TEST ERROR ANALYSIS IN MATHEMATICS EDUCATION:

A MIXED METHOD STUDY

by

LOIS KATHLEEN HOLLISTER DAYMUDE

(Under the Direction of John Olive)

ABSTRACT

Students' low test scores in mathematics can be disappointing for students, teachers, and parents. Data from analyses of errors on mathematics tests have the potential to inform students, teachers, and parents about improving the processes of teaching and learning mathematics. Maximizing performance on tests enhances students' academic success and opportunities. This study addressed high school students' and their teacher's analysis of errors on mathematics tests using a theoretical perspective of pragmatism. I coached 43 ninth-grade students in the fall of 2009 in the use of a tool to aid student metacognition, student test performance, and student learning through more informed teaching. This mixed methods study used qualitative and quantitative methods to answer these questions:

1. What effects does the use of the test-error analysis tool have on students' mathematics test-related behavior (i.e., preparing for and taking tests) and outcomes (e.g., errors made, points lost, test scores)?
2. What benefits and drawbacks do my students, their parents, and I perceive from the use of the test error analysis tool with mathematics tests? In particular, what do the students and I learn from the analysis?

3. What are the most common types of errors, according to the analysis, in a mathematics course? How does this information inform the students and me to promote the learning of mathematics?
4. What groupings, patterns, and trends can be observed from test error analysis data? For example, do the frequencies of some error types decrease? If latent groups are identified, what are their characteristics and what are the probabilities that students move from one group to another?

The study used Excel, MPlus, Fathom, and Minitab for quantitative analysis and coding for qualitative analysis, integrating the results for conclusions. The most common error types were the following: *not knowing how*, *knowing how but forgetting*, *making arithmetic errors*, and *running out of time*. Testing process errors tended to improve; mathematical content errors worsened slightly as the content got more difficult over the semester. Students cumulative test scores were better than their unit test scores, indicating a possible benefit of the test error analysis process. Students whose grades were in the middle of the class tended to benefit more from the analyses than struggling or excelling students. Information for the parents of struggling students and for the teacher for future instruction and assessment was very helpful.

INDEX WORDS: Test error analysis, Student analysis, Assessment, Algebra, Error types, Improving instruction, Parent communication, Metacognition, Mixed methods, Mathematics education, Secondary, High school, Latent class analysis, MPlus, Fathom, Minitab

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DEDICATION

This dissertation is dedicated to the love of my life, Tom Daymude, to our inspiring children Barbie and Chip, and to my parents: Barbara Packard, who excelled as my first teacher; Lloyd Hollister, Jr., who always wanted me to be a doctor, and Herman Packard, who helped me start my college education.

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I owe the very existence of this study to my students who thoughtfully analyzed, documented, and summarized their test errors so that we could learn from their efforts. I thank you all!

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BACKGROUND AND RATIONALE

For most students, testing is an integral part of school, whether it be standardized tests or subject tests, government-mandated tests, or a teacher's course-specific tests. Many people are blocked from professional and personal opportunities because they fear or perform poorly on mathematics tests (Tobias, 1993). With the national emphasis on education and related accountability, there has been an enhanced focus on student testing. "In order to provide a quality education for every child in America, we must first test them to find out which children are not learning at the level or pace necessary to keep up" (U.S. Department of Education [DOE], 2004, para. 13).

Experience has convinced me that students' low mathematics test scores can be disappointing for students, teachers, and parents. Parents seem particularly interested in improving students' mathematics testing abilities because they understand the impact test averages have on overall averages and the impact that related standardized tests have on students' opportunities.

In the mathematics classroom, any student's test score that is less than perfect indicates that the student's performance has room for improvement. When graded tests are returned, some students informally analyze what happened, inquire about errors made, and make mental adjustments to do better the next time. Other students mentally or physically discard the test, trying to minimize negative feelings. I wanted to see what my students and I could learn from their errors on mathematics tests. I believe that "awareness of one's intellectual behavior is a prerequisite for working to change it" (Schoenfeld, 1987, p. 199). I wanted to serve as a cognitive coach to help my students "recognize weaknesses and make improvements" (Maher, Davis, & Alston, 1992, p.261).

Metacognition is "awareness and monitoring of one's own cognitive state or condition; knowledge about one's own cognitive processes and memory system" (Ashcraft, 1994, p. 676). I believe that "metacognitive knowledge has the capacity to play a critical role in children's academic achievement" (Carr, Alexander, & Folds-Bennett, 1994, p. 583). I wondered if improving metacognitive skills in relation to students' learning of mathematics could help them improve their abilities to prepare for and take mathematics tests. I wanted to use concrete data from my students' errors to inform the students in their test preparation and test-taking skills while also informing me in developing my teaching strategies (NCTM, 1995).

Teachers realize that testing can take time away from learning, but there is a trend toward learning during and from testing and other forms of assessment (e.g., Balanced Assessment in Mathematics Project, 1995; Pandey, 1990; Stenmark, 1991). One of the National Council of Teachers of Mathematics' (NCTM, 2000) six principles was that "assessment should support the learning of mathematics and *should furnish useful information to both teachers and students* [italics added]" (p. 11). I explored the use of a test error analysis tool to accelerate harvesting, processing, and sharing that information. NCTM also advises that:

> The most effective assessment of all is that of one's own learning. One of the most valuable lifelong skills students can acquire is the ability to look back and reflect on what they have done and what they still need to do. Students who develop a habit of self-assessment will also develop their potential for continued learning. (Stenmark, J., 1991, p.6)

I wanted students to assess their mathematical content and testing error types and learn from those errors as well as learning a process that could be applied to other types of learning.

The McAcy (1993) process that spawned this study was intended to analyze only careless mistakes, not failure to prepare or to understand a topic. I thought that expanding her tool (see Appendix A) to show gaps in knowledge and retrieval weaknesses might help students prepare for future tests (including the final exam and end-of-course test) as well as perform better in subsequent testing situations. I wanted to see how cumulative results from the process could inform my planning and instruction.

The book *Study Skills That Stick* (Nuzum, 2001) includes a similar tool. Nuzum recommended the following usage:

> Have students analyze their performance. Students need truly to understand how they did on a test, and why. Did they know the material? Did they prepare for the test effectively? How well did they perform during the test? (In other words, they need to know that they are responsible for their performance and the grade that they received.) After each examination, have each student fill out a test-analysis questionnaire that asks these questions and any others that you think are pertinent. Ask them whether there is anything they would do differently if they had to study for and take the test again. This is a powerful tool for helping students improve their test-taking abilities. (p. 75)

A middle school teacher in New York modified the tool for her students to use on each of her tests (Menechella, 2004). Her tool is shown in Appendix B.

According to "How Teachers and University Faculty Perceive the Need for and Importance of Professional Development in Performance-based Assessment" (Johnson, Thompson, Wallace, Hughes, & Manswell Butty, 1998),

> Teachers frequently are called upon to construct and use assessments to appraise the learning that takes place in their classrooms. They are also expected to understand the results of those assessments sufficiently enough to interpret them for students and parents, and to plan instructional programs that meet the needs identified by those results. However, as several researchers have noted, the background knowledge many teachers have for these tasks is limited (Impara, 1995). Over half the teachers in the United States have never completed a course in educational measurement, and fewer than one-third of

> all states require such course work for initial certification (Boothroyd, McMorris, & Pruzek, 1992). (p. 197)

For many years, I have been interested in learning more from my students' errors. It was often difficult for me to understand the reason a student may have missed a test item, especially if the item was left blank. Did the student accidentally skip the item? Did the student mean to come back to that item and forget to do so? Did the student have trouble understanding the item? Or could the student not recall the answer? I realized that these were questions the students themselves could answer best. I tried to incorporate those ideas into the version of the tool that I designed (see Appendix C).

I wanted to see if test error analysis could help me better understand my students' assessment results, not only by improving my understanding of individual student test errors, but also by analyzing cumulative data from all students involved in the analysis. I also wanted to enhance student metacognition in their mathematics education. I wanted to promote *learning from* testing.

Research Questions

Much research has been done on student learning and student assessment in mathematics (e.g., Nuzum, 2004; Tobias, 1993; Vislocky & Leslie, 2002). However, I have not seen any research evaluating a metacognitive tool tailored to high school mathematics tests encouraging and empowering students to manage their own test preparation and performance processes.

With its coaching, informing, and self-improvement features, such a tool has the potential to be a very dynamic aid for student and teacher learning about test performance, but how helpful is it? Specifically,

1. What effects does the use of a test-error analysis tool have on students' mathematics test-related behavior (i.e., preparing for and taking tests) and outcomes (e.g., errors made, points lost, test scores)?
2. What benefits and drawbacks do my students, their parents, and I perceive from the use of a test error analysis tool with mathematics tests? In particular, what do the students and I learn from the analysis?
3. What are the most common types of errors, according to the analysis, in a mathematics course? How does this information inform the students and me to promote the learning of mathematics?
4. What groupings, patterns, and trends can be observed from test error analysis data? For example, do the frequencies of some error types decrease? If latent groups are identified, what are their characteristics and what are the probabilities that students move from one group to another?

The purpose of the study was to evaluate a test error analysis process and associated tool with respect to learning and teaching high school mathematics.

Description of the Test Error Analysis Process and Tool

Karen McAcy (1993) created the test error analysis tool in Appendix A and used it in her middle school mathematics classes. Inspired by her idea, I modified her process to include what I thought might be the most common errors for my ninth-grade mathematics classes as shown in the Test Error Analysis Worksheet tool in Appendix C. I placed types of testing errors that related to the testing process (not specific to the subject of mathematics) in the top portion of the form and types of errors more related to mathematics content in the bottom portion of the form.

A separate summary sheet for each student accommodated multiple tests for trend data as shown in the Test Error Analysis Summary in Appendix D.

The Test Error Analysis Worksheet is a matrix in which students record their points lost by error type for each test. I considered using other representations, such as recording numbers of incidents of each error type instead of points lost, which would have been easier for the students. But I decided to use points lost for three reasons:

1. It weighted error types commensurate to the way points were marked off, so minor errors (such notation errors) counted less in cumulative results than more significant errors (such as leaving a test item blank).
2. If the total points lost in the matrix matched the total points taken off the test, then it was likely that everything missed had been accounted for. This procedure provided a check that the analysis was thorough.
3. If points lost were recorded on the form instead of numbers of incidents, those data could be converted to numbers of incidents, but if only incidents were recorded, they could not be converted to points lost. Hence recording points lost allowed more flexibility for data analysis later.

In the matrix, rows represent error types; columns represent test items on which students made errors. Test items eliciting student responses could be in the form of test questions, statements, or instructions. The matrix was horizontally divided into testing process errors (such as *didn't follow directions*) and mathematics content errors (like *forgot vocabulary*). Content issues were further documented in an open-ended way by listing the concepts for the student to review on the back of the form.

The matrix was tailored to the course with the error types that I anticipated occurring the most often. However, the testing process and content error sections each had categories called *other* for students to add their own error types if none of the prescribed ones fit a particular error. This flexibility allowed the tool to promote its "self-improvement" in that I could revise the tool for future classes by adding error types that repeatedly appeared in the *other* categories and deleting those error types that were not used. I used a similar version of the tool with my Integrated Mathematics I classes in the 2008–2009 school year and revised it based on student responses to create this one for the classes in the present study.

I split *arithmetic error* into two parts to help distinguish between arithmetic errors with fractions and arithmetic errors without fractions to provide more differentiation of arithmetic error types. I was interested in quantifying this distinction because of learning about ground-breaking research on children's learning of fractions (e.g., Steffe, 2003; Olive & Steffe, 2002) and realizing from experience that many high school students have yet to master fractions.

The common algebraic misconception that $(a + b)^2 = a^2 + b^2$ is considered an over-generalization of the Distributive Property (Maurer, 1987, p.169). I referred to this binomial multiplication error as *freshman's dream* (Hungerford, 1997) in class and on the form hoping that using a memorable name would help students recognize the error and avoid it.

I used shading of rows and columns for ease of recording and entering data. Directions at the top were created as a short check-list for students to be sure they were thorough in completing their analyses. The open-ended area on the reverse side allowed students to summarize specific content areas to review.

I developed a Test Error Analysis Summary tool like the one shown in Appendix D for the students to summarize information on their Test Error Analysis Worksheets for each test.

Columns on the summary tool represent error type totals for each test for the semester. The eight individual Test Error Analysis Worksheets (for the seven unit tests and midterm) and the Test Error Analysis Summary sheets together documented the analysis of the student test errors.

CHAPTER 2

THEORETICAL PERSPECTIVES AND LITERATURE REVIEW

Theoretical Perspectives

This study was based on a perspective of pragmatism, linking a constructivist theoretical perspective for student learning, an interpretivist perspective of students' analysis of their testing errors, and a positivist perspective of my quest for mathematical truth in grading student test papers. Pragmatism supported a blending of different worldviews.

> Pragmatism is typically associated with mixed methods research. The focus is on the consequences of research, on the primary importance of the question asked rather than the methods, and multiple methods of data collection inform the problems under study. Thus it is pluralistic and oriented toward "what works" and practice. (Creswell & Plano Clark, 2007, p. 23)

I believe that each student constructs his or her own mathematical model mentally, based on interpretation of mathematical and social experiences. Various forms of communication and assessment help me understand what that model looks like. As a teacher, I strive to foster an environment that is rich in experiences to help all of the students confirm, correct, and grow their mental mathematical constructions. Throughout this paper, when I refer to teacher "instruction," I mean fostering that environment with student mathematical "construction" as its goal. In the classroom, I try to use a variety of assessment techniques, including computer and classroom labs, daily practice assignments, independent and group assessments, and assessments with and without support materials (e.g., calculators, computers, notes, or books). In general, I consider this approach to be constructivist, with each student constructing an internal model of mathematics.

However, when grading assessments, I use my own model of mathematics, which is based on a quest toward a mathematical truth that I believe exists and to which I aspire. If my model of mathematics and a student's model disagree, points are deducted from the student's score. Partial credit may be given if I deem the student's model to be close to my "true" mathematics model, that is, if I see evidence of learning that goes significantly beyond a missing answer. These grading decisions are admittedly subjective, and subject to change, even after the grade is posted, if I can be convinced of the merit of an answer provided that was not given full credit.

This study also borrows ideas from a perspective of symbolic interactionism, in that I asked the students to use the tool to symbolize the meanings they placed on their errors, by type. This request was to help me see the students' "definition of the situation, what they take into account, and how they interpret" (Schwandt, 2001, p. 245) their test results. This interaction, in turn, could help both the student and me interpret the results to plan improvements in learning, teaching, and testing.

As is common with mixed methods studies, multiple theoretical perspectives seemed to apply. So an umbrella of pragmatism incorporated the different worldviews into a foundation for the research.

I created the conceptual framework diagram in Figure 2.1 to provide a model for this research. It is similar to one used by Izsák (2000). This conceptual framework and its interpretivist perspective reflect the meaning the students placed on their analysis and their development of personal solutions to promote their own improvement. This student self-improvement process is represented by the student's metacognitive loop at the top. The triangle below it represents the communication patterns between the student, the teacher, and the parents.

The metacognitive loop on the right is for what I learned from the analysis to provide input to my teaching processes. The test analysis tool supported analysis and communication at a detailed level between my students and me as well as optionally between the students and their parents and between the parents and me. Secondarily, a metacognitive loop could be added for the parents showing any guidance gained by the tool for use in parenting processes. Note that although there was some student-to-student interaction in brainstorming advice and in classroom learning activities, it was not encouraged for the test error analysis process to protect student privacy (see Principle 11 below) and is, therefore, not depicted in the diagram.

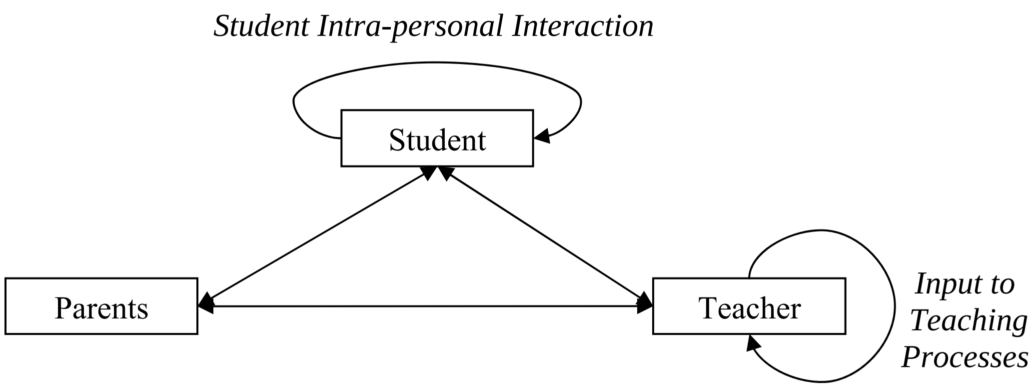


Figure 2.1. Triangle of student-teacher-parents interaction with metacognitive loops.

I used the following principles to guide me in designing the study:

1. Students construct their mathematics based on their experiences in the classroom and elsewhere (Piaget, 1964, von Glasersfeld, 1992).
2. Although multiple forms of assessment are used in the mathematics classes where the study was conducted, our written tests were assessments given at reasonably regular intervals and were similar in difficulty for students who have kept up with their daily

assignments. (The students took these tests without the assistance of notes, books, or other people. The tests were graded based on the teacher's mathematics.)

3. Teacher observation and interpretation may be inadequate to correctly and completely understand student errors without additional student input (Clements, 1980; Newman, 1977). (I recognize that even though this test error analysis may facilitate that understanding, neither the teacher nor the student may ever fully understand why a student missed a particular test item.)
4. Students conscientiously analyze their testing errors when coached to do so with the tool.
5. Tests are graded, returned, and gone over as soon after the test as possible to promote accuracy of student recollection and metacognitive results.
6. I have a (perhaps unattainable) goal to treat every student equitably without prejudice or perceived favoritism.
7. Any additional student time required for the study would be kept to a minimum.
8. Students and parents trust me enough to openly share their insights on the benefits and drawbacks of the tool.
9. Parents are interested in student performance.
10. The level of communication between students and their parents varies widely.
11. A student's grades are considered to be confidential, accessible only to the student, his or her parents, and school staff, unless the student or parents deem otherwise.

I periodically refer back to these principles to provide rationale for methodological choices made in the research.

Location in the Literature

As I mentioned above, much research has been done on student learning and student assessment in the field of mathematics (e.g., Lin, 2006; Nuzum, 2004; Tobias, 1993; Vislocky & Leslie, 2002). Teachers have analyzed tests and test items using various methods, estimating difficulty and discrimination indices, reliability, and validity (Clark, 1992; Thorndike, 2005). Errors students make that are specific to mathematics (e.g., Clements, 1980, 1982; Radatz, 1979; Ashlock, 1976; Lankford, 1972) have been identified.

Student metacognition in mathematics instruction (e.g., Carr, et al., 1994; Peterson, Swing, Braverman, & Buss, 1982; Schoenfeld, 1987) and student mathematics anxiety (e.g., Ho, et al, 2000; Ma, 1999; Tobias, 1993) have been studied. Schoenfeld reported that:

> Research on metacognition has focused on three related but distinct categories of intellectual behavior:
>
> 1. Your knowledge about your own thought processes. How accurate are you in describing your own thinking?
> 2. Control or self-regulations. How well do you keep track of what you're doing …?
> 3. Beliefs and intuitions. What ideas about mathematics do you bring to your work in mathematics …? (Schoenfeld, 1987, p.190)

Prior research has focused on instructional metacognition, having students monitor their thought processes during mathematical exercises (e.g., Kramaraski, Mevarech, & Arami, 2002). This present study used a retroactive metacognitive approach, asking students after assessments to look back over their previous thought processes during the assessments and reflect on reasons for errors in an attempt to promote learning from mistakes and prevention of error recurrence.

Borasi (1994) studied the use of errors from many sources (the research subjects' own errors as well as errors made by other students) as learning opportunities for high school students, using the errors to expand student content knowledge. In the present study, students were asked to analyze their own errors, identify reasons for the errors, document cumulative results, and strive to prevent commission of those errors on future assessments.

Research about high school students' analysis of their test errors, which can be viewed as a subset of both assessment in mathematics and student metacognition (as shown in Figure 2.2), seems to be missing.

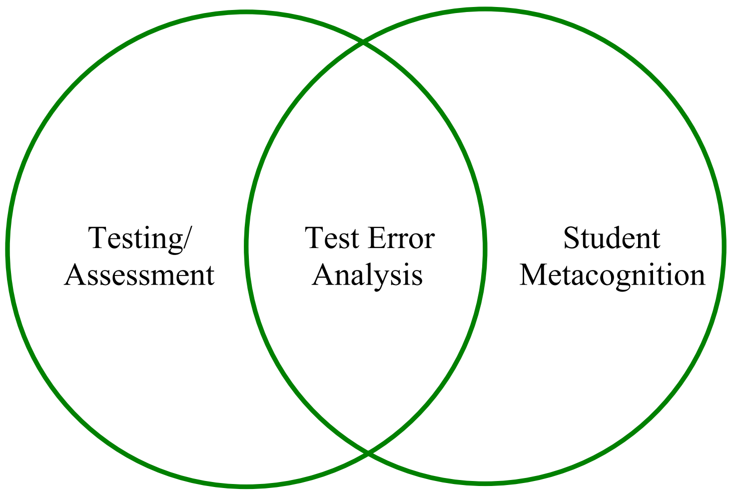


Figure 2.2. Venn diagram of the relationship between assessment and metacognition.

I wanted to address this missing component. According to the U.S. Department of Education:

> A good evaluation system provides invaluable information that can inform instruction and curriculum, help diagnose achievement problems and inform decision making in the classroom, the school, the district and the home. Testing is about providing useful information and it can change the way schools operate. (U.S. DOE, 2004, para. 32)

I aimed to see how much my students, their parents, and I could learn from test error analysis. I hoped that I might also spawn further research in evaluating metacognitive processes to empower students to better manage their test preparation and performance.

CHAPTER 3

METHODS AND RESEARCH DESIGN

Type of Study

The test error analysis tool was embedded within an instructional process that involved instruction, learning, testing, test error analysis, teacher assessment of analysis results, and subsequent planning for further instruction in a cyclic pattern throughout the semester. A simplified version of the process is shown in Figure 3.1.

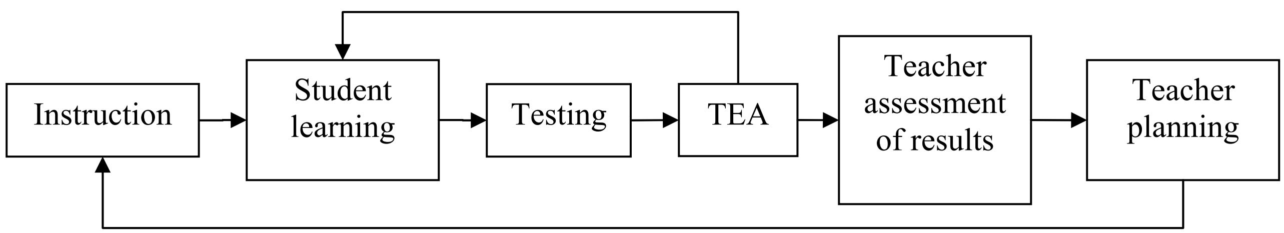


Figure 3.1. Simplified instructional process with test error analysis.

My basic research framework was an embedded mixed method design, a design in which quantitative data from the test error analysis tool played a core role. Interview, questionnaire, and observation data from the test error analysis processes provided additional information to help evaluate use of this student test error analysis tool with respect to its use in learning and teaching high school mathematics.

Therefore, I considered this research to be an exploratory embedded mixed methods design with a quantitative evaluation of the aggregate student test error analysis results embedded in an analysis of the overarching test error analysis process based on qualitative input from students, their parents, and me. Figure 3.2 shows a simplified diagram of an embedded mixed methods design (Creswell & Plano Clark, 2007):

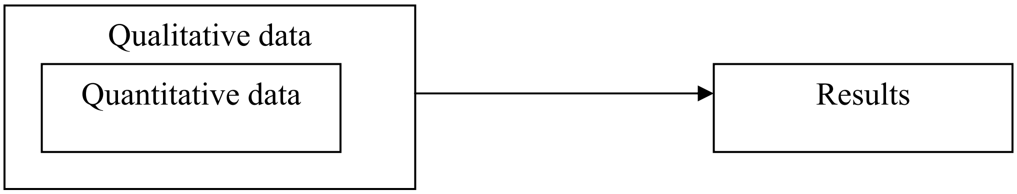


Figure 3.2. Relationship of embedded mixed methods design with quantitative data embedded in a qualitative framework.

The central premise of mixed methods research is that the combination of quantitative and qualitative analysis provides a better understanding than either one alone. Whereas quantitative studies might overlook participants' voices and researchers' biases might not be disclosed, qualitative studies have inherent researcher interpretations and difficulty in generalizing findings to large groups. Mixed methods research provides more comprehensive evidence because researchers use qualitative and quantitative tools. It helps answer questions that cannot be answered by either design alone; allows collaboration across sometimes adversarial research strategies; encourages use of multiple worldviews, including pragmatism; and combines inductive and deductive reasoning for a more persuasive rationale than either quantitative or qualitative research provide in isolation (Creswell & Plano Clark, 2007).

I chose a mixed methods design to address the metacognitive concept of student test error analysis so that I could investigate not only the main errors my students were making but also their insights into the process, the tool, and its helpfulness, summaries of what I learned from the analysis, and the opinions of the parents involved. The reason for collecting the two types of data was to gather input on (a) the types of errors identified and (b) the overall perceived benefits of the tool for the students, parents, and me.

Therefore, the purpose of using mixed methods for this study was mostly complementarity, but also initiation (Greene, 2007). In other words, qualitative and quantitative methods might tell more about student test error analysis together than either one would alone (complementarity). The novelty of the test error analysis process and the present study's results might foster other studies providing further evaluation of the concept (initiation).

There are many different ways to categorize mixed methods designs. Creswell and Plano Clark (2007) espouse four main types of mixed methods designs: triangulation, embedded, explanatory, and exploratory. The planned process for the present study involved concurrent and sequential data collection and analysis from several sources as summarized in Figure 3.3.

Data source	Students				Parents	Teacher/ researcher
Research question	1, 2, 3, 4	1, 2		1, 2	2	1, 2, 3, 4
Data collection	Tool	Questionnaire		Student-constructed advice	Interviews, questionnaire	Teacher perspective
		Likert	Open-ended			
Data analysis	Latent class analysis/ linear regression	Frequency/ latent class analysis	Coding		Coding	
Data interpretation	Quantitative		Qualitative			
Conclusions	Integration of results					

Figure 3.3. Flow of data from collection to conclusions with reference to research questions.

I considered it to be primarily an embedded concurrent design with a qualitative priority (Creswell, 2003). Data from the use of the tool, parent input, and teacher input were concurrently collected. At the end of the semester, student questionnaires and summative teacher observations were analyzed, providing a secondary sequential element to the design. Integration primarily occurred at the conclusion of the study analysis, as shown in Figure 3.3.

The study used frequency analysis, linear regression, and latent class analysis (LCA) to analyze student testing errors. I gathered questionnaire and qualitative interview data exploring the value of the test error analysis for students, parents, and me in this application. Frequency analysis and LCA were used for the Likert scale data in the questionnaires.

Site of Research

I conducted the research in three gifted mathematics classes at a large public high school where I taught five classes each day. My other two classes were elective classes, as opposed to core mathematics classes, so they were not included in the study. The school was in a suburb of a large metropolitan area in Georgia. It had Grades 9 through 12, about 3500 students, and 220 faculty and staff members, 35 of whom taught mathematics. I chose this site for my direct access to it and to its student participants; additional benefits included multiple races, cultures, and socioeconomic student backgrounds, the school's focus on academics and continuous improvement, and its favorable reputation in the community. Table 3.1 shows the demographics of the school and of the participants in the study.

Table 3.1
Site and Participant Demographics

Demographics		
Characteristic	School 2009–2010	Study Participants
Enrollment	3469	43
American Indian/Alaskan Native	1%	0%
Asian	7%	5%
Black/African American	13%	5%
Hispanic or Latino (any race)	13%	12%
Native Hawaiian/Pacific Islander	0%	0%
White	61%	79%
Multiracial	4%	0%
Special Education	8%	0%
ESOL	12%	0%
Gifted	23%	100%
Free/Reduced Lunch	22%	21%

Since all of my students returned their signed parent/student consent forms, the study began with all 48 students in three classes of Accelerated Integrated Mathematics I in the fall of 2009. Four students transferred out of the class during the semester, and one student lost some of his tests and test error analysis forms. Eliminating these five students left 43 students (20 females and 23 males; 17 in first period, 10 in second period, and 16 in third period) with complete data.

Data Collection and Analysis Procedures

Instruction

The fall semester curriculum was based on the algebra portion of the Georgia Department of Education Accelerated Mathematics I Standards listed in Appendix E, with one exception. We delayed covering the step and piecewise functions until spring semester, when the focus would change to geometry, data analysis, and probability. Four other teachers and I taught the fall course at our school and covered the first semester curriculum at roughly the same pace.

Test Construction

Our tests were designed primarily for accountability (to see how well students had mastered mathematical curricula) and to inform instruction (Balanced Assessment in Mathematics Project, 1995). Our school district provided the multiple-choice midterm. We five teachers took turns designing the unit tests, which we modified sometimes as a group and sometimes individually to meet our assessment needs. Although we were not required to give identical tests, we tried to maintain consistency in the level of difficulty of assessments between teachers. We collaborated to create the free-response performance exam and the multiple-choice final exam which were administered at the end of the semester.

Our lessons spanned all four of Webb's (2002) Depth of Knowledge Levels, but the rush to cover the cover the curriculum pressed us to limit assessment time. These time constraints generally restricted the level of items on our tests to the first three levels since we wanted to be sure all major concepts were assessed in an assessment period.

Test item formats for the seven lesson-specific unit tests included multiple-choice, short answer, and brief essay. The multiple-choice items, while not recommended by reformers (Mathematical Sciences Education Board, 1993), gave the students practice with the format required for the multiple-choice part of the final exam and for the end-of-course test given the following semester (and therefore not included in the study). The multiple-choice and short-answer items also allowed for efficiency of grading and, therefore, quicker feedback to the students. I strove to provide prompt feedback, not only to promote better learning of the mathematics and a better student-teacher relationship, but also because I believed it to be instrumental to the students' ability to recall their thought processes for better accuracy in test error analysis. Short-answer and essay items provided assessment of students' mathematical communication in addition to computation skills.

The performance exam that we gave two weeks before the end of the semester was different from the other tests. The performance exam consisted of four more complex Webb Level 3 and 4 items of a comprehensive nature. Students were to choose two of the four items to answer in a class period. Student responses to these items helped to assess mathematical communication skills as well as problem solving, reasoning, and mathematical content knowledge. These exams were graded on a rubric and then converted to a 100-point scale. Although these tests were returned to the students when graded, they were not included in the test error analysis process as points lost could not be directly identified from the scoring system.

The multiple-choice final exam was given on the last day of the semester, thereby allowing no time for student test error analysis, as class rosters changed for the second semester. Scores for the performance exam and final exam were included as data even though student errors on those two assessments were not analyzed.

Test Reliability

My goal in assessment was for the tests to provide an accurate measurement of student learning or depth of knowledge. There are several reasons that a test might not be an accurate reflection of student mathematics achievement. One is low reliability. Reliability "refers to the accuracy or precision of a measurement procedure. Indices of reliability give an indication of the extent to which the scores produced by a particular measurement procedure are consistent and reproducible" (Thorndike, 2005, pp. 110–111).

There are many ways to measure a test's reliability, but I was somewhat limited by the small number of items (at most 20) and the small number (43) of students. I first used the split-half reliability coefficient, calculating even- and odd-numbered item scores. I found the Pearson correlation coefficient for the linear regression (r) and used it to calculate the reliability estimate for the test, $2r/(1 + r)$, using the Spearman-Brown prophecy formula (Thorndike, 2005).

For comparison, I then used the more consistent coefficient α method to measure the test's reliability:

$$\alpha = n/(n - 1)\ (SD^2_X - \sum SD^2_i)/SD^2_X,$$

where n represents the number of items in the test (20 for Test 1), SD^2_X is the variance in the test scores ($\approx$ 48.8 for Test 1), and $\sum SD^2_i$ is the sum of variance values for each item ($\approx$ 27.32 for Test 1). So for Test 1,

$$\alpha = 20/19\ (48.8 - 27.32)/48.8 \approx .46.$$

These estimates of reliability for the nine tests varied, as shown in Table 3.2. It should be noted that while both reliability estimates for the 2-question performance exam were dismally low, the Kuder-Richardson Formula 21 (KR-21) estimate, which assumes the test items to be of comparable difficulty (an assumption which could not be made on the other tests), yielded an estimate of .49. Overall, I deemed the test reliability to be moderate, but not stellar. Better test reliability could have strengthened the results of the study.

Table 3.2
Summary of Test Reliability Estimates for the Eight Tests

Test	Reliability estimate	
	Even-odd	α
1	.69	.46
2	.65	.62
3	.90	.71
4	.65	.68
Midterm	.54	.36
5	.74	.67
6	.79	.56
7	.82	.75
Performance	.05	.08
Final	.51	.30

Test Validity

The second reason a test might not be an accurate reflection of student learning or depth of knowledge is low validity or "construct validity" (Lesh, 1992, p.5), and it is the most important reason. How valid a test is tells "whether the test measures what we want to measure, all of what we want to measure, and nothing but what we want to measure" (Thorndike, 2005, p. 145). My four colleagues and I drafted and edited the tests to contain what we considered to be the most significant skills of which the students needed to demonstrate mastery. We reached consensus that the tests contained what we wanted to measure. This consensus provided a practical and convenient check of intended test validity. Additionally, in general, student responses seemed to validate empirical test validity, with exceptions on a few items.

Other Testing Issues

In addition to reliability and validity, there are two ways a student's test score can belie the student's learning or knowledge. Let us sort the possibilities into Type I (false positive) and Type II (false negative) errors.

Let a Type I error represent the case where the student gets an item right on the test but does not have a solid grasp of the concept. There are several ways that can happen even if the test has favorable reliability and validity. On a test with four-choice items, a student who knew none of the content could guess correctly on an average of one fourth of those items; this scenario was not mitigated in my research. Alternatively, a student might try many computations until one surfaces as a choice provided on the test. We tried to minimize this practice by providing distracters (incorrect choices) that included the most common errors we anticipated the students making. The number of multiple-choice items for each test is shown in Table 3.3. It is interesting to note that the four tests with multiple-choice items had the lowest even-odd reliability estimates. In fact, linear regression shows a strong negative correlation of

$$y \approx -0.28x + 0.81 \text{ with } r \approx -.91,$$

where y represents the even-odd reliability estimate and x represents the percentage of multiple-choice items. A significant issue in this relationship is that I awarded no partial credit for incorrect multiple-choice responses, but partial credit was possible with responses to most other categories of items. With 20 or fewer items on the tests with multiple-choice items, that would have increased the disparity in scoring between correct and partially incorrect responses.

Table 3.3
Number of Multiple-Choice Items for Each Test

Test	Multiple-choice items	Total no. of items
1	6	20
2	11	20
3	0	20
4	10	20
Midterm	20	20
5	0	20
6	0	14
7	0	11
Performance	0	2
Final	50	50

Another way for a Type I error to occur is for a student to copy answers from another student's paper. Two parallel versions of the test were used in the classroom. Copies of one version of each test used in the study are provided in Appendix F. Tests were distributed in striped (alternate classroom rows of students had Version A; others had Version B) arrangements, in checkerboard arrangements, or randomly to discourage this type of cheating. Additionally, other forms of cheating could have been used (such as use of notes, books, or cell phones), but I did not detect any of them on these tests.

Let a Type II error represent the case where a student lost points on an item in spite of mastery of the concepts. This phenomenon could occur for many reasons: (1) The test item may have been biased (Dyer, 1994); (2) The student did not understand the item because of low English proficiency (LEP, Secada, 1994); (3) I graded the test incorrectly; (4) The student may have been distracted by other issues, not feeling well, or overcome with test anxiety.

My colleagues and I tried to prevent Type II errors by remaining multi-culturally sensitive and minimize testing bias. None of these students were of LEP. Students who thought I had taken off points unfairly seemed to be comfortable defending their stances; if I agreed, the grade was adjusted. If not, I explained further. I tried to infuse some light-hearted humor into the tests in an attempt to alleviate stress, although use of humor in tests to reduce test anxiety is not necessarily supported by research (Berk, 2000; McMorris, 1985). For example, in Test 1, Item 16 has Han Sum being photographed and the name of the school in Item 17 is Aiming High; on Test 2, the answer box contains a message intended to induce a smile.

At the end of several lessons (about every 2 weeks), I administered these tests to my students in a 53-minute period. The tests were taken individually without the assistance of notes, books, or other people. Graphing calculators were allowed on the tests, and I provided them to the students upon request.

Test Error Analysis

In every case, I graded and returned the tests to the students the next school day to maximize student memory of their thought processes that had generated the previous day's responses. The students filled out a blank Test Error Analysis Worksheet as we went over the test in class, analyzing their test errors and documenting them on the tool, recording numbers of points lost for each error. Sometimes more than one error type applied to points lost on a test item, and the students either split the points lost over more than one error type or chose a primary one.

Students added categories or error types as necessary in rows marked *other* for any errors that did not fall into the given categories. This process was repeated after each test until the end of the semester for each of the unit tests and the midterm. Students summarized cumulative results on the Test Error Analysis Summary tool. Going over the first test, educating the students on use of the tool, and coaching them through the analysis took most of the 53-minute period for all three classes. Subsequent tests required about half of the period to go over the tests and complete the test error analysis. A sample student Test Error Analysis Worksheet for Test 4 is shown in Appendix G.

I reviewed each student's test and corresponding tool for consistency and sensibility, checking with the student and making adjustments as necessary. If I had questions about a student's error analysis (because of missing information or mismatched totals, for example), I asked follow-up questions of the student to better understand the testing or analysis processes and ensure completion of the data. I analyzed and summarized results by frequency (of points lost by error type), student groupings (using LCA), and trends to see changes over time.

The test error analysis on the first two tests was considered to be instructional from a grading perspective. Students were graded on the quality of their test error analysis for all subsequent tests (3–7 and the midterm) except for the performance and final exams. These test error analysis grades were minor grades (less than 1% of the overall grade) in the daily assignment category. Students were allowed to earn full credit on those daily grades if they corrected any discrepancies I found in their test error analysis. This practice provided them with a small incentive to be sure their test error analysis was thorough, sensible, and (I hoped) thoughtful.

I considered validity of the tool to be enhanced by the categories listed as *other* and the open-ended responses on the back of the form. Reliability of the tool was encouraged in several ways:

1. I put comments on students' graded papers and suggestions of possible error types to promote consistency of error types chosen in the closed- and open-ended parts of the form (*other* and listings on back).

2. I identified test error analysis forms with total points lost that did not match the total points missed (entry A2 on the form) and asked the students to rectify the discrepancies.
3. Any error types identified by the students for which I did not mean to deduct points for that test were addressed by inquiring with the student and recategorizing the error. Sometimes this recategorizing was required because students were unclear on my reasons for the point deductions; sometimes students and I attached different meanings to a particular error type.
4. I compared test error analysis forms to student tests for compatibility, asking the students to clarify or make corrections as necessary, thereby rectifying discrepant findings.
5. Some errors could be coded in multiple ways; when applicable, I made changes in the coding of error types to promote consistency across the data set.
6. I clarified any testing errors listed in the *other* category by asking the student if necessary.
7. I ensured that vocabulary and concept errors were specified on the back of the form.
8. I randomly selected one male and one female from each class to go over their test error analysis forms with their tests in informal interviews about the process and validate or correct any assumptions I had made.

Student-Constructed Advice

After the students analyzed the first test, I asked them to brainstorm prescriptive ideas for each category of error in small groups (3 or 4 students each). Groups were formed as follows: Students blindly chose a card from a modified deck with three suits; aces worked together, twos, and so on (providing four to six groups of 3 students each). The handout in Appendix H was used. For example, the aces group covered the first error classification; the twos group, the second; and so on until all classifications, including any added *other* classifications, were addressed. Groups took turns presenting their recommendations for certain error types to the rest of the class. After each group presented advice for preventing a given error type, other students were asked if they had advice to add. In a few cases, I added additional suggestions. Students kept a copy of their lists of ideas (augmented with any additional suggestions from the class) for their notebooks and turned in one copy (for each group) to me. I summarized the ideas from all three classes and gave the summary to the students verbally and as a handout, thanking them all for their insights. Although this student-constructed advice was created only once in the semester, the students were encouraged throughout the year to refer to their lists for ideas to improve their testing performance and to add any other ideas that they deemed to be helpful. The advice was analyzed for common themes and creative suggestions to be highlighted in this report.

Student Questionnaire

I asked each student to fill out a questionnaire (Appendix I) at the end of the semester to get student perspectives on the helpfulness of the tool and its related student-generated advice in improving test results and facilitating communication with parents and me about testing. The Likert scale allowed me to quantify the usefulness of the test error analysis tool. I coded and analyzed the open-ended qualitative data. Frequency analysis, including graphs, and LCA provided comparison results for each item of the questionnaire.

Latent Class Analysis

I used frequency analysis to quantify aggregate points lost by error type for each test and at the end of the semester. I also used LCA to investigate patterns and trends among groups of students choosing certain error types. LCA is a statistical method used to identify subtypes of behaviors based on response patterns on items or indicators. For example, students can be categorized based on their error type classifications (observations) into different types of test takers (subtypes). This information could help tailor instruction to improve students' learning behavior and metacognition.

Measurement Software

I used Microsoft Excel, Fathom, Mplus, and Minitab software to analyze the quantitative data.

Unstructured Interviews and Surveys of Parents

During the semester, only two students' parents asked for conferences. In each conference, I discussed the test error analysis process, went over the student's test error analysis forms, and invited the parent to comment on the helpfulness of the information. At the end of the semester, I solicited input from parents of all participating students using a survey by e-mail to see whether parents had seen the tool and what value it might have had for them. These questions are provided in Appendix J. All responses were analyzed.

Teacher Perspective

I provided a teacher perspective as a participant-researcher. General questions to address were listed in Appendix K, but as the data were gathered and analyzed, additional information was included.

Other Considerations

At the beginning of the semester, I shared the tool and a brief overview of the process with my 35 colleagues in the Mathematics Department, hoping that I could include data from other teachers as part of the process. However, none of my colleagues shared any results to include. When I asked them for specifics, they responded that they did not try it because they thought it would take too much time. However, three of them reported that the idea had merit and that they would try it the following semester.

I considered requiring my students to keep journals, but that seemed to violate Principle 7 (Any additional student time required for the study alone should be kept to a minimum), so journals were not included in the design.

Putting It All Together

Several types of quantitative data were analyzed for the present study:

1. Frequency analysis of points lost by error type.
2. Trends over the year of average student test scores using a run chart (a connected comparison of average test scores on the *y*-axis with time on the *x*-axis).
3. Multiple scenarios of LCA of students using points lost by error type
4. Student end-of-semester test error analysis evaluation (e.g., Likert questionnaire results)

Multiple qualitative results were coded and analyzed; the types of analysis used varied on the quantities and types of data collected. Question-to-methods relationships are summarized in Table 3.4.

Table 3.4
Summary of Methods by Research Question

Research question	Data collection method	Data analysis method
1. What effects does the use of the test-error analysis tool have on students' mathematics test-taking behavior and outcomes?	Test error analysis tool	Frequencies/means of points lost by error type over time Error type trends Test grade trends
2. What benefits and drawbacks do students, parents, and I perceive from the use of the test error analysis tool with mathematics tests?	Student questionnaire Likert scale items Student questionnaire open-ended items Parent interviews/ surveys	Coding Teacher perspective
3. What are the most common types of errors, according to the students, for a given mathematics course?	Test error analysis tool	Frequencies and means of points lost by error type over time
4. What groupings, patterns, and trends can be observed from the data?	Test error analysis tool	Latent class analysis Linear regression

Schedule. Table 3.5 shows the timeline for the study.

Table 3.5
Research Timeline

Schedule	
Learn curriculum and finalize forms.	Fall 2008
Obtain IRB and school district approvals.	Fall 2009
Collect data. Week 1: Solicit student and parent approvals. Week 2: Administer, grade, and return first test. Introduce tool. Week 3: Gather error analysis data from first test. Conduct brainstorming for student recommendations. Weeks 4–18: Continue testing and test error analysis for each test. Record any relevant parent comments. Week 18: Conduct student and parent surveys with questionnaires.	Fall 2009
Analyze data.	Fall 2009/ Spring 2010
Write up results.	Summer/Fall 2010

Subjectivities, Anticipated Risks, Benefits, and Limitations

I am a 53-year old Caucasian female. I have been teaching high school mathematics for 16 years. Before that, I was a quality manager for AT&T (formerly known as American Telephone and Telegraph). I have a B.A. in mathematics, an M.S. in engineering, and an Ed.S. in mathematics education. I am in my 7th year of doctoral study.

This study was undoubtedly influenced by my relationships with my students. Students with more affinity for me may have recorded more positive responses to the tool and, inversely, students who had a less positive relationship with me may have found less favorable results in using the tool. This possibility should be kept in mind when considering generalizability.

One of the risks of this study was that it focused on students' test errors. For some students, this focus may have seemed like a negative emphasis on their mistakes. To overcome this risk, I reminded them of the positive goals (improving test preparation and test-taking skills) we had in mind. "It's all about the learning," I admonished. Another risk was that the time involved getting student input could have detracted from time spent on the curriculum and testing. I tried to manage this time efficiently, with an extra half-period spent on learning the process the first week, and then having students complete their test error analysis while we were going over the tests in class. Students were generally able to complete the forms when we were discussing test items they did not miss, unless they were absent when we went over the test.

I envisioned the following as potential benefits: This metacognitive tool could help to reduce mathematics test errors, which prevent some students from achieving their mathematical potential and, consequently, their academic and career goals. It could provide insights into obstacles to testing success and foster ideas for performance improvement. It could likewise help to improve the communication within the student-teacher-parents triangle. Data from the tool could be used to inform teacher lesson preparation and instruction. The tool could be modified and extrapolated to assist students in other subject areas and future academic situations. Many students seemed unfamiliar with the concept of a spreadsheet, where row totals and column totals should match; this learning application provided a secondary benefit.

Limitations of the study were also considered. The study was only as good as its principles and implementation. Several major limitations were highlighted:

1. Principle 4: Students conscientiously analyze their testing errors. If students did not take the analysis seriously, the results might not be trustworthy.
2. Principle 8: Students and parents trust me enough to openly share their insights on the benefits and drawbacks of the tool. If they did not feel comfortable sharing their input, that knowledge would be, at best, lost, and at worst, misleading.

3. This was my first mixed methods study, so as a novice researcher, I am confident that it had limitations.
4. I did not anticipate that the study would be immediately generalizable; sample sizes were relatively small, test error analysis tools would need to be tailored to each course, related processes would need to be revised to meet each teacher, student, and course combination.

CHAPTER 4

TESTING RESULTS

The data collection began with testing. The mean and standard deviation of student scores for each test, in the order administered, are shown in Table 4.1. Unit tests were numbered 1 through 7. The cumulative midterm exam was given between Tests 4 and 5. The semester concluded with a performance exam and a comprehensive final exam. All of the concepts covered in the first test and most of those in the second test were review for the students from the previous year.

Table 4.1
Mean and Standard Deviation for the 100-Point Tests for the 43 Students

	Test											
Statistic	1	2	3	4	Midterm	5	6	7	Perf	Final	M	SD
M	90	84	76	74	92	79	79	76	86	82	82	6
SD	7	11	13	13	7	11	10	16	5	6	10	

Figure 4.1 shows a run chart of the means of students' test scores (from Table 4.1) in chronological order. Mean test scores of unit tests did not display an upward trend. The cumulative midterm, performance, and final exam scores were higher than the results for Tests 3–7 for three possible reasons: (1) a lower level of difficulty, (2) their homogenous multiple-choice (for the midterm and final exam) or essay (for the performance exam) format, or (3) students' improvement in content knowledge. In the last case, analysis of test errors from previous tests might have contributed to better performance when the content was revisited on the cumulative exams.

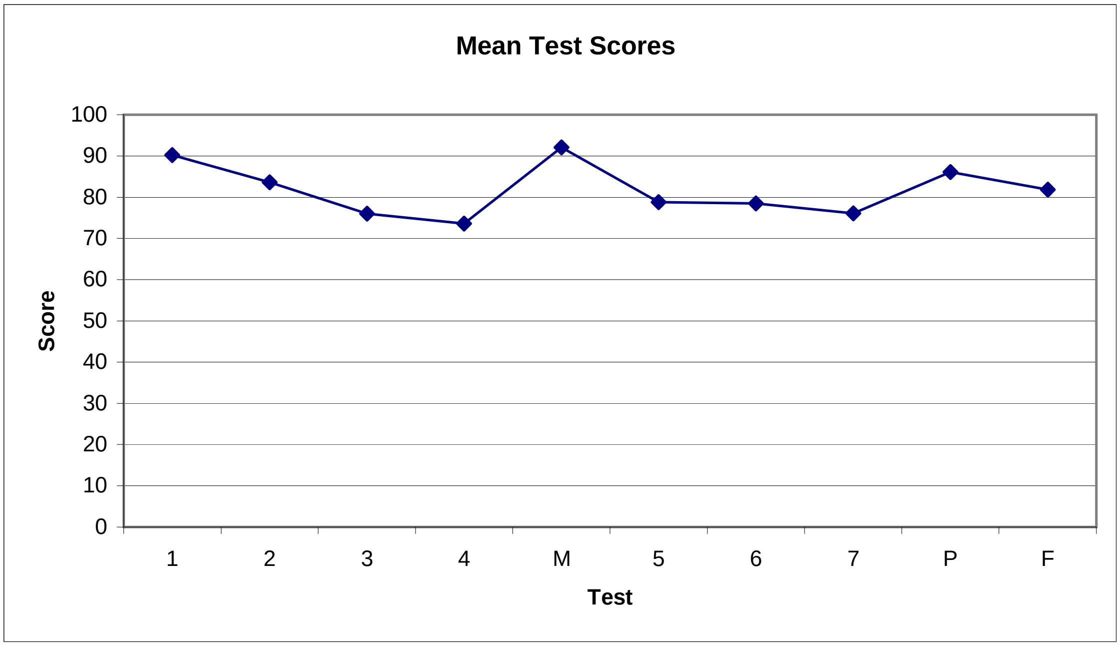


Figure 4.1. Student mean test scores on the 7 unit tests, the midterm, the performance exam, and the final exam in chronological order.

These grades may seem low for gifted students. One reason is that the most gifted ninth graders at the school were already a year ahead of these students, having mastered this accelerated material in eighth grade. Another reason is the richness of the curriculum, which has proved challenging to ninth graders statewide (Dodd & Perry, 2010). A third reason is that many students were not faithful about practicing their mathematics in or out of the classroom, as evidenced by their overall 79% daily average. These daily grades were based on assignments that I collected several days after the assignments were given and we had gone over them in class. I typically graded for accuracy of steps; all answers had already been provided. I would, therefore, have expected students who kept up with daily practice to have averages near 100%.

I calculated correlations in Minitab between test scores by student using the Pearson *r*, as shown in Table 4.2 below. The earlier test is shown with the later test to see how well success on one test would predict success on the next test. Test correlations statistically different from zero, based on *p*-values of .05 or lower, are indicated in bold. As might be expected, no *r*-values show a statistically significant negative correlation, indicating that students who did well on one test tended to do well on the others. However, some of the correlations are very weak or non-existent, as exemplified by the Test 1 to Test 6 and Test 1 to Test 3 *r*-values. Since Test 1 should have been review for these students from their eighth-grade classes, its correlation strength with tests that contained new material was not as strong as tests of new material correlating with each other.

Table 4.2
Pearson r *- values of Correlations of Student Test Scores from Test to Test*

Subsequent Test	Prior Test								
	1	2	3	4	M	5	6	7	P
2	.141								
3	.031	.299							
4	.088	**.337**	**.519**						
Midterm	.162	.155	.295	**.312**					
5	.200	.150	.228	**.430**	**.504**				
6	.016	**.383**	**.393**	**.559**	**.404**	**.503**			
7	.029	**.539**	**.381**	**.374**	**.280**	**.455**	**.359**		
Performance	–.078	.133	**.343**	.290	.067	.169	.253	**.382**	
Final	.060	**.347**	**.324**	.256	.116	.240	.181	**.443**	**.415**

I was intrigued by the differences in correlation between the lesson tests and the final exam. Thinking that it might be caused by the number of items the final exam contained from each unit test's content area, I created Table 4.3

Table 4.3
Number of Final Exam Items by Unit Test Content

	Test						
	1	2	3	4	5	6	7
Number of Items	10	10	2	5	11	7	5

The number of items from each unit test failed to explain why Tests 2, 3, and 7 had a stronger correlation with the final exam than Tests 4, 5, and 6. I determined that Tests 4, 5, and 6 had common content in square roots in addition to their obvious sequential relationship, while Tests 2, 3, and 7 shared concepts in quadratics.

CHAPTER 5

TEST ERROR ANALYSIS

Students analyzed their testing errors on the seven unit tests and on the midterm. In general, the students seemed to strive for accuracy in reflecting their testing errors, often asking how to best code a given error as we went over a test.

I interviewed the six randomly selected students with each of their tests and tools to be sure I understood their analyses. This was a helpful process because it allowed me to understand the students' mathematical processes as well as improving consistency of coding.

Sometimes when I asked a student for more information about their mathematical thinking or error coding, they agreed that the coding should be changed. A few examples (using pseudonyms) follow. Please note that some of these items may differ slightly from those in the Appendix F due to different versions of the tests. On Item 8 of Test 2, Serena responded with $x^3 + x^2y + xy^2 + y^3$ when finding the volume of a cube with side length $x + y$. She attributed this to an *arithmetic error – no fractions*, but when we looked at her steps, we agreed with my assessment that she *didn't distribute properly*. On Item 2 Test 3, Andy gave –1 (instead of $x = –1$) as the equation for the axis of symmetry of the parabola. He coded this error *other (math): forgot equation*, but agreed that it could be considered an *incorrect notation* error for consistency. On Item 13 of Test 5, Moe correctly set $–14 = 4y$, but then said $y = 3\ 1/2$ instead of –3 1/2. She decided that omitting the negative sign was *missing/incorrect units*, but confirmed that it was an *arithmetic error – fractions* since no units of measure were involved. On Item 12 of Test 4, Emma correctly set up $f(–x) = 2(–x)^4 – 3(–x)^2 – 5$ to justify that $f(x) = 2x^4 – 3x^2 – 5$ was an even function. She then simplified it to $f(-x) = 2x - 3x - 5$. She said she "distributed the exponents to the signs only" and surmised that she *didn't distribute properly*. When we revisited the Distributive Property, she confirmed that this was an *arithmetic error – no fractions*.

My assessments of the students' mathematical thinking were not always accurate, and the interviews helped me to see the students' perspectives more clearly. For example, on Item 10 of Test 3, Serena had used $x = -b/(2a) = 5/(2*3)$ to see if $x = 2$ was a solution to the quadratic equation. I suspected that she had confused two concepts (finding the vertex of a parabola with finding the solutions to a quadratic equation). She had indicated that she had *misread the question*. When I asked her about it, she said that she had read the item too quickly and thought she was to confirm whether or not the given value of x would identify the vertex of a related quadratic function. However, the interviews confirmed that the vast majority of my assessments of the students' error types were accurate, which gave me confidence to make adjustments in other students' error coding to help improve the consistency and accuracy of the data.

The following error types were eliminated because no students lost points for those reasons: *Missing/incorrect units*, *other 2 (testing)*, *multiplied instead of using exponents*, and *spelling*. *Arithmetic error* categories (with and without fractions) were augmented to include simple algebraic errors as well, so they became *arithmetic/algebraic error* types. *Graphing error* was augmented to include *graph-reading* as well. Two error types were renamed: *Illogical or incomplete proof* was changed to *incomplete explanation*. *Didn't provide final answer* was changed to *didn't factor completely* and was subsequently moved to the Content Errors Section. *Incorrect notation* was also moved to the Content Errors Section. Based largely on student errors coded to *other (math)*, the following content error types were added: *Omitted one of multiple solutions*, *included an extraneous solution*, and *'cancelled' wrongly*.

Table 5.1

Total Points Lost for each Error Type by Test, Including Mean and Standard Deviation

Error type	Test								Total	Mean	SD
	1	**2**	**3**	**4**	**M**	**5**	**6**	**7**			
Mean score	90	84	76	74	92	79	79	76		81	6
Testing points (generic – could apply to any subject)											
Ran out of time	1	29	78	187	5	12	43	35	390	49	57
Misread the question	52	9	30	31	20	9	6	13	170	21	15
Didn’t follow directions	2	28	71	13	0	23	10	4	151	19	22
Accidentally skipped the question	31	10	31	10	0	2	7	9	100	13	11
Other (testing)	10	5	1	10	20	4	7	19	76	10	6
Misread my own writing	3	3	1	6	0	4	9	4	30	4	3
Didn’t show steps	5	6	0	1	0	2	0	5	19	2	2
Unclear communication	0	0	4	6	0	6	2	0	18	2	3
Total testing points lost	104	90	216	265	45	62	84	89	1152	119	76
Content points (math specific)											
Didn’t know how	45	133	226	315	50	324	225	371	1689	211	117
Knew how, but forgot	29	51	79	116	65	60	45	80	525	66	25
Arithmetic/algebraic error – no fractions	10	61	23	57	15	150	114	41	471	59	46
Didn’t understand question	54	24	30	71	35	20	118	23	375	47	31
Confused two concepts	49	39	83	58	30	28	10	20	317	40	22
Didn’t distribute properly	6	69	13	41	10	21	53	67	280	35	24
Graphing/graph-reading error	46	0	97	52	30	23	2	0	250	31	31
Didn’t simplify	5	37	3	2	5	90	15	66	223	28	31
Used the wrong formula	16	15	75	14	0	20	82	0	222	28	30
Arithmetic/algebraic error – fractions	7	7	22	14	10	29	44	60	193	24	18
Didn’t combine known concepts	6	27	20	18	20	37	41	16	185	23	11
Omitted one of multiple solutions	0	101	7	0	0	0	25	25	158	20	32
Incorrect notation	14	0	36	15	0	10	48	14	137	17	16
“Cancelled” wrongly	0	0	0	0	0	0	0	132	132	17	44
Forgot vocabulary	27	0	21	15	0	12	11	0	86	11	10
Calculator usage error	0	5	54	14	0	5	6	0	84	11	17
Other (math)	0	30	15	21	0	0	1	1	68	9	11
Didn’t factor completely	0	0	10	17	25	0	0	9	61	8	9
Incomplete explanation.	4	1	0	32	0	1	0	0	38	5	10
Freshman’s dream (binomial	0	15	0	0	0	13	0	0	28	4	6
Included an extraneous solution	0	0	0	0	0	5	0	13	18	2	4
Total content points lost	318	615	814	872	295	848	840	938	5342	693	239
Total points lost	422	705	1030	1136	340	910	924	1027	6494	812	276

Forty-three students took each of the 100-point tests. Total points lost for each of the 43 students on each of the eight tests with test error analysis data is shown in Table 5.1, including means and standard deviations across the eight tests on which students analyzed their testing errors. The average points lost per student across all eight tests included 16 content points (693/43):

– 5 for *didn't know how*,
– 2 for *knew how, but forgot*,
– 1 for *arithmetic/algebraic error – no fractions*,
– 1 for *didn't understand question*,
– 1 for *confused two concepts*,
– 1 for *didn't distribute properly*,
– 1 for *graphing error*,
– 1 for *didn't simplify*,
– 1 for *used the wrong formula*,
– 1 for *arithmetic/algebraic error – fractions*, and
– 1 for *didn't combine known concepts*,

and 3 testing points (119/43):

– 1 for *ran out of time*,
– < 1 for *misread the question*, and
– < 1 for *didn't follow directions*.

About 15% of the errors were testing errors and 85% were mathematical content errors.

These results do not conflict with prior cognitive analysis of middle school students' problem solving behaviors where students committed reading-related errors on 22% of word problems (Pope, 2004). Similarly, Newman's research showed that 35% of low achieving sixth graders' errors were reading-related (Newman, 1977), and Clements found that 11% of errors committed by seventh graders were reading-related (Clements, 1980). In the 13 applied mathematics items contained in the seven unit tests of the present study, *didn't understand the question* caused errors 5% of the time and *misread the question* occurred 2% of the time, leading to a 7% reading-related error rate on the applied mathematics items. This reading-related error rate is lower than the prior research mentioned; I expected gifted ninth graders to be better readers than these younger subjects, even with more challenging items.

As a teacher, I was interested in both the quantities of points lost by test item and the number of students who had problems with each test item. In addition to many students losing many points, an item may surface in the most penalized listing by points lost by a few students losing many points or by many students losing a few points. Therefore, I included both total points lost by item and the number of students over whom those points were distributed. The graphs are included for quick visual comparison of these results for each item. (A wide disparity between points lost and number of students indicates the case of a few students losing many points, while close association between points lost and number of students may depict many students losing few points.) Additional details for selected test items are shown in Appendix L.

Test 1 Results

Table 5.2 shows Test 1 points lost by test item; this relationship is graphed in Figure 5.1. Two students earned perfect scores; the remaining 41 students lost a total of 422 points on the 20-item test. The items with the most points lost were Items 20 (on which 62 points lost were spread over 25 students), 19 (with 41 points distributed over 12 students), and 15 (39 points over 12 students).

Table 5.2
Summary of Points Lost by Item for Test 1

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	20	15	20	19	3	20	20	11	15	7	19	20	20	15	39	3	26	27	41	62	422
Students	4	3	4	11	3	4	4	5	3	6	15	6	16	12	12	2	18	10	12	25	

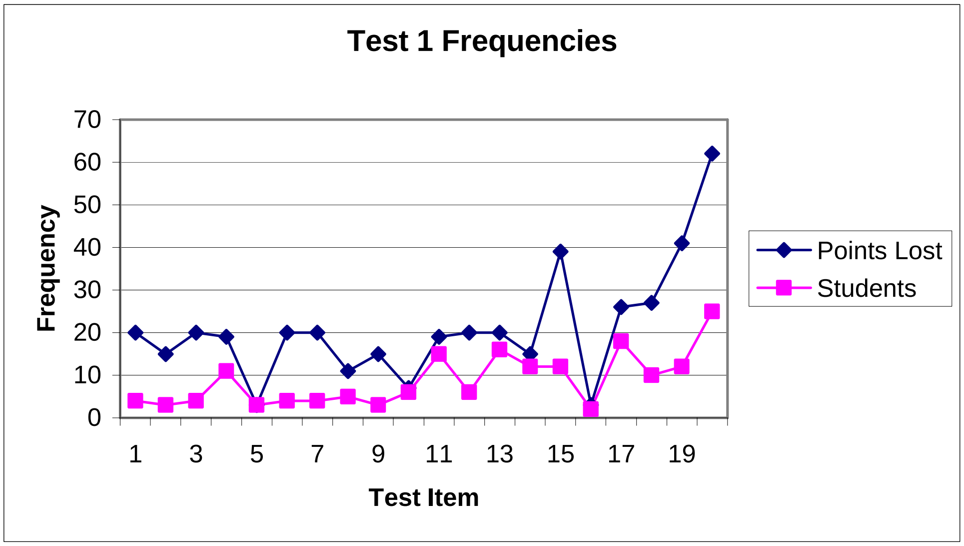


Figure 5.1. Points lost by test item for Test 1

Item 20 (with 62 points lost distributed over 25 students) follows:

20. Consider the following functions:

$m(x) = -4x + 3$ $a(x) = 4x + 3$ $t(x) = -4x - 3$ $h(x) = 4x - 3$

Which one is a reflection of *m* in the *y*-axis? _________

Which one is a reflection of *t* in the *x*-axis? _________

The students' largest hurdles were *not understanding the item* (with 16 points distributed over 5 students) and *misreading the item* (with 13 points distributed over 5 students). Function notation was confusing to some of the students. I was surprised that no students coded this error as being caused by *using the wrong formula*, even though part of the lesson was that $f(-x)$ would reflect $f(x)$ in the y-axis and $-f(x)$ would reflect it in the x-axis.

Twelve students lost a total of 41 points on Item 19, which read:

Compare each function below to its parent function $f(x) = x$ by **circling** the appropriate transformations **and filling in** the applicable **detail blanks,** if any.

	Function	Transformation	Details
19.	$h(x) = 1/4\ x$	vertical shift	_______ units up
		vertical stretch	_______ units down
		vertical shrink	with scale factor ____

The wording of the item may have seemed awkward to the four students who *misread* or *did not understand the item*. I had designed the item for ease of grading; it was less time-consuming for me to circle the proper transformation and fill in the appropriate blank than to write out the correct answer. In grading, I tried to write corrections, not just mark answers wrong. It is also possible that with better test preparation, the students who missed the item would have been able to respond correctly.

No students coded this error as being caused by using the wrong formula, even though they could have considered it a variation of $f(x) = a\ f(x - h) + k$ where $a = 1/4$, $h = 0$, and $k = 0$ with $|\ a\ |$ representing the scale factor of the vertical stretch or shrink, h representing the horizontal shift, and k representing the vertical shift. (Horizontal stretches and shrinks were not covered in this lesson.)

The third most difficult item on the first test was Item 15 on which 39 lost points were spread over 12 students. The item and a discussion of its test error analysis follow.

15. Find $f(4)$ if $f(x) = 4x + 9$.

In this item, all errors were attributed to content issues. Four students lost a total of 12 points for *not understanding the question*; 3 students lost 9 points because they *knew how, but forgot*. Two students lost 6 points for *confusing the two concepts* x and $f(x)$.

Additionally, 20 points spread over 16 students were lost on Item 13; the most common reason was failure to understand the parent function comparison (transformation) part of the item. Nineteen points lost were distributed over 15 students on Item 11, primarily for misreading the graph. This was partially due to poor item design; it was difficult to identify the slope in the figure. Overall, the most difficult concepts for students on the first test were function notation and transformation.

Test 2 Results

Table 5.3 shows Test 2 points lost by item; this relationship is graphed in Figure 5.2. A total of 705 points lost were distributed over the 43 students on the 20-item test. The most penalty points were for Items 17 (89 points), 16 (87 points), 11 (75 points), and 15 (74 points).

Table 5.3
Summary of Points Lost by Item for Test 2

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	35	0	10	30	5	10	5	45	5	60	75	19	37	25	74	87	89	26	47	21	705
Students	7	0	2	6	1	2	1	9	1	12	15	10	15	15	37	40	22	14	12	9	

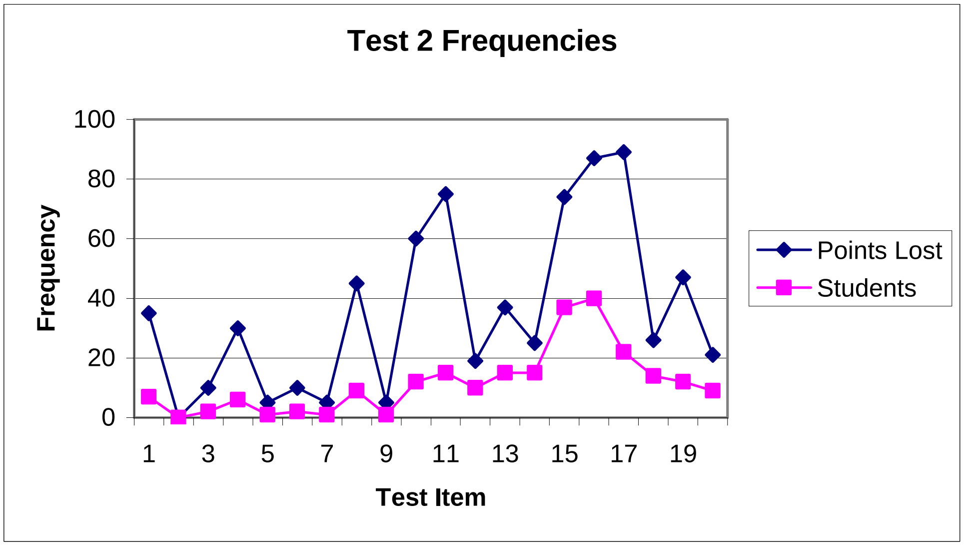


Figure 5.2. Points lost by test item for Test 2.

Item 17 (with 89 points lost distributed over 22 students) is shown below:

17. A rolling kickball is kicked into the air with an initial vertical velocity of 12 feet per second. How long will the ball be in the air, using the vertical motion function $h(t) = -16t^2 + vt + s$?

Nine students lost a total of 40 points because they *did not know how* to perform the application item. The remaining points lost on this item were distributed over 1 or 2 students per error type.

Item 16 follows on which 40 students lost at least one point (87 points lost distributed over the 40 students):

16. Solve by factoring: $w^3 + 4w^2 = 4w + 16$

Sixteen students *omitted one of multiple solutions*, accounting for 20 of the lost points. Some of these students used "guess and check" instead of factoring. Ten points lost were distributed over five students who *did not follow directions*. The five students who *confused two concepts* typically confused "solve by factoring" with "factoring"; they factored but did not solve.

Fifteen students lost a total of 75 points on Item 11, which read as follows:

11. Find the coefficient of x^3 in $(x - 6)^4$.

Twenty-five of the 75 points lost were due to *not simplifying*, typified by students who used only the entry (4) from Pascal's triangle, and did not include the $(-6)^3$ as part of the coefficient. Fifteen more points lost were spread over three students who *didn't know how*.

Thirty-seven students lost a total of 74 points on Item 15, which follows:

15. Solve by factoring: $x^3 - 19x^2 + 84x = 0$

Nineteen points were lost by 16 students for *omitting a solution*, typically $x = 0$. Most of the rest of the lost points were attributed to students who *did not know how* or committed *arithmetic or algebraic errors*.

Based on the Test 2 results, I concluded that the most difficult concepts for the students were applications, binomial expansion, and solving by factoring.

Test 3 Results

Table 5.4 shows Test 3 points lost by item; this relationship is graphed in Figure 5.3. A total of 1030 points were lost by the 43 students on the 20-item test. The most penalty points were for Items 20 (96 points), 18 (89 points), and 8 (84 points).

Table 5.4
Summary of Points Lost by Item for Test 3

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	13	31	55	71	36	50	48	84	34	26	74	32	70	23	61	51	37	89	49	96	1030
Students	12	11	23	35	20	23	20	28	10	8	26	10	26	11	16	13	9	34	12	23	

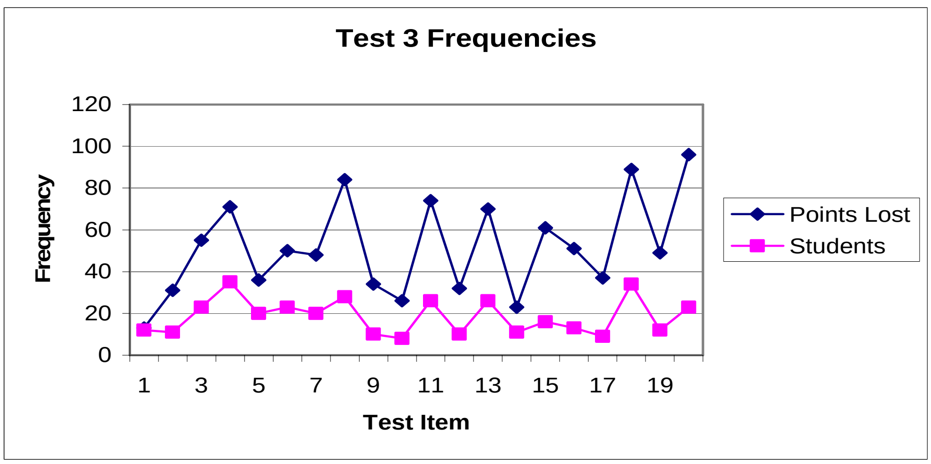


Figure 5.3. Points lost by test item for Test 3.

Twenty-three students lost a total of 96 points on Item 20 which read as follows:

20. The daily profit *y* of Quad Ratix, Inc., depends on how many of their trademark Para Bolas (*x*) are made. The profit can be found from the equation:

$$y = -\frac{1}{2}x^2 + 20x + 230$$

How many Para Bolas need to be made for a *maximum* profit? How much profit will that yield?

More than half of the points lost were for the 11 students who said they *did not know how* or *ran out of time*.

Thirty-four students lost a total of 89 points on Item 18, which read as follows:

18. A drink bottle is dropped off the top of a set of tall bleachers. The height y in feet above the ground can be given by $y = -16t^2 + 400$ where t is the number of seconds since the bottle was dropped. What is an appropriate domain and range for this relationship (in the context of this problem)?

Seven students lost 22 points because they *did not know how* to respond to the item. Eleven students lost 16 points for *notation* errors, typified by providing only integer values for the domain ({0, 1, 2, 3, 4, 5} instead of $0 \leq x \leq 5$) or giving a range of $400 \leq y \leq 0$ or $0 \leq x \leq 400$ instead of $0 \leq y \leq 400$.

Based on points lost, the third most difficult item was Item 8, which read as follows:

8. Graph the following. Include your vertex and two points on each side of it. Plot the axis of symmetry with a dashed line.

$$y = -\frac{1}{2}|x-1| + 5$$

Of the 24 students who had errors on Item 8, 12 had graphing errors, six *did not follow directions*, and three *did not know how*. Students who lost points by not following directions failed to include two points on each side of the vertex or omitted the axis of symmetry.

Additionally, 71 points lost were distributed over 35 students on Item 4, primarily because they did not make the connection between identifying the vertex on a graph and creating a quadratic equation in vertex or standard form.

Test 4 Results

Table 5.5 shows Test 4 points lost by item; this relationship is graphed in Figure 5.4. A total of 1136 points were lost by the 43 students on the 20-item test. The most penalty points were for Items 16 (123 points), 12 (116 points), 17 (84 points), and 19 (84 points).

Table 5.5
Summary of Points Lost by Item for Test 4

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	15	25	25	65	75	80	75	70	30	30	57	116	30	29	51	123	84	39	84	33	1136
Students	3	5	5	13	15	16	15	14	6	6	28	38	17	13	32	42	29	9	35	17	

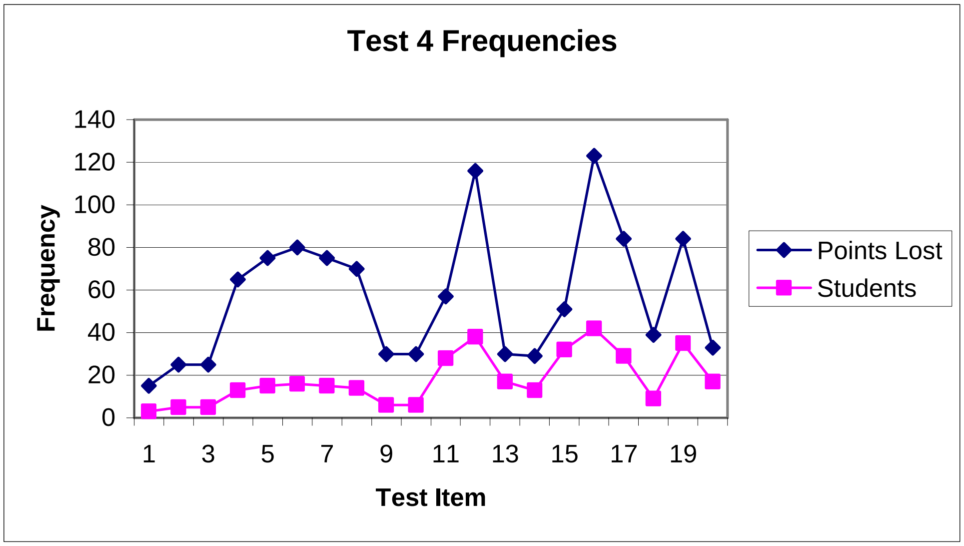


Figure 5.4. Points lost by test item for Test 4.

Item 16 (with 123 points lost distributed over 42 students) is shown below:

16. For the rational function $y = \frac{3}{x-2} - 1$, state the transformations from the parent function, determine the asymptotes, and state the domain and range.

The primary reasons given were *knew how, but forgot*; *didn't know how*; and *ran out of time*. Although eight students claimed to have *run out of time*, which is classified as a *testing* issue, better preparation for the test could have streamlined answering the items and improved the *content* of the responses.

Item 12 (with 116 points lost distributed over 38 students) follows:

12. Is $f(x) = 2x^4 - 3x^2 - 5$ even, odd, or neither? Justify your answer algebraically.

Twelve students said they lost points for *not knowing how*; five *knew how, but forgot*; and four *did not understand the question*. The four other students who *confused two concepts* typically confused odd and even functions. Few remembered how to show that if $f(-x) = f(x)$, the function was even and that if $f(-x) = -f(x)$, the function was odd. Many used only the exponents of x to determine that the function was even and failed to justify it algebraically.

Item 17 (with 84 points lost distributed over 29 students) read as follows:

17. The height of a football after t seconds that has been kicked is given by the function $h(t) = -16t^2 + 48t$.

    a. What is the maximum height of the football? ____________ Steps:

    b. If the crossbar of a goal post is 10 feet high, and it will take the ball 2 seconds to get to the goal post, will the ball still be high enough to go over the crossbar? ____________ Steps:

The primary error codes were *did not know how* and *ran out of time*. Some students lost points for *not showing steps*, but these errors were coded as *not following directions* since showing steps was explicit in the item.

Item 19 (with 84 points lost distributed over 35 students) is shown below:

19. Factor $-2x^2 + 2$: _________________________

Sixteen students lost a point each for *not factoring completely*. Most of the others said they *ran out of time* or *didn't distribute properly*.

Additionally, on Item 15, 32 students together lost 51 points due to the many opportunities for error in graphing a system of quadratic inequalities.

Based on error analyses of Test 4, primary content weaknesses were in differentiating between odd and even functions, factoring, and application.

Midterm Results

Table 5.6 shows midterm points lost by item; this relationship is graphed in Figure 5.5. The most penalty points were for Items 5 (80 points), 7 (55 points), and 16 (40 points). I was denied permission to include any midterm items but describe the most difficult ones by concepts assessed.

Table 5.6
Summary of Points Lost by Item for the Midterm

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	10	20	10	15	80	15	55	20	35	0	5	15	0	5	5	40	0	0	0	10	340
Students	2	4	2	3	16	3	11	4	7	0	1	3	0	1	1	8	0	0	0	2	

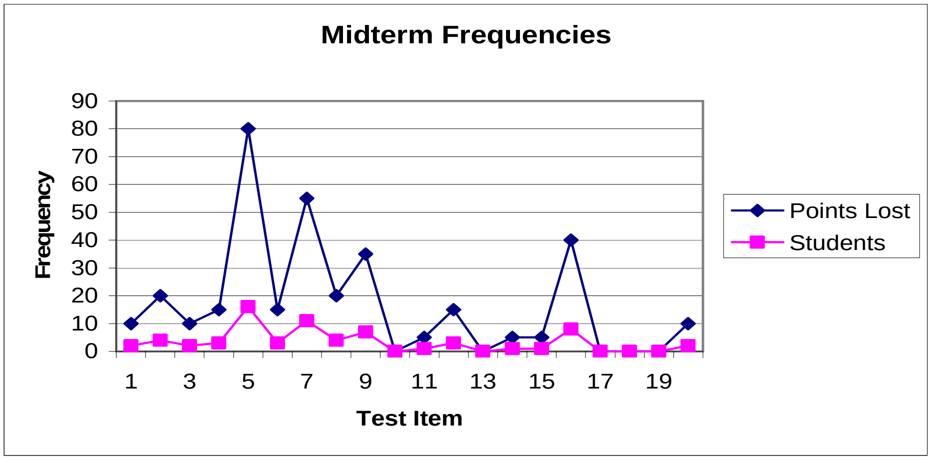


Figure 5.5. Points lost by test item for the midterm.

Item 5 dealt with domain and range for a rational function. Five students lamented that they *did not know how* to respond; three students *did not understand the question*; three students made *graphing errors*.

Item 7 was the second most-missed item; it asked for the expansion of a binomial cubed. Five of the 11 students who missed it claimed to have *known how, but forgotten*.

The third most difficult item was Item 16, which asked for complete factorization of a binomial. The students' most prevalent error type was *not factoring completely*.

Furthermore, seven students lost 35 points on Item 9, primarily because they *knew, but forgot how* to factor a polynomial that was the cube of a binomial. Rational functions, binomial expansion, and factorization surfaced as the students' weakest areas halfway through the semester.

Test 5 Results

Table 5.7 shows Test 5 points lost by item; this relationship is graphed in Figure 5.6. The most penalty points were for Items 11 (127 points), 13 (83 points), and 10 (73 points).

Table 5.7
Summary of Points Lost by Item for Test 5

	Item																				Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Points	30	25	23	44	14	43	42	42	11	73	127	64	83	60	40	48	42	20	39	40	910
Students	9	10	13	26	9	22	12	26	9	29	39	15	23	34	17	24	17	7	13	10	

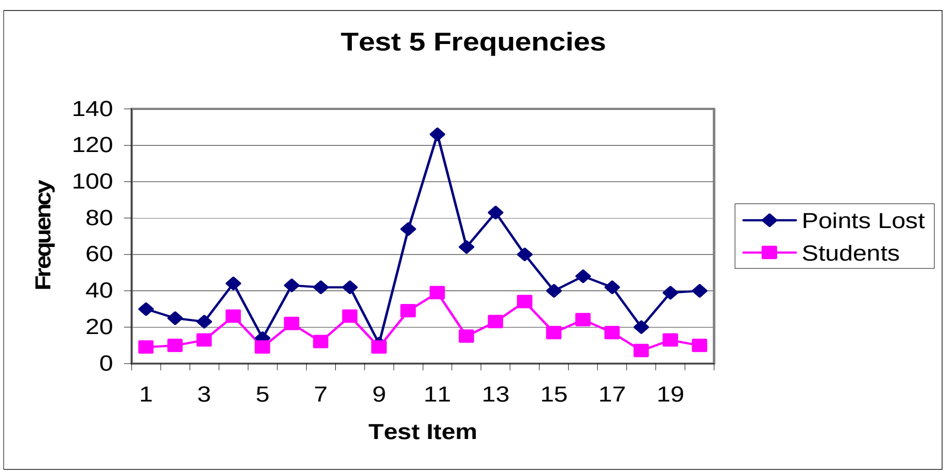
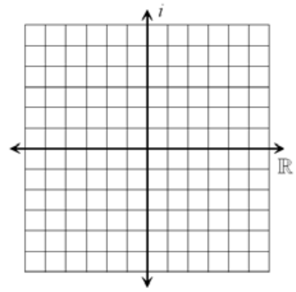


Figure 5.6. Points lost by test item for Test 5.

Item 11 (with 127 points lost distributed over 39 students) is shown below:

Graph the following complex numbers in the complex plane, and then find the absolute value of each number:

11. $B: 4 + 5i^2$ ____________________



This item proved to be more taxing than I expected because many students did not realize the benefits of simplifying i^2 to -1 before graphing. Twelve students blamed their errors on *not simplifying*; eleven on *not knowing how*.

Item 13 (with 83 points lost distributed over 23 students) follows.

13. Solve for x (and y if possible): $15i - 14 = 4y + 3xi$

Nine students lost 35 points for *not knowing how*.

Item 10 (with 73 points lost distributed over 29 students) had the same directions as Item 11 above and read as follows:

10. A: $3 + -5i$

Additionally, 60 lost points were spread over 34 students on Item 14, primarily for thinking that the imaginary square root of -1, i, was a variable because it has some similarities with variables; for example, we use a letter to represent it. In general, complex numbers were new to these students and the errors on this test reflected gaps in their knowledge.

Test 6 Results

Table 5.8 shows Test 6 points lost by item; this relationship is graphed in Figure 5.7. The most penalty points were for Items 5 (180 points), 4 (117 points), 12 (93 points), and 14 (90 points).

Table 5.8
Summary of Points Lost by Item for Test 6

	Item														Total
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Points	61	26	14	117	180	51	10	46	59	87	34	93	56	90	924
Students	33	12	7	24	36	24	7	26	31	22	23	34	16	30	

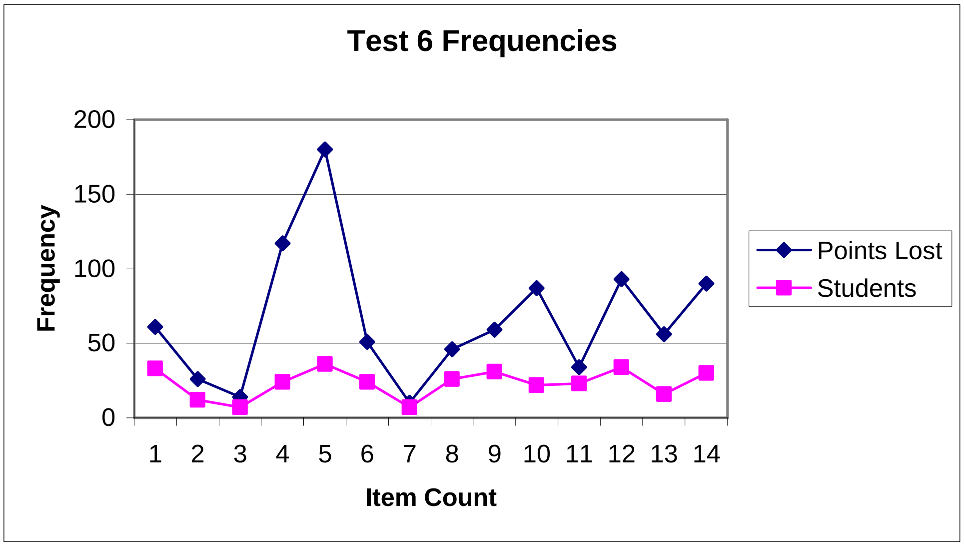


Figure 5.7. Points lost by test item for Test 6.

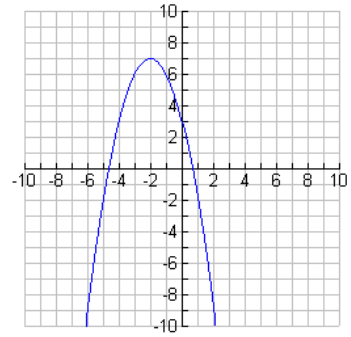
Item 5 (with 180 points lost distributed over 36 students) follows:

5. Frank's job pays him based on the number of sales he makes. His pay *per sale* is equal to the number of sales plus 20. He made $800 last week. Write an equation to determine the number of sales he made last week and solve.

Twenty-seven students said they *did not understand the item* or *did not know how*. This item seems unrealistic and irrelevant to the students' everyday lives, and I would not use it with future classes, since I think more engaging questions would enhance learning and testing results (e.g., Mathematical Sciences and Education Board, 1990, 1993).

Not knowing how to respond and *not understanding the item* were also the main reasons for missing Item 4 (with 117 points lost distributed over 24 students), which follows:

4. The graph of a quadratic function is provided. Use the graph to describe what you know about the discriminant of its quadratic equation.



This item prompted a class discussion of the difference between a quadratic equation, a quadratic function, and the quadratic formula which seemed to help students clarify the vocabulary.

The third most difficult item was Item 12 (with 93 points lost distributed over 34 students). It reads as follows:

12. Complete the square to put in vertex form: $y = 4x^2 - 8x + 23$. Find the vertex.

Not knowing how, *using the wrong formula*, and *arithmetic* errors prevented 17 students from answering Item 12 correctly.

Item 14 (with 90 points lost distributed over 30 students) follows:

14. A quadratic function, $f(x)$, has an average rate of change of -3 over the interval $-4 \le x \le -1$ and an average rate of change of 3 over the interval $-1 \le x \le 2$. Which way does the parabola open? What is its axis of symmetry?

The predominant reason for missing Item 14 was *not knowing how*.

In addition, 61 points lost were spread over 33 students who had trouble writing a quadratic equation in standard form with given solutions, primarily because they *did not know how*. Student knowledge gaps for Test 6 included discriminants, completing the square, average rates of change, axes of symmetry, and application of the concepts. *Not knowing how* was the primary reason for making errors on the assessment and told me that I had not facilitated enough student construction of these mathematical concepts.

Test 7 Results

Table 5.9 shows the breakdown of 1,027 points lost by the 43 students on the 11-item test, as graphed in Figure 5.8. The most penalty points were for Items 10 (176 points), 1 (138 points), and 11 (119 points).

Table 5.9
Summary of Points Lost by Item for Test 7

	Item											Total
	1	2	3	4	5	6	7	8	9	10	11	
Points	138	66	99	76	64	41	65	100	83	176	119	1027
Students	23	22	26	23	31	16	18	19	17	40	34	

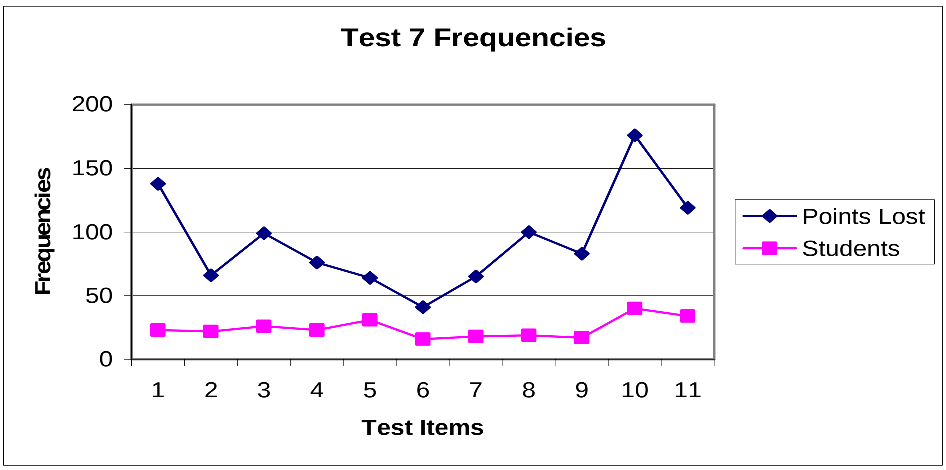


Figure 5.8. Points lost by test item for Test 7.

Item 10 (with 176 points lost distributed over 40 students) follows:

10. Solve for x: $\dfrac{x+2}{x-4} - \dfrac{2x}{x-1} = \dfrac{18}{x^2-5x+4}$

The most significant reasons for missing Item 10 were *not knowing how* and *forgetting*. In addition, ten students *included an extraneous solution*.

Twenty-three students lost a total of 138 points on Item 1, which read:

1. Simplify the expression. State the excluded value: $\dfrac{28x}{7x-21}$

All errors for this item were coded as content errors. *Not knowing how* and *"canceling" wrongly* were the two most significant reasons.

The third most difficult item on the last test was Item 11 (with 119 points lost distributed over 34 students), which follows.

> 11. You have a rectangular deck that has length that is 3 times as big as the width. If *x* is the width of the deck write an expression for the ratio of the perimeter to the area of the deck. Simplify if possible.

The primary reasons for losing points on Item 11 were: *not knowing how*, *not understanding the question*, and *not simplifying*.

Thirty-one students lost at least one point on Item 5, which asked for long division of a binomial into a trinomial; seven students used *incorrect notation* when expressing the remainder, four *didn't distribute properly* when subtracting, and three *didn't know how*.

In general, missed points on Test 7 were due to *not knowing how*, *forgetting*, and *"canceling" wrongly*, indicating that rational expressions still posed problems for students. *Arithmetic/algebraic error – fractions* did not surface as a major problem for students in the test error analysis. However, I believe that better understanding of fractions and their operations might have improved student scores on Test 7.

Of the 18 errors coded to *Arithmetic/algebraic error – fractions*, the most common error, occurring three times, was treating a rational sum or difference like a proportion by cross-multiplying instead of finding a common denominator or eliminating the common denominator. This finding differs from prior research (Lankford, 1972), which found that the most common error among seventh-grade students when adding fractions was to add the denominators as well as the numerators. I attribute the difference to our emphasis on solving rational equations, including proportions, in addition to operations on rational expressions.

Summary of Most Common Errors

Recall that the third set of research questions was as follows: "What are the most common types of errors, according to the analysis, in a mathematics course? How does this information inform the students and me to promote the learning of mathematics?" While I expected content topics such as quadratics, complex numbers, and rational equations to surface in the highlights of points lost, common themes emerged as well: transformations, factoring, and application items. Errors in these areas spanned multiple tests and content areas.

In addition, the analysis gave me insights into the assessment process to better understand the students' perspective of the test items. This analysis was more helpful than I expected, not only in going over tests with these students but also in planning for future classes. Since so many of the error types were coded to *not knowing how* and *forgetting*, it was clear which topics most needed to be revisited.

I used the items with the most points lost and the most students affected to develop a review for the students for their final exam in the first semester and for my spring classes for their end-of-course test, which was comprehensive for the year at the end of the second semester. Since I will be teaching the same course in the future, I will be better prepared to emphasize key points for learning in class, to discuss likely testing pitfalls with students before assessment, and to prepare more student-centered assessments (using *did not understand the question* data to identify and improve problematic test items).

Recall that the most common testing error type was *ran out of time*. Students who were ill-prepared often used inefficient strategies, such as guess-and-check to arrive at solutions. They spent inordinate amounts of time on a few items at the expense of addressing the remaining items on the tests.

Using the data from Table 5.1, I calculated correlations between tests based on frequencies of each error type using the Pearson *r* correlation coefficient, as shown in Table 5.10 below. The 29 error types created the 29 cases for the correlation statistic. The earlier test is shown with the later test to see how well points lost to a given error type on one test would predict points lost to that error type on the next test. Test correlations statistically different from zero, based on *p*-values of .05 or lower, are indicated in bold. The *Total* in Table 5.10 refers to the total number of errors for each error type across all eight tests (the *Total* column in Table 5.1.

Table 5.10
Pearson r*-Values Showing Correlations of Points Lost by Error Type from Test to Test*

Subsequent Test	Prior Test							
	1	2	3	4	Midterm	5	6	7
1								
2	0.18							
3	**0.54**	**0.55**						
4	**0.42**	**0.64**	**0.84**					
Midterm	**0.64**	**0.43**	**0.58**	**0.62**				
5	0.32	**0.73**	**0.72**	**0.77**	**0.52**			
6	**0.39**	**0.69**	**0.67**	**0.76**	**0.50**	**0.84**		
7	0.26	**0.68**	**0.67**	**0.77**	**0.49**	**0.86**	**0.73**	
Total	**0.47**	**0.77**	**0.86**	**0.92**	**0.66**	**0.92**	**0.88**	**0.90**

It is interesting to note that the first two tests and the midterm had less correlation than the others with the total points lost by error type. This could be due to the fact that the grades were higher on these tests and, therefore, these tests had fewer points lost to errors.

The three pairs of tests with the highest correlation were Tests 5 and 7, Tests 3 and 4, and Tests 5 and 6. Tests 5 (on radical expressions) and 7 (on rational expressions) had extremely high correlation as they focused on symbolic algebra, such as order of operations, but not spatial relationships (although graphing could have been used for some solutions). Tests 3 and 4 shared an emphasis on quadratics. Tests 5 and 6 both featured complex numbers.

Grouping and Trends

I was particularly interested in trends of errors my students made. Did students lose fewer points on some error types as the semester progressed? I had originally planned to conduct latent transition analysis on the eight tests, but that proved unrealistic with n = 43 students (L. K. Muthén, personal communication, June 20, 2010).

I did not identify any steady reductions or increases in points lost per error type for any of the error types other than the spike from 0 on the first seven tests to 132 on the last test for *"cancelling" wrongly*. I decided to look at the mean points lost from the first half of the semester to the second half of the semester. Since opportunities for testing errors (which were theoretically not content specific) should have been relatively consistent across all eight tests, I used total errors for Tests 1–4 to evaluate errors for the first half of the semester and I used the midterm and Tests 5–7 for the second half of the semester. This relationship is shown in the top portion of Table 5.11.

The following testing error types seemed to show significant reduction of points lost over the course of the semester:

– *accidentally skipped the question*

– *ran out of time*

– *didn't follow directions*

– m*isread the question*

In general, it seemed that students grew more careful about preventing these errors over the course of the semester. In particular, learning to take tests in a 53-minute period instead of roughly the 90-minute periods some of the students had in eighth grade was an adjustment. Additionally, I allowed no extra time for testing unless there were extenuating circumstances;

many students had been allowed to finish tests after school or the next day in middle school. The fact that we did not retest in high school was also a surprise for some of the students. These factors combined to influence many students to learn and to practice better time-management skills during testing. While the test error analysis should have reinforced the need for these changes, I think it is likely that students would have improved their testing efficiency without it. The substantial reductions in *running out of time* and the other testing error types highlighted contributed to an overall decrease in total testing points lost from the first half to the second half of the semester (from 766 to 386, a 50% reduction).

I took a different approach to compare content error points lost during the two halves of the semester. Since neither the first test nor the midterm contained new content, I eliminated them from the total points lost for each semester half for the comparison. The total points lost for content error types for Tests 2–4 and 5–7 are shown in the bottom portion of Table 5.11.

Table 5.11

Comparison of Semester Halves' Total Points Lost for Each Error Type

Testing error type	Total points lost for tests		Change	Percent change from 1st to 2nd half of semester
	1–4	M–7		
Ran out of time	295	95	–200	–68
Misread the question	122	48	–74	–61
Didn't follow directions	114	37	–77	–68
Accidentally skipped the question	82	18	–64	–78
Other (testing)	26	50	24	92
Misread my own writing	13	17	4	31
Didn't show steps	12	7	–5	–42
Unclear communication	10	8	–2	–20
Total testing points lost	766	386	–380	–50
Content error type	**Total points lost for tests**		**Change**	**Percent change from 1st to 2nd half of semester**
	2–4	5–7		
Didn't know how	674	920	246	36
Knew how, but forgot	246	185	–61	–25
Arithmetic/algebraic error – no fractions	141	305	164	116
Didn't understand question	125	161	36	29
Confused two concepts	180	58	–122	–68
Didn't distribute properly	123	141	18	15
Graphing/graph-reading error	149	25	–124	–83
Didn't simplify	42	171	129	307
Used the wrong formula	104	102	–2	–2
Arithmetic/algebraic error – fractions	43	133	90	209
Didn't combine known concepts	65	94	29	45
Omitted one of multiple solutions	108	50	–58	–54
Incorrect notation	51	72	21	41
"Cancelled" wrongly	0	132	132	Undefined
Forgot vocabulary	36	23	–13	–36
Calculator usage error	73	11	–62	–85
Other (math)	66	2	–64	–97
Didn't factor completely	27	9	–18	–67
Incomplete explanation.	33	1	–32	–97
Freshman's dream (binomial multiplication)	15	13	–2	–13
Included an extraneous solution	0	18	18	Undefined
Total content points lost	2223	2545	322	14

I noticed several changes with this comparison. For example, students tended to fare better in avoiding the following error types:

– *Graphing and graph-reading error*

– *Confused two concepts*

– *Calculator usage error*

– *Omitted one of multiple solution*

They tended to lose more points on the following error types the second half of the semester:

– *Didn't know how*

– *Didn't simplify*

– *Arithmetic/algebraic error – no fractions*

– *Arithmetic/algebraic error – fractions*

– *Included an extraneous solution*

– *"Cancelled" wrongly*

The students lost more content points over time (from 2223 in the first half of the semester to 2545 in the second half, a 14% increase) on the tests with new content. Some of these changes in points lost per error type were strongly influenced by the test content. For example, students had considerably more opportunities to *include extraneous solutions* or to *"cancel" wrongly* in Test 7.

Overall, students' unit test scores tended to decline as the semester progressed. I attribute this trend, in part, to the high grades on the first two tests, which had more review items from their eighth-grade curriculum, and in part to the increasing difficulty of the material, with new mathematical concepts frequently building upon previous ones. In general, students got better at avoiding testing errors, but their content errors increased over time.

Test error analysis could have partially explained the students' success on the comprehensive midterm and finals; these cumulative tests were their most significant chance to retest any content errors and the students averaged 12 points higher on the midterm (92), 6 points higher on the performance exam (86), and 2 points higher on the final exam (82) than on the unit tests (mean: 80). Any students who truly learned from their content errors should have excelled on these summative exams. Alternatively, the formats of these comprehensive exams were different: the homogenous multiple-choice midterm and final exams might have been easier for the students, especially those inclined to use a "guess and check" strategy, than the unit tests. Likewise, the performance exam, with its emphasis on communication might have provided an advantage to the more linguistically talented students.

Latent Class Analysis of Testing Error Types

Latent class analysis (LCA) is a statistical method used to separate observed data into latent (unobserved) groups or classes. I used MPlus statistical software to conduct LCA. I started with the 43 students' total responses over eight tests using the errors coded to the eight testing error types. I isolated testing error types for two reasons. (1) Using only testing error types kept the number of error types analyzed at one time smaller since the number of cases n was 43. (2) Opportunities for testing problems could be considered constant across the eight tests, but content changed throughout the semester and, therefore, opportunities for different types of content errors were more variable.

Because of the large number of zeros in the data, I considered them to be count data approximating a Poisson distribution. The first challenge was to determine into how many classes to split the students. Two indices were used to decide the number of latent classes: the Akaike Information Criterion (AIC; Akaike, 1987) and Bayesian Information Criterion (BIC;

Schwartz, 1978). The model that yields the smallest values on these indices indicates the best-fitting model. Both AIC and BIC favored the 3-class model, as shown by their lower values in Table 5.12. The 1-class model failed to terminate normally due to an insufficient number of expectation (E) steps, even with the number of iterations for the expectation maximization (EM) algorithm set to 1000, as did the 4-Class model (due to an ill-conditioned, non-positive definite Fisher information matrix and other errors, even with 4000 initial stage starts and 1000 final stage optimizations) so they were not considered viable.

Table 5.12
Summary of Information Criteria to Select Number of Classes for Testing Errors

Information criterion	Model			
	1-class	2-class	**3-class**	4-class
AIC	Model did not terminate normally.	1834.66	**1746.76**	Model did not terminate normally.
BIC		1864.61	**1792.55**	

The 3-class model grouped 18 students into Class 1, 8 students into Class 2, and 17 students into Class 3. The run chart in Figure 5.9 shows the estimated means on the eight error types. The values on the *x*-axis represent the eight error types as shown in the key. The values on the *y*-axis represent total points lost by error type for the eight tests. The error types that most defined the separation in the classes were: *accidentally skipping the question, not following directions, misreading the question,* and *running out of time*.

Class 1 students were the most successful, making few errors of any type. Class 2 students had high incidences of *accidentally skipping items*, but tended to be quite successful in other areas. Based on follow-up questions, these students often planned to come back to answer an item, but failed to do so or did not realize that they had only answered part of an item. Class 3 students were most likely to *run out of time*, *misread the item*, or *not follow directions*.

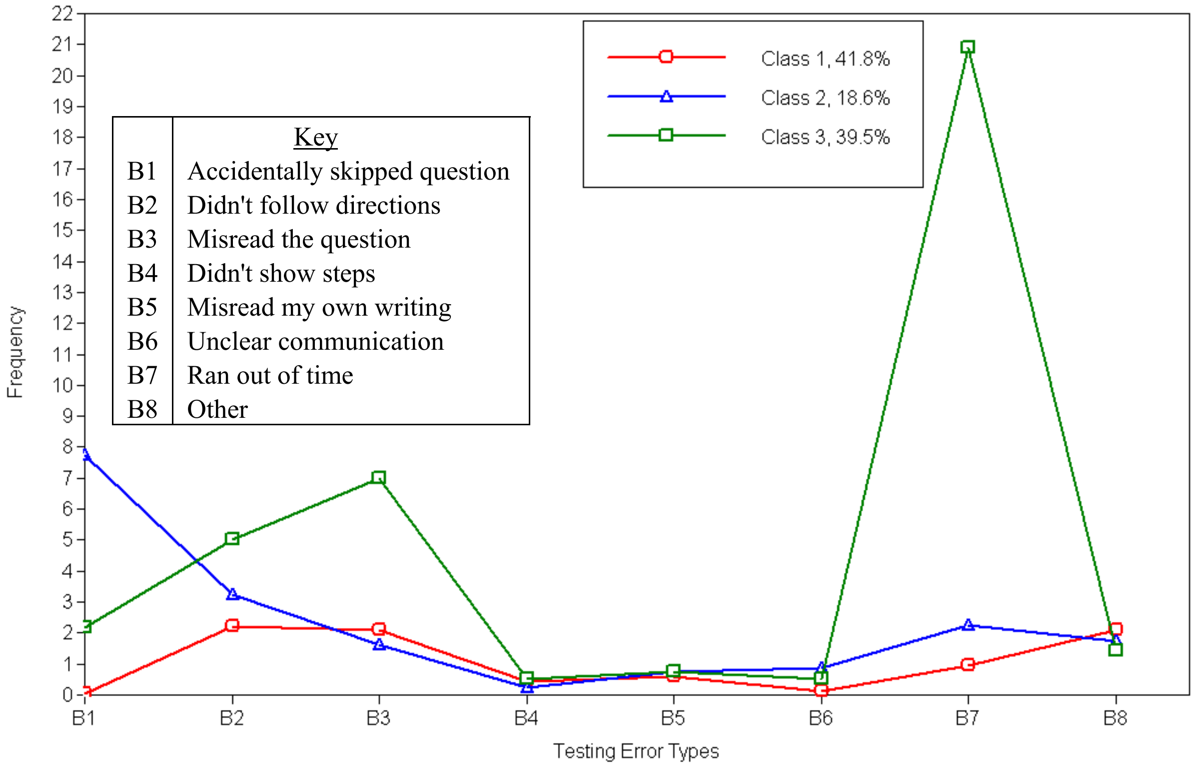


Figure 5.9: LCA of testing error types on the 8 tests with 3 classes

The most prominent differentiator overall was *ran out of time* which set Class 3 apart from the others. Follow-up questions with students found that many students who ran out of time were not using the most expedient methods of solving, but instead were using inefficient strategies like "guess and check." That especially made identifying fractional and irrational solutions difficult and time consuming, not only affecting the item in question but also taking up valuable time from other items on the test.

LCA of Content Error Types

I approached the LCA of the content errors a little differently from the testing errors. First, there were 21 different error types classified as content errors. I could tell from the LCA of the testing errors that the error types with low frequencies left little room to differentiate classes of students. So I chose the nine error types that had frequencies higher than 200. Second, since the numbers were so much larger for the content errors than for the testing errors (because students lost over 5 times as many points for content errors as testing errors), the data were considered to be continuous (instead of count), as encouraged for more reliable computations by the MPlus software.

For the above reasons, I conducted LCA of the 43 students' total responses over eight tests using the nine content error types that had the highest amounts of points lost. Since the BIC numbers were so close for the 1-, 2-, and 3-class models, I determined that the AIC and BIC agreed on the 3-class model, as shown by the values in Table 5.13. MPlus suggested that the 4-Class model might not be trustworthy (because of a non-positive definite first-order derivative product matrix), so it was not considered. Figure 5.10 shows the estimated means for the 3-class model.

Table 5.13
Summary of Information Criteria to Select Number of Classes for Content Errors

Information Criterion	Model			
	1-class	2-class	3-class	4-class
AIC	2737.32	2722.56	2707.03	Model was deemed untrustworthy.
BIC	2769.02	2771.87	2773.95	

The 3-class model grouped ten students in Class 1, seven students in Class 2, and 26 students in Class 3. In the 3-class model, Class 1 was least likely to say they *did not know how*, but most likely to have *confused two concepts* or *made graphing errors*. Class 2's students were most likely to *not know how*, make *arithmetic errors*, and *not distribute properly*. Class 3's students were more likely to claim that they *forgot* or *did not simplify*; they bested their peers at arithmetic and distributing. Table 5.14 shows the student quantities and first eight test score means for each class.

Table 5.14
Number of Students (and First Eight Test Score Means) for Testing and Content Classes

Content Classes	Testing Classes		
	1	2	3
1	3 (87)	2 (76)	5 (78)
2	0	4 (76)	3 (75)
3	15 (86)	2 (82)	9 (79)

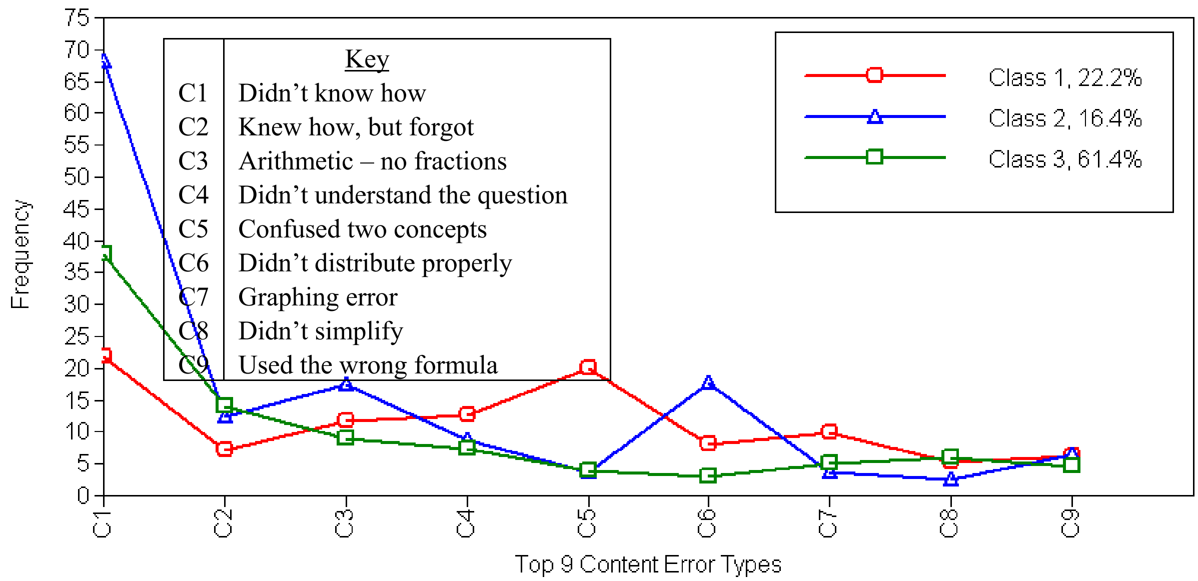


Figure 5.10: LCA of content error types on the 8 tests with 3 classes

The top two error types overall, by points lost, were *did not know how* and *knew how, but forgot*. I was particularly interested in this distinction because it seemed to indicate a difference between lacking a mathematical construction of concepts and challenges in retrieving acquired knowledge. I conducted LCA for each of the tests using only these two error types. Table 5.15 shows information criteria used to select the number of classes. AIC and BIC disagreed on the number of classes. I chose the more conservative 2-class approach which would allow for simpler computation over the eight tests. MPlus failed to provide two classes for the midterm, so it was omitted.

Table 5.15
Summary of Information Criteria to Select Number of Classes for Top Two Errors

Information Criterion	Model			
	1-class	2-class	3-class	4-class
AIC	755.06	745.02	742.82	**738.87**
BIC	762.10	**757.35**	760.43	761.77

Table 5.16 shows the frequency of each pattern for the 43 students over the remaining seven tests. In each case, a *1* represents the class that scored lower in the *did not know how* category; a *2* represents joining the class that scored higher in *did not know how*. Any pattern that is not listed did not occur. From the table, I determined that nine students consistently *knew how* as evidenced by their permanent stay in Class 1; four students only joined Class 2 for one test; and 11 students only joined Class 2 for two tests.

Table 5.16
Frequency Student Patterns between Classes over the Seven Tests

Pattern	Frequency
1 1 1 1 1 1 1	9
1 1 1 1 1 1 2	1
1 1 1 1 1 2 1	2
1 1 1 1 1 2 2	1
1 1 1 1 2 1 2	2
1 1 1 1 2 2 1	1
1 1 1 1 2 2 2	2
1 1 1 2 1 1 1	1
1 1 2 2 1 1 1	1
1 1 2 2 1 2 2	1
1 1 2 2 2 2 2	1
1 2 1 1 1 1 2	2
1 2 1 1 1 2 2	1
1 2 1 1 2 1 1	1
1 2 1 1 2 2 1	1
1 2 1 2 1 1 1	2
1 2 1 2 1 2 1	1
1 2 1 2 1 2 2	2
1 2 1 2 2 1 1	1
1 2 1 2 2 2 2	1
1 2 2 1 1 1 1	1
1 2 2 2 1 1 1	1
1 2 2 2 1 2 2	1
1 2 2 2 2 2 2	1
2 1 2 1 1 1 2	1
2 2 1 1 1 1 1	1
2 2 1 1 1 1 2	1
2 2 2 2 1 2 2	1
2 2 2 2 2 2 2	1

Since n = 43 did not provide enough cases for MPlus to complete a latent transitional analysis, I used the table to calculate the transitional probabilities of moving between classes from test to test manually (see Table 5.17). One should be mindful that these empirical probabilities are based on the limited number of cases.

Table 5.17
Probabilities of Staying or Moving between Classes from Test to Test

Probabilities of Staying or Moving between Classes from Test to Test												
Test 1	Stay/ Move	**Test 2**	Stay/ Move	**Test 3**	Stay/ Move	**Test 4**	Stay/ Move	**Test 5**	Stay/ Move	**Test 6**	Stay/ Move	**Test 7**
Class 1	.58 .42	**Class 1**	.83 .17	**Class 1**	.76 .24	**Class 1**	.74 .26	**Class 1**	.68 .32	**Class 1**	.72 .28	**Class 1**
Class 2	.80 .20	**Class 2**	.30 .70	**Class 2**	.80 .20	**Class 2**	.31 .69	**Class 2**	.67 .33	**Class 2**	.72 .28	**Class 2**

Table 5.17 shows how likely a student in one class was to move to the other from one test to the next. For example, a student in Class 1 with the low incidence of *not knowing how* on Test 3 had a 76% chance of staying in that class on Test 4. What was interesting about this to me was how much more stable the Class 1 students were (how much more likely they were to stay in Class 1) than the Class 2 students. It was comforting to see that the chances of moving from Class 2 to Class 1 were good from Test 2 to Test 3 (which both focused on quadratics) and from Test 4 to Test 5 (which both shared square root relationships).

Student-Constructed Advice to Prevent Errors

After analyzing test errors from the first test, each of the three classes brainstormed their advice, as designed. I was pleased with the creativity and practicality of their ideas. The following three suggestions highlight the potential of their insights.

To help prevent *freshman's dream*, students recommended use of an area model such as the one shown in Figure 5.11.

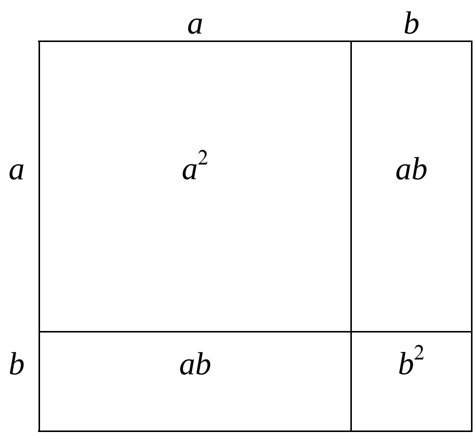


Figure 5.11 Area model representation of $(a + b)^2$.

This model helps students to synthesize the geometric relationship of binomial multiplication with the symbolic relationship, providing a new way of thinking about $(a + b)^2$ (Tillema, 2009).

Students also recommended memory devices to counter problems with *incorrect notation* and *not knowing how*. They offered similar assistance to *freshman's dream* sufferers: As a way to remember to multiply all term combinations of the product of two binomials $(a + b)^2 = (a + b)(a + b)$, they suggested First ($a^2$), Outside ($ab$), Inside ($ab$), and Last ($b^2$), aka FOIL, a mnemonic recognized as being related to mathematical content (Kilpatrick, 1985).

Studying and practicing was a common theme for preventing most error types. Third period was particularly energized by a term they coined that they thought would work for most of the error types: "Studytize!"

My original plan was to summarize the advice from each class and have three different lists. However, I did not want to deprive one class of the insights of another, so I consolidated all of the advice from the three classes, paraphrasing but trying to capture the essence of each class's ideas. As I presented the consolidated advice to each class, I asked them to make sure that the list accurately represented their thoughts. Some students in third period expressed disappointment that I had paraphrased their coined word “studytize” into boring verbs like “study” and “practice.” I was excited that they had taken so much ownership in their creation of the advice. I apologized for misrepresenting their idea and encouraged them to write it in on their copies in bold print. The consolidated student-constructed advice for all three classes is shown in Appendix M.

CHAPTER 6

STUDENT QUESTIONNAIRE RESPONSES

All students responded to the end-of-semester questionnaire (see Appendix N), although some items were left unanswered. In the questionnaire, I asked the students for their insights into the test error analysis processes. It should be noted that this questionnaire was not anonymous. I required the students to put their names on their responses to be sure I had a response from each student, to allow possible correlation of responses with other data, and to be able to follow up with questions if a response was unclear to me. That requirement may have introduced bias in the responses of students who wanted to impress me in a positive (or negative) way. The coefficient α estimate of reliability for the quantitative responses to the questionnaire was $\alpha = .73$

Section A

The four items in Section A were designed to get information about how the students prepared for their tests. For each of these items, I asked the students to circle 0, 1, 2, 3, or 4 or write in a response for a number more than four. All 43 students responded; some depicted non-integer values, which were recorded as indicated.

The first item was as follows:

> A1. On average, I spent ___ hours outside of class preparing for each test (beyond daily assignments).

Table 6.1 provides the frequency of the responses and the mode for the first item. I knew how well my students used their class time. I also knew who spent time on daily assignments outside of class. This item was designed to help me evaluate other effort that may have influenced the students' test scores. The responses seemed to validate and complement my prior knowledge of students' efforts.

Table 6.1
Summary of Questionnaire Responses for Item A1

	Response										Mode
Measure	0	0.25	0.5	1	1.5	2	3	4	5	6	
Frequency	8	1	6	13	1	8	2	1	2	1	1

The 43 responses to the next three items are summarized in Table 6.2.

Table 6.2
Summary of Questionnaire Responses for Items A2–A4

Item	Response Frequency						Mode
	0	1	2	3	4	5	
A2. I came to Ms. Daymude for extra help before or after school about ____ times a week.	32	8	3	0	0	0	0
A3. I was tutored by someone other than Ms. Daymude about ____ times a week.	29	8	3	1	0	2	0
A4. I reviewed my test error analysis data about ____ times this semester.	27	9	3	2	2	0	0

Although I encouraged students to come in for extra help before or after school, most students did not take advantage of that offer. This is substantiated by the questionnaire responses to Item A2.

Based on student comments during the semester, some students had paid tutors; others were helped by friends and relatives, including older siblings. Most students, however, did not get much help from others, as the responses to Item A3 indicate.

Responses to Item A4 indicate that most students did not review their test error analysis data. This may explain why many did not benefit from it.

Section B

The second section of the questionnaire contained 11 statements. The first nine were in a Likert format. I asked the students to consider each statement provided and respond with *1* for *Strongly Disagree*, *2* for *Disagree*, *3* for *Neutral*, *4* for *Agree*, and *5* for *Strongly Agree*. All 43 students responded to the first eight items. Forty-two students responded to the ninth item numerically; the other student did not circle a response, but noted that her "grades went up & down."

Student responses to these statements and the mean response are summarized in Table 6.3. Statements 4 and 9 were negatively worded, so the reverse mean is shown after the standard mean.

Table 6.3
Summary of Questionnaire Responses for Items B1–B10

Statement	Frequency per Response					M/Rev
	1	2	3	4	5	
1. I prepared well for my tests.	2	5	23	10	3	3.2
2. Test error analysis helped me learn from my testing errors.	8	14	12	8	1	2.5
3. Test error analysis helped me prepare for subsequent tests.	6	20	13	4	0	2.3
4. Test error analysis took too long.	1	20	11	8	3	2.8/3.2
5. The class's advice in preventing testing errors was helpful to me.	2	13	15	12	1	2.9
6. Test error analysis helped me explain my testing performance to my parents.	7	16	14	4	2	2.5
7. As the semester progressed, I got more comfortable taking tests.	8	7	12	16	0	2.8
8. As the semester progressed, I learned to prepare better for tests.	2	11	13	17	0	3.0
9. As the semester progressed, I became less satisfied with my mathematics testing performance.	1	11	9	13	8	3.4/2.6
10. In general, I would say that over the course of the semester, my test performance _________.	7	7	26	3	0	2.6

Responses to Items B2, B3, and B6 led me to believe that most students did not find test error analysis helpful in preparing for tests or in communicating with their parents about their testing performance. Responses to Item B4 suggested that test error analysis was not too time consuming.

The mode in Item B7 differed from the mean and I was surprised by the mode of *agree* since the mathematics concepts tended to build on previous information and my perception was that it was getting more difficult, which could have influenced test-taking comfort in a negative way.

I asked students to respond to the tenth statement in Section B with *1* for *got much worse*, *2* for *got worse*, *3* for *stayed about the same*, *4* for *got better*, and *5* for *got much better*. These results are also included in Table 6.3. Three students (7%) felt their test performance had improved, while a third of the students felt their test performance got worse. This qualitative data paralleled the quantitative trending of test scores, especially considering the number of review items in the first two tests.

I asked students who responded to Item 10 with other than a *3* to complete the following statement:

11. Changes in my test performance over time were mostly due to:

Twenty-one students did not respond. The other results are summarized in Table 6.4. I judged seven (21%) of the 33 comments to be positive and 26 (79%) to be negative as indicated in the pole column; note that some of the 22 students made more than one comment.

Table 6.4
Summary of Questionnaire Responses for Item B11

Response	Pole	Frequency
Lack of preparation/studying	–	8
Not understanding concepts	–	5
Not understanding the teacher	–	3
Preparation/studying	+	2
Understanding the teacher	+	2
Tests getting harder	–	2
Not paying attention in class	–	2
Time spent on other subjects	–	2
Not doing well on tests	–	1
Understanding the concepts	+	1
Concepts getting easier	+	1
Tasks (collaborative exercises exploring/applying the concepts in class)	+	1
Health/medical issues	–	1
Test anxiety	–	1
Studying the wrong material	–	1

It appeared from the responses that many of the students realized that lack of preparation was detrimental to their understanding and to their test performance.

Section C

Section C of the questionnaire asked six open-ended questions about testing and test error analysis. Student responses are given verbatim in Appendix N. The first question follows.

1. What did you learn from your test error analysis?

A summary of the 39 students' responses follows in Table 6.5. Including the four blank responses, I judged 40 (73%) of the 55 responses to be positive and 15 (27%) to be negative as indicated in the pole column; note that some of the students made more than one comment. These responses seemed to conflict with the responses to Items B2 and B3 of the questionnaire, heralding a benefit of open-ended questions on questionnaires. Many of these responses may seem similar, but since there could be distinctions between them, I coded and counted them separately.

Table 6.5
Summary of Questionnaire Responses for Item C1

Response	Pole	Frequency
I learned about my mistakes.	+	8
Nothing.	–	7
I make dumb mistakes.	+	7
Not much.	–	4
I learned about my weaknesses.	+	4
I learned what I needed to review.	+	3
I know the material.	+	2
I learned to watch for careless errors.	+	2
I learned to check back over my work.	+	2
I learned that I did not correct my errors.	+	1
I learned what I needed to spend more time on.	+	1
I learned how to do the math.	+	1
I learned to ask for help more.	+	1
I learned that I need to pay more attention.	+	1
I learned that those – 1's and – 2's can kill your grade.	+	1
I learned how to analyze my errors and learn from my mistakes.	+	1
I learned what I need to do better.	+	1
Most of my mistakes are testing, not content errors.	+	1
I learned what types of mistakes I make most frequently.	+	1
I learned how to correct my mistakes.	+	1
I learned to not make the same mistakes over again on other tests.	+	1

The second question in Section C follows.

2. What did you learn from the advice the class generated?

Student responses were coded and grouped for summary in Table 6.6. Eighteen students (42%) did not seem to value the student-generated advice, including 4 who did not respond, 8 who said they learned nothing, 5 who said they did not learn much, and 1 who did not think the class had given advice. The other 25 students (58%) seemed to see some value in the advice.

Table 6.6
Summary of Questionnaire Responses for Item C2

Response	Pole	Frequency
Nothing.	–	8
Not much.	–	5
I learned to check over my work more carefully.	+	5
I learned that studying is helpful.	+	4
I learned how to study better.	+	4
I learned what other people did that I do.	+	2
I learned what to do/what not to do.	+	2
It's usually different from reasons I didn't do well.	+	1
I learned that everyone makes different mistakes.	+	1
I learned how to stop making mistakes.	+	1
I knew most of the advice, but it reminded me.	+	1
I learned that if I didn't do well on the test, that I wasn't alone.	+	1
I learned to change my perspective and be more attentive in class.	+	1
I learned how to get a better grade.	+	1
I learned that I should read more carefully.	+	1
I learned a lot; it was very helpful.	+	1
I learned to study methods, not vocabulary or strategies.	+	1
I learned to take my time on tests.	+	1
The class never gave advice.	–	1
I learned a little more than from the test error analysis.	+	1

The third question in Section C follows.

3. What do you plan to do next year to maximize your test performance?

Student responses were coded and grouped for summary in Table 6.7. One student left this question blank. Of particular significance was that more than half of the students planned to study more.

Table 6.7
Summary of Questionnaire Responses for Item C3

Response	Frequency
Study more; practice longer.	25
Ask questions.	3
Pay more attention in class.	2
Nothing.	2
Do homework.	2
Study in groups.	1
Continue to study hard.	1
Look over my test error analysis and try not to make the same mistakes.	1
Turn in assignments.	1
Keep using a tutor.	1
Watch for mistakes.	1
Check my work better.	1
Change to a non-accelerated class.	1
Review harder problems so I don't panic on the hard ones.	1
I don't know.	1
Take notes in class.	1
Make sure I understand everything.	1
Wing it.	1
I don't know if I'll pass.	1

The fourth question in Section C was as follows:

4. What advice would you give to future students to help them test better?

Responses from the 41 students who did not leave the question blank were coded and grouped for summary in Table 6.8.

Table 6.8
Summary of Questionnaire Responses for Item C4

Response	Frequency
Study.	26
Pay attention in class.	10
Do your homework.	5
Ask questions.	5
Come in if you need extra help.	3
Check your work.	2
Don't take accelerated math.	2
Take notes.	2
Study in groups.	1
Don't cram.	1
Turn in assignments.	1
Use your time wisely.	1
Get a good night's sleep.	1
Relax.	1
Learn on your own.	1
Don't get lazy; don't get behind.	1
Watch your signs.	1
Ask for calculator tricks.	1

The fifth question in Section C follows.

5. What changes would you recommend to the test error analysis process or the forms that we used?

Student responses were coded and grouped for summary in Table 6.9.

Table 6.9
Summary of Questionnaire Responses for Item C5

Response	Frequency
None	14
Add more choices.	11
Don't do it.	4
Add "stupid mistake."	2
Make them optional.	2
Sometimes it's hard to identify an error type.	1
Use it for quizzes, so it will help our tests.	1
Use it to study.	1
Alternate coloring between light and dark.	1
Reduce the number of choices.	1

In addition to the 14 students who explicitly said they would not recommend any changes, 6 students may have been implying that they did not recommend any changes by leaving the response area blank. Explanatory comments ranged from "it was set up well, and professionally" and "very good, no changes" (which were included in the *none* responses) to "rid the earth of them, they didn't help" (which was one of the *don't do it* responses). Although 11 students wanted more choices added to the form, 1 recommended reducing the number of choices.

The sixth question in Section C follows.

6. What other comments would you like to add?

Student responses were coded and grouped for summary in Table 6.10

Table 6.10
Summary of Questionnaire Responses for Item C6

Response	Frequency
TEA didn't help.	2
I liked the TEA.	2
Make worksheets similar to tests/quizzes instead of giving book work.	2
Thanks for allowing friends to sit together.	2
I really want to pass; it's hard.	2
Thanks for being a great teacher and explaining material to me.	1
Pay attention.	1
Hope your professor likes this ☺	1
I'm glad I was in your class.	1
I learned a lot.	1
I enjoyed the group work.	1
Study.	1
Peace.	1
Merry Christmas.	1
Most of my errors were concepts, which changed with each test. I still don't know if I get it.	1

Some of the responses related to test error analysis and some did not.

LCA of the Questionnaire Responses

I performed an LCA on the quantitative questionnaire results. Note that Items B4 and B9 were reverse coded in the analysis since they were negatively worded. The 2-class model fit best as shown in Table 6.11. The 3-class model was deemed untrustworthy, even with large start values because of a non-positive definite first order derivative product matrix.

Table 6.11
Summary of Information Criteria to Select LCA Model for Questionnaire Responses

Information criterion	Model		
	1-class	2-class	3-class
AIC	1750.66	1684.60	Model was deemed untrustworthy.
BIC	1799.98	1760.34	

The 2-class model divided the students rather evenly with 22 students in Class 1 and 21 in Class 2, as seen in Figure 6.1. The most distinguishing items were the following:

B2: Test error analysis helped me learn from my testing errors.

B3: Test error analysis helped me prepare for subsequent tests.

B5: The class's advice in preventing testing errors was helpful to me.

A4: I reviewed my test error analysis about ___ times this semester.

B6: Test error analysis helped me explain my testing performance to my parents.

Each of these items had responses that were at least a unit higher for Class 2 students than for Class 1 students. In light of Item A4 (how often students reviewed their test error analysis), it appears that, in general, the more the students used the test error analysis, the more they felt they got out of it; inversely, the less they used the analysis, the less they gained. This result makes sense and provides support for the process.

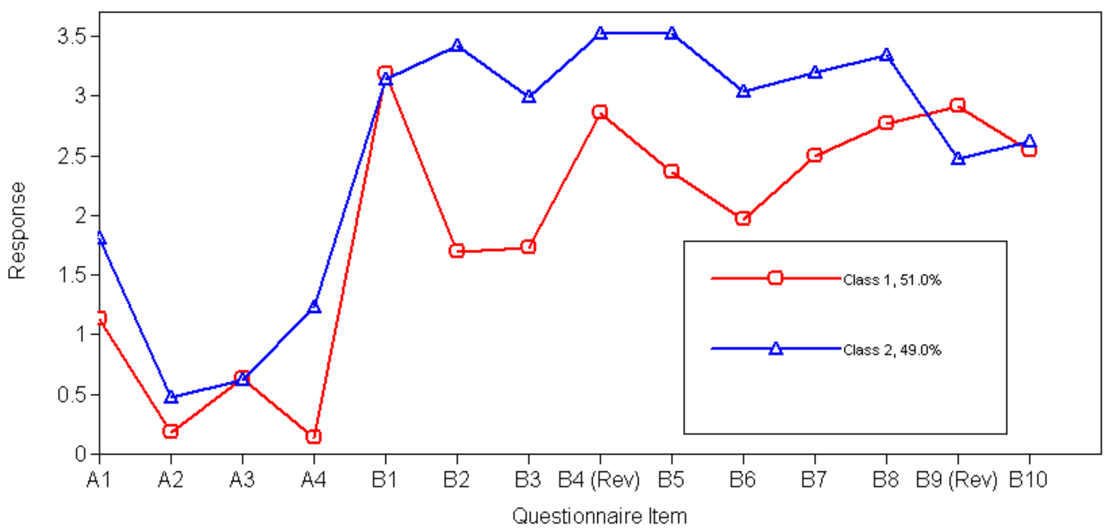


*Figure 6.*1: LCA of quantitative questionnaire results

To summarize the questionnaire results, responses to the process and its usefulness were mixed. Some students said they benefited from the process, and others did not. These results seem to mirror McAcy's (1993) findings that students' responses were "somewhat mixed; however, students who took the time to use the list effectively showed improvement" (p. 298).

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CHAPTER 7

PARENT AND TEACHER PERSPECTIVES

Parent Perspective

As mentioned previously, two paths were designed to solicit parent input on the test error analysis process: informal discussions during parent conferences in a self-selected sample and the e-mail questionnaire surveying parents at the end of the semester.

Parent Conferences

Parents of both students who asked for conferences were concerned about low grades, including test grades. In each of the two conferences, I discussed the test error analysis process, went over the child's test error analysis forms, and invited the parent to comment on the helpfulness of the information.

The father of the first student was especially impressed with the potential benefits of his son's test error analysis results. "This is great! We're going to get him a tutor for extra one-on-one help. I can give this to the tutor, and she'll know just what he needs!" This allusion to the value of the tool to a tutor really caught me by surprise; I had not anticipated the tool having value to anyone other than students, parents, and teachers. Furthermore, this was the father of the student who later said "rid the earth of them, they didn't help" when asked what changes he would recommend to the test error analysis process or the forms that we used. The mother of the second student said her daughter's test error analysis was "helpful," but I did not detect the high level of enthusiasm that the first parent had shown.

Parent Survey

Both responses to the end-of-semester parent survey are included in Appendix O. Neither of the responses were from the two students' parents who had provided feedback on the process during the conferences, so these responses raised to four the number of students whose parents provided input.

The first parent implied that she had not needed to discuss the tool with her son, so it had not been helpful to her: "I do not discuss Austin's approach to math as he knows more than I do!! I only check to make sure he has prepared his work."

The second parent had not seen the tool but asked her daughter Evelyn about it. She decided that it was "somewhat helpful" in understanding her child's test performance and communicating with her child. Evelyn then noted that, "It was kind of hard to use it when you had a high grade." (Her test average was a 96.) This small sample of four seemed to indicate that the tool was helpful for struggling students whose parents wanted to be more involved in coaching their children, but that it ranged between somewhat helpful and unnecessary for others.

There were no responses from 39 out of the 43 sets of parents. This lack of parental input could have been due to satisfaction with their students' testing performance, a low interest in the research, an unwillingness to share opinions that may have been negative, busy schedules, or other reasons.

Although disappointed by the low response rate from parents, I consider the parental input from the present study to parallel the results that McAcy reported: "The parents' reaction has been positive. It gives concrete support for earned grades. It gives parents something to act on." (1993, p. 298).

Teacher/Researcher Perspective

My intention in this chapter is to provide constructed knowledge in mathematics education from this study in the spirit inspired by Hatch and Shiu (1998). However, one of my goals as a teacher/researcher is to critically reflect on my teaching and researching practices (Jaworski, 1998).

I found the test error analysis to be very helpful in assessing student errors, not only for the students who completed the analysis, but also for future classes. It helped me develop review material for the final exam and the end-of-course test and informed my teaching to better emphasize potential pitfalls and improve assessments (tests, quizzes, and other forms). It also helped me improve my instruction and assessment for the following year. It helped me make instructional decisions as well as monitor student progress and achievement (NCTM, 1995).

As we went through the test error analysis process, I found out that my comments and point deductions on the graded papers were not as clear as I thought they were. Sometimes students thought I was deducting points for one reason (or even one item) when, in fact, it was for another.

Knowing that about 15% of the students' errors were testing process errors and not mathematical content errors led me to use some caution in making assumptions about the reliability of any assessment's measurement of student knowledge of mathematics. The margins of error of these assessments' measures of student content knowledge were larger than I might have assumed before the study began.

I was surprised that *didn't know how* beat out *knew how, but forgot* in points lost. I knew we had covered all of the material in class. I thought that the concepts were at least comprehended and retained, even if students had trouble retrieving the information during assessment. So I expected *knew how, but forgot* to be the reason they lost the most points. Analyzing the data told me that my students were not having retrieval problems as much as gaps in their construction of the mathematics. My challenge is to find ways to improve students' mathematical constructions, particularly in the content areas identified in the analysis.

I found the tool to be tremendously beneficial in communicating with the two students' parents who asked about their children's testing or improvement opportunities. These parents quickly grasped the usefulness of the tool and appreciated its content. I did not foresee what one parent pointed out: that he could hand the test error analysis summary form over to a tutor for a quick reference of strengths and weaknesses in planning additional one-on-one support for his son. In general, discussing the tool seemed to help satisfy parents who were frustrated with their child's performance and wanted information that they could use to help coach their child.

I have been using versions of the test error analysis tool for my classes for 5 years, modifying it each year, but this is the first year I have done a detailed analysis with it. Before I started using the tool, some of the students who did poorly would hide their test or dispose of it, even before we went over it. However, when they were expected to fill out a test error analysis form, they were more attentive when we went over the test and asked more questions about items they had missed. Some students seemed doubly punished if they did poorly on a test; not only did their grade suffer, but they had to work harder to complete the form! Since I considered myself to be their personal trainer for mathematics, I was not surprised by occasional resistance to an exercise such as test error analysis that I thought would benefit my students even though it required more thought or work on their part. It is natural for students to resist work and change, but I considered it my job to take them a little further than they thought they could go mathematically. I thought the more attentive listening when going over tests in order to fill out the form had merit in helping the students grow. I felt that the less prepared a student was for a test, the more he or she could benefit from an analysis of the errors on it.

I was also impressed with the consideration and creativity that the students used in developing their advice for the different error types. They seemed to enjoy the process and took pride in their ideas.

If I were to conduct this study again, I would do a better job of summarizing the error type findings for the class after each test. Typically, it took me 3 to 8 hours to grade the tests on the night of the testing so that I could return the test to the students the next school day. I gave the grading a high priority because it allowed us to go over the test and analyze the errors while the test was still fresh in the students' minds. Then, with the full-time teaching job, I was too exhausted in subsequent days to review the forms; verify, correct, and input the errors; and present the summary to the students. Much of my analysis, including the LCA, took place after the semester was over. Therefore, the students did not benefit as much as they could have if I had shared their aggregate information in a timelier manner. This would have promoted more use of selected errors as "springboards for inquiry," as heralded by Borasi (1987). Also, sharing cumulative results would have allowed me to review their advice for the most prevalent error types, reinforce that part of their labor on behalf of test error analysis, and encourage them to use the newfound knowledge to broaden their content knowledge and better their testing performance. I believe that these improvements to the test error analysis process could have improved student learning from their test results.

Although the student questionnaire responses could be generalized as weakly in opposition to test error analysis, some students clearly credited the tool with helping them learn from their testing errors. This learning may have influenced the reduction in testing process errors over time. The increase in content points lost could be attributed to an increase in difficulty of the mathematics as the course progressed.

It is possible that test error analysis helped to better student performance on the cumulative tests. The improvements in cumulative test scores, in turn, provided an extra boost to most students since I replaced the lowest test grade on Tests 1–4 with the midterm grade if it helped the student's average and likewise replaced the lowest test grade on Tests 5–7 with the final exam score. The opportunity for this grade replacement gave my students and me optimism about their finishing the semester with better grades.

Thorough analysis of the data is very time-consuming. This is a process that could be mechanized (e.g., by using an electronic spreadsheet specially designed for student input of the test error analysis data) for more accurate coding by students, automatic total and summary computations, easy revisions, and faster data analysis.

Although the students and I strived for objectivity in categorizing the test error analysis data, some errors could be categorized in multiple ways, and there was admittedly subjectivity involved in some of these judgments. For example, at the end of the semester one student admitted that she hesitated to say *didn't know how* on the form because then she had to think of something to write on the back. She tended to gravitate toward *didn't understand the question* and *ran out of time*.

Student input and test error analysis results gave me insights into potential improvements to the test error analysis process. I recommend the following changes to the form for any future uses:

– Omit *missing/incorrect units* and *spelling* on first semester's form (They apply to the geometry in second semester.)

– Replace *didn't provide final answer* with *didn't factor completely* and move it to Content Section

– Move *incorrect notation* to the Content Section

– Add *order of operations* to the Content Section

– Add *omitted one of multiple solutions* to the Content Section

– Add *included an extraneous solution* to the Content Section

– Add *'cancelled' wrongly* to the Content Section

Additionally, perhaps some of the content error types could be split a different way, providing a new dimension of error types, (i.e., a *reason* for the error types). For example, I might distinguish knowledge level error types such as *didn't know how*, *knew how but forgot*, and a new category of *knew better* from content error types like *arithmetic errors* and *not distributing properly*. Students would have to record their scores in *both* dimensions. This would allow me to better understand the depth of their knowledge (from the reasons) and the content they were missing separately.

I would have liked for the test error analysis processes to show more dramatic results in academic performance improvement and in student perceptions of their benefits. There could be several reasons why more positive results did not materialize. If students considered the analysis to be more of a chore to get done than an opportunity to enhance their learning, the potential benefits of the analysis could have been wasted. I think I could have minimized this phenomenon with more prompt summative feedback to the class; I could have emphasized my learning from the data, which could have conveyed some (perhaps) contagious excitement about the results.

In addition, it is possible that the accelerated pace of the curriculum of this course (Dodd & Perry, 2010) combined with academic or social pressures from other teachers or from parents allowed student anxiety to fester. I believe this added anxiety may have interfered with open-minded learning and construction of mathematics (Tobias, 1993; Ma, 1999; Ho et al., 2000), even after the assessment.

Although the test error analysis results got mixed reviews from the students, it was beneficial to me directly in informing my teaching, and I believe it was beneficial, at least indirectly, to my students.

CHAPTER 8

INTEGRATION OF RESULTS, SUMMARY, AND IMPLICATIONS

Integration of Results

I integrated the qualitative and quantitative results in several ways to synthesize the data from the mixed methods study: comparing questionnaire responses to test scores, comparing questionnaire responses to measures of student motivation, and investigating case studies of students whose grades put them in jeopardy of failing the course.

I compared the open-ended responses to Item C1 on the questionnaire of students who indicated they learned nothing from the test error analysis to students who learned something. This comparison is shown in Table 8.1.

Table 8.1

Summary of Mean Test Scores by Student Responses to Questionnaire Item C1

C1 response	Number of students	Mean score of tests			Improvement from unit to cumulative tests
		All 10	7 unit	3 cumulative	
Nothing	15	78	75	84	10
Something	28	84	82	88	6

I investigated correlations between the 43 students' responses to Item A4 on the questionnaire, x, (how often students reviewed their test error analysis) and the students' unit test averages u. The linear regression equation for this relationship was $u = –1.53x + 80.56$ ($r = –.22$, $r^2 = .05$). Likewise, I considered the Item A4 responses x and the cumulative test averages c across the 43 students and found $c = –.60x + 87.15$ ($r = –.16$, $r^2 =.03$). I had expected a positive correlation for both of these, which would have shown that the more the students reviewed their test error analysis, the better they did on unit and cumulative tests, so this negative correlation was surprising to me. However, based on Evelyn's comment ("It was kind of hard to use it when you had a high grade.") in her mom's response to the parent survey, I realized that the students with the highest grades did not have as great a need to use their test error analysis. That led me to believe that a quadratic model might fit the data better. I found these quadratic relationships: $u = -1.72x^2 + 4.06x + 79.64$ ($r^2 = .13$) and $c = -.88x^2 + 2.27x + 86.67$ ($r^2 = .10$). The slightly higher r^2 values with the negative coefficients of x^2 led me to believe that the most successful students and the least successful students gained the fewest benefits from the test error analysis. That made sense to me because the best students did not see as much need for the test error analysis and the least successful students were struggling largely because they did not use good study habits, including using the test error analysis.

I wondered if there was a correlation between Item A4 responses x and the gains students made from their unit test averages to their cumulative test averages ($c - u$). Linear regression yielded $(c - u) = .93x + 6.58$ ($r = .17$, $r^2 = .03$) while quadratic regression yielded $(c - u) = .84x^2 - 1.79x + 7.03$ ($r^2 = .06$). I considered these to be negligible correlations.

I performed latent class analysis, matching responses to Item A4 with the average test scores, but found no significant relationship.

Since studies on "how motivational factors work in tandem with metacognitive knowledge to influence performance within the domain of mathematics" are limited (Carr, et al., 1994, p. 584), I compared student daily averages (which included homework and classwork, but not test error analysis grades) to Questionnaire Item A4 responses, but found no statistically significant relationship.

As mentioned before, test error analysis may have contributed to success on the cumulative exams. At the end of the semester, two students' averages were teetering on failing the course. Stuart had a 68 average going into the final exam. He scored a 79 on the performance exam and a 78 on the final exam. His mean test score for the first eight tests was 66. His father was the parent who was impressed with the test error analysis and said he would share the information with the tutor. Stuart's response to Questionnaire Item A3 indicated that he had used a tutor about once a week. However, Stuart's response to Questionnaire Item C1 indicated that he gave no credit to the test error analysis; he said he had learned nothing from it. In response to Item C5, he said the test error analysis process could be improved if we "rid the earth of them. They didn't help." Since I replaced his lowest test score (a 38 on Test 7) with the final exam grade, which also counted as 15% of his average, he passed the class with a 71.

The second student in danger of failing was Dean. He had a 71 average going into the final exam. With a 76 on the performance exam and a 72 on the final exam, he passed the class with a 72. In response to Questionnaire Item C1, he said he learned "what he needed help with" from the test error analysis.

Success on the cumulative exams for even the weakest students ensured that all 43 students passed the class in spite of the rigorous curriculum. Part of the credit for this phenomenon may be due to student analysis of their testing errors or the quality of the review for the cumulative exams that included the most missed items based on the test error analysis results. Alternatively, it may have been the extra motivation of the students at risk at the end of the semester, extra pressure from the parents, the format of the test, or a perceived lower difficulty level of the test.

Summary and Implications

In response to the first research question, "What effects does the use of the test-error analysis tool have on students' mathematics test-related behavior (i.e., preparing for and taking tests) and outcomes (e.g., errors made, points lost, test scores)?", the data show mixed results. Unit test scores did not show an increase over time, implying that student test preparation may not have improved. The cumulative tests scores were higher than the unit test scores that assessed new content, indicating that students lost fewer points on those tests and may have learned from their errors on the unit tests.

Next, consider the second question set, "What benefits and drawbacks do students, parents, and I perceive from the use of the test error analysis tool with mathematics tests? In particular, what do the students and I learn from the analysis?" Overall, the student reaction was mixed, but some students indicated that they learned about their mistakes from their analyses. In the two parent conferences of struggling students, the parents responded that they found it helpful. I found the manual summary of the data to be time-consuming, but very useful in analyzing student errors, informing my instruction and assessment preparation, and communicating with parents. I learned vital information about my students, my instruction, and my assessments.

For the third question, "What are the most common types of errors, according to the analysis, for a given mathematics course?" the data showed (1) *didn't know how*, (2) *knew how but forgot*, (3) *arithmetic/algebraic error – no fractions*, and (4) *ran out of time* as the most predominant errors by points lost. Regarding "how does this information inform the students and me to promote the learning of mathematics?" students performed better on cumulative tests than on the unit tests with new material, indicating that they may have learned from analyzing their errors. I learned about student and teaching weaknesses to inform my instructional and assessment processes.

Finally, "what groupings, patterns, and trends can be observed from the data? For example, do the frequencies of some error types decrease? If latent groups are identified, what are their characteristics and what are the probabilities that students move from one group to another?" Students fared better on cumulative tests than unit tests. Testing process errors declined from the first half of the semester to the second half, but content errors rose slightly as the content grew more difficult. The more the students used the tool, the more favorably they ranked it. Students whose grades were in the middle of the class tended to benefit more from the analyses than struggling or excelling students. There were insufficient data to definitively identify probabilities of moving between latent classes.

I think the test error analysis would have had more positive results if I had been able to do the aggregate analysis more promptly to share it with the students in a more timely manner. The analysis proved labor-intensive and time-consuming without a mechanized system of student data entry.

Even though results of this research were mixed, it was clear that the process helped to promote learning for me and some of the students and parents. I offer the following in conclusion.

Implications for Teachers

This tool and its related processes have potential to assist teachers in instruction, planning, reviewing, and parental contact. The test error analysis tool needs to be tailored to each course. Teachers need to develop the tool based on error types they anticipate occurring most often.

Successful implementation of this process requires energy and commitment. Teachers need to commit to returning graded tests to the students as soon as possible so the students can accurately recollect their thought processes in the testing. Teachers need to be willing to explain how a paper or electronic spreadsheet works and to coach the students through the process of completing the tool, especially after the first test. To maximize student interest in the process, teachers should give feedback to their classes on aggregate findings after each test. A mechanized spreadsheet with hints for students and automatic totals could streamline data completion, verification, and analysis.

Implications for Students

Test error analysis can be used to assist students in managing, preparing for, and taking tests, and to assist in parental communication. Disappointment from losing points can be channeled into the constructive process of documenting, analyzing, and summarizing errors to turn them into learning opportunities. Revisiting test error analysis results periodically can help to reinforce learning of testing and content skills.

Even if a teacher does not coordinate a test error analysis form and process, students can develop a form or template for their own use in other subjects, creating their own error types as they emerge.

I hope this study sparks interest in, use of, and further research on student test error analysis. I believe the test error analysis tool and related processes have merit and hope that other researchers will find ways to overcome the limitations of this study and use its strengths as “springboards for inquiry” (Borasi, 1994) to further our knowledge of students’ analysis of their testing errors.

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APPENDIX A: KAREN MCACY'S TEST ERROR ANALYSIS TOOL

Name																		
Semester																		
Test			Score															
		Added Wrong	Multiplied Wrong	Used Wrong Operation	Dropped a Negative Sign	Did not Distribute Number	Did not Distribute Negative One	Copied Incorrectly	Used Wrong Slope	Mistake in Formula	Did Not Follow Directions	Did Not Use Requested Method	Incomplete	Forgot the O.I. In F.O.I.L	Cancelled Wrong	Factored Wrong	Skipped the Problem	Other (Indicate on the back)

From "Sharing Teaching Ideas: Careless Mistakes," by K. McAcy, 1993, *Mathematics Teacher, Volume 86*, pp. 298–299.

APPENDIX B: LYNNE MENECHELLA'S TEST ERROR ANALYSIS TOOL

Mathematics Test Analysis Sheet

Name of Test: ________________________________

Grade I expected to earn: ______   Grade I actually earned: ______

What I used/did to study (√):   my notes ____ my homework ___ my textbook ____ my review sheet ______

other: ______________________________________

Write the number of each problem missed in the Q# box. Check (√) the box that best describes why you missed each problem.

Question number ⇒ Type of error ⇓	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #	Q #
Never learned it – absent from class																				
Never understood it – did not get help																				
Did not study it																				
Did not understand or did not follow the directions																				
Did not understand the question																				
Did not show enough work to communicate how I solved it																				
Used the wrong operation or rule																				
Made an arithmetic or calculator error																				
Other (explain below)																				

Other type(s) of errors: ____

Major strengths? ____

Major weakness? ____

A realistic plan I will use to improve my understanding and test performance is _________________________

____

____

<u>*On your own sheet of paper*</u>, correct each problem you missed. Be sure to show all your work (including work for multiple choice questions). Staple your own paper to this paper and your test. Have a parent/guardian sign below.

Parent/Guardian: I understand my child will receive half-credit back for each problem corrected for a maximum grade of 85.

Signature: ______________________________ Date: ____________________

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APPENDIX C: Test Error Analysis Worksheet

Name ___________________ Test _______ Period ______ Date _________

Please analyze the points lost for this test, starting with a column for the first question you had an error on. You need not include the negative symbol or zero fill. Be sure to include specifics ("careless mistake" is not specific) for anything attributed to "other", write vocabulary and concept lists on the back, and make sure your totals match up. The goal is to learn from each test and improve your performance on math tests.

A. Overall														
1. Grade														
2. **Total points missed** (100 – A1)														
Error Type \ Question Number														**Sum**
B. Testing Points (Generic – could apply to any subject)														
1. Accidentally skipped the question														
2. Didn't follow directions														
3. Misread the question														
4. Didn't show steps														
5. Misread my own writing														
6. Missing/incorrect units														
7. Didn't provide *final* answer														
8. Unclear communication														
9. Incorrect notation														
10. Ran out of time														
11. Other (Testing):														
12. Other (Testing):														
13. Total Testing Points Lost														
C. Content Points (Math specific)														
1. Didn't understand question														
2. Forgot vocabulary (List on back.)														
3. Didn't know how (List on back.)														
4. Didn't combine known concepts														
5. Didn't simplify														
6. Confused two concepts (List on back.)														
7. Used the wrong formula (List on back.)														
8. Knew how, but forgot (List on back.)														
9. Arithmetic error – fractions (+, –, *, /)														
10. Arithmetic error – no fractions (+, –, *, /)														
11. Didn't distribute properly														
12. Multiplied instead of using exponents														
13. Graphing error														
14. Freshman's dream (binomial mult.)														
15. Illogical or incomplete proof														
16. Calculator usage error (List on back.)														
17. Spelling:														
18. Other (Math):														
19. Other (Math):														
20. Total Content Points Lost														
Total Points Lost (B13 + C20) (Sum should match points missed in A2.)														

Test Error Analysis Worksheet

Name ______________________ Test _______ Period ______

List vocabulary to review:

List concepts to review:

APPENDIX D: Test Error Analysis Summary

Name ____ Semester __________ Period ______

Please summarize the points lost on your Test Error Analysis Worksheets (you need not include the negative symbol or zero fill) for each of your individual tests. Be sure to include: specifics ("careless mistake" is not specific) for anything attributed to "other", vocabulary and concept lists on the back, and make sure your totals match up. The goal is to learn from each test and improve your performance on math tests.

Error Type \ Test										**Avg**
A. Overall										
1. Grade										
2. Total points missed (100 – A1)										
										Sum
B. Testing Points (Generic – could apply to any subject)										
1. Accidentally skipped the question										
2. Didn't follow directions										
3. Misread the question										
4. Didn't show steps										
5. Misread my own writing										
6. Missing/incorrect units										
7. Didn't provide *final* answer										
8. Unclear communication										
9. Incorrect notation										
10. Ran out of time										
11. Other (Testing): ________________________										
12. Other (Testing): ________________________										
13. Total Testing Points Lost										
C. Content Points (Math specific)										
1. Didn't understand question										
2. Forgot vocabulary (List on back.)										
3. Didn't know how (List on back.)										
4. Didn't combine known concepts										
5. Didn't simplify										
6. Confused two concepts (List on back.)										
7. Used the wrong formula (List on back.)										
8. Knew how, but forgot (List on back.)										
9. Arithmetic error – fractions (+, –, *, /)										
10. Arithmetic error – no fractions (+, –, *, /)										
11. Didn't distribute properly										
12. Multiplied instead of using exponents										
13. Graphing error										
14. Freshman's dream (binomial mult.)										
15. Illogical or incomplete proof										
16. Calculator usage error (List on back.)										
17. Spelling: ________________________										
18. Other (Math): ________________________										
19. Other (Math): ________________________										
20. Total Content Points Lost										
Total Points Lost (B13 + C20) **(Should match points missed in A2.)**										

Test Error Analysis Summary

Name ______________________ Semester _______ Period ______

List vocabulary to review by test:

List concepts to review by test:

APPENDIX E: GEORGIA DEPARTMENT OF EDUCATION ACCELERATED MATHEMATICS I STANDARDS

Accelerated Mathematics 1

This is the first in a sequence of mathematics courses designed to prepare students to take AB or BC Advanced Placement Calculus. It includes radical, polynomial and rational expressions; functions and their graphs; quadratic and radical equations; fundamentals of proof; properties of polygons, circles and spheres; coordinate geometry; sample statistics and curve fitting. (*Prerequisite: Successful completion of 8th Grade Mathematics.*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic methods. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

NUMBER AND OPERATIONS

Students will use the complex number system.
MA1N1. Students will represent and operate with complex numbers.

a. Write square roots of negative numbers in imaginary form.
b. Write complex numbers in the form $a + bi$.
c. Add, subtract, multiply, and divide complex numbers.
d. Simplify expressions involving complex numbers.

ALGEBRA

Students will explore functions, solve equations and operate with radical, polynomial and rational expressions.
MA1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

a. Represent functions using function notation.
b. Graph the basic functions $f(x) = x^n$, where $n = 1$ to 3, $f(x) = \sqrt{x}$, $f(x) = |x|$, and $f(x) = 1/x$.
c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the x- and y-axes.
d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
f. Recognize sequences as functions with domains that are sets of whole numbers.
g. Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.
h. Determine graphically and algebraically whether a function has symmetry and whether it is even, odd, or neither.
i. Understand that any equation in x can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

MA1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

a. Simplify algebraic and numeric expressions involving square root.
b. Perform operations with square roots.
c. Add, subtract, multiply, and divide polynomials.
d. Add, subtract, multiply, and divide rational expressions.
e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$
$(x + y)(x - y) = x^2 - y^2$
$(x + a)(x + b) = x^2 + (a + b)x + ab$
$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

f. Use area and volume models for polynomial arithmetic.

MA1A3. Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.

a. Convert between standard and vertex form.
b. Graph quadratic functions as transformations of the function $f(x) = x^2$.
c. Investigate and explain characteristics of quadratic functions, including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, intervals of increase and decrease, and rates of change.
d. Explore arithmetic series and various ways of computing their sums.
e. Explore sequences of partial sums of arithmetic series as examples of quadratic functions.

MA1A4. Students will solve quadratic equations and inequalities in one variable.

a. Solve equations graphically using appropriate technology.
b. Find real and complex solutions of equations by factoring, taking square roots, and applying the quadratic formula.
c. Analyze the nature of roots using technology and using the discriminant.
d. Solve quadratic inequalities both graphically and algebraically, and describe the solutions using linear inequalities.

MA1A5. Students will investigate step and piecewise functions, including greatest integer and absolute value functions.

a. Write absolute value functions as piecewise functions.
b. Investigate and explain characteristics of a variety of piecewise functions including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, points of discontinuity, intervals over which the function is constant, intervals of increase and decrease, and rates of change.
c. Solve absolute value equations and inequalities analytically, graphically, and by using appropriate technology.

Georgia Department of Education
Kathy Cox, State Superintendent of Schools
7/19/2005

APPENDIX F: TESTS 1–7

Name: ________________________ Date: ___________ Period: ______ AIAI Test 1: 1.2-7

1. A bag of chips costs $2.33. Your total grocery bill, *b*, is a function of the number of bags of chips, *n*, you purchase. Write an equation to represent this function.

a. $b = \frac{2.33}{n}$ c. $b = 2.33n$

b. $n = 2.33b$ d. $\frac{b}{2.33} = n$

2. Decide whether the information defines a function. If it does, state the domain of the function.

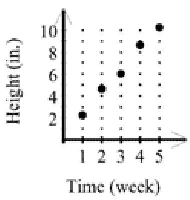
Input	0	1	2	3	3
Output	1	2	3	2	1

Domain, if yes: ___________________________

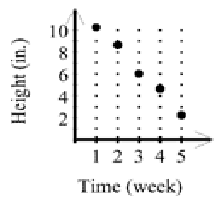
3. The table at the right shows the height of a plant over time. Find the scatter plot that shows the relationship between time and the height of the plant.

Bamboo Height	
Time (Week)	Height
1	2.25
2	4.63
3	6.00
4	8.63
5	10.25

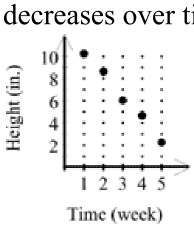
a. The height of the plant decreases over time.



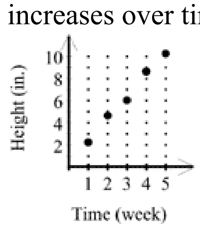
c. The height of the plant increases over time.



b. The height of the plant decreases over time.



d. The height of the plant increases over time.

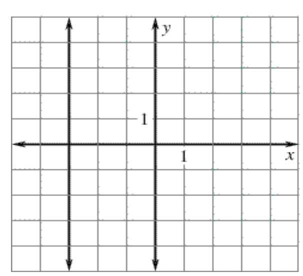


4. Find the slope of the line passing through the points *A* (10, –1) and *B* (–3, –6).

5. Find the *y*-intercept of the line with the equation $4x - 3y = 12$. .

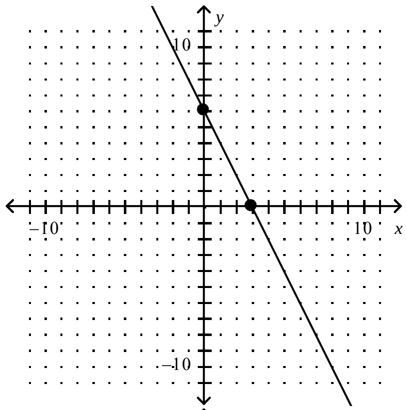
6. Determine the slope of the line in the graph at the right.

a. undefined b. 3 c. 1/3 d. 0

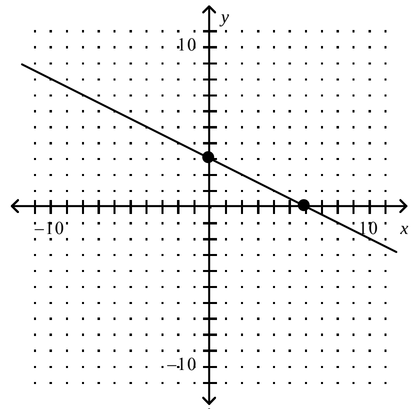


7. Graph the linear equation $3x + 6y = 18$ by finding the *x*- and *y*-intercepts.

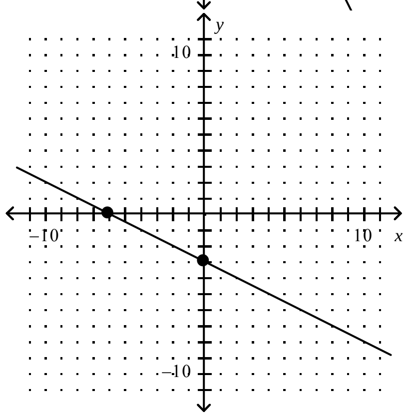
a.



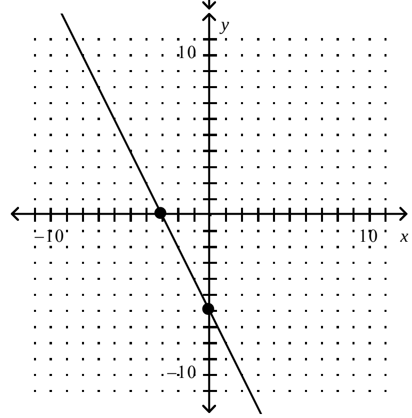
c.



b.



d.

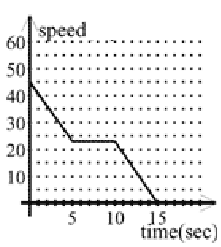


8. Determine if the line $-7x + 6y = 3$ is parallel to the line $y = \frac{7}{6}x + 1$. Explain your answer.

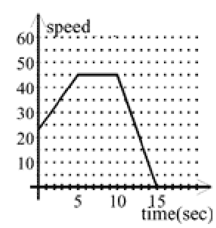
____

9. Which graph below would match the situation described? A car traveling at 23 mi/h accelerates to 45 mi/h in 5 seconds. It maintains that speed for the next 5 seconds, and then slows to a stop during the next 5 seconds.

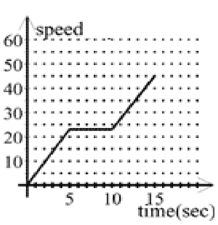
a.



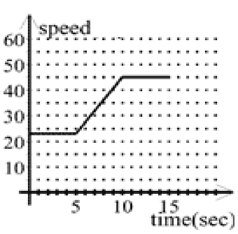
b.



c.



d.



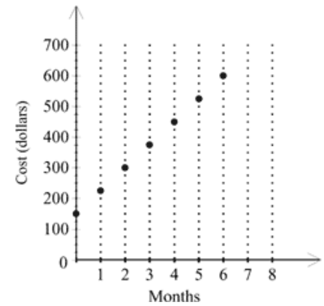
10. Find the slope and y-intercept of the line with the equation $2x - 8y = -20$.

m: _____ b: _____

11. The cost to install and use a premium satellite-television service in a particular city is shown in the graph. Find the slope and y-intercept of a line joining the points on the graph and explain what the slope and y-intercept represent.

Slope: __________ Explanation:

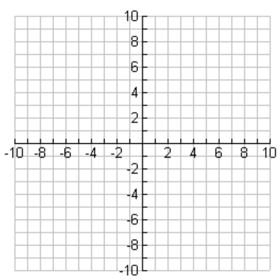
y-intercept: ________ Explanation:



12. Let $h(x) = -4x + 16$. Find x when $h(x) = 48$.

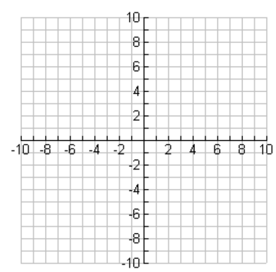
Graph the function. Compare the graph with the graph of $f(x) = x$.

13. $g(x) = \frac{3}{4}x$



Transformation:

14. $g(x) = x - 4$



Transformation:

15. Find $f(4)$ if $f(x) = 4x + 9$.

16. When Han Sum had his picture taken, the photographer charged a \$11 sitting fee and \$5 for each sheet of pictures purchased.

a. Write the function for the price of the pictures, where x is the number of sheets purchased.

b. What would his total cost be to purchase 7 sheets of pictures?

17. Class start times for the seven periods of Aiming High School are as follows:

7:23, 8:22, 9:21, 10:20, 11:19, 12:18, 1:17.

Consider only the first six periods. Make a table of the hours and minutes of these bell times:

Period	1st	2nd	3rd	4th	5th	6th
Hour						
Minute						

If it is a function, write a rule to show minute times in terms of the hour times. ______________

If it is not a function, explain why not. ____

What is its domain? ____

What is its range? ____

Compare each function below to its parent function $f(x) = x$ by **circling** the appropriate transformations **and filling in** the applicable **detail blanks,** if any.

Function	Transformation	Details
18. $g(x) = x - 5$	vertical shift	________ units up
	vertical stretch	________ units down
	vertical shrink	with scale factor _______
19. $h(x) = 1/4x$	vertical shift	________ units up
	vertical stretch	________ units down
	vertical shrink	with scale factor _______

20. Consider the following functions:

$m(x) = -4x + 3$ $a(x) = 4x + 3$ $t(x) = -4x - 3$ $h(x) = 4x - 3$

Which one is a reflection of m in the y-axis? _________

Which one is a reflection of t in the x-axis? _________

Accelerated Integrated Algebra I: Test 2: 2.1–2.9 /100

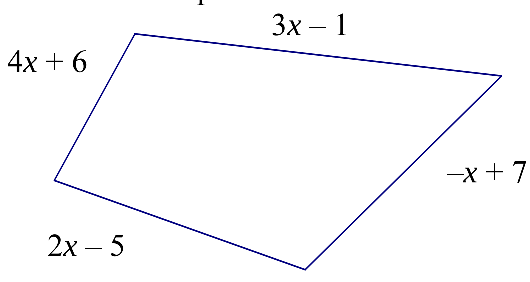
Name ____________________ Pd _____ Date _____________

Select the best answer. Shade it in the box provided at the right.

Answer Box

1. M A T H
2. I S F U
3. N ! M A
4. T H I S
5. K U L !
6. I T ’ S
7. T H E B
8. E S T P
9. A R T O
10. F S C H
11. O o L !

1. The perimeter of this quadrilateral is 27. Solve for x.



M. $x = 2.4$ **A.** $x = 2.5$ **T.** $x = 2.6$ **H.** none of these

2. Find the product: $2x^2y\,(4x^4y^2 + x^3y^3 - y)$.

I. $8x^6y^3 + 2x^5y^4 - 2x^2y^2$ **F.** $8x^6y^3 + 2\,x^3y^3 - 2y$

S. $6x^6y^3 + 3x^5y^4 + x^2y^2$ **U.** $8x^8y^2 + 2\,x^6y^3 - 2x^2y$

For the next two questions, let $f(x) = 2x^2 - 3x + 7$ and $g(x) = 5x^2 - 3x + 4$.

3. Find the sum of $f(x)$ and $g(x)$.

N. $7x^4 - 6x^2 + 11$ **!.** $10x^2 - 9x + 28$ **M.** $7x^2 + 11$ **A.** $7x^2 - 6x + 11$

4. Find the difference, $f(x) - g(x)$.

T. $-3x^2 + 3$ **H.** $-3x^2 - 6x + 11$ **I.** $-3x^2 - 6x + 3$ **S.** $-3x^2 + 11$

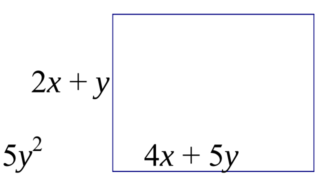
5. Find the product, $f(x) \cdot g(x)$ if $f(x) = 3x^2 - 2x$ and $g(x) = 4x^2 - 5$.

K. $12x^4 + 10x$ **U.** $12x^4 - 10x$ **L.** $12x^4 - 8x^3 - 15\,x^2 + 10x$ **!.** $12x^4 - 8x^3 - 15\,x^2 - 7x$

6. $(6x - 5y)^2 =$

I. $36x^2 - 60xy + 25y^2$ **T.** $36x^2 - 25y^2$ **’.** $36x^2 + 25y^2$ **S.** $36x^2 - 22xy + 25y^2$

7. Find the area of the rectangle with the given length and width.



T. $8x^2 + 5y^2$ **H.** $6x + 6y$ **E.** $12x + 12y$ **B.** $8x^2 + 14xy + 5y^2$

8. Find the volume of a cube with side length $x + y$.

E. $x^3 + y^3$ **S**. $x^3 + x^2y + xy^2 + y^3$ **T**. $x^3 + 3x^2y + 3xy^2 + y^3$ **P**. $x^3 + 2x^2y + 2xy^2 + y^3$

9. Factor the polynomial: $9x^2 - 100y^2$:

A. $(3x - 10y)(3x + 10y)$ **R**. $(3x - 10y)(3x - 10y)$
T. $(3x + 10y)(3x + 10y)$ **O**. $(x^2 - y^2)(9 - 100)$

10. Solve by factoring: $10x = 2x^2$

F. $x = 1/5$ **S**. $x = 0$ **C**. $x = 0$ or $x = 5$ **H**. $x = 5$

11. Find the coefficient of x^3 in $(x - 6)^4$.

O. 4 **o**. 6 **L**. 24 **!**. –24

12. Find the zeros of the function: $f(t) = t^2 - 4t - 21$. ________________

13. Factor: $4y^2 - 20yz + 25z^2$. ________________

For the next three questions, solve by factoring:

14. $x^2 = 100$ ________________

15. $x^3 - 19x^2 + 84x = 0$ ________________

16. $w^3 + 4w^2 = 4w + 16$ ________________

17. A rolling kickball is kicked into the air with an initial vertical velocity of 12 feet per second. How long will the ball be in the air, using the vertical motion function $h(t) = -16t^2 + vt + s$?

Review.

18. Find the slope of a line that contains the points (–2, 4) and (2, 4). ____________

19. Given $10y + 20 = 6x$ and $5y = -3x + 15$, determine if their lines are parallel, perpendicular or neither.

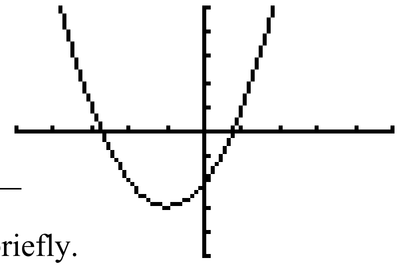
20. Find the x- and y-intercepts for $7x + 5y = -35$. x-int: ____________

y-int: ____________

Accelerated Integrated Algebra | Inspiration to: ________________

Test 3: M1 1.9, 2.10-12, M2 2.3-4 | Date ____________ Pd _________

Consider the graph shown at the right.



1. What is the vertex of this parabola? __________________

2. What is the equation for its axis of symmetry? ______________

3. Is this relation a one-to-one function? ___________ Explain briefly.

____

4. What is the equation for the above parabola? Assume no stretching is involved.

____

Find the vertices for the following parabolas. Indicate if the vertex is a maximum or a minimum.

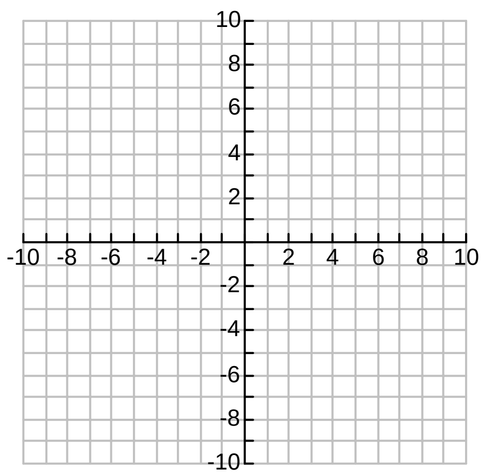
5. $y = 2x^2 - 8x + 3$ Vertex _________
Circle one: min max

6. $y = -3(x^2 + 4x - 6)$ Vertex _________
Circle one: min max

Graph the following. Include your vertex and two points on each side of it. Plot the axis of symmetry with a dashed line.

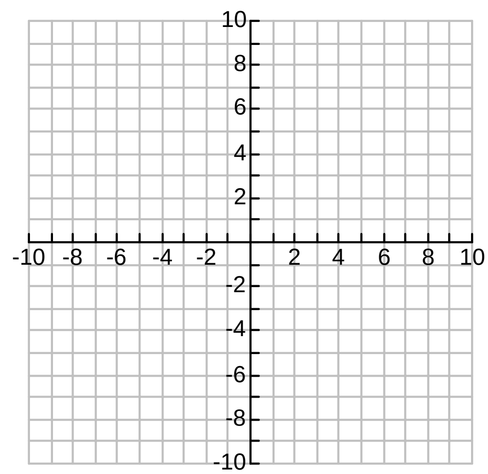
7. $y = x^2 - 4x + 3$

x					
y					



8. $y = -\frac{1}{2}|x - 1| + 5$

x					
y					



Determine whether the following value is a solution to the equation given. Show calculation.

9. $x^2 - 4x - 12 = 0; x = 2$ yes no

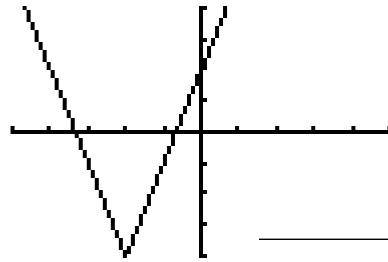
10. $3x^2 - 5x - 2 = 0; x = 2$ yes no

Tell the values of *x* for which the function is increasing and decreasing in the graph below.

11. Increasing: ________________

12. Decreasing: __________________

13. What is the equation of this function?
(Hint: It has been stretched vertically by a factor of 3.)



____

Solve (if possible) by graphing with a calculator.

14. $x^2 - x - 6 = 0$

15. $4x^2 - 5 = -8x$

16. $-\frac{1}{2}x^2 + 2x = 6$

Solution(s):__________ Solution(s): __________ Solution(s): __________

A drink bottle is dropped off the top of set of tall bleachers. The height *y* in feet above the ground can be given by $y = -16t^2 + 400$ where *t* is the number of seconds since the bottle was dropped.

17. How far was the drink bottle off the ground at 3 seconds? _______________

18. What is an appropriate domain and range for this relationship (in the context of this problem)?

Domain: ____________________________ Range: ______________________________

19. How many seconds before the bottle hits the ground? (Hint: It can be factored.)

The daily profit *y* of Quad Ratix, Inc., depends on how many of their trademark Para Bolas (*x*) are made. The profit can be found from the equation: $y = -\frac{1}{2}x^2 + 20x + 230$

20. How many Para Bolas need to be made for a *maximum* profit? How much profit will that yield?

Para Bolas needed: ________________ Maximum profit: ________________

AIA Test 4 Name: ______________________________

M2 3.2,9 M1 3.1,3,6,12 Date: _____________ Period: _____

1–10: Multiple-Choice: Please select the best answer.

1. What is the axis of symmetry of $y = -4(x+2)^2 - 5$?
   a. $x = -2$ b. $x = 2$ c. $x = -5$ d. $x = 5$

2. What is the vertex of $y = 2(x-3)^2 - 6$?
   a. (3, –6) b. (–3, 6) c. (–3, –6) d. (3, 6)

3. Which of the following extrema exist for the quadratic equation $y = -\frac{1}{2}(x-1)^2 + 4$?
   a. max of 4 b. min of 4 c. max of 1 d. min of 1

4. What are the domain and range of $y = -(x-2)(x+2)$?
   a. D: all real numbers, R: $y \le -4$ c. D: all real numbers, R: $y \ge -4$
   b. D: all real numbers, R: $y \ge 4$ d. d: all real numbers, R: $y \le 4$

5. The end behavior of $f(x) = -x^3 - 2x^2 + x - 3$ is
   a. As $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$.
   b. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
   c. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.
   d. As $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$.

6. The standard form of the quadratic equation $y = -\frac{1}{2}(x-2)^2 + 5$ would be
   a. $y = -\frac{1}{2}x^2 + 3$ b. $y = -\frac{1}{2}x^2 + 2x + 7$ c. $y = -\frac{1}{2}x^2 + 2x + 3$ d. $y = -\frac{1}{2}x^2 - 2x + 7$

7. A quadratic function has *x*-intercepts of 2 and –4 and a vertex of (–1, 3). Find *a* for the function.
   a. 2 b. $\frac{1}{2}$ c. $-\frac{1}{3}$ d. 7

8. Solve: $x^2 + 3x - 18 \le 0$
   a. $-6 \le x \le 3$ b. $x \le -6$ or $x \ge 3$ c. $x \le -3$ or $x \ge 6$ d. $-3 \le x \le 6$

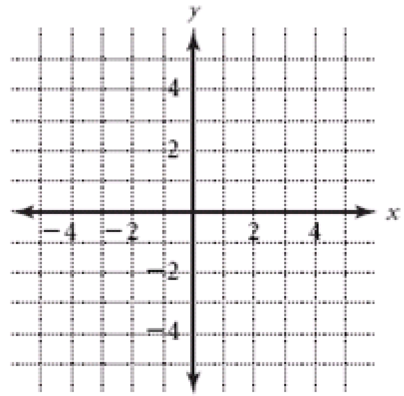
9. The *x*- and *y*-intercepts of the line $2x - 6y = 18$ are:
   a. (9, 0) and (0, –3) c. (–3, 0) and (0, 9)
   b. (9, 0) and (0, 3) d. (–9, 0) and (0, –3)

10. The zeros of $y = x^2 - 4x - 32$ are:
   a. 4, 8 b. –4, 8 c. –4, –8 d. 4, –8

11. State the transformations from the parent function for the cubic equation $y = -\frac{3}{5}(x-2)^3 + 1$.

12. Is $f(x) = 2x^4 - 3x^2 - 5$ even, odd, or neither? Justify your answer algebraically.

13. State the transformations from the parent function, give the domain and range, and sketch the graph of $y = \sqrt{x-3} - 2$.



Transformations:

Domain: ____________

Range: _____________

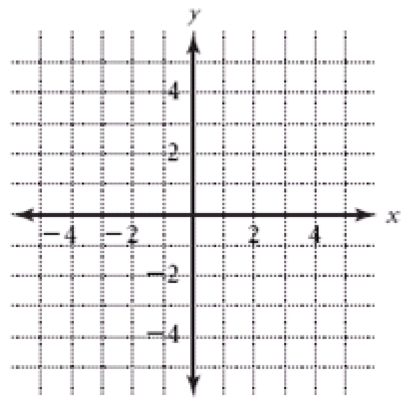
14. Find the average rate of change of the function from x_1 to x_2.

$f(x) = x^2 - 2x - 3$ $x_1 = 0$ $x_2 = 3$ ________________

15. Graph the system of quadratic inequalities:

$y \le -x^2 + 2x + 1$

$y < x^2 - 1$



16. For the rational function $y = \frac{3}{x-2} - 1$, state the transformations from the parent function, determine the asymptotes, and state the domain and range.

a. Transformations:

b. Vertical Asymptote: _____________ Horizontal Asymptote: ______________

c. Domain = ____________________ Range = _____________________

17. The height of a football after t seconds that has been kicked is given by the function $h(t) = -16t^2 + 48t$.

a. What is the maximum height of the football? ____________ Steps:

b. If the crossbar of a goal post is 10 feet high, and it will take the ball 2 seconds to get to the goal post, will the ball still be high enough to go over the crossbar? ____________ Steps:

18. Factor: $x^2 - 13x - 48$ _______________ 19. Factor: $-2x^2 + 2$ ______________________

20. What transformation must be done to the parent function, $f(x) = x^2$, for the function to have a maximum?

AIA UNIT 5 TEST
3.4-3.5 (M1), 1.1-1.4 (M2)

NAME________________________
DATE_____________PERIOD____

Write the expression as a complex number in standard form. Show steps.

1. $(9-8i)-(11-9i)$

2. $5i-(3+4i)+6$

3. $5-\sqrt{-80}$

1.____________________ 2.___________________ 3._______________

4. $\dfrac{-4+\sqrt{-60}}{2}$

5. $(3+2i)(-4+i)$

6. $\dfrac{6+2i}{4+8i}$

4.___________________ 5.___________________ 6._____________

7. $\dfrac{1+2i}{1-3i} \div \dfrac{1+3i}{1-2i}$

8. $5i(2i^2)+3i^3$

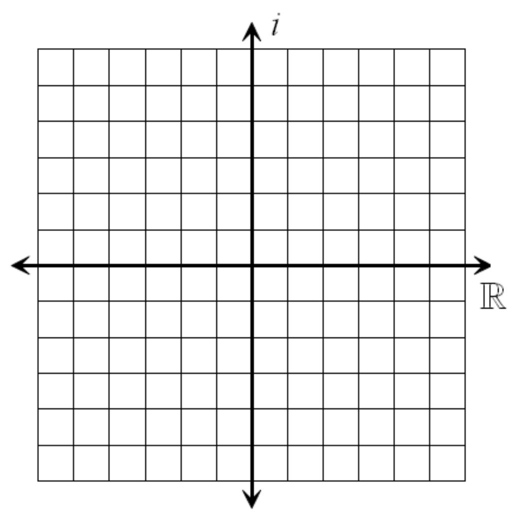
9. $\sqrt{\dfrac{-50}{64}}$

7.___________________ 8.___________________ 9._____________

Graph the following complex numbers in the complex plane below, and then find the absolute value of each number:

10. $A: 3+-5i$ ______________________

11. $B: 4+5i^2$ ______________________



Solve for x (and y if possible).

12. $-2x + 4yi = 18 + 28i$

13. $15i - 14 = 4y + 3xi$

x = ______ y =_______

x = _____ y = _____

14. a. Is it possible to solve $i^x = -1$ for x? How many solutions are there? Explain.

b. In relation to what we have learned this chapter, is i a variable? Explain.

SIMPLIFY:

15. $\sqrt{12x^5} \bullet \sqrt{6x}$ = _______________

16. $\sqrt{\dfrac{13}{2y^2}}$ = ____________________

SOLVE:

17. $\sqrt{30 - x} = x$

18. $\sqrt{2x - 10} = \sqrt{x + 7}$

17.________________________

18.______________________

19. $3\sqrt{x - 6} + 8 = 32$

20. $\sqrt{x - 15} - \sqrt{x - 7} = 0$

19.__________________________

20. ________________________

Test 6: M2 3.3-3.8 Name ___________________________

Accelerated Integrated Algebra I Date ______________ Period _____

1. Write a quadratic equation in standard form that has the following solutions.
   a. $x = 5, -2$ a. ___________

   b. $x = 6i, -6i$ b. ___________

Find the discriminant and determine the # of solutions.

2. $3x^2 - 6x + 3 = 0$

2. Discriminant: ___________

real solns: ______________

imaginary solns: _________

3. $-5x^2 + 4x = 0$

3. Discriminant: ___________

real solns: ______________

imaginary solns: _________

4. Below is the graph of a quadratic function. Use the graph to describe what you know about the discriminant of its quadratic equation.

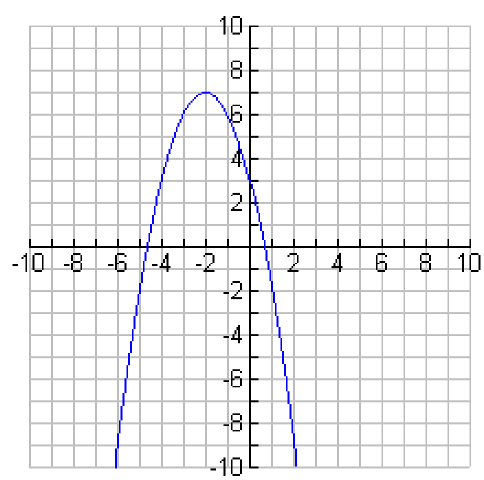
____

____

____

____

____



5. Frank's job pays him based on the number of sales he makes. His pay *per sale* is equal to the number of sales plus 20. He made $800 last week. Write an equation to determine the number of sales he made last week and solve.

Equation: ____________________________

Solution: _____________________

Solve for x using any method. Show steps. Answers should be in simplest exact (not rounded) form.

6. $7x^2+6x+2=0$

7. $x^2-10x+21=0$

6. ________________

7. ________________

8. $1.5x^2+3x-6=0$

9. $2(x^2-5)=-x^2-1$

8. ________________

9. ______________

10. $\frac{x}{3}+\frac{2}{3x}=1$

11. $x^2-6x+4=6x-2$

10. ________________

11. ________________

12. Complete the square to put in vertex form: $y = 4x^2 - 8x + 23$.
Find the vertex.

12. ________________

Vertex: ___________

Calculate the average rate of change over the given interval.

13. $f(x)=2x^2-5x+3, 0\le x\le 3$

13. ________________

14. A quadratic function, $f(x)$, has an average rate of change of -3 over the interval $-4\le x\le -1$ and an average rate of change of 3 over the interval $-1\le x\le 2$.
Which way does the parabola open?

14. _______________

What is its axis of symmetry?

____

Accelerated Integrated Algebra 1 Test 7: 3.7–3.11 Name____________________

Show steps for all responses. Date __________ Period _____

Simplify the expression. State the excluded value(s).

1. $\dfrac{28x}{7x-21}$

Simplified________________________

Excluded value(s)__________________

2. $\dfrac{x^2+13x+42}{x^2-2x-63}$

Simplified________________________

Excluded value(s)__________________

Find the product or quotient.

3. $\dfrac{x^2-1}{2x^2-3x+1} \bullet \dfrac{4x-2}{3x+18}$

3. __________________________

4. $\dfrac{x^2+3x-10}{3x^2-3x} \div \dfrac{x^2-8x+12}{x-1}$

4.__________________________

Use long division.

5. $(24x^2-19x+6)\div(8x-1)$

5. _________________________

6. $(12x^2+10x-11)\div(3x+4)$

6. _________________________

Find the sum or difference

7. $$\frac{5x+7}{3x-4}-\frac{2x-9}{3x-4}$$

7. ______________________________

8. $$\frac{7}{18x^2}+\frac{12}{9x^3}$$

8. ______________________________

Solve for *x*.

9. $$\frac{8}{x-5}=\frac{x}{3}$$

9. ______________________________

10. $$\frac{x+2}{x-4}-\frac{2x}{x-1}=\frac{18}{x^2-5x+4}$$

10. ______________________________

11. You have a rectangular deck that has length that is 3 times as big as the width. If *x* is the width of the deck write an expression for the ratio of the perimeter to the area of the deck. Simplify if possible.

11. ______________________________

APPENDIX G: SAMPLE STUDENT TEST ERROR ANALYSIS

Test Error Analysis Worksheet

Name (withheld) Test A1A Test 4 Period 2 Date 10-22-09

Please analyze the points lost for this test, starting with a column for the first question you had an error on. You need not include the negative symbol or zero fill. Be sure to include specifics ("careless mistake" is not specific) for anything attributed to "other", write vocabulary and concept lists on the back, and make sure your totals match up. The goal is to learn from each test and improve your performance on math tests.

A. Overall	
1. Grade	82
2. Total points missed (100 – A1)	18

Error Type \ Question Number	4	12	14	15	16	17	19							**Sum**
B. Testing Points (Generic – could apply to any subject)														
1. Accidentally skipped the question														
2. Didn't follow directions														
3. Misread the question														
4. Didn't show steps														
5. Misread my own writing														
6. Missing/incorrect units														
7. Didn't provide *final* answer														
8. Unclear communication														
9. Incorrect notation														
10. Ran out of time														
11. Other (Testing):														
12. Other (Testing):														
13. Total Testing Points Lost														
C. Content Points (Math specific)														
1. Didn't understand question														
2. Forgot vocabulary (List on back.)														
3. Didn't know how (List on back.)														
4. Didn't combine known concepts														
5. Didn't simplify														
6. Confused two concepts (List on back.)					2	1								3
7. Used the wrong formula (List on back.)							5							5
8. Knew how, but forgot (List on back.)		3												3
9. Arithmetic error – fractions (+, –, *, /)														
10. Arithmetic error – no fractions (+, –, *, /)	5		1											6
11. Didn't distribute properly														
12. Multiplied instead of using exponents														
13. Graphing error				1										1
14. Freshman's dream (binomial mult.)														
15. Illogical or incomplete proof														
16. Calculator usage error (List on back.)														
17. Spelling:														
18. Other (Math):														
19. Other (Math):														
20. Total Content Points Lost	5	3	1	1	2	1	5							18
Total Points Lost (B13 + C20) (Sum should match points missed in A2.)	5	3	1	1	2	1	5							18

Daymude

Test Error Analysis Worksheet

Name ______________________ Test _______ Period ______

List vocabulary to review:

List concepts to review:

even, odd, neither questions
asymtotes

APPENDIX H:
STUDENT-CONSTRUCTED ADVICE HANDOUT

Group Members: ________________________
____
____
____

What advice would you give to students having the following test errors? Give two or three suggestions for each. Use the back if necessary.

B 1. Accidentally skipped the question

B 2. Didn't follow directions

B 3. Misread the question

B 4. Didn't show steps

B 5. Misread my own writing

B 6. Missing/incorrect units

B 7. Didn't provide final answer

B 8. Unclear communication

B 9. Incorrect notation

B 10. Ran out of time

C 1. Didn't understand question

C 2. Forgot vocabulary

C 3. Didn't know how

C 4. Didn't combine known concepts

C 5. Didn't simplify

C 6. Confused two concepts

C 7. Used the wrong formula

C 8. Knew how, but forgot

C 9. Arithmetic error (+, -, *, /)

C 10. Didn't distribute properly

C 11. Multiplied instead of using exponents

C 12. Graphing error

C 13. Freshman's dream (binomial mult.)

C 14. Illogical or incomplete proof

C 15. Spelling

APPENDIX I: END OF SEMESTER STUDENT TESTING QUESTIONNAIRE

So…how did the testing go?!!!

Name ___________________________ *Date* ____________ *Period* _________

Dear Students,

Thank you for all of your hard work in preparing for, taking, and analyzing your tests this semester! I am interested in gaining your insights about these processes. Please respond to the following questions so that I can understand your perspectives. Your input will be kept confidential, but may be used in my research on test error analysis.

Fondly, Ms. D.

A. These first questions help me understand how you prepared for your tests. There is no need to exaggerate or understate what you did. Just circle the response that you feel is most accurate for you.

	0	*1*	*2*	*3*	*4*	*Write the number if more than 4.*
1. On average, I spent ____ hours outside of class preparing for each test (beyond daily assignments).	*0*	*1*	*2*	*3*	*4*	_____
2. I came to Ms. Daymude for extra help before or after school about ____ times a week.	*0*	*1*	*2*	*3*	*4*	_____
3. I was tutored by someone other than Ms. Daymude about ____ times a week.	*0*	*1*	*2*	*3*	*4*	_____
4. I reviewed my test error analysis data about ____ times this semester.	*0*	*1*	*2*	*3*	*4*	_____

< over >

B. Please consider each of the following statements. Indicate how much you agree or disagree with them by circling the number that corresponds to your response.

	Strongly Disagree	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
1. I prepared well for my tests.	*1*	*2*	*3*	*4*	*5*
2. Test error analysis helped me learn from my testing errors.	*1*	*2*	*3*	*4*	*5*
3. Test error analysis helped me prepare for subsequent tests.	*1*	*2*	*3*	*4*	*5*
4. Test error analysis took too long.	*1*	*2*	*3*	*4*	*5*
5. The class's advice in preventing testing errors was helpful to me.	*1*	*2*	*3*	*4*	*5*
6. Test error analysis helped me explain my testing performance to my parents.	*1*	*2*	*3*	*4*	*5*

Now think back to how you felt on those first tests at the beginning of the semester. Then consider how your testing progressed as time went on.

	Strongly Disagree	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
7. As the semester progressed, I got more comfortable taking tests.	*1*	*2*	*3*	*4*	*5*
8. As the semester progressed, I learned to prepare better for tests.	*1*	*2*	*3*	*4*	*5*
9. As the semester progressed, I became less satisfied with my mathematics testing performance.	*1*	*2*	*3*	*4*	*5*

Please summarize the change, if any, in your testing performance over the semester.

	got much worse.	*got worse.*	*stayed about the same.*	*got better.*	*got much better.*
10. In general, I would say that over the course of the semester, my test performance _____.	*1*	*2*	*3*	*4*	*5*

If you answered 1, 2, 4, or 5 to question 10 above, please answer question 11. If you answered 3, (stayed about the same), please skip to Section C.

11. Changes in my test performance over time were mostly due to:

C. Please answer the following open-ended questions about testing and test error analysis.

1. What did you learn from your test error analysis?

2. What did you learn from the advice the class generated?

3. What do you plan to do next year to maximize your test performance?

4. What advice would you give to future students to help them test better?

5. What changes would you recommend to the test error analysis process or the forms that we used?

6. What other comments would you like to add?

Thank you very much for your insights! I look forward to reading them and considering them for next semester and for my research. If you think of anything you'd like to add later, feel free to let me know. Enjoy your break! ☺

Best wishes, Ms. D.

APPENDIX J: PARENT QUESTIONNAIRE E-MAIL

Dear Parents,

Thank you for letting me include your child's data in my research on test error analysis. I would like to get your input if you are familiar with the test error analysis process that your child has used. So, the purpose of this e-mail is to give you an opportunity to share your insights as parents of the student participants in the study through this optional questionnaire.

Your child has developed a summary sheet of errors made on our tests throughout the semester. A blank Summary of Errors form is posted on my website in my handouts. I have also posted the Student-Generated Advice and the Preliminary Findings handouts.

Please answer the following questions if you would like to share your perspective:

- Has your child shared his/her test error analysis with you?
- If not, you may stop here, or ask your child to share it with you and continue:
- If so, what have you learned from your child's test error analysis?
- How helpful is it in understanding your child's test performance and communicating with your child?

Feel free to share any additional comments on the test error analysis process.

Thank you once again,

Kathy Daymude

APPENDIX K: QUESTIONS FOR TEACHER ASSESSMENT OF TOOL

Name ______________

Date ______________

How helpful was the test error analysis in assessing reasons for student testing errors?

What did test error analysis tell you about students' mathematics learning difficulties?

What did it tell you about mathematics teaching?

How did it affect student test performance?

What trends did you notice?

How helpful was it in communicating with parents about student testing performance?

APPENDIX L: ADDITIONAL DETAILS FOR SELECTED TEST ITEMS

This appendix contains additional test error analysis data for selected test items. Please note that some students split coding of errors to more than one type, so student totals indicated in some of the item tables may differ from straight sums of student numbers.

Test 1

Item 20

Error type	Points	Students
Testing		
Misread the question	13	5
Total testing errors	**13**	**5**
Content		
Didn't understand the question	16	5
Didn't know how	9	3
Confused two concepts	6	3
Knew how, but forgot	8	4
Didn't distribute properly	2	1
Graphing error	8	4
Total content errors	**49**	**20**
Total	**62**	**25**

Of the four students who attributed their errors to *graphing* or *graph-reading* errors, subsequent interviews revealed that these errors tended to be made by students who graphed the equations on their calculators.

Item 19

Error type	Points	Students
Testing		
Accidentally skipped the question	12	3
Misread the question	1	1
Total testing errors	**13**	**4**
Content		
Didn't understand the question	8	3
Forgot vocabulary	4	1
Didn't know how	10	2
Confused two concepts	6	2
Graphing error	1	1
Total content errors	**29**	**9**
Total	**41**	**12**

Item 15

Error type	Points	Students
Content		
Didn't understand the question	12	4
Didn't know how	5	1
Didn't combine known concepts	4	1
Confused two concepts	6	2
Knew how, but forgot	9	3
Arithmetic/algebraic error – no fractions	3	1
Total	**39**	**12**

Test 2

Item 16

Error type	Points	Students
Testing		
Didn't follow directions	10	5
Ran out of time	10	2
Total testing errors	**20**	**7**
Content		
Didn't know how	17	4
Confused two concepts	11	5
Arithmetic/algebraic error, no fractions	1	1
Didn't distribute properly	4	2
Didn't factor completely	8	4
Other	6	2
Omitted one of multiple solutions	20	16
Total content errors	**67**	**33**
Total	**87**	**40**

Two students' responses were coded as *other*. One did not use the Zero Product Property. The other had the right answer for the wrong version of the test; this student may have copied her neighbor's paper, but she denied it and was, therefore, not charged with cheating.

Item 17

Error type	Points	Students
Testing		
Didn't show steps	6	2
Misread my own writing	3	1
Ran out of time	9	2
Total testing errors	**18**	**5**
Content		
Didn't understand the question	9	2
Didn't know how	40	9
Didn't combine known concepts	10	2
Confused two concepts	3	1
Arithmetic/algebraic error, no fractions	2	1
Didn't distribute properly	2	1
Incomplete answer	5	1
Total content errors	**71**	**17**
Total	**89**	**22**

Item 11

Error type	Points	Students
Testing		
Misread the question	5	1
Other (didn't read all the answers)	5	1
Total testing errors	**10**	**2**
Content		
Didn't understand the question	5	1
Didn't know how	15	3
Didn't combine known concepts	5	1
Didn't simplify	25	5
Knew how, but forgot	10	2
Didn't distribute properly	5	1
Total content errors	**65**	**13**
Total	**75**	**15**

The student who classified the answer as *other* had "24" instead of "–24", saying he "didn't read all the answers." This error could also have been considered an *arithmetic error with no fractions*, which would have changed it to a content error.

Item 15

Error type	Points	Students
Testing		
Didn't follow directions	7	3
Misread the question	2	2
Ran out of time	5	1
Total testing errors	**14**	**6**
Content		
Didn't know how	24	6
Confused two concepts	2	2
Arithmetic/algebraic error, no fractions	12	5
Didn't factor completely	2	2
Other (divided by zero)	1	1
Missing solution	19	16
Total content errors	**60**	**31**
Total	**74**	**37**

The students who confused two concepts confused “solve by factoring” with “factoring” and factored without solving for x. The student who listed *other* as her reason divided by x, effectively “dividing by zero,” then failed to include 0 as a solution. This could have been included with the *missing solution* errors.

Test 3

Item 20

Error type	Points	Students
Testing		
Accidentally skipped the question	5	1
Misread the question	8	2
Ran out of time	25	5
Total testing errors	**38**	**8**
Content		
Didn't understand the question	15	3
Didn't know how	24	6
Used the wrong formula	10	2
Arithmetic/algebraic error – fractions	2	1
Arithmetic/algebraic error – no fractions	2	2
Calculator usage error	**5**	1
Total content errors	**58**	**15**
Total	**96**	**23**

Students who used the wrong formula failed to find the vertex at $x = -\frac{b}{2a}$, finding the right-most *x*-intercept instead; these errors could also have been considered confusing two concepts.

Item 18

Error type	Points	Students
Testing		
Accidentally skipped the question	12	3
Misread the question	2	1
Ran out of time	10	2
Total testing errors	**24**	**6**
Content		
Didn't know how	22	7
Didn't combine known concepts	4	2
Confused two concepts	5	2
Knew how, but forgot	10	4
Graphing error	**5**	2
Didn't factor completely	3	1
Notation	16	11
Total content errors	**65**	**29**
Total	**89**	**34**

Item 8

Error type	Points	Students
Testing		
Didn't follow directions	8	6
Total testing errors	**8**	**6**
Content		
Didn't understand the question	5	1
Didn't know how	13	3
Confused two concepts	6	2
Used the wrong formula	8	2
Knew how, but forgot	5	2
Arithmetic/algebraic error – fractions	4	1
Arithmetic/algebraic error – no fractions	4	2
Graphing error	31	12
Total content errors	**76**	**24**
Total	**84**	**28**

The two concepts that were confused were graphing quadratic functions and graphing absolute value functions.

Test 4

Item 16

Error type	Points	Students
Testing		
Misread the question	2	1
Ran out of time	32	8
Total testing errors	**34**	**9**
Content		
Didn't understand the question	15	4
Didn't know how	26	8
Confused two concepts	6	3
Knew how, but forgot	28	11
Incomplete answer	2	2
Calculator usage error	**7**	2
Notation	5	5
Total content errors	**89**	**35**
Total	**123**	**42**

One of the two students who confused two concepts interchanged the horizontal and vertical asymptotes; the other confused the domain and range.

Item 12

Error type	Points	Students
Testing		
Accidentally skipped the question	5	1
Didn't follow directions	4	2
Misread my own writing	1	1
Ran out of time	5	1
Total testing errors	**15**	**5**
Content		
Didn't understand the question	16	4
Didn't know how	43	12
Confused two concepts	14	4
Used the wrong formula	8	3
Knew how, but forgot	11	5
Arithmetic/algebraic error – no fractions	**2**	2
Omitted one of multiple solutions	7	3
Total content errors	**101**	**33**
Total	**116**	**38**

Item 17

Error type	Points	Students
Testing		
Didn't follow directions	9	4
Ran out of time	29	6
Total testing errors	**38**	**10**
Content		
Didn't understand the question	7	2
Didn't know how	28	8
Didn't combine known concepts	2	1
Confused two concepts	2	2
Knew how, but forgot	2	1
Arithmetic/algebraic error – no fractions	1	1
Graphing error	1	1
Incomplete answer	1	1
Calculator usage error	2	2
Total content errors	**46**	**19**
Total	**84**	**29**

Students who confused two concepts failed to correctly differentiate between *t* and *h*(*t*).

Item 19

Error type	Points	Students
Testing		
Ran out of time	39	8
Total testing errors	**39**	**8**
Content		
Didn't know how	10	2
Didn't simplify	1	1
Confused two concepts	1	1
Used the wrong formula	5	1
Knew how, but forgot	5	1
Didn't distribute properly	6	5
Incomplete answer	1	1
Didn't factor completely	16	16
Total content errors	**45**	**28**
Total	**84**	**35**

Midterm

Item 5

Error type	Points	Students
Testing		
Misread the question	5	1
Total testing errors	**5**	**1**
Content		
Didn't understand the question	15	3
Didn't know how	25	5
Didn't combine known concepts	5	1
Confused two concepts	5	1
Knew how, but forgot	5	1
Arithmetic/algebraic error – no fractions	5	1
Graphing	15	3
Total content errors	**75**	**15**
Total	**80**	**16**

The student whose error was coded as an *arithmetic error – no fractions* had the wrong sign in the denominator; her error was wholly contained in the denominator.

Item 7

Error type	Points	Students
Testing		
Other (bubbled in wrong answer)	5	1
Total testing errors	**5**	**1**
Content		
Didn't understand the question	5	1
Didn't know how	10	2
Confused two concepts	5	1
Knew how, but forgot	25	5
Arithmetic/algebraic error – no fractions	5	1
Total content errors	**50**	**10**
Total	**55**	**11**

The error coded as *other* was identified as bubbling in the wrong answer. The student who claimed to have *confused two concepts* listed binomial expansion and Pascal's triangle as the two concepts he confused.

Item 16

Error type	Points	Students
Testing		
Didn't factor completely	20	4
Ran out of time	5	1
Total testing errors	**25**	**5**
Content		
Didn't simplify	5	1
Confused two concepts	5	1
Didn't distribute properly	5	1
Total content errors	**15**	**3**
Total	**40**	**8**

Test 5

Item 11

Error Type	Points	Students
Testing		
Misread the question	2	1
Ran out of time	5	1
Total testing errors	**7**	**2**
Content		
Didn't understand the question	5	1
Forgot vocabulary	1	1
Didn't know how	39	11
Didn't combine known concepts	6	2
Didn't simplify	27	12
Confused two concepts	5	2
Used the wrong formula	2	1
Knew how, but forgot	14	4
Arithmetic/algebraic error – no fractions	5	2
Graphing	16	8
Total content errors	**120**	**37**
Total	**127**	**39**

The student who used the wrong formula used $\sqrt{a^2 + (bi)^2}$ instead of $\sqrt{a^2 + b^2}$ to find $|a + bi|$.

Item 13

Error type	Points	Students
Testing		
Misread my own writing	1	1
Total testing errors	**1**	**1**
Content		
Didn't know how	45	9
Didn't combine known concepts	6	2
Didn't simplify	4	1
Confused two concepts	4	1
Used the wrong formula	5	1
Knew how, but forgot	10	2
Arithmetic/algebraic error – fractions	**2**	2
Arithmetic/algebraic error – no fractions	5	3
Notation	1	1
Total content errors	**82**	**22**
Total	**83**	**23**

Item 10

Error type	Points	Students
Testing		
Accidentally skipped the question	2	1
Ran out of time	2	1
Total testing errors	**4**	**2**
Content		
Didn't understand the question	2	1
Didn't know how	37	15
Didn't combine known concepts	3	1
Used the wrong formula	8	4
Knew how, but forgot	5	2
Arithmetic/algebraic error – no fractions	9	4
Graphing error	6	3
Total content errors	**70**	**28**
Total	**74**	**29**

The student who confused two concepts interchanged the real and imaginary axes and missed the absolute value of the complex number.

Test 6

Item 5

Error type	Points	Students
Testing		
Ran out of time	11	2
Total testing errors	**11**	**2**
Content		
Didn't understand the question	65	12
Didn't know how	80	15
Didn't combine known concepts	10	2
Knew how, but forgot	6	1
Arithmetic/algebraic error – fractions	5	1
Arithmetic/algebraic error – no fractions	2	2
Notation	1	1
Total content errors	**169**	**34**
Total	**180**	**36**

The student who recorded a notation error provided an expression ($x^2 + 20x - 800$) instead of an equation ($x^2 + 20x - 800 = 0$).

Item 4

Error type	Points	Students
Testing		
Unclear communication	2	2
Ran out of time	7	1
Total testing errors	**9**	**3**
Content		
Didn't understand the question	35	6
Forgot vocabulary	11	2
Didn't know how	33	8
Didn't combine known concepts	12	2
Confused two concepts	4	1
Used the wrong formula	6	1
Knew how, but forgot	7	1
Total content errors	**108**	**21**
Total	**117**	**24**

The student who used the wrong formula used the quadratic formula instead of the formula for the discriminant. The student who confused two concepts confused the quadratic formula with a quadratic equation and confused the vertex of the parabola with its *y*-intercept.

Item 12

Error type	Points	Students
Testing		
Didn't follow directions	4	1
Misread my own writing	2	1
Total testing errors	**6**	**2**
Content		
Didn't understand the question	8	2
Didn't know how	24	6
Didn't simplify	1	1
Confused two concepts	5	1
Used the wrong formula	19	6
Knew how, but forgot	7	2
Arithmetic/algebraic error – fractions	1	1
Arithmetic/algebraic error – no fractions	7	5
Didn't distribute properly	3	1
Notation	12	10
Total content errors	**87**	**32**
Total	**93**	**34**

The student who *did not follow directions* did not demonstrate completing the square. *Incorrect notation* was coded for a solution of $y = 4(x – 1) + 19$ instead of $y = 4(x – 1)^2 + 19$.

Item 14

Error type	Points	Students
Testing		
Misread the question	3	1
Ran out of time	14	2
Total testing errors	**17**	**3**
Content		
Didn't understand the question	3	1
Didn't know how	31	5
Didn't combine known concepts	8	2
Used the wrong formula	3	1
Knew how, but forgot	6	2
Arithmetic/algebraic error – no fractions	6	1
Graphing error	2	1
Notation	14	14
Total content errors	**73**	**27**
Total	**90**	**30**

Notation errors included identifying the axis of symmetry as –1 instead of $x = -1$.

Test 7

Item 10

Error type	Points	Students
Testing		
Didn't show steps	5	1
Misread my own writing	2	1
Other (Miscopied a step)	3	1
Total testing errors	**10**	**3**
Content		
Didn't know how	76	10
Didn't combine known concepts	2	1
Confused two concepts	3	1
Knew how, but forgot	28	5
Arithmetic/algebraic error – fractions	7	1
Arithmetic/algebraic error – no fractions	4	3
Didn't distribute properly	12	4
Omitted one of multiple solutions	4	1
Included an extraneous solution	11	10
"Cancelled" wrongly	19	3
Total content errors	**166**	**37**
Total	**176**	**40**

The student who confused two concepts confused factoring with solving. The student who *omitted one of the multiple solutions* identified the extraneous solution 4 as a possibility but failed to include the correct solution, 5.

Item 1

Error type	Points	Students
Content		
Didn't know how	56	9
Didn't simplify	13	2
Confused two concepts	7	1
Knew how, but forgot	2	1
Arithmetic/algebraic error – fractions	4	2
Arithmetic/algebraic error – no fractions	5	4
Didn't distribute properly	7	1
"Cancelled" wrongly	44	8
Total content errors	**138**	**23**
Total	**138**	**23**

The student who confused two concepts multiplied the expression by $7x - 21$ as if she were solving an equation instead of simplifying an expression.

Item 11

Error type	Points	Students
Testing		
Accidentally skipped the question	1	1
Didn't follow directions	3	2
Misread the question	12	2
Total testing errors	**16**	**5**
Content		
Didn't understand the question	23	3
Didn't know how	48	7
Didn't combine known concepts	2	1
Didn't simplify	19	18
Confused two concepts	10	2
Notation	1	1
Total content errors	**103**	**31**
Total	**119**	**34**

The student who confused two concepts interchanged addition and multiplication. She correctly identified the perimeter as $3x + 3x + x + x$ and the area as $3x * x$, but she added the $3x$ and x for the area, then inverted the expression to find a ratio of 1:2. The student with the notation error also had a simplification error, responding with $8x = 3x^2$ instead of $8:3x$.

APPENDIX M: STUDENT-CONSTRUCTED ADVICE

Student-Constructed Advice by Test Error Type

B 1. Accidentally skipped the question
Check over your test when you're finished.
B 2. Didn't follow directions
Read the question multiple times, slowly and carefully.
Double check your answer.
B 3. Misread the question
Read the question multiple times, slowly and carefully.
Double check your answer.
B 4. Didn't show steps
Show all work on paper; don't do it in your head.
B 5. Misread my own writing
Take your time; write neatly.
B 6. Missing/incorrect units
Reread the question.
Check the problem for units and make sure to keep them in your answer.
Practice including units on daily assignments.
B 7. Didn't provide final answer
Read the directions.
Check your answers.
B 8. Unclear communication
Show all your work clearly.
Use arrows from equation to equation.
Use proper math terms and symbols.
Circle/box in your answer.
B 9. Incorrect notation
Check your answers.
Study notation; use memory devices.
Use proper math terms and symbols.
Take your time.
B 10. Ran out of time
Use time wisely, pace yourself.
Study more.
You don't have to answer the questions in order: Skip the most difficult ones and come back to them.
C 1. Didn't understand question
Study notes before the test.
Reread the question.
Ask the teacher for clarification during the test.
C 2. Forgot vocabulary
Study vocabulary before the test; use flashcards.
Look for content clues on the test for definition.
Use synonyms, if possible.
C 3. Didn't know how
Practice possible test questions.
Participate in class; ask questions.
Study; use memory devices.
C 4. Didn't combine known concepts
Study more.
Review concepts the night before and right before the test so you remember them.
Pay attention in class; ask questions.
Check work.
Come back to the question later; you might figure it out while you're answering other questions.
Do the math; go step by step.
C 5. Didn't simplify
Simplify; carry out expression.
Take your time.
Double check your answer.

C 6. Confused two concepts

Study to make sure you know the concepts and can distinguish between them.
Come up with ways to remember the concepts.

C 7. Used the wrong formula

Study formulas and material.
Reread the question.

C 8. Knew how, but forgot

Know all the formulas and material.
Practice.
Move on to other questions and see if you can remember later.

C 9. Arithmetic error – fractions (+, – , *, /)

Check fractions.
Know the right formulas.
Take your time.
Double check answer: Plug it back in to see if it works.

C 10. Arithmetic error – no fractions (+, –, *, /)

Go slower.
Check work with or without a calculator.
Practice.

C 11. Didn't distribute properly

Carry out the expression; learn to distribute.
Double check your work.
Use the right property.
Practice distributing.

C 12. Multiplied instead of using exponents

Pay attention to the problem.
Make sure to multiply if you see a multiplication sign or parentheses; use exponents if the number is small and raised.
Remember exponent rules.
Use flashcards.
Practice.

C 13. Graphing error

Check your work.
Use equal intervals on your scales.
Label axes.
Remember "y to the sky".
Graph it again and compare.

C 14. Freshman's dream (binomial mult.)

Pay attention in class.
Write it out.
Use the area model.
FOIL it.
Multiply back out to see if you get the right answer.

C 15. Illogical or incomplete proof

Work through proof thoroughly.
Read all directions.
Show all work.
Check work.
Learn all proof concepts; review theorems, postulates, definitions, and properties.

C 16. Calculator usage error (List on back.)

Practice skills on calculator.
Do the problem twice.
Press the right buttons.
Don't rush.

C 17. Spelling

Spell correctly.
Study math vocabulary ten minutes every night.
Look for the word elsewhere on the test.
Use flashcards.

APPENDIX N: STUDENT QUESTIONNAIRE OPEN-ENDED RESPONSES

What follows are students' coded verbatim responses to the open-ended items of the questionnaire. I applied pseudonyms to any names that were used. Capitalization and punctuation are identical to student responses. A dash "–" indicates no verbal response. Apparent spelling and grammatical errors are noted with "[*sic*]".

The coding key is provided with quantities of similar responses for each reason listed in parentheses.

Item B11. Changes in my test performance over time were mostly due to:

<u>Coding Key: Response (frequency)</u>

A. lack of preparation/studying (8)
B. preparation/studying (2)
C. not doing well on tests (1)
D. not understanding teacher (3)
E. understanding teacher (2)
F. not understanding concepts (5)
G. understanding concepts (1)
H. changes in level of difficulty of concepts (easier) (1)
I. changes in level of difficulty of concepts (harder) (0)
J. changes in level of difficulty of tests (easier) (0)
K. changes in level of difficulty of tests (harder) (2)
L. tasks (collaborative exercises exploring and applying the concepts in class) (1)
M. health/medical issues (1)
N. test anxiety (1)
O. not paying attention in class (2)
P. time spent on other subjects (2)
Q. studying the wrong material (1)
blank or "?" (21)

Coded Verbatim Responses to Item B11

Student	Response	Code
1	–	
2	Not preparing for the test	A
3	doing okay on assignments and bombing tests	B, C
4	–	
5	–	
6	Inability to understand what you say	D

7	No understanding because of unmedicated ADHD	M
8	Not studying or not understanding the material	A, F
9	Inability to grasp explanations of concepts	D
10	I think I was confused with the material. I asked for help in the afternoons sometimes and in the mornings. Talking/asking for help from Ms. Daymude helped me. I think I have test anexiety [*sic*] b/c tests really stress me out.	F, E, N
11	–	
12	–	
13	Easier stuff, tasks.	H, L
14	–	
15	–	
16	–	
17	–	
18	lack of studying	A
19	studying	B
20	?	
21	–	
22	–	
23	Not getting the material, and/or the difficulty of the questions.	F, K
24	* note: my testing average did go from high to low to high to medium over the semester.	
25	–	
26	A [*sic*] didn't understand the new material because it was taught to me in a way i normally didn't math. [*sic*]	F, D
27	Not paying attention in class	O
28	Had to put more effort into other classes, took time away from this one.	A, P
29	Getting a better understanding for the material and my teacher.	G, E
30	–	
31	Difficulty of test	K
32	–	
33	lack of studying and not understanding the information	A, F
34	–	
35	I had a greater work load [*sic*] as the semester progressed and I spent less time studying & trying to fully understand the concept	A, P
36	–	
37	–	
38	Lack of studing [*sic*]/preperation [*sic*]	A
39	–	
40	–	
41	not paying attention in class	O
42	Studying wrong matirial [*sic*].	A, Q
43	–	

Item C1. What did you learn from your test error analysis?

Coding Key: Response (frequency)

A. I learned about my mistakes. (8)
B. I learned that I did not correct my errors. (1)
C. Nothing. (7)
D. Not much. (4)
E. I learned what I needed to review. (3)
F. I learned what I needed to spend more time on. (1)
G. I know the material. (2)
H. I made dumb mistakes. (7)
I. I learned how to do the math. (1)
J. I learned to ask for help more. (1)
K. I learned about my weaknesses. (4)
L. I learned to watch for careless errors. (2)
M. I learned to check back over my work. (2)
N. I learned that I need to pay more attention. (1)
O. I learned that those – 1's and – 2's can kill your grade. (1)
P. I learned how to analyze my errors and learn from my mistakes. (1)
Q. I learned what I need to do better. (1)
R. Most of my mistakes are testing, not content errors. (1)
S. I learned what types of mistakes I make most frequently.(1)
T. I learned how to correct my mistakes.(1)
U. I learned to not make the same mistakes over again on other tests.(1)

Coded Verbatim Responses to Item C1.

Student	Response	Code	Rating
1	My mistakes	A	1
2	That I did not correct my errors	B	1
3	nothing really. they just made mistakes evident.	C, A	0
4	Not much	D	0
5	Test error analysis helped me learn what I needed to review and spend more time on.	E, F	1
6	Nothing	C	0
7	–		0
8	I usually know the material but make dumb mistakes.	G, H	1
9	nothing	C	0
10	I learned to know how to do the problem & to ask for help more. I realized that I needed to cut back on careless errors.	H, I, J	1
11	My mistakes and weaknesses	A, K	1
12	–		0
13	Not much	D	0
14	anot [*sic*] much I just made stupid choices.	D, H	0
15	to watch for careless errors.	L	1

16	I learned to check back over my work.	M	1
17	I learned to pay attention to careless errors.	L	1
18	What [*sic*] I need to pay more attention.	N	1
19	I make a lot of easy mistakes	H	1
20	Nothing	C	0
21	I learned exactly what I did on my tests.	A	1
22	Practically nothing	C	0
23	–		0
24	I learned why I was making mistakes on tests. I was making some stupid mistakes, but the test error analysis helped me learn which concepts I didn't fully grasp so that I could review them.	A, H, E	1
25	I learned that I made some simple mistakes that I should have gotten right.	H	1
26	I didn't learn much from them	D	0
27	–		0
28	I need to double check my work.	M	1
29	Where my weaknesses are.	K	1
30	Those little – 1, – 2 seriously can kill your grade.	O	1
31	Where my mistakes were	A	1
32	I learned how to analyze my errors and learn from my mistakes	P	1
33	What I needed to review and what I need to do better	E, Q	1
34	That I made simple mistakes.	H	1
35	my specific errors	A	1
36	Most of my errors have nothing to do with content and are just testing errors.	G, R	1
37	my weak points	K	1
38	Nothing	C	0
39	What types of mistakes I male [*sic*] most frequently, and how to correct them.	S, T	1
40	To not make the same mistakes over again on other tests.	U	1
41	What I needed help with	K	1
42	Nothing	C	0
43	my errors	A	1

Item C2. What did you learn from the advice the class generated?

Coding Key: Response (frequency)

A. How to study better. (4)
B. It's usually different from reasons I didn't do well. (1)
C. Nothing. (8)
D. I learned to check over my work more carefully. (5)
E. I learned that studying is helpful. (4)
F. I learned that everyone makes different mistakes. (1)
G. I learned what other people did that I do. (2)

H. I learned how to stop making mistakes. (1)
I. Not much. (5)
J. I knew most of the advice, but it reminded me. (1)
K. I learned that if I didn't do well on the test, that I wasn't alone. (1)
L. I learned to change my perspective and be more attentive in class. (1)
M. I learned how to get a better grade. (1)
N. I learned that I should read more carefully. (1)
O. I learned what to do/what not to do. (2)
P. I learned a lot; it was very helpful. (1)
Q. I learned to study methods, not vocabulary or strategies. (1)
R. I learned to take my time on tests. (1)
S. The class never gave advice. (1)
T. I learned a little more than from the test error analysis. (1)

Coded Verbatim Responses to Item C2.

Student	Response	Code
1	Tips on rembering [*sic*] subjects.	A
2	How to study better	A
3	it's usually totally different from reasons I don't do well.	B
4	Nothing	C
5	The advice the class generated gave me ideas on how to study and review concepts.	A
6	Nothing	C
7	–	
8	check over my work more carefully	D
9	Studying = helpful	E
10	Everyone makes different mistakes and it helped to know what other people did that I do and helped me to know how to stop it.	F, G, H
11	To check over my work	D
12	–	
13	didn't help much	I
14	I know most of the advice, but it reminded me.	J
15	nothing.	C
16	I didn't learn anything from the class, but I learned a little from the test analysis	C
17	Nothing	C
18	To "studyfy"	E
19	not a lot	I
20	Studytize	E
21	I didn't really learn anything.	C
22	Not that much	I
23	–	
24	I learned that, if I didn't do well on the test, I probably wasn't alone. After one test that most everyone in the class failed, we changed our perspective and became more attentive in class.	K, L

25	Look over/check work.	D
26	I didn't learn much from it.	I
27	–	
28	Not much	I
29	How to get a better grade.	M
30	That I need to study more.	E
31	Check carefully	D
32	How to study and to prepare for the tests.	A
33	that a lot of people have the same problems I do	G
34	That I missed problems because I didn't read it carefully enough.	N
35	what to do/not to do.	O
36	nothing	C
37	a lot, very helpful	P
38	Nothing new.	C
39	To study the methods, not study vocab or strategies.	Q
40	To take my time during a test and look back over my test	R, D
41	never really gave advice	S
42	A little more than the test error anyalsis [*sic*] but relitivly [*sic*] the same.	T
43	What to do and not to do	O

Item C3. What do you plan to do next year to maximize your test performance?

Coding Key: Response (frequency)

A. Study in groups. (1)
B. Study more; practice longer. (25)
C. Continue to study hard. (1)
D. Ask questions. (3)
E. Pay more attention in class. (2)
F. Look over my test error analysis and try not to make the same mistakes. (1)
G. Turn in assignments. (1)
H. Keep using a tutor. (1)
I. Watch for mistakes. (1)
J. Check my work better. (1)
K. Change to a non-accelerated class. (1)
L. Nothing. (2)
M. Review harder problems so I don't panic on the hard ones. (1)
N. I don't know. (1)
O. Take notes in class. (1)
P. Do homework. (2)
Q. Make sure I understand everything. (1)
R. Wing it. (1)
S. I don't know if I'll pass. (1)

Coded Verbatim Responses to Item C3.

Student	Response	Code
1	Study in groups	A
2	Study & practice longer	B
3	Study even more I guess	B
4	Study more	B
5	I plan to continue to study hard	C
6	Ask Questions	D
7	pay more attention in class	E
8	look over my test error analyses and try not to make the same mistakes	F
9	Study harder	B
10	STUDY!!! and ask for help when I don't understand	B, D
11	Study more	B
12	Study	B
13	Study more	B
14	Turn in assignments.	G
15	Keep using a tutor and watch for mistakes	H, I
16	Study more and ask questions	B, D
17	Check my work better and pay more attention.	J, E
18	Study more	B
19	Study more	B
20	drop accelerated	K
21	Study every night, and study for longer.	B
22	Nothing – it's already good	L
23	–	
24	Next year, I plan to review harder problems before a test. That way, when the test has really hard questions we might not necessarily have gone over, I won't panic.	M
25	Study & practice	B
26	I don't kno [*sic*]	N
27	Study more and take notes in class	O
28	Study 10 times as much	B
29	Study more and review	B
30	STUDY	B
31	Study more	B
32	STUDY! & Do homework.	B, P
33	STUDY!	B
34	Study	B
35	STUDY MORE! & make sure I understand everything fully	B, Q
36	I plan to go on taking my tests the way I always have.	L
37	finish homework completely	P
38	"wing it" on every test	R
39	To actually study for at least 30 min per test.	B
40	Study more	B

41	study alot [*sic*] more	B
42	I'm at a place where I don't know if I'll pass.	S
43	Study more, do more review problems	B

Item C4. What advice would you give to future students to help them test better?

Coding Key: Response (frequency)

A. Study in groups. (1)
B. Don't cram. (1)
C. Study. (26)
D. Come in if you need extra help. (3)
E. Pay attention in class. (10)
F. Do your homework. (5)
G. Ask questions. (5)
H. Turn in assignments. (1)
I. Use your time wisely. (1)
J. Check your work. (2)
K. Don't take accelerated math. (2)
L. Get a good night's sleep. (1)
M. Relax. (1)
N. Learn on your own. (1)
O. Don't get lazy; don't get behind. (1)
P. Watch your signs. (1)
Q. Ask for calculator tricks. (1)
R. Take notes. (2)

Coded Verbatim Responses to Item C4.

Student	Response	Code
1	Study in groups	A
2	Don't cram	B
3	–	
4	Study outside of class	C
5	Study! It is very important to study, and come in if you need extra help.	C, D
6	listen hw study ?'s	E, F, C, G
7	pay attention in class	E
8	–	
9	Study	C
10	Study material, pay attention, and ask for help.	C, E, G
11	Study & do all your homework	C, F
12	– Study – do the homework – ask questions	C, F, G, D

	– come in for help	
13	Study on your own time	C
14	Turn in assignments.	H
15	Study & use your time wisley [*sic*].	C, I
16	Pay attention and study the material.	E, C
17	Check your work thoroughly.	J
18	Study, reveiw [*sic*], do the homework	C, F
19	Make sure you are prepared.	C
20	Don't take accelerated	K
21	Study more	C
22	Pay attention in class	E
23	–	
24	I would advise future students to be prepared for the test. Study as much as you need to the night and days before (which could be 15 minutes or two hours). Get a good night's sleep and relax. When the teacher hands you the test, don't panic. It may look intimidating, but chances are, it's not impossible.	C, L, M
25	Practice problems a lot!	C
26	Learn more on their own	N
27	Pay attention in class	E
28	Don't get lazy, don't get behind.	O
29	Review and if you don't understand ask someone for help.	C, G
30	* watch your signs * study (at least a little bit) * ask Mrs. Daymude for all of the calculator tricks at the Beginning of the year	P, C, Q
31	Ask questions	G
32	STUDY! & Do your homework.	C, F
33	Study! And PAY ATTENTION!	C, E
34	Pay attention in class	E
35	make sure you know the concept well – pay attention.	C, E
36	Double check. You can catch some simple mistakes.	J
37	review chapter	C
38	Take this class to improve your math and get into a good colledge [*sic*], but don't take this class if you want to have a high GPA.	K
39	To study, most gifted students don't.	C
40	Study, listen in class, go in for help if you need it.	C, E, D
41	always take notes and study	R, C
42	Find matirial [*sic*] to study.	C
43	use the notebook wisely	R, C

Item C5. What changes would you recommend to the test error analysis process or the forms that we used?

Coding Key: Reason (frequency)

A. Add more choices. (11)
B. Add "stupid mistake". (2)
C. None. (14)
D. Sometimes it's hard to identify an error type. (1)
E. Use it for quizzes, so it will help our tests. (1)
F. Use it to study. (1)
G. Don't do it. (4)
H. Make them optional. (2)
I. Alternate coloring between light and dark. (1)
J. Reduce the number of choices. (1)

Coded Verbatim Responses to Item C5.

Student	Response	Code
1	more options	A
2	Add more categories	A
3	give a spot for stupid mistakes	B
4	None	C
5	maybe some more catigories.? [*sic*]	A
6	100 more categories	A
7	–	
8	Sometimes it is hard to figure out which category to put.	D
9	more categories	A
10	More categories	A
11	More mistake choices	A
12	–	
13	use it on quiz, so we can review it for test	E
14	More choices.	A
15	nothing.	C
16	Use it to study.	F
17	It should have a careless mistake section.	B
18	–	
19	–	
20	Don't do it	G
21	To not make them mandatory.	H
22	None	C
23	–	
24	– More options for missed points.	A
25	I'm ok with it ☺	C
26	Well it didn't help me	G, H
27	Alternate the coloring between light and dark	I

28	Stay the same	C
29	I think it was set up well, and professionally.	C
30	People start to lose interest with how many options there are.	J
31	Nothing	C
32	Very good! No changes.	C
33	none	C
34	n/a	C
35	–	
36	None.	C
37	More "other:" blanks	A
38	Don't change the test error analysis.	C
39	More math content.	A
40	I wouldn't change it, I think	C
41	Don't have any	G
42	rid the earth of them, they didn't help.	G
43	no changes	C

Item C6. What other comments would you like to add?

Coding Key: Reason (frequency)

A. TEA didn't help. (2)
B. Thanks for being a great teacher and explaining material to me. (1)
C. Make worksheets similar to tests and quizzes instead of giving book work. (2)
D. Thanks for allowing friends to sit together. (2)
E. Pay attention. (1)
F. Hope your professor likes this ☺ (1)
G. I'm glad I was in your class. (1)
H. I learned a lot. (1)
I. I enjoyed the group work. (1)
J. Study. (1)
K. Peace. (1)
L. Merry Christmas. (1)
M. I liked the test error analysis. (2)
N. I really want to pass/it's hard. (2)
O. Most of my errors were concepts, which changed with each test. I still don't know if I get it. (1)

Coded Verbatim (with Pseudonyms) Responses to Item C6.

Student	Response	Code
1	–	
2	N/A	
3	–	
4	The TEA didn't help me.	A

5	–	
6	none	
7	–	
8	–	
9	–	
10	Thanks for being a great teacher and explaining material to me.	B
11	–	
12	–	
13	Make worksheets instead of book. It helps a lot. Kind of make it similar to test or quiz.	C
14	Nothing	
15	Thank you for letting me sit next to linda [*sic*] all semester ☺ we really appriciate [*sic*] it.	D
16	PAY ATTENTION	E
17	Thank you for letting me & Katie sit next to each other all year. We really appreciate it! ☺	D
18	–	
19	–	
20	None	
21	none	
22	None	
23	–	
24	– Hope your professor likes this ☺	F
25	☺ I'm glad I was in your class. I learned a lot.	G, H
26	None	
27	I enjoyed the group work	I
28	–	
29	Study! Study! Study!	J
30	< peace symbol >	K
31	–	
32	Merry Christmas?	L
33	liked test error analysis	M
34	n/a	
35	–	
36	Nothing.	
37	–	
38	Passing is hard! Ms. is a nice teacher [*sic*], however if you can give students more worksheets for classwork grades.	N, C
39	Very Good Idea for Test Error Analysis.	M
40	–	
41	the test error analysis didn't really help because most of my errors were concepts and none of our next grades were over those concepts so I still don't even know if I have it right now	A, O
42	I really want to pass. ☺	N
43	??	

APPENDIX O: PARENT RESPONSES TO QUESTIONNAIRE

Response from First Parent

Mrs. Daymude,

Concerning the questionnaire, I do not discuss Austin's approach to math as he knows more than I do!! I only check to make sure he has prepared his work.

Thank you and have a great vacation!!

Nina

Response from Second Parent

– Has your child shared his/her test error analysis with you?

No

– If not, you may stop here, or ask your child to share it with you and continue:

– If so, what have you learned from your child's test error analysis?

That most of the mistakes she makes are careless mistakes.

– How helpful is it in understanding your child's test performance and communicating with your child?

Somewhat helpful

Feel free to share any additional comments on the test error analysis process.

"It was kind of hard to use it when you had a high grade"
– quote from Evelyn [student]