PEDAGOGICAL CONTENT KNOWLEDGE OF PRESERVICE SECONDARY MATHEMATICS TEACHERS

by

HULYA KILIC

(Under the Direction of Denise S. Mewborn)

ABSTRACT

The purpose of this study was to investigate how a methods course and its associated field experience supported the development of pedagogical content knowledge for preservice secondary mathematics teachers. I also investigated what course topics contributed to that development from the preservice teachers' perspective. The data were collected in the form of interviews, observations, questionnaire and class artifacts and were analyzed according to the pedagogical content knowledge framework developed for this study. Six preservice teachers participated in the study, and each was interviewed three times during the semester. All documents produced by the preservice teachers or distributed in the course were collected to gain a better understanding of the nature of the course topics and preservice teachers' experiences with them.

I defined pedagogical content knowledge as having four components: knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum. Knowledge of subject-matter refers to knowing mathematical concepts, facts, and procedures and the relationships among them. Knowledge of pedagogy encompasses knowledge of planning a lesson and teaching strategies. Knowledge of learners entails knowledge of students' common difficulties, errors, and misconceptions. Finally, knowledge of curriculum includes knowledge of learning goals for different grade levels and instructional materials such as technology, manipulatives, and textbooks.

The preservice teachers' knowledge of subject-matter was influential on the other components of their pedagogical content knowledge. The preservice teachers' ability to make appropriate connections among mathematical concepts, to generate different solutions and representations for problems, to address students' difficulties and misconceptions effectively, and to choose appropriate examples to teach a particular topic were largely based on the depth of their subject-matter knowledge. However, the field experiences contributed to their repertoire of examples of students' difficulties and misconceptions as well as instructional strategies and materials. Although the preservice teachers thought that course topics contributed to their pedagogical content knowledge, they were weak in applying their knowledge when they were asked to design a hypothetical lesson or help a hypothetical student who was struggling to understand particular mathematical concepts. The findings of this study imply that teacher education programs need to offer content courses that provide preservice teachers with opportunities to review fundamental topics taught in secondary school mathematics classes.

INDEX WORDS: pedagogical content knowledge, preservice teachers, secondary, mathematics, methods course, field experiences

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DEDICATION

I dedicate this work to my family for their endless love and support.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

CHAPTER 1

INTRODUCTION

Preservice mathematics teachers deal with different aspects of learning, teaching, and curricular issues in their teacher education programs. Teacher education programs provide several content, general pedagogy, and content-specific methods courses to support the development of professional knowledge for teaching. In these courses, preservice teachers are expected to construct and improve different knowledge domains for effective teaching.

Over two decades, educational researchers and policy makers have been discussing what knowledge a teacher should possess and how that knowledge is constructed and developed in a teacher education program or through experience in the field (Borko & Putnam, 1996; Fennema & Franke, 1992; Shulman, 1986; Wilson, Cooney, & Stinson, 2005). An immediate answer to the question “What knowledge?” is thought to be subject-matter knowledge. A teacher should have in-depth knowledge of what he or she is supposed to teach and also strong conceptual understanding of a topic and its relationships with other topics.

Research on teachers’ subject-matter knowledge indicates that many teachers lack conceptual understanding of their subject-matter (e.g., Ball, 1990a; Ball, Lubienski, & Mewborn, 2001; Brown & Borko, 1992; Even & Lappan, 1994; Gess-Newsome, 1999a). Teachers fail to explain the meaning of a mathematical concept and its relation with other concepts and to generate explanations or other representations for that concept. Although they are able to explain the procedural steps in an algorithm, they cannot explain the reasons for certain mathematical procedures. Furthermore, teachers’ knowledge about particular mathematical concepts often is not complete or correct. Having strong subject-matter knowledge is essential to becoming a teacher, but it is not sufficient for effective teaching (Ball, 1991; Ball & Bass, 2000; Borko & Putnam, 1996; Feiman-Nemser & Buchmann, 1987; Grouws & Schultz, 1996). Teachers should know how to teach that subject-matter and also be aware of other factors such as curriculum, students, and teaching strategies that might influence their teaching.

In the document *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics [NCTM], 1991), the importance of different knowledge domains for mathematics teachers is emphasized. In addition to possessing general knowledge of subject-matter, of pedagogy, of learning and learners, and of curriculum, teachers should have context-specific knowledge, which includes knowing how to teach a particular mathematical concept to particular students, how to represent a particular mathematical idea, how to respond to students' questions, and what tasks to use to engage students in a new topic. Furthermore, teachers' confidence in their knowledge of mathematics affects their choice of mathematical tasks and the kinds of learning environments they create.

Shulman (1987) identified seven knowledge domains for teachers: namely, subject-matter knowledge; general pedagogical knowledge; pedagogical content knowledge; knowledge of learners and learning; curriculum knowledge; knowledge of educational contexts; and knowledge of educational philosophies, goals, and objectives. According to Shulman, a teacher should know the content, pedagogy, curriculum, and the interaction between them. I believe that one of the most important aspects of being a teacher is to know how to orchestrate the learning environment to facilitate students' understanding of a particular concept and to contribute to their intellectual development. Shulman (1986) named this kind of knowledge "pedagogical content knowledge." He identified pedagogical content knowledge as "the ways of representing and formulating the subject that make it comprehensible to others” (p. 9). He stated that pedagogical content knowledge includes teachers’ knowledge about specific topics that might be easy or difficult for students and possible conceptions or misconceptions that student might have related to the topic.

My interest in teachers’ pedagogical content knowledge has emerged from my own experiences as a mathematics teacher. Before getting started in the doctoral program in mathematics education, I taught for five years in middle schools. My first year of teaching was full of disappointment. I tried to learn about effective ways of managing the classroom, presenting a task, and assessing students’ understandings. I assumed that if I presented the mathematical concept clearly by using different examples, all students would understand that concept and be able to do all the homework problems. However, it was not the case; some students were performing very well, whereas others were failing to accomplish the given tasks. I did not know much about how to handle students’ misconceptions and failures and began to think that I was not an appropriate person for being a teacher even though I knew my subject-matter very well and I knew about the psychology of learning. During the second and the third year of my teaching, I realized that the reason for the ineffective teaching practices in my first year was my lack of knowledge of how to transform my subject-matter knowledge, pedagogical knowledge, and contextual knowledge into acts of teaching, that is, a lack of pedagogical content knowledge. I expected that my students would achieve my goals and disregarded the fact that they might or might not have the necessary background knowledge to achieve them. I did not have any idea which concepts might be difficult or confusing for them to understand or how I could make those mathematical concepts more meaningful and accessible for them. I asked challenging questions but disregarded the way they might influence my students’ motivation for learning mathematics. Unfortunately, I was not introduced to pedagogical content knowledge as a domain of teacher knowledge in my methods course.

The harmonization of all types of teacher knowledge might yield effective teaching practices. However, a teacher does not immediately achieve that harmony among all types of knowledge that would facilitate their teaching practices as well as enhance their students' learning. It requires continuous efforts to balance among content, students, curriculum, educational goals, and assessment tools. I believe that pedagogical content knowledge is essential to establishing such balance because the knowledge of content, students, and curriculum is embedded in that knowledge (Gess-Newsome, 1999a; Grossman, 1990).

Although pedagogical content knowledge is assumed to be developed as teachers gain more experience in teaching (Borko & Putnam, 1996; Calderhead, 1996), I believe that preservice teachers should know about the notion of pedagogical content knowledge and try to make sense of it through their methods courses and field experiences in order to be ready for their first year of teaching. Studies of preservice mathematics teachers' knowledge and skills related to teaching have revealed that methods courses and field experiences are likely to contribute to the development of pedagogical content knowledge to some extent (Ball, 1991; Grossman, 1990). Therefore, I sought to study to what extent a methods course addressed pedagogical content knowledge.

Background

Agreement that subject-matter knowledge is not enough for being a good teacher has led researchers to investigate what knowledge and skills are required for effective teaching. Many researchers noted that teachers should definitely possess knowledge of subject-matter, students, pedagogy, and curricular issues (e.g., Ball, 1990a; Brown, Cooney, & Jones, 1990; Feiman-

Nemser & Buchmann, 1987) and also be able to interweave them effectively when planning for instruction as well as when teaching in the classroom (e.g., Ball, 1990a; Borko & Putnam, 1996; Fennema & Franke, 1992; Shulman, 1986). The term *pedagogical content knowledge* refers to such ability and knowledge (Brown & Borko, 1992; Gess-Newsome, 1999a; Grouws & Schultz, 1996).

However, the ambiguity of what constitutes pedagogical content knowledge has led to difficulty in conducting studies on the pedagogical content knowledge of teachers (Brown & Borko, 1992; Marks, 1990; Mewborn, 2000). Because pedagogical content knowledge is directly related to acts of teaching, studies of teachers' content knowledge or pedagogical knowledge are likely to provide information about teachers' pedagogical content knowledge. In fact, some researchers have investigated the relationships between pedagogical content knowledge and other knowledge domains (e.g., Even 1993; Kinach, 2002). Yet, more studies are needed to understand the nature of pedagogical content knowledge and how it is developed through teacher education programs and through field experiences (Grouws & Schultz, 1996).

Among the limited number of studies on pedagogical content knowledge, many of them concern elementary school teachers. Furthermore, most of these studies have limited the scope of the topics for which they investigated teachers' knowledge and practices. Marks (1990) investigated the components of pedagogical content knowledge by interviewing elementary teachers about their practices of teaching equivalent fractions. He suggested four major areas of pedagogical content knowledge: subject-matter for instructional purposes, students' understanding, media for instruction (e.g., materials), and instructional processes. Similarly Carpenter and his colleagues (Carpenter, Fennema, Peterson, & Carey, 1988) studied elementary teachers' pedagogical content knowledge. They attempted to clarify the nature of teachers' knowledge of students’ solutions of addition and subtraction problems. They found that teachers did not have a rich knowledge domain to draw from to plan for instruction based on the assessment of the processes that students use to solve problems.

Moreover, Ball and Wilson (1990) investigated the mathematical understanding and pedagogical content knowledge of preservice elementary teachers on certain topics such as division, place value, fractions, area, and perimeter. They noted that many preservice teachers lacked conceptual understanding of these topics and were therefore they unable to represent them in meaningful ways to teach for understanding. Even (1993) also attempted to describe the relationship between subject-matter knowledge and pedagogical content knowledge in the context of functions. She noted that preservice teachers’ pedagogical decisions about teaching functions were limited to their own understanding of the concept. They tended to explain the procedures to solve the problems without justifying the reasons behind them. Furthermore, Kinach (2002) asked her preservice teachers to develop instructional explanations for addition and subtraction of integers. She found that preservice teachers’ conceptions of mathematics were likely to influence their decisions about how to teach mathematics. They conceived of “knowing mathematics” as getting the answer. Therefore, they emphasized learning the rules and applying them in given problems.

A few studies have provided evidence that novice teachers’ pedagogical content knowledge improved through teaching and preparing to teach (Brown & Borko, 1992). Ball (1990a) designed her methods course for elementary preservice teachers to help them learn to teach mathematics. She noted that a few of them understood that knowing mathematics entails not only knowing how to carry out the algorithms but also knowing the reasoning behind the procedures and rules. Likewise, Philipp et al. (2007) indicated that the preservice teachers’ views of mathematics, learning, and teaching changed after taking a methods course in which they intensively investigated students' mathematical thinking. They noted that the changes mostly occurred in the preservice teachers' beliefs rather than their mathematical knowledge. Overall, however, the intervention was successful in raising preservice teachers' awareness of critical issues of teaching mathematics.

Grossman (1990) stated that field experiences provide opportunities for preservice teachers to develop their knowledge of students' understanding, knowledge of specific content, and knowledge of curriculum. She also noted that preservice teachers might struggle with representations of the subject-matter; however, their initial teaching experiences help them to construct their own pedagogical content knowledge.

Although pedagogical content knowledge is accepted to be a special knowledge domain for teaching (Brown & Borko, 1992), it still does not have a unique definition that all researchers agree upon. There are a limited number of studies of pedagogical content knowledge of teachers (Mewborn, 2000). The existing studies either address a few aspects of pedagogical content knowledge or investigate the nature of pedagogical content knowledge that emerges from practices of teaching particular mathematical concepts. In my study, I decided to investigate different aspects of preservice secondary mathematics teachers' pedagogical content knowledge. Furthermore, the tasks that I used to investigate the nature and the development of preservice teachers' pedagogical content knowledge involved various secondary school mathematics concepts.

Research Questions

The purpose of this research study was to learn about how a methods course and its associated field experience supports the development of pedagogical content knowledge for preservice secondary mathematics teachers. Grossman (1990) proposed that field experiences and content-specific methods courses contribute to the development of pedagogical content knowledge. Methods courses enable preservice teachers to learn about the overarching purposes for teaching specific subject-matter, and strategies and techniques to teach that subject. Through field experiences, preservice teachers are given opportunities to make connections between what they have learned so far and what a real classroom environment looks like. They can also improve their repertoire of teaching strategies and students' misconceptions during field experiences. Therefore, methods courses and field experiences help preservice teachers to develop their knowledge of teaching a particular subject-matter, pedagogy, and students.

Furthermore, Tamir (1988) suggested that the instructor of a methods course might help the development of pedagogical content knowledge of preservice teachers by providing opportunities for microteaching. For a microteaching activity, preservice teachers need to prepare a lesson plan in which they describe what they will teach and how they will teach it. In attempting to find appropriate answers for those questions, preservice teachers will use their knowledge of content, knowledge of learners, knowledge of curriculum, knowledge of pedagogy, and pedagogical content knowledge. After the implementation of the lesson, preservice teachers evaluate their teaching practices and learn from their experiences. Because preservice teachers tend to rely on their own experiences while they are teaching (e.g., Ball, 1988; Calderhead & Robson, 1991; Even 1993; Foss & Kleinsasser, 1996), different microteaching experiences can contribute to the development of their knowledge domains.

Based on the literature about the development of preservice teachers' pedagogical content knowledge, the research questions that guided my study were as follows:

1. What aspects of preservice teachers' pedagogical content knowledge are developed in a methods course and its associated field experience?
2. What course experiences provide for this development from the preservice teachers' perspectives?

Although pedagogical content knowledge is assumed to be developed as teachers gain more experience in teaching, I believe that preservice teachers should possess some level of pedagogical content knowledge and improve it while gaining experience in the field. Hence, I observed what issues were discussed in the methods and field experience course and how preservice teachers made sense of them through their class activities and field experiences.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Among other factors, teachers' knowledge of subject-matter, knowledge of pedagogy, knowledge of learning and learners, and knowledge of the contexts of classroom, school and society influence their teaching practices, which in turn are likely to influence students' learning (Ball & McDiarmid, 1990; Fennema & Franke, 1992; Gess-Newsome, 1999a; Shulman, 1987). Several studies have been conducted on the nature of teachers' knowledge and its impact on their practices and students' learning. Because the focus of this study was pedagogical content knowledge, in this section I present different views of what constitutes pedagogical content knowledge and how it is related to other knowledge domains. I also illustrate studies on pedagogical content knowledge and related knowledge domains. Then, I discuss how teacher education programs contribute to the development of preservice teachers' professional knowledge and skills, and in particular, their pedagogical content knowledge.

Pedagogical Content Knowledge

Definition of Pedagogical Content Knowledge

Different perspectives about what constitutes teachers' knowledge domains have led to different definitions for pedagogical content knowledge and various descriptions about its nature. When Shulman first introduced the term in 1986, he defined it as knowing how to represent the subject-matter to facilitate students' understanding. However, as a result of arguments about teacher knowledge domains, pedagogical content knowledge is either accepted as a distinct knowledge domain for teaching or not. Not only is identifying pedagogical content knowledge as a knowledge domain controversial, but what constitutes that knowledge is also debated by scholars.

Shulman (1987) identified seven domains of teacher knowledge, one of which is pedagogical content knowledge. He explained why he identified pedagogical content knowledge as a knowledge domain for teachers as follows:

> Among those categories, pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 8)

Shulman claimed that pedagogical content knowledge is a distinct body of knowledge even though knowledge of content and knowledge of pedagogy contribute to it. He also noted that pedagogical content knowledge includes knowledge of learners, knowledge of educational context, and knowledge of instructional materials.

Tamir (1988), however, made a sharper distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. He claimed that each type of knowledge is composed of four categories-namely, student, curriculum, instruction, and evaluation- but they have different meanings in each domain. He provided examples for each category to reveal the distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. For instance, for the student category, knowing about Piaget’s developmental levels is related to general pedagogical knowledge, whereas knowing about specific common conceptions and misconceptions in a given topic is related to subject-matter-specific pedagogical knowledge. Furthermore, he identified teachers’ skills in diagnosing students’ conceptual difficulties in a given topic and their knowledge about effective use of instructional tools as subject-matter-specific pedagogical knowledge.

Similarly Ball and Bass (2000) identified teachers’ knowledge of students’ difficulties and appropriate teaching strategies to eliminate those difficulties as part of teachers’ pedagogical content knowledge. They defined pedagogical content knowledge as follows:

> Pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy. These bundles offer a crucial resource for teaching mathematics, for they can help the teacher anticipate what students might have trouble learning, and have ready alternative models or explanations to mediate those difficulties. (p. 88)

However, they stated that there is no way of adequately anticipating what students might think or whether a new representation or explanation for a familiar topic is needed; therefore, teachers should decide how they will orchestrate activities according to the nature of current circumstances.

Moreover, Hill, Ball, and Schilling (2008) proposed a model of mathematical knowledge for teaching in which subject-matter knowledge and pedagogical content knowledge are two major domains contributing to it. In that model, knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum are defined to be included in pedagogical content knowledge (see Figure 1). Thus, teachers’ pedagogical content knowledge entails the knowledge of how students think about, know or learn particular content and what teaching strategies and curriculum materials can be used to teach that content.

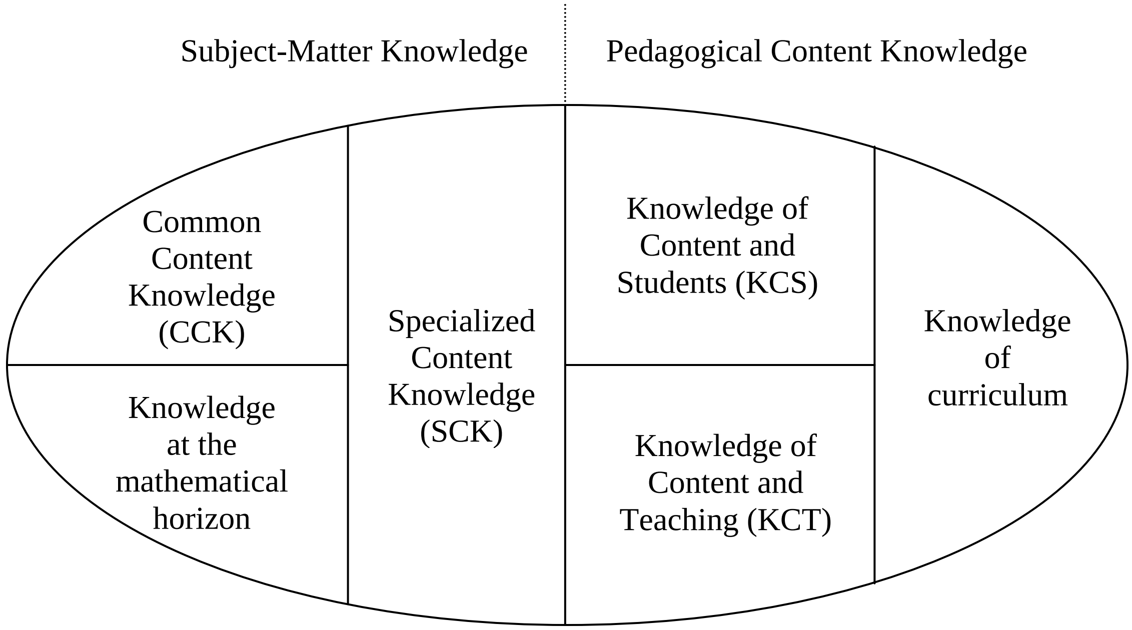

Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377).

In contrast, Grossman (1990) identified four knowledge domains for teachers: general pedagogical knowledge, subject-matter knowledge, pedagogical content knowledge, and knowledge of context. The model shown in Figure 2 represents what is included in each knowledge domain and how they are related to each other. She proposed that pedagogical content knowledge is composed of four central components. She identified the first component as knowledge and beliefs about the purposes for teaching a subject at different grade levels. She noted that knowledge of students' understanding, conceptions, and misconceptions of particular topics in a subject is a component of pedagogical content knowledge. Moreover, she stated that knowledge of curriculum materials available for teaching particular subject-matter and knowledge of instructional strategies and representations for teaching particular topics are components of pedagogical content knowledge.

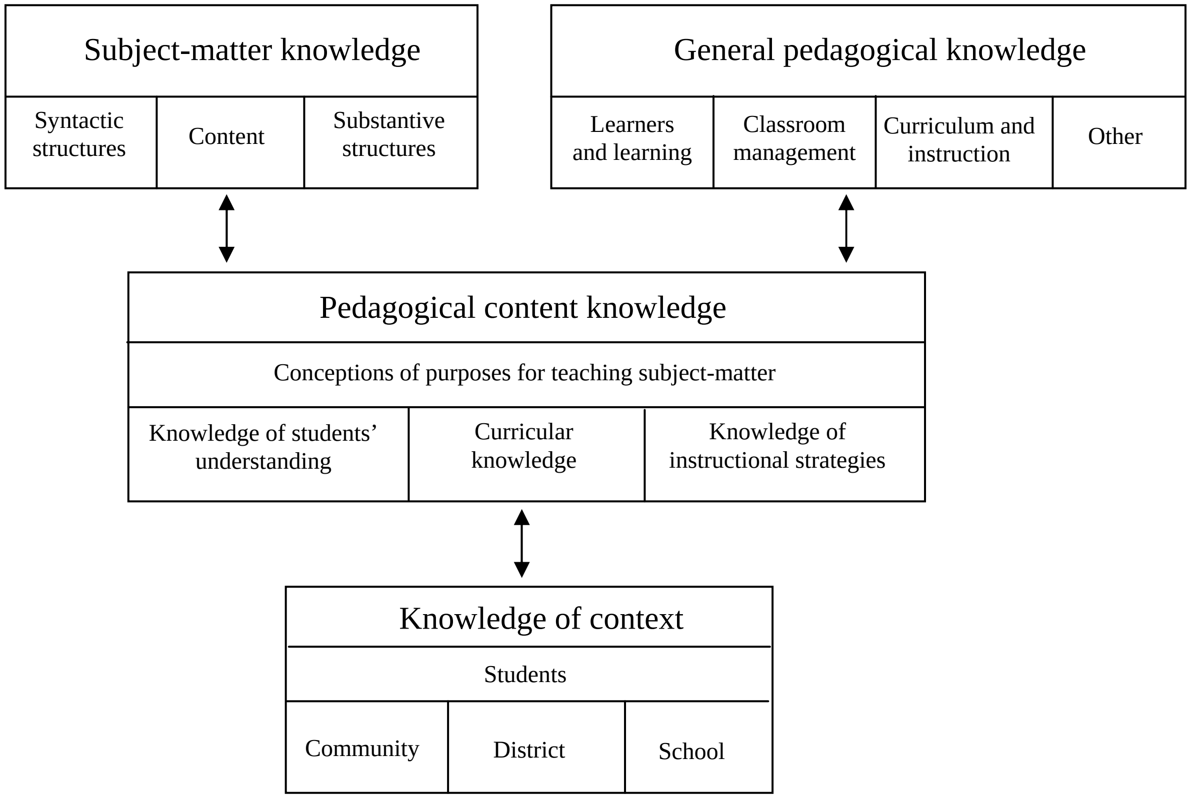

Figure 2. Model of teacher knowledge (Grossman, 1990, p. 5).

Based on his study with elementary teachers Marks (1990) suggested three different derivations for pedagogical content knowledge. First, pedagogical content knowledge is rooted in subject-matter knowledge. The transition from subject-matter knowledge to pedagogical content knowledge is achieved through transforming a particular piece of content to make it comprehensible for specific learners. Second, pedagogical content knowledge is primarily derived from general pedagogical knowledge, which preservice teachers acquire from courses where they learn about students' learning processes and teaching strategies. However, teachers have to apply those ideas to particular content and particular students when they are teaching. Third, pedagogical content knowledge is derived from the combination of subject-matter knowledge and general pedagogical knowledge or from previous construction of pedagogical content knowledge. Teachers make decisions about what learning activities and teaching strategies to use when teaching a particular topic by depending on their previous experiences with teaching that topic.

Similarly, Grouws and Schultz (1996) perceived pedagogical content knowledge as the subset of content knowledge that "has particular utility for planning and conducting lessons that facilitate student learning" (p. 444). In fact, they defined pedagogical content knowledge as the knowledge base including, but not limited to, useful representations and analogies, clarifying examples and counterexamples, and connections among ideas. Hence, they noted that pedagogical content knowledge is "content knowledge that is useful for teaching" (p. 444).

Carlsen (1999) discussed pedagogical content knowledge from structural and poststructural point of views. He stated that from a structural point of view, pedagogical content knowledge is a form of teacher knowledge distinct from other forms and defined by its relationship to those forms. He defined five domains of teacher knowledge as knowledge about general educational context, knowledge about specific educational context, general pedagogical knowledge, subject-matter knowledge, and pedagogical content knowledge. He emphasized that from a structural point of view pedagogical content knowledge is established through its relationship to and difference from other knowledge domains. On the other hand, poststructuralists reject the view of knowledge that is fixed and systematic. They think that knowledge might be formed in a different way in each new educational context. A poststructural view also accepts that definitions of pedagogical content knowledge may vary with respect to who defines it; that is, the definition of pedagogical content knowledge may not be the same for a mathematician, a mathematics teacher, or a mathematics teacher educator. Therefore, the main criticism by the poststructuralist of the structural approach is that the structuralist represents knowledge as fixed and external to teachers and students and disregards the relationship between power and knowledge.

Gess-Newsome (1999a) classified views about the nature of pedagogical content knowledge under two models: the integrative model and the transformative model (see Figure 3). She stated that the integrative model is based on the idea that pedagogical content knowledge is not a separate knowledge domain for teachers; rather, it emerges as an integration of subject-matter knowledge, pedagogy, and context during the act of teaching. In that model, teaching is conceived as the presentation of content to students using appropriate forms of instruction. The task of the teacher in that model is to integrate subject-matter, pedagogy, and context in accordance with the purpose of the lesson to create effective learning opportunities for the students. However, the integration depends on the purpose of the lesson; therefore, teachers should integrate all types of knowledge specifically for each topic taught. In the integrative model, expertise in teaching is defined as possessing well-organized individual knowledge bases and the ability to move smoothly from one knowledge base to the next base.

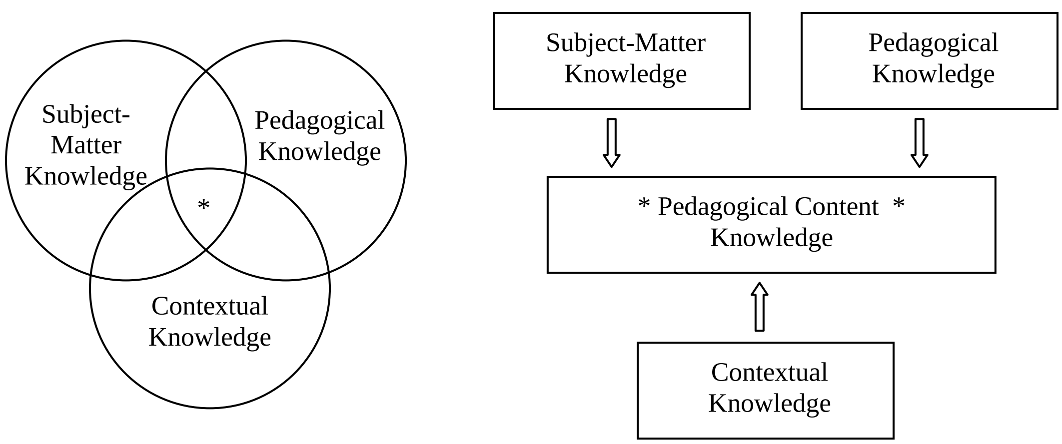

Figure 3. Two models of teacher knowledge (* = knowledge needed for classroom teaching) (Gess-Newsome, 1999a, p. 12).

The second model for pedagogical content knowledge is the transformative model. In this model, pedagogical content knowledge is conceived of as a synthesized knowledge base for teaching. It is accepted that knowledge bases for subject-matter, pedagogy and context exist, but they are useful only when transformed into pedagogical content knowledge. In the transformative model, effective teaching is possible when teachers possess well-structured and easily accessible pedagogical content knowledge for all topics taught.

In terms of implications for teacher preparation, the integrative model suggests that knowledge bases can be taught separately or integrated, and teaching experiences support the development, selection, integration and use of knowledge bases. On the other hand, the transformative model suggests that knowledge bases are best taught in an integrated fashion, and teaching experiences support the development, selection, and use of pedagogical content knowledge.

Moreover, Gess-Newsome (1999a) identified the potential risks for both models. In the integrative model, teachers may fail to see the importance of content over pedagogy, and they may pay little attention to content structure or contextual factors. In the transformative model, teachers may ignore context but focus on some common teaching practices that exist for given topics and are specific to grade level. However, she positioned herself between these two extremes and recognized knowledge bases of subject-matter, pedagogy, and context and their reciprocal relationship with pedagogical content knowledge. She stated that pedagogical content knowledge "is a unique domain that does not totally subsume all other knowledge, allowing for distinctions within and across domains" (p. 13). She accepted that new knowledge gained through teacher education programs and teaching experiences increases the depth of pedagogical content knowledge.

Morine-Dershimer and Kent (1999) identified sources of pedagogical content knowledge and how they interact with each other. The model they suggested for interaction between pedagogical content knowledge and other knowledge domains is shown in Figure 4. They believed that knowledge of pedagogy, knowledge of learning and learners, knowledge of curriculum, knowledge of content, knowledge of specific context, and knowledge of instructional issues contribute to pedagogical content knowledge. Additionally, the interactions between those knowledge domains are represented in their model.

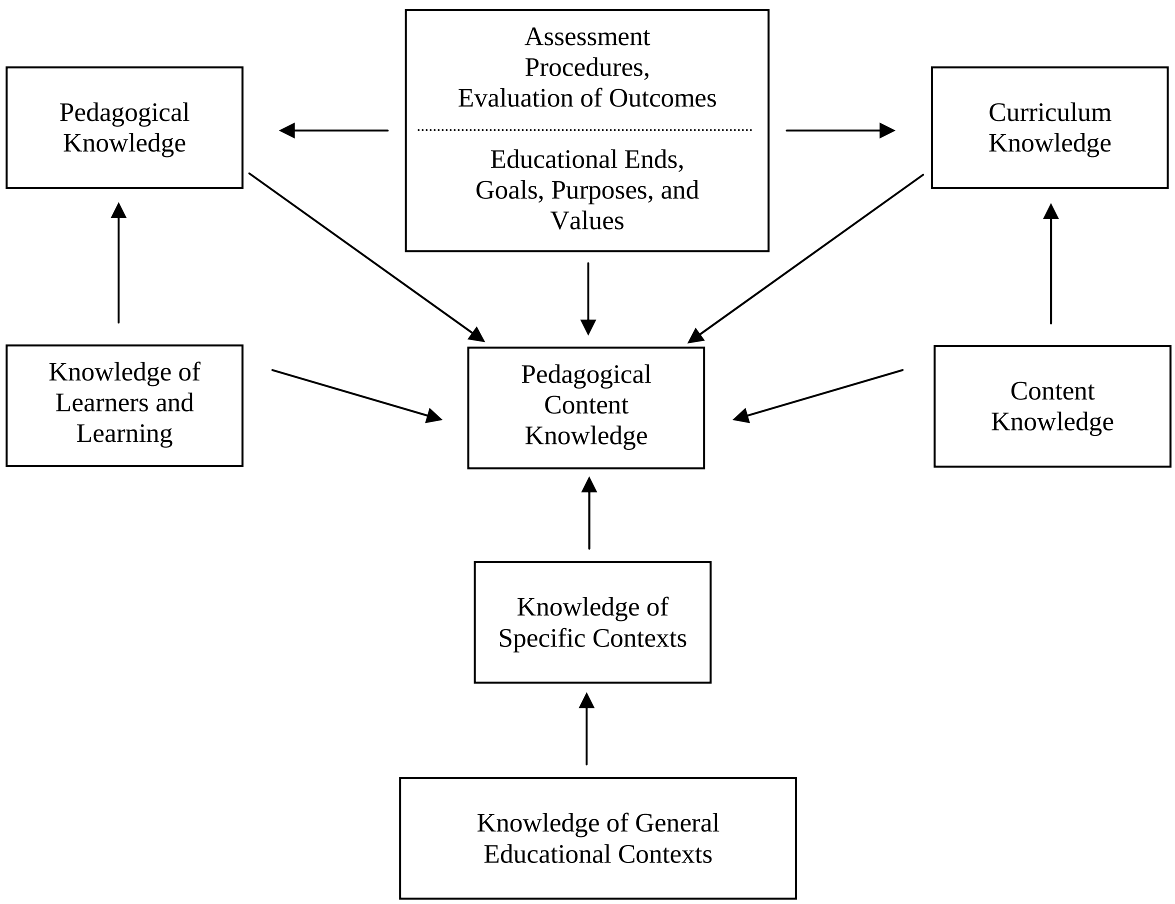

Figure 4. Knowledge domains contributing to pedagogical content knowledge (Morine-Dershimer & Kent, 1999, p. 22).

Although there are a variety of the views about what constitutes pedagogical content knowledge in the literature, all of the scholars cited here agree that pedagogical content knowledge interacts in some way with other knowledge domains. Many researchers agree that pedagogical content knowledge includes knowledge of subject-matter, knowledge of pedagogy, and knowledge of learners.

The Nature of Pedagogical Content Knowledge

Teaching entails various knowledge, skills, and abilities that enable teachers to create a learning environment that supports students' intellectual and social development (Fennema & Franke, 1992; Franke, Kazemi, & Battey, 2007). Even though there are ambiguities about what knowledge teachers should possess for effective teaching (Fennema & Franke, 1992), researchers have conducted studies of teachers' knowledge of and beliefs about subject-matter, pedagogy, students' learning, and curriculum (Brown, Cooney, & Jones, 1990; Calderhead, 1996). There is a limited number of studies focusing specifically on pedagogical content knowledge. Because pedagogical content knowledge is related to subject-matter knowledge (Grossman, Wilson, & Shulman, 1989; Grouws & Schultz, 1996), it is impossible to study pedagogical content knowledge without a context. Therefore, some researchers prefer to investigate teachers' content knowledge and its impact on teachers' pedagogical and instructional decisions. Therefore, the studies presented in this section are examples of both types of research.

One of the earliest studies on pedagogical content knowledge was conducted by Carpenter and his colleagues (Carpenter, Fennema, Peterson, & Carey, 1988). They defined pedagogical content knowledge as the knowledge of what students already know about a topic, what misconceptions about the topic that they may have developed, how they move from the state of little understanding to the state of mastery, how to assess students' understanding, how to diagnose and eliminate misconceptions, and what instructional strategies facilitate connections between what students are learning and they already know. Using their definition, Carpenter et al. investigated elementary teachers' pedagogical content knowledge of children's solutions to addition and subtraction problems. They found that the teachers were successful at predicting students' performance on problem-solving tasks, but they were unable to predict what strategies the students would use to solve the problems. They noted that this result could be evidence that the teachers did not make their instructional decisions based on the strategies that students use to solve problems, but they did pay attention to the difficulty level of the problems.

Grossman (1990) investigated the nature of the pedagogical content knowledge of novice English teachers. She noted that the teachers' knowledge and beliefs about the purposes for teaching a subject had an impact on their instructional decisions. For instance, one of the participants viewed teaching literature as explaining the given text in depth, whereas another participant put emphasis on helping students relate the text to their own lives. The two participants used different instructional materials and activities to pursue their goals. Similarly, Ball (1990a) noted that preservice teachers' conceptions about teaching mathematics influenced their pedagogical decisions. They perceived that knowing mathematics means knowing how to carry out procedures; therefore, they attempted to tell or show students how to solve algorithms step by step to help them learn mathematics.

Even (1993) investigated preservice secondary teachers' knowledge of functions and its connections to their pedagogical content knowledge. Her data were based on a questionnaire completed by 152 preservice teachers and interviews conducted with 10 of them. She found that the preservice teachers tended to rely on their previous learning about functions rather than blending their previous knowledge with new ideas learned in college. They did not know the modern definition of function and relied on the “vertical line test” to determine whether a given relation is a function or not. However, the modern definition would have enabled them to recognize that some relations are functions even though they fail the vertical line test. Therefore, when teaching functions, these teachers said, they would just emphasize using the vertical line test without providing the reasoning behind the test. Consequently, they would likely mislead students about what a function is. Even noted that the preservice teachers were unable to generate effective ways to teach for understanding.

Even and Tirosh (1995) examined teachers’ presentations of certain content in terms of their knowledge of subject-matter and students. Their study was premised on the idea that to generate appropriate representations and explanations for a concept, teachers should not only know the facts, rules, and procedures but also know why they are true. For instance, one participant knew that 4 divided by 0 is undefined but did not know why. Therefore, this participant would tell students that it is one of the mathematical axioms that should be memorized. Additionally, Even and Tirosh noted that the preservice teachers were unable to address students’ misconceptions effectively. Given two cases of incorrect solutions for 4 divided by 0 (e.g., $4 \div 0 = 0$ and $4 \div 0 = 4$), they preferred to suggest their own answers rather than attempting to understand the students’ reasoning. Thus, Even and Tirosh concluded that teachers’ knowledge of subject-matter and students’ thinking had a strong influence on their pedagogical decisions.

Similarly, Kinach (2002) indicated that preservice teachers’ inabilities to unpack mathematical ideas influenced their abilities to teach them effectively. She asked preservice teachers to explain addition and subtraction operations with integers in three contexts: selfchosen, number line, and algebra tile. She noted that the preservice teachers attempted to give the rules and say how to execute the operations but not explain why the algorithm works. She stated that their pedagogical content knowledge was instrumental (Skemp, 1978) because they viewed teaching as giving rules, showing students how to use them, and then making students practice with them. She also noted that to generate effective explanations, the preservice teachers needed to strengthen their conceptual knowledge of mathematics.

Grossman and her colleagues (Grossman, Wilson, & Shulman, 1989) suggested four dimensions of subject-matter knowledge for teaching: content knowledge, substantive knowledge, syntactic knowledge, and beliefs about subject-matter. They defined content knowledge as knowing the facts, concepts, and procedures. They noted that content knowledge certainly influences instruction because teachers need to decide what students should know in order to perform well in the subject-matter. Substantive knowledge refers to knowing how the concepts and facts are organized; therefore, it is influential in curricular decisions. They described syntactic knowledge as knowing the syntactic structures that guide inquiry in the discipline and noted that such knowledge enables teachers to be critical about the legitimacy of new information in their discipline. Overall, they proposed that all components of subject-matter and beliefs about subject-matter affect teaching. When teachers know the subject-matter thoroughly, they are able to make connections between topics and provide conceptual explanations for procedures. However, they also noted that “the ability to transform subject-matter knowledge requires more than knowledge of the substance and syntax of one’s discipline: it requires knowledge of learners and learning, of curriculum and context, of aims and objectives, of pedagogy” (p. 32).

Similarly, Ball (1990a) stressed the importance of subject-matter knowledge for teaching. She investigated preservice elementary teachers' knowledge of division with fractions based on the data collected for a longitudinal study called The Teacher Education and Learning to Teach (TELT) study. She noted that the preservice teachers had difficulty explaining the meaning of division with fractions. She indicated that the preservice teachers' substantive knowledge of mathematics was based on memorization. They were unable to generate a representation for the problem or explain the reasoning behind their calculations even though they could perform them correctly. In other words, they lacked knowledge of the connections between mathematical concepts. She also pointed out that their beliefs about mathematics were influential in their approaches to the problems. They perceived mathematics as set of rules and facts, doing mathematics as following procedures to arrive an answer, and knowing mathematics as knowing how to do it. Therefore, they would teach students how to carry out algorithms rather than teaching about the underlying reasoning that makes algorithms work as they do.

The research on teaching and teacher knowledge reveals that teachers definitely need to know their subject-matter in depth, but they also need to know how to teach it to a particular group of students. Therefore, effective teaching entails an integration of different knowledge domains. In fact, there are reciprocal relationships between knowledge domains (Gess-Newsome, 1999a; Morine-Dershimer & Kent, 1999). Therefore, the studies on a particular type of knowledge will inform the study of other knowledge needed for teaching.

Teacher Education

Many researchers stress that teacher education programs need to help preservice teachers improve their knowledge of and skills for effective teaching through coursework and practice (e.g., Barnes, 1989; Borko & Putnam, 1996; Calderhead & Robson, 1991; Fennema & Franke, 1992; Philipp et al., 2007). The findings of studies on preservice teachers support the recommendation that content knowledge, pedagogical content knowledge, and pedagogical reasoning should be central foci of teacher education programs (Brown & Borko, 1992; Grouws & Schultz, 1996). Studies also show the impact of coursework and field experiences on preservice teachers' knowledge of and beliefs about teaching mathematics and provide suggestions for further studies in this area.

Graeber (1999) investigated aspects of pedagogical content knowledge that should be included in a mathematics methods course. Using Shulman's definition of pedagogical content knowledge, she identified five "big ideas" and discussed how each of them could be incorporated into a methods course. The big ideas she proposed were (1) understanding students' understanding is important, (2) students knowing in one way do not necessarily know in the others, (3) intuitive understanding is both an asset and a liability, (4) certain characteristics of instruction appear to promote retention, and (5) providing alternative representations and recognizing and analyzing alternative methods are important. For the first and the third items, she suggested that preservice teachers should be given different examples of students' misconceptions and asked to analyze students' thinking and generate a way of eliminating such misconceptions. She noted that the instructor of a methods course could show videos of teachers who attempt to rectify students' misconceptions. For the second item, preservice teachers could be given examples in which getting the right answer does not necessarily imply conceptual understanding or vice versa. For the fourth and fifth items, the instructor should emphasize alternative ways of teaching a particular concept such as using manipulatives to allow students to explore mathematical ideas. Also, preservice teachers should be given opportunities to examine student-generated algorithms in order to decide on the validity and generalizability of such algorithms. In fact, Graeber's suggestions are common practices in many methods courses, but preservice teachers may benefit from those experiences differently.

A longitudinal study conducted at Michigan State University (Schram, Wilcox, Lanier, & Lappan, 1988) aimed to investigate the nature and the extent of the changes in preservice elementary teachers' beliefs and knowledge about mathematics and teaching and learning mathematics as a result of a series of innovative mathematics content courses, a mathematics methods course and a curriculum seminar. The data were collected from 24 preservice teachers during their two-year teacher preparation program and their first year of teaching. The preservice teachers took three content courses that were specifically oriented to exploring ideas about numbers, geometry, probability, and statistics as well as the relationships between them. The instructor attempted to create a learning environment for preservice teachers in which they could work in groups to explore ideas, discuss the solutions to the problems, generate different representations, and make connections among mathematical ideas. Schram et al. noted that at the end of the courses, the preservice teachers' views about mathematics had changed; initially they thought that mathematics was a meaningless series of symbols and rules, but by the end of the courses they appreciated the value of conceptual understanding of mathematics. Furthermore, they liked the way the instructor set up the learning environment. However, they were unable to transfer what they experienced in the courses to their own instruction. Some of them still held their traditional view of mathematics and emphasized procedural knowledge rather than conceptual understanding when teaching mathematical facts and procedures.

Similarly, Feiman-Nemser and Buchmann (1987) found that preservice teachers had difficulty making inferences from their learning experiences during the teacher preparation program and applying them when teaching in a classroom. Although they were able to follow some routines to keep students engaged, they were unable to assess students' needs and understanding or modify their instruction accordingly. Feiman-Nemser and Buchmann indicated that mentor teachers and university supervisors should give student teachers more explicit feedback about their practices and instructional decisions so that they might learn from their experiences and improve their pedagogical skills.

Ball (1988) indicated that teacher educators should provide opportunities for preservice teachers to evaluate their own understanding and knowledge of teaching and learning mathematics during their teacher preparation programs. She designed her introductory methods course for elementary preservice teachers with that intention. She assigned a permutation project for the preservice teachers in which they first tried to learn about permutations themselves, then watched a teacher (Deborah Ball) helping a student to explore the idea, and finally tried to help a child or an adult learn about permutations. She asked preservice teachers to pay attention to what they were thinking, doing, and feeling during each phase of the project. She introduced the topic with a challenge and then let them work with manipulatives to explore the permutation concept. Then she used several tasks and established questioning techniques to teach the concept of permutations to a child. She noted that many of the preservice teachers tried to model her when teaching that concept to someone else. In the end, she noted that the preservice teachers became aware that knowing mathematics for themselves is different from knowing it to teach others, and they learned that there is more than one way to represent or explain a mathematical concept.

Ball and Wilson (1990) analyzed data from the TELT study, which aimed to investigate what teachers are taught and what they learn in 11 different preservice, inservice, induction, and alternative-route programs. Ball and Wilson investigated the nature of pedagogical content knowledge and perceptions about mathematics of beginning teachers enrolled in an alternative route program and a standard teacher education program. They explored the teachers' ideas and understanding about mathematics, teaching and learning mathematics, and students in the specific contexts of place value, fractions, division and multiplication, proportion, theory and proof, area and perimeter, and variables. None of the students in either program was well-prepared for unpacking meanings of mathematical ideas they had studied. The participants' mathematical knowledge was mostly procedural, and their conceptual understanding of some concepts such as division by zero was weak. Furthermore, most of them conceived of teaching as telling and showing how to perform operations or apply the rules. In fact, many of them lacked a repertoire of different representations of mathematical concepts. Therefore, Ball and Wilson concluded that teacher education courses had little impact on prospective teachers' knowledge and skills. They suggested that teacher educators must pay attention to the content and pedagogy of teacher education.

Philipp and his colleagues (Philipp et al., 2007) investigated whether preservice elementary teachers' content knowledge and beliefs improved if they were given opportunities to learn about students' mathematical thinking as they were learning the mathematics they would teach. They collected data from 159 preservice teachers enrolled in their first mathematics content course, which focused on whole number and rational number concepts and operations. The instructional materials were designed to support preservice teachers' conceptual understanding of those topics, and the preservice teachers were given examples of students' ways of solving problems. Philipp et al. randomly assigned the preservice teachers to one of four treatment groups, each with a different way of interacting with children's mathematical knowledge, and a control group. The preservice teachers' beliefs about teaching and learning mathematics changed, but there were no significant changes in their knowledge. However, the treatment group in which the preservice teachers watched and analyzed videos of students solving problems and then worked with a student themselves was the most effective one in contributing to preservice teachers' knowledge of students' thinking. The preservice teachers expressed positive feelings about working with students, even though some of them found it challenging. Therefore, Philipp et al. suggested that preservice teachers should be given opportunities to work with children in the early years of their training.

The literature on teacher education programs reveals that the coursework and field experiences have an impact on preservice teachers' conceptions about and knowledge of content, teaching, learning, and students (Borko & Putnam, 1996) but in different ways and to different degrees. Most of the research leads to the conclusion that teacher education programs should be revised to better support the development of knowledge bases for effective teaching. The courses offered in teacher education programs should enable preservice teachers to improve their knowledge in a specific knowledge base and provide opportunities to relate or apply what they learn in these courses to the practice of teaching.

Theoretical Perspective

Pedagogical content knowledge is a unique knowledge domain for teachers and refers to teachers' knowledge of how to organize and represent particular topics or issues to facilitate students' understanding and learning (Ball & Bass, 2000; Borko & Putnam, 1996; Carpenter et al., 1988; Shulman, 1986, 1987). Therefore, teachers are expected to know how mathematical concepts are developed and the connections between them, teaching goals for different grade levels, the needs of their students, and appropriate teaching strategies for them.

My review of the literature on the definition of pedagogical content knowledge revealed an interaction between pedagogical content knowledge and other knowledge domains. Hence, for the purpose of my study, I assumed that pedagogical content knowledge includes knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum. Furthermore, I adopted Shulman's (1986, 1987) ideas about pedagogical content knowledge and defined pedagogical content knowledge as the ways of knowing how to represent a topic effectively to promote students' understanding and learning and being able to diagnose and eliminate students' misconceptions and difficulties about that topic. I also agree with Gess-Newsome (1999a) and Morine-Dershimer and Kent (1999) that there is a reciprocal relationship between pedagogical content knowledge and other knowledge domains. In particular, I believe that knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum are essential to pedagogical content knowledge.

Some research about teachers' knowledge of subject-matter, students, or pedagogy also involves teachers' beliefs about teaching and learning, the relationship between knowledge and beliefs, and how beliefs influence teachers' practices. Many studies have revealed that students come to their preservice teaching programs with beliefs about teaching and learning that are shaped by their own school experience and are hard to change through teacher education courses (Ball, 1988; Borko & Putnam, 1996; Even, 1993; Foss & Kleinsasser, 1996). Therefore, in my study I tried to be aware of the preservice teachers' beliefs about teaching and learning mathematics but did not attempt to assess those beliefs. Instead, I tried to learn about their knowledge of mathematics, pedagogy, students, and curriculum through interviews, observation, and examination of their written work.

In my definition of pedagogical content knowledge, knowledge of subject-matter refers to knowledge of mathematical facts and concepts and the relationships among them. I define strong mathematical knowledge as knowing how mathematical concepts are related and why the mathematical procedures work. Subject-matter knowledge also influences teachers' instruction and students' learning (e.g., Ball, 1990a; Ball & Bass, 2000; Borko & Putnam, 1996; Ma, 1999; Thompson, 1992). Therefore, subject-matter knowledge includes being able to relate a particular mathematical concept with others and explain or justify the reasons behind the mathematical procedures explicitly to promote students' understanding.

Knowledge of pedagogy covers knowledge of planning and organization of a lesson and teaching strategies. Teachers who have strong pedagogical knowledge have rich repertoires of teaching activities and are able to choose tasks, examples, representations, and teaching strategies that are appropriate for their students. In addition, they know how to facilitate classroom discourse and manage time for classroom activities effectively.

Knowledge of learners refers to knowing students' common difficulties, errors, and misconceptions. Teachers who posses a strong knowledge base in this domain know what mathematical concepts are difficult for students to grasp, which concepts students typically have misconceptions about, possible sources of students' errors, and how to eliminate those difficulties and misconceptions.

Finally, knowledge of curriculum includes knowledge of learning goals for different grade levels and knowledge of instructional materials. Teachers with strong knowledge in this area know the state and NCTM standards for teaching mathematics identified for different grade levels and plan their teaching activities accordingly. They choose appropriate materials (e.g., textbooks, technology, and manipulatives) to meet the goals of the curriculum and use them effectively. The summary of the components of pedagogical content knowledge that I used in my study is presented in Figure 5.

Pedagogical Content Knowledge

Knowledge of subject-matter	Facts, concepts, and the relationships among them
Knowledge of pedagogy	Planning and organizing a lesson
	Teaching strategies
Knowledge of learners	Students’ common difficulties, errors, and misconceptions
Knowledge of curriculum	Learning goals for different grade levels
	Instructional materials

Figure 5. The components of pedagogical content knowledge.

CHAPTER 3

METHODOLOGY

This study was designed to investigate the nature of pedagogical content knowledge developed in a methods course and its associated field experience in a group of preservice secondary mathematics teachers. I observed the methods course Teaching and Learning Secondary Mathematics and its associated field experience course Secondary School Mathematics Field Experience in fall 2008 at the University of Georgia. I wanted to understand the variety and the extent of the issues discussed in these courses and how preservice teachers could benefit from those discussions and field experiences. I decided to conduct a qualitative study because I was “concerned with process rather than simply with outcomes or products” (Bogdan & Biklen, 1998, p. 6). Also, I wanted to “obtain in-depth understandings about the way things are, why they are that way, and how the participants in the context perceive them” (Gay & Airasian, 2003, p. 13).

Furthermore, a qualitative research design provided me with the flexibility to modify the data collection instruments in the process (Bogdan & Biklen, 1998). I used multiple sources for collecting data, including interviews, observations, a questionnaire, and written documents, and I modified the questions in the second and third interviews with respect to the preservice teachers’ previous answers. Then, I attempted to triangulate all data to reduce the risk of the biases and the limitations of a specific data source (Bogdan & Biklen, 1998; Cohen, Manion, & Morrison, 2007; Maxwell, 2005). Maxwell (2005) noted that interviews help a researcher understand the participant’s perspective, but observations enable the researcher to draw inferences about that perspective that cannot be obtained from interview data. Therefore, I specifically paid attention to the preservice teachers' performance in course activities and interactions with their classmates and the instructors in the class. Additionally, I asked them to elucidate their answers for the questionnaire items to reduce misinterpretation of their perceptions.

Participant Selection

This study took place in the methods course and field experience course for preservice secondary mathematics teachers at the University of Georgia. The number of students enrolled in the methods course was 30. Two of the students were participating in year long, half day internships, so they were not required to take the field experience course. However, one of the interns enrolled in the field experience course and attended the class for eight weeks. After eight weeks, he decided to drop the field experience course but remained enrolled in the methods course. Thus, initially there were 29 students in the field experience course, but eventually there were only 28 enrolled. Of the 30 students, 25 were undergraduates, and 5 were pursuing post-baccalaureate certification through a masters degree program. There were 6 male and 24 female students in the methods course, and 4 of them were African American, 2 of them were Korean, and 24 of them were White. The ages of the students ranged between 20 and 34.

I chose 6 students to participate in this study from among those who were enrolled in both courses. The selection of participants was purposeful (Bogdan & Biklen, 1998; Cohen, Manion, & Morrison, 2007). I tried to ensure that the participants were representative of the students in the methods course in terms of initial levels of pedagogical content knowledge. Therefore, at the beginning of the semester I administered a questionnaire to all students to learn about their knowledge of teaching mathematics as well as how they perceived their knowledge (see Appendix A).

The questionnaire consisted of 13 items; 8 of them were multiple-choice, 1 was Likert-type and 4 were short-answer question. The questionnaire items were written to address the components of pedagogical content knowledge I identified in my theoretical framework in Chapter 2. Each multiple-choice item was aligned to one knowledge type. For instance, Items 1 and 6 were aligned with knowledge of subject-matter, Items 2 and 5 were aligned with knowledge of pedagogy, Items 3 and 7 were aligned with knowledge of curriculum, and Items 4 and 8 were aligned with knowledge of learners. The short-answer questions involved multiple knowledge types. For instance, Item 10 entailed knowledge of subject-matter, pedagogy, and learner. The alignment of each questionnaire item with aspects of pedagogical content knowledge is presented in Table 1.

I assigned points to each item to decide the preservice teachers' knowledge level, and then I used the overall score for the categorization of the preservice teachers in terms of their initial knowledge levels. For multiple-choice items I assigned 1 point for "disagree," 2 points for "somewhat agree," and 3 points for "agree." The Likert-type question had a 4-point scale with 1 point for "not adequate," 2 points for "adequate," 3 points for "competent," and 4 points for "very good." The points given for each knowledge level were added to the overall score of the individuals. For short-answer questions I read the answers for each item and then developed a rubric according to the depth and the clarity of the explanations. The scale for the rubric varied between 0 and 3 points, with 0 points given for no answer, 1 point given for vague answers or answers without explanations, 2 points given for answers without justifications or answers with minor mathematical errors, and 3 points given for valid explanations or justification. The total scores ranged between 29 and 43. I discussed the ratings for each answer with a peer and we had .90 inter-rater reliability (Cohen, Manion, & Morrison, 2007) on the scores. In cases where we disagreed on a rating, we discussed what points to assign those answers and agreed on the final scores.

Table 1

The Alignment of Questionnaire Items with Pedagogical Content Knowledge

	Questionnaire item	Aspects of Pedagogical Content Knowledge			
		KSM	KP	KL	KC
1	At the end of my degree program I will have taken enough content courses to be an effective mathematics teacher in grades 6-12.	x			
2	At the end of my degree program I will have taken enough courses about teaching mathematics to be an effective mathematics teacher in grades 6-12.		x		
3	I know what mathematics content is to be addressed in each year of the 6-12 mathematics curriculum.				x
4	I know possible difficulties or misconceptions that students might have in mathematics in grades 6-12.			x	
5	I have a sufficient repertoire of strategies for teaching mathematics.		x		
6	I know how mathematical concepts are related.	x			
7	I know how to integrate technology in mathematics lessons.				x
8	I know how to diagnose and eliminate students' mathematical difficulties and misconceptions.			x	
9	Read the definitions of four knowledge bases. How do you perceive your knowledge level in each knowledge base?	x	x	x	x
10	Look at the student work given below. How can you explain to the student that his or her solution is incorrect? $\sqrt{9x^2 + 25y^4} = 3x + 5y^2$	x	x	x	
11	Assume that you will introduce "inverse functions". Make a concept map for inverse functions showing which mathematical concepts or facts relate to inverse of functions.	x			x
12	If you were introducing how to factor trinomials, which of the following trinomials would you use first? Explain your reasoning. $2x^2 + 5x - 3$, $x^2 + 5x + 6$, $2x^2 - 6x - 20$	x	x	x	
13	Assume that you will teach the following topics in a semester. In which order would you teach them to build on students' existing knowledge? Explain your reasoning. Polynomials, trigonometry, factorization, quadratic equations	x			x

Note. KSM: Knowledge of subject-matter, KP: Knowledge of pedagogy, KL: Knowledge of learners, KC: Knowledge of curriculum

For selection purposes the preservice teachers were classified into the categories of high, medium, and low based on their professed knowledge about teaching mathematics as well as their answers to short-answer items in the questionnaire. The purpose of the categorization was to follow the changes, if any, in their pedagogical content knowledge throughout the semester. Ten students with scores between 29 and 35 were categorized as having a low level of knowledge; the next 10 students with scores between 36 and 38 were categorized as having a medium level of knowledge; and the last 9 students with scores between 39 and 43 were categorized as having a high level of knowledge. Then, I chose two preservice teachers from each group as the participants of this study. The distribution of the scores is presented in Figure 6, and the study participants are identified.

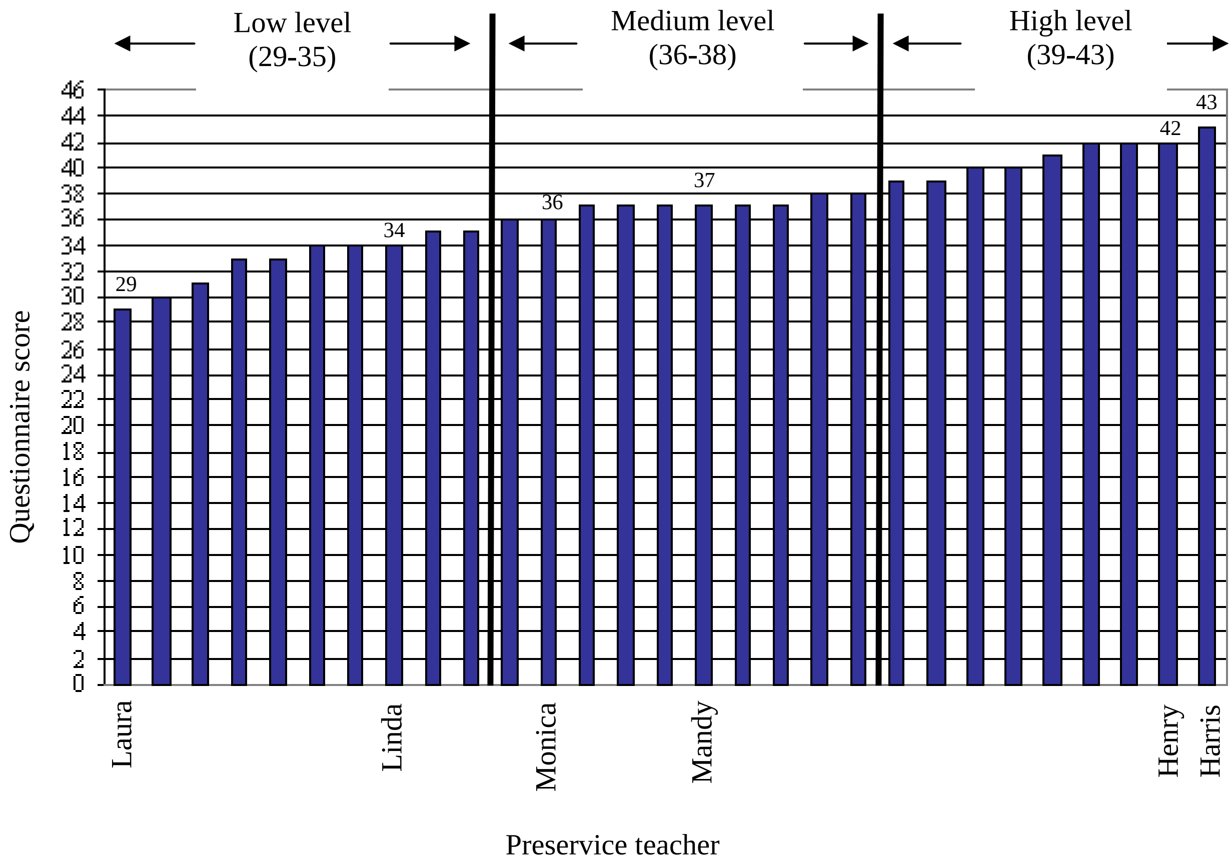

Figure 6. The distribution of the questionnaire scores.

Based on the analysis of questionnaire data, 2 male and 4 female students were chosen as the participants in the study. Laura and Linda (pseudonyms) were categorized as having a low level of pedagogical content knowledge with overall scores of 29 and 34, respectively. Laura was 21 years old, White, and a senior. Linda was 21 years old, White, and a senior. Monica and Mandy (pseudonyms) were categorized as having a medium level of pedagogical content knowledge with overall scores of 36 and 37, respectively. Monica was 20 years old, African American, and a senior; she was pursuing a double major in mathematics and mathematics education. Mandy was 34 years old, White, and a senior. Henry and Harris (pseudonyms) were categorized as having a high level of pedagogical content knowledge with overall scores of 42 and 43, respectively. Henry was 26 years old, White, and a graduate student. Harris was 22 years old, White, and a senior. The choice of pseudonyms of the participants was purposeful such that the initial letter of the pseudonym represents the participant's initial level of pedagogical content knowledge (L for low, M for medium, and H for high).

Data Collection

I attempted to use different sources to collect data in order to gain insight into the students' development of pedagogical content knowledge during the methods and field experience courses and to identify specific activities and events that contributed to that development. The main sources of data were a questionnaire, observations, students' written work and interviews. I was a participant-observer in all class sessions in both classes and took field notes. I collected any artifacts used in the methods and field experience courses and examined participants' assignments and the midterm exam. I conducted three interviews with each participant and one interview with the instructor of the methods course and the instructor of the field experience course. The data sources are explained in detail below.

Observation

I attended in all class sessions and took notes about classroom activities and the preservice teachers' participation. The preservice teachers were usually working in groups to discuss given tasks, and then they shared their ideas with the rest of the class. Because it was difficult to follow the discussion of each group thoroughly in a single lesson, I tried to identify three participants out of the six for each lesson and observe them while they were working in groups or individually. I took extensive notes about their performance on the given tasks and their participation in the class. I also took notes about what the 6 participants said during whole class discussions. Furthermore, I collected any artifacts (e.g., handouts, multimedia presentations, and journal articles) discussed in the class in order to make inferences about the goals of that particular lesson and how they were manifested.

Students' Work

In the methods course, the preservice teachers were given assignments either as preparation for the next class period or as the extension of or reflection on the issues discussed in the class. Furthermore, they were given a midterm exam and asked to prepare a portfolio as the final product of the course. In the field experience course, the preservice teachers were given assignments as preparation for the next class period, and they were required to write four field reports during their time in schools. For each field report, the preservice teachers were expected to reflect on a major issue discussed in the class. The preservice teachers wrote field reports on: (1) teachers' questioning techniques, (2) mathematical tasks used by teachers, (3) a written assessments used by teacher, and (4) students' mathematical thinking and understanding. I examined all assignments and the exam completed by the 6 participants to gain a better understanding of their experiences in the methods course and their reflections on the field experiences.

Interview

I conducted three interviews with each participant and one interview with the instructor of the methods course and the instructor of the field experience course. The first interview was held during the third week of the semester. It consisted of eight questions. I mainly asked what the participants expected to learn in the methods and field experience course that would contribute to their pedagogical content knowledge. I also gave them some cases to analyze to learn how well they could articulate the aspects of pedagogical content knowledge to generate ideas about the given cases.

I conducted the second interview during the eighth week of the semester just after their second field experience. In the second interview, I asked seven questions that led them to reflect on the concepts and issues discussed in the methods and the field experience courses and how they contributed to their pedagogical content knowledge. During the second interview I gave them cases similar to the ones given in the first interview in order to learn about the development of their pedagogical content knowledge.

The third interview was held during the last week of the classes, and its format was similar to the second interview. I asked 11 questions in the third interview. At the beginning of the interview, I gave them a shortened version of questionnaire including Items 1 through 9 to see how they perceived their knowledge levels at the end of the semester. Furthermore, I asked them to make an overall evaluation of the methods and field experience course in terms of their gains from these courses. All student interview protocols are in Appendix B.

I interviewed the instructor of the methods course and the instructor of the field experience course at the beginning of the semester and asked about their goals for these courses and their expectations of the preservice teachers (see Appendix C). The instructor of the methods course was Ashley (pseudonym). She was a doctoral student in mathematics education, and she had taught high school mathematics for 11 years. Her course goals included raising preservice teachers' awareness of various issues that would have an impact on how they teach mathematics. She stated that she would especially focus on different teaching strategies, using instructional materials, mathematical content knowledge, and assessment. The instructor of field experience course was Kevin (pseudonym), and he was also a doctoral student in mathematics education. Kevin had 8 years of teaching experience in middle and high schools. He wanted his students to look at a mathematics classroom from different perspectives and mainly concentrate on the teacher, the students, and mathematics.

Data Analysis

I used all data collected to answer both of my research questions. Because I used the questionnaire to select my participants I scored all of the answers given in the questionnaire. However, for interview transcripts I used the framework of pedagogical content knowledge that I developed for this study and analyzed the data accordingly. I used the field notes and student work to have a better understanding of the nature of the courses and the participants' experiences in these courses.

Analysis of the Interviews, Field Notes and Student Work

I used the pedagogical content knowledge framework developed for this study to analyze the interview transcripts, field notes, and students' written work. I defined pedagogical content knowledge as having four components: knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum. The preservice teachers' answers to given mathematical problems, thoughtfulness of their justification for the answers, and validity of their explanations about how mathematical concepts are related or why a particular solution is incorrect were counted as the indicators of their subject-matter knowledge. The preservice teachers' knowledge of pedagogy was identified in terms of reasonableness of their choice of teaching activities, tasks, examples, and representations, comprehensiveness of their lesson plans and creativity of their ideas about motivation and promoting classroom discourse. Their repertoire of students' possible difficulties and misconceptions in mathematics and their ability to identify and eliminate such difficulties, errors, and misconceptions was coded as their knowledge of learners. Because knowledge of curriculum refers to knowing learning goals for different grade levels and how to use different instructional materials in mathematics lessons, the preservice teachers' curriculum knowledge was assessed in terms of their ability to identify a reasonable order of mathematical concepts to be taught in a semester, to differentiate learning goals for different grade levels, and to choose appropriate instructional materials such as textbooks, technology, and manipulatives to meet those goals.

I read through each students' work, transcripts, and daily field notes to get familiar with the content. I read each transcript to code each participant's answers in terms of the type of knowledge demonstrated in the questions, and then I compared the answers to similar types of questions to determine any change in their knowledge level of that particular knowledge domain. I read the field notes and student work to learn about the scope and diversity of the course topics and how the preservice teachers perceived these topics. After completing the coding of each transcript I read through all student work, transcripts, and field notes for a final time to check whether the alignment of each category was reasonable.

Using my field notes, the artifacts distributed in the class, and the homework assignments, I made a list of the fundamental issues discussed in the methods course and field experience course. There were some issues, like classroom management and assessment, that were discussed in both courses. The preservice teachers were asked to work on a mathematics problem at the beginning of each session of the methods course. The instructor chose the problems of the day either from the course textbook or other resources. The preservice teachers shared their experiences in the field experience course after each field experience and wrote a field report. During the field experiences, most of the preservice teachers had the opportunity to observe classrooms in which new Georgia high school mathematics curriculum (the Georgia Performance Standards or GPS) was being implemented. The new curriculum is based on an integrated approach to high school mathematics courses rather than separate courses in algebra, geometry, and trigonometry. The new curriculum consists of a series of four mathematics courses called Math I, Math II, Math III, and Math IV. The curriculum is intended to be "de-tracked" but does allow an accelerated option for students wishing to take Advanced Placement courses in their senior year. In the accelerated track, students take Accelerated Math I, Accelerated Math II, and Accelerated Math III. Although the state prescribed names for the courses, some school districts used different names for these new types of courses, such as Advanced Mathematics. The academic year of 2008-2009 was the first year of the implementation of the curriculum with ninth-grade students.

During the semester, the preservice teachers discussed learning theories, standards-based curricula and textbooks, motivation, promoting communication in classroom, manipulatives, planning instruction, effective questioning, cognitive demand of a task, classroom management, and assessment and rubrics in one course or the other. They watched videos about teachers' questioning techniques and students’ problem-solving skills. In the methods course, the preservice teachers were assigned to prepare a lesson and implement 15 minutes of that plan in front of their classmates (microteaching). The instructors videotaped each presentation and gave it to the presenter for self-reflection. In addition, the preservice teachers were asked to teach a lesson during their last field experience. However, some of them did not have an opportunity to teach at all because their mentor teachers did not arrange for it.

CHAPTER 4

DATA ANALYSIS

In this study I aimed to investigate what aspects of preservice teachers' pedagogical content knowledge (PCK) developed in a mathematics methods course and its associated field experience. I also examined what course topics contributed to the development of each aspect of PCK from the preservice teachers' perspectives. For this study I defined PCK as consisting of knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum. The preservice teachers' ability to solve given mathematics problems, justify their reasoning, and explain how mathematical concepts are related were conceived of as the indicators of the level of their knowledge of subject-matter. Their abilities to prepare comprehensive lesson plans and be critical when choosing teaching strategies, activities, tasks, examples, and representations were considered to be related to their knowledge of pedagogy. The preservice teachers' repertoire of examples of students' possible difficulties and misconceptions in mathematics and their abilities to identify and eliminate were used as indicators of their knowledge of learners. Finally, knowledge of curriculum encompassed their abilities to identify the order in which mathematical concepts should be taught in a course, to differentiate learning goals for different grade levels, and to choose appropriate instructional materials such as textbooks, technology, and manipulatives to meet such goals. The research questions that guided this study were as follows:

1. What aspects of preservice teachers' pedagogical content knowledge are developed in a methods course and its associated field experience?

2. What course experiences provide for this development from the preservice teachers' perspectives?

The answers to both research questions are discussed together by knowledge domain below. In each section I first present the findings from content-specific questions and then explain how the preservice teachers' viewed the development of their knowledge in each domain. Before discussing the findings of this study, I introduce each participant briefly in sequential order based on their initial level of pedagogical content knowledge.

Participants

Laura

Laura was a senior in the mathematics education program. She did not have tutoring experiences other than helping her friends with their homework. Although she engaged in the given tasks in the class and completed all assignments, she was reluctant to share her ideas in the small group activities or participate in whole class discussions. She observed lessons in seventh- and eighth-grade classes and also Mathematics 1, Advanced Mathematics 1, and Accelerated Mathematics 1 classes. She taught a lesson in a seventh-grade class during her last field experience.

Laura expected to learn how to prepare a lesson plan, how to manage a classroom, and how to integrate technology into instruction during her methods course and field experiences. She also wanted to develop her skills of predicting students' difficulties. She noted that the field experiences helped her to improve her repertoire of teaching strategies, instructional materials, and examples of students' errors and misconceptions. By the end of the semester, Laura was still not confident in her teaching ability and expressed great concern about classroom management. She wanted to have more teaching experiences in the field and learn more about how to deal with disruptive behavior in the classroom. She thought that a teacher fails to teach despite strong knowledge of content, pedagogy, and curriculum if she or he lacks effective classroom management skills. Therefore, she wanted to apply to a school system that had a teacher mentor program where experienced teachers coach novice teachers.

Laura's knowledge of subject-matter had a significant impact on her pedagogical content knowledge. She knew the mathematical rules, facts, and procedures; however, she lacked a deeper understanding of why they work. Her view of mathematics as a collection of rules and facts was reflected in her view of how to teach mathematics. She said she would emphasize the rules, procedures, and facts when teaching mathematics, and she would repeat those rules and procedures when addressing the students' difficulties or misconceptions. On the other hand, she said she would prepare her lesson plan thoroughly, use various teaching strategies including group work and individual work, and use various instructional materials including technology, manipulatives, and textbooks. Therefore, I infer that the field experiences contributed to her pedagogical content knowledge; however, she needed to develop her conceptual understanding of mathematics to improve her knowledge of learners and planning instruction.

Linda

Linda was a senior in the mathematics education program. She had some tutoring experiences with middle school and high school students, and she was tutoring a seventh grader when this study was conducted. She was an active participant in the small group and whole class discussions. She observed two eighth-grade classes in the middle school, and Mathematics 1, AP Calculus, Geometry, and AP Statistics courses in the high school. During her last field experience she taught a lesson in an AP Statistics class.

Linda wanted to be a teacher who knows her subject-matter thoroughly and plans her lesson in a way that students not only understand the mathematical rules, procedures, or facts but also can conceptualize why they work. She said she would try to use effective questioning as a medium of assessing students' understanding and encouraging them to think about a particular topic more deeply. Furthermore, she said she would try to differentiate teaching activities according to the needs of the students when planning lessons. During the methods course and field experience she wanted to improve her repertoire of examples of students' difficulties and misconceptions in mathematics and learn various ways to address those difficulties.

By the end of the semester Linda was aware that all aspects of pedagogical content knowledge are essential for effective teaching practice. There were some improvements in her pedagogical content knowledge. For instance, she was able to predict students' possible difficulties and misconceptions even though she needed to improve her repertoire of how to eliminate those difficulties. Similarly, she was able to describe instructional materials she might use to achieve her goals. Furthermore, when planning lessons Linda tried to start with an easy example to show how the rule, fact, or procedure worked and then increased the difficulty level of the examples gradually. She emphasized teaching the procedures and facts before teaching underlying concepts because she thought that conceptual understanding follows from understanding how the procedures and facts work.

Monica

Monica was a senior pursuing a dual degree in mathematics and mathematics education. She had some tutoring experiences with middle school and high school students; however, she was not tutoring when this study was conducted. She was intrinsically motivated to learn about issues discussed in these courses and generally remembered the details of the reading assignments and classroom discussions. However, she rarely volunteered to share her ideas during small group and whole class discussions. During her field experiences, she observed a sixth- and an eighth-grade class and also Mathematics 1, Accelerated Mathematics 1 and an AP Statistics courses. She taught a lesson in AP Statistics in her last field experience.

Monica thought that the methods course and field experiences contributed to her knowledge of teaching and said she would definitely apply the ideas and activities that she learned in the courses and observed in the field when she teaches. She planned to use effective questioning techniques as a form of informal assessment and group work as a platform to enable students to discuss the mathematical concepts and terminology that they are learning. She noted that she wanted to create various activities that intrinsically motivate students to learn mathematics and engage in lessons.

The methods course and field experiences contributed to the improvement of some aspects of Monica’s pedagogical content knowledge. Observing students and teachers raised her awareness about several issues that influence the effectiveness of the instruction. For example, she began to think about what might be difficult for students to grasp, how she could eliminate those difficulties, and what materials she could use to achieve her goals. She said she would use manipulatives, visual aids, or real-life examples to explain mathematical facts or concepts. However, she often overestimated what students might know about a mathematical topic. For instance, she failed to generate simple examples when introducing a new concept because she assumed that the students would know previous concepts in depth and be able to make the connections between the concepts immediately. Monica liked the integrated nature of the new state curriculum and was enthusiastic about teaching with it.

Mandy

Mandy was a senior in the mathematics education program. She changed her career from accounting to teaching mathematics; therefore, she did not have any previous teaching or tutoring experiences. She rarely participated in whole class discussions even though she shared her ideas in the small group discussions. During her field experiences she observed a sixth-grade class, an eighth-grade class and an Algebra 2 class. She also taught a lesson in the eighth-grade class.

Because teaching mathematics was Mandy’s second career, she thought that she needed to improve her knowledge in each aspect of pedagogical content knowledge. She noted that the field experiences raised her awareness about several issues of teaching mathematics. She realized that a teacher should not only know his or her content thoroughly but also know how to teach a certain topic to a particular group of students. She stated that she needed to refresh her memory about mathematical facts and concepts and the relationships between them. She noted that she had difficulty understanding algebra because it entails memorization of rules and formulas. However, she liked geometry because she was a visual learner and she could visualize and understand geometric concepts easily. Therefore, she said she would prefer to teach a geometry course rather than an algebra course. Mandy also wanted to improve her ability to address students’ difficulties and misconceptions and choose appropriate instructional materials that would enhance students’ understanding.

Having weak conceptual understanding of mathematics hampered the development of Mandy’s pedagogical content knowledge. She failed to remember some mathematical concepts and facts, and therefore she could not generate a plan for teaching them. She mostly attempted to address students’ difficulties by telling them the procedures or facts without justifying the reasoning behind them. When she had a better command of a topic, she was able to generate representations or real-life examples to explain it. Although she thought that her field experiences were beneficial, she thought that she needed more practice in the field to improve her pedagogical skills. Furthermore, her repertoire of examples of students' possible difficulties and misconceptions was limited to those noticed during her field experiences. Because she did not observe a class where the new state curriculum was being implemented, she did not feel ready to teach with the new curriculum. By the end of the semester, there was not much improvement in Mandy's pedagogical content knowledge.

Henry

Henry was a graduate student in the mathematics education program. He was an active participant in the small group and whole class discussions. He was completing an internship in which he spent half a day every day in a high school for the entire year. He was observing and helping students in Mathematics 1 and Geometry classes. He taught several times when his mentor teacher was absent. However, he implemented his mentor teacher's lesson plans and tried to mimic his style of teaching instead of developing his own plans. Initially, Henry attended both the methods course and field experience courses; however, after 8 weeks he dropped the field experience course.

Henry thought that the effectiveness of a lesson depended on a teacher's classroom management skills because the teacher could not achieve the goals for the lesson nor could the students benefit from the instruction without an appropriate teaching environment created by the teacher. He stated that he needed to improve his skills of presenting ideas to enable students to understand them clearly and make relevant connections with other ideas. Also, he wanted to find various ways of motivating students to learn mathematics.

Observing a single teacher and the same courses and students throughout the semester had negative and positive effects on the development of Henry's pedagogical content knowledge. Although spending a longer period of time in a particular class helped him to learn more about students and classroom routines, his gains from his field experience were limited to his mentor teacher's view of teaching mathematics in the context of two different classes. He did not have the opportunity to observe other teachers or other grade levels that would help him to evaluate different teaching practices and develop his own philosophy of teaching. Furthermore, he was deprived of an opportunity to improve his skills of writing lesson plans and developing teaching activities because he used his mentor teacher's lesson plans when he taught. On the other hand, having extended experiences in Mathematics 1 and Geometry classes helped him remember the basics of the subject-matter discussed in those courses as well as learn about students' possible difficulties and misconceptions with the context. Therefore, by the end of the semester some aspects of his pedagogical content knowledge were developed, while others were still weak.

Harris

Harris was a senior in the mathematics education program. He had some tutoring experiences in algebra, precalculus, and statistics, but he was not tutoring when this study was conducted. He was an active participant in the small group and whole class discussions. He observed a sixth-grade and an eighth-grade class in the middle school and also Mathematics 1, Advanced Mathematics 1, Algebra 2, and AP Calculus courses in the high school. Although he wanted to teach a lesson during the last field experience, his mentor teacher did not give him the opportunity to do so.

Harris wanted to teach in a middle school rather than in a high school because he thought that younger students were more easily motivated to learn mathematics when the teacher creates a learning environment where students have fun with mathematics. He liked to use discovery and competition problems to motivate students and increase their participation in the class. In addition, he preferred using real-life examples to motivate students to learn mathematical concepts. He thought that middle school students would be motivated to pay attention in class when teachers use technology to teach certain mathematical concepts. Furthermore, he wanted to establish good relationships with his students but still be seen as an authority figure in the classroom.

Throughout the semester some aspects of Harris's pedagogical content knowledge developed. He noted that the methods course and the field experiences contributed to his knowledge of pedagogy and knowledge of learners, in particular. He indicated that his knowledge of subject-matter and knowledge of curriculum were based solely on what he knew from the past. He stated that being in the field contributed to his repertoire of teaching strategies and examples of students' difficulties and misconceptions in mathematics. In fact, Harris began to differentiate what teaching strategies and instructional materials were more appropriate when teaching a particular topic to a particular group of students by the end of the semester. For instance, he noted that in the student-centered classroom where he was placed for a field experience, the students seemed to understand the mathematics better because the teacher encouraged them to explain and justify their reasoning for their solutions. However, in another classroom where he was placed, the teacher was not aware of what the students were struggling with understanding and simply assigned more problems for practice. Although Harris did not know the specific learning goals for each grade level, he was aware that he could use visual and concrete aids such as interactive white boards and manipulatives when teaching particular concepts. Harris's knowledge of subject-matter had an impact on his way of teaching a particular topic or addressing students' difficulties and misconceptions. When he knew the subject-matter thoroughly, he was able to make better connections with other topics and use more appropriate representations and examples than when he did not.

Knowledge of Pedagogy

Knowledge of pedagogy encompasses knowledge of students, planning instruction, teaching strategies, and assessment (Borko & Putnam, 1996). Teachers with strong pedagogical knowledge establish a well-structured learning environment and sustain effective teaching practices to promote students' engagement and understanding. Having a rich repertoire of teaching strategies enables teachers to meet the needs of different types of students by choosing appropriate examples, tasks, and representations to build on students' existing knowledge and facilitate their understanding. The findings of this study supported the fact that a teacher's ability to teach for understanding is based not only on his or her pedagogical knowledge but also his or her content knowledge (Ball, 1988; Fennema & Franke, 1992). Knowing the conceptual foundations of topics and the relationships between the concepts allow a teacher to develop teaching activities that enhance students' understanding of the subject-matter and enable students to make such connections for themselves.

Repertoire of Teaching Strategies

Having a deeper understanding of a particular topic enabled the preservice teachers to justify the reasoning behind mathematical procedures and facts by using visual or concrete representations or by making connections with other concepts. When they lacked a deep understanding, they simply explained how to carry out the procedures or apply a mathematical fact to the given problem. In the first interview, I asked the preservice teachers how they could help a student who was confused about getting $2 = 0$ as the solution of a system of linear equations, namely $2x - y = -1$ and $2y = 4x$ (see Figure 7).

Interview 1: Solving systems of linear equations

Assume that one of your students got confused when he or she found $2 = 0$ as the result of the solution of a system of linear equations. How do you explain to him or her the meaning of this result?

Sample student work:

$$\left.\begin{array}{l} 2x - y = -1 \longrightarrow 2x + 1 = y \\ 2y = 4x \longrightarrow 2y = 4x \end{array}\right\} \longrightarrow 2\cdot(2x+1) = 4x \longrightarrow 4x + 2 = 4x \longrightarrow \begin{array}{l} 2 = 4x - 4x \\ 2 = 0 \end{array}$$

Figure 7. The solving systems of linear equations task.

Henry and Mandy were unable to recognize that the solution 2 = 0 meant that there was no solution of the system or that the lines did not have a point of intersection. Henry thought that "it means you divided by zero or did some kind of illegal maneuver." He suggested writing the equations in the slope-intercept form to find the wrong step, but he did not explain further how it would help him to detect the error. Likewise, Mandy said "Whenever you get something like $2 = 0$ or $7 = 3$, somewhere along the line here you didn't follow the mathematical rule." She rewrote the second equation as $y = 2x$ but did not continue on working this question. Mandy failed to realize that the lines have the same slope and are therefore parallel, even though she wrote the equations of the lines in slope-intercept form. It is unclear whether she did not know that the slopes of lines provide information about the relationship between (i.e., parallel lines have the same slope) or whether she was simply unable to recall and apply this knowledge at the time of the interview. However, neither preservice teacher was able to reason about the task by thinking about what a solution to a system of linear equations represents (a point of intersection of the lines). Neither one suggested using visual aids such as graphs to investigate the given case and help students understand the context better; rather these participants said they would explain the procedural steps for solving the system of equations to students.

In contrast, the other participants said they would graph the lines to show students that they would not intersect. Linda noted that getting such an answer would indicate that there is no *x* value that satisfies both equations for a specific *y* value. Then she said, “Graphing it would be the easiest way because…if you give them a picture they can understand a lot better.” Linda said she would graph the equations to support her explanations and foster students’ understanding.

Laura stated that she would ask the student to check the calculations first. If the student got the same answer, then she would tell her that “this *x* in the first equation is probably not equal to this *x* in the second equation.” Then, she would graph both equations to show that the graphs would not intersect. She suggested using graph paper or a graphing calculator to sketch the graphs. She would also talk about parallel lines because “when lines do not intersect that means they have the same slope and further they are parallel.” Thus, her reason for graphing the equations was twofold: to address the student’s difficulty in understanding systems of linear functions and to make connections with other concepts such as parallelism and slope. Although Laura’s inference about nonintersecting lines was not valid for 3-dimensional space, it is valid for the given context.

Harris also said he would suggest checking the answer for accuracy and then he would talk about what it means to get no solution as the result of systems of linear equations. He would relate that discussion to the idea of independent lines, and then he would graph the lines to show that getting $2 = 0$ means that there is no solution and the lines are independent, that is, they are not intersecting. It was evident that he would graph the lines to support his explanations and help students understand the given case better.

Monica said she would prefer to talk about all possible cases of the solution of systems of linear equations. She would rewrite the given equations in the slope-intercept form and then graph them to show that the graphs are not intersecting. Then she would give examples of other two cases and graph them to show how the solution of the system relates to the graphs of the lines on the coordinate plane. It seemed that Monica's goal was to put this particular example in a larger context by providing examples of each case: A unique solution means the lines intersect, no solution means the lines are parallel, and infinitely many solutions means the lines coincide. By approaching the problem in this manner, Monica was trying to help the student make sense of systems of linear equations more generally rather than just in the given case.

In the first interview I also asked the preservice teachers how they could help a student who was having difficulty in multiplying binomials. Most of them said they would explain the procedure for using the "FOIL method" to multiply binomials. FOIL is a mnemonic used for multiplying the terms of two binomials in an order such that first terms, outer terms, inner terms, and last terms are multiplied and then simplified to find the result of the multiplication. The preservice teachers did not attempt to justify the reasoning behind the procedure, but some of them indicated that they were applying the distributive law when multiplying binomials. They assumed that applying the distributive law after separating the terms would help students understand the multiplication of the binomials. However, the students might not understand the distributive law and just try to memorize the procedure. The preservice teachers failed to mention several other approaches that were more conceptual. For instance, a teacher could work with small numbers to show how the distributive law works. For instance, one could create a simple word problem to show that $3 \cdot 7 = 3 \cdot (2+5) = 3 \cdot 2 + 3 \cdot 5$. Similarly, it is possible to use an area model to explain the multiplication of binomials in the form of $ax+b$. Given two binomials $ax+b$ and $cx+d$, draw a rectangle having these binomials as the dimensions and then construct four small rectangles with dimensions $(ax)\times(cx)$, $(ax)\times d$, $(cx)\times b$, and $b \times d$. The sum of the areas of all of the rectangles gives the area of the original rectangle, which is a visual illustration of the multiplication of binomials. Also, using algebra tiles would allow students to find the area of a rectangle as the sum of partial areas in a manner similar to the area model just described.

In another task, I asked the preservice teachers how to help a student who simplified a rational expression inappropriately by using “canceling” as shown in Figure 8. Most of the preservice teachers started by saying they would explain the procedure of simplifying rational expressions.

Interview 1: Simplifying rational expressions

Look at the student work given below. How can you explain to the student that his or her solution is incorrect?

$$\frac{2x^3y^2 - 6xy}{3xy^2 - x^3y^3} = \frac{2x^3\cancel{y}^2 - 6\boxed{x}y}{3\boxed{x}\cancel{y}^2 - x^3y^3} = \frac{2-6y}{3-y^3}$$

Figure 8. The simplifying rational expressions task.

Mandy and Henry were unsure how to clarify the student’s misconception. Mandy said that she would tell the student that the numerator and denominator are a unit, and therefore she cannot randomly cancel out the terms. She stated that the rules for multiplication of exponents are different from the rules for addition; however, she did not give examples of such rules or explicitly relate them to this task. She suggested using the idea of a complex conjugate to get rid of the denominator, but then she realized that she could not use a complex conjugate in the context of real numbers. Although she was aware of that the student's solution was incorrect, she could not recognize that the numerator and denominator should be written in factored form before simplifying the terms. Hence, she failed to generate an effective way to approach the student's misconception and help her to understand how to simplify rational expressions.

Similarly, Henry said he would tell the student that a term cannot be simplified when it is associated with another term through addition or subtraction. However, he did not explain what he would do to clarify such misconception. Instead, he said that explaining why the solution is incorrect is harder than solving the problem.

In contrast, some participants mentioned that they would show the student how to factor the given expressions and then simplify them. Laura, Linda, and Monica said they would explain how to factor the numerator and denominator and then cancel out common terms. Laura would tell the student that "when we want to cancel out we need to remember that we are taking away every term in our numerator and every term in our denominator." Then she would show how to factor the numerator and denominator and then simplify them. She also said, "Being able to explain is tricky." She noted that she would emphasize the idea of factoring and try to make sure that the student understood it. Similarly Linda would show how to factor the terms step by step, first working on the x terms and then the y terms. She said that she did not know whether there is an easier way to explain it. Monica said she would talk about the division and multiplication rules of exponents. However, she did not explain how these rules would be helpful.

Harris also would explain how to factor the numerator and the denominator. However, first, he would try to convince the student that his or her reasoning was invalid by rewriting the given expression as the sum of two fractions, that is, $\frac{a}{c+d}+\frac{b}{c+d}$ and then applying the student's method to the fractions such that for each fraction, he would simplify the single term in the numerator with one of the term in the denominator. Thus, he would show that the answer obtained in this way was different from the student's answer in the example. While Harris's explanation would help the student realize her mistake, it would not necessarily help her to understand *why* she needs to factor the expressions.

Although Laura, Linda, and Monica explained how to factor, this might not be convincing for the student because it does not include a rationale for why it is necessary to find common terms in the numerator and denominator and then cancel them. They did not clarify the reasoning behind writing the numerator and the denominator in factored form rather than leaving them as they are, that is in the form of $\frac{a+b}{c+d}$. Again, there are a number of more conceptual approaches that the preservice teachers could have mentioned but did not. For instance, using particular numerical examples would show that the student's reasoning was invalid. For instance, if the 2s are canceled in $\frac{2+4}{5-2}$, the answer is $\frac{4}{5}$, but the correct answer is 2. The order of operations could be used to explain this task as well, noting that when the numerator or denominator of a fraction involves more than one term, they are assumed to be inside parentheses. Because the division operation does not precede parentheses, simplification cannot be applied randomly over the single terms. Furthermore, the idea of equivalent fractions and simplification could be applied in this situation. For instance, showing that $\frac{6}{8}=\frac{2\cdot 3}{2\cdot 4}=\frac{3}{4}$ and then extending the analogy to examples with variables would show how these concepts are related to the given problem.

The preservice teachers were inclined to tell the procedures and facts when they were asked to explain a particular example to the students. When they knew the subject-matter in depth, they were able to explain why the procedure or fact works, but they rarely attempted to use visual aids or manipulatives to facilitate the students' understanding unless the context lent itself to the use of representations, as in the case of functions.

Choices of Teaching Activities, Tasks and Examples

The preservice teachers' choices of teaching activities, tasks, and examples depended on their views of teaching and learning mathematics. Collectively, the preservice teachers viewed mathematics as the set of rules, procedures, and facts. When asked to teach a particular topic, they mostly stated mathematical facts and described how to carry out the procedures or apply a rule. Given a set of examples and asked to place them in the order in which they would teach them, some of the preservice teachers ordered them to illuminate the general form or ideas of the topic such as the slope-intercept form of linear equations. Furthermore, the reasonableness and thoroughness of their explanations depended on the strength of their subject-matter knowledge. Their decisions about what examples to use to teach a given topic revealed that they did not necessarily pay attention to how the examples would facilitate students' understanding; rather, they looked at their surface features such as the number of terms involved in a given equation or number of steps to solve that equation.

During the second interview I asked preservice teachers in which order they would use the given examples to introduce how to graph linear functions (see Figure 9). Linda, Mandy and Harris put the examples in the same order, starting with $y = x + 5$ because it is in slope-intercept form followed by $2x + 3y = 6$ and $3x - 8y + 12 = 0$. They said they would rewrite the equations in slope-intercept form because they wanted their students to convert the given equations into that form before graphing them. Then they would show how to graph $y = 5$ because it is a special case; that is, it has zero slope.

Interview 2: Graphs of linear equations

Assume that you will introduce how to graph linear functions. Here are some examples of linear equations. In which order you would like to use these equations? Tell me your reasoning.

$2x+3y=6$ $\quad y=5$ $\quad y=x+5$ $\quad 3x-8y+12=0$

Figure 9. The graphs of linear equations task.

In contrast, Monica and Henry said they would show that special case at the beginning and then continue with examples having slope and *y*-intercept. Monica said the following:

> It would be interesting to see the difference between $y=5$ and $y=x+5$ because with $y=x+5$ you have the slope of 1 and *y*-intercept of 5, but with $y=5$ just have your line. So, I guess it would be easiest to start with $y=5$ because that becomes slope-intercept of $y=x+5$, you know getting students to see that they're similar in that respect.

Although Monica's second statement is ambiguous, it seems that she compared two equations in terms of their slopes and *y*-intercepts. Both lines have the same *y*-intercept, but they have different slopes. The slope of the line $y=x+5$ is 1, but the slope of $y=5$ is 0. However, students may fail to make connections between the equation of the line and its graph. They might visualize that horizontal lines have zero slope, but they might not recognize that $y=5$ represents a horizontal line. Henry said he would introduce $y=5$ as the collection of all points such that the "*y* intercept is 5." Then, he would introduce $y=x+5$ in the same manner and talk about dependent and independent variables. In that respect, Henry's approach to the problem was different from the other preservice teachers' approaches. He interpreted the equations as the collection of points satisfying certain relations to foster students' understanding of linear equations. However, some students may still have difficulty in conceptualizing that $y=5$ is a horizontal line because they may misconstrue that the equation of a line should involve two variables, namely x and y.

Laura said she would first introduce $y = x + 5$ to show the slope-intercept form and then continue with $y = 5$ as a special case where there is no slope. However, she did not explain how she would ensure that students would understand the connections between two cases.

Although there is no correct way of ordering the given linear equations, the preservice teachers indicated that they would explain the slope-intercept form of linear equations and encourage their students to rewrite the linear equations given in other forms (e.g., standard form) in the slope-intercept form. Therefore, except Monica and Henry, the preservice teachers preferred to start with $y = x + 5$ to show how to graph a linear equation given in slope-intercept form. Although starting with $y = 5$ was not wrong, Monica and Henry needed to plan for potential student difficulties with understanding what $y = 5$ represents. Based on their explanations I inferred that that they were not aware of these difficulties. The other preservice teachers did not clarify how they would ensure that students would understand the graph of $y = 5$, either. They only indicated that they would not start with that example because it is a special case.

During the third interview I gave four examples of rational equations and again asked preservice teachers in which order they would use them with students (see Figure 10). Laura, Harris, and Monica preferred to start with $\frac{3}{x+1} = \frac{x}{2}$ because it seemed easier than the others. Laura said that "it would not be easiest but the simplest for students understand." Similarly, Harris noted that although solving the equation by cross-multiplication leads to a quadratic equation, students would assume that it is an easy question because each individual term looks simple. Monica stated that she would start with this example because students would need to use the distributive property only once in contrast to the other examples. These preservice teachers analyzed the examples based on how they would be perceived by students rather than their mathematical demands.

Interview 3: Solving rational equations

Assume that you will introduce solving rational equations. Here are some examples of rational equations. In which order would you like to use these equations? Tell me your reasoning.

$$\frac{2}{x(x-2)} = \frac{1}{x-2} \qquad \frac{1}{x-4} = \frac{2}{3x+1} \qquad \frac{5}{x} + \frac{x}{x-1} = 1 \qquad \frac{3}{x+1} = \frac{x}{2}$$

Figure 10. The solving rational equations task.

Linda and Henry stated that they would start with $\frac{1}{x-4} = \frac{2}{3x+1}$ because it would yield a linear equation in the end. Linda stated that even though $\frac{3}{x+1} = \frac{x}{2}$ seemed straightforward at first glance, students would need to solve a quadratic equation to find the answer. In contrast, they could solve the chosen example easily because “it is almost like a review and just solving for simple *x*.” Linda and Henry made their decisions about ordering examples by looking at the mathematics involved in the equations. They preferred to start with the one that would yield a linear equation rather than a quadratic equation.

Mandy said she would first introduce $\frac{5}{x} + \frac{x}{x-1} = 1$ because “a lot of students would be comfortable with the idea of common denominator.” Indeed, students are likely to not only be familiar with the idea of finding a common denominator but also cross multiplication in order to solve the equation. Mandy may have been considering the order in which students learn particular topics in the curriculum and basing her choice of starting example on the fact that students learn about adding fractions earlier than they learn about proportions and cross multiplication. However, students might struggle with solving that particular example because it entails knowledge of distribution and solving quadratic equations in addition to knowledge of adding fractions. In fact, all other participants except Laura said that they would use that example as the last one because it was different from the others. They indicated that to solve that equation, students would need to find the common denominator, whereas others could be solved by cross-multiplying the terms. Laura used that equation as her third example because she did not want to present all of the examples that could be solved by cross multiplication together so as not to lead students to think that was the only way to start such problems.

I asked the preservice teachers how they could motivate students to learn trigonometry. Some of them suggested using the unit circle to show trigonometric ratios, and others said they would use real-life applications. For instance, Laura said she would focus on finding trigonometric ratios and the relationships between them by using the unit circle. Linda said she would show how to find the trigonometric ratios of special angles by using an isosceles right triangle and an equilateral triangle after introducing the unit circle. Laura and Linda put their emphasis on finding trigonometric ratios on the unit circle because they thought that students would not need to memorize the ratios; rather, they could derive these ratios by using the unit circle. Monica said she would show students how to find the trigonometric ratios on a right triangle.

Mandy, Harris, and Henry all said they would start teaching trigonometry with real-life applications. Mandy noted that trigonometry is the study of angles for her. Therefore, she would give examples of careers where trigonometry is used, such as surveyors and architects using trigonometry to determine angle of elevation. Harris would also try to give various examples that would attract students' attention such as discussing why the length of the shadow of an object changes during a day or how the wavelengths of sound work. Henry said that he would use an example from physics because it is the subject in which he used trigonometry the most. He gave an example of a ball rolling down a ramp and splitting the components of the forces on the ball to show how to use trigonometry to determine the horizontal and vertical forces. It was evident that preservice teachers' experiences with the subject-matter as learners would influence their ideas about how to teach it.

In summary, the preservice teachers' views of mathematics and their experiences with the subject-matter were influential on their decisions about how to teach that subject-matter. They tended to emphasize teaching mathematical facts and algorithms and chose examples that would serve that purpose. Some preservice teachers looked at the surface features of examples rather than the mathematical thinking elicited by those examples. However, some preservice teachers tried to enrich their teaching practices with real-life examples.

Preservice Teachers' Perceptions about the Development of Knowledge of Pedagogy

The preservice teachers thought that the methods course and field experiences contributed to the development of their knowledge of pedagogy because they discussed some pedagogical issues in the courses and they observed different teachers in the field. At the beginning of the semester they indicated that they expected to learn how to write lesson plans and discuss different teaching strategies. By the end of the semester, they noted that they learned the basics of planning instruction, but they still needed to improve their ability to write effective lesson plans.

Level of knowledge. At the beginning of the semester the preservice teachers filled out a questionnaire that included questions about their knowledge levels as well as content-specific questions. The main purpose of the questionnaire was to select the participants. However, the first nine items were given to the participants again at the end of the semester to detect changes in their perceptions of knowledge levels. Items 2 and 5 were aligned to knowledge of pedagogy. Item 2 says "At the end of my degree program I will have taken enough courses about teaching mathematics to be an effective mathematics teacher in grades 6-12" and item 5 says "I have a sufficient repertoire of strategies for teaching mathematics." The ninth item was a 4-point Likert-type question asking them to rank their knowledge level for each aspect of pedagogical content knowledge. The scores of their perceived level of knowledge of pedagogy before and after the methods course and field experiences are presented in Table 2.

Table 2

The Preservice Teachers' Perceived Level of Knowledge of Pedagogy

	Before		After		Before	After
Participants						
	Item 2	Item 5	Item 2	Item 5	Item 9	Item 9
Laura	2	2	2	2	1	2
Linda	2	1	3	3	2	3
Monica	2	2	2	3	2	3
Mandy	2	2	2	2	3	2
Henry	3	3	3	2	2	2
Harris	3	2	3	2	3	3

Note. Scale for Item 2 and Item 5. 1: Disagree, 2: Somewhat Agree, 3: Agree
Scale for Item 9. 1: Not Adequate, 2: Adequate, 3: Competent, 4: Very Good

Monica, Laura, and Linda thought that their knowledge of pedagogy improved during the semester, and Harris and Henry noted no change (Item 9). Mandy rated her level of knowledge as decreasing, which perhaps indicates that she became more aware of the knowledge base she needed and had overestimated her knowledge level at the beginning of the semester (Item 9).

I also interviewed each preservice teacher about his or her answers to the questionnaire items at the beginning and end of the semester to provide some context for the numbers in the table above. At the beginning of the semester, Laura stated that she wanted to learn how to write lesson plans. Although she was given opportunity to write lesson plans in the methods course she had a limited number of opportunities to implement her plans in classrooms with students. Therefore, she did not think there was much improvement in her knowledge of pedagogy compared with other knowledge domains even though it improved with respect to the beginning of the semester. In contrast, Linda thought that the field experiences and methods course contributed to growth in her knowledge of pedagogy because she realized the importance of planning a lesson and choosing appropriate examples when introducing a new concept.

Although Harris and Henry indicated that their knowledge of pedagogy improved throughout the semester, their overall score for knowledge of pedagogy remained constant. Harris noted that he had better understanding of what might be difficult for students. Henry thought that he sometimes failed to transfer what he learned in the course to the secondary classroom because he was concerned mostly about classroom management and keeping students engaged. Mandy lowered her self-assessment of her knowledge of pedagogy even though she thought that methods course and field experiences contributed to the development of her knowledge of pedagogy. She realized that she would need to differentiate her teaching strategies to meet the needs of her students. For example, she noted that when planning her lesson she would need to think of how to involve kinesthetic, visual, or shy students in classroom activities.

The contribution of the methods course topics. At the end of the semester I gave the preservice teachers a list of major topics covered in the methods and field experience courses, and I asked them to evaluate how each of these topics contributed to each aspect of their pedagogical content knowledge. They indicated that they benefited from all of them at certain levels, and all participants agreed on particular course topics that contributed to the development of each specific knowledge domain. The preservice teachers' perceptions about which course topics contributed to the improvement of their knowledge of pedagogy are presented in Table 3.

All the preservice teachers agreed that discussion of motivation, manipulatives, planning instruction, and the cognitive demand of tasks contributed to their knowledge of pedagogy. The preservice teachers learned that they need to plan for motivating their students by using different teaching strategies or challenging problems to attract students' attention and increase their engagement. Harris noted that because he was intrinsically motivated to learn mathematics, he did not consider that motivating students to learn mathematics is such "a big issue." Therefore, he was pleased to learn about different ideas to motivate students in the methods course.

The preservice teachers thought that manipulatives facilitate students' understanding of mathematics and motivate them to engage in learning the content. Henry said he would use manipulatives to enrich his instruction as well as meet the needs of different types of learners. Linda stated that when planning her lessons she would consider what manipulatives would be useful for teaching that particular topic and likely to facilitate students' understanding.

Table 3

The Contribution of the Course Topics to the Development of Knowledge of Pedagogy

Topic	**Participants**					
	Laura	Linda	Monica	Mandy	Henry	Harris
Problem of the day	x	x	x			
Learning theories		x	x	x	x	
Curriculum and textbooks		x	x	x		
Motivation	x	x	x	x	x	x
Promoting communication	x	x	x			x
Manipulatives	x	x	x	x	x	x
Planning instruction	x	x	x	x	x	x
Microteaching	x	x	x		x	
Effective questioning	x	x	x		x	x
Cognitive demand of tasks	x	x	x	x	x	x
Classroom management	x	x	x	x		
Assessment			x	x		

The preservice teachers acknowledged that planning effective instruction entails strong knowledge of pedagogy. They indicated that preparing a lesson plan for microteaching after the discussion of planning instruction was a good practice for them because they needed to pay attention to several issues including activities, examples, materials, and assessment when writing a lesson plan. In addition, Mandy stressed that she would plan for using different teaching strategies for different types of students.

Moreover, the preservice teachers stressed that teachers should be critical when preparing tasks for students. Henry noted that the tasks should not be too easy or too hard but should enable students to generate solutions depending on what they have learned. He stated that when the task is too hard, students are likely to give up before finding the answer. Laura also pointed out that hard questions decrease students' motivation. For example, if difficult problems are placed in the end of a test, students may give up answering the rest of the test after answering easy ones given at the beginning. Linda noted that when preparing a task she needed to ensure that students possess the necessary knowledge to solve it. Otherwise, she may need to give some additional information about the task and lower its cognitive demand. Similarly, Monica would prepare tasks that are appropriate for her students and try to maintain the cognitive demand of tasks during instruction.

Additionally, the preservice teachers stressed the importance of planning for effective questioning in the classroom. Monica said that it is "a way of informal assessment…a good way to get students involved and keep them engaged." She also noted that when planning lessons, she needed to write several questions that she would ask in the class. Likewise, Linda stated that she would write the questions that she would ask in the class in her lesson plan because she thought that creating an interactive class environment would enable her to address students' difficulties more effectively.

The preservice teachers thought that most of the course practices contributed to the development of their knowledge of pedagogy. They had also opportunities to observe some of those issues in the field and make connections between what they discussed in the methods course and the teachers' actions and also distinguish between effective and less effective teaching practices.

Contribution of field experiences. The preservice teachers went to the field four times throughout the semester. The first three field experiences were scheduled for 3 days each, and the last one was a 2-week experience during which the preservice teachers were expected to teach a lesson. Although they were given specific themes that they should be looking for during the experiences, they had the opportunity to evaluate overall classroom practices from a prospective teacher's point of view. For instance, Monica noted that being in the classroom enabled her "to see how different teachers wrote up lesson plans and how different teachers implement them." I asked the preservice teachers to reflect specifically on teachers' practices and students. I also asked them what their plan would be for the next lesson if they were to teach the classes they observed the following day. Their responses suggested that they would apply some ideas they discussed in the methods course.

The preservice teachers noted that teachers followed certain routines in each lesson. At the beginning of the lesson the teachers reviewed the homework assignment and solved some of the problems that students had failed to understand. Then they presented the main activity and asked students to work on problems either individually or as a group. They checked students' work by circulating around the classroom. Finally, they explained the homework assignment and closed the lesson.

When I asked the preservice teachers what they would have done in the next lesson if they were teaching it, they told me that they would start either with checking the homework assignment or clarifying some issues from the previous lesson. Laura and Henry would check homework problems and solve some of them. Harris said that he did not want to spend more time on solving homework problems; therefore, he would limit the number of the problems for each lesson and solve the ones that the majority failed to answer. Mandy would review the quiz given to the students in a particular lesson because no one was able to finish it during the time allotted.

Next, the preservice teachers explained what they would do in the lesson as the main activity. The preservice teachers focused on motivating students to engage in the activity and planning for addressing their difficulties. Harris stated that in one of the classes he observed the students were struggling with solving systems of equations. Therefore, he decided to prepare three problems varying in difficulty level and assign each problem to the groups that he formed previously. He would ask them to explain their solutions step by step in front of the class and encourage the rest of the class give them feedback. Thus, he “would let the students to teach each other.” Linda observed a teacher-centered class during her field experience. Hence, she would aim to “get more student involvement” instead of doing everything herself. She would lead a discussion about the context and try to promote students’ understanding through effective questioning.

It seemed that preservice teachers’ major concerns were keeping students motivated and fostering their understanding. To address these concerns, they would have students work in groups and learn from each other. They would also use effective questioning techniques to increase students’ participation and but also to assess their understanding. Indeed, their suggestions revealed that they tried to apply what they learned in the methods course. The topics of cooperative learning, effective questioning, and motivation had been discussed in the methods course by the time this interview was conducted. Furthermore, their explanations were compatible with their thoughts about what course practices contributed to the development of their knowledge of pedagogy. As presented in previous section, they said that they benefited from the discussion of these issues in the methods course.

Summary

Knowledge of pedagogy can be conceived of as a mixture of various knowledge, skills, and dispositions that enable teachers to make subject-matter understandable for all students. To achieve desirable learning outcomes teachers should create a learning environment so that different types of students can benefit from the instruction. The students should be given opportunities to make sense of the subject-matter through representations, manipulatives, or real-life examples rather than merely memorization of the rules or procedures. The tasks and examples should be appropriate for their students and allow for building on their prior knowledge. There is an extensive literature base about preservice teachers' knowledge of pedagogy, and the findings of my study are generally consistent with the findings of other studies.

Preservice teachers lack knowledge of instructional strategies and representations (Ball, 1990a; Grossman, 1990). They perceive teaching as telling the rules, showing students how to use them, and then having students practice them (Kinach, 2002). Furthermore, preservice teachers' pedagogical decisions are influenced by their subject-matter knowledge (Ball, 1988; Borko & Putnam, 1996; Fennema & Franke, 1992; Foss & Kleinsasser, 1996). As Brophy (1991) noted

> Where their [teachers'] knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond more fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text for the content, deemphasize interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static factual knowledge. (p.352)

Additionally, lacking conceptual knowledge of mathematics hinders preservice teachers in making connections between mathematical concepts or justifying the reasoning behind the rules and algorithms. Therefore, they are inclined to explain how the procedures and rules work and choose problems for students to practice applying the rules and procedures.

Ball (1988, 1991) also pointed out how preservice teachers' subject-matter knowledge affects their ways of teaching. She stated that when preservice teachers' conceptual knowledge of mathematics was strong, they put more emphasis on explaining why procedures work and how mathematical concepts are related. She also noted that preservice teachers mostly viewed mathematics as an abstract set of symbols and rules to be memorized. Similarly, Foss and Kleinsasser (1996) found that preservice teachers manifested their conceptions of mathematics in the lessons they taught in the field. Preservice teachers viewed mathematics as numbers, arithmetic operations, or computational skills, and their lesson plans were mostly oriented toward fostering students' procedural skills.

Moreover, Fennema and Franke (1992) identified knowledge of mathematical representations as a knowledge domain for teachers because teachers need to know how to translate mathematics into representations that enable students to understand it. Teachers need to know how to interpret and represent mathematical topics to facilitate learning with understanding. They noted that use of real-life situations, concrete representations, manipulatives, and pictorial representations help students develop better understanding of abstract ideas. However, Ball (1990a) noted that preservice teachers had difficulty in generating appropriate representations for solving problems even though they could perform the operations correctly.

The findings of my study were consistent with the findings of other studies regarding the relationship between teachers' pedagogical decisions and subject-matter knowledge. The preservice teachers' choices of teaching strategies and examples were influenced by their content knowledge and views about teaching and learning mathematics. They thought that mathematics is composed of rules, procedures and facts to be memorized, and this view had an impact on their decisions about how to teach particular topics. They emphasized teaching how to carry out procedures carefully or apply a rule in the given context. However, a few of them attempted to justify their explanations through representations. In some cases, they were unable to generate different ways of teaching a particular topic because they did not know much about it. Decisions on what examples to use when introducing a topic were not necessarily based on the difficulty level of the examples or how well it was representative of the context. Some of the preservice teachers made their decisions by looking at the surface features of the examples.

However, the preservice teachers' reflections on the course practices and field experiences revealed that they tried to plan for fostering students' understanding. They said they would use various instructional materials to help students understand the subject-matter better, and they would try to keep them engaged through activities done individually or as a group. They would be sensitive to the cognitive demand of the tasks, would ensure that the tasks were appropriate for their students, and would not lower the cognitive demand of tasks during the instruction. In addition, having observed different teachers and students in the field enabled them to learn about different classroom practices.

In summary, the preservice teachers' knowledge of pedagogy emerged from their knowledge of subject-matter and their experiences. Their view of teaching mathematics was echoed in their decisions about the examples that they would use when teaching a particular topic. They paid attention to either the mathematical thinking elicited or how well it would fit the generic form or how students would perceive it. Their approaches to students' errors and difficulties provided insight about the depth of their subject-matter as well as the depth of their repertoire of teaching strategies. They tended to tell how to carry out the procedure or apply the fact without justifying their reasoning. However, they also indicated that they would use questioning techniques to understand the flaws in students' thinking and address their misconceptions effectively. When asked to think of or write a lesson plan on a particular concept or fact, they were inclined to introduce it through questioning and then assign group work that allow students to explore more about the issue. Therefore, I conclude that the methods course and field experiences raised the preservice teachers' awareness about effective teaching practices; however, they need to improve their content knowledge and also internalize those experiences through more practice in the field.

Knowledge of Learners

Knowledge of learners is generally defined as knowing about the characteristics of a certain group of students and establishing a classroom environment and planning instruction accordingly to meet the needs of these students (Fennema & Franke, 1992). Because the preservice teachers in this study did not have intimate knowledge of a group of learners, I assessed their knowledge of learners by looking at their ability to anticipate students' possible difficulties and misconceptions about the subject-matter and their ability to explain how to address them effectively. Hence, knowledge of learners entails knowing the subject-matter as well as effective teaching strategies.

Teachers not only need to be able to help students when mistakes arise but also need to craft their lesson plans to either avoid or deliberately elicit common student errors. Moreover, teachers need to be able to determine the source of students' difficulties and errors in order to correct them effectively. For instance, a student's difficulty in solving a geometry problem might not necessarily be due to not knowing the geometric concept but may be due to a lack of arithmetic or algebraic skills. Therefore, teachers need to know the subject-matter to analyze students' errors and misconceptions.

Identification of Source of Students' Difficulties and Errors

When given examples of students' errors and asked how to address them, the preservice teachers tended to repeat how to carry out the procedures or explain how to apply a rule or fact to solve the problem. In some cases, they would first ask students to explain their solutions to help students assess their own understanding and realize their mistakes. However, then, they would explain how to solve the problem procedurally.

Preservice teachers were unable to analyze the reasons behind students' errors or difficulties. They usually came up with a reason, which was apparent and procedural. However, they did not state how flaws in students' conceptual understanding would likely lead to failure in generating a correct solution. For example, when I asked them how they could help a student who was having difficulty in multiplying binomials, preservice teachers explained how to use the distributive law and assumed that students knew why the distributive law works. They thought that students had difficulty in multiplying binomials simply because they did not know how to distribute the terms correctly. They did not consider that students might know how to apply the distributive law but fail to multiply variables or negative integers correctly. For instance, students might think that $2x \cdot 5x = 10x$ or $-2(x-3) = -2x-6$. Laura and Henry did point out that students might struggle with multiplying variables and adding similar terms, but they did not explain how they would clarify those issues for the students.

In the case of simplifying rational expression (see Figure 8), the preservice teachers said they would explain to students how to factor the numerator and denominator before canceling out the common terms. They noted that the student failed to simplify the given expression because she did not know how to factor variable expressions. However, another reason underlying the error might be weakness of the student's knowledge of exponents and operations with them. Although Monica stated that she would review the properties of exponents, such as showing that $x^3 = x \cdot x \cdot x$ or $\frac{x^3}{x} = x^2$, she did not state explicitly how she would relate these properties to the idea of simplifying the terms or writing the expressions in factored form. As indicated in the pedagogical knowledge section, the preservice teachers were not able to clarify the reasoning behind simplifying terms before showing how to carry out the procedure.

During the second interview I showed preservice teachers student work where the student found the solution of the equation $2x^4 - 18x^2 = 0$ to be ± 3 by taking $18x^2$ to the other side of equation and then dividing both sides by $2x^2$ (see Figure 11). I asked them how they could explain that the solution is invalid.

Interview 2: Solving polynomial equations

Look at the student work given below. How can you convince your student that his/her answer is invalid?

$$2x^4 - 18x^2 = 0$$
$$2x^4 = 18x^2$$
$$\frac{2x^4}{2} = \frac{18x^2}{2}$$
$$x^4 = 9x^2$$
$$\frac{x^4}{x^2} = \frac{9x^2}{x^2}$$
$$x^2 = 9$$
$$x = \mp 3$$

Figure 11. The solving polynomial equations task.

With the exception of Henry, the preservice teachers were unable to recognize the student's error. They stated that they would tell the student that factoring is a better way to solve that equation because it will help you find all of the solutions, including zero. For instance, Monica said "you just have to remind them that there are other ways of solving the problem, and this is one way she didn't necessarily get every solution." It was evident that she did not notice the student's error and therefore did not recognize that her explanation would not help the student understand why her method was incorrect. Henry also said he would explain how to factor the given equation; however, he would first tell the student that when dividing with x^2 she needs to make sure that x is not zero. Thus, he was able to identify and clarify the student's confusion about why her method did not work. The preservice teachers' approaches to this problem revealed that they were unable to recognize the gap in students' understanding of solving polynomial equations. Instead, they merely focused on the procedural steps and suggested another method that they were sure would yield all solutions.

The preservice teachers commonly attributed students' errors and misconceptions to their inability to remember and correctly perform procedures. Therefore, they emphasized improving students' procedural skills. It seemed that the preservice teachers' perceptions of mathematics as well as their subject-matter knowledge had an impact on how they would approach problems. If they knew the concept in depth, then they were able to detect the flaws in students' understanding and address them effectively.

Preservice Teachers' Perceptions about the Development of Knowledge of Learners

The preservice teachers thought that the methods course and field experiences contributed to their knowledge of learners. At the beginning of the semester, they noted that they did not know much about how they could help students struggling with understanding certain topics.

They hoped to improve their repertoire of examples of students' possible difficulties and misconceptions during their field experiences. The preservice teachers identified two experiences in particular where they developed this repertoire. First, when solving problems of the day in the methods course they discussed possible student errors that might arise. Second, during the field experiences they had opportunities observe mathematical ideas that seemed to be difficult for students to grasp.

Level of knowledge. In the questionnaire item 4 and item 8 were aligned to knowledge of learners. Item 4 was "I know possible difficulties or misconceptions that students might have in mathematics in grades 6-12" and item 8 was "I know how to diagnose and eliminate students' mathematical difficulties and misconceptions." As shown in Table 4 the preservice teachers' conceptions about the level of their knowledge of learners changed during the semester.

Table 4

The Preservice Teachers' Perceived Level of Knowledge of Learners

	Before		After		Before	After
Participants						
	Item 4	Item 8	Item 4	Item 8	Item 9	Item 9
Laura	2	1	3	2	1	3
Linda	2	1	2	2	2	2
Monica	3	3	3	2	3	3
Mandy	3	2	1	2	3	3
Henry	2	2	2	3	3	3
Harris	3	2	3	2	4	3

Note. Scale for Item 4 and Item 8. 1: Disagree, 2: Somewhat Agree, 3: Agree
Scale for Item 9. 1: Not Adequate, 2: Adequate, 3: Competent, 4: Very Good

Laura’s rating showed the greatest growth of any participant (Item 9). She attributed her growth to class discussions about different types of learners but indicated that she still needed to improve her ability to understand what students are thinking. Harris stated that his knowledge of learners improved during the semester because he had better understanding of what might be difficult for students and where they might make mistakes when solving problems. However, he seemed to overestimate his knowledge of learners at the beginning of the semester because his rating at the end of the semester was lower than the one at the beginning, despite his statement in the interview that his knowledge improved. He perceived his knowledge of learners at the “competent” level at the end of the semester.

Although Linda noted that she learned more about students’ difficulties by being in the field, she ranked her knowledge of learners as “adequate” at the end of the semester because she could figure out where students mess up but she might not anticipate students’ possible difficulties or misconceptions about a topic beforehand. Henry noted that before interning in a class he had no idea about whether students would have problems. However, at the end of the semester he knew more about the sources of their misconceptions.

The contribution of the methods course topics. The preservice teachers valued the discussions of students’ difficulties and errors in the methods course; however, they thought that the field experiences contributed to their knowledge of learners more than the methods course. The course topics that they identified as contributing to their knowledge of learners are presented in Table 5.

Apparently, none of the preservice teachers thought that the discussions on the curriculum, promoting communication, and classroom management contributed to their knowledge of learners. However, they mostly agreed that the problems of the day, planning instruction, and assessment did raise their awareness about students' thinking.

Table 5

The Contribution of the Course Topics to the Development of Knowledge of Learners

Topic	**Participants**					
	Laura	Linda	Monica	Mandy	Henry	Harris
Problem of the day	x	x	x		x	x
Learning theories						x
Curriculum and textbooks						
Motivation		x				
Promoting communication						
Manipulatives			x	x		
Planning instruction	x	x			x	x
Microteaching				x		
Effective questioning		x	x			
Cognitive demand of tasks	x	x				
Classroom management						
Assessment	x	x				x

The preservice teachers noted that the problems of the day helped them realize where students make mistakes and what might be difficult for them to understand. However, Mandy thought that she would have benefited from the problems of the day more if a few examples of student work accompanied the problems so that she could generate better ideas about how she could eliminate those difficulties. Moreover, the preservice teachers indicated that when planning their instruction they need to foresee what might be difficult and confusing for students and plan for eliminating these difficulties. Linda stated that when preparing assessment tools and rubrics she would consider what is difficult or confusing for students and make sure that she has already clarified that issue before the test.

Furthermore, the preservice teachers thought that manipulatives could foster students' understanding. Monica stated that manipulatives could be used to show how algorithms work, such as addition of negative and positive integers. Similarly, Mandy would use manipulatives to eliminate students' misconceptions. Linda indicated that she would use effective questioning techniques to figure out the flaws in students' understanding. Then she would plan for addressing those deficiencies.

Contribution of field experiences. The preservice teachers stressed that the field experiences helped them improve their repertoire of examples of students' difficulties and misconceptions. Mandy stated that she observed a class where students struggled with the idea of the equations of horizontal and vertical lines and failed to understand what $y = 2$ represents. Laura mentioned that high school students had difficulty in solving real life problems involving factoring trinomials. Henry realized that some students failed to understand a new topic because they did not possess sufficient prior knowledge.

When asked to plan for teaching the next lesson for the class that they observed, the preservice teachers said they would open the lesson by addressing students' difficulties that they observed in the previous lessons. For instance, Laura noticed that in the seventh grade class the students had difficulty in understanding adding and subtracting integers. Therefore, she would use manipulatives such as colored coins to represent positive and negative integers to facilitate students’ understanding of the procedures. Similarly, Henry said he would use models of geometric solids to help students who were struggling with how to find the surface area and volume of 3-dimensional solids. In Linda’s class the students were struggling with adding and multiplying polynomials because they lacked knowledge of basic operations with exponents. Therefore, she would encourage the students to figure out the rules. She would show them how to derive the rules by asking their input in each step. Then, she would make a list of the rules for the students who still had difficulty remembering them and let them use the list as a guide when solving problems.

Observing different grade levels and mathematics topics helped the preservice teachers realize which topics were likely to be difficult for a particular group of students to grasp. Yet, they need to enrich their repertoire of how to clarify some issues for students and eliminate their difficulties. It was evident that they knew that using visual or concrete aids could facilitate students’ understanding, but they needed to practice with them in order to use them effectively. Furthermore, it seemed that they would try to make students realize their own errors through questioning and then they would emphasize the correct steps of algorithms.

Summary

The preservice teachers’ knowledge of learners was intertwined with their knowledge of subject-matter and pedagogy. Oftentimes they were unable to notice what conceptual knowledge the students were lacking that caused them to fail to solve the given problem. They assumed that students did not know the mathematical fact or rule that they need to apply to the given problem or they did not follow the procedure carefully. Therefore, they would eliminate students’ difficulties or misconceptions by explaining how to solve the problem procedurally.

The findings of other studies reported similar results to those reported here. For example, studies on teachers' knowledge of learners have shown that beginning teachers lack knowledge of students' mathematical thinking (Feiman-Nemser & Parker, 1990; Fennema & Franke, 1992). They do not know much about what problems students may encounter when learning a specific topic. Moreover, they do not have a rich repertoire of strategies for presenting the material in a way that facilitates students' understanding or for eliminating students' misconceptions effectively.

Even and Tirosh (1995) investigated teachers' knowledge of students and found that teachers were reluctant to make an attempt to understand the source of students' responses even though they needed this information in order to make appropriate instructional decisions to help the students learn. When students gave incorrect answers they tended to explain the correct answer rather than asking the students how they found that answer. Thus, they missed an opportunity to detect the gaps in students' mathematical understanding and help them to construct their mathematical knowledge.

Even and Tirosh (1995) also noted that teachers' ability to detect and eliminate students' misconceptions about certain topics is related to their knowledge about those topics. For instance, they gave a sample of student work to teachers including both the solution and student's explanation of the solution. The student made an assumption that the slope of a line is directly proportional to the angle between the line and x-axis when finding the equation of a line passing through a given point (e.g., the angle between the line $y = 2x$ and the x-axis is twice that of the line $y = x$). The teachers did not recognize that misconception and instead said that they would tell the student to be careful when making estimations. This approach was comparable to how the preservice teachers in my study attempted to answer the question of solving polynomial equations. Almost all of them failed to recognize the student's error and instead of dealing with the error suggested another method to solve the problem.

Although the preservice teachers were unsure how to address the flaws in students' thinking, observing students in the field raised preservice teachers' awareness about students' possible difficulties and misconceptions. Grossman (1990) also noted that field experiences contribute to preservice teachers' repertoire of students' difficulties and misconceptions more than the methods courses. The only way they knew to resolve difficulties was to tell students mathematical facts and show them how to apply them to the given problem. Yet, depending on the context and their content knowledge, some preservice teachers also considered using visual and concrete aids to support their explanations.

Knowledge of Curriculum

Knowledge of curriculum refers to knowing the learning goals for different grade levels and how to use different instructional materials to accomplish those goals (Grossman, 1990; Shulman, 1987). Teachers not only need to know the learning goals for the specific grade level that they are teaching but also how a particular topic is discussed in previous or later grade levels and how it relates to ideas in other subjects. With this knowledge they can develop a plan for teaching that topic that builds on students' previous learning and facilitates students' understanding of later topics. Also, teachers need to know how to use instructional materials including visual and concrete aids, textbooks, and technology to facilitate students' understanding. They should decide which instructional tool is more helpful to teach a particular concept or works better for a particular group of students.

Ordering Topics Based on Mathematical Relationships

During the second and the third interviews I gave preservice teachers a set of mathematics topics and asked them to put the topics in order so that each topic would build on students' existing knowledge in a meaningful way. The preservice teachers either said they would teach the prerequisites first or that they would introduce a new concept when it came out of the discussion of another concept.

During the second interview I gave the following topics to preservice teachers: imaginary numbers, exponents, trigonometry, and quadratic functions. Laura and Monica said that they would teach trigonometry before imaginary numbers. Laura stated that knowing trigonometry would enable her "to work with converting imaginary numbers as polar to Cartesian coordinates." Most probably, she meant that she would show students how an imaginary number could be written in the form of $r(\cos\alpha + i\sin\alpha)$ where $x = r\cos\alpha$ and $y = r\sin\alpha$ for any $z = x + iy$. Thus, students need to know trigonometric ratios in order to understand that representation. Monica noted that "imaginary numbers go hand in hand with trigonometry in a way especially with the unit circle….because you can look at different roots of unity." In order to find the roots of unity one needs to write the imaginary numbers in polar form. Therefore, she would teach trigonometry before imaginary numbers. However, Henry said he would teach imaginary numbers before trigonometry in order to discuss the trigonometric form of imaginary numbers in the context of trigonometry. He said that he did not remember exactly what that relation referred to but he thought that trigonometry should follow the imaginary numbers.

Linda stated that she would teach imaginary numbers before teaching quadratic functions because she would use them in quadratic functions. However, Mandy and Henry would teach imaginary numbers after quadratic functions because while discussing the roots of quadratic functions they would show examples of functions that do not have real roots but imaginary roots. Additionally, all participants except Linda said they would teach quadratic functions just after exponents because students need to be familiar with the x^2 term to understand quadratics. Linda would teach imaginary numbers after exponents because “it involves doing exponents and you can use them in quadratic functions.” Laura, Linda, and Monica tried to put the topics in order from the perspective of how well the prerequisites are satisfied before teaching a new topic. However, Mandy and Henry preferred to teach a new topic when they needed to discuss it in the context of another topic.

Laura, Linda, and Henry were consistent in their way of ordering topics during the third interview. Given ellipses, quadratic formula, transformations, and parabolas Laura and Linda said they would teach the quadratic formula before parabolas because students would need to find the intercepts of parabolas. However, Henry said he would start with parabolas because he would like to teach the parabola as the shape that “comes out of any quadratic function.” Then he would show the quadratic formula to find the roots of the parabola. Likewise, Monica said she would teach parabolas before the quadratic formula because she would first show different transformations of parabolas and then discuss finding the roots of them. Mandy was initially undecided about the order of teaching the quadratic formula and parabola. She said, “The quadratic formula comes out of the ideas of parabolas, but sometimes you need the quadratic formula to find the *x*-intercepts.” Ultimately, she decided to show the quadratic formula before teaching parabolas; however, she stated that she would probably follow the order of her textbook when teaching these topics.

The preservice teachers’ view of teaching mathematics and their own content knowledge were influential on their decisions about the arrangement of the topics in a semester. Some of them emphasized teaching all prerequisites before introducing a new topic whereas others would introduce new issues when they come across them in another context. Both approaches could yield effective outcomes provided that the teachers present the material in a way that allows students to understand the connections among the topics.

Ordering Topics Based on the Teacher's Content Knowledge

The preservice teachers' knowledge of subject-matter had an impact on their decisions about how to put topics in order. When they were unsure about the relationships between the given topics or did not know much about the topic, they said they would teach it last or separately.

Harris stated that he struggled with imaginary numbers and therefore would teach just the basic ideas after teaching other topics. Similarly, Mandy said she would teach trigonometry separately because she did not feel comfortable with it. She said, "You need the idea of function in order to do trigonometry and imaginary numbers, but I'm not sure where to stick the trigonometry." It seemed that she was able to make connections between imaginary numbers and quadratic functions but was not sure how imaginary numbers and trigonometry are related because her knowledge of these topics was not thorough. Finally, Linda said that she had no idea about the ellipses except their shapes. She said that she had never learned ellipses, and therefore she would teach them separately from other topics.

Preservice Teachers' Perceptions about the Development of Knowledge of Curriculum

The preservice teachers thought that the issues they discussed in the methods course and their field experiences added to their knowledge of curriculum even though at the beginning of the semester, Laura, Mandy, Henry, and Monica noted that they did not expect to learn anything new in terms of curricular issues. They indicated that they were familiar with using different computer software or manipulatives. However, Linda said that she knew how to use technological devices and manipulatives as a student, but she needed to learn how to use them as a teacher. And Harris said that he was not familiar with integrated curriculum and wanted to learn more about it.

Level of knowledge. The third and the seventh items of the questionnaire were aligned to curriculum knowledge. Item 3 says “I know what mathematics content is to be addressed in each year of the 6-12 mathematics curriculum,” and item 7 says “I know how to integrate technology in mathematics lessons.” Because the preservice teachers observed different grade levels and courses in different school settings, their perceptions about the improvement in their curriculum knowledge varied during the semester as shown in Table 6.

Table 6

The Preservice Teachers’ Perceived Level of Knowledge of Curriculum

	Before		After		Before	After
Participants						
	Item 3	Item 7	Item 3	Item 7	Item 9	Item 9
Laura	2	3	3	3	3	3
Linda	3	1	3	3	2	3
Monica	2	2	2	2	3	3[a]
Mandy	1	2	1	2	2	2
Henry	2	3	1	2	3	2[b]
Harris	3	2	3	2	3	2

Note. Scale for Item 3 and Item 7. 1: Disagree, 2: Somewhat Agree, 3: Agree
Scale for Item 9. 1: Not Adequate, 2: Adequate, 3: Competent, 4: Very Good
[a] Rated as 3.5 [b] Rated as 1 or 2.

Monica, Laura, and Linda all thought that their knowledge of curriculum had improved. Monica said that curricular and content-related issues were emphasized more in the methods course, referencing discussion about how to use manipulatives, technology, and textbooks in different grade levels. At the beginning of the semester Monica stated that she did not expect the courses to lead to improvement in her knowledge of curriculum, but apparently the methods course and field experiences raised her awareness about issues that she did not pay attention to initially. Laura also noted that during her field experiences she was exposed to several textbooks and worksheets prepared by the teachers. She used the teacher's edition of one of the textbooks when preparing a lesson plan for her teaching experience in the field. Linda also noted that she realized how effective use of technology and manipulatives would promote students' understanding and motivation.

Henry and Harris lowered their ratings of their knowledge of curriculum at the end of the semester, perhaps because they overestimated their knowledge level at the beginning (Item 9). Henry did not feel that his knowledge of curriculum was strong enough because his knowledge was limited to the particular courses he observed during his internship and the instructional tools available in that classroom. Harris stated that he observed how teachers were making connections between mathematical concepts and implementing the new curriculum; however, he did not think that these issues were discussed in depth in the methods course. Mandy agreed with this assessment, noting that she expected to discuss the new curriculum in detail in the methods course. She was not satisfied with the time devoted to discussing the new curriculum and therefore did not see much improvement in her knowledge of curriculum.

The contribution of the methods course topics. The preservice teachers saw the benefit of the discussions of curriculum and textbooks, manipulatives, planning instruction, and assessment (see Table 7) with respect to increasing their knowledge of curriculum. The only major course topic that no one identified as contributing to their knowledge of curriculum was learning theories.

Table 7

The Contribution of the Course Topics to the Development of Knowledge of Curriculum

Topic	**Participants**					
	Laura	Linda	Monica	Mandy	Henry	Harris
Problem of the day		x				
Learning theories						
Curriculum and textbooks	x	x	x			x
Motivation		x				
Promoting communication		x				
Manipulatives	x	x		x		x
Planning instruction	x	x	x	x		
Microteaching		x	x			
Effective questioning	x	x				x
Cognitive demand of tasks	x	x				
Classroom management		x				
Assessment		x	x		x	x

Monica noted that at the beginning of the semester they talked about the NCTM standards and the Georgia Performance Standards and compared them. They were also given an opportunity to look at a few textbooks which were written as the supplementary materials of standards-based curricula to see how the topics were presented and organized in each series of textbooks. Linda learned that she could meet the learning goals identified in a single standard in different lessons rather than attempting to address them in one lesson. Similarly, Laura and Mandy would plan for a series of lessons to cover everything needed to achieve the standards. Laura also noted that she would use textbooks or online resources when planning her instruction.

Mandy stated that she would plan for using manipulatives in her class when appropriate. Harris noted that he realized that a manipulative could be used for different purposes. For instance, he was familiar with base ten blocks because his mentor teacher used them to explain the volume, but he did not know the name of these blocks. He realized that he could use those blocks to explain either the base-ten system or volume or exponential growth. All of the preservice teachers emphasized that the learning goals specific to the grade level should be considered when preparing assessment tools. For instance, Monica stated that the test needs to cover "what they [students] had done in class like what you think students have learned just in this class." Henry also noted that the test should assess what is stated in the curriculum.

The preservice teachers' evaluations of course topics in terms of their contribution to their knowledge of curriculum differed. However, they were aware that they need to know the standards for each grade level they could teach and that they would need to enrich their instruction by using various instructional materials. They were also aware of the need to plan for assessing how well the students attained the learning goals identified for a particular grade level.

Contribution of field experiences. The preservice teachers had opportunities to observe how teachers used instructional materials and how they attempted to achieve the learning goals for particular lessons. They indicated that in some of the classrooms they saw the standards to be covered that day or for the entire unit posted on the board. The teachers were using different instructional tools including textbooks, worksheets, interactive white boards, and graphing calculators.

Laura observed two ninth-grade classes where the new curriculum was being implemented and noted that, as suggested in the new curriculum, the teachers had students work in groups and then present their findings to the class. She also saw how teachers were using textbooks and worksheets as instructional tools. Monica and Harris also noted that they saw several textbooks during their field experiences. Harris stated that he was overwhelmed when he saw the content to be covered in a semester. He was surprised that some topics that he learned in Algebra 2 were now included in the ninth-grade mathematics course. However, Monica said that she was surprised when she saw the textbook used in the ninth-grade class because “it did not seem very much different than other textbooks. It still has a theme, practice problems, and exercise type things.” She was expecting to see some activities for group work because it is emphasized in the new curriculum. Linda and Harris observed how the teachers were using the interactive white board to teach certain topics, noting it not only helped students visualize the content better but also motivated them to engage in the lesson.

The preservice teachers indicated that they would use technology and manipulatives when I asked them what they would do if they were supposed to be the teacher of the classes that they observed. Harris noted that he would plan for an activity on the interactive white board because the students “seemed they had fun with using the SmartBoard” in the previous lessons. Therefore, he would allow each student to manipulate irregular geometric shapes on the interactive white board to find their areas. Linda said she would use graphing calculators to show the difference between the graphs of the first and second derivatives of functions. Similarly, Monica said she would use graphing calculators to show how factoring trinomials helps with finding the roots of quadratic functions.

It seemed that field experiences helped preservice teachers learn about instructional tools that they could use when teaching a particular topic. They planned to use them to facilitate students' understanding of some mathematical topics as well as to attract students' attention to the lesson. Some of them realized that using manipulatives without adequate guidance would not help students understand the mathematical idea underlying the activity. It was evident that field experiences raised preservice teachers' awareness about effective use of instructional tools.

Summary

Having different experiences in the secondary school influenced the preservice teachers' thoughts about what is covered in particular grade levels. The recent change in Georgia high school mathematics curriculum led to confusion about the distribution and arrangement of the topics in each grade level. Although the preservice teachers knew how to find online curriculum resources, they were unsure about how to plan effectively to achieve the goals of the new curriculum. However, given a set of mathematics topics, they were able to sequence them so that each topic was related to the others in certain ways. Their decisions about the order emerged from their perceptions of what would be effective for students' understanding as well as their own experiences with the subject-matter.

The preservice teachers' ideas about horizontal and vertical arrangement of topics were limited by their own experiences as students and their content knowledge. They could make connections among given topics, but in some cases they were unable to support their claims. Furthermore, some of them stated that they were not good at certain topics or had not been taught about them in the high school. Their content knowledge and their views of mathematics were influential in their decisions. Some of them preferred to accomplish all prerequisites before teaching a new topic while others would introduce a new concept when they needed to use it in the context of another topic. It is difficult to judge the superiority of one approach over the other because the effectiveness of these approaches depends on how well the preservice teachers would execute their plans.

The methods course and field experiences raised their awareness about planning for using instructional tools in the lessons effectively. They realized that they could use different materials to meet the needs of different types of students. They thought that using graphing calculators or different applets would help students visualize the mathematical ideas and also provide an opportunity for students to test different cases in order to understand the mathematical ideas and the connections between them more easily. They also noted that technology and manipulatives could be used as incentives because they observed that students were more likely to participate in lessons when dynamic software was used. One thing that was missing from the methods course was the opportunity to critically analyze textbooks or other supplementary materials.

Even though Grossman (1990) identified that knowledge of curriculum entails knowing how the topics and concepts in a subject are organized and structured both horizontally and vertically, it is hard to find studies specifically designed to investigate preservice teachers' decisions about the order of topics to be taught in a course. Instead, the researchers investigated preservice teachers' curriculum knowledge during methods courses that were specifically designed to improve preservice teachers' awareness about curriculum materials.

Castro (2006) investigated the change in preservice elementary teachers' conceptions of mathematics curriculum materials and the perceived role of these materials in the classroom. She collected her data from a methods course that was designed to help preservice teachers develop their skills of using curriculum materials effectively. She found that preservice teachers began to put more value on using manipulatives, the teachers' guide, and assessment resources as instructional tools by the end of the course.

Lloyd and Behm (2005) explored how preservice elementary teachers analyzed a textbook and what their criteria were for their analyses. They chose some activities from a traditional and a reform-oriented textbook and asked preservice teachers to analyze them. They did not give any structure for the analysis because they wanted preservice teachers to develop their own ideas about what to look for when analyzing instructional materials. They concluded that preservice teachers' prior experiences in traditional classrooms played a crucial role in their analysis of textbooks. The preservice teachers stated that they liked the activities from the traditional textbook because it was similar to the way they were taught.

Content knowledge has also been shown to affect how teachers critique textbooks and select materials to use to teach (Grossman, Wilson, & Shulman, 1989). Teachers with weak content knowledge may fail to recognize what mathematical thinking is elicited by the activities and how the activities facilitate conceptual understanding of the subject-matter. Furthermore, when they do not feel comfortable with their knowledge of certain topics they try to avoid teaching them. This finding is consistent with the findings of this study because the preservice teachers indicated that they would postpone teaching some topics to the end of the semester because they did not know much about them. The preservice teachers' experiences as students in high school as well as in the methods courses and their own conceptual understanding of mathematics impacted their curricular decisions. However, their field experiences and classroom practices contributed to their repertoire of instructional materials that could be used when teaching a particular subject-matter.

Knowledge of Subject-Matter

The depth and accuracy of teachers' mathematical knowledge has an impact on their teaching effectiveness (e.g., Ball, 1990a; Brown & Borko, 1992; Gess-Newsome, 1999a). For example, teachers not only need to know how and when to apply mathematical rules, procedures, and facts but also have a deeper understanding of why they work so that they can enhance students' conceptual understanding of mathematics by providing the reasoning behind those rules and procedures. Furthermore, they need to know their subject-matter well in order to decide what prior knowledge is required, what examples and tasks are appropriate, and what types of representations could be used to teach a particular topic. With this type of deep and well-connected knowledge of mathematics, teachers can develop effective plans to teach for understanding.

Procedural Knowledge Limited to Memorization of the Rules, Facts, and Procedures

As noted above, when given examples of students' errors or difficulties in understanding a particular topic, the preservice teachers attempted to explain the procedures, facts, or rules that would either yield the correct answer or clarify the ambiguity. However, most of the time they did not justify the reasoning behind the procedures at all or they failed to explain it clearly.

When I asked preservice teachers how they could help a student who had difficulty in multiplying binomials, most of them told me that they would show her how to use the FOIL method. Laura, Linda, and Mandy did not state that FOIL is a mnemonic that helps students remember how to distribute the terms over binomials; rather they just explained how to carry out the procedure to find the answer. However, Monica stated that the distributive law and FOIL are equivalent methods because FOIL "is just an acronym that…you make sure that you do multiply through, by all the different terms." She said she would separate the terms of the first binomial and then apply the distributive law over the second binomial. Similarly, Henry noted that he would start with the distributive law and apply it twice and then tell students about the FOIL method because the distributive law underlies the FOIL method. On the other hand, Harris stated that he would explain how to use the distributive law by separating the terms of the first binomial and showing how to distribute them over the second binomial and then combining the results to get final answer. He noted that if the student had trouble doing the distributive laws, he would work on "just the basic formula of $a(b+c)$ just say, 'All right, remember from your basic law that equals $a \cdot b + a \cdot c$, now apply it.'"

Although Monica, Henry, and Harris were aware that the distributive law is what allows one to multiply two binomials, their approaches to the student's difficulty were procedural because they did not clarify why the distributive law works. They assumed that applying the distributive law after separating the terms would help students understand multiplication of the binomials. However, as I indicated in the knowledge of pedagogy section, this approach is problematic because it is likely to lead to students memorizing the FOIL procedure without understanding why it works.

During the third interview, I asked the preservice teachers how they could help a student who made a mistake when solving inequalities by not changing the direction of the inequality after dividing the coefficient of the x term by a negative number (see Figure 12). All participants stated that they would tell the student that when dividing by a negative number you need to flip the inequality sign. To convince the student that the answer was incorrect they would ask her to check the reasonableness of the result by assigning a value from the solution set to x. Furthermore, all of them were aware of that there was a mathematical explanation for why they need to change the inequality sign; however they failed to state it clearly.

Interview 3: Solving inequalities

Look at each of the student work given below. How can you explain to the student that his or her solution is incorrect?

$$\begin{aligned}-2x+5 &\le x-1\\ -2x-x &\le -1-5\\ -3x &\le -6\\ x &\le 2\end{aligned}$$

Figure 12. The solving inequalities task.

Laura said that there should be a mathematical explanation for flipping the sign, she did not know it. Mandy, Monica, and Henry suggested graphing the given inequality to justify changing the direction of the inequality. Monica did not explain what she meant by graphing, but Mandy stated that she would "start graphing them out, shading the sides, and show them that way where the solution comes from." I presume she meant that both sides of the inequality could be thought of as two separate inequalities and could be solved as systems of inequalities. Likewise, Henry suggested setting up $y=-3x$ and $y=-6$ to investigate common solution as if they were inequalities. His explanation was as follows:

> Draw that line $y=-6$ and then draw $y=-3x$ ….So we have this, it is equal $x=2$. So when $x=2$, $y=-6$. And then we want to see where if [it is] less than or equal to -6. It is going to go in that area [he shades underneath of $y=-6$]. So we pick a point on this graph that can fall down in the shaded region say '*x* is greater than or equal to 2.'

Henry's reasoning was vague because he did not identify the inequalities clearly. Based on his explanations, I concluded that he assumed that $y \le -6$. However, it was not clear whether he thought $y \le -3x$ or $y \ge -3x$ because he did not shade the common region. To obtain the answer as "*x* is greater than or equal to 2" he probably considered the latter inequality, but he did not state it explicitly.

Harris also failed to explain clearly the reason for reversing the inequality. He stated that "you switch the sign because you are dealing with two negatives and kind of they cancel each other and you have to move the other direction that on, that is what happens." It seemed that he borrowed the idea of getting a positive number as the result of dividing two negative numbers to explain why the sign should be switched. However, his reasoning would not work in the case of having a positive number on the other side of the inequality. For instance, Harris's reasoning does not address the solution of $-3x \le 6$. Therefore, Harris's explanation was incomplete and not mathematically valid.

In contrast, Linda explained the reason behind the procedure clearly. She stated the following:

> It is more of in my head type thing that if a negative number times *x* is less than another negative number then *x* by itself should be greater. Like if -3*x* is less than -6 that means it has to be still a negative number. Because if it is positive it is going to be greater than [-6].

Linda pointed out the fact that if a number is less than a negative number, then it is itself a negative number. Thus, she concluded that -3*x* is a negative number. Then she referred back to the multiplication of integers and noted that the product of two numbers is negative if and only if one of the numbers is negative and the other is positive. Therefore, *x* would be a positive number and also greater than or equal to 2 because when -3 is multiplied by a number greater than 2, say 5, it should be still less than -6. Also, she implicitly stated that *x* cannot be a negative number; otherwise the inequality would not be valid because -3*x* would be a positive number.

The examples given above showed that preservice teachers' knowledge of subject-matter was mostly procedural. Even though they were aware that there were conceptual foundations behind the procedures, they were unable to remember or explain them clearly. Hence, in order to explain to their future students why procedures, facts or rules work, they will need to spend time reviewing the ideas, analyzing how and why they work, and thinking about ways to explain the concepts to students.

Depth of Understanding

The preservice teachers' answers to the content-specific questions revealed that they lacked conceptual understanding of some topics. Some preservice teachers stated that they did not know much about ellipses while others were unable to explain the difference between permutations and combinations. Furthermore, they sometimes failed to justify their reasoning about how two topics are related to each other.

In the third interview, I asked the preservice teachers in which order they would teach parabolas, transformations, ellipses and the quadratic formula. Henry and Linda stated that they did not know about ellipses. Henry noted that he recently saw the formula of an ellipse, but he did not remember it. However, Linda said that she had not ever learned about ellipses. Laura stated that she would teach ellipses last because she thought that it was a more advanced topic than the others. However, she did not explain what she meant by being "more advanced." She might know that an ellipse is an example of a second-degree function with two variables, which makes it a more advanced topic compared to parabolas. Because she neither wrote the general equation of an ellipse nor explained the relationship with other topics given in the list, it was difficult to assess her knowledge of ellipses.

On the other hand, Mandy said the following:

> Because I know ellipses come last. If they don't know this stuff, they are not going to get in ellipse because ellipses come out of the quadratic formula almost. Because you are just manipulating that $ax^2 + bx = c$ and you just play around with the whole thing.

Mandy's reasoning was ambiguous because it was not clear what she meant by manipulation of the equation of quadratic functions and "playing around with the whole thing." She did not specifically talk about finding the roots of ellipses, but she attempted to relate the quadratic formula and ellipses in some way. It was unclear whether she used the term “quadratic formula” on purpose or whether she meant the standard form of quadratic functions. Her claim that ellipses come out of the quadratic formula is not a reasonable argument. Therefore, it is difficult to claim that she had a conceptual understanding of ellipses.

Monica and Harris held a misconception that ellipses can be formed by two parabolas. Harris explained his thoughts this way:

> Ellipses, you are manipulating the quadratic formula and parabola formulas. You’re seeing a different view. … And then the formula for ellipse builds and the shape of an ellipse builds on both of these visually. An ellipse is kinda like two parabolas kinda connected like this [connects his fingers in a shape of an ellipse] and you end up with, you get a stretched out circle…it’s something where students can kinda see ‘All right, it’s not, you know what the parabola, you don’t really have this perfect arc it’s kinda stretched out, all right, well ellipses are also stretched out.’

Similarly, Monica said the following:

> Ellipses I think are good when you’re looking at transformations, but also you can relate the formula for an ellipse with some of the graphs of your parabolas when you rotate them a certain way they would look like a certain portion of an ellipse and you can connect them that way to show…this parabola, what portion of an ellipse would this or how could you use the formula of this parabola to create the ellipsoid that you know that corresponding to the ellipsoid and that would start some interesting [discussion].

In both cases it was evident that preservice teachers’ inferences about the relationship between parabolas and ellipses were based on their shapes. Although an ellipse could be visualized as a combination of two parabolas, it is a mathematically invalid argument. They most probably assumed that the boundary of a closed region between parabolas that have the same roots would be an ellipse. For instance, they might assume that if one of the parabolas passes through (3, 0), (-3, 0), and (0, 2) while the other one passes through (3, 0), (-3, 0), and (0, -2) then the boundary of the region between them would represent an ellipse passing through (3, 0), (-3, 0), (0, 2), and (0, -2). However, if they checked this claim for accuracy, they would see that any point lying on the boundary of the region (except the vertices) would not satisfy the equation of the ellipse. In fact, ellipses and parabolas are two types of conic sections, and some mathematicians accept that a parabola is an ellipse having one of the foci at infinity. Therefore, ellipses and parabolas are not totally unrelated concepts, but relating them in terms of their graphs is unreasonable. Furthermore, Monica was confused about the concepts of ellipsoids and parabolas. She said that parabolas could be used to create an ellipsoid based on her assumptions about ellipses and parabolas. However, an ellipsoid is a quadratic surface composed of ellipses or circles rather than parabolas. The explanations provided by the preservice teachers revealed that their understanding of ellipses was quite shallow.

When I asked the preservice teachers how they would clarify the difference between permutations and combinations for their students, they either told me that they did not know the difference themselves or gave me some examples that they would use to make the distinction. However, some of those examples were not completely valid. Laura stated that she did not know anything about permutations and combinations. Harris, Henry, and Monica noted that they would try to explain the difference by relating the meaning of the word with the actions to be taken.

Harris gave real-life examples of a “combo menu” for combinations and “perming hair” or “permit” for permutations. He said that he would emphasize that the order of the actions is not important in combinations just as eating the foods on menu can be done in any order. He said that following a precise order when perming hair or getting a permit is important in order for it to work correctly. Although the words may help students remember the difference, the examples for permutations may lead to a misconception that there is a unique solution for permutation questions. He also did not clarify how his examples would help students distinguish the differences in the formulas for permutations and combinations.

Similarly, Henry stated that he would use word “mutation” to explain permutations because “mutating” the digits in a number would give a different number. He said he would use the handshake problem to explain combinations. However, he noted that he did not know how to clarify the distinction between the formulas because he was still confusing the formulas himself. Monica also said she would explain that combination refers to combining some things, but permutation refers to changing the order. She did not mention how she would address students’ difficulties in figuring out the formula of each.

Mandy said that she would use the example of combining 2 pairs of pants and 4 sweaters to explain combinations and ordering books in a shelf to explain permutations. She stated that the formula for permutations is simpler than the formula for combinations, but she did not write them. Linda failed to distinguish the examples for combinations and permutations. She stated that ordering index cards for the word “RAIN” is an example of a permutation, and finding the number of two-letter words derived from the word “BOOT” would be an example of a combination. She noted that because there are two O’s, changing the place of them would not make any difference. Although her explanation was true, the problem itself was still a permutation problem because she would find the number of different words rather than different combinations of two letters from the word “BOOT.” She probably confused the fact that the number of two-letter words with repeating letters is the same as the number of combinations of two letters out of four letters. Therefore, she used the formula for combinations to solve the “BOOT” problem.

The preservice teachers had limited knowledge of some mathematical concepts. Some of them said that they had not been taught these concepts in depth in high school, whereas others were able to remember some examples that were used to explain some concepts. They were unable to provide mathematically valid arguments to justify their claims because their inferences often relied on the surface features of the topics such as the shape of the graph of an ellipse.

Relationships Between Mathematical Ideas

The preservice teachers' abilities to make meaningful connections between mathematical concepts and explain the reasoning behind algorithms and rules depended on the robustness of their conceptual knowledge of mathematics. For example, they were asked to identify the prior knowledge that students would need in order to learn quadratic functions and logarithms. The preservice teachers' suggestions were reasonable but depended on either their own preferences of how to introduce the topics or their experiences with them.

In the first interview, I asked the preservice teachers what students should know before learning about quadratic functions and how they would introduce quadratic functions to students. Laura stated that she would expect students to know linear functions thoroughly as well as know how to graph lines. She would emphasize how to graph quadratic functions and relate their graphs to the general equation of quadratic functions. That is, she would show how the graph of the function changes with respect to the coefficients of the terms; thus, she would introduce the transformations of functions. Laura's assessment of the necessary prior knowledge matched her approach to introducing the topic.

Mandy also said she would start with graphing quadratic functions. Therefore, she thought that students would know linear equations, their graphs, and the meaning of slope. She said that she would start with the graph of x^2 and then continue with $x^2 + 2$ and $(x + 2)^2$ because she had been taught in that way. Even though she did not state explicitly what she would try to teach by showing the graphs of different functions, she said that she would discuss the ideas of domain and range. Therefore, I assume that she aimed to teach domain and range of functions through their graphs.

Linda and Monica said they would start with teaching how to find the roots of quadratic functions. Therefore, Linda suggested that students should know lines, equations of lines, and exponents. She did not expect them to know factorization of trinomials because she would teach that in the context of quadratic functions. After working on some examples she would show the quadratic formula and explain how to use it. Likewise, Monica would focus on teaching the quadratic formula. However, she said that students would need to know how to graph functions, specifically the meaning of finding x-intercepts, how to factor trinomials, and they should be aware that quadratic functions are second-degree polynomial functions. She said she would ask students to factor given quadratic functions to find the x-intercepts. Then, she would give an example where it would be difficult to find the factors by trial and error to introducing using the quadratic formula to find the roots of quadratic functions. In both cases, the preservice teachers' main goal was to teach how to find the roots of quadratic functions using the quadratic formula. Their approaches to teaching quadratic functions seemed to be meaningful because in order to graph functions student would need to know their intercepts.

Harris said he would try to relate the graphs of quadratic functions to their equations. Therefore, he identified exponents, solving equations, factoring, the distributive law, and graphing linear functions as essential prerequisite knowledge. His ultimate aim was to show that the graphs of quadratic functions look like a curve because they have two roots whereas the graphs of linear functions look like a straight line because they have at most one root. Although he explained how he would engage prior knowledge when introducing quadratic functions, his assumptions about the relationship between linear and quadratic functions were not completely valid because he disregarded that there are infinitely many lines passing through a single point. He thought that the lines passing through the points (3, 0) and (-3, 0) were $y = x - 3$ and $y = x + 3$, respectively. Therefore, the equation of the quadratic function passing through those points would be $(x-3)(x+3)$, that is $y = x^2 - 9$. His explanations revealed that he merely focused on an isolated example and failed to evaluate whether his arguments were mathematically valid in a more general sense.

Henry talked about how his mentor teacher taught quadratic functions. His mentor teacher started with engaging students' prior knowledge of x^2; namely, that the square of a number and the square of its opposite are equal to each other (e.g., $7^2 = (-7)^2$). Then his mentor wrote the squares of whole numbers less than 10 to show the fact that squared numbers increase in a non-linear fashion. Finally, the mentor also showed how the graphs of functions derived from a parent function differ; that is, he mentioned transformations of functions. Henry noted that he would teach quadratic functions in a similar way.

Preservice teachers' suggestions about what topics should be covered before teaching logarithms were self-evident. They were all aware that students should have a deep understanding of exponentials. Harris, Henry, Mandy, Monica, and Laura emphasized knowing how to graph because the graphs of logarithmic functions are special. They also noted that logarithmic functions are inverses of exponential functions and that their graphs are symmetric with respect to $y = x$ line to justify their reasoning for why students need to know exponentials before logarithms. Furthermore, Linda and Laura stated that students should know what solving equations means if they are going to understand the fact that logarithms are used for solving the equations of exponential functions.

In discussing prior knowledge needed for logarithms, Linda, Monica, and Henry all mentioned the number *e* explicitly. Linda and Monica noted that students can understand the properties of natural logarithms better if they are familiar with the number *e*. Henry said he would introduce number *e* in the context of natural logarithms and then show how it is used when solving compound interest problems. Both Mandy and Harris said students would have already been introduced to the number *e* in the context of exponential growth and decay because it is taught before logarithms. Mandy stated that she would discuss what Pe^{rt} means in the context of exponential growth or decay, but she did not explicitly identify the role of the number *e* in working with logarithms. Harris did not explain how knowing about exponential growth or decay would help develop an understanding of logarithms.

The preservice teachers were aware that mathematics topics are related to each other and that one concept could be built on another concept or fact. Based on their mathematical knowledge and experiences, they were able to identify some connections between given topics. However, they were unable to justify their reasoning if they did not know the subject in depth.

Preservice Teachers' Perceptions about the Development of Knowledge of Subject-Matter

The preservice teachers thought that they did not develop new content knowledge in the methods course or during their field experiences; rather, they said they were given an opportunity to remember some mathematical facts that they had not seen recently. They noted that they had already taken many content courses and were still taking content courses at the time this study was conducted. Therefore, they seemed to think that their content knowledge was being developed outside of and largely prior to the methods course.

Level of knowledge. The preservice teachers' answers to Items 1 and 6 on the questionnaire provided information about their perceived level of knowledge of subject-matter.

Item 1 was “At the end of my degree program I will have taken enough content courses to be an effective mathematics teacher in grades 6-12” and Item 6 was “I know how mathematical concepts are related.” They were also asked to rank their subject-matter knowledge level in the ninth item. As presented in Table 8, four of them ranked their content knowledge as “very good.” The preservice teachers perceived an increase in their knowledge of subject-matter not necessarily because of the methods course but because of the content courses they were taking during the semester. However, analysis of the content-specific questions that were asked during the interview revealed that preservice teachers lacked conceptual understanding of some mathematical facts and concepts.

Table 8

The Preservice Teachers’ Perceived Level of Knowledge of Subject-Matter

	Before		After		Before	After
Participants						
	Item 1	Item 6	Item 1	Item 6	Item 9	Item 9
Laura	2	2	3	3	2	4
Linda	3	2	3	2	3	4
Monica	2	2	3	3	2	3[a]
Mandy	3	2	3	2	3	3
Henry	3	3	3	3	4	4
Harris	3	3	2	3	3	4

Note. Scale for Item 1 and Item 6. 1: Disagree, 2: Somewhat Agree, 3: Agree
Scale for Item 9. 1: Not Adequate, 2: Adequate, 3: Competent, 4: Very Good
[a] Rated as 3.5.

At the beginning of the semester Harris noted that he wanted to improve his knowledge of how mathematical concepts are related. However, he noted that he still relied on his previous knowledge when he was asked to connect mathematical concepts. Monica stated that she saw an improvement in her subject-matter knowledge because of the content courses but also some methods course practices such as the problem of the day. Although Mandy's scores did not change by the end of the semester, she said that the methods course and field experiences contributed to her subject-matter knowledge.

The contribution of the methods course topics. The preservice teachers did not think that the methods course contributed to development of their knowledge of subject-matter. However, they thought that problems of the day, microteaching, and the discussion on the cognitive demand of tasks helped them to remember some mathematical ideas they had not seen for awhile (see Table 9). For instance, Harris stated that he liked to see how his classmates approached the problems of the day because it improved his repertoire of solution strategies for a particular type of problem. Monica stated that having the opportunity to microteach encouraged her to choose the topic she wanted to teach and review it while planning for the lesson. Linda thought that knowledge of subject-matter is essential to preparing cognitively demanding tasks because teachers need to know a particular topic in depth and its connections to other topics.

It is not surprising that the preservice teachers did not perceive any improvement in their subject-matter knowledge due to the course topics. However, they were aware that they need to know their content thoroughly in order to develop a comprehensive lesson plan and be able to address students' difficulties effectively.

Table 9

The Contribution of the Course Topics to the Development of Knowledge of Subject-Matter

Topic	**Participants**					
	Laura	Linda	Monica	Mandy	Henry	Harris
Problem of the day	x	x	x	x	x	x
Learning theories						
Curriculum and textbooks						
Motivation						
Promoting communication						
Manipulatives						
Planning instruction						
Microteaching	x	x	x			x
Effective questioning						
Cognitive demand of tasks	x	x		x		x
Classroom management						
Assessment						

Contribution of field experiences. The preservice teachers noted that field experiences helped them refresh their memories about mathematical facts and concepts. In some cases they needed to review topics being taught in the class in order to understand ongoing discussions better. For instance, Linda stated that she observed a lesson about finding the maximum and minimum points of functions and realized that she forgot some of the related mathematical ideas. Therefore, she reviewed the material before going to class the following day. The preservice teachers reviewed the subject-matter they were supposed to teach during their field experiences. In particular, Monica noted that she did not know much about statistics, but she had to teach a lesson in an AP Statistics class. Therefore, she studied statistics and learned about blocked designs and matched pair designs.

Summary

Knowledge of subject-matter is a critical component of pedagogical content knowledge. Having a deep understanding of mathematical concepts, facts, and algorithms is essential to effective teaching for understanding (Borko & Putnam, 1996). When teachers know the connections between various mathematical concepts and the reasoning behind the procedures or facts, they are likely to design their instruction to foster students' mathematical thinking (e.g., Ball & McDiarmid, 1990; Even & Tirosh, 1995; Fennema & Franke, 1992).

The results of the study revealed that the preservice teachers' knowledge of subject-matter consisted of memorized rules, facts, and procedures and poorly organized mathematical concepts. They lacked knowledge of the conceptual foundations of certain topics. The deficiencies in their conceptual understanding were evident in their arguments about the connections between mathematical ideas. Either they were unsure about the relationship between the given concepts or they made inferences about the surface features of the concepts, which were not necessarily valid.

The studies of preservice teachers' subject-matter knowledge support the finding of this study that the preservice teachers lacked conceptual understanding of mathematics and had a tendency to rely on their previous knowledge about the content (Ball, 1990a; Even, 1993; Even & Lappan, 1994; Foss & Kleinsasser, 1996; Gess-Newsome, 1999b). Ball (1990a) found that many preservice teachers did not possess conceptual understanding of mathematical ideas and procedures. She stated that preservice teachers' mathematical knowledge was primarily based on mathematical rules, and they were not able to make connections between mathematical ideas. Most of them perceived mathematics as an abstract and meaningless set of rules to be memorized. Gess-Newsome (1999b) also noted that preservice teachers' subject-matter knowledge was based on algorithms and facts learned in high school. She stated that preservice teachers were confident in their subject-matter knowledge even though they lacked conceptual understanding of the content that they were supposed to teach. That is, they were not aware of their weaknesses in subject-matter knowledge.

Furthermore, content-specific studies showed that studying a concept intensively might not necessarily improve one's conceptual understanding of that concept. Wilson (1994) investigated the development of a preservice teacher's knowledge of function as she participated in a course that emphasized mathematical and pedagogical connections of the function concept. Wilson stated that at the beginning of the semester the preservice teacher's knowledge was limited and extremely fragmented. Although she was better in solving problems related to real-world functional situations by the end of the semester, she was still unsure about how to use graphs of functions in other contexts such as solving equations and inequalities.

In summary, the preservice teachers' subject-matter knowledge was mostly procedural and fragmented. The validity of their explanations for why certain procedures or rules work or how mathematical concepts and facts are related depended on the robustness of their mathematical knowledge.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Teachers need to possess various knowledge and skills to plan for and implement effective teaching practices that promote students' understanding and learning. However, they not only need to have in-depth knowledge of the subject-matter, pedagogy, curriculum, and students but also need to be able to deploy this knowledge effectively while teaching. Pedagogical content knowledge is a special knowledge base for effective teaching that involves interweaving various knowledge and skills. Content-specific methods courses and field experiences are conceived of as arenas for teachers to develop their pedagogical content knowledge. Therefore, I aimed to investigate what aspects of preservice secondary mathematics teachers' pedagogical content knowledge developed in a secondary mathematics methods course and its associated field experiences. I also sought to determine what course topics contributed to the development of pedagogical content knowledge from the preservice teachers' perspective.

I observed the mathematics methods course for preservice secondary teachers and its associated field experience in fall 2008 at the University of Georgia. Twenty-five undergraduate and five graduate students were enrolled in the methods course. In the methods course, the preservice teachers discussed several issues about teaching and learning mathematics such as planning instruction, promoting discourse, using manipulatives, and assessment. Their field experience consisted of four visits to schools, and they wrote field reports on teachers' questioning techniques, the cognitive demand of the tasks, the assessment tools, and students' mathematical thinking.

I used a qualitative design in my study. I observed the methods course and field experience course and took field notes about course topics and the preservice teachers' engagement. From the 30 preservice teachers, I purposefully chose 6 representative students as my participants based on a questionnaire administered at the beginning of the semester. According to their answers on the questionnaire, I categorized the preservice teachers as having high, medium, or low levels of pedagogical content knowledge and chose 2 preservice teachers from each category as the participants of the study. I conducted three interviews with each participant throughout the semester to learn about the development of their pedagogical content knowledge using tasks such as error analysis. I also asked them what course topics contributed to their pedagogical content knowledge. In addition, I shortened the initial questionnaire and gave it to my participants at the end of the semester to detect changes in their perceived knowledge levels. I collected all artifacts distributed in the courses and looked at the students' assignments to gain a better understanding of the course topics and students' thoughts and reflections about those topics. At the beginning of the semester I interviewed the instructor of each course to learn about their goals for the course.

I defined pedagogical content knowledge as having four components: knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum. The correctness of the preservice teachers' answers to given mathematical problems and the validity of their explanations about how mathematical concepts are related or why a particular solution is incorrect were counted as indicators of their knowledge of subject-matter. The level of their knowledge of pedagogy was assessed by the reasonableness of their choice of teaching activities, tasks, examples, and representations and the comprehensiveness of their lesson plans. The preservice teachers' repertoire of examples of students' difficulties and misconceptions and their ability to identify and address those difficulties and errors were taken as indicators of their knowledge of learners. Finally, the preservice teachers' knowledge of curriculum was assessed in terms of their ability to identify a reasonable order of mathematical concepts to be taught in a semester, to differentiate learning goals for different grade levels, and to choose appropriate instructional materials such as textbooks, technology, and manipulatives to meet those goals.

The analysis of data revealed that there were similar patterns in each aspect of the preservice teachers' pedagogical content knowledge. Two findings about their knowledge of pedagogy were salient. First, the preservice teachers' repertoire of teaching strategies was limited by the robustness of their subject-matter knowledge. Second, the preservice teachers' choices of teaching activities, tasks, and examples depended on their views of teaching and learning mathematics. The preservice teachers were able to justify the reasoning behind mathematical facts by using visual or concrete representations or by making connections with other concepts when they had a solid understanding of a given topic. Otherwise, they simply explained how to carry out the procedures or apply a mathematical fact to the given problem. Because they viewed mathematics as a set of rules, procedures, and facts, they were mostly inclined to tell procedures and rules when asked how they would teach a particular topic. That is, their view of teaching mathematics was compatible with their view of mathematics. However, they thought that the methods course and field experiences contributed to the development of their knowledge of pedagogy. They stressed that the field experiences helped them to improve their repertoire of teaching strategies.

The most significant finding about the preservice teachers' knowledge of learners was their lack of ability to identify correctly the source of students' difficulties and errors. They thought that students fail in mathematics because they do not know the procedures or rules to be applied or they apply them incorrectly. Therefore, they were inclined to address students' errors by repeating how to carry out the procedures or explaining how to apply a rule. In some cases, they said they would ask the students to explain their solutions first in order to help students assess their own understanding and realize their mistakes. The preservice teachers noted that the methods course and field experiences raised their awareness about students' thinking. They realized that they needed to plan for addressing students' difficulties when teaching a particular topic. They also observed which concepts were difficult for students to grasp during their field experiences.

There were two common features of the preservice teachers' knowledge of curriculum. First, the preservice teachers' decisions about the order of the topics to be taught in a course were based on their perceptions of how the mathematical topics are related to each other. Second, the preservice teachers' ideas about how to sequence topics were influenced by their content knowledge. Given a set of mathematics topics to be put in order, the preservice teachers said they would either teach the prerequisites first or introduce a new concept when it came out of the discussion of another concept. Furthermore, they said they would teach the topics that were less familiar to them at the end or separately from the others. However, they thought that some of the course practices and field experiences raised their awareness about curricular issues. For instance, they had the opportunity to observe how instructional materials could be incorporated into lesson during their field experiences.

Finally, three issues were significant in terms of the preservice teachers' knowledge of subject-matter. First, the preservice teachers' knowledge of mathematics was procedural and was grounded in their memorization of the rules, facts, and procedures. Second, the preservice teachers lacked a deep understanding of some mathematics topics. Third, the preservice teachers' knowledge of how mathematical ideas are related to each other was limited to their conceptual understanding of the given topics. The preservice teachers' answers to content-specific questions revealed that their mathematical knowledge was mostly procedural, and they were unable to explain the reasoning behind the procedures or rules. The preservice teachers indicated that they did not learn any new mathematical content in either the methods course nor during the field experiences, but they had opportunity to refresh their memories about some mathematical ideas.

Conclusions

I identified 4 salient features of the nature and the development of preservice teachers' pedagogical content knowledge. First, knowledge of subject-matter is a crucial component of pedagogical content knowledge and influences the quality of the other aspects of pedagogical content knowledge. The preservice teachers' ability to make appropriate connections among mathematical concepts, justify the reasoning behind mathematical procedures, generate different solutions and representations for problems, address students' difficulties and misconceptions effectively, and choose appropriate examples to teach a particular topic was largely based on the depth of their knowledge of subject-matter.

The preservice teachers lacked a conceptual understanding of some areas of mathematics, and they relied largely on their procedural knowledge when answering content-specific questions. This procedural nature of their subject-matter knowledge was echoed in their ways of teaching mathematics and helping students to understand mathematics as they perceived teaching as telling the procedures and rules (Ball, 1990a). Thus, their repertoire of teaching strategies and representations was limited. When they knew the conceptual foundations of particular mathematical ideas they attempted to use representations or real-life examples. However, most of the real-life examples were based on their own experiences as students (Calderhead & Robson, 1991); therefore, they failed to explain how those representations or examples help students develop a conceptual understanding of the mathematical concepts or facts that they were trying to exemplify. Furthermore, when I asked them what prior knowledge students should have before learning a particular topic or how to put mathematical topics in an order to build on students' existing knowledge, they provided explanations that were either self-evident or vague. A similar pattern was observed when they attempted to address students' difficulties and errors. Their lack of subject-matter knowledge hindered their ability to generate appropriate strategies and representations to eliminate students' difficulties with certain topics (Borko & Putnam, 1996), as they generally indicated that they would tell students the rules and procedures to help them solve problems correctly. Additionally, the preservice teachers were unaware of the sources of flaws in students' mathematical thinking and assumed that students had weaknesses in their procedural knowledge.

Second, the course practices and field experiences raised preservice teachers' awareness of some issues of teaching and learning mathematics; however, they were not able to apply this knowledge. In the methods course, the preservice teachers discussed several cases of planning instruction, using teaching strategies, assessing students' understanding, and using manipulatives. During the field experiences, they observed different classroom environments and teaching practices and saw how teachers were pacing the lesson, how they were using instructional materials, and how students were performing on the tasks. Thus, they had an opportunity to make connections between the issues they discussed in the methods course and how those issues played out in a classroom environment. However, they were unable to make inferences from their course practices and field experiences when they were asked to help a hypothetical student who was struggling to understand particular mathematical concepts. The preservice teachers generally said they would emphasize how to carry out algorithms and apply rules with little or no reference to using visual or concrete manipulatives to explain a mathematical concept (unless the topic inherently lent itself a visual representation as in the case of graphing functions). It appeared that the preservice teachers did not internalize their experiences in the methods course and field experiences and use them an asset for their teaching (Gess-Newsome, 1999b). Perhaps they need more experiences in the field as an observer and as a practitioner to transfer their learning from the course to the act of teaching. It is also possible that they would demonstrate their knowledge differently in an actual classroom situation than they did in an interview situation with hypothetical students.

On the other hand, the field experiences did contribute to the preservice teachers' repertoire of students' difficulties and misconceptions and instructional materials. They realized that students might struggle with understanding some mathematical facts and concepts such as slope, factoring trinomials, and multiplying binomials that they did not initially think would be problematic. Furthermore, they observed how teachers incorporated instructional tools in their lessons and how these tools were likely to enhance students' understanding and motivation. All participants agreed that when teachers used interactive white boards for exploration of mathematical ideas, not only were students more engaged but they also seemed to understand the mathematics better.

Third, the preservice teachers benefited from the course practices and field experiences to varying degrees. I categorized the preservice teachers according to their pedagogical content knowledge levels at the beginning of the semester based on questionnaire responses. The findings revealed that the preservice teachers who were categorized as having a low level of pedagogical content knowledge experienced the most improvement in their pedagogical content knowledge. They perceived that almost all course practices contributed to the development of their knowledge of pedagogy, students, and curriculum to some extent. In fact, their answers to the content-specific questions indicated that by the end of the semester they were more thoughtful about what is difficult for students to grasp and how they could use visual or concrete aids to help them understand. Furthermore, all the preservice teachers perceived an improvement in their knowledge of subject-matter not because of the methods course and field experiences but because of the content courses they were taking during the semester. However, they noted that the course practices and field experiences helped them to remember some mathematical ideas that they had not been dealing with recently.

Fourth, the preservice teachers generally overestimated the level of their knowledge of each aspect of pedagogical content knowledge. Their answers to the questionnaire items (Items 1 through 9) were inconsistent with how they answered the content-specific questions. For instance, most of the preservice teachers said that they knew how mathematical concepts are related (Item 6) and ranked their knowledge of subject-matter as “very good” (Item 9) even though they lacked conceptual understanding of some mathematical topics such as ellipses. Furthermore, the preservice teachers’ perceptions about the level of their knowledge of subject-matter, pedagogy, learners, and curriculum influenced their ideas about how the course experiences contributed to the development of each of knowledge type. For instance, some of the preservice teachers thought that they had enough knowledge of curriculum and technology because they had taken a curriculum course and a technology course in previous semesters. They, therefore, did not expect an improvement in their knowledge of curriculum during the methods course. Although some course practices, such as microteaching, provided an opportunity for them to apply and improve their knowledge of curriculum, a few of them indicated that such practices contributed to their knowledge.

Limitations of the Study

This study has three limitations. The first limitation was the preservice teachers' familiarity with and knowledge of the content used in the interview tasks. The second limitation stems from the preservice teachers' perceived level of each component of pedagogical content knowledge. Finally, the third limitation was that each preservice teacher had different field experiences.

The tasks I designed for the questionnaire and interviews involved secondary school mathematics content, and I used different items in each interview. This was problematic because if the preservice teachers did not have a strong conceptual understanding of the content involved in an item, the item revealed their content knowledge rather than another aspect of their pedagogical content knowledge. Because I used different items throughout the study, I was not able to detect improvement in their pedagogical content knowledge, because their content knowledge was the overriding determinant of their success in answering the questions. For instance, at the beginning of the semester some of the preservice teachers were able to address a particular student error effectively because they knew the content involved, but at the end of the semester they performed poorly on a similar item involving different content because they did not know much about the content.

I administered a portion of the questionnaire in a pre-post fashion to detect changes in the preservice teachers' perceptions about their pedagogical content knowledge. This portion of the questionnaire asked them to rate their level of knowledge for each component of pedagogical content knowledge. At the end of the semester, some of the preservice teachers rated their knowledge as lower than their knowledge at the beginning of the semester even though they indicated in interviews that their knowledge had improved. This finding suggests that the preservice teachers initially overestimated their knowledge levels in certain domains. Their inability to assess their own knowledge level accurately likely had two consequences. First, I may have miscategorized the pool of participants in terms of their initial level of pedagogical content knowledge. Second, the comparison of the knowledge levels on the pre- and post-questionnaire items does not reflect their gains from the methods course and their field experiences. Although their ratings suggest that the methods course and field experiences did not contribute to their pedagogical content knowledge, it was evident that the preservice teachers became more critical when assessing their knowledge.

The preservice teachers observed different teachers and students in different grade levels. Therefore, their gains from the field experiences were varied. Observing different classroom settings seemed to influence their ideas about and knowledge of teaching mathematics (Ball, 1988; Borko & Putnam, 1996). Furthermore, their experiences contributed differentially to their repertoire of teaching strategies and examples of students' difficulties and misconceptions. For instance, some of them observed teachers who were effectively incorporating technology in their lessons and decided to use similar types of activities in their lessons, and others observed how teachers were dealing with low-achieving students and realized that they would need to differentiate their teaching practices according to their students.

Implications

This study investigated how preservice teachers' pedagogical content knowledge was developed in the methods course and its associated field experiences. The findings revealed that the methods course and field experiences were essential but not enough to prepare preservice teachers for their future careers because some aspects of their pedagogical content knowledge were still weak. Although the preservice teachers participated in four different field experiences, most of them were for a short duration, and the preservice teachers taught at most one lesson in the schools. Having fewer field experiences where the preservice teachers spent more time in the same classroom might have afforded them an opportunity to develop a richer knowledge base about a particular set of students and content. Having more opportunities to teach lessons or implement activities they designed might have helped them transfer what they learned in the methods course to the practice of teaching. Grossman (1990) noted that methods courses are likely to provide an opportunity for preservice teachers to understand the purpose of teaching a particular subject-matter and acquire knowledge of teaching strategies that are the most appropriate to achieve that purpose. She also stressed that field experiences help preservice teachers learn about students' misconceptions, prior knowledge of particular topics, and the curriculum. Therefore, she suggested that the methods course should be accompanied by field experiences to enable preservice teachers to think critically about teaching and learning practices. Because finding quality field placements is challenging and the logistics of arranging field placements are time consuming, alternatives to traditional field experiences could afford preservice teachers with opportunities to apply what they are learning in a methods course to the practice of teaching. For example, Tamir (1988) found that microteaching activities help preservice teachers improve their pedagogical content knowledge because they need to think about what strategies, examples, representations and materials they will use to teach a particular topic and how they will assess students' understanding. The preservice teachers in this study had only one opportunity to engage in microteaching and found it valuable, so the practice of microteaching, which was popular in the 1980s but has waned in popularity recently, might be used more extensively in methods courses. In addition, analyzing student work and watching videotapes showing students discussing the solution of a problem could also provide authentic contexts in which preservice teachers could develop their pedagogical content knowledge.

Many preservice teachers perceive the methods course as a platform where they will learn how to teach mathematics (Ball, 1990b; Grossman, 1990) and expect to learn how they can help students to understand and to do mathematics. Ball (1990b) described the methods course as "about acquiring new ways of thinking about teaching and learning. But it is also about pedagogical ways of doing, acting, and being as a teacher" (p. 10). Graeber (1999) noted that methods courses address pedagogical content knowledge and curricular knowledge by their very nature because preservice teachers focus on planning instruction to facilitate students' understanding of the subject-matter. However, preservice teachers may fail to understand the purpose of specific course practices and their relation to developing pedagogical content knowledge. Therefore, exposing preservice teachers to the elements of pedagogical content knowledge and being explicit about which aspects of their knowledge are being developed by specific course activities could be useful. Further, asking preservice teachers to reflect on course activities and on their knowledge development periodically could give instructors a better sense of how the course is being perceived by preservice teachers.

The preservice teachers in this study lacked a conceptual understanding of some high school mathematics topics despite having taken a number of advanced mathematics content courses (such as linear algebra, abstract algebra, and statistics) and three courses designed to allow them to explore high school content from an advanced perspective. The preservice teachers tended to rely on their memories of their high school mathematics courses rather than drawing on their more recent experiences in college courses when answering content-specific questions during the interviews. Because they did not take advantage of their more recent mathematics experiences, their responses did not suggest effective ways to address students' errors or appropriate plans to teach a concept. These preservice teachers needed opportunities to activate and deploy their knowledge in the service of instructional activities.

Ball (2003) noted that increasing the number of mathematics courses that preservice teachers take may not increase the quality of their teaching unless they are equipped with mathematical knowledge and skills that enable them to teach mathematics effectively. In addition, Ball and her colleagues (Ball, Thames, & Phelps, 2008) pointed out that

> Unfortunately, subject-matter courses in teacher preparation programs tend to be academic in both the best and worst sense of the word, scholarly and irrelevant, either way remote from classroom teaching….Although there are exceptions, the overwhelming majority of subject-matter courses for teachers, and teacher education courses in general, are viewed by teachers, policy makers, and society at large as having little bearing on the day-to-day realities of teaching and little effect on the improvement of teaching and learning. (p. 404)

Ferrini-Mundy and Findell (2001) mentioned three approaches that could be used to create connections between undergraduate mathematics and high school mathematics: mathematical, integrative, and emergent. The mathematical approach refers to providing opportunities for preservice teachers to study high school mathematics from an advanced standpoint. They pointed out that this approach runs the risk of providing a limited opportunity for development of pedagogical content knowledge because the main goal is to support preservice teachers' subject-matter knowledge rather than their knowledge of teaching mathematics. In an integrative approach the goals of content and pedagogy courses are intertwined to enable preservice teachers to see the connections between them better. In the emergent approach preservice teachers analyze an act of teaching using of videos of classrooms, student work, or written cases and determine what mathematical knowledge teachers need in that situation. The teacher education program in which preservice teachers in this study were enrolled used the mathematical approach, albeit apparently with limited success. The other two approaches would be worthy of consideration as well.

Suggestions for Further Studies

In this study it was difficult to make inferences about the level of a person's pedagogical content knowledge when the person did not have robust content knowledge of the mathematics embedded in a task. Therefore, researchers might limit the level of the mathematics involved in questions such as analyzing student errors. For instance, for secondary preservice teachers, tasks might involve knowledge of integers, irrational numbers, variable expressions, functions and their graphs, and trigonometry rather than complex numbers or other topics for which they are less likely to have robust conceptual understanding.

As noted in the limitations section, I was not able to make claims about growth in preservice teachers' pedagogical content knowledge because that knowledge was mitigated by their content knowledge. Thus, in future studies researchers might use the same tasks as the beginning and end of the study to allow for an analysis of the development of knowledge. In particular, using the same tasks would allow researchers to determine whether the preservice teachers are able to enrich their repertoire of teaching strategies and improve their ability to address students' errors over the course of the study.

Another study similar to this one might be conducted over a longer period of time, such as during the methods course and also during the student teaching period. Because pedagogical content knowledge is dynamic (Borko & Putnam, 1996), we would expect preservice teachers' knowledge to grow and change as they have more opportunities to plan and teach lessons. Seeing preservice teachers deploy their pedagogical content knowledge in planning for and teaching students in an authentic setting would give researchers more opportunities to see knowledge use, growth, and development.

Concluding Remark

The major goal of teaching is to enhance students’ understanding and learning. Teachers need to be equipped with various knowledge and skills to establish and maintain effective teaching environments that enable them to achieve that goal. Therefore, teacher education programs should provide opportunities for preservice teachers to develop their knowledge of and skills for effective teaching. Preservice teachers need to learn the conceptual foundations of the subject-matter and how to tailor their instruction to a particular group of students. That is, they need to learn how students learn, what teaching strategies facilitate students’ learning and understanding, and what instructional tools help them to prepare effective lessons. But at the same time they have to manage the classroom and keep students engaged. Therefore, not only the methods course but also other courses (e.g., content, pedagogy, curriculum) offered in the teacher education program should address those issues. Finally, preservice teachers need opportunities to spend time in classrooms as observers and practitioners to learn more about students and teaching and to internalize and actualize their gains from methods courses.

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APPENDIX A

Questionnaire

Instruction: For each of the following items choose the response that best fits you.

1. At the end of my degree program I will have taken enough content courses to be an effective mathematics teacher in grades 6-12.
    a. Agree
    b. Somewhat agree
    c. Disagree
2. At the end of my degree program I will have taken enough courses about teaching mathematics to be an effective mathematics teacher in grades 6-12.
    a. Agree
    b. Somewhat agree
    c. Disagree
3. I know what mathematics content is to be addressed in each year of the 6-12 mathematics curriculum.
    a. Agree
    b. Somewhat agree
    c. Disagree

4. I know possible difficulties or misconceptions that students might have in mathematics in grades 6-12.
    a. Agree
    b. Somewhat agree
    c. Disagree
5. I have a sufficient repertoire of strategies for teaching mathematics.
    a. Agree
    b. Somewhat agree
    c. Disagree
6. I know how mathematical concepts are related.
    a. Agree
    b. Somewhat agree
    c. Disagree
7. I know how to integrate technology in mathematics lessons.
    a. Agree
    b. Somewhat agree
    c. Disagree
8. I know how to diagnose and eliminate students’ mathematical difficulties and misconceptions.
    a. Agree
    b. Somewhat agree
    c. Disagree

9. Read the definitions of the following Knowledge Bases:

Knowledge of subject-matter: To know mathematical concepts, facts, and procedures, the reasons underlying mathematical procedures and the relationships between mathematical concepts.

Knowledge of pedagogy: To know how to plan a lesson and use different teaching strategies.

Knowledge of learners: To know possible difficulties, errors, and misconceptions that students might have in mathematics lessons.

Knowledge of curriculum: To know learning goals for different grade levels and how to use different instructional materials (e.g., textbook, technology, manipulatives) in mathematics lessons.

How do you perceive your knowledge level in each knowledge base identified above? Use the following scale: 1-not adequate 2-adequate 3-competent 4-very good

Knowledge of subject-matter: ……

Knowledge of pedagogy: ……

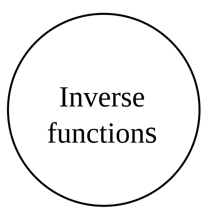
Knowledge of learners: ……

Knowledge of curriculum: ……

10. Look at the student work given below. How can you explain to the student that his or her solution is incorrect?

$$\sqrt{9x^2 + 25y^4} = 3x + 5y^2$$

11. Assume that you will introduce "inverse functions". Make a concept map for inverse functions showing which mathematical concepts or facts relate to inverse of functions.



12. If you were introducing how to factor trinomials, which of the following trinomials would you use first? Explain your reasoning.

    $2x^2 + 5x - 3$, $x^2 + 5x + 6$, $2x^2 - 6x - 20$

13. Assume that you will teach the following topics in a semester. In which order would you teach them to build on students' existing knowledge? Explain your reasoning.

    Polynomials, trigonometry, factorization, quadratic equations

APPENDIX B

Interview Protocols for Preservice Teachers

Protocol for Interview 1

1. What do you expect to learn in the methods course and your field experience that will contribute to your four knowledge bases: knowledge of subject-matter, knowledge of pedagogy, knowledge of learners, and knowledge of curriculum?
2. What kind of difficulties or misconceptions do you think a secondary school student might have in algebra? What might be the reasons for such difficulties or misconceptions?
3. Would you give some examples of integrating technology into mathematics lessons?
4. Assume that you will teach quadratic functions.
    - What do you expect that your students should already know?
    - How do you introduce quadratic functions to your students?
5. Assume that one of your students asks for your help in multiplying binomials. How do you help him or her?
6. Students may confuse similar and congruent triangles. What kind of examples do you give them to clarify the differences between similar and congruent triangles?
7. Assume that one of your students got confused when he or she found $2 = 0$ as the result of the solution of a system of linear equations. How do you explain to him or her the meaning of this result?

8. Look at the student work given below. How can you explain to the student that his or her solution is incorrect?

$$\frac{2x^3y^2 - 6xy}{3xy^2 - x^3y^3} = \frac{2x^3\cancel{y}^2 - 6\boxed{x}y}{3\boxed{x}\cancel{y}^2 - x^3y^3} = \frac{2-6y}{3-y^3}$$

Protocol for Interview 2

1. Think about the concepts you have discussed so far in your methods course and field experience.
    - How do they contribute to your knowledge of subject-matter?
    - How do they contribute to your knowledge of pedagogy?
    - How do they contribute to your knowledge of learners?
    - How do they contribute to your knowledge of curriculum?
2. Would you please briefly summarize each of your field experiences?
    - What was the grade level?
    - Which topics were discussed?
    - Was there anything interesting in terms of students' mathematical thinking or misconceptions?
    - If you were supposed to teach the following lesson, what might be your plan for that lesson?
3. For each day you were in the field would you tell me about what teaching actions including questioning techniques and the tasks discussed in the class, that your mentor teachers took were particularly noteworthy? Why?

4. Assume that you will introduce how to graph linear functions. Here are some examples of linear equations. In which order you would like to use these equations? Tell me your reasoning.

   $2x + 3y = 6 \qquad y = 5 \qquad y = x + 5 \qquad 3x - 8y + 12 = 0$

5. Assume that you will teach the following topics in a semester. In which order would you teach them to build on students' existing knowledge? Tell me your reasoning.

   Imaginary numbers, exponents, trigonometry, quadratic functions

6. Look at the student work given below. How can you convince your student that his/her answer is invalid? Why being able to solve such equations is important—what is the relation to other math topics?

$$
\begin{aligned}
2x^4 - 18x^2 &= 0 \\
2x^4 &= 18x^2 \\
\frac{2x^4}{2} &= \frac{18x^2}{2} \\
x^4 &= 9x^2 \\
\frac{x^4}{x^2} &= \frac{9x^2}{x^2} \\
x^2 &= 9 \\
x &= \mp 3
\end{aligned}
$$

7. What else do you want to learn about and discuss more in your methods and field experience course?

Protocol for Interview 3

1. Think about the concepts and issues you have covered in your methods course and field experience course. Below, you are given a list of some of those issues. For each of the item please discuss:

  - How does it contribute to your knowledge of subject-matter?
  - How does it contribute to your knowledge of pedagogy?
  - How does it contribute to your knowledge of learners?
  - How does it contribute to your knowledge of curriculum?

Some of the issues and activities covered in the methods course:

- Problems of the day
- Learning theories (behavioral theories, social cognitive theory, constructivism; cooperative learning)
- Standards-based curricula--textbooks
- Motivation
- Promoting communication in classroom (student-talk vs teacher-talk; discourse)
- Manipulatives
- Planning instruction (traditional vs inquiry based vs developmental; lesson planning)
- Microteaching

Some of the issues and activities covered in the field experience:

- Effective questioning
- Cognitive demand of a task
- Classroom management
- Assessment (rubrics)
- Field experience

Knowledge of subject-matter: To know mathematical concepts, facts, and procedures, the reasons underlying mathematical procedures and the relationships between mathematical concepts.

Knowledge of pedagogy: To know how to plan a lesson and use different teaching strategies.

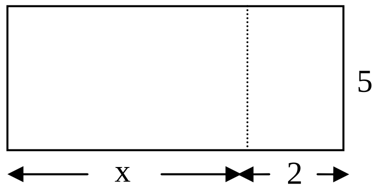
Knowledge of learners: To know possible difficulties, errors, and misconceptions that students might have in mathematics lessons.

Knowledge of curriculum: To know learning goals for different grade levels and how to use different instructional materials (e.g., textbook, technology, manipulatives) in mathematics lessons.

2. Would you please briefly summarize the process of how you were prepared for the whole-class teaching experience?
   - What factors did you specifically pay attention to when planning your lesson? (e.g., students, timing, management of class, and your mentor teachers' suggestions)
   - Did you use any concepts that you discussed in the methods course when preparing your plan?
   - Was there anything interesting in terms of students' mathematical thinking or misconceptions?
   - What would you do differently if you were to teach the lesson again?
   - What might be your plan for a follow up lesson?
3. Look at each of the student work given below. How can you explain to the student that his or her solution is incorrect?

a)
$$
\begin{aligned}
-2x+5 &\le x-1 \\
-2x-x &\le -1-5 \\
-3x &\le -6 \\
x &\le 2
\end{aligned}
$$

b)



Student's answer:

Perimeter = 2 x (length + width)

$2(2x + 5) = 4x + 10$

Perimeter: $4x + 10$

4. Give me at least three examples that you might use when you introduce addition and subtraction with radicals. Why do you choose those examples?
5. Assume that you will introduce solving rational equations. Here are some examples of rational equations. In which order would you like to use these equations? Tell me your reasoning.

   a) $\dfrac{2}{x(x-2)} = \dfrac{1}{x-2}$ b) $\dfrac{1}{x-4} = \dfrac{2}{3x+1}$ c) $\dfrac{5}{x} + \dfrac{x}{x-1} = 1$ d) $\dfrac{3}{x+1} = \dfrac{x}{2}$

6. Assume that you will teach the following topics in a semester. In which order would you teach them to build on students' existing knowledge? Tell me your reasoning.

   Ellipses, quadratic formula, transformations, parabolas

7. Tell me three topics/mathematical concepts that should be covered before teaching logarithms. Explain your reasoning.
8. Assume that you are preparing a lesson plan for teaching trigonometry. How do you motivate students to learn trigonometry?
9. Students may confuse permutation and combination. What kind of examples do you give them to clarify the differences between permutation and combination?
10. How do the methods course and field experience contribute to your repertoire of students' difficulties and misconceptions in algebra?
11. What else do you think should have been discussed in the methods course and field experience course?

APPENDIX C

Interview Protocol for Instructors

1. What are the topics you plan to cover in the methods course / field experience course?
2. What are your goals for each topic? Tell me in detail.
    - Why do you think that it should be discussed in the methods course / field experience course?
3. What does a typical lesson look like?
4. How do you choose the materials or tasks for the course?
5. What are your criteria for assessment?