GRAPHIC ORGANIZERS APPLIED TO SECONDARY ALGEBRA INSTRUCTION

FOR STUDENTS WITH LEARNING DISORDERS

by

ROBERT TODD IVES

(under the direction of Noel Gregg)

ABSTRACT

Students who have particular difficulty in mathematics are a growing concern for educators. These students may struggle with mathematics because of mathematics specific disorders as well as oral language, reading disorders, and attention disorders. In spite of this growing concern, there is little research on teaching methods and techniques to help these students. This lack of empirical evidence for intervention is increasingly apparent for secondary the mathematics. Empirical evidence does support that students with language and reading disorders appear to benefit from methods and techniques that incorporate concrete and spatial representations of concepts in the learning of basic mathematics skills. However, there is little research to determine if similar efforts would be effective for more advanced mathematics content such as secondary algebra. Graphic organizers, which have been widely used and documented for improving reading comprehension, may be a technique that can be modified for upper level secondary mathematics content. The purpose of this investigation was to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts involved in solving systems of linear equations by students identified as having learning disabilities or attention disorders. Two replications of the application of a two group comparison of means design were carried out. In each replication, one group was taught to solve systems of linear equations through direct and strategy instruction. The other was taught with the same methods into which a graphic organizer was incorporated. Results of immediate posttests indicated that in both replications the students who worked with the graphic organizers demonstrated better performance in solving systems of equations as well as in understanding the concepts that justify the process for solving these systems. The difference in understanding concepts was maintained on a posttest after two to three weeks, but the difference in ability to solve systems of equations was not.

INDEX WORDS: Algebra, Dyscalculia, Graphic organizers, Intervention, Learning disabilities, Learning problems, Mathematics disorders, Regular and special education relationship, Secondary mathematics, Social validity, Attention, Mathematics skills

GRAPHIC ORGANIZERS APPLIED TO SECONDARY ALGEBRA INSTRUCTION FOR STUDENTS WITH LEARNING DISORDERS

by

ROBERT TODD IVES

B.A., Rice University, 1977

M.Ed., University of St. Thomas, 1993

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2002

GRAPHIC ORGANIZERS APPLIED TO SECONDARY ALGEBRA INSTRUCTION

FOR STUDENTS WITH LEARNING DISORDERS

by

ROBERT TODD IVES

Approved:

Major Professor: Noel Gregg

Committee: David Gast
Cheri Hoy
Jeremy Kilpatrick
Stephen Olejnik

Electronic Version Approved:

Gordhan L. Patel
Dean of the Graduate School
The University of Georgia
August 2002

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

> Many important mathematicians and scientists have had great difficulty with lower mathematics but excel at higher mathematics – which is less mechanical, less memory based – but often more visual, more logical based, more conceptual, more philosophical. Einstein had such difficulties – as did Stephen Wolfram, the founder of Wolfram Research and the inventor of the high level, general purpose professional mathematics software program called "Mathematica." (West, 2000, p. 25)

History provides examples of several distinguished mathematicians for whom basic mathematics skills, such as memorization of mathematics facts and rote application of algorithms, were both tedious and difficult. In contrast, these scholars were much more successful with the flexible, sometimes more spatial, thinking required for more advanced mathematics problem solving. In six years of experience as a secondary mathematics teacher of students with learning disabilities, I have seen the same pattern in many of those students. Quite a few of those students performed within the average range across curricula such as algebra, precalculus, and even calculus. Yet these same students often had significant deficits in language and reading skills, and struggled desperately to memorize basic mathematics facts.

Roughly 6% of students in the United States have some form of calculation disorder (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996). Similar figures have been found in the former Czechoslovakia, Europe, and Israel as well (Gross-Tsur et al., 1996; Sharma, 1986). Reading disorders appear to be comorbid with calculation disorders in roughly 40-50% of these students (Badian, 1983; Gross-Tsur et al., 1996). In fact, typical approaches for subtyping mathematics disabilities consistently identify some types of calculation deficits as analogous to reading and other language problems (Badian, 1983; Geary, 1993, 2000; Hécaen, Angelergues, & Houillier, 1961; Kosc, 1974; Marolda & Davidson, 2000; Padget, 1998; Rourke, 1989; Rourke, 1993; Rourke & Conway, 1998; Silver, Pennett, Black, Fair, & Balise, 1999). Some authors have suggested that mathematics and reading disabilities might have similar etiologies (Geary, 2000; Rourke, 1993) or that the calculation deficits are secondary to the reading problems (Padget, 1998). Further investigation of the bi-directional influence of reading and mathematics abilities is a critical need in the literature pertaining to students with learning disabilities.

In spite of the prevalence of mathematics calculation disorders, researchers have not addressed calculation problems as extensively as reading disorders (Geary, 2000; Kulak, 1993; Padget, 1998; Rourke & Conway, 1998). The lack of attention to mathematics and learning disabilities is evident within the intervention literature. For example, Mastropieri, Scruggs, and Shiah (1991) carried out an extensive search for studies of interventions in mathematics instruction for students with learning disabilities and found a total of only thirty over a 14 year period (1975-1988).

Although research on interventions for basic mathematics skills is limited, empirical studies focused on interventions for higher level mathematics skills with students with learning disabilities are almost nonexistent. For example, a more recent review of intervention studies targeting algebra skills in students with learning disabilities turned up only seven studies over a 27 year period (1970-1996) (Maccini, McNaughton, & Ruhl, 1999). A hand search covering 10 years (1990-1999) of four journals (*Journal of Learning Disabilities*, *Learning Disability Quarterly*, *Learning Disability Research & Practice*, *Exceptional Children*) identified 13 studies investigating interventions for basic calculation skills, and 10 studies investigating interventions to improve problem solving based on basic mathematics skills, but only one that addressed algebra or any other higher level mathematics skills, for a total of 24 studies on mathematics interventions (Ives, 2000). These numbers contrast with 30 studies addressing interventions for basic reading and another 32 studies addressing interventions for reading comprehension, for a total of 62 studies on reading interventions based on the same search. Not only are there far fewer studies addressing disabilities in mathematics, but also not a single one of these studies attempted to address aptitude-treatment interactions across subtypes of mathematics disabilities. Further, texts intended to survey intervention approaches in general for students with learning disabilities and mild disabilities, as well as those specifically targeting secondary students with disabilities and teaching mathematics to students with disabilities, fail to even mention these higher level mathematics skills (Bender, 1996; Bley & Thornton, 1995; Jones, Wilson, & Bhojwani, 1998; Masters, Mori, & Mori, 1993; Mastropieri & Scruggs, 1994; Meese, 1994; Retish, Hitchings, Horvath, & Schmalle, 1991). This dearth of research on interventions for upper level mathematics exacerbates the challenge of working with students who have a poor grasp of mathematics facts as well as poor reading or language skills.

Mainstream classroom instruction in mathematics assumes adequate learner language and reading competence (Bley & Thornton, 1995; Moses & Cobb, 2001;

Rivera, 1998). This reliance presents a challenge for students who demonstrate learning disorders. Over the years, teachers and researchers have developed several techniques to illustrate mathematics concepts that are not dependent solely on reading and language comprehension (Harris, Miller, & Mercer, 1995; Jitendra & Hoff, 1996). These techniques include the use of concrete objects and spatial representations of relationships that may not rely as heavily on language and reading skills. One technique is the concrete-semiconcrete-abstract (CSA) sequence for mathematics instruction. CSA has shown positive results in teaching basic mathematics to students identified with learning disabilities in general, and mathematics disabilities in particular (Harris et al., 1995; Marzola, 1987; Miller & Mercer, 1993; Miller, Mercer, & Dillon, 1992; Peterson, Mercer, & O'Shea, 1988). The CSA sequence begins with the use of concrete objects (manipulatives) to demonstrate mathematics concepts and relationships. Instruction then moves from these concrete tools to the abstract symbols of conventional mathematics. This sequence is designed to create a bridge between concrete and abstract representations (Heddens, 1986). As such, the sequence may help introduce students to the concepts and relationships being represented and drawing less heavily on language skills during this introduction.

Although a valuable technique for mathematics instruction, the CSA sequence cannot be readily applied to all types of mathematics. In particular, higher level mathematics concepts and relationships often do not lend themselves to concrete models. As a result, the literature in mathematics intervention provides limited support for teachers who want to help students with these higher level mathematics concepts and relationships when those students demonstrate significant language and reading deficits.

A promising technique from the reading comprehension literature might provide assistance in the teaching of higher level mathematics to students with language and reading deficits. Students who are trying to understand concepts and relationships in their reading have been given the opportunity to apply graphic organizers in a variety of ways to improve their reading comprehension (Alvermann & Swafford, 1989; Dunston, 1992; Moore & Readance, 1984; Rice, 1994; Robinson, 1998; Swafford & Alvermann, 1989). In these studies, graphic organizers are spatial arrangements of words, phrases, and sentences, and they may also include graphic elements such as arrows, and boxes. The spatial and graphic features are intended to indicate relationships between the verbal elements. The concept of graphic organizers can be expanded and modified to apply to mathematics content. In particular, the verbal elements could be replaced by mathematical symbols, expressions, and equations. In this way graphic organizers may be useful for helping students understand concepts and relationships that involve these mathematical symbols, expressions, and equations and that can be represented spatially.

Statement of Need

Some techniques for teaching mathematics include concrete and spatial elements that may reduce, or provide alternatives to, the reliance of instruction on reading and language comprehension. However, these techniques have not been applied to higher level secondary mathematics and may not be applicable to some of this content. Consequently, there is a need to develop instructional techniques for secondary mathematics, such as the use of graphic organizers, that may provide a means of making some upper level secondary mathematics concepts and relationships more accessible to students, including those with language and reading disorders.

Statement of the Problem

The purpose of this investigation is to address the question of whether integrating graphic organizers into instruction that already incorporates strategy instruction and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts involved in solving systems of linear equations by students identified as having learning disabilities or attention disorders. Solving systems of linear equations is typically first covered in a first year algebra course (Larson, Kanold, & Stiff, 1993a; McConnell et al., 1993; Saxon, 1993), when students are taught to solve systems of two linear equations with two variables through several methods. The topic is then revisited in a second year of algebra (Coxford & Payne, 1990; Larson, Kanold, & Stiff, 1993b; Nichols et al., 1986; Saxon, 1997; Senk et al., 1993) when systems of three linear equations with three variables are solved.

The mathematical topic of systems of equations was selected for two reasons. First, solving these systems requires ongoing application of several basic algebraic concepts. The graphic organizer and instruction were designed to reinforce those concepts. Second, the topic requires that students carry out a relatively long, multi-step process. Thus the graphic organizer was designed to provide guidance through the complex procedure as well as facilitate application of concepts.

Definitions of Terms

This section specifies the definition of several terms as they are applied specifically in the present studies.

Algebra

According to Karush (1962), algebra, as seen in the secondary curriculum can be described as "the study of operations and relations among numbers through the use of variables" (p. 4).

Attention-Deficit/Hyperactivity Disorder

The American Psychiatric Association (2000) described Attention-Deficit/Hyperactivity Disorder (ADHD) as "a persistent pattern of inattention or hyperactivity-impulsivity that is more frequently displayed and more severe than is typically observed in individuals at a comparable level of development" (p. 85).

Dyscalculia

The definition that Kosc (1974) proposed for developmental dyscalculia is as follows:

> Developmental dyscalculia is a structural disorder of mathematical abilities which has its origins in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions. (p. 47)

Kosc's definition is applied in this dissertation. He stressed that dyscalculia is a disorder specific to mathematics rather than the result of general cognitive deficits. He further indicated that dyscalculia is often associated with other problems in symbolic processing, such as dyslexia. Kosc also was careful to exclude underachievement in mathematics that was attributable to external factors such as poor instruction or health problems.

Graphic Organizer

For purposes of this dissertation, a graphic organizer is a display that presents information using verbal or mathematical symbols and also visual/spatial elements (Alvermann, 1981). The visual/spatial elements indicate relationships between the symbolic elements. Examples of graphic organizers with verbal symbols include tables, outlines, and idea webs. When applied to mathematics, graphic organizers include mathematical symbols, expressions, and equations; they do not include words, phrases, and sentences.

Learning Disability

"Learning disabilities is a general term that refers to a heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning, or mathematical abilities" (National Joint Committee on Learning Disabilities, 1990, January).

Mathematics Disorder

According to the American Psychiatric Association (2000), Mathematics Disorder is evidenced by "mathematical ability that falls substantially below that expected for the individual's chronological age, measured intelligence, and age-appropriate education" (p. 53). This definition includes calculation as well as mathematics reasoning.

Representational System

A representational system is a set of interrelated and internal forms of information used in memory (Sadoski & Paivio, 2001). In less formal terms, a representational system describes how information occurs in our minds so that we can remember it and think about it.

Social Validity

Social validity, in the context of educational interventions, is the value or importance of that intervention as perceived by the consumers of that intervention (Schwartz, 1996). Interventions consumers may include students, teachers, parents, administrators and any other interested parties.

Research Questions

Q1 Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?

Q2 Will the difference in performance cited in the first research question be maintained for two to three weeks after instruction and immediate posttesting are completed?

Q3 Will the use of graphic organizers to teach secondary students with learning disabilities or attention disorders to solve systems of equations with two linear equations in two variables demonstrate social validity?

Q4 Will the findings of the first and third questions be replicated when graphic organizers are used to teach secondary students with learning disabilities or attention disorders to solve systems of three linear equations in three variables?

Overview of Methods

The research questions were addressed through two replications. Each replication was a quantitative study comparing two groups of secondary students with learning disabilities or attention disorders. In each study one group received instruction in a topic in algebra that included application of a graphic organizer designed to provide spatial associations to the material being taught. The other group in each study received instruction that was verbally equivalent but without the graphic organizer. Results included quantitative outcome measures of mathematics achievement, as well as qualitative measures of social validity.

Research on the effectiveness of graphic organizers applied to upper level mathematics instruction must first determine if they contribute to conventional instructional approaches for students with learning disabilities. The approach using graphic organizers is designed for group instruction, and the students are typically taught in either inclusive settings or mixed disability groups. Group design is appropriate for these research questions. Questions about aptitude-treatment interactions across subtypes of dyscalculia, individual differences, student and teacher attitudes, etc., are important to the future of this line of investigation, and some of these questions might be addressed through other methods. However, addressing such questions is currently premature.

CHAPTER 2

REVIEW OF RELEVANT LITERATURE

Students who have difficulties with language can struggle with mathematics instruction in a variety of ways (Bley & Thornton, 1995; Miller & Mercer, 1998). For example, they may have difficulty following or understanding instruction, understanding mathematical terms, reading or understanding word problems, recognizing variations on tasks, verbalizing what they know, or identifying irrelevant information. These students are found in classrooms along with other student who do not have to face these language challenges. In an effort to support these students, the purpose of this investigation was to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts involved in solving systems of linear equations by students identified as having learning disabilities or attention disorders.

This chapter begins with a review of specific instructional methods and techniques that have received recent attention in the literature pertaining to students with learning disabilities. A critique of the strengths and weaknesses of these methods and techniques is presented. Specific attention is focused on the use of graphic organizers in the teaching of higher level secondary mathematics concepts and procedures to students with learning disorders (i.e., learning disabilities and ADHD).

The use of graphic organizers to teach higher level secondary mathematics concepts and procedures to students with special learning needs is justified in light of current research exploring subtypes of mathematics disabilities. Finally, the Dual Coding Theory (Paivio, 1986) is presented as the theoretical framework for the studies.

Instructional Methods and Techniques for Teaching Mathematics

Three instructional methods (i.e., strategy instruction, direct instruction, and schema-based instruction) and three instructional techniques (i.e., manipulatives, the concrete-semiconcrete-abstract sequence, and graphic organizers) are critiqued as to their strengths and weaknesses in the teaching of higher level secondary mathematics concepts and procedures to students with specific needs. The methods and techniques reviewed were selected based on their strong empirical base for students with disabilities.

Strategy Instruction

The strategy deficit model is a relatively recent approach to describing learning disabilities that developed out of the work of Deshler (Deshler & Lenz, 1989; Deshler & Schumaker, 1993) and Swanson (Swanson, 1989, 1993; Swanson, Christie, & Rubadeau, 1993; Swanson & Cooney, 1985; Swanson & Rhine, 1985). Swanson, Hoskyn, and Lee (1999) described a strategy as "made up of two or more goal-oriented tactics and sequential methods" (p. 24). These strategies are directed towards improving higher level thinking processes such as problem solving, comprehending, and learning rules or algorithms. According to Swanson's model, students with learning disabilities have difficulty with various thinking tasks because they lack certain effective strategies, choose inappropriate strategies, and fail to self-monitor their efforts. Strategy instruction has been widely applied to mathematics topics, especially to problem solving (Hutchinson, 1993; Keeler & Swanson, 2001; Maccini & Hughes, 2000; Montague, 1992, 1998; Montague, Applegate, & Marquard, 1993).

Swanson et al (1999) noted, “Strategy instruction is distinguished from other approaches because of instructions directing students to access information from long-term memory about procedural knowledge” (p. 24). Specific instructional practices that Swanson attributed to strategy instruction include (a) daily review, (b) statements of instructional objectives, (c) teacher presentation of new material, (d) guided practice, (e) independent practice, (f) formative evaluation, (g) verbal modeling by the teacher of the steps or processes, (h) elaborate explanations to guide task performance, (i) reminders to use strategies or procedures, (j) multistep instructions, and (k) verbal dialogue. Many of these elements rely heavily on the oral and written language skills of the students, both receptive and expressive. For example, daily review, presentation of material, verbal modeling, explanations, reminders, instructions, and dialogue all often take place through oral language, and sometimes written language. Evaluation and practice typically include much written language as well. Students with language difficulties would be at a disadvantage in lessons taught using strategy instruction alone because many elements of this method rely on effective language skills in the student.

Direct Instruction

Direct instruction is another teaching approach that appears to rely heavily on oral language skills. Swanson et al (1999) noted that direct instruction shares some common characteristics with strategy instruction, including (g) verbal modeling by the teacher of the steps or processes, (h) elaborate explanations to guide task performance, (i) reminders to use strategies or procedures, (j) multistep instructions, and (k) verbal dialogue. As noted, these elements generally are carried out through oral and written language. In addition, Swanson also noted some elements of direct instruction that were not included in his definition of strategy instruction. These unique elements included (l) modeling of the skill by the teacher, (m) breakdown of the task into smaller steps, (n) repeated probes and feedback, (o) prescribed material at a rapid pace, and (p) directed questions related to skills. Task breakdown, probes and feedback, and questioning all depend on the use of language between the teacher and the students. As with strategy instruction, students with language difficulties may be at a disadvantage in lessons taught solely using direct instruction.

Schema-based Instruction

Jitendra and colleagues (Jitendra et al., 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999) have used a schema based approach to improving word problem solving abilities in students who are at risk and students who have learning disabilities. Generally students were trained to distinguish between three types of word problems, and then were taught to use specific diagrams, provided by the investigators, to solve each type of problem. Each diagram consisted of a few geometric shapes and a few words. The diagrams were designed to indicate specific places for entering data from the problem, and thereby indicate the relationships between those data, which would in turn indicate what actions to take to solve the problem. This work is quite similar to earlier work (Fuson & Willis, 1989; Willis & Fuson, 1988) with second grade students in general education classes who were learning to solve one-step addition and subtraction problems. This approach is based on the theory that successful problem solving derives from the effective selection and application of schemata that represent the elements of the problem in a meaningful way, and that these schemata can be taught directly. As with concrete manipulatives, students with relative weaknesses in language skills may particularly benefit from schema-based approaches to problem solving that take advantage of spatial skills for understanding relationships.

Manipulatives and Concrete-Semiconcrete-Abstract Sequencing

Stellingwerf and Van Lieshout (1999) defined manipulatives as “concrete external representations” of mathematics problems (p. 459). They are often used in the first phase of a three phase teaching sequence for representing problems that is known as the concrete-semiconcrete-abstract (CSA) sequence (Miller & Mercer, 1993). In the concrete phase of instruction the skills are presented using physical objects. These objects are manipulated to perform the required task. The objects may also be paired with abstract representations (numbers, signs, etc.) from the beginning, or the abstract representations may be introduced later. In the semiconcrete phase the skills are performed using drawings or some other iconic representation of the manipulatives rather than the manipulatives themselves. At this point the abstract representations are paired with the semiconcrete representations. Finally, in the abstract phase, students practice the skills using only the abstract representations. This sequence is based on the belief that students first learn mathematics through manipulating concrete objects, which prepares them to learn through pictures, which in turn prepares them to learn through abstract representations (Miller & Mercer, 1993).

The CSA sequence assumes that using concrete representations facilitates the acquisition of conceptual understanding. In contrast, strategy and direct instruction are not based on a sequence of representations. There is no requirement that concrete or semiconcrete representations be incorporated into instruction for either of these methods.

It should be evident that manipulatives and the CSA sequence specify elements that rely on spatial skills more than the strategy instruction and direct instruction approaches do.

The use of manipulatives and applying the CSA sequence to mathematics instruction are widely advocated for basic skills instruction. For example, Miller and Mercer (1993) identified materials for CSA sequencing in learning basic operations and place value, money, time, weight and measurement, fractions, decimals, percents, and geometry. Quite a few studies support the use of manipulatives in general, and the CSA sequence in particular, for instruction in these areas of mathematics for students with and without identified disabilities (Cain-Caston, 1996; Fueyo & Bushell, 1998; Harris et al., 1995; Maccini & Hughes, 2000; Marzola, 1987; Miller & Mercer, 1993; Miller et al., 1992; Peterson et al., 1988; Stellingwerf & Van Lieshout, 1999). Students with relative weaknesses in language skills may particularly benefit from opportunities to work with concrete materials to build their understanding of mathematics. Unfortunately, these concrete materials may not be readily applied to some higher level secondary mathematics topics such as imaginary numbers or logarithmic functions.

Graphic Organizers

As students move into higher levels of mathematics, concrete and spatial/visual teaching techniques are not only less well documented, they are also more difficult to devise. Manipulatives and the CSA sequence used during mathematics instruction are readily applied to elementary level mathematics skills such as performing basic operations, using money, and using place value. Unfortunately, there is an intervention vacuum, in research based concrete and spatial/visual techniques for teaching much of secondary algebra, functional analysis, or calculus. In fact, post-secondary texts intended to survey intervention approaches in general for students with learning disabilities and mild disabilities, as well as those specifically targeting secondary students with disabilities and teaching mathematics to students with disabilities, fail to even mention these higher level mathematics skills and concepts (Bender, 1996; Bley & Thornton, 1995; Jones et al., 1998; Masters et al., 1993; Mastropieri & Scruggs, 1994; Meese, 1994; Retish et al., 1991). Relevant concrete manipulatives for advanced mathematics concepts are hard to imagine and visual representations do not convey the meaning of the concepts.

Students with poor language skills would benefit from a visual approach for displaying complex mathematics relationships. Researchers in the field of reading have been looking at an approach to improving comprehension using visual displays to represent relationships between the pieces of information in a text. Perhaps these graphic organizers can be borrowed and applied to teaching higher-level mathematics concepts.

Reading Comprehension

The evolution and effectiveness of graphic organizers applied to reading comprehension is well supported in the literature (Alvermann & Swafford, 1989; Dunston, 1992; Moore & Readance, 1984; Rice, 1994; Robinson, 1998; Swafford & Alvermann, 1989). Ausubel recommended the use of advanced organizers as a means of improving comprehension of classroom learning tasks. Advanced organizers are short prose passages intended to provide scaffolding based on prior knowledge to facilitate incorporating new knowledge (Ausubel, 1960). Subsequent investigators suggested modifying the advanced organizer by using key vocabulary and short phrases rather than prose, and arranging these verbal elements in a visual/spatial configuration that would represent the relationships between the verbal elements (Earle, 1969; Estes, Mills, & Barron, 1969). These displays became known as structured overviews. However, research failed to consistently support the effectiveness of either advanced organizers or structured overviews, at least as they were applied to reading comprehension.

Arguing that the problem lay in student engagement rather than theory, subsequent researchers have attempted to show that graphic organizers can improve reading comprehension when the students are actively engaged in working with, or creating them. One approach has been to give students incomplete graphic organizers prior to reading. Students are required to complete the graphic organizer as they read. Alvermann and others have demonstrated fairly consistent, but small, effects of graphic organizers using this approach to engage the students (Alvermann, 1981, 1982; Alvermann & Boothby, 1983, 1986; Alvermann, Boothby, & Wolfe, 1984; Barron & Schwartz, 1984; Boothby & Alvermann, 1984). A second approach is to train students to create their own graphic organizers (in this case called graphic postorganizers), give them a novel text to read, and assess their comprehension (Barron & Schwartz, 1984; Bean, Singer, Sorter, & Frazee, 1986; Davidson, 1982; Dunston & Ridgeway, 1990; Griffin & Tulbert, 1995; Holley & Dansereau, 1984; Long & Adersley, 1984; Novak, Gowin, & Johansen, 1983; Vaughan, 1982). These studies have often found somewhat larger effect sizes and more consistently positive results for post organizers as opposed to preorganizers (Dunston, 1992; Griffin & Tulbert, 1995; Moore & Readance, 1984; Rice, 1994).

One criticism of these studies relates to method. Published studies have often not been specific about the depth and type of instruction that students received (Dunston, 1992; Robinson, 1998), making evaluation and replication difficult. In particular, it is often not clear whether, or how much, explicit instruction students have received concerning the relationships between verbal elements in organizers, how to use the organizers to learn the information, what information the graphic elements of the organizer represent, or how to construct organizers of their own. This kind of information is critical to an understanding of what makes graphic organizers effective.

A second concern is that these studies typically assess vocabulary and factual units, instead of relationships, as their dependent variables (Robinson, 1998). In spite of the fact that graphic organizers are intended to convey relationships visually, the learning of this information is often not evaluated. A revealing exception to this pattern is a study by Kiewra using researcher created organizers with forty-four college students (Kiewra, Dubois, Christian, & McShane, 1988). Kiewra compared the effectiveness of an outline, which is one-dimensional, with that of a matrix organizer, which is two-dimensional. Both organizers had identical content. Kiewra found that students using the matrix organizer, compared with students using the outline, were significantly more able to remember the relationships highlighted by the horizontal dimension of the matrix, which was not available to the latter students (Kiewra et al., 1988).

Mathematics

Graphic organizers rely on visual/spatial reasoning skills more than conventional teaching approaches do, and may be applied to the teaching of higher level mathematics. To this end, three important modifications to the use of graphic organizers are recommended here. First, the content of the graphic organizers in mathematics would no longer be verbal elements such as words, phrases, and sentences. Rather the content would be mathematical analogues to these verbal elements such as symbols (for numbers, variables, operations, inequalities, etc.), expressions, and equations.

Second, it is important to keep in mind that although acquiring basic mathematics skills often involves learning facts, higher level skills are concerned with concepts, patterns, and processes. As such, the goal in using graphic organizers for higher level mathematics is not to learn the mathematical elements. There is no point in memorizing the numbers, expressions, and equations in a task. The goal is to recognize and learn the patterns that relate these elements. This means that in the graphic display, the spatial arrangement of the mathematical elements, instead of the symbols, expressions, and equations, carries the information to be learned.

The results of a study by Earle (1969) are particularly relevant here. Earle worked with pairs of matched seventh and ninth grade mathematics classes. One class in each pair was designated as an experimental group and the other as a control group. Teachers were coached in creating structured overviews for a unit of study for their classes. The structured overviews consisted of key terms relevant to the units (e.g. polynomials, distributive property, completing the square) arranged in a diagram that spatially represented the hierarchical relationships between the terms. Instruction was carried out over a 16 day period and was designed to be equivalent in each paired set of classrooms except that instruction in the experimental groups referred to the structured overviews. A posttest of the terms that constituted the content of the structured overviews yielded no statistically significant differences between groups. However ad hoc follow-up tests of the relationships between the terms, as represented by the organizers, showed statistically significant differences in both pairs in favor of the experimental groups. Correlational effect sizes of these differences based on the *t*-scores and degrees of freedom (Friedman, 1968) were in the medium to large range based on Cohen's (1988) guidelines. These results suggest that incorporating graphic organizers into instruction should have an impact on assessment as well. If graphic organizers are used, then teachers should be sure to assess for understanding of the relationships represented by the spatial and visual elements of the organizer. In reading comprehension teachers often want students to learn the declarative elements in an organizer (e.g., character names in a novel), as well as the relationships between these elements (e.g., who is father to whom, who killed whom, etc.). However, the declarative knowledge is not always important in upper level mathematics. That is, we are not usually interested in having students memorize equations. Instead we would like students to learn the procedures for solving equations, for example, as well as the concepts that make sense of those procedures.

The third recommendation for applying graphic organizers to higher level mathematics instruction is that graphic organizers should be an integral part of good instruction, not a substitute for instruction. Graphic organizers should be incorporated into lessons such that the relationships students are to learn are explicitly taught and connected to the graphic organizers. Strategy instruction has been shown effective in helping students with learning disabilities to learn, and generalize strategies across a variety of subjects (Deshler, Ellis, & Lenz, 1996; Deshler & Lenz, 1989; Deshler & Schumaker, 1993), as well as in mathematics in particular (Mercer & Miller, 1992). Further, in a meta-analysis of studies of interventions for students with learning disabilities, approaches that included direct instruction or strategy instruction, or both, generally produced greater effect sizes than approaches without these features (Swanson et al., 1999). With respect to graphic organizers applied to mathematics, effective approaches may include not only instruction in the visual elements of the display, but also the use of the organizer as a learning guide, and the construction of organizers, when appropriate for the instructional objectives. The present studies were intended to test the effectiveness of using graphic organizers to teach higher level mathematics concepts and procedures as well as the stability over time, social validity, and replicability of those effects.

Mathematics Disabilities Subtypes and Mathematics Instruction

The literature in the field of learning disabilities pertaining to subtyping of mathematics disabilities based on student performance crossing neuropsychology with achievement tasks has limited application to understanding the learning of higher level mathematics to students with mathematics disabilities because the achievement tasks focus on recall of mathematics facts and calculation skills. Rourke and his colleagues (Ozols & Rourke, 1988; Rourke, Dietrich, & Young, 1973; Rourke & Finlayson, 1978; Rourke & Strang, 1978; Rourke, Young, & Flewelling, 1971; Strang & Rourke, 1983) identified two broad categories of learning disabilities: one verbal and one nonverbal. According to their findings, children in the nonverbal learning disabilities group performed poorly on basic arithmetic, although not showing deficits in spelling or word recognition. The children with verbal learning disabilities had less difficulty with basic arithmetic (although still performing below standardized norms) and performed much more poorly on spelling and word recognition tasks. Taken together these findings might suggest that children with verbal learning disabilities have difficulty with basic arithmetic because of some underlying cognitive deficit that also causes difficulty with spelling and word recognition, whereas children with nonverbal learning disabilities have difficulty with basic arithmetic because of cognitive deficits that do not impact spelling and word recognition.

Kosc's (1974; Sharma, 1986) system for subtyping developmental mathematics disabilities, or dyscalculia, includes six different types that may manifest separately or in combination. Three of these – the verbal, lexical, and graphical dyscalculias, - clearly parallel language disabilities. Verbal dyscalculia is evidenced by deficits in the ability to verbally name mathematical terms and relations orally or in writing. Lexical dyscalculia refers to difficulties with reading mathematical symbols. Graphical dyscalculia is a deficit in manipulating mathematical symbols in writing. Sharma (1986) has noted that these forms of developmental dyscalculia often occurs in combination with reading and written language difficulties.

Kosc's other three types of developmental dyscalculia do not have such clear connections to language problems. Practognostic dyscalculia is a disability that interferes with mathematical manipulation of concrete and pictorial representations. Ideognostical dyscalculia refers to difficulty with understanding mathematical concepts and calculations. Operational dyscalculia refers to a disturbance in the ability to carry out mathematical operations. The connections between these three subtypes and language disabilities, if any, are not clear.

Geary (1993; 2000) has proposed three subtypes of developmental mathematics disabilities. He attributes one subtype to deficits in semantic memory and also notes (Geary, 2000) that it "appears to occur with phonetic forms of reading disorder" (p. 6). Geary's second mathematical disorder subtype involves problems with procedural skills.

The third subtype in Geary's scheme reflects visuospatial deficits and is evidenced by misalignment of numerals, misinterpretation of place value, and difficulties with geometry. Geary reports that there is no clear relationship between these latter two subtypes of mathematical disorders and reading.

All three of these efforts to subtype developmental mathematics disabilities suggest that some students have difficulty with arithmetic because of language related problems, whereas others may have difficulty with arithmetic for reasons unrelated to language skills. However, these conclusions need to be qualified. First all three of these investigators identified students with mathematics disabilities through tests of calculation skills. Any generalization to problem solving or other higher level reasoning related to mathematics would be speculative. Second, there is no research to support a claim for aptitude treatment interactions based on any of these subtyping systems.

Paivio's Dual Coding Theory

If teaching techniques that incorporate the use of concrete objects and spatial representations of relationships facilitate learning by students with language and reading deficits, there may be some means of storing and processing information that are not based in language representations. These two types of representation would have to have some level of independence, but at the same time be mutually interactive. A theory of cognition that incorporates both language based and image based representation and processing was helpful for framing the rationale for the present studies as well as the results.

Paivio (1986) has proposed a general theory of mental representation and cognition that features a verbal versus nonverbal distinction. According to his Dual

Coding Theory (DCT), the two primary symbolic systems of cognition are language and imagery, which are essentially verbal and nonverbal, respectively. The logogen is a basic verbal unit, much like a node, in the verbal associative network. An imagen fills the same role in the nonverbal associative network. More specifically, they are modality specific units in their respective associative systems. Imagens may occur in any sensory modality, although visual images are more familiar, and more important for educational applications. These units can be activated by an external stimulus, or through their connections to other, previously activated, unit. Although Paivio does not specify a size for logogens or imagens, "logogens are word-like and imagens are object- or scene-like" (Sadoski & Paivio, 2001, p. 47). This arrangement leads to three types of connections or associations. There are representational connections between sensory systems and the two associative systems (verbal and nonverbal). There are associative connections within each system that connect logogens to each other in the verbal system and imagens to each other in the nonverbal system. Finally, there are also referential connections that connect imagens to logogens, and vice versa, between the two associative systems. These connections permit activation of logogens and imagens by external stimuli, other representations within their own system, and other representations from the other system.

The verbal system is a network of logogens. The strengths of connections are probabilistic and dynamic. They change based on frequency of associations and also depending on similarities between logogens. By contrast, the nonverbal system is a nested associative network. It is less constrained than the verbal system, and can be both continuous and discontinuous. It can be sequential or simultaneous. These differences in the networks derive from the characteristics of their elements. Logogens and imagens are assumed to “retain properties derived from perceptions in our various sensory modalities” (p. 4) even after they have become associated with other representations. For example, verbal representations (logogens) continue to be sequential, whereas spatial representations (imagens) can be parallel or simultaneous (Clark & Paivio, 1991). Thus we can “zoom” in or out on a mental image to answer a question about a detail of a scene or an overall view.

The DCT also accounts for individual differences in learning. Different individuals may have had different experiences that result in not only different associations and content, but also different preferences for storage. In other words, as a result of different experiences some people may prefer verbal storage and others may prefer nonverbal storage. Of course, people may also store information differently as a result of more immediate variables, such as different instructions and context (Clark & Paivio, 1991; Sadoski & Paivio, 2001). In addition, there may be innate differences that lead to preferred modes of representation (Sadoski & Paivio, 2001).

The DCT can be used to explain the effectiveness of graphic organizers for reading comprehension. With effective instruction, the graphic organizers provide nonverbal imagens and referential connections to logogens, thereby improving retrieval. More specifically, graphic organizers improve learning through elaboration, and through the representation of nonsequential relationships (such as part-whole) (Clark & Paivio, 1991).

In fact, Sadoski and Paivio (2001) have provided an extensive review of research in reading and written expression to show that the DCT explains many widely recognized results in these areas. For example, concrete presentations of information yield better recall than abstract presentations of the same material. According to DCT, concreteness increases the probability of imagery, thus providing more referential connections for recall. The fact that pictures with text typically make content more memorable than concrete text without pictures, which is in turn more memorable that abstract text, also suggests that both verbal and nonverbal encodings are at work and that they are additive. According to the DCT model, teachers can improve instruction by strengthening both associative and referential connections through images, concrete language, and verbal associations (Clark & Paivio, 1991). Unfortunately, the DCT has not been applied as thoroughly to the teaching and learning of mathematics as it has been to the teaching and learning of reading and written expression in the professional literature.

Summary

Students who demonstrate learning disorders may have difficulty in mathematics classes for a variety of reasons. They may have difficulty understanding the language directed towards them from teachers and texts, as well as struggle to express what they know or articulate their questions orally or in writing. They may also have difficulty using internal language to make associations, monitor their progress, or follow procedures. Attention problems may also interfere with self-monitoring, as well as effective encoding, planning, and other cognitive processes. Instructional techniques such as strategy and direct instruction are common in mathematics classes but rely significantly on the use of oral language, thus putting students with language disorders at a disadvantage. Other instructional techniques, including the use of manipulatives, the concrete-semiconcrete-abstract sequence, and diagrams to guide problem solving, explicitly incorporate elements that do not rely as heavily on language. However, these techniques do not address higher level secondary mathematics, and may not be readily applicable to these higher level topics. Research on the use of graphic organizers to improve reading comprehension suggests that they can help students understand relationships represented in their reading. Because graphic organizers represent these relationships through spatial relationships, they may also be helpful for students trying to understand relationships in mathematics.

CHAPTER 3

METHODS

Research indicates that direct and strategy instruction, both separately and together, are effective when teaching a variety of academic content to students with learning problems (Swanson et al., 1999). The purpose of this investigation was to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts by students identified as having learning disabilities or attention disorders. Specifically, these studies address skills and concepts related to solving systems of linear equations. To this end a two group comparison of means experimental design was used. The investigation included two systematic replications of the design in which both the participants and instructional content differed.

Group Design

One of the challenges of using group designs, especially in the field of special education, is designing an investigation that will have adequate statistical power when a limited number of participants is available. In these situations a single-subject design is often recommended (Tawney & Gast, 1984). Given the question being addressed, a comparative single-subject research design might have been appropriate. However, comparative designs such as the multitreatment design and the alternating treatments design require that the behaviors being taught were reversible (Holcombe, Wolery, &

Gast, 1994). In the case of instructional interventions, such as those applied in this investigation to teach students to solve systems of linear equations, the behaviors being taught were not considered reversible. An alternative single-subject approach would be to use an adapted alternating treatments design or a parallel treatments design. In these two cases, multiple equivalent and independent behaviors are taught rather than one reversible behavior (Holcombe et al., 1994). However, higher level mathematics skills generally share common basic skills and can not be considered independent. Further, their equivalent difficulty is difficult to substantiate. As a result of these problems, group design was selected instead of a single-subject design.

To address the limitations that a relatively small sample size places on statistical power, the comparison was restricted to two groups, as fewer groups yields greater power, other things being equal. In addition, an alpha level of .1 was selected to determine statistical significance. These design elements are discussed in more detail in the data analysis section for Study 1.

Study 1

The first study was designed to apply a graphic organizer to teaching secondary students with learning difficulties to solve systems of two linear equations with two variables. The task requires some complex decision making in the context of multiple steps. There are many routes to a correct solution for these systems, although in practice choices are typically made from a much narrower selection of practical alternatives. For example, the process often requires finding common multiples of the coefficients of some terms in the equations. For any two coefficients there are an infinite number of common multiples, any of which could be used to carry out that step of the process. However, typically the least common multiple is used, or else the two coefficients are multiplied by each other to find a common multiple.

Setting

All student participants in both studies attended a private school in Georgia that is dedicated to students with learning disabilities and attention disorders. The school provides programs for grades 6-12 and has a total enrollment of about 200 students. The high school (grades 9-12) is separate from the middle school (grades 6-8). This site was chosen in part because the school offers an environment in which all students in every class have been identified as having learning problems. This identification process is described in the participants section that follows. The impact of a classroom intervention on students with learning difficulties can be assessed more efficiently in an environment in which all students in every class have been identified as having learning problems than in inclusive settings where classrooms would typically have very few such students. That is, because the typical inclusive setting may only have a few students with learning problems, more classes would have to be included in the study in order to reach the same number of students with learning problems as can be reached in this more specialized setting.

The school uses the series of mathematics textbooks published by McDougal Littel. For the Algebra I classes the text is entitled *Algebra I* (Larson, Boswell, Kanold, & Stiff, 2001a). The scope and sequence of the course adheres quite closely to the text. The most notable modification of instruction at the school, compared with typical general education classes covering the same content, is the small class size. With rare exceptions, class sizes are less than 10 students in the high school. This small class size permits much more individualized instruction than would be available in a typical general education classroom.

Participants

Participants included students in five Algebra I classes as well as the two teachers for those classes. Characteristics of the student participants are detailed first. These data are followed by information about the teacher participants.

Student Participants

Descriptive data for the students were taken from the school files after appropriate student and parent permissions were granted. Copies of informed consent forms for parents and informed assent forms for students are attached in Appendix A. Each student with a learning disability must provide a current psychoeducational profile to the school from a qualified diagnostician and that profile must identify cognitive processes impacting learning for that student. Therefore guidelines from both Georgia state eligibility and the Diagnostic and Statistical Manual of Mental Disorders (American Psychiatric Association, 2000) were used for identifying learning disabilities and attention-deficit/hyperactivity disorder (ADHD). However, in some cases the profile may cite “characteristics of” a learning disability without making a definitive diagnosis. Students admitted with attention deficit disorders are required to provide, at minimum, a letter from a physician confirming the diagnosis, as well as some documentation of a prior history of attention difficulties. The school does not admit students with primarily emotional or behavioral problems, although some students with a history of such problems are admitted provisionally if those problems are believed to be a secondary consequence of the learning problems. School policy requires that psychological and educational documentation of learning problems be submitted for each student upon application to the school and updated at least every three years. Student data from these assessments were considered current if they had been acquired within the last three years.

All the participating students for this study were in one of five sections of Algebra I. Four of the sections were in the high school, and the fifth section was in the middle school. Students were assigned to the four high school sections according to criteria not related to their mathematics abilities. Their choices among available electives were the most influential determinants of which section of Algebra I they were assigned to. There were 26 students in these four sections and 8 more students in the middle school section. All of these students were invited to participate. One high school student elected not to participate, and two others did not have complete consent forms. One middle school students was not eligible to participate because of absences. This left a total of 30 participants.

Each of the four high school classes was assigned to either the graphic organizer (GO) condition or the control (CO) condition such that there are two classes in each condition, and the number of student participants in each condition was as nearly equal as possible. This yielded two groups, one including 11 students in sections of 7 and 4 each, and one including 12 students in sections of 6 each. The first group was randomly assigned the GO condition, and the other group became the CO group. The participants from the middle school class were randomly assigned to one of two groups and their class schedules were altered during the week of the study so that each group could work separately with me. In this way 3 middle school students became part of the GO group and 4 middle school students became part of the CO group. Table 1 shows the relationships between teachers, students and group assignments for Study 1.

Table 1

Number of Student Participants by Section That Were Assigned to the Graphic Organizer (GO) and Control (CO) Groups in Study 1

Teacher	Number of Students	Group Assigned
High School	7	GO
	4	GO
	6	CO
	6	CO
Middle School	3	GO
	4	CO

Of the 14 students in the GO group, 10 (71%) were male, and 4 (29%) were female. This distribution compares with that of the CO group of 16 in which 11 (69%) were male and 5 (31%) were female. The ages of the GO group ranged from 13.6 to 19.3 years and averaged 15.9 years (SD = 1.3). For the CO group the age range was 14.7 to 17.9 years with a mean of 15.8 (SD = 0.9). There was one Asian-American student in the GO group. All other students were Caucasian-American. English was the first language for all students. The intelligence (IQ) scores of the GO group, expressed as standard scores, ranged from 85 to 136 and averaged 100 (SD = 15). For the CO group the IQ range, in standard scores, was 80 to 143 with a mean of 102 (SD = 18).

Table 2 reports socioeconomic status, grade level, and diagnoses for both groups. Socioeconomic status was estimated as the highest educational degree completed by either parent. Diagnoses sum to more than 100% because many students have multiple diagnoses. Other sums may not be exactly 100% because of rounding. Of the 30 student participants, 16 had initial diagnosis information in their school files. Of these 16, 14 were first diagnosed in early elementary school – first to third grade. One was diagnosed in kindergarten, and one was diagnosed in seventh grade.

Table 2

Characteristics of Student Participants in Study 1

	Graphic Organizer Group	Control Group
Highest Parent Degree as N (%):		
HS	1 (7%)	0 (0%)
Assoc.	1 (7%)	2 (13%)
BA/BS	8 (57%)	9 (56%)
Master’s	2 (14%)	5 (31%)
Doctoral	2 (14%)	0 (0%)
Grade Level as N (%):		
7	1 (7%)	0 (0%)
8	2 (14%)	4 (25%)
9	6 (43%)	7 (44%)
10	3 (21%)	4 (25%)
11	1 (7%)	1 (6%)
12	1 (7%)	0 (0%)

Diagnoses as N (%):

ADHD	11 (79%)	12 (75%)
LD/Reading	5 (36%)	6 (38%)
LD/Language	3 (21%)	3 (19%)
LD/Mathematics	3 (21%)	2 (13%)
LD/Written	2 (14%)	2 (13%)
Tourette's	2 (14%)	1 (6%)
OCD	0 (0%)	2 (13%)

Teacher Participants

One teacher taught all four sections in the high school, and a different teacher taught the middle school section. Both of these teachers participated in the study. The high school teacher was certified in both mathematics and music education. After two years of teaching music at the elementary level in public schools, the high school teacher began teaching secondary mathematics. This teacher had five years of experience teaching a variety of high school algebra classes, including three years specifically with students with learning problems in a private school. The middle school teacher had about 25 years of teaching experience at secondary levels. Seventeen of those years were in public schools. In public schools this teacher taught both mathematics and science. The teacher's private school teaching experience was primarily in mathematics. Content ranged from seventh grade mathematics to precalculus. The teacher was certified in both special education and gifted education.

Instruments

The graphic organizer itself was the critical instructional tool being tested in this study. Sources of data included the outcome variable data used to test the statistical significance of the two group comparisons. In addition, data were collected through questionnaires and interviews to analyze the social validity of the intervention. Procedural fidelity was assessed by the teachers observing the instruction in their classes.

Graphic Organizer

A graphic organizer was defined as a display that presents information using verbal or mathematical symbols as well as visual/spatial elements, where the visual/spatial elements indicate relationships between the symbolic elements. In discussing maps and diagrams in particular, Winn (1991) noted that the visual/spatial elements may include the relative positions of the symbolic elements, but they may also include the relationship of the symbolic elements to the frame within which the symbolic elements are placed. The graphic organizer for this study included both of these representations of relationships, as shown in Figure 1 and described here. Figure 1 shows an example of a completed graphic organizer as a two by three (two rows and three columns) array of rectangular cells with Roman numeral column headings. This organizer was used in its entirety in Study 2. However, in Study 1 only columns II and I were used for these smaller systems of equations.

III	II	I
$2x + 4y + 2z = 16$ $-2x - 3y + z = -5$ $2x + 2y - 3z = -3$	$y + 3z = 11$ $-y - 2z = -8$	$z = 3$
$2x + 4(2) + 2(3) = 16$ $2x + 14 = 16$ $2x = 2$ $x = 1$	$y + 3(3) = 11$ $y + 9 = 11$ $y = 2$	$z = 3$

Figure 1. A completed graphic organizer for solving systems of linear equations in three variables.

The lines constituting the borders of the rectangles serve to divide the symbolic content into cells based on meaningful distinctions. They also serve to emphasize the relative positions of various symbolic content elements. In a typical system of equations, the solving of the system (finding the roots) involves working from cell to cell in a clockwise direction starting with the top left cell. The top row is used to combine equations in order to eliminate variables until an equation in one variable is produced. Once this equation is found, the bottom row serves to guide the finding of successive roots until the entire system is solved.

Each column is headed by a Roman numeral, with the Roman numerals in descending order from left to right: III, II, I. Although the Roman numerals are certainly symbolic elements, they are elements of the frame of the graphic organizer, and not symbolic elements of the content of the systems to be solved using the graphic organizer. The left column is labeled “III.” Equations in three variables are placed in the top cell of this column, equations in two variables are placed in the top cell of the middle column, and equations in only one variable are placed in the top cell of the right column. Thus each Roman numeral indicates the number of variables in the equations below it. The relative lateral position of equations also indicates more or fewer variables: equations with more variables are to the left and equations with fewer variables are to the right. Again, both the relative position of symbolic content elements and their position relative to the frame indicate relationships between the elements.

Test of Prerequisite Skills

I constructed a test of prerequisite skills that was administered to all students in this study. It can be found in Appendix B. The results of this test were used, as necessary, to modify lessons to ensure that both groups were familiar with prerequisite skills relevant to solving systems of linear equations by using linear combinations. Parts of lessons that specifically addressed prerequisite skills were the same for both groups and did not include the use of graphic organizers.

Four items of the instrument test the first prerequisite skill: (a) solving linear equations in one variable. For this skill two difficulty levels were identified. The lower difficulty level was a one-step equation, and the higher difficulty level was a two-step equation. Four other prerequisite skills are also assessed. The skills are as follows: (b) substituting a value in place of variable in linear equations of two variables so that they can be solved for the remaining variable (2 items), (c) combining (adding) linear equations with two variables, (d) multiplying linear equations in two variables by a constant (2 items), and (e) finding common multiples for two numbers (4 items). All of these skills had been taught in lower level mathematics classes or earlier in the Algebra I course.

Test of Content Skills

I designed and constructed a test to measure the procedures and concepts to be taught in the intervention phase of the study. The content skills test has two sections. The first section is a group of three short-answer questions designed to assess how well students understand the concepts that can be used to justify the procedures for solving systems of equations in two variables. These concepts are related to the coding categories that were used to classify instructional statements and questions as described in the Procedural Fidelity section that follows. The first question relates directly to the idea that linear equations in one variable have a unique solution, even though those in more than one variable do not. The next two questions address the need to eliminate variables so as to produce a uniquely solvable equation in one variable, and whether this can be done in a specific example. These questions were reviewed and revised twice by an experienced mathematics educator.

Following the first three questions are four systems of equations to be solved. The first system requires no multiplying of equations and begins with two equations in two variables. The second system requires multiplying equations but still begins with two equations in two variables. The third system requires multiplying equations and begins with one equation in two variables and one equation in one variable. The fourth system involves generalization. Three linear equations in three variables are given, but no multiplication of equations is required, and all three equations contain all three variables.

All of these systems were taken from popular textbooks. They were selected to be typical of the kinds of systems solved by students in general education classes when they are being taught to solve systems of linear equations. All of the selected systems had integer coefficients and solutions.

The two versions of this test were generated by creating twice as many items as needed for a single test, and randomly assigning items by type to create two equivalent versions. One version of the test was used as an immediate posttest and the other was used as a test of maintenance administered two to three weeks later. Both versions of the content skills test for this study are contained in Appendix B, along with scoring keys.

Teacher Generated Assessment

Two weeks before instruction for the study began, the teachers provided to me a test of the material covered in class during Study 1 classes. These tests were typical of the classroom assessment that would be used for that material if no study were being carried out. The teacher generated test reflected the teachers' expectations for the students regarding content and difficulty of the material taught. Inclusion of this instrument was important to the study for three reasons. First, the fact that the test reflected the terachers' performance expectations for the students meant that this measure incorporated one type of social validity. Second, this test was used as an outcome variable to test for group differences in mean scores. Third, the content of the test was used to modify lesson plans in advance, as appropriate, to ensure that the lessons covered all of the expected material.

Teacher Interview

I interviewed each teacher using a protocol of open-ended questions. The purpose of the interview was to collect data on the social validity of the graphic organizer. The questions were based on Wolf's (1978) description of three basic components of social validity. This approach to social validity continues to be widely recognized (Carpenter, Bloom, & Boat, 1999; Gresham & Lopez, 1996; Schwartz, 1999; Storey & Horner, 1991). Wolf's three components are (a) the social importance of the goals of the intervention, (b) the social acceptability of the intervention procedures, and (c) the social importance of the results. Storey and Horner (1991) recommended structured approaches to gathering social validity data that yield quantitative data that can be correlated across participants. However, in this study only two teachers were involved. The open-ended structure of the interview was intended to provide opportunities for the teachers to offer responses that might not have occurred to me and thus would not appear in a structured questionnaire. The interview was recorded and transcribed.

The questions that provided the frame of the interview are listed in Appendix C. The first two questions asked about the challenges of teaching this material to students with learning problems, and about the importance of the material. These questions were designed to address the social importance of the goals of the study. The next two questions asked about how the graphic organizers were helpful to students and whether they seemed to be more helpful to some students than to others. These questions were designed to probe the importance of the results of the study. Finally, the last three questions asked if the teacher would consider using a similar approach, how graphic organizers might generalize to other content in the course, and what disadvantages to graphic organizers were identified. These three questions were designed to investigate the third area of social validity identified by Wolf, that of the social acceptability of the procedures. The results were analyzed qualitatively as described later.

Student Questionnaire

The student participants in both groups completed questionnaires providing social validity feedback. The questions were based on the same three components of social validity as the questions framing the teacher interview were. Questions for the graphic organizer group addressed the use of the graphic organizer specifically, and the questions for the control group were more general. The student questionnaire for each group is contained in Appendix D.

As with the teacher interview questions, the questions on the student questionnaires were designed to address each of the three elements of social validity discussed by Wolf - the social importance of the goals of the intervention, the social acceptability of the intervention procedures, and the social importance of the results. Each questionnaire included six questions with responses in the form of a five level Likert-type scale ranging from “Very” through “Somewhat” to “Not at all.” The first two questions asked about the importance of solving systems of equations in the context of algebra and in the context of everyday life. These questions were designed to address the social relevance of the goals of the intervention. The next two questions asked whether the instruction was helpful for learning the material, and whether students would like to learn about using a similar approach for other content. These questions should have uncovered views related to the social validity of the results of the study. The last two questions asked how easy it is to use the approach and how likely it is that the student would use the approach in the future. These questions probed the social validity of the procedures. The data from these questionnaires have been analyzed quantitatively as described later.

Procedures for Study 1

This section first addresses how student, teacher, and school confidentiality were protected. Next the advance preparation for the study is described, including the review of student files, lesson plans, and classroom acclimation. In-class procedures then follow, and the last major division of this section is a description of data analyses as they apply to the research questions.

Confidentiality

Each student participant was assigned a unique number for the study. Once the data collection was complete, the students' names were removed from all data records and the numbers were used to identify records. No separate documentation matching names to participant numbers was retained. The teacher participants' names were also removed from all data records once data collection was completed. Any presentation or publication of the study, including this dissertation, will not include specific identifying details of individual participants or the school involved.

Advance Preparation

Advance preparation for this study included collection of descriptive data for the students and teacher participants. In addition, I prepared lesson plans in advance, spent some time in the classroom before the study began to acclimate the students to my presence.

Review of Student Files. After appropriate written informed consent was obtained, I reviewed the school files of each student participant to identify demographic and educational information as described in the Participants section above.

Lesson Planning. Two weeks before the instructional phase of the experiment, the teacher provided the teacher generated test to assess performance on the material covered in this study. Based on this test, and the teacher's estimate of the amount of class time that would typically be spent on the material, I planned the lessons. The lessons were constructed to cover all of the skills and difficulty levels represented on the teacher generated tests. Lessons included elements of both strategy and direct instruction as defined and shown by Swanson et al (1999) to be effective for students with learning disabilities. They found that instruction that included elements of both strategy and direct instruction tended to be more effective, based on quantitative data on a variety of achievement measures, than instruction including elements of only one approach. As noted earlier, both strategy and direct instruction place great reliance on language skills. By including both of these approaches in all lessons, I hoped to provide a more stringent test of the additional contribution to achievement of the graphic organizer than if less effective approaches to instruction had been used for comparison.

Appendix E describes a five or six day series of lessons. In this study the lessons were designed to teach students to solve systems of two linear equations with two variables. The first lesson was a review and assessment of prerequisite skills. The second lesson presented relatively simple examples of systems of equations, and the next two lessons introduced variations, including equations in only one variable, and equations that require multiplication by constants before they can be combined to eliminate variables. The content of the lessons was modified on an ad hoc basis during the study in response to the progress that students were making with the material. This flexibility was intended to preserve a realistic teaching experience that recognized the importance of student needs. Responsible teachers routinely adjust lessons to fit the needs of students. However, adjustments to the lesson plan content were carried out in all six sections of the course. Thus the only systematic difference between sections was the use of the graphic organizer.

Classroom Acclimation. I attended all five sections of the course every day for at least one week prior to beginning the instructional phase of the experiment. During this time I occasionally provided tutoring and support characteristic of a teaching assistant. My objective was to give the students time to become accustomed to my presence and authority.

In-class Procedures and Instruction

On the first day of instruction, the students completed the test of prerequisite skills. I read the instructions and problems aloud, and students were reassured that they were not expected to be able to do all of the problems but were encouraged to attempt as much of the test as they could. They were given ample time to complete whatever problems they were able to complete. Once the prerequisite tests were completed and collected, instruction began with a review of the prerequisite skills. Prepared lesson plans were carried out at the conclusion of the prerequisite skills review.

On the last day of instruction, the students completed one version of the content skills test. The students also completed the questionnaire. The teacher administered the teacher generated test whenever it had been planned in the normal course of the classes. This occurred within a week of completion of the instructional phase of the study. Between two and three weeks after the instructional phase of the study was completed the students completed the second version of the content skills test.

Data Analysis

Statistical significance can impel an investigator to reject the null hypothesis that two groups are equal. However, this approach alone is not enough to demonstrate that an intervention is effective in a useful way. The social validity of an intervention can provide additional information for weighing its practical value. In addition, statistical significance does not necessarily indicate replicability. For this reason, the investigation was designed to include systematic replication.

After efforts to document and analyze procedural fidelity are described, the process of data analysis is described in terms of the research questions. The question of replicability would be supported by comparing results across the two studies. The data analysis relevant to the other three questions is presented in the same order that the questions were listed in Chapter 1.

Procedural fidelity. Procedural fidelity is particularly critical to ensure that the verbal instruction provided to students is comparable across conditions in order to test the specific influence of the graphic organizer on the outcome variables. For this reason, the teacher categorized verbal instruction statements and these results were evaluated following each day’s instruction. This evaluation was used to adjust the following day’s instruction to ensure that the verbal instruction was roughly equivalent across the two groups. Once the data collection was completed, the total number of statements identified in each of the four categories, summed across the control classes was compared with the same totals summed across the experimental groups.

While I carried out classroom instruction, the teacher categorized each statement during instruction. The statements were categorized into one of four categories as indicated below. The teacher was trained to carry out this categorization during the three days of instruction of new material. The steps for training the teacher to record data for procedural reliability are listed in Appendix F and described here. During the training session, the teacher was provided with definitions and examples for each of the four categories. The teacher was encouraged and given ample time to discuss and ask questions about these definitions and examples with the goal of constructing a common understanding of the four categories. Then the teacher rated each of the statements in the sample transcript of the first model problem in Appendix F. I had already rated the statements in this. The two sets of ratings were compared. Discrepancies were discussed, and interrater agreement was calculated. Interrater reliability was estimated as the percent of exact matches when at least one of the raters scored an entry as belonging to one of the four categories. If interrater reliability had been less than 90%, a second round of ratings would have been done on a second model transcript of a lesson. However, both teachers produced interrater reliabilities of over 90% on their first attempts.

The four categories are also summarized in Appendix F, and described here. The first entry category included any entry that indicates or asks the number of different variables in one or more equations. The number of variables in an equation is an important question to answer when solving these systems because the intent of doing successive linear combinations in the problem is to generate new equations in fewer variables until an equation in only one variable has been derived. This equation can then be solved for the value of that one remaining variable. For the experimental group only, the number of variables in any equation was to be explicitly matched to the Roman numeral column headings of the graphic organizer as well. Thus the sequence of equations with fewer variables could be associated with the left to right sequence of columns in the graphic organizer.

The second category of entries included any entry that addresses the question of whether items in two different equations match, or are equal, in some way. The need for matching both variables and numerical values arises in these systems. First, in order for two equations to be combined to yield a third that has fewer variables than either of the first two, the variables must be the same for both equations. For example, two equations in the same three variables can typically be combined to yield a third equation in two variables, although it might have only one variable or even no variables at all. On the other hand, two equations in two variables each, but having only one variable in common, will typically yield another equation in two variables. In addition, the values of the coefficients of corresponding variables in different equations must be equal in order for them to add to zero, assuming that the signs of the two coefficients are opposite. There is no point in combining equations if no variable will be eliminated through the canceling of opposite coefficients. For the experimental group only, this matching process was reinforced by the fact that new equations with fewer variables were placed in columns in the organizer that are different from those containing the equations that were combined.

The third category of entries included entries that question or state whether an equation is solvable. Generally a linear equation has a unique solution if it contains only one variable. If it has more than one variable it usually has an infinite number of roots. These entries reinforce the idea that the goal of eliminating variables is to derive reduced equations in only one variable so that specific roots can be found. They also reinforce the idea that during the substitution phase of solving these systems, enough substitution has to be done so that only one variable remains in an equation, or it can not be solved. For the experimental group, the number of variables in an equation is tied to the Roman numeral heading of the column containing that equation.

The fourth category of entries included any statement or question that involves the number of equations being addressed. To solve these systems of equations students need to learn that two equations in the same variables must be combined to yield a new equation in fewer variables. This concept becomes particularly important for systems in which some equations do not have all three variables to begin with.

These four categories of statements relate to concepts involved in understanding the steps for solving systems of equations using linear combinations. Particularly in the first three categories, these concepts are tied to the column headings, and to the fact that the columns of the graphic organizer are arranged in descending numerical order from left to right. Thus the graphic organizer was designed to represent those patterns and relationships between equations that are important to these concepts.

Data analysis for Question 1: Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?

Statistical significance testing has been criticized, in part, because researchers typically apply a conventional alpha level of .05 or .01 without considering the implication or consequences of their selection. As Clark (1999) pointed out, “a conscientious researcher should select an alpha that minimizes the potential impact of either incorrectly rejecting or failing to reject the null hypothesis" (p. 283). In other words, researchers should consider factors that influence the importance of both Type I and Type II errors and determine a balance that is appropriate for the particular situation. For example, larger alpha levels may be justified for field based research, unavoidably small N, large variability of participants, and data collection problems (Sutlive & Ulrich, 1998).

There were at least three reasons for considering an alpha level for this study that is larger than the usual conventions. One was that this study investigated the effectiveness of an intervention for which little, if any research existed. As a result the cost of a Type II error was increased. If the study data failed to indicate that graphic organizers were effective, even though they actually were, then further investigation would be less likely and a useful intervention might be lost. In contrast, if graphic organizers were found to be effective according to the data, even though they actually were not, efforts to replicate an intervention that at first appeared to be promising would uncover this error. A second reason was that this was a field based investigation rather than a laboratory study. As such, there was likely to be greater error variance as a result of uncontrolled intervening variables. Third, as is often the case in the field of special education, the sample size for this study was limited.

When Earle (1969) applied a graphic organizer to teaching the vocabulary of mathematics he found little or no effect for the definitions themselves. However, he found medium to large effect sizes, according to Cohen's (1988) guidelines, for the relationships between the terms. This results contrasts with the Swanson et al (1999) observation of relatively small effect sizes for mathematics interventions in general that are used with students with learning disabilities. On the other hand, Swanson et al also found larger effect sizes when investigators carried out interventions themselves, and they found that experimental outcome measures yielded larger effect sizes than standardized tests did. For the present study, both of these findings would predict larger effect sizes. For these reasons, an alpha level of .10 was used and power analysis was based on medium to large effect sizes.

The question to be addressed here is whether use of the graphic organizer improves the performance of these students on solving systems of linear equations in two variables. This question was addressed statistically for both the teacher generated test and the content skills test.

Each teacher scored and assigned grades for the teacher generated tests. In both cases, the teachers included the content for the study in a test that covered additional material as well. The two tests did not include identical items. However, both teachers only included systems of two linear equations in two variables in which all coefficients were single digit integers, and all solutions were made up of integers. For these reasons the difficulty level was considered equivalent for the two tests and their results were combined for statistical analysis.

Only the points each student received on the questions related to the study content were used in the analysis. These points were converted into percentages of the number of available points for those problems. After appropriate checks of the normality and homogeneity of variance assumptions, the means of these grades were compared across the two conditions using a one-way analysis of variance (ANOVA) with an alpha level of .10. Results are presented in terms of effect size and statistical significance.

The content skills test resulted in two scores. The first three questions on the test were designed to test for understanding of the concepts behind the solution process. For each student, the percent of the total possible score on the first three questions was used to determine whether there was a statistically significant difference between groups in the mean scores. This analysis was carried out in the same way as the analysis on the scores from the teacher-generated test, as described above, for both the posttest and the maintenance test. See the scoring guides that follow both versions of the test in Appendix B for details concerning their scoring.

The last four questions on the content skills test required that the student solve systems of equations. Because the process of solving these systems of equations involved multiple steps, these systems were graded to allow partial credit. For each system, a point was earned for each new equation generated that contributed to the solution of the system. An additional point was earned for each correctly assigned value in the final solution. Refer to the scoring guides that follow both versions of the test in Appendix B for more details concerning their scoring. These total scores for each version were analyzed using the same approach as that used for the scores on the teacher generated test.

Both versions of the content skills test (posttest and maintenance) were graded only after they had been coded by participant number, names removed, and the tests shuffled so that the grader was blind to group membership.

In addition, an error analysis was carried out on the student responses for the systems on the content skills tests. Initially errors were identified as calculation errors or errors in concept/procedure. If a system was not attempted that was considered a conceptual error, as students were given ample time to complete the tests.

Data analysis for Question 2: Will the difference in performance cited in the first research question be maintained for two to three weeks after instruction and immediate posttesting are completed?

The data from the content skills maintenance test were analyzed using the same approach that was used for the immediate posttest data. In addition, these data were compared descriptively with the immediate posttest data.

Data analysis for Question 3: Will the use of graphic organizers to teach secondary students with learning disabilities or attention disorders to solve systems of equations with two linear equations in two variables demonstrate social validity?

Conventional statistical significance testing has been criticized for failing to identify results that are meaningful in a practical sense, as opposed to statistically significant. Wolf (1978) addressed this problem in describing social validity in terms of three concerns. They are the social significance of the goals, the social importance of the results, and the social acceptability of the procedures. To this list Storey and Horner (1991) added socially optimal levels of performance. Schwartz (1996) clarifies the point that social validity is a matter of perception and more specifically the perception of the consumers of the intervention. Consumers of educational interventions would include students and teacher, of course, but may also include parents, administrators, and others.

Some procedural elements of this study were designed to reinforce social validity. The teachers created graded tests of the relevant material that were typical of the expectations for these students. Thus, those tests supported the validity of the performance standards to which the students were being held. Further, the content of the plans was based on this test, supporting the claim for the social importance of the goals. The fact that the lesson plan procedures were based on approaches to instruction that have been shown generally effective via effect size meta-analysis (Swanson et al., 1999) supports a claim for socially important effects. In addition, the teacher interviews and the student questionnaires were designed to specifically address the social validity of this study.

The teacher interviews were transcribed and analyzed in several ways. Statements that supported or questioned the social validity of the use of graphic organizers were identified. Any statements reflecting on the application, generalizability, or limitations of the use of graphic organizers were considered. Transcripts of all teacher interviews are attached in Appendix G. The student questionnaires provided quantitative data on social validity. These data were also summarized.

Study 2

This second experiment was designed to apply a graphic organizer to teaching secondary students with learning difficulties to solve systems of three linear equations with three variables. The task requires some complex decision making in the context of multiple steps, generally requiring more steps and more decision making than with a system of two linear equations with two variables. In both cases there are many routes to a correct solution for these systems, although in practice choices are typically made from a much narrower selection of practical alternatives. For example, the process often requires finding common multiples of the coefficients of some terms in the equations. For any two coefficients there are an infinite number of common multiples, any of which could be used to carry out that step of the process. However, typically the least common multiple is used, or else the two coefficients are multiplied together to find a common multiple. These larger systems also are more likely to have other variations that may be recognized and influence decision making. For example, some of the equations may not have all three variables, or sometimes two variables are eliminated when two equations are combined.

There were some important differences between the procedures of Study 1 and the procedures of Study 2. Study 2 included a much smaller number of student participants. Statistical analysis was not expected to produce statistically significant results because of the loss of power. However, the same statistical analyses were planned. Thompson's (1993; 1996) recommendation to following up statistical significance tests with a "what if" analysis was applied. That is, for results that were not statistically significant, if effect size measures were comparable to or greater than those found in Study 1, the number of participants necessary to achieve statistical significance was estimated. This was done with the assumption that the effect size would remain constant.

Study 2 included data from the content skills as an immediate posttest. In addition, the student questionnaires and teacher interview to address social validity were included. However, because of practical considerations, no follow up test for maintenance was included. Nor were the results of a teacher generated test included in the study. As a result, Question 2 was not tested for Study 2.

Setting

All student participants in both studies attended a private school in Georgia, that is dedicated to students with learning disabilities and attention disorders. The school provides programs for grades 6-12 and has a total enrollment of about 200 students. The high school (grades 9-12) is separate from the middle school (grades 6-8). This site was chosen in part because the school offers an environment in which all students in every class have been identified as having learning problems. This identification process is described in the participants section that follows. The impact of a classroom intervention on students with learning difficulties can be assessed more efficiently in an environment in which all students in every class have been identified as having learning problems than in inclusive settings where classrooms would typically have very few such students. That is, because the typical inclusive setting may only have a few students with learning problems, more classes would have to be included in the study in order to reach the same number of students with learning problems as can be reached in this more specialized setting.

The school uses the series of mathematics textbooks published by McDougal Littel. For the Algebra II classes the text is entitled *Algebra II* (Larson, Boswell, Kanold, & Stiff, 2001b). The scope and sequence of the course adheres quite closely to the text. The most notable modification of instruction at the school, compared with typical general education classes covering the same content, is the small class size. With rare exceptions, class sizes are less than 10 students in the high school. This small class size permits much more individualized instruction than would be available in a typical general education classroom.

Participants

Participants included students in the Algebra II classrooms as well as the teachers. Characteristics of the student participants are detailed first. These data are followed by information about the teacher participants.

Student Participants

Descriptive data for the students were taken from the school files after appropriate student and parent permissions were granted. Copies of informed consent forms for parents and informed assent forms for students are attached in Appendix A. Each student with a learning disability must provide a current psychoeducational profile to the school from a qualified diagnostician and that profile must identify cognitive processes impacting learning for that student. Therefore guidelines from both Georgia state eligibility and the Diagnostic and Statistical Manual of Mental Disorders (American Psychiatric Association, 2000) were used for identifying learning disabilities and attention-deficit/hyperactivity disorder (ADHD). However, in some cases the profile may cite “characteristics of” a learning disability without making a definitive diagnosis. Students admitted with attention deficit disorders are required to provide, at minimum, a letter from a physician confirming the diagnosis, as well as some documentation of a prior history of attention difficulties. The school does not admit students with primarily emotional or behavioral problems, although some students with a history of such problems are admitted provisionally if those problems are believed to be a secondary consequence of the learning problems. School policy requires that psychological and educational documentation of learning problems be submitted for each student upon application to the school and updated at least every three years. Student data from these assessments were considered current if they had been acquired within the last three years.

All the participating students for this study were in one of three sections of Algebra II. Students were assigned to the three high school sections according to criteria not related to their mathematics abilities. Their choices among available electives were the most influential determinants of which section of Algebra II they were assigned to. Each of the three classes was assigned to either the graphic organizer (GO) condition or the control (CO) condition such that the number of student participants in each condition was as nearly equal as possible. In this case the results of this assignment were a group of two classes with 3 and 4 students each for a total of 7, and a second group of one class with 8 students. Of these 15 students, 2 elected not to participate and 3 others did not have complete consent forms. This left 5 student participants in each of the GO and CO groups. Table 3 shows the relationships between teachers, students and group assignments for Study 1.

Table 3

Number of Student Participants by Section That Were Assigned to the Graphic Organizer (GO) and Control (CO) Groups in Study 2

Teacher	Number of Students	Group Assigned
High School	3	GO
	2	GO

5	CO

All 10 participants in both groups were male. The ages of the GO group ranged from 16.9 to 19.3 years and averaged 17.6 years (SD = 0.4). For the CO group the age range was 17.2 to 18.6 years with a mean of 17.8 (SD = 0.3). There was one Asian-American student in the GO group. All other students were Caucasian-American. English was the first language for all students. The intelligence (IQ) scores of the GO group, expressed as standard scores, ranged from 96 to 130 and averaged 107 (SD = 14). For the CO group the IQ range, in standard scores, was 91 to 124 with a mean of 100 (SD = 16). Each group included one senior and four juniors.

Table 4 reports socioeconomic status, and diagnoses for both groups. Socioeconomic status was estimated as the highest educational degree completed by either parent. Diagnoses sum to more than 100% because many students have multiple diagnoses. Other sums may not be exactly 100% because of rounding.

Table 4

Characteristics of Student Participants in Study 2

Graphic Organizer Group	Control Group

Highest Parent Degree as N (%):		
HS	1 (20%)	0 (0%)
Assoc.	0 (0%)	1 (20%)
BA/BS	3 (60%)	1 (20%)
Master’s	1 (20%)	3 (60%)
Doctoral	0 (0%)	0 (0%)
Diagnoses as N (%):		
ADHD	3 (60%)	3 (60%)
LD/Reading	4 (80%)	1 (20%)
LD/Language	1 (20%)	1 (20%)
LD/Mathematics	1 (20%)	2 (40%)
LD/Written	2 (40%)	1 (20%)
Nonverbal	0 (0%)	1 (20%)

Instruments

As in Study 1, the graphic organizer itself was the critical instructional tool being tested in this study. Sources of data included the content skills test of concepts and system solving used to compare group performance. In addition, data were collected to analyze the social validity of the intervention, as well as the procedural fidelity.

Graphic Organizer

As in Study 1, a graphic organizer was defined here as a display that presents information using verbal or mathematical symbols as well as visual/spatial elements, where the visual/spatial elements indicate relationships between the symbolic elements. In discussing maps and diagrams in particular, Winn (1991) noted that the visual/spatial elements may include the relative position of the symbolic elements, but they may also include the relationship of the symbolic elements to the frame within which the symbolic elements are placed. The graphic organizer for this study is shown in Figure 1. The difference between the graphic organizers for the two studies was simply that the graphic organizer for Study 2 included the third column to the left that the graphic organizer for Study 1 did not use. That means the graphic organizer for this study was a two by three (two rows and three columns) array of rectangular cells with Roman numeral column headings.

The lines constituting the borders of the rectangles serve to divide the symbolic content into cells based on meaningful distinctions. They also serve to emphasize the relative positions of various symbolic content elements. In a typical system of equations, the solving of the system (finding the roots) involves working from cell to cell in a clockwise direction starting with the top left cell. The top row is used to combine equations in order to eliminate variables until an equation in one variable is produced. Once this equation is found, the bottom row serves to guide the finding of successive roots until the entire system is solved.

Each column is headed by a Roman numeral, with the Roman numerals in descending order from left to right: III, II, I. Although the Roman numerals are certainly symbolic elements, they are elements of the frame of the graphic organizer, and not symbolic elements of the content of the systems to be solved using the graphic organizer. The left column is labeled “III.” Equations in three variables are placed in the top cell of this column, equations in two variables are placed in the top cell of the middle column, and equations in only one variable are placed in the top cell of the right column. Thus each Roman numeral indicates the number of variables in the equations below it. The relative lateral position of equations also indicates more or fewer variables: equations with more variables are to the left and equations with fewer variables are to the right. Again, both the relative position of symbolic content elements and their position relative to the frame indicate relationships between the elements.

Specific values for each variable are found for across the bottom of the graphic organizer. The first value is found in the bottom right cell, whose column is headed by the Roman numeral "I." The second value is found in the middle cell of the bottom row, and the third variable is found in the bottom left cell. Thus the Roman numeral headings coincide with the n^{th} variable being solved and the values are found in a lateral sequence from right to left. Again both relative positioning of symbolic content elements and the position of these elements relative to the frame indicate relationships between the elements.

Test of Prerequisite Skills

I constructed four items to test each of the following prerequisite skills: (a) solving linear equations in one variable, (b) substituting values in place of variables in linear equations until they can be solved for one remaining variable, (c) combining (adding) linear equations, (d) multiplying linear equations by a constant, and (e) finding common multiples. The prerequisite skills test for Study 2 covered the same set of skills as those covered on the prerequisite skills test for Study 1. However, some of the tasks included higher difficulty levels for Study 2.

All of these skills are typically covered in lower level mathematics classes. The first four are included in Algebra I curriculum, and the fifth is seen repeatedly in both elementary and middle school mathematics courses. The third, fourth, and fifth prerequisite skills are used as intermediate steps in solving systems of linear equations with three variables. Solving a system of equations by linear combination involves combining equations together in pairs such that one variable at a time is eliminated from each combined equation. To accomplish this, it is often necessary to multiply equations by constants to ensure that the coefficients for a particular variable in the two equations will be opposites. Identifying possible choices for these coefficients requires being able to determine common multiples. The first two prerequisite skills are used to solve for specific values of the variables. The first value is found by solving an equation in one variable, and subsequent values are found by substituting the solutions into equations and then solving them for other variables.

For each of the first four prerequisite skills listed above, two difficulty levels were identified and I wrote two questions for each difficulty level. For Skill (a), the lower difficulty level was a one-step equation, and the higher difficulty level was a two-step equation. For Skills (b) to (d), the lower difficulty level involved applying the skill to an equation with two variables, and the higher difficulty level required applying the same skill to an equation with three variables. In each case, tasks at both levels of difficulty may appear in any given system.

Test of Content Skills

The content test followed a format parallel to that of Study 1. The test included two sections. The first section was a group of six short-answer questions designed to assess how well students understand the concepts that justify the procedures for solving these systems of equations. These concepts are related to the coding categories that were used to classify instructional statements and questions as described in the Procedural Fidelity section that follows. The questions relate directly to the idea that linear equations in one variable have a unique solution, and those in more than one variable do not. One question applies that concept to assessing a specific situation in which substitution may or may not produce an equation in only one variable. Two questions address the need to eliminate variables so as to produce a uniquely solvable equation in one variable, and whether that elimination can be done in a specific example. One question addresses the understanding that a solvable system must have at least as many equations as there are variables in the system. These questions were reviewed and revised twice by an experienced mathematics educator.

Following the first six questions are four systems of equations to be solved. The first system requires no multiplying of equations and begins with three equations in three variables. The second system requires multiplying equations but still begins with three equations in three variables. The third system requires multiplying equations and begins with one equation in three variables and one equation in two variables. The fourth system involves a generalization. Four linear equations in four variables requiring no multiplication of equations are given, and all four initial equations contain all four variables. All of these systems were taken from popular textbooks. They were selected to be typical of the kinds of systems solved by students in general education classes when they are being taught to solve systems of linear equations. All of the selected systems had integer coefficients and solutions. The prerequisite skills test and the content skills test for this study are contained in Appendix H, along with a scoring key for the content skills test.

Teacher Interview

I interviewed the teacher in Study 2 and recorded and transcribed the interview. The questions that provided the frame of the interview are the same as those listed in Appendix C for Study 1. The questions were based on Wolf's (1978) description of three basic components of social validity. This approach to social validity continues to be widely recognized (Carpenter et al., 1999; Gresham & Lopez, 1996; Schwartz, 1999; Storey & Horner, 1991). Wolf's three components were (a) the social importance of the goals of the intervention, (b) the social acceptability of the intervention procedures, and (b) the social importance of the results. Storey and Horner (1991) recommended structured approaches to gathering social validity data that yield quantitative data that can be correlated across participants. However, in this study only one teacher was involved so correlation is not possible across participants. Further, the open-ended structure of the interview was intended to provide opportunities for the teacher to offer responses that might not have occurred to me and thus would not have appeared in a structured questionnaire.

The first two questions ask about the challenges of teaching this material to students with learning problems, and about the importance of the material. These questions were designed to address the social importance of the goals of the study. The next two questions ask about how the graphic organizers were helpful to students and whether they seemed to be more helpful to some students than to others. These questions were designed to probe the importance of the results of the study. Finally, the last three questions ask if the teacher would consider using a similar approach, how graphic organizers might generalize to other content in the course, and what disadvantages to graphic organizers were identified. These three questions were designed to investigated the third area of social validity identified by Wolf, that of the social acceptability of the procedures. The results were discussed qualitatively as described later.

Student Questionnaire

The student participants in both groups for Study 2 completed a questionnaire providing social validity feedback based on the same three components of social validity as used previously. As with Study 1, the questionnaire for students in the GO group included specific questions about the graphic organizer, and the questionnaire for the CO group included more general, but parallel questions. These student questionnaires were the same as those used for Study 1, and are contained in Appendix D. The questions on these questionnaires were designed to address each of the three elements of social validity discussed by Wolf - the social importance of the goals of the intervention, the social acceptability of the intervention procedures, and the social importance of the results. Each questionnaire includes six questions with responses in the form of a five level Likert-type scale ranging from “Very” through “Somewhat” to “Not at all.” The first two questions ask about the importance of solving systems of equations in the context of algebra and in the context of everyday life. These questions were designed to address the social relevance of the goals of the intervention. The next two questions ask whether the instructions was helpful for learning the material, and whether students would like to learn about using a similar approach for other content. These questions were intended to uncover views related to the social validity of the results of the study.

The last two questions ask about how easy it is to use the approach and how likely it is that the student will use the approach in the future. These questions probed the social validity of the procedures. The data from these questionnaires were reported and then discussed quantitatively as described later.

Procedures for Study 2

As with Study 1, this section will first address how student, teacher, and school confidentiality was protected. Next the advance preparation for the study is described, including the review of student files, lesson plans, and classroom acclimation. In-class procedures then follow, and the last major division of this section is a description of data analyses as they apply to the research questions.

Confidentiality

Student and teacher participant confidentiality, as well as the confidentiality of the school's identity were preserved as indicated in Study 1. Each student participant was assigned a unique number for the study. Once the data collection was complete, the students' names were removed from all data records and the numbers were used to identify records. No separate documentation matching names to participant numbers was retained. The teacher participant's name was also removed from all data. Any presentation or publication of the study will not include specific identifying details of individual.

Advance Preparation

Advance preparation for this study included collection of descriptive data for the students, teacher, and school. In addition, I prepared lesson plans in advance, and spent some time in the classroom before the study began to acclimate the students my presence.

Review of Student Files. Demographic and educational information on student participants were collected as in Study 1. Once appropriate written informed consent was obtained, I reviewed the school files of each student to identify demographic and educational information as described in the participants section above.

Lesson Planning. The specific lessons used in Study 2 were similar to those in Study 1, except that the lessons addressed systems of three linear equations with three variables. Procedures for generating the lessons were essentially the same. Two weeks before the instructional phase of the experiment, the teacher provided samples of items that assessed performance of the material covered in this experiment. Based on these samples, and the teacher's estimate of the amount of class time that would typically be spent on the material, I planned the lessons. The lessons were constructed to cover all of the skills and difficulty levels represented on the samples items provided by the teacher. Lessons included elements of both strategy and direct instruction as defined and shown to be effective for students with learning disabilities by Swanson et al (1999). They found that instruction that included elements of both strategy and direct instruction tended to be more effective, based on quantitative data on a variety of achievement measures, than instruction including elements of only one approach. As noted earlier, both strategy instruction and direct instruction place great reliance on language skills. By including both of these approaches in all lessons, I hoped to provide a more stringent test of the additional contribution to achievement of the graphic organizer, than if less effective approaches to instruction had been used for comparison.

These two approaches to instruction have some overlapping elements as well as some that are unique to each approach. Common elements of both strategy and direct instruction that will be incorporated into the lesson plans are (a) daily review, (b) statements of instructional objectives, (c) teacher presentation of new material, (d) guided practice, (e) independent practice, and (f) formative evaluation. In addition, elements unique to strategy instruction that will be included in the lesson plans are (a) verbal modeling by the teacher of the steps or processes, (b) elaborate explanations to guide task performance, (c) reminders to use strategies or procedures, (d) multistep instructions, and (e) verbal dialogue. Elements unique to direct instruction that will be included in the lesson plans are (a) modeling of the skill by the teacher, (b) breakdown of the task into smaller steps, (c) repeated probes and feedback, (d) prescribed material at a rapid pace, and (e) directed questions related to skills. The first lesson presented relatively simple examples of systems of equations, and successive lessons introduced variations, including equations that require multiplication by constants before they can be combined to eliminate variables and systems that included equations with less than three variables. The text of Appendix I describes the content of the lessons in more detail. Lessons were adjusted to the progress of the students, but were consistent across all three sections of the class, with the exception of the use of the graphic organizer.

Classroom Acclimation. The investigator attended all four sections of the course every day for at least one week prior to beginning the instructional phase of the experiment. During this time I provided tutoring and support characteristic of a teaching assistant. My objective was to give the students time to become accustomed to my presence and authority.

In-class Procedures and Instruction

The first day of instruction began with the test of prerequisite skills. I read the instructions aloud, gave the students adequate time, and assured them that they should expect to find some items that are difficult. Once the test of prerequisite skills was completed and collected, instruction began with a review of these skills and then proceeded according to the lesson plans. The content skills test was given on the last day of instruction, as was the social validity questionnaire

Data Analysis

Data analysis for the second study will closely parallel that applied to the first study. As noted in Study 1, statistical significance can impel an investigator to reject the null hypothesis that two groups are equal. However, this approach alone is not enough to demonstrate that an intervention is effective in a useful way. The social validity of an intervention can provide additional information for weighing its practical. In addition, statistical significance does not necessarily indicate replicability. For this reason, the investigation is designed to include systematic replication.

After efforts to document and analyze procedural fidelity are described, the process of data analysis is described in terms of the research questions. The question of replicability would be supported by comparing results across the two studies. The data analysis relevant to the other three questions are presented in the same order that the questions were listed in Chapter 1.

Procedural fidelity. Procedural fidelity was investigated through a process much like that in Study 1. After similar training and testing of interrater reliability, the teacher categorized the statements of the investigator during classroom instruction according to the same four category system.

Procedural fidelity is particularly critical to ensure that the verbal instruction provided to students is consistent across conditions in order to test the specific influence of the graphic organizer on the outcome variables. For this reason, the teacher categorized verbal instruction statements and these results were evaluated following each day's instruction. This evaluation was used to adjust the following day's instruction to roughly equalize the verbal instruction across the two groups. Once the data collection was completed, the total number of statements identified in each of the four categories, averaged across the control classes, was compared with the same totals averaged across the experimental groups.

While I carried out the classroom instruction, the teacher categorized each statement. The statements were categorized into one of four categories as indicated below. The teacher was trained to carry out this categorization during the lessons. The steps for training the teacher to record data for procedural reliability are listed in Appendix F and described here. During the training session, the teacher was provided with definitions and examples for each of the six categories. The teacher was encouraged and given ample time to discuss and ask questions about these definitions and examples with the goal of constructing a common understanding of the six categories. Then the teacher rated each of the statements in the sample transcript of the model lesson transcript in Appendix F. I had already rated the statements in this transcript. The two sets of ratings were compared and interrater agreement was calculated. Interrater reliability was estimated as the percent of exact matches when at least one of the raters scored an entry as belonging to one of the four categories. If interrater reliability were less than 90%, a second round of ratings and reliability calculations would have been carried out on a second model problem. In fact, the initial interrater reliability was over 90%, so this additional training was not considered necessary.

The four categories are also summarized in Appendix F, and described here. The first entry category included any entry that indicates or asks the number of different variables in one or more equations. The number of variables in an equation is an important question to answer when solving these systems because the intent of doing successive linear combinations in the problem is to generate new equations in fewer variables until an equation in only one variable has been derived. This equation can then be solved for the value of that one remaining variable. For the experimental group only, the number of variables in any equation was to be explicitly matched to the Roman numeral column headings of the graphic organizer as well. Thus the sequence of equations with fewer variables could be associated with the left to right sequence of columns in the graphic organizer.

The second category of entries included any entry that addresses the question of whether items in two different equations match, or are equal, in some way. The need for matching both variables and numerical values arises in these systems. First, in order for two equations to be combined to yield a third that has fewer variables than either of the first two, the variables must be the same for both equations. For example, two equations in the same three variables can typically be combined to yield a third equation in two variables, although it might have only one variable or even no variables at all. On the other hand, two equations in two variables each, but having only one variable in common, will typically yield another equation in two variables. In addition, the values of the coefficients of corresponding variables in different equations must be equal in order for them to add to zero, assuming that the signs of the two coefficients are opposite. There is no point in combining equations if no variable will be eliminated through the canceling of opposite coefficients. For the experimental group only, this matching process was reinforced by the fact that new equations with fewer variables are placed in columns in the organizer that are different from those containing the equations that were combined.

The third category of entries included entries that question or state whether an equation is solvable. Generally a linear equation has a unique solution if it contains only one variable. If it has more than one variable it usually has an infinite number of roots. These entries reinforce the idea that the goal of eliminating variables is to derive reduced equations in only one variable so that specific roots can be found. They also reinforce the idea that during the substitution phase of solving these systems, enough substitution has to be done so that only one variable remains in an equation, or it can not be solved. For the experimental group, the number of variables in an equation is tied to the Roman numeral heading of the column containing that equation.

The fourth category of entries included any statement or question that involves the number of equations being addressed. To solve these systems of equations students need to learn that two equations in the same variables must be combined to yield a new equation in fewer variables. This concept becomes particularly important for systems in which some equations do not have all three variables to begin with.

These four categories of statements relate to concepts involved in understanding the steps for solving systems of equations using linear combinations. Particularly in the first three categories, these concepts are tied to the column headings, and to the fact that the columns of the graphic organizer are arranged in descending numerical order from left to right. Thus the graphic organizer was designed to represent those patterns and relationships between equations that are important to these concepts.

Data analysis for Question 1: Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?

These data were analyzed in the same way as the achievement data from Study 1. As noted earlier, statistical significance testing has been criticized, in part, because researchers typically apply a conventional alpha level of .05 or .01 without considering the implication or consequences of their selection. As Clark (1999) pointed out, “a conscientious researcher should select an alpha that minimizes the potential impact of either incorrectly rejecting or failing to reject the null hypothesis” (p. 283). In other words, researchers should consider factors that influence the importance of both Type I and Type II errors and determine a balance that is appropriate for the particular situation. For example, larger alpha levels may be justified for field based research, unavoidably small N, large variability of participants, and data collection problems (Sutlive & Ulrich, 1998).

There were at least three reasons for considering an alpha level for this study that is larger than the usual conventions. One was that this study investigated the effectiveness of an intervention for which little, if any research existed. As a result the cost of a Type II error was increased. If the study data failed to indicate that graphic organizers were effective, even though they actually were, then further investigation would be less likely and a useful intervention might be lost. In contrast, if graphic organizers were found to be effective according to the data, even though they actually were not, efforts to replicate an intervention that at first appeared to be promising would uncover this error. A second reason was that this was a field based investigation rather than a laboratory study. As such, there was likely to be greater error variance as a result of uncontrolled intervening variables. Third, as is often the case in the field of special education, the sample size for this study was limited.

When Earle (1969) applied a graphic organizer to teaching the vocabulary of mathematics he found little or no effect for the definitions themselves. However, he found medium to large effect sizes, according to Cohen's (1988) guidelines, for the relationships between the terms. This results contrasts with the Swanson et al (1999) observation of relatively small effect sizes for mathematics interventions in general that are used with students with learning disabilities. On the other hand, Swanson et al also found larger effect sizes when investigators carried out interventions themselves, and they found that experimental outcome measures yielded larger effect sizes than standardized tests did. For the present study, both of these findings would predict larger effect sizes. For these reasons, an alpha level of .10 was used and power analysis was based on medium to large effect sizes.

The question to be addressed here was whether use of the graphic organizer improved the performance of these students on the content skills test. After appropriate checks of normality and equal variance assumptions were carried out, the means of these grades were compared across the two conditions using a one-way analysis of variance (ANOVA) with an alpha level of .10. Results were discussed in terms of effect size and statistical significance.

The content skills test yielded two scores. The first six questions on the test were designed to test for understanding of the concepts behind the solution process. The total score for each student on these first six questions was used to determine if a statistically significant relationship exists between group membership and the means of these scores. The last four questions on the content skills test required actually solving systems of equations. Because the process of solving these systems of equations involves multiple steps, these systems were graded to allow partial credit. For each system, a point was earned for each new equation that contributes to the solution of the system. An additional point was earned for each correctly assigned value in the final solution. Refer to the scoring guides that follow the test in Appendix H for more details concerning the scoring. These total scores were analyzed using the same approach as that used for Study 1. The content skills tests were graded only after they have been coded by participant number, names have been removed, and the tests have been shuffled so that the grader is blind to group membership.

In addition, an error analysis was carried out on the student responses for the content skills tests. Initially errors were identified as calculation errors or errors in concept/procedure.

Data analysis for Hypothesis 3: Will the use of graphic organizers to teach secondary students with learning disabilities or attention disorders to solve systems of equations with two linear equations in two variables demonstrate social validity?

The same procedural elements that support social validity in Study 1 are also incorporated into Study 2. In addition, the teacher interview and student questionnaires will be handled as in Study 1.

Conventional statistical significance testing has been criticized for failing to identify results that are meaningful in a practical sense, as opposed to statistically significant. Wolf (1978) addressed this problem in describing social validity in terms of three concerns. They are the social significance of the goals, the social importance of the results, and the social acceptability of the procedures. To this list Storey and Horner (1991) added socially optimal levels of performance. Schwartz (1996) rightly clarifies the point that social validity is a matter of perception and more specifically the perception of the consumers of the intervention. Consumers of educational interventions would include students and teacher, of course, but may also include parents, administrators, and others.

Some procedural elements of this study reinforced social validity. The teacher provided samples of test items for the relevant material that were typical of the expectations for these students. These items supported the validity of the performance standards to which the students were being held. Further, the content of the lesson plans were based in part on these items, supporting the claim for the social importance of the goals. The fact that the lesson plan procedures were based on approaches to instruction that have been shown generally effective via effect size meta-analysis (Swanson et al., 1999) supports a claim for socially important effects. In addition, the teacher interview and the student questionnaire were designed to address the social validity of this study.

The teacher interview was transcribed and analyzed in several ways. Statements that support or question the social validity of the use of graphic organizers were identified. Any statements reflecting on the application, generalizability, or limitations of the use of graphic organizers were considered. The student questionnaires provided quantitative data on social validity. These data were summarized.

CHAPTER 4

RESULTS

Results for statistical analyses for each study are presented separately. These sections also include interrater reliability results of training for the coding done by the teachers, and the results of that coding in tabular form for each study. Analysis of variance (ANOVA) was used for each comparison of means. ANOVA assumes that data are distributed normally for each group and that variances are equal across groups being compared. These assumptions were tested prior to running the ANOVAs. To test the assumption of normality, skewness and kurtosis values were calculated. All skewness and kurtosis values were within the range of ±1.0. On the basis of these results I assumed there was an adequate approximation to normal distributions. Levene's Statistic (Huck & Cormier, 1996) was used to test the assumption that variances were equal across groups for the outcome variable in each comparison. The values for the Levene's Statistics ranged from .14 to 1.00. These values did not justify rejecting the null hypotheses that the variances were equal in each case. Results of ANOVAs are reported in terms of probability levels, and effect size based on an alpha level of 0.10. Along with the results of the statistical analyses for each study, the data from the student questionnaires are summarized in tabular form for both studies. Comparisons of data across studies to address the fourth question follow the results for each individual study. Results from the teacher interviews are discussed as appropriate in Chapter 5, and the transcripts of the interviews are found in Appendix G.

Statistical Analyses from Study 1

As stated earlier, the purpose of these studies is to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts by students identified as having learning disabilities or attention disorders. Specifically, these studies address skills and concepts related to solving systems of linear equations. The purpose of this particular study, was to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction further contributes to the acquisition of mathematics skills and concepts related to solving systems of two linear equations with two variables by students identified as having learning disabilities or attention disorders.

The two teachers were trained to code statements and questions used during instruction. The training process and the coding categories are described in the Procedural Fidelity section of Chapter 3, and they are both outlined in Appendix F as well. After initial training in the coding procedure, the high school teacher's interrater reliability with the sample precoded by the investigator was 93%. For the middle school teacher the interrater reliability was 96%. Both of these values exceeded the 90% criterion determined to be adequate for the study.

The coding done by both Algebra I teachers during the instruction of new material (Days 2, 3, and 4 of the lesson plans) was averaged by class period for each group. Table 5 shows these averages for both the graphic organizer (GO) group and the control (CO) group. Abbreviations of the coding categories are indicated in the left column. Totals are vulnerable to rounding error. The last coding category is an issue in solving systems of more than two equations and two variables because decisions must be made about how many and which equations to combine. For systems of two equations and two variables the only possibility is to combine the two equations that are given. That is why there are no entries for that category in Study 1. The values are reasonably similar across the two groups, supporting the claim that verbal instruction was comparable for both groups.

Table 5

Class Period Averages for Graphic Organizer and Control Groups on Verbal Coding Categories in Study 1

Coding Category	Graphic Organizer	Control
Number of Variables	7.7	6.7
Matching or Equal Items	3.3	3.7
Solvable?	3.7	4.0
Number of Equations	0	0
Totals	14.7	14.3

Data analysis for Hypothesis 1: Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?

Each teacher generated test was graded by the teacher. Scores on items related to the content covered in the study were pulled out for each participant and transformed into percents of the total number of points available for those items. The mean and standard deviation of these scores for each group are reported in Table 6 along with the number in each group (n), significance level (p), and effect size (η^2) of the ANOVA. Based on an alpha level of .10 this result was statistically significant. The effect size falls within the medium to large range suggested by Cohen (Cohen, 1988).

Table 6

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on Teacher Generated Test in Study 1

	N	Mean	SD	F(1, 28)	p	η^2
Control	16	63.07	32.15	3.14	.087	.101
Graphic Organizer	14	81.84	24.7			

The content skills test resulted in two scores for the immediate posttest and two scores for the maintenance test. The first three questions on the tests were designed to test for understanding of the concepts behind the solution process. The percent received out of the total possible score for each student on these first three questions was used to determine if means for groups were different. These analyses were carried out in the same way as the analysis on the scores from the teacher-generated test, as described above, for both the immediate posttest and the maintenance test. The means and standard deviations of these scores for each group are reported in Tables 7 and 8 along with the number in each group (n), F ratios, significance levels (p), and effect sizes (η^2) of the ANOVA. Based on an alpha level of .10 both of these results were statistically significant. Both effect sizes fall above the large value suggested by Cohen (1988).

Table 7

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the Concept Section of the Content Skills Immediate Posttest in Study 1

	N	Mean	SD	F(1, 28)	p	η^2
Control	16	61.26	5.62	7.86	.009	.219
Graphic Organizer	14	84.28	6.00			

Table 8

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the Concept Section of the Content Skills Maintenance Posttest in Study 1

	N	Mean	SD	F(1, 28)	p	η^2
Control	16	62.60	21.76	6.11	.020	.179
Graphic Organizer	14	82.80	23.34			

The last four questions on the content skills test required the students to solve systems of equations. They were similar to the systems on the teacher generated test. Total system solving scores in the content skills test for the immediate posttest and maintenance adminstrations were analyzed using the same approach as that used for the scores on the conceptual sections of the tests. The means and standard deviations of these scores for each group are reported in Tables 9 and 10 along with the number in each group (n), F ratios, significance levels (p), and effect sizes (η^2) of the ANOVA. Remarkably, the means were identical for the two groups on the immediate posttest. Based on an alpha level of .10 neither of these results was statistically significant. Both effect sizes fall below the small value suggested by Cohen (1988).

Table 9

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the System Solving Section of the Content Skills Immediate Posttest in Study 1

	N	Mean	SD	F(1, 28)	p	η^2
Control	16	38.43	16.26	.19	.664	.007
Graphic Organizer	14	41.86	26.11			

Table 10

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the System Solving Section of the Content Skills Maintenance Posttest in Study 1

	N	Mean	SD	F(1, 28)	p	η^2
Control	16	28.57	11.37	.00	1.000	.000
Graphic Organizer	14	28.57	16.57			

In addition, an error analysis was carried out on the student responses for the content skills tests. Errors were identified as calculation errors or errors in concept/procedure. If a system was not attempted this was considered a conceptual error, as students were given ample time to complete the tests. 83% of errors were attributed to calculation rather than conceptual/procedural problems.

Data analysis for Question 2: Will the difference in performance cited in the first research question be maintained for two to three weeks after instruction and immediate posttesting are completed?

The results of the concept sections of the content skills tests are compared over time for the two groups in Figure 2. Scores on the concept sections were converted to percents of the total available points averaged for each group. Figure 2 shows that the control group averaged between 60 and 65 percent of the available points at the immediate posttest and the two to three week follow up on this section of the content skills test. On the same sections the graphic organizer group earned between 80 and 85 percent of the available points. The GO group scores are higher than the CO group scores at both testings as indicated by the statistical tests above, and neither group shows an obvious loss of conceptual understanding over time. This result indicates that differences are maintained over time.

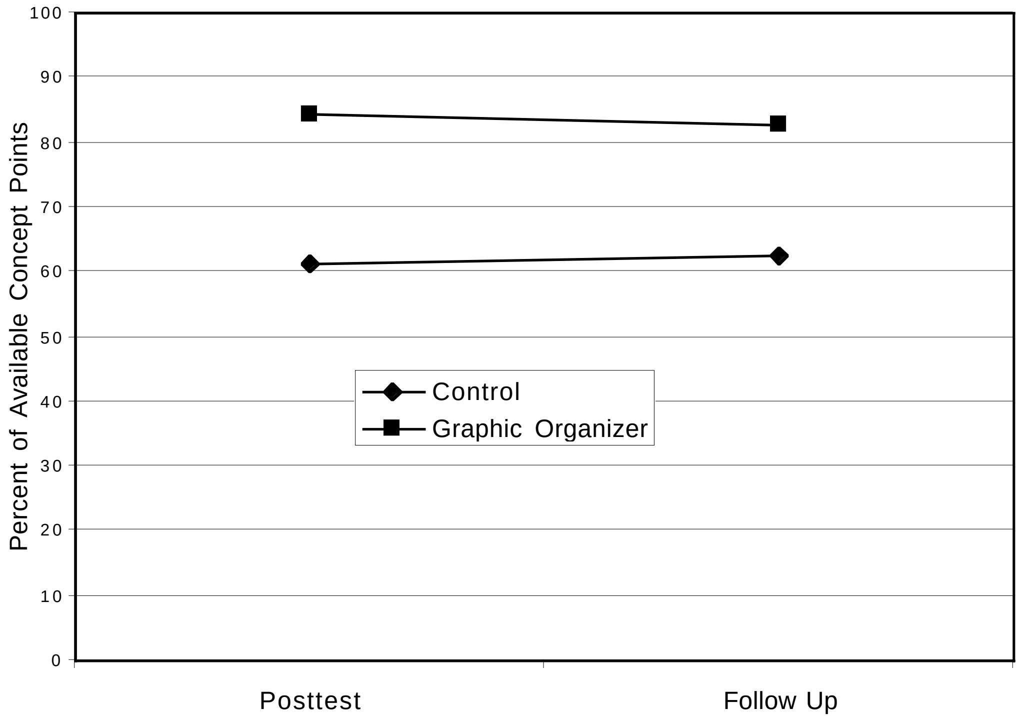

Figure 2. Comparison of Concept Section Scores Between Control and Graphic Organizer Groups and Over Time for Study 1.

The results of the concept sections of the content skills tests are compared over time for the two groups in Figure 3. In contrast to the results for the conceptual sections of the tests, these data not only show that the differences between the two groups are minimal,they also show that both groups did more poorly over time. This result fails to show that differences are maintained over time except in the sense that minimal differences continue over time.

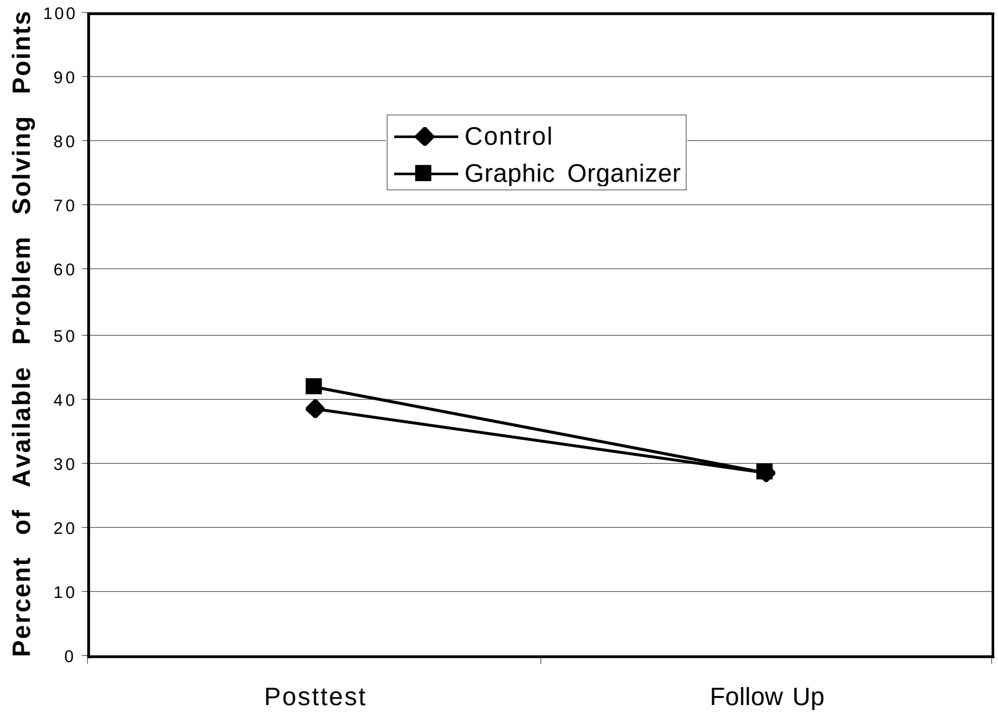

Figure 3. Comparison of System Solving Section Scores Between Control and Graphic Organizer Groups and Over Time for Study 1.

Data analysis for Question 3: Will the use of graphic organizers to teach secondary students with learning disabilities or attention disorders to solve systems of equations with two linear equations in two variables demonstrate social validity?

Transcripts of the teacher interviews are attached in Appendix G. They are discussed in Chapter 5 of this dissertation and excerpts from the interviews are quoted there as appropriate. The student questionnaires provided quantitative data on social validity. The questionnaires appear in Appendix D and are described in detail in the Instruments section above. Average scores for each group on each question and differences between these averages are summarized in Table 11 with abbreviated descriptions of the questions. Differences may reflect rounding error.

Table 11

Student Questionnaire Data for Study 1

	Graphic Organizer	Control	Difference
#1 Important in algebra?	4.01	4.19	-.18
#2 Important in life?	4.01	3.19	.83
#3 Instruction helpful?	4.25	4.69	-.44
#4 Wish to learn more?	4.06	3.94	.13
#5 Easier with or without?	4.07	4.00	.07
#6 Will use again?	3.55	4.25	-.70

On a scale of one to five these means are generally quite high. Only three of the twelve averages are below four, and none is below three. This suggests that the data suffer from ceiling effects and may not discriminate social validity effectively. The two largest differences between the two groups occurred for Questions 2 and 6. For Question 2, the GO group rated solving systems of equations more important in daily life than the CO group did. This difference is related to the fact that the GO group gave the same average rating to the importance of solving systems of equations in algebra class. In contrast, the CO group rated the importance of solving systems of equations as lower in everyday life than in algebra class. In Question 6 the GO group reported being less likely to use the graphic organizer again on a similar system and the CO group reported being relatively more likely to use "the same approach" in the future for similar systems. These data do not give strong support for group differences based on the third research question.

Statistical Analyses from Study 2

As stated earlier, the purpose of these studies is to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts by students identified as having learning disabilities or attention disorders. Specifically, these studies address skills and concepts related to solving systems of linear equations. The purpose of this particular study then, is to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of mathematics skills and concepts related to solving systems of three linear equations with three variables by students identified as having learning disabilities or attention disorders.

The teacher was trained to code statements and questions used during instruction. The training process and the coding categories are described in the Procedural Fidelity section of Chapter 3, and they are both outlined in Appendix F as well. The coding done by the Algebra II teacher during the instruction of new material (days 2, 3, and 4 of the lesson plans) was averaged per class period for each group. Table 12 shows these averages for both the GO group and the CO group. Abbreviations of the coding categories are indicated in the left column. Total are vulnerable to rounding error. The values are reasonably similar across the two groups, supporting the claim that verbal instruction was comparable for both groups.

Table 12

Averages per Class Period for Verbal Coding Categories in Control and Graphic Organizer Classes in Study 2

Coding Category	Graphic Organizer	Control
Number of Variables	13.3	9.3
Matching or Equal Items	9.7	9.7
Solvable?	6.0	6.7
Number of Equations	2.0	3.3
Totals	31.0	29.0

Data analysis for Question 1: Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?

The content skills test resulted in two scores for the immediate posttest. The first six questions on the tests were designed to test for understanding of the concepts behind the solution process. The percent received out of the total possible score for each student on these first six questions was used to determine if a statistically significant relationship existed between group membership and the means of these scores. These analyses were carried out in the same way as the analyses on the scores from Study 1, as described above. The means and standard deviations of these scores for each group are reported in Table 13 along with the number in each group (n), F ratio, significance level (p), and effect size (η^2) of the ANOVA. Based on an alpha level of .10 this result was not statistically significant. The effect size falls between the medium and large values suggested by Cohen (1988).

Table 13

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the Concept Section of the Content Skills Immediate Posttest in Study 2

	N	Mean	SD	F(1, 8)	p	η^2
Control	5	74.55	19.71	1.09	.327	.120
Graphic Organizer	5	87.27	18.85			

Given the small number of participants in Study 2, it is not surprising that the results are not statistically significant. However, the difference in means is the same direction as that from Study 1, and consistent with the first question. Further, the effect size is quite similar in magnitude (.120 compared to .101). Holding the effect size constant, the estimated number of participants necessary to make this result statistically significant is 26, or 13 in each group. This compares quite well with the results of Study 1. In this sense, these results support the first question.

The last four questions on the content skills test required actually solving systems of equations. Total system solving scores in the content skills test for the immediate posttest and were analyzed using the same approach as that used for the scores on the conceptual section of the test. The means and standard deviations of these scores for each group are reported in Table 14 along with the number in each group (n), F ratio, significance level (p), and effect size (η^2) of the ANOVA. Unexpectedly, these results were statistically significant based on an alpha level of .10. In addition, the effect size is quite large. This result supports the first question for system solving in Study 2, in contrast to the lack of support from the system solving results of the content skills test in Study 1. An error analysis on the student responses for the system solving section of the content skills tests showed that 88% of errors were attributed to calculation rather than conceptual/procedural problems.

Table 14

Results of ANOVA Comparing Control Versus Graphic Organizer Group Means on the System Solving Section of the Content Skills Immediate Posttest in Study 2

	N	Mean	SD	F(1, 8)	p	η^2
Control	5	25.19	19.49	11.26	.0.10	.585
Graphic Organizer	5	62.22	15.14			

Data analysis for Question 3: Will the use of graphic organizers to teach secondary students with learning disabilities or attention disorders to solve systems of equations with two linear equations in two variables demonstrate social validity?

Transcripts of the teacher interviews are attached in Appendix G. They are discussed in Chapter 5 of this dissertation and excerpts from the interviews are quoted there as appropriate. The student questionnaires provided quantitative data on social validity. The questionnaires appear in Appendix D and are described in detail in the Instruments section above. Average scores for each group on each question and differences between these averages are summarized in Table 15 with abbreviated descriptions of the questions. Differences may reflect rounding error.

Table 15

Means and Differences Between the Control and Graphic Organizer Groups from Student Questionnaire Data for Study 2

	Graphic Organizer	Control	Difference
#1 Important in algebra?	4.60	4.00	.60
#2 Important in life?	3.60	3.40	.20
#3 Instruction helpful?	4.60	4.40	.20
#4 Wish to learn more?	4.60	3.80	.80
#5 Easier with or without?	4.20	3.40	.80
#6 Will use again?	4.20	4.40	-.20

On a scale of one to five these means are generally quite high, as they were for Study 1. This time four of the twelve averages are below four, and none are below three. This suggests that the data suffer from ceiling effects and may not discriminate social validity effectively. The three largest differences between the two groups occurred for questions one, four, and five. In all three cases the GO group rated their experiences more favorably than did the CO group. For question one, the GO group rated solving systems of equations more important in algebra class than did the CO group. In question four the GO group reported being more enthusiastic about learning more applications for graphic organizers than did the CO group for learning more applications for “this approach.” In question five the GO group reported that it was easier to solve systems of equations with the graphic organizer than without it compared to the CO group's views about solving systems of equations with and without "this approach." These data do provide some support for the third question.

Data Analysis Across Studies

Data analysis for Question 4: Will the findings of the first and third questions be replicated when graphic organizers are used to teach secondary students with learning disabilities or attention disorders to solve systems of three linear equations in three variables?

Figure 4 compares the results for the content skills immediate posttests in both studies. Both studies produced differences favoring the GO groups on the concept sections of the tests. This is seen when comparing the first two columns of the two groups of four columns. This comparison supports the fourth question. In contrast, only Study 2 resulted in a difference between the groups on the system solving portions of the content skills tests. This difference does favor the GO group, thus supporting the first question. However, because Study 1 did not support the first question for the system solving section of the content skills test, these results can not support the fourth question. This comparison can be seen in the last two columns of each group of four. Of course, the first study did find a statistically significant difference favoring the GO group on the teacher generated test results. This indicates some partial support for the fourth question with respect to solving the systems.

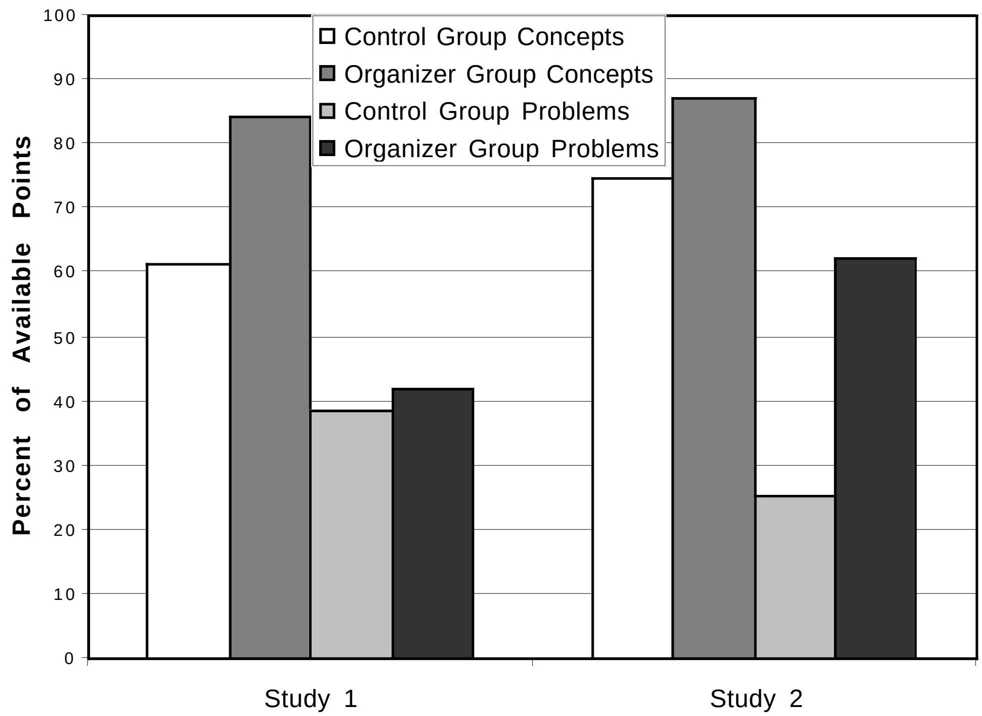

Figure 4. Comparison of Content Skills Test Results for Control and Graphic Organizer Groups Across Both Studies.

Although the fourth question is supported with respect to the data on conceptual understanding, and at least partially supported for the data on system solving, the student questionnaire data on social validity do not support the fourth question. The questionnaire data support a claim for social validity for Study 2 but not for Study 1.

CHAPTER 5

DISCUSSION

The purpose of this investigation was to address the question of whether integrating graphic organizers into instruction that already incorporates strategy and direct instruction, further contributes to the acquisition of higher level mathematics skills and concepts by students identified as having learning disabilities or attention disorders. Specifically, these studies addressed skills and concepts related to solving systems of linear equations. To this end four research questions were posited. The results of the two studies are summarized in this chapter as they relate to the four questions and to show how they may be framed within Dual Coding Theory (DCT) (Paivio, 1986). The results of this exploratory investigation suggest many unanswered questions and opportunities for further investigation. These questions and opportunities are reviewed, particularly as they relate to the responses on the social validity measures of the two studies. Finally, the results were discussed in the context of implications for reform in mathematics instruction as well as high stakes testing.

Summary of Results

The first quesstion for these studies was that students who have been taught to solve systems of equations with graphic organizers will perform statistically significantly better on outcome measures than students instructed on the same material without graphic organizers. The outcome measures were designed to measure two constructs. One was the ability of the students to solve systems of linear equations. In Study 1, results from the teacher generated test for solving systems of equations yielded a statistically significant mean difference favoring the group that worked with the graphic organizer (GO) over the control group (CO). Results from the systems to be solved on the content skills test in Study 2 yielded a statistically significant mean difference and large effect size in spite of the small number of participants. Both of these findings support the first question. However, results from the systems to be solved on the investigator generated instrument in Study 1 did not yield statistically significant mean differences for the immediate posttest. This last finding does not support the first question.

The second construct that the outcome measures were designed to assess was the conceptual understanding of the process of solving systems of linear equations. Results from the conceptual questions on the content skills test from Study 1 yielded a statistically significant mean difference in favor of the graphic organizer group for the immediate posttest. Results from the conceptual questions on the content skills test from Study 2 did not yield a statistically significant mean difference for the immediate posttest. However the direction of the difference was consistent with the first question and the effect size was similar. Because Study 2 had a small number of participants, the results were projected to a larger number of participants when the effect size was held constant. This analysis found that the results would have been statistically significant if the number of participants had been equal to that in Study 1. This result is supportive of the first question. Therefore the results from the conceptual understanding scores in both studies support the first question. Across both constructs and both studies, four of five sets of data from both studies support the first question.

The second question predicted that the difference in performance cited in the first question favoring students taught with the graphic organizer would be maintained two weeks after instruction and immediate posttesting are completed. The results of both sections of the follow up administration of the content skills test in Study 1 address this question. These results are indicated in Figures 2 and 3. Results from the conceptual questions on the content skills test from Study 1 yielded a statistically significant mean difference in favor of the GO group for the follow up posttest. However, as with the immediate posttest, results from the systems to be solved on the follow up administration of the content skills test in Study 1 did not yield statistically significant mean differences. The second question is supported by the data on conceptual understanding, but not by the data on solving systems of equations.

According to the third question, the use of graphic organizers would demonstrate social validity. A review of the results from the student questionnaires suggests that the data suffer from ceiling effects. In Study 1 the students in the GO group did not seem to value the use of the graphic organizer according to their reports on the questionnaires. This impression is supported by the fact that very few of these students (2 out of 14) used the graphic organizer to solve systems of equations on either the teacher generated tests or the content skills tests. In contrast, the GO participants in Study 2 tended to report more favorable feelings about the graphic organizer. In addition, 4 out of 5 of these students also used the graphic organizer to solve systems of equations on the content skills test. All three teachers were consistently enthusiastic about using the graphic organizer themselves in the future and also about the perceived positive impact the graphic organizer had on their students' understanding and performance. There is some support for the third question in these results.

The fourth question predicts that the findings of the first and third questions will be replicable across students and lesson content. Both studies yielded similar results for the conceptual sections of the content skills posttests. The fourth question is supported by these results. Study 1 had two tests of the students' ability to solve systems of equations – one from the teacher generated test and one from the content skills test. Only the teacher generated test yielded a result consistent with the first question. In Study 2,results from the system solving section of the content skills test also supported the first question. Taken together, these results provide some support for the fourth question with respect to the students' ability to solve systems of equations.

The fourth question receives less support with respect to the social validity data. The interviews indicate that the teachers were uniformly supportive of the use of graphic organizers in both studies. However, the responses from the student questionnaires seemed to be supportive of the use of graphic organizers only from Study 2.

The fact that the conceptual understanding of the GO groups was better than that of the CO groups in both studies, and that this difference was maintained over time in Study 1 is important. Sadoski and Paivio (2001) defined a concept as "a category that belongs to one or more superordinate categories, has defining characteristics, and has examples and possibly nonexamples available" (p. 94). One important concept that guides the process of solving systems of linear equations is the idea that linear equations in one variable can be solved for a unique solution, and linear equations in more than one variable have infinitely many solutions. More specifically, linear equations in one variable may be considered as a category that is subordinate to categories such as equations with a finite number of solutions, linear equations, or mathematical equations in general. Defining characteristics might include having an equals sign, having exactly one variable, and having exactly one solution. Both examples and nonexamples of linear equations in one variable occur each time a system of linear equations is solved. The graphic organizers in this study provide some cues to the elements of this concept. For example, the equations in one variable appear in the organizers with equations in more than one variable, demonstrating the categorical relationships of the equations. The equations in only one variable are always placed in the right side column of the organizer. This placement provides a spatial cue that the equations in one variable are examples of something that the equations in the other columns are not, and vice versa. In this way associative connections are reinforced between these nonverbal representations.

Recalling Winn's (1991) distinction between relative positions of elements of the graphic organizer to each other, and relative positions of elements of the graphic organizer to the frame of the organizer provides another view of these results. This time the inconsistent differences between the two groups in Study 1 on the system solving sections of the investigator tests, as well as the teacher generated tests, may be explained. Consider again the concept of linear equations in one variable being solvable for unique single solutions, and linear equations of more than one variable have infinite solutions. As noted above, this concept was represented by the relative positions of equations to each other. The frame of the graphic organizer was not necessary to make this distinction clear. Only equations in the right side columns are solvable, and equations to the left of these were not. In contrast, another objective of instruction with the graphic organizers was to help students follow the complex and multi-step procedures of solving these systems of equations. The steps were tied to the frame of the graphic organizer. Each column was headed by a Roman numeral. The columns were in descending order from left to right. Students in the GO group could have represented the columns or their headings as imagens (basic imagery unit, in the nonverbal associative network). The Roman numerals could be seen as iconic indicators of the number of variables in the equations below them. Two “I’s” would indicate equations with two variables. This representation would allow for additional associative and referential connections and should have led to better performance in carrying out the steps of the systems.

However, the system solving scores for the GO groups were not consistently stronger than those for the CO groups. Perhaps the column headings were more readily coded as logogens (basic verbal unit in the verbal associative network) for many of the students. Two “I’s” would be coded as the logogen “two.” If so, they would have provided more opportunity for associative connections between logogens for the GO group than for the CO group, they would not have provided an opportunity for additional referential connections between logogens and imagens. Thus the frame may not have provided as rich a source for additional connections to reinforce the procedures as the relative positions of elements of the organizer did for the concepts.

Of course, there are other possible explanations for the inconsistent results on the system solving measures. The GO group in the Algebra I study did statistically significantly better than the CO group on the teacher generated test systems, but this was not true for system solving items on the content skills test. One possible explanation is that the content skills test listed the conceptual questions before the systems. These questions may have produce a priming effect, reminding the student about eliminating variables for example, that could have eliminated performance differences that might have arisen without this priming. DCT incorporates this priming effect by interpreting it in terms of activation of parts of the associative systems. A second explanation may be simply that the students knew they would be graded on the teacher generated test and not on the content skills test. This would be an example of the influence of prior instructions on motivation and performance. In terms of DCT, students motivated by concern for their grades would have stronger activation of connections within and between the coding systems, thereby improving performance

In the context of the inconsistent results for the system solving in Study 1, it may be helpful to consider the results from the system solving section of the content skills test in Study 2. Unlike Study 1, this comparison of means yielded a statistically significant difference in spite of the fact that there were only five participants per group. This unexpected result can at least in part be attributed to an important difference between solving systems of equations with two variables as opposed to systems with three variables.

The last system on the content skills tests for both studies was a generalization system. For Study 1, students who had been practicing solving systems of two variables were challenged with a system in three variables. For Study 2, students who had been practicing solving systems of three variables were challenged with a system in four variables. Almost half of the difference between the mean system solving scores for the two groups in Study 2 came from differences in performance on this system, and very few students in either group from Study 1 earned any points at all on their last system. No participant in Study 1 was able to earn more than one point based on their independent work, although three of the students were later guided through the process to final solutions. This difference between the studies may be because of the fact that systems of three equations in three variables provide more opportunity for unambiguous recognition of patterns in the system solving process than do systems of two equations with two variables.

Error analysis of these final questions supports this conclusion. In Study 1 six students attempted to combine equations to eliminate a variable in the final system. Five of those students approached this task by trying to combine all three equations simultaneously. Four of those actually succeeded in combining all three equations after multiplying one or more by a constant such that a variable was eliminated and the new equation of only two variables was consistent with the system. However, after getting this far, they did not know how to get a second equation with two variables. There is nothing mathematically invalid about combining all three equations. The point here is that these students had been solving systems of two equations by combining both equations to get one solvable equation in one variable. They apparently generalized this into the inference that they needed to combine all the equations at the same time, regardless of how many equations there were in the system.

In contrast, the four students in Study 2 who tackled their last system all attempted to combine the equations in pairs. This allowed them to explore several alternatives for coming up with three consistent equations with three variables from the four original equations with four variables. These students never attempted to combine all the original equations at once.

A simpler mathematical example may make this distinction clearer. Consider a sequence of numbers beginning 1, 2, . . . The third number in this sequence could be derived in several ways. It may be an arithmetic sequence found by adding one to each number to yield 1, 2, 3, 4, 5, . . . Alternatively, it could be a geometric sequence found by doubling the previous entry to yield 1, 2, 4, 8, 16, . . . A third possibility is that the sequence is similar to the Fibonacci Sequence. Entries are equal to the sum of the previous two entries and in this case would yield 1, 2, 3, 5, 8, . . . In this case having more numbers in the sequence narrows the possible ways that the sequence can be constructed. If it begins with 1, 2, 3, . . . then it is not geometric. If it begins with 1, 2, 3, 4 . . . then it is not like the Fibonacci sequence. Similarly, students in Study 2 were able to work with systems with three equations in which the equations were combined in pairs, but not combined in triplets. Thus they had the opportunity to eliminate combining all equations as an approach to solving the systems. The students in the GO group in Study 2 would have done better than the CO group on the generalization system because the graphic organizer provided a source for imagens that reinforced this idea of combining equations in pairs.

Social Validity

Social validity was measured by two products. Each of the teacher participants was interviewed and those interviews were recorded and transcribed. In addition, each of the student participants completed a questionnaire. Given the relatively narrow range of the group means on the six questions in both studies (ranging from 3.19 to 4.69) differences of 0.5 or more are considered to warrant discussion. Responses to both of these instruments are discussed in the context of the three types of social validity described by Wolf (1978). In addition, additional research questions are raised in each of these three areas.

One area of social validity is whether the goals of the study are valuable to the participants. Students were asked how important solving systems of equations was for their algebra class and for their lives. In Study 1 the CO group rated systems of equations a full point less important in their lives than in algebra class. This difference also explained why the CO group's rating of importance in their lives was .83 points lower than that of the GO group. In contrast there was no difference in ratings of the GO group for algebra class importance versus life importance. It is possible that the GO group valued systems of equations more in life because the graphic organizer enhanced their self-efficacy for solving these systems. In their interviews all three teachers reported hearing students in the GO groups making comments like, "This is easy!" In Study 2 the results from the students on these two questions were somewhat different. Both groups reported that solving these systems was more important in algebra class than in life by margins of over half a point. In addition, the GO group valued the importance in algebra class .6 points more than the CO group did. This last difference may also be attributable to self-efficacy. For example, one of the GO group members reported, "Bob, your boxes stick in my head . . . so I don't have to think." On looking over the generalization system at the end of the test, another student in the same group said, "I *know* I can do this."

The teachers consistently supported the value of the goals of the study. They all reported that solving systems of equations was very important for students and emphasized the practical applications of this type of system solving. They also considered solving systems of equations to be among the average or more difficult topics for their students to learn. They saw the material as important for their students and felt that the challenges of teaching this material justified exploring different approaches for teaching the material more effectively.

These results suggest some unanswered questions regarding the value of the goals of these studies. One is whether the use of graphic organizers can be generalized to other topics in mathematics. This is obviously an opportunity for further study. However, the teacher interviews offer some encouraging clues. The teachers were asked if they felt that the graphic organizers could be applied to other topics, and if so, could they offer an example. All teachers responded positively to these questions and two teachers offered specific examples. One example was using a graphic organizer to help guide the process of solving equations that require several steps. The teacher emphasized procedural knowledge, referring to problems with a “sequence component” or a “step component” to them. Another teacher suggested using a graphic organizer to help clarify differences between conic sections. This teacher was more concerned with a conceptual understanding of differences rather than a process for making them comparable. These two suggestions could be pursued by following up on the observation made earlier and based on Winn’s (1991) distinction between relative positions of elements to each other as opposed to relative positions of elements to a frame. A frame may be devised to reinforce the procedures of the first suggestion, and the relative positions of exemplars of different conic sections may be used to indicate their differences conceptually.

Another open question implicit in the theoretical frame of these studies is whether there is a relationship between the students’ nonverbal reasoning skills and the degree to which they benefit from graphic organizers. In these studies results were averaged across heterogeneous groups of students. One way to approach this question would be to use Rourke's (Rourke et al., 1973; Rourke et al., 1971) verbal versus performance intelligence score discrepancy to see if students with discrepancies in favor of performance scores benefit more than students with discrepancies in favor of verbal scores. If so, then this discrepancy could be used as a diagnostic tool to identify students for whom nonverbal instructional approaches, including graphic organizers, should be emphasized.

The second area of social validity to be discussed is whether the method used was socially acceptable for the participants. Among the student responses to the two questions addressing this issue a large difference between a GO group and a CO group occurred only once. When asked if they thought the instruction was helpful the GO group in Study 2 averaged .6 points higher than the CO group. A review of the system solving sections of the content skills tests in both studies as well as the teacher generated test in Study 1 support this result. In Study 1 only two students out of fourteen in the GO group tried to use the graphic organizer consistently. In contrast, four of the five students in the GO group from Study 2 used the graphic organizer throughout the system solving section of the content skills test. This difference may be the result of system complexity. As systems get larger their complexity grows exponentially. Students in Study 1 may have seen the systems as manageable without using the graphic organizer, and students tackling the more complex systems in Study 2 may have seen more benefit to the approach.

All three teachers said that they planned on using the graphic organizers the next time they were teaching this same material. The teachers were also asked if they thought the graphic organizer would be helpful for all students. All three felt that at worst the use of graphic organizers would do no harm. They also reflected on visual versus auditory learning styles, with two of the teachers taking the view that even students who were primarily auditory learners would still benefit from using graphic organizers.

Because this is a preliminary study into the effectiveness of using graphic organizers to teach advanced mathematics, there are many unanswered questions concerning the social acceptability of the approach. Some have to do with teacher implementation. Will these teachers actually use this approach to teach this content next year? Can teachers generate their own effective graphic organizers for other topics? How can teachers be trained to both create and use graphic organizers in their instruction? All of these questions are amenable to investigation.

The third type of social validity to be discussed here is whether the results were important to the participants. The results of the system solving tests can be discussed in terms of percent grades to provide another view of this question. In Study 1 the CO students earned an average of 63% of the available points on the teacher generated test. If this were their final score they would not have passed this test. In contrast the GO students earned 82% of the available points, which translates into a perfectly respectable grade report. On the other hand, the system solving section of the content skills test would have given the CO group and the GO group 38% and 42% respectively. These scores would be disappointing on a grade report and the difference would not be considered important. In Study 2 the system solving section of the content skills test yielded percent scores of 25% and 62% for the CO group and the GO group respectively. Although neither of these would be considered passing grades, the difference is dramatic in practical terms.

Some questions about whether the results are important relate to the use of language in the classroom. These studies were set in small classes where every student was identified with a learning or attention problem. However, in a public school setting the classes would likely be larger and comprised mostly of students without identified disabilities. This environment may have unknown implications for the effectiveness of instruction with graphic organizers, especially for these students. For example, larger class sizes mean less individual attention for each student. Each student will receive a smaller proportion of the instruction that is directed at individuals. This may be particularly a problem for students with attention problems who need to be actively engaged to avoid getting lost. In addition, these studies did not attempt to monitor or control for the language of the students in the classroom. The extent to which students interact with each other and the teacher may impact the effectiveness of the instruction. In addition, the kinds of questions that students without disabilities ask in class may be different than those asked by student with disabilities, or they may be pitched at a different level. If so, the responses to those questions may not be particularly helpful to the students with disabilities. Further, students with mild learning disabilities have been described as inactive learners in the classroom (Hallahan & Bryan, 1981). This characteristic may reduce even further the chances that they will get their questions answered.

Related to this question is the issue of gender differences. Just as students with learning disabilities have been described as inactive in classrooms, girls have also been described this way in mixed gender classroom (Fredericksen, 2000; Grober & Mewborn, 2001), although perhaps for different reasons. Boys seem to dominate the attention of the teacher. The student participants in these dissertation studies were overwhelmingly male. This suggests that the results of these studies may not generalize to female students, or to female students with learning problems.

One additional issue regarding the social validity of the results should be mentioned. The experience of teaching some classes with a graphic organizer and some without a graphic organizer in the same day for several days revealed an observation that should be addressed into future studies. The graphic organizer provided a kind of structure to the instruction such that verbal elements of strategy and direct instruction described earlier were effortlessly carried out. This was particularly evident when reviewing the codings done by the teachers during instruction. The investigator had to make deliberate effort to increase the use of questions and statements in the classes without graphic organizers in order to more closely balance the instructional language being used in the classes with the graphic organizers. This phenomenon suggests that an unanticipated goal to be investigated is that graphic organizers contribute a structure to the instruction. How might the nature of the language in the classroom change when graphic organizers are being used if no effort is being made to control it? Would these changes improve instruction beyond any benefits accrued from the use of a graphic organizer?

The Reform Movement in Mathematics Education

Mathematics educators have used *Principles and Standards for School Mathematics* (NCTM Standards) (National Council of Teachers of Mathematics, 2000), published by the National Council of Teachers of Mathematics, as the centerpiece for advocating reform in mathematics instruction. This document, and its earlier versions, advocate for mathematics instruction that emphasizes understanding over rote memorization, and learning through open-ended problem solving. The goals of this reform include helping students to become more flexible problem solvers who feel empowered by their own mathematics skills.

Special educators (Cawley, Parmar, Foley, Salmon, & Roy, 2001; Chard & Kameenui, 1995; Hofmeister, 1993) have been critical of this movement for several reasons. One concern is that the movement has been based on theory without adequate supporting data. According to this argument, the risk of losing opportunities to take advantage of tried and true approaches in order to introduce the more constructivist approaches being recommended is too great for students with disabilities who are already losing ground to the general education population. Strategy and direct instruction, for example, are typically not seen as constructivist in nature, but there is quite a bit of empirical evidence for their effectiveness (Swanson et al., 1999). Of course, it can be argued that the outcome measures in this research primarily assess calculation skills and word problem solving through well defined steps. As such, they are not addressing the kind of flexible problem solving that mathematics educators are concerned about.

Relatively recent data seem to support the use of curricula consistent with the NCTM Standards. For example, in 1997 the United States Department of Education appointed a Mathematics and Science Expert Panel to evaluate mathematics and science curricula. Five mathematics curricula were found to be exemplary, in part because they were consistent with the NCTM Standards, but also because a body of research had accumulated that demonstrated the effectiveness of these programs. The reports on these curricula were produced in 1999, and are available at the Web site for the Eisenhower

National Clearinghouse (Mathematics and Science Expert Panel, 1999). Four of the exemplary mathematics curricula were designed for secondary students. These reports generally show that students in these programs consistently perform better than other students on measures of flexible problem solving. On the other hand, data on more structured tasks such as calculation and problems typical of standardized tests did not show consistent differences between the groups. Of course, the students in the more progressive curricula are doing no worse on these tasks than other students, but it is not clear that they are doing better.

Boaler (1999) found a similar pattern of results when comparing secondary students in two British schools over a two year period. One school adopted a nontraditional mathematics curriculum that involved learning through open-ended problem solving, and the other school stayed with more traditional approaches. Here again, assessment of basic skills showed the two groups to be similar overall, and the students in the progressive curriculum seemed to be stronger in flexible problem solving.

These results suggest that curricula consistent with the NCTM Standards do have some advantages over conventional instruction when unconventional assessment is used to uncover those advantages. However, these data do not specifically address the performance of students with disabilities. Perhaps students with disabilities would not gain the same advantages.

The results of the current studies are interesting in the context of this discussion. The instruction would not typically be described as constructivist. Elements of strategy and direct instruction were used to indicate a fairly specific procedure for solving systems of equations for both groups. The students did not construct their own procedures for solving these systems. On the other hand, the instruction was deliberately focused on helping students understand the concepts behind the process. The data from these studies showed that the graphic organizer seemed to contribute to conceptual understanding of the process for both studies. From this it appears that conceptual understanding can be fostered through instructional approaches that have been shown effective for teaching basic skills and procedures to students with disabilities.

Understanding of mathematics is one goal of the NCTM Standards. Another is flexible problem solving. Here the data from the dissertation studies are less clear. The generalization systems at the end of each of the content skills tests are the closest thing to flexible problem solving that was assessed. Students had to generalize their understanding of smaller systems so that they could applied those ideas to larger systems. As noted earlier, the Study 1 results showed no evidence of differences between groups on this generalization system. On the other hand, the Study 2 results showed an unexpectedly strong difference in favor of the students in the GO group.

In general the dissertation data seem to show a pattern of results consistent with the goals of the NCTM Standards even though the instructional approaches were not. This finding supports the belief that students with disabilities can acquire conceptual understanding of the mathematics they are engaged in. In addition, they support the concerns of special educators that the NCTM Standards may be overlooking some approaches that could help achieve those goals. It should be noted that the NCTM Standards state that “there is no one *right way* to teach” (p. 18). Identifying the NCTM Standards with constructivist approaches to teaching comes from impressions based on curricula derived from the NCTM Standards, as well as examples and guidance given in the NCTM Standards and supporting materials. Nevertheless, the NCTM Standards are widely seen as supporting a constructivist approach to instruction (Grobecker, 1999).

High Stakes Testing

Educators have criticized high stakes testing on several fronts. One criticism is that these tests are typically standardized tests using multiple choice questions. Obviously these questions are not posing open-ended problems. This type of question does not lend itself to assessing critical thinking and flexible problem solving. As a result teachers are forced to teach to the test and the kinds of skills that are not being assessed are neglected (Rotberg, 2001; Stoskopf, 2001; Thomas, 2001).

In this context the data from this dissertation and the data supporting the use of curricula consistent with the NCTM standards are enlightening. These data show that instruction that focuses on conceptual understanding of mathematics does not lead to poorer performance on the kinds of skills typically assessed in high stakes testing. Of course this instruction may not lead to higher performance on these tests either. The comparison between results on content skills tests and teacher generated tests in the present studies and those in high stakes testing must be made carefully. The format of the tests in these studies was different from those typically found on high stakes tests. For example, there were no multiple choice questions in the tests used in these studies. Nevertheless, many of the facts and procedural skills necessary for solving systems of equations on these tests would also be applied by students taking high stakes tests over the same material.

The data from this dissertation do suggest that instruction with graphic organizers can lead to better conceptual understanding of the process, although application of that conceptual understanding did not seem to follow for the participants in the GO group of Study 1. Data on curricula consistent with the NCTM Standards does seem to lead to better flexible problem solving skills, and the same may be said about the dissertation data in Study 2 where the systems were more complex.

These data collectively seem to suggest that the impulse to “teach to the test” as a response to high stakes testing is unjustified. Students taught through curricula that have higher level thinking skills in mind do just as well on the assessment of declarative and procedural knowledge on these tests as do students taught this knowledge as the primary goal. This generalization leads to two recommendations. The first is that teachers and administrators should take a “leap of faith” to adopt curricula that focus on concepts, critical thinking, and flexible problem solving. Their tests scores should not suffer, and the students should benefit in other ways. The second is that assessment of these higher level thinking skills should be included as measures of student progress.

Conclusion

The results of this study suggest that using graphic organizers to teach higher level mathematics to students with learning and attention problems leads to improved conceptual understanding of that mathematics content. The use of graphic organizers may also lead to improved system solving when the systems become complex enough to challenge the ability of students to keep the process organized without the organizers. These exploratory results are encouraging enough to warrant further investigation of the applicability of graphic organizers to other topics, and other classroom settings. In addition, teacher implementation questions need to be answered to determine if teachers can generate and use graphic organizers independently. A seed has been planted which promises to bring forth much fruit.

REFERENCES

Alvermann, D. E. (1981). The compensatory effect of graphic organizers on descriptive text. *Journal of Educational Research, 75*, 44-48.

Alvermann, D. E. (1982). Restructuring text facilitates written recall of main ideas. *Journal of Reading, 25*, 754-758.

Alvermann, D. E., & Boothby, P. R. (1983). A preliminary investigation of the differences in children's retention of "inconsiderate" text. *Reading Psychology: An International Quarterly, 4*, 237-246.

Alvermann, D. E., & Boothby, P. R. (1986). Children's transfer of graphic organizer instruction. *Reading Psychology: An International Quarterly, 7*, 78-100.

Alvermann, D. E., Boothby, P. R., & Wolfe, J. (1984). The effect of graphic organizer instruction on fourth graders' comprehension of social studies text. *Journal of Social Studies Research, 8*, 13-21.

Alvermann, D. E., & Swafford, J. (1989). Do content area strategies have a research base? *Journal of Reading, 32*, 388-394.

American Psychiatric Association. (2000). *Diagnostic and statistical manual of mental disorders* (4th ed.). Washington, DC: Author.

Ausubel, D. P. (1960). The use of advanced organizers in the learning and retention of meaningful verbal behavior. *Journal of Educational Psychology, 51*, 267-272.

Badian, N. A. (1983). Dyscalculia and nonverbal disorders of learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. V, pp. 235-264). New York: Grune & Stratton.

Barron, R. F., & Schwartz, R. M. (1984). Graphic postorganizers: A spatial learning strategy. In C. D. Holley & D. F. Dansereau (Eds.), *Spatial learning strategies* (pp. 275-289). New York: Academic Press.

Bean, T. W., Singer, H., Sorter, J., & Frazee, C. (1986). The effect of metacognitive instruction in outlining and graphic organizer construction on students' comprehension in a tenth-grade world history class. *Journal of Reading Behavior, 18*, 153-169.

Bender, W. N. (1996). *Teaching students with mild disabilities*. Boston: Allyn & Bacon.

Bley, N. S., & Thornton, C. A. (1995). *Teaching mathematics to students with learning disabilities* (Third Edition ed.). Austin: Pro-Ed.

Boaler, J. (1999). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open University Press.

Boothby, P. R., & Alvermann, D. E. (1984). A classroom training study: The effects of graphic organizer instruction on fourth graders' comprehension. *Reading World, 23*, 325-339.

Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology, 23,* 270-274.

Carpenter, C. D., Bloom, L. A., & Boat, M. B. (1999). Guidelines for special educators: Achieving socially valid outcomes. *Intervention in School and Clinic, 34,* 143-149.

Cawley, J. F., Parmar, R., Foley, T. E., Salmon, S., & Roy, S. (2001). Arithmetic performance of students: Implications for standards and programming. *Exceptional Children, 67*, 311-328.

Chard, D. J., & Kameenui, E. J. (1995). Mathematics instruction for students with diverse learning needs: Heeding the message of the Cheshire Cat. *Focus on Learning Problems in Mathematics, 17*(2), 24-38.

Clark, C. M. (1999). Further considerations of null hypothesis testing. *Journal of Clinical and Experimental Neuropsychology, 21*, 283-284.

Clark, J. M., & Paivio, A. (1991). Dual coding theory and education. *Educational Psychology Review, 3*, 149-210.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (Second ed.). Hillsdale, NJ: Lawrence Erlbaum.

Coxford, A. F., & Payne, J. N. (1990). *HBJ Algebra 2*. Orlando: Harcourt Brace Jovanovich.

Davidson, J. L. (1982). The group mapping activity for instruction in reading and thinking. *Journal of Reading, 26*, 52-56.

Deshler, D. D., Ellis, E. S., & Lenz, B. K. (1996). *Teaching adolescents with learning disabilities*. Denver: Love Publishing.

Deshler, D. D., & Lenz, B. K. (1989). The strategies instructional approach. *International Journal of Disability, Development and Education, 36*, 203-224.

Deshler, D. D., & Schumaker, J. B. (1993). Strategy mastery by at-risk students: Not a simple matter. *Elementary School Journal, 94*(2), 153-167.

Dunston, P. J. (1992). A critique of graphic organizer research. *Reading Research and Instruction, 31*(2), 57-65.

Dunston, P. J., & Ridgeway, V. G. (1990). The effect of graphic organizers on learning and remembering information from connected discourse. *Forum for Reading, 22*(1), 15-23.

Earle, R. A. (1969). Use of the structured overview in mathematics classes. In H. L. Herber & P. L. Sanders (Eds.), *Research in reading in the content areas: First year report* (pp. 49-58). Syracuse, NY: Syracuse University, Reading and Language Arts Center.

Estes, T. H., Mills, D. C., & Barron, R. F. (1969). Three methods of introducing students to a reading-learning task in two content subjects. In H. L. Herber & P. L. Sanders (Eds.), *Research in reading in the content areas: First year report* (pp. 40-47). Syracuse, NY: Syracuse University, Reading and Language Arts Center.

Fredericksen, E. (2000). Muted colors: Gender and classroom silence. *Language Arts, 77*, 301-308.

Friedman, H. (1968). Magnitude of experimental effect and a table for its rapid estimation. *Psychological Bulletin*, *70*, 245-251.

Fueyo, V., & Bushell, D., Jr. (1998). Using number line procedures and peer tutoring to improve the mathematics computation of low-performing first graders. *Journal of Applied Behavior Analysis, 31*, 417-430.

Fuson, K. C., & Willis, G. B. (1989). Second graders' use of schematic drawings in solving addition and subtraction word problems. *Journal of Educational Psychology*, *81*, 514-520.

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, *114*, 345-362.

Geary, D. C. (2000). Mathematical disorders: An overview for educators. *Perspectives, 23*(3), 6-9.

Gresham, F. M., & Lopez, M. F. (1996). Social validation: A unifying concept for school-based consultation research and practice. *School Psychology Quarterly*, *11*, 204-227.

Griffin, C. C., & Tulbert, B. L. (1995). The effect of graphic organizers on students' comprehension and recall of expository text: A review of the research and implications for practice. *Reading & Writing Quarterly: Overcoming Learning Difficulties*, *11*, 73-89.

Grobecker, B. (1999). Mathematics reform and learning differences. *Learning Disability Quarterly*, *22*, 43-58.

Grober, D. A., & Mewborn, D. S. (2001). Promoting equity in mathematics classrooms. *Middle School Journal, 32*(3), 31-35.

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25-33.

Hallahan, D. P., & Bryan, T. H. (1981). Learning disabilities. In J. M. Kauffman & D. P. Hallahan (Eds.), *Handbook of Special Education*. Englewood Cliffs, NJ: Prentice-Hall.

Harris, C. A., Miller, S. P., & Mercer, C. D. (1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research & Practice, 10*, 180-195.

Hécaen, H., Angelergues, R., & Houillier, S. (1961). Les variétés cliniques des acalculies au cours des lésions rétrorolandiques: Approches statistiques du problème. *Revue Neurologique*, *105*, 85-103.

Heddens, J. W. (1986). Bridging the gap between the concrete and the abstract. *Arithmetic Teacher, 33*(6), 14-17.

Hofmeister, A. M. (1993). Elitism and reform in school mathematics. *Remedial & Special Education, 14*(6), 8-13.

Holcombe, A., Wolery, M., & Gast, D. L. (1994). Comparative single-subject research: Description of designs and discussion of problems. *Topics in Early Childhood Special Education*, *14*(1), 119-145.

Holley, C. D., & Dansereau, D. F. (1984). Networking: The technique and the empirical evidence. In C. D. Holley & D. F. Dansereau (Eds.), *Spatial learning strategies* (pp. 81-108). New York: Academic Press.

Huck, S. W., & Cormier, W. H. (1996). *Reading statistics and research* (Second ed.). New York: Harper Collins.

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16*, 34-63.

Ives, B. (2000). [Empirical studies on math interventions in journals addressing learning disabilities]. Unpublished raw data.

Jitendra, A. K., Griffith, C. C., McGoey, K., Gardill, M. C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *Journal of Educational Research, 91*, 345-355.

Jitendra, A. K., & Hoff, K. (1996). The effects of schema-based instruction on the mathematical word-problem-solving performance of students with learning disabilities. *Journal of Learning Disabilities*, *29*, 422-431.

Jitendra, A. K., Hoff, K., & Beck, M. M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial & Special Education, 20*, 50-64.

Jones, E. D., Wilson, R., & Bhojwani, S. (1998). Mathematics instruction for secondary students with learning disabilities. In D. P. Rivera (Ed.), *Mathematics education for students with learning disabilities* (pp. 155-176). Austin: Pro-Ed.

Karush, W. (1962). *The crescent dictionary of mathematics*. Palo Alto, CA: Dale Seymour Publications.

Keeler, M. L., & Swanson, H. L. (2001). Does strategy knowledge influence working memory in children with mathematical disabilities? *Journal of Learning Disabilities, 34*, 418-434.

Kiewra, K. A., Dubois, N. F., Christian, D., & McShane, A. (1988). Providing study notes: Comparison of three types of notes for review. *Journal of Educational Psychology, 80*, 595-597.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities*, *7*, 164-177.

Kulak, A. G. (1993). Parallels between math and reading disability: Common issues and approaches. *Journal of Learning Disabilities, 26*, 666-673.

Larson, R. E., Boswell, L., Kanold, T. D., & Stiff, L. (2001a). *Algebra I*. Evanston, IL: McDougal Littel.

Larson, R. E., Boswell, L., Kanold, T. D., & Stiff, L. (2001b). *Algebra II*. Evanston, IL: McDougal Littel.

Larson, R. E., Kanold, T. D., & Stiff, L. (1993a). *Algebra 1*. Lexington, MA: D. C. Heath.

Larson, R. E., Kanold, T. D., & Stiff, L. (1993b). *Algebra 2*. Lexington, MA: D. C. Heath.

Long, G., & Adersley, S. (1984). Networking: Application with hearing-impaired students. In C. D. Holley & D. F. Dansereau (Eds.), *Spatial learning strategies* (pp. 109-125). New York: Academic Press.

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice, 15*, 10-21.

Maccini, P., McNaughton, D., & Ruhl, K. L. (1999). Algebra instruction for students with learning disabilities: Implications from a research review. *Learning Disability Quarterly, 22*, 113-126.

Marolda, M. R., & Davidson, P. S. (2000). Mathematical learning profiles and differentiated teaching strategies. *Perspectives*, *23*(3), 10-15.

Marzola, E. S. (1987). Using manipulatives in math instruction. *Journal of Reading, Writing, and Learning Disabilities International*, *3*, 9-20.

Masters, L. F., Mori, B. A., & Mori, A. A. (1993). *Teaching secondary students with mild learning and behavior problems*. Austin: Pro-Ed.

Mastropieri, M. A., & Scruggs, T. E. (1994). *Effective instruction for special education* (Second ed.). Austin: Pro-Ed.

Mastropieri, M. A., Scruggs, T. E., & Shiah, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research & Practice*, *6*, 89-98.

Mathematics and Science Expert Panel. (1999). *Exemplary and promising mathematics programs*. U. S. Department of Education. Retrieved March 8, 2002, from the World Wide Web: http://www.enc.org/professional/federalresources/exemplary/promising/document.shtm?input=CDS-000496-496_toc

McConnell, J. W., Brown, S., Eddins, S., Hackworth, M., Sachs, L., Woodward, E., Flanders, J., Hirschhorn, D., Hynes, C., Polonsky, L., & Usiskin, Z. (1993). *University of Chicago School Mathematics Project Algebra*. Glenview, IL: Scott, Foresman.

Meese, R. L. (1994). *Teaching learners with mild disabilities: Integrating research and practice*. Pacific Grove, CA: Brooks/Cole.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education*, *13*(3), 19-35, 61.

Miller, S. P., & Mercer, C. D. (1993). Using data to learn about concrete-semiconcrete-abstract instruction for students with math disabilities. *Learning Disabilities Research & Practice*, *8*, 89-96.

Miller, S. P., & Mercer, C. D. (1998). Educational aspects of mathematics disabilities. In D. P. Rivera (Ed.), *Mathematics education for students with learning disabilities* (pp. 81-95). Austin: Pro-Ed.

Miller, S. P., Mercer, C. D., & Dillon, A. S. (1992). CSA: Acquiring and retaining math skills. *Intervention in School and Clinic, 28*, 105-110.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities*, *25*, 230-248.

Montague, M. (1998). Cognitive strategy instruction in mathematics for students with learning disabilities. In D. P. Rivera (Ed.), *Mathematics education for students with learning disabilities* (pp. 177-199). Austin: Pro-Ed.

Montague, M., Applegate, B., & Marquard, K. (1993). Cognitive strategy instruction and mathematical problem-solving performance of students with learning disabilities. *Learning Disabilities Research & Practice, 8*, 223-232.

Moore, D. W., & Readance, J. E. (1984). A quantitative and qualitative review of graphic organizer research. *Journal of Educational Research, 78*, 11-17.

Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Math literacy and civil rights*. Boston: Beacon Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Joint Committee on Learning Disabilities. (1990, January). Learning disabilities: Issues on definition. (Retrieved from http://www.ldonline.org/njcld/defn_91.html)

Nichols, E. D., Edwards, M. L., Garland, E. H., Hoffman, S. A., Mamary, A., & Palmer, W. F. (1986). *Holt Algebra 2 With Trigonometry*. New York: Holt, Rinehart and Winston.

Novak, J. D., Gowin, D. B., & Johansen, G. T. (1983). The use of concept mapping and knowledge vee mapping with junior high school science students. *Science Education, 67,* 625-645.

Ozols, E. J., & Rourke, B. P. (1988). Characteristics of young learning-disabled children classified according to patterns of academic achievement: Auditory-perceptual and visual-perceptual abilities. *Journal of Clinical Child Psychology, 17*(1), 44-52.

Padget, S. Y. (1998). Lessons from research in dyslexia: Implications for a classification system for learning disabilities. *Learning Disability Quarterly, 21*, 167-178.

Paivio, A. (1986). *Mental representations: A dual coding approach*. New York: Oxford University Press.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4*(1), 52-56.

Retish, P., Hitchings, W., Horvath, M., & Schmalle, B. (1991). *Students with mild disabilities in the secondary school*. New York: Longman Publishing Group.

Rice, G. E. (1994). Need for explanations in graphic organizer research. *Reading Psychology: An International Journal*, *15*, 39-67.

Rivera, D. P. (1998). Mathematics education and students with learning disabilities: Introduction. In D. P. Rivera (Ed.), *Mathematics education for students with learning disabilities* (pp. 1-31). Austin: Pro-Ed.

Robinson, D. H. (1998). Graphic organizers as aids to text learning. *Reading Research and Instruction, 37*(2), 85-105.

Rotberg, I. C. (2001). A self-fulfilling prophecy. *Phi Delta Kappan, 83*, 170-171.

Rourke, B. P. (1989). *Nonverbal learning disabilities: The syndrome and the model*. New York: Guilford Press.

Rourke, B. P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. *Journal of Learning Disabilities, 26*, 214-226.

Rourke, B. P., & Conway, J. A. (1998). Disabilities of arithmetic and mathematical reasoning: Perspectives from neurology and neuropsychology. In D. P. Rivera (Ed.), *Mathematics education for students with learning disabilities* (pp. 59-79). Austin: Pro-Ed.

Rourke, B. P., Dietrich, D. M., & Young, G. C. (1973). Significance of WISC Verbal-Performance discrepancies for younger children with learning disabilities. *Perceptual and Motor Skills, 36*, 275-282.

Rourke, B. P., & Finlayson, M. A. J. (1978). Neuropsychological significance of variations in patterns of academic performance: Verbal and visual-spatial abilities. *Journal of Abnormal Child Psychology, 6*(1), 121-133.

Rourke, B. P., & Strang, J. D. (1978). Neuropsychological significance of variations in patterns of academic performance: Motor, psychomotor, and tactile-perceptual abilities. *Journal of Pediatric Psychology, 3*(2), 62-66.

Rourke, B. P., Young, G. C., & Flewelling, R. W. (1971). The relationships between WISC Verbal-Performance discrepancies and selected verbal, auditory-perceptual, visual-perceptual, and problem-solving abilities in children with learning disabilities. *Journal of Clinical Psychology, 27*, 475-479.

Sadoski, M., & Paivio, A. (2001). *Imagery and text: A dual coding theory of reading and writing*. Mahwah, NJ: Lawrence Erlbaum Associates.

Saxon, J. H., Jr. (1993). *Algebra 1: An Incremental Development* (Second ed.). Norman, Oklahoma: Saxon.

Saxon, J. H., Jr. (1997). *Algebra 2: An Incremental Development* (Second ed.). Norman, Oklahoma: Saxon.

Schwartz, I. S. (1996). Expanding the zone: Thoughts about social validity and training. *Journal of Early Intervention, 20*, 204-205.

Schwartz, I. S. (1999). Controversy or lack of consensus? Another way to examine treatment alternatives. *Early Childhood Special Education, 19*(3), 189-193.

Senk, S. L., Thompson, D. R., Viktora, S. S., Rubenstein, R., Halvorson, J., Flanders, J., Jakucyn, N., Pillsbury, G., & Usiskin, Z. (1993). *University of Chicago School Mathematics Project Advanced Algebra*. Glenview, IL: Scott, Foresman.

Sharma, M. C. (1986). Progress of Dr. Ladislav Kosc's work on dyscalculia. *Focus on Learning Problems in Mathematics, 8*(3 & 4), 47-119.

Silver, C. H., Pennett, H. D.-L., Black, J. L., Fair, G. W., & Balise, R. R. (1999). Stability of arithmetic disability subtypes. *Journal of Learning Disabilities, 32*, 108-119.

Stellingwerf, B. P., & Van Lieshout, E. C. D. M. (1999). Manipulatives and number sentences in computer aided arithmetic word problem solving. *Instructional Science, 27*, 459-476.

Storey, K., & Horner, R. H. (1991). An evaluative review of social validation research involving persons with handicaps. *Journal of Special Education*, *25*, 352-401.

Stoskopf, A. (2001). Reviving Clio: Inspired history teaching and learning. *Phi Delta Kappan*, *9*, 468-473.

Strang, J. D., & Rourke, B. P. (1983). Concept-formation/non-verbal reasoning abilities of children who exhibit specific academic problems with arithmetic. *Journal of Clinical Child Psychology, 12*(1), 33-39.

Sutlive, V. H., & Ulrich, D. A. (1998). Interpreting statistical significance and meaningfulness in adapted physical activity research. *Adapted Physical Activity Quarterly*, *15*(2), 103-118.

Swafford, J., & Alvermann, D. E. (1989). Postsecondary research base for content strategies. *Journal of Reading, 33*, 164-169.

Swanson, H. L. (1989). The effects of central processing strategies on learning disabled, mildly retarded, average, and gifted children's elaborative encoding abilities. *Journal of Experimental Child Psychology*, *47*, 370-397.

Swanson, H. L. (1993). An information processing analysis of learning disabled children's problem solving. *American Educational Research Journal, 30*, 861-893.

Swanson, H. L., Christie, L., & Rubadeau, R. J. (1993). The relationship between metacognition and analogical reasoning in mentally retarded, learning disabled, average, and gifted children. *Learning Disabilities Research & Practice, 8*, 70-81.

Swanson, H. L., & Cooney, J. B. (1985). Strategy transformation in learning disabled and nondisabled students. *Learning Disability Quarterly, 8*, 221-230.

Swanson, H. L., Hoskyn, M., & Lee, C. (1999). *Interventions for students with learning disabilities: A meta-analysis of treatment outcomes*. New York: Guilford Press.

Swanson, H. L., & Rhine, B. (1985). Strategy transformations in learning disabled children's math performance: Clues to the development of expertise. *Journal of Learning Disabilities, 18*, 596-603.

Tawney, J. W., & Gast, D. L. (1984). *Single subject research in special education*. Columbus, OH: Merrill.

Thomas, P. L. (2001). Standards, standards everywhere, and not a spot to think. *English Journal, 91*(1), 63-67.

Thompson, B. (1993). The use of statistical significance tests in research: Bootstrap and other alternatives. *Journal of Experimental Education, 61*, 361-377.

Thompson, B. (1996). AERA editorial policies regarding statistical significance testing: Three suggested reforms. *Educational Researcher, 25*(2), 26-30.

Vaughan, J. L., Jr. (1982). Use of the construct procedure to foster active reading and learning. *Journal of Reading, 25*, 412-422.

West, T. G. (2000). Math for dummies. *Perspectives, 26*(4), 25.

Willis, G. B., & Fuson, K. C. (1988). Teaching children to use schematic drawings to solve addition and subtraction word problems. *Journal of Educational Psychology, 80*, 192-201.

Winn, W. (1991). Learning from maps and diagrams. *Educational Psychology Review, 3*, 211-247.

Wolf, M. M. (1978). Social validity: The case for subjective measurement or How applied behavioral analysis is finding its heart. *Journal of Applied Behavior Analysis*, *11*, 203-214.

APPENDIX A

INFORMED CONSENT AND ASSENT FORMS

INFORMED CONSENT FOR TEACHERS

I agree to take part in a research study titled “Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disabilities,” which is being conducted by Bob Ives, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, home: 706-559-7565, office: 706-542-4571, e-mail: rives@coe.uga.edu, under the direction of Dr. Noel Gregg, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, 542-4597. I do not have to take part in this study ; I can stop taking part at any time without giving any reason, and without penalty. I can ask to have information related to me returned to me, removed from the research records, or destroyed.

The reason for the study is to test the prediction that students learn algebra better when instruction includes visual/graphic presentation. Research has already shown that some people have stronger visual skills, while others have stronger verbal skills. However, the idea that visual presentation might improve how well a student learns algebra, has not been researched.

I will not benefit directly from this research beyond that fact that the study will be covering algebra material normally covered during the course anyway, and this material will be taught by an experienced, effective, certified teacher. However, my participation in this research may lead to teaching techniques that are more individualized to the way that students learn algebra. I also may observe approaches to teaching algebra that I would want apply in the future.

The procedures are as follows:

1) Participation in the study will last approximately two weeks during regular algebra class times.
2) I will be asked to construct and provide a test of the material being covered during the study that is typical of a test I would ordinarily give students for that material.
3) I will be asked to administer and grade the test that I construct, and share the results with the investigator.
4) I will be asked to observe the classes that the investigator is teaching for the duration of the study and complete a checklist while observing.
5) I will be asked to participate in an interview at the end of the study which will give me an opportunity to contribute my views about the study.

No discomforts or stresses are expected beyond those typically experienced during the algebra class.

No risks are expected.

No deception is involved in this study.

While the study is going on the only people who will know that I am a research participant are members of the research team, the school staff, and students. Any information that is obtained in connection with this study and that can be identified with me will remain confidential and will be disclosed only with my permission or as required by law. The investigator will audiotape the interview. Tapes may be stored for up to three years before being destroyed. No future publication or presentation of information gathered from this study will include details that would identify individual participants.

The researcher will answer any further questions about the research, now or during the course of the project, and can be reached by at home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu.

I understand the procedures described above. My questions have been answered to my satisfaction, and I agree to participate in this study. I have been given a copy of this form.

_______________________________________ Signature of Researcher and Date

_______________________________________ Signature of Teacher and Date

For questions or problems about your rights please call or write: Chris A. Joseph, Ph.D., Human Subjects Office, University of Georgia, 606A Boyd Graduate Studies Research Center, Athens, Georgia 30602-7411; Telephone (706) 542-6514; E-Mail Address IRB@uga.edu.

INFORMED PARENT/GUARDIAN CONSENT FOR PARTICIPATION

I agree to permit my child ________________________________ , to take part in a research study titled "Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disabilities," which is being conducted by Bob Ives, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu, under the direction of Dr. Noel Gregg, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, 542-4597. I do not have to allow my child to take part in this study; I can stop my child's participation at any time without giving any reason, and without penalty. I can ask to have information related to my child returned to me, removed from the research records, or destroyed.

The reason for the study is to test the prediction that students learn algebra better when instruction includes visual/graphic presentation. Research has already shown that some people have stronger visual skills, while others have stronger verbal skills. However, the idea that visual presentation might improve how well a student learns algebra, has not been researched.

The study will be covering algebra material normally covered during the course, and this material will be taught by an experienced, effective, certified teacher. My child may also benefit to the extent that the visual/graphic approach adds to the effectiveness of conventional instruction. In addition, my child's participation in this research may lead to teaching techniques that are more individualized to the way that students learn algebra.

The procedures for this study are as follows:

1) Participation in the study will last approximately two weeks during regular algebra class times.
2) My child will take a test of prerequisite skills related to the material being covered in class.
3) My child will participate in algebra class activities and assignments as usual.
4) My child will take a test on the material covered in class that has been created by the regular classroom teacher.
5) My child will take two tests on the material covered in class that have been created by the investigator. These tests will have no impact on my child's grades in the course.
6) My child will complete a questionnaire about the study.

No discomforts or stresses are expected beyond those typically experienced during the algebra class.

No risks are expected beyond those typically experienced during the algebra class.

No deception is involved in this study.

While the study is going on, the only people who will know that my child is a research participant are members of the research team, the school staff, parents and classmates. Any information that is obtained in connection with this study and that can be identified with a specific student will remain confidential and will be disclosed only with my permission or as required by law. Once data collection is completed, each participant will be assigned a participant number and names will be removed from the data files such that no record matching names to participant numbers or names to data will exist. No future publication or presentation of information gathered from this study will include details that would identify individual participants. Neither parents nor students will have access to individual results from this study. Parents, students, and staff for the entire school will be invited to attend a presentation of the overall findings of the study, during which confidentiality of participants will be preserved.

The researcher will answer any further questions about the research, now or during the course of the project, and can be reached by at home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu.

I understand the procedures described above. My questions have been answered to my satisfaction, and I agree to allow my child to participate in this study. I have been given a copy of this form.

____ Signature of Researcher and Date

____ Signature of Parent/Guardian and Date

For questions or problems about your rights please call or write: Chris A. Joseph, Ph.D., Human Subjects Office, University of Georgia, 606A Boyd Graduate Studies Research Center, Athens, Georgia 30602-7411; Telephone (706) 542-6514; E-Mail Address IRB@uga.edu.

INFORMED PARENT/GUARDIAN CONSENT FOR ACCESS TO RECORDS

I agree to permit access to school records of my child ________________________________ at The XXXXXXX School, by Bob Ives, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu, as part of a research study titled "Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disabilities," which is being conducted under the direction of Dr. Noel Gregg, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, 542-4597. I do not have to allow access to my child's school records; I can stop my child's participation at any time without giving any reason, and without penalty. I can ask to have information related to my child returned to me, removed from the research records, or destroyed.

The reason for the study is to test the prediction that students learn algebra better when instruction includes visual/graphic presentation. In order to study the impact of visual presentation on student achievement, the investigator will need to gather information on other variables that may also influence achievement. These types of information are listed in the next paragraph. My child may benefit from participation because the study will be covering algebra material normally covered during the course, and this material will be taught by an experienced, effective, certified teacher. My child may also benefit to the extent that the visual/graphic approach adds to the effectiveness of conventional instruction. In addition, my child's participation in this research may lead to teaching techniques that are more individualized to the way that students learn algebra. Analysis of the study results will be clearer and more specific when compared to the information recorded from the student files. The information collected from those files will remain confidential. No presentation or publication of the results of this study will include information that identifies individual students.

The following types of information may be recorded from my child's school files:

1) Descriptive information including my child's birthdate and age, grade in school, ethnicity, first language, and gender.
2) Diagnoses from psychoeducational profiles that are relevant to school performance, including learning disabilities, and attention deficits.
3) Medications my child is currently taking that may influence to school performance.
4) Scores from standardized academic achievement tests.
5) Scores from standardized cognitive skills and achievement tests.
6) Report card term grades from mathematics classes taken prior to the current grading period.
7) History of participation in Special Education programs, and other academic interventions.

No discomforts or stresses are expected because of accessing these records.

No risks are expected because of accessing these records.

No deception is involved in accessing these records.

Any information that is obtained in connection with this study and that can be identified with my child will remain confidential and will be disclosed only with my permission or as required by law. Once data collection is completed, my child will be assigned a participant number and names will be removed from the data files such that no record matching my child's name to his/her participant number will exist. No future publication or presentation of information gathered from this study will include details that would identify individual participants. Parents, students, and staff for the entire school will be invited to attend a presentation of the overall findings of the study, during which confidentiality of participants will be preserved.

The researcher will answer any further questions about the research, now or during the course of the project, and can be reached by at home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu.

I understand the procedures described above. My questions have been answered to my satisfaction, and I agree to allow my child to participate in this study. I have been given a copy of this form.

____ Signature of Researcher and Date

____ Signature of Parent/Guardian and Date

For questions or problems about your rights please call or write: Chris A. Joseph, Ph.D., Human Subjects Office, University of Georgia, 606A Boyd Graduate Studies Research Center, Athens, Georgia 30602-7411; Telephone (706) 542-6514; E-Mail Address IRB@uga.edu.

INFORMED STUDENT ASSENT FOR PARTICIPATION

I, _________________________________ agree to take part in a research study titled "Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disabilities," which is being conducted by Bob Ives, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu, under the direction of Dr. Noel Gregg, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, 542-4597. I do not have to take part in this study; I can stop taking part at any time without giving any reason, and without penalty. I can ask to have information related to me returned to me, removed from the research records, or destroyed.

The reason for the study is to see if students learn algebra better when instruction includes diagrams. Research has already shown that some students learn better with diagrams and some students learn better with words. However, this difference has not been shown for algebra yet.

I understand that the study will be covering algebra material normally covered during the course. The diagrams may help me to learn the material better. In addition, my participation in this research may help other students to learn algebra better in the future.

The procedures for this study are as follows:

1) Participation in the study will last approximately two weeks during regular algebra class times.
2) I will take a test of prerequisite skills related to the material being covered in class.
3) I will participate in algebra class activities and assignments as usual.
4) I will take a test on the material covered in class that has been created by the regular classroom teacher.
5) I will take two tests on the material covered in class that have been created by the investigator. These tests will have no impact on my grades in the course.
6) I will complete a questionnaire about the study.

This study does not involve anything more uncomfortable, stressful, or risky than my normal experience in algebra class.

No deception is involved in this study.

While the study is going on, the only people who will know that I am a research participant are members of the research team, the school staff, parents and classmates. Any information about me in this study will remain confidential and will be disclosed only with my permission or as required by law. My parents and I will not have access to my results from this study. Parents, students, and staff for the entire school will be invited to attend a presentation of the overall findings of the study, but no specific information about me will be presented there either.

The researcher will answer any further questions about the research, now or during the course of the project, and can be reached by at home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu.

I understand the procedures described above. My questions have been answered to my satisfaction, and I agree to participate in this study. I have been given a copy of this form.

____ Signature of Researcher and Date

____ Signature of Participant and Date

For questions or problems about your rights please call or write: Chris A. Joseph, Ph.D., Human Subjects Office, University of Georgia, 606A Boyd Graduate Studies Research Center, Athens, Georgia 30602-7411; Telephone (706) 542-6514; E-Mail Address IRB@uga.edu.

INFORMED STUDENT ASSENT FOR ACCESS TO RECORDS

I, ________________________________ , agree to permit access to my school records at The XXXXXXX School by Bob Ives, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu, as part of a research study titled “Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disabilities,” which is being conducted under the direction of Dr. Noel Gregg, Department of Special Education, University of Georgia, 537 Aderhold Hall, Athens, Georgia 30602-7153, 542-4597. I do not have to allow access to my school records; I can stop my participation at any time without giving any reason, and without penalty. I can ask to have information related to me returned to me, removed from the research records, or destroyed.

The reason for the study is to see if students learn algebra better when instruction includes diagrams. Analysis of the study results will be clearer when compared to information from my school files. The information to be recorded from my files is listed in the next paragraph. I may benefit from participation because the study will cover algebra skills I would normally have to cover anyway, and the material will be taught by an experienced, effective, certified teacher. I may also benefit further from the visual/graphic approach. My participation in this research may lead to teaching ideas that help other students learn algebra.

The following types of information may be recorded from my school files:

1) Descriptive information including my birthdate and age, grade in school, ethnicity, first language, and gender.
2) Diagnoses from psychoeducational profiles that are relevant to school performance, including learning disabilities, and attention deficits.
3) Medications I am currently taking that may influence to school performance.
4) Scores from standardized academic achievement tests.
5) Scores from standardized cognitive skills and intelligence tests.
6) Report card term grades from mathematics classes taken prior to the current grading period.
7) History of participation in Special Education programs, and other academic interventions.

Allowing access to my school records for this study should cause no unusual discomforts, stresses, or risks.

No deception is involved in accessing these records.

Any information about me that is collected for this study will remain confidential and will be disclosed only with my permission or as required by law. No future publication or presentation of information gathered from this study will include details that would identify me specifically. Parents, students, and staff for the entire school will be invited to attend a presentation of the overall findings of the study, during which confidentiality of participants will be preserved.

The researcher will answer any further questions about the research, now or during the course of the project, and can be reached by at home: 706-559-7565, office: 706-542-4237, e-mail: rives@coe.uga.edu.

I understand the procedures described above. My questions have been answered to my satisfaction, and I agree to allow access to these records. I have been given a copy of this form.

____ Signature of Researcher and Date

____ Signature of Participant and Date

For questions or problems about your rights please call or write: Chris A. Joseph, Ph.D., Human Subjects Office, University of Georgia, 606A Boyd Graduate Studies Research Center, Athens, Georgia 30602-7411; Telephone (706) 542-6514; E-Mail Address IRB@uga.edu.

APPENDIX B

PREREQUISITE SKILLS TEST, CONTENT SKILLS TESTS, AND SCORING GUIDES FOR STUDY 1

Prerequisite Skills Test for Study 1

1) Solve for *x*:

$$7x = 42$$

2) Solve for *x*:

$$-3x = 18$$

3) Solve for *x*:

$$2x - 5 = 3$$

4) Solve for *x*:

$$-3x + 8 = 14$$

5) Assume $x = 3$ and solve for y:

$$2x - 4y = 2$$

6) Assume $x = -4$ and solve for y:

$$-3x - 6y = 0$$

7) Add these two equations:

$$2x - 4y = 6 \qquad\qquad 4x + 3y = -2$$

8) Add these two equations:

$$-7x - 2y = 5 \qquad\qquad x + 2y = 11$$

9) Multiply this equation by a factor of 3:

$$2x - 5y = 8$$

10) Multiply this equation by a factor of -2:

$$-3x + y = 4$$

11) Find a common multiple for 2 and 5:

12) Find a common multiple for 3 and 9:

13) Find a common multiple for 4 and 6:

14) Find a common multiple for 8 and 12:

Content Skills Test for Study 1 (Version A)

1) How many solutions does each equation have?

$5x = 35$

$3x - y = 16$

2) Is it possible to change these two equations so they can be combined to create a new equation with only ONE variable in it? If yes, how would you do it? If no, why not?

$2x - 3y = -5$ $4x - 5y = 23$

3) Why is it important to eliminate variables when solving systems of equations?

4) On a separate sheet of paper solve this system of equations:

$2x - y = 19$

$-2x + 4y = -4$

5) On a separate sheet of paper solve this system of equations:

$3x - 2y = -11$

$2x + y = -5$

6) On a separate sheet of paper solve this system of equations:

$-2x - y = 14$

$3x = -27$

7) On a separate sheet of paper solve this system of equations:

$2x - y + 3z = 7$

$-2x + 4y - 5z = -3$

$2x - 7y + 8z = 0$

Scoring Guide for Content Skills Test for Study 1 (Version A)

1) One point for indicating that the first equation has only one solution, or for solving the first equation, and one point for any response indicating that the second equation has multiple answers without indicating a specific number. Examples include "an infinite number," "a lot," "too many to count," etc.

2) One point for indicating that it is possible, and one point for indicating that the process requires multiplying or getting coefficients to add to zero. Full credit should also be given if a student correctly accomplishes these goals without describing them.

3) One point for indicating that equations with only one variable can be solved while those with more variables can not be solved.

4) One point for a consistent and independent equation with one variable

One point for each correct value in the final solution: (12, 5)

Maximum of 3 points

5) One point for a consistent and independent equation with one variable

One point for each correct value in the final solution: (–3, 1)

Maximum of 3 points

6) One point for each correct value in the final solution: (–9, 4)

Maximum of 2 points

7) One point each up to a maximum of two points for consistent and independent equations with the same two variables

One point for one consistent equation with one variable

One point for each correct value in the final solution: (3, 2, 1)

Maximum of 6 points

Content Skills Test for Study 1 (Version B)

1) How many solutions does each equation have?

$5x + 2y = 35$

$-4y = 16$

2) Is it possible to change these two equations so they can be combined to create a new equation with only ONE variable in it? If yes, how would you do it? If no, why not?

$x - 3y = -7$ $\qquad$ $3x - 2y = 14$

3) Why is it important to eliminate variables when solving systems of equations?

4) On a separate sheet of paper solve this system of equations:

$3x + 5y = -7$

$x - 5y = 11$

5) On a separate sheet of paper solve this system of equations:

$3x + 2y = -5$

$-2x - 5y = -4$

6) On a separate sheet of paper solve this system of equations:

$x - 3y = 13$

$2y = -10$

7) On a separate sheet of paper solve this system of equations:

$2x + 2y - z = 1$

$3x - 2y + 4z = 28$

$-5x + 2y - 7z = -45$

Scoring Guide for Content Skills Test for Study 1 (Version B)

1) One point for any response indicating that the second equation has multiple answers without indicating a specific number. Examples include “an infinite number,” “a lot,” “too many to count,” etc. One point for indicating that the second equation has only one solution, or for solving the second equation.

2) One point for indicating that it is possible, and one point for indicating that the process requires multiplying or getting coefficients to add to zero. Full credit should also be given if a student correctly accomplishes these goals without describing them.

3) One point for indicating that equations with only one variable can be solved while those with more variables can not be solved.

4) One point for a consistent and independent equation with one variable

One point for each correct value in the final solution: (1, –2)

Maximum of 3 points

5) One point for a consistent and independent equation with one variable

One point for each correct value in the final solution: (–3, 2)

Maximum of 3 points

6) One point for each correct value in the final solution: (–2, –5)

Maximum of 2 points

7) One point each up to a maximum of two points for consistent and independent equations with the same two variables

One point for one consistent equation with one variable

One point for each correct value in the final solution: (4, –2, 3)

Maximum of 6 points

APPENDIX C

FRAMING QUESTIONS FOR TEACHER INTERVIEWS

Framing Questions for Teacher Interviews

1) In your experience, how challenging has it been to teach your students to solve systems of equations?

2) How important do you feel it is for your students to learn how to solve these systems of equations?

3) How do you feel the graphic organizer was helpful for students learning this material?

4) Do you feel that all students benefit equally from using the graphic organizer? Why or why not? Can you give a specific example?

5) Would you try a similar approach to this material when you are teaching it? Why or why not?

6) What other topics do you think might be effectively taught using some kind of graphic organizer? How would you approach it, and what might the organizer look like?

7) What kinds of disadvantages do you see in using a graphic organizer to teach algebra?

APPENDIX D

STUDENT QUESTIONNAIRES

Graphic Organizer Student Questionnaire

For each question, circle the number of the response that best describes your answer.

		Very		Somewhat		Not at all
1)	How important is it in your algebra class to learn how to solve systems of equations?	5	4	3	2	1
2)	How important is it in everyday life to learn how to solve systems of equations?	5	4	3	2	1
3)	Was the graphic organizer helpful for learning this material?	5	4	3	2	1
4)	Would you be interested in learning about graphic organizers for other kinds of problems?	5	4	3	2	1
5)	Is it easier to solve these problems with the graphic organizer than it would be without the graphic organizer?	5	4	3	2	1
6)	Are you likely to use the graphic organizer the next time you have one of these					

problems?	5	4	3	2	1

Control Student Questionnaire

For each question, circle the number of the response that best describes your answer.

		Very		Somewhat		Not at all
1)	How important is it in your algebra class to learn how to solve systems of equations?	5	4	3	2	1
2)	How important is it in everyday life to learn how to solve systems of equations?	5	4	3	2	1
3)	Was the instruction helpful for learning this material?	5	4	3	2	1
4)	Would you be interested in learning about using this approach for other kinds of problems?	5	4	3	2	1
5)	Is it easier to solve these problems with the same kind of instruction than it would be without this instruction?	5	4	3	2	1
6)	Are you likely to use the same approach the next time you have one of these					

problems?	5	4	3	2	1

APPENDIX E

LESSON DESCRIPTIONS FOR STUDY 1

Lesson Plans for Study 1

Day 1

The primary purpose of the first day of instruction is to review the prerequisite skills assessed on the pretest of prerequisite skills. These skills include a) solving linear equations in one variable, b) solving linear equations that have one variable remaining after substituting a value for a second variable, c) multiplying linear equations by constants, d) adding linear equations, and e) finding common multiples.

Prior to addressing these prerequisite skills, the topic of systems of two linear equations in two variables will be introduced with a hypothetical problem. Students will be asked to imagine that they have each voted on which student in the class is most likely to become President of the United States. The teacher has tallied the votes. The problem is to determine how many votes each of two students received based on a pair of clues. Two specific students will be identified in the class to make this problem more concrete. For purposes of this description I will call them Al Gore, and Writh M. The clues are:

1) Twice Al's votes added to three times Writh's votes totals 22 votes. This will be written as the equation $2x + 3y = 22$.

2) Negative two times Al's votes minus five times Writh's votes makes –30. This clue yields the equation $-2x - 5y = -30$.

Once these two equations have been posted for the class we will discuss the fact that solving them by substitution would lead to equations containing fractional coefficients. Certainly these coefficients can be changed to integers with the multiplication of appropriate factors. Nevertheless, the instructor will let the class know that on the following day they will solve this system without having to deal with fractional coefficients. For the rest of this lesson they will be reviewing and practicing some skills they already have done before that they will need to solve that problem. The students will be asked to solve independently some simple one-step linear equations with integer solutions and involving either addition/subtraction or multiplication/division. These students have worked with these skills before in Algebra I. This skill is necessary to solve the more complex systems, and the review is probably particularly valuable for students with learning problems. Once solutions have been checked and procedures have been discussed, a few of examples of two-step linear equations with integer solutions will be solved, checked, and discussed the same way. Discussion of solutions will include the fact that these solutions are unique. This should cover prerequisite skill a) noted above.

Next the equation $x + y = 12$ will be posted. Students will be asked how this equation is different from the ones they have just solved. One response, which will be emphasized, is that this equation has two variables. Students will be encouraged to think of any two numbers that add to 12. This generative process will extend to include negative integers and perhaps fractions and decimals. Eventually this discussion will lead to the conclusion that there are an infinite number of possible solutions to this equation, some of which are posted. Next, the equation $2x – y = 12$ will be posted. Again multiple possible solutions will be generated, and again the students should come to the conclusion that an infinite number of solutions is possible.

Having arrived at the generalization that equations with one variable can be solved, while those with more variables can not, prerequisite skill b) will be addressed as follows. Returning to the equation $x + y = 12$, students will be given a few values for x and asked to solve for y. They will also be given a few values for y and asked to solved for x. The same will be done for the equation $2x - y = 12$. The emphasis of this practice will be to recognize that when a value is substituted into one of these equations, for either variable, only one variable remains. Therefore the equation can be solved for the remaining variable. This should cover prerequisite skill b).

At this point we will return our attention to one of the one variable equations, such as $2x - 3 = 11$. We will have already determined that the solution to this equation is 7. Working with partners, each student will be asked to multiply the entire equation by any number they wish. They will be encouraged to use numbers that are relatively easy to work with. Students will check their resulting equations with their partners, and then share them with the teacher who will post them, while checking them for accuracy and noting the multipliers used. Next, each student will be asked to solve his or her own new equation and then check answers with a partner. Solutions will then be shared with the entire class. The purpose of this exercise is to demonstrate that multiplying an equation by a constant yields an equivalent equation with the same solution set as the original equation. We will also discover by example that using the number zero as a factor is an exception to this rule. This covers prerequisite skill c).

For the next prerequisite skill we will look at the equivalent equations we created for the equation $2x - 3 = 11$ as well as the equation $x + y = 12$. In each case we will select pairs of equivalent equations and add them by combining like terms. Then we will explore solutions to show that any solution for both of the original equations will also be a solution for the combined equation. A third example, that of a two variable linear system will also be used. In this case, $x + y = 12$ and $2x - 3y = -6$ will be used to show that the solution (6, 6), which works in both equations, also solves the combined equations, while solutions that work for only one of the starting equations, will not work in the combined equation. This covers prerequisite skill d).

Finally, we will need to review briefly some ways of finding common multiples. Least common multiples of integers (ignoring negative signs which can be corrected by multiplying by –1) are typically found by looking at the prime factorization of the original integers or by serially multiplying each of the original integers by 1, 2, 3, . . . until the least common multiple is found. Technically, solving these systems can be done with any common multiples of coefficients, and sometimes students are given the option of simply multiplying two integers together to find a common multiple. For this lesson students will be asked to identify common multiples of pairs of integers and to describe how those common multiples were found. In this way we will be able to build on what the students already know about common multiples rather than imposing a prescribed procedure. This approach will review prerequisite skill e).

Once the prerequisite skills have been reviewed, the assessment instrument for prerequisite skills will be distributed. Students may begin this test in class and will be permitted to complete it as best they can as homework. The instructor will stipulate that this instrument is for assessment only and not for a grade. Students will be encouraged to do the best they can without outside assistance.

Day 2

At the beginning of class, four items will be posted at the front of the room. The first item is the system of two equations used to introduce this topic on the first day. The second item is a linear equation in two variables with a list of possible solutions, also from the previous day’s lesson. The purpose of this item is to remind students of the prerequisite concept that equations with more than one variable do not have unique solutions, while those with only one variable can typically be solved. The third item is an example of two equations being added together, drawn from the previous day's lesson. The fourth item is a blank graphic organizer as illustrated in Figure 1, except that the left column is removed as this column is not used for systems of two equations with two variables. The lesson will proceed in a highly interactive question/answer format as modeled here. The sequence of the lesson will be a review of prior material and prerequisite skills, modeling of new skills, guided practice, and independent practice. While students are encouraged to participate, the teacher will also call on specific students to ensure that less active students are still following, and engaging in, the lesson. Comments referring specifically to the graphic organizers and its elements will only be included in the lessons for the experimental group. Otherwise the two groups will have essentially the same lesson format and content. Italicized type suggests student responses, while Roman type indicates what the teacher would say.

> Yesterday I told you that we were going to solve this system of equations in a new way today (indicating the posted system). Actually you are going to solve this system yourselves. All I'm really going to do is ask you some easy questions, because you already know enough mathematics to solve this system of equations. But before you do that, let's make sure we're clear about an idea we looked at yesterday. How many possible solutions are there to this equation (indicating the posted two variable equation)?
>
> *Lots. An infinite number. More than you can count.*

Can you give me another example of a solution for this equation other than the ones already posted?

Several students will be given an opportunity to suggest additional solutions, including solutions containing negative integers.

Great. Now look at these equations here (indicating the two equations added to make a third). How did we get this third equation?

We added the other two

That's right. So where did this first term come from?

We added the first term from each of the other two equations. (They may be named specifically.)

Why didn't we add the *x* term of this equation with the *y* term of the second one?

Because you have to add like terms. They have to have the same variable.

Alright. Now let's look at this drawing I have on the board (indicating the blank graphic organizer). I call it a graphic organizer. We're going to use this as a tool to help solve this system of equations. In fact, I don't even want you to take any notes yet. For this first problem you're just going to talk me through it. Now, how many columns are there in this graphic organizer?

Two

Good. How many rows are in this graphic organizer?

Two

Alright. Now, what is this Roman numeral at the top of the left column (pointing) of the graphic organizer?

Two

Now, here's a really tough question. What is the Roman numeral at the top of the right column (pointing)?

One

Great. Now, take a look at the top equation in this system. How many variables are in this top equation?

Two

Can we solve it?

No

Why not?

It has too many variables. We can only solve equations with one variable.

Exactly. Now, we have a column with Roman numeral One on top, and a column with Roman numeral Two on top (indicating throughout). If you were going to put this equation with two variables into one of these columns, which column would you put it in?

Column two

Okay, here goes. (The first equation is rewritten in the top left cell of the graphic organizer.)

This sequence of questions and answers about the first equation is repeated for the second equation so that both have been entered into the graphic organizer. The reason for emphasizing the connection between the number of variables in an equation and the column into which it is entered, is that this concept anticipates the Day 4 lesson where one equation may have only one variable.

Okay, now we have both equations entered into the graphic organizer. Remind me again, can we solve any of these equations as they are now?

No

Would it help if we could create some equations with fewer variables?

Yes

Let's take a look at the x-terms in the first two equations. What would we get if we added 2x and –2x?

Nothing. Zero. They cancel.

If we add the first two equations together will we have any x's left?

No

Does that mean our new equation will have fewer variables than the two we start with?

Yes

Let's do it. What do we get when we combine 2x and –2x?

Nothing. Zero. They cancel.

What do we get when we combine 3y and –5y?

–2y

What about when we combine 22 and –30?

–8

What is the new equation we have created?

–2y = –8

How many variables are in the new equation?

One

Which column did I put that equation in?

Column one

What Roman numeral is at the top of that column?

One

Coincidence?

I think not

Does this new equation have the same number of variables as the first two?

No

How is it different?

It only has one variable. Less variables.

Let's take a look at this new equation you made. Is it possible to solve this equation?

Yes

Okay. Well I've been saving the bottom row for answers, so I'm going to bring this straight down into the bottom row. Go ahead and solve it. . . . What did you get?

Four

OK. It looks like y equals four. How many variables have we solved for so far?

One

What Roman numeral is at the top of the column above the solution?

One

Coincidence?

I think not

Well, now we know what y equals. We still have to find a value for x. Tell me, how many variables are in each equation of the left column?

Two

If I substitute the ‘4’ for the ‘y’ in the first equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variable in it?

Yes

If I substitute the ‘4’ for the ‘y’ in the second equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variables in it?

Yes

Which of those equations do you think will be easier to solve?

The first one

Both equations will lead to the same solution. This is an opportunity to suggest a self-monitoring question that could save a problem solver some extra effort.

Bring that equation straight down the column to the lower box, substitute the ‘4’ for the ‘y’ in the equation, and solve it. Let’s do it. If I substitute the ‘4’ for the ‘y’ in the first equation in column two what do I get?

$2x + 3(4) = 22$

Good. Now what would you do first to solve this?

Multiply 3 times 4

And what do you get?

12

What's our new equation?

2x + 12 = 22

What's next?

Subtract 12 from both sides. Move 12 to the other side and make it a minus 12.

Okay. So 12 minus 12 cancels. What is 22 minus 12?

10

Great. Now what?

Divide by 2. Divide both sides.

So 2 divided by 2 is?

One. Cancels.

And 10 divided by 2?

Five.

Good. Now what do we do?

We're done

So what does x equal?

5

Looks like we have both parts of the solution now. y equals 4 and x equals 5.

Let's try something else, just for grins.

At this point we would follow the same procedure indicated here when we solved for x, except we would substitute the y value into the second equation in the second column.

The intention is to demonstrate that both equations will yield the same solution. In practice we would not solve both of them.

So, does it matter which equation you choose to solve for y?

No

Would you rather choose an easier equation, or a harder one?

Easier

Me too.

All that is left is to point out that conventionally the solution is written as an ordered pair with the variables in alphabetical order. The solution will be briefly related to the original problem as described in the Day 1 plan. The next phase of the lesson is two guided practice examples. These systems are selected so that variables are eliminated, and solved for, in a different order, and so that negative integers appear in the solution. Students should not assume that the x is always eliminated first for example. Further, students need to realize that, while these problems are designed to yield integer solutions, practical applications of this process often do not. The guided practice systems are as follows:

$3x - y = 0$

$-4x + y = 1$

and

$-5x + 3y = 14$

$5x + y = -2$

Because these problems are used as guided practice, the basic instructional sequence will be the same for solving these systems as that for the model problem above. A blank graphic organizer will be filled in for each as before. However, in this case the model problem will remain in sight for students to refer to, and students will draw and fill in their own graphic organizers as they follow the example. Further, as much of the verbal scaffolding as possible will be removed. Specifically, students will be asked to identify and describe each step themselves while being directed to the first model when appropriate, and they will perform the calculations (adding and solving equations) themselves. Each of these steps will be checked with the group before we proceed to the next step. In the end, they will have a completed graphic organizer for these systems, which they can use as a model for independent practice problems, and correct solutions: (–1, –3) and (–1, 3), respectively. As independent practice, five new systems will be given for homework. In each case both equations will have both variables, and they can be solved without multiplying any equation by a constant. Students will also receive the solution to one of the systems in a sealed envelope, which they may open when they believe they have the solution. All of these systems will be checked on the following day in class. This should not take a great deal of time as the steps will generally be the same for all students.

Day 3

The primary goal of this day's instruction is to teach students to work with systems in which one or more of the equations may have to be multiplied by a constant in order for terms to sum to zero so that variables can be eliminated. Before pursuing this goal, the independent practice assignment from the previous day will be reviewed. After this review, the instructor will draw students' attention to four items that will be posted at the front of the room. The first item is a new system of two equations that requires the multiplication of equations by constants as noted for the day's goal.

$3x - 2y = 20$

$2x + 5y = -12$

The second posted item is a couple of examples of common multiples that the students found on Day 1. The third item is a couple of examples of equations multiplied by constants, also from Day 1. The fourth item is another blank graphic organizer as illustrated in Figure 1. Through questioning, prior knowledge relating to the first three items will be activated. After a reminder that these ideas will be important for solving the system on the board, the instruction of application of these ideas to systems of equations begins. The two equations are placed in Column II, on the left side of the organizer. However, this system can not be solved simply be adding equations together, so something like the following model of a dialogue would occur.

Take a look at this new system of equations. If we add the first two equations, which variable will be eliminated?

None

How is this system different from any of the systems you have solved so far?

The variables do not cancel

Can we solve this system if we don't eliminate any variables?

No

If we multiply these equations by other numbers, will that change the answers to the system?

No

Take a look at the examples we have over here (indicating the common multiples). Could common multiples help us to get coefficients that are opposites?

Yes

Okay. Let's try to use common multiples to get some variable coefficients to cancel. Take a look at the coefficients for x. They're 3, and 2 (indicating). The coefficients for y are –4, and 5. Which set of coefficients do you think will be easiest to work with?

x (Students may make another choice, and the system can be solved regardless of the choice they make.)

Great. Let's see if we can eliminate the x variable. How would you multiply those equations so that the x's cancel?

Multiply the top equation by 2 and the bottom by 3. Change all the signs in on equation.

Alright. Let's see what happens. What do we get when we multiply the top equation by 2?

$6x - 4y = 40$

Good. How many variables are in that equation?

Two

So which column are we going to put it in?

The left column. Column II.

If we add the new equation to the original bottom equation will the z's cancel?

No

Let's multiply the bottom equation by 3. What do we get?

$6x + 15y = -36$

Good. Now, how many variables are in that equation?

Two

So which column are we going to put it in?

The left column. Column II.

If we add our two new equations together will the x's cancel?

No

Why not?

They're both positive. They have to be opposites.

How can we fix that?

Multiple one equation by –1. Change all the signs on one equation.

Let's do it. If we do that with the first equation, what is our new equation?

$-6x + 4y = -40$

Now, if we combine this equation with the other new one, will the x's cancel?

Yes

Go ahead and do. Let's see what you get.

$19y = -76$

Did it work?

Yes

How many variables does our new equation have?

One

What column does it go in?

The right column. Column I.

Can we use this new equation to solve for a variable?

Yes

At this point it should be clear how the lesson would proceed. They solve for one variable, and substitute this solution into an equation with two variables to solve for the other variable. This will lead to the final solution of (4, –4).

As with the previous day's lesson, this model would remain posted as we work through two examples for guided practice. The scaffolding would be reduced for these examples and students will identify the steps and carry out the calculations themselves. They would then receive five problems for homework, along with the solution to one of them in a sealed envelope.

Day 4

The primary goals for this day's lesson are to introduce systems in which one of the equations has only one variable, and to provide guided mixed practice. This time there are no new prerequisite skills involved. After reviewing the homework problems, students will address a new problem that is posted.

$-5x + 3y = 19$

$2x = -4$

Students should have no difficulty recognizing that this problem is different from those they have solved before because one of the equations only has one variable rather than two. Instruction begins as it has with the dialogue in the other models, placing the equations into the graphic organizer. In this case the equation with only one variable will be placed in the right column. Once both equations have been placed in the graphic organizer, the dialogue turns to how to handle this system in which the one of the equations has only one variable.

Do we have an equation with one variable that we can solve (indicating the right column)?

Yes

Do we have two equations with two variables that we can combine together to get rid of a variable?

No

Go ahead and bring the equation with one variable down to the bottom box and solve it. What do you get?

$y = -4$

From here the lesson would proceed as in the previous day's lesson. The value for y is substituted into the equation with two variables in order to solve for the x value. In this case the solver does not have the option of choosing one of two equations with two variables. This model would be followed by three problesm for guided practims. One problem will be like this day's model in that one equation will have one variable and the other will have two. Both of the other two problems will have two equatios with two variables each. One of these will requiring multiplying of equations by constants before combining equations, and the other will not. The dialogue will focus on recognizing the differences between these three systems and the approaches for solving them. As before, this guided practice will be followed by an independent assignment of five problems, this time including all three types. The solution to one of them will be provided in a sealed envelope.

Day 5+

The first task of this day is to review the homework assignment from the previous day. Students will then be given the teacher generated content test. After students have completed the teacher generated content test, and time permitting, they will take the content skills content test and complete the social validity questionnaire. It is unlikely that all of these tasks can be completed in one day. A more likely scenario is that the teacher generated test will be completed on Day 5 but the investigator generated test and the questionnaire will be completed on the following day.

APPENDIX F

PROCEDURAL RELIABILITY DOCUMENTS - DESCRIPTION OF TRAINING, CATEGORY LIST, AND SAMPLE TEXT FOR INTERRATER RELIABILITY

Training of the Classroom Teacher to Record Procedural Reliability Data

1) The investigator will provide the teacher with definitions and examples for each of the four categories.

2) The teacher and the investigator will discuss the definitions and examples. The teacher's questions will be addressed. The teacher will create new examples that fit into each category.

3) The teacher will rate each statement in the first lesson script of this appendix according to the four categories.

4) The teacher's ratings will be compared to prior ratings of the same statements by the investigator.

5) Interrater reliability will be estimated based on the codings of the four categories.

6) Discrepancies will be discussed.

7) If interrated reliability is less than 90% (expressed as the percent of exact matches when at least one rater scored the entry as belonging to one of the four categories), steps 3 through 6 will be repeated for the second lesson script in this appendix.

Categories of Verbal Instruction Statements

Category 1	Number of variables in one or more equations
Category 2	Matching variables Matching coefficients
Category 3	Solvability of an equation
Category 4	Number of equations

Sample Lesson Text for Coding Practice

Now, how many columns are there in this graphic organizer?

Three

Good. How many rows are in this graphic organizer?

Two

Alright. Now, what is this Roman numeral at the top of the left column (pointing) of the graphic organizer?

Three

Okay. What's the Roman numeral at the top of the middle column (pointing)?

Two

Now, here's a really tough question. What is the Roman numeral at the top of the right column (pointing)?

One

Great. Now, take a look at the top equation in this system. How many variables are in this top equation?

Three

Can we solve it?

No

Why not?

It has too many variables. We can only solve equations with one variable.

Exactly. Now, we have a column with Roman numeral One on top, a column with Roman numeral Two on top, and a column with Roman numeral Three on top (indicating throughout). If you were going to put an equation with three variables into one of these columns, which column would you put it in?

Column three

Okay, here goes. (The first equation is rewritten in the top left cell of the graphic organizer.)

*****

Okay, now we have all three equations entered into the graphic organizer. Remind me again, can we solve any of these equations as they are now?

No

Would it help if we could create some equations with fewer variables?

Yes

Let's take a look at the x-terms in the first two equations. What would we get if we added 2x and –2x?

Nothing. Zero. They cancel.

If we add the first two equations together will we have any x's left?

No

Does that mean our new equation will have fewer variables than the two we start with?

Yes

Let's do it. What do we get when we combine 2x and –2x?

Nothing. Zero. They cancel.

What do we get when we combine 4y and –3y?

y or 1y

And if we combine 2z and z?

3z

What about when we combine 16 and –5?

11

What is the new equation we have created?

$y + 3z = 11$

How many variables are in the new equation?

Two

Which column did I put that equation in?

Column two

What Roman numeral is at the top of that column?

Two

Coincidence?

I think not

Does this new equation have the same number of variables as the first three?

No

How is it different?

It only has two variables. Less variables.

Take a look back at the original equations again. What is the first term of the middle equation?

$-2x$

What is the first term of the bottom equation?

$2x$

What would we get if we combine those two terms?

They cancel. Nothing. Zero.

Let's try adding these two equations and see what happens. What do we get when we combine 2x and –2x?

Nothing. Zero. They cancel.

What do we get when we combine –3y and 2y?

–y or –1y

And if we combine z and –3z?

–2z

What about when we combine –5 and –3?

–8

What is the new equation we have created?

–y – 2z = –8

How many variables are in the new equation?

Two

Which column did I put that equation in?

Column two

What Roman numeral is at the top of that column?

Two

Coincidence?

I think not

Now take a look at our two new equations. Can you name the two variables in the top one?

y and z

What are the two variables in the second one?

y and z

Are they the same?

Yes

*****

Let’s take a look at these new equations you made. Is it possible to solve the first equation?

No

Why not?

Too many variables. Need one variable.

Can we solve the second equation?

No

Why not?

Too many variables. Need one variable.

What is the first term of the first equation?

y

Okay. What’s the first term of the second equation?

–y

What would happen if we added them together?

They would cancel.

Let’s try adding these two equations together. What do you get when you add y and –y?

Zero. Nothing. Zero y.

How about when you add 3z and –2z?

z. One z.

And what is 11 and –8?

3

What's our new equation?

z = 3

How many variables are in that equation?

One

Can we solve it?

Yes. It's already solved.

Okay. Well I've been saving the bottom row for answers, so I'm going to bring this straight down into the bottom row. How many variables have we solved for so far?

One

What Roman numeral is at the top of the column above the solution?

One

Coincidence?

I think not

Well, now we know what z equals. We still have to find values for y and x. Tell me, how many variables are in each equation of the left column?

Three

If I substitute the ‘3’ for the ‘z’ in the first equation in the left column, how many variables will be left in the equation?

Two

Can we solve an equation with two variables?

No

How many variables will I have left if I substitute the ‘3’ for the ‘z’ in the second equation in the left column?

Two

Will we be able to solve it?

No

If I substitute the ‘3’ for the ‘z’ in the third equation in the left column will I be able to solve it?

No

Why not?

Two variables. Too many variables.

Well then it looks like knowing that z equals 3 doesn’t help us with the first three equations. Let’s look at the two equations you created in column two. Tell me, how many variables are in each equation of the middle column?

Two

If I substitute the ‘3’ for the ‘z’ in the first equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variable in it?

Yes

If I substitute the ‘3’ for the ‘z’ in the second equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variables in it?

Yes

Which of those equations do you think will be easier to solve?

The first one

*****

Bring that equation straight down the column to the lower box, substitute the ‘3’ for the ‘z’ in the equation, and solve it. Let’s do it. If I substitute the ‘3’ for the ‘z’ in the first equation in column two what do I get?

$y + 3(3) = 11$

Good. Now what would you do first to solve this?

Multiply 3 times 3

And what do you get?

9

What’s our new equation?

$y + 9 = 11$

What’s next?

Subtract 9 from both sides. Move 9 to the other side and make it a minus 9.

Okay. So 9 minus 9 cancels. What is 11 minus 9?

2

Great. Now what?

We're done

So what does y equal?

2

Looks like we have two parts of the solution now. y equals 2 and z equals 3.

Before we try to find x, let's try something else, just for grins.

*****

So, does it matter which equation you choose to solve for y?

No

Would you rather choose an easier equation, or a harder one?

Easier

Me too. Okay now we still need to find a value for x.

APPENDIX G

TRANSCRIPTS OF TEACHER INTERVIEWS

Study 1: Transcript of Middle School Algebra I Teacher Interview

Can you first give me a real quick summary of your teaching experience?

I’ve been in education about twenty-seven years. Seventeen years in public schools. That would include twelve both science and math. Math experience is from seventh grade math through precalculus. Been in the private sector about seven years or eight years, and worked with LD students to gifted students and have background and educational certification in each of those.

Great. OK, in your experience how challenging has it been to teach students to solve systems of equations?

In many situations it’s quite challenging, due to a variety of reasons, some of them being the internal processing for the individual child and how they approach the problem.

So relative to other topics in algebra would you say solving systems of equations was fairly typical in terms of difficulty or more difficult than average or . . .

I would probably place it in the average category with factoring being the most challenging for Algebra I students.

OK. How important do you think it is for students to learn to solve systems of equations?

Extremely important because that is something that actually has application in the real world.

Do you feel like the graphic organizer was helpful for the students in those groups to learn that material?

Yes, it was very helpful. There are many students that are very visual learners and that is especially true of your LD population because a significant number of students with LD diagnosis are not auditory learners and the school system is primarily set up for auditory learners.

And based on your experience sitting in on those classes while I was teaching them, do you recall any situations where kids said or did anything that would support the idea that it was helpful for them?

Yes. There was a couple of situations and I'm not sure whether they occurred in class or outside of class, but "I like the way it's set up. This is easy." And for an LD child to say math is easy some of those same children would not have referred to it as being easy and it varied from class to class. In reviewing for a test two days ago which included the material, the ones in the graphic organizer, "Oh, is that the type of problems we used the box with?"

You've sort of addressed this issue already. Do you feel that all students benefit equally from using the graphic organizers and if so why, and if not why not?

It gives an additional method of presenting the material and I think that the more ways that content can be presented the higher the likelihood that the children will be successful. It adds an additional presentation.

So do you think that basically all the kids will benefit from . . .

Yes. I think that all children could benefit because even if they are auditory learners it provides an additional reinforcement for the content. And you can't specify a person as being just one type of learner because in a given situation you don't know in advance which method is going to work for an individual child. You can make a guess at it.

The next time that you're teaching systems of equations would you try a similar approach yourself?

Definitely, yeah.

Do you think there are other topics that might effectively be taught in algebra using graphic organizers, and if so, does any in particular come to mind?

I think in the factoring, I can see ways that factoring could utilize graphic organizers. It would give them a definite sequence and also even in Algebra I, not just in Pre-algebra, on anywhere that they have a situation in which there is a situation where there is a sequence of events that needs to be followed, such as combining like terms on the same side of the equal mark, and then going from that to when you're solving for unknowns, and then going from that to moving things from one side of the equation to the other side of the equation. You know, various options. So anything that has a sequence component to it or step component to it I think a graphic organizer seems to be very effective.

Finally, what kinds of disadvantages do you see to using graphic organizers for teaching algebra topics?

The disadvantage, and this would be the only one that I could see, is that the information is presented so . . . in such a structured format that it reduces the amount of creativity or problem solving abilities that they may have . . . that an individual child may develop.

Great. Any other comments that you would like to make with respect to the value, or lack thereof, of this experience?

I think it was very beneficial both for the students and for me and I really enjoyed it.

Good. Alright. Thank you very much.

Study 1: Transcript of High School Algebra I Teacher Interview

First can you give me a very brief summary of your teaching experience, what you've taught and for how long and what kind of students.

I have taught on different, many different levels. I'm a private tutor in music and have a background in music education, and I'm certified in mathematics, and I also got certified in middle school. I taught in the public schools, in a private school . . . several small schools like this. I started out as a music ed person and taught in Philadelphia in a minority school - north Philadelphia for a year. I was teaching music there. Came to Georgia. Taught in two schools. Music at the elementary level. Then went back to school for engineering. Went for three years. Got a job at another elementary school teaching music. Then from there I progressed more into the mathematics arena. Got certified in mathematics seven through twelve, and got a job in Clayton county at a high school teaching music and drama. Then I switched to music and math, and then for the last year - I was there for two years - the last year I was just exclusively teaching math. And then I got a job here teaching math, and I'm also the chorus teacher here, but math is my full time participation here. Music is more an elective. I'm teaching algebra. So I've got sort of a mixture of elementary, middle school, and high school, and more leaning towards the high school right now. You know, teaching Algebra I, II, and III as a tutor to after school - for after school for credit kids here at the school, and then during the day I teach Algebra I.

And how long have you been teaching here?

This is my third year. I taught general - what's call Applied Mathematics. You know, Julie teaches that now - Julie and Patty.

Right. OK. Now in your experience teaching math how difficult is it for students to learn to solve systems of equations as compared to other material in an algebra classroom? Is it typical, more difficult than most, or less?

I think it's probably more difficult. Think that - (cough) Excuse me. - I think that what's most difficult about specifically solving systems of equations is the sequencing, and, which includes presenting the three methods of solving systems of equations at once so that they're not just solving the systems of equations but they're also remembering specific steps that go with the specific method of solving systems of equations. I think there's a lot of information that they have to draw on that confuses them in addition to the math anxiety about looking at the thing and saying, "Wait a second. There's a lot going on here." That - I don't know if that makes sense but . . .

Yeah. How important do you feel it is for students to learn how to solve systems of equations?

I think in the scheme of things, solving systems of equation is very practical. I think it's practical from the standpoint that kids are going to have to learn to manage numbers and manage several accounts at the same time. I mean bills, paying bills, having repairs done to your car, having estimates and all, and I think that solving systems of equations really helps to structure your thinking in a way that goes beyond just memorization. That it, it's important from the standpoint that it really - let's see if I can word this correctly - it develops the concept of doing more than one thing at once and also then - There's a word for this. I don't know what the word is, but thinking about what you're dong while you're doing it - sort of tracking yourself while you're doing it so that by the time you go from the beginning to the end you've gone through so many steps it's, unless you're watching where you're going along the way, it's easy to sort get an answer and then go "That's it," regardless of whether it's right or wrong if you're not tracking yourself. And I think that solving systems of equations is very very practical in the life scheme of things but also in an algebra sense it's very important because it requires that sort of self-analysis of your work which is important for kids.

Do you feel that the graphic organizer was helpful for students learning how to solve systems of equations, and if so, how do you think it was helpful for them?

I think that it was helpful because it allowed the kids to have a - I like to call things a jumping off point. A lot of the kids that we teach here have difficulty starting something. They have difficulty starting the processes, so they're given a problem. They'll look at it and be confused but as soon as you say, "Oh, remember we're starting here," they can - once they get that jumping off, they can generally work through the problems. But they have trouble kind of pulling up that initial step, and I think the graphic organizer - I noticed several people say, "Well, should we write that box down? You know, that you had put on the paper." And I also felt like as I was viewing it, it was helpful for me to organize what, what we're, you know, how better to, to at least approach - because I find in algebra, even if the kids are sort of savvy enough to not have to be directed every single step in the process - they can maybe skip a step, "Oh I know that five and two is seven so I'm just gonna skip that step of writing it down. I'm just gonna write seven here." That even if they're doing that it's still going on in their heads, so they're still organizing things in their head, and I think the graphic organizer really helped a lot of people. Unfortunately I don't think that in this particular setting it helped quite as much as I would -I would have liked it to because it’s time. You know time constraints. I mean if we had, you know, two or three days and gone back and had just specifically tested on that. But that’s true of a lot of stuff in algebra, I mean a lot of stuff we do just because of the nature of the kids, you know. It’s - if I could like just test each concept it probably would, you know, would have helped to solidify it a little bit. I don’t know if the kids are gonna be able to draw, you know, when they give - when they’re given the test of the final at the end of the year whether they’re gonna be able to remember, “I yeah, I gotta draw the boxes.” You know what I’m saying?

That’s a good question, yeah. Do you feel that all of the students benefited equally from using the graphic organizer?

All the students that used it?

Right.

Well, I can’t really say because, you know, kids have different - different ways of learning. Gosh that’s a tough question. My - my gut reaction would be “no” because I think they were given the same opportunity to understand it, but - but some of the kids - I mean it’s just the kids are so different. I can’t really answer. I’m sorry.

So what might be characteristic of a kid who might not benefit from the graphic organizer? How would - How might those kids be different from the ones -

Maybe someone who is more of an auditory learning or somebody who is more visual in the sense that they’re transferring that onto a piece of paper. That they might see it and that it might make more sense to them when they see it but they might not be able to reproduce it, you know, when they go, like I said, when they’re going to take the final at the end of the year they might not be able to reproduce it on the paper - draw it out. Just somebody who might have problems writing, like (name deleted) for example, who takes a long time to write. He might have problems, you know, drawing it, and then squaring it, and then putting this here and this here, ‘cause he kind of goes out of the boxes and stuff. But that in no way diminishes what, you know - how it was put together because it think it was very well organized, and I’m not saying that it could -he could learned it better any other way. It’s just that it’s hard to tell.

OK. The next time that you’re teaching systems of equations would you consider trying a similar approach?

Sure.

OK. Are there other topics in algebra that you think might be effectively taught using some kind of graphic organizer? And if so, are there anything - any in particular that come to mind?

Well, I’m not sure that - I mean, are you asking that any concepts can be taught using the graphic organizer. Is that - is that the statement that you’re making? And here is the topic that I would like to teach with the graphic organizer, because there are several things that are not real clear. For example, parabolas and hyperbolas and, you know, conic sections and the formulas. It’s not real clear. Especially trying to - once you manipulate the formulas and all, it becomes clear but it doesn’t really become clear when you’re just presenting it to the kids. They’re sort of struggling with, you know, how does the circle - and translating the circle around the graph differ from the, you know, the hyperbola and translating it around on the graph, and you know that kind of thing.

So, are you saying that perhaps some kind of graphic organizer might help to organize that information so that it would make more sense. They'd be able to relate those conic sections to each other?

Right. Right. Exactly. Because I've found that - I've - I've found that really if you go back to the circle - to the formula for the circle and relate it to the other things, the other drawings, the other drawings are really manipulations of that basic formula. So, yeah.

OK. And what kinds of disadvantages do you see to using some kind of a graphic organizer to teach topics in algebra? (long pause) You're allowed to say none. (both laugh)

Well, let's see. You see, I think that regardless of - regardless of what is on - what is put on paper you're always formulizing in math. You're always organizing in math. You're always organizing something in your head or on paper, but regardless there's always some kind of organization that needs to take place. It think it would help to, like I said, come and give somebody somewhere to jump off, especially when you're just learning it for the first time, because it's like learning a different language. You know, it's - you - you have to have some kind of construct or, you know, the anxiety takes over. Especially in a place like this. You know, in a school setting where there's - there's kids with disabilities that there's a lot of anxiety that doesn't have anything to do with math. It's just the anxiety of it, and, you know, kids say, "I'm gonna fail. I'm gonna fail." Because they don't have that sort of sense of like, well you know, this has to make sense because, you know, take for example adding positive and negatives, you know. Kids come to learn a lot of rules but until they make that construct of direction - this negative is going to the left and positive is going to the right and they're canceling each other out, or whatever it is - until they learn that, they have that anxiety about “Am I gonna remember the rule? Am I gonna remember the rule?” So I think it’s very - I’m sorry. What was the question again?

Disadvantages . . .

Disadvantages? I don’t think there are any disadvantages to it. I guess the only thing that might be a disadvantage is if the kids are being required to sort of do a lot of manipulation before getting to the actual meat of the discussion whether, you know, whatever the topic is. They might get a little sort of sidetracked a little bit from the meaning of the, what you’re asking them to do, and they might not be able to transfer to the next step. I don’t know. I mean I’m just talking off the top of my head. I haven’t a clue.

You’re allowed to talk off the top of your head.

That’s the only thing I can think maybe, you know.

Any other comments about this experience, graphic organizers in particular, or participating in the study, or anything that - that you want to share for better or for worse?

I - I think you’re a, you know, very well organized and have a lot of knowledge, bring a lot of knowledge here, and I’ve enjoyed talking with you about a lot of issues, you know, concerning disabilities, and math, and - and I think that this particular - this particular experience of teaching math to these kids has - has seen a growth in me as a math teacher that goes from the general to the specific. You know I think we teach a lot of general concepts to kids because that’s the way we teach in America generally. You know, we’re given a class of kids that maybe have a portion of the kids that excel and a portion of the kids that - that are slower learners and we have the - kind of the average student in the middle and that’s who we teach to. You know, we teach that general concept and the way that I used to teach algebra was more the general concept of “Hey, you’ve got a and b and a times b is, you know,” using the -the variables. But now I teach specifically - specific with numbers and - and because of the nature of the kids - and it seems to work really well and I - I think that the - the idea of making some kind of organization evident to the kids is very very important. I think in a - in a - in a public school setting as well I think we don’t do that as much as we need to, and I appreciate the learning experience that this has brought.

Good. Thank you for all your time and help.

Study 2: Transcript of High School Algebra II Teacher Interview

Well, first of all, can you give me a brief summary of your teaching experience – what subjects you have taught, what kinds of students, and for how long?

Well, let's see. Of course I've been teaching horseback riding for years. I guess I started teaching math when I got here. I've taught everything in high school - Algebra I, Algebra II, Precalculus, Calculus, some remedial classes, Applied Math, Consumer Math – um, I guess that's it. I haven't taught Geometry.

How long have you been here?

Oh. I've been here six years.

Are you certified?

I have a BA in Economics with a minor in Accounting, and a Master's degree in Accounting. I just finished the requirements to be certified in math, but I don't have the certificate yet.

In your experience, how challenging has it been to teach your students to solve systems of equations?

Oh it's quite challenging. They have a tough time because the problems are so long. It's easy for them to get lost. Plus it combines so many different things – multiplying equations, solving equations, substitution – they have to keep track of when to do what.

So compared to other topics in Algebra II, would you say that solving systems of equations was one of the most challenging things you cover, about average, or . . .

Oh it's definitely one of the most difficult things we do. I just wish we had more time. If I spent a week on the graphing method, and then a week on substitution - I teach all three methods - and then another week on the addition method maybe they would do better. It just seems like there's so much to cover.

How important do you feel it is for your students to learn how to solve these systems of equations.

Oh it's very important. The thinking involved is very challenging for them. They have to keep track of a lot of different things, and put them together, and sort of plan ahead. The students struggle to do all that at the same time. I think it's something they should be able to do better.

Are these kinds of thinking skills important outside of school, as well as in an algebra class?

Definitely. There's lots of situations when – in life – when we have to coordinate different kinds of information, and know when to do what.

How do you feel the graphic organizer was helpful for students learning this material?

Of course, this isn't the way I teach it. It seemed like some of the kids really understood it well with your boxes. Like (name deleted). He seemed really excited and confident about using them. I teach a lot of things by writing down each step in words and then doing it. The kids seem to really like that and it helps them. So this was very different from what they're used to. I start with triangular systems that they can solve by substitution. Then we introduce longer equations that require addition and eliminating variables. That way it's connected to something they have already done instead of being something totally new.

Why do you think that (name deleted) seemed excited about it?

I don't know. I think some kids learn differently from others – some with pictures, some with reading, some by lecture. Maybe he just connects better to pictures.

Okay. Do you feel that all students benefit equally from using the graphic organizer, of those that were in that group?

Well, it's hard to say. It depends on if they were making an effort. A lot of them didn't do the homework that week, which surprised me. I wouldn't expect them to do well if they didn't do the homework. Probably they could all get some benefit out of it though.

Would you try a similar approach the next time you're teaching this material?

No, probably not. It's just not my style. They seem to learn the steps pretty well when I write out each step and then do it. I guess maybe if I had a student who was not getting it that way, I might try your way to see if that worked better for him.

Can you think of other topics in math that could be taught with a graphic organizer?

Not really. Maybe my brain just doesn't think that way. I just write out the steps in order and show the students how to do them.

What kinds of disadvantages do you see in using the graphic organizer?

Well, it seemed like your system worked fine for regular problems. You know, when nothing unusual comes up. You did lots of examples where all three equations had all of the variables, and then the variables are eliminated one at a time. It seems like it doesn't work so well when the problems aren't so neat. Like when some of the equations don't have all of the variables, or what happens when two variables are eliminated at the same time? I mean, I know you did some examples of that, but it just didn't seem to work as well.

Good. Anything else?

Not really.

Okay. Is there anything else you'd like to add about graphic organizers, or participating in the study?

Well, it's really interesting to see how someone else teaches – different approaches. I enjoyed that.

Good, I'm glad. I certainly appreciate your time and support in the project.

APPENDIX H

PREREQUISITE SKILLS TEST, CONTENT SKILLS TEST, AND SCORING GUIDE FOR STUDY 2

Prerequisite Skills Test for Study 2

1) Solve for *x*:

$$7x = 42$$

2) Solve for *x*:

$$-3x = 18$$

3) Solve for *x*:

$$2x - 5 = 3$$

4) Solve for *x*:

$$-3x + 8 = 14$$

5) Assume $x = 3$ and solve for *y*:

$$2x - 4y = 2$$

6) Assume $x = -4$ and solve for y:

$$-3x - 6y = 0$$

7) Assume $x = 5$, $y = 2$ and solve for z:

$$x - 3y + 2z = 3$$

8) Assume $x = -1$, $z = 3$ and solve for y:

$$-5x + 2y + 2z = -1$$

9) Add these two equations:

$$2x - 4y = 6 \qquad\qquad 4x + 3y = -2$$

10) Add these two equations:

$-7x - 2y = 5$　　　　$x + 2y = 11$

11) Add these two equations:

$5x - 2y + z = 7$　　　　$2x + 6y - 3z = 20$

12) Add these two equations:

$-7x - y + 4z = -9$　　　　$2x + y + 2z = 13$

13) Multiply this equation by a factor of 3:

$2x - 5y = 8$

14) Multiply this equation by a factor of -2:

$-3x + y = 4$

15) Multiply this equation by a factor of -1:

$$3x - 2y + 4z = -8$$

16) Multiply this equation by a factor of 7:

$$-x + 5y - 3z = -8$$

17) Find a common multiple for 2 and 5:

18) Find a common multiple for 3 and 9:

19) Find a common multiple for 4 and 6:

20) Find a common multiple for 8 and 12:

Content Skills Test for Study 2

1) How many solutions does each equation have?

$5x = 35$

$3x – y = 16$

2) Can you combine these two equations to create a new equation with only ONE variable? If yes, how would you do it? If no, why not?

$5x – 2y = 8$ $\qquad$ $2y + 7z = 9$

3) Can you change these two equations so that you can combine them to create a new equation with only ONE variable in it? If yes, how would you do it? If no, why not?

$2x – 3y = –5$ $\qquad$ $4x – 5y = 23$

4) Why is it important to eliminate variables when solving systems of equations?

5) If you know that **$x = -2$**, can you find a value for y using this equation? Explain your answer.

$$-3x + y - 4z = -6$$

6) Is it possible to solve this system of equations? Explain your answer.

$$2x - 3y + z = 9$$

$$x + 7y - 4z = -13$$

7) On a separate sheet of paper solve this system of equations:

$$2x - y + 3z = 7$$

$$-2x + 4y - 5z = -3$$

$$2x - 7y + 8z = 0$$

8) On a separate sheet of paper solve this system of equations:

$$x + 3y - 2z = -10$$

$$2x - 2y + z = 13$$

$$-3x + y - z = -15$$

9) On a separate sheet of paper solve this system of equations:

$$-3x - y + 2z = -4$$

$$-2x + 4y + z = -24$$

$$y - 3z = -28$$

10) On a separate sheet of paper solve this system of equations:

$$2w + 5x - y - 2z = -1$$

$$-2w - 2x + 4y - 3z = 14$$

$$2w - x - 5y + 7z = -21$$

$$-2w + 4x + 7y - 10z = 28$$

Scoring Guide for Content Skills Test for Study 2

1) One point for any answer indicating multiple answers without specifying how many. Examples include “an infinite number,” “a lot,” “too many to count,” etc.

2) One point for indicating that it is NOT possible.

3) One point for indicating that the values of coefficients for matching variables must be the same, and one point for indicating that the signs of those coefficients must be different. Credit should also be given if a student correctly accomplishes these goals without describing them.

4) One point for indicating that equations with only one variable can be solved while those with more variables can not be solved.

5) One point for indicating that solving is not possible, and one point for indicating that the equation would still have more than one variable after the substitution.

6) One point for indicating that solving is not possible, and one point for indicating that a this system does not have enough equations.

7) One point each up to a maximum of two points for consistent and independent equations with the same two variables

One point for one consistent equation with one variable

One point for each correct value in the final solution: (3, 2, 1)

Maximum of 6 points

8) One point each up to a maximum of two points for consistent and independent equations with the same two variables

One point for one consistent equation with one variable

One point for each correct value in the final solution: (3, –1, 5)

Maximum of 6 points

9) One point for one consistent equation with the two variables y and z

One point for one consistent equation with one variable

One point for each correct value in the final solution: (8, –4, 8)

Maximum of 5 points

10) One point each up to a maximum of three points for consistent and independent equations with the same three variables

One point each up to a maximum of two points for consistent and independent equations with the same two variables

One point for one consistent equation with one variable

One point for each correct value in the final solution: $(1, -1, 2, -2)$

Maximum of 10 points

APPENDIX I

LESSON DESCRIPTIONS FOR STUDY 2

Lesson Plans for Study 1

Day 1

The primary purpose of the first day of instruction is to review the prerequisite skills assessed on the pretest of prerequisite skills. These skills include a) solving linear equations in one variable, b) solving linear equations that have one variable remaining after substituting given values for one or more other variables, c) multiplying linear equations by constants, d) adding linear equations, and e) finding common multiples.

Prior to addressing these prerequisite skills, the topic of systems of three linear equations in three variables will be introduced with a hypothetical problem. Students will be asked to imagine that they have each voted on which student in the class is most likely to become President of the United States. The teacher has tallied the votes. The problem is to determine how many votes each of three students received based on a set of clues. Three specific students will be identified in the class to make this problem more concrete. For purposes of this description I will call them Al, Geb, and Rah. The clues are:

1) Twice Al's votes added to four times Geb's votes and twice Rah's votes totals 16 votes. This will be written as the equation $2x + 4y + 2z = 16$.
2) –2 times Al's votes minus three times Geb's votes plus Rah's votes makes –5. This clue yields the equation $-2x - 3y + z = -5$.
3) The final clue is that twice Al's votes plus twice Geb's votes minus three times Rah's votes equals –3. This statement leads to the equation $2x + 2y - 3z = -3$.

Once these three equations have been posted for the class they will be set aside with the explanation that this problem will be solved on the following class day. First they will be reviewing and practicing some skills they already have done before that they will need to solve that problem. The students will be asked to solve independently some simple one-step linear equations with integer solutions and involving either addition/subtraction or multiplication/division. Students in Algebra II should find these to be unchallenging, but this skill is necessary to solve the more complex systems, and the review is probably particularly valuable for students with learning problems. Once solutions have been checked and procedures have been discussed, a few of examples of two-step linear equations with integer solutions will be solved, checked, and discussed the same way. Discussion of solutions will include the fact that these solutions are unique. This should cover prerequisite skill a) noted above.

Next the equation $x + y = 12$ will be posted. Students will be asked how this equation is different from the ones they have just solved. One response, which will be emphasized, is that this equation has two variables. Students will be encouraged to think of any two numbers that add to 12. This generative process will extend to include negative integers and perhaps fractions and decimals. Eventually this discussion will lead to the conclusion that there are an infinite number of possible solutions to this equation, some of which are posted. Next, the equation $2x - y = 12$ will be posted. Again multiple possible solutions will be generated, and again the students should come to the conclusion that an infinite number of solutions is possible.

Having arrived at the generalization that equations with one variable can be solved, while those with more variables can not, prerequisite skill b) will be addressed. Returning to the equation $x + y = 12$, students will be given a few values for x and asked to solve for y. They will also be given a few values for y and asked to solved for x. The same will be done for the equation $2x - y = 12$. The emphasis of this practice will be to recognize that when a value is substituted into one of these equations, for either variable, only one variable remains. Therefore the equation can be solved for the remaining variable. This process will be generalized to include a couple examples of linear equations in three variables such that values for two of the variables are given for substitution. Students will be led to noting that if only one substitution is made these equations will still have two variables remaining and will not be solvable. This should cover prerequisite skill b).

At this point we will return our attention to one of the one variable equations, such as $2x - 3 = 11$. We will have already determined that the solution to this equation is 7. Working with partners, each student will be asked to multiply the entire equation by any number they wish. They will be encouraged to use numbers that are relatively easy to work with. Students will check their resulting equations with their partners, and then share them with the teacher who will post them, while checking them for accuracy and noting the multipliers used. Next, each student will be asked to solve his or her own new equation and then check answers with a partner. Solutions will then be shared with the entire class. The purpose of this exercise is to demonstrate that multiplying an equation by a constant yields an equivalent equation with the same solution set as the original equation. In abbreviated form, as a group, we will go through the same process with the two variable equation $x + y = 12$, showing by examples and counterexamples that solutions to equivalent equations work in all of the equivalent equations, and nonsolutions to equivalent equations fail to work in any of the equivalent equations. This covers prerequisite skill c).

For the next prerequisite skill we will look at the equivalent equations we created for the equation $2x - 3 = 11$ as well as the equation $x + y = 12$. In each case we will select pairs of equivalent equations and add them by combining like terms. Then we will explore solutions to show that any solution for both of the original equations will also be a solution for the combined equation. A third example, that of a two variable linear system will also be used. In this case, $x + y = 12$ and $2x - 3y = -6$ will be used to show that the solution (6, 6), which works in both equations, also solves the combined equations, while solutions that work for only one of the starting equations, will not work in the combined equation. This covers prerequisite skill d).

Finally, we will need to review briefly some ways of finding common multiples. Least common multiples of integers (ignoring negative signs which can be corrected by multiplying by –1) are typically found by looking at the prime factorization of the original integers or by serially multiplying each of the original integers by 1, 2, 3, . . . until the least common multiple is found. Technically, solving these systems can be done with any common multiples of coefficients, and sometimes students are given the option of simply multiplying two integers together to find a common multiple. For this lesson students will be asked to identify common multiples of pairs of integers and to describe how those common multiples were found. In this way we will be able to build on what the students already know about common multiples rather than imposing a prescribed procedure. This approach will review prerequisite skill e).

Once the prerequisite skills have been reviewed, the assessment instrument for prerequisite skills will be distributed. Students may begin this test in class and will be permitted to complete it as best they can as homework. The instructor will stipulate that this instrument is for assessment only and not for a grade. Students will be encouraged to do the best they can without outside assistance.

Day 2

At the beginning of class four items will be posted at the front of the room. The first item is the system of three equations used to introduce this topic on the first day. The second item is a linear equation in two variables with a list of possible solutions, also from the previous day's lesson. The purpose of this item is to remind students of the prerequisite concept that equations with more than one variable do not have unique solutions, while those with only one variable can typically be solved. The third item is an example of two equations being added together, drawn from the previous day's lesson. The fourth item is a blank graphic organizer as illustrated in Figure 1. The lesson will proceed in a highly interactive question/answer format as modeled here. The sequence of the lesson will be a review of prior material and prerequisite skills, modeling of new skills, guided practice, and independent practice. While students are encouraged to participate, the teacher will also call on specific students to ensure that less active students are still following the lesson. Comments referring specifically to the graphic organizers and its elements will only be included in the lessons for the experimental group. Otherwise the two groups will have essentially the same lesson format and content. Italicized type suggests student responses, while Roman type indicates what the teacher would say.

> Yesterday I told you that we were going to solve this system of equations today (indicating the posted system). Actually you are going to solve this system yourselves. All I'm really going to do is ask you some easy questions, because you already know enough mathematics to solve this system of equations. But

before you do that, let's make sure we're clear about an idea we looked at yesterday. How many possible solutions are there to this equation (indicating the posted two variable equation)?

Lots. An infinite number. More than you can count.

Can you give me another example of a solution for this equation other than the ones already posted?

Several students will be given an opportunity to suggest additional solutions, including solutions containing negative integers.

Great. Now look at these equations here (indicating the two equations added to make a third). How did we get this third equation?

We added the other two

That's right. So where did this first term come from?

We added the first term from each of the other two equations. (They may be named specifically.)

Why didn't we add the *x* term of this equation with the *y* term of the second one?

Because you have to add like terms. They have to have the same variable.

Alright. Now let's look at this drawing I have on the board (indicating the blank graphic organizer). I call it a graphic organizer. We're going to use this as a tool to help solve this system of equations. In fact, I don't even want you to take any notes yet. For this first problem you're just going to talk me through it. Now, how many columns are there in this graphic organizer?

Three

Good. How many rows are in this graphic organizer?

Two

Alright. Now, what is this Roman numeral at the top of the left column (pointing) of the graphic organizer?

Three

Okay. What’s the Roman numeral at the top of the middle column (pointing)?

Two

Now, here’s a really tough question. What is the Roman numeral at the top of the right column (pointing)?

One

Great. Now, take a look at the top equation in this system. How many variables are in this top equation?

Three

Can we solve it?

No

Why not?

It has too many variables. We can only solve equations with one variable.

Exactly. Now, we have a column with Roman numeral One on top, a column with Roman numeral Two on top, and a column with Roman numeral Three on top (indicating throughout). If you were going to put an equation with three variables into one of these columns, which column would you put it in?

Column three

Okay, here goes. (The first equation is rewritten in the top left cell of the graphic organizer.)

This sequence of questions and answers about the first equation is repeated for the second and the third equations until all three have been entered into the graphic organizer. The reason for emphasizing the connection between the number of variables in an equation and the column into which it is entered, is that this concept anticipates the Day 4 lesson where one or more equations may have less than three variables.

Okay, now we have all three equations entered into the graphic organizer. Remind me again, can we solve any of these equations as they are now?

No

Would it help if we could create some equations with fewer variables?

Yes

Let’s take a look at the x-terms in the first two equations. What would we get if we added 2x and –2x?

Nothing. Zero. They cancel.

If we add the first two equations together will we have any x’s left?

No

Does that mean our new equation will have fewer variables than the two we start with?

Yes

Let’s do it. What do we get when we combine 2x and –2x?

Nothing. Zero. They cancel.

What do we get when we combine 4y and –3y?

y or 1y

And if we combine 2z and z?

3z

What about when we combine 16 and –5?

11

What is the new equation we have created?

$y + 3z = 11$

How many variables are in the new equation?

Two

Which column did I put that equation in?

Column two

What Roman numeral is at the top of that column?

Two

Coincidence?

I think not

Does this new equation have the same number of variables as the first three?

No

How is it different?

It only has two variables. Less variables.

Take a look back at the original equations again. What is the first term of the middle equation?

–2x

What is the first term of the bottom equation?

2x

What would we get if we combine those two terms?

They cancel. Nothing. Zero.

Let's try adding these two equations and see what happens. What do we get when we combine 2x and –2x?

Nothing. Zero. They cancel.

What do we get when we combine –3y and 2y?

–y or –1y

And if we combine z and –3z?

–2z

What about when we combine –5 and –3?

–8

What is the new equation we have created

–y – 2z = –8

How many variables are in the new equation?

Two

Which column did I put that equation in?

Column two

What Roman numeral is at the top of that column?

Two

Coincidence?

I think not

Now take a look at our two new equations. Can you name the two variables in the top one?

y and z

What are the two variables in the second one?

y and z

Are they the same?

Yes

Students need to learn that the variables typically should be the same if we wish to combine two equations to produce a new equation with fewer variables than the original equations.

Let's take a look at these new equations you made. Is it possible to solve the first equation?

No

Why not?

Too many variables. Need one variable.

Can we solve the second equation?

No

Why not?

Too many variables. Need one variable.

What is the first term of the first equation?

y

Okay. What's the first term of the second equation?

–y

What would happen if we added them together?

They would cancel.

Let’s try adding these two equations together. What do you get when you add y and –y?

Zero. Nothing. Zero y.

How about when you add 3z and –2z?

z. One z.

And what is 11 and –8?

3

What’s our new equation?

z = 3

How many variables are in that equation?

One

Can we solve it?

Yes. It’s already solved.

Okay. Well I’ve been saving the bottom row for answers, so I’m going to bring this straight down into the bottom row. How many variables have we solved for so far?

One

What Roman numeral is at the top of the column above the solution?

One

Coincidence?

I think not

Well, now we know what z equals. We still have to find values for y and x. Tell me, how many variables are in each equation of the left column?

Three

If I substitute the ‘3’ for the ‘z’ in the first equation in the left column, how many variables will be left in the equation?

Two

Can we solve an equation with two variables?

No

How many variables will I have left if I substitute the ‘3’ for the ‘z’ in the second equation in the left column?

Two

Will we be able to solve it?

No

If I substitute the ‘3’ for the ‘z’ in the third equation in the left column will I be able to solve it?

No

Why not?

Two variables. Too many variables.

Well then it looks like knowing that z equals 3 doesn’t help us with the first three equations. Let’s look at the two equations you created in column two. Tell me, how many variables are in each equation of the middle column?

Two

If I substitute the ‘3’ for the ‘z’ in the first equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variable in it?

Yes

If I substitute the ‘3’ for the ‘z’ in the second equation in column two, how many variables will be left in the equation?

One

Will we be able to solve that equation if it has one variables in it?

Yes

Which of those equations do you think will be easier to solve?

The first one

Both equations will lead to the same solution. This is an opportunity to suggest a self-monitoring question that could save a problem solver some extra effort.

Bring that equation straight down the column to the lower box, substitute the ‘3’ for the ‘z’ in the equation, and solve it. Let’s do it. If I substitute the ‘3’ for the ‘z’ in the first equation in column two what do I get?

y + 3(3) = 11

Good. Now what would you do first to solve this?

Multiply 3 times 3

And what do you get?

9

What’s our new equation?

y + 9 = 11

What’s next?

Subtract 9 from both sides. Move 9 to the other side and make it a minus 9.

Okay. So 9 minus 9 cancels. What is 11 minus 9?

2

Great. Now what?

We're done

So what does y equal?

2

Looks like we have two parts of the solution now. y equals 2 and z equals 3.

Before we try to find x, let's try something else, just for grins.

At this point we would follow the same procedure indicated here when we solved for y, except we would substitute the z value into the second equation in the second column. The intention is to demonstrate that both equations will yield the same solution. In practice we would not solve both of them.

So, does it matter which equation you choose to solve for y?

No

Would you rather choose an easier equation, or a harder one?

Easier

Me too. Okay now we still need to find a value for x.

The process of solving for x is parallel to that of solving for y. The most significant difference is that we must substitute both the value for y and the value for z into one of the original three equations. This eliminates two variables to leave an equation in one variable that can be solved in the lower left cell of the graphic organizer. The final solution, as it would appear in the completed graphic organizer, is shown in Figure 1. All that is left is to point out that conventionally the solution is written as an ordered triplet with the variables in alphabetical order. The solution will be briefly related to the original problem as described in the Day 1 plan. The next phase of the lesson is a guided practice example. This system is selected so that variables are eliminated, and solved for, in a different order, and so that negative integers appear in the solution. Students should not assume that the x is always eliminated first for example. Further, students need to realize that, while these problems are designed to yield integer solutions, practical applications of this process often do not. The guided practice system is as follows:

$$5x - 3y - 2z = 29$$

$$-x + 3y - z = -5$$

$$-x - 3y + 4z = -13$$

Because this problem is used as guided practice, the basic instructional sequence will be the same for solving this system as that for the model problem above. A blank graphic organizer will be filled in as before. However, in this case the model problem will remain in sight for students to refer to, and students will draw and fill in their own graphic organizers as they follow the example. Further, as much of the verbal scaffolding as possible will be removed. Specifically, students will be asked to identify and describe each step themselves while being directed to the first model when appropriate, and they will perform the calculations (adding and solving equations) themselves. Each of these steps will be checked with the group before we proceed to the next step. In the end, they will have a completed graphic organizer for this system, which they can use as a model for independent practice problems, and a correct solution: (3, –2, –4). As independent practice, two new systems will be given for homework. In both cases all three equations will have all three variables, and they can be solved without multiplying any equation by a constant. Students will also receive the solution to one of the systems in a sealed envelope, which they may open when they believe they have the solution. Both of these systems will be checked on the following day in class. This should not take a great deal of time as the steps will generally be the same for all students.

Day 3

The primary goal of this day's instruction is to teach students to work with systems in which one or more of the equations may have to be multiplied by a constant in order for terms to sum to zero so that variables can be eliminated. Before pursuing this goal, the independent practice assignment from the previous day will be reviewed. After this review, the instructor will draw students' attention to four items that will be posted at the front of the room. The first item is a new system of three equations that requires the multiplication of equations by constants as noted for the day's goal.

$-3x + 4y - z = -65$

$x - y - 2z = -15$

$-2x - 3y - 2z = -35$

The second item is a couple of examples of common multiples that the students found on Day 1. The third item is a couple of examples of equations multiplied by constants, also from Day 1. The fourth item is another blank graphic organizer as illustrated in Figure 1. Through questioning, prior knowledge relating to the first three items will be activated. After a reminder that these ideas will be important for solving the system on the board, the instruction of application of these ideas to systems of equations begins. The three equations are placed in Column III, on the left side of the organizer.

However, this system can not be solved simply be adding equations together, so something like the following model of a dialogue would occur.

Take a look at this new system of equations. If we add the first two equations, which variable will be eliminated?

None

Which variable will be eliminated if I add the bottom two equations?

None

What if I add the to and the bottom equations?

None

How is this system different from any of the systems you have solved so far?

The variables do not cancel

Can we solve this system if we don't eliminate any variables?

No

Take a look at the examples we have over here (indicating the common multiples). Could common multiples help us to get coefficients that are opposites?

Yes

If we multiply these equations by other numbers, will that change the answers to the system?

No

Okay. Let's try to use common multiples to get some variable coefficients to cancel. Take a look at the coefficients for x. They're –3, 1, and –2 (indicating). The coefficients for y are 4, –1,and –3. The coefficients for z are –1, –2, and –2. Which set of coefficients do you think will be easiest to work with?

z (Students may make another choice, and the system can be solved regardless of the choice they make.)

Great. Let's see if we can eliminate the z variable. Which two equations do you want to work with first?

The bottom two

How would you multiply those equations so that the z's cancel?

Multiply the middle equation by –1. Change all the signs in the middle equation.

Alright. Let's see what happens. What do we get when we multiply the middle equation by –1?

$–x + y + 2z = 15$

Good. How many variables are in that equation?

Three

So which column are we going to put it in?

The left column. Column III.

If we add the new equation to the original bottom equation will the z's cancel?

Yes

Let's do it. What is our new equation?

$–3x – 2y = = –20$

Did it work?

Yes

How many variables does our new equation have?

Two

What column does it go in?

The middle column. Column II.

Can we use this new equation to solve for a variable?

No

Why not?

It has too many variables

Would it help if we had another equation with just the same two variables?

Yes.

Why?

We could combine them to get an equation with only one variable

Okay. Back to the original equations. We have already combined the bottom two equations. Is there a way to eliminate the z variable by combining two other equations? . . .

At this point it should be clear how the lesson would proceed. Students would find a second equation with two variables that match the one they have already derived, after determining how to find a common multiple for the z coefficients. Then they would carry out the same analysis on the two equations with two variables – selecting a variable to eliminate, finding a common multiple of the coefficients, multiplying the equation(s) and combining them. Then the solution process proceeds as before. They solve for one variable, substitute this solution into an equation with two variables to solve for a second variable, and substitute for both of these variables in an equation with three variables to solve for the third variable. This will lead to the final solution of (10, –5, 15).

As with the previous day's lesson, this model would remain posted as we work through a second example for guided practice. The scaffolding would be reduced for the second example and students will identify the steps and carry out the calculations themselves. They would then receive two problems for homework, along with the solution to one of them in a sealed envelope.

Day 4

The primary goal for this day's lesson is to introduce systems in which one or more of the equations have less than three variables. This time there are no new prerequisite skills involved. After reviewing the homework problems, students will address a new problem that is posted.

$2x - 3y + z = -1$

$x + 2y = -13$

$3x - y - 4z = -23$

Students should have no difficulty recognizing that this problem is different from those they have solved before because one of the equations only has two variables rather than three. Instruction begins as it has with the dialogue in the other models, placing the equations into the graphic organizer. In this case the equation with only two variables will be placed in the middle column. Once all three equations have been placed in the graphic organizer, the dialogue turns to how to handle this system in which the one of the equations has only two variables.

Do we have an equation with one variable that we can solve (indicating the right column)?

No

How do we usually create an equation with one variable that we can solve?

Add two equations with two variables

Do we have two equations with two variables?

No

Do we have one?

Yes

How can we get another?

Combine the two equations that have three variables

Good. Now which variables are in the equations with two variables that we already have?

x and y

Which two variables does our new equation have to have?

x and y

Right. So which variable do we have to eliminate when we combine the two equations with three variables?

z

Great. If we add the two equations with three variables as they are now, will the z variable be eliminated?

No

So how can we eliminate the z variable?

Multiply. Find a common multiple. Multiply the first equation by 4.

From here the lesson would proceed as in the previous day's lesson. Again, this model would be followed by another example. The second example would include an equation of only one variable, as well as two equations with three variables each. This second example would be solved with less scaffolding and with the students doing and checking the calculations themselves. They will also be recording this example in a graphic organizer in their notes. As before, this second example will be followed by an independent assignment of two problems. The solution to one of them will be provided in a sealed envelope.

Day 5+

The first task of this day is to review the homework assignment from the previous day. Students will then be given the teacher generated content test. After students have completed the teacher generated content test, and time permitting, they will take the content skills test and complete the social validity questionnaire. It is unlikely that all of these tasks can be completed in one day. A more likely scenario is that the teacher generated test will be completed on Day 5 but the investigator generated test and the questionnaire will be completed on the following day.