MATHEMATICS TEACHERS’ USE OF MATHEMATICAL KNOWLEDGE IN DESIGNING LESSONS ON SYSTEMS OF LINEAR EQUATIONS

by

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(Under the Direction of Denise A. Spangler)

ABSTRACT

The purpose of this study was to explore what teachers’ knowledge is used when they design instructions and plan lessons on a unit of systems of linear equations. All the participating teachers had taught mathematics for at least 4 years, including a year of teaching algebra. Participants were recruited from public schools in the Southeastern United States. By using a sequencing activity and a semi-structured interview, I examined what subdomains of pedagogical content knowledge (PCK) were used and what knowledge was used within each subdomain. All four participant teachers considered all the subdomains of PCK when they designed lessons on a unit of systems of linear equations. They also considered other types of knowledge, such as level of students and time frame, that could not be categorized into the subdomains. All four participant teachers started with a graphing method for teaching systems of linear equations, but only one teacher proposed to teach elimination prior to substitution. The results of this study allow teacher educators to proceed from a more informed use of teachers’ knowledge in designing and facilitating teacher education programs and professional development.
INDEX WORDS: Inservice teachers, Secondary mathematics, Algebra, System of linear equations, Teachers’ knowledge, Pedagogical content knowledge, Lesson planning, Teacher education
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CHAPTER 1

INTRODUCTION

Nearly all high schools in the United States offer at least one formal or college-preparatory mathematics course, such as Algebra I, II, or Geometry (Smith, 2013). Among those courses, Algebra I is the most commonly offered. However, this course became much more difficult under the Common Core State Standards Initiative (2010). The New York State Report Card reported that only 63% of students who took the Algebra I test passed the first common-core-aligned exam in 2015; compared with the 72% pass rate in the previous year, this was a dramatic drop (Gewertz, 2015). Likewise, though Algebra I is a core subject in the high school curriculum, many students struggle with it.

Among many topics in Algebra I, systems of linear equations are considered particularly difficult because of the new concepts on different types of solutions and methods used for solving such systems. Typically, three types of solutions for systems of linear equations are introduced in mathematics textbooks: one solution, no solution, and infinitely many solutions. For solving the systems of linear equations, three different solving methods are presented: graphing, substitution, and elimination. As there are so many new concepts introduced in this unit, algebra teachers often teach systems of linear equations procedurally rather than conceptually (Afamasaga-Fuata‘i, 2006). As a result, the topic requires more teaching skills and strategies than most other topics in Algebra I.

Systems of linear equations is a fundamental concept for advanced mathematics, such as linear algebra. As it is a key concept for these later subjects, the mathematics teacher’s role in
presenting and delivering this concept is critical. Just as textbooks use different strategies to introduce the concept of system of linear equations, teachers have their own teaching strategies to introduce the concept. In other words, each teacher develops his or her own order of tasks and examples based on his or her experience and knowledge to maximize students’ understanding of systems of linear equations.

According to Shulman (1986), teachers’ knowledge plays a critical role when they teach and plan lessons. In 2008, Ball and her colleagues modified Shulman’s (1986, 1987) notion of teacher’s knowledge based on the problems that arise in teaching mathematics, which they identified as mathematical knowledge for teaching (MKT). They proposed three subdomains within pedagogical content knowledge (PCK)—knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC)—and three within subject matter knowledge—common content knowledge (CCK), specialized content knowledge (SCK), and horizon knowledge (Ball, Thames, & Phelps, 2008). In addition, McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) found that secondary school algebra teachers decompress their knowledge when they design lessons and choose and sequence algebra tasks. Likewise, recent studies on teachers’ knowledge revealed that teachers have and utilize such different types of knowledge in order to teach mathematics effectively (Hill et al., 2008; McCrory et al., 2012; Remillard & Kim, 2017).

**Purpose and Research Question**

Although considerable research studies have been done regarding teachers’ knowledge, there appears to be a lack of certainty with respect to how and which domains of knowledge are decompressed or used by teachers when they plan lessons, especially when they make a choice in sequencing example problems. This lack of certainty further suggests a need to explore their
knowledge that is interactively used in designing instruction. Therefore, for this study I investigated teachers’ pedagogical content knowledge (PCK), specifically related to students, curriculum, and teaching, which is actively used when they design lessons for systems of linear equations. Based on such needs and promise, the following research question was developed:

What PCK of mathematics teachers is used when they design lessons on a unit of systems of linear equations?

**Researcher Perspective**

Although there is an increasing need for teaching and learning algebra in secondary school mathematics, the preparation for teaching algebra has not been studied widely in the mathematics education field (Stein, Kaufman, Sherman, & Hillen, 2011). I anticipated that the knowledge generated from this study would afford new insights and so inform secondary mathematics education practice. More specifically, I expect this study to contribute to the development of teacher preparation programs and broaden our understanding of teachers’ knowledge and lesson design. The next chapter begins with an overview of the context and background that frames the study and the research methods that were followed.
CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to explore what teachers’ pedagogical content knowledge (PCK) is used and how teachers activate that knowledge when they design lessons on a unit of systems of linear equations. To frame this study, I selectively and critically reviewed the following topics: (a) teachers’ knowledge, (b) algebra in secondary mathematics, and (c) systems of linear equations. A review of the literature on these three topics provides an understanding of the context of secondary mathematics education and subdomain categories of PCK.

I made extensive use of online databases, such as Google Scholar, JSTOR, and ERIC. The following keywords were used to locate articles: teachers’ knowledge, pedagogical content knowledge, lesson planning, lesson design, algebra, teaching algebra, systems of linear equations, and sequencing tasks.

**Teachers’ Knowledge**

Shulman (1986) proposed the concept of PCK as a blind spot of research on teacher education and teaching. In the following year, he introduced seven different categories of teachers’ knowledge for effective teaching, and pedagogical content knowledge was one of them (Shulman, 1987). In his study, he described PCK as a “special amalgam of content and pedagogy” (Shulman, 1987, p. 8). Since then, his conceptualization of teacher knowledge has broadly influenced the field of mathematics education (Hill, Sleep, Lewis, & Ball, 2007). However, after Shulman introduced the concept of PCK, a number of studies criticized the concept for different reasons (Depaepe, Verschaffel, & Kelchtermans, 2013). For example, Ball,
Thames, and Phelps (2008) pointed to the lack of a definition and an empirical foundation of PCK. Ball and her colleagues modified Shulman’s (1986, 1987) notion of teacher’s knowledge based on the problems that arise in teaching mathematics. They identified mathematical knowledge for teaching, and they proposed three subdomains within PCK—knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC).

KCS, which is the first subdomain of PCK, is “knowledge that combines knowing about students and knowing about mathematics” (Ball et al., 2008, p. 401). Also, Hill, Ball, and Schilling (2008) defined KCS as “content knowledge intertwined with knowledge of how students think about, know, or learn a particular content” (p. 375). KCS is used when teaching encompasses specific content and particular facts about learners (Ball et al., 2008; Hill et al., 2008). Hill and her colleagues (2008) found that their mathematical items to measure teachers’ KCS fell into four categories—common student errors, students’ understanding of content, student developmental sequences, and common student computational strategies.

The second subdomain, KCT, is knowledge that “combines knowing about teaching and knowing about mathematics” (Ball et al., 2008, p. 401). According to Ball et al., KCT includes mathematical knowledge for designing instruction, such as sequencing mathematical tasks and evaluating instructional representations and procedures. Because lesson planning usually includes such knowledge, KCT would play an important role in lesson planning, even it is not solely for planning lessons.

The last subdomain, KCC, originated from Shulman’s (1987) curriculum knowledge (Ball et al., 2008). Shulman defined curriculum knowledge as “particular grasp of the materials and programs that serve as ‘tools of the trade’ for teachers” (Shulman, 1987, p. 8). Ball and her
colleagues (2008) temporarily placed Shulman’s curriculum knowledge as “knowledge of content and curriculum” within their PCK, but they were not sure about its separation from other subdomains in PCK. I used Depaepe et al.’s (2013) definition of KCC—“knowledge about instructional materials and programs” (p. 14)—in their systematic review of PCK in mathematics education because their KCC definition followed Shulman’s (1987) and Ball et al.’s (2008).

McCrory et al. (2012) proposed three categories of knowledge of algebra for teaching: knowledge of school algebra, knowledge of advanced mathematics, and algebra-for-teaching knowledge. In their study, they found that secondary school algebra teachers decompress their mathematical knowledge when they design lessons and choose and sequence algebra tasks. However, they did not specifically focus on the teachers’ PCK but more on their knowledge of mathematics content.

Recently, Remillard and Kim (2017) proposed a framework—knowledge of curriculum embedded mathematics (KCEM)—for identifying the mathematical knowledge that teachers activate when using curriculum resources. The framework includes four categories: foundational mathematical ideas, representations and connections among these ideas, relative problem complexity, and mathematical learning pathways. Remillard and Kim described how their four knowledge domains fit together with the subdomains within Ball et al.’s (2008) MKT. However, their study was conducted only about curriculum resources, so that there is a need for exploring the teachers’ knowledge used in other instructional stages, such as designing lessons and assessing students.

Panasuk, Stone, and Todd (2002) found that well-organized lesson plans helped not only teachers to efficiently manage their classroom time and but also students to understand better the connections between mathematical concepts and ideas. Lesson planning is an important
instructional decision-making process for effective teaching, but little is known about the components of this process, such as teachers’ planning strategies, priorities, and possible variables.

**Algebra in Secondary Mathematics**

Algebra has been a gatekeeper to advanced mathematics courses and entering technical jobs (Stein, Kaufman, Sherman, & Hillen, 2011). Algebra I and II courses in high school provide fundamental concepts and processes for taking advanced mathematics courses (McCrorry et al., 2012).

According to Smith’s (2013) national survey, almost all high schools in the United States offer Algebra I and Algebra II courses, and furthermore nearly all the students in these schools are able to access these courses. As a result, there has been a dramatic increase in enrollment in algebra courses. However, the number of students who fail in algebra has also increased (Stein et al., 2011). Likewise, though Algebra I is a core and predominant subject in the high school curriculum, many students struggle with it.

**Systems of Linear Equations**

According to Kirvan, Rakes, and Zamora (2015), understanding and solving systems of linear equations is a major theme in secondary mathematics education because it is based on early concepts of functions and the meaning of variables and because it supports future learning of algebra. The Common Core State Standards for Mathematics (CCSSI, 2010) specifically stated the following standards for systems of linear equations:

- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions,
• Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (pp. 65–66)

Among many topics in Algebra I, systems of linear equations are considered particularly difficult because of the complexity of the multi-step solving process and the new concepts regarding different types of solutions. Blume and Heckman (1997) found that most of their eighth and twelfth graders struggled with “solving equations and inequalities other than fairly simple ones” (as cited in McCrory et al., 2012, p. 586).

Typically, most mathematics textbooks introduce several solving methods—graphing, substitution, and elimination—and three types of solutions—one solution, no solution, and infinitely many solutions—in different orders. Some textbooks first deal with one solution system of linear equations with all three different solving methods (e.g., graphing, substitution, and elimination) and later introduce the other types of solutions (i.e., no solution and infinitely many solutions) as special cases. In contrast, other textbooks introduce all types of solutions for each solving method. Just as textbooks use different strategies to introduce the concept of systems of linear equations, teachers also have their own teaching strategies to deliver this concept. In other words, each teacher develops his or her own order of tasks and examples based on his or her experience and knowledge to maximize students’ understanding of systems of linear equations. Therefore, a better understanding of teachers’ PCK when they design instructions for a unit of systems of linear equations is needed for developing students’ mathematical proficiency.

Researchers have made great efforts to conceptualize the domain of teachers’ knowledge for teaching mathematics. However, little is known about the interactions among different types
of teachers’ knowledge, and especially in the setting of designing instructions and planning lessons.

Thus, based on the literature on the topic of teachers’ knowledge, algebra in secondary mathematics, and systems of linear equations, I explored what teachers’ knowledge is activated and how their knowledge is used when they design instructions or lessons for a unit of systems of linear equations.
CHAPTER 3

METHODOLOGY

The purpose of this study was to explore, using a sample of inservice public high school mathematics teachers, the use of their pedagogical content knowledge (PCK) when they plan lessons about systems of linear equations. I believed that a better understanding of these knowledge domains would allow educators to proceed from a more informed use of teachers’ knowledge in designing and facilitating teacher education programs and professional development. In seeking to understand these teachers’ knowledge, the study addressed a research question: What PCK is used by inservice mathematics teachers when they design instruction and plan lessons for a unit of systems of linear equations? This chapter describes the study’s research methodology and includes discussions of the following areas: (a) description of the research participants, (b) methods of data collection, (c) analysis of data, and (d) limitations of the study.

Description of the Research Participants

When selecting the participants of this study, I sought to locate individuals with various years of teaching experience. Thus, I used a network, whereby participants were asked to refer other individuals whom were teaching or had taught Algebra I. The criteria for selection of participants were as follows:

- All participants had taught high school mathematics at least 4 years, and
- All participants had taught Algebra I at least a year.

I decided on a delimiting time frame of 4 years to ensure that the participants had adequate teaching experience in high school mathematics.
All the participant teachers were recruited from public schools in the Southeastern United States. Mike (like all participants, a pseudonym) was the first teacher who was interviewed. He was a mathematics teacher in a public school. He majored in mathematics education for his undergraduate and graduate degrees, and education specialist was the highest degree that he earned. While acquiring his degrees, he took ten mathematics education courses. Mike had 20 years of teaching experience and had taught algebra for 14 years. He was teaching precalculus to tenth, eleventh, and twelfth graders at the time he was interviewed.

The second participant teacher, Carl, majored in mathematics education for his bachelor’s degree and in post-secondary education for his master’s degree. The highest degree that he earned was an education specialist in instructional technology. He took two mathematics education courses during his undergraduate degree program. Carl had taught algebra for 10 years. He was teaching Algebra I and Algebra I with Support to ninth-grade students at the time he was interviewed.

Jan, who was the third participant teacher, majored in child development and early childhood education for her undergraduate degree. She took one mathematics education course and eight early childhood education courses. She had taught algebra for 6 years and was teaching Algebra I with Support and Accelerated Algebra I to ninth graders.

The last participant teacher, David, majored in mathematics and statistics for his bachelor’s degree and in mathematics education for his doctoral degree. He took about ten mathematics education courses while acquiring his degrees. He was fairly new to teaching algebra; he had taught algebra for 2 years. At the time of the interview, he was teaching Algebra II and College Readiness Mathematics to eleventh- and twelfth-grade students.
Methods of Data Collection

Employing multiple methods and triangulation is critical in attempting to obtain an in-depth understanding of how teachers’ PCK is used when they design lessons on a unit of systems of linear equations. In order to add to the trustworthiness of the study, I employed a number of different data-collection methods: survey, activity, and interviews.

Phase I: Survey

The four teachers who agreed to participate were asked to complete a survey before the sequencing activity. The survey included nine questions, which were designed to collect demographic and contextual information, such as classes currently being taught. The survey appears as Appendix A.

Phase II: Activity

As soon as the participants finished the survey, they were asked to do a sequencing activity with 12 mathematics problems involving systems of linear equations. The problems were taken from one of the Algebra I textbooks based on the Common Core State Standards for Mathematics (CCSSI, 2010). The problems were worked examples from the unit on systems of linear equations in the textbook. The problems included two forms of linear equations: slope intercept form and standard form. Also, some of the problems included fractions and decimals as coefficients of the variables, x and y. The participants were asked to put them in the order that they would use in their lessons. This sequencing activity was adopted and modified from McCrory et al.’s (2012) study. The activity was designed to explore teachers’ knowledge and how teachers use it when selecting and sequencing the mathematics problems of systems of linear equations. The activity appears as Appendix B.
Phase III: Interview

Creswell (2007) stated that a major benefit of collecting data through individual, in-depth interviews is that they offer the potential to capture a person’s perspective of an event or experience. Thus, the interview was selected as the primary method for data collection in this research because it has the potential to elicit rich descriptions of how and why teachers use their PCK when planning lessons for systems of linear equations. In this study, the interview was a typical one-on-one semi-structured interview. Five questions (Appendix C) in total were asked of the participants during the interview, which lasted about 30-40 minutes for each participant.

Analysis of Data

The challenge throughout data collection and analysis was to make sense of large amounts of data and identify significant patterns. The formal process of data analysis began by assigning the teachers’ responses to the knowledge domains based on Ball et al. (2008). The responses were color coded, and each color identified the domains and subdomains of PCK. As the process of coding proceeded, new colors were added to capture other knowledge that could not be put in the categories. In addition, peer debriefing was conducted to confirm the trustworthiness of the data.

Limitations of the Study

This study contains certain limiting conditions. A major limitation of this study is that the number of research participants was restricted. There were only four participant teachers in this study, which restrained the diverse environment of planning lessons. Thus, generalizing this study to other mathematics teachers, other schools, and other mathematics topics might be inappropriate. However, this study is a pilot study of understanding mathematics teachers’ PCK
in planning lessons. I believe that this study will guide the future research on understanding of teachers’ PCK in designing lessons.
CHAPTER 4

FINDINGS

The purpose of this qualitative study was to investigate the teachers’ pedagogical content knowledge (PCK) that is used when they plan lessons about systems of linear equations. I believe that this study can allow mathematics educators to proceed from a more informed perspective. It can suggest a knowledge domain that is useful for supporting a teacher education program and professional development as teachers decompress different domains of knowledge when they design a lesson, especially when sequencing mathematical tasks (McCrory et al., 2012). This chapter presents the key findings obtained from four interviews with inservice high school mathematics teachers who had taught mathematics for at least 4 years, including at least a year of teaching algebra. This study was particularly focused on the three subdomains of PCK, as the goal of the study was to understand what PCK is possessed and how it is used by algebra teachers when they design lessons. In this chapter, the findings are presented in terms of the three subdomains of PCK that Ball et al. (2008) suggested. Then, other findings that caught my attention are articulated.

Teachers’ Knowledge Within the Subdomains of PCK

Surprisingly, all four participant teachers considered all three subdomains of PCK proposed by Ball et al. (2008) when they planned lessons for systems of linear equations. In the following section, the teacher responses are presented according to the subdomains of PCK: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC).
Teachers’ Knowledge of Content and Students (KCS)

KCS is “content knowledge intertwined with knowledge of how students think about, know, or learn a particular content” (Hill, Ball, & Schilling, 2008, p.375). KCS is used when teaching encompasses specific content and particular facts about learners (Ball et al., 2008; Hill et al., 2008). Hill and her colleagues found that the mathematical items they used to measure teachers’ KCS fell into four categories—common student errors, students’ understanding of content, student developmental sequences, and common student computational strategies (Hill et al., 2008). In this study, I was able to find similar categories from the teachers’ responses for planning lessons for systems of linear equations.

All the participant teachers in this study used KCS very often when they planned a lesson. Two teachers, David and Carl, considered common student errors when they planned. In the following response, David identified a common error that students make related to the concept of solutions of systems:

So, being able to check the solutions could be a factor once you get to being able to solve it and get the solution, and also being able to put the solutions in the correct order and making sure that your x isn’t any further than your y because that can come into play when you’re dealing with positive and negative numbers. Because some students just assume the lowest number goes first, the higher number goes second. As odd as that may be, they do, a lot of them do a sum of that, without really understanding what they are actually solving for. (David, personal communication, March 30, 2017)

According to this response, students often write solutions of systems of linear equations in incorrect order. For example, some students write the two numbers of the solution in the order of quantity, smaller number first and larger number later, rather than putting the coordinates in the order of x and y. Other students even add the two solution values. Later in his interview, David indicated how such common student errors can be corrected and how that impacted his planning of the unit; he suggested going back to the meaning of the solutions and the coordinates:
It will [impact lessons] and again it goes back to being able to understand what their solution means. That is a coordinated point, and your coordinated points do go in a particular order, which goes back before we even got to graphing lines. (David, personal communication, March 30, 2017)

Carl also mentioned a similar student error in the meaning of solution to systems of linear equations. He added that explaining what an ordered pair solution to a system is could help to reduce common student errors related to the meaning of solution of a system. The following response by Carl illustrates how he managed the errors that students make related to the solutions of systems:

One thing that students struggle with a lot is, what a solution is to an equation. So, we talk about, “what is a solution?” We’ve just come off graphing, and then we can talk about what is a solution to a system of equations in that ordered pair, [which] is a solution to both equations at the same time.” (Carl, personal communication, March 29, 2017)

Because Carl and David acknowledged the common student error regarding the meaning of the solution to a system of linear equation, they included in their planning a question or a time to ask about and revisit the meaning of the solutions and the coordinates. These responses from the teachers imply that they consider students’ common errors in planning lessons on systems of linear equations.

Students’ understanding of content is one of four categories that Hill and her colleagues (2008) found in their study. In the present study, Jan and Mike also considered this kind of knowledge when they planned lessons. Both teachers described students’ surprise when they encounter “strange” answers, where both $x$ and $y$ disappear in the process of solving systems of linear equations by substitution or elimination.

I would sort of allow the special cases to come up, and then they would have to try to figure out what that meant, once they got that strange answer and decide, “Uh oh, what do I do here? Looks like I don’t have $x$ equals something, I have this weird equation at the end. What am I going to do?” And they would have to struggle with that and figure out how to do that. (Jan, personal communication, March 29, 2017)
Jan’s detailed description of the students’ surprise and their thinking process reflects that she acknowledges the students’ understanding of those “strange” answers. Unfortunately, she did not specify how she would help students find the solutions of systems in her response. However, Mike’s response illustrates how he would scaffold his students to understand those “strange” answers:

They do the substitution, and they get something like 2 equals negative 1, and then I'll say, “Well, is that true?” You know, we'll speculate, and then I'll have them, “Well, did we make a mistake?” You know, and catch misconceptions like somebody will see this and they'll think, “Oh, really this should have been \( x \) over here, or I should have subtracted 2.” And we'll get out all of those. But once we finally get down to this, then I'll say, “Well, is that true?” They'll say, “No,” and then I'll say, “Well, what does it mean?”—and if they can get somewhere, I'll say, “Let's go back to this. What are the three possibilities for these lines? One solution, because they intersect; no solution, because they're parallel or infinite solutions, because they are the same line.” And when they see this, and they say, “Oh, it's false.” Then I'll say, “Well, since it's not that one, which one of these two would it be?” “Well, no solutions, because this is not a true equation.” (Mike, personal communication, March 28, 2017)

Mike’s response demonstrated how he would lead the lessons based on students’ answers to his step-by-step questions on the “strange” solutions. This response implies that teachers may prepare a variety of scenarios of lessons based on student’s progress in content understanding.

**Teachers’ Knowledge of Content and Teaching (KCT)**

KCT is mathematical knowledge that is used when designing a lesson, such as sequencing mathematical content and tasks, or evaluating instructional representations, methods, or procedures used in instruction (Ball et al., 2008). Similarly, Remillard and Kim (2017) proposed four dimensions of *knowledge of curriculum embedded mathematics* (KCEM). Among these four dimensions, *relative problem complexity* is revealed to rely on both KCT and KCS from Ball et al.’s (2008) study. All four participant teachers showed their possession of this knowledge. They identified the mathematical problems’ relative complexity and ordered tasks by increasing complexity.
All four teachers distinguished the level of problems. They all suggested starting from the easy examples and gradually increasing the complexity. Mike particularly emphasized starting his lessons with easy examples for each solving method (i.e. graphing, substitution, and elimination) for systems of linear equations.

So, each day when I'm planning my lessons, I'm going to have start with [problems] easy to graph and then more complex, more complex. Start with easy to substitute, then more complex, more complex. Start with easy to eliminate, and they get more complex and more complex. Never been harder. Just we need to start off with something we can do, and then build from there. (Mike, personal communication, March 28, 2017)

Jan also implied that she considered the complexity of the problems. She pointed out that she started with specific examples that did not include other algebraic processes than graphing, substitution, or elimination. In other words, in order to help her students understand the authentic meaning of each solving method, she reduced the steps that could distract them:

I would begin with the ones that are already solved for y and the slope intercept forms, so they don’t have to initially do that [changing the equations to slope intercept form]. So, we would focus on the graphing part itself and the solutions, [...] we would begin with the two where they are already solved for the variables, so you’re just plugging in that value to the other equations. [...] I would probably, with the regular class, begin with the one where you would automatically eliminate, so you’re only left with one variable. (Jan, personal communication, March 29, 2017)

Whereas Jan considered the distracting steps as a standard for identifying complexities in the examples, David reflected on the student's' progress. The following response illustrates how he considered such progress when selecting examples:

I would teach this one first, the \( y = 2x + 2 \) and \( y = 2x - 1 \), to start off with. What I would say is a simpler form of equations. [...] So I guess when I say it’s simple, I really meant that was based on what their strengths were from the prior lesson or wherever I started teaching graphical linear equations. (David, personal communication, March 30, 2017)

The last teacher, Carl, also focused on reducing distracting factors by canceling the variables with opposite coefficients when teaching the elimination method. For example, he
suggested starting with an example of $2x + 5y = 7$ and $6x - 5y = -9$ because the $y$ variable disappears easily when using the elimination method. The following response by Carl illustrates the reason for starting with such example and his teaching strategy developed through his experience:

We would go to simple elimination, simple elimination here, with coefficients. We already have opposite coefficients here. Once we are done with our opposite coefficients, we’ll work into where we have to multiply one of the equations by a constant, determine what will give us opposite coefficients for one of the variables, so that we can add. And at that point we will work up to, possibly multiplying both equations by something to give us opposite coefficients. Now, I prefer to use addition. Get opposite coefficients and just add everything, because I find that, the students have found more success just focusing on adding, rather than making the decision to add or subtract. (Carl, personal communication, March 29, 2017)

All four teachers showed that they consider the complexity of the examples when they select or sequence them. This result reflects that some teachers plan their lessons by starting with simple or easy examples in order to focus more on the characteristics of each solving method and the concept of solutions of systems of linear equations.

**Teachers’ Knowledge of Content and Curriculum (KCC)**

KCC is “knowledge about instructional materials and programs” (Depaepe et al., 2013, p. 14). All four teachers considered their knowledge of content and curriculum. Specifically, three of the teachers, Jan, David, and Mike, mentioned that they usually consider students’ prior knowledge when planning lessons, such as what students have learned in the previous units or courses and what they know about systems of linear equations.

When I asked Jan what factors she considered when she planned lessons for systems of linear equations, she specifically pointed out students’ prior knowledge. She emphasized that knowledge by describing how she would find out what students had learned in earlier curriculum related to the topic:
Certainly, their prior knowledge. So I usually forget what they’ve done in middle school, so I might look that up in the standards and see exactly what they did in eighth grade or contact the eighth-grade teachers and ask them. So, prior knowledge, and then basically it’s just kind of doing this, figuring out what examples I’m going to use, figuring out which methods, in which order, although I do have my order kind of set now. (Jan, personal communication, March 29, 2017)

David also indicated that his example choices would depend a lot on students’ prior knowledge. Unlike Jan, he explained that he considered prior knowledge in order to provide students appropriate levels of problems for effective teaching.

The first one [thing] I’ll have to consider is the prior knowledge. Because the problems that I would present would have to [be] pretty much based on either what they learned before they got to me or the previous unit that I’ve taught. [...] I have to consider the prior knowledge before I decide what kind of equations I want to give them because I’m not going to give them just the basics. I’m not going to give them just the [middle] of the road type of problems. I’m going to give them all Level 5 problems, so to speak. But I have to know what their prior knowledge is, so I’ve got to know where to begin and where I think they should be. (David, personal communication, March 30, 2017)

Both Jan and David listed students’ prior knowledge as an important factor for planning. However, their reasoning was different. David considered the prior knowledge to provide students appropriate levels of problems, while Jan supported figuring out the order of examples and methods.

The third teacher, Mike, mentioned that he would use what students already know and what they learned last year when he planned the unit on systems of linear equations. Though he listed the students’ prior knowledge as a factor that affects his lesson planning, he did not specify the reason for it.

But I am really big on what you [students] knew, what you learned last year, and where you are going to go with this, what we're going to do with this. (Mike, personal communication, March 28, 2017)

In addition to students’ prior knowledge, Jan and Carl pointed out that they would consider visual representations of systems of linear equations. The following responses from Jan
and Carl illustrate why they used a variety of visual representations and how they would implement that decision in their classrooms:

Because I’m a visual learner, and I, since I’m egocentric, I assume that a lot of the world is also visual learners, so I like for them to see the solution before they have to algebraically figure [it] out without the graph. […] I want them to see it, to visually be able to picture it, to say, “Oh, look, they are not meeting, there’s no solution there, because they’re never meeting there,” before they have to do it strictly. (Jan, personal communication, March 29, 2017)

If we have access to technology such as TI-84, calculators, graphing calculators, we can use those graphing calculators to get a different visual representation of the ordered pair. (Carl, personal communication, March 29, 2017)

Jan thought the visual representations of systems would help students see what a solution of systems is and the three different types of solutions. Carl mentioned that he would use graphing calculators to show students a different visual representation of a solution, an ordered pair.

The responses from these participants reveal that they had sufficient knowledge for teaching mathematics in terms of content, students, teaching, and curriculum. Furthermore, all that knowledge appeared to influence their lesson planning. For example, they activated the knowledge of common student errors when they present the concept of solutions of systems of linear equations.

**Teachers’ Knowledge Outside the Subdomains of PCK**

While analyzing the data, I recognized knowledge that I could not categorize into the subdomains of PCK that Ball et al. (2008) suggested. Three of the four teachers stated other types of knowledge, such as time frame, when planning their lessons. In the following section, other types of knowledge that teachers considered are illustrated.

All four participant teachers employed knowledge, such as class size, that was outside the subdomains of PCK when they planned lessons on systems of linear equations. When they were
asked to prioritize that knowledge, three of the four teachers explicitly mentioned the following two factors: level of students and time frame. I was unable to match these factors to the categories of PCK that Ball et al. (2008) proposed.

Carl and Jan placed level of students as a key factor in planning lessons on systems of linear equations. This factor is distinguished from the KCS in that it does not involve the students’ understanding of the content. This knowledge of level of students is a holistic understanding of their mathematical proficiency. The following responses from Carl and Jan present how their knowledge of students’ proficiency affected planning the lessons.

The level of the student and the course, because I’m currently teaching a, an Algebra I with Support class, which is a lower level class. Students need more scaffolding; students need more review of previous skills, whereas [in] an advanced class, we would spend much less time, and we would go straight to the, more theory. We would talk about, “What is the system? Why is it a solution?” And when we could [go] straight to that, an advanced course. We’d really have to go back and build our way up, in [a] lower level course. (Carl, personal communication, March 29, 2017)

Although Carl mentioned only allotting more time for the lower level students, Jan specifically explained how she planned to differentiate the lessons for the advanced students.

The accelerated students, I would have a great time throwing the methods together a little bit with a task, where they sort of have to go through and, you know, kind of discover for themselves or remember from middle school, because they have done it [in] middle school, but sort of discover it themselves how to use those methods and solve it. It would not be nearly this formulaic for the accelerated students, but for the on level and the support [students] it is definitely more intentional, I guess, as far as the examples go. (Jan, personal communication, March 29, 2017)

In addition to the level of students, time frame was considered by Carl and Jan. However, their reasoning was different. Carl referred to the time frame based on students’ progress. As students change every year, the students’ progress and the speed of understanding the content are different. Thus, he stated that he would manage the time that was left for him to introduce the real-world implications of systems of linear equation. In other words, Carl considered the time
frame as a factor for his short-term planning of lessons as he wanted to introduce more applications.

Obviously, the more time we have, the more we can get into applications. The more we can get into real world uses of systems, which is really meaningful to these kids. They may not understand it, and they may not write a system of equations for it. But it is possible that they use a system of equations in their lives, and I think relating that to them and allowing them to see, “Hey, I do use this” or “Hey, that is a system,” I think it engages them a little bit more. So, that’s, that would be something that we would finish up with, as we do have more time. Because it’s very important to have the skill, but it’s also very important to understand how to apply the skill. Then that gives it some context for them. As far as, how much we have, because we have so many standards in Algebra I, and there are many standards that are much more complex. (Carl, personal communication, March 29, 2017)

Jan also considered time frame as a huge factor in planning the unit on systems of linear equations. Unlike Carl, Jan focused on allocating enough time for practicing each method. The following response by Jan illustrates how she thought about the allowance of time for students to practice:

So, I guess the thing with this particular unit is factoring in enough practice time for them. With the on-level [students], I mean they are going to need some straight practice once we’ve done the graphing. Straight practice once we’ve done substitutions. Straight practice once we’ve done eliminations. And then probably a day of practice where they have more overlap and are working on all the methods. So, time would be a huge factor in the planning and what type of examples to use. (Jan, personal communication, March 29, 2017)

The results in this section reveal that the participant teachers activated knowledge that cannot be categorized within the subdomains of PCK, such as level of students and time frame. They also show how that knowledge affected their lesson planning. Even the same knowledge, regarding time, was considered with different reasons: one to introduce real world application of systems of linear equations and the other to provide enough time for practicing each solving method.
Different Sequencing of the Methods for Solving Systems

All four participant teachers started with the graphing method for solving systems of linear equations. While three of the four teachers taught the unit in the same order—graphing, substitution, and elimination—only Carl chose to teach the solving methods in the order of graphing, elimination, and substitution.

Mike gave a reason for starting with graphing when teaching systems of linear equations. He believed that graphing not only connects systems of linear equations with previous units but also helps students in later courses such as Algebra II:

Because typically when I do systems of equations, we have done graphing previous to that. Like usually in an algebra course, we would solve linear equations, and then we graph lines, and then we emphasize that that solution that we found was where we cross the x-axis and then to me that leads up to systems of equations. I mean we could go the other way around, but I think it's easier while graphing is fresh on their mind and for Algebra I students I'm trying to beat into their head how great [the] slope intercept form is, because they're going to use that in Geometry, they're going to use that in Algebra II. and they're going to use that … all along. (Mike, personal communication, March 28, 2017)

Two other teachers, David and Carl, supported teaching graphing prior to other methods as it is directly related to the previous unit, which introduces the graphing of a linear equation and the slope intercept form. The following responses from David and Carl articulate why they wanted to start with graphing:

As far as graphing linear equations was done before this particular lesson. Um, I would like to have that talk if it wasn’t taught. I would like to teach that first before I get into systems. To give them an idea of what the solution would actually mean and to talk about intersection between two lines. (David, personal communication, March 30, 2017)

Because of how the course builds, we have typically just finished with graphing, slope intercept form, point intercept form, standard form of linear functions. We’ve just finished solving equations, multi-step equations. So, that in itself gives me an idea of where we are. (Carl, personal communication, March 29, 2017)
Although those three teachers mentioned only the previous unit, Jan viewed the issue more broadly. She wanted to teach the graphing method first because it connects Algebra I and the mathematics topics in the middle school. She also pointed out that substituting cannot be taught before graphing because it is not covered in eighth-grade mathematics:

*I guess for me the graphing is also a bit of a bridge between the middle school and high school, because they are supposed to come in knowing how to graph lines. It’s still a struggle for some of them, because they forget, or they weren’t quite sure how to do it in middle school. But to me the graphing is kind of a bridge. [...] So, I kind of like to start with the graphing just so it kind of moves them into middle school. I mean from middle school, to high school, if that makes sense.* (Jan, personal communication, March 29, 2017)

All four teachers began with the graphing method because their students had just learned about graphing linear equations in the previous unit or had learned related concepts in middle school. This decision shows that they tended to put topics first that were familiar to students and new topics later in their teaching.

In contrast to Mike, Jan, and David, who chose to teach the methods in the order of graphing, substitution, and elimination, Carl chose to teach elimination prior to substitution. He explained that substitution requires more steps to solve systems than elimination.

*Once we eliminate a variable, it becomes a one-step equation, at which time we can solve for that. The big thing that we run into there is, once we eliminate a variable, some students may struggle with the substitution of that back into one of the equations. Just simple, even if [it is] as simple as addition and subtraction at times, in ninth grade, depending on the level of the student, [substitution] can be complicated. So, there are times when we use a warm up, use a warm up to go in and address substituting in a number to solve or to simplify an equation. [...] Typically, we’ll spend a day or two on elimination and then we’ll go to substitution. Substitution is all based on their ability to solve an equation. So, before we get too deep into the substitution method, we have to review solving multistep equations. Because again, if you don’t have the foundation of being able to solve an equation, when we use the substitution method, and we have to simplify multistep equations, we really struggle with doing that.* (Carl, personal communication, March 29, 2017)
Though Jan also indicated that students struggle the most with substitution, unlike Carl, she taught substitution before elimination. Although she said that the order of substitution and elimination can be interchangeable, Jan supported the sequence she used in terms of students’ preference for solving methods. In other words, if students learned elimination first, they would resist using other solution methods.

These [substitution and elimination] are kind of interchangeable, really. Probably, I would do it because this is everyone’s favorite method, the elimination method, maybe not everyone, but most of the students thought by the end, they preferred the elimination method and so, I don’t want them to find a method that they like best and then want to use that here and not be willing to work for the substitution part. Because to me, the students struggle most with substitution out of these three, no question about it. […] [Also] when they have to do more than one thing, they get discouraged easily. So if they first solve for a variable and then substitute it in and then solve the system, to them that feels like a lot of steps, and they might give up too easily, because they don’t want to put in the time. (Jan, personal communication, March 29, 2017)

This preference for elimination directly relates to students’ motivation. They were not willing to solve systems using other methods, especially substitution. In addition to that, Jan believed putting elimination at the end would be a positive motivation for students because it is a simpler method to use.

The question is, should I do substitution or elimination? And I’m going to do the more difficult one first and save the, the one where they can sort of breathe a little easily, more easily for last. So, it’s just a personal preference. There’s probably some researched way perhaps that we are supposed to be doing it, but I just do it sort of based on how I have experienced things with the students before. (Jan, personal communication, March 29, 2017)

Although Algebra I textbooks handled a unit on systems of linear equations at least two different ways as described in the previous chapters, all four participant teachers agreed on teaching all types of systems of linear equations before they finished with the graphing method. Carl and Mike articulated the reason for discussing all types of systems on the same day when graphing was taught.
It’s also important at that point [end of teaching graphing] to discuss the different types. We have coincident lines, where there’s an infinite number of solutions. Parallel lines, do not share any common points and those are important things that also gives us a good understanding of systems of equations and the different types of solutions that we can get from systems of equations. (Carl, personal communication, March 29, 2017)

Mike insisted on showing students the three possible relations of the two linear equations on the coordinate plane. The following response by Mike illustrates why he wanted to deal with all different types of systems while teaching graphing as a solving method for systems of linear equations.

So, I probably do intersections first, but on that same day of graphing do the parallel ones and do those the last—do the one that's the same and then I mix that order up….And then I just give them a list, and we talk about how we could determine just by looking at the equations whether they're going to intersect, are parallel, or they're the same. And then from there, that's probably when I go after that. That [would] be a couple of days at least. (Mike, personal communication, March 28, 2017)

Chapter Summary

This chapter reported three findings from this study. Through the sequencing activity and one-on-one semi-structured interview, I investigated the research question: What PCK do mathematics teachers use when they design lessons on a unit of systems of linear equations?

The primary finding of this study was that all four participant teachers considered all the subdomains of PCK proposed by Ball et al. (2008) when they designed lessons on systems of linear equations. However, there was some knowledge that could not be categorized into the subdomains, such as the level of students and the time frame. Finally, in the sequencing activity, all four participant teachers started with the graphing method. Furthermore, while three of the four teachers agreed to teach the remaining methods of solving systems in the order of graphing, substitution, and elimination, one teacher chose to teach the methods in the order of graphing, elimination, and substitution.
CHAPTER 5

CONCLUSIONS

The purpose of this study was to explore mathematics teachers’ use of their pedagogical content knowledge (PCK) when they plan lessons on systems of linear equations. By using a sequencing activity and a semi-structured one-on-one interview, I collected data from four public high school mathematics teachers and analyzed it with a framework that Ball, Thames, and Phelps (2008) proposed: *mathematical knowledge for teaching*.

In the previous chapter, the following three findings were discussed: (a) all four participant teachers considered all the subdomains of PCK proposed by Ball et al. (2008) when they designed lessons on a unit of systems of linear equations; (b) they also considered other types of knowledge, such as the level of students and the time frame, which could not be categorized into the subdomains; and (c) all four participant teachers started with the graphing method for teaching systems of linear equations because their students had just learned how to graph linear equations on the coordinate plane, which increased their familiarity with graphing. Only one teacher proposed to teach the solving methods in the order of graphing, elimination, and substitution, while the other teachers chose to teach the remaining solving methods in the order of graphing, substitution, and elimination.

Based on these findings, I concluded that various types of teachers’ knowledge are interactively used when teachers design their lessons on a unit of systems of linear equations. However, this knowledge was not limited to the subdomains of the PCK that Ball et al. (2008) suggested. Teachers also consider the teaching environments and the students’ motivations. In
addition, because of the overlap of the characteristics and the features of the subdomains, I could not determine which specific knowledge influences teachers’ lesson-planning process most or least.

**Implications**

The study can allow educators to proceed from a more informed use of teachers’ knowledge in designing and facilitating teacher education programs and professional development. Because the study identified some of the knowledge and the factors that inservice teachers use when they plan lessons, such knowledge could be reflected in teacher education programs. For example, because the study revealed that inservice teachers consider the relative complexity of the problems they might use when planning lessons for systems of linear equations, the teacher education program can develop tasks that allow prospective teachers to practice implementing this knowledge when writing lesson plans. Providing such opportunities to prospective teachers during the program can be an authentic experience for them. Eventually, this experience could contribute to smoothing teachers’ transition from teacher education programs to mathematics classrooms by reducing some of the difficulties that might come from a lack of experience in classrooms.

Understanding teachers’ knowledge used when designing lessons could also contribute to professional development by suggesting ideas for instruction. In this view, professional development is expected to function in a learning community in which teachers learn what knowledge other teachers implement while planning lessons and why they consider it. Teachers can share their instructional ideas or knowledge about content, students, teaching, and curriculum during department meetings or team meetings.
Future Research

This study assumed that teachers teach traditional on-level Algebra I classes. Because all the participant teachers preferred a teacher-centered classroom, a typical lecture-based classroom, there was little variance in their lesson planning or styles of teaching. Thus, I would like to further the study by recruiting diverse teachers who teach systems of linear equations in different instructional settings, such as a student-centered classroom or a flipped classroom (Kirvan et al., 2015).

Teachers’ understanding of students’ diversity is known to be a critical factor in teaching. However, because the focus of the present study was not on diversity, I did not refer to this issue. Perhaps as a result, none of the participant teachers mentioned the students’ diversity, such as language diversity, when planning lessons. Thus, future research might be conducted under the assumption of the presence of English Language Learners or English as Second Language students in the algebra classes. More specifically, the research can further explore the knowledge teachers use when they design lessons for linguistically heterogeneous classes and how it differs from linguistically homogeneous classes.

In order to generalize the understanding of mathematics teachers’ knowledge used in planning, we should also take a look at their knowledge in other topics in algebra or other mathematics subjects, such as geometry and precalculus. I believe that this would bring more general understanding of what teachers’ knowledge is activated and how they use their knowledge when they design lessons for teaching mathematics.
REFERENCES


APPENDICES

Appendix A. Demographic Survey

All of your responses are strictly confidential. Please answer the following questions and check the one that best describes you.

1. What is your gender? Female _____ Male _____

2. What is your ethnicity?
   Caucasian _____
   African-American _____
   Hispanic _____
   Asian _____
   Other (Please specify) ________________

3. What is your highest degree earned?
   BA/BS _____ MS/MA _____ Specialist _____ Doctorate _____

4. What was your major in your undergraduate and graduate studies (if applicable)?
   (Please specify for each degree)
   Undergraduate ________________ Graduate______________

5. How many mathematics education courses did you take? _______

6. How did you get your teaching certificate?
   ___________________________________________________

7. How many years have you taught Algebra 1? _______ years

8. What grade are you teaching now?
   Grade 9 _____ Grade 10 _____ Grade 11 _____ Grade 12 _____

9. What mathematics class are you currently teaching?
   ___________________________________________________
Appendix B. Sequencing Activity

The following examples are from Algebra I textbooks that you can commonly see in public high schools. Select and place these examples in order that you would teach for solving systems of linear equations for the average level of high school students. When you are done with ordering, we will talk about why you put them in such an order. You may take as much time as you need.

<table>
<thead>
<tr>
<th>Example A.</th>
<th>Example B.</th>
<th>Example C.</th>
<th>Example D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x + 2$</td>
<td>$2y - x = 2$</td>
<td>$y = 2x + 2$</td>
<td>$y = 3x$</td>
</tr>
<tr>
<td>$y = 3x - 2$</td>
<td>$y = \frac{1}{2}x + 1$</td>
<td>$y = 2x - 1$</td>
<td>$x + y = -32$</td>
</tr>
<tr>
<td>Example E.</td>
<td>Example F.</td>
<td>Example G.</td>
<td>Example H.</td>
</tr>
<tr>
<td>$3y + 4x = 14$</td>
<td>$x = -2y + 4$</td>
<td>$y = 3x - 11$</td>
<td>$2x + 5y = 17$</td>
</tr>
<tr>
<td>$-2x + y = -3$</td>
<td>$3.5x + 7y = 14$</td>
<td>$y - 3x = -13$</td>
<td>$6x - 5y = -9$</td>
</tr>
<tr>
<td>Example I.</td>
<td>Example J.</td>
<td>Example K.</td>
<td>Example L.</td>
</tr>
<tr>
<td>$-2x + 15y = -32$</td>
<td>$3x + 2y = 1$</td>
<td>$2x + 6y = 18$</td>
<td>$9x + 8y = 15$</td>
</tr>
<tr>
<td>$7x - 5y = 17$</td>
<td>$4x + 3y = -2$</td>
<td>$x + 3y = 9$</td>
<td>$9x + 8y = 30$</td>
</tr>
</tbody>
</table>

Appendix C. Interview Questions

The interview will assume that you are teaching a general Algebra I class (average level of students).

1. Can you explain why you placed them in such an order?
2. What factors do you consider when you plan lessons on systems of linear equations?
3. How do those factors affect your lesson planning?
4. Among the factors that you just mentioned, which one do you consider the most when you plan lessons on systems of linear equations? What is the reason for it?
5. Are there other factors that you might want to consider but could not have included in designing lessons on systems of linear equations?