ABSTRACT

Existing literature does not provide an analysis of an economy containing both risk averse and risk neutral individuals who are faced with a lottery that finances a public good. We provide a thorough analysis of individual behavior and strategic interactions between both types of individuals. We find that voluntary giving is crowded out completely, if preferences are separable. As an explanation for voluntary contributions, we introduce the notion of impure warm glow giving. We show that impure warm glow givers contribute more to the public good than without the warm glow. Furthermore, we show that neutrality of public-good provision with respect to the distribution of income does not hold in this setup.

INDEX WORDS: public good, warm glow, lottery, charity, philanthropy, voluntary contributions
GIVING WITH IMPURE WARM GLOW: AN APPLICATION TO LOTTERY PLAYING

by

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Chapter 1

Introduction

This thesis develops a model of charitable fundraising. We assume that the charity supplies a public good and uses a lottery to finance it. In particular, we analyze individuals’ behavior when facing a charity which offers a lottery. Individuals are assumed to maximize their utility by taking the actions of others as given. Existing models such as Morgan’s (2000) are altered by relaxing the assumption of risk-neutrality. Particularly, we are interested in the impact of adding risk-averse individuals to the existing model. We analyze how risk-averse individuals behave in this model and how both groups, risk-averse and risk-neutral individuals, interact. Furthermore, we relax the assumption that individuals can decide only whether or not to play the lottery. We show that donating to the prize pool will induce other individuals to wager more, thereby increasing the prize. Additionally, we allow individuals to contribute directly to the public good via donations, since we know empirically that most charities rely on voluntary donations even when they offer a lottery.

The link between lotteries and the provision of a public good can easily be seen in the existence of state-run lotteries. The State of Georgia offers a lottery where the revenue is used to fund education. Several other states also offer lotteries to fund public goods, most commonly education. Morgan (2000) points out that “state-run lotteries are operated in 36 U.S. states. In fiscal year 1995, revenues from these amounted to just under $32 billion” [Morgan (2000, p.763)]. He also shows that lotteries are an important instrument for private charities and non-profit organizations to raise funds. “In Britain, private charities raise about 8% of their income
through lotteries. In the U.S. in 1992, among 26 reporting states, about $6 billion was raised by private charities through lotteries” [Morgan (2000, p. 761)]. Furthermore, Morgan (2000) emphasizes that the link between public goods and lotteries affects ticket sales. The payout rates are higher (about 80%) when the link to a public good is absent, like in casino-run bingo and keno games, than in state- or charity-run lotteries (where it is about 50%). Lotteries are, therefore, commonly used instruments to fund public goods and individuals derive utility from the link to a public good since it affects ticket sales.

By allowing individuals to play other strategies, such as donating to the prize pool, we discover that voluntary giving is crowded out when a lottery exists. Morgan (2000) obtained the same result for risk-neutral individuals. But in an economy consisting of risk-averse individuals, voluntary contributions to a public good cannot easily be explained theoretically. To overcome such difficulties, we use Andreoni’s (1989) model of warm-glow giving. Individuals derive utility not only from the public good, but also from the act of giving. The warm-glow can be interpreted as social esteem for being known as a generous giver or as satisfaction of one’s own moral constraints. We alter the Andreoni (1989) model to develop a better explanation for charitable giving. The utility gain from giving depends not only on one’s own contributions, but also on other individuals’ contributions. This theory of impure warm-glow provides not just an explanation for the existence of voluntary contributions, but also generalizes the lottery model. In this case, the warm-glow reflects the chance of winning the prize.

The next chapter provides a short review of some of the major theoretical findings in the literature on charitable giving. In Chapter 3, we introduce a model in which a public good is funded by a charity offering a lottery. We assume risk-averse individuals, and allow them to donate to the charity as well as to buy lottery tickets. Chapter 4 analyzes the model by showing
how risk-averse individuals behave and what impact their behavior has on the economy. In Chapter 5, we introduce impure warm-glow giving. We compare impure warm-glow givers with non-warm-glow givers, and investigate whether neutrality holds in this model. Chapter 6 summarizes the major findings of this thesis.
Chapter 2

Literature Review

There exists a vast literature on charitable giving and the means by which funds are raised for charities. One of the basic papers on charitable giving is Warr (1982). For simplicity, he analyzes an economy with two rich individuals and one poor individual, where the consumption of the poor individual yields positive marginal utility for the rich. Hence, the poor individual’s consumption is a public good for the rich. The rich person maximizes utility by taking the other rich individual’s contribution to the poor person’s consumption as given. Warr shows that this “voluntary contributions game” leads to inefficient underprovision of charity. Furthermore, he shows that the existence of a government collecting lump sum taxes from the rich in order to increase the amount of the public good leads to crowding out of the rich individuals’ contributions to the public good, dollar for dollar, as long as the taxes are less than the optimal contributions. Fiscal redistribution of income is neutral and does not lead to a Pareto improvement. This conclusion is known in the literature as a neutrality result. However, Warr shows that taxing the rich individual’s consumption of the private good and using the proceeds to subsidize donations increases the supply of the public good and can make both poor and rich individuals better off.

Sudgen (1982) discusses three key assumptions for the theory of private philanthropy that are adopted in the existing literature and the problems they cause theoretically:

(1) The publicness assumption: donors are assumed to derive utility from the amount of the public good they provide.
(2) The utility maximizing assumption: individuals are assumed to be utility maximizing; hence, the amount of their donations is determined by the solution to a utility maximizing problem.

(3) The Nash conjectures assumption: the individuals solve their maximization problem by taking the contributions of other individuals as given.

Sudgen (1982) shows that this theory is not applicable to large charities. Empirical evidence reveals that large charities consist of many donors each of whom gives a relatively small amount, and these individual donations cumulate to a large amount of money raised for the public good. The doubt Sudgen expresses in his paper concerns the benefit an individual receives from giving to a large charity. Encountering an already high amount of contributions (several million dollars, for instance), why should one donate a small amount ($50, for instance)? Why does a large change in other individuals’ contributions not lead to changes in a given donor’s decision? With the three assumptions above, Sudgen concludes that nobody would contribute. He suggests dropping one of these assumptions; in particular, he suggests replacing publicness with the assumption that, instead, individuals derive utility from the act of giving to the charity. This suggestion has been taken up by Andreoni (1989), who extended it to the warm-glow theory, which will be discussed in detail later in this chapter.

Sudgen (1984) further revised the theory of privately funded public goods such as charities. As a solution to the problems posed by the three assumptions, Sudgen presents the “reciprocity principle”. This principle is a weaker form of the “Kantian principle”, which assumes that individuals contribute because of a moral constraint. In particular, one individual contributes the amount other people are expected to contribute. Sudgen finds that this principle is too strong. If

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1 See Sudgen (1982)
2 See Laffont (1975) and Collard (1983)
nobody else contributes, it is hard to believe that an individual would contribute just because he expects others to do as he does. The principle of reciprocity Sudgen introduces is the following: suppose every member of a group contributes at least a certain amount of effort (money, work, etc.); then each individual in that group is induced to contribute at least that level, whether through moral constraints or social prestige. He shows that, under this assumption, an equilibrium exists in which the free-rider problem is solved, but this is not the only equilibrium consistent with the model. An equilibrium where a desirable public good is entirely unfunded could also exist. Generally, there is still undersupply of the public good, but with the reciprocity principle there will be more of the public good provided than without it.

Cornes and Sandler (1984) introduce a joint public-private good, which they call an impure public good. Their model works the following way: suppose we have two goods, $c$ and $q$. By purchasing $c$, $c$ units of a characteristic are produced (which is the consumption of the good). The commodity produces no other characteristics for this individual or for any other person and can be thought of as a private good. By purchasing a market commodity $q$, $\beta$ units of a second characteristic, $x_i$, are produced and $\gamma$ units of a third characteristic, $z_i$, are produced. The utility function is defined as:

$$ U_i = U_i(c_i, x_i, Z) $$

where $Z = z_1 + z_2 + \ldots + z_n$. Hence, $Z$ is a public characteristic. We can think of $q$ as a joint public-private, or impure, public good. By purchasing $q$, individual $i$ contributes to the public good and also benefits by consuming the private characteristic $x_i$. The utility function is assumed to be increasing in all of its arguments and is quasiconcave. The individual maximizes utility subject to two constraints. $c_i$ enters the utility function as numeraire, so the budget constraint is given by:
where $I_i$ is individual $i$'s income. The other constraints are the production constraints:

$$x_i = \beta q_i, \quad z_i = \gamma q_i.$$  

The utility function can be transformed into an indirect utility function of market commodities:

$$U(c_i, x_i, Z_{-i} + z_i) = U(c_i, \beta q_i, Z_{-i} + \gamma q_i)$$

$$= V(c_i, q_i; Z_{-i})$$

where $Z_{-i}$ are the contributions to the public characteristic by all other individuals. $V$ retains all of the properties of the utility function $U$. In their analysis of this model, Cornes and Sandler find that an increase in the size of the group does not necessarily increase inefficiency or free riding. This follows because the jointly produced private output can serve a privatizing role, not unlike the establishment of property rights. Complementarity between the joint products brings out this privatizing effect. They also find that the stability of a Nash equilibrium depends not only on the income effect, but also on the substitution effect. Furthermore, they show that Nash behaviour does not necessarily imply inefficiency. Lastly, the joint-product model need not satisfy the neutrality theorem, thereby providing a role for tax and redistribution policy. Their paper does not specify a particular utility function and is, therefore, a very general analysis of the impure public good model.

In a subsequent paper, Cornes and Sandler (1994) derive additional comparative statics properties from the model. They find that an exogenous change in money income $I_i$ has an ambiguous effect on contributions. Assuming quasilinear preferences, $U_i(c_i, x_i, Z) = c_i + F(x_i, Z)$, a change in virtual income, $I_i + Z_{-i}$, would change $c_i$ one for one, but leave $x_i$ and $Z_i$ unaffected. This assumption of quasilinearity of the utility function is a commonly exploited device for removing income effects. With quasilinear preferences, the
comparative statics effect of a change in $Z_i$ depends on whether characteristic $x$ and $Z$ are complements or substitutes. For instance, if they are substitutes, consumption of the public good rises with an increase in income, but contributions, together with the jointly produced private characteristic, fall. The effect of an increase in the parameter $\beta$, assuming quasilinear preferences, again depends on the substitutability between $x$ and $Z$. The demand for $x$ rises unambiguously, while the demand for $Z$ may rise or fall, if $x$ and $Z$ are substitutes. Hence, the comparative statics results for a change in $\beta$ are ambiguous. If $x$ and $Z$ are complements, the effect of change in $\beta$ is generally ambiguous, but under certain conditions the demands for $x$ and $Z$ unambiguously rise. An increase in $\gamma$ has an ambiguous effect on the demand for $x$, but unambiguously increases the demand for $Z$ if $x$ and $Z$ are substitutes. Hence, the overall effect on $q$ is ambiguous. If they are complements, there is a tendency for the demand for both to fall.

Cornes and Sandler suggested many applications of these findings to charitable giving. Several interpretations of the general impure public good model have been offered to explain charitable giving. Applying the pure public good model to charities leads to unsatisfactory predictions, including those discussed by Sudgen (1982) and the neutrality result discovered by Warr (1982). The theory of public goods and the assumption of utility maximization, together with Nash conjectures, predict a crowding-out effect of government giving and lump sum taxes analogous to Ricardian equivalence. However, empirical results\(^3\) show that crowding out is quite small and that neutrality does not hold. Thus, there must be different explanations for charitable giving. The Kantanian and reciprocity principles and the impure public good model developed

\(^3\) See Clotfelter (1985) and Schiff (1985) for empirical evidence of incomplete crowding out. In addition to these, there are experimental studies that demonstrate that the behavioral assumptions that are necessary for crowding out are contradicted in the laboratory [see Dawes and Thaler (1988) and Andreoni (1988)].
by Cornes and Sandler provide alternatives in which, for example, neutrality does not necessarily hold.

One possible application of the Cornes and Sandler model is Andreoni’s theory of warm-glow giving. People are assumed to give to a charity for two reasons. First, they care about the public good provided by the charity, an assumption which is present in every setup dealing with charities. Additionally, Andreoni assumes that people also derive utility from the act of giving. This can be interpreted as an increase in utility for being known as a generous giver or simply the good feeling one gets from giving. If this warm-glow is zero, then the individual cares nothing about the amount he has contributed to the public good, and the model reduces to the basic voluntary contributions game. Andreoni calls that case pure altruism. At the other extreme, a purely egoistic individual does not care about the public good itself, but values only the warm-glow of giving. Notice that the latter is exactly what Sudgen suggested as one solution to the free-rider problem. The preference model Andreoni (1989) presents is the following:

\[ U_i = U_i(x_i, Y, g_i) \]

where:

- \( x_i \): the private good
- \( Y \): the total amount of the public good \( (Y_i = \sum_{i=1}^{n} g_i) \)
- \( g_i \): individual \( i \)'s contribution to the public good

Hence, each individual \( i \) maximizes utility subject to the following constraints:

\[ x_i + g_i = w_i - \tau_i \]

\[ G_{-i} + g_i + T = Y \]
where:

\( w_i \): individual \( i \)'s income

\( \tau_i \): lump sum tax on individual \( i \)

\( G_{-i} \): all others’ contributions to the public good (\( \sum_{j \neq i} g_j \))

\( T \): total government contribution financed by taxes (\( T = \sum_{i=1}^n \tau_i \))

It’s easy to see that this model is simply the Cornes-Sandler model of an impure public good, where \( \beta = \gamma = p = 1 \). Andreoni’s analysis is concerned with the altruism coefficient \( \alpha_i \), which is derived by substituting the constraints into the utility function and solving for \( Y \):

\[
Y = f^i(w_i + Y_{-i}, Y_{-i} + \tau_i)
\]

where:

\( y_i \): individual \( i \)'s total contribution to the public good (\( y_i = g_i + \tau_i \))

\( Y_{-i} \): all others’ contributions to the public good (\( \sum_{j \neq i} y_j \)).

The altruism coefficient is defined as follows:

\[
0 < \alpha_i = \frac{f_1^i}{f_1^i + f_2^i} = -\left. \frac{dY_{-i}}{dw_i} \right|_{w_i} < 1
\]

where:

\( f_1^i \): derivative of \( f^i \) with respect to its first argument,

\( f_2^i \): derivative of \( f^i \) with respect to its second argument.

This altruism coefficient serves to index the degree of altruism. For instance, if an individual is purely altruistic and does not derive utility from the act of giving, then \( f_2^i = 0 \) and, hence,
Thus, the more altruistic an individual is, the higher the value of $\alpha_i$. With this model set up, Andreoni shows that neutrality does not hold, in general, for a redistribution of income from individual 1 to 2:

$$dY = c(\alpha_2 - \alpha_1)d\tau$$

where:

$$d\tau = d\tau_1 = -d\tau_2$$

$c$: constant

In particular, redistributing income to more altruistic individuals provides more of the public good. Neutrality only holds when individuals are equally altruistic. Hence, the model is capable of explaining the empirical regularity that the pure altruism model fails to explain, namely that crowding out is not complete. Moreover, by assuming that people derive utility from the act of giving, the explanation is very natural and intuitive.

Andreoni (1990) extends his analysis of the warm-glow giving model, investigating the implications of alternative tax and subsidy policies. He finds that an increase in the lump sum tax for individual $i$, holding the taxes paid by others constant, increases the total provision of the public good if $\alpha_j < 1$ for at least one individual $j$. Furthermore, an increase in the subsidy rate for one individual $i$, holding the subsidy rates for the others constant, increases the total provision of the public good if $\alpha_j < 1$ for at least one individual $j$. He concludes that these results are consistent with empirical evidence he got from an experiment. Furthermore, the introduction of subsidies in his model allows Andreoni to characterize the optimal tax treatment of charities.

Morgan’s (2000) analysis of lotteries as an instrument for raising funds for the provision of public goods represents an alternative interpretation of the Cornes and Sandler model. Private charities, along with states, often use lotteries to increase the provision of a public good. My
thesis is based on Morgan’s approach to fund-raising by means of lotteries. Lotteries are viewed as a practical means of trying to overcome the free-rider problem, and are often used by institutions lacking the power to tax. Morgan assumes quasilinear preferences and neutrality with respect to income risk. I extend his analysis to risk-averse individuals, and show how adding risk-aversion affects the model.

Morgan’s main analysis deals with fixed-prize raffles. Individuals can purchase lottery tickets to increase their chances of winning the prize, which is a fixed sum of money. The number of tickets purchased minus the amount of the prize (equal to the lottery revenue) goes towards the provision of the public good. The raffle is not held if the total amount of tickets purchased does not lie within $\delta$ of the prize. Perfect information and Nash conjectures, along with utility maximization and publicness, are assumed in this model. Individuals maximize expected utility with respect to their wagers, provided that the raffle is held:

$$EU_i = w_i - x_i + \frac{x_i}{x(N)} R + h_i(x(N) - R)$$

where:

- $w_i$: individual $i$’s endowment of the private good
- $x_i$: individual $i$’s wager (amount of lottery tickets purchased)
- $x(N)$: sum of all individuals’ wagers
- $R$: amount of the prize
- $G = x(N) - R$: amount of the public good provided
- $h_i(.)$: the amount of utility gain for any $G$. $h_i(.)$ is assumed to be twice differentiable, continuous, strictly monotonic, and concave.
It is straightforward to see that this model of a fixed-prize raffle is an application and extension of the Cornes and Sandler model of an impure public good. By purchasing lottery tickets, an individual benefits both from increasing the amount of the public good, which can be linked to the public characteristic, and from increasing the chances of winning the prize, which can be interpreted as the private characteristic in the Cornes and Sandler model. However, there is an additional feature not present in the Cornes and Sandler model insofar as an individual, by purchasing lottery tickets, lowers the chance of others winning the prize. Hence, there is also a negative externality built into the model, which is not found in the Cornes and Sandler model.

Morgan (2000) proves the existence of an unique equilibrium for the fixed-prize raffle. He also finds that the fixed-prize raffle provides more of the public good than does voluntary contributions. One implication of the model is that fixed-prize raffles completely crowd out voluntary giving. However, first-best prize raffles must provide less than the first-best levels of the public good provision. Nonetheless, Morgan shows that for any $\varepsilon > 0$ there exists an economy of size $\sum_{i=1}^{n} w_i^*$ and a raffle with prize $R^*$ such that the public-goods provision induced by the raffle lies within $\varepsilon$ of the first-best outcome. Furthermore, the raffle provides positive amounts of the public good if and only if the public good is socially desirable. Hence, lotteries as a means of raising funds for public goods are efficient instruments. Neutrality with respect to outside donations does not hold, that is, crowding out is less than one for one, and donations to the prize pool provide more of the public good than do direct contributions. Finally, note that all the results Morgan derives are based on the specific assumptions of quasilinear and risk-neutral preferences.
Chapter 3

The Model

We consider a charity that raises funds by offering a lottery. In order to do so, we use a model that is similar to Morgan’s. Individuals are assumed to be expected utility maximizers, the good funded by the charity is assumed to be a public good, and individuals are assumed to maximize utility, taking the actions of others (wagers and donations) as given. People are assumed to have perfect information. However Morgan’s analysis only deals with risk-neutral individuals. We extend his model to include agents with risk-averse utility functions and analyze the interactions between risk-neutral and risk-averse individuals.

There is an economy with one private good and one public good that is funded by a private charity without taxing power. We assume that the economy consists of \( n \) individuals, some who are risk-neutral and some who are risk-averse, with the following respective utility functions:

\[
U_k(x_k, G) = x_k + h_k(G)
\]

\[
U_j(x_j, G) = v_j(x_j) + h_j(G)
\]

with \( k \) being risk-neutral and \( j \) being risk-averse. \( x \) is the private good and \( G \) is the public good. \( v \) and \( h \) are assumed to be twice differentiable, strictly monotonic and quasiconcave. When playing the lottery, the two types of individuals are faced with the following utility maximization problems:

\[
\max_{y_i} \left\{ EU_k = w_k - y_k + \frac{y_k}{y(N)} R + h_k(x(N) - R) \right\}
\]
\[
\max_{y_j} \left\{ EU_j = (1 - \frac{y_j}{y(N)}) y_j (w_j - y_j) + \frac{y_j}{y(N)} y_j (w_j - y_j + R) + h_j(y(N) - R) \right\}
\]

where:

\( y_j \): individual \( i \)'s wager

\( y(N) \): the sum of all individuals' wagers (\( y(N) = \sum_{i=1}^{n} y_i \))

\( w_i \): individual \( i \)'s endowment of the private good

\( R \): prize of the raffle

The term \( \frac{y_j}{y(N)} \) is the probability of winning the prize. It follows that \( \frac{y_k}{y(N)} R \) enters the utility function of the risk-neutral individuals, since they are indifferent between the expected value of the lottery and the lottery itself. Risk-averse individuals are faced with two wealth situations. The utility when not winning the prize is multiplied by the probability of not winning, and the utility when winning the prize is multiplied by the probability of winning. Furthermore, since we want to compare risk-averse and risk-neutral individuals, we make the following assumption without loss of generality:

\[
v'_j(w_j) = 1 \quad \forall j \in \{1,...,n\}
\]

Note that the amount of the public good provided is the sum of all wagers minus the prize, \( G = x(N) - R \). The first-order conditions for the optimal choice of \( y \), denoted \( y^* \), are the following, conditional on making a positive bet:

\[
\frac{\partial EU_k}{\partial y_k} = -1 + \sum_{i=k}^{n} \frac{y_i}{y^*(N)} R + h'_k(G^*) = 0
\]
\[
\frac{\partial EU_i}{\partial y_j} = -v_j'(w_j - y_j^*) + \frac{y_j^*}{y(N)} \left[ v_j'(w_j - y_j^*) - v_j'(w_j - y_j + R) \right]
\]
\[
\sum_{j \neq i} \frac{y_i}{y(N)} \left[ v_j(w_j - y_j) - v_j(w_j - y_j + R) \right] + h_j'(G)
\]

Morgan has shown that, in the case of quasilinear preferences, fixed-prize raffles crowd out voluntary giving completely. However, empirically, even with an existing lottery, donations to the public good are observed. Therefore, we want to allow individuals to play two more strategies than just buying lottery tickets. On the one hand, agents can give directly to the public good via voluntary contributions. For any function \( v_i \), whether linear or concave, the utility function would then become:
\[
U_i^C = v_i(w_i - g_i) + h_i(y(N) + g_i - R),
\]
where \( g_i \) is individual \( i \)’s contribution to the public good. The first-order condition for the optimal choice of \( g_i \) denoted \( g_i^* \), when giving a positive amount, is the following:
\[
\frac{\partial U_i^C}{\partial g_i} = -v_i'(w_i - g_i^*) + h_i'(y(N) + g_i^* - R) = 0.
\]
In addition, individuals can donate to the prize pool to increase the size of the prize, along with other people’s wagers, and thereby increase provision of the public good. The maximization problem is
\[
\max_{D_i} U_i^p = v_i(w_i - D_i) + h_i(y(N, D_i) + D_i - \hat{R}),
\]
where \( y(N, D_i) \) denotes the sum of all individuals’ wagers as a function of \( i \)’s donation to the prize pool, and \( \hat{R} = R + D_i \) is the prize plus individual \( i \)’s contribution to it. Individual \( i \) can induce other people to wager more by increasing his donation to the prize. Since Morgan has
shown that the sum of all wagers increase \( \frac{\partial y(N, D_i)}{\partial D_i} > 0 \), when the prize increases individual \( i \) might choose to donate to the prize pool instead of wagering or giving directly to the public good. The first-order condition for the optimal choice of \( D_i \), denoted \( D_i^* \), assuming donations to the prize pool are positive, is the following:

\[
\frac{\partial U_i^o}{D_i} = -v_i'(w_i - D_i) + h_i'(y(N, D_i) - R)y'(N, D_i).
\]
Chapter 4

The Effect of Risk Aversion on Playing the Lottery

In this setup, the model differs from the Morgan model in several respects. We allow the economy to have risk-averse people, and we allow the individuals to play other strategies, namely contributing to the public good directly and donating to the prize pool. How do individuals interact? Do the results obtained by Morgan carry over to the more general model? First, I establish two important results:

Lemma 1: An equilibrium exists in the fixed-prize raffle.

Proof: Since the set of each individual’s actions is compact and convex, and payoffs are continuous and concave, the standard existence conditions are satisfied. [See Theorem 20.3 Osborne and Rubinstein (1994)]. QED

Lemma 2: Having at least one risk-neutral individual with quasilinear preferences in the economy, there is no equilibrium set of wagers \( \{y_1^*, ..., y_N^*\} \) such that \( y(N) - R < -\delta \), provided the individual’s endowment is sufficiently large.

Proof: Suppose the contrary is true. Then there is an equilibrium where the raffle is called off; each wager would be refunded. Thus, in this equilibrium, each bettor’s utility is simply \( w_k \) or \( v_j(w_j) \). Now consider a deviation in the wager by the risk-neutral individual \( k \).

Suppose that \( k \) alters his bet so that
\[ y(N) - R = -\delta \]

Then the raffle is held and \( k \) realizes the expected utility

\[ U_k = w_k - y_k + \frac{y_k}{y(N)} R = w_k + y_k (\frac{R}{R - \delta} - 1) \]

which exceeds \( w_k \), since \( \delta > 0 \). Thus, this deviation is profitable for risk-neutral individuals, which contradicts the supposition. \( QED \)

Therefore, assuming a mixed economy of risk-neutral and risk-averse individuals and that the public good is funded by a charity which offers a lottery, we conclude that the lottery always will be held.

What can we say about the interaction of risk-neutral and risk-averse individuals? Intuitively, risk-averse people are less inclined to play the lottery, since they prefer the expected value of a lottery to the lottery itself, whereas risk-neutral individuals are indifferent between them. Hence, if the only thing that differs among individuals is the degree of risk aversion, then one might guess that a risk averter would wager a smaller amount or would not play the lottery at all. To establish this result, we set up a model with risk-averse and risk-neutral “twins”, defined as people who have identical preferences for the public good, the same amount of endowed wealth, and the same marginal utility of income evaluated at the wealth endowment, so that \( v_j'(w_j) = 1 \) for all \( j \). The \( n \) pairs of twins differ only in their degree of risk aversion, so that individual \( k \) is risk-neutral and individual \( j \) is risk-averse. The twins assumption is made for computational convenience. Can we say that risk-averse individuals in this model do not play the lottery? We cannot, in general. However, Theorem 1 holds in this economy:
Theorem 1: Consider an economy consisting of $n$ twins, one risk-neutral one risk-averse. If a lottery exists to fund a public good, more risk-neutral individuals play the lottery than risk-averse people.

Proof: Let

$$\Psi_j = \frac{\partial EU_j}{\partial y_j}_{y_j=0}$$

be the marginal benefit for an individual to change the bet from zero to a small amount. In order for individual $j$ to bet a positive amount, $\Psi_j > 0$ has to hold. The marginal benefit of entering the lottery for a risk-neutral individual is

$$\Psi_k(G) = -1 + \frac{R}{R+G} + h'_k(G),$$

where $G = y(N) - R$ denotes the provision of the public good enabled by the wagers of all others. For a risk-averse individual entering the lottery, the marginal benefit becomes

$$\Psi_j(G) = -v'_j(w_j) + \frac{1}{y(N)}[v(w_j + R) - v(w_j)] + h'_j(y(N) - R)$$

$$= -1 + \frac{R}{G + R} v'_j(\xi_j) + h'_j(G)$$

for $\xi_j \in (w_j, w_j + R)$, since by the mean value theorem

$$\frac{v_j(w_j + R) - v_j(w_j)}{w_j + R - w_j} = v'_j(\xi_j)$$

for at least one $\xi_j \in (w_j, w_j + R)$.

$v_j$ being strictly monotonic and concave and $v'_j(w_j) = 1$ implies $0 < v'_j(\xi_j) < 1$. Notice that $\Psi_j$ is everywhere decreasing in $G$ for all individuals. Let $G_i : \psi_i(G_i) = 0 \ \forall i \in \{1, ..., 2n\}$.

Then the following Lemma holds, which is used to prove the Theorem:
Lemma 3: Any equilibrium generating $G$ of the public good consists of bets $y_i > 0$ for all $i$ such that $G_i > G$ and zero bets by all others.

Proof: Suppose there exists an equilibrium in which an individual with $G_i > G$ does not play the lottery; i.e., $y_i = 0$. But then $\Psi_i(G) > \Psi_i(G_i) = 0$ and it is profitable for this individual to enter the lottery. On the other hand, suppose there exists an individual $i$ with $G_i \leq G$. Then $\Psi_i(G) \leq \Psi_i(G_i) = 0$ and it is not profitable to enter the lottery. QED

We continue to prove the Theorem. With $j$ being risk-averse and $k$ being risk-neutral, we have the following:

$$\Psi_j(G) = -1 + \frac{R}{G+R} \left[ v_j'(\xi_j) \right] + h'(G) < -1 + \frac{R}{R+G} + h'(G) = \Psi_k(G)$$

which implies $G_j < G_k$ for all twins, where $k$ is risk-averse and $j$ is risk-neutral. From Lemma 3 we can conclude that, for any equilibrium $G$, more of the risk-neutral individuals play the lottery than risk-averse individuals. QED

Does the degree of risk aversion affect individuals’ behavior in this model? Intuitively, the higher the degree of risk aversion the less inclined an individual would be to play the lottery. We get indeed this result when proceeding the same way as before by setting up the “twins model”.

Corollary 1: Consider an economy with $n$ pairs of individuals, each pair consisting of two identical individuals except that one is more risk-averse, denoted with subscript $l$, than the other, denoted with subscript $j$. The individuals with the higher degree of risk aversion are less likely to play the lottery.
Proof: Let individual $j$’s utility function be defined as above. For $l$ we get:

$$U_l = v_l(x_l) + h_l(G),$$

where $w_l = w_j, h_l = h_j$, and $v_l$ is a concave transformation of $v_j$, with $v_l(x_l) = C(v_j(x_j))$

for some concave function $C$, with $C'(v_j(w_j)) = 1$. Hence, we get for $l$

$$\frac{v_l(w_l + R) - v_l(w_l)}{R} = \frac{C(v_j(w_l + R)) - C(v_j(w_l))}{R} \frac{v_j(w_l + R) - v_j(w_l)}{R}$$

$$= C'(\chi_l) \frac{v_j(w_l + R) - v_j(w_l)}{R}$$

$$= C'(\chi_l)v_j'(\xi_j)$$

for at least one $\chi_l \in (v_j(w_j), v_j(w_j + R))$, by the mean value theorem, implying that

$$C'(\chi_l) < 1.$$ Hence, $G_j > G_i$ since

$$\Psi_i(G) = -1 + \frac{R}{R + G} \frac{v_j(w_j + R) - v_j(w_l)}{R} + h_l'(G)$$

$$= -1 + \frac{R}{R + G} C'(\chi_l)v_j'(\xi_j) + h_l'(G)$$

$$< -1 + \frac{R}{R + G} v_j'(\xi_j) + h_j'(G)$$

$$= \Psi_k(G)$$

By Lemma 3, we get the desired result. QED

In this model, we are not able to show that all risk-averse individuals choose not to play the lottery, although it is an intuitive way to think about this result. Therefore, we want to investigate under what conditions no risk-averse individual plays the lottery. We arrive at these conditions as a Corollary of Theorem 1:
Corollary 2: If, for all twins $k$ and $j$, where individual $k$ is risk-neutral and individual $j$ is risk-averse, $v_j' (\xi_j) \leq 1 - \frac{y^*_k}{G + R}$ holds, then no risk-averse individual plays the lottery.

Proof: From the first-order conditions for the risk-neutral people we obtain:

$$-1 + \frac{\sum y_i}{y(N)^2} R + h'_k (y(N) - R) = 0$$

$$\Leftrightarrow -1 + \frac{y(N) - y^*_k}{y(N)^2} R + h'_k (\sum y'_i + y^*_k - R) = 0$$

$$\Leftrightarrow -1 + \frac{R}{R + G^*}(1 - \frac{y^*_k}{G + R}) + h'_k (G^*) = 0$$

For the equilibrium amount of the public good, denoted $G^*$, the above equations hold.

Assuming now, for each pair of individuals $k$ and $j$, $v_j' (\xi_j) \leq 1 - \frac{y^*_j}{G + R}$, we get the following

$$\Psi_j (G^*) = -1 + \frac{R}{R + G^*} v_j' (\xi_j) + h'_j (G^*)$$

$$\leq -1 + \frac{R}{R + G^*}(1 - \frac{y^*_k}{G + R}) + h'_j (G^*)$$

$$= 0$$

Hence, it is not profitable for any risk-averse individual $j$ to enter the lottery and we have the desired result. QED

The condition for risk-averse people not to play the lottery is not really strict. Especially in big lotteries, where the prize is large, the revenue is high and $G^* = Y^*(N) - R$ is very large. In a large economy, the number of people participating in the lottery is high, so that $y^*_k$ is small relative to $G^* + R$, implying that $1 - \frac{y^*_k}{G + R}$ is very close to 1. Thus, we conclude that risk-averse people do not play the lottery if the prize and the number of twins are sufficiently large.
4.1. The Effect on an Individual’s Behavior of Allowing Individuals to Donate

Now, we want to allow individuals to play different strategies, as mentioned in the beginning. People can donate to the prize pool to increase the amount of the public good or they can contribute to the public good directly. These strategies might be reasonable, since Morgan has shown that a donation to the prize pool induces other individuals to wager more and a direct contribution to the public good does not decrease other individuals’ wagers one for one. However, we have to drop one strategy, since it is strictly dominated by the other two:

**Theorem 2:** For all individuals, whether they are risk-averse or risk-neutral, if a lottery exists to fund a public good, then playing the lottery or donating to the prize pool strictly dominates contributing to the public good directly.

**Proof:** From Lemma 2 we know that, if a charity offers a lottery, the lottery will always be held if at least one risk-neutral individual belongs to the economy. Even if all individuals decide to donate to the prize pool or contribute to the public good, there will always be at least one individual who plays the lottery and wagers a sufficiently large amount so that the raffle will be held (see Lemma 2). Hence, playing the lottery cannot be dominated by either of the other two strategies for all individuals.

To prove Theorem 2, we continue to focus on the other strategies. As Morgan has shown [see Corollary 2, Morgan (2000)], “small donations to the prize pool provide more of the public good than direct contributions”. We can show that, whether an individual is risk-averse or risk-neutral, a donation to the prize pool is strictly preferred to contributing to the public good; that is,

\[ U_i^D = v_i(w_i - D_i) + h_i(y(N,D_i) + D_i - \hat{R}) > v_i(w_i - D_i) + h_i(y(N,0) + D_i - R) = U_i^C \]
for any function $v_i$ being linear or concave, since $y(N, D) > y(N) + D$ [see Morgan (2000), Proposition 3].

Furthermore, playing the lottery is strictly preferred to contributing to the public good, since

$$EU_i = \left(1 - \frac{y_i}{y(N, 0)}\right)v_i(w_i - y_i) + \frac{y_i}{y(N, 0)}v_i(w_i - y_i + R) + h_i(y(N, 0) - R)$$

$$> \left(1 - \frac{y_i}{y(N, 0)}\right)v_i(w_i - y_i) + \frac{y_i}{y(N, 0)}v_i(w_i - y_i) + h_i(y(N, 0) - R)$$

$$= v_i(w_i - y_i) + h_i(y(N, 0) + y_i - R) = U_i^C$$

for any function $v_i$ being linear or concave. Hence, nobody in the economy chooses to contribute directly the public good, since playing the lottery and donating to the prize pool are strictly preferred to direct contributions. QED

But how then do we explain direct donations to the public good or to the charity? We address this question in the next chapter, where we try to find explanations for an individual’s choosing to contribute, even though it is not reasonable in this setup.

Recall that we started our analysis of an economy consisting of risk-neutral and risk-averse agents by allowing them to decide only whether or not to play the lottery. Originally, we wanted to analyze the effect of adding risk-averse individuals to the economy. Let’s assume for simplicity that these risk-averse individuals have risk-neutral twins. As we have shown, risk-averse individuals are less inclined to play the lottery. Especially, if $n$ and $R$ are large, virtually no risk-averse individual plays the lottery. Hence, they would either contribute nothing, donate to the prize pool, or give directly to the public good. The question we are interested in is the following: do the risk-neutral individuals prefer to have additional risk-neutral individuals or
would they prefer to have additional risk-averse individuals in the economy? Unfortunately, we cannot make a general statement about what kind of people risk-neutral individuals would prefer. The reason is that we cannot even state a proposition about how risk-averse individuals behave. We know that, in general, not all risk-averse individuals would play the lottery, but they could also donate to the prize pool or contribute to the public good. However, conditional on playing the lottery, we can say that risk-neutral individuals would prefer to have individuals who give voluntarily to the public good or donate to the prize pool over individuals who play the lottery.

**Theorem 3:** Risk-neutral individuals with quasilinear preferences who play the lottery prefer to have individuals in the economy who give to the public good or donate to the prize pool rather than to have people who play the lottery.

**Proof:** Let individual \( i \in \{1, \ldots, n\} \) play the lottery and have quasilinear preferences defined by

\[
EU_i = w_i - y_i + \frac{y_i}{y(N)} R + h_i(y(N) - R).
\]

Let individual \( j \) choose only between the pure strategies. Hence, he has the opportunity to play the three strategies as stated in the hypothesis.

(1) If individual \( j \) plays the lottery, individual \( i \)'s expected utility becomes:

\[
EU_i^1 = w_i - y_i^1 + \frac{y_i^1}{y(N) + y_j} R + h_i(y(N) + y_j - R).
\]

(2) If individual \( j \) gives to the public good directly, individual \( i \)'s expected utility becomes:

\[
EU_i^2 = w_i - y_i^2 + \frac{y_i^2}{y(N)} R + h_i(y(N) + g_j - R).
\]
(3) If individual \( j \) increases the prize, individual \( i \)'s expected utility becomes:

\[
EU_i^3 = w_i - y_i^3 + \frac{y_i^3}{y(N, D_j)}\hat{R} + h_i(y(N, D_j) + D_j - \hat{R})
\]

Trivially, (2) is preferred to (1) by bettor \( i \) if \( g_j = y_j \), which we assume for comparison.

Hence, we are left to show that (3) is also preferred to (1). Let \( y(N) = y(N, 0) \) and \( y(N, D_j) \) be equilibrium total wagers for cases (3) and (1). Again, assume for comparison that \( y_j = D_j \). We already obtained \( y(N, D_j) > y(N, 0) + D_j \). By assuming \( y_j = D_j \), we get:

\[
y_j < y(N, D_j) - y(N, 0).
\]

To get equality, we define \( \epsilon \) such that it satisfies \( y_j + \epsilon = y(N, D_j) - y(N, 0) \), implying \( \epsilon > 0 \). Consider the following cases:

(a) \( y_i^l \geq \epsilon \), then \( EU_i^3 \) evaluated at \( y_i^3 = y_i^l - \epsilon \) exceeds \( EU_i^3 \), since

\[
EU_i^3 = w_i - y_i^3 + \frac{y_i^3}{y(N) + y_j + \epsilon}(R + y_j) + h_i(y(N) + y_j + \epsilon - R)
\]

\[
= w_i - y_i^l + \epsilon + \frac{y_i^l - \epsilon}{y(N) + y_j} (R + y_j) + h_i(y(N) + y_j - R)
\]

\[
= w_i - y_i^l + \frac{y_i^l}{y(N) + y_j} R + h_i(y(N) + y_j - R) + \epsilon(1 - \frac{R}{y(N) + y_j}) + y_j(\frac{y_i^l - \epsilon}{y(N) + y_j})
\]

\[
> EU_i
\]

The inequality holds, since \( y(N) + y_j > R \) (see Lemma 2) and \( y_i \geq \epsilon \) by assumption.

(b) \( y_i^l \in [0, \epsilon) \), then \( EU_i^3 \) evaluated at \( y_i^3 = 0 \) exceeds \( EU_i^3 \), since

\[
EU_i^3 = w_i - y_i^l (1 - \frac{R}{y(N) + y_j}) + h_i(y(N) + y_j - R)
\]

\[
< w_i + h_i(y(N) + y_j + (\epsilon - y_i^l) - R) = EU_i^3
\]

The inequality holds, since analogous to (a) \( y(N) + y_j > R \) (see Lemma 2) and \( y_i^l < \epsilon \).
The cases above are exhaustive for all possible choices of $y_i^1$. Now, let $y_i^3$ be optimal for individual $i$ in an equilibrium when individual $j$ chooses to participate in the lottery and $y_i^3$ be optimal for individual $i$ in an equilibrium when individual $j$ chooses to donate to the prize pool. We have established that $EU_i^3$ evaluated at

$$\hat{y}_i^3 \equiv \max \{0, \bar{y}_i^1 - \varepsilon\}$$

is strictly greater than $EU_i^1$ evaluated at its maximizer $\bar{y}_i^1$, while, by definition of a maximizer, $EU_i^3$ evaluated at $\bar{y}_i^3$ is at least as great as $EU_i^3$ evaluated at $\hat{y}_i^3$. Thus, a risk-neutral individual who plays the lottery prefers to have individuals in the economy who, instead of playing the lottery, give directly to the public good or donate to the prize pool.

**QED**

Having shown that, in a big charity, virtually no risk-averse individual would play the lottery, we may find that some of these individuals choose to donate to the prize pool or give to the public good directly. If so, then risk-neutral individuals would benefit from having risk-averse individuals in the economy. Can we say the same for risk-averse individuals contributing to the public good or donating to the prize pool? Obviously, people who increase the prize pool need individuals who play the lottery; otherwise, they would not benefit from increasing the prize pool. On the other hand, individuals who contribute voluntarily to the public good prefer to have risk-neutral individuals in the economy, since playing the lottery provides more of the public good than does voluntary contributions [see Morgan (2000), Theorem 1]. Hence, in a mixed economy consisting of risk-neutral and risk-averse individuals, both groups benefit from having the other type of individual in the economy.
Having analyzed the interactions between risk-neutral and risk-averse individuals exhaustively, we now apply ourselves to the question: why do people would give directly to the public good? Theoretically, we have shown that giving to the public good is strictly dominated by the other strategies.
Empirically, even with an existing lottery, there are people who give directly to the charity instead of playing the lottery. Charities often raise funds not just via lotteries, but also by relying on donations. However, theoretically, nobody would contribute to the charity voluntarily in the setup of Chapter 4 if a lottery exists to fund the public good. Lotteries are used more as an additional method for raising funds. Note that we invoked three assumptions in this model:

1. The publicness assumption: individuals are assumed to derive utility from the amount of the public good provided and try to increase the amount of the public good, whether via people playing the lottery, donating to the prize pool, or contributing directly to the public good.

2. The utility maximizing assumption: individuals are assumed to be utility maximizing, hence, the amount of their contributions is determined as the solution to a utility maximization problem.

3. The Nash conjectures assumption: the individuals solve their maximization problem by taking the amount of the public good provided by other individuals as given (through wagers or donations).

These assumptions concerning a charity that offers a lottery are the same that Sudgen (1982) criticizes when considering voluntary contributions to big charities. Recall that Sudgen suggests dropping or altering at least one of these assumptions. He recommends dropping the publicness
assumption. “The most obvious way of doing this is to make the philanthropist’s utility a function of his own gift rather than of the extent of the charitable activity as a whole” (Sudgen 1982). One attempt to do this is provided by Andreoni (1989) with his theory of warm-glow giving. In Andreoni (1989) individuals are assumed to derive utility not just from the public good, but also from the act of giving. Different authors have taken up this model of warm-glow giving; among these are Engers and McManus (2004), who it to charitable auctions. One can think of a rationale for warm-glow giving as valuing the social esteem generated by being known as a generous contributor. Charities often make big donations more visible; for example, concert and theater programs commonly contain lists of sponsors and benefactors. If social esteem was not an issue for some people, the charities would not care about these effects to publicize their donations. Applying this to the lottery model, we use Andreoni’s model of warm-glow giving as an explanation for direct contributions to the public good.

Note that the warm-glow explanation can be also applied to lottery playing, where the warm-glow comes from the chance of winning the prize, instead of moral or social concerns. Hence, the warm-glow giving theory is both a generalization of the lottery and a model for voluntary giving. In Andreoni’s model, the amount of one’s own contribution enters the individual’s utility function. However, since we can think about the warm-glow as reflecting either social esteem or the chance of winning the lottery prize, we shall alter Andreoni’s model in the following way: The utility an individual receives from the act of giving depends not only on the amount given, but also depends on the amount other individuals give. For example, if we think of the warm-glow as deriving from social esteem, the prestige would be higher if one’s contribution is higher, on average, than others’. That is, others would think of one as being generous for contributing more than average, and would think of one as being niggardly for contributing less than average.
Also, when we think of warm-glow deriving from the chance of winning the lottery prize, one’s chance of winning would be higher if others’ wagers were relatively low, and vice versa. Therefore, we extend Andreoni’s model to incorporate an “impure” warm-glow giving model.

Consider an economy with one private good and one public good funded by a charity. For simplicity, we assume individuals have quasilinear preferences that are neutral with respect to income risk. Moreover, perfect information, utility maximization, Nash conjectures and publicness are assumed. Thus, the utility function for each individual \( i \in \{1,\ldots,n\} \) can be written as:

\[
U_i = x_i + h'(G) + \phi^i(g_i, G_{-i})
\]

where, again, \( x_i \) is the amount of the private good consumed and \( G \) is the amount of the public good. \( g_i \) denotes individual \( i \)'s contribution to the public good and \( G_{-i} = \sum_{j \neq i} g_j \) represents the sum of all other individuals’ contributions. The function \( \phi^i(g_i, G_{-i}) \) is the impure warm-glow function, which is assumed to have the following properties:

\[
\begin{align*}
\phi^i_1 &\equiv \frac{\partial \phi^i}{\partial g_i} > 0, \\
\phi^i_{11} &\equiv \frac{\partial^2 \phi^i}{\partial g_i^2} < 0, \\
\phi^i_2 &\equiv \frac{\partial \phi^i}{\partial G_{-i}} \leq 0, \\
\phi^i_{12} &\equiv \frac{\partial^2 \phi^i}{\partial g_i \partial G_{-i}} < 0
\end{align*}
\]

These assumptions capture the ideas discussed above. \( \phi^i(g_i, G_{-i}) \) is monotonic and concave in its first argument but exhibits diminishing marginal utility of \( g_i \) in \( G_{-i} \). Furthermore, \( \phi^i(g_i, G_{-i}) \) is assumed to be decreasing in its second argument. Holding one’s own contribution constant, an increase in other individuals’ contributions decreases the marginal value of the warm-glow.
Note that if we have \( \varphi^i = \frac{g_i}{g_i + G_{-i}} R \), we get exactly the same utility maximization problem as the lottery playing maximization problem in Chapter 4. The impure warm-glow theory can be seen as a generalization of the lottery model. Moreover, the impure warm-glow theory of contributing to the public good provides an explanation for the existence of direct contributions to the public good when a lottery exists to fund a public good. Playing the lottery or donating to the prize pool no longer dominates voluntary contributions. Especially risk-averse individuals will tend to contribute voluntarily if they get a warm-glow from giving, since their benefit from playing the lottery is less than the benefit derived by risk-neutral individuals. From now on, we shall treat the impure warm-glow giving model separately. But since it is very general, it can be applied to the lottery model.

The utility maximization problem for contributing directly to the public good becomes the following for each individual, where \( G_{-i} \) is exogenous:

\[
\max_{g_i} \left\{ U_i = w_i - g_i + h_i'(G_{-i} + g_i) + \varphi_i'(g_i, G_{-i}) \right\},
\]

which implies the following first-order condition for the optimal choice of \( g_i \), denoted as \( g_i^* \), conditional on contributing a positive amount

\[
\frac{\partial U_i}{\partial g_i} = -1 + h_i'(G_{-i} + g_i^*) + \varphi_i'(g_i^*, G_{-i}) = 0.
\]

The condition for individuals to contribute can be derived in a manner that parallels the method used in Chapter 4. Again, let

\[
\Psi_i = \frac{\partial U_i}{\partial g_i} \bigg|_{g_i=0}
\]

be the marginal benefit of changing the contribution from zero to a small amount. Hence, we get:
\[
\Psi_i(G) = -1 + h_i'(G) + \phi_i'(0, G)
\]

Thus, if \( \Psi_i > 0 \) it is profitable for the individual to contribute a positive amount \( g_i > 0 \).

As in Chapter 4, we can establish the existence of an equilibrium (see Lemma 1), since each individual’s strategy set is compact and convex and the payoffs are continuous and quasiconcave. Since \( \Psi_i(G) \) is everywhere decreasing in \( G \), we define \( G_i \) such that \( \Psi_i(G_i) = 0 \).

Next, we can obtain the same result derived in Chapter 4 (see Lemma 3), that any equilibrium generating \( G^* \) of the public good consists of contributions \( g_i > 0 \) for all \( i \) such that \( G_i > G^* \) and zero contributions from all other individuals. The proof is completely analogous.

5.1. Comparing Impure Warm-Glow Giving to Non-Warm-Glow Giving

Intuitively, one would think that warm-glow givers contribute more to the public good than non-warm-glow givers, since they derive utility not only from the public good but also from the amount contributed to it. Andreoni (1989) and Cornes and Sandler (1984) show that this is indeed the case. Can we obtain the same result for impure warm-glow givers? Morgan (2000) has shown that more of the public good is provided when financed by a lottery than when financed through voluntary contributions. Voluntary contributions, in this case, means giving without any kind of warm-glow. Since the lottery model is just a special case of the impure warm-glow model, we have to prove the following:

Theorem 4: Impure warm-glow givers with quasilinear preferences provide more of the public good than non-warm-glow givers, who are otherwise identical.

Proof: Let there be two economies, the first one, denoted with superscript A, consisting of non-warm-glow givers, the second one, denoted with superscript B, consisting of impure
warm-glow givers. The individuals in both economies are identical otherwise. In both economies, the provision of the public good is left to a voluntary contributions game. Note that the individuals in economy A provide a unique equilibrium set of contributions \( \{ g_1^A, ..., g_n^A \} \), since preferences are quasilinear.

Suppose the contrary of the proposition is true. Hence, there exists an equilibrium set of contributions \( \{ g_1^B, ..., g_n^B \} \) in economy B, such that

\[
\sum_{i=1}^{n} g_i^B = G^B \leq G^A = \sum_{i=1}^{n} g_i^A .
\]

For an equilibrium in economy A, the following first-order conditions must hold

\[
-1 + h_i'(G^A) = 0 , \quad \forall g_i^A > 0 .
\]

But then for all \( i \) such that \( g_i^A > 0 \), we have

\[
0 = -1 + h_i'(G^A) \leq -1 + h_i'(G^B) < -1 + h_i'(G^B) + \varphi_i'(g_i^B, G^B) = \frac{\partial U_i^B}{\partial g_i^B} ,
\]

since individuals are identical. Hence, it is profitable for the individuals in the second economy to contribute more. Therefore, \( \{ g_1^B, ..., g_n^B \} \) cannot be an equilibrium set of contributions, contrary to the supposition. It follows that impure warm-glow givers provide more of the public good than non-warm-glow givers. \( QED \)

Impure warm-glow givers provide more of the public good than non-warm-glow givers if preferences are quasilinear. What is left to charities is to find an optimal way to give donors a warm-glow. One way of doing this is to offer lotteries. Another way is to make outstanding donations more visible to the public. The optimal way for each charity depends on the preferences of the individuals and the cost conditions of the charity.
5.2. Neutrality

We next ask whether the government can pursue policies to increase the amount of the public good. Warr (1982) finds that lump sum taxes levied to increase the amount of the public good crowd out voluntary giving dollar for dollar and are, therefore, neutral. Cornes and Sandler (1984), Andreoni (1989), and Morgan (2000) presented different models of charities where neutrality does not hold. Does neutrality also apply for the impure warm-glow model with quasilinear preferences?

Let us change our model by introducing government. Suppose the government imposes lump sum taxes on individuals and uses the revenue to supply a public good. Hence, each individual’s utility function becomes

\[ U_i = w_i - g_i - t_i + h^i(G_{-i} + g_i + T) + \phi^i(g_i, G_{-i}) , \]

where \( t_i \) denotes the tax collected from individual \( i \), and

\[ T = \sum_{i=1}^{n} t_i \]

denotes the total revenue collected from all individuals.

The first-order condition for the optimal choice of contributions to the public good, conditional on making a positive contribution, is

\[ \frac{\partial U_i}{\partial g_i} = -1 + h^i(G^* + T) + \phi^i(g_i^*, G_{-i}) = 0 . \]

Since we want to analyze whether neutrality holds in this model, we consider changes in the taxes imposed by the government. First, note that if the total tax revenue does not change \( (dT = 0) \), the first-order conditions stay the same and hence the utility maximization problem, along with the equilibrium amount of the public good, does not change. Tax policies that simply redistribute income among individuals have no effect on the total provision of the public good, as
long as $t_i \leq w_i - g_i^*$ for all $i$. The last assumption is reasonable, since individuals’ contributions are usually relatively small compared to their total wealth. We conclude that a change in tax policy that does not change the total amount of tax revenue collected is neutral in our model. This is because we assumed quasilinear preferences. Specifying a model with quasilinear preferences is a convenient way to remove income effects. If we have individuals with more general preferences, a redistribution of income would not be neutral. As an example, suppose we assume individuals have risk-averse utility functions, as in Chapter 4. The first-order conditions then become:

$$\frac{\partial U_i}{\partial g_i} = -\nu_i'(w_i - g_i^* - t_i) + h_i'(G^* + T) + \varphi_i'(g_i^*, G_{-i}) = 0$$

Since $t_i$ appears in the first-order conditions, a redistribution of income (e.g. $dt_i = -dt_j$), where the total amount of taxes remains unchanged, would not be neutral in general. However, with our specific assumption on individuals’ preferences, we get neutrality of redistributions of income.

What will happen if the tax revenue changes ($dT \neq 0$)? In the case of lottery playing with quasilinear preferences, Morgan (2000) finds that outside donations do not crowd out lottery wagers completely. Hence, an outside donation would increase the equilibrium amount of the public good and is, therefore, not neutral. Since we remove the income effect by assuming quasilinear preferences, a change in the tax revenue has the same effect as a change in outside donations. In the special case of the lottery warm-glow, neutrality does not hold. However, the analysis of a more general model of impure warm-glow is more complex. Unlike Morgan’s analysis, we cannot sum over the first-order conditions to remove the variable $G_{-i}$, which is treated exogenously for each individual but is obviously affected by a change in $T$. 
Thus, we proceed by totally differentiating the first-order conditions for each individual

\[ h_{i}^{\prime}(G_{-i}^{*} + g_{i}^{*} + T)(dg_{i}^{*} + dT) + \phi_{11}^{i}(g_{i}^{*}, G_{-i}^{*})dg_{i}^{*} + \phi_{12}^{i}(g_{i}^{*}, G_{-i}^{*})dG_{-i}^{*} = 0 \]

and rewrite this equation by using the variable \( G - G_{-i} \) in place of \( g_{i} \). We get the modified first-order condition:

\[ h_{i}^{\prime}(G^{*} + T)(dG^{*} + dT) + \phi_{11}^{i}(G^{*} - G_{-i}^{*}, G_{-i}^{*})dG^{*} + \phi_{12}^{i}(G^{*} - G_{-i}^{*}, G_{-i}^{*})dG_{-i}^{*} = 0. \]

Rearranging this equation yields

\[ \frac{dG^{*}}{dT} = \frac{-h_{i}^{\prime}(G^{*} + T) - \frac{dG_{-i}^{*}}{dT}(\phi_{11}^{i}(G^{*} - G_{-i}^{*}, G_{-i}^{*}) - \phi_{12}^{i}(G^{*} - G_{-i}^{*}, G_{-i}^{*}))}{h_{i}^{\prime}(G^{*} + T) + \phi_{11}^{i}(G^{*} - G_{-i}^{*}, G_{-i}^{*})}. \]

By inspecting the equation, we can’t make a definite statement about the sign of \( \frac{dG^{*}}{dT} \), since it contains the variable \( \frac{dG_{-i}^{*}}{dT} \). Intuitively \( \frac{dG_{-i}^{*}}{dT} \) should be close to 1, implying that \( \frac{dG^{*}}{dT} \geq -1 \), since the numerator would be smaller than the denominator. However, we have to examine it mathematically. From this expression, we can state the following theorem:

**Theorem 5:** In the model of impure warm-glow giving with quasilinear preferences, an increase in tax financing of the public good increases its supply.

**Proof:** Consider the above expression for \( \frac{dG^{*}}{dT} \). First, note that \( 0 > \frac{dG_{-i}^{*}}{dT} \geq -1 \) implies, for all individuals \( i \) who contribute a positive amount,
\[
\frac{dG^*}{dT} = -\frac{h'_{i1}(G^* + T) - \frac{dG^*_i}{dT}(\varphi'_{i1}(G^* - G^*_{-i}, G^*_i) - \varphi'_{i2}(G^* - G^*_{-i}, G^*_i))}{h'_{i1}(G^* + T) + \varphi'_{i1}(G^* - G^*_{-i}, G^*_i)} \\
\geq \frac{-h'_{i1}(G^* + T) + (-1)\varphi'_{i1}(G^* - G^*_{-i}, G^*_i) - \frac{dG^*_i}{dT}\varphi'_{i2}(G^* - G^*_{-i}, G^*_i))}{h'_{i1}(G^* + T) + \varphi'_{i1}(G^* - G^*_{-i}, G^*_i)} \\
= -1 - \frac{\varphi'_{i2}(G^* - G^*_{-i}, G^*_i) - \frac{dG^*_i}{dT}}{h'_{i1}(G^* + T) + \varphi'_{i1}(G^* - G^*_{-i}, G^*_i)} \frac{dG^*_i}{dT} \\
> -1
\]
assuming \(\frac{dG^*_i}{dT} < 0\). In this case we get the desired result, since an increase in \(T\) decreases contributions less than dollar for dollar. Note that if, for only one individual \(i\) who contributes positively \(0 > \frac{dG^*_i}{dT} \geq -1\) holds, we get the desired result. On the other hand, if for all \(i\) \(\frac{dG^*_i}{dT} > 0\), which is arguably not reasonable, then \(\frac{dG^*_i}{dT} \geq 0 \geq -1\). Hence, this case also yields the proposition.

Thus, we only have to rule out the case that all individuals who contribute positively are faced with \(\frac{dG^*_i}{dT} < -1\). Suppose the contrary is true, so that there exists an equilibrium set of contributions \(\{g^*_1, ..., g^*_n\}\) such that \(\frac{dG^*_i}{dT} < -1\) for all individuals who contribute positively. Recall the first-order conditions for these individuals must hold:

\[
\frac{\partial U_i}{\partial g_i} = -1 + h'_{i1}(g^*_i + G^*_i + T) + \varphi'_{i1}(g^*_i, G^*_i) = 0 .
\]

Consider an increase in \(T\). Totally differentiating the first-order condition yields:

\[
0 = h'_{i1}(G^*_i + g^*_i + T)(dG^*_i + dg^*_i + dT) + \varphi'_{i1}(g^*_i, G^*_i)dg^*_i + \varphi'_{i2}(g^*_i, G^*_i)dG^*_i .
\]
This is equivalent to the following expression:

\[
0 = h'(G^*_i + g^*_i + T)(1 + \frac{dG_i}{dT}) + \phi_l(G^*_i, G^*_j) \frac{dG_i}{dT} + \left[ h'_1(G^*_i + g^*_i + T) + \phi'_1(G^*_i, G^*_j) \right] \frac{dG_i}{dT},
\]

where the first two addends are positive, since \(\frac{dG_i}{dT} < -1\). In order for the equation to hold,

\[
\frac{dg^*_i}{dT}
\]

must be greater than zero for all \(i\) who contribute positively. Hence, all individuals who are faced with \(\frac{dG_i}{dT} < -1\) give more than before, implying that \(\frac{dG_i}{dT} < -1 \Rightarrow \frac{dg^*_i}{dT} > 0\).

Since \(\frac{dG_i}{dT} < -1\) for all \(i\), but at the same time \(\frac{dg^*_i}{dT} > 0\) for all \(i\), a set of contributions \(\{g^*_1, \ldots, g^*_n\}\) such that \(\frac{dG_i}{dT} < -1 \ \forall i: g^*_i > 0\) cannot constitute an equilibrium. As a matter of fact, it can analogously be shown that \(\frac{dG_i}{dT} > 0\) for all \(i\) who contribute positively cannot be an equilibrium either. The first two addends become negative, implying that

\[
\frac{dg^*_i}{dT} < 0 \text{ for all } i \text{ who contribute positively, contradicting } \frac{dG_i}{dT} > 0 \text{ for all } i. \text{ Hence for any equilibrium } G^* \text{ an increase in } T \text{ provides more of the public good, since } \frac{dG^*}{dT} > -1. \ QED.
\]

Analogous to the warm-glow model of Andreoni (1989) and to the lottery model of Morgan (2000), we also find that neutrality does not hold. Government can increase the amount of the public good by collecting taxes from individuals and use the revenue to supply a public good. Hence, we get results similar to the warm-glow model of Andreoni. Since we assumed
quasilinear preferences, we get independence of the public good provision from the distribution of income. Hence, this model is capable of explaining the empirical regularities that the non-warm-glow giving model fails to explain, namely that crowding out is incomplete. Moreover, it provides a better explanation of giving than the warm-glow model by Andreoni since, arguably, the warm-glow also depends on others’ contributions.
The purpose of this thesis was to investigate the effect of adding risk-averse people to an economy with risk-neutral individuals who play a lottery to fund a public good. We were able to show that risk-averse people are less inclined to play the lottery; the more risk-averse an individual is, the less inclined he is to play the lottery. In particular, when the number of people in the economy and the prize are both large, we show that virtually no risk-averse individuals play the lottery. However, we introduced two other possible strategies, which individuals can choose: increasing the prize pool and giving to the public good directly. We found that lottery players prefer to have donors in the economy and vice versa. We concluded that both types of individuals (risk-neutral and risk-averse) benefit from having the other type in the economy, since risk-averse people are less inclined to play the lottery and, hence, are more inclined to donate to the public good. Furthermore, we saw that giving directly to the public good is strictly dominated by the other two strategies. However, we know empirically that, even with an existing lottery, voluntary giving is not crowded out completely.

Hence, we tried to find an explanation for this behavior. We used Andreoni’s model of warm-glow giving and altered it to a model of impure warm-glow. The amount of utility gain from giving depends not only on the amount of one’s own contribution but also on others’ contributions. The marginal utility gain is decreasing in others’ contributions to the public good, holding constant the total amount of the public good provided.
Moreover, we pointed out that the impure warm-glow giving model is not only an explanation for the existence of voluntary contributions to a charity that offers a lottery to fund a public good, but also provides a generalization of the “lottery warm-glow”. The utility gain from giving can also be interpreted as increasing the chance of winning the prize in a lottery. We have shown that people who give because of an impure warm-glow provide more of the public good than people who do not get a warm-glow. Furthermore, we showed that neutrality does not hold in this model even though we assumed quasilinear preferences. A government can increase the provision of the public good by collecting lump sum taxes and using the revenue to finance the public good. This theory of impure warm-glow appears to be a better explanation for charitable giving. No matter what the interpretation of warm-glow may be, social esteem, satisfying moral constraints or playing a lottery, the warm-glow depends on other individuals’ contributions.
References


