CREATING OPTIMAL MATHEMATICS LEARNING ENVIRONMENTS: COMBINING ARGUMENTATION AND WRITING TO ENHANCE ACHIEVEMENT

by

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(Under the direction of Paul A. Schutz)

ABSTRACT

The issue of mathematics underachievement among students has been an increasing national concern over the last few decades. Research suggests that academic success can be achieved by focusing on both the individual and social aspects of learning by creating a bridge between cognitive and socio-cultural views of knowledge and learning. Within the area of mathematics education, the development of metacognitive skills and the incorporation of discourse in classroom instruction has resulted in deeper conceptual understandings and increased mathematical achievement. However, studies in this field tend to focus on the effects of these practices separately, making research that seeks to harness the potential of both quite rare. The study described in this paper attempted to address this gap in the literature by examining the effects of writing and argumentation on achievement. Two hundred and eleven students and five teachers participated in this multi-methods study, which investigated the effects of three treatment conditions on mathematical achievement. These conditions were writing alone, argumentation alone, and writing and argumentation combined. Recognizing that teacher factors are also important in the developing and sustaining of effective learning environments and acknowledging the influence of teachers' beliefs on any educational initiative, the study also sought to describe the beliefs of these five ninth grade mathematics teachers and investigated how consistently these beliefs were manifested in practice.

INDEX WORDS: Mathematical argumentation, Writing, Teachers' beliefs, Mathematical beliefs, Belief Change

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CHAPTER 1

INTRODUCTION

Mathematics achievement has been an increasing national concern, specifically in relation to public education (Schmidt, Wang, & McKnight, 2005). This concern is in response to fairly negative national and international reports regarding US students' mathematical performance. For example, reports from the Trends in International Mathematics and Science Study (TIMSS) (National Center for Educational Statistics, 1999, 2003) revealed deficiencies in students' understanding and fluency in mathematics. These gaps were associated with curriculum focus and design and also with the quality of instruction, especially in the middle grades (Boe & Shin, 2005). This lack of achievement, relative to other industrialized nations, fosters doubt about the United States' future economic competitiveness and students' abilities to compete on the global market, and so stakeholders (including parents, policymakers, administrators) are becoming more involved in efforts to find a solution.

With the increasing demands on teachers and administrators to improve understanding and achievement for all students (with the passing of the No Child Left Behind Act), we need to holistically address the issue of mathematical under-achievement. This involves the possible restructuring of mathematical learning environments and a thorough examination of the factors that influence how teachers organize and maintain these environments. These initiatives tend to be quite effortful and laborious, and so for them to be successful careful and critical investigations need to be done of the constitutive components of these environments. These components include the techniques and strategies used within them and also the attitudes and perceptions of those in charge of their functioning, the teachers. The articles included in this document attempt to address both these issues - increasing mathematical achievement through the implementation of strategies that promote mathematical fluency and conceptual understanding, as well as exploring the factors that influence teachers' abilities to structure and maintain successful mathematical learning environments.

Researchers (Cobb et al., 1991; Lesh, Doerr, Carmona, & Hjalmarson, 2003; Pontecorvo, 1993; Schoenfeld, 1992) suggest that academic success can be achieved by focusing on both the individual and social aspects of learning. In this regard, within the area of mathematics education, the development of metacognitive skills and the incorporation of discourse in instruction has resulted in deeper conceptual understandings and increased mathematical achievement (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Garii, 2002; Kramarski, Mevarech, & Arami, 2002a; M. K. Stein, 2001). However, researchers have tended to focus on the effects of these practices separately, making studies that seek to harness the potential of both these strategies rare (Nasir, 2005). Article one seeks to address this gap in the literature by reporting on a study that investigated the effect of two such strategies on mathematics achievement. These strategies were writing, an activity designed to encourage metacognitive thinking, and mathematical argumentation, an activity designed to engage students in shared knowledge construction. The 211 ninth grade, Algebra 1 students and the five teachers who were participants in this study, engaged in discourse and writing activities over the course of 10 weeks. Each activity was designed to engage students in knowledge building by channeling the cognitive resources activated when involved in individual and collective knowledge construction, as it was believed that maximizing students' opportunities to learn would enhance mathematical understanding and achievement.

Engaging students in these types of activities required not only the use of appropriately challenging tasks but also that the teacher actively facilitated student involvement. Essentially, the role of the teacher is crucial in promoting focused involvement and understanding as students engage in any form of knowledge construction. Teachers, beyond the facilitation of these specific tasks, are integral to the learning process as they are the ones in charge of the daily structuring and monitoring of the instructional activities aimed at building understanding and overall academic success. Teachers' decision making processes regarding how they plan, teach, interact with students, reflect, and evaluate their daily teaching experiences are considered to be a product of their beliefs about the nature of the subject, pedagogy and student learning. Much research has been done in this area over the last few decades because of the observed intimate relationship between teachers' beliefs about teaching and student learning and their approaches to instruction (Nespor, 1987; Pajares, 1992; Woolfolk-Hoy, Davis, & Pape, 2006).

Specifically, within the field of mathematics education, researchers have documented the relationship between teachers' beliefs and their pedagogical decision-making, stating that these beliefs have tremendous influence over their daily practice (Cooney, 2003; Handal, 2003; Raymond, 1997). As such, any educational initiative that requires teachers to modify their teaching practices should investigate their beliefs about the nature of mathematics, its pedagogy and students' learning in the domain. In so doing, they can better assess how well these beliefs align with or are in conflict with the objectives of these initiatives in efforts to better understand their impact on the initiative's success. The second article attempted to do this by describing the mathematical beliefs of the five teachers involved in incorporating reform-oriented practices and activities in their classroom. Additionally, it investigated the degree of alignment between the beliefs and their instructional practice and how consistent this alignment was over time and in attempts to incorporate these reform-oriented practices.

Beliefs, because of their nature as well as how they develop and are organized, tend to be pervasive and highly resistant to change. This resistance often becomes problematic when efforts are made to incorporate and sustain educational innovations and ultimately any reform initiatives. In this regard it is often necessary, although not always enacted, to engage teachers in the process of belief change. These efforts are primarily made with pre-service teachers before they formally enter the profession, but there is also an apparent need for more of these attempts to be made with in-service teachers as well (see article 2). Within mathematics teacher education many of these attempts at belief change employ cognitively-based strategies which are often unsuccessful in producing long-term belief modification. Despite the potential value of these strategies it is hypothesized in the third article that they are often unsuccessful because they emphasize only the individual, psychological aspects of belief development and organization. With the increasing recognition of the social influences on learning, some researchers within the field have embraced the notion that although beliefs are deeply personal they are also a product of the culture and norms held by the larger community (Llinares, 2002). This conceptualization of beliefs goes beyond the more individualistic, cognitive point of view to one that is more situative and contextual, recognizing the importance of the community in the formulation of beliefs. Article three examines this issue of belief change, the need for belief change, approaches taken toward belief change and the degrees of success in engaging teachers in this process. Connections are made between the successes and failures of these approaches as a function of the underlying theoretical framework. It concludes with a discussion of the lessons learned from previous belief change efforts and makes suggestions for future research and initiatives in this area.

To address these issues research from several bodies of literature were used to inform the design of these studies. This literature includes research on teachers' beliefs in general and also domain-specific mathematics teachers' beliefs. This section distinguishes between general epistemological beliefs and teachers' individual beliefs about mathematics as a field of study focusing on the latter. Additionally, descriptions of the theories and research underlying individual and social knowledge construction are also provided incorporating what has been written about metacognition and discourse. In talking about metacognition and discourse, the research on writing and mathematical argumentation is included to provide information about strategies that were considered useful in improving students' mathematical understanding and achievement. A review of this literature is presented in the next section.

CHAPTER 2

REVIEW OF THE LITERATURE

Individual Knowledge Construction - Metacognition

Increasingly the field of mathematics education has become concerned with students' abilities to engage in critical thinking and higher order reasoning; this concern being the impetus to several reform initiatives as it is believed that without the development of these skills our students will continue to underachieve (Garii, 2002; NCTM, 2000). In this regard, numerous studies (Brown, 1987; Carr & Biddlecomb, 1998; Cornoldi & Lucangeli, 1997; Garii, 2002; Kramarski et al., 2002a; Zan, 2000) have been done in the area of cognition and metacognition examining how these processes work together to enhance problem solving skills and effective strategy use. Many (Mayer, 1998; Schoenfeld, 1987; Silver, 1985, 1987) have concluded that both cognitive and metacognitive processes work together in problem solving and that these processes are better facilitated with the mastery of basic mathematical skills, an integral component in the development of mathematical expertise. The move within this domain towards a more constructivist approach has provided the necessary scaffolding of these goals, as students have increased opportunities to engage in meaningful knowledge construction fostering deeper understandings and communicative competence. This ability to effectively communicate and solve problems within the domain is considered to be largely dependent on the level of cognitive processing and metacognitive abilities.

However, in order to discuss metacognition and its relation to learning and developing mathematical expertise we must differentiate between cognition and metacognition. Garofalo and Lester, similar to Silver (1982) and Schoenfeld (1992) makes a distinction between what is considered cognitive from that metacognitive and states, "Cognitive is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (1985, p.164). This distinction categorizes the link between metacognition (choosing, planning and monitoring) and success in mathematical problem solving (doing) in that the metacognitive informs the cognitive. To avoid the possible confusion in past years about the definition of metacognition most theorists now generally accept Flavell's (1976)[1] description stating:

> Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them…..refers, among other things to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective. (p. 232)

Essentially metacognition involves the conscious use and control individuals have over their own cognitive functions (Cornoldi & Lucangeli, 1997). It refers to thinking about your own thoughts and incorporates the ability to assess your own thoughts and mental state and evaluate how that state will influence the present and the future (Meichenbaum & Biemiller, 1998). This assessment includes the individual bringing to the forefront knowledge and strategies specific to the task and also knowledge about themselves as learners and their own cognitive processes. It involves both awareness of the factors that contribute to and hinder our learning and the

[1] Flavell's work on metacognition can be viewed as an advancement of Jean Piaget's work on human cognitive development, specifically the deliberate, goal-directed thinking embedded in Piaget's conceptualization of formal operations (Hacker, 1998).

regulation of these factors to achieve our goals[2] (Schoenfeld, 1987; Silver, 1987; Wilson & Clarke, 2002). These factors also include the individual's beliefs and judgments about themselves and their affective states concerning their knowledge, abilities and motivation as learners (Hacker, 1998). Despite the varying conceptualizations of metacognition over the years, most theorists reference two main aspects: knowledge of cognition and regulation of that cognition; these are then subdivided into separate but related processes or stages (Brown, 1987; Schoenfeld, 1987; Silver, 1985).

Studies in the area of mathematics, specifically in relation to metacognition and problem solving have categorized the metacognitive behaviors that students engage in while completing mathematical tasks. Some present a process model (Polya, 1957), and others categorize them in stages (Garofalo & Lester, 1985; Simon, 1987) or components (Jacobs & Paris, 1987). The first stage is often referred to as the planning stage and involves the selection of appropriate strategies and the allocation of resources. Individuals assess and try to understand the problem by analyzing the information and conditions surrounding the task. They gauge their familiarity with the task by activating prior knowledge and assessing the level of difficulty. Setting strategic goals and evaluating the probability of arriving at a solution are also included in this stage. The regulation stage follows and involves monitoring actions and the application of skills. Students often spend a significant portion of time in this phase as it requires making predictions, strategy sequencing and testing and retesting procedures. The third and final stage is evaluation, which includes the appraising of decisions and processes employed throughout the activity and the

[2] These descriptions fall within Flavell and Wellman's (1977) categorization of metacognitive knowledge in to person, task and strategy factors that contribute to overall task performance.

result of these executed plans[3]. These processes interweave both the knowledge and regulation of cognition (Garofalo & Lester, 1985).

This coherence within the research regarding the definition of the term and description of its stages or components came after years of confusion about the exact meaning and composition of *metacognition*. Contributing to this confusion were the similarities of the concept to other terms which also encompassed the idea of examining one's own knowledge and thoughts. Garofalo & Lester (1985) identifies Skemp's (1979) notion of 'reflective intelligence' and Piaget's (1976) 'reflexive abstraction' and Silver (1987) mentions Atkinson & Schiffrin's (1968) 'control processes' as such terms. They are not synonymous with metacognition but refer to individuals being conscious of the mechanisms for knowledge construction and the mental processes that support them. These constructs began to develop and take form out of Flavell's (1976) initial notion that cognitive systems that were functioning effectively, not only learns and operates but is also cognizant of the factors that help it to function well. Further, during the early 1970s three separate domains recognized the importance of self-regulation in the cognitive processes and contributed significantly to the field, these included: artificial intelligence, developmental psychology and mathematics education (Schoenfeld, 1987, 1992). Following that, several other studies were done in the 1980s (Garofalo & Lester, 1985; Lesh, 1985; Silver, 1982) and focused on the usefulness of the construct for mathematics education, specifically in relation to effective and resourceful problem-solving behavior.

Research in the field of metacogntion and mathematics generally focus on three areas (Schoenfeld, 1987; Silver, 1987); first, knowledge and awareness individuals have over their own cognitive processes, second how well individuals control or regulate their metacognitive

[3] These stages are labeled differently with respect to the researcher and some include a fourth stage. However, they generally converge on the idea that both knowledge and regulation occurs at all stages and they all include the actions mentioned above.

behaviors and third, beliefs individuals have about the subject and how that influences the way they 'do' mathematics and their problem-solving performance. Silver (1987) elaborates on this latter aspect stating that no model of problem solving can be complete without an adequate account of the role of metacognition and belief systems[4].

Research in the first area generally conclude that awareness of metacognitive processes usually appear later in the developmental process and that young children have great difficulty in describing their own mental processes. Much of this early research (Cavanaugh & Borkowski, 1980; Kreutzer, Leonard, & Flavell, 1975) was done in the area of metamemory, assessing how well people could judge their memory of events and concluded that the relationship between metacognition and cognitive processes is a function of age and task variables. Research conducted by Karmiloff-Smith (1979) demonstrated that only older children as opposed to four and five year olds demonstrated evidence of systematic planning before engaging in a mathematical task. They concurred along with others (Bjorklund, 2005; Dominowski, 1998; Kreutzer et al., 1975) that the ability and inclination to plan, execute the plan and evaluate the appropriateness of the employed strategies developed with age. However, other studies (Hacker, 1998; Wilson & Clarke, 2002) recognized that younger children often have limited linguistic skills or simply lack the vocabulary needed to describe their thought processes, but still engage in some metacognitive activity. To alleviate these problems current researchers employ multiple techniques and strategies to garner this information. Wilson & Clarke (2002) combined the use of video stimulated recall and 'thinking cues' along with a card-sorting procedure. This card-sorting procedure, conducted following engagement in the task allowed the students to reconstruct their thought processes by choosing the card with the appropriate statement. Using

[4] This latter aspect will be covered in the next section.

multiple techniques also allowed them to address concerns of authenticity (Nisbett & Wilson, 1978) and criticisms of 'putting words into the mouths' of the students.

Researchers in this field must also contend with issues of measurement as it is often difficult to monitor or assess metacognition. Brown (1987) states that the knowledge component of metacognition, otherwise referred to as declarative metacognition (Kluwe, 1982; Schneider & Lockl, 2002)[5], is often easier to measure because they are usually state-able in written or verbal form while the regulation component is often unconscious as the processes involved become highly automated (Brown, 1987). Wilson & Clarke (2002) suggests that for this reason, the automaticity of some procedures, the techniques used to assess metacognition are often unreliable. Early studies that utilized techniques such as self-reporting may not have been successful as students were often unable to recall their thought processes. The reason for this inability to report was often difficult to ascertain as it could have been due to the actually lack of metacognitive behaviors. Alternatively, the problem solving processes could have been automatised (Dominowski, 1998; Wilson & Clarke, 2002). Additionally, participants tend to reconstruct inaccurate memories, invent memories and have spontaneous lapses in memory when asked to report their thoughts. Attempts to remedy these problems include applying some of the techniques mentioned earlier. Dominiowski (1998) added to this area of research specifically with regard to the effect of verbalization on cognitive processes. In his analysis of the effects of verbalization on problem solving he concluded that depending on the type and quality of the verbal utterances they could have no effect on task performance or they could lead to improved performance. Specifically, verbalization was not necessary to improve problem solving ability but tend to improve task performance when they involved focusing on metacognitive processes.

[5] Schneider & Lockl (2002) defines declarative metacognition as the explicit, conscious and factual knowledge a person has about the characteristics of the task being performed.

Researchers (Garofalo & Lester, 1985; Schoenfeld, 1987, 1992; Silver, 1982) have also commented on the correspondence between the metacognitive behaviors, described above, and the conceptions and models of problem solving (Polya, 1957) and conclude that together they equal mathematical success. Mayer (1998) concurred and added that successful problem solving depends on three components: skill, metaskill and will and that each of these components are influenced by instruction. Skill involves the individual's knowledge of the basic skills and their ability to execute these skills with mastery. In addition to possessing these domain-specific skills, it is important that the individual effectively manage these skills knowing when, where and how to monitor them when engaging in a task; referred to as 'metaskill'.

Schoenfeld (1987) concluded from his research that the difference between success and failure in problem solving is how the problem solvers make use of what they know. Expert mathematicians tend to efficiently self-monitor and self-regulate, carefully analyzing the problem and the possible strategies and so spend less time actually working on the solution. The acquisition of these expert 'skills' is thought to be an important component to mathematical success hence the push for them to be incorporated into everyday instruction. This also relates to the last criteria, will, which refers to the motivational aspects of the learning process. Flavell and Wellman (1977) alluded to this with reference to the influence of the *person* factors on performances. Similar to *will* these incorporate an individual's belief about themselves as learners, the nature of the task, judgments about their capabilities to attain success and the attributions they make following completion of the task. Within the domain of mathematics these also include beliefs about the nature of mathematics and mathematics learning, as an individual may have the necessary knowledge to successfully complete the task but their beliefs prevent them from applying it (Schoenfeld, 1987).

Kramarski et al. (2002) and Schoenfeld (1992) reported on the positive effects of metacognitive instruction on mathematical performance. Additionally Kramarski et al. (2002) stated that metacognitive instruction lead to greater achievement regardless of whether it was administered individually or in a group. These students consistently performed better on mathematically authentic tasks than those who did not receive such metacognitive guidance or support. Lucangeli & Cornoldi (1997) provided support for this in their research demonstrating higher metacognitive awareness when guided in the process, in this particular case with the use of suggestive questioning. They also concluded that higher metacognitive awareness was influenced by higher mathematical competence in that, students who had clear knowledge of procedures were better able at plan their solution and evaluate the appropriateness of the strategy and accuracy of the solution (Lucangelli & Cornoldi, 1997). This was particularly true with mathematical tasks where the solution required a standard algorithm, which the student either had knowledge of or not. This suggested that metacognitive processes of prediction, planning, monitoring and evaluating were not particularly critical for automatised skills. Both Wilson and Clarke (2002) and Kramarski et al. (2002) also observed a similar phenomenon in their study where there was a greater reliance on metacognitive skills for non-routine problems than on routine problems for students who received metacognitive instruction.

Ultimately the results revealed that elements of metacognitive activity were related to mathematical performance and this relationship was closer when the tasks required less automated processes. This research has implications for both teaching and task selection in that for students to develop the necessary metacognitive skills they need to have the relevant basic mathematical knowledge and be exposed to rich mathematical tasks that require application of problem solving techniques (as outlined by Schoenfeld (1983)). Later research (Wilson & Clarke,

2002) also supports these findings. Wilson and Clarke (2002) in their study stated that problem solving involves the complex interplay between cognition and metacognition. Success was both a function of the students' knowledge base and also the metacognitive behaviors employed; but whilst the use of metacognition did not guarantee successful completion of the problem it did encourage perseverance and the application of more effective solution strategies.

Research on metacognition in the domain of mathematics overlaps with findings in the area of reading (Palinscar & Brown, 1984) and writing (Bereiter & Scardamalia, 1987). They all describe similar metacognitive behaviors observed in experts in the domain, emphasize the need for metacognitive instruction and report on the positive impact of having a repertoire of these behaviors on achievement. Undoubtedly, if we intend for our students to achieve mathematically, or rather academically it is essential that we provide learning environments that support both the acquisition and development on these mental processes across domains. In this regard it is important that we incorporate learning strategies and activities that seek to develop metacognitive skills as they are considered integral to mathematical success. Writing is an activity that has been demonstrated within the domain (Pugalee, 2001) and otherwise (Rivard & Straw, 2000; Scardamalia & Bereiter, 1986) to help students focus and consolidate their thinking. As such, promoting the incorporation of writing in the mathematics classroom appears to be a useful strategy for enhancing students' mathematical understanding and achievement.

Writing in Mathematics

Researchers tend to concur that successful mathematical problem solving is a function of metacognitive awareness and the application of these skills. It follows then that students' development of these skills should be an integral part of mathematics instruction. The National Council of Teacher of Mathematics *Principles and Standards for Teachers* (NCTM, 1989, 1991,

2000) supports the view and emphasize the use of both communication and reflection in the study and teaching of mathematics. This comes as a larger call for mathematical literacy in which communication is seen as playing an essential role in clarifying and developing understanding. Writing is considered an important component of this drive for increased literacy and its incorporation into daily practice is encouraged. Writing is seen as a tool to help students consolidate their thinking and also aids in reflection and clarification of students' mathematical thoughts and ideas (NCTM, 2000). It also sustains the development of reasoning and communication skills and the ability to make connections; ultimately contributing to the enhancement of metacognitive behaviors (Pugalee, 2001).

Vygotsky (1986) who is primarily known for his notion of learning as context-bound cultural activity also considered writing an important activity for knowledge construction. Writing, he theorized necessitates deliberate and focused analytical action and it requires the individual to organize and compact their inner thoughts (speech) into a comprehensible whole (Pugalee, 2001). It helps to tie down ideas and link old ideas to new concepts engaging the student in analytical thinking and reflection, making the shift from a passive to an active learner (Kasperek, 1996; Porter & Masingila, 2000; Vygotsky, 1962). Within the mathematics classroom, writing ranges from informal, unstructured journal writing to formal assessments of mathematical reasoning (Baxter, Woodward, & Olson, 2005). Similarly research conducted in the field of mathematics education also includes many forms of writing, including journaling (Baxter et al., 2005), problem creation and development (Porter & Masingila, 2000), portfolios and writing for concept development (Kasperek, 1996).

These writing efforts on the part of teachers are often incorporated to address low mathematics achievement resulting from lack of basic skills and students' misconceptions of mathematics and mathematical concepts (Porter & Masingila, 2000; Schoenfeld, 1992). Students often hold naïve and narrow beliefs about the nature of mathematics which affect the way they think about mathematics and problem solving and ultimately their procedural ability (Schoenfeld, 1987). One way that researchers have sought to counteract this phenomenon is through the use of writing-to-learn mathematics (Ganguli & Henry, 1994; Kasperek, 1996; Porter & Masingila, 2000). *Writing-to-learn mathematics* is a philosophy outlined by Countryman (1992), who posits that:

> Knowing mathematics is doing mathematics. We need to create situations where students can be active, creative, and responsive to the physical world. I believe that to learn mathematics, students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short by being active in the world. Writing is an ideal activity for such processes (p. 2)

Stehney (1990) took this a bit further and purported that writing is an essential tool for learning and not just a means of expression and displaying learnt concepts. Engaging in the writing process requires the students to analyze, compare and synthesize information, creating a clear conceptual picture through written words (Kroll & Halaby, 1997). It also provides opportunities for review, reflection and evaluation of the problem-solving process, working as a tool to improve conceptual understanding, metacognition and communication (Kasperek, 1996; Kroll & Halaby, 1997; Pugalee, 2001). Writing they believe reinforces metacognitive behaviors and as Schoenfeld (1992) and Silver (1982) both suggest pedagogy that allows students to become more aware of their own thought processes should be encouraged.

Bereiter and Scardamalia (1987) in their research on the metacognitive aspects of writing differentiated between writing as knowledge-telling and writing as knowledge-transforming.

Knowledge-transforming is described as a strategy demanding mental effort, engaging the writer in metacognitively guided planning, diagnosis and problem solving. This description accurately describes the processes students would undergo in writing focused on concept development in the domain of mathematics. This is further supported by Pugalee (2001) who concluded that a metacognitive framework was present in the writing of students and generally followed the stages mentioned earlier. He suggests that writing has the potential to function as a vehicle in developing and supporting metacognitive skills and behaviors critical for mathematical problem solving.

Researchers agree that to experience continual success in mathematical problem solving the individual must exercise metacognitive behaviors. As demonstrated, the act of writing does facilitate this awareness and provides opportunities for the students to practice and improve these skills. It therefore suffices as an appropriate means to supplement metacognitive instruction. This study which is described in this document specifically proposes the incorporation of 'problem-focused writing' through which the students demonstrate their understanding of concepts underlying the given problem. Through writing the students provide descriptions of these concepts, the procedures necessary to arrive at a solution and how to apply these procedures appropriately and accurately. Engaging in 'problem-focused writing' will allow the students to develop metacognitive and problem solving skills leading to increased conceptual understanding and greater mathematical achievement.

Social Construction of Knowledge

The success of any reform movement in the field of mathematics education will be influenced by the conception of mathematics held by those involved in the initiative (Borko & Putnam, 1997; Buttery, 1990). These views are often quite shallow ranging from mathematics being a body of facts and procedures to be memorized and reproduced to a science based on pattern-seeking (Schoenfeld, 1992). Conceptualizing mathematics in this way tends to trivialize it and so most modern mathematicians hold the view of mathematics as “a social activity where a ‘community of learners’ engage in practices based on study, observation and experimentation to determine the nature of principles of theoretical or ‘real-world’ systems. It is an exploratory, dynamic and evolving system involving the formulation of conjectures, exploration of patterns and finding solutions” (p. 335) (Schoenfeld, 1992). This idea of mathematics learning being an inherently social and constructive activity has been the focus of several researchers in recent decades (Forman, 1989; Goos, Galbrath, & Renshaw, 2002; Lesh et al., 2003). This discussion is informed primarily by the work of Vygotsky (1978) and socio-cultural theory. This theory extends the notion of learning beyond individual cognition to the social realm.

Socio-cultural theory states that through the process of socialization or enculturation we develop skills of interpretation and meaning construction (Vygotsky, 1986). Specifically within the field of mathematics, thinking mathematically or rather having a mathematical point of view is essentially, envisioning the world as mathematicians do (Schoenfeld, 1992). Knowledge is considered to be culturally shaped and defined, developing understandings by our interactions and participation within the ‘community of practice’ (Case, 1996). Mathematical competence or proficiency is characterized by an individual becoming more expert in the practices of the mathematical community. This view of knowing and learning is aligned with mathematical reform initiatives and with curricular trends that emphasize communication, problem solving and mathematical reasoning (NCTM, 2000). Additionally, they focus on students becoming active participants in the mathematical community. However, any initiative with these goals in mind undoubtedly requires that some teachers change or adjust their pedagogical styles. These changes must be justified by improving our understanding of how students learn in these social contexts (Goos et al., 2002). There are several perspectives that inform the social nature of knowledge construction all born out of Vygotsky's early work. They include *distributed cognitions* (Cole & Engestrom, 1993), *situation cognitions* (Brown, Collins, & Duguid, 1989) and the *social constructivist theory* (Forman, 1989; Stone & Wertsch, 1984). Although these perspectives are quite useful in discussing the social aspects of learning, I will focus on the sociocultural theory as it encompasses all the relevant tenets underlying social knowledge construction. It will also highlight how metacognition is nascent in this theory.

The sociocultural perspective deviates from the cognitive perspective by focusing less on knowledge and learning as primarily internal involving complex cognitive processes, to a more contextualized view. The cognitive view holds the individual as the primary unit of analysis while the sociocultural focuses on the group and the initiation of the individual into group practices. From this perspective, knowledge is seen as constructed by the group through continuous interaction in daily practices which transforms the individual's current environment (Case, 1996). The specific context for this discussion is the mathematics classroom with each student along with the teacher comprising this community of learners (Brown et al., 1993). An integral component of the daily practices within this community is the collective engagement in discourse. Within the field of mathematics this is commonly referred to as mathematical argumentation (Cobb et al., 1991; Leonard, 2000; Stein, 2001). Language and various cultural tools are crucial to these learning environments as they facilitate the individual's increasing ability to effectively engage in the community's practice.

Research supports these ideas, as collective discourse is considered fundamental to students' construction of meaning (Cobb et al., 1991; Forman et al., 1998; Pontecorvo, 1993;

Rogoff, 1990). Consequently, classroom discussions where students are able to make worthwhile contributions, ask questions or have their ideas evaluated and receive immediate feedback is considered one of the more effective tools for knowledge construction (Inagaki, Hatano, & Morita, 1998). This forum tends to increase in its effectiveness for knowledge building when these students are situated in groups engaged in authentic problem situations (Kramarski et al., 2002a). Within these environments the teacher's role is crucial, not as the repository of knowledge but as the one who initiates and guides the students in these 'community' practices. The groups would engage in mathematical discourse around the problem or concepts and the process would be continuously scaffolded by the teacher, guiding the students towards expertise. This coaching and scaffolding by the teacher is considered vital to creating new levels of student understanding (O'Flahavan & Stein, 1992).

Scaffolding and guidance by a more knowledgeable individual is seen as a necessary component of the learning environment as it provides the initiation into the community's praxis. This scaffolding by the more expert other allows the students to progress from an assisted position to a more independent self-regulated mode (Brown et al., 1993; Wertsch, 1984). These ideas developed out of Vygotsky's notions of the zone of proximal development (ZPD), which describes individuals being engaged in equal status partnerships interacting above their own individual knowledge level according to more general social scripts (Goos et al., 2002). It defines the distance between current levels of comprehension and levels that can be accomplished in collaboration with people (Brown et al., 1993). This notion can be extended to incorporate the view that within the educational environment there is learning potential among individuals of varying levels of expertise in group interactions; however without this collaboration this knowledge will not be attained. This peer collaboration embodies a reciprocal process where each member of the group is free is share their thoughts and have the opportunity to explore each other's reasoning in order to come to a shared understanding. Throughout this process the individual must also assess their own thoughts in relation to the viewpoints offered by their peers in order to make meaningful contributions to the group. In the event of conflicting or contradictory opinions each student must re-assess their response comparing one's own ideas with that of the group. This reciprocal process of co-constructing meaning engages the individual in mental processes of both a cognitive and metacognitive nature. It externalizes the processes of self-argumentation that characterizes individual mental processes thereby supporting the development of metacognitive skills (Schoenfeld, 1987).

Another interesting perspective on this social construction of knowledge that is of relevance to this discussion is outlined by the social constructivist theory. This theory also purports that independent cognitive functioning is initially acquired in a social/communicative context (Forman, 1989). Of primary interest however is the notion of 'proleptic instruction' used to classify teaching and learning within the ZPD (Stone & Wertsch, 1984). This is defined as the communicative process where an individual develops a better understanding of the task situation by creating the necessary background knowledge through responding to the directives or instructions of others. Simply put, within a collaborative environment the cognitive benefits of the interpersonal interaction would be maximized if the students create a bi-directional ZPD through the interchanging of teacher (expert) and student (novice) roles.

Forman (1989) in her research of two students engaging in a collaborative problem-solving task observed that the opposing stances of the two students on the task helped them to develop and incorporate new planning and reasoning strategies. Throughout the interaction process the students interchanged roles of 'knower' and 'learner' and helped each other in monitoring and regulating strategies. She also noted that the success of these types of interactions depends on a certain level of interdependence between the students. This she states develops over time in a reciprocal manner and initially these interactions may not go smoothly. Similar to Schwarz, Neuman, Gil and Ilya (2003), Forman (1989) and others (Salomon & Globerson, 1989) also commented on the fact that although there was evidence of learning within the group, the external dynamics influenced the learning outcome of these students. Influential group attributes included, the value the students placed on each other's opinions, the individual perspectives and power they entered the collaboration with, level of domain knowledge possessed by each student and the individual identities the group members actualized. These factors seemingly affected the development of the bi-directional ZPD between the students and hence the learning outcome.

Of special note is that researchers in this area examine two different phenomena, depending on the theoretical framework in which the research is embedded. For example, research by Artzt and Armour-Thomas (1992) examined the individual contributions to collaborative metacognition, their focus being on the individual (Artzt & Armour-Thomas, 1992). Alternatively, Goos, Galbraith and Renshaw (2002) sought to capture the interactive nature of the groups' metacognitive monitoring and regulation, focusing on the group product. Differentiating between these two phenomena presents research with somewhat conflicting results. Goos et al (2002) concluded that group interactions can both hinder or enhance metacognitive activity depending on the flexibility of the students in their metacognitive roles throughout the engagement. Additionally, the group product was often diminished due to the students' lack of monitoring and evaluation, processes considered essential aspects of metacognitive awareness that contribute to successful problem solving (Schoenfeld, 1987; Silver, 1982). However, Artzt and Armour-Thomas (1992) stated that grouping students for mathematical problem solving does promote discourse with students as it allows them to express their own and listen to the ideas of others. This interplay of ideas helps to support the development of metacognitive awareness. They also added that these discursive opportunities within the small group also encouraged social and cognitive behaviors in ways that promoted and constrained successful problem solving. These constraints were partially attributed to the perceptions students had of themselves and their group members. These two studies explicate the potential difficulties in assessing research in this area as the researchers differentiate between measurement of the individual metacognitive benefits of collaborative activity and the effects of socially mediated metacognition.

However, regardless of the focus of the researcher these studies provide evidence that students do learn through their interaction with others. Vygotsky (1978) proposed that higher order thinking originates in and develops through the internalization of our interaction with others and whilst the cognitive perspective somewhat conflicts with this view it does hold that peer grouping facilitates learning. In light of this it is imperative that as educators we capitalize on the potential of instructional strategies from both paradigms.

Teachers' Mathematical Beliefs

Teachers' beliefs within the domain are often classified in two categories: beliefs about the nature of mathematics and beliefs about mathematics teaching and learning (Cooney, 2003; Thompson, 1992). Researchers (Cooney, 1985; Lesh, 1985; Polya, 1957; Schoenfeld, 1987; Silver, 1982, 1985; Thompson, 1992) have documented that individuals have different and varying conceptions about the nature and meaning of mathematics. These perspectives tend to be quite dominant and so often influence teachers' conceptions of mathematics teaching and learning. These views and conceptions, referred to as beliefs in the literature tend to pervade the very essence of teaching. They reflect how teachers view the portrayal of roles in the classroom for both teacher and student, their choice of classroom activities and the instructional strategies they use in the classroom. These views also relate to how teachers characterize mathematical understanding and knowledge, how students learn mathematics and the purpose of schools in general (Thompson, 1992). These beliefs are central to the way teachers' conceptualize and actualize their role in the mathematics classroom, and so they are also integral to any discussion of student learning within the domain.

Several theorists have discussed the varying conceptions of teachers' beliefs about mathematics (Cooney, 1985; Ernest, 1989a, 1998). In particular, Ernest (1998) identified and distinguished between three different views about the nature of mathematics. These distinctions can be aligned to Kuhs and Ball's (1986) classification of 'dominant views of how mathematics should be taught' (p.2). The 'problem solving view' conceptualizes mathematics as a dynamic and continually expanding field of human creation and invention encompassing a process of inquiry and coming to know. The constructivist view of mathematics learning can be aligned with this perspective as it purports the establishment of a learner-focused environment (Cobb & Steffe, 1983; P. Thompson, 1985; von Glasersfeld, 1987). Within this environment, students are actively engaged in the learning process where they are encouraged and allowed to generate and develop their own ideas. The teacher's role is that of facilitator, continuously guiding and challenging the students. They assist the students in developing ideas, highlighting inadequacies in their thinking and helping them develop expertise in supporting and defending their conclusions (Kuhs & Ball, 1986).

Another perspective about the nature of mathematics, the Platonist view, holds mathematics as a static but unified body of knowledge that is there to be discovered, not created. This view states that mathematics consists of the accumulation of facts, rules and skills to be used in the process of achieving an external end. A classroom established around this conception differs from the former in that it has a dual focus on both the content and student understanding. In that while its primary focus is the content it also emphasizes knowledge and understanding of the facts and procedures underlying the content; this understanding is seen as constructed by the student (Kuhs & Ball, 1986; A. Thompson, 1992).

Lerman (1983) discussed two additional but competing views, the fallibilist and absolutist conceptions of the nature of mathematics. The fallibilist perspective considers mathematics an uncertain discipline developed through conceptions, proofs, and refutations. Conversely, from the absolutist perspective, mathematics is a certain and absolute body of knowledge based on universal and absolute foundations (Lerman, 1983). Connections were made between this latter view of mathematics teacher-focused style of teaching which emphasized the transmission of facts and right answers, promoting a single approach and path to a specific solution. Within this learning environment the teacher is the individual who possesses the knowledge. The teacher is responsible for defining, demonstrating and explaining the content. Additionally, emphasis is placed on practice and didactic interactions and students are required to adhere to established algorithms and produce correct answers in an effort to demonstrate mathematical knowledge and fluency (A. Thompson, 1992). The students are often passive in the learning process; student exploration and investigation are seldom and often not incorporated into activities. This kind of routine often leaves students with a distorted view of mathematics. This perspective and the teaching strategies derived from them are and have been considered problematic for decades as they present a naïve and shallow view of mathematics and encourage conceptual misunderstandings (Cobb et al., 1991; Schoenfeld, 1992).

The relationship between teachers beliefs and their instructional practice is not considered linear, not a direct cause and effect association. However, this association is often quite salient. It is described as a complex and reciprocal relationship and so researchers must go beyond examination of teachers' professed beliefs when investigating conceptions (Charalambous, Phillippou, & Kyriakides, 2002; Raymond, 1997; A. Thompson, 1992). To get an adequate assessment of any teacher's conception of a domain the researcher should engage in repeated observation of the learning environment and the instructional practice. Through these examinations researchers (McGilliard, 1983; A. Thompson, 1992) have consistently observed inconsistencies between teacher's stated views and their practice. However, making dual examinations of both the verbal and observational data will provide explanations about the external factors that may adversely influence their practice resulting in the subordination of their beliefs (A. Thompson, 1992a). Additionally, it is also prudent to determine whether these teachers are aware of this disconnect and the explanations they provide for them.

CHAPTER 3

CREATING OPTIMAL MATHEMATICS LEARNING ENVIRONMENTS: COMBINING ARGUMENTATION AND WRITING TO ENHANCE ACHIEVEMENT [6]

[6] Cross, D.I. (To be submitted to Journal for Research in Mathematics Education)

Abstract

The issue of mathematics underachievement among students has been an increasing national concern over the last few decades. Research suggests that academic success can be achieved by focusing on both the individual and social aspects of learning. Within the area of mathematics education, the development of metacognitive skills and the incorporation of discourse in classroom instruction has resulted in deeper conceptual understandings and increased mathematical achievement. However, studies in this field tend to focus on the effects of these practices separately, making research that seeks to harness the potential of both quite rare. This paper reports on a study which is aimed at addressing this gap in the literature by examining the effects of writing and argumentation on achievement. Two hundred and eleven students and five teachers participated in this multimethod study, which investigated the effects of three treatment conditions on mathematical achievement. These conditions were writing alone, argumentation alone, and writing and argumentation combined. Analysis of covariance revealed significant differences between the groups and tests of the contrasts showed that students who engaged in both argumentation and writing had greater knowledge gains than students who engaged in argumentation alone or neither.

Introduction

Mathematics achievement has been an increasing national concern, specifically in relation to public education (Schmidt et al., 2005). Reports from the Trends in International Mathematics and Science Study (TIMSS) (National Center for Educational Statistics, 1999, 2003) revealed deficiencies in students' understanding and fluency in mathematics and associated these gaps both with curriculum focus and design and also with the quality of instruction, especially in the middle grades (Boe & Shin, 2005). This lack of achievement relative to other industrialized nations fosters doubt about the United States' future economic competitiveness and its ability to compete on the global market, and so stakeholders (including parents, policymakers, administrators) are becoming more involved in efforts to find a solution.

To address this issue of mathematics underachievement, in recent decades there has been increased interest in mathematics education reform with reference to the departure from traditional methods of teaching mathematics. Focus is being placed on the role of discourse in classroom instruction and the need for increased problem solving and critical thinking skills (Cobb et al., 1991; Leonard, 2000; NCTM, 2000; Schoenfeld, 1998; Stein, Smith, Henningsen, & Silver, 2000). These efforts are supported by researcher who suggest the development of metacognitive skills and the incorporation of mathematical argumentation in instruction has resulted in students having deeper conceptual understandings subsequently increasing mathematical achievement (Forman et al., 1998; Garii, 2002; Kramarski, Mevarech, & Arami, 2002b; Stein, 2001). As such, focusing on both the individual and social aspects of learning seem to be the key to academic success (Cobb et al., 1991; Lesh et al., 2003; Pontecorvo, 1993; Schoenfeld, 1992). However, in the past researchers have tended to focus on the effects of practices derived from either the cognitive or the socio-cultural domains, making studies that seek to harness the potential of both these strategies rare (Nasir, 2005). This study attempts to address this gap in the literature.

Theoretical Framework

This study is informed by insights that have amassed from both the cognitive and socio-cultural perspectives. Most modern theories of learning promote the view that meaningful learning cannot occur with students passively receiving information from teachers (Brown et al., 1989; Lesh et al., 2003) and promote the idea of 'the individual being actively engaged in knowledge construction' as an integral part of learning. However these perspectives differ with regard to the primary medium through which this knowledge is constructed. Within a strict cognitive tradition, knowledge is constructed by the mind and learning is considered an internal process of assimilating new information or experiences in an effort to understand it (Case, 1996; Prawat, 1996). Alternatively, from a more socio-cultural perspective less focus is placed on these internal processes, and knowledge is seen as constructed through the engagement in the social practices of a particular group (Case, 1996; Rogoff, 1995).

Although these perspectives are seemingly contradictory, in the strict sense a bidirectionality is assumed in the relationship between the individual and his socio-cultural contexts. This relationship is such that the individual, his thoughts, beliefs and actions are influenced by his environmental and cultural contexts, and reciprocally these contexts are defined through the individual's position in and contribution to it (Nasir, 2005). In this regard, both socio-cultural and individual forces are considered essential to cognitive change and so an integration of the two should maximize the opportunities for students to learn (Cobb, 1994; Hatano & Inagaki, 2003). Therefore an environment incorporating strategies aligned with both perspectives may lead to greater achievement than an environment built on just one (Nasir, 2005).

This study aimed to investigate this phenomenon and to determine if a learning environment specifically designed to support strategies and techniques born out of both cognitive and socio-cultural perspectives will yield higher mathematics achievement than those supporting just one perspective or neither.

Specifically, I examined the effects of combining the social and individual influences on learning by incorporating activities that supported the development of metacognitive skills and discourse in the classroom. Within the mathematics education community these learning strategies are considered effective techniques in developing mathematical expertise. An overview of the research done on metacognition and discourse in the domain of mathematics will be provided followed, by a discussion of the specific activities used to develop these skills.

Individual Knowledge Construction - Metacognition

The field of mathematics education has continually investigated the factors surrounding students' abilities to engage in higher order reasoning. In this regard numerous studies have been done in the area of cognition and metacognition examining how these processes work together to enhance problem solving skills and effective strategy use (Brown, 1987; Carr & Biddlecomb, 1998; Cornoldi & Lucangeli, 1997; Garii, 2002; Kramarski et al., 2002; Zan, 2000). Researchers have concluded that along with the mastery of basic mathematical skills, both cognitive and metacognitive abilities are crucial to improvement in problem solving ability (Mayer, 1998; Schoenfeld, 1987; Silver, 1985, 1987) . The move within this domain toward a more constructivist approach has provided the necessary scaffolding of these goals, as students have increased opportunities to engage in meaningful knowledge construction fostering deeper understandings and communicative competence. This ability to effectively communicate within the domain is believed to be largely dependent on the level of cognitive processing and metacognitive abilities.

Essentially metacognition involves the conscious use and control individuals have over their own cognitive functions (Cornoldi & Lucangeli, 1997). It refers to thinking about your own thoughts and incorporates the ability to assess your own ideas and mental state and evaluate how that state will influence the present and the future (Meichenbaum & Biemiller, 1998). This mental assessment includes the individual bringing to the forefront knowledge about the task and knowledge about strategies relevant to the task. Additionally, of importance is that the individual activates knowledge about himself as a learner and his own cognitive processes. Metacognition also involves both awareness of the factors that contribute to and hinder our learning and the regulation of these factors to achieve our goals[7] (Schoenfeld, 1987; Silver, 1987; Wilson & Clarke, 2002). These factors also include the individuals' beliefs and judgments about himself and his affective states concerning his knowledge, abilities and motivation as a learner (Hacker, 1998). Despite the varying conceptualizations of metacognition over the years, most theorists reference two main aspects as integral components: knowledge of cognition and regulation of that cognition (Brown, 1987; Schoenfeld, 1987; Silver, 1985).

Distinctions have been made between what is considered cognitive from that which is metacognitive. Garofalo and Lester (1985), similar to Silver (1982) and Schoenfeld (1992) agree that cognitive is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done (Garofalo & Lester, 1985, p.164). This distinction categorizes the link between metacognition (choosing, planning and monitoring) and success in mathematical problem solving (doing) in that the metacognitive informs the cognitive.

[7] These descriptions fall within Flavell and Wellman's (1977) categorization of metacognitive knowledge into person, task and strategy factors that contribute to overall task performance.

Researchers found that students who received metacognitive instruction consistently performed better on mathematically authentic tasks than those who did not receive such metacognitive guidance or support (Kramarski et al., 2002; Schoenfeld, 1992). These researchers along with others (Wilson & Clarke, 2002) concluded that problem solving essentially involves a complex interplay between cognition and metacognition and so success is considered as both a function of the students' knowledge base and also the metacognitive behaviors employed.

The development of metacognitive skills appears to be crucial in enhancing students' problem solving abilities and developing mathematical expertise. If we intend for our students to achieve mathematically, then it is important that we engage them in activities that support the acquisition and development of these mental processes. Writing, although more common in other academic domains, is considered an activity that forces students to engage in self-argumentation and reflection thereby making it suitable for promoting metacognitive thinking in mathematics.

Writing for Knowledge Construction. Researchers agree that successful mathematical problem solving is a function of metacognitive awareness and the application of these skills. It follows then that students' development of these skills should be an integral part of mathematics instruction (Garofalo & Lester, 1985; Mayer, 1998; Schoenfeld, 1987, 1992; Silver, 1985). Coupled with this notion, the *Principles and Standards for Teachers* (NCTM, 1989, 1991, 2000) emphasize the use of both communication and reflection in the study and teaching of mathematics. This comes as a larger call for mathematical literacy in which communication is seen as playing an essential role in clarifying and developing understanding. As such writing is considered an important component of this drive for increased literacy and its incorporation into daily practice is encouraged. Writing is seen as a tool to help students consolidate their thinking and also aids in reflection and clarification of their mathematical thoughts and ideas . It also sustains the development of reasoning and communication skills and the ability to make connections; ultimately contributing to the enhancement of metacognitive behaviors (Pugalee, 2001).

Vygotsky (1986) who was primarily known for his notion of learning as context-bound cultural activity also considered writing an important activity for knowledge construction. Writing, he theorized, necessitates deliberate and focused analytical action and it requires the individual to organize and compact his inner thoughts (speech) into a comprehensible whole (Pugalee, 2001). It helps to consolidate ideas and link old ideas to new concepts (Vygotsky, 1962), engaging the student in analytical thinking and reflection, thereby making the shift from a passive to an active learner (Kasperek, 1996; Porter & Masingila, 2000). One way that researchers have sought to incorporate this kind of activity in the mathematics classroom is through the use of writing-to-learn mathematics (Ganguli & Henry, 1994; Kasperek, 1996; Porter & Masingila, 2000). *Writing-to-learn mathematics* is a philosophy outlined by Countryman (1992) in her book (of the same title), who posits that:

> Knowing mathematics is doing mathematics. We need to create situations where students can be active, creative, and responsive to the physical world. I believe that to learn mathematics, students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short by being active in the world. Writing is an ideal activity for such processes" (p. 2)

Prior to this publication, Stehney (1990) purported that writing is an essential tool for learning and not just a means of expression and displaying learned concepts. Similarly, Bereiter and Scardamalia (1987) and others (Kroll & Halaby, 1997; Pugalee, 2001) advocated for writing being an effective means for knowledge building. In their research on the metacognitive aspects of writing, they differentiated between writing as knowledge-telling and writing as knowledge-transforming. Knowledge-transforming is described as a strategy demanding mental effort, engaging the writer in metacognitively guided planning, diagnosis and problem solving. Expanding on this, Kroll and Halaby (1997) stated that engaging in the writing process requires the students to analyze, compare and synthesize information, creating a clear conceptual picture through written words. It also provides opportunities for review, reflection, and evaluation of the problem-solving process, working as a tool to improve conceptual understanding, metacognition, and communication (Kasperek, 1996; Kroll & Halaby, 1997; Pugalee, 2001). Writing reinforces metacognitive behaviors, and pedagogy that allows students to become more aware of their own thought processes should be encouraged.

Social Construction of Knowledge

The success of any reform movement in the field of mathematics education will be influenced by the beliefs about mathematics held by those involved in the initiative (Borko & Putnam, 1997; Buttery, 1990). These views are often quite diverse ranging from mathematics being a body of facts and procedures to be memorized and reproduced to a science based on pattern-seeking (Schoenfeld, 1992). Conceptualizing mathematics as simply a body of facts and procedures is considered quite shallow and also tends to oversimplify and trivialize the domain. Most modern mathematicians and mathematics educators purport the view of mathematics as "a social activity where a 'community of learners' engage in practices based on study, observation, and experimentation to determine the nature of principles of theoretical or 'real-world' systems. It is an exploratory, dynamic, and evolving system involving the formulation of conjectures, exploration of patterns, and finding solutions" (Schoenfeld, 1992, p 335). This idea of mathematics learning being an inherently social and constructive activity has been the focus of several researchers in recent decades (Forman, 1989; Goos et al., 2002; Lesh et al., 2003) being influenced primarily by the work of Vygotsky (1978) and proponents of socio-cultural and situated theories that promote the notion of learning beyond individual cognition to the social realm.

Socio-cultural theory asserts that through the process of socialization and enculturation we develop skills of interpretation and meaning construction (Vygotsky, 1986). Specifically within the field of mathematics, thinking mathematically or rather having a mathematical point of view, is essentially envisioning the world as mathematicians do (Schoenfeld, 1992). Knowledge is considered to be culturally shaped and defined and we develop understandings through our interactions and participation within the 'community of practice' (Case, 1996). Mathematical competence or proficiency is characterized by an individual becoming more expert in the practices of the mathematical community. Language and various cultural tools are crucial within these communities as they facilitate the individual's increasing ability to effectively engage in the community's practice. This view of knowing and learning is aligned with mathematical reform initiatives (similar to those stated in the *Principles and Standards for School Mathematics*) and recent curricular trends as they focus on students becoming active participants in the mathematical community, emphasizing communication, problem solving and mathematical reasoning (NCTM, 2000). However, any initiative with these goals in mind undoubtedly requires that some teachers change or adjust their pedagogical strategies. These changes will be better facilitated by improving our understanding of how students learn in these social contexts (Goos et al., 2002).

In a sense, socio-cultural factors help to create an environment that stimulate or induce the desired mechanisms for knowledge construction (Hatano & Inagaki, 2003). To aid in this knowledge building, classroom communities (comprising the individual students along with the teacher) develop various social norms that represent ways of thinking and doing within this environment. These social norms include discourse elements characterized by the type of talk individuals in the community engage in and how the rules of this engagement are defined. In the mathematics classroom this discourse is commonly referred to as mathematical argumentation and is characterized by the sharing, explaining, and justifying of mathematical ideas. (Cobb et al., 1991; Leonard, 2000; Stein, 2001)

Argumentation for Knowledge Sharing. Classroom discussions where students are able to make worthwhile contributions, ask questions, have their ideas evaluated, and receive immediate feedback are considered one of the more effective strategies for knowledge construction (Inagaki et al., 1998). These discussions are considered most useful for knowledge building when the students are working collaboratively addressing problem situations (Kramarski et al., 2002). This peer collaboration embodies a reciprocal process where each member of the group is free to share his thoughts and have the opportunity to explore each other's reasoning in order to come to a shared understanding. Throughout this process the individual must also assess his own thoughts in relation to the viewpoints offered by his peers in order to make meaningful contributions to the group. In the event of conflicting or contradictory opinions each student must re-assess his response comparing his own ideas with that of the group. This reciprocal process of co-constructing meaning engages the individual in mental processes of both a cognitive and metacognitive nature. It externalizes the processes of self-argumentation that characterizes individual mental processes providing support for and reinforcing metacognitive skills (Schoenfeld, 1987).

Research supports these ideas, as collective discourse is considered fundamental to students' construction of meaning (Cobb et al., 1991; Forman et al., 1998; Pontecorvo, 1993; Rogoff, 1990). Additionally, previous reform efforts (Ball, 1991; Cobb et al., 1991; Forman, 1989; Knuth & Peressini, 2001; Stein, 2001), have highlighted the effects of mathematical discourse and its great potential for enhancing students' communication skills and knowledge within the domain of mathematics. Therefore, incorporating activities within the classroom that allow students to engage in this form of discourse is essential to the development of students' critical thinking skills and mathematical understandings.

Teacher Role

Within these learning environments the teacher's role is crucial, not as the repository of knowledge, but as the one who initiates and guides the students in 'community' practices. Maximizing the effectiveness of these classrooms through their transformation into environments of inquiry requires that the teacher take on the role of 'facilitator' and not 'transmitter of knowledge' (Cobb et al., 1991). In so doing, while the students are collaboratively engaging in argumentation around mathematical ideas and concepts they should be continuously scaffolded by the teacher, guiding the students towards expertise. The teacher is ultimately responsible for creating an intellectual environment in the classroom where serious engagement in mathematical thinking is the norm, by selecting and using suitable curricular materials and appropriate instructional tools and techniques to support learning.

This is a deviation from the regular chalk and talk, teacher-centered classroom where the responsibility for thinking and reasoning tends to be removed from the students and the intellectual authority placed on the teacher. For most teachers this is often a change in the way they structure their lessons and relate to their students as they are required to take a step back from the traditional role in the classroom and place the focus on the student (Handal, 2003). It often requires a dramatic shift in the instructional practices of the teacher as the classrooms should be dominated by both student-student and teacher-student discourse characterized by metacognitive guidance and scaffolding (Cobb et al., 1991; Hmelo-Silver, 2002; Leonard, 2000; Stein, 2001).

The success of these classrooms depends on how well the teacher develops and continually negotiates classroom norms to enable quality teacher-student and student-student engagement in meaningful discourse. Whole class discussions should consist of the teacher asking open questions where students can explore and generate defensible ideas. Additionally, teachers should encourage students to independently solve problems, pose conjectures and establish a safe forum where students feel free to formulate their own and critique each other's mathematical arguments (McClain, McGatha, & Hodge, 2000). Modeling these practices within the whole-class is crucial for students to be able to enact them in their own small groups. As students engage in these conversations, the teacher should facilitate by listening to their responses, helping students to recognize conflicts between alternative explanations and solutions and emphasizing the importance of valid justifications, guiding the development of diverse but accurate ways of thinking (Cobb et al., 1991).

Purpose

The purpose of this study was to examine the effect of mathematical argumentation and writing on the mathematical achievement of ninth-grade Algebra 1 students. It was thought that students engaged in discursively-rich and cognitively-enhanced learning environments will demonstrate greater understanding and achievement of mathematical concepts than those students within a more traditional classroom. Additionally, the combination of both sets of strategies will produce greater achievement than the use of just one or neither. This study served to answer the following research questions:

a) Will student's understanding of mathematical concepts as measured by an external assessment tool be greater having been immersed in a learning environment combining mathematical argumentation and writing as opposed to those students *within a traditional classroom*?
b) Will students' understanding of mathematical concepts as measured by an external assessment tool be greater having been immersed in a learning environment combining mathematical argumentation and writing as opposed to those students *who only engage in writing*?
c) Will students' understanding of mathematical concepts as measured by an external assessment tool be greater having been immersed in a learning environment combining mathematical argumentation and writing as opposed to those students *who only engage in mathematical argumentation*?

Additionally, a qualitative component to the study served to examine the effects of both argumentation and writing on the students' learning. Specifically, (i) how did students' talk within their groups impact their knowledge of the concepts being discussed? and (i) how did the writing activities consolidate the students' thinking independently and following their engagement in argumentation?

Methodology

Method

A quasi-experimental design was adopted for this study. The students were already in intact classrooms and so treatment conditions were randomly assigned to the three groups. One group served as the control group. All the students undergoing the treatment were placed in groups of four by their teachers. The teachers were asked to organize the groups so each group would have a mix of gender and ability. Two groups of students (groups of four) were randomly selected from both the argumentation and writing (AW) groups and the argumentation only (A-only) group for video-taping in order to provide a more in-depth analysis and to enhance the discussion of the quantitative results.

Participants

The participants in this study were 211 ninth-grade Algebra 1 students in eight intact classrooms across two high schools in a southeastern suburban community. The high schools consisted of grades 9 – 12 and had a minority population of over 70 % from middle class to working class income levels. There were five Algebra 1 teachers who took part in this study. They varied in ages, the number of years taught, and in their degree of certification to teach at the high school level. The teachers were distributed across the groups so that each of the four groups had two different teachers.

Data Collection

Instrumentation

Pre-post Assessment. A 19-item test was developed to measure the students' learning gains. Over the 10 week period of the intervention the students were expected to cover Algebra 1 content aligned to 18 state standards. An item pool was created from state released tests items that aligned to each of these standards. Approximately five items were selected for each of the 18 standards (some questions comprised more than one standard). From this set of five, one item was selected that aligned to each of the standards. All items were multiple-choice and the test was administered in a pre-post format (see Appendix F).

Classroom Activities. Over the 10-week period the students covered seven concept areas in the Algebra 1 curriculum. Each concept area took about a week to cover with some taking a few days less or more. The teacher would cover the content and the students would take their in-class assessment. On the day following the in-class assessment, the students engaged in the argumentation and writing activities. The activities covered major areas in the curriculum: 1) solving linear equations 2) solving multi-step linear equations 3) formulas, functions and relations 4) the slope of a line 5) graphing linear equations 6) writing linear equations in different forms and 7) solving and graphing linear inequalities. Each activity consisted of two questions that were very different from the questions in their text and the problems they solved during their regular classes (see appendices A - E). These questions focused on the 'big ideas' surrounding the content, the key ideas for which the students were expected to develop enduring understandings (Wiggins & McTigue, 2005). In other words, the questions were designed so the students would be encouraged to explore the underlying conceptual meaning beyond the application of formulas and algorithms. The questions were a mixture of open-ended and more closed items. The open-end items had multiple solution strategies and answers and the more closed items were focused on the students explicating their understanding of the particular concepts. Several (3 to 5) questions were drafted initially and then were examined by three experienced mathematics teachers for feedback on the level of difficulty and conceptual relevance. It was considered important that questions be sufficiently complex to evoke some cognitive struggle in efforts to extend their knowledge. All treatment groups received the same questions but how they were required to address the questions was different. These treatment conditions are described as follows:

1) activities structured around mathematical argumentation
2) activities structured around writing
3) activities structured around mathematical argumentation and writing

Treatment

Each concept area was covered by the group through teacher instruction, and after the students had completed their formal in-class assessment the students engaged in the following treatment conditions (see table 3.1).

Argumentation Only Group. The students were instructed to get in their groups of four to discuss the questions. These groups were fairly heterogeneous, with a mix of both gender and ability. Each student was required to follow a three-step routine: 1) read through the questions individually to gather initial thoughts 2) each student in the group should state his response and explain why his response is valid 3) defend his answers in light of other responses. While each group engaged in the argumentation routine the teacher was encouraged to facilitate. After each activity the students engaged in a whole class discussion about the questions. This discussion which was led by the teacher required that each group share its answers and talk about any disagreements they had during argumentation and how they were resolved. The class also came to a consensus about the correct approach(es) for the questions and plausible solution strategy(ies).

Teacher Facilitation. Teachers who volunteered for the study worked with the researcher to develop the activities around the curriculum and also to discuss how to best facilitate the treatment conditions. This facilitation was considered important, especially in the initial activities, as students often have difficulty engaging in talk around mathematical activities (Rittenhouse, 1998). The general guidelines for the teachers included ensuring that the students were following the routine and also ensuring that the students were making sense of the questions and developing better understandings of the mathematical content. This involved the teacher approaching the groups, listening to the discussion, and encouraging the students to support their statements through mathematical reasoning. Where necessary the use of graphs, diagrams and formulas was encouraged as ways of providing additional support and justification for their statements. The teachers were also directed to use questioning to push the students towards thinking deeply and more critically about their ideas and statements. In the case where the students were stuck or confused, teachers were encouraged to provide suggestions or hints that served to redirect the students' thinking or guide them toward alternative strategies or solutions. The teachers were informed that these hints however, should not include the answers but be used as a way of guiding students towards constructing better understandings and generating additional but useful thoughts about the concepts.

Writing Only Group. Prior to engaging in the first writing activity the researcher engaged the students in a discussion about the purpose of the activity and how they were to address the questions. The students were told that explaining why and how were the key features that they were expected to address (the questions were also phrased in a way to incorporate these scaffolds). Also, they were instructed to rely on algorithms and formulas only as necessary and if used, to explicate the reasoning behind their application. The students were instructed to read and answer the questions individually. The teacher would then collect the papers, review the questions, and provide written feedback. This feedback did not focus on the correct answer but addressed possible misconceptions the student may have had and outlined the thinking that underlay the question.

Argumentation and Writing Group. The students in the combined treatment group engaged in both routines. For the first question they engaged in the argumentation routine and then were required to produce a written response for the question. Then, for the second question they engaged in the writing routine.

Observations

During the implementation period I observed both the teacher and students in all four groups (AW, A-only, W-only and comparison) twice per week. My observations focused on how well the teacher facilitated the mathematical discourse and used the writing activities to both stimulate and advance the students' mathematical reasoning and problem solving skills. My observation of the students focused on their participation and engagement in the discursive activities and how well they developed over time. In the comparison group my observations served to document the extent to which the teacher encouraged student-student and teacher-student discourse.

Results

Quantitative Analysis

The pretest was administered to all four groups (three treatment groups and one control group) of students one week prior to the beginning of the treatment (see table 3.2). The posttest was administered to all groups within one week of the students engaging in the seventh and final activity. Analysis of Covariance was conducted on the data using the pretest scores as the covariate. Hypothesis testing included only the posttest scores for the students from all groups. Table 3.3 shows the adjusted means and standard deviations for the posttest scores by treatment.

Analysis of the adjusted means for all the students showed that the students in the argumentation and writing group scored higher (10.17) that any of the other treatment groups and the comparison group. The writing only group scored the second highest (9.72), then the argumentation only group (9.00) with the comparison group scoring the lowest of all the groups (8.66). To address the three research questions posed the analyses were based on the posttest scores for each group and then post hoc pairwise comparisons were also examined. For the posttest scores there were significant differences found for the main effects for treatment, $F(3, 206) = 4.284$, $p = 0.006$, $\eta^2 = 0.059$. These results indicated that there were differences between the performances of the students for the different groups. To specifically identify where the differences in the groups were and to answer the remaining research questions post hoc comparisons were done.

Research Question 1. The first research question sought to examine if there would be significant differences in students' achievement for students who engaged in both argumentation and writing activities (AW) over those students who did not engage in either type of activity (C). Post hoc analyses of pairwise comparison revealed that there were significant differences ($p < 0.05$) between these two groups, AW > C, $p=0.001$, $\eta^2 =0.050$. This suggests that students who engaged in argumentation followed by individually reasoning through writing about mathematical problems showed better achievement than those who did neither.

Research Question 2. The second research question investigated students' understanding of mathematical concepts for students who engaged in combined mathematical argumentation and writing versus students' who engaged in writing only. Post hoc analyses of pairwise comparison revealed that there were no significant differences ($p < 0.05$) between these two groups, AW > W, $p= 0.34$, $\eta^2 = 0.004$.

Research Question 3. Research Question 3 examined the differences in achievement of students who experienced both argumentation and writing activities in comparison to students who only engaged in argumentation. Post hoc analyses of pairwise comparison reflected that there were significant differences ($p < 0.05$) between these two groups, AW > A, $p = 0.019$, $\eta^2 = 0.026$. These results suggest that students who engaged in both argumentation and writing had greater achievement than students who only engaged in argumentation as demonstrated by their significantly greater learning gains.

Discussion of Quantitative Results

The results of the analyses suggested that engaging students in activities where they have the opportunity to engage in shared or individual meaning making leads to greater achievement than engaging in neither. From the analyses of the means we also see that students who engaged in both activities did reflect increased scores over students who worked individually on similar tasks or those who collaboratively discussed these tasks. However, while all these differences were not significant it does suggest that engaging students in both types of activities provides greater opportunity for learning the content. Further analyses of the pairwise comparisons revealed additional but peculiar information in that although the means of these individual activity groups were higher than the comparison group only the difference between the comparison group and the writing only group was significant, A> C, $p= 0.498$, $\eta^2 = 0.002$ and W > C, $p=0.028$, $\eta^2 = 0.023$.

The lack of significant gains of the argumentation only group was considered odd because of the perceived affordances of engaging in mathematical talk. Having the opportunity to discuss their responses and have them assessed and critiqued by one’s peers and receive feedback should have provided opportunities for the students to both consolidate their current understandings and extend their knowledge about the concepts. Additionally, argumentation is also presumed to provide students with additional opportunities to identify possible misconceptions, which may lead to the development of new understandings. These opportunities to engage in talk did not appear to have enhanced the students' understanding greatly as their gains were minimal. Further analysis of the conversation of two groups of students and writing samples may provide some insight into how the discursive engagements and writing activities impacted the assessment scores.

Qualitative Analysis of Transcripts

An in-depth analysis was conducted on the transcripts taken from the students' argumentation around the fourth activity (see Appendix A) in an attempt to gain insight into how argumentation may have enhanced the students' understanding of the targeted concepts and the role that writing played in consolidating this understanding. Additionally, the papers of the writing-only students were analyzed to illuminate their thinking about the concepts and also to provide greater insight and possible explanations for the quantitative results. Specifically, this analysis was designed to provide a clearer picture of why the students in the combined group (AW group) had significant learning gains over the control group and the argumentation only (A-only) group but non-significant gains over the writing only (W-only) group. Additionally, this analysis should provide possible explanations for the minimal gains of the A-only group over the control group.

Argumentation. The transcripts I focus on for this part of the analysis surround the fourth classroom activity in which the students were engaged. These transcripts were chosen as the focus for several reasons: i) this activity was about midway in the intervention when the students were beginning to engage in the practices of argumentation such as stating and warranting their responses and providing a defense in the face of counterargument, ii) it highlighted the importance of teacher facilitation in serving as a guide for the discussions, identifying errors and channeling students' thinking towards resolving those errors, and iii) they included collectively several features that were present and observed in the argumentation around other activities (such as the presentation of an alternative perspective leading to further investigation). The activity required that students determine whether two formulas provided for the slope would result in the same number value and in light of that discuss the validity of the two formulas (see Appendix A). Argumentation was considered a useful learning strategy for providing students with opportunities to share their own ideas and understandings, exposing them to the ideas and thinking of others, and allowing them to consider alternative perspectives. These alternative perspectives encourage students to consider possibilities that they may not have if left on their own. This excerpt from the transcript of students in the AW group provides an example of this:

Kevin: What did you say?
Deana: Abel use these points with this formula and Cain uses these points with this formula. Are they both correct? Use the points to support your answer.
Deana: Yes, I think it's yeeeess…yeees
Winsome: No, I put no
Kevin: I put no
Deana: I put Abel is correct because that is the only formula I was taught how to use
Winsome: Is it this one [points to Abel's formula]…I thought they were the same
Deana: No
Eddie: No ..it's $y_2 - y_1$
Deana : Yeah, that's the right formula
Kevin: Yeah that one
Eddie: Yeah that's the real formula
Winsome: Yeah but when you think about it when you plug in the numbers I'm pretty sure it would work either way.

After examining the formulas each student states their response. Initially Deana states that she thinks both formulas would work and then further states that she thinks Abel is correct, contradicting her original statement. Her original hesitation in saying ‘yes’ seems to imply that she is unsure. However, when her group members give an opposing response it clarifies her thinking about the claim she is making. Her statement about Abel’s formula being correct and her negative response to Winsome’s statement about the formulas being the same would support the notion that she thinks both formulas would not result in the same value for the slope and that she misspoke earlier. Three of the students seem to agree that only one formula is correct (Abel’s) because that is the formula they were taught and that is the only one they have ever used. However, Winsome disagrees with their claim and suggests an alternative viewpoint and a valid means of testing her hypothesis. It is interesting to note that although the question did instruct the students to verify their responses by using the ordered pairs they did not; they were apparently confident in their answers because of prior knowledge and experience (inferred from Deana’s statement in line 12 and Eddie’s in line 19). Having been presented with an opposing view the students were then encouraged to investigate the idea thereby creating an opportunity for learning. Small group or peer interaction that supports argumentation often places the student in a situation where he experiences some cognitive disequilibrium through the presentation of an alternative perspective that challenges his own conceptions. Through this process he has the opportunity to extend his own thinking and understanding about concepts and also to clarify points of difficulty or misunderstanding (Glaser, 1991).This was clearly the case in this discussion because without being presented with an opposing view it appeared that the other students would have been satisfied with believing the two formulas would lead to different answers. The discussion continued:

22 Kevin: I’m gonna try it.
23 Eddie: Me too.
24 Winsome: But I put no just because that was the formula [pointing to Abel’s formula].

25 Deana: But it might be yes,
26 Kevin: What is -2 -3
27 Eddie: -6 .. I mean -5
28 Deana : -5 yeah
29 Winsome: You get the same thing except positive.
30 Deana: So they both can’t be correct then.
31 Kevin: Which one did you get correct though
32 Deana: This is Cain.
33 Winsome: I don’t know.
34 Kevin: You get the same numbers but you don’t get the…
35 Winsome: Same sign.

Winsome’s suggestion leads the students to actually use the ordered pairs provided to investigate. However, due to computation errors they arrive at the incorrect answer and revert to their original conclusion. This interaction between the students demonstrates how discourse provides the opportunity for the distribution and sharing of different perspectives and the potential of this knowledge sharing to encourage conjecturing and further exploration. It forced them to collectively evaluate their previous understanding of how to calculate the slope, to actually try to verify what they had been taught.

All four students getting the same incorrect answer spawned further investigation and after examination of their papers I observed that the students did get the same number value but had failed to simplify one of their answers. After calculating the slope all four students had the following responses: using Cain’s formula they got -1/-5 and then for Abel’s formula they got 1/5. They did not recognize that simplifying -1/-5 would give you 1/5 which would possibly have led them to conclude that both formulas produced the same numeric result. Later in the discussion the teacher questioned the students about their discussion and what they concluded:

42 Teacher: Well, what did you guys put?
43 Eddie: Well I think we all thought it was yes for the second part.
44 Deana: Yes and also for the first part.
45 Deana: Well almost everybody put no and then we came to yes.
46 Kevin: Well I put no and stuck with it.

47 Deana : Well all of us really put no.
48 Teacher: Well you can't just change your answer arbitrarily. You must have a reason.
49 Kevin: I did, I say no because if you plug in the numbers you will get the same
50 number but not the same sign.
51 Teacher: Really, let me see [teacher looks at Kevin's paper].
52 Eddie: Yeah it was like -1 and 1 and…
53 Kevin: And then you have a -5 and a -5
54 Teacher: So what is that equal to if you have a -1 over a -5?
54 Eddie: Isn't it just 1 over 5?

In this excerpt the teacher tries to get the students to talk about their answers and to explain why and how they arrived at their conclusions. After a bit of indecision by Deana, Kevin steps up and explains why they concluded that both formulas were not correct. After the teacher guided them towards examining their calculations more closely Eddie recognizes that -1/-5 is equal to 1/5 and so both formulas *did* actually produce the same value. Here the teacher identified that they had made an error and prompted them to reexamine their thinking through questioning. By not providing the correct answer the teacher placed the responsibility on the students for their own thinking, forcing them to reassess their initial ideas and allowing them to creating new knowledge for themselves. In this instance the questioning by the teacher was important for them to take a second look and evaluate the accuracy of their conclusion. For inexperienced students the role of the teacher in this regard is crucial to encourage the students to critically examine their responses and solutions. The teacher, however, did not stop there but further encouraged the students to explore the reasoning behind this new emergent knowledge that Winsome had alluded to earlier.

69 Teacher: So what does that mean?
70 Eddie: Oh yeah ..I see…
71 Teacher: So do you know why that is though?

[about 5 seconds later]

72 Winsome: Because it's just the variables that you switch around so it doesn't

matter.
Teacher: Can you explain that again and tell everybody?
Winsome: Well because you could have chosen any number to be y_1 either 4 or 5 or
you could have label any of the points [pointing to the x –values] x_1 or x_2
so it would work out to be the same
Deana: Oh…I see
Eddie: Yeah
Winsome: So it doesn't matter
Teacher: So do you really understand [looking at Deana]? Explain it to me.
Deana: Yeah I get because you usually ask us which one do we want to be y1 and
y2 and we usually choose the first one but if we switched it around it
would be Abel's formula.

The teacher emphasized by asking them to explain, that understanding why both formulas worked was important. Also, that it was not enough to rely on Winsome's reasoning, but they had to come to an understanding for themselves, making the knowledge their own. The conversation continued with the students trying to apply this newfound reasoning to the second part of the question. While the students seemed to understand computationally why both formulas would work the teacher could have extended the discussion toward a more conceptual discussion. It was also important for the students to understand that the slope represented the ratio of the vertical change and the horizontal shift producing a measure of the steepness of the line. Therefore, the positioning of the x (x_1 and x_2) and y (y_1 and y_2) variables was not important because given computational accuracy and consistency in ordering the variables they will produce the correct value for the slope. This understanding was important for students to grasp in order to enhance the usefulness of the second question and to appropriately address it. Additionally, while procedural knowledge of this concept may be sufficient to accurately answer most questions that appear on standardized tests at this level, having a more robust understanding of the slope is important for higher level mathematics.

At the end of this conversation it appeared that all the students had come to a fair understanding of why both formulas will work to produce the value of the slope. The discussion not only allowed them to talk about what they knew about the formula for the slope, but the presentation of an opposing view engaged them in further exploration leading to an extension of their understanding.

Not all the discourse groups engaged in this type of shared knowledge construction; there were differences in the quality of argumentation in the different groups. With this second group of students all four students are engaged in the discussion, but the onus was placed on one student to provide an answer and defend his position.

Toni: He's wrong…this one [pointing to Abel's formula]…
Brad: Who's wrong?
Toni: You wanna bet.
Brad: What are you talking 'bout? Cain did it different from Abel.
Amber: I know that…this one is right [pointing to Cain's formula].
Toni: You should go y_1 first.
Amber: Prove it.
Brad: Yes you can .
Amber: No you can't.
Brad: Look…you can go up 5 and over 1 or up 1 and over 5. Can't you do it like that both ways?
Susie: Would it be the same thing?
Brad: No.
Toni & Amber: [laughs] ok

Although it is difficult to understand from his discourse, Brad starts out by saying that the formulas will produce the same value although they are different. He attempts to provide evidence for his reasoning by plotting the ordered pairs and determining the value of the slope by estimating the vertical distance between both points and then the horizontal distance. He makes a mistake with his method and so does not get the same value. However, he is convinced that they should work and persists following a suggestion from one of the other group members.

Brad: Wait...let me think
Susie: Use the numbers.
Amber: I don't know...let me try it.
Brad: Ok there you go...yes they do, they do equal the same thing
Amber: Don't be yelling!

Toni: You got 1 over 5 and 5 over 1
Brad: But they equal the same thing.
Amber: 1 over 5 is not the same as 5 over 1.
Susie: So the answer is yes then.
Brad: Yes that is what I put, because both formulas can be used to find the point.
Susie: Can you explain it?
Amber: Is it right?
Toni: I quit.
Brad: Ok let me show you...look, look. You take the 3 and then the 5, this is right here and then this is x1 and then you do this [labeling the next set of points]. Then you subtract this and then you go like this 3 minus -2 which is 3 plus 5, 3 plus 2 which is 5 so...over then you go 5 – 4 is 1 so that's your fraction right there.
Susie: so is it the same thing?
Amber: aaaaaaaaaaaaaghhh

Brad's previous method did not work for him so he now uses the values from the ordered pairs to prove his hypothesis that the formulas will produce the same answer. He demonstrates to the other group members through calculations that he is correct in his hypothesis, but they do not seem to follow his argument. One proceeds to ask if the formulas are the same and the other lets out a gasp of seeming frustration. Although Brad has provided them with an alternative view they are not encouraged to investigate the possibility on their own but rely on Brad to provide proof. Whilst the first group engaged in knowledge sharing and developing mutual understanding these students were satisfied to rely on the conclusion of Brad if he could produce a valid proof. A few minutes later the teacher tries to have them explain their conclusion:

Teacher: So what do you notice?
Amber: They are the same
Teacher: So this is the one we used in class right …what's wrong?
Amber: Because it is not suppose to be right.
Teacher: Why not?
Toni: [laughs]
Amber: Ok.

From their responses to the teacher it appears that they are still not sure why both formulas produce the same answer, and it is the cause of some frustration for the students.

Following Brad's explanation the other group members are willing to accept that the formulas produce the same value but are confused because it contradicts what they have been taught in class. The teacher-student interaction is brief, and no guidance is provided to help the students think more critically about the question. The students are in a position of cognitive disequilibrium but are provided with no guidance about how to resolve this issue. Also, it is unclear from Brad's explanation whether or not he knows why the order of the variables in the formula is not integral to the correct calculation of the slope.

While this conversation may have been beneficial to Brad, it appeared that the other group members did not make sense of the question or come to any better understanding of the underlying concepts. In this regard, the conversation did not provide them with additional opportunities to increase their knowledge beyond their regular classroom instruction. As such, while they had increased exposure to these concepts, the students' opportunities for knowledge building were not significantly different from those students in the control group.

Rivard and Straw (2000) suggested that there are four mechanisms important in group discussions for promoting understanding and for mobilizing the conversation. They are asking questions, conjecturing, formulating ideas together and explaining (p. 580). There were differences in the way the two groups actualized these mechanisms. For both groups at least one student hypothesized that the formulas would produce the same value but how the other group members responded was different. In the first group Winsome's conjecture was taken up by the other students as a challenge that all attempted to investigate. During this process they were questioning each other about the procedures they were employing and explaining their thoughts and conclusions. This ultimately led them to conclude that the formulas would not produce the same answer. In Group 2 while all the group members engaged in questioning, Brad made the hypothesis and was the only group member who tried to explain his reasoning. There was no knowledge sharing or collective formulation of ideas and so at the end of the discussion some were still confused about why and how both formulas resulted in the same, correct answer. The differences in the quality of the argumentation in which the students engaged appeared to have had an impact on their overall understanding of the concepts and were reflected in their test scores. Group 1 had an average of 8 out of 19 items correct on the pretest and 12.7 out of 19 correct on the post-test, an increase of 4.7. However, group 2 had an average of 7.3 out of 19 items correct on the pretest and 9.3 on the post-test, and increase of 2. These results suggest that the shared meaning-making processes impacted the students' mathematical understanding reflected in them having a larger increase of 2.7 from pretest to post-test. From examination of the other video-tapes and observations made throughout the implementation these differences in the quality of discourse between the two groups were fairly reflective of the degrees of difference in the conversations across all the four-member student groups in the both AW group and A-only group.

Writing. The writing activity appeared to have helped students organize their thoughts and consolidate their thinking, allowing them to better express their understanding of the concepts. An example of this is demonstrated below in the writings of the AW students for part b of the question following their discussion. Part b is shown because it provides a good example of how the students articulated their reasoning following talk:

Winsome: Yes, as long as you don't switch around the numbers in the ordered pair. You subtract the y's on top and the x's on the bottom. It works because any of the ordered pairs can be $x_2\ y_2$ and $x_1 y_1$.

Kevin: Yes if you plug in the numbers in the formulas you will get the same answers because any of the points can be x_1y_1.

Eddie: Yes, it doesn't matter if you switch the y_1 and the y_2 as long as you do the same with x_1 and x_2.

Deana: Yes, as long as you use the formulas correctly and watch your signs. But as long as you don't do y_1 on top and y_2 on the bottom it wouldn't work. Keep y's together and x's together.

From the transcript both Deana and Winsome had provided explanations of why the different formulas would lead to the same answer. While talking lead the students to share their ideas and clarify meanings and understandings, writing allowed the students to focus and to organize their thoughts in a more concise and coherent manner. From these writings one has a better picture of the meaning that the students took from the activity as it allowed then to organize and refine their ideas and produce a more structured response.

For the group that engaged in argumentation and then writing both activities appeared to have complimented each other allowing the students to have produced a more comprehensive product. Writing tends to force the individual to converge and focus their ideas to produce a more organized and coherent response following the more open and unstructured discussion. However, even without the benefit of generating and sharing ideas with peers, the students in the writing only group seemed to have used the writing tasks to also organize and consolidate their thinking about the two formulas for the slope. Examples of their responses are presented:

- Yes, because the x's and the y's are together no matter what ways it goes. They are the same just numbers on different sides.

- Yes, because the x’s and the y’s together so you will get the same thing no matter which way it goes it will still have the y’s on top and the x’s on the bottom.
- Yes, because the formulas are really the same thing. For Abel you will get 1/5 and for Cain you will get 1/5 just that the numbers start being switched around
- Yes, they are the same. Either way you label the points you get 1/5

From the writing samples both groups of students came to the correct conclusion but both sets of writing lacked conceptual depth. In that, both sets of responses focused on the procedural aspect of the concept and did not reveal much about why they thought this occurred. However, there was a significant difference between the writing samples from the writing only students and the students in the combined activity group. A larger percentage of the writing only students used the values provided to support their answer; 87% of the writing only students used the values (from part a) to support their conclusions while only 54% of the argumentation and writing students did. This may indicate that the AW students after their discussions thought more conceptually about the relationship between the two formulas *or* following part a they saw no reason to provide additional support for this claim. On the other hand, having engaged in the activity individually and not being allowed the opportunity to try out their ideas on others may have forced the W-only students to resolve any confusion or indecision independently and perhaps generated the need to prove their claim to themselves leading to the use of the ordered pairs as a means of justification. This motivation to justify and explain their responses may have caused these students to activate additional cognitive resources engaging in more intense analysis of the question, ultimately leading to increased understanding of the concepts.

Overall the writing activities seemed to provide the students with the opportunity to make sense of the question, reflect on and organize their thoughts about the concepts, and structure their ideas to produce a meaningful response. While neither of the groups came to a more robust understanding of the slope most students seemed to have concluded that both formulas would lead to the same answer. Having to focus so intently on articulating their understanding seemed to have forced them to invest more cognitive resources into making sense of the problem, which appeared to have enhanced their conceptual understanding as the two groups that engaged in writing activities had the highest means (AW and W-only groups).

Discussion

Mathematical learning is an interactive as well as an individual, constructive activity (Cobb et al., 1991). As such a variety of opportunities to construct mathematical knowledge exist in both collaborative and individual settings. These rich learning opportunities arise when students attempt to resolve or address conflicting viewpoints or arguments that come about when engaged in discussion with others or provided with challenging tasks. As such being able to generate and articulate thoughts and ideas and to reconstruct understandings are fundamental to students' mathematical reasoning and development. Both argumentation and writing are activities that seem to activate the cognitive resources necessary to develop rich understandings of mathematical content.

This study sought to investigate the combined and individual effects of these activities on students' understanding of Algebra 1 concepts. The analyses produced two sets of notable results. First, through examination of the means, the results revealed that engaging in either activity lead to greater achievement than none at all and that combined, these activities were more beneficial for students' understanding than individually or none at all. Second, post hoc analyses revealed significant results for two of three research questions investigated. Students who engaged in writing activities, either alone or combined with argumentation (AW and W-only), had significantly greater learning gains than the students who engaged in neither activity (C). Additionally, engaging in combined activities (AW) led to significantly greater increases in mathematical achievement than engagement in argumentation only (A-only). However, there were no significant increases in the learning gains between the AW and W-only groups, nor between the A-only and comparison groups. The qualitative analyses served to both complement and provided explanations for these findings.

In the qualitative finding, argumentation was found to be a useful strategy in promoting the generation and sharing of ideas. This collective sharing allowed students to hear the ideas of others, which helped in highlighting misconceptions and in confirming their own thinking about concepts. In some cases it produced a necessary conflict with the student's current understandings, thereby forcing the student to make attempts to resolve it and so creating an opportunity to eliminate a misconception and enhance his knowledge about the concept. Being presented with opposing views in some instances, lead to conjecturing and exploration, providing opportunities for the students to generate new knowledge for themselves. This was evident in both discourse groups although the level of knowledge sharing and collaboration was varied. The analyses of group conversations illuminated features of the conversations that appeared to generate knowledge building and growth as well features that did not lead to enhancement of the students' understanding.

In the first group (AW) all members were actively engaged in working through and talking about their responses. With the presentation of a conflicting viewpoint they collectively formulated an approach to investigate this opposing view. Although they arrived at the incorrect response, the discussion forced the group members to think more deeply and reflect on their initial assumptions and to further investigate the validity of their original ideas. When asked by other group members they were expected to explain their reasoning forcing them to think more critically about the concepts. Some of these features were present in the second group (A-only) although the knowledge generation and sharing was only evident in one group member (Brad). Although the over-reliance of the other group members on their 'prior understandings' seemed to hinder their learning, it served as a point of challenge for Brad to investigate and provide proof of his own conjecture. Additionally, although the other members of this group appeared to not have come to an understanding of the underlying concept it did provide some disequilibrium, which could have encouraged further exploration. An important feature however, was that although they were confused by Brad's explanation they did not unquestioningly accept Brad's idea. The propensity of group members to unquestioningly accept claims by seemingly more knowledgeable students is often a problem that decreases the effectiveness of argumentation in groups, as these students rely on the knowledge of their higher achieving peers and do not try to construct their own understandings or making sense of the concepts for themselves (Rivard & Straw, 2000).

In light of this, it is imperative for teachers to emphasize that students should not privilege coming to a consensus over their own individual understanding. As such, while students become better at argumentation through increased participation, the teacher should ensure that within these discourse groups, especially heterogeneous ability groups, that student are holding each other accountable for justifying their responses and defending the positions in light of others. It is through being forced to elaborate one's thinking that thoughts become more refined and synthesized leading to deeper conceptual understandings. In the absence of this the potential of discourse groups for negotiating and refining collective understanding is minimized, nullifying the learning experiences. These occurrences may explain the minimal learning gains of the A only group over the control group as the conversations in the A only group seem to lack the features that encouraged sense-making leading to increased learning.

Although the students' argumentation did improve somewhat over the course of the implementation, the overall process was rather challenging for both the students and the teachers. It was very difficult initially to get the students to talk about the concepts and to have the teachers serve as guides rather than the knowledge bearers. Over time the students improved in their participation; however, the content in the majority of their statements reflected minimal understanding of the prerequisite mathematical concepts. So although the students improved in their readiness to participate in the conversations, the content of the responses often reflected minimal understanding. This appeared to be due to two factors - the students' prior knowledge and understanding of prerequisite concepts appeared to be low (the mean of pretest scores for the discourse groups was 8.3 of a total of 18 items) and classroom social norms for discourse were not sufficiently established in these classes.

Classroom social norms are characterized by explanation, justification and argumentation (Yackel & Cobb, 1996). These norms of rational argument are not specific to mathematics classrooms but also occur in science, literature, and social sciences classrooms where students are responsible for justifying their responses and making sense of the ideas of others. Mathematical argumentation takes these norms a step further because integral to high quality argumentation is the validity and mathematical sophistication included in the students' statements. Specifically, it requires the student to be cognizant of what counts as evidence in mathematics and what is regarded as a mathematically valid justification or warrant. However, if these basic social norms are not established, then the difficulty in engaging students in mathematical argumentation becomes compounded.

While there was an increase in the amount of talk that took place in these groups, many of these conversations still lacked the mathematical rigor that would push the students to extend their thinking. Namely, students tended not to hold their peers accountable for producing a defensible argument for their statements. An example of this is seen in group one's conversation. Initially the students stated that the two formulas were fundamentally different' therefore, they would not result in the same numeric value. Their justification for this was that Abel's formula was the one they were taught and so the other formula must be incorrect. Although prompted to use the ordered pairs as a means of gathering evidence to support their claim, the students did not take up this avenue for justification until challenged by Winsome's statement. The students did not appear to differentiate between valid mathematical evidence and 'my teacher said so' as plausible means of justification.

Although time was taken to orient and engage students in these discursive practices in an attempt to establish these classroom norms, additional support needs to be provided through modeling and teacher monitoring and facilitation. Additionally, it is necessary to instruct students about the nature of argument within the domain of mathematics. Specifically, *how* do you provide support for your claims, and *what* constitutes mathematical evidence or a valid justification within the field. Of equal or maybe more importance is to identify for the students what does *not* qualify as a reasonable or valid warrant in mathematics.

The writing activities tended to set the stage for students to first think about the task, generate ideas about appropriate strategies to address the problem, and then evaluate the effectiveness of the approach. They presented a good way for students to consolidate their knowledge about different concepts and how they related and to synthesize these ideas to produce an organized and coherent response. Having engaged in talk about the tasks prior to engaging in the writing activities may have removed some of the cognitive burden of generating the thoughts individually and also allowed for greater meaning making collectively. It also appeared that engaging in writing in some way forced the students to internalize this argumentation; this may have allowed the students to engage their metacognitive skills when approaching the questions. This opportunity to interact individually with the content and to develop deeper understandings was clearly beneficial to the students and so while the mean scores were lower than the students in the combined activities group, the difference was not significant. From the analyses, writing appears to be a very useful strategy for possibly engaging metacognitive thinking, and helping students to better refine and synthesize their ideas, ultimately leading to greater understandings. For future research it will be important to come to a better understanding of the cognitive processes students activate when engaged in writing activities. Specifically, future research needs to investigate the existence and nature of the self-argumentation process that students seem to engage in when writing. Additionally, by gaining greater insight into what students perceive as substantial warrants and into the decision-making process underlying students justifying their responses, we can provide better support and guidance for students with regard to how to adequately warrant their claims. Finally, it would be useful to incorporate more scaffolds within the tasks to guide the student towards thinking more deeply about the concepts.

Although individually and combined these activities enhanced student learning, both activities seemed to have their limitations. Both types of activities have the potential to help students develop better understandings and identify and clarify misconceptions, but without the necessary prior knowledge to engage productively in them, learning will be hindered. For example, the lack of prior knowledge appeared to have been a barrier to student participation and understanding. In the AW group the students came to the incorrect conclusion because they did not know or recognize that dividing two negatives results in a positive number making the answer from the use of both formulas the same. With the use of questioning the teacher was able help them identify their error and guide them towards the correct conclusion. However, in contrast with A-only group the students seemed to be unable to follow Brad's reasoning. He was aware of alternative methods to calculate the value of the slope but had difficulty explaining it to the students because this method was unfamiliar to them. This inability to follow Brad's reasoning appeared to hinder the effectiveness of the routine and the subsequent understanding of the students. In this case the presence of the teacher would have been invaluable to help guide the students toward a solution.

This raises the issue of the teacher's role in the facilitation of these activities. Of critical importance to the success of these activities is the teacher developing classroom social norms for both group interaction and writing tasks. When engaging in individual writing activities it is necessary that students are encouraged to persist when solving challenging problems, and also to initially think divergently about the task generating multiple strategies and then selecting the correct and most efficient path to the solution. In converging on a solution strategy students should also have clear understandings of why the rejected strategies are not viable. Prior to engaging in group discussions teachers should orient students on how to engage in collective discussion by first modeling it in whole class discussions. Ensuring that students explain and justify their solutions, listening to and making sense of the ideas and solutions of their peers and recognizing and explaining divergence in solutions are all necessary steps in making discussions constructive. Students will undoubtedly have difficulty engaging in argumentation and so as students are developing these skills it is necessary for teachers to observe these interactions, assessing students' solutions and reasoning, and guiding them towards developing rich, robust understandings.

The teachers in the study facilitated the groups to different degrees, but it was evident that for some groups the activities that we were trying to engage the students in violated present classroom norms. From the examples provided above it is clear that students had difficulty articulating their reasoning and understanding, both verbally and in print. In addition to this not being a common practice in their classrooms this may be as a result of other factors, including literacy/reading levels of the students, degree of prior knowledge of the students, and the unfamiliarity of the expectations of the task. Additionally, it is important that the teacher redefines his/her role within the classroom. In several of the classrooms, for example group one's classroom, the teacher was seen as the knowledge bearer and so information provided by the teacher was used as justification by the students. Initially it appeared that students perceived this information as off limits for questioning or challenge. This perceived and enacted role of the teacher often prevents the students from constructing their own understandings of mathematical concepts and deprives them of authentic mathematical learning experiences. While a full discussion of the role of the teacher in the mathematics classroom is beyond the scope of this discussion it is important for further research in this area to acknowledge the impact teachers have on the success of these activities and take suitable steps to adequately address the issue.

The results from this research suggest that engaging in activities reflective of both cognitive and socio-cultural views of knowledge and learning does lead to increased understanding and achievement. It also appears that engaging in both is better than engaging in argumentation only. However, we must be cautious in making grand claims in this regard as the qualitative analysis does demonstrate that the extent to which these students engaged in the discursive activities as designed was minimal. Having had sufficient time to establish classroom social norms around discourse, students in both discourse groups may have been able to harness the full potential of these activities. The teacher's role is also important in both modeling these practices and guiding the students during their conversations. While the writing activities may have served as a heuristic for the AW group, as a tool to further encourage them to make meaning of the concepts, for the W-only group it was the means through which they were able to transform their current knowledge into more robust understandings. These activities however, seem to be more useful for knowledge building having followed opportunities to collaborate discursively with peers.

This study is significant in several ways. Research has demonstrated that both mathematical discourse (Cobb et al., 1991; Inagaki et al., 1998; Yackel & Cobb, 1996) and writing (Baxter et al., 2005; Porter & Masingila, 2000; Pugalee, 2001) have been effective in improving student understanding. However, this study investigated how both strategies can be merged to produce increased learning and understanding and revealed that together they result in greater achievement than individually. There are several features of the study that make it easily transferable into practice. The study designed activities around important units of content in the Algebra 1 curriculum and implemented them as the concepts unfolded during regular instruction. Incorporating the practices in this way adds authenticity as the activities were effective despite the often tumultuous nature of regular schooling. Additionally, the teachers were incorporated into the design of the activities minimizing the difficulty in transferring it to regular classroom practice.

Table 3.1: Activities for each group

Treatment Group	Questions on Activities	Activity
Argumentation & Writing (AW)	1	Argumentation Writing
	2	Writing
Argumentation Only	1 & 2	Argumentation
Writing Only	1& 2	Writing
Comparison	None	None

Table 3.2: Timeline for assessments and activities

Time	Week 1	Weeks 2-9	Week 10
Assessments	Pre-Test	Activities 1-7	Posttest
Group	All groups	All groups	All groups

Table 3.3: Means and Standard Deviations of the four groups.

		Pretest		Posttest	
Treatment	N	Means	Standard Deviations	Means (adjusted)	Standard Deviations
Argumentation Only	43	7.60	2.779	9.00	3.169
Writing Only	51	7.64	2.629	9.72	3.040
Argumentation and Writing	62	8.64	2.797	10.17	2.827
Comparison	55	8.24	2.589	8.66	2.618

CHAPTER 4

THE INFLUENCE OF ALGEBRA TEACHERS' MATHEMATICAL BELIEFS ON THEIR INSTRUCTIONAL PRACTICE[8]

[8] Cross, D. I. (To be submitted to Journal of Mathematics Teacher Education.)

Abstract

This collective case study reports on an investigation into how algebra teachers describe their beliefs about mathematics, mathematics pedagogy and student learning and the extent to which these beliefs are manifested in practice. It also examined the pervasiveness of these beliefs in the face of efforts to incorporate reform-oriented classroom materials and instructional strategies. Five high school teachers of ninth-grade algebra at different stages in their teaching career were interviewed and observed over a 10-week period. Analysis of the data revealed that in general their beliefs were very influential on their daily pedagogical decisions and also that their beliefs about the nature of mathematics served as a primary source of their beliefs about pedagogy and student learning. The results of this investigation provides useful insights for the mathematics education community as it presents models of the teachers' beliefs demonstrating the diversity among the in-service teachers, the role and influence of beliefs about the nature of mathematics on the belief structure, how the teachers designed their instructional practices to reflect them and suggestions for belief change efforts.

Introduction

Over the last few decades more emphasis has been given to the role of teachers in the learning process and their influence on how the learning context is shaped and structured. Recognizing that the context in which learning takes place has enormous influence on what is being taught and learned, the mathematics education community began to invest more time and resources into research about teachers. Specifically, researchers in the field of mathematics education and others including educational psychology and teacher education have become increasingly aware of the influence of teachers' beliefs on their pedagogical decisions and classroom practice (Clarke & Peterson, 1986; Cobb et al., 1991; Lumpe, Haney, & Czerniak, 2000; Nespor, 1987; Pajares, 1992; Raymond, 1997; Thompson, 1992b; Torff, 2005; Wilson & Cooney, 2002). In this regard, it is believed that for there to be improvement in mathematics achievement, classroom practices must reflect reform recommendations. This would require a change in the instructional practices of many mathematics teachers at all levels, a change that can only be actualized if we come to a better understanding of not only the types of beliefs these teachers have, but how these beliefs are related to each other and practice, and the implications of these for change.

Beliefs are defined as embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it developed through membership in various social groups that are considered by the individual to be true (Cross, in preparation). They describe the organization and content of a person's thinking that are presumed to drive that person's actions (Bryan & Atwater, 2002). They are personal, stable and often reside at a level beyond the individual's immediate control or knowledge. They are considered to be very influential in determining how individuals frame problems and structure tasks and are considered strong predictors of behavior (Nespor, 1987; Rimm-Kaufman & Sawyer, 2004). Research has also stated that beliefs are often precursors to action, specifically with regard to teachers' pedagogical decisions and tend to be unaffected by educational attainment or teaching experience (Lumpe et al., 2000; Pajares, 1992; Torff & Warburton, 2005). This relationship is not considered linear as there can be multiple factors that ultimately contribute to an individual's action (Skott, 2001). Beliefs are not thought to be isolated but tend to cluster in an organized system classified by their order and strength. Consequently, due to this complex nature of belief systems, they are often difficult to measure and change (Ambrose, Clement, Phillip, & Chauvot, 2004; Green, 1971).

As a result of their tendency to remain implicit and the highly resistant nature, pre-service teachers often leave their teacher education programs and begin practice with beliefs about knowledge, teaching and student learning that they accumulated throughout their own student experience (Borko & Putnam, 1997; Pierce & Kalkman, 2003; Rimm-Kaufman & Sawyer, 2004). Because these views are often quite pervasive and tacit, they tend to remain intact and unchanged unless there are focused and deliberate efforts to change those beliefs. This process of belief change is considered important because beliefs are considered to be closely aligned to practice and therefore if teachers are to implement reform-oriented curricula their beliefs should reflect more constructivist views.

This article reports on an investigation into how Algebra 1 teachers describe their beliefs about mathematics, mathematics pedagogy and student learning and the extent to which these beliefs are manifested in practice. Specifically, I investigated the relationship among the beliefs of these teachers and how they structured their classrooms, interacted with their students, and assessed their students' learning. The results of this investigation adds to the literature as it directly portrays the diversity in these in-service teachers beliefs, how these beliefs were organized and how the teachers designed their instructional practices to reflect them. Much of the research done on mathematics teachers' beliefs has been conducted with pre-service teachers in attempts to examine and if needed change their beliefs before they begin their teaching practice. This study takes a different focus as the participants are all in-service teachers at different stages in their teaching career and examines the pervasiveness of their beliefs in the face of efforts to incorporate more reform-oriented strategies.

Teachers' Mathematical Beliefs

Teachers' beliefs within the domain of mathematics are often classified in two categories: beliefs about the nature of mathematics and beliefs about mathematics teaching and learning (Cooney, 2003; A. Thompson, 1992). Researchers have documented that individuals have different and varying beliefs about the nature and meaning of mathematics (Cooney, 1985; Lesh, 1985; Polya, 1957; Schoenfeld, 1987; Silver, 1982, 1985; A. Thompson, 1992). These individual perspectives tend to be quite dominant and often influence teachers' conceptions of mathematics teaching and learning. They reflect how teachers conceptualize teacher and student roles within the classroom, their choice of classroom activities and the instructional strategies they use. These beliefs also relate to how teachers characterize mathematical understanding and knowledge, how students learn mathematics and the purpose of schools in general (A. Thompson, 1992). They are considered central to the way teachers' conceptualize and actualize their role in the mathematics classroom, and so they are integral to any efforts to improve student learning.

Several theorists have discussed the varying conceptions of teachers' beliefs about mathematics (Cooney, 1985; Ernest, 1989a, 1998). In particular, Ernest (1988) identified and distinguished between three different views about the nature of mathematics; they include the instrumentalist view, the Platonist view and the problem solving view. These distinctions can be aligned to Kuhs and Ball's (1986) classification of 'dominant views of how mathematics should be taught' (p.2). They are also reflective of Dionne's (1984) three basic perspectives which are the traditional, formalist and constructivist perspectives (as cited in Liljedahl, Rolka & Rosken, 2007). However, for the purposes of this discussion I will use Ernest's categorizations as they provide a more holistic description of these views of mathematics. The 'problem solving view' conceptualizes mathematics as a dynamic and continually expanding field of human creation and invention encompassing a process of inquiry and coming to know. The constructivist view of mathematics learning can be aligned with this perspective as it purports the establishment of a learner-focused environment (Cobb & Steffe, 1983; P. Thompson, 1985; von Glasersfeld, 1987). Within this environment, students are actively engaged in the learning process where they are encouraged and allowed to generate and develop their own ideas. Teachers who hold this view would consider their role to be that of facilitator, continuously guiding and challenging the students. They would assist the students in developing their own ideas, highlighting inadequacies in their thinking and helping them develop expertise in supporting and defending their conclusions (Kuhs & Ball, 1986).

The instrumentalist view of mathematics holds that mathematics consists of a collection of facts, procedures and skill sets to be used in the process of achieving an external end, often the solution to a problem. From this perspective learning is seen as the mastery of specific skills that facilitates the accurate solving of problems. A teacher who holds this conception of mathematics would view his/her role as being the source of knowledge, instructing students on how to appropriately and accurately apply these rules and procedures. The classroom environment would perhaps mirror what educators refer to as teacher-focused, a style of teaching that focuses on the content of teaching and what the teacher is doing (Lindblom-Ylanne, Trigwell, Nevgi, &

Ashwin, 2006). The emphasis is placed on the transmission of facts and right answers, promoting a single approach and path to a specific solution. Within this environment the teacher is the individual who possesses the knowledge and is responsible for defining, demonstrating and explaining the content. Additionally, there tends to be an emphasis on practice and didactic interactions and students are required to adhere to established algorithms to produce correct answers in an effort to demonstrate mathematical knowledge and fluency (A. Thompson, 1992a). The students are often passive in the learning process; student exploration and investigation are seldom used. This perspective and the teaching strategies derived from it are and have been considered problematic for decades within the field of mathematics education as they are thought to present a naïve and shallow view of mathematics and often leave students with a distorted view of mathematics (Cobb et al., 1991; Schoenfeld, 1992).

Another perspective about the nature of mathematics, the Platonist view, holds mathematics as a static but unified body of knowledge that is there to be discovered, not created. It is a consistent, stable and objective structure where the teacher is the explainer focusing on conceptual understanding of this knowledge. A classroom established around this conception differs from the former in that it has a dual focus on both the content and student understanding. While its primary focus is the content, it also emphasizes knowledge and understanding of the facts and procedures underlying the content; learning is seen as the student's reception of the knowledge (Kuhs & Ball, 1986; A. Thompson, 1992a).

Although it is perceived that how a teacher conceptualizes mathematics has direct impact on her teaching, the relationship between a teacher's beliefs and her instructional practice is not considered linear. However, while the relationship cannot be accurately described as a direct cause and effect association, it is often quite salient. It is described as a complex and reciprocal relationship and so researchers must go beyond examination of teachers' professed beliefs in order to get an adequate assessment of any teacher's conception of a domain (Charalambous et al., 2002; Raymond, 1997; A. Thompson, 1992a). A thorough investigation of the phenomenon would require the researcher to engage in repeated observation of the learning environment and the instructional practice. Making dual examinations of both the verbal and observational data would provide more information about the alignment between beliefs and practice and also provide useful data about other factors that may adversely influence their practice resulting in the subordination of their beliefs (A. Thompson, 1992a).

I used these insights in investigating how in-service teachers describe their beliefs about the domain of mathematics, its teaching and learning and the degree of alignment of these professed beliefs and their daily instructional practice. I believed that investigating this phenomenon will provide a clear picture of how these beliefs are organized and linked to instruction and ultimately how this information can be used to initiate the course of reconstructing these beliefs if necessary. Specifically, I sought to answer the following questions:

a) How do teachers describe their beliefs about the nature of mathematics, student learning and mathematics pedagogy?

b) To what degree are teachers' beliefs be aligned with their instructional practice?

Investigating these questions prior to and as teachers were beginning the process of incorporating reform-oriented instruction and materials was considered important as previous research reported on both the struggle teachers have in changing their practice and also how beliefs often serve as a hindrance to the actualization of these practices.

Research Methods

Participants

The study was a part of a larger project focusing on the effects of mathematical argumentation and writing and the achievement of Algebra 1 students. Eleven teachers were originally contacted to be a part of the study, however, only five agreed to be participants; the remaining teachers stated insufficient time and the low academic level of their students as reasons they were unable to participate in the project. As participants in the project the teachers were provided with materials and resources to incorporate mathematical argumentation and writing into their classroom activities. The following are brief descriptions of these five teachers:

Mr. Brown. Mr. Brown was in his first year of formal teaching. At the time of the interview his teaching experience included being a substitute teacher at both middle and high school during the previous semester and tutoring chemistry and calculus in college. He had passed the content examinations required to teach in the state and currently had a provisional teaching certificate. Mr. Brown planned on enrolling in a masters degree program in the following year and obtaining certification while completing the program.

Ms. Jones. A veteran teacher of 30 years, Ms. Jones had taught all the mathematics courses offered at high school with the exception of statistics. In her early years she taught computer courses but since then she has only been involved in the mathematics department. She has been at her current school for 25 years but taught at two other schools previously. Following high school she completed a bachelor’s degree in mathematics education, followed by a masters degree and then a specialist degree in mathematics education.

Ms. Reid. For Ms. Reid teaching is a second career. She left her job as a management information systems (MIS) specialist three years prior to the study to become a teacher. Her preparation for teaching included successfully completing the content examinations required to teach in the state as well as an alternative teacher preparation program. She, however, considers the best preparation for being a teacher is being a mother.

Mr. Henry. Mr. Henry was in his third year teaching but had only taught Algebra 1 and pre-algebra at the high school level. He had taught a course in research methods to college students for a year prior to becoming a high school teacher. He passed the required content examinations to teach high school but had not yet completed the requirements for certification. Mr. Henry enjoyed teaching but intended to return to school within the next few years to pursue a doctoral degree.

Mr. Simpson. After 18 years Mr. Simpson considers teaching his chosen and desired career path. He began his academic career in the field of accounting but found it to be quite boring and so under the influence of his parents and in-laws embarked on a career in mathematics education. After earning a bachelor's degree in this area he began teaching mathematics and then soon after went back to university to pursue a masters degree in mathematics education. He has taught all the high school mathematics classes except calculus.

Schools

The two schools in this study were located in a suburban county in the southeastern United States. They are traditional high schools covering the grades nine through 12 curricula and each had a minority population of over 50 %. One school had a ninth grade academy, which was an effort by the school to have all ninth graders take classes with only their peers. The other school did not have an official ninth grade academy, but they made efforts to keep all ninth graders together.

Methodology

The particular methodology that this research applies is that of case study (Stake, 2000). It would best be described as a collective case study as a number of cases are examined to investigate a particular phenomenon. The cases are used jointly to investigate the similarities, differences and variety of the data collected in the sample to provide an in-depth view of the complexity of the phenomenon under investigation. Specifically for the study, the intent is to provide insight into the teachers' mathematical beliefs about the subject, its teaching and learning. In this instance the individual cases are not of primary interest but help to provide a context through which the phenomenon under investigation can be studied and better understood. These cases will all have some characteristics in common but will also differ on other factors, however, these cases were considered integral in providing a better understanding, possibly theorizing about a greater set of cases (Crabtree & Miller, 1999; Stake, 2000).

The case, the unit of analysis in this study is the Algebra 1 mathematics teacher. In particular, I am interested in the systems of beliefs that each teacher held regarding the domain of mathematics. In order to gather comprehensive, in-depth and systematic information (Patton, 2002) several methods of data collection were used, including interviews, observations and document collection and analysis.

Data Collection and Analysis

As mentioned earlier, this study was embedded within a larger project investigating the impact of mathematical argumentation and writing on achievement. The intervention was quasi-experimental and focused on engaging students in argumentation and writing activities over the course of 10 weeks. All five teachers engaged in a 45 minute semi-structured interview conducted by the researcher prior to the start of the intervention. The interview questions focused on the teachers' views about mathematics as a domain of knowledge (e.g., If you were to think of four words you thought were closely related to mathematics, what would they be?), mathematics pedagogy (e.g., How would you describe your role in the classroom?) and student learning (e.g., How do you think students learn mathematics best?) (see Appendix G). Interviews were used as they were considered to be the best instrument to allow teachers the opportunity to describe in detail their views about mathematics providing a clearer window and better access to their beliefs. Prior to the semi-structured interview, two formal observations were conducted of each teacher. After the interview weekly observations (10 weeks) were done of each class for which detailed field notes were taken. These observations were followed by informal discussions with the teachers to elicit their thought processes related to specific actions observed and decisions related to specific pedagogical decisions made. Notes were taken of the discussions to record the teachers' statements. The observations were done on different days each week to try to get a broad picture of the each teacher's practice. Additionally, rapport was developed with the teacher to allow for unannounced observations so several of the observations were done without notifying the teacher in advance.

The recorded interviews were transcribed and the transcripts thoroughly read and compressed into briefer statements by use of meaning condensation, a method involving condensing "natural units" (the subjects responses) into "central themes" (Kvale, 1996). I adapted Strauss and Corbin's (1990) open coding as a technique for coding participants' statements relevant to their beliefs about the nature of mathematics, mathematics pedagogy and student learning. These initial codes converged into themes categorizing the teachers' beliefs about the subject and how students learn it best. The development of categories was an ongoing iterative process where themes were allowed to emerge. As each transcript was coded, both the codes and the categories were reevaluated and refined to mirror participants' description of their beliefs. Within these broader categories there were predominant themes that emerged from the data that will be presented as metaphors representing how these teachers described their beliefs. The observation data will also be included to provide a picture of how these beliefs were manifested in the teachers' daily classroom practice. In describing these teachers' practices three areas will be highlighted, as these are often reflective of the types of beliefs teachers' hold (Ernest, 1998). These areas are i) the classroom environment, ii) the teacher-student and student-student discourse and interactions and, iii) the types of assessments and how they were used.

Results

How a teacher conceptualizes the nature of mathematics tends to influence the teaching strategies used and activities he/she designs for the students. As such, it is often difficult to determine specifically whether an observed behavior within the classroom is a manifestation of a subject domain belief, pedagogical belief or learning belief. To clarify this, informal interviews were conducted with the teachers to elicit this information and triangulate the results. In presenting the results, I will foreground one particular teacher as an example of how the beliefs are interconnected and to demonstrate also how one particular behavior can seemingly be reflective of multiple beliefs. Next, the beliefs of the other participants will be presented within broader categories capturing the compartmentalization of teachers' beliefs into views about the nature of the subject, its related pedagogy and the process of learning. Within this section of the results I use metaphors as a way of providing vivid imagery and illustrations of the teachers' beliefs. This metaphorical language came from the teachers' own descriptions and so provides a clearer understanding of how they conceptualize their beliefs related to mathematics. I begin with the complete case of Mr. Simpson and then present the descriptions of the other teachers to discuss how they conceptualized mathematics, teaching and learning and its relation to their instruction.

The Case of Mr. Simpson

Mr. Simpson was a veteran teacher of 18 years and had taught at the same high school for the duration of his teaching career. He has both a bachelors and masters degree in mathematics education and has taught all of the math courses at the secondary level with the exception of calculus. Mr. Simpson enjoys teaching and considers teaching his chosen career path. At the time of the interview he was seated at the desk in his classroom and had just finished viewing his students' 8th grade standardized test scores. He commented that he often used these scores not as a way of assessing the student's mathematical ability but to give him a better understanding of the student's mathematical history and the motivational and confidence issues the students bring with them to his class. Mr. Simpson stated,

> ...they have confidence problems… either they haven't done well in the past or things like that. CRCT is your basic mathematics and obviously they know they didn't pass it and so there is a confidence issue there, that they maybe hadn't done well in middle school and they get passed on to me anyway. So confidence especially in algebra 1 is important because this class is sort of the basis for all the mathematics now at the high school.

For him knowing the students was an important part of designing suitable and effective activities to satisfy their learning needs. He put a lot of thought into planning his lessons, taking into account the knowledge of his students, who they are as learners and considering the most effective ways to guide them along the learning path. Here, he describes some of the factors he considers when evaluating the effectiveness of a lesson,

> …the way the lesson flowed ahm… what I try and do especially if I am changing gears I try to choose problems from where I am to where I want to be, instead of just saying ok this is what we did last time… we were working doing positive and negative numbers and now we are doing logarithms. I think about if there is a way I can transition into that where it doesn't look like I am doing anything new but I really am so they are comfortable doing something new … keep the comfort and then say, 'by the way we are doing something new' and it is not to trick them but get them to realize that by the end of the class everything we do you can do.

This was evidenced in several observations of Mr. Simpson's class, in the way he introduced concepts and led students toward understanding. It was important to him that the students be given increasingly difficult problems gradually, to build their confidence in their ability to approach them correctly and also for them to see the underlying conceptual relationship between these problems. He, however, did not settle for superficial associations, but it was important for students to explain what they meant when they used different terms, especially commonly used mathematical terms and those used in the text. His verbal interactions with the students during instructional time largely involved questioning, but the questions that focused more on the students demonstrating that they really had a thorough grasp of the concept rather than on producing a correct answer.

For example, when introducing the concept of the slope it was clear that he wanted the students to understand what the slope represented and not just be able to recall the formula. One attempt to do this was through allowing the students to explore the value of the slope in relation to different kinds of lines. In a follow up activity students were asked to determine whether the slope of a particular line was one (m =1) by looking at it and defending their position. Several of the students responded by saying the slope of the line was negative (and therefore not one) and warranted that response by saying that the line was decreasing. Mr. Simpson encouraged the students to explain what they meant by decreasing in terms of the direction of the slope and with respect to the changing values of the x and y and also to make the connection with the formula. He also challenged them to find other ways to prove their claim. Students readily responded to the challenge and explained their ideas to their peers. During this interchange, Mr. Simpson stood to the side and allowed the students to explore other options, questioning the students about their rationales when appropriate. This interaction between the students was quite typical of the class and also the way in which Mr. Simpson took on more of a participant role within the classroom.

This example not only reflects how he viewed himself as a teacher and the role he played in the classroom but also what he believed was an effective way for students to learn mathematics. When asked how he would describe a mathematics teacher he responded by saying, "I think a mathematics teacher is like a guide; I want to guide them through the learning process more so than tell them, more than being a dictator." Mr. Simpson did not want to place himself in authority over of the content; he did not believe that he should position himself to be in absolute power or control over the mathematical knowledge. Rather, he wanted to create an environment in which the students could generate and take ownership over their own ideas and develop their own understandings of the concepts that they were exposed to in the classroom. One way in which he did this was to have the students create their own definitions for concepts; they were not allowed to use the definitions from the text. The students were encouraged to define the terms out of their own experiences with the concepts. This was one of the ways that he felt he was able to guide the students in the learning process, by designing activities that allowed the students to see ‘the concept in action’ and then through discussions help them to tailor an appropriate definition that had ‘real meaning’ for the them. Here he describes what he means by guiding the students,

> …I go back and say that [to the students] you can solve this problem, then you can solve this more difficult problem, and so I want to kind of lead them through the mathematics…as I said I want to be a guide and being a guide means ahm- … doesn’t necessarily mean giving them a lot of freedom but letting them express what they know.

Mr. Simpson tried to provide multiple opportunities for the students to express what they know. It was not only important for them to demonstrate their learning to him but also to the other members of their classroom community. He encouraged student-student discourse within the classroom so that the students would rely less on him for verification of their answers. He talks about how he encourages that,

> …but the more I do the less I am involved in the process, I can try and back away and let them go at it and then one of the big things I do in my class is that they consult a neighbor and before they can ask me a question you have to ask somebody around you.

Mr. Simpson’s views about mathematics teaching and student learning were rooted in his views about the nature of mathematics, how he conceptualized mathematics as a domain of knowledge. In describing mathematics he stated four words that he thought were closely related to the word, they were “numbers, ahm… multiplication…ahm… I think solving…thinking.” In thinking about the association between those words and mathematics he provided a brief exposition about what he tried to impress on his students about mathematics. He articulated, “I try to talk to the kids a lot about why they do mathematics and we talk about it not being a way of doing something but it’s a way of thinking about something.” In his view, mathematics was not simply the knowledge of formulas and procedures and accurate computation skills. It was more about the thinking process one engaged in when addressing a problem and so he structured his classes around developing this kind of thinking. He elaborated,

> …so we try and get them into a mind set where they not only solve problems in class but we give them a broad base... ahm… and the broad knowledge to solve problems outside of the classroom like how much did I pay for this shirt? For this much dinner how much tip should I leave? and those kinds of things... Like right now in my ACP algebra 2 class and most of the kids they know everything we've done…and we got into imaginary numbers which is not something familiar to them and I said this is like going in and trying to replace a faucet and you haven't done that before so lets work through the process of how to do it and then we can go in and fix the faucet, which in this case is solving problems with imaginary numbers. So thinking has a lot to do with my class and they get mad at me sometimes because they will go, what's the answer? ... and I will say that I don't care what the answer is--how do you solve this problem? ... and how do you solve this problem? ...more so than what's the answer to this problem and what's the answer to this problem, which is what they have seen over and over and over …I try to emphasize that a little bit.

Mr. Simpson does not view engaging in mathematics as simply applying the appropriate formula to obtain a correct answer but more as an approach to problem situations both within and outside the classroom. Mathematics he believed, incorporates reading the problem, assessing what the problem is asking and seeking an appropriate path to arrive a solution, which is not necessarily the 'easiest' way - in this case referring to simply using a formula. As he spoke about how he viewed mathematics it became clear that he defined mathematics by associating it with what he believed were elements of mathematical expertise. In other words, one way in which he conceptualized mathematics was within the actions that mathematicians engage in when they do mathematics. He stated,

> ...I get back to the idea of thinking, they look at a problem and they think about how to solve the problem before they do anything… I talk all the time to my kids that to be a mathematician… a mathematician doesn't rush into a problem. A mathematician is going to look at it and say that the best way to solve this is to do this, I can do it like this.

He explained how he encouraged his students to be mathematicians, think like mathematicians and so approach problem situations like mathematicians do; this involved the students "looking above the problem", "seeing multiple ways to solve", "thinking", "processing". Mr. Simpson explained;

> …I want them to be mathematicians and I think that what mathematicians do is that they see a problem and they kind of look above it and say I can probably go this way, and this is probably the best route to solve this problem instead of saying well I know this formula lets try this or lets try that just because that is the easy way to do it or that is what comes to my mind first.

Here he described the connection between how he views mathematics, or what he thought the nature of mathematics was and what constituted mathematical expertise. For Mr. Simpson his views about mathematics pedagogy and student learning were directly linked to his ideas about mathematics as a subject and the practices of mathematicians. These views were quite evident in the way he structured his classroom, having the students take charge of their learning, encouraging them to maximize their learning experiences and to continuously examine their own thinking and the thinking of their peers. He continuously engaged the students in discourse and was comfortable to initiate the conversation, then step back and let the students take charge, guiding them appropriately. While correct answers were important what was integral was the understanding and the meaning the students ascribed to those answers, because for Mr. Simpson without that the answers were meaningless.

I use Mr. Simpson's case to demonstrate how teachers' beliefs about teaching and learning mathematics are often rooted in their conceptions of mathematics. Additionally, to illustrate how a teachers' classroom behavior can be seen as a manifestation of their inner beliefs. For the other teachers their beliefs about mathematics, though quite different from Mr. Simpson, were also a driving force in how they designed their classroom activities and made pedagogical decisions. I now present the views or the other teachers as themes that emerged from the analysis of the interviews.

Nature of Mathematics

Many people, including teachers, do not view mathematics as fallible, a product of human invention. Rather they see mathematics, its foundation and derived concepts as the ultimate authority, not something to be questioned, and not consisting of understandings grounded in our own experiences. It is perceived not as a matter of the mind but of memory (Cooney, 2003). With this perspective of the nature of mathematics, individuals often associate mathematics with the memorization and application of formulas with *one* specific path to *one* correct solution. Alternatively, individuals who hold a more constructivist view see mathematics as a human creation developed through active discourse and participation in the community of practice. Mathematical laws, principles and concepts were formulated through a process of critical thought and analysis and through discourse with others in the community. Conjectures were made and refuted and those that withstood were accepted on the validity of the claims.

These views of mathematics tend to influence how we approach the subject and for teachers the way they design their learning activities and orient their students to the content. These varying conceptions of mathematics are presented here through the teachers' descriptions.

Mathematics as 'Computation'. Some of the teachers, in talking about how they viewed mathematics, seemed to think about the subject in terms of formulas, procedures and calculations. When asked to describe mathematics using four words, Mr. Henry, a high school mathematics teacher for 3 years stated, "addition, subject, school... and multiplication… ." He further elaborated that, in thinking about a subject that was most like mathematics, "…I would say my research classes because we use a lot of statistics, science because we use a lot of calculations." For him mathematics constituted a subject that students did in school involving computations and calculations. As such, he viewed its relatedness or association with other domains of knowledge on the basis of the amount of calculations involved. This was evidenced in the way he organized his classroom activities and interacted with his students. During the period of observation, the class did not engage in any group discussion or organized collaborative activity (outside of those designed for the intervention). At times the students would talk with each other to check their answers, but for the majority of the instructional time they sat at their individual desks facing the white board at the front of the classroom from which they were instructed. The teacher-student interaction was of two forms. During instruction the discourse was very didactic, with Mr. Henry asking questions, the students responding followed by an evaluative remark from the teacher. On the other hand, when he wasn't instructing the teacher-student conversations were far less formal and were often quite humorous.

In Mr. Henry's class it was very important that you knew when and how to use the correct formula and that you got the right answer. While he never stated that understanding why a formula was appropriate in a particular instance was not important, he appeared to be more focused on the students being able to recall the formula and them working towards a correct answer. This was emphasized on multiple occasions when students got incorrect answers and instead of trying to help them figure out why conceptually their approach was incorrect, he would reduce the significance of their errors to incorrect calculations or misuse of the formula. On several occasions, after being told their answers were incorrect and seemingly misunderstanding why this was, the students would proceed to again apply the incorrect procedures to similar questions. Having not understood conceptually why their responses were incorrect initially they were unable to apply appropriate procedures to new but conceptually similar problems.

Mr. Brown and Ms. Reid also responded similarly to these interview questions. Mr. Brown was in his first year of teaching and was only beginning the process toward teacher certification. Mr. Brown responded by saying that when he thinks about mathematics the first thoughts that come to mind are "... addition, subtraction, multiplication, division…" He explained that those came to mind quickly because those are the absolute basic operations, "… that is as basic as all math breaks down to, at least in my mind… ." Mr. Brown thought about mathematics in terms of solving problems with the use of the four basic mathematical operations, and when necessary with the appropriate formula. He expressed this view in the way he related math to other subjects,

> The sciences... maybe chemistry was very related to math, physics even more so … I think that one is mainly all math, it is math… you know ahm… with all the formulas…

Ms. Reid, a certified teacher in her second year of teaching, had left a career in managing information systems to become a teacher. Ms. Reid's response to the question was also similar as she thought the subject most closely related to mathematics was chemistry because "…most of that was dealing with numbers and equations and calculating." However, she contextualized mathematics a bit differently in that her associations were made within the area of algebra. Ms. Reid teaches three algebra classes so algebra is the domain of mathematics that she is most familiar with and constantly exposed to. She states that if she were to use four words to describe mathematics they would be, " ... probably equation, problem, solve ...variable." She admits that her view is a bit narrow and explains that the words that come to mind quickly are those that she uses everyday, those associated with algebra. She explains, "Notice that all of them have to do with algebra because that is what I deal with and not just mathematics." Whilst she recognizes that mathematics constitutes more than just algebra she admits that it is difficult to think beyond that because that is all that exists in her teacher world. When asked how she would describe mathematics to a layman, she pauses for a few moments and then responds with shrugged shoulders,

> I am more stuck in the algebra world, so when you say math that is what I think of because that is what I teach… so I am kinda in my own little bubble as far as what it is that I do five days a week all year long. Algebra 1 is the one subject I teach all year long and so when I think about mathematics I think about linear equations, algebraic expressions and solving for variables and things of that nature. I don't think of times tables and long division. I'm in my own little world.

Conceptualizing mathematics as computation places emphasis on a correct solution and may reduce mathematics to merely the pursuit of the right answer. However, this objectivity, the notion that within mathematics there is a clear right and wrong is comforting for some. Within this algebra world that Ms. Reid lived she was comforted by the absoluteness and seeming objectivity of the subject. She talks about this when considering a subject she views as least like mathematics - english,

> I like math because there is a right and a wrong and language arts was always so iffy to me. It's about how the teacher felt that day, if she felt I answered correctly, if she felt I did a good job. Math is $x = 5$ and it either is or it isn't and I like that certainty, this is right and English is as far away from that as is humanly possible… you see that [language arts] has to do with opinion and feeling and you can argue your way through it and in language arts and in world history you can discuss things and situations and you can give me your opinion and quite possibly change my mind. You can give me your opinion from now until the cows come home and if the answer is $x = 4$ and you give me $x = 2$ you are not going to change my mind.

She confirms this view when she talks about her initial thoughts in thinking about mathematics. Ms. Reid states, "RIGHT! getting the right answer, that there is a black and a white a right and a wrong, that's what I like about it... that's what I think of." For Ms. Reid mathematics is certain, it is unambiguous and unquestionable and she likes that about the subject. She also considers it an important feature of the subject and aligns her teacher role with this internalized view of mathematics. Ms. Reid considers the mathematics teacher unique and doesn't think she could teach any other subject because,

> I cannot visualize myself teaching anything but mathematics. I think it's a different kind of individual…you are dealing with a different kind of subject matter. If the answer is $x = 4$ and you come up with $x = 2$. I can't look at you and say that this is really cool how did you come up with that? It doesn't equal two it equals four… .

Ms. Reid, Mr. Brown and Mr. Henry all seemingly had quite traditional (Ernest, 1989a; Kuhs & Ball, 1986; Raymond, 1997) conceptualizations of the nature of mathematics, which appeared to influence how they approached teaching the subject and assessing their students' knowledge. All three teachers had a routine way of introducing concepts and assessing their students. For a particular concept or topic there was a way or set of ways that students were assessed (specific types of questions). The students were taught how to answer these types of questions; first, identify what the question is asking, then apply the proper procedure and perform the calculations accurately. In the event that the student was asked to explain how he/she arrived at the answer, a response detailing the procedure step by step or indicating the use of the appropriate formula was often sufficient. Students were rarely expected to justify their solutions by explaining their reasoning or understanding of the concepts. It appeared that these teachers associated understanding with the appropriate use of procedures and formulas, and computational accuracy.

Mathematics as 'a way of thinking'. Not all the teachers held this view of mathematics rooted in computations and formulas. Ms. Jones, a veteran, certified mathematics teacher of 30 years expressed different views in relation to mathematics. When asked to describe mathematics, she responds with the following, "it is problem solving but it is also about teaching students how to think and that is often difficult…" Mr. Simpson also spoke similarly about mathematics being a thinking process focusing on how an individual approaches and navigates their way through problem situations rather than about the calculations and correct answers. However, although their perspectives seemed to be aligned in their descriptions of their approach to teaching, how these views were manifested was different. While Mr. Simpson designed activities and taught in a way to elicit this kind of thinking regardless of the type of student and the content Ms. Jones believed that this form of analytical thought was more applicable depending on the context. She further stated,

> …it is not difficult when you are teaching algebra because you are given an algorithm to follow but when you are teaching geometry when you are teaching students how to prove things it is about teaching students how to think about things in a logical, systematic manner and that is very difficult.

In her view, there were certain subjects within the domain of mathematics that lent themselves better to incorporating this type of thinking and learning.

Mathematics as 'a foreign language'. Some of the teachers identified one feature of mathematics that they believed made it different from most other bodies of knowledge. They made reference to the idea that mathematics involved the process of building on, that whilst other subjects were an accumulation of somewhat isolated chunks of information, mathematics has a form of foundational structure where developing expertise within the domain requires continuous expansion and development of basic mathematical knowledge and skills. Therefore, becoming more advanced in mathematics requires the continuous use of basic skills and knowledge of previously learnt concepts. Both Mr. Simpson and Ms. Jones likened this feature to that of language, specifically a foreign language, requiring a foundation of basic knowledge and skills in order to develop expertise within the domain. Mr. Simpson in his explanation of why he perceived that learning a foreign language was most like learning mathematics stated,

> …basically because it builds on what you have previously learned. You see you start with the rudimentary things like nouns and pronouns and then you build up to short sentences and then to longer sentences and then to paragraphs and then you build up to me talking to you and so it is always building on what you have learned in the past.

To emphasize his point he presented history as a subject that contrasted this feature of mathematics,

> ...it's not that building process like in mathematics where the kids need that foundation of good mathematics like you need a good foundation in foreign language where you need to know the roots of words and how the words act and that kind of thing.

Ms. Jones expressed similar views that within the domain of mathematics, to develop expertise, to solve more complex problems it is necessary for the individual to be able to understand fundamental concepts and perform the basic operations. She states,

> Mathematics is like learning a language and we know that when students are learning mathematics they can not do the more difficult problems until they have a basic foundation and the same is true with a foreign language because you must learn the small words before you can make a sentence with those words. The same is true with mathematics you have to learn how to do things like add, subtract, multiply and divide before you can develop an equation and then solve it.

This belief guided the way in which she introduced her students to new concepts and structured her activities. She would "take small pieces of material" and then "try to build on that material within the lesson". The students would then practice that for a while and then "give them something a little more challenging" until they were able to complete similar problems on their own.

All the teachers believed that it was important for their students to have the necessary conceptual foundation for them to do well in algebra. They explained how difficult it was for them to teach their students because they lacked knowledge of working with integers and fractions and that often times they fell behind in their teaching because they had to deviate to review or teach these concepts. Some of the teachers felt that it was their responsibility to ensure that the students had enough of this foundation before they could move on to Algebra 1 concepts. Ms. Reid was one of those teachers and stated, "…everything builds and if I am just putting stuff on top of something that you don't understand then we are all gonna come tumbling down." She would often stop her lesson and spend chunks of time going over and explaining elementary concepts to students before actually covering the lesson she had planned. These deviations from the scheduled lesson would set the class back but in conversation with Ms. Reid, she expressed that while she was concerned about the loss of time she knew it was necessary for the students to understand the basic concepts.

Mathematics Teaching

Teachers' mental models of mathematics teaching and learning tends to shape the way in which they establish, organize and maintain their classroom environments, how they interact with their students and how they perceive their teaching role. Specifically in relation to the mathematics teaching, the beliefs influence teacher actions and the type and variety of activities they incorporate in the classroom and how they position themselves within these activities. Here we see how these teachers conceptualized mathematics teaching.

*Teaching as Demonstration***.** Many of the teachers had very positive experiences of learning mathematics throughout their student lives and for some these experiences influenced their decisions to become teachers. Mr. Henry finds his experiences as a teacher very different from those he had in school which he attributes to the fact that he doesn't teach advanced classes. He finds it difficult to come up with ways to get his students to understand and finds it harder to explain things. He however has a class that he enjoys and he refers to it as favorite class because during that block he gets to *teach*. He describes what it is like to teach this class,

> [I] show them how to do things and have them sit and watch and pay attention and then practice and try and ask questions if they don't understand. Like what a teacher looks like on tv, that's my television class.

Mr. Henry generally finds his daily job quite difficult because of the academic level of his students. He believes they are not to blame for their lack of basic knowledge and skills and is determined to teach them as much as he can in the manner that he believes they will learn best.

Mr. Brown in describing the way he organizes his class likened it to his own experiences in middle and high school. He too was in advanced classes and considered them very structured and similar to the way his classes are now, they would "work and cover a particular lesson" which was followed by "practice and homework" and then the teacher would "ensure you mastered it" before moving on. They were encouraged to keep a notebook which was checked periodically to ensure they had copied all information necessary for understanding. Mr. Brown also shared that in high school they had a lot of open discussions and there were times his class would spend significant amounts of time on a particular problem and would remain engaged. He thought that this was possible because it was an advanced class where the students "were more willing to learn" and "engaged in the discussion, [they were] more eager". In his own classes, however, he admitted that they pretty much followed a routine everyday, "we [he and the students] get some notes, work on some examples and then having them practice and then reviewing their practice". This style of showing students what to do and then having them repeat it or follow was a common teaching technique among some of the teachers. Not surprisingly, it seemed to align well with what they believed was important about mathematics, emphasizing the correct use of procedures and accurate computations. Having had a good model in their teachers the students would be able to duplicate what the teachers did and thereby be successful. Ms. Reid expressed it well when she described what she thought was good teaching,

> I had excellent math teachers in high school. They left you a little bit more to figure things out on your own… but as the general rule, the same cycle, introduce the concept, show how to do something let the class play with it a little bit, try to make sure everyone has a handle on it, assign homework come back the next day, make sure that everyone understood the concept when they were trying to do it on their own and then build on that concept to move on to the next one.

These teachers from their own experiences believed that this was an effective way of covering the content and ensuring that their students learnt. They were very committed to this particular teaching approach and throughout the 10 weeks of observation they didn't deviate much from it. Their students were so programmed in this routine they had been established that any deviation caused fierce questioning from the students. In one particular observation of Ms. Reid's class, which usually began with a review of the homework assignment, she decided to change the routine not only in order but in how she assessed their knowledge. She started a new topic and decided to leave the review until after the lesson. This decision was rewarded with a barrage of questions from the students, "are we going over the homework?", "when are you going over the homework?" which went on for several minutes making it difficult to begin the lesson. Giving in, Ms. Reid reviewed the assignment and the students were so focused on getting the right answer that it was difficult for her to get them to explain how they came to that answer. I use this example for two reasons; first to demonstrate that the classes were so routinely structured that it was difficult to motivate the students to focus and engage in anything different. Second, because of the emphasis placed on getting the correct answer, the students often adhered to a particular procedure without being able to articulate why they chose it or how it related to the overall concept.

*Teaching as Guiding***.** Several teachers spoke about the notion of "guiding" their students, but more in a directive way. Mr. Simpson focused on giving his students enough freedom to explore and come to understandings on their own but the others talked about guiding more in terms of ensuring that their students stayed on the correct path, which was in this case the teacher's path. Ms. Jones elaborated on what she meant by guiding her students,

> Well I have to be the one to provide, I have to be the one to guide them through, guide them through the material and to answer their questions, to be knowledgeable enough about the material to answer their questions. And to not only tell them that they are wrong but tell them why they are wrong and to help them find their mistakes and correct them.

This difference in meaning ascribed to the notion of guiding was evidenced in the way the teachers engaged the students during classroom instruction. An example of this was seen in how two different teachers introduced their students to the concept of a function. Mr. Simpson presented the students with tables of values in which some of the values had a linear relationship and others did not. He then asked his students to examine each table of values and to write down what they observed. After he had given them some time to do this individually they were told to exchange their ideas with their colleagues. Following this collaboration the students were then asked to share their ideas with the class. Mr. Simpson wrote the ideas on the board and then asked the students to check their ideas with the different tables on the board. When they had sufficiently explored the values he asked them if they could define the relationship between the numbers. This led the students to representing the relationships in words which eventually led to algebraic expressions. At the end of this interchange the students were then introduced to the word “function” which they were able to associate with their own understanding of a particular numeric relationship and for which they had already constructed their own definition. In another class the approach was rather different. The lesson began with the teacher placing the topic “Functions” on the white board and written below it was the definition. The students were then given examples of functional relationships starting with an algebraic expression and an input-output table. They were given questions on this for practice followed by reverse examples, where they were given the input-output table and then asked to produce the algebraic expressions that would define those values.

These classes were different in two distinct aspects; one, the overall design of the learning activities and the role of the teacher the instructional setting and second, the general behavior of the students during the activities. In Mr. Simpson’s class the students were very active, shouting out answers, moving around and talking with each other. Although the students would not necessarily be characterized as disruptive, it was very difficult for Mr. Simpson to keep them on task and although they were participating it took almost the entire 90 minutes to get them through the activity and complete a class work assignment. In contrast, Mr. Henry’s class was much more quiet, most of the students were taking notes and although they were shouting out answers all the students were in their seats and seemingly took turns in responding to the teacher’s questions. There were a few students who got off task but it was much easier for Mr. Henry to identify who was being disruptive and refocus them.

Although Mr. Simpson’s class was much louder and there were instances of off-task behavior he was satisfied that they had engaged in an authentic learning experience. He however admitted and I observed that his classes were not always like this. Sometimes they fell behind and he had to resort to the more traditional forms of instruction to catch up. For several of the teachers, classroom management was very important for them to the extent that it would sometimes cripple them in creating more engaging experiences for their students. Control was very important for them and so they were very concerned about the noise level, student movement and the level of off-task behavior. Mr. Simpson was also concerned about these issues but with 18 years of experience he was confident in his teaching ability and knew that these were often not good indicators of the quality of the learning experience.

Mr. Brown similarly embraced the notion of guiding but used the analogy of a football coach. As the junior varsity football coach he likened a lot of aspects of teaching mathematics to coaching his football players. He believed that like coaching, teaching required a lot of discipline in two specific aspects. To develop expertise in mathematics one had to be disciplined in learning and practicing it and also the teacher had to be very structured and disciplined in the way they presented the material. He explained by saying,

> …math is one thing that not everyone understands but that everyone can understand if they have lots of practice and repetition. Math is an actual functional thing that you do and so in order to get better at it most people could just work real hard and get better, that's one thing I see with my students... ah... the same thing as far as athletics….

As a mathematics teacher he felt that is was necessary to be rigorous in presenting the material and to not make assumptions about the students knowledge or understanding; like coaching you have to start by teaching the fundamentals and then moving on a little bit further each day,

…. and having to be rigorous to… not skipping over the little things… me as a coach personally for our players we don't just assume that they are all great at this aspect of the sport we have to coach them each day in the little bitty fundamental kinds of things and I think math is the same way.

Ms. Jones also likened teaching to coaching, in that she felt she had to prepare her students in a disciplined and rigorous way to perform at their best level. This included "being knowledgeable" about the subject so you could structure their activities and "guide them through the material", and also be able to "help them find their mistakes and correct them". Similar to a coach, she knew that there would be obstacles and times of defeat and in those moments her responsibility was to "inspire, encourage and motivate my students to do." Both Mr. Brown and Ms. Jones adopted this persona in their daily practice. Ms. Jones in particular was always up-beat and cheerful and knew exactly what to say to convince her students to complete one more problem or to make another attempt after they had failed. She was always praising her students saying "good job", "good try" with a pat on the back or head; when they didn't do well on a particular task she would let them know that the problem was difficult and they had made a good attempt (if they did) and that *we* would go over it and try again the next time. Both teachers considered themselves a part of the team with the enormous responsibility of ensuring everyone's success. Teachers often adopt different teaching roles depending on the perceived learning or emotional needs of their students (Williams, Cross, Hong, Aultman, Osbon & Schutz, in press).

Mr. Brown also felt an enormous responsibility to help his team win; in this case the team was his algebra 1 students. Mr. Brown believed that his job extended beyond just teaching them the content, "but it's also kind of my job to show them how to be disciplined in a way that they'll know they need to take things seriously. I personally take responsibility for how they do and how they do not do." For Mr. Brown he was preparing his team, his students for the game of life, ensuring that they were prepared for what the real world entailed.

Mr. Simpson had similar convictions concerning what he considered teaching to be. He had a philosophy of teaching that went beyond just delivering the content, he wanted his students to have options and so it was necessary for him to provide them with a broad knowledge base so that they were sufficiently equipped to make the life choices that best suited them. This included helping them to think about problems that arose, within the context of the mathematics classroom or otherwise, in a systematic and analytical way, choosing the best and most efficient path to a solution. For him, mathematics was a way of thinking and so we engaged in 'mathematics every single day'.

Student Learning

As teachers, creating a learning environment where student understanding and achievement is maximized is paramount. One key feature that underlies how these environments are structured and maintained is the beliefs teachers hold about how students learn mathematics best.

Learning as Repetition/Practice. Many of the teachers emphasized the importance of practice in the development of expertise in mathematics. They thought it was an integral part of the learning process and so their students routinely engaged in heavy doses of class-work and homework assignments, which were considered imperative. As mentioned earlier Ms. Reid associated good teaching with teachers who had organization and structure; for her this meant there should be classroom routines that function like well oiled machines serving the students' learning needs. This included providing sufficient opportunities for them to engage in the content, which meant ensuring the students had enough daily assignments, enough practice. Her convictions in this regard came from her own negative experiences with her daughters' algebra teachers and their lack of structure, which proved to be detrimental for her children. Here she describes the experience and the attributions she made for her children's low performance in algebra,

> I watched my daughters struggle with math in school because they were taught completely different from what I was used to… I can honestly tell you that in a one year period of time these two girls, same math teacher back to back taking algebra, and in a one year period of time those girls probably had homework, and I am really stretching it here, maybe 20 times the whole year.

Similar to Ms. Reid, Mr. Henry thought that practice was an important part of becoming 'good' at mathematics. He too, intertwined practice into the process of learning and places practice above the process of coming to know. Mr. Henry states,

> …practice, the way they learn it is probably less important than their practicing it. I can show them, they can come and show me, they can do it together, they can read it, they can write it. I think that if they don't go home and practice it they don't learn it.

He makes a distinction here between learning a concept and understanding a concept, in that understanding can only come through practice. In his view a student can learn a concept but if he or she does not follow up with practice the knowledge will disappear. Mr. Henry adds, "I don't care how well they get it at school if they don't go home and practice it, it all just goes away". This view of learning can be linked to how these teachers viewed mathematics. Conceptualizing mathematics as a set of facts and procedures to be followed places the students at the receptive end of the knowledge, passively engaging in the practice for the mastery of skills.

Mr. Brown also believed that practice was an important part of developing expertise in the domain. When asked how he thought students learnt mathematics best he responded, “lots of practice… I tell them everyday because a lot of the kids don't want to do their homework and when they don’t do their homework obviously they don’t learn from it.” This belief stemmed from how he viewed mathematical ability and from his own early experiences with the subject. He explained that,

> …like when I went through calculus I didn't really understand why, why the different concepts, what made them true or false I didn’t really know the why of the problems but I knew the how exactly and that was enough for me to continue to learn and now being a teacher I can look back and now I understand the how but I got the fundamentals of the process of it and in my opinion if they don’t understand the why but they do lots of practice then they shouldn't have any problems with how to do it.

In his own experiences as a student he had difficulty making meaning of the concepts and so practicing lots of problems helped him to be successful. In turn, he believed that if he can get his students to adopt the methods he used then they too could progress in their mathematics. Mr. Brown also concluded from his own experiences that mathematical ability was not necessarily inherent but that an individual could become proficient if they engaged in enough practice. He elaborated, “I feel that everybody even if you are minded that way or not you can still work real hard or practice the problems to become good at them.”

*Getting it Right vs Learning as Understanding***.** The teachers in discussing student learning discussed both how they thought students learned best and also what they thought were good indicators of learning. Most of the teachers agreed that one of the most efficient ways to evaluate learning was by using some form of assessment, it could be informal like oral questioning or a paper and pencil test. However, there were differences in way they interpreted the results of these assessments. For Mr. Simpson these formal or informal assessments were an opportunity for the students to tell him what they knew and he was looking for evidence that the students were thinking about the concepts correctly and not necessarily focused on the right answer. He explained, "… so it's the process more so than the answer… and as you get better and better we'll worry about the answer but the process is really important right now." In contrast, for Mr. Henry getting the correct answer was important; this was how he evaluated whether or not the students were getting it, "because the grades matter the most to me. It's great if you are excited about learning and I'll try to talk that up if I can, but the bottom line is you gotta be right, you gotta get it right." Whilst he did talk about it being necessary for his students to understand, this understanding was equated to getting the right answer. His perceived role in the classroom was to be the person there to make sure the students "get it", but for him "getting it" meant that the students had to get the correct answer every time.

Learning is 'not a spectator sport'. All of the teachers agreed that students needed to get involved in the learning process; however, there was disparity in how they characterized this involvement and the reasons for which this involvement was necessary. I discussed earlier that both Mr. Henry and Mr. Brown encouraged this kind of involvement in the form of practicing math problems. Ms. Jones on the other hand, although she wanted participation and involvement in her students, conceptualized it differently. Ms. Jones wanted her students to get involved and to be excited and engaged about learning. She encouraged her students to ask questions, share their thoughts and ideas and challenge her responses if necessary. These behaviors were encouraged in the various activities she engaged her students in such as using the mini dry erase boards where they would work out questions on their boards and then share them with the class by holding them up. She also had a PRS (public response system) in her class for which each student had a remote control. Ms. Jones would then place a question on the projector or the board and the students could buzz in with what they thought was the right answer. She enjoyed engaging in these activities because it gave the students the opportunity to actively participate and it also provided instantaneous feedback. She explains her reasons for engaging her students in these activities,

> I think students learn by doing... mathematics is not a spectator sport you have got to actually get in there and do it… unless you do it for your self you are not going to know if you really can do it so you have to be involved, you have to participate.

Ms. Reid had similar views about student learning, she too believed that students had to get involved and participate in order to learn. She shared,

> I like for them to come up and work on the smart board for them to get in front of the class and teach the problem…I think in order to learn you have to do it yourself. You have got to try it yourself.

Both Ms. Reid and Ms. Jones wanted their students involved, in both a physically and a mentally active way. They both spoke about the students' participation in the form of them adopting a teacher role. They believed that if the student was able to teach the content to someone else then that was evidence of their own understanding. Mr. Simpson explained it best when discussing the ways that he thought facilitated mathematical learning,

> …it does get them to think about the mathematics and if this person can explain to this other person how they got it a lot of times it will stick with them a lot better. So a lot of

time the doing part and the explaining to someone else kinda goes hand it hand and if I can explain it to you, I can be more proficient with it and I am gonna remember it better.

Overall, all the teachers tried to ensure that the students were somehow involved in the learning process. The varied ways in which they achieved this was a reflection of how they seemingly conceptualized mathematics and their mental model of students' learning.

Discussion

Cohesiveness of Teachers' Beliefs

Ernest (1998) in his piece on the impact of teachers' beliefs on the teaching of mathematics talked about the relationship between how an individual teacher conceptualizes mathematics as a domain of knowledge and the influences of that on their views about how to teach mathematics and the learning of mathematics. He posits that there is somewhat of a hierarchical structure where the beliefs about teaching and learning are rooted in their view about the nature of mathematics. In other words, fundamentally views about mathematics teaching and learning are derived from or are closely link to their ideas about mathematics. Others (Green, 1971; Rokeach, 1968)) have theorized about the structure of the belief systems and provide support for this structural order. Green (1971) purports that beliefs tend to organize in clusters and have a fairly illogical order where there are primary and derivative beliefs. Primary beliefs are those that do not derive from any other belief and derivative beliefs are those that sprout from or are held on the basis of another belief (Fig. 1). So we can say that belief 3 is held on the basis of belief 2 and belief 2 is held on the basis of belief 1. This he refers to as the quasi-logical order in which beliefs are held meaning that they are structure not based on objective premises but on the thinking of the individual and how these beliefs are valued by them. In examining how the teachers described their beliefs these theories provide a framework from which to discuss the organization of their beliefs (see Figure 4.1)

From the analysis of the interviews with the teachers it appears that they all held fairly strong ideas about what constituted mathematics. Although there were differences among the five teachers all had strong conceptions that stemmed from their own early experiences with the subject. On one hand, several of the teachers had beliefs about mathematics that are not considered conducive to the type of mathematics teaching and learning supported by the National Council of Teachers of Mathematics (see *Principles and Standards for School Mathematics*, NCTM, 2000). Some of these beliefs include 1) mathematics is computation and 2) the goal of doing mathematics problems is to obtain the correct answer (Frank, 1988).

Namely, Mr. Henry, Mr. Brown and Ms. Reid in thinking about mathematics seemed to focus more on the skills and procedural aspect of the subject and less so on the cognitive processes. In other words there was less emphasis placed on how to critically approach problem situations, engaging in subsequent analysis and finding suitable paths to resolving the problem. This was articulated in their descriptions of mathematics, in the words they thought were closely associated with the subject and how they described it in relation to other domains of knowledge. For them the salient features of mathematics were the formulas, procedures, rules and seeming objectivity intrinsic within the subject that set it apart from others. With this view of the mathematics, seeing it as set of facts, rules and procedures to be used in the pursuit of the correct answer tend to shape how they approached the design of their instructional activities, in the tasks they engaged their students in, the quality of interaction they encouraged in the classroom and the types of evaluation methods they employed. In other words, their beliefs about teaching and learning appeared to be derived from their beliefs about the nature of mathematics (Green, 1998).

It seems fairly intuitive that if you believe that mathematics constitutes a set of facts and rules that expertise in the domain would be expert knowledge of these rules and skill sets, including how and when to apply them appropriately. As such these teachers (Mr. Henry, Ms. Reid and Mr. Brown) in attempts to develop mathematical expertise in their students would see their role as that of provider of this knowledge, whether it be by positioning themselves as the repository of this knowledge or by providing the resources for them to acquire this knowledge. These thoughts were expressed by the teachers in talking about what they envisioned a mathematic teacher to be and how they described their role and responsibilities in the classroom. For these three teachers their role in the classroom was to provide the students with the mathematical content, ensuring that they created opportunities for students to store this knowledge. This process of 'storage' often being memorization achieved through repeated practice of these procedures. They conceived of learning as applying the correct procedures in the right context while maintaining computational accuracy and that mathematical understanding came through practice. All these beliefs formed a cohesive network or structure that served to support and sustain itself (see Figure 4.2).

Although Mr. Simpson's beliefs about mathematics differed considerably from the other teachers, overall they did cluster in a similar way to the others. His more constructivist-oriented beliefs saw mathematics as a thinking and problem solving activity, where he prioritized meaning making of the problem situation and finding suitable approaches over coming to a correct answer. As such, his beliefs about teaching and learning were shaped and molded by his mathematical perspective. Holding mathematics as a constructive process, he views it not as isolated bits of facts and concepts but an interconnecting and evolving set of relationships from which individuals construct personal meaning. Mr. Simpson consistently espoused these views in conversations he had with his students and how he described orienting his students to the subject. He constantly emphasized the notion of mathematics being a process of engaging in critical thinking and that understanding came through one's own personal experiences within the domain. In light of this he saw his role as the person who needed to design activities for the students so they could engage in these learning experiences focusing on this constructive meaning-making process. He did not see this necessarily as an individual process but acknowledged the need for collaboration both in the learning process and as a source of verification and evaluation. As a guide he is able to help them embark on this process and help them successfully navigate through it. These ideas and the role that he took on in the classroom were a reflection of his conception of mathematics demonstrating again the cohesiveness and clustering nature of their beliefs. (See Figure 3)

Ms. Jones on the other hand held a view of mathematics that was manifested in different teaching roles across her classes. Ms. Jones's conceptualization of mathematics reflected elements of both constructivist perspectives and instrumentalist views. She believed that mathematics was about problem solving and learning to think critically but she also held that mathematics constituted a huge bank of knowledge that was rooted in numbers. Similar to the other teachers her views about teaching and learning were related to how she viewed mathematics as a domain of knowledge. These different perspectives were reflected in how she perceived her teaching roles and exposed her students to mathematics. Ms. Jones believed that her role as a teacher was to guide her students towards understanding and to encourage, motivate and support them. Simultaneously she also believed that it was her responsibility to have the knowledge base so she could show the students how to solve problems, identify student errors and show the students how to correct them. These differing views on mathematics pedagogy were apparently linked to the specific areas of mathematics and they seemingly did not present any internal conflict. As these beliefs are organized according to how the individual sees their connections these two beliefs may have been held simultaneously without conflict. This is possible because they are often held apart by another belief; in this case perhaps, mathematics is not a cohesive domain of knowledge (See Fig. 4). However, whilst her mathematical beliefs do not fully reflect any one perspective, overall they were more aligned with a Platonist view as she emphasized the importance of both the content and understanding.

These teachers' beliefs which all tended to lie within one of the three categories, beliefs about the nature of mathematics, mathematics teaching and mathematics learning were also organized in a derivative manner where beliefs about teaching and learning appeared to sprout from beliefs about mathematics as a domain. However, although all the teachers taught the same subject and had similar mathematical knowledge the content of their beliefs aligned with perspectives ranging from the problem-solving view to the instrumentalist view. It is the content of these beliefs that appeared to have the greatest influence on their instructional practice. The alignment between the two will be discussed in the next section.

Alignment between beliefs and practice

Some of the literature in recent years have focused on the inconsistency between the mathematical beliefs of teachers and their practice (Chapman, 2002; Ernest, 1989a; Raymond, 1997; A. Thompson, 1984). Specifically, these studies discussed the misalignment of classroom practice when teachers espouse constructivist-oriented beliefs and then their practice is to the contrary. In contrast, the five teachers presented here described beliefs that range from an instrumentalist or more traditional to a problem-solving or more constructivist view and observations of their instructional practice evidenced that there was fairly close alignment.

Earlier I established the connection between beliefs about learning and teaching, and the nature of mathematics and so given their derivative nature I will focus here on the former (holding that they are a reflection of the latter). Mr. Henry, Ms. Reid and Mr. Brown all had earlier student experiences that shape their beliefs about mathematics, its teaching and student learning in the domain. They described their role within the classroom as the person with the knowledge, who would show the students how to use the procedures, how to do the math and then provide opportunities for them to practice. All three classrooms on repeated observation matched this format exactly, with very little deviation. The students were exposed to concepts through demonstration from the teacher, with the teacher at the front of the room and the students listening and taking notes.

While knowledge construction does take place in different kinds of settings, these students were not usually provided with opportunities to make sense of the material independently; whatever meaning they seemed to ascribe to the concepts were the meanings the teachers provided. Emphasis was placed on knowing the procedures and the appropriate formulas and student explanations tend to be limited to when and how to use them. After being introduced to the concept, 'understanding' was perceived to be attained and reinforced through practice of textbook-like problems. This understanding was evaluated primarily with respect to how well the student was able to apply the appropriate skill sets and also heavily on the correctness of the final answer. There was little student-student discourse of a conceptual nature with these conversations consisting mainly of off-task behavior and answer checks. A majority of the teacher-student talk was conducted in an IRE (initiate-respond-evaluate) manner where the teacher would initiate the questions, the students would respond and then an evaluation was done of this response. In situations where the teachers were required to engage in reform-oriented practices, namely facilitating student discourse, they still defaulted into the IRE pattern although they had been taught different techniques. These teachers' actions resembled very closely behavior that is indicative of more traditional beliefs or one holding an instrumentalist view of mathematics. In line with this perspective these teachers seemed to place themselves in a position of authority over the content taking on the role of deliverer of knowledge and not necessarily that of a participant along with their students in the knowledge construction process. This teacher role was seemingly reflective of the procedural and skill-oriented conceptualization of mathematics that these teachers appeared to hold.

Similarly Mr. Simpson's classroom practice was for the most part quite reflective of his view of mathematics teaching and student learning. Being from a family of mathematicians, he also had early influences that seemed to help shape these notions. Standing back and allowing his students to develop their own personal meaning and providing opportunities for students to tell him what they knew were the major principles that governed how Mr. Simpson organized his classroom. In addition to Mr. Simpson, Ms. Jones also believed that students' individual thoughts and personal meaning-making were important components of developing deeper conceptual understandings. In this regard they both emphasized the importance of student ownership over their own ideas. As such they tried to position themselves more as guides for their students allowing the students to develop their own individual understanding of the content, a feature they considered vital to student learning.

I would, however, highlight that evidence of this in Ms. Jones's practice was fairly inconsistent. With her younger ninth grade students, while she didn't necessarily provide the same freedom to explore as Mr. Simpson did, she did use different forms of questioning techniques with her students that often forced them to think beyond her own instruction. Ms.

Jones, similar to the three teachers discussed earlier did routinely assign rather straightforward, procedural questions. However, she had a combined focus on getting the correct answer and on understanding. It was equally important for students to get the right answer and for them to understand what the solution meant in the context of the problem. Of importance however was the disparity in the organization and structure of Ms. Jones's classroom activities in her different classes which seemed to be dependent on the type of students and the subject. She did allude to this in a statement she made in the interview where she discussed the differences between teaching Algebra 1 and geometry. She made a distinction between teaching geometry where she taught students how to think, as opposed to algebra where you could show them an algorithm and have them follow it. Observations were made of a higher level class and there were clear differences in the way she structured both classes and interacted with the students. Ms. Jones seemingly had conflicting beliefs about mathematics in general, student learning and mathematics teaching. Her conception of mathematics and the way that it is best taught seemed to be influenced by the type of students and the particular subject area of mathematics she was teaching. As mentioned earlier, individuals may hold beliefs that are contradictory because they are not perceived by the individual to be contrasting views. They tend to remain intact because of another belief (Green, 1971). In the case of Ms. Jones this belief may have be domain-related (believing that the conceptual nature of algebra and geometry are fundamentally different) or student related (believing that underachieving students learnt best through direct instruction). Additional questioning and observation would be needed to identify the specific belief. With regard to student learning, researchers have observed that these beliefs can differ with regard to specific groups of students (Fuchs, Fuchs, Hamlett, & Karns, 1998; Torff, 2005; Torff & Warburton, 2005). For example, teachers often believe that tasks that are considered to be at a high cognitive level should be assigned to high achieving students as students who are deemed to have low cognitive or critical thinking skills are unable to adequately address or successfully complete these tasks (Torff, 2005). This may have been a factor in the pedagogical decisions Ms. Jones made about her different classes of students.

Conclusions

In this study, beliefs appeared to play a significant role in the pedagogical decisions the teachers made and the way they structured their classrooms. From this analysis there was greater alignment than misalignment between these beliefs and practice, which indicates that for these teachers beliefs served as a fairly reliable predictor of the type of instruction that took place in the classroom. It must be made clear that although the majority of these teachers did not express beliefs that are reflective of mathematics teaching and learning proposed in Principles and Standards for School Mathematics (NCTM, 2000) they were all diligent, hardworking and dedicated teachers. These teachers did express extreme affection for their students and were also quite passionate about the subject and their students' success. They in effect were using the resources available to them to ensure their students' success. While I argue that some of the beliefs they held did limit the availability and use of their pedagogical resources, I do not question their dedication to trying to maximize their students' success.

This collective case study of these five ninth grade Algebra 1 teachers provides useful insights about the larger population of algebra teachers. These insights are particularly useful as teachers of Algebra 1 often serve as the gatekeepers for students progressing to higher level mathematics. As such, they provide for the students an integral and significant portion of the conceptual foundation, and an introduction into the level and type of thinking that is required for more advanced courses. This study confirms the notion that teachers' beliefs about mathematics greatly influence the way teachers organize and structure their lessons but also provides new insights that have implications for teacher education and the mathematics education community. It presents teachers' mathematical beliefs as a cohesive structure where the beliefs are connected in a derivative order with pedagogical and student learning beliefs stemming from beliefs about the nature of mathematics. However, although the types of beliefs and the general structure appeared to be consistent across teachers the content of beliefs tended to differ among the teachers. This has important implications for both teacher education programs and professional development. It is generally agreed within the mathematical community that constructivist-oriented beliefs (aligned with Ernest's (1989) problem solving view) are more desirable and conducive to the development of problem solving, learner-centered classroom environments (Mewborn & Cross, 2007). Therefore, teachers who do not hold beliefs that are aligned with this view would perhaps need to be engaged in programs that would help them first become more aware of these beliefs and then proceed to modify or change these beliefs. Holding that these beliefs form a cohesive unit where teaching and learning beliefs are derived from teacher's conceptions of mathematics, it suggests that if the beliefs about the nature of mathematics change then those derivative beliefs would also begin to be reconstructed. In this regard, the belief change process may have greater success if it were organized around re-conceptualizing teachers' views about the nature of mathematics. In so doing it would also perhaps be useful if prospective teachers' beliefs were targeted primarily in the mathematics content courses in efforts to mold them within a more constructivist frame.

Along similar lines in efforts to incorporate reform-oriented practices with in-service teachers an approach that focuses primarily on the views of mathematics that the initiative wants students to have may be more prudent. Then aligning these views with the actual practices they want teachers to incorporate in the classroom would possibly make it clearer to teachers how their own conceptions of mathematics differ with those of the initiatives. From the results, namely Mr. Simpson and Ms. Jones we have seen how reform-oriented practices can be derived from opposing beliefs. In the case of Mr. Simpson because his instructional techniques were derived from more constructivist beliefs they were enacted quite consistently. In contrast Ms. Jones's classroom practices were fairly inconsistent as they were aligned with views that were not solely reflective of constructivist principles. In this regard, organizing professional development with emphases on the mathematical ideas may help reduce the incidence of teachers filtering these new teaching practices into their old molds of instruction. However, as a caveat to this notion, beliefs also tend to be organized in clusters and the findings of this research presented a model of how the mathematical beliefs of these five teachers may be structured, but there are undoubtedly other clusters of beliefs formed through their personal experiences that may also impact their daily classroom decisions and practices.

Another important finding of this research was that while beliefs have enormous influence on teaching practice that there were other factors that influenced how these teachers envisioned and enacted their roles within the classroom. All five teachers spoke extensively about the constraints that existed which prevented them from having their ideal classroom. Mr. Simpson was fairly consistent in maintaining the classroom environment he envisioned for his students but admitted that he frequently fell behind in the prescribed curriculum and had to resort to more traditional methods to cover the content. Additionally, the teachers, primarily the ones who were early in their teaching careers were very concerned about classroom management and commented that they may lose control of the class if they engaged in more 'alternative' methods of teaching. Preserving their teacher identity, how they perceive themselves and also how they wish others to perceive them in their teacher role, is a factor that also influences how teachers interact with their students and structure their classrooms (Williams et. al., in press). Initiatives and mandates that serve to threaten this teacher identity may cause anxiety and may also be rejected consciously or unconsciously by teachers. As teacher educators and researchers we need to ask ourselves how teachers perceive change and whether or not they find it important. It is clear that for these teachers in addition to their beliefs being a constraint to them adopting more constructivist methods there were also institutional factors that served as deterrents.

It is apparent that there is not a clear linear relationship between beliefs and practice and that other factors do influence how teachers perceive and enact their roles in the classroom. These factors include internal psychological constructs such as self-concept, teacher identity and teacher efficacy (Gregoire, 2003; Schutz, Cross, Hong, & Osbon, 2007; Williams et al., in press; Woolfolk-Hoy et al., 2006) and also external factors such as school and department culture, curriculum mandates, class sizes etc (Hallam & Ireson, 2005; Hart, 2002). Additional research needs to be done in this area to detail and describe these perceived constraints and the extent to which they directly impact the instructional decisions teachers make.

Lastly, the results also have implications for belief change efforts within mathematics teacher education. Much has been written about the process of belief change within the field with the presumption that if teachers held constructivist beliefs this would lead to more effective instruction resulting in increased mathematical understanding and achievement. Although the belief change process is considered extremely difficult, it is considered vital towards our efforts of eliminating mathematics underachievement. The findings from this research provide a glimpse into the previous success of these efforts for pre-service teachers and the level of difficulty of potential efforts for in-service teachers. Three of the five participants had completed a mathematics teacher education program but only one had fully embraced reform-oriented principles and practices demonstrating that there is still much room for improvement in this regard. Additionally, these five participants across only two schools produced three different models of beliefs; the number of different models most likely will increase as the pool of teachers gets larger from a mathematics department to an entire school district. Considering the diversity of the potential models it begs the question of whether efforts towards belief change that assumes that one approach may work for all teachers are prudent. In this regard, the study of this phenomenon needs to be done extensively with both our prospective and practicing teachers. As such developing a better understanding of the role of belief structure in the support and maintenance of beliefs and how the organization of beliefs impacts teacher behavior will serve as an aid in making the process of belief change more successful.

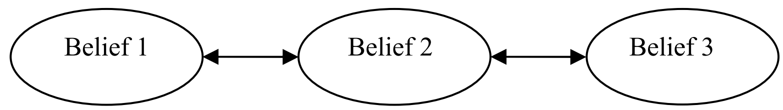


Figure 4.1: Green's Quasi-logical Structure of Beliefs

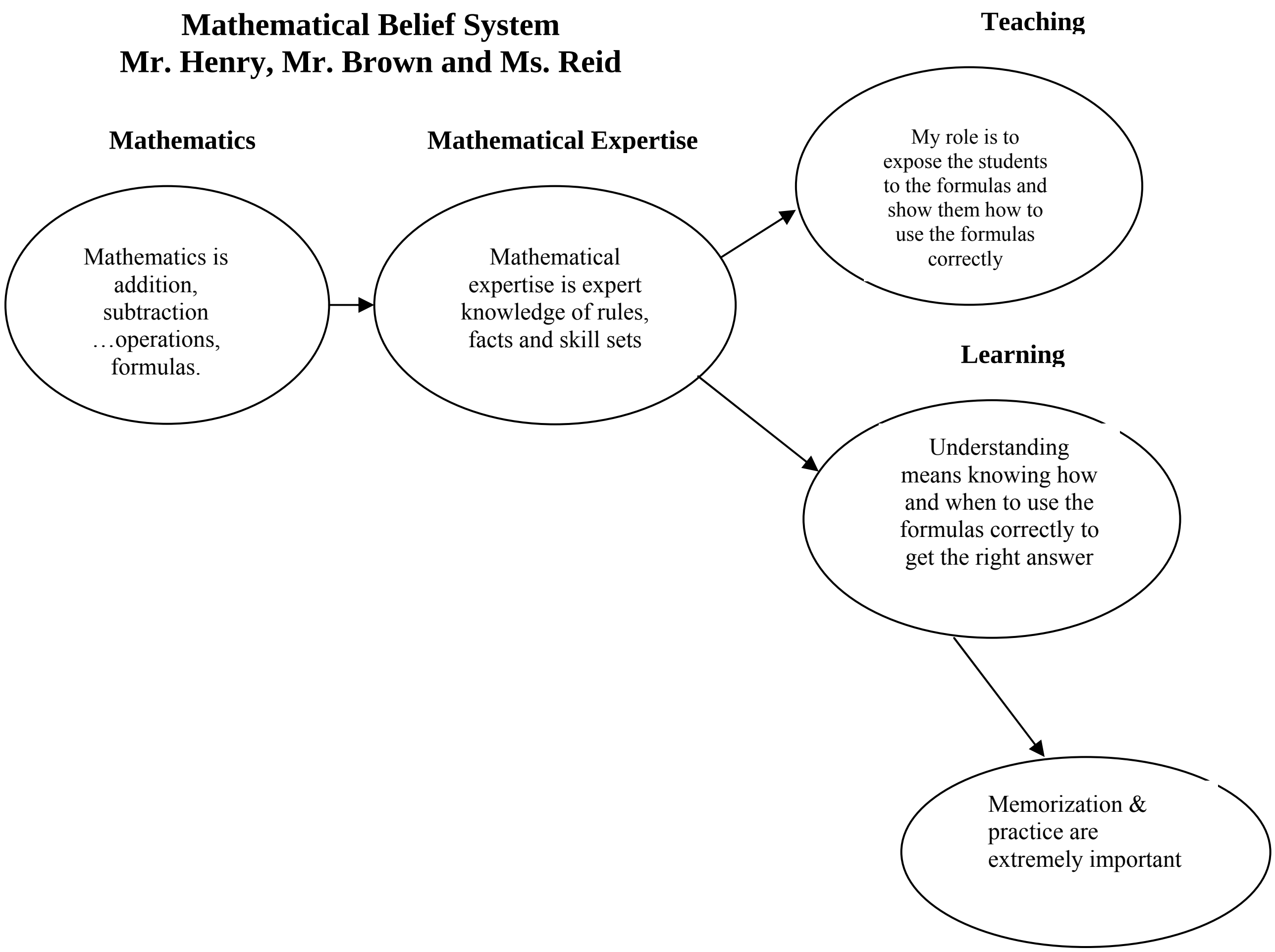


Figure 4.2: Mathematical Belief System of Mr. Henry, Mr. Brown and Ms. Reid

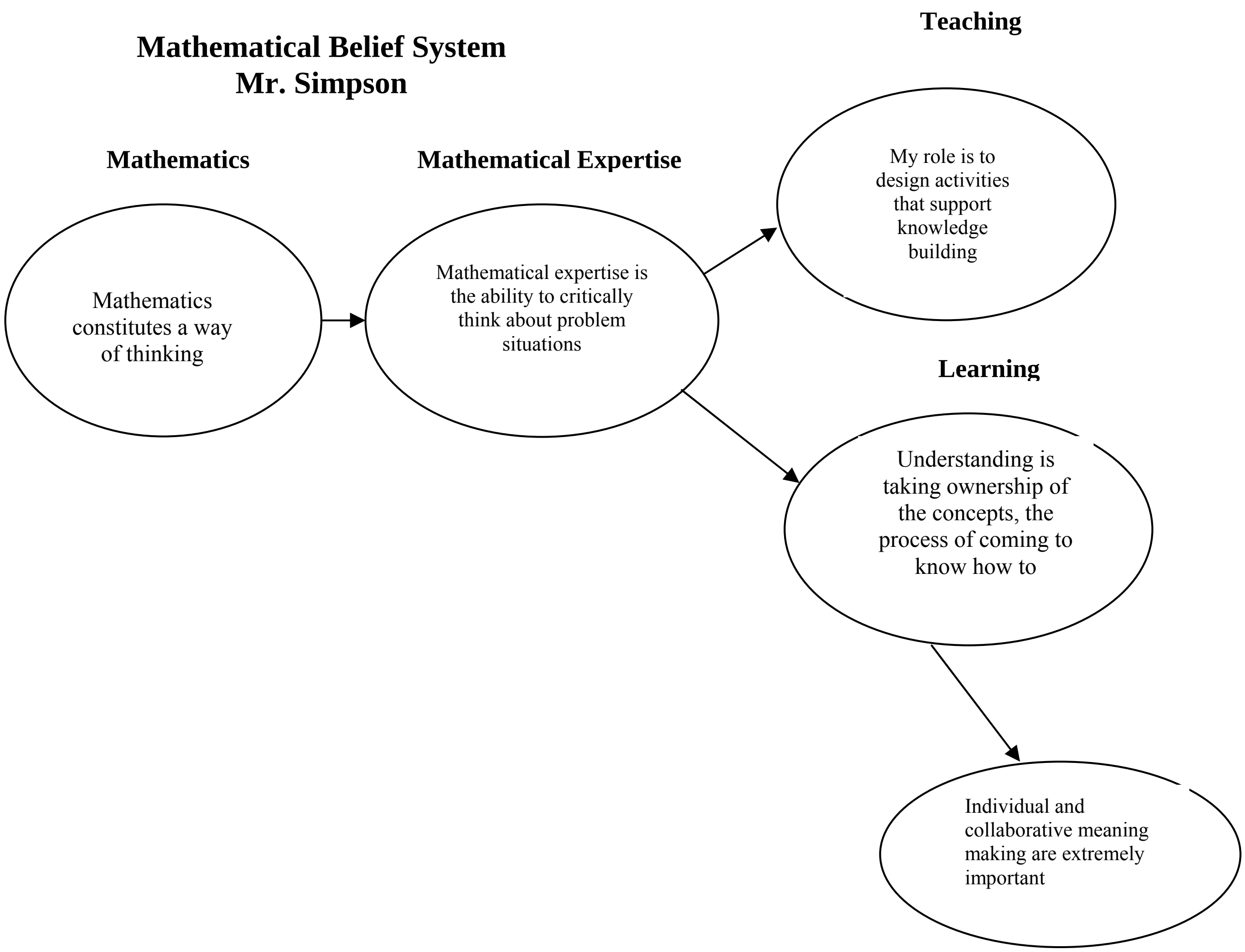


Figure 4.3: Mathematical Belief System of Mr. Simpson

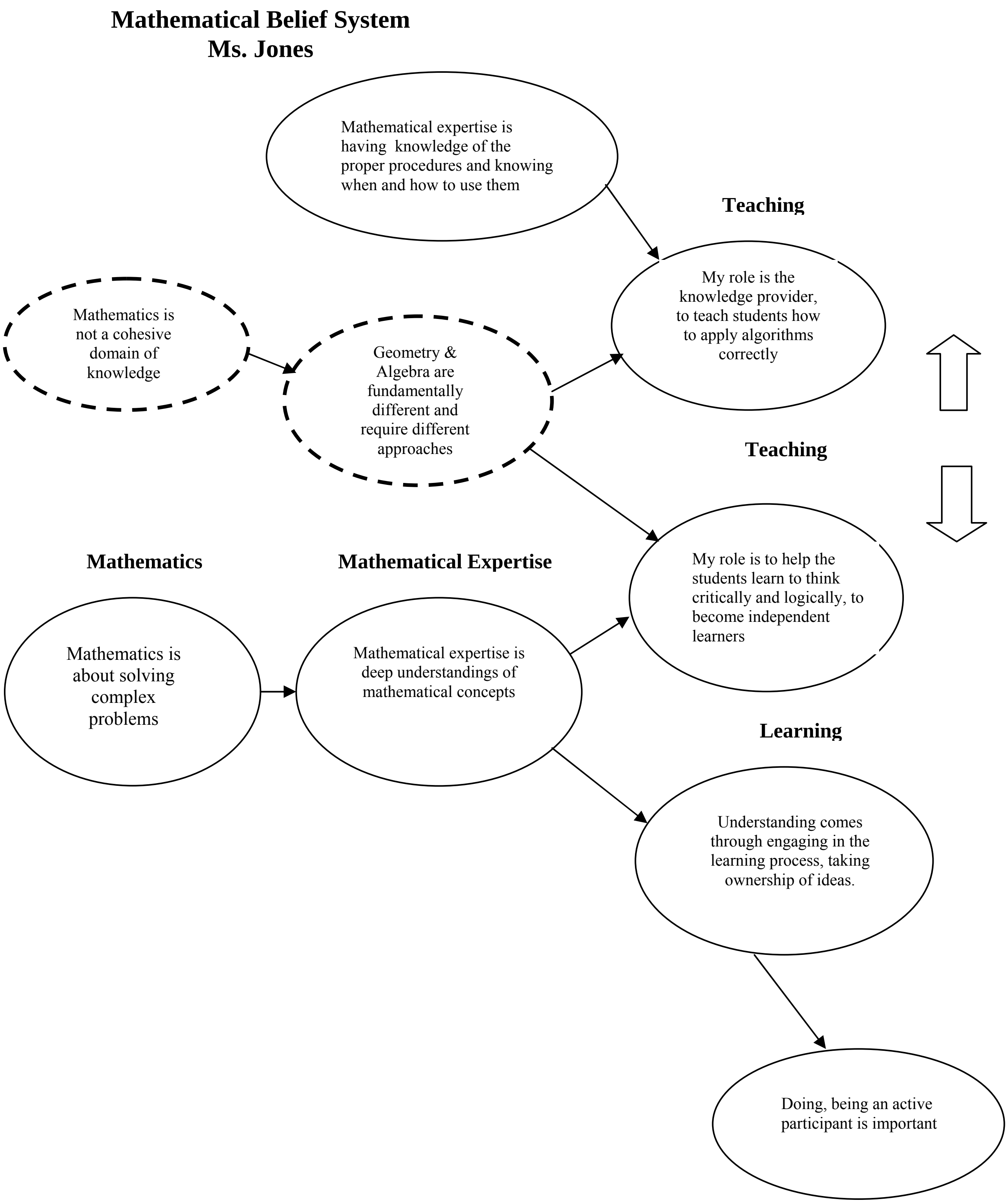


Figure 4.4: Mathematical Belief System of Ms.Turner

CHAPTER 5

BELIEF CHANGE: A DUAL APPROACH[9]

[9] Cross, D. I. (To be submitted to International Journal of Science and Mathematics Education)

Abstract

Beliefs are considered an important factor in determining the pedagogical decisions teachers make, the way they frame tasks and how they interact with their students. These personal dispositions are often quite resilient in the face of reform and so tend to interfere with efforts to change teachers' practices. In light of recent reform efforts in mathematics education the issue of belief change has become increasingly important as there is a need for teachers to transform their current practices to those that reflect the objectives of the initiative. However, these efforts in the past have employed mainly cognitive-oriented strategies with limited success. This article attempts to address these limitations by suggesting a complementary approach acknowledging both the individual and social aspects of belief formation and enactment. In approaching belief change from this perspective we will hopefully address some of major hindrances to success in previous efforts and begin to employ strategies that may lead to long term belief modification.

Introduction

Research concerning beliefs with respect to teacher education has become increasingly popular since the advent of cognitive psychology in the late 70s (Richardson, 2003; Woolfolk-Hoy et al., 2006). Along with other cognitive constructs, including emotions and motivation, the study of beliefs has provided insight into the mental lives of those more intimately involved in the educational process, teachers and students. This interest in the inner workings of the minds of teachers provides a window into their thought processes as they plan, teach, interact with students, reflect, and evaluate their daily teaching experiences. The insight this type of research provides has become more important over the last decade because of the observed intimate relationship between teachers' beliefs about teaching and student learning and their approaches to instruction (Nespor, 1987; Pajares, 1992; Woolfolk-Hoy et al., 2006). Beliefs are thought to be quite influential with respect to the decisions people make on a daily basis. Specifically within the lives of teachers, they tend to directly impact decisions about teachers' instructional practices. These beliefs are considered to be quite pervasive and very resistant to change. Consequently, this connection between beliefs and practice is often considered a hindrance to education reform initiatives that seek to change classroom teaching from a transmission model to a more constructivist model because without a change in beliefs, there is rarely any change in classroom instruction (Richardson, 2003).

Working from the assumption that beliefs are an important factor in understanding and predicting human action and behavior, programs that seek to prepare teachers for the profession have sought to engender in their students beliefs that are considered conducive to academic success. This is often a difficult process, as it requires engaging the student teachers in the process of belief change. For in-service teachers, professional development seminars and courses tend to be the forum that would serve this group of teachers in a similar manner, as a means of access to the thought processes and the inner workings of the minds of those already in the profession. However, this is rarely the primary objective, rather they tend to focus on promoting and orienting teachers to new educational innovations. With this realization researchers are now investigating how these forums can be used to directly and indirectly address issues of teachers' beliefs and how to better align them with the goals of reform initiatives (Gates, 2006; Hart, 2002; Lloyd, 2002). These efforts reflect the notion that the practice of teaching is in itself an ongoing learning process and that teacher education programs are only one avenue to address this issue of misalignment between teachers' beliefs and reform initiatives.

In targeting beliefs and ultimately belief change as a part of the teacher education and teacher development process, it is presumed that the quality of teaching and learning will improve (Azjen, 1991; Chapman, 2002; Ernest, 1989b). This hope, however, seems to be somewhat unstable as studies that would serve as confirmation of this notion, the idea that teachers who hold reform-oriented beliefs are better able to develop and maintain effective learning environments, are not conclusive (Wilson & Cooney, 2002). From the literature, studies that investigate the relationship between mathematics teachers' beliefs and their instructional practice report mixed results; some state that beliefs and practice are often misaligned and so teachers who profess reform-oriented beliefs tend not to actualize them in practice (Handal, 2003; Raymond, 1997), while others state that there is congruence between beliefs and practice (Peterson, Fennema, Carpenter, & Loef, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001). As researchers within the educational community we are beginning to gain greater insight into the true nature, origins and structure of teachers' belief systems, how and if these beliefs translate into effective modes of practice and if so, how to better create programs to cultivate these beliefs.

This article attempts to examine the issue of teacher belief change from two perspectives - beliefs as an individual, cognitive construct and beliefs as a collective phenomenon (Goddard, Hoy, & Woolfolk-Hoy, 2004; Llinares, 2002). While the paper broadly addresses the issue of belief change, specific focus will be placed on the phenomenon within the mathematics education community. The study of beliefs within mathematics education has been primarily from the perspective that a belief is a cognitive construct that resides within the individual mind and so research within this domain seeks to define and investigate the nature and workings of this construct as a study of the individual within a larger community (see A. Thompson, 1992 for a review). However, with increasing recognition of the social influences on learning, some researchers within the field have embraced the notion that although beliefs are deeply personal they are also a product of the culture and norms held by the larger community (Llinares, 2002).

To examine these issues this article will provide a brief overview of beliefs, including a definition that attempts to capture its individual and social nature, its origins and how these factors influence the belief structure. Included in this section is also a discussion of the theoretical framework underlying the cognitive and situated nature of beliefs and the way that beliefs are organized. From this discussion, the groundwork will be laid to begin to talk about belief change, the need for belief change, approaches taken toward belief change and the degrees of success in engaging teachers in this process. Here I make a connection between the successes and failures in the belief change process as a function of the underlying theoretical framework of the approach used. I conclude by discussing the lessons learned from previous belief change efforts and make suggestions for future research and initiatives in this area.

Beliefs – a construct with duality

Much of the literature on beliefs highlights the difficulty in finding an appropriate definition for the construct (Hofer & Pintrich, 1997; Pajares, 1992). This difficulty partially results from the term being used in many domains, including theology, philosophy and educational research. This situation becomes more problematic when the study of beliefs is coupled with other cognitive constructs (such as motivation and emotions), and also in trying to make the distinction between knowledge and beliefs (Murphy & Mason, 2006; Woolfolk-Hoy et al., 2006). As a result, many educational researchers do not make a clear distinction between the two, and so either use both terms interchangeably or refer specifically to knowledge *or* beliefs (Murphy & Mason, 2006). Both constructs have been studied from both a philosophical and psychological perspective and attempts have been made to distinguish them with regard to their focus, and the need for and criteria for justification. I seek to avoid this confluence of the terms and so will define beliefs as embodied conscious and unconscious ideas and thoughts about oneself, the world and one's position in it developed through membership in various social groups, and considered by the individual to be true. To situate the remaining discussion two main features will be highlighted - beliefs as socially constructed and individually enacted.

Beliefs as socially constructed

Socio-cultural perspectives see individuals as constitutive of social and cultural practices and not simply influenced or affected by them. While individuals have physically developed and independent bodies and brains, what constitutes the 'person' are dispositions acquired through their interactions within different social contexts and are not merely natural endowments (Martin, 2006; Op'Teynde, DeCorte, & Verschaffel, 2002). The knowledge and beliefs that people hold are constructions born out of particular historical traditions and are reflective of the internalizations of the social and cultural practices of the community with which they identify (Gates, 2006). In essence, we can say that humans are socially constituted beings whose development is a process involving life events and social interactions through which they adopt the artifacts and practices of a particular culture or community (Martin, 2006). Essentially, individuals are the products of their pasts and the influences of the different societal norms that they have been a part of, out of which they have developed particular ways of being and thinking.

Researchers from various fields of study have examined this phenomenon, specifically the social foundations of individual learning and the development of beliefs (Llinares, 2002; Walsh & Charalambides, 2001). I focus on two main perspectives as they provide a useful framework from which to discuss beliefs as a socially constructed phenomenon: the social psychological theory of habitus and situated learning perspectives incorporating communities of practice. One perspective grounded in the social psychology literature describes these enduring dispositions as habitus, which is developed through our early socializations and familial interactions (Bourdieu & Wacquant, 1992). Because of the often confining nature of these early interactions and the lack of outsider influences, there are limitations on its development and so habitus generally embodies only the past and current conditions of that particular society. Our later experiences within the educational context are influenced by these dispositions, and so we find that individuals tend to behave and think in ways that are reflective of their own early experiences layered with the effects of schooling. Habitus is considered to be quite pervasive as it manifests itself in almost every area of our lives; from the type and use of language to the way we situate ourselves within our communities and the wider society. It shows itself at various levels (at the inter- and intrapersonal, individual or social level) and should not be considered a distinct, observable phenomenon but rather an interdependent and interconnecting network of dispositions reflective of the relationship between the individual and his/her immediate social context (Gates, 2006).

Through social engagements these deep-seated dispositions are concretized into what is referred to as ideologies; these are more hidden systems of thoughts and ideas, which tend to manifest themselves in social activity. They can be seen as more structured and organized components of our habitus through which we ascribe meaning, construct our reality, and position ourselves in the world. These ideologies become visible and are often evident through the particular discourse in which individuals are engaged. In professional communities such as teaching, these discourses are shaped by the commonality of the experiences - the language and the conceptualizations that the individual members of the community hold.

In our early years our predominant social groups are situated in the home and at school, however, as we age we become members of many communities. Not all these groups are professional or well organized, and the degree or level of participation differs from group to group. Within these communities we collectively engage in the pursuit of different ventures. For example, within the mathematics education community we make efforts to increase our knowledge and understanding of how students learn and the features of the environments that supports this kind of learning. Through the social relations and interactions generated and developed through these pursuits certain common practices emerge. These practices become representative of the particular social group or community, referred to as a ‘community of practice’ (Wenger, 1998). These communities are defined by and are reflective of the joint enterprise that is continually negotiated by its members, the mutual engagement that preserves its social identity, and the collection of resources the members have developed over time. They embody more than just a group of people who converge around a shared set of knowledge and skills but they are communities that have common interests, shared ideas and commitments. In addition, they engage in particular kinds of practice, discourse, ways of thinking and doing, and develop resources that embody the accumulated knowledge of the specific community (Greeno, Collins, & Resnick, 1996).

How we learn within these communities is akin to the notion of habitus, the way humans learn and develop particular ideologies. This process is referred to as enculturation described within situated learning perspectives. With immersion into a particular social context and through observation and participation individuals tend to adopt the behavior, language and evident norms of that particular community. This assimilation of the practices and norms of the community tends to happen with relative ease and without explicit teaching. Enculturation describes this process through which individuals consciously and unconsciously adopt the behavior and belief systems of new social groups (Brown, Collins, & Duguid, 1989).

Both the concept of 'communities of practice' and 'enculturation' are nested within a broader theoretical approach to learning known as situated perspectives, which highlights the social influences on human learning and development. These theories suggest that knowledge is contextually situated, influenced by the context, activity and the culture in which it is being used (Brown et al., 1989). Central to this perspective are two assumptions; one is that the social and physical contexts surrounding an activity are a part of the activity and that the activity is integral to the learning that can be achieved through engagement in that activity. These constructs are not separate entities but form an intricate network embodying a dynamic, interdependent relationship integrating the context, the activity and in this case the teacher self. This theory of learning becomes relevant in this context because of the nature of beliefs and how they are generated. Beliefs are thought to be constitutive of information derived from the individual's own experiences which they regard as true, and so beliefs are in a sense personalized forms of knowledge. The development of beliefs can be thought of as a process of acquiring information through our interactions with the environment and others; so in a sense it can be described as learning, where learning is viewed as a process of enculturation into a particular community (Brown et al., 1989; Greeno et al., 1996). As such examining ideas about thinking and learning from a situated perspective provides new conceptual and analytical constructs with which to study the role of teachers' beliefs and belief change (Ambrose, 2004; Goos, 2005; Llinares, 2002; Walsh & Charalambides, 2001).

Beliefs in this regard can be considered a product of the activity and situational context in which they are developed. They are not static but will evolve with each new experience as our interactions and negotiations within the context will undoubtedly reshape and restructure it into a new, more intricate form. Therefore, beliefs can be viewed as always under construction because an individual's experiences have the power to either reinforce or modify his existing beliefs; in the later case they can become the impetus for the process of change. Change from this perspective is also considered a product of learning and development and so some researchers who address belief change from this viewpoint describe or refer to this process as teacher change, teacher learning or teacher development.

Within the context of teacher education, student teachers would acquire the beliefs of the of the community as they become increasingly adept participants in the social and cultural practices of that community through 'legitimate peripheral participation' (Lave & Wenger, 1991). Specifically, student teachers in their teacher education programs would progressively move from novice or peripheral positions to more central or expert forms of participation in the specific community. This is usually specific to the particular community of practice; for example math teachers becoming more expert participants in the mathematics education community. This requires that students be initiated and guided through these practices by being involved in authentic tasks and developing new understandings that are integral to the identity of the community. Because these activities and constructs do not exist in isolation the meanings that a student teacher derives from engaging in the teacher learning process is related to the focus and the portrayal of these constructs (in this case beliefs) in the learning process.

Beliefs as individually enacted

The belief concept has been studied across many domains such as philosophy, psychology, broadly defined teacher education and domain-specific teacher education, to name a few. Within most of these fields, particularly mainstream psychology, beliefs are considered to be a mental representation, an integral component of an individual's thoughts. Along with other constructs (such as motivation, goals and emotions) beliefs describe the organization and content of an individual's thoughts and are presumed to be precursors for the individual's actions (Bryan & Atwater, 2002). As such they are considered one of the best predictors of human behavior (Pajares, 1992; Rokeach, 1968).

These dispositions and views about the world are developed over time through our interactions with the world and others. Individuals, as they grow become members of different social contexts, the home, school, church and the wider society to name a few (Op'Teynde et al., 2002). Our early socializations, upbringing, experiences and interactions with others form an intricate network of influences that shape our own 'unique' way in which we see the world. From these signals and messages that we constantly receive through our interactions with the world we draw conclusions based on our interpretations of these messages, which develop into our own individual belief systems (Furinghetti & Pehkonen, 2002; McLeod, 1992). These experiences begin very early in our lives and we orient ourselves to the world by accepting and believing that almost all sensory information we receive, for example what we see and hear, is true. These beliefs in a sense belong to the individual as they become the only referents for its truthfulness, making these beliefs quite personal. This uncritical acceptance tends to go unchallenged until the individual is presented with new situations that contradict what he/she previously believed and held as true. Because it is the holder of the belief that ascribes the truth value, change or modification of a belief is solely dependent on how the individual interprets this new information. These mental representations provide a framework from which we view and interpret the world. They influence our perceptions and judgments and serve as the basis for our actions. Because of their nature and organization they are considered to be quite personal, stable and often quite tacit, often existing below the individual's level of awareness or immediate control.

Their resolute nature can be somewhat attributed to a particular feature of beliefs, their organization as belief systems. I examine this feature of beliefs in an attempt to provide insight into their stable and resistant nature, which will serve as a framework for the discussion on difficulties with current belief change models.

Belief Systems. Green (1971), building on the work of Rokeach (1968) provides a well-developed and detailed outline of the organization of belief systems. He used this model to present a unique perspective on the role of teaching and its relation to beliefs. The concept of belief system is a metaphor used to describe the organization of individual beliefs (Green, 1971). Green (1971) distinguished between two kinds of beliefs - beliefs about the world and beliefs about beliefs. Beliefs about the world are established as conclusions logically deduced from factual premises. A belief about a belief is the particular attitude an individual takes toward a belief, which he states has the potential to become a belief in itself, often referred to as values. This distinction is classified as the difference between what we believe, the content of the belief and how we believe it, the psychic strength with which we hold the belief.

Quasi-logical structure. This first dimension of the belief system purports that beliefs cannot be held in total isolation or independence of other beliefs. They are connected in a similar way to that of premises and conclusions. This systematic structure of beliefs is organized within a framework of logic (or rather quasi-logic), consisting of basic or *primary* beliefs and their *derivatives* (see figure 4.1). They are considered quasi-logical because there is no stable or fixed order in which these beliefs are connected from primary to derivative. Logical order would be based on the content of the belief but this structure is associated with *how* these beliefs are held. For example, a teacher enters the profession with very successful student experiences believing that 'all students can learn; they just learn differently' (Belief 1). Green (1971) would regard this as a basic or primary belief as it is not derived from any other belief. Additionally, the same teacher also holds that she needs to provide for her students multiple opportunities to learn in a variety of formats (Belief 2). This belief would be regarded as a derivative belief as it followed from her primary belief, mentioned earlier. This teacher supplements all her lessons with visual aids, provides hands-on activities and opportunities for students to engage in discourse because she believes that lecturing is a limiting instructional strategy (Belief 3). This order of beliefs can be seen as a pattern of questions and answers, Belief 2 is the reason for Belief 3 and Belief 1 the reason for Belief 2 (see Figure 4.1). Green (1971) also refers to some beliefs being so basic that they are not derived from other beliefs. I would argue that often these 'basic' beliefs are derived from a combination of personal experience and ideologies formed from social and cultural influences. Research (Borko & Putnam, 1997; Rimm-Kaufman & Sawyer, 2004) has informed us that teachers often enter the profession with beliefs about knowledge, teaching and student learning from their own educational experiences. According to Green's description, these basic beliefs would spawn derivative beliefs all emerging from that same basic premise.

Psychological Strength. The ease with which a belief is modified or changed is dependent on its psychological centrality, meaning the strength with which these beliefs are held. The degree of psychological strength of a belief is described as its spatial location and similar to its status, this feature is also related to *how* beliefs are held and not the content of the belief. Those beliefs that are held with great psychological strength are called central or *core* beliefs and the others are referred to as *peripheral* beliefs. This feature of belief systems is considered separate from its status (primary or derivative) and the system is so structured that these two features can vary independently of one another. In sum, because of these two mutually exclusive characteristics we find that belief systems have the potential to be quite illogical in organization and that individuals can hold two incompatible, inconsistent beliefs without internal conflict, given that they are never required to examine them concurrently.

Belief Clusters. There is a third dimension to this system; the way individuals cluster their beliefs. This grouping process provides protection and support for their incompatibility and inconsistencies. Because of the 'protective shield' that we provide these clusters it is possible to hold conflicting core beliefs. This segregation of beliefs is often upheld by a belief itself. For example, let's say that a teacher believes that 'schools should be an environment where students are provided with all opportunities to excel' and she also holds the belief that 'students who are not in the gifted classes should not be recommended for advanced math courses.' The teacher holds these two seemingly incompatible beliefs and for her there is no apparent contradiction. These two beliefs are held at bay by another belief - 'ability is fixed'. Believing that ability is fixed she perhaps thinks that if they have not demonstrated their aptitude for mathematics by being in the gifted class, then they will not be able to handle the more difficult advanced classes.

Beliefs systems, despite the seeming rigidity of their three dimensional organization, are not fixed and are susceptible to change. This idea is paramount as the changeable nature of this system is advantageous, lending itself to continuous improvement. This process is likened to that of education, as learning is essentially the process of formulating and modifying our beliefs; therefore, if these systems were not transformable then the entire education process would be futile. All these characteristics and their propensity for movement and fluctuation have implications for learning, specifically in the way individuals absorb and assimilate new information and relate new experiences to familiar ones. In essence, teaching should be aimed at formulating belief systems that minimize the quantity of core beliefs and belief clusters and maximizes the relationships between them. In minimizing the quantity of core beliefs we can reduce the number of beliefs that are highly resistant to change allowing for modification when faced with contradictory facts.

Evidential and non-evidential beliefs. Another feature of beliefs is in the evidential and non-evidential ways in which some beliefs are held, the difference being whether there is evidence in support of the belief or if it is held on the basis of ideology. Non-evidential beliefs are difficult to change or modify as they are *not* held on the basis of rational argument; therefore, being confronted with evidence to the contrary would perhaps do little in bringing about a belief change. This distinction between evidential and non-evidential beliefs is useful in the way we look at social ideologies developed through immersion in a particular culture or community. While there may be evidentiary reasons behind a community's way of thinking and doing, often new members are initiated into the cultural practices without acknowledgement of these underlying reasons. Additionally, over time these practices may have diminished in their useful but are still upheld because they have not been sufficiently challenged. Given the dimensional organization of beliefs and the way in which beliefs are held, Green (1971) recommended that individuals minimize their of core beliefs because they are so strongly and passionately held, they are highly resistant to change. Essentially, the only beliefs that should be held so intensely are those that continually allow us to ask questions and leave other beliefs open to examination and inquiry - beliefs that have a 'due regard for truth.'

Beliefs and Belief Change

Teachers' Beliefs

Studies in the field of educational psychology and teacher education have reported on the influence of teachers' beliefs on their pedagogical decisions and classroom practice (Clarke & Peterson, 1986; Cobb et al., 1991; Lumpe et al., 2000; Pajares, 1992; Torff, 2005). They agree that beliefs are extremely influential with regard to teachers' instructional decisions but also contend that there are other factors that may ultimately contribute to any individual's action (Lumpe et al., 2000; Pajares, 1992; Raymond, 1997; Skott, 2001). This influence beliefs have on teachers' instructional practice is linked to how beliefs originate and develop through an individual's own experiences over the course of a lifetime. Richardson (2003) stated that there are three sources out of which beliefs about knowledge and teaching are developed. They are personal experience, experiences in schooling and other forms of instruction, and experience with formal knowledge including academic subjects and pedagogical knowledge. In this regard, the study of beliefs becomes quite important in education and specifically in relation to teachers, as unlike other professions, novices within the teaching profession enter with experiences from their student careers (Pajares, 1992; Richardson, 1996). They can be considered 'insiders' as they already have deep and intimate knowledge about school and the education process and tend to view their new experiences through their 'old eyes.' These beliefs are so pervasive that they seem to remain intact despite the aim of teacher education programs to transform these often tacit and unexamined beliefs into objectively reasonable and evidentiary ones (Ball, 1988; Feiman-Nemser, 1983; Fenstermacher, 1979). Specifically, studies of teachers' beliefs about critical thinking activities and the pedagogical decision-making process concluded that these beliefs were quite stable and were unaffected by educational attainment or teaching experience (Torff & Warburton, 2005).

Pre-service teachers often enter their teacher education programs and subsequently the profession with beliefs about knowledge, teaching and student learning from their own educational experiences. Because they are deeply held and often implicit beliefs also often serve as filters through which information on education reform and curriculum mandates are sieved (Yerrick et. al, 1997). As a result implementing reform of any sort is difficult, as teachers tend to assimilate portions of the reform visions in ways that support their own beliefs (Hollon & Anderson, 1987). As such, these pre-service teachers often leave their teacher education programs and begin practice with the same beliefs that they accumulated throughout their lives as students (Rimm-Kaufman & Sawyer, 2004). Similarly, in-service teachers often return to their classrooms following engagement in professional development efforts having assimilated the new information into their already established ways of teaching. Therefore, reform initiatives that are dependent on teacher involvement for successful implementation often fail to meet their objectives, suggesting that for reform initiatives to be successful they should first consider addressing the issue of teacher belief change.

Teacher Belief Change

With the constant implementation of reform efforts teachers are often faced with the task of changing their current teaching practices and beliefs to align them with the goals of the new initiative. This process can be laborious and is often unsuccessful because teachers' current beliefs are closely related to practice and these beliefs are often not aligned with reform objectives. In light of this, educational researchers and teacher educators have sought to address this issue by targeting belief change, primarily at the onset, before pre-service teachers begin their teaching careers.

In the past few years, several models of belief change have been investigated in attempts to transform teachers' current beliefs to those more conducive to reform-oriented teaching practices. However, despite their underlying conceptual framework researchers have agreed that belief change is a difficult if not an impossible process (Chapman, 2002; Ernest, 1989b; Pajares, 1992; Richardson, 2003). It requires the recognition that a change is needed, the desire and readiness to change, and an environment that will stimulate and support the change. Many of these approaches toward belief change take a very individualistic stance, viewing belief systems as a purely internal, psychological construct. As such they incorporate techniques that rely on pre - and in-service teachers engaging in cognitively oriented processes including metacognitive awareness, self reflection and reflective discourse. Although these conceptual models of belief change differ, there is some agreement and commonality on two main issues. One, for belief change to be possible individuals must become more aware of the content of their beliefs, and second, they must have the opportunity to engage in experiences that produce cognitive disequilibrium forcing them to examine their beliefs and reflect on them (Gregoire, 2003; Murphy & Mason, 2006; Wilcox, Schram, Lappan, & Lanier, 1991).

Many teacher education programs in efforts to address belief change design activities to incorporate these strategies. These efforts have been reported in several studies over the years, predominantly with pre-service teachers (Ambrose et al., 2004; Anderson & Piazza, 1996; Gunstone & Northfield, 1992; Lloyd, 2002; Middleton, 2002; Wilkins & Brand, 2004; A. Wilson, 2006). Initially, within most of these programs the issues were addressed through two main avenues: introductory education courses promoting constructivist ideology and through the practice of student teaching (Ambrose, 2001). However, success through these avenues was varied, with some studies revealing that frequently these experiences tend to solidify the teachers' initial beliefs rather than change them (Ambrose, 2004; Zeicher, Tabacknick, & Densmore, 1987). To improve the rate of success of these efforts it was suggested that courses in the teacher education program be geared towards achieving dissonance (Tatto & Coupland, 2003).

Cognitive dissonance in this regard would entail engaging students in experiences so that their new understandings from the teacher perspective conflicts with those experienced in their prior student role. The idea here is that pre-service teachers' beliefs were developed during an engaged setting and are often implicitly held so they are difficult to change from an academic, propositional approach (Richardson, 2003). In other words, because these beliefs were developed through active experience they will be difficult to change through a passive approach, namely sitting in a classroom receiving didactic instruction. Another factor that seems to play a role in the level of success of these efforts is time, in that these beliefs are so powerful and deeply entrenched following over 12 years of student experiences that the teacher education program is too short (on average 3 years) to accomplish this (Richardson, 2003; Weber & Mitchell, 1996).

Designing activities for student teachers to experience dissonance is not a simple endeavor. Toward this effort researchers have proposed different models of belief change to address this need (Ambrose, 2001; Anderson & Piazza, 1996; Dole & Sinatra, 1998; Gregoire, 2003; Kagan, 1992; Wilkins & Brand, 2004). One such model proposes a view of belief change as conceptual change (Gunstone & Northfield, 1992; Kagan, 1992). The model is based on constructivist theories, the underlying thesis being that individuals construct knowledge through their interactions with the environment and their everyday experiences. Individuals form mental representations of what they know and interpret new information through these already existing structures. Often these existing representations (sometimes referred to as misconceptions) are limited or incorrect and stand in opposition to the information being presented serving as a hindrance to the individual processing new information (Mestre, 1986; Murphy & Mason, 2006; Resnick, 1983). In this regard we can view the traditional beliefs that some teachers hold on entering teacher education programs as misconceptions about pedagogy and students' learning.

Conceptual change in this instance would entail a reorganization or reformulation of mental representations in light of new educational experiences. In presenting the students with information that directly contradicts their previously held conceptions a cognitive conflict is produced (Posner, Strike, Hewson, & Gertzog, 1982). This presentation of contradictory evidence is considered necessary to facilitate knowledge reconstruction as the individual recognizes that his existing representations are no longer sufficient to explain the new phenomena. Prior to and during these experiences it is important that the teachers are engaged in supplemental activities to make their beliefs less tacit. This may allow them to think more deeply about their current beliefs, identify how these new experiences contradict these beliefs, and make conscious decisions about the need for modification (Gregoire, 2003; Middleton, 2002; Woolfolk-Hoy et al., 2006).

In addition to this, student teachers need to be engaged in practical application of the reform-oriented academic theories and ideas and be allowed to examine and reflect on them in relation to their own understandings. This approach has been applied in different ways with some success; through the examination of the student teachers' own teaching on videotapes, followed by critique from peers and themselves (Manning & Payne, 1989; Stones, 1986); through the use of case studies examining the teaching of others (Carter & Richardson, 1989); and through strict classroom supervision where current practices were challenged with limited time delay (Kagan, 1992). An important factor that appeared to contribute to the level of success was the timing and order of these experiences as it was observed that the process was more effective when teachers engaged in a formal reflection process following their practical experience rather than before (Tilemma, 2000).

Any process of self-examination and change will undoubtedly involve the interplay of other psychological constructs. Belief change is no different and so we find that constructs such as emotions, motivation and identity also play a role. Specifically, teachers' perceptions of themselves and of belief change may be possible hindrances to successful modification. An important question to consider in this regard is "to what extent does endorsing a new belief structure, or modifying existing knowledge, threaten or align with teachers' developing understandings of who they were, or will be in their classrooms?" (Woolfolk-Hoy et al., 2006, p 278). In examining the process of belief change it is necessary to consider the personal relevance of these innovations to teachers' constructions and preservations of their teacher identities (Gregoire, 2003; Vulliamy, 1997). At inception, reform initiatives may threaten how teachers view themselves or their roles in the classroom and so regardless of whether they see and understand the perceived value of the reform, it may be difficult for them to embrace it as it challenges their sense of self, or rather teacher self (Hargreaves, Earl, Moore, & Manning, 2001; Lasky, 2005; Sachs, 2000; Wheatley, 2002). Additionally, depending on the level of teacher efficacy, which is the teacher's belief in his success in his educational responsibilities, these initiatives may cause stress and threat appraisals (Goddard, Hoy, & Woolfolk-Hoy, 2000; Gregoire, 2003). Coupled with perceived insecurities, teachers may lack the necessary motivation for belief change; this may occur regardless of how efficacious they feel in the profession (Mahurt, 1994).

In light of this, several researchers have concluded that the process of belief change involves both cognitive and affective processes (Dole & Sinatra, 1998; Gregoire, 2003; Hargreaves et al., 2001). As such, efforts have been made to modify teachers' beliefs factoring in the perceived role of affect and other motivational constructs (e.g., Gregoire's (2003) Cognitive-Affective Model of Conceptual Change and Dole and Sinatra's (1998) Cognitive Reconstruction of Knowledge model (CRKM)). These models of belief change suggest that emotions play an important and significant role in influencing cognition and beliefs, and that both cognitive processing and affective forces need to be engaged for there to be knowledge revision. They hold that individuals will be receptive to new information or ideas and possibly alter their mental representations if other factors are present, such as interest, familiarity or understanding of the new material. If the teachers do not perceive the new objectives or experiences as valuable or useful there will be no incentive to pursue change (Dole & Sinatra, 1998). Further, for this change to be lasting, individuals must be equipped with the necessary metacognitive skills to process the new information effectively and to be able to adequately engage in reflection. If this cognitive engagement is low, then there will be little or no change in their beliefs and often individuals will revert to their previous conceptions.

In addition to the motivational and cognitive factors deemed necessary for change it is also proposed that 'true' belief change is dependent on the particular emotion that is elicited by a particular event (Gregoire, 2003). When new information is presented, for example, policies for a reform initiative or reform-oriented beliefs, it often requires teachers to examine their current teaching practices, which may bring to the surface their perceptions of their identities and roles as teachers. Teachers whose perceptions of themselves and their teaching appear to be parallel to the initiative make positive judgments or appraisals eliciting positive emotions, which lead to less systematic processing. On the other hand, others, perhaps due to motivational or efficacy factors, perceive the required changes as a threat or challenge. These challenge appraisals are those hypothesized to lead to change or acceptance of the reform mandates as they create the cognitive dissonance that is considered integral to belief change.

Both the Cognitive-Affective Model of Conceptual Change (Gregoire, 2003) and the Integrated Model of Belief Change (Ashton & Gregoire, 2003) highlight this interaction between cognition, beliefs and emotions. The core tenet of these models is that negative emotions or feelings of dissatisfaction produced by cognitive disequilibrium tend to stimulate thoughts related to the new information. These thoughts elicit further emotions, which serve as the impetus for belief change (Ashton & Gregoire, 2003). Consequently, this change will also bring forth emotional responses that will ultimately lead to the actualization of the reform objectives or the embracing of the new beliefs. However, this dissatisfaction does not always elicit emotions that are amenable to change. Alternatively, emotions may arise that do not stimulate the cognitive or motivational factors that induce change and so the individual may hold on to their old beliefs because of low systematic processing.

Problems with current models. These approaches toward change encompass a similar sequence of strategies (awareness, reform-promoting teaching experiences, and reflection), which have demonstrated varying levels of success. The limited success of these strategies may be due to several reasons including the degree of effectiveness of each step of the process. For example, if the student teaching experiences and other activities pre-service teachers are engaged in are not perceived by them as sufficiently relevant or contradictory they may not produce the necessary cognitive disequilibrium. Similarly, if they become more aware of their beliefs but they do not perceive that their current beliefs are in conflict with those they are being encouraged to adopt, then change may not occur. These instances along with others have been discussed as barriers to the belief change process (Hallam & Ireson, 2005; Hayney & McArthur, 2002). In this regard, Green's (1971) philosophical model of belief systems becomes useful for explaining the cause of these hindrances and subsequently the limitations of the proposed strategies.

Green states that beliefs are held either evidentially or non-evidentially, meaning as a matter of evidence or ideology, respectively. Depending on how beliefs are held an individual may be open to change through the acknowledgement of a good counter argument or evidence to the contrary. Otherwise, the individual, in this case a teacher, may hold a belief 'without regard to evidence, or contrary to evidence or apart from good reasons' (Green, 1971, p.48). In the case where the beliefs are non-evidentially held the teacher will accept information because it supports the belief and reject information that opposes it. As such, experiences that are intended to cause dissonance would be perceived in a way that supports the existing belief or would be disregarded as possibly irrelevant. In this case, other strategies would need to be employed.

On the other hand, students who enter education programs often have already established beliefs about teaching and learning, most of which could be regarded as evidentially held beliefs. These beliefs are evidentially held in that, despite the disparity between these beliefs and reform-oriented/constructivist beliefs, they are held on the basis of their own experiences. For example, a student entering college in a teacher education program has presumably done well academically in her previous school career. If that student experienced primarily traditional, didactic instruction and succeeded along with the majority of her peers, she may hold the view that it works well all the time and for all subjects and students. The student holds this belief because it is supported by evidence (i.e., her own and the experiences of her peers). Although this belief may be tacit, all new information and experiences tend to be filtered through this already established view. Because it is evidentially held, the sequence of strategies could be employed and would perhaps be successful. However, this method is not without its limitations, because despite these attempts, the two contradictory views could be held together seemingly without contradiction (Green, 1971). Alternative strategies would also need to be employed in this case.

While this review was not exhaustive it does highlight the essential features of the models that have been proposed and the strategies employed in efforts focused on teacher belief change. The organization of beliefs in systems and the features of these systems provide possible explanations for the limited success and open the door for discussion of other approaches. As such approaching belief change from a socially-oriented perspective may address these limitations and ultimately lead to long-term success.

Combining Cognitive and Sociocultural Perspectives on Belief Change

There have been several strategies proposed to address the issue of teacher belief change, all with varying degrees of success and apparent limitations. From a cognitive perspective there are several elements that seem to be crucial in the process, they include increasing teachers' individual awareness of their beliefs, engaging them in experiences that will lead to cognitive dissonance and then providing opportunities for reflection. Additionally, in the last decade increased attention has been placed on the role of motivational and affective factors in this process. Research (Dole & Sinatra, 1998; Gregoire, 2003) in this area has had some success with approaches designed to address the influences of these factors demonstrating that constructs such as teacher identity, emotions and the personal value ascribed to the change are also important factors to consider for successful belief change.

It is apparent that while these models and approaches have reflected some success, these successes were short-lived, as recent studies still report difficulty in implementing reform practices citing teachers' beliefs as a barrier (Cross, in preparation). One reason for this may be that most of the models and approaches designed to effect teacher belief change have emerged from a cognitive perspective, which in a sense, only addresses one aspect of the nature of beliefs. These approaches align well with the conceptual representation of beliefs as solely individually constructed and enacted thoughts or ideas. They tend to deemphasize the fact that individuals are social beings and that the social network with which we identify has tremendous influence in the formation and sustaining of the individual's belief system. As such this influence can be harnessed and through efforts mimicking enculturation be applied to the belief change process. These two perspectives are not perceived as contradictory but rather complementary as they address both the social and cognitive dimensions of learning embodied within the process of belief change.

The findings of research serve as evidence to this complementary relationship between both perspectives and highlight their convergence in several areas (Llinares, 2002; Skott, 2001). One area where they are fully aligned is in the conclusion that the process of belief change is arduous and time-consuming and requires external support to be both initiated and sustained. From a more cognitive-based belief change perspective time is an important component because in a sense the process seeks to try and undo the effects of 12 plus years of traditional schooling. The idea here is that student teachers developed their beliefs through being actively engaged in a particular school culture as students. They embraced a way of thinking through being a participant in this culture, which now seemingly conflicts with the ideas they are expected to acknowledge and accept in the teacher learning process. This way of thinking in effect must be transformed through developing new conceptualizations of teaching and learning.

The process however first requires that these old beliefs be transformed, through engagement in activities that promote belief awareness, and exposure to contradictory evidence coupled with reflection. Only then can these new conceptions begin to take shape. Because of the seemingly three-part sequential nature of this process, time is crucial. Two important factors serve as impediments to the success of this three-part process. First, it is assumed that the sources of evidence will be sufficiently contradictory to stimulate change (discussed earlier). Additionally, while this approach has its value it does not account for the possible counteracting influence of the teaching culture in which the student is currently immersed, in that the current environment may not necessarily embody the beliefs the initiative wishes the teacher to embrace.

In this regard, while the experiences may be producing cognitive conflict on the one hand, the nature of environment is providing support for the existing beliefs on the other.

To address both the issue of time and the counteracting effect of negative experiences we can harness the powerful effect of cultural and social influences in generating and sustaining of beliefs. In this regard change or the cultivation of the new, desired beliefs would perhaps be more successful if a dual approach was applied. So, in addition to being engaged in reflection and positive teaching experiences teachers would be additionally immersed in a culture constitutive of the beliefs they are now expected to adopt – the constructivist-oriented beliefs of the mathematics education community (for example those referred to in the *Principles and Standards for School Mathematics,* NCTM, 2000).

On entering teacher education programs, student teachers are positioned to become new members of a community. The 'community of practice' of which mathematics educators are a part has shared ideas, common interests, ways of thinking and doing that are representative of that community. Ideally, these communities should exist within the teacher education programs in which pre-service teachers are enrolled and attributes of this 'practice' should be evidenced in these communities so entering students as new members become initiated into the practice of mathematics educators. Through the process of enculturation these new members will adopt the particular norms, behavior and discourse of mathematics educators. This means adopting the mathematics education community's way of thinking and being and incorporating the ideas, thoughts and discourse of the community into their own teacher identity (Goos, 2005). Having these community practices authentically embodied in the culture of the program of which these student teachers become a part is the first step in this socially-oriented process of belief change. Within this larger social group, the individual's private world would be exposed opening the door for a new way of thinking, where the learner's currently held views would begin to be reshaped by ideologies of the new community, in effect creating a new habitus (Wilcox et al., 1991). Similar to the ways that individuals acquire the ideologies of a society, through the sharing of ideas, thoughts and feelings, so these teachers would adopt the beliefs and values of the mathematics teaching community.

This approach to belief change however operates on its own assumption - one being that these communities exist and also that these new conceptualizations will become evidentially held. It therefore requires that these communities are engaging in authentic mathematical practices reflecting their proposed ideologies, practices and ideologies that student teachers will adopt. For complete success, these 'communities' must also be present within the schools where these teachers will begin their practice and serve as legitimate extensions of those in the teacher education programs; only then will these new beliefs be sustained and the issue of time be adequately addressed.

Immersing student teachers in mathematical communities of practice also addresses the problem of the seeming temporary effect of some belief change efforts. Following an intervention (geared towards belief change) there was evidence of initial change, but this change was apparently not sustained as the beliefs were not actualized in practice. A possible reason for this may be in the current ways that researchers assess belief change; there may be a need for improvement of these measures. This may also be as a result of the infrastructure and culture of the school environment making it difficult for these beliefs to be actualized. As such, successful teacher education programs and professional development seminars do not see the fruits of their success because these teachers enter or return to communities, which now embody conflicting belief systems. Additionally in instances where the teachers within a school have established a ‘community of practice’ they may be competing with school cultures that have different and opposing sets of principles. The importance of the larger social context for both the change process and for belief manifestation must be emphasized and as such the school community must be a support system working concurrently with efforts toward teacher change for these initiatives to be successful.

Another perspective raised by Ambrose (2001) is the notion of belief extension rather than change. She proposed that beliefs are often extended but not changed. As stated by Green (1971) individuals can hold contradictory beliefs within their belief systems with no apparent internal conflict. This occurs because these two beliefs are held apart by another belief (Green, 1971). Recognizing the possibility that an opposing belief can hinder one from being actualized, it is important to identify and address these beliefs that support this extension as they block the progression of the belief change process. In this regard, awareness, examination and reflection have been described as somewhat effective activities at the individual level, but to increase the success of belief change these practices should also co-exist within an environment embodying the culture of the desired community. It is important that there exists a forum where these teachers are able to reflect and make meaning of their experiences. Meaning making in this instance is aligned with Wenger’s (1998) view, where meaning making is considered a process of negotiation such that the meaning is situated in the process.

This process of negotiating meaning involves both participation and reification. There is somewhat of a dialectic relationship between the two and neither can be isolated for learning to take place. Participation is characterized as being actively involved in the community and being engaged in its social practices; for student teachers this would mean being engaged in the authentic practices of and building relationships within the mathematics education community.

Reification is the process of placing form on what is considered abstract (e.g. thoughts, ideas), putting it in a 'congealed' form. In the learning to teach process, which is the primary context for any belief change activity with teachers, both these processes must be engaged specifically targeting the beliefs that teachers hold. Efforts toward belief change in teacher education programs are often indirect. For example, the teaching experiences in which student teachers are engaged are often primarily to have them learn about how students think mathematically; the main objective is not to change beliefs but it is rather a hopeful by-product of these engagements. Often these learning experiences are followed up with discussions targeting the experience and not necessarily the *beliefs* that student teachers hold concerning the experiences. It is only when teachers' beliefs are the direct focus of these discursive activities and they become open to questioning that there becomes the possibility that they might be reified (Llinares, 2002; Richardson, 1996). There must be a distinction made between the role of beliefs as a reference for garnering meaning from a situation (namely classroom learning experiences) and beliefs being the target for learning. It is through making beliefs the target of learning in these discursive interactions that teachers can fully engage in reification of their new emerging beliefs.

Reification of beliefs should be an important part of the enculturation process. As student teachers adopt these new practices and ways of thinking reification becomes integral to the development of evidentially held beliefs. The process of reification goes beyond simply having the teachers become more aware or making the beliefs less tacit; it encourages the teacher to transform these intangible dispositions into objectified existence, creating points of focus around which meaning will be ascribed (Wenger, 1998). Making their beliefs an object of reflection may influence the ways these teachers adopt the community's ways of thinking and new understandings as their own, hopefully making them more evidentially-held (Llinares, 2002; McLellan, 1996).

Conclusion

This article attempted to address the issue of belief change by providing an overview of the research that has been done in the area highlighting the features of these efforts that make success problematic and presenting alternatives strategies that align with a more socially-oriented view of beliefs. The aim here was not to devalue the usefulness of these models but to point out the value of methods and strategies that emphasize the social aspects of beliefs. Holding that beliefs are a socially constructed, individually enacted phenomenon requires that as researchers and teacher educators we design belief change efforts to reflect all relevant aspects of beliefs. In doing so, we address the major hindrances to success in previous efforts and begin to employ approaches that may lead to long term belief modification.

CHAPTER 6

CONCLUSION

Learning by definition is a process of knowledge building that can be maximized by employing both social and individual mechanisms through which we receive and interpret information. Mathematics learning is no different and is both a collaborative and individual, constructive activity (Cobb et al., 1991). Holding this as true, learning environments designed to use activities and employ strategies that allow students access to these avenues for learning should yield greater knowledge gains than those which do not. The articles in this document sought to investigate this notion holistically by examining two specific aspects of this environment; one, the effect of combined strategies on mathematics achievement and the teacher factors that contribute to the maintenance of this environment.

The effect of incorporating mathematical argumentation and writing as learning strategies in Algebra 1 classrooms, detailed in the first article, revealed that students who engage in combined strategies had significantly greater knowledge gains than students who engaged in argumentation only, and those students who engaged in neither activity. This suggests that designing activities where students are able to generate and articulate thoughts and ideas and come to their own understandings are fundamental to students' mathematical reasoning and development. When combined, both argumentation and writing seem to activate the cognitive resources necessary to develop rich understandings of mathematical content. However, while these activities together appeared to help students construct understandings and consolidate their knowledge better than engaging in argumentation only, they only had a slightly greater impact on achievement than engaging only in writing activities.

From the qualitative analyses of the data it was observed that there were differences in the quality of argumentation that the students engaged in, which seemingly impacted the knowledge they gained from the activity. Quality in this sense was determined by three criteria - the mathematical validity of the justifications students provided for their responses, the degree to which each group member participated in the discussion, and the mathematical understandings students gained from the activity (judged by their final answer). Students in discourse groups that were high on those criteria had higher means on the post assessment than those who were low on the criteria, indicating that the quality of argumentation was related to their mathematical understanding. Additionally, engaging in writing activities following argumentation appeared to help the students consolidate their thinking and deepening their own personal understandings of the concepts. This perceived benefit of the writing activities to enhance students' understanding was also evidenced with the students who engaged in writing alone as the writing samples from both groups (W only and AW) were of similar quality. This would suggest that while the discursive activities allowed students to share their own ideas and hear the ideas of others, writing activities were crucial for making these understandings their own. Further, while there were clearly benefits to writing and argumentation individually, combining both activities seemed most beneficial for students as it yielded greater mathematical achievement.

There were also other factors that appeared to impact the success of the activities. These included the prior knowledge of the students, the degree to which the teacher facilitated the activities, and the learning trajectory of both teachers and students with respect to engagement and facilitation of both activities. We need to come to a better understanding of the degree to which the level of prior knowledge impacts a student's ability to engage in mathematical discourse thereby resolving the issue of whether argumentation leads to greater understandings or whether rich understandings leads to effective argumentation. Additionally, further research in this area should also ensure that prior to engagement in discursive activities students are knowledgeable of what constitutes a valid argument and the criteria for justification within the mathematical context.

Of critical importance to the effectiveness of these types of activities in enhancing students' mathematical understanding is the degree to which teachers develop classrooms norms that promote focused engagement in both writing and discursive activities. Also, as students are developing their skills in talking and writing about their ideas, teachers need to use appropriate techniques to guide the students. It was observed that teachers' inability to facilitate student learning in these ways were both a result of inexperience with these techniques and also their beliefs about the nature of mathematics, and mathematics teaching and learning. Teachers' beliefs seemingly had tremendous influence on how they structured their classrooms, interacted with their students, and how they both incorporated and facilitated the reform-oriented strategies. It was also observed that these sets of beliefs formed a cohesive unit where beliefs about mathematics pedagogy and learning were derived from their beliefs about mathematics as a domain of knowledge. However, while these teachers' beliefs cohered in similar structures, the content of the beliefs formed different belief models. These models are hypothesized to be integral in any attempts to modify or restructure these teachers' beliefs to ones that reflect more constructivist, reform-oriented mathematical beliefs.

Beliefs are clearly an important construct in teacher education research as they are thought to be quite influential in teachers' decision- making process and specifically they tend to directly impact teachers' daily instructional practices. Consequently, this connection between beliefs and practice is often considered a hindrance to education reform initiatives that seek to transform classroom teaching to more constructivist models because research has demonstrated that without a change in beliefs, classroom instruction often remains the same. Working from the assumption that a change in pedagogy will lead to increased achievement, teacher belief change in mathematics education is of critical importance. However, efforts in the past have not been successful or have only resulted in a temporary change. These efforts have primarily been from the perspective that a belief is a psychological construct that resides within the individual mind and so mainly cognitively-oriented strategies have been employed. This perspective tends to negate the social dimensions of belief change, which is an important factor in how beliefs are developed and organized. Holding that beliefs are a socially constructed, individually enacted phenomenon provides us with new avenues through which to explore the concept of beliefs and belief change. The aim here was not to devalue the usefulness of these cognitive models but to point out the value of methods and strategies that emphasize the social aspects of beliefs. By incorporating these methods we address the major deterrents to success in previous efforts and begin to employ approaches that may lead to long term belief modification.

In sum, if our goal within the mathematics education community is to improve mathematics achievement, then we must design learning environments that promote critical and analytical thinking and that provide students with opportunities to engage in activities that aid in the construction of enriched mathematical understandings. These environments are constitutive of three main components that are separate but must cohere in a manner that is conducive to knowledge construction and conceptual understanding. They are 1) the students and their learning needs 2) the teacher and the factors influencing instructional practice and 3) the activities or tasks assigned to the classroom community. The articles in this document touch on each of these components and provided insights that may serve to enrich both teacher education and mathematics education as a whole.

REFERENCES

Ambrose, R. (2001). *Catalyzing change in preservice teachers' beliefs: Effects of the mathematics early field experience.* Paper presented at the Annual Meeting of the American Educational Research Association, April 10-14, 2002, Seattle, WA.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7*, 91-119.

Ambrose, R., Clement, L., Phillip, R., & Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104*(2), 56-69.

Anderson, D., & Piazza, J. (1996). Changing beliefs: Teaching and learning mathematics in constructivist pre-service classrooms. *Action in Teacher Education, 18*, 51-62.

Artzt, A., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem-solving in small groups. *Cognition and Instruction, 9*, 137-175.

Ashton, P., & Gregoire, M. (2003). At the heart of teaching: The role of emotion in changing teachers' beliefs. In J. Raths & A. McAninch (Eds.), *Teacher beliefs and classroom performance: The impact of teacher education* (Vol. 6, pp. 99-122). Norwood NJ: Ablex.

Atkinson, R. C., & Schriffin, R. M. (1968). Human memory: A proposed system and its control processes. In K. W. Spence & J. T. Spence (Eds.), *The Psychology of Learning and Motivation* (Vol. 2). London: Academic Press.

Azjen, I. (1991). The theory of planned behavior. *Organizational Behavior & Human Decision Processes, 50*, 179-211.

Ball, D. (1988). Unlearning to teach mathematics. *For the learning of mathematics, 8*(1), 40-48.

Ball, D. (1991). "Implementing the *Professional Standards for Teaching Mathematics*: What's all this talk about discourse?" *Arithmetic Teacher* (39), 44-48.

Baxter, J., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communicating for academically low-achieving students. *Learning Disabilities Research and Practice, 20*(2), 119-135.

Bereiter, C., & Scardamalia, M. (1987). *The psychology of written composition*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Bjorklund, D. (2005). *Children's thinking: Cognitive development and individual differences* (4th ed.): Thomson Wadsworth.

Boe, E., & Shin, S. (2005). Is the United States really losing the international horse race in academic achievement? *Phi Delta Kappan, 86*(9).

Borko, H., & Putnam, R. (1996). Learning to teach. In R. Calfee & D. Berliner (Eds.), *Handbook of Educational Psychology* (pp. 673-708). New York: Macmillan Library Reference USA: Simon & Schuster Macmillan.

Bourdieu, P., & Wacquant, L. (1992). *An invitation to reflexive sociology*. Chicago: University of Chicago.

Brown, A. (1987). Metacognition, executive control, self-regulation and other more mysterious mechanisms. In F. Weinert & R. Klume (Eds.), *Metacognition, motivation and understanding* (pp. 65-116). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Brown, A., Ash, D., Rutherford, M., Nakagawa, K., Gordon, A., & Campione, J. C. (1993). Distributed expertise in the classroom. In G. Salomon (Ed.), *Distributed cognitions* (pp. 188-228). Cambridge: Cambridge University Press.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.

Bryan, L., & Atwater, M. (2002). Teacher beliefs and cultural models: A challenge for science teacher preparation programs. *Science Education, 86*(6), 821-839.

Buttery, T. (1990). First annual ATE teacher survey of critical issues in teacher education. *Action in Teacher Education, 12*(2), 1-7.

Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics: From a constructivist perspective. In D. Hacker, J. Dunlosky & A. Graesser (Eds.), *Metacognition in educational theory and practice* (pp 69-92). Mahweh, NJ: Lawrence Erlbaum & Associates.

Carter, K., & Richardson, V. (1989). A curriculum for an initial-year-of-teaching program. *Elementary School Journal, 89*, 405-419.

Case, R. (1996). Changing views of knowledge and their impact on educational research and practice. In D. R. Olson & N. Torrance (Eds.), *The handbook of education and human development: New models of learning, teaching and schooling* (pp. 75-99): Blackwell Publishers.

Cavanaugh, J., & Borkowski, J. (1980). Searching for metamemory-memory connections: A developmental study. *Developmental Psychology, 16*, 441-453.

Chapman, O. (2002). Belief structure and in-service high school mathematics teacher growth. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 177-193). Dordrecht: Kluwer Academic Publishers.

Charalambous, C., Phillippou, G., & Kyriakides, L. (2002). *Towards understanding teachers' philosophical beliefs about mathematics*: Department of Education, University of Cyprus.

Clarke, C., & Peterson, P. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (3rd Edition) (pp 255-296). New York: MacMillan.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*, 13-20.

Cobb, P., & Steffe, L. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*, 83-94.

Cobb, P., Yackel, E., Wood, T., Nicholls, J., Wheatley, G., Trigatti, B., et al. (1991). Assessment of a problem-centered second-grade mathematics project. *Journal of Research in Mathematics Education, 22*(1), 3-29.

Cole, M., & Engestrom, Y. (1993). A cultural-historical approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp 47-87). Cambridge University Press.

Cooney, T. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16*(5), 324-336.

Cooney, T. (2003). *Mathematics teacher education in rural communities: Developing a foundation for Action.* Paper presented at the ACCLAIM Research Symposium, McArthur, OH.

Cornoldi, C., & Lucangeli, D. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition, 3*(2), 121-139.

Countryman, J. (1992). *Writing to learn mathematics*. Portsmouth, NH: Heinemann.

Crabtree, B., & Miller, W. (1999). Researching practice settings: A case study approach. In B. Crabtree & B. Miller (Eds.), *Doing qualitative research* (2nd ed., pp. 293-312). Newbury Park, CA: Sage.

Cross, D. (in preparation). Creating optimal mathematics learning environments: Combining argumentation and writing to enhance achievement.

Dionne, J. J. (1984). *The perception of mathematics among elementary school teachers.* Paper presented at the Sixth Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education, University of Wisconsin, Madison.

Dole, J., & Sinatra, G. (1998). Reconceptualizing change in the cognitive construction of knowledge. *Educational Psychologist, 32*, 109-128.

Dominowski, R. (1998). Verbalization and problem solving. In D. Hacker, J. Dunlosky & A. Graesser (Eds.), *Metacognition in educational theory and practice* (pp 25-46). Mahwah, NJ: Lawrence Erlbaum Associates.

Ernest, P. (1989b). The knowledge, beliefs and attitudes of the mathematics: A model. *Journal of Education for Teaching, 15*, 13-33.

Ernest, P. (1998). *The impact of beliefs on the teaching of mathematics.* Paper presented at the ICME IV, Budapest, Hungary.

Feiman-Nemser, S. (1983). Learning to teach. In L. Shulman & G. Sykes (Eds.), *Handbook of teaching and policy* (pp 139-151). New York: Longman.

Fenstermacher, G. D. (1979). A philosophical consideration of recent research on teacher effectiveness. In L. S. Shulman (Ed.), *Review of research in education* (Vol. 6, pp. 157-185). Itasca, IL: Peacock.

Flavell, J., & Wellman, H. (1977). Metamemory. In R. Kail & J. Hagen (Eds.), *Perspectives on the development of memory and cognition* (pp. 3-33). Hillsdale, NJ: Lawrence Erlbaum and Associates.

Forman, E. (1989). The role of peer interaction in the social construction of mathematical knowledge. *International Journal of Educational Research, 13*, 55-70.

Forman, E., Larreamendy-Joerns, J., Stein, M. K., & Brown, C. (1998). "You're going to want to find out which and prove it': Collective argumentation in a mathematics classroom. *Learning and Instruction, 8*(6), 527-548.

Frank, M. (1988). Problem solving and mathematical beliefs. *Arithmetic Teacher, 35*, 32-34.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Karns, K. (1998). High-achieving students' interactions and performance on complex mathematical tasks as a function of homogeneous and heterogeneous pairings. *American Educational Research Journal, 35*, 227-268.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 39-57). Dordrecht: Kluwer Academic Publishers.

Ganguli, A., & Henry, R. (1994). *Writing to learn mathematics*: Center for Interdisciplinary Studies in Writing.

Garii, B. (2002). *That 'Aha' experience: Meta-cognition and student understanding of learning and knowledge.* Paper presented at the American Educational Research Association, New Orleans, LA.

Garofalo, J., & Lester, F. (1985). Metacognition, cognitive monitoring and mathematical performance. *Journal of Research in Mathematics Education, 16,* (163-176).

Gates, P. (2006). Going beyond belief systems: Exploring a model for the social influence on mathematics teacher beliefs. *Educational Studies in Mathematics, 63*, 347-369.

Glaser, R. (1991). The maturing of the relationship between the science of learning and cognition and educational practice. *Learning and Instruction, 1*, 129-144.

Goddard, R., Hoy, W., & Woolfolk-Hoy, A. (2000). Collective teacher efficacy: Its meaning, measure, and impact on academic achievement. *American Educational Research Journal, 37*(2), 479-507.

Goddard, R., Hoy, W., & Woolfolk-Hoy, A. (2004). Collective efficacy beliefs: Theoretical developments, empirical evidence, and future directions. *Educational Researcher, 33*(3), 3-13.

Goos, M. (2005). A sociocultural analysis of the development of pre-service and beginning teachers' pedagogical identities as users of technology. *Journal of Mathematics Teacher Education, 8*, 35-59.

Goos, M., Galbrath, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics* (49), 193-223.

Green, T. (1971). *Teaching and the formation of beliefs*. Troy, NY: Educator's International Press Inc.

Greeno, J. G., Collins, A. M., & Resnick, L. B. (1996). Cognition and learning. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pg 15-46). New York: McMillan.

Gregoire, M. (2003). Is it a challenge or a threat? A dual process model of teachers' cognition and appraisal processes during conceptual change. *Educational Psychology Review, 15*, 147-179.

Gunstone, R., & Northfield, J. (1992). Conceptual change in teacher education: The centrality of metacognition. *Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA.*

Hacker, D. (1998). Definitions and empirical foundations. In D. Hacker, J. Dunlosky & A. Graesser (Eds.), *Metacognition in educational theory and practice* (pp 1-23). Mahwah, NJ: Lawrence Erlbaum Associates.

Hallam, S., & Ireson, J. (2005). Secondary school teachers' pedagogic practices when teaching mixed and structured ability classes. *Research Papers in Education, 20*(1), 3-24.

Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator, 13*(2), 47-57.

Hargreaves, A., Earl, L., Moore, S., & Manning, S. (2001). *Learning to change teaching beyond standards*. San Francisco, CA: Jossey-Bass.

Hart, L. (2002). A four year follow-up study of teachers' beliefs after participating in a teacher enhancement project. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 161-176). Dordrecht: Kluwer Academic Publishers.

Hatano, G., & Inagaki, K. (2003). When is conceptual change intended? A cognitive sociocultural view. In G. Sinatra & P. Pintrich (Eds.), *Intentional conceptual change* (pp. 407-427). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Hayney, J. J., & McArthur, J. (2002). Four case studies of prospective science teachers' beliefs concerning constructivist teaching practices. *Science Education, 86*(6), 783-802.

Hmelo-Silver, C. (2002). *Facilitating collaborative knowledge construction.* Paper presented at the 36th Hawaii International Conference on System Sciences, Hawaii.

Hofer, B., & Pintrich, P. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of Educational Research, 67*(1), 88-140.

Hollon, R. E., & Anderson, C. W. (1987). *Teachers' beliefs about students' learning processes in science: Self-reinforcing belief systems*. Unpublished manuscript, Washington, DC.

Inagaki, K., Hatano, G., & Morita, E. (1998). Construction of mathematical knowledge through whole-class discussion. *Learning and Instruction, 8*(6), 503-526.

Jacobs, J., & Paris, S. (1987). Children's metacognition about reading: Issues in definition, measurement and instruction. *Educational Psychologist* (22), 255-278.

Kagan, D. (1992). Implications of research on teacher belief. *Educational Psychologist, 27*(1), 65-90.

Karmiloff-Smith, A. (1979). Micro- and macro-development changes in language acquisition and other representations systems. *Cognitive Science, 3*, 91-118.

Kasperek, R. (1996). *Effects of using writing-to-learn mathematics.* Paper presented at the American Educational Research Association, New York.

Kluwe, R. (1982). Cognitive knowledge and executive control: Metacogntion. In D. Griffin (Ed.), *Animal Mind-Human Mind*. New York: Springer-Verlag.

Knuth, E., & Peressini, D. (2001). Unpacking the nature of discourse in the mathematics classroom. *Mathematics Teaching in the Middle School, 6*(5), 320-325.

Kramarski, B., Mevarech, Z., & Arami, M. (2002a). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*, 225-250.

Kramarski, B., Mevarech, Z., & Arami, M. (2002b). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*, 225-250.

Kreutzer, M., Leonard, C., & Flavell, J. (1975). An interview study of children's knowledge about memory. *Monographs of the Society for Research in Child Development, 40*.

Kroll, L., & Halaby, M. (1997). Writing to learn mathematics in the primary school. *Young Children, 3*(2), 54-60.

Kroll, L., & Halaby, M. (1997). Writing to learn mathematics in the primary school. *Young Children, 3*(2), 54-60.

Kuhs, T., & Ball, D. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills and dispositions*. East Lansing: Michigan State University, Center of Teacher Education.

Kvale, S. (1996). *Inter views: An introduction to qualitative research interviewing*. Thousand Oaks, CA: Sage.

Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education, 21*(8), 899-916.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Leonard, J. (2000). Let's talk about the weather: Lessons learned in facilitating mathematical discourse. *Mathematics Teaching in the Middle School, 5*(8), 518-523.

Lerman, S. (1983). Problem solving or knowledge centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology, 14*(1), 59-66.

Lesh, R. (1985). Conceptual analyses of problem solving performance. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp 309-330). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., Doerr, H., Carmona, G., & Hjalmarson, M. (2003). Beyond constructivism. *Mathematical Thinking & Learning, 5*(2/3).

Liljedahl, P., Rolka, K., & Rosken, B. (2007). Affecting affect: The reeducation of preservice teachers' beliefs about mathematics and mathematics learning and teaching. In W. G. Martin, M. E. Strutchens & P. C. Elliott (Eds.), *The Learning of Mathematics: Sixty-ninth Yearbook* (pp. 319-330). Reston, VA: National Council of Teachers of Mathematics.

Lindblom-Ylanne, S., Trigwell, K., Nevgi, A., & Ashwin, P. (2006). How approaches to teaching are affected by discipline and teaching context. *Studies in Higher Education, 31*(3), 285-298.

Llinares, S. (2002). Participation and reification in learning to teach: The role of knowledge and beliefs. In G. C. Leder, E. Pehkonen & T. G. (Eds.), *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer Academic Publishers.

Lloyd, G. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 149-159). Dordrecht: Kluwer Academic Publishers.

Lucangelli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition, 3*, 121-139.

Lumpe, A., Haney, J., & Czerniak, C. (2000). Assessing teachers' beliefs about their science teaching context. *Journal of Research in Science Teaching, 37*(3), 275-292.

Mahurt, S. (1994). Teacher in transition: A model of one teacher's change process. *Paper presented at the Annual Meeting of the National Reading Conference, San Diego, CA.*

Manning, B., & Payne, B. (1989). A cognitive self-direction model for teacher education. *Journal of Teacher Education, 40*, 27-32.

Martin, J. (2006). Social cultural perspectives in educational psychology. In P. Alexander & P. Winne (Eds.), *Handbook of Educational Psychology* (2nd ed.)(pp 595-614). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Mayer, R. (1998). Cognitive, metacognitive and motivational aspects of problem solving. *Instructional Science*(26), 49-63.

McClain, K., McGatha, M., & Hodge, L. (2000). Improving Data Analysis Through Discourse. *Mathematics Teaching in the Middle School, 5*(8), 548-553.

McGilliard, W. (1983). Selected factors in the conceptual systems of geometry teachers: Four Case Studies. *Dissertations Abstracts International, 44* (1364A).

McLellan, H. (1996). Situated learning: Multiple perspectives. In H. McLellan (Ed.), *Situated learning perspectives* (pp 5-18). New Jersey: Educational Technology Publications.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575-596). New York: Macmillan.

Meichenbaum, D., & Biemiller, A. (1998). *Nurturing Independent Learners*. Cambridge, MA: Brookline Books.

Mestre, J. (1986). Teaching problem solving to bilingual students: What do research results tell us? *Journal of Mathematics Education in Science and Technology, 17*, 393-401.

Mewborn, D., & Cross, D. (2007). Mathematics teachers' beliefs about mathematics and links to students' learning. In W. G. Martin, M. Struchens & P. Elliott (Eds.), *The Learning of Mathematics: Sixty-Ninth Yearbook* (pp. 259 -270). Reston, VA: NCTM.

Middleton, V. (2002). Increasing pre-service teachers' diversity beliefs and commitment. *The Urban Review, 34*, 343-361.

Murphy, P. K., & Mason, L. (2006). Changing knowledge and beliefs. In P. Alexander & P. Winne (Eds.), *Handbook of educational psychology* (2nd ed., pp. 305-324).

Nasir, N. (2005). Individual cognitive structuring and the sociocultural context: Strategy shifts in the game of dominoes. *The Journal of the Learning Sciences, 14*(1), 5-34.

National Center for Educational Statistics. (1999). *Trends in International Mathematics and Science Study*. Reston, VA.

National Center for Educational Statistics. (2003). *Trends in International Mathematics and Science Study*. Reston, VA.

National Center for Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The Council.

National Center for Teachers of Mathematics (1991). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The Council.

National Center for Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19*, 317-328.

Nisbett, R., & Wilson, T. (1978). Telling more than we know: Verbal reports on mental processes. *Psychological Review, 84*, 231-259.

O'Flahavan, J. E., & Stein, C. (1992). In search of the teacher's role in peer discussion about literature. *Reading in Virginia, 17*, 34-42.

Op 'T eynde, P., DeCorte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 13-37). Dordracht: Kluwer Academic Publishers.

Pajares, F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Palinscar, A., & Brown, A. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction, I*, 117-175.

Patton, M. (2002). *Qualitative research & evaluation methods* (3rd ed.). Thousand Oaks: Sage.

Peterson, P., Fennema, E., Carpenter, T., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*(1), 1-40.

Piaget, J. (1976). *The child and reality: Problems of genetic psychology*. New York: Penguin Books.

Pierce, J., & Kalkman, D. (2003). Applying learner-centered principles in teacher education. *Theory into Practice, 42*(2), 127-132.

Polya, G. (1957). *How to solve it*. Princeton, NJ: University Press.

Pontecorvo, C. (1993). Social interaction in the acquisition of knowledge. *Educational Psychology Review, 5*(3), 293-310.

Porter, M., & Masingila, J. (2000). Examining the effects of writing on conceptual and procedural knowledge in calculus. *Educational Studies in Mathematics* (42), 165-177.

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education, 66*(2), 211-227.

Prawat, R. S. (1996). Constructivisms, modern and postmodern. *Educational Psychologists, 31*, 215-225.

Pugalee, D. (2001). Writing, mathematics and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5).

Raymond, A. (1997). Inconsistencies between a beginning elementary teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550-576.

Resnick, L. (1983). Mathematics and science learning. A new conception. *Science, 220*, 477-478.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula, T. Buttery & E. Guyton (Eds.), *Handbook of research on teacher education*. New York: Simon & Schuster Macmillan.

Richardson, V. (2003). Preservice teachers' beliefs. In J. Raths & A. McAninch (Eds.), *Teacher beliefs and classroom performance: The impact of teacher education* (Vol. 6, pp. 1-22). Greenwich, CN: Information Age Publishing Inc.

Rimm-Kaufman, S., & Sawyer, B. (2004). Primary-grade teachers' self efficacy beliefs, attitudes toward teaching, and discipline and teaching practice priorities in relation to the responsive classroom approach. *The Elementary School Journal, 104*(2), 321-341.

Rittenhouse, P. (1998). The teacher's role in mathematical conversation: Stepping in and stepping out. In M. Lampert & M. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 163-189). Cambridge: University press.

Rivard, L., & Straw, D. (2000). The effect of talk and writing on learning science: An exploratory study. *Science Education, 84*(5), 566-593.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.

Rogoff, B. (1995). *Observing sociocultural activity in three planes: Participatory appropriation, guided participation, and apprenticeship*. Cambridge, UK: Cambridge University Press.

Rokeach, M. (1968). *Beliefs, attitudes & values: A theory of organization & change*. San Francisco: Jossey-Bass.

Sachs, J. (2000). Rethinking the practice of teacher professionalism. In C. Day, A. Fernandez, T. Hauge & J. Moller (Eds.), *The life and work of teachers: International perspectives on changing times* (pp. 109-129). London: Falmer Press.

Salomon, G., & Globerson, T. (1989). When teams do not function the way they ought to. *International Journal of Educational Research, 13*, 89-99.

Scardamalia, M., & Bereiter, C. (1986). Research on written composition. In M. Wittrock (Ed.), *Handbook of research on teaching* (pp. 778-803). New York: MacMillan.

Schmidt, W., Wang, H., & McKnight, C. (2005). Curriculum coherence: An examination of US mathematics and science content standards from an international perspective. *Journal of Curriculum Studies, 37*(5), 525-559.

Schneider, W., & Lockl, K. (2002). The development of metacognitive knowledge in children and adolescents. In T. Perfect & B. Schwartz (Eds.), *Applied metacognition* (pp 224-247). Cambridge: Cambridge University Press.

Schoenfeld, A. (1987). What's all the fuss about metacognition? In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp 189-215). Hillsdale, NJ: Lawrence Erlbaum Associates Inc.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on teaching and mathematics learning* (pp 334-370). New York: Macmillan Publishing Company.

Schoenfeld, A. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*, 1-94.

Schutz, P., Cross, D., Hong, J., & Osbon, J. (2007). Teacher understandings, thoughts and beliefs about emotions in the classroom. In P. Schutz & R. Pekrun (Eds.), *Emotions in education* (pp 223-242) . San Diego, CA: Elsevier Inc.

Schwarz, B., Neuman, Y., Gil, J., & Ilya, M. (2003). Construction of collective and individual knowledge in argumentative activity. *The Journal of the Learning Sciences, 12*(2), 219-256.

Silver, E. A. (1982). *Thinking about problem solving: Toward an understanding of metacognitive aspects of problem solving.* Paper presented at the Paper presented at the Conference on Thinking, Suva, Fiji.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. A. Silver (Ed.), *Teaching and learning problem solving: Multiple research perspectives (pp247-266)*. Hillsdale, NJ: Lawrence Erlbaum Publishers.

Silver, E. A. (1987). Foundations of cognitive theory and research for mathematics problem-solving instruction. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp 33-60). Hillsdale, NJ: Lawrence Erlbaum Associates.

Simon, P. (1987). *Individual differences in the self-regulation of learning, emerging from thinking aloud protocols.* Paper presented at the American Educational Research Association, Washington, DC.

Skemp, R. (1979). *Intelligence, learning and action*. New York: John Wiley & Sons.

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4*, 3-28.

Stake, R. (2000). Case studies. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (pp. 435-454). Thousand Oaks, CA: Sage.

Stehney, A. (1990). Mathematicians write: Mathematics students should too. In A. Sterrett (Ed.), *Using writing to teach mathematics* . Washington DC: The Mathematical Association of America.

Stein, M., Smith, M., Henningsen, M., & Silver, E. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. Reston, VA: National Council of Teachers of Mathematics, Inc.

Stein, M. K. (2001). Mathematical argumentation: Putting umph into classroom discussions. *Mathematics Teaching in the Middle School, 7*(2), 110 - 112.

Stipek, D., Givvin, K., Salmon, J., & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213-226.

Stone, C., & Wertsch, J. (1984). A social interactional analysis of learning disabilities remediation. *Journal of Learning Disabilities, 17*(4), 194-199.

Stones, E. (1986). Towards a systemic approach to research in teaching: The place of investigative pedagogy. *British Educational Research Journal, 35*(6), 28-42.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park: Sage.

Tatto, M., & Coupland, D. (2003). Teacher education and teachers' beliefs. In J. Raths & A. McAninch (Eds.), *Teacher beliefs and classroom practice: The impact of teacher education* (Vol. 6). Greenwich, CN: Information Age Publishing.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Education Studies in Mathematics, 15*, 105-127.

Thompson, A. (1992a). Teachers' beliefs and conceptions: A synthesis of research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: MacMillan Publishing Company.

Thompson, A. (1992b). Teachers' beliefs and conceptions: A synthesis of research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Thompson, P. (1985). Experience, problem solving, and learning mathematics: Considerations in developing mathematics curricula. In E. Silver (Ed.), *Teaching and learning mathematics problem solving: Multiple research perspectives* (pp. 189-236). Hillsdale, NJ: Lawrence Erlbaum.

Tilemma, H. (2000). Belief change towards self-directed learning in student teachers immersion in practice or reflection on action. *Teaching and Teacher Education, 16*(5-6), 575-591.

Torff, B. (2005). Developmental changes in teachers' beliefs about critical thinking activities. *Journal of Educational Psychology, 97*(1), 13-22.

Torff, B., & Warburton, E. C. (2005). Assessment of teachers' beliefs about classroom use for critical thinking activities. *Educational and Psychological Measurement, 65*, 155-179.

von Glasersfeld, E. (1987). Learning as a constructivist activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3-17). Hillsdale, NJ: Lawrence Erlbuam.

Vulliamy, G. (1997). Teacher identity and curriculum change: A comparative case-study analysis of small schools in England and Finland. *Comparative Education, 33*(1), 97-115.

Vygotsky, L. (1962). *Thought and language*. Cambridge, MA: MIT Press.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Vygotsky, L. (1986). *Thought and Language* (2nd ed.). Cambridge, MA: MIT Press.

Walsh, J., & Charalambides, L. (2001). Individual and social origins of belief structure change. T*he Journal of Social Psychology, 130*(4), 517-532.

Weber, S. J., & Mitchell, C. (1996). Betwixt and between: The culture of student teaching. In Z. Moore (Ed.), *Foreign language teacher education: Multiple perspectives* (pp. 301-316). Lanham, Maryland: University Press of America.

Wenger, E. (1998). *Communities of Practice: Learning, meaning and identity*. New York: Cambridge University Press.

Wertsch, J. (1984). The zone of proximal development: Some conceptual issues. In B. Rogoff & J. Wertsch (Eds.), *Children's learning in the zone of proximal development* (pp. 7-18). San Francisco: Jossey-Boss.

Wheatley, K. (2002). The potential benefits of teacher efficacy doubts for educational reform. *Teaching and Teacher Education, 18*, 5-22.

Wiggins, G., & McTigue, J. (2005). *Understanding by design*. Alexandria, VA: Association for Supervision and Curriculum Development.

Wilcox, S., Schram, P., Lappan, G., & Lanier, P. (1991). *The role of a learning community in changing pre-service teachers' knowledge and beliefs about mathematics education*. East Lansing, MI: National Center for Research on Teacher Education.

Wilkins, J., & Brand, B. (2004). Change in pre-service teachers' beliefs: An evaluation of a mathematics methods course. *School Science and Mathematics, 104*(5), 226-232.

Williams, M., Cross, D., Hong, J., Aultman, L., Osbon, J., & Schutz, P. (in press). There are no emotions in math: How teachers approach emotions in the classroom. *Teacher College Record* (Manuscript submitted).

Wilson, A. (2006). *Assessing change in preservice teachers' beliefs about learning and teaching mathematics.* Paper presented at the Society for Information Technology and Teacher Education International Conference 2006, Chesapeake, VA.

Wilson, J., & Clarke, D. (2002). *Monitoring mathematical metacognition.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Wilson, M., & Cooney, T. (2002). Mathematics teacher change and development. The role of beliefs. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-147). Dordrecht: Kluwer Academic Publishers.

Woolfolk-Hoy, A., Davis, H., & Pape, S. (2006). Teacher knowledge and beliefs. In P. Alexander & P. Winne (Eds.), *Handbook of Educational Psychology* (2nd ed., pp. 715-738).

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yerrick, R., Parke, H., & Nugent, F. (1997). Struggling to promote deeply rooted change: The 'filtering effect' of teacher's beliefs on understanding transformational views of teaching science. *Science Education, 81*(2), 137-160.

Zan, R. (2000). A metacognitive intervention in mathematics at university level. *International Journal of Mathematical Education in Science and Technology, 31*(1), 143-150.

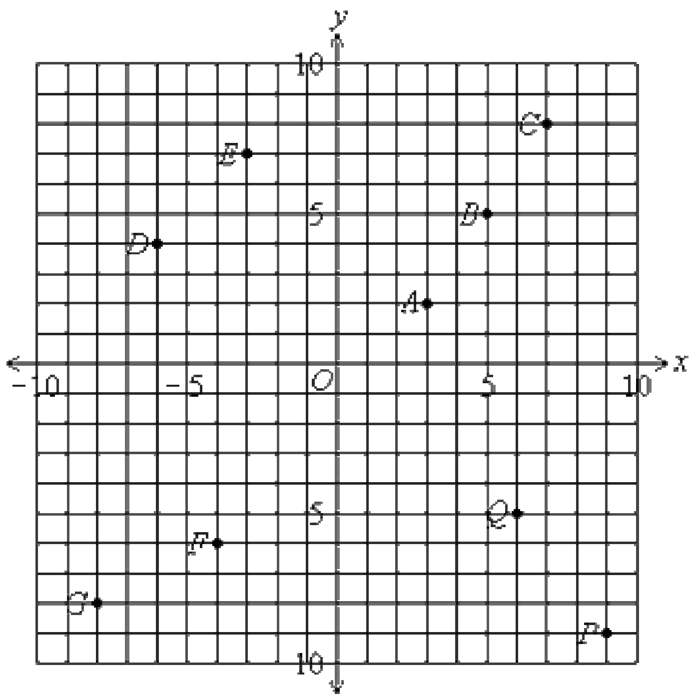
Zeicher, N., Tabacknick, R., & Densmore, K. (1987). Individual, institutional and cultural influences on the development of teachers' craft knowledge. In J. Calderhead (Ed.), *Exploring teacher's thinking* (pp. 21-59). London: Cassell.

APPENDICES

Appendix A

Activity 1

1) Using the diagram below to answer the following questions:



a) Write the following points as ordered pairs:

B ___________ F ____________

Q ___________ D ___________

Explain how you found the ordered pairs.

b) What is common about the ordered pairs of the following points?
Hint : Look at the x-values and the y-values

(i) points in the 1st quadrant

(ii) points in the 3rd quadrant

(iii) Do you think you will find similarities for points in the 2nd and the 4th quadrant as well? Why?

c) Cita wrote down the ordered pairs for the following points. Identify the ordered pairs she wrote incorrectly and state why they are wrong.

(i) A (3,2) ______________________________

(ii) F (-6,-4) ______________________________

(iii)P (9,9) ______________________________

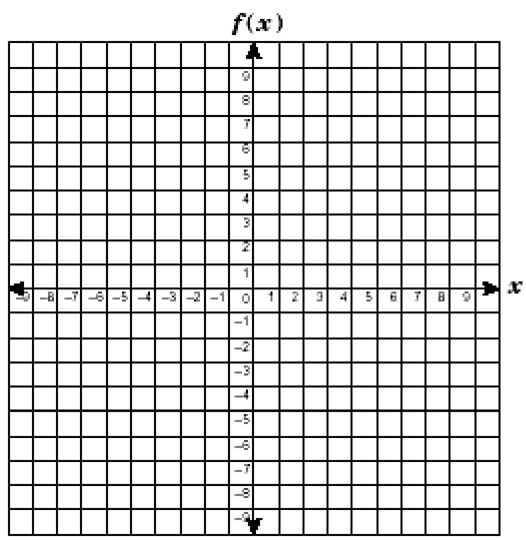
(iv)D(-6,4) ______________________________

(v) Q (5,6) ______________________________

2)

a) Write the following values as ordered pairs and plot them on the graph below.

x	-2	-1	0	1	2
y	-3	0	3	6	9



Connect the points.

b) Write down the ordered pair of the point on the line that crosses the x-axis.

c) Write down the ordered pair of the point on the line that crosses the y-axis.

d) What do you notice about these ordered pairs?

Appendix B

Activity 2

1) Points (3,5) and (-2, 4) lie on a line.

Cain use the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

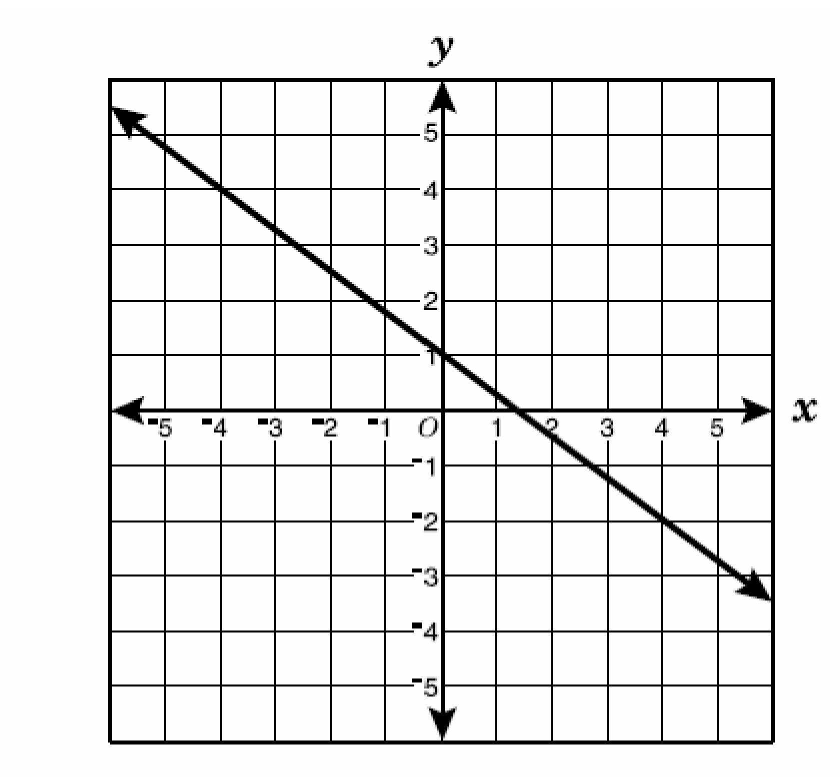
Abel uses the formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

a) Can both Cain and Abel be correct?
   Use the points given above to support your answer.

b) Will this (your answer for part a) be true for any 2 points? Why?

2)



John calculates the value of the slope of the line above and says m = 1. Sue takes a look at the line and without any calculations says he is incorrect.

a) Explain how Sue knows this without calculating the value of the slope herself?
b) Prove to John (through calculation or otherwise) that he is incorrect.

Appendix C

Activity 3

1)

Mr. Smith gives his class the following problem to solve:

$$4x - 2(3 + 2x) = 3(2x + 2) - 6x$$

John attempts the problem:

$4x - 2(3 + 2x)$	$= 3(2x + 2) - 6x$	
$4x - 6 + 4x$	$= 6x + 6 - 6x$	(Step 1)
$4x + 4x - 6$	$= 6x - 6x + 6$	(Step 2)
$8x - 6$	$= 6$	(Step 3)
-6	$= -6$	(Step 4)
$8x$	$= 0$	(Step 5)
x	$= 0$	(Step 6)

Jane checks his work and said he made errors and his final answer is not correct. Jane works out the problem and says it has no solution.

a) Where did John make errors? Name the steps.

b) If you were Jane, how would you explain to John what his errors were and why they are wrong?

c) Explain to John how he could have checked his answers.

2)

David's brother is four times his age and his sister is three years older than he is. If you double the combined ages of his siblings it would equal 46 years. How old is David?

Susan begins the problem like this.

David's age = x
David's brother's age = 4x
David's sister's age = 3x

$$2(x + 4x + 3x) = 46$$

a) Do you agree with the way Susan began the problem? Why or why not?

b) If you disagree with Susan how would you have done it differently? If you agree how would you complete the problem?

c) Explain how Susan can check to see if her answer is correct?

Appendix D

Activity 4

1) **3x + 2y = 6**

Jodi wants to graph the above equation. She knows she must start by choosing a few values for x.
First, Jodi chooses x = -2, 0 and 2 and put them in a table.

a) Complete Jodi's table.

x	-2	0	2	
y				-3

How was finding the first 3 missing values in the table (corresponding values for x = -2, 0, 2) different than finding the last missing value (corresponding value for y = -3)?

b) Explain (in words) what Jodi would do next to graph the equation.

c) Jodi says that the point (1, 3) lies on the graph of 3x + 2y = 6. Explain to Jodi, **using 2 methods**, how she can check if she is correct.

2)

Jason uses the following lines to make a shape on his graph paper.

y = -4
x = 5
y = 0
x = -3

a) What shape is on Jason's graph paper? Make a sketch to support your answer.

b) What equations would you use to make a square on your graph paper where the origin, the point (0,0) is at the center?

Appendix E

Activity 5

1)

Mrs. Brown gives her Algebra 1 class the following question:

Solve $3x + 2 > 11$

John solves the problem and shows his work.

	$3x + 2 > 11$
Step 1	$-2 \quad -2$
Step 2	$3x < 9$
Step 3	$x < 3$
Step 4	$x = 3$

Kim checks John's work and tells him that he has made 2 mistakes in his work.

a) Where do you think Kim thinks John made the mistakes? Name the steps.

b) Describe the mistakes (What is John doing that is incorrect?)

c) If you were Kim how would you explain to John why those steps were incorrect?
Hint: (i)You may want to differentiate between a linear equation and a linear inequality
(ii) Explain when and why the inequality sign changes

2)

Shawn's Work	Cameron's Work
a) $28 \geq 7x$ $\frac{28}{7} \geq \frac{7x}{7}$ (Step 1) $4 \geq x$ (Step 2)	a) $28 \geq 7x$ $7x \geq 28$ (Step 1) $\frac{7x}{7} \geq \frac{28}{7}$ (Step 2) $x \geq 4$ (Step 3)

Examine both Shawn's work and Cameron's work.

a) Can you identify any errors in their work? Name the person and the step(s).

b) Describe the error (s) that was made.

c) Explain how you would help the person correct it.

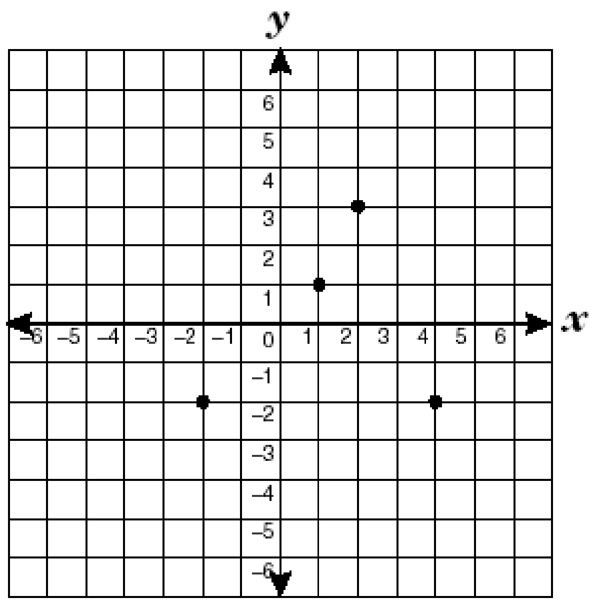
Appendix F

Algebra 1 A Test

1) Simplify the algebraic expression
$5(x - 2) + 2(3x - 12x + 12)$

a) $-x + 10$
b) $-13x + 14$
c) $6x^2 - 7x + 10$
d) $6x^2 - 19x + 14$

2)



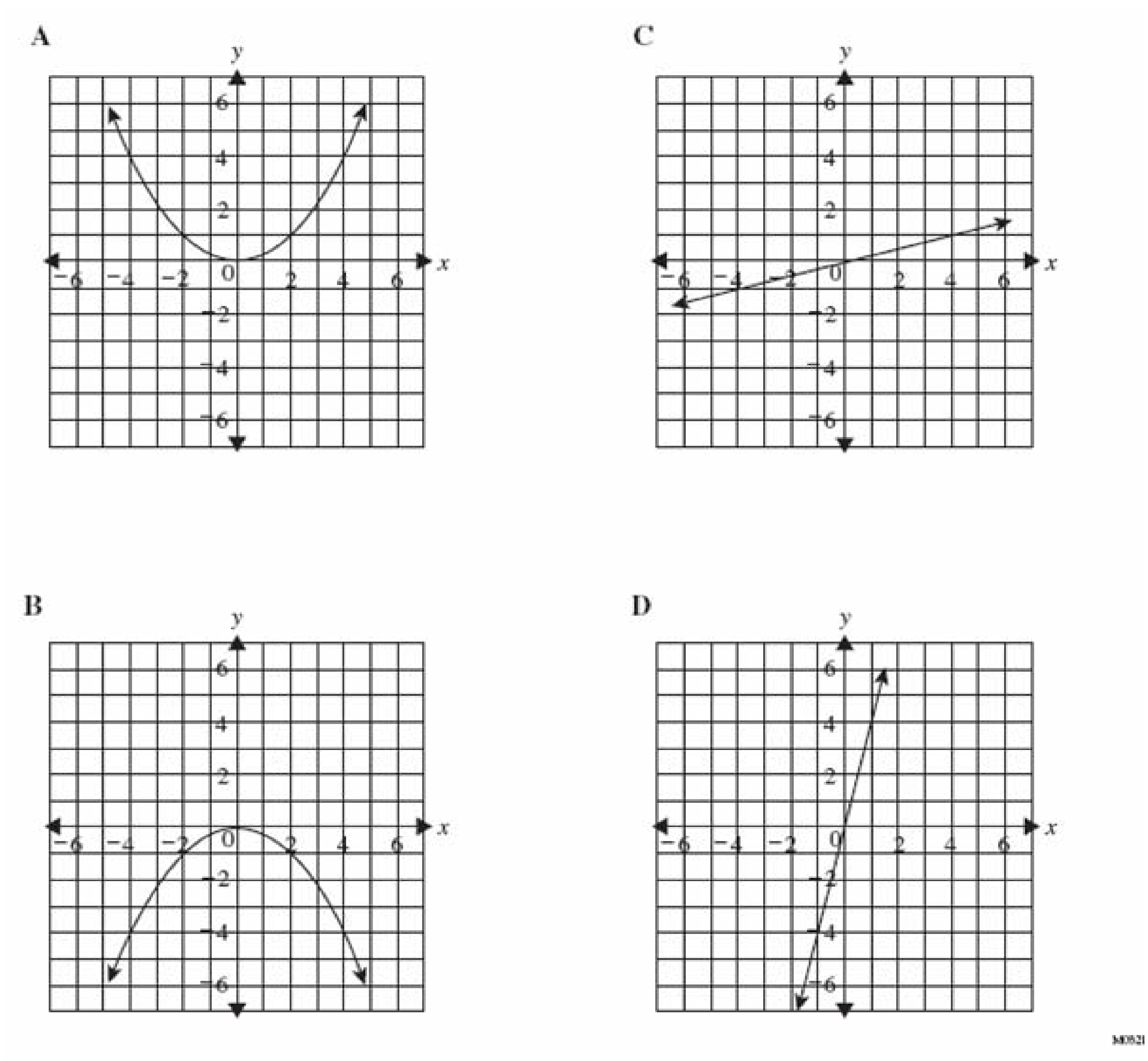
What is the apparent range of the relation shown on the grid?

a) (-2,1,2,4)
b) (-2,1,3)
c) (1,2,3,4)
d) (-2, 2,3, 4)

3) Rico's first three test scores in biology were 65, 90 and 73. What is the mean score?

a) 65
b) 73
c) 76
d) 90

4) Which of the following is the graph of y = ¼ x?

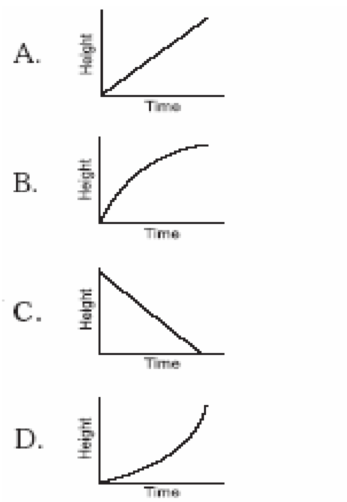


5) Solve for x.

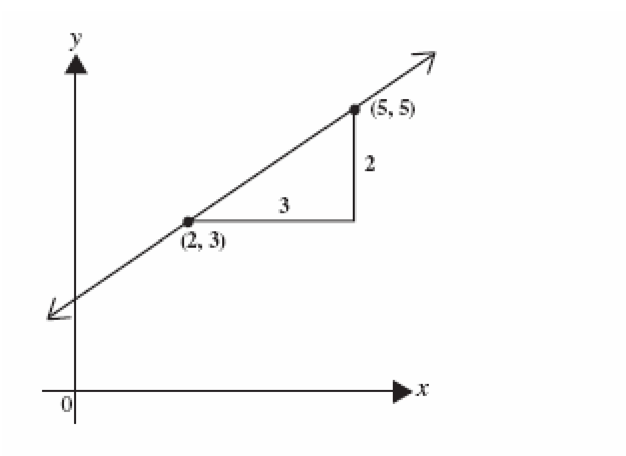
$2x - 3 = 7$

a) -5
b) -2
c) 2
d) 5

6) Kendra is observing how the height of a tall, thin cylindrical candle changes over time as it burns. Which graph best represents the changes in the height of the candle over time?



7) What is the slope of the line below?



a) – 3/2
b) -2/3
c) 2/3
d) 3/2

8) Which of the following is equivalent to 9-3x > 4(2x -1)

a) 13< 11x
b) 13> 11x
c) 10> 11x
d) 6x > 0

9) Which two lines are parallel?

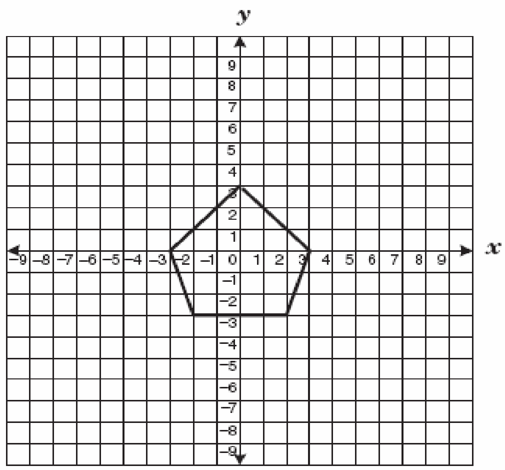
a) $2x + 5y = 6$ and $5x + 2y = 10$
b) $3x + 4y = 12$ and $6x + 8y = 12$
c) $2x + 5y = 6$ and $-2x + 5y = 14$
d) $3x + 4y = 12$ and $6x – 8y = 20$

10) Which of the following numEddieal expressions results in a negative number?

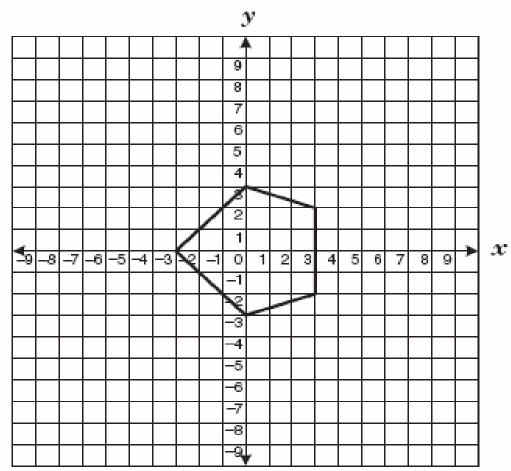
a) (-7) + (-3)
b) (-3) + (7)
c) (3) + (7)
d) (3) + (-7) + (11)

11) If the vertices of a polygon are (-2,3), (2,3) (3,0), (0, -3) and (-3,0), which graph best represents the polygon?

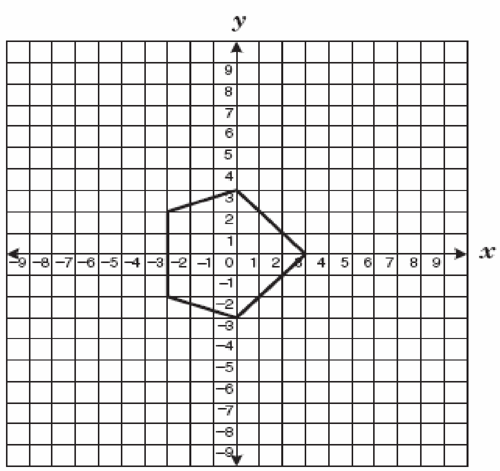
F



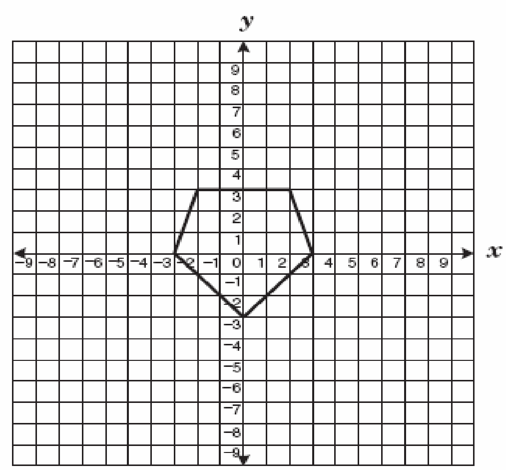
H



G



J



12) What is the y-intercept of the line 2x-3y =12

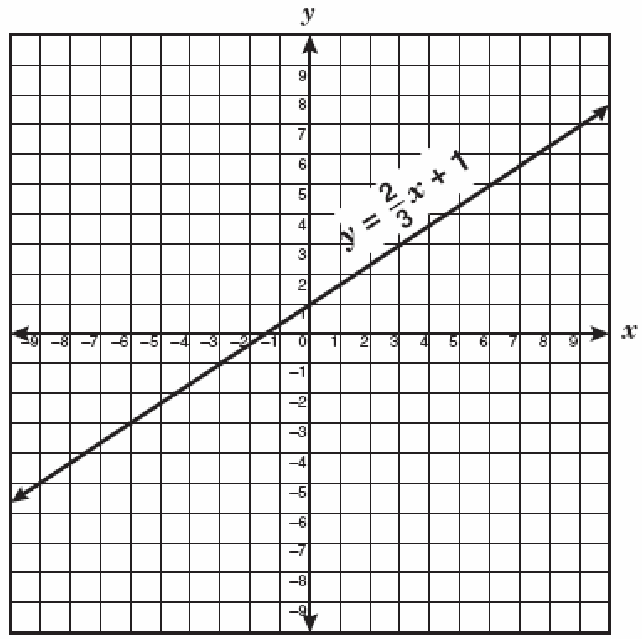
a) (0,-4)
b) (0, -3)
c) (2,0)
d) (6,0)

13) If a is a positive number and b is a negative number, which expression is always positive?

a) a - b

b) a + b
c) a x b
d) a ÷ b

14) Use the graph of y = 2/3 x + 1 to solve the equation for x when y = -3



a) x = -6
b) x = -1
c) x = 1
d) x = 3

15) Which function represents the line that contains the point (2,12) and has a slope of
-3?
a) f(x) = -3x + 6
b) f(x) = -3x + 18
c) f(x) = -3x + 34
d) f(x) = -3x + 38

16) What is the solution of the system of equations shown below?
y= 3x – 5
y = 2x

a) (1, -2)
b) (1, 2)
c) (5, 10)
d) (-5, -10)

17) Solve for n

$$2n + 3 < 17$$

a) $n < 2$
b) $n < 3$
c) $n < 5$
d) $n < 7$

18) What is the zero of the function $f(x) = 3x – 21$?

a) -7
b) 0
c) -21
d) 7

19) Which of the following is equivalent to the equation shown below?

$$\frac{20}{x} = \frac{4}{x-5}$$

a) $x(x-5) = 80$
b) $20 (x-5) = 4x$
c) $20 x = 4(x – 5)$
d) $24 = x + (x – 5)$

Appendix G
Interview Questions

Introduction

1) Are you currently a full time student or are you teaching?

2) How long have you been a student? What is your program/research interests? OR How long have you been teaching?

3) What grades and courses have you taught? Where have you taught?

4) Do you teach subjects other than mathematics? If so, what other subjects? For how long and at what levels?

5) How did you prepare yourself for this occupation?

6) Will you return to teaching? When?

Beliefs about mathematics

Preliminary Activity

1) If you were playing Taboo and wanted a friend to guess the word “mathematics,” what four one-word clues would you give?

2) What other subject(s) do you think mathematics most like? Why? Least like? Why?

4) Complete this phrase:

“ A mathematics teacher is like a ____________________”

5) ‘Draw a mathematician’ Task

Where is the mathematician? What is the mathematician doing? What kinds of tools or materials is the mathematician using?

Questions

1) What is the first thought that comes to mind when you think of mathematics?

2) How would you describe mathematics if you were asked to do so by one of your students?

3) Describe your experiences as a mathematics student in:

   a) elementary school

   b) middle school

   c) high school

4) Describe some of your favorite activities with your students.

5) What do you think mathematicians do when they do mathematics?

6) What do you think is the best way that students learn mathematics? What do you think are the most important aspects of mathematics teaching?

7) How do you know when you have had a good mathematics lesson?

8) If you had to write a paper about the nature of mathematics, how would you begin to answer the question, "what is mathematics?"

9) a) Considering this view that you hold of mathematics, how would you design a classroom activity to reflect this view?

   b) Could you provide an example (s). *[10] (specific activity)

10) a) How would you describe your role during this activity?

    b) How would your describe your responsibilities within the learning environment in general?*

11) How would you describe the responsibilities of the student?

12) Do you think there are constraints within the school environment that impede how you would like to structure your classroom activities?

13) What are some of these constraints?

[10] Questions identified with a * are dependent on the response to the previous question.

14) If there were no constraints attached to teaching, what would your ideal class period look like?

15) What would you say are the main characteristics of this ideal classroom?

16) Why are they of primary importance?

17) If you were speaking to a pre-service teacher, what advice would you give them on how to be an effective or successful mathematics teacher?