [p^2 + (df_h)^2 - 5]}$

$u = \frac{s\,[2df_e + df_h - p - 1] - p(df_h) + 2}{2\, p(df_h)}$

df_e = the degrees of freedom for the error SSCP matrix

df_h = the degrees of freedom for the hypothesis

The formula using the F value and u is defined as:

$$\omega^2_{mult} = 1 - \frac{N\,u^s}{(N\text{-}K)\,F(+u)^s + u^s}$$

The formulas using Λ and the F value obtained the exact same value of the strength of association (Sachdeva, 1973).

Hotelling and Rozeboom index

Hotelling (1936) and Rozeboom (1965) proposed a multivariate measure of association as a generalization of the squared multiple correlation coefficient in the multivariate regression context: the function of the canonical correlation (Cramer and Nicewander, 1979). The Hotelling and Rozeboom's index is defined as:

$$R^2_{mult}(HRI) = 1 - \frac{|\mathbf{S}_{error}|}{|\mathbf{S}_{total}|} = 1 - \Pi^{r}_{j=1}\,(1-\rho^2_j)$$

where,

$\mathbf{S}_{error}$ = error sum of squares and cross products (SSCP) matrix,

$\mathbf{S}_{total}$ = total sum of squares and cross products (SSCP) matrix,

r = the number of dependent variables (assumes $p \leq q$, where p = the number of dependent variables and q = the number of independent variables),

ρ^2_j = the squared canonical correlation, where $\rho^2_j = \lambda_j/(1+\lambda_j)$.

It is analogous to the Wilks index and may be interpreted as one minus the proportion of unexplained, generalized variance.

$$R^2_{mult}(HRI) = \eta^2_{mult}(WI) = 1 - \Lambda = 1 - \Pi^{r}_{j=1}\,(1-\rho^2_j)$$

Cramer and Nicewander Index

In 1979, Cramer and Nicewander proposed several multivariate measures of association in the multivariate regression context "derived using other generalizations of the squared multiple correlation coefficient" (Cramer and Nicewander, p.49). Two of them are defined as:

$$R^2_{mult}(CNI) = 1 - \frac{|\mathbf{S}_{error}|^{1/p}}{|\mathbf{S}_{total}|^{1/p}} = 1 - [\,\Pi^{r}_{j=1}(1-\rho^2_j)\,]^{1/p}$$
$$= 1 - (\Lambda)^{1/p}$$

and

$$R^2_{mult}(CNI1) = \frac{Tr(\mathbf{S}^{-1}_{total}\mathbf{S}_{reg})}{Tr(\mathbf{S}^{-1}_{total}\mathbf{S}_{total})} = \frac{\sum^{r}_{j=1}\rho^2}{p}$$

where,

$\mathbf{S}_{error}$ = error sum of squares and cross products (SSCP) matrix,
$\mathbf{S}_{total}$ = total sum of squares and cross products (SSCP) matrix,
$\mathbf{S}_{reg}$ = regression sum of squares and cross products (SSCP) matrix,
p = the number of dependent variables (assumes $p \leq q$, where q = the number of independent variables),
ρ^2_j = the squared canonical correlation.

The $R^2_{mult}(CNI)$ is "equal to one minus the geometric mean of the $1-\rho^2_j$, and which has a proportion of variance interpretation" (Cramer and Nicewander, 1979, p.49). The $R^2_{mult}(CNI1)$ is the arithmetic average of the squared canonical correlation for the separate linear combinations of two sets of variables.

Serlin Index

Serlin (1982) examined the utility of an average squared canonical correlation ($R^2_{multCNI1}$) in the discriminant analysis context. In the discriminant analysis context, "the interpretation of $R^2_{multCNI1}$ can be closely aligned to that of Fisher's correlation ratio, in that it can be shown to equal a ratio of between group and total sums of squared deviations" (Serlin, 1982, p.414). When there are r discriminant functions, where r is the min(p, q), p is the number of dependent variables and q is the number of groups minus one, "a sum of squares between groups can be associated with each discriminant function and is equivalent to the corresponding Roy's criterion,

Θ, the sum of squares total for each discrimiant function is unity" (Serlin, 1982, p.415). That is,

$$SS_{Between} = \sum_{j=1}^{r} SS_{Between_j} = \sum_{j=1}^{r} \Theta_j$$

$$SS_{Total} = \sum_{j=1}^{r} SS_{Total_j} = r$$

where:

$$\Theta_j = \lambda_j/(1 + \lambda_j), \quad \lambda_j = SS_{Between_j} / SS_{Within_j}$$

The ratio of the overall between group and total sum of squares is,

$$\frac{SS_{Between}}{SS_{Total}} = \frac{\sum_{j=1}^{r} \Theta_j}{r} = \frac{\sum_{j=1}^{r} \rho^2_j}{r}$$

It is the average of the squared canonical correlations between the set of dependent variables and a set of dummy variables, and same as Cramer and Nicewander index (CNI1) in the multivariate regression context. The $\sum_{j=1}^{r} \Theta_j$ is the Pillai –Bartlett MANOVA test criterion, U, thus effect-size is defined as:

$$\eta^2_{mult}(SEI) = \frac{SS_{Between}}{SS_{Total}} = \frac{U}{r}$$

Adjusting the MANOVA Measures of Association

Tatsuoka Adjustment

In 1973, Tatsuoka found that ω^2_{mult} is highly positively biased when the number of variables is large, the sample size is small, and the population value of ω^2_{mult} is small. Therefore, he decided to develop an adjustment formula to reduce the bias in ω^2_{mult}. After reviewing the sampling distribution of ω^2_{mult}, he observed that the amount of bias seemed to be a linear function of $1 - \omega^2_{mult}$ for fixed p and N, where p is the number of dependent variables and N is total sample size. That is, the amount of bias equaled $m(1 - \omega^2_{mult})$. From this equation, an adjusted value of

ω^2_{mult} was computed as:

$$(\omega^2_{mult})_{adj} = \omega^2_{mult} - m(1 - \omega^2_{mult})$$

He then determined that "m was approximately inversely proportional to N and roughly directly proportional to p" (Tatsuoka, 1973, p.18). Tatsuoka estimated m:

$$m = cM^aQ^b$$

where, c, a, and b were to be determined on a least-squares basis,
$M = N – 1 – (p + K)/2$,
$Q = p(K – 1)$.

He found c, a, and b using special equation (see, Tatsuoka, 1973, p.19): c = .3680, a = -1.0677, and b = 1.3631. And the adjustment equation defined as:

$$(\omega^2_{mult})_{adj} = \omega^2_{mult} - .368\,[N - 1 - (p + K)/2]^{-1.0677}[\,p(K - 1)]^{1.3631}\,(1 - \omega^2_{mult})$$

Tatsuoka considered several estimators of M and Q were tried out. Three of the most promising estimators led to following values for c, a, and b:

$M = N - 1 - (p + K)/2,\ Q = p^2 + (K - 1)^2$:	c = .2801,	a = -1.0692,	b = 1.1343
$M = N,\ Q = p(K - 1)$:	c = .4358,	a = -1.1048,	b = 1.3899
$M = N,\ Q = p^2 + (K - 1)^2$:	c = .3041,	a = -1.1066,	b = 1.1579

Tatsuoka determined that $M = N$ and $Q = p^2 + (K – 1)^2$ was the most effective combination for adjustment procedure. Observing further that the value of c was close to 1/3, a was close to –1, and b was close to 1, he proposed "alternative, simpler formula, rule- of - thumb correction" (Tatsuoka, 1973, p.24).

The rule- of - thumb correction formula is defined as:

$$(\omega^2_{mult})_{adj} = \omega^2_{mult} - \frac{p^2 + q^2}{3N}(1 - \omega^2_{mult})$$

where,

p = the number of variables,

q = the degree of freedom for hypothesis,

N = the sample size.

Tatsuoka believed that this formula was adequate when "$p*q \leq 49$ and $75 \leq N \leq 2000$" (Tatsuoka, 1973, p.31) and that this adjustment "will suffice for all practical purpose" (Tatsuoka, 1973, p.31) when used with Wilks index ($\eta^2_{mult\text{-}WI} = 1 - \Lambda$) and Hsu index ($\eta^2_{mult\text{-}HI} = V / 1 + V$). That is,

$$(\eta^2_{mult\text{-}WI})_{adj} = \eta^2_{mult\text{-}WI} - \frac{p^2 + q^2}{3N}(1 - \eta^2_{mult\text{-}WI}),$$

$$(\eta^2_{mult\text{-}HI})_{adj} = \eta^2_{mult\text{-}HI} - \frac{p^2 + q^2}{3N}(1 - \eta^2_{mult\text{-}HI}).$$

Huberty (1994, p.195) applied the Tatsuoka formula to adjust the ω^2_{mult}, the Shaffer-Gillo index, the Cramer-Nicewander index, and the Serlin index.

Serlin Adjustment

According to Serlin (1982), the η^2_{mult}(SEI) is a biased estimator because "the expected value of η^2_{mult}(SEI) is nonzero when the null hypothesis is true" (p.414). In other words, the Serlin index is a measure of the strength of association in the sample not in the population. When there is zero association in the population, the expected value of $\eta^2_{mult\text{-}SEI}$ is

$$E(\eta^2_{\text{mult-SEI}}) = \frac{b}{N-1}$$

where, b = max(p, q), where p is the number of variables and q is the number of group minus 1.

It is similar to the expected value for the multiple R^2:

$$E(R^2) = \frac{p}{N-1}$$

where, p = the number of independent variables in the multiple regression.

Therefore, Serlin (1982) proposed the adjustment for $\eta^2_{\text{mult-SEI,}}$ which is parallel to the R^2 adjustment. It was originated by Ezekiel (1930). The adjustment is defined as:

$$R^2_E = 1 - \frac{N-1}{N-p-1}(1-R^2),$$

$$(\eta^2_{\text{mult-SEI}})_{\text{adj}} = 1 - \frac{N-1}{N-b-1}(1-\eta^2_{\text{mult-SEI}}).$$

where, b = max(p, q), where p is the number of variables and q is the number of groups minus 1.

Although the two adjustment procedures have been recommended to reduce bias in multivariate effect size estimators, no study evaluating them has been identified. In this study, two adjustment procedures are used with the eight of multivariate measures of association- suggested by Wilks (WI), Hsu (HI), Stevens (SI), Shaffer-Gillo (SGI), Serlin (SEI), Tasuoka-Sachdeva (TSI), Hotelling-Rozeboom (HRI), and Cramer-Nicewander (CNI)- under the planned sampling conditions using SAS/IML (SAS Institute INC, version 8, 1999-2001).

Related Study

As indicated above, several researchers have proposed indices of measure of association in the MANOVA context. However, few studies have been conducted to examine the distributional properties these measures. One exception was Tatsuoka (1973) who examined the statistical properties (mean) of TSI by computer simulation study.

According to Tatsuoka, TSI was highly positively biased when the number of variables is large, the sample size is small, and is especially biased for population sets with low effect sizes when the ratio N/p (of total sample size to number of variables) was any lower than 40 or so. To reduce the bias in TSI, he (1973) developed an adjustment formula. He maintained that this adjustment formula for TSI suffices in the case of $p*(k-1) \leq 49$ and $75 \leq N \leq 2000$ and it could be used with WI and HI, as well as with TSI.

CHAPTER 3

METHODS

The purposes of this study are: 1) to examine degree of bias and precision in eight multivariate measures of association and 2) to evaluate the effectiveness of the Tatsuoka and Serlin procedures for adjusting eight effect-size measures. SAS/IML (SAS Institute INC, version 8, 1999-2001) is used to generate normal random numbers by the rannor function and to compute the descriptive statistics (means and standard deviation) for the following eight effect-size measures: suggested by Wilks (WI), Hsu (HI), Stevens (SI), Shaffer-Gillo (SGI), Serlin (SEI), Tasuoka-Sachdeva (TSI), Hotelling-Rozeboom (HRI), and Cramer-Nicewander (CNI). In addition, each of the effect-size measures is adjusted using the methods suggested by Tatsuoka (TA) and Serlin (SA). In this chapter the data generation procedure used is described along with the specific sampling conditions.

Sampling conditions

Four factors are manipulated for the present study when the multivariate assumptions are met: 1) the number of populations compared (k), 2) sample size (n), 3) the number of response variables (p), and 4) effect size (η^2). Three sets of populations (k) were considered: 2, 3, and 5. For each population set, equal samples of two sizes were drawn: n = 10, and 50, it yielded total sample sizes of N = 20, 30, 50, 100, 150, and 250. Three levels are used for the number of variables (p): 3, 5, and 10. Additionally, four levels of effect-size are considered: η^2_m =0, .1, .3, and .5. There are a total of 3*2*3*4 = 72 sampling conditions. When Tatsuoka (1973) examined the sampling distribution of omega squared and developed a correction formula for the bias of

TSI, he considered comparisons involving 5 populations, three sample sizes (n = 15, 30, and 60), three variable sets (3, 5, and 10), five effect- size levels (.1, .3, .5, .7, and .9), and two conditions of average intercorrelations among variables (low: .10 - .30, moderate: .40 - .60). In this study, the factor of the intercorrelations among variables is not considered but confined as zero. Tatsuoka (1973, p.13) indicated that the magnitude of average intercorrelations among variables had "virtually no effect on the sampling distribution" of Tatsuoka-Sachdeva index (TSI).

Generating the populations

When effect-size is zero, the null case, each of the k populations has a normal distribution with a mean of 0 and variance of 1 for each of the p variables. In cases where effect-size is not zero, data are generated as in the null case, but a constant is added to each observation in one sample on each of the p variables. The constant corresponds to the desired population mean. The constants are chosen to meet the specified relationship (eta squared) between the grouping factor and the dependent variables. The eta squared means that the proportion of generalized variance or total variance of the dependent variable accounted for by membership in the different populations. The formula for the population eta squared provided by Tatsuoka (1973) is defined as:

$$\eta^2_m = 1 - \frac{|\boldsymbol{\Sigma}|}{|\boldsymbol{\Sigma} + \alpha\alpha'/k|} \qquad (3.1)$$

where, $\boldsymbol{\Sigma}$ = common variance covariance matrix,

k = number of groups,

α = α_{jk} (j = 1,2,…, p; k = 1,2, …, k), where j = the number of dependent variables, k = the number of groups ,

α_{jk} = $\mu_{jk} - \mu_{j.}$, where μ_{jk} = population mean for variable j and population k, μ_j = grand mean for variable j.

For the present study, p*p identity matrix is used for common variance covariance matrix (**Σ**) because there were no intercorrelations among variables. SAS/IML was used to determine the population means to meet the various preassigned values of eta squared effect sizes. The complete computer program for determining the population means is shown in appendix A for the case of 2 populations and 5 variables when the desired effect-size is .1.

The way the population means were determined was described below. When 2 populations are compared and 5 variables are considered under the desired effect-size (eta squared) is .1, α =

$$\begin{vmatrix} .1492 & -.1492 \\ .1492 & -.1492 \\ .1492 & -.1492 \\ .1492 & -.1492 \\ .1492 & -.1492 \end{vmatrix}$$

With this matrix the population eta squared effect-size formula provided by Tatsuoka (1973), effect-size equals .1001556. The solution was checked by generating a half million observations for each group and computing η^2 (eta squared). The results of generating a half million observations for each group and computing η^2 yielded same as population eta squared in rounded four decimal places at all sampling conditions. Although there are many alternative combinations of population means that would lead to the same η^2, it was decided to consider situations that one population's means was not zero and all variables had same means. The population means assigned to all variables in one group for the various combinations of k and p to achieve the desired effect size, η^2_m, are presented in Table 3.1. All other population means were set equal to zero.

Table 3.1

One non-zero population mean vector of each sampling condition

k	η^2_m	p		
		3	5	10
2	.100	.385	.2984	.211
	.300	.756	.586	.4141
	.500	1.155	.895	.633
3	.100	.409	.317	.224
	.300	.802	.6214	.4393
	.500	1.225	.949	.671
5	.100	.482	.373	.264
	.300	.946	.732	.518
	.500	1.444	1.119	.791

However, eta squared population effect-size formula (3.1) does not provide a population effect-size for SGI, SEI, and CNI. These effect size indices are based on different definition of effect size. The SGI represents a weighted average of the estimated correlation ratios for each of the discriminant functions. The SEI is the arithmetic average of the squared canonical correlation for the separate linear combinations of two sets of variables. And the CNI is equal to one minus the geometric mean of the $1-\rho^2_j$, where ρ^2_j is the squared canonical correlation between grouping variables and j^{th} linear discriminant function (LDF). As a result in the non-null case SGI, SEI, and CNI have different meaning of η^2. The relationship between SGI, SEI, and CNI and η^2_m is a function of the number of discriminant functions, r.

The SGI effect-size is computed as:

$$\zeta^2 = \frac{V}{r + V} \qquad (3.2)$$

where, V = sum of the eigenvalues of $\mathbf{H}^* \mathbf{E}^{-1}$

The relationship between ζ^2 and η^2 can be formed based on Hsu's statement that $V = \eta^2/1-\eta^2$. Substituting this definition of V in equation 3.2, ζ^2 is defined as:

$$\zeta^2 = \frac{\eta^2/1-\eta^2}{r + \eta^2/1-\eta^2} = \frac{\eta^2/1-\eta^2}{(\eta^2 - r\,\eta^2 + r)\,/1-\eta^2}$$

Thus,

$$\zeta^2 = \frac{\eta^2}{\eta^2 - r\,\eta^2 + r}$$

where, η^2 = population eta squared,
r = min(p,q), where p is the number of variables and q is the number of group minus one.

The population effect-size for SEI is defined as:

$$\xi^2 = \frac{SS_{Between}}{SS_{Tottal}} = \frac{\sum_{j=1}^{r} \Theta_j}{r} \qquad (3.3)$$

Because $\Theta_j = \lambda_j/(1+\lambda_j)$ and $\lambda_j = SS_{Betweenj} / SS_{Withinj}$,

$$\sum_{j=1}^{r} \Theta_j = \sum_{j=1}^{r} (SS_{Betweenj} / SS_{Withinj}) / (1 + SS_{Betweenj} / SS_{Withinj})$$

$$= \sum_{j=1}^{r} (SS_{Betweenj}) / (SS_{Withinj} + SS_{Betweenj})$$

$$= \sum_{j=1}^{r} (SS_{Betweenj}) / (SS_{Totalj})$$

$$= (SS_{Between}) / (SS_{Total})$$

Eta squared, η^2, is former defined as $\eta^2 = SS_{Between}/ SS_{Total}$, so $\sum_{j=1}^{r} \Theta_j = \eta^2$.

and

$$\xi^2 = \frac{\eta^2}{r}$$

where : η^2 = population eta squared,

$r = \min(p,q)$, where p is the number of variables and q is the number of group minus one.

SEI is therefore the average contribution each discriminant function makes to η^2.

The formula for the population CNI can be derived simply as shown below.

$$\tau^2 = 1 - [\, \Pi^r_{j=1} (1-\rho^2_j) \,]^{1/r}$$

Because $\Pi^r_{j=1} (1-\rho^2_j) = \Lambda$ and $\eta^2 = 1 - \Lambda$,

$$\tau^2 = 1 - [\Lambda]^{1/r}, \; \Lambda = 1 - \eta^2$$

Thus,

$$\tau^2 = 1 - (1 - \eta^2)^{1/r}$$

where : η^2 = population eta squared,
$r = \min(p,q)$, where p is the number of variables and q is the number of group minus one.

From the above it is shown that SGI, SEI, and CNI provide different definitions of effect size when $\eta^2 > 0$.They are all influenced by r, where $r = \min(p,q)$. When $r = 1$, they are the same. Table 3.2 provides parameters values rounded to three decimal places for SGI (ζ^2), SEI (ξ^2), and CNI (τ^2) that correspond to eta squared index (WI, HI, SI, TSI, and HRI). The means of the sampling distribution for SGI, SEI, and CNI were compared to these values to estimate the degree of bias associated with these three effect-size indices.

Table 3.2
Parameter values measures of effect-size

r	η^2_m	ζ^2	ξ^2	τ^2
2	.100	.053	.050	.051

	.300	.176	.150	.163
	.500	.333	.250	.293
3	.100	.033	.033	.035
	.300	.125	.100	.112
	.500	.250	.167	.206
4	.100	.027	.025	.026
	.300	.097	.075	.085
	.500	.200	.125	.159

r=min(p,q)

Generating the Samples

Data for each group (k = 1, . . . , K) were generated using the following linear model:

$$\mathbf{y}_{ij} = \boldsymbol{\mu}_j + \boldsymbol{\varepsilon}_{ij}$$

where, $\boldsymbol{\varepsilon}_{ij} \sim N(0,\mathbf{I})$,
$\boldsymbol{\mu}_j$ = vector of p population means for the group j.

The error component $\boldsymbol{\varepsilon}_{ij}$ was generated using the rannor function in SAS/IML. The $\boldsymbol{\mu}_j$ were taken from tables 3.1. The computer program for generating samples and computing statistics is shown appendix B.

For each condition, 10,000 replications were generated. For each replication, values for WI, HI, SI, SGI, SEI, TSI, HRI, and CRI were calculated. The means and standard deviations of each statistic were computed across the 10,000 replications. The bias was estimated by subtracting the population effect-size from mean of each effect-size index. In this study, difference between the mean effect-size and the parameters identified in Table 3.2 that was 0 to two decimal places was considered acceptable.

CHAPTER 4

RESULTS

The purposes of this study are: 1) to examine the degree of bias and precision in eight multivariate measures of association and 2) to evaluate the effectiveness of the Tatsuoka and Serlin procedures for adjusting the eight effect-size measures. In the previous chapter, the method used to generate the sampling distributions of Wilks index (WI), Hsu index (HI), Stevens index (SI), Shaffer-Gillo index (SGI), Serlin index (SEI), Tasuoka-Sachdeva index (TSI), Hotelling-Rozeboom index (HRI), and Cramer-Nicewander index (CNI) was described. Each of the effect-size measures was adjusted using the methods suggested by Tatsuoka (TA) and Serlin (SA). The bias was estimated by subtracting population effect-size from mean of each effect-size index.

In this chapter the results of the study are presented. First, the degree of bias associated with each index is presented. Second, the effectiveness of the two adjustment procedures is evaluated. And third, the precision with which adjusted and unadjusted measures of association estimate the effect-sizes is considered. The chapter ends with a summary of the research findings.

Bias in Unadjusted Measures of Effect Size

The results indicate that all of the unadjusted effect-size measures were biased to some degree and the amount of bias was affected by the number of populations compared, sample sizes, the number of response variables, and effect size. The pattern of results was similar for all eight indices, but the magnitude of the bias varied among the indices. The complete results are presented in Appendix C, but to facilitate the understanding the main factors affecting bias results are presented in several smaller tables which highlight the effect of 1) magnitude of the effect-size, 2) sample size, 3) number of variables, and 4) number of populations compared.

Effect size

Table 4.1 presents the bias of the unadjusted effect-size indices as the population effect size increased. As shown in Table 4.1, for all of the effect-size measures bias decreased as the population effect size increased. For example, considering the Wilks index (WI) when p=5, n=10, and k=2, the bias was .260, .235, .181, and .131 for η^2 = 0 .1, .3, and .5, respectively. The same pattern is apparent for all eight indices and for group sizes of 2, 3, and 5.

Table 4.1

Bias of the unadjusted effect-size indices as population effect-size increases

p	n	k	η^2_m	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
5	10	2	0	.260	.260	.195	.260	.260	.211	.260	.260
			.1	.235	.235	.159	.235	.235	.189	.235	.235
			.3	.181	.181	.090	.181	.181	.141	.181	.181
			.5	.131	.131	.040	.131	.131	.100	.131	.131
		3	0	.321	.306	.257	.185	.173	.265	.321	.179
			.1	.287	.266	.213	.177	.161	.234	.287	.170
			.3	.222	.195	.136	.160	.141	.179	.222	.151
			.5	.160	.132	.074	.135	.122	.128	.160	.130
		5	0	.347	.319	.290	.107	.098	.285	.347	.102
			.1	.315	.278	.246	.108	.095	.258	.315	.101
			.3	.244	.197	.162	.106	.087	.199	.244	.096
			.5	.173	.123	.090	.098	.079	.139	.173	.088

Sample size

Table 4.2 presents the bias of the unadjusted effect-size as sample size increased. As can be seen in Table 4.2, for all of the effect-size measures the bias was much greater when sample size was small (n = 10) than when sample size was large (n = 50). Considering the Wilks index

(WI) when k=3, p=5, $\eta^2 = 0$, and n=10, the bias was .321. But when n was increased 50, the bias of WI was .065. These results also demonstrate that even with a relatively large sample size (N = 150) all eight measures of association over-estimated the relationship between the grouping variable and outcome measures to an unacceptable degree.

Table 4.2

Bias of the unadjusted effect-size indices as sample size increases

k	p	η^2_m	n	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	5	0	10	.321	.306	.257	.185	.173	.265	.321	.179
			50	.065	.065	.062	.033	.033	.052	.065	.033
		.1	10	.287	.266	.213	.177	.161	.234	.287	.170
			50	.059	.057	.051	.033	.031	.047	.059	.032
		.3	10	.222	.195	.136	.160	.141	.179	.222	.151
			50	.046	.040	.031	.030	.027	.036	.046	.029
		.5	10	.160	.132	.074	.135	.122	.128	.160	.130
			50	.033	.026	.016	.025	.023	.025	.033	.024

Number of variables

Table 4.3 presents the bias of the unadjusted effect-size indices as the number of outcome variables increased from 3 to 10. As shown in Table 4.3, for all of the effect-size measures the bias increased as the number of variables was increased. For example, considering the Wilks index (WI) when k=3, n=50, and $\eta^2 = 0$, the bias was .040, .065, and .129 for p=3, 5, and 10, respectively.

Table 4.3

Bias of unadjusted effect-size indices as the number of variables (p) increases

k	n	η^2_m	p	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	50	0	3	.040	.039	.038	.020	.020	.026	.040	.020

	5	.065	.065	.062	.033	.033	.052	.065	.033
	10	.129	.126	.117	.067	.066	.117	.129	.067
.1	3	.035	.034	.031	.020	.018	.023	.035	.019
	5	.059	.057	.051	.033	.031	.047	.059	.032
	10	.117	.110	.097	.065	.063	.105	.117	.065
.3	3	.027	.025	.019	.019	.016	.017	.027	.017
	5	.046	.040	.031	.030	.027	.036	.046	.029
	10	.090	.078	.060	.058	.055	.081	.090	.057
.5	3	.020	.017	.010	.017	.013	.012	.020	.015
	5	.033	.026	.016	.025	.023	.025	.033	.024
	10	.064	.050	.031	.047	.048	.057	.064	.048

Number of populations

Table 4.4 presents the bias of the unadjusted effect-size indices as the number of populations compared increases. As shown in Table 4.4, the bias all of the effect-size indices except SGI, SEI, and CNI increased as the number of populations increased. For SGI, SEI, and CNI bias decreased as the number of populations increased. For example, considering the Wilks index (WI) when p=5, n=10, and η^2=0, the bias was .260, .321, and .347 for k=2, 3, and 5, respectively. On the other hand, considering Shaffer-Gillo index (SGI) when p=5, n=10, and η^2=0, the bias was .260, .185, and .107 for k=2, 3, and 5, respectively.

Table 4.4

Bias of the unadjusted effect-size indices with regard to the number of population compared

p	n	η^2_m	k	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
5	10	0	2	.260	.260	.195	.260	.260	.211	.260	.260
			3	.321	.306	.257	.185	.173	.265	.321	.179
			5	.347	.319	.290	.107	.098	.285	.347	.102
		.1	2	.235	.235	.159	.235	.235	.189	.235	.235
			3	.287	.266	.213	.177	.161	.234	.287	.170

		5	.315	.278	.246	.108	.095	.258	.315	.101
	.3	2	.181	.181	.090	.181	.181	.141	.181	.181
		3	.222	.195	.136	.160	.141	.179	.222	.151
		5	.244	.197	.162	.106	.087	.199	.244	.096
	.5	2	.131	.131	.040	.131	.131	.100	.131	.131
		3	.160	.132	.074	.135	.122	.128	.160	.130
		5	.173	.123	.090	.098	.079	.139	.173	.088

Comparing two populations

When two populations were compared, SI and TSI had less bias than WI, HI, SGI, SEI, HRI, and CNI regardless of sample size, the number of response variables, and effect size. The Wilks index (WI) and the Hotelling-Roseboom index (HRI) were the most biased indices under most sampling conditions (see results in Table 4.5). Table 4.5 presents the unadjusted effect-size bias when 2 populations are compared.

Table 4.5

Bias of the unadjusted effect-size indices when 2 populations are compared

k	η^2_m	n	p	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	0	10	3	.157	.157	.129	.157	.157	.106	.157	.157
			5	.260	.260	.195	.260	.260	.211	.260	.260
			10	.526	.526	.321	.526	.526	.489	.526	.526
		50	3	.030	.030	.028	.030	.030	.020	.030	.030
			5	.051	.051	.048	.051	.051	.041	.051	.051
			10	.101	.101	.091	.101	.101	.091	.101	.101
	.1	10	3	.140	.140	.101	.140	.140	.091	.140	.140
			5	.235	.235	.159	.235	.235	.189	.235	.235
			10	.474	.474	.264	.474	.474	439	.474	.474
		50	3	.027	.027	.022	.027	.027	.017	.027	.027
			5	.045	.045	.038	.045	.045	.035	.045	.045
			10	.090	.090	.073	.090	.090	.081	.090	.090

	.3	10	3	.109	.109	.055	.109	.109	.065	.109	.109
			5	.181	.181	.090	.181	.181	.141	.181	.181
			10	.365	.365	.156	.365	.365	.336	.365	.365
		50	3	.020	.020	.011	.020	.020	.011	.020	.020
			5	.034	.034	.021	.034	.034	.025	.034	.034
			10	.069	.069	.043	.069	.069	.061	.069	.069
	.5	10	3	.078	.078	.020	.078	.078	.043	.078	.078
			5	.131	.131	.040	.131	.131	.100	.131	.131
			10	.264	.264	.073	.264	.264	.242	.264	.264
		50	3	.015	.015	.005	.015	.015	.008	.015	.015
			5	.025	.025	.010	.025	.025	.018	.025	.025
			10	.050	.050	.021	.050	.050	.043	.050	.050

Comparing three or five populations

A similar pattern of results were obtained when three or five populations were compared. To present this pattern Table 4.6 summarizes the results for a comparison of three populations. For no or small effects (η^2= 0 or .1) SGI, SEI, and CNI were less biased than WI, HI, SI, TSI, and HRI. For moderate or large effects (η^2= .3 or .5) the results frequently revealed a different pattern. When k=3, n=10, p=5, 10, and η^2= .3; k=3, n=10 (50), p=3, 5, 10, and η^2=.5; k=5, n=10, p=3, 5, 10, and η^2= .5, the Stevens index (SI) was the least biased measure of association. The Serlin index (SEI) was the least biased index when three or more populations compared under the most sampling conditions except for conditions stated above.

Table 4.6

Bias of the unadjusted effect-size indices when 3 populations are compared

k	η^2_m	n	p	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	0	10	3	.198	.193	.170	.110	.102	.135	.198	.106
			5	.321	.306	.257	.185	.173	.265	.321	.179
			10	.580	.533	.409	.370	.346	.541	.580	.358

	50	3	.040	.039	.038	.020	.020	.026	.040	.020
		5	.065	.065	.062	.033	.033	.052	.065	.033
		10	.129	.126	.117	.067	.066	.117	.129	.067
.1	10	3	.178	.169	.141	.108	.096	.120	.178	.102
		5	.287	.266	.213	.177	.161	.234	.287	.170
		10	.520	.468	.344	.351	.325	.484	.520	.339
	50	3	.035	.034	.031	.020	.018	.023	.035	.019
		5	.059	.057	.051	.033	.031	.047	.059	.032
		10	.117	.110	.097	.065	.063	.105	.117	.065
.3	10	3	.138	.125	.088	.101	.082	.089	.138	.092
		5	.222	.195	.136	.160	.141	.179	.222	.151
		10	.404	.348	.227	.310	.288	.375	.404	.300
	50	3	.027	.025	.019	.019	.016	.017	.027	.017
		5	.046	.040	.031	.030	.027	.036	.046	.029
		10	.090	.078	.060	.058	.055	.081	.090	.057
.5	10	3	.100	.085	.047	.087	.069	.062	.100	.080
		5	.160	.132	.074	.135	.122	.128	.160	.130
		10	.287	.234	.124	.252	.251	.266	.287	.253
	50	3	.020	.017	.010	.017	.013	.012	.020	.015
		5	.033	.026	.016	.025	.023	.025	.033	.024
		10	.064	.050	.031	.047	.048	.057	.064	.048

In sum, even when arelatively large sample size (n=50), all of the unadjusted effect-size measures were biased an unacceptable degree. Therefore, adjustment procedures suggested by Tatsuoka (1973) and Serlin (1982) need to reduce a bias in all of the unadjusted effect-size measures presented in this study. In the subsequence part, the amount of adjusted bias of eight effect-size measures using the Tatsuoka and the Serlin procedures are described. In addition, the effectiveness of these procedures is evaluated.

The additional means of the sampling distributions of the 8 unadjusted effect-size indices are reported in Appendix C for comparisons of 2, 3, and 5 populations, involving 3, 5, and 10 measures, with sample sizes of 10 and 50 when the population effect sizes are 0, .1, .3, and .5, respectively.

Bias in Adjusted Measures of Effect Size

Comparing two populations

In this study, the bias was estimated by subtracting population effect-size from mean of each effect-size index across 10,000 replications. The difference between the mean effect-size and the parameters that was 0 to two decimal places was considered acceptable. The bold number indicates the acceptable degree of bias.

Serlin Adjustment

The results indicate (see Table 4.7) that when two populations are compared, the Serlin adjustment provides an appropriate adjustment for all measures of effect-size except the Stevens index (SI) and the Tatsuoka-Sachdeva index (TSI) under most conditions. Table 4.7 provides the bias of adjusted effect-size using the Serin adjustment when two populations are compared. Applying the Serlin adjustment to the SI and TSI indices over-corrects for bias and the relationship is underestimated. These results were consistent for all effect sizes, and number of variables considered here.

Table 4.7

Bias of the adjusted effect-size indices using the Serin adjustment when 2 populations are compared

k	η^2_m	n	p	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	0	10	3	**-.000**	**-.000**	-.033	**-.000**	**-.000**	-.060	**-.000**	**-.000**
			5	**-.003**	**-.003**	-.091	**-.003**	**-.003**	-.069	**-.003**	**-.003**
			10	**.001**	**.001**	-.432	**.001**	**.001**	-.078	**.001**	**.001**
		50	3	**-.000**	**-.000**	**-.001**	**-.000**	**-.000**	-.010	**-.000**	**-.000**
			5	**.000**	**.000**	**-.002**	**.000**	**.000**	**-.009**	**.000**	**.000**
			10	**.000**	**.000**	-.011	**.000**	**.000**	-.010	**.000**	**.000**

.1	10	3	**-.001**	**-.001**	-.047	**-.001**	**-.001**	-.060	**-.001**	**-.001**
		5	**-.001**	**-.001**	-.105	**-.001**	**-.001**	-.064	**-.001**	**-.001**
		10	**.002**	**.002**	-.442	**.002**	**.002**	-.071	**.002**	**.002**
	50	3	**.000**	**.000**	**-.004**	**.000**	**.000**	-.010	**.000**	**.000**
		5	**.000**	**.000**	**-.007**	**.000**	**.000**	-.010	**.000**	**.000**
		10	**.000**	**.000**	-.019	**.000**	**.000**	-.010	**.000**	**.000**
.3	10	3	**-.001**	**-.001**	-.065	**-.001**	**-.001**	-.053	**-.001**	**-.001**
		5	**-.003**	**-.003**	-.126	**-.003**	**-.003**	-.058	**-.003**	**-.003**
		10	**-.005**	**-.005**	-.446	**-.005**	**-.005**	-.067	**-.005**	**-.005**
	50	3	**-.000**	**-.000**	**-.009**	**-.000**	**-.000**	-.010	**-.000**	**-.000**
		5	**-.000**	**-.000**	-.014	**-.000**	**-.000**	-.010	**-.000**	**-.000**
		10	**-.000**	**-.000**	-.030	**-.000**	**-.000**	-.010	**-.000**	**-.000**
.5	10	3	**-.000**	**-.000**	-.068	**-.000**	**-.000**	-.041	**-.000**	**-.000**
		5	**.000**	**.000**	-.123	**.000**	**.000**	-.042	**.000**	**.000**
		10	**.002**	**.002**	-.399	**.002**	**.002**	-.044	**.002**	**.002**
	50	3	**.000**	**.000**	-.010	**.000**	**.000**	**-.007**	**.000**	**.000**
		5	**.000**	**.000**	-.015	**.000**	**.000**	**-.007**	**.000**	**.000**
		10	**.000**	**.000**	-.031	**.000**	**.000**	**-.007**	**.000**	**.000**

Tatsuoka Adjustment

Table 4.8 provides the bias associated with the eight effect-size indices after using the Tatsuoka adjustment. The results indicate that Tatsuoka procedures typically over-adjusts the sample values and under-estimates the population parameter. Only when sample size was large (n=50) and the number of variables was small (p=3), the Tatsuoka adjustment provides an appropriate adjustment for some measures of effect size. The Tatsuoka adjustment for TSI did not provide an appropriate adjustment under most sampling conditions when two populations were compared. Table 4.8 provides the bias of adjusted effect-size indices using the Tatsuoka adjustment when two populations were compared.

Table 4.8

Bias of the adjusted effect-size indices using the Tatsuoka adjustment when 2 populations are compared

k	η^2_m	n	p	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	0	10	3	.017	.017	-.015	.017	.017	-.042	.017	.017
			5	-.059	-.059	-.152	-.059	-.059	-.129	-.059	-.059
			10	-.269	-.269	-.821	-.269	-.269	-.370	-.269	-.269
		50	3	**-.002**	**-.002**	**-.003**	**-.002**	**-.002**	-.012	**-.002**	**-.002**
			5	-.031	-.031	-.034	-.031	-.031	.041	.031	.031
			10	-.201	-.201	-.214	-.201	-.201	-.214	-.201	-.201
	.1	10	3	.013	.013	-.031	.013	.013	-.043	.013	.013
			5	-.052	-.052	-.161	-.052	-.052	-.118	-.052	-.052
			10	-.240	-.240	-.805	-.240	-.240	-.335	-.240	-.240
		50	3	**-.001**	**-.001**	**-.006**	**-.001**	**-.001**	-.011	**-.001**	**-.001**
			5	-.028	-.028	-.036	-.028	-.028	-.039	-.028	-.028
			10	-.181	-.181	-.204	-.181	-.181	-.194	-.181	-.181
	.3	10	3	.010	.010	-.051	.010	.010	-.040	.010	.010
			5	-.043	-.043	-.173	-.043	-.043	-.101	-.043	-.043
			10	-.196	-.196	-.757	-.196	-.196	-.275	-.196	-.196
		50	3	**-.002**	**-.002**	-.011	**-.002**	**-.002**	-.011	**-.002**	**-.002**
			5	-.022	-.022	-.037	-.022	-.022	-.032	-.022	-.022
			10	-.142	-.142	-.177	-.142	-.142	-.153	-.142	-.142
	.5	10	3	**.008**	**.008**	-.058	**.008**	**.008**	-.032	**.008**	**.008**
			5	-.027	-.027	-.158	-.027	-.027	-.073	-.027	-.027
			10	-.132	-.132	-.643	-.132	-.132	-.191	-.132	-.132
		50	3	**-.000**	**-.000**	-.011	**-.000**	**-.000**	**-.008**	**-.000**	**-.000**
			5	-.015	-.015	-.032	-.015	-.015	-.023	-.015	-.015
			10	-.100	-.100	-.139	-.100	-.100	-.109	-.100	-.100

Comparing three or five populations

Serlin adjustment

Table 4.9 provides the bias associated with the eight effect-size indices after using the

Serlin adjustment when the sample size was large (n=50). The results indicate that the Serlin procedure appropriately adjust for bias in SGI, SEI, and CNI when the sample size was large (n=50). However, this procedure generally under-adjust the amount of bias in WI, HI, SI, TSI, and HRI under all sampling conditions when three or more populations were compared.

Table 4.9

Bias of the adjusted effect-size indices using the Serlin adjustment under the selected conditions

k	n	p	η^2_m	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	50	3	0	.020	.020	.019	**.000**	**.000**	**.006**	.020	**.000**
			.1	.017	.016	.013	**.000**	**-.000**	**.005**	.017	**.000**
			.3	.014	.011	**.005**	**.002**	**.001**	**.003**	.014	**.001**
			.5	.011	**.007**	**.000**	**.004**	**-.001**	**.002**	.011	**.001**
		5	0	.033	.032	.030	**.000**	**-.000**	.019	.033	**.000**
			.1	.030	.027	.022	**.001**	**-.000**	.018	.030	**.001**
			.3	.023	.017	**.008**	**.003**	**-.001**	.012	.023	**.001**
			.5	.017	.010	**-.000**	**.003**	**-.001**	**.008**	.017	**.001**
			0	.066	.063	.054	**.000**	**-.000**	.053	.066	**.000**
			.1	.061	.053	.040	**.002**	**.000**	.048	.061	**.001**
		10	.3	.047	.033	.014	**.003**	**-.001**	.036	.047	**.001**
			.5	.033	.017	**-.002**	**.003**	**-.001**	.025	.033	**.000**
5	50	3	0	.031	.031	.030	**.000**	**-.000**	.016	.031	**.000**
			.1	.028	.026	.024	**.003**	**.000**	.014	.028	**-.000**
			.3	.022	.017	.013	**.002**	**-.000**	.010	.022	**.000**
			.5	.016	**.009**	**.005**	**.002**	**-.001**	**.007**	.016	**.000**
		5	0	.058	.056	.055	**.000**	**-.000**	.042	.058	**-.000**
			.1	.052	.047	.044	**.000**	**-.000**	.038	.052	**.000**
			.3	.040	.030	.024	**.001**	**-.001**	.028	.040	**.000**
			.5	.029	.017	.010	**.002**	**-.001**	.020	.029	**.000**
		10	0	.117	.109	.103	**.000**	**.000**	.102	.117	**.000**
			.1	.104	.090	.081	**.001**	**-.000**	.090	.104	**.000**
			.3	.082	.058	.047	**.002**	**-.000**	.070	.082	**.001**
			.5	.058	.031	.020	**.003**	**-.001**	.049	.058	**.000**

Table 4.10 present the bias in adjusted SGI, SEI, and CNI using the Serlin adjustment when the sample size was small. As shown in those two tables, when the sample size was small, the appropriateness of the Serlin adjustment for SEI, SGI, and CNI depends on the sampling conditions. However, it worked better for SEI than for SGI and CNI.

Table 4.10

Bias of the adjusted SGI, SEI, and CNI using the Serlin adjustment

k	p	η^2_m	SGI	SEI	CNI
3	3	0	**.007**	**-.000**	**.003**
		.1	.011	**-.002**	**.005**
		.3	.018	**-.005**	**.006**
		.5	.020	**-.008**	**.007**
	5	0	.015	**.001**	**.008**
		.1	.017	**-.002**	**.007**
		.3	.022	**-.006**	**.009**
		.5	.024	**-.008**	.010
	10	0	.036	**.001**	.021
		.1	-.038	**-.002**	.018
		.3	-.039	**-.006**	.018
		.5	.034	**-.010**	.015
5	3	0	**.006**	**.000**	**.003**
		.1	.011	**-.001**	**.003**
		.3	.013	**-.000**	**.003**
		.5	.018	**-.007**	**.005**
	5	0	**.006**	**-.004**	**.000**
		.1	**.010**	**-.004**	**.002**
		.3	.015	**-.007**	**.003**
		.5	.018	**-.010**	**-.002**
	10	0	.025	**-.000**	.012
		.1	.028	**-.001**	.013
		.3	.031	**-.005**	.012
		.5	.035	**-.008**	.012

Tatsuoka adjustment

Table 4.11 provides the bias associated with the eight effect-size indices after using the Tatsuoka adjustment. The Tatsuoka adjustment did provide an appropriate adjustment for some effect-size indices when sample size was large, the number of variables was small, and population effect-size was large but frequently it could either under-adjust or over-adjust the magnitude of the effect.

As seen in Table 4.11, the Tatsuoka adjustment for TSI provided an appropriate adjustment under most presented sampling conditions, especially when the sample size was large. But, when the sample was small (n=10), the Tatsuoka adjustment for TSI frequently did not provide an appropriate adjustment (see Appendix C). According to Tatsuoka (1973), the adjustment formula for TSI suffices in case of $p*(k-1) \leq 49$ and $75 \leq N \leq 2000$. However, it also appeared works outside these limits. For example, Tatsuoka adjustment for TSI could provide an appropriate adjustment when k=3, n=10, p=3, and η^2= .1, .3, .5 ($N \leq 75$). But, it did not provide an appropriate adjustment although these constraints were satisfied, when k=3, n=50, p=10, and η^2= 0, .1, .3, .5 ($p*(k-1) \leq 49$ and $75 \leq N \leq 2000$). Even though Tatsuoka (1973) believed that TA would provide a valid adjustment in WI and HI, the results presented indicate that it depends on the sampling conditions. Table 4.12 provides the bias of the adjusted effect-size indices using the Tatsuoka adjustment under the selected sampling conditions.

Table 4.11

Bias of the adjusted effect-size indices using the Tatsuoka adjustment under the selected conditions

k	n	p	η^2_m	WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	50	3	0	.012	.012	.011	**-.007**	**-.008**	**-.001**	.012	-007
			.1	.010	**.009**	**.005**	**-.006**	**-.008**	**-.002**	.010	**-.007**

			.3	**.008**	**.005**	**-.000**	**-.003**	**-.008**	**-.002**	**.008**	**-.005**
			.5	**.007**	**.003**	**-.003**	**-.001**	**-.007**	**-.001**	**.007**	**-.004**
		5	0	**.005**	**.004**	**.002**	-.028	-.028	**-.008**	**.005**	-.028
			.1	**.005**	**.002**	**-.002**	-.025	-.027	**-.007**	**.005**	-.026
			.3	**.004**	**-.001**	-.011	-.020	-.025	**-.006**	**.004**	-.022
			.5	**.003**	**-.003**	-.014	-.015	-.023	**-.005**	**.003**	-.019
			0	-.071	-.075	-.085	.147	-.148	-.087	-.071	-.148
			.1	-.063	-.072	-.087	-.138	-.141	-.077	-.063	-.139
		10	.3	-.049	-.065	-.087	-.118	-.127	-.062	-.049	-.122
			.5	-.036	-.053	-.077	-.095	-.113	-.045	-.036	-.104
5	50	3	0	.016	.015	.014	-.016	-.016	**-.000**	.016	-.016
			.1	.014	.012	.010	-.012	-.015	**-.000**	.014	-.015
			.3	.011	**.006**	**.002**	-.012	-.015	**-.000**	.011	-.014
			.5	**.008**	**.002**	**-.002**	**-.009**	-.015	**-.000**	**.008**	-.012
		5	0	.027	.025	.023	-.033	-.033	.010	.027	-.033
			.1	.024	.018	.015	-.031	-.033	**.009**	.024	-.032
			.3	**.004**	**-.001**	-.011	-.020	-.025	**-.006**	**.004**	-.022
			.5	.013	.001	-.005	-.023	-.031	**.004**	.013	-.027
		10	0	.021	.013	**.006**	-.107	-.108	**.004**	.021	-.107
			.1	.018	**.002**	**-.006**	-.104	-.106	**.002**	.018	-.105
			.3	.015	-.011	-.023	-.094	-.101	**.002**	.015	-.097
			.5	.010	-.018	-.031	-.082	-.096	**.001**	.010	-.090

Comparing the Tatsuoka and the Serlin adjustment

Table 4.12 compares bias of adjusted effect-size indices using the Serin and the Tatsuoka adjustments under selected sampling conditions. As shown in Table 4.12 when three or more populations are compared, the Tatsuoka adjustment adjusts the bias of WI, HI, GI, TSI, and HRI more effectively in comparison to the Serlin adjustment; the Serlin adjustment adjusts bias of SGI, SEI, and CNI more appropriately in comparison to the Tatsuoka adjustment except 4 sampling conditions (k=3, n=50, p=10, and $\eta^2 = 0, .1, .3,$ and .5).

Table 4.12

Bias of the adjusted effect-size indices using the Serlin and the Tatsuoka adjustments under the selected conditions

k	p	n	η^2_{multi}		WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	5	10	0	TA	.102	.082	.018	-.076	-.093	.028	.102	-.084
				SA	.180	.161	.103	.015	**.001**	.112	.180	**.008**
		50	0	TA	**.005**	**.004**	**.002**	-.028	-.028	**-.008**	**.005**	-.028
				SA	.033	.032	.030	**.000**	**-.000**	.019	.033	**.000**
	10	10	.1	TA	.081	-.029	-.298	-.336	-.395	**.003**	.081	-.364
				SA	.320	.242	.051	-.038	**-.002**	.265	.320	.018
		50	.1	TA	-.063	-.072	-.087	-.138	-.141	-.077	-.063	-.139
				SA	.061	.053	.040	**.002**	**.000**	.048	.061	.001
5	5	10	.3	TA	.120	.059	.015	-.111	-.141	.062	.120	-.127
				SA	.192	.139	.101	-.015	**-.007**	.142	.192	**.003**
		50	.3	TA	.018	**.007**	**.002**	-.028	-.032	**.006**	.018	-.030
				SA	.040	.030	.024	**.001**	**-.001**	.028	.040	**.000**
	10	10	.5	TA	.083	.020	-.021	.093	-.137	.041	.083	-.117
				SA	.136	.080	.043	.018	-.010	.098	.136	**-.002**
		50	.5	TA	.010	-.018	-.031	-.082	-.096	**.001**	.010	-.090
				SA	.058	.031	.020	**.003**	**-.001**	.049	.058	**.000**

TA = TA adjusted effect-size

SA = SA adjusted effect size

Precision

Table 4.14 presents the standard deviations of unadjusted and adjusted effect-size indices across the 10,000 replications under selected sampling conditions. The results indicate that the unadjusted effect-size measures had smaller standard deviations than either the Tatsuoka or the Serlin adjusted effect-size measures. For example, considering the Wilks index (WI) when k=3, p=5, n=10, and $\eta^2 = 0$, the standard deviations are .1138, .1505, and .1375 for unadjusted WI, adjusted by TA, and adjusted by SA, respectively. The difference in precision is greatest when sample sizes is small. The precision of the Serlin adjusted effect-size measures is always greater than the precision of the Tatsuoka adjusted effect-size measures. The difference in precision is typically small and cannot compensate for the difference in bias associated with the eight effect size indices. The standard deviations of all sampling conditions are presented in Appendix C.

Table 4.13

Standard deviations of effect-size indices across the 10,000 replications

k	p	n	η^2_m		WI	HI	SI	SGI	SEI	TSI	HRI	CNI
3	5	10	0	ES	.1138	.1050	.0950	.0758	.0666	.1205	.1138	.0711
				TA	.1505	.1388	.1256	.1003	.0880	.1593	.1505	.0940
				SA	.1375	.1269	.1148	.0916	.0804	.1456	.1375	.0859
		50	0	ES	.0282	.0275	.0265	.0148	.0144	.0284	.0282	.0146
				TA	.0300	.0293	.0283	.0158	.0154	.0302	.0300	.0156
				SA	.0291	.0285	.0275	.0154	.0149	.0294	.0291	.0152
	10	10	.1	ES	.1194	.1135	.1058	.0791	.0659	.1261	.1194	.0724
				TA	.1367	.1299	.1210	.0906	.0754	.1443	.1367	.0829
				SA	.1332	.1266	.1180	.0883	.0735	.1407	.1332	.0808
		50	.1	ES	.0522	.0493	.0471	.0310	.0284	.0527	.0522	.0297
				TA	.0643	.0606	.0580	.0382	.0350	.0649	.0643	.0365
				SA	.0560	.0528	.0505	.0332	.0304	.0565	.0560	.0318
5	5	10	.3	ES	.0709	.0664	.0683	.0594	.0304	.0776	.0709	.0415
				TA	.0903	.0845	.0870	.0756	.0388	.0989	.0903	.0528
				SA	.0790	.0739	.0761	.0661	.0339	.0865	.0790	.0462
		50	.3	ES	.0443	.0426	.0422	.0194	.0121	.0449	.0443	.0154
				TA	.0467	.0449	.0445	.0205	.0128	.0474	.0467	.0162
				SA	.0452	.0434	.0431	.0198	.0123	.0458	.0452	.0157
	10	10	.5	ES	.0535	.0523	.0579	.0611	.0379	.0589	.0535	.0458
				TA	.0949	.0928	.1027	.1085	.0672	.1045	.0949	.0812
				SA	.0672	.0658	.0727	.0768	.0476	.0740	.0672	.0575
		50	.5	ES	.0360	.0344	.0345	.0250	.0118	.0366	.0360	.0171
				TA	.0415	.0397	.0399	.0289	.0136	.0422	.0415	.0198
				SA	.0375	.0358	.0360	.0261	.0122	.0381	.0375	.0178

ES = Unadjusted effect Size

TA = TA adjusted effect-size

SA = SA adjusted effect size

Summary

In sum, when two populations are compared, the Serlin adjustment provides an appropriate adjustment for all measures of effect-size except SI and TSI. However, the Sserlin procedure could adjust the bias of SI and TSI more appropriately than the Tatsuoka adjustment. When three or five populations were compared, the results indicated that the Tatsuoka adjustment adjusted bias of WI, HI, GI, TSI, and HRI more effectively than the Serlin adjustment; the Serlin adjustment adjusted bias of SGI, SEI, and CNI more appropriately in comparison to the Tatsuoka adjustment. The Tatsuoka adjustment reduced the bias of TSI appropriately when the sample size was large and the number of variable was small. The Serlin adjustment for SEI provided an appropriate adjustment under most sampling conditions presented in this study.

With regard to precision, the unadjusted effect-size measures had smaller standard deviations than either the Tatsuoka or the Serlin adjusted effect-size measures. The difference in precision was greatest when sample sizes are small. The precision of the Serlin adjusted effect-size measures was always greater than the precision of the Tatsuoka adjusted effect-size measures.

The means and standard deviations of the sampling distributions of the 16 statistics (8 effect sizes adjusted by Tatsuoka adjustment and 8 effect sizes adjusted by Serlin adjustment) are reported in Appendix C for the comparisons of 2, 3, and 5 populations, involving 3, 5, and 10 measures, with sample sizes of 10 and 50 when the population effect sizes are zero, .1, .3, and .5, respectively.

CHAPTER 5

DISCUSSION

Researchers have been strongly encouraged to assess and report effect-size estimates as a supplement to statistical hypothesis tests. The reporting of an effect-size measure is currently required by several prominent education journals. For this requirement to be useful the effect-size measure reported should be unbiased and estimated with precision.

However, although the multivariate effect size measures are known to be biased, many researchers are unaware of this bias and are unaware of procedures that are available to adjust these effect-size measures. Multivariate effect-size measures suggested in many textbooks and those currently reported on computer output provide biased estimates population differences.

In the current study, the degree of bias and precision in eight multivariate measures of association were examined and the effectiveness of the Tatsuoka and the Serlin procedures for adjusting the eight effect-size measures were evaluated. The sampling distributions of the unadjusted measures of association and measures of association adjusted by the Tatsuoka and the Serlin procedures were investigated by a computer simulation technique under certain conditions. The eight multivariated effect size measures studied included: Wilks index (WI), Hsu index (HI), Stevens index (SI), Shaffer-Gillo index (SGI), Serlin index (SEI), Tasuoka-Sachdeva index (TSI), Hotelling-Rozeboom index (HRI), Cramer-Nicewander index (CNI). The SGI, SEI, and CNI effect size measures are routinely reported on the SPSS output for multivariate analyses. In addition, each of the eight effect-size measures was adjusted using the methods suggested by Tatsuoka (TA) and Serlin (SA).

The current results involving the unadjusted measures of effect-size (TSI) are compatible with those reported by Tatsuoka (1973) who examined the sampling distribution of TSI with respect to the number of variables, total sample size, and effect size. The results of the present study showed that all of the unadjusted effect-size measures were biased to some degree and the amount of bias was affected by the number of populations compared, sample size, the number of response variables, and effect-size. For all of the effect-size measures, the bias could be substantial when sample sizes were small, the number of variables was large, and population effect-size was small. For all of the effect-size measures except SGI, SEI, and CNI the bias could be substantial when the number of populations was large. But for the SGI, SEI, and CNI effect-size measures bias decreased as the number of groups increased.

When the two adjustment procedures were used to reduce the bias in each effect size, the effectiveness of procedures depended on the number of populations compared and the effect size measures used. When two populations were compared, the Serlin adjustment reduced the bias of all eight effect size measures more effectively than the Tatsuoka adjustment and the precision of the Serlin adjusted effect-size measures was always greater than the precision of the Tatsuoka adjusted effect-size measures. Based on these results, the Serlin adjustment is recommended for reducing the bias for all measures of effect-size presented in this study except for SI and TSI. For the SI and TSI effect size measure the Serlin procedure underestimated the strength of relationship between the grouping variable and the outcome variables.

When three or more populations are compared, the Tatsuoka adjustment reduced the bias of WI, HI, GI, TSI, and HRI more effectively than the Serlin adjustment. The Serlin adjustment however reduced the bias of SGI, SEI, and CNI more effectively than the Tatsuoka adjustment. Furthermore, the Tatsuoka adjustment reduced the bias more effectively in TSI than in WI, HI, GI, and HRI. The Serlin adjustment reduced the bias more effectively in SEI than in SGI and CNI. Although the Tatsuoka adjustment for TSI could provide an appropriate adjustment when sample size is large and the number of variables is small, the Serlin adjustment for SEI more frequently provides an unbiased effect-size index. In addition, the precision of the Serlin adjusted effect-size measures was always greater than the precision of the Tatsuoka adjusted effect-size measures.

When a researcher wants to report an effect-size measure in a MANOVA context when three or more populations are compared, the SEI index adjusted by the Serlin procedure can be recommended based on reduced bias and increased precision. However, this recommendation cannot be made for all conditions. Because, as stated in Chapter 3, "different interpretations of shared variation are reflected across the indices" (Huberty, 1983, p.712): WI, HI, SI, TSI, and HRI represent the proportion of generalized variance or total variance of among the dependent variables accounted for by the grouping variable. On the other hand, SGI represents a weighted average of the estimated correlation ratios for each of the discriminant functions, SEI is the arithmetic average of the squared canonical correlation for the separate linear combinations of two sets of variables, and CNI is equal to one minus the geometric mean of the $1-\rho^2_j$. Choosing an effect-size measure depends on how a researcher defines the parameter of interest in addition to the bias and precision of the estimator. As suggested by Huberty (1983, p. 710), choosing a multivariate measure of effect-size "may be based on a researcher's preference."

A recommendations based on the researcher's preference of measures of effect-size can be made as follow, if a researcher prefers WI, HI, SI, TSI, or HRI to SGI, SEI, and CNI, the TSI adjusted by the Tatsuoka procedure can be recommended, provided that the sample size is greater then 75 and the product of the number of variables and the grouping variable degrees of freedom are less than 49. If a researcher prefers SEI, SGI, or CNI to WI, HI, SI, TSI, and HRI, the Serlin adjustment procedure for these effect-size can be recommended.

This study has some limitations. First, this study is limited to the one-way MANOVA context. Further study should include the two-way MANOVA context and more complex designs so as to get more generalizable results. Second, this study is limited to conditions that all assumptions for MANOVA are met. The situations where the assumptions are violated to some degree should be examined in future studies.

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APPENDICES

Appendix A. The SAS Program for determining the population mean

```
proc iml;
p=5;
k=2;

u11=.2984;
u12=.2984;
u13=.2984;
u14=.2984;
u15=.2984;
u21=0;
u22=0;
u23=0;
u24=0;
u25=0;
u1=(u11+u21)/2;
u2=(u12+u22)/2;
u3=(u13+u23)/2;
u4=(u14+u24)/2;
u5=(u15+u25)/2;
print u1 u2 u3 u4 u5;

d11=u11-u1;
d12=u12-u2;
d13=u13-u3;
d14=u14-u4;
d15=u15-u5;
d21=u21-u1;
d22=u22-u2;
d23=u23-u3;
d24=u24-u4;
d25=u25-u5;

d1=d11//d12//d13//d14//d15;
d2=d21//d22//d23//d24//d25;
```

```
print d1 d2;

SSCPH1=d1*d1`;
SSCPH2=d2*d2`;
SSCPH=SSCPH1+SSCPH2;
print SSCPH SSCPH1 SSCPH2;

x={1 0 0 0 0,
   0 1 0 0 0,
   0 0 1 0 0,
   0 0 0 1 0,
   0 0 0 0 1};
print x;

y=x+(SSCPH/k);
dety=det(y);
detx=det(x);
print y dety detx;

Pomegas=1-detx/dety;
print Pomegas;
run;
```

Appendix B. The SAS program for generating samples and computing statistics

```
proc iml;
n=10;
G=3;
p=5;

mu=.6214;
ef=j(n,p,1);
effect=mu*ef;
Pmean=j(3,8,.300);

rep=10000;
val=j(p,1,0);
ES=j(rep,8,0);
TAES=j(rep,8,0);
SAES=j(rep,8,0);
Do I=1 to rep;

X1=rannor(j(n,p,0))+effect;
X2=rannor(j(n,p,0));
X3=rannor(j(n,p,0));

X1bar=X1[:,];
X2bar=X2[:,];
X3bar=X3[:,];

m11=X1bar[,1];
m12=X1bar[,2];
m13=X1bar[,3];
m14=X1bar[,4];
m15=X1bar[,5];
m21=X2bar[,1];
m22=X2bar[,2];
m23=X2bar[,3];
m24=X2bar[,4];
```

```
m25=X2bar[,5];
m31=X3bar[,1];
m32=X3bar[,2];
m33=X3bar[,3];
m34=X3bar[,4];
m35=X3bar[,5];

A=j(n,p,1);
X11=m11*A[,1];
X12=m12*A[,2];
X13=m13*A[,3];
X14=m14*A[,4];
X15=m15*A[,5];
X21=m21*A[,1];
x22=m22*A[,2];
X23=m23*A[,3];
x24=m24*A[,4];
X25=m25*A[,5];
X31=m31*A[,1];
x32=m32*A[,2];
X33=m33*A[,3];
x34=m34*A[,4];
X35=m35*A[,5];


P1=X11||X12||X13||X14||X15;
P2=X21||X22||X23||X24||X25;
P3=X31||X32||X33||X34||X35;

D1=x1-p1;
D2=x2-p2;
D3=x3-p3;

SSCPE1=D1`*D1;
SSCPE2=D2`*D2;
SSCPE3=D3`*D3;
```

```
SSCPE=SSCPE1+SSCPE2+SSCPE3;
determinantSSCPE=DET(SSCPE);
TX=X1//X2//X3;
mTX=TX[:,];
mmTX=mTX//mTX//mTX;
mX1X2=X1bar//X2bar//X3bar;
DH=mX1X2-mmTX;
SSCPH=n*DH`*DH;

SSCPT=SSCPE+SSCPH;
determinantSSCPT=DET(SSCPT);

inverseE=INV(SSCPE);
eigvals=EIGVAL(inverseE*SSCPH);
eig=eigvals[,1];
reigvals=RANK(eig);

val[1,1]=reigvals[1,1];
val[2,1]=reigvals[2,1];
val[3,1]=reigvals[3,1];
val[4,1]=reigvals[4,1];
val[5,1]=reigvals[5,1];

do jj=1 to p;
if val[jj,1]=5 then first=jj;
if val[jj,1]=4 then sec=jj;
if val[jj,1]=3 then trd=jj;
if val[jj,1]=2 then forth=jj;
if val[jj,1]=1 then fifth=jj;
end;

  eigval1=eig[first,1];
  eigval2=eig[sec,1];
  eigval3=eig[trd,1];
```

```
W=(1/(1+eigval1))#(1/(1+eigval2));
R=eigval1;
HL=eigval1+eigval2;
BP=(eigval1/(1+eigval1))+(eigval2/(1+eigval2));

q=G-1;
S=HL#((3#n-3)-p-1)/(3#n-3);
r=min(p,q);
rr=1/r;
k=g;
b=max(p,q);

WI=1-W;
HI=HL/(1+HL);
SI=S/(1+S);
SGI=HL/(r+HL);
SEI=BP/r;
TSI=((3#n-k)-(3#n-1)#W)/((3#n-k)+W);
HRI=1-W;
CNI=1-W##rr;

TAWI=WI-(p##2+q##2)#(1-WI)/(3#3#n);
TAHI=HI-(p##2+q##2)#(1-HI)/(3#3#n);
TASI=SI-(p##2+q##2)#(1-SI)/(3#3#n);
TASGI=SGI-(p##2+q##2)#(1-SGI)/(3#3#n);
TASEI=SEI-(p##2+q##2)#(1-SEI)/(3#3#n);
TATSI=TSI-(p##2+q##2)#(1-TSI)/(3#3#n);
TAHRI=HRI-(p##2+q##2)#(1-HRI)/(3#3#n);
TACNI=CNI-(p##2+q##2)#(1-CNI)/(3#3#n);

SAWI=1-(3#n-1)/(3#n-b-1)#(1-WI);
SAHI=1-(3#n-1)/(3#n-b-1)#(1-HI);
SASI=1-(3#n-1)/(3#n-b-1)#(1-SI);
SASGI=1-(3#n-1)/(3#n-b-1)#(1-SGI);
SASEI=1-(3#n-1)/(3#n-b-1)#(1-SEI);
SATSI=1-(3#n-1)/(3#n-b-1)#(1-TSI);
```

```
SAHRI=1-(3#n-1)/(3#n-b-1)#(1-HRI);
SACNI=1-(3#n-1)/(3#n-b-1)#(1-CNI);

ES[i,1]=WI;
ES[i,2]=HI;
ES[i,3]=SI;
ES[i,4]=SGI;
ES[i,5]=SEI;
ES[i,6]=TSI;
ES[i,7]=HRI;
ES[i,8]=CNI;

TAES[i,1]=TAWI;
TAES[i,2]=TAHI;
TAES[i,3]=TASI;
TAES[i,4]=TASGI;
TAES[i,5]=TASEI;
TAES[i,6]=TATSI;
TAES[i,7]=TAHRi;
TAES[i,8]=TACNI;

SAES[i,1]=SAWI;
SAES[i,2]=SAHI;
SAES[i,3]=SASI;
SAES[i,4]=SASGI;
SAES[i,5]=SASEI;
SAES[i,6]=SATSI;
SAES[i,7]=SAHRi;
SAES[i,8]=SACNI;
END;

mES=ES[:,];
mTAES=TAES[:,];
mSAES=SAES[:,];

ESs=ES[+,];
```

```
ESss=ES[##,];
ESsq=(ESs##2)/rep;
ESssq=ESss-ESsq;
ESv=ESssq/(rep-1);
ESsd=sqrt(ESV);

TAESs=TAES[+,];
TAESss=TAES[##,];
TAESsq=(TAESs##2)/rep;
TAESssq=TAESss-TAESsq;
TAESv=TAESssq/(rep-1);
TAESsd=sqrt(TAESV);

SAESs=SAES[+,];
SAESss=SAES[##,];
SAESsq=(SAESs##2)/rep;
SAESssq=SAESss-SAESsq;
SAESv=SAESssq/(rep-1);
SAESsd=sqrt(SAESV);

mean=mES//mTAES//mSAES;
sd=ESsd//TAESsd//SAESsd;
bias=mean-Pmean;
title "ES(.3)10n3g5pN";
print Bias mean sd;
run;
```

Appendix C.1 Bias and Standard Deviations of unadjusted Effect Size, Effect-size adjusted by TA, Effect-size adjusted by SA when population effect-size is zero

k	n	p			WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	10	3	Bias	ES	.157	.157	.129	.157	.157	.106	.157	.157
				TA	.017	.017	-.015	.017	.017	-.042	.017	.017
				SA	-.000	-.000	-.033	-.000	-.000	-.060	-.000	-.000
			SD	ES	.1109	.1109	.0954	.1109	.1109	.1132	.1109	.1109
				TA	.1294	.1294	.1113	.1294	.1294	.1320	.1294	.1294
				SA	.1317	.1317	.1133	.1317	.1317	.1344	.1317	.1317
		5	Bias	ES	.260	.260	.195	.260	.260	.211	.260	.260
				TA	-.059	-.059	-.152	-.059	-.059	-.129	-.059	-.059
				SA	-.003	-.003	-.091	-.003	-.003	-.069	-.003	-.003
			SD	ES	.1346	.1346	.1119	.1346	.1346	.1386	.1346	.1346
				TA	.1929	.1929	.1604	.1929	.1929	.1987	.1929	.1929
				SA	.1827	.1827	.1519	.1827	.1827	.1881	.1827	.1827
		10	Bias	ES	.526	.526	.321	.526	.526	.489	.526	.526
				TA	-.269	-.269	-.821	-.269	-.269	-.370	-.269	-.269
				SA	.001	.001	-.432	.001	.001	-.078	.001	.001
			SD	ES	.1544	.1544	.1390	.1544	.1544	.1628	.1544	.1544
				TA	.4143	.4143	.3731	.4143	.4143	.4370	.4143	.4143
				SA	.3259	.3259	.2935	.3259	.3259	.3438	.3259	.3259
	50	3	Bias	ES	.030	.030	.028	.030	.030	.020	.030	.030
				TA	-.002	-.002	-.003	-.002	-.002	-.012	-.002	-.002
				SA	-.000	-.000	-.001	-.000	-.000	-.010	-.000	-.000
			SD	ES	.0240	.0240	.0232	.0240	.0240	.0241	.0240	.0240
				TA	.0248	.0248	.0239	.0248	.0248	.0249	.0248	.0248
				SA	.0248	.0248	.0239	.0248	.0248	.0248	.0248	.0248
		5	Bias	ES	.051	.051	.048	.051	.051	.041	.051	.051
				TA	-.031	-.031	-.034	-.031	-.031	.041	.031	.031
				SA	.000	.000	-.002	.000	.000	-.009	.000	.000
			SD	ES	.0313	.0313	.0297	.0313	.0313	.0314	.0313	.0313
				TA	.0340	.0340	.0322	.0340	.0340	.0341	.0340	.0340
				SA	.0330	.0330	.0312	.0330	.0330	.0330	.0330	.0330

		10	Bias	ES	.101	.101	.091	.101	.101	.091	.101	.101
				TA	-.201	-.201	-.214	-.201	-.201	-.214	-.201	-.201
				SA	.000	.000	-.011	.000	.000	-.010	.000	.000
			SD	ES	.0426	.0426	.0388	.0426	.0426	.0427	.0426	.0426
				TA	.0569	.0569	.0519	.0569	.0569	.0570	.0569	.0569
				SA	.0473	.0473	.0432	.0473	.0473	.0475	.0473	.0473
3	10	3	Bias	ES	.198	.193	.170	.110	.102	.135	.198	.106
				TA	.082	.076	.051	-.018	-.026	.010	.082	-.022
				SA	.106	.100	.075	.007	-.000	.035	.106	.003
			SD	ES	.0995	.0951	.0868	.0612	.0535	.1045	.0995	.0573
				TA	.1138	.1088	.0993	.0700	.0613	.1196	.1138	.0656
				SA	.1109	.1061	.0968	.0682	.0597	.1166	.1109	.0639
		5	Bias	ES	.321	.306	.257	.185	.173	.265	.321	.179
				TA	.102	.082	.018	-.076	-.093	.028	.102	-.084
				SA	.180	.161	.103	.015	.001	.112	.180	.008
			SD	ES	.1138	.1050	.0950	.0758	.0666	.1205	.1138	.0711
				TA	.1505	.1388	.1256	.1003	.0880	.1593	.1505	.0940
				SA	.1375	.1269	.1148	.0916	.0804	.1456	.1375	.0859
		10	Bias	ES	.580	.533	.409	.370	.346	.541	.580	.358
				TA	.096	-.005	-.273	-.356	-.409	.012	.096	-.382
				SA	.359	.288	.098	.039	.001	.300	.359	.021
			SD	ES	.1133	.1014	.0985	.0952	.0837	.1220	.1133	.0890
				TA	.2443	.2186	.2123	.2052	.1805	.2630	.2443	.1920
				SA	.1730	.1548	.1503	.1453	.1278	.1862	.1730	.1359
	50	3	Bias	ES	.040	.039	.038	.020	.020	.026	.040	.020
				TA	.012	.012	.011	-.007	-.008	-.001	.012	-007
				SA	.020	.020	.019	.000	.000	.006	.020	.000
			SD	ES	.0223	.0221	.0215	.0116	.0113	.0225	.0223	.0115
				TA	.0230	.0227	.0222	.0119	.0116	.0232	.0230	.0118
				SA	.0228	.0225	.0220	.0118	.0115	.0230	.0228	.0117

		5	Bias	ES	.065	.065	.062	.033	.033	.052	.065	.033
				TA	.005	.004	.002	-.028	-.028	-.008	.005	-.028
				SA	.033	.032	.030	.000	-.000	.019	.033	.000
			SD	ES	.0282	.0275	.0265	.0148	.0144	.0284	.0282	.0146
				TA	.0300	.0293	.0283	.0158	.0154	.0302	.0300	.0156
				SA	.0291	.0285	.0275	.0154	.0149	.0294	.0291	.0152
		10	Bias	ES	.129	.126	.117	.067	.066	.117	.129	.067
				TA	-.071	-.075	-.085	-.147	-.148	-.087	-.071	-.148
				SA	.066	.063	.054	.000	-.000	.053	.066	.000
			SD	ES	.0375	.0357	.0337	.0205	.0199	.0379	.0375	.0202
				TA	.0462	.0439	.0414	.0252	.0246	.0466	.0462	.0249
				SA	.0402	.0382	.0361	.0220	.0214	.0406	.0402	.0217
5	10	3	Bias	ES	.231	.220	.204	.088	.082	.160	.231	.085
				TA	.102	.090	.072	-.063	-.070	.020	.102	-.067
				SA	.161	.150	.133	.006	.000	.084	.161	.003
			SD	ES	.0809	.0741	.0704	.0351	.0308	.0869	.0809	.0329
				TA	.0943	.0864	.0821	.0410	.0359	.1014	.0943	.0384
				SA	.0861	.0789	.0750	.0374	.0328	.0926	.0861	.0350
		5	Bias	ES	.347	.319	.290	.107	.098	.285	.347	.102
				TA	.169	.133	.096	-.136	-.148	.090	.169	-.142
				SA	.273	.242	.209	.006	-.004	.204	.273	.000
			SD	ES	.0851	.0737	.0698	.0329	.0274	.0919	.0851	.0300
				TA	.1084	.0938	.0889	.0419	.0350	.1170	.1084	.0382
				SA	.0948	.0820	.0777	.0366	.0306	.1023	.0948	.0334
		10	Bias	ES	.611	.530	.461	.224	.203	.571	.611	.213
				TA	.310	.167	.045	-.376	-.411	.240	.310	-.394
				SA	.511	.409	.323	.025	-.000	.462	.511	.012
			SD	ES	.0818	.0667	.0665	.0467	.0388	.0894	.0818	.0424
				TA	.1452	.1184	.1179	.0828	.0689	.1585	.1452	.0752
				SA	.1028	.0838	.0835	.0587	.0488	.1123	.1028	.0532

	50	3	Bias	ES	.047	.047	.046	.016	.016	.032	.047	.016
				TA	.016	.015	.014	-.016	-.016	-.000	.016	-.016
				SA	.031	.031	.030	.000	-.000	.016	.031	.000
			SD	ES	.0189	.0184	.0182	.0066	.0064	.0191	.0189	.0065
				TA	.0195	.0191	.0188	.0068	.0066	.0197	.0195	.0067
				SA	.0191	.0187	.0184	.0067	.0065	.0193	.0191	.0066
		5	Bias	ES	.077	.075	.074	.020	.019	.062	.077	.020
				TA	.027	.025	.023	-.033	-.033	.010	.027	-.033
				SA	.058	.056	.055	.000	-.000	.042	.058	-.000
			SD	ES	.0231	.0221	.0216	.0062	.0060	.0234	.0231	.0061
				TA	.0243	.0233	.0228	.0066	.0063	.0246	.0243	.0065
				SA	.0235	.0225	.0221	.0064	.0061	.0238	.0235	.0062
		10	Bias	ES	.152	.145	.139	.041	.040	.138	.152	.040
				TA	.021	.013	.006	-.107	-.108	.004	.021	-.107
				SA	.117	.109	.103	.000	.000	.102	.117	.000
			SD	ES	.0310	.0282	.0273	.0089	.0086	.0314	.0310	.0088
				TA	.0357	.0326	.0316	.0103	.0100	.0362	.0357	.0101
				SA	.0322	.02947	.0285	.0093	.0090	.0327	.0322	.0091

ES = Unadjusted effect Size

TA = TA adjusted effect-size

SA = SA adjusted effect size

Appendix C.2 Bias and Standard Deviations of unadjusted Effect Size, Effect-size adjusted by TA, Effect-size adjusted by SA when population effect-size is .1

k	n	p			WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	10	3	Bias	ES	.140	.140	.101	.140	.140	.091	.140	.140
				TA	.013	.013	-.031	.013	.013	-.043	.013	.013
				SA	-.001	-.001	-.047	-.001	-.001	-.060	-.001	-.001
			SD	ES	.1408	.1408	.1252	.1408	.1408	.1447	.1408	.1408
				TA	.1643	.1643	.1461	.1643	.1643	.1688	.1643	.1643
				SA	.1672	.1672	.1487	.1672	.1672	.1718	.1672	.1672
		5	Bias	ES	.235	.235	.159	.235	.235	.189	.235	.235
				TA	-.052	-.052	-.161	-.052	-.052	-.118	-.052	-.052
				SA	-.001	-.001	-.105	-.001	-.001	-.064	-.001	-.001
			SD	ES	.1492	.1492	.1300	.1492	.1492	.1546	.1492	.1492
				TA	.2139	.2139	.1863	.2139	.2139	.2217	.2139	.2139
				SA	.2025	.2025	.1764	.2025	.2025	.2099	.2025	.2025
		10	Bias	ES	.474	.474	.264	.474	.474	439	.474	.474
				TA	-.240	-.240	-.805	-.240	-.240	-.335	-.240	-.240
				SA	.002	.002	-.442	.002	.002	-.071	.002	.002
			SD	ES	.1489	.1489	.1429	.1489	.1489	.1576	.1489	.1489
				TA	.3995	.3995	.3837	.3995	.3995	.4231	.3995	.3995
				SA	.3143	.3143	.3018	.3143	.3143	.3329	.3143	.3143
	50	3	Bias	ES	.027	.027	.022	.027	.027	.017	.027	.027
				TA	-.001	-.001	-.006	-.001	-.001	-.011	-.001	-.001
				SA	.000	.000	-.004	.000	.000	-.010	.000	.000
			SD	ES	.0579	.0579	.0562	.0579	.0579	.0581	.0579	.0579
				TA	.0598	.0598	.0581	.0598	.0598	.0600	.0598	.0598
				SA	.0597	.0597	.0579	.0597	.0597	.0599	.0597	.0597
		5	Bias	ES	.045	.045	.038	.045	.045	.035	.045	.045
				TA	-.028	-.028	-.036	-.028	-.028	-.039	-.028	-.028
				SA	.000	.000	-.007	.000	.000	-.010	.000	.000
			SD	ES	.0587	.0587	.0562	.0587	.0587	.0589	.0587	.0587
				TA	.0638	.0638	.0611	.0638	.0638	.0640	.0638	.0638
				SA	.0618	.0618	.0592	.0618	.0618	.0620	.0618	.0618

		10	Bias	ES	.090	.090	.073	.090	.090	.081	.090	.090
				TA	-.181	-.181	-.204	-.181	-.181	-.194	-.181	-.181
				SA	.000	.000	-.019	.000	.000	-.010	.000	.000
			SD	ES	.0624	.0624	.0580	.0624	.0624	.0626	.0624	.0624
				TA	.0834	.0834	.0775	.0834	.0834	.0837	.0834	.0834
				SA	.0694	.0694	.0645	.0694	.0694	.0697	.0694	.0694
3	10	3	Bias	ES	.178	.169	.141	.108	.096	.120	.178	.102
				TA	.074	.064	.031	-.012	-.026	.007	.074	-.019
				SA	.095	.085	.053	.011	-.002	.030	.095	.005
			SD	ES	.1194	.1135	.1058	.0791	.0659	.1261	.1194	.0724
				TA	.1367	.1299	.1210	.0906	.0754	.1443	.1367	.0829
				SA	.1332	.1266	.1180	.0883	.0735	.1407	.1332	.0808
		5	Bias	ES	.287	.266	.213	.177	.161	.234	.287	.170
				TA	.089	.062	-.007	-.070	-.092	.020	.089	-.080
				SA	.159	.134	.070	.017	-.002	.096	.159	.007
			SD	ES	.1218	.1122	.1041	.0865	.0735	.1295	.1218	.0798
				TA	.1611	.1484	.1377	.1144	.0972	.1713	.1611	.1055
				SA	.1472	.1356	.1258	.1045	.0888	.1565	.1472	.0964
		10	Bias	ES	.520	.468	.344	.351	.325	.484	.520	.339
				TA	.081	-.029	-.298	-.336	-.395	.003	.081	-.364
				SA	.320	.242	.051	-.038	-.002	.265	.320	.018
			SD	ES	.1105	.0995	.1001	.0978	.0858	.1193	.1105	.0913
				TA	.2383	.2146	.2158	.2109	.1850	.2572	.2383	.1968
				SA	.1687	.1520	.1528	.1493	.1310	.1821	.1687	.1394
	50	3	Bias	ES	.035	.034	.031	.020	.018	.023	.035	.019
				TA	.010	.009	.005	-.006	-.008	-.002	.010	-.007
				SA	.017	.016	.013	.000	-.000	.005	.017	.000
			SD	ES	.0478	.0471	.0462	.0274	.0244	.0482	.0478	.0259
				TA	.0492	.0484	.0475	.0282	.0251	.0496	.0492	.0266
				SA	.0488	.0480	.0471	.0280	.0249	.0492	.0488	.0264

		5	Bias	ES	.059	.057	.051	.033	.031	.047	.059	.032
				TA	.005	.002	-.002	-.025	-.027	-.007	.005	-.026
				SA	.030	.027	.022	.001	-.000	.018	.030	.001
			SD	ES	.0496	.0482	.0468	.0287	.0258	.0500	.0496	.0272
				TA	.0528	.0513	.0499	.0306	.0274	.0532	.0528	.0290
				SA	.0513	.0498	.0485	.0297	.0267	.0518	.0513	.0282
		10	Bias	ES	.117	.110	.097	.065	.063	.105	.117	.065
				TA	-.063	-.072	-.087	-.138	-.141	-.077	-.063	-.139
				SA	.061	.053	.040	.002	.000	.048	.061	.001
			SD	ES	.0522	.0493	.0471	.0310	.0284	.0527	.0522	.0297
				TA	.0643	.0606	.0580	.0382	.0350	.0649	.0643	.0365
				SA	.0560	.0528	.0505	.0332	.0304	.0565	.0560	.0318
5	10	3	Bias	ES	.205	.188	.170	.089	.077	.140	.205	.081
				TA	.089	.070	.048	-.056	-.070	.013	.089	-.065
				SA	.144	.126	.106	.011	-.001	.052	.160	-.003
			SD	ES	.0943	.0860	.0827	.0456	.0370	.1016	.0943	.0411
				TA	.1100	.1004	.0965	.0533	.0432	.1186	.1100	.0479
				SA	.1004	.0916	.0881	.0486	.0394	.1083	.1004	.0437
		5	Bias	ES	.315	.278	.246	.108	.095	.258	.315	.101
				TA	.155	.108	.067	-.128	-.145	.083	.155	-.136
				SA	.248	.207	.171	.010	-.004	.185	.248	.002
			SD	ES	.0918	.0789	.0759	.0397	.0313	.0994	.0918	.0351
				TA	.1169	.1005	.0967	.0506	.0399	.1266	.1169	.0448
				SA	.1023	.0879	.0846	.0442	.0349	.1107	.1023	.0391
		10	Bias	ES	.550	.463	.394	.221	.197	.515	.550	.209
				TA	.281	.125	.004	-.360	-.403	.217	.281	-.382
				SA	.461	.351	.265	.028	-.001	.416	.461	.013
			SD	ES	.0790	.0651	.0660	.0492	.0400	.0864	.0790	.0441
				TA	.1401	.1155	.1170	.0873	.0710	.1532	.1401	.0782
				SA	.0993	.0818	.0829	.0619	.0503	.1086	.0993	.0554

	50	3	Bias	ES	.043	.040	.038	.019	.015	.028	.043	.015
				TA	.014	.012	.010	-.012	-.015	-.000	.014	-.015
				SA	.028	.026	.024	.003	.000	.014	.028	-.000
			SD	ES	.0371	.0361	.0357	.0148	.0128	.0376	.0371	.0137
				TA	.0383	.0373	.0369	.0153	.0132	.0388	.0383	.0142
				SA	.0375	.0365	.0361	.0150	.0129	.0380	.0375	.0139
		5	Bias	ES	.069	.064	.061	.020	.019	.055	.069	.019
				TA	.024	.018	.015	-.031	-.033	.009	.024	-.032
				SA	.052	.047	.044	.000	-.000	.038	.052	.000
			SD	ES	.0385	.0366	.0360	.0120	.0103	.0391	.0385	.0111
				TA	.0406	.0386	.0379	.0127	.0109	.0412	.0406	.0117
				SA	.0393	.0373	.0367	.0123	.0105	.0399	.0393	.0113
		10	Bias	ES	.136	.122	.114	.040	.038	.123	.136	.039
				TA	.018	.002	-.006	-.104	-.106	.002	.018	-.105
				SA	.104	.090	.081	.001	-.000	.090	.104	.000
			SD	ES	.0404	.0364	.0355	.0132	.0116	.0409	.0404	.0124
				TA	.0466	.0421	.0410	.0153	.0135	.0473	.0466	.0143
				SA	.0420	.0380	.0370	.0138	.0121	.0426	.0420	.0129

ES = Unadjusted effect Size

TA = TA adjusted effect-size

SA = SA adjusted effect size

Bias = Estimated effect-size – Population effect size

Appendix C.3 Bias and Standard Deviations of unadjusted Effect Size, Effect-size adjusted by TA, Effect-size adjusted by SA when population effect-size is .3

k	n	p			WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	10	3	Bias	ES	.109	.109	.055	.109	.109	.065	.109	.109
				TA	.010	.010	-.051	.010	.010	-.040	.010	.010
				SA	-.001	-.001	-.065	-.001	-.001	-.053	-.001	-.001
			SD	ES	.1528	.1528	.1444	.1528	.1528	.1591	.1528	.1528
				TA	.1782	.1782	.1684	.1782	.1782	.1857	.1782	.1782
				SA	.1814	.1814	.1714	.1814	.1814	.1890	.1814	.1814
		5	Bias	ES	.181	.181	.090	.181	.181	.141	.181	.181
				TA	-.043	-.043	-.173	-.043	-.043	-.101	-.043	-.043
				SA	-.003	-.003	-.126	-.003	-.003	-.058	-.003	-.003
			SD	ES	.1521	.1521	.1446	.1521	.1521	.1596	.1521	.1521
				TA	.2180	.2180	.2073	.2180	.2180	.2287	.2180	.2180
				SA	.2064	.2064	.1962	.2064	.2064	.2166	.2064	.2064
		10	Bias	ES	.365	.365	.156	.365	.365	.336	.365	.365
				TA	-.196	-.196	-.757	-.196	-.196	-.275	-.196	-.196
				SA	-.005	-.005	-.446	-.005	-.005	-.067	-.005	-.005
			SD	ES	.1338	.1338	.1483	.1338	.1338	.1428	.1338	.1338
				TA	.3591	.3591	.3981	.3591	.3591	.3834	.3591	.3591
				SA	.2825	.2825	.3132	.2825	.2825	.3016	.2825	.2825
	50	3	Bias	ES	.020	.020	.011	.020	.020	.011	.020	.020
				TA	-.002	-.002	-.011	-.002	-.002	-.011	-.002	-.002
				SA	-.000	-.000	-.009	-.000	-.000	-.010	-.000	-.000
			SD	ES	.0700	.0700	.0690	.0700	.0700	.0705	.0700	.0700
				TA	.0724	.0724	.0713	.0724	.0724	.0728	.0724	.0724
				SA	.0722	.0722	.0711	.0722	.0722	.0727	.0722	.0722
		5	Bias	ES	.034	.034	.021	.034	.034	.025	.034	.034
				TA	-.022	-.022	-.037	-.022	-.022	-.032	-.022	-.022
				SA	-.000	-.000	-.014	-.000	-.000	-.010	-.000	-.000
			SD	ES	.0703	.0703	.0688	.0703	.0703	.0707	.0703	.0703
				TA	.0764	.0764	.0747	.0764	.0764	.0769	.0764	.0764
				SA	.0740	.0740	.0724	.0740	.0740	.0745	.0740	.0740

		10	Bias	ES	.069	.069	.043	.069	.069	.061	.069	.069
				TA	-.142	-.142	-.177	-.142	-.142	-.153	-.142	-.142
				SA	-.000	-.000	-.030	-.000	-.000	-.010	-.000	-.000
			SD	ES	.0703	.0703	.0679	.0703	.0703	.0708	.0703	.0703
				TA	.0940	.0940	.0908	.0940	.0940	.0947	.0940	.0940
				SA	.0782	.0782	.0756	.0782	.0782	.0788	.0782	.0782
3	10	3	Bias	ES	.138	.125	.088	.101	.082	.089	.138	.092
				TA	.057	.042	.000	-.002	-.028	.001	.057	-.014
				SA	.074	.059	.018	.018	-.005	.019	.074	.006
			SD	ES	.1241	.1193	.1158	.0980	.0704	.1323	.1241	.0842
				TA	.1420	.1366	.1326	.1121	.0806	.1514	.1420	.0964
				SA	.1384	.1331	.1292	.1093	.0786	.1475	.1384	.0940
		5	Bias	ES	.222	.195	.136	.160	.141	.179	.222	.151
				TA	.069	.033	-.045	-.052	-.086	.011	.069	-.069
				SA	.123	.090	.018	.022	-.006	.071	.123	.009
			SD	ES	.1183	.1107	.1086	.0987	.0759	.1269	.1183	.0869
				TA	.1565	.1464	.1436	.1305	.1004	.1678	.1565	.1149
				SA	.1430	.1338	.1312	.1193	.0917	.1533	.1430	.1050
		10	Bias	ES	.404	.348	.227	.310	.288	.375	.404	.300
				TA	.062	-.057	-.319	-.283	-.360	-.000	.062	-.319
				SA	.248	.163	-.021	-.039	-.006	.204	.248	.018
			SD	ES	.0966	.0900	.0978	.0981	.0832	.1049	.0966	.0895
				TA	.2083	.1940	.2110	.2115	.1795	.2262	.2083	.1930
				SA	.1475	.1373	.1494	.1497	.1271	.1601	.1475	.1367
	50	3	Bias	ES	.027	.025	.019	.019	.016	.017	.027	.017
				TA	.008	.005	-.000	-.003	-.008	-.002	.008	-.005
				SA	.014	.011	.005	.002	.001	.003	.014	.001
			SD	ES	.0566	.0559	.0554	.0401	.0290	.0572	.0566	.0346
				TA	.0582	.0575	.0570	.0412	.0299	.0588	.0582	.0356
				SA	.0577	.0571	.0565	.0409	.0296	.0584	.0577	.0353

		5	Bias	ES	.046	.040	.031	.030	.027	.036	.046	.029
				TA	.004	-.001	-.011	-.020	-.025	-.006	.004	-.022
				SA	.023	.017	.008	.003	-.001	.012	.023	.001
			SD	ES	.0571	.0550	.0551	.0408	.0300	.0577	.0571	.0354
				TA	.0607	.0595	.0587	.0434	.0319	.0614	.0607	.0377
				SA	.0590	.0578	.0570	.0422	.0310	.0597	.0590	.0366
		10	Bias	ES	.090	.078	.060	.058	.055	.081	.090	.057
				TA	-.049	-.065	-.087	-.118	-.127	-.062	-.049	-.122
				SA	.047	.033	.014	.003	-.001	.036	.047	.001
			SD	ES	.0576	.0550	.0539	.0419	.0322	.0583	.0576	.0370
				TA	.0709	.0677	.0663	.0516	.0397	.0717	.0709	.0455
				SA	.0617	.0590	.0578	.0449	.0345	.0625	.0617	.0396
5	10	3	Bias	ES	.159	.134	.112	.083	.068	.106	.159	.076
				TA	.069	.040	.014	-.048	-.069	.007	.069	-.059
				SA	.110	.083	.059	.013	-.000	.052	.110	.003
			SD	ES	.0947	.0879	.0866	.0593	.0393	.1027	.0947	.0483
				TA	.1104	.1025	.1011	.0692	.0459	.1198	.1104	.0564
				SA	.1008	.0936	.0923	.0632	.0419	.1094	.1008	.0515
		5	Bias	ES	.244	.197	.162	.106	.087	.199	.244	.096
				TA	.120	.059	.015	-.111	-.141	.062	.120	-.127
				SA	.192	.139	.101	-.015	-.007	.142	.192	.003
			SD	ES	.0897	.0798	.0792	.0518	.0333	.0977	.0897	.0411
				TA	.1143	.1016	.1009	.0660	.0425	.1244	.1143	.0524
				SA	.0999	.0888	.0882	.0577	.0371	.1088	.0999	.0458
		10	Bias	ES	.427	.333	.267	.209	.184	.399	.427	.196
				TA	.216	.050	-.066	-.326	-.388	.166	.216	-.358
				SA	.357	.239	.156	.031	-.005	.322	.357	.012
			SD	ES	.0688	.0606	.0639	.0555	.0399	.0754	.0688	.0461
				TA	.1220	.1075	.1134	.0985	.0708	.1338	.1220	.0818
				SA	.0864	.0761	.0803	.0698	.0501	.0948	.0864	.0579

	50	3	Bias	ES	.033	.028	.024	.016	.013	.021	.033	.014
				TA	.011	.006	.002	-.012	-.015	-.000	.011	-.014
				SA	.022	.017	.013	.002	-.000	.010	.022	.000
			SD	ES	.0443	.0434	.0431	.0239	.0154	.0449	.0443	.0194
				TA	.0457	.0448	.0445	.0247	.0160	.0464	.0457	.0200
				SA	.0448	.0439	.0436	.0241	.0156	.0455	.0448	.0196
		5	Bias	ES	.053	.043	.038	.019	.017	.042	.053	.018
				TA	.018	.007	.002	-.028	-.032	.006	.018	-.030
				SA	.040	.030	.024	.001	-.001	.028	.040	.000
			SD	ES	.0443	.0426	.0422	.0194	.0121	.0449	.0443	.0154
				TA	.0467	.0449	.0445	.0205	.0128	.0474	.0467	.0162
				SA	.0452	.0434	.0431	.0198	.0123	.0458	.0452	.0157
		10	Bias	ES	.106	.084	.073	.038	.036	.096	.106	.037
				TA	.015	-.011	-.023	-.094	-.101	.002	.015	-.097
				SA	.082	.058	.047	.002	-.000	.070	.082	.001
			SD	ES	.0432	.0400	.0395	.0198	.0131	.0439	.0432	.0160
				TA	.0499	.0461	.0456	.0229	.0151	.0507	.0499	.0185
				SA	.0450	.0416	.0412	.0207	.0136	.0457	.0450	.0167

ES = Unadjusted effect Size

TA = TA adjusted effect-size

SA = SA adjusted effect size

Bias = Estimated effect-size – Population effect size

Appendix C.4 Bias and Standard Deviations of unadjusted Effect Size, Effect-size adjusted by TA, Effect-size adjusted by SA when population effect-size is .5

k	n	p			WI	HI	SI	SGI	SEI	TSI	HRI	CNI
2	10	3	Bias	ES	.078	.078	.020	.078	.078	.043	.078	.078
				TA	.008	.008	-.058	.008	.008	-.032	.008	.008
				SA	-.000	-.000	-.068	-.000	-.000	-.041	-.000	-.000
			SD	ES	.1295	.1295	.1315	.1295	.1295	.1370	.1295	.1295
				TA	.1511	.1511	.1534	.1511	.1511	.1599	.1511	.1511
				SA	.1538	.1538	.1562	.1538	.1538	.1627	.1538	.1538
		5	Bias	ES	.131	.131	.040	.131	.131	.100	.131	.131
				TA	-.027	-.027	-.158	-.027	-.027	-.073	-.027	-.027
				SA	.000	.000	-.123	.000	.000	-.042	.000	.000
			SD	ES	.1233	.1233	.1303	.1233	.1233	.1312	.1233	.1233
				TA	.1767	.1767	.1868	.1767	.1767	.1881	.1767	.1767
				SA	.1673	.1673	.1768	.1673	.1673	.1781	.1673	.1673
		10	Bias	ES	.264	.264	.073	.264	.264	.242	.264	.264
				TA	-.132	-.132	-.643	-.132	-.132	-.191	-.132	-.132
				SA	.002	.002	-.399	.002	.002	-.044	.002	.002
			SD	ES	.1036	.1036	.1374	.1036	.1036	.1118	.1036	.1036
				TA	.2782	.2782	.3688	.2782	.2782	.3000	.2782	.2782
				SA	.2189	.2189	.2901	.2189	.2189	.2360	.2189	.2189
	50	3	Bias	ES	.015	.015	.005	.015	.015	.008	.015	.015
				TA	-.000	-.000	-.011	-.000	-.000	-.008	-.000	-.000
				SA	.000	.000	-.010	.000	.000	-.007	.000	.000
			SD	ES	.0611	.0611	.0612	.0611	.0611	.0618	.0611	.0611
				TA	.0632	.0632	.0632	.0632	.0632	.0638	.0632	.0632
				SA	.0630	.0630	.0631	.0630	.0630	.0637	.0630	.0630
		5	Bias	ES	.025	.025	.010	.025	.025	.018	.025	.025
				TA	-.015	-.015	-.032	-.015	-.015	-.023	-.015	-.015
				SA	.000	.000	-.015	.000	.000	-.007	.000	.000
			SD	ES	.0605	.0605	.0606	.0605	.0605	.0611	.0605	.0605
				TA	.0658	.0658	.0659	.0658	.0658	.0664	.0658	.0658
				SA	.0637	.0637	.0638	.0637	.0637	.0644	.0637	.0637

		10	Bias	ES	.050	.050	.021	.050	.050	.043	.050	.050
				TA	-.100	-.100	-.139	-.100	-.100	-.109	-.100	-.100
				SA	.000	.000	-.031	.000	.000	-.007	.000	.000
			SD	ES	.0586	.0586	.0590	.0586	.0586	.0593	.0586	.0586
				TA	.0784	.0784	.0789	.0784	.0784	.0792	.0784	.0784
				SA	.0652	.0652	.0657	.0652	.0652	.0659	.0652	.0652
3	10	3	Bias	ES	.100	.085	.047	.087	.069	.062	.100	.080
				TA	.042	.025	-.017	.004	-.028	-.000	.042	-.010
				SA	.053	.037	-.004	.020	-.008	.012	.053	.007
			SD	ES	.1035	.1011	.1029	.1009	.0626	.1116	.1035	.0824
				TA	.1185	.1158	.1178	.1155	.0716	.1277	.1185	.0944
				SA	.1155	.1128	.1148	.1125	.0698	.1245	.1155	.0920
		5	Bias	ES	.160	.132	.074	.135	.122	.128	.160	.130
				TA	.051	.013	-.062	-.035	-.079	.008	.051	-.055
				SA	.089	.055	-.014	.024	-.008	.050	.089	.010
			SD	ES	.0964	.0931	.0975	.0991	.0683	.1043	.0964	.0833
				TA	.1275	.1231	.1289	.1311	.0903	.1379	.1275	.1101
				SA	.1165	.1125	.1178	.1198	.0825	.1261	.1165	.1006
		10	Bias	ES	.287	.234	.124	.252	.251	.266	.287	.253
				TA	.042	-.072	-.308	-.226	-.324	-.003	.042	-.270
				SA	.176	.094	-.072	.034	-.010	.143	.176	.015
			SD	ES	.0746	.0739	.0882	.0912	.0748	.0814	.0746	.0807
				TA	.1608	.1594	.1901	.1966	.1613	.1756	.1608	.1740
				SA	.1139	.1129	.1346	.1392	.1142	.1243	.1139	.1232
	50	3	Bias	ES	.020	.017	.010	.017	.013	.012	.020	.015
				TA	.007	.003	-.003	-.001	-.007	-.001	.007	-.004
				SA	.011	.007	.000	.004	-.001	.002	.011	.001
			SD	ES	.0493	.0491	.0491	.0447	.0256	.0500	.0493	.0357
				TA	.0508	.0505	.0505	.0460	.0263	.0515	.0508	.0367
				SA	.0504	.0501	.0501	.0456	.0261	.0511	.0504	.0364

		5	Bias	ES	.033	.026	.016	.025	.023	.025	.033	.024
				TA	.003	-.003	-.014	-.015	-.023	-.005	.003	-.019
				SA	.017	.010	-.000	.003	-.001	008	.017	.001
			SD	ES	.0482	.0476	.0477	.0439	.0261	.0489	.0482	.0354
				TA	.0513	.0507	.0507	.0467	.0278	.0521	.0513	.0376
				SA	.0499	.0493	.0493	.0454	.0270	.0506	.0499	.0366
		10	Bias	ES	.064	.050	.031	.047	.048	.057	.064	.048
				TA	-.036	-.053	-.077	-.095	-.113	-.045	-.036	-.104
				SA	.033	.017	-.002	.003	-.001	.025	.033	.000
			SD	ES	.0473	.0462	.0465	.0440	.0278	.0480	.0473	.0359
				TA	.0582	.0569	.0572	.0541	.0342	.0591	.0582	.0442
				SA	.0507	.0495	.0498	.0471	.0298	.0514	.0507	.0385
5	10	3	Bias	ES	.114	.087	.065	.077	.061	.075	.114	.069
				TA	.050	.018	-.006	-.034	-.067	.004	.050	-.050
				SA	.081	.051	.027	.018	-.007	.038	.081	.005
			SD	ES	.0791	.0759	.0769	.0690	.0359	.0863	.0791	.0503
				TA	.0923	.0886	.0897	.0805	.0419	.1007	.0923	.0587
				SA	.0842	.0809	.0819	.0735	.0382	.0920	.0842	.0536
		5	Bias	ES	.173	.123	.090	.098	.079	.139	.173	.088
				TA	.083	.020	-.021	-.093	-.137	.041	.083	-.117
				SA	.136	.080	.043	.018	-.010	.098	.136	-.002
			SD	ES	.0709	.0664	.0683	.0594	.0304	.0776	.0709	.0415
				TA	.0903	.0845	.0870	.0756	.0388	.0989	.0903	.0528
				SA	.0790	.0739	.0761	.0661	.0339	.0865	.0790	.0462
		10	Bias	ES	.305	.215	.156	.191	.171	.284	.305	.181
				TA	.154	-.003	-.108	-.278	-.372	.118	.154	-.328
				SA	.255	.143	.068	.035	-.008	.229	.255	.012
			SD	ES	.0535	.0523	.0579	.0611	.0379	.0589	.0535	.0458
				TA	.0949	.0928	.1027	.1085	.0672	.1045	.0949	.0812
				SA	.0672	.0658	.0727	.0768	.0476	.0740	.0672	.0575

	50	3	Bias	ES	.023	.018	.014	.014	.012	.015	.023	.013
				TA	.008	.002	-.002	-.009	-.015	-.000	.008	-.012
				SA	.016	.009	.005	.002	-.001	.007	.016	.000
			SD	ES	.0379	.0375	.0375	.0292	.0135	.0386	.0379	.0207
				TA	.0392	.0388	.0388	.0302	.0139	.0398	.0392	.0214
				SA	.0384	.0380	.0380	.0296	.0137	.0390	.0384	.0210
		5	Bias	ES	.038	.026	.020	.018	.015	.029	.038	.017
				TA	.013	.001	-.005	-.023	-.031	.004	.013	-.027
				SA	.029	.017	.010	.002	-.001	.020	.029	.000
			SD	ES	.0378	.0370	.0370	.0253	.0107	.0384	.0378	.0168
				TA	.0398	.0390	.0391	.0267	.0113	.0405	.0398	.0178
				SA	.0385	.0378	.0378	.0258	.0109	.0392	.0385	.0172
		10	Bias	ES	.075	.050	.039	.035	.033	.068	.075	.034
				TA	.010	-.018	-.031	-.082	-.096	.001	.010	-.090
				SA	.058	.031	.020	.003	-.001	.049	.058	.000
			SD	ES	.0360	.0344	.0345	.0250	.0118	.0366	.0360	.0171
				TA	.0415	.0397	.0399	.0289	.0136	.0422	.0415	.0198
				SA	.0375	.0358	.0360	.0261	.0122	.0381	.0375	.0178

ES = Unadjusted effect Size

TA = TA adjusted effect-size

SA = SA adjusted effect size

Bias = Estimated effect-size – Population effect size