STUDENT INTERACTIONS DURING ASYNCHRONOUS PROBLEM SOLVING IN COLLEGE ALGEBRA USING A COMMUNICATION TENSOR

by

THOMAS EDMOND COOPER III

(Under the Direction of Jeremy Kilpatrick)

ABSTRACT

The goal of this study was to investigate the quality and nature of the students' interactions and the students' perceptions of factors influencing their participation and interactions in two sections of College Algebra that I taught using two approaches to asynchronous collaboration. Both sections were facilitated with a tool known as a communication tensor, which allows students to see (or not) each other's work on each assignment. In a shared-work section, students worked individually on assignments for a designated time, at which point each student's work was shared via the communication tensor. Each student in the shared-work section was responsible for constructing his or her own answers. The other section used a small-groups approach, working together in groups of three to four students on the assignments.

To address the nature of the interactions, I used the archived student work to adapt a coding scheme by Stacey and Gooding (1998) to fit the asynchronous setting. A common form of interaction identified by the scheme was when a student posted a proposed solution after viewing a classmate's work. For such responses, I developed a refined coding scheme to determine whether a student used classmates' work, the level of individual contribution, and the relative correctness of the solutions. In both sections, the most common behavior was students working independently and using classmates' work to compare answers or get assistance, but the small-groups section displayed a higher level of participation and interaction, including traditional forms of interaction such as asking questions and responding to group members.

To investigate student perceptions of factors influencing participation and interactions, I used a series of open-ended surveys and student interviews. Student responses indicated a wide range of factors including problems accessing or using technology, the difficulty of the assignments, the asynchronous format, situational factors such as busy schedules, and dispositional factors such as a strong preference for paper-and-pencil work.

INDEX WORDS: College Algebra, Mathematics, Online, Asynchronous, Communication Tensor, Problem Solving, Collaboration, Interaction

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DEDICATION

This dissertation is dedicated to my wife, Mary, for all of her support during the process of balancing a full-time career and doctoral studies.

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CHAPTER 1

INTRODUCTION

As technology has advanced over the last several decades, educators and researchers have explored ways to utilize the new tools for educational purposes. The Internet is now a driving force behind distance education, and many teachers are incorporating it into face-to-face classes as well. A clear sign of this increased use is that nearly all major textbook companies are now promoting Internet-based courseware packages along with their books. Examples of this type of software include MyMathLab (Pearson Education, 2006) and CengageNOW (Cengage Learning, 2008). A primary selling point of these systems is the software's ability to instantly generate and grade countless routine practice problems. Certain tutorials can even solve the randomly generated examples and show step-by-step algorithmic procedures. This form of technology has definite promise in helping students develop procedural fluency with certain types of problems. However, there is more to mathematics than applying step-by-step procedures to solve routine problems. The American Mathematical Association of Two-Year Colleges (AMATYC, 2006) has published standards for the first 2 years of college mathematics. This document contains standards for content, pedagogy, and intellectual development. The intellectual development standards emphasize such skills as modeling, problem solving, reasoning, communicating, and linking multiple representations. To master such a diverse set of skills, students need more than repetitive drill and practice of routine procedures. This is one reason that most courseware packages also come with various communication tools, which range from real-time discussions with text, audio, video, or combinations to various forms of asynchronous communications.

Thomas Banchoff (personal communication, June 17, 2008) has noted that there is no consensus on the use of the term asynchronous, and he has used the term *semi-synchronous* to describe assignments for which “The students responded to the same materials at about the same time period, then received timely responses in the form of individual commentary or through the medium of a solution key.” For the purpose of this study, I use synchronous and asynchronous as dichotomous terms for describing instances of communication. An instance of communication is synchronous if and only if individuals participate at the same time, either face-to-face or through some form of technology, otherwise it is asynchronous. Students might interact synchronously and asynchronously to complete an assignment, but any given instance of communication between one or more people either occurs in real-time or not. When I refer to computer conferencing or online assignments as asynchronous, I mean that the software was not specifically designed to facilitate real-time discussions and that communication occurs through messages that are stored and accessed through the software.

The use of computer conferencing involving asynchronous discussions has slowly started to gain the attention of practitioners and researchers as a way to promote online mathematical discourse and problem solving. With the advent of a large research base on collaborative learning in mathematics (E. G. Cohen, 1994; Springer, Stanne, & Donovan, 1999), attention has turned to collaboration using the new medium provided by the World Wide Web. Although references to the potential of online collaboration are abundant (Harasim, 1986; Klemm, 1998; Lamb & Klemm, 1997; Smogor, 2002), many authors have acknowledged that promoting meaningful participation in asynchronous problem solving can be a daunting task. For example, a group of authors from the Georgia Institute of Technology discuss unsuccessful attempts to promote online collaboration in mathematics, engineering, and science classes (Guzdial, Lodovice, Realf, Morley, & Carroll, 2002). They report that even in this academically competitive environment, many students elected to receive grades of zero rather than participate in asynchronous collaborative problem solving. In her doctoral study, Myers (2002) found that when instructors gave students an option to collaborate asynchronously via discussion threads or synchronously via chat room tools, very little use was made of the asynchronous tools. A lack of real-time interaction and immediate feedback seem to be major limiting factors in the effectiveness of asynchronous collaboration in mathematics courses.

A few researchers such as Myers (2002) and Kosiak (2004) have begun to explore the student interactions that take place in online collaborative problem solving, but the literature remains sparse. Some researchers who have reported on mathematical problem solving in computer conferencing have focused on implementations that rely on the use of a single discussion forum shared by small groups or whole classes (Hurme & Jarvela, 2005; Kosiak, 2004; Simonsen & Banfield, 2006). A novel approach to Internet-based problem solving is an organizational tool known as a communication tensor designed by Thomas Banchoff (2005) and his students at Brown University.

Banchoff's Communication Tensor

To facilitate online mathematical problem solving, I use a course management system that I have programmed based on Thomas Banchoff's (2005) communication tensor. Banchoff uses the term *tensor* because the software organizes the students' work in multidimensional arrays of hyperlinked squares. When a user accesses the tensor, the initial screen contains an array of hyperlinked squares with a row for each assignment and a column for each student, as well as columns for hints and solution keys (see Figure 1). If a user clicks on an assignment name in the assignment array, the tensor displays a similar array broken down by problem for

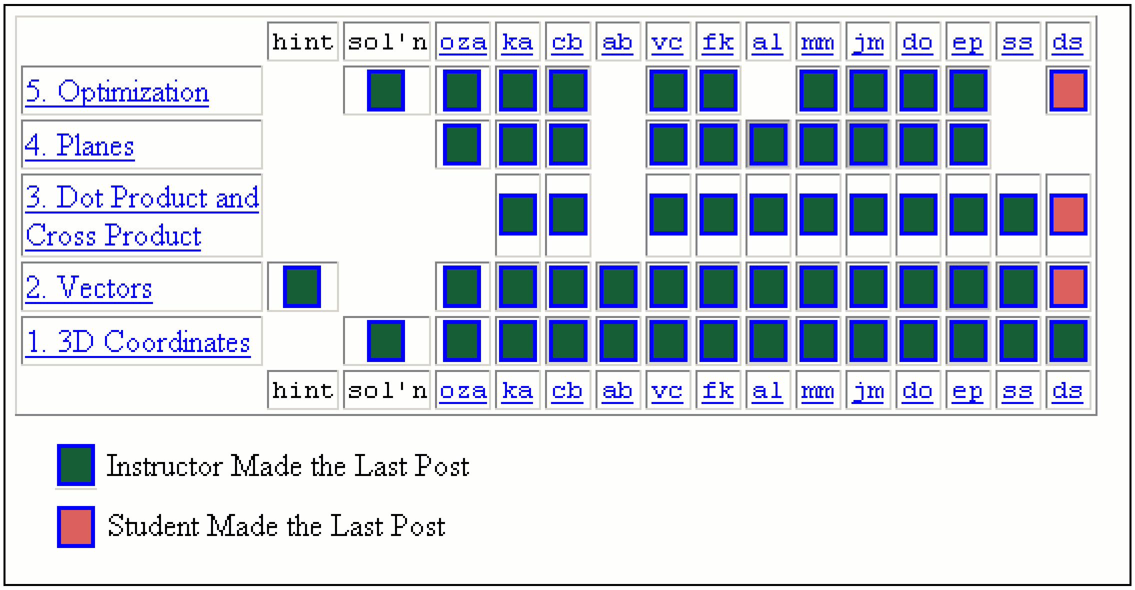


Figure 1. An instructor's view of the assignment array in Banchoff's communication tensor for a class with 13 students and five assignments. Blank spaces indicate that no posts have been made.

that assignment (see Figure 2). I used Banchoff's tensor as a student at the University of Georgia (UGA) and later in my own teaching. In both cases, I found the layout advantageous because I could access the work of any particular student on any assignment quickly without searching through a large collection of messages. By clicking on the hyperlinked square in a row for an assignment or problem, a user gains access to the work space belonging to the student whose initials head the column containing that square. Banchoff uses the term *tensor square* to refer both to the hyperlinked squares contained in the tensor and to the students' work spaces to which the squares are linked. Each tensor square operates like a traditional discussion board, allowing users to add posts using basic text or any Web page authoring language. The use of Web page authoring languages allows for the incorporation of any type of multimedia that can be placed on a Web page including images, mathematical symbols, and interactive Java applets.

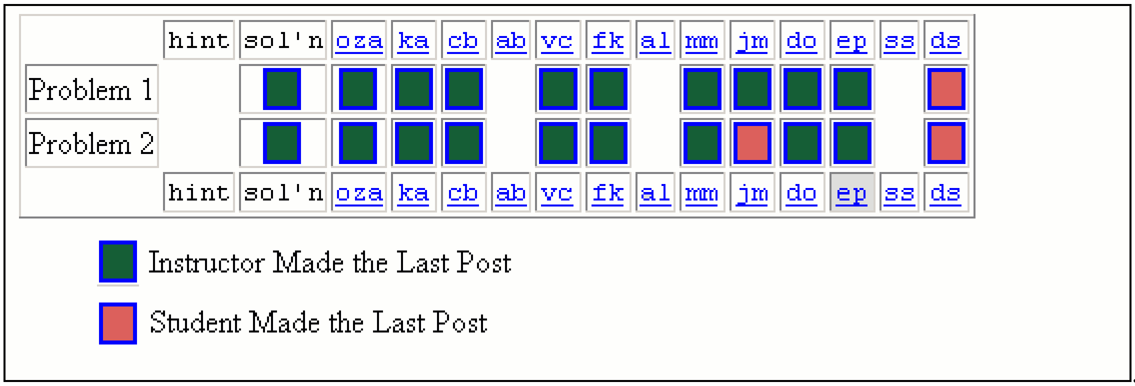


Figure 2. An instructor's view of the problem array in Banchoff's communication tensor for an assignment with two problems for a class with 13 students. Blank spaces indicate that no posts have been made.

Another feature of Banchoff's tensor that assists users in navigating the numerous squares is a color-coding scheme that toggles based on the last type of user to make a post in an assignment space. A tensor square does not appear until a student has posted at least one message for his or her assignment. A tensor square appears red when a student was the last person to make a post in an assignment space, and a tensor square appears green when the instructor was the last person to make a post. For example, consider the column headed by the initials *ds* in Figure 1. The student with this username has not posted any messages for the assignment on planes; the instructor was the last person to post a message in this student's space for the assignment on 3D coordinates; and the last message posted in each of the other tensor squares associated with student ds was made by student ds or a classmate. The same color-coding scheme is used, broken down by problem, when a user clicks on an assignment label (see Figure 2). The colors are a time-saving tool, allowing the instructor to see which squares need attention and allowing the students to know which squares have received feedback.

The arrays of hyperlinked squares and color codes are helpful organizational tools, but the most important feature of Banchoff's tensor is a time-controlled lock that can be used to permit or deny access to students' work. When an instructor creates an assignment, he or she can set a date and time for the assignment to unlock. When an assignment is locked, a student can view only his or her own work in the tensor. Figure 3 shows a screenshot of the main assignment array from the perspective of student ds. The first four assignments are unlocked, and the student can access his or her classmates' work by clicking on the squares. The optimization assignment, however, is locked and student ds can see only his or her own square for that assignment. This locking feature allows instructors to divide assignments into two phases, placing more responsibility on each student than a traditional group discussion does. During the initial phase, students are required to work individually, while possibly receiving feedback or hints from the instructor, and during the second phase, students can continue to work on their assignments with access to each other's work. The initial phase reduces the likelihood that a student would simply copy a classmate's work, and it gives each student a chance to contribute. The second phase allows students to interact and improve their work.

An additional feature of Banchoff's tensor that I have found helpful as an instructor is the ability to make private posts. Whenever a message is created, a user has the option to make the post private or public. If a private post is made by a student or the instructor in a tensor square, the contents of the post will only be visible to the instructor and the student who owns the square. If a student accesses a classmate's tensor square that contains a private post, he or she will see a message that a private post has been made. This feature allows the students to post messages for the instructor that they do not want classmates to read. I have also used private

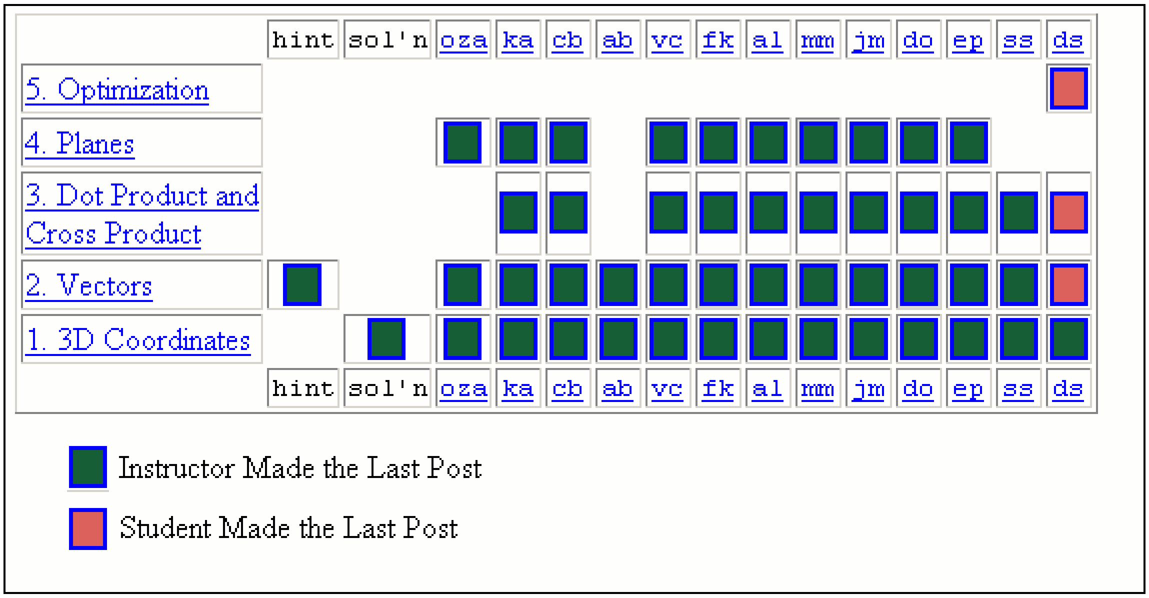


Figure 3. The view of the assignment array in Banchoff's communication tensor for a class with five assignments and 13 students from the perspective of a student with the username ds. The assignment labeled Optimization is locked and the other assignments are unlocked.

posts to make comments to students that I did not want their classmates to read, such as details about grades.

The Development of a Three-Level Communication Tensor

After taking a course taught by Thomas Banchoff with his communication tensor at UGA, I was eager to try the software with my community college students. I used the software from UGA's Web server to supplement a Calculus III class and as a major component of two hybrid (partially online and partially face-to-face) sections of Precalculus. I soon learned that teaching with this software and motivating student participation and interaction are not easy tasks. As Artigue (2002) points out, researchers have often underestimated the complexity of using technology with students. Despite my efforts to promote interaction, most students focused on their own assignments. If they did look at other students' work, very few made direct references or posted in another student's assignment square. This lack of documented interaction left me with no way to know whether students were getting the advantages of seeing each other's work. One way that I found to remedy this lack of interaction with some success in the Precalculus sections was to split each class into small groups and require the students in each group to work together in a single tensor square.

Feeling a desire to tweak a few of the features for pedagogical and research purposes, I developed my own version of Banchoff's communication tensor that can be implemented through my Web directory. Using the fundamental characteristics of Banchoff's tensor, I created a course management system that arranges students' work in an array of hyperlinks with a time-controlled lock to divide assignments into individual and whole-class phases, and I included the ability to create private messages. Like Banchoff's tensor, my software creates rows for each assignment and columns for each student, as well as columns for hints and class discussions. When writing my own software, I decided not to include a feature allowing users to view a problem array for each assignment. Despite this difference, I still call my array of hyperlinked squares a tensor because clicking on an individual hyperlink within the array leads to an assignment space containing work on multiple problems. An aesthetic difference between my software and Banchoff's is that my hyperlinks are actually rectangles; however, I prefer to use Banchoff's terminology and refer to the hyperlinks and the assignment spaces as tensor squares.

In addition to several minor programming changes, there are three notable differences between my software and Banchoff's software: a logging feature, a different color-coding scheme, and a lock to facilitate group work. I added a logging feature that records a time-stamped entry each time that a student clicks on a tensor square so that instructors can determine which tensor squares each student has accessed. The information contained in the logged files could be helpful to an instructor, and my students' logged files were essential for my research on student interactions.

Like Banchoff, I incorporated a color-coding scheme for the tensor squares, but I used a different set of colors. In my software, a white tensor square is created for each student when an assignment is written. One advantage that this has over Banchoff's software is that the instructor or classmates can post a message for a student without having to wait for the student to make an initial post. My coding-scheme also includes red, green, and blue squares. I use red and green similar to the way Banchoff does, but I have reversed the coloring. Thinking in terms of an instructor who is going to grade the assignments in a tensor and associating red with stop and green with go, I wrote my software to turn tensor squares green when a student makes a post and red when the instructor makes a post. I also wanted a student to be able to see when a classmate had left a message, so my software turns a tensor square blue when a student makes a post in a classmate's square. Figure 4 shows a sample screen shot from my software of the instructor's view for a class with two assignments and five students. It is evident that no one has made a post on the second assignment because all of the squares are white. For the first assignment, no posts have been made in the class square or in the squares belonging to Students 2 and 5. The red squares indicate that the instructor was the last person to make a post in the hints column and in Student 1's square; the green square indicates that Student 3 was the last person to make a post in his or her square; and the blue square indicates that a classmate was the last person to make a post in Student 4's square.

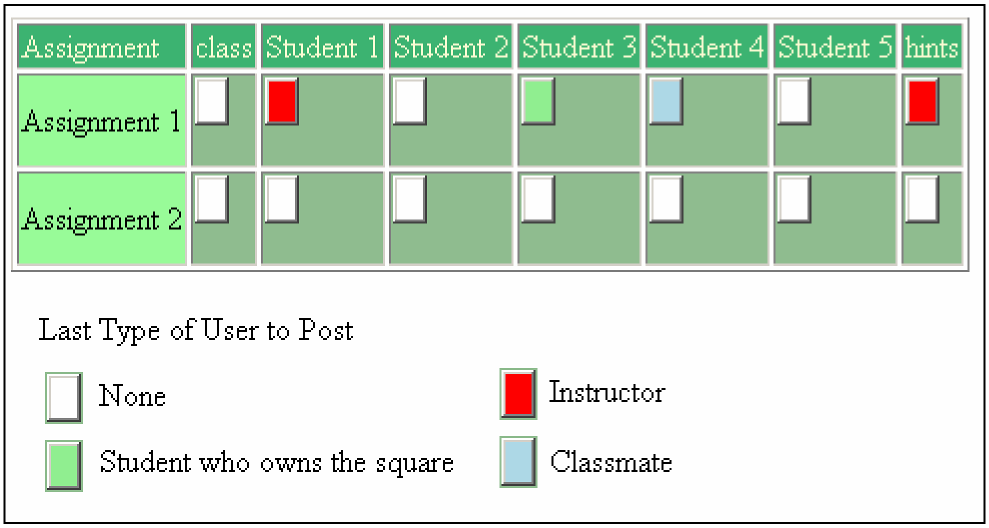


Figure 4. The instructor's view of the communication tensor in the author's software for a class with two assignments and five students.

The most significant difference between my software and Banchoff's is that I created an additional group-level lock. When a student is added to a section in the software, he or she is assigned a group number. When the group lock is open, a student can access any tensor square associated with his or her group number. Using this feature, an instructor can allow students to work in groups by restricting each student's access to his or her group members' squares, and I have found it helpful to create single group squares for the students to use for group work. Table 1 contains a summary of the five possible ways to lock my tensor, and the following sections describe two specific approaches to using the software that I implemented in this study. Unless otherwise noted, the term tensor hereafter refers to my software and not Banchoff's software.

The Shared-Work Approach

One goal of this study was to investigate the way students use a communication tensor and classmates' work on assignments administered through the approach that Banchoff (2005) has used with his students. I call this approach to online problem solving a *shared-work*

Table 1
Levels of Access to Assignments

Lock setting	Student access
All locked	None
Only individual unlocked	Own assignment
Only group unlocked	Group square
Individual and group unlocked	Each group member's square
Class unlocked	Everyone's square

approach because unlike traditional forms of cooperative or collaborative learning, students working in a shared-work approach are not assigned to specific groups. Instead, each student has access to a space where he or she is responsible for constructing responses to the online assignments. During an initial phase, a student's work is accessible only to that student and the instructor. Then comes a secondary phase in which students can continue to work on the assignments while having access to classmates' work through the communication tensor. Figure 5 shows a sample screenshot of the tensor as viewed by Student 1 in a class with five students, using a shared-work approach with my software. The first assignment is unlocked with hyperlinked squares available for each student, and the second assignment is locked limiting Student 1's access to his or her own work, a hint square, and a class square that the instructor can use for solutions or whole-class discussions. In this shared-work approach, each individual is responsible for completing his or her own assignment, but the tensor allows access to the work posted by all students. It is the responsibility of each student to decide which tensor squares to explore and how to use the viewed work. One of my main goals in this study was to see what use students make of their classmates' work during the secondary phase and to develop an appropriate coding scheme to analyze the data archived by a communication tensor.

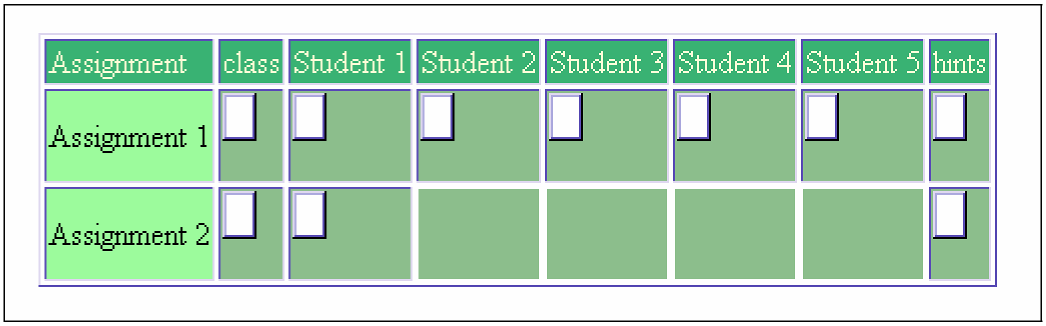


Figure 5. Student 1's view of the tensor in a class with two assignments and five students, operating with a shared-work approach. Assignment 1 is unlocked at the class level and Assignment 2 is locked for an individual phase.

The Small-Groups Approach

A traditional way to get students to work together and share learning experiences is through group work. To provide a comparison for the shared-work approach in the present study, I implemented a second approach that I call a *small-groups approach.* In a small-groups approach, each student is assigned to a group of three or four students. The instructor may choose to have students submit answers in individual tensor squares prior to group work, but at some point, students working under a small-groups approach are required to work together in a single group square.

Figure 6 displays a sample screenshot that shows the view of Student 1 who is a member of Group 1 along with Student 2. Students 3 and 4 make up Group 2. In this figure, the first assignment has been unlocked, allowing full access to all of the work done by the class. The second assignment is set at the group level, allowing Student 1 to view his or her own square, the other group member's square, and the Group 1 square along with the class and hints squares.

Assignment	class	Group 1	Group 2	Student 1	Student 2	Student 3	Student 4	hints
Assignment 1	☐	☐	☐	☐	☐	☐	☐	☐
Assignment 2	☐	☐		☐	☐			☐
Assignment 3	☐			☐				☐

Figure 6. Student 1's view of the tensor in a small-groups section. Assignment 1 is unlocked at the class level; Assignment 2 is locked at the class level but unlocked for groups; and Assignment 3 is locked for an individual phase.

The third assignment is set for an initial individual phase, and Student 1 has access only to his or her own square, the hint square, and the class square.

Purpose of the Study

With the benefits of time and space independence, there is an increasing demand for asynchronous learning activities in higher education. Banchoff's communication tensor and my adaptation provide new possibilities for facilitating such activities. As communication shifts from face-to-face discussions to online forms of communication, there is a need to investigate the ways that students interact. A few researchers have investigated face-to-face collaborative problem solving in mathematics and developed potential frameworks or coding schemes (Artzt & Armour-Thomas, 1992; Stacey & Gooding, 1998). In reviewing the literature, I found only two researchers (Kosiak, 2004; Myers 2002) who attempted to develop or apply coding schemes to online mathematical communications and discuss the quality and nature of online interactions.

The purpose of this study was to teach two sections of College Algebra with my software, using a shared-work approach with one and a small-groups approach with the other and to investigate and compare the quality and nature of student interactions in those sections. In addition, because I had difficulty getting some students to participate in the past, I aimed to investigate factors that influenced participation and interactions.

Research Questions

With these goals in mind, I taught sections of College Algebra using the shared-work and small-groups approaches to online assignments. The courseware that I designed allowed for the implementation of both approaches via a communication tensor. Using data from these two sections, I addressed the following research questions:

1. How does the nature of collaboration and interaction in asynchronous mathematical problem solving using a communication tensor compare between the shared-work and the small-groups approaches?
2. How does the quality of collaboration and interaction in asynchronous mathematical problem solving using a communication tensor compare between the shared-work and the small-groups approaches?
3. What factors influence student participation and interaction in the shared-work and the small-groups approaches?

CHAPTER 2

LITERATURE REVIEW

Research on the topic of online education has boomed over the past two decades with the establishment of such publications as the *Journal of Distance Education* and the *American Journal of Distance Education*. There are quantitative, qualitative, and mixed-method studies on a wide variety of topics. There is a wealth of literature on computer conferencing as well as collaborative learning in mathematics, but I have uncovered a much smaller number of studies focusing on the use of computer conferencing to create collaborative problem-solving environments in mathematics.

In this chapter, I review the literature that is most relevant to the present study. I begin with an overview of research on collaborative learning. In particular, I address literature relating to the rationale behind a push for collaborative learning and empirical evidence concerning its effectiveness. I then review literature relating to the nature of interactions during collaborative problem solving in mathematics, including proposed coding schemes for analyzing interactions. Following the discussion of collaboration, I turn to online mathematics and attempt to show what has been done on asynchronous mathematics learning, recommended best practices, and the use of computer conferencing to facilitate online collaborative problem solving in mathematics. I conclude the chapter with a discussion of research into factors influencing participation in computer conferencing.

Collaborative Problem Solving

A huge amount of literature is available on collaborative or cooperative learning in mathematics and other subjects. Davidson, Reynolds, and Rogers (2001) point out that the terms *collaborative* and *cooperative* do not necessarily mean the same thing. For some authors collaborative learning is less structured than cooperative learning, with more responsibility shifted from the teacher to the students. However, other authors use the terms interchangeably to describe any setting in which students work together on a common task. Three references provide general overviews of collaborative learning.

Rogers, Reynolds, Davidson, and Thomas (2001) edited a volume for the Mathematical Association of America (MAA) that discusses the use of collaborative learning in undergraduate mathematics. The volume contains advice and recommendations from professors who have used collaborative methods in their courses. In the first chapter, Davidson, Reynolds, and Thomas (2001) provide a general overview of collaborative mathematics, including a rationale, a brief summary of literature, and descriptions of several major efforts to implement collaborative learning in college mathematics. I found these authors' listed reasons for using collaborative mathematics helpful in the development of my goals for asynchronous problem solving. Specific benefits include deepening understanding through explanations to others, experience with nonroutine and open-ended problems, exposure to alternative ways of thinking, and opportunities to judge for oneself the validity of mathematical arguments.

Springer, Stanne, and Donovan (1999) conducted a meta-analysis to explore the effectiveness of using small groups in undergraduate courses in science, technology, engineering and mathematics (STEM). They found 383 reports dated 1980 or later on small-group learning in undergraduate STEM courses. They reduced the set to 39 studies by applying two additional criteria: The studies had to be conducted in a classroom setting instead of a laboratory, and the researchers had to report enough statistical information for Springer et al. to compute the standard mean difference effect sizes (J. Cohen, 1988). Cohen's effect size statistic, denoted *d*, can take on values from 0 to 1, and Cohen suggested that .2, .5, and .8 represent small, medium, and large differences, respectively.

The meta-analysis aimed to address two sets of research questions. The first set concerned the effectiveness of collaborative learning over noncollaborative methods with respect to achievement, attitudes, and persistence in STEM courses. The results of the meta-analysis were quite positive in favor of collaboration, showing increases in achievement, attitudes, and persistence in STEM courses. The effect size reported for student achievement based on 37 studies was $d = .51$. The effect size for persistence in STEM courses based on 9 studies was $d = .46$, and the effect size for attitudes based on 11 studies was $d = .55$. All of these differences were in favor of collaborative learning.

Springer, et al.'s (1999) second set of research questions focused on what they call moderators of small-group learning. They found that the achievement effect sizes were higher in studies in which the researcher was the instructor and in studies using experimental and control groups versus single sample designs. But in all cases, the results favored the collaborative approaches. They also compared results at 4-year colleges versus 2-year colleges and found higher achievement effect sizes at 4-year colleges. In particular, they note that the 2-year colleges had an effect size of only .21, which was not statistically significant. They found no differences in effect sizes when comparing genders or academic fields. They found that small-groups were more beneficial to groups that were predominately African American or Hispanic than for predominately White or heterogeneous groups. The most notable result across all of the comparisons was that although the effect sizes varied, collaboration was always favored over noncollaborative methods of instruction.

E. G. Cohen (1994) provides a conceptual review of literature relating to the use of small groups in various subjects. Her goal was to investigate factors that influence success in groups, and therefore she did not look at studies comparing collaborative approaches to whole class or individual approaches. Cohen's primary finding was that the role of group interactions is highly dependent on the nature of the tasks. Her review of the literature revealed that the frequency of interaction within groups rarely correlates with achievement when the groups are working on routine procedural tasks. Interaction does correlate with achievement in many studies when groups are given open-ended, nonroutine tasks. In these studies, the important predictor of success seems to be the level of elaborate explanations given by group members. Cohen also warns that students need help learning to work in groups and that teachers need to be explicit about expectations.

Coding Schemes and Frameworks for Collaboration

In an effort to describe the nature of student interactions that take place in collaborative problem-solving, researchers have developed coding schemes and analytical frameworks to classify student behavior. A framework by Schoenfeld (1985) divides the problem-solving process into episodes during which students work on a particular type of task. Schoenfeld's episodes include *read*, *analyze*, *explore*, *plan/implement*, and *verify*. Artzt and Armour-Thomas (1992) built upon Schoenfeld's work to develop a protocol analysis framework to differentiate between cognitive and metacognitive behaviors during different episodes of the problem-solving process. The Artzt and Armour-Thomas framework divides the problem-solving process into eight potential episodes and designates a predominant cognitive level for each episode. The episodes consist of the five described by Schoenfeld with *implement* and *plan* separated into different episodes and the additional episodes *explore* and *watch and listen*. Artzt and Armour-Thomas classify the *read* episode as predominantly cognitive. They classify the episodes *understand*, *analyze*, and *plan* as predominantly metacognitive and the episodes *explore*, *implement*, and *verify* as a combination of metacognitive and cognitive behavior. They do not assign a predominant cognitive level to the episode called *watch and listen*.

Artzt and Armour-Thomas (1992) applied their framework to the behavior of 27 seventh-grade students who were videotaped during an individual problem-solving session. The researchers based their analysis on the student's achievement test score, the videotaped session, and a stimulated-recall interview. The framework was applied to the students' behavior as observed over 1-minute intervals. The coding resulted in 442 behaviors, with 39% coded metacognitive, 36% coded cognitive, and 25% coded watch and listen. Of the metacognitive behaviors, 36% occurred during exploring and 32% occurred during understanding. Of the cognitive behaviors, 60% occurred during exploring and 24% occurred during reading. Exploring was the most commonly coded episode. In their conclusion, Artzt and Armour-Thomas emphasize the importance of the watch and listen episode noting, "The degree of watching and listening behaviors of students may be the defining variable of whether students are engaged in group interaction at all" (p. 162). If that conclusion is true, one cannot discount the value of what is commonly referred to as "lurking" in asynchronous communication, during which students read the work of others without creating their own posts.

Stacey and Gooding (1998) developed a coding scheme to analyze student behavior in small-group settings by observing fifth- and sixth-grade students working on problems designed to reduce misconceptions concerning fractions. Using turn taking as the unit of analysis, Stacey and Gooding developed a coding scheme with categories labeled *asking questions*, *responding*, *directing*, *explaining with evidence*, *thinking aloud*, *proposing ideas*, *commenting*, and *refocusing discussions*. Using a pretest and posttest, Stacey and Gooding looked for patterns of behavior leading to improvements in achievement. They identified five out of seven groups as effective because all group members improved their scores.

The two ineffective groups had the fewest numbers of turn-taking behaviors and less interaction. The ineffective groups had 56% of their turn taking coded as thinking aloud, as opposed to 36% in the effective groups. The ineffective groups had lower percentages than the effective group in every other category except commenting, in which they had 2.3% as opposed to 1.6% for the effective groups. Stacey and Gooding (1998) also found that individual improvers in each group had higher instances of explaining with evidence and repeating each other's statements.

Myers (2002) offers an analytical framework specifically designed for online collaboration. The framework contains participative, social, interactive, and heuristic dimensions. Because messages can contain multiple themes or ideas, Myers analyzed *units of meaning*. A unit of meaning is "a consistent idea or theme in a message" (Kosiak, 2004, p. 68). For the participative dimension, Myers counted the total number of posts and units of meaning per student and identified whether or not each unit was task specific. For the social dimension, Myers simply noted whether a unit of meaning was social instead of directly relating to problem solving. She denoted a unit of meaning interactive if a student made the unit in response to another student's post or if another student responded to the unit, and she coded the units with Stacey and Gooding's (1998) scheme. Finally, for the heuristic dimension, Myers used a classification of episodes devised by Garofalo and Lester (1985): *orientation*, *organization*, *execution*, and *verification*.

Myers (2002) applied her coding scheme to the behavior of four small groups, each containing three middle school teachers taking an online mathematics course for teachers organized by a nonprofit organization in New York. The groups were assigned to work on a problem using their choice of synchronous chat rooms or asynchronous discussion boards. Myers's analysis of the groups' activities revealed wide variety in terms of medium selected, quantity of posts, and nature of interactions. One group's total activity consisted of posting three asynchronous messages. Not surprisingly, that group received the lowest score of 13 out of 20 on the assignment. Another low-scoring group only used the chat rooms but spent most of their time on social discussions not related to the problem. A third group appeared to work independently and used asynchronous and synchronous communication to coordinate their efforts. This group was successful on the assignment, earning 19 out of 20 points. Finally, the only group to earn a perfect 20 out of 20 made extensive use of both communication tools and displayed a high level of online interactions. In her conclusions, Myers argues that the asynchronous discussion boards did not produce interactive discourse and did little to advance the problem-solving process.

Suggested Best Practices for Asynchronous Learning

Several authors, including Chickering and Ehrmann, (1996) and Sands (2002), offer advice for the use of online instruction but provide no empirical evidence for their claims. Chickering and Ehrmann give seven principles for success: Good practice encourages contact between students and faculty, develops reciprocity and cooperation among students, uses active learning techniques, gives prompt feedback, emphasizes time on tasks, communicates high expectations, and respects diverse talents and ways of learning. Specifically addressing hybrid classes, Sands offers advice on making connections between the online and the face-to-face components. He stresses that the online component needs to receive more attention as it is the novel experience for students and teachers. Sands recommends that instructors start small and work backwards from a set of goals, aim for interactivity, prepare for a loss of power and distribution of time demands throughout the week, plan to spend class time connecting to the online work, and be prepared to help students adjust to working online. These are recommendations that I tried to keep in mind as I incorporated online components into my courses.

Studies of Asynchronous Undergraduate Mathematics

There is little literature on the use of asynchronous communication tools in mathematics courses. In a few studies, the researchers assumed that online mathematics should provide interaction and collaboration among students, and in others, the researchers take the stance that online mathematics should be more independent, with students learning from algorithmically generated assignments and assessments.

Satler (2005) provides an enlightening look at distance education used for remedial courses at 2-year colleges. She presents results from a national survey completed by members of the American Mathematical Association of Two-Year Colleges (AMATYC), who had taught online or hybrid remedial mathematics courses. In addition to demographic data showing that the average teacher was female and nearly 50 years old, Satler’s survey examined the types of software and interactions used by the participants as well as their perceptions of its usefulness. Most teachers reported that they primarily used quizzes, exams, textbooks, and software tutorials. They reported using the tools more often and perceived them as more useful than communication tools such as chat rooms and discussion boards. These results support my view that many teachers of distance education courses approach them more as independent study in which students are presented with information, lacking the rich collaboration among students and between students and teachers that distance education researchers recommend.

Allen (2001) presents an overview of the development of an online Calculus I course at Texas A&M University and includes a discussion of the activities he used and a section on evaluating students' perceptions. He states that online mathematics needs to be task oriented and "for now quizzes and worksheets seem to be working" (p. 21). He asserts that the online quizzes can be randomly generated and provide immediate feedback, making them superior to assignments in traditional classes. Over several semesters, the developers surveyed students to seek opinions on several aspects of the course ranging from comprehension and comfort level to course management and delivery. Allen claims that the project was successful because the students in the online courses did well on traditional exams and in successive calculus courses. Despite being online, the courses had designated times for students to meet together in a computer lab. The model presented by Allen is based on a traditional lecture format with no mention of discussions or chat. The students met in a lab and learned from the computer. A teacher was present to answer questions, grade assignments, and hold office hours. Thus, this course was not a typical example of distance education, and it did not represent a collaborative approach. Interestingly, Allen reports that most students began to form collaborative groups in the lab on their own.

Rouhani (2005) and Rho (2000) present case studies of students' understanding of the concept of functions when taught in an online environment. Rouhani describes an online college algebra course at a 2-year college. She reports that there was minimal use of online discussions and a low level of interactions. In fact, she recommends that future research should look at ways to encourage more interactions in online mathematics courses. Rho looked at the use of computer conferencing but in an atypical environment. She recruited four students to study functions in a virtual environment. They worked in pairs on a series of problems designed around common misconceptions. Rho reports that the students used synchronous and asynchronous communication. Each of the students showed considerable improvement in his or her conceptual understanding of functions. Rho argues that the study shows that students can learn mathematics in a virtual environment; however, the study was limited in that there was such a focused effort on teaching one concept to four students.

McCallum (2005) compared three forms of college mathematics course delivery: standard 15-week classes, condensed classes that met twice as long for 8 weeks, and accelerated classes that met for 8 weeks with half of the work done online. McCallum used observations and interviews with students and instructors to create descriptions of the courses. His research questions focused on differences in student learning, student satisfaction, and instructional satisfaction, and he reports that each delivery method had its own strengths and weaknesses. The most relevant result for the present study was that the accelerated class that McCallum observed was supposed to have a strong online component but did not. The teacher tried to fit as much material into the face-to-face class as possible, contrary to Sands’s (2002) recommendation that the online components receive primary attention.

Hodges (2006) investigated self-efficacy in an asynchronous college algebra and trigonometry course at the Virginia Polytechnic Institute and State University. He used an experimental and control group to test the effect of motivational e-mails from the instructor on self-efficacy. In addition, Hodges looked for correlations between self-efficacy and performance. The e-mail messages had no significant impact on self-efficacy. A regression model analysis of self-efficacy versus achievement revealed a significant positive correlation between the two factors, but the effect was small, with self-efficacy explaining only 9% of the variance in achievement.

Computer Conferencing in K-12 Mathematics

A few researchers have explored using computer enhanced collaboration with elementary and junior high school students. Nagai, Okabe, Nagata, and Akahori (2000) report on an experiment conducted with students from three junior high schools in Japan. A control group meeting face-to-face and a group using computer conferencing completed an assignment based on two open-ended problems. For each problem, the students were to work individually for 1 hour, followed by 1 hour of collaboration. For one problem, there was no significant difference, but for the other, the students using the computer discussion boards for collaboration found 14 solutions versus the control group’s 4. Nagai, Okabe, et al. conclude that computer conferencing is beneficial for dealing with problems that have multiple solutions and strategies.

Nagai, Shiraki, Koshikawa, and Akahori (2001) report a follow-up study in which the software had a feature described as a knowledge map. The knowledge map allows students to create graphic maps that show the connections and relationships between messages, whereas in ordinary discussion forums, messages can become diffused. To evaluate their software, Nagai, Shiraki, et al. used two second-grade classes. Students in one classes worked mathematics problems with the knowledge map software, and the other class used a more typical tree-structured discussion board. Achievement was the same in each group. Student comfort was less with the new software, but the students using the knowledge map created much larger sets of related messages. Nagai, Shiraki, et al. believe that students need time to get used to the software. This observation provides a strong reason to study a course for an entire semester as I did in the present study.

De Corte, Verschaffel, Dhert, and Vandeput (2002) present a study similar to that of Nagai, Okabe, et al. (2000). De Corte et al. describe a controlled experiment to promote collaborative learning with the computer-conferencing software Knowledge Forum. The software primarily uses discussion threads. De Corte et al. examined two fifth-grade and two sixth-grade classes. Teachers taught the lessons, and students worked together using the software in the experimental classes, whereas the control classes worked face-to-face. After a pretest and posttest, the sixth graders showed a positive effect on their problem-solving skills, but the fifth graders did not. De Corte et al. found no significant increase in the students' pleasure and persistence in solving mathematics application problems or in their attitudes toward the teaching and learning of mathematical problem solving. The students did show a significant increase in their attitudes toward collaboration in general and toward computers.

Hurme and Jarvela (2005) also report on a study involving the Knowledge Forum software. The purpose of their study was to investigate the use of metacognition by 16 thirteen-year-old Finnish ninth graders on problem-solving activities administered with Knowledge Forum. The assignments investigated for the study included a geometry task and a probability task. The data consisted of the notes posted by students in the software. Of 188 notes, Hurme and Jarvela classified 72 as mathematical knowledge, 17 as mathematical questions, 33 as relevant, and 66 as trivial. They also classified the notes based on the different aspects of problem solving present and metacognition used. They report evidence that the students regulated their problem solving showing evidence of metacognition and used a large number of trivial notes for social regulation. An interesting question posed for future study was “Do students utilise their peers’ thinking while solving problems” (p. 70). I addressed this question in my study.

Pragnell, Roselli, and Rossano (2006) describe a controlled experiment designed to see if fifth-grade students could learn geometry as well in an e-learning environment as in a face-to-face setting. Both the control group and the experimental group followed a model in which students worked in small groups, and the group grade was determined by adding the individual grades. The control group worked face-to-face with a teacher present for guidance. The experimental group used computer software that offered tutorials as well as allowing for collaboration. The experimental group was completely self-contained, getting all instruction and assessment from the computer and a virtual tutor. The primary conclusion was that the computer-based group achieved just as well and had collaboration that was more active.

Computer Conferencing in Undergraduate Mathematics

Only after an extensive search of literature was I able to uncover a small number of reports focusing on the use of computer conferencing in undergraduate mathematics. I have already discussed Myers’ (2002) study of small-group collaboration and her analytical framework for analyzing online discussions. In this section, I highlight the other studies on the use of computer conferencing in undergraduate mathematics courses.

One report comes from Banchoff (2005) in the form of a paper presented at a conference in Daejeon, Korea. This paper describes the communication tensor, which I introduced in chapter 1, and an accompanying Java applet for creating dynamic two-dimensional and three-dimensional graphs. The majority of the paper describes Banchoff’s use of the software in his courses, and it concludes with some remarks on evaluation. Every semester, Banchoff collects student responses to open-ended evaluation questions about the course and the software. He presents samples of student communication from the courseware and selected responses to the evaluation questions in an appendix to the article. Student responses to the question “How often did you look at the work of other students?” were fairly evenly distributed between usually, occasionally, and rarely in two of the three multivariable calculus classes. In a 2003 class, 61% responded with occasionally. In all three classes, the majority of the students responded that they did look at some students’ work more than others did and that they were comfortable sharing their work online. One question of interest in my study was how students find and use information in the tensor. Banchoff provides verbatim feedback from a few students on how they decided which students’ work to use. Responses ranged from searching the tensor from left to right to figuring out who was most likely to have a correct response.

In a presentation at the National Distance Education Conference, Lamb and Klemm (1997) gave several compelling reasons to use computer conferencing in undergraduate mathematics. They argue that computer conferencing can be structured to ensure that students interact with peers. They claim that computer conferencing is more tangible and visible than face-to-face interaction, it lessens the feelings of embarrassment that some students might have about asking questions, and it makes thoughts more explicit through writing. Lamb and Klemm add that computer conferencing allows for more time for “research, reflection, and integration of mathematical concepts” (p. 90). I am skeptical about the claims that computer conferencing lessens feelings of embarrassment and can be structured to ensure student interaction. In a report addressing the use of asynchronous computer conferencing at the Georgia Institute of Technology, Guzdial, Lodovice, Realf, Morley, and Carroll (2002) report that participation in many mathematics, science, and engineering courses has been low, with a large percentage of students choosing to take a grade of zero instead of participating. I do agree that the capabilities for collaboration along with time for research and reflection are major strengths of this form of communication, but participation is a challenging issue faced by facilitators.

In another article, Klemm (1998) presents advice on increasing participation in computer conferencing. He writes that various psychological and social factors influence student participation, but teachers must not allow students to become “lurkers” by reading the work of others and not participating. His eight suggestions to increase participation are the following: require participation, form learning teams, make the activity interesting, do not settle for just opinions, structure activity, require a hand-in assignment, know what you are looking for and involve yourself to make it happen, and use peer grading.

Like Klemm, Smogor (2002) calls for the use of computer conferencing in mathematics without offering empirical evidence. Smogor points out advantages and pitfalls from personal experience. The advantages include making students more active learners, enforcing reflection through writing, being available around-the-clock, and learning from others. In addition, he believes that a lack of mathematics symbols can potentially make students more aware of the importance of terminology and symbolization. This heightened awareness of terminology and symbolization requires students to think of mathematics as a way of communicating with others. Potential pitfalls or possible problems include student resistance to nontraditional methods, discomfort or unfamiliarity with computers, and apprehensions toward making work public.
In addition to pointing out advantages and potential problems, Smogor lists keys for successful implementation. He asserts that teachers must acknowledge student effort in a tangible way such as grades. He also suggests providing a model of what a good conference posting looks like. He also suggests that instructors provide occasional feedback on student performance, and he warns that the instructor should not dominate the discussions.

Simonsen and Banfield (2006) investigated the role of the instructor in promoting mathematical discourse in an asynchronous statistics course for teachers. They analyzed the transcripts to classify types of interventions used by the instructor and to determine which types were more effective. The classification of interactions included resolve, validate, redirect, expand, and withhold. Simonsen and Banfield recommend that instructors withhold as much as possible so that students can take responsibility for the discussions. There seems to be a delicate balance for instructors. If instructors do too much, participants may not contribute in meaningful ways. If instructors do too little, participants may feel neglected or view the discussions as insignificant.

Kosiak's (2004) dissertation, which was directed by Simonsen, provides a detailed investigation into the use of computer conferencing for mathematics. Kosiak studied a course in which students completed online problem-solving activities modeled after the Treisman workshops (Fullilove & Treisman, 1990) and a control group that completed the same activities individually with paper and pencil. The workshops were designed by Uri Treisman at the University of California at Berkeley to improve the performance of African American students in calculus courses. As part of the workshops, students worked in groups of 5 to 7 students on "worksheets containing carefully constructed, unusually difficult problems" (p. 468). Kosiak argues that she tried to construct problems similar in nature to those used by Treisman. According to Fullilove and Treisman, these types of problems include common exam problems that rarely appear in homework, problems designed to reveal deficiencies or misconceptions, problems that introduced motivating examples or counterexamples, problems designed to improve the use of mathematical language, and problems designed to master "the computational tricks and shortcuts known to many of the best students but which are neither mentioned in the textbook nor taught explicitly by the instructor" (p. 468).

Kosiak asked two main types of research questions. First, she wanted to explore the nature of interaction in the small online groups. She coded the online discussions using the scheme developed by Stacey and Gooding (1998). Kosiak found that most of the messages were thinking aloud. Other types of messages she identified included responding, explaining with evidence, and questioning. Kosiak also coded the messages using a model for interaction developed by Gunawardena, Lowe, and Anderson (1997). This coding scheme looks for evidence of co-construction of knowledge. Kosiak reports that in 19 of 24 problem-solving episodes there was such evidence. In addition to looking at the nature of collaboration, Kosiak ran a series of statistical tests on achievement. Out of 12 hypothesis tests, only the correlation between a student's number of high-level messages on the Gunawardena et al. scale and the student's final exam score was significant. This correlation failed to be significant when Kosiak applied a Bonferroni method to account for multiple comparisons.

Factors Influencing Participation in Online Education

One of the research questions in this study concerned factors that influence student participation and behavior in online assignments, and I therefore conclude this review of literature by looking at what some others have had to say about online participation in general. Although I found no studies that directly addressed this question in mathematics, researchers studying other subjects have addressed the topic.

For more than two decades, Linda Harasim (1986) has been a strong supporter of using computer conferencing in higher education. She discusses a pilot program at the Ontario Institute for Studies in Education in which 20 graduate students and 21 practicing teachers took an online course on sex equity and gender issues in relation to computers in education. Describing factors listed by the learners as advantages and disadvantages to using the computer conferencing, she lists ten advantages and five disadvantages. Some of the advantages included the removal of distance and time constraints, 24-hour access, and links between work and learning. Other advantages included equal opportunities to contribute to discussions without being dominated by a few individuals, a focus on content over the person presenting it, opportunities to react spontaneously, selective access to a wealth of information from the groups, group support including moral boosting, input from diverse sources, and archival of the transcripts. The disadvantages cited by Harasim included a loss of visual cues, the difficulty and time needed for reaching a group consensus online, health problems such as eyestrain and backstrain related to using computers, the cost of the necessary technology, and a variety of technical problems.

Bullen (1998) investigated student perceptions of factors influencing participation in an online computer science ethics course at a Canadian university. From a review of the literature, including Harasim (1986), Bullen developed a conceptual framework asserting that factors influencing online participation fall into four categories: attributes of the medium, design of the learning activities, student dispositional factors, and student situational factors. He used this framework to analyze and organize the results obtained from interviewing 13 students.

Under attributes of the medium (Bullen, 1998), the students discussed several factors that they viewed as having both positive and negative effects on participation. For instance, time-independence was a positive factor allowing students to work at their own pace on their schedule, but it was also a negative factor because they had to manage their time effectively. The text-based nature of the communication was also seen by the students as a positive and negative factor. Some students liked the opportunity to compose their thoughts and reread them before posting, whereas others found communicating through text impersonal and felt detached from their classmates. As for the permanence of the postings, some students liked having the ability to look back at the discussions, whereas others did not like the fact that everything they posted would remain available throughout the semester.

The students in Bullen's (1998) study also reported that certain aspects of the course had positive and negative effects on their participation. The instructor required the students to participate by counting participation as 15% of their total grade. Some students admitted that this requirement was a motivating factor, but others said that it occasionally led to participation that was not meaningful. Bullen writes, "They felt that they were often simply restating what had already been said by other students" (p. 171). This complaint is certainly a potential problem when students are working in groups for problem solving. Once someone has presented a correct solution, students may feel that there is nothing left to contribute.

The students in Bullen's (1998) study cited a number of situational and student dispositional characteristics that they perceived as influencing participation. Situational characteristics included other classes and an unsuitable home study environment. Dispositional characteristics included a strong preference for face-to-face learning, being introverted, and feeling a need for more teacher direction.

Zafeiriou, Nunes, and Ford (2001) present a study of student perceptions similar to Bullen's (1998). They selected undergraduate and graduate students who were taking courses at the University of Sheffield in Information Studies and Management. All of the participants were required to have used synchronous and asynchronous modes of computer conferencing and to have experience with online group-based projects. Using qualitative analysis from semi-structured interviews, Zafeiriou et al. report on students' perceptions of participation. The students reported that judging group members' participation was more challenging in an online setting than in a face-to-face setting. They argued that meaningful participation required a combination of quantity and quality. Several students mentioned that they preferred short posts with a few useful comments to long rambling messages.

Zafeiriou et al. (2001) present eight factors that they call intervening conditions. Among the intervening factors, most are student characteristics, including familiarity with computers, familiarity with the subject, familiarity with the particular software, typing skills, and level of interest. The others include group size, technical problems, and group attendance. Obviously, a student has a greater chance of success and a higher likelihood of participating if he or she is comfortable with the technology and the course content, and having to type all responses can be time consuming and problematic in mathematics. As for the group factors, Zafeiriou et al. merely note that large groups can be difficult to manage online, and most students mentioned that at least one group member did not participate in many of the online activities.

The common theme among these three reports (Bullen, 1998; Harasim, 1986; Zafeiriou et al., 2001) is that there are many factors that influence and potentially hinder participation in online learning activities. Problems related to accessing and using technology are obviously going to influence students who are using computers and the Internet to complete assignments, and these factors are mentioned in all three reports. Attributes of the medium and attributes of the students were also common factors reported as influencing participation. One factor that the literature has not addressed that could influence participation in online learning activities is the nature of the course. Using an online discussion board to discuss a piece of literature in an English class is a very different process than using a communication tensor to facilitate mathematical problem solving. Even within a subject, such as mathematics, there is much diversity across courses. For these reasons, it is important to consider the nature of the course when investigating factors that influence participation.

Summary

The review of literature provided evidence that collaboration is beneficial in mathematics courses (Springer, Stanne, & Donovan, 1999) and that there are many factors that influence participation in online activities (Bullen, 1998; Harasim, 1986; Zafeiriou, Nunes, & Ford, 2001). Several studies of online mathematics courses indicate that instructors have used an individualized approach with a lack of interactions (Allen, 2001; McCallum 2005; Rouhani 2005; Satler 2005). Rho (2000) looked at computer conferencing, but her study did not take place in the context of a course. A few studies consisted of short-term experiments with elementary and junior high school students (Nagai, Okabe, Nagata, & Akahori, 2000; Nagai, Shiraki, Koshikawa, & Akahori, 2001; De Corte, Verschaffel, Dhert, & Vandeput, 2002), and only a handful of empirical studies have been conducted to investigate the use of computer-conferencing software in undergraduate mathematics, with the most notable studies being two dissertations from Montana State University (Myers, 2002; Kosiak, 2004). The students in Myers' study mainly collaborated through a synchronous chat tool. Kosiak's dissertation provided the deepest look into the use of asynchronous discussions for collaborative mathematics that I could find, but she leaves much room for investigation. In particular, Banchoff's (2005) communication tensor provides a novel approach to online collaboration that needs to be explored. The mathematics, mathematics education, and distance education communities could certainly benefit from more studies on the use of computer conferencing in the teaching and learning of mathematics.

CHAPTER 3

METHOD

This study was designed to investigate the quality and nature of student work and interaction, as well as factors influencing participation and interaction, during online mathematics assignments administered with a course management tool called a communication tensor. In this chapter, I discuss the methodology used to address the research questions. I begin with a discussion of the college and the College Algebra course used in the study. Then I describe the participants and explain the data collection and analysis methods, including coding schemes I developed for analyzing asynchronous mathematical problem solving.

Setting and Course Description

This study involved two sections of College Algebra that I taught during the fall semester of 2007 at a large multi-campus 2-year college located in a metropolitan area of the southeastern United States. The college serves a diverse population of commuter students with a variety of associate degree and transfer programs. During the study, the college had approximately 21,000 students, with over 13,000 minority students and over 3000 international students representing 147 countries. The campus that served as the setting for this study had approximately 4500 students, and just over 400 of them were enrolled in 16 sections of College Algebra. Fourteen of those sections met in a traditional face-to-face setting, and two were hybrids that met one day a week and had online components.

College Algebra is one of two introductory, three-semester hour courses offered at the college. Students majoring in a field requiring Precalculus or other advanced mathematics courses are required to take College Algebra, whereas other students are encouraged to take Introduction to Mathematical Modeling. Students can enroll in College Algebra by achieving a score of 37 or better on the Computerized-Adaptive Placement Assessment and Support System (COMPASS) algebra examination developed by the American College Testing Program (ACT, 2008). For students in need of remediation, the college offers three noncredit mathematics courses: Essential Algebra, Beginning Algebra, and Intermediate Algebra. To be allowed to register for College Algebra, any student scoring below 37 on the placement exam must pass Intermediate Algebra. Intermediate Algebra is the only remedial course required for students scoring between 30 and 36, whereas a student scoring below 30 must take either Essential Algebra or Beginning Algebra as a prerequisite. Essential Algebra contains all of the content covered in Beginning Algebra and a review of basic arithmetic. Students scoring between 17 and 29 can take Beginning Algebra and avoid the arithmetic review. Thus, there are a variety of ways that a student can satisfy the prerequisites for College Algebra.

The college catalogue describes College Algebra as a functional approach to algebra, focusing on linear, quadratic, polynomial, rational, exponential, and logarithmic functions. Each College Algebra instructor is responsible for constructing his or her own course syllabus, assignments, and assessments, but all instructors are required to cover the topics on a common course outline. At the time of this study, instructors could select from two textbooks (Blitzer, 2007; Hornsby, Lial, & Rockswold, 2007). I used the textbook by Blitzer.

The Participants

To ensure the implementation of the shared-work and small-groups approaches, this study involved two face-to-face sections of College Algebra for which I was the instructor. Both sections met on Mondays, Wednesdays, and Fridays from August 20 through December 9. One section met from 8:00–8:50 am, and the other met from 12:00–12:50 pm. The mathematics department chair developed the class schedule, and students were allowed to register for any of the 16 sections of College Algebra until each class was filled. When I began this study, I was especially interested in investigating the ways individual students use the contents of their classmates' tensor squares; therefore, I decided to use the shared-work approach with the 8:00 am section, which had more students.

In the shared-work section, one student withdrew without ever logging into the online software, leaving a section of 27 students, which are referred to in this study by the pseudonyms SW1 through SW27. Twenty-two students completed the course by taking the final exam. Four students officially withdrew, and 1 quit attending class or participating in the online assignments. Each of these 5 participated in at least one online assignment, and they were included in the study when possible. Of the 27 students, 11 were female, and 16 were male. The students' ages ranged from 18 to 31, with an average age of 21. All 22 students who completed the class returned a demographic survey. According to that survey, the average student worked 26 hours a week while taking four to five classes at the college, and all but 4 of the students reported having a computer with Internet access at home.

The small-groups section started with 24 students, referred to in this study by the pseudonyms SG1 through SG24. Eighteen students completed the course by taking the final exam. Three students officially withdrew, and 3 quit attending class or participating in the online assignments. Twenty-one students contributed at least one online post and were included in the study when possible. The section had 15 females and 9 males, and the students' ages ranged from 17 to 35, with an average age of 20. The 17-year-old students were joint-enrollment students, taking the course for college and high school credit simultaneously. Of the 18 students who completed the course, 15 returned a demographic survey. According to that survey, the average student worked 22 hours a week while taking four to five classes, and all but 3 reported having a computer with Internet access at home.

Some of the students in this study had taken mathematics courses, including remedial courses and introductory level collegiate courses, prior to enrolling in my class. Table 2 contains descriptions of the students' most recent post secondary mathematics experience. It should be clear from the previous descriptions and the information in Table 2 that a large percentage of the students in both sections had previously been unsuccessful in mathematics courses and were balancing busy schedules.

Table 2

Percent of Students by Section with Recent Post-Secondary Mathematics Experience

Previous experience	Shared work (n=27)	Small groups (n=24)
None	30	33
Passed Intermediate Algebra[a]	33	33
Failed to earn a C or better in College Algebra	26	25
Failed to earn a C or better in Introduction to Mathematical Modeling	4	0
Earned a C or better in Introduction to Mathematical Modeling	4	8
Earned a C or better in Quantitative Reasoning at another college	4	0

[a]To pass Intermediate Algebra, a student must earn a C or better in the course and pass an exit exam.

Online Facilitation

The approach to the online assignments was the only intentional difference in the facilitation of the two sections of College Algebra used in this study. I conducted both sections with similar lectures and face-to-face activities. I assigned the same homework from the textbook and used similar quizzes and exams. Both sections were assigned the same set of online problems (Appendix A), but the approach to collaboration was different. In the shared-work section, I created individual tensor squares for each student (see p. 11), and I used the time lock to divide the assignments into an initial and secondary phase. For the initial phase, I typically gave students 1 week to work on assignments privately. During the secondary phase, which also typically lasted 1 week, students could continue to work on their assignments with access to classmates' work in the tensor.

The facilitation of the small-groups approach evolved over the semester. Initially, I created only group squares in the tensor and gave the students approximately 2 weeks to work on the assignments in groups. After the first five assignments, I decided that I had to do something to increase participation. As in the shared-work section, I created individual squares and required students to post privately during an initial phase. In the small-groups section, however, students worked together in group squares during the secondary phase.

I also altered the group composition twice during the semester (Table 3). Using the initial roster, I divided the section into seven groups of 3 or 4 students each. During the first class, I offered the students a chance to change into groups of their choice. Three students took that offer, and I placed them in the same group. After the first three assignments, I knew who had likely withdrawn, and I rearranged the students into five new groups in an attempt to have as many participating students in each group as possible. In the new arrangement, I kept the 3 students who had asked to work together in a group, and I kept the 4 joint-enrollment students together. The students worked in those five groups for Assignments 4 to 13. For Assignments 14 to 16, I reorganized the students into pairs because, like Myers (2002), I had noticed that in most of the groups, pairs of students seemed to be collaborating more with each other than with other

Table 3
The Composition of the Small Groups

Group Number	Member Pseudonyms
	Assignments 1 to 3
1	SG3, SG5, SG6, SG8
2[a]	SG9, SG10, SG11, SG16
3	SG1, SG13, SG14, SG15
4	SG2, SG4, SG7, SG21
5	SG18, SG20, SG22, SG24
6[b]	SG12, SG17, SG19, SG23
	Assignments 4 to 13
1	SG8, SG18, SG24
2	SG5, SG6, SG22
3	SG10, SG11, SG16, SG4
4	SG7, SG13, SG14, SG15
5[b]	SG12, SG17, SG19, SG23
	Assignments 14 to 16
1	SG4, SG7
2	SG5, SG23
3	SG6, SG12
4	SG8, SG24
5	SG10, SG16
6	SG11, SG13
7	SG15, SG19
8	SG17, SG18

[a]Students SG10, SG11, and SG16 were the students who asked to be in a group together.
[b]The 4 joint-enrollment students.

group members. With only two students in each group, I also removed the initial phase of the assignment.

The Online Tasks

With a set of goals for the course that included communicating, sharing ideas, and working with multiple representations, I used a variety of mathematical tasks in the online assignments. I assigned 40 problems over 16 assignments, presented as four sets. Assignments 1 to 5 were intended as an introduction to functions; Assignments 6 to 9 focused on linear functions; Assignments 10 to 13 focused on quadratics, polynomials, and rational functions; and Assignments 14 to 16 focused on logarithmic and exponential functions. The complete collection of assignments is in Appendix A.

O'Daffer, Charles, Cooney, Dossey, and Schielack (2005) define a problem as a situation that meets the following conditions:

a. It involves a question that represents a challenge for the individual.
b. The question cannot be answered immediately by some routine procedure known to the individual.
c. The individual accepts the challenge. (p. 36)

The second condition, in particular, is not met by many tasks that we have commonly come to call problems. For instance, solving a quadratic equation does not meet this criterion if a student knows how to use the quadratic formula. I attempted to create a set of problems that met these three conditions as well as I possibly could.

The types of problems that I used in the assignments can be described as fitting six general categories. Eight of the problems were contextual, using real-world phenomena such as projectile motion. Most of these problems were closed-ended with single correct answers, but they required more than the use of a single memorized procedure. Most of the contextual problems also required students to deal with a variety of representations including equations, graphs, and tables. For example, the first problem of Assignment 7 required students to solve an applied word problem using numerical, algebraic, and graphical methods. I also asked the students to explore multiple representations on five closed-ended noncontextual problems. These problems required a student to perform tasks similar to those found in many traditional textbooks, but they also emphasized viewing concepts through more than one representation. For example, the problems in Assignment 4 asked the students to investigate translations by completing numerical tables and plotting the results. Two other problems, 5.1 and 5.2, were closed-ended and noncontextual without emphasizing multiple representations.

Two of the problem types required the students to deal with open-ended situations. One type of problem asked the students to respond to questions with multiple correct answers and solution strategies. For instance, several assignments asked the students to produce reasonable formulas for given graphs. Because the graphs did not contain scales, infinitely many solutions were possible. Five of these open-ended assignments, Problems 6.1, 6.2, 6.3, 10.1, and 10.2, were taken from Cooney, Sanchez, Leatham, and Mewborn (n.d.). Another type of open-ended assignment used in this study was problem posing. For instance, in the first problem of Assignment 11, I asked the students to solve a contextual problem involving the area of a rectangle. After working on the problem, the students were supposed to pose a similar problem and solve a classmate's problem.

Many of the problems required the students to construct and post graphs with a Java applet that I wrote for use with the course management system, and I designed five of the problems specifically to explore concepts with parameters that the students could adjust dynamically in the graphing applet with sliders. For example, Assignment 2 asked the students to explore graphs and determine the domain and range of $y = mx + b$ and $y = \sqrt{x - a} + b$. The Java applet allows users to alter the parameters while viewing the effects on the graphs. A complete solution would include generalizations covering all cases of the parameters.

Eight of the problems fit a category that I call *individualized parameters*. For those assignments, the students worked similar problems, but each student used formulas made by inserting his or her own unique set of parameters. For many problems, I asked the students to replace the parameter *n* in formulas with the sum of the last two digits of their student identification numbers. I also used the students' birth months to assign parameters. This procedure created a situation in which each student had to solve his or her own unique problem with access to classmates' work on similar problems.

Data Collection

The sources of data for this study included the student work archived in the communication tensor, the log files containing information on which links a student followed in the tensor, open-ended questionnaires, and interviews with 13 students. In this section, I provide a rationale for and description of each source of data. Table 4 shows a timeline for the data collection.

Table 4

Data Collection Timeline

Month	Activity
August	Assignments 1 to 3 completed
September	Brief survey administered
	Assignments 4 to 9 completed
October	Assignments 10 to 13 completed
	Extended survey administered
November	Assignments 14 to 16 completed
	Demographic survey administered
	Interviews conducted

The Online Data

The first research question explored in this study concerned the nature of interactions by students using the tensor, and a major goal was to develop a coding scheme to analyze the archived data and address that question. The second research question was similar but focused on the quality of the interactions and student work. As I discussed in chapter 2, several researchers have worked to develop analytical frameworks or coding schemes to describe interaction and collaboration in face-to-face (Artz & Armour-Thomas, 1992; Stacey & Gooding, 1998) and online settings (Myers, 2002), but the communication tensor and the shared-work approach provide a different setting than other forms of communication. To my knowledge, no one has attempted a detailed analysis of the student work and viewing patterns for a course using a communication tensor and a shared-work approach. To address these issues, I needed to make a detailed analysis of the student work and viewing logs. The archived material included the transcripts from each student's tensor square for 16 assignments and a log file for each student that contained time-stamped entries showing each time that a student clicked on a tensor square or created, modified, or deleted a post. A separate log file also stored the content of each post, which provided access to any modified or deleted data.

Open-Ended Surveys

The archived data from the online software provided considerable evidence about what occurred in the two classes, but it provided little information from the students' perspectives. To gain additional information about the way the students used the software and to address the question of what factors influenced the nature of interaction and participation, I asked the students to complete a series of open-ended surveys.

After the first three assignments had been completed, the students were given a three-question survey, referred to as the brief survey. The questions differed for each section because of the different approaches to the assignments. Students in the shared-work section were asked to describe how they did or did not find viewing classmates' work helpful, why they did not attempt an assignment if applicable, and why they might have failed to meet initial due dates

(Appendix B). Students in the small-groups section were asked to explain whether they felt their group was successful, what I could do to increase participation and performance, and why they might not have done an assignment (Appendix C). Twenty-two students completed the brief survey in the shared-work section, and 15 students completed the survey in the small-groups section.

Following Assignment 13, I gave all students an extended survey concerning their participation and experiences with the tensor. As with the brief survey, the different sections had slightly different questions tailored to the approach to collaboration implemented in the class (Appendices B & C). In total, 18 students completed the extended survey in the shared-work section, and 15 students completed the extended survey in the small-groups section. I also administered a brief demographic survey to obtain information about computer access, hours worked, and other classes taken (Appendices B & C).

Student Interviews

Using the guides in Appendices D and E, I conducted semi-structured interviews with 13 students on their experiences with the online assignments. I applied purposeful sampling to select participants at various levels of ability and participation. After the classes had completed the first half of their assignments, I used the online work to select students with a range of scores obtained using the rubrics in Appendix F. I had originally intended to interview 6 students from each class grouped into high, middle, and low performance categories, but one low performance student who had previously been unable to schedule an interview was later able to participate, providing 7 interviewees in the shared-work section. I interviewed each selected participant in my office, and to accommodate the scheduling needs of the students, I interviewed students in pairs on three occasions. The pairs of students interviewed included SW25 with SW18, SW2 with SW12, and SG6 with SG12.

All participants in the study agreed to participate by giving signed consent. I informed all students that their participation or failure to participate in the study would have no impact on their course grade. The online assignments were a part of ordinary classroom procedures required by all students, but I informed each student that he or she did not have to agree to allow me to use his or her data in the study. No additional benefits were given to the students who participated in the interviews.

Table 5 contains the age, hours employed per week, and the number of courses that each interviewee was taking. Table 5 also contains performance information including the number of problems attempted, the number of total posts made by the student during the semester, and the student's average score per problem.

Table 5
Descriptive Data for the Interviewees

Student	Age (years)	Hours worked	No. of courses	No. of problems attempted[a]	No. of posts made	Average score
SG24	19	20	5	30	37	3.23
SG8	20	35	6	28	41	3.25
SG19	17	0	3	37	52	4.68
SG7	19	38	3	40	92	4.75
SG12	17	0	5	40	110	5.48
SG6	21	0	4	39	85	5.63
SW2	19	20	4	27	32	2.73
SW17	20	20	4	32	38	2.80
SW12	23	25	6	34	45	2.93
SW18	22	40	5	37	58	3.63
SW4	24	30	4	34	45	4.10
SW27	25	25	4	39	65	4.70
SW25	28	0	5	39	89	4.73

[a]There were 40 problems total.

Small-Groups Section Interviewees

Low Performance

Student SG24 said that mathematics was one of her favorite subjects and one of her strongest. She said that she had taken algebra, trigonometry, and calculus in high school; however, she made a low score on the COMPASS algebra exam and was required to take Beginning Algebra and Intermediate Algebra before entering College Algebra. SW24 said that she used computers often, but the online assignments and the tensor were new experiences.

Student SG8 said that she had always made around a 70 to 72 in high school and "would always try to figure out how to work the problem, never try to understand it more than that." She placed into Intermediate Algebra and completed that course before entering College Algebra. SG8 was the only student interviewed in the small-groups section who did not have a computer with Internet access at home, and she stated that she only used a computer when required for school.

Middle Performance

Student SG19 was one of the joint-enrollment students. She said that she had always been an A or B student in mathematics, and she placed directly into College Algebra. Her responses during the interview indicated that she was experienced and comfortable using computers for school and personal use. Although SG19's scores were not among the highest in her class, she played an important role in her group, posting a large number of proposed solutions.

Student SG7 had previously taken Intermediate Algebra and passed Introduction to Mathematical Modeling. During her interview, SG7 said that she was comfortable using the Internet and that she often used her computer to watch movies and listen to music. She was highly active in the online assignments, making 92 posts during the semester, which was the second highest total in either class.

High Performance

Student SG12 was the most active student in either section, making 110 posts. Like the other joint-enrollment students, SG12 placed directly in College Algebra and reported having no job other than going to school. She said that mathematics was not her favorite or strongest subject but that she usually did well in mathematics classes. She said that she did not like computers very much, but obviously those feelings did not keep her from being an active participant.

Student SG6, who placed directly into College Algebra, consistently scored at the top of the class on all assignments, online or otherwise. She did not have a job outside of school and was married with two children. Despite her high performance in the course, SG6 claimed that she had never liked mathematics. She said that she had gone as far as trigonometry in high school and that she thought her teachers had not explained concepts well. She also said that she was comfortable with computers, using them often for school work.

Shared-Work Section Interviewees

Low Performance

Student SW2 said that he had taken College Algebra at another college and had not done well. He said that he had trouble understanding the teacher's lessons and that he was nervous coming into my class. He expressed a high level of comfort with computers using the term *computer literate* several times.

Student SW17 was a Hispanic male who spoke English as a second language. That fact is worth noting because the online assignments required a considerable amount of reading and writing. SW17 placed directly into College Algebra and said that he liked mathematics and usually learned things quickly. He was a computer science major and expressed a strong interest in computers.

Student SW12 had an average score per problem that was slightly above the class average, but he consistently failed to meet deadlines. He often viewed at least one other classmates' work before constructing his first post, which was not necessarily a bad thing. One goal of the online assignments was for students to learn from their classmates. SW12 passed the course, and I am convinced that he did learn from viewing the work of others. During his interview, SW12 said that mathematics was not his favorite subject but that he saw a big need for it. When asked about his experiences in mathematics classes, he said that he only took one mathematics course in high school and that was during his first year. Not surprisingly, he was required to take Beginning Algebra and Intermediate Algebra at the college. He said that he was very comfortable with computers and commented, "I Google everything."

Middle Performance

Student SW18 was the only student interviewed in the shared-work section who reported not having Internet access at home. SW18 also had one of the busiest reported schedules, but she still managed to have one of the higher levels of participation in the online assignments with 58 posts. She had taken Beginning Algebra and Intermediate Algebra, and she said that mathematics had always been difficult for her. When students' online performance was compared with their overall performance in the class, SW18 was an anomaly. Her average score per problem of 3.63 was a relatively strong performance compared to her classmates' scores, but she did poorly on exams and earned a D in the course.

Like SW17, Student SW4 spoke English as a second language. SW4 said that she liked mathematics but had struggled with it in the past. She placed into Beginning Algebra and completed the sequence of remedial courses. She expressed a strong interest in computers and told me after the formal interview that she was working with a company that designed Web pages.

High Performance

Student SW27 was one of the few students in the shared-work section who tested directly into College Algebra. Though he joked on the surveys about having weak basic skills, he performed well in every aspect of the course. He said that he liked mathematics, talked about its usefulness, and said that it was a subject requiring constant practice. He said that he had always struggled with modeling and applying mathematics to real-life situations and that he liked the opportunity to work on those skills in the online assignments. He seemed very comfortable with computers and said that he saw no excuse for people not having access to computers today.

Student SW25 had the highest average score per problem in the shared-work section. Although, she did not work outside the home, she had demanding responsibilities to a husband, two children, and five classes. SW25 said that she liked mathematics but did not like computers. Like many of her classmates, she placed into Beginning Algebra and had to complete a two-course sequence before starting College Algebra. She said that the first course was not bad, but the second course was challenging because it was online. When asked why she took an online class if she did not like computers, she said that it was the only way that she could fit it into her summer schedule and still have time for her children.

Data Analysis

Each research question required a different form of analysis. To address questions concerning the quality and nature of interactions, I needed to develop an appropriate set of analytical tools for investigating the archived student work. First, I used the archived data to gather simple information on participation. In each class, I counted the online messages to determine the number of posts that I made and the number of posts that the students made. In the shared-work section, I had to use the log files to determine a student's viewing and posting behaviors. For each of the 432 total tensor squares in the shared-work section, I checked the student's log to see if the student opened the assignment, posted to the assignment, opened a post with instructor feedback, posted after instructor feedback, or posted after opening another student's work. I performed a similar analysis for the small-groups section. This analysis made it possible to identify when a student participated and when there was potential interaction. To gain a more meaningful understanding of the student behavior and to make comparisons between the classes, I developed a general coding scheme for asynchronous collaboration and a refinement for the specific types of behavior that occurred with the communication tensor.

The Asynchronous Interaction Coding Scheme

As discussed in chapter 2, several researchers, including Stacey and Gooding (1998), have worked to develop frameworks and coding schemes to analyze the nature of student interactions during collaborative problem solving. Kosiak (2004) used Stacey and Gooding's coding scheme, which was developed for face-to-face group work, to analyze student interactions during asynchronous online problem solving. Kosiak's participants were students in College Algebra classes similar to the sections used in my study. The course described by Kosiak was similar to my course, including a similar edition (Blitzer, 2003) of the textbook that I used

(Blitzer, 2007). Because of the similarities between the two courses, I began analyzing the data with Stacey and Gooding's coding scheme.

Like Myers (2002) and Kosiak (2004), I used a unit of meaning as the unit of analysis. For instance, in a single message, many students in my small-groups section both proposed a solution and asked the other group members whether they agreed. Such a message contains a unit of meaning for proposing the solution and a unit of meaning for asking a question. If a message contained proposed solutions to several parts of a multipart problem, I combined the responses into a single unit of meaning.

When I tried to apply Stacey and Gooding's (1998) coding scheme, it became obvious that I needed to make some modifications. I decided to remove two categories and add four of my own. Table 6 contains the ten resulting categories, their abbreviations, and a brief description of each. Appendix G contains a complete version of the coding scheme with examples, and Appendix H contains a sample, with codes, of a problem-solving episode from a small group. I made the changes to the coding scheme to provide a better description of the asynchronous data. The original coding scheme contained categories called *thinking aloud* and *proposing ideas*. In a face-to-face setting, thinking aloud can occur when a student verbally expresses his or her thought process aloud while in the presence of group members. For example, a student would be thinking aloud if he or she verbalized a computation or read a problem aloud without directing the statement at other group members. In an online setting, especially asynchronous, this activity is much less likely to occur because a student has to type all responses. Holding firmly to Stacey and Gooding's scheme, Kosiak (2004) coded messages as thinking aloud when a student presented an answer with no explanation or posted a full answer key because in her opinion these

Table 6

The Asynchronous Interaction Coding Scheme

Category	Abbreviation	Description
Proposing solutions	PS	The unit of meaning contains a proposed solution to all or part of a problem.
Proposing solutions in response	PSR	The unit of meaning contains a proposed solution to all or part of a problem, and the student made the unit after viewing another student's proposed solution.
Responding to the instructor	RI	The unit of meaning contains a response to a question made by the instructor or a proposed solution that obviously used the instructor's feedback.
Commenting	C	The content of the unit of meaning is not directly related to solving the mathematical task. This is the default category for units of meaning that do not fit into other categories.
Asking questions	Q	The unit of meaning contains one or more questions concerning content, correctness of proposed solutions, or logistics directed to other students or the instructor.
Responding	R	The unit of meaning contains a response to a question, an expression of agreement, or an expression of disagreement.
Reposting	RP	The unit of meaning contains the same content as a prior post made by the same student.
Directing	D	The unit of meaning contains a request or directs classmates to do something.
Refocusing the discussion	RD	The unit of meaning contains an attempt to change the focus of the discussion.
Explaining with evidence	EE	The primary purpose of the unit of meaning was to explain to classmates the content of a previous post using mathematical evidence.

messages represent a student's thoughts on a task without the use of interactive dialogue. I thought it more appropriate to replace thinking aloud and proposing ideas with a category called *proposing solutions* (PS). An attempt to present any partial or complete solution to a mathematical task can be coded in this category.

In addition to combining two categories into the single PS category, I found it necessary to create three additional categories to describe the nature of the messages that the participants created. First, the presence of instructor feedback given during the problem-solving process made it necessary to create a category called *responding to the instructor* (RI). In addition, the way that I administered the assignments led to a number of postings that were essentially reposts of prior messages. In the first five assignments, this reposting occurred often in the small-groups section because I required the groups to post a final response in a separate tensor square after working on the problems in their group squares. On later assignments, reposting occurred when I required the students to make initial posts in private tensor squares before working in groups. Occasionally a student made the same post two or more times in a row, which was likely due to hitting the "submit" button repeatedly. I coded these units of meaning, which contained no new information, as *reposting* (RP). Finally, to draw out the similarities between the shared-work and small-groups approaches, I created a category called *proposing solutions in response* (PSR). This category was for messages that contained a proposed solution made after viewing a classmate's proposed solution, a common occurrence when using a communication tensor.

After much consideration, I decided to maintain Stacey and Gooding's (1998) category called *explaining with evidence* (EE), even though I found only one unit of meaning that I would code in that category. Kosiak (2004) writes that she found it difficult to distinguish between thinking aloud and EE in the asynchronous setting. It is only appropriate to assign the code EE if a unit of meaning clearly shows interaction between students. As a teacher, I expect solutions to come with explanations; therefore, many of the responses that I coded as proposing solutions or proposing solutions in response contained explanations with evidence. If the purpose of a response was to present a complete or partial solution to a problem with or without explanation, I coded the unit as PS or PSR. To receive a code of EE, a response had to have been made with the intent to further explain previously proposed solutions.

In addition to the categories previously described, I retained five other categories from Stacey and Gooding's (1998) coding scheme: *asking questions* (Q)*, directing* (D)*, refocusing the discussion* (RD)*, responding* (R)*,* and *commenting* (C). Table 6 contains descriptions of each of these categories.

Intercoder Reliability of the Asynchronous Interaction Coding Scheme

To provide evidence that my coding was consistent, I had a second coder use the asynchronous interaction coding scheme to analyze a portion of the data. The second coder was a professional educator with two graduate degrees in science education and 6 years experience as a secondary science teacher. To train the second coder in using the coding scheme, I had her read descriptions similar to those found in this chapter, and I demonstrated the scheme by coding a few examples with her. I then asked the second coder to analyze each message from both classes for one assignment from each of the four assignment sets. Specifically, these were Assignments 2, 7, 11, and 15 (Appendix A). These assignments contained 244 posts from the small-groups section and 248 posts from the shared-work section. Because a message could contain multiple units of meaning, it was not surprising that the second coder's analysis resulted in a different number of units of meaning than my own. In the shared-work section, I recorded 277 units of meaning, whereas the second coder recorded 275. In the small-groups section, I recorded 299 units of meaning, and the second coder recorded 316. Table 7 shows the average number of units of meaning per message found by each coder.

Table 7
Average Number of Units of Meaning per Post by Section

Coder	Shared Work	Small Groups
Researcher	1.12	1.23
Second coder	1.11	1.30

The different numbers of units of meaning made it more challenging to quantify the intercoder reliability. For instance, J. Cohen's (1960) Kappa coefficient assumes that each coder assigns a single code to the same number of items. Instead of using that statistic, I computed the percentage of agreement out of total units coded. If I agreed with the second coder on a unit of meaning and its code, I recorded that as an agreement. For instance, on one message we both coded PS, C, and Q. That message resulted in three agreements. I recorded a disagreement if we gave different codes to a unit of meaning or if one of us coded a unit of meaning not coded by the other. For instance, if I coded an entire message as RI and the second coder coded it as PS, I counted this as one disagreement. If I coded a message as PS, and the second coder coded it as PS and C, I counted this as one agreement and one disagreement. I computed the percentage of agreements using the sum of recorded agreements and disagreements for the total number of units coded (Table 8).

Table 8
Intercoder Agreements and Disagreements

Section	Agreement	Disagreement	Percent agreement
Shared Work	232	57	80
Small Groups	269	54	83

Many researchers seem to agree that coder agreement rates around 80% demonstrate a reasonable level of reliability. Myers (2002), for instance, reported an agreement rate of 81% computed from 50 units of meaning. My second coder had the added difficulty of identifying units of meaning. Even with that extra challenge, we had at least 80% agreement in each section.

By far, most of the intercoder disagreements concerned the RI category. The RI code was involved in 41 of the 57 disagreements in the shared-work section and 24 of the 54 disagreements in the small-groups section. I probably did not explain the category to the second coder as well as I could have, and it was not always easy to distinguish when a student was proposing a solution or incorporating feedback. In most of the disagreements, I coded a unit of meaning as RI, whereas the second coder used PS. If the student addressed a direct question that I asked in a feedback post or built upon a hint that I gave, such as pointing out a computational mistake or setting up an equation, the response should have been coded RI. In other words, if a student clearly used information that I made in a post to construct his or her next post, the unit of meaning should have been coded as RI instead of PS. This distinction required careful analysis and comparison of the students' posts and my posts.

Most of the disagreements not centering on the RI category occurred because one of the coders identified a unit of meaning that the other did not, such as a comment or question embedded in a proposed solution. With a large data set, it is reasonable to expect coders to miss some of the less noticeable units of meaning. To make the most use of the second coder's results, I reanalyzed all disagreements and made changes in my coding when I found it appropriate. This reanalysis resulted in my changing the codes or adding additional codes in 18 cases of the 111 disagreements. I used my coding for the data analysis.

Individual Student Interactions

The asynchronous interaction coding scheme did not tell much about the ways individual students behaved. For example, one student might have contributed a large number of PS units of meaning and few other units, whereas a second student might have contributed a large number of PSR units and very few PS units. To address this issue, I looked at the number of units of meaning coded in each category for each student. By examining the data for individual students, I developed seven interaction types (Table 9). I defined high and low participation based on the average number of posts per student made in each class, and I defined a nonparticipant as anyone who withdrew or made less than 1% of the total posts. I used the characteristics in Table 9 and the results of the coding scheme to assign an interaction type to each student in this study. In the small-groups section, I examined the results based on the group structure for Assignments 4 to 13, and I compared those findings to the individual results in the shared-work section. Because there was limited subjectivity in applying these interaction types based on the results of the asynchronous coding, I did not use a second coder to check reliability.

A Refined Coding Scheme for PSR

To gain a better understanding of the ways the students used one another's work and the quality of their interactions, I further examined each unit of meaning that I coded as PSR. I carefully read each of these units of meaning and all posts viewed prior to their creation in an attempt to discover themes and develop a refined coding scheme. I had some questions that the coding scheme had to address. For instance, I wanted a scheme that could identify when a student had simply copied a classmate's work or incorporated a classmate's work into his or her own response. I also wanted the scheme to keep track of when a student created a more correct or less correct response than any he or she had viewed. In other words, I wanted the scheme to

Table 9
Student Interaction Types

Interaction Type	Characteristics
Leader	The student made more posts than the class average, with more PS units than PSR and R units, and the student had a large number of student-to-student interactions coded R, C, Q, RD, D, or EE.
Active independent	The student made more posts than the class average, with more PS units than PSR and R units, and the student had a small number of student-to-student interactions coded R, C, Q, RD, D, or EE.
Active follower	The student made more posts than the class average, with more PSR and R units than PS units, and the student had a large number of student-to-student interactions coded R, C, Q, RD, D, or EE.
Noninteractive follower	The student made more posts than the class average, with more PSR and R units than PS units, and the student had a small number of student-to-student interactions coded R, C, Q, RD, D, or EE.
Low-participating independent	The student made fewer posts than the class average and had more units coded PS than PSR and R.
Low-participating follower	The student made fewer posts than the class average and had more units coded PSR and R than PS.
Nonparticipant	The student contributed less than 1% of the units of analysis or withdrew from the course.

help reveal when a student improved upon the work of others or viewed a correct response and was unable to incorporate it into his or her own work. After some initial analysis and consultation with my major professor, I decided that a scheme designed to interpret the PSR units of meaning needed to be divided into multiple dimensions. The full coding scheme can be found in Appendix I and is described below. Appendix J contains examples of the coding scheme applied to student work.

Dimension 1: Incorporation of Classmates' Work

The first dimension of the coding scheme for the PSR units of meaning is used to decide whether a unit of meaning incorporates the work of one or more classmates. The possible outcomes are *yes* (Y), *no* (N), *similar problem* (SP), and *other* (O). In order for a response to be coded Y, the response had to display some obvious similarity to a viewed proposed solution. A student's copying or paraphrasing a classmate's work was an obvious example of a unit of meaning that would be coded Y. There were also less-obvious cases, such as a student catching a mistake or viewing a partially correct response and correcting or completing it. Several of the problems, especially those that I called individualized parameters, resulted in students using the same approach as a classmate to solve a similar problem or presenting an alternate but conceptually equivalent solution. For these types of responses, I used the code SP. Two of the assignments asked students to create their own problems and solve a classmate's problem. Attempting a classmate's problem was coded PSR, and in the refined coding scheme I coded these as O for Dimension 1 and did not apply the other dimensions, which were not applicable for this type of activity.

Dimension 2: Individual Contribution

The second dimension of the refined coding scheme was used only for messages coded Y or SP on the first dimension. For messages that showed some incorporation of classmates' work, the second dimension assessed the level of individual contributions made by the student. The possible outcomes of this dimension were *copied*, *reworded, minor contribution*, and *major contribution*. I used the code *copied* when a student copied another student's work verbatim, and I used the code *reworded* when a student gave a response that was essentially the same as a classmate's response but presented or worded differently. The code *minor contribution* was used for messages that were highly similar to a classmate's response but contained minor contributions by the student such extra explanations or graphs, correction to minor mistakes, or direct attempts to explain a classmate's post. The code *major contribution* was used when students incorporated some part of a classmate's post but made major individual contributions by adding essential details, correcting major mistakes, or adding small details or corrections to one's own prior posts. For messages coded SP, the individual contribution was coded *major contribution* unless the student's solution or individualized problem contained portions identical to a classmate's.

Dimension 3: Relative Correctness to Others

The first two dimensions of the coding scheme say nothing about the correctness of students' contributions. It was quite possible for a student to add content to a solution and make it less correct. I also wanted to see how students' responses compared with those that were viewed and not used. These are some of the reasons that I included a dimension to compare relative correctness between a student's post and those viewed by the student prior to making the post.

When considering the relative correctness of a student's post compared to those viewed in the tensor, it is important to keep in mind that the students had different goals in the two implementation approaches. In the small-groups section, the groups were supposed to work together to create solutions, and in that setting, correctness of a solution depended on the collective responses of the group members. In the shared-work section, each student was supposed to create a complete response in his or her individual tensor square; therefore, a post containing a response to only a portion of a problem could have been interpreted as improving a group's response in the small-groups section, whereas a similar response could have been seen as less correct than a viewed solution in the shared-work section. For that reason, it was tempting to create separate coding schemes for the two sections, but I preferred to create a single scheme.

After working with the student data, I decided on five categories. I describe each in this section, but full interpretation requires consideration of the other dimensions and the class setting. I used the code *improvement* for student responses that when combined with any prior posts by that student form a more correct solution than any solution viewed in the tensor. I use the code *regression* for student responses that were less correct than the most accurate response viewed in the tensor when comparing the portion of a question that the response addressed. The category did not include a student correctly answering less of a question than a classmate had. Other categories were designed to capture that behavior. If a student made a response to a question that was similar to a classmate's and that I believed had the same level of correctness, I used the code *equivalent*. If a student made a response that was equivalent to a portion of a classmate's work but missing some details found in the other student's work, I use the code *equivalent/partial*. Note that an *equivalent/partial* response would be graded as less correct in the shared-work section, but not in the small-groups section. Finally, I used the category *mixed* for messages that contained improvements on some parts and regressions on different parts of a question. I also use the *mixed* category for messages that improved the solution to a portion of a problem and did not address other portions considered by classmates, because the solution was more correct on one portion and less correct by omission on another portion.

Dimension 4: Relative Correctness to Prior Posts

I added a final dimension to the coding scheme to reveal when a classmate's posts influenced a student into changing the correctness of his or her response. This dimension used the same categories as *relative correctness to others*, but I compared the student's work to his or her own prior post instead of to that of his classmates. This dimension was not applicable in many cases, because not all PSR units of meaning followed other posts made by the student.

Intercoder Reliability of the PSR Coding Scheme

As with the general coding scheme, I had a second coder use the PSR scheme on a portion of the units of meaning to check for intercoder reliability. I trained the same coder in the same manner as I did for the first coding scheme. I first had her read descriptions of the dimensions and categories, and then I demonstrated the coding scheme with a few PSR examples. The coder used the PSR scheme to code 50 randomly selected units of meaning, 25 from each section. To check reliability, I used the same approach for multi-dimensional coding as Myers (2002). I assigned each code a value of one point. Each unit of meaning was worth at most four points, depending on the number of applicable dimensions. Dividing the total number of agreements by the total number of possible agreements from the 50-unit sample, I found an agreement percentage of 80%.

Online Assignment Scores

One of my research questions called for comparing the quality of the interactions in the online assignments. The coding scheme for the PSR units of analysis was one way to address quality. Another was to analyze the overall correctness of the student responses. Comparing scores on the online assignments between sections was problematic because of the different approaches that I used. I attempted to create comparable and fair scoring guides (Appendix F) that aimed to promote participation in each approach. In both sections, I graded each problem out of 6 possible points, 4 points for the overall response and 2 points for individual contributions. In the shared-work sections, overall responses consisted of all work created by each individual student in his or her tensor square. In the small-groups section, overall responses consisted of the work collectively produced by the group. In both sections, I graded the overall responses after the final due dates, using the 4-point rubric (Appendix F). In the shared-work section, I expected students to get each problem at least halfway correct during the initial phase. Therefore, I assessed individual contributions after the initial phase due dates using the rubric but recording a maximum of 2 points. A response worth 2, 3, or 4 points on the rubric received the full 2-point credit for the initial phase. In the small-groups section, the students earned up to 2 points based on their contribution to the group with the added stipulation that they would receive no credit at all on an assignment if they did not participate.

I recorded the individual scores for each student out of 6 possible points in both sections. I also recorded the overall score out of 4 points for each student in the shared-work section and each group in the small-groups section. Because the results for the 4- and 6-point scales were extremely similar, I decided to report the results of the 6-point scale only. Using the students who did not withdraw and contributed at least 1% of the units of meaning, I computed the average score per student for each of the 40 problems. I then computed the overall class average per student and the number of times that each section outscored the other. These data provided information on the quality of the work produced in each class and allowed for some comparisons across approaches.

Analysis of Student Interviews and Questionnaires

I used the interviews and questionnaires to gain insight into the students' perspectives concerning the online assignments. To analyze these data, I used a constant comparative method (Glaser, 1965). I read each response several times and then began grouping similar statements together in search of patterns and themes. I focused specifically on data that I believed addressed one of my research questions, ignoring other data that I found helpful as a teacher but considered outside of the scope of the study. Like Rouhani (2005) "my goal was to establish categories that (a) reflected the purpose of the study, (b) were exhaustive and reflected all relative data, and (c) were mutually exclusive" (p. 31). After identifying categories from the data, I analyzed them further to develop explanations and address my research questions. For factors influencing participation, I used Bullen's (1998) conceptual framework, discussed in chapter 2. I slightly altered Bullen's labeling, using *attributes of the medium*, *attributes of the learning activities*, and *attributes of the student*.

Validity and Reliability

Every researcher wants readers to accept his or her research as trustworthy and believable. Quantitative researchers have well-defined constructs with statistical measurements to provide evidence to readers that their work is valid and reliable. Huck (2004) writes that the closest synonym for *reliability* is *consistency*. That is, reliability refers to the extent to which the findings of a research study can be replicated. I have already discussed intercoder reliability for the two coding schemes that I used. Another way to address reliability for qualitative studies is to provide sufficient descriptions of the methodology and enough details for the reader to judge whether the conclusions are reasonable and make sense. I have addressed reliability by providing complete descriptions of my methodology including the data collection and analysis methods.

Huck (2004) says that for quantitative researchers, the closest synonym for *validity* is *accuracy*, and "a measuring instrument is valid to the extent that it measures what it purports to measure" (p. 88). Patton (1990) argues that in qualitative studies, "the researcher is the instrument" (p. 14), and therefore, the believability of a qualitative study hinges on the credibility of the researcher. A common way to strengthen the credibility of qualitative studies is to use triangulation or multiple forms of data collection and data analysis. Such multiple forms might collectively account for possible weaknesses in the individual components. As Patton puts it, "Studies that use only one method are more vulnerable to errors linked to that particular method than studies that use multiple methods in which different types of data provide cross-data validity checks" (p. 248). I sought to achieve triangulation by using the data collection and analysis methods described in this chapter, including analysis of student work, interviews, and open-ended surveys.

Whereas quantitative researchers strive for results that they can generalize from their sample to a larger population, qualitative researchers are typically more concerned with understanding a case or phenomenon in detail. If the reader has sufficient details, he or she can judge the relevance of the results and decide whether they might be applicable in other situations. Lincoln and Guba (1985) use the term *transferability* for research that can be useful to others who are investigating similar phenomena under similar conditions. I aimed for transferability in this study by providing detailed descriptions of the software, the course, the participants, the methodology, and the results.

CHAPTER 4

RESULTS

In this chapter, I discuss the results of my investigation into the use of a communication tensor to facilitate online mathematical problem solving. As previously stated, the three research questions focused on the nature of student interactions, the quality of student interactions and online work, and factors that influence student participation and interactions. I begin by discussing some overall participation results. Then I address the research questions in order.

General Participation

Over the semester, students in both College Algebra sections in this study were required to complete 40 online problems in 16 assignments. Appendix K shows the number of assignments that each student participated in by making at least one post, the number of assignments for which a student viewed classmates' work, and the total number of posts made by each student.

The 22 students who completed the shared-work section by taking the final exam made 826 posts, with an average of 37.5 posts per student and 2.3 posts per assignment per student. The number of posts made by individual students ranged from a low of 6 to a high of 89. Providing feedback and making comments, I posted 1,231 messages. It is not surprising that my posts outnumbered those of the students as I followed the majority of the problems with additional graded feedback after the due dates.

The 18 students who completed the small-groups section by taking the final exam made 867 posts, with an average of 48.2 posts per student and 3.0 posts per assignment per student.

The number of posts made by these 18 students ranged from a low of 0 to a high of 110. Providing feedback and making comments, I posted 473 messages, which was dramatically fewer than the number of messages I posted in the shared-work section. Clearly, monitoring and providing feedback to 27 individual students placed greater time demands on me than keeping up with a small number of groups.

Several posts contained more than one unit of meaning (Table 10). In the shared-work section, 27 students made 924 posts with 1,032 units of meaning, an average of 1.12 units per message. In the small-groups section, 21 students made 878 posts with 1,040 units of meaning, an average of 1.18 units per message. Thus, the small-groups section used slightly more units of meaning per message on average than the shared-work section, but the average message in each section contained a single unit of meaning.

Table 10

Frequency by Section of Number of Units of Meaning per Message

Units of meaning per message	Shared work (n = 27)	Small groups (n = 21)
1	832	739
2	76	117
3	16	21
4	0	1

The Nature of the Asynchronous Interactions

To begin addressing the research question concerning the nature of collaboration and interaction, I used the logged records to determine the instances in which students had viewed the work of others. In the shared-work section, there were 188 instances, approximately 68% of the instances of assignments attempted, in which students viewed the work of others. In the small-groups section, one might have expected that viewing classmates' work would have been more frequent inasmuch as the students were assigned to groups, and indeed, there were 204 instances in that section, approximately 90% of the instances of assignments attempted, in which students viewed the work of others. To understand the nature of the interactions, I applied the asynchronous interaction coding scheme, and in the interviews and surveys, I asked the students about their experiences. In the remainder of this section, I describe results on the overall nature of the interactions in each section, compare and contrast those results, and discuss interactions by individual students.

The Shared-Work Section

I began the interviews by asking the students to describe how they had typically worked on the online assignments. SW4 and SW18 said that they had often sought help at the campus tutorial center. The five other interviewees each described working on the problems individually during the initial phase, aided by the textbook, class notes, and any hints that I had placed in the tensor. These students explained that they would do as much as they could on their own and then wait for the tensor to open. According to the interviews and the surveys, what the students did once the tensor unlocked often depended on how successful they had been during the initial phase. Table 11 contains the frequencies of the students' responses to the question of how they had found it helpful to view classmates' work in the tensor. The majority of the responses indicated that the students mainly used the work of their classmates to get help when they could not solve a problem on their own or to check their work. A few students responded that it was helpful to see different ways to solve problems, and a few others said that they preferred not to look at their classmates' work. These responses provided some initial information about the nature of the interactions that occurred through the tensor, and the asynchronous interaction coding scheme provided more details.

Table 11

Frequency of Responses to Uses of Classmates' Work by Data Source for the Shared-Work Section

Use	Brief survey (n = 22)	Extended survey (n = 18)	Interview (n = 7)
To get help completing or starting a problem	6	7	7
To check solutions	7	4	1
To see alternate solutions	2	3	0
Prefer not to look	3	3	0

Using the results of the asynchronous interaction coding scheme, I computed the percentage of units of meaning in each category by dividing the number of units of meaning given that code by the total number of units of meaning coded (Table 12). The results show that student-to-student interactions were limited mainly to PSR. More than 86% of the units were coded PS, PSR, and RI, and creating a PS or RI unit did not require student-to-student interactions. There were units coded C and Q, but most of those were directed at me in the form of content questions or comments stating something about the technology not working or that the student did not know how to start a problem. The units coded R can be attributed to the requirements of the questions. All but 3 of those units occurred on Assignment 6, which contained problems asking students to discuss a classmate's work. The other units coded R were posted by a student on Assignment 7, apparently continuing the behavior from Assignment 6.

Appendix L contains the distribution of the coding scheme results by problem. With a few exceptions, the interaction patterns were quite stable over time. I have already mentioned that all of the units coded R were posted for Assignments 6 and 7. I also tried to encourage interaction on Problems 7.3 and 11.1 by asking students to solve a problem written by a classmate. These two problems provided 11 and 10 units coded PSR, respectively, and those totals were among the highest per problem.

Table 12
Percent of Asynchronous Interaction by Coding Scheme Category in the Shared-Work Section

Category	Percent
Proposing solutions	53.8
Proposing solutions in response	18.4
Responding to the instructor	14.0
Commenting	7.6
Asking questions	2.7
Responding	1.9
Reposting	1.5
Directing	0
Refocusing the discussion	0
Explaining with evidence	0

I have already noted that several students indicated that they mainly looked at classmates' work to get help when they were stuck or to check their work. Moreover, some students indicated that they did not look at other students' work when they knew that they had solved a problem correctly. Asked on the extended survey why they might have rarely viewed classmates' work, 3 students said that they did not look at classmates' work when they understood a problem, and 5 students made similar statements during interviews. The results of the coding provided some evidence to support this claim. There were significant negative correlations between the percentage of PS units posted per problem and the percentage of units posted in categories C ($r = -.59$, $p < .01$), Q ($r = -.40$, $p < .05$), RI ($r = -.41$, $p < .01$), and PSR ($r = -.55$, $p < .01$). When the students were confident in their solutions, they tended to post PS units and move on to other assignments.

The Small-Groups Section

According to the online data and the students' responses to the interviews and surveys, the small-groups section operated in a shared-work mode with increased interaction. Asked on the extended survey whether they had worked with anyone offline, 10 of the 15 students in the section who responded said that they worked alone. One of those 10 students said that she had occasionally exchanged text messages with her group members, but none of her group members mentioned this form of communication. Two students said that they mainly worked alone but that they had met with group members on occasion to discuss the problems. Three students said that they had not worked offline with their group members but that they had received help from friends. Four of the interviewees affirmed their responses to the survey question, but two did not. SG8, who said on the survey that she had worked alone, told me during her interview that she and SG24 had met frequently during the second half of the semester, and SG7, who also said on the survey that she had worked alone, said during her interview that she had occasionally compared answers with her group members before class.

On the extended survey, I asked each student in the section if his or her group had interacted to solve problems. One student did not respond, and one student said yes. Seven students said that they "did their own thing," using the wording of my question, and seven students gave responses indicating that they had not collaborated in a manner typically associated with face-to-face group work but that they had shared and compared answers and exchanged some interactive messages. With the exception of SG8 and SG24, the interviewees gave similar description of working independently and comparing answers. The results of the asynchronous interaction coding scheme, discussed below, provide more evidence that most students tried to solve the problems for themselves and then shared and compared the results using the group squares, displaying more interaction than the shared-work section.

It is impossible to interpret the results of the asynchronous interaction coding scheme in the small-groups section without taking into account the organizational changes that I made following Assignments 5 and 13 (see pp. 40–41). Appendix M shows the distribution of the asynchronous interactions by problem and student, and Table 13 shows the percent of units coded in each category for the assignment sets corresponding to the major organizational changes. The main reason that I started requiring students to make initial posts in individual tensor squares was because a few students had participated in the first 5 assignments by posting "I agree" and nothing more. As I had hoped, adding the requirement of an initial post reduced the number of units coded R, which included statements such as "I agree," and dramatically increased the percentage of units coded PS on Assignments 6 to 13. There was also an increase in units coded C as the students began making comments to me during the initial phase and continued commenting during the group phase.

Table 13
Percent of Asynchronous Interaction by Category and Assignment Sets Corresponding to Organizational Changes in the Small-Groups Section

Category	1 to 5	6 to 13	14 to 16	1 to 16
Proposing solutions	29.5	45.8	38.3	40.3
Proposing solutions in response	24.1	11.5	34.7	19.0
Responding to the instructor	4.2	6.7	6.6	6.1
Commenting	7.3	12.7	11.7	11.2
Asking questions	9.6	5.8	5.6	6.7
Responding	13.8	5.8	2.6	7.2
Reposting	10.3	10.8	0.5	8.8
Directing	0.4	0.7	0	0.5
Refocusing the discussion	0.4	0.2	0	0.2
Explaining with evidence	0.4	0	0	0.1

An unexpected result of requiring initial posts in the small-groups section was a sizable decrease in the percent of units coded PSR. When I placed students into pairs and removed the initial phase for Assignments 14 to 16, the percent of PSR units dramatically rebounded and the percent of PS units decreased. As in the shared-work section, when considering all 16 assignments, there was a significant negative correlation between the percent of PS and PSR units per problem in the small-groups section ($r = -.45$, $p < .01$). It seems that in the asynchronous setting, the easiest way for students to participate was by posting their solutions, even when those solutions agreed with something that had already been posted. Any proposed solution posted in a group square following a PS post became a PSR unit by default. In the absence of individual squares, the students had fewer opportunities to create PS units. This argument is strengthened by the fact that the students posted 115 more PS units and 64 fewer PSR units for the 8 assignments containing an initial phase (Assignments 1 to 5 and 14 to 16) than they did for the other 8 assignments.

The assignment structure also led to a high percentage of RP in the small-groups section that greatly decreased following Assignment 9. For Assignments 1 to 5, I required each group to create final posts summarizing the group's work. The students frequently reposted one of the PS units as a final post. For Assignments 6 through 13, the students had to make initial posts in their individual squares before working in groups. This requirement led to a number of students reposting their initial responses in the group squares. By Assignment 10, I had programmed the software to automatically copy the initial posts into the group squares, and the RP essentially stopped.

Comparing the Nature of the Asynchronous Interactions

As one might expect, the small-groups section displayed more diverse interactions than the shared-work section. In both sections, PS units occurred at the highest rate with PSR a distant second. In the shared-work section, PS occurred most frequently for all but 3 of the 40 problems. In the small-groups section, PS occurred most frequently for all 19 of the problems that required initial posts, but PSR occurred more frequently than PS for 11 of the 21 remaining problems. Most of the interaction in the shared-work section other than PSR occurred between me and the students in the form of RI, C, and Q. The shared-work section did have 20 units coded R, but these were a result of the requirements of Assignment 6. The small-groups section had several instances of C, Q, and R between students. The small-groups section also had a small number of units coded D, RD, and EE, whereas the shared-work section had none.

Perhaps the most informative comparisons between the two sections can be made on Assignments 6 to 13. For those assignments, students in both sections were required to make initial posts in individual squares, with the difference being that the second phase consisted of group work in the small-groups section, whereas students worked individually with access to classmates' work in the shared-work section. The organizational structure led to a large percentage of RP in the small-groups section. Because reposting does not add any new information on an assignment, it is also informative to look at the percentages without RP (Table 14).

Table 14
Percent of Asynchronous Interaction by Coding Scheme Category and Section for Assignments 6 to 13

	Including RP		Excluding RP	
Category	Shared Work	Small Groups	Shared Work	Small Groups
Proposing solutions	50.6	45.8	51.6	51.3
Proposing solutions in response	15.4	11.5	15.7	12.9
Responding to the instructor	15.9	6.7	16.2	7.5
Commenting	9.2	12.7	9.4	14.2
Asking questions	3.4	5.8	3.5	6.5
Responding	3.6	5.8	3.7	6.5
Reposting	1.8	10.8	–	–
Directing	0	0.7	0	0.8
Refocusing the discussion	0	0.2	0	0.2
Explaining with evidence	0	0	0	0

Excluding RP, the two sections had essentially the same percent of units coded PS, and the small-groups section had a slightly smaller percent of PSR units with more student-to-student interaction than the shared-work section. As with all of the problems, the majority of the units in the shared-work section were PS, PSR, and RI. I made fewer posts in the small-groups section, and the students responded with fewer RI units and more units coded C, Q, R, D, and RD. Placing the students into groups for the second phase reduced the amount of work for me and increased the quantity and variability of student-to-student interactions.

Individual Student Interactions

To determine the asynchronous interaction patterns for individual students, I examined the distribution of the results of the asynchronous coding scheme by student (Appendices N and O). Using those results, I assigned each student an interaction type (Table 9).

Small-Groups Section

Not surprisingly, the small-groups section provided more diversity in the types of interactive student behavior than the shared-work section. I classified 8 of the 24 original students in the small-groups section as nonparticipants. They contributed only 12 of the 1,040 units of meaning. I was able to determine that the majority of the nonparticipants had withdrawn by the end of the first three assignments. Using that information, I formed the five groups that worked together on Assignments 4 to 13 (see Table 3). Because students worked in those groups for a majority of the problems, I used those groups to organize this portion of the analysis.

Group 1. In the first group, I classified SG18 as an active independent. She created a high number of PS units and did little interacting. On several assignments, she posted an initial response before anyone else and never made another post. I classified SG8 as an active follower and SG24 as a low-participating follower. They each had more PSR and R units than PS units, working off SG18's posts. SG8 and SG4 also had modest levels of interaction according to the asynchronous interaction coding scheme; however, they both participated in interviews and reported that they often worked together in the computer lab, preferring face-to-face interaction.

Group 2. Unfortunately SG22 became a nonparticipant. He remained in the class, but he participated on only one assignment, posting "I agree" on each problem. The other two students acted as a leader, SG6, and an active follower, SG22. SG6 led off most assignments by proposing solutions, and she frequently made comments, questions, and responses directed at SG5. SG5 had more R and PSR units than PS units, but he usually contributed to the assignments with those types of units.

Group 3. The three students who asked to be in this group all had a low level of participation. They would frequently make comments in class about disliking working on the computer and forgetting assignments. With the inconsistent participation from his group members, SG4 became an active independent and solved many of the problems on his own. When asked how comfortable he was working in groups on a survey, SG4 wrote, "Initially, I did not like working in groups online. However, I realized that if I paid more attention in class, I would not need help from other group members. Now I enjoy doing it."

Group 4. SG13 was a low-participating independent with 62% of her posts coded PS, and SG15 was a low-participating follower with 62% of his posts coded as PSR or R. SG7 was an active independent, making the second highest number of posts in either class. She had a large number of posts coded PS, RI, and PSR. If she had made more posts directed at her group members, I would have classified her as a leader.

Group 5. This group was by far the most active and successful group. I classified SG12 and SG19 as leaders for interacting frequently and contributing a large number of PS units. SG23 operated as an active independent. She contributed to the group with a large number of PS units, but she did not interact frequently. The other member, SG17, operated as a low-participating follower, producing many more PSR and R units than PS units.

The Shared-Work Section

Because the primary form of interaction found in the shared-work section was PSR units, there were no students from the shared-work section that I classified was leaders. The most active students in the shared-work section operated as active independents, with 12 of the 27 students classified as this type. Eight of the students became nonparticipants. Five of the nonparticipants withdrew and the other three contributed 26 of the 1,032 units of meaning. One student, SW12, was a noninteractive follower, participating often but creating more PSR than PS units. Three students were low-participating independents, and 3 were low-participating followers.

The Quality of the Asynchronous Interactions

The second research questions called for a comparison of the quality of the interactions and collaboration between the two sections. To address that question, I used the multi-dimensional coding scheme for PSR units and the scores from the online work. First, I present the results and make comparisons for each dimension of the coding scheme. Then I address the scoring results.

Results of the PSR Refined Coding

In chapter 3, I described a multi-dimensional coding scheme that I developed to gain more information about the units of meaning coded as PSR. The unit of analysis for this portion of the study was a student's response to a problem posted after viewing a classmate's proposed solution. Occasionally, a student made multiple posts, dividing pieces of his or her response. For example, a student might have posted an algebraic portion of a solution in one post and followed that immediately with a post containing a graph. When multiple posts occurred, I combined them into single units of analysis. In all, I coded 185 units of analysis in the shared-work section, and 186 units of analysis in the small-groups section.

Dimension 1: Incorporation of Classmates' Work

I used the first dimension of the PSR coding scheme to determine whether a student used a classmate's work in constructing his or her own response. The possible outcomes are *yes* (Y), *no* (N), *similar problem* (SP), and *other* (O). Table 15 shows the percentage of the total units of analysis in each section coded for each outcome. The results show that the two sections behaved similarly. In each section, just over half of the units of analysis showed evidence that the student had incorporated a classmate's work, and just over a fourth of the units of analysis were SP. The two sections also had similar percents of units coded N and O.

Table 15

Percent of Responses Incorporating Classmates' Work by Section

Section	Y	N	SP	O
Shared work	51.4	14.1	28.6	5.9
Small groups	51.6	16.1	26.9	5.4

Dimension 2: Individual Contribution

I coded each of the messages coded Y or SP with the second dimension of the scheme to assess the individual contribution made by the student. Table 16 shows the percent of units coded at each level for the two sections.

Although the percentages of units coded at each level differed across sections, the ordering was the same. In both sections, most of the messages were coded as *major*

Table 16
Percent of Individual Contributions by Section

Section	Copied	Reworded	Minor contribution	Major contribution
Shared work	6.8	35.1	18.2	39.9
Small groups	10.3	24.7	16.4	48.6

contributions, but that was more common in the small-groups section, and *reworded* messages were a much closer second in the shared-work section than in the small-groups section. The percentage of posts displaying some original contribution was higher in the small-groups section, with 65% percent of the units coded *minor* or *major contribution* as compared with 58.1% in the shared-work section. The biggest difference between the sections in Table 16 occurred for *reworded*, with students more likely to have paraphrased in the shared-work section. In the shared-work section, the logged data were required to determine what a student had viewed, whereas in the small-groups section, it was usually obvious when a post followed another directly in the group square. Perhaps students were more likely to paraphrase in the shared-work section than in the small-groups section, hoping to pass off the work as their own.

Dimension 3: Relative Correctness to Others

Table 17 shows the percent of units of analysis coded in each category for the two sections. In both sections, the most common occurrence was an equivalent post, which can be interpreted as either agreeing or copying. In the shared-work section, the equivalent code occurred 94 times, with 38 of those being paired with *rewording* on Dimension 2, and 8 paired with *copied*. These pairings represent 46 out of 185 or 24.9% of the PSR units coded. A negative interpretation is that the students were copying, and a positive interpretation would be that they were showing agreement. In the small-groups section, the equivalent code occurred 76 times, with 20 of those being paired with *rewording*, and 11 paired with *copied*. These 31 pairings

Table 17
Percent of Relative Correctness of Responses by Section

Section	Improvement	Regression	Equivalent	Equivalent/ partial	Mixed
Shared work	11.5	26.4	54.0	5.7	2.3
Small groups	26.1	11.9	43.2	13.6	5.1

represent 16.7% of the total 186 PSR units coded in the small-groups section. This difference in percents clearly indicates that students were less likely to create posts that were essentially the same as a classmate's in the small-group setting. When this activity occurred in the small-group section, it was much easier to interpret it as agreeing. For instance, student SG12 reposted a group member's work as a final response for her group seven times, accounting for most of the *copied* codes in the small-groups section.

Many of the equivalent posts occurred on units of analysis coded SP. On this type of problem, students viewed a classmate's solution and created an alternate solution or solved a similar problem with a different set of parameters. I coded each of these messages as a *major contribution* for Dimension 2 unless the student's solution or problem had portions that were identical to something in classmates' work. Units coded SP and *major contribution* occurred 24 times in the shared-work section and 30 times in the small-groups section.

One interesting result in Table 17 concerns the improvement and regression categories. The percentages are almost exactly the opposite between the two sections, with the shared-work section creating more posts coded *regression*. One explanation may lie in the way that I facilitated the courses. In a group setting, students may have been more likely to keep quiet or ask a question if they did not understand, whereas each student had to post an individual answer in the shared-work section to receive any credit.

For the equivalent/partial category, the entries in Table 17 also favor the small-group section. Without the equivalent/partial category, I would have coded even more units of analysis as regression in the shared-work section because the less-complete posts were less correct. In the small-groups section, however, I would have coded the equivalent/partial units as equivalent because they maintained the same level of correctness for the group as a whole.

Both sections had a small number of units coded as mixed. There were four such units in the shared-work section, and all of them contained a combination of improvements and regressions. In the small-groups section, nine units were mixed. Five of the nine contained mixtures of improvements and regressions, and four contained improvements while omitting needed portions, an understandable behavior for group work.

Dimension 4: Relative Correctness to Prior Posts

The final dimension compared the relative correctness of a PSR unit to a student's own prior posts. This dimension was applicable in only 37 cases for the shared-work section and 50 cases for the small-groups section. The percents of the applicable cases for each section are shown in Table 18.

Table 18

Percent of Relative Correctness to Prior Posts by Section

Section	Improvement	Regression	Equivalent	Equivalent/ partial	Mixed
Shared work	75.7	2.7	21.6	0	0
Small groups	66.0	12.0	20.0	2.0	0

Fortunately, in both sections, improvement was the most common occurrence. Mixed responses involving improvements and regressions did not occur at all, and there was only one equivalent/partial code. The biggest difference was the larger percentage of regressions in the small-groups section. A *regression* code means that a student had a prior post and changed to something that was less correct, but it does not include simply omitting details. The code means that something was actually changed from correct to incorrect. Regression occurred once in the shared-work section and six times in the small-groups section. It seems like a reasonable assumption that the latter students were not very confident in their prior posts, and likely wanted to conform to the group.

Online Assignment Scores

Appendix P shows the average score per student by problem on the 6-point rubric (Appendix F), and Table 19 shows the overall average score for each section and the number of problems on which each section outscored the other. The averages were computed by first averaging the scores on the 40 problems for each participating student and then averaging those results. I defined a participating student as one who did not withdraw from the course and participated by contributed more than 1% of the units of meaning coded with the asynchronous coding scheme.

Table 19
Class Average Score per Problem by Section and Number of Times Each Section Outscored the Other

Section	Average score	Outscored other section
Shared work	2.88	7
Small groups	3.78	33

Although the scores came from different grading methods and different situations, Table 19 shows that the small-groups section earned higher scores on most of the problems. One could argue that stronger students can pull up weaker students' scores in a group setting. If a student made some small contribution to a problem in the small-groups section, and another group member solved the problem correctly, the first student received a score of 5 points and might not have understood the solution. A similar situation could have occurred in the shared-work section. A student might have made a minimal attempt at a problem during the initial phase and then copied a classmate's response during the second phase to earn 5 points. This anomaly is one thing that the PSR coding scheme was devised to catch by identifying cases of copying and paraphrasing. This study was primarily intended to take an in-depth look at the interactions that occurred through the use of the communication tensor. I believe the results of the coding schemes and Table 19 suggest that it is worth conducting further studies to see if the small-groups approach, which was actually a blend of the shared-work approach and traditional group work for Assignments 6 to 13, can consistently lead to higher levels of interaction and better results than an individualized approach.

Factors Influencing Participation and Interactions

In this section, I discuss the third research question, which concerns the factors influencing student participation or interactions. I present the results from each section and then discuss the differences and similarities. The reader should note that all tables in the remainder of this chapter contain the numbers of students giving responses in certain categories and that some students gave multiple responses or no responses to certain questions.

Shared-Work Section

Factors Influencing Participation

The factors that I identified as influencing student participation in the online assignments can be broadly classified as attributes of the student, the medium, and the learning activities. On the surveys and during the interviews, the students gave several responses corresponding to factors in each of these categories (Table 20).

Table 20
Frequency of Student Responses to Questions Concerning Factors Influencing Participation in the Online Assignments by Data Source for the Shared-Work Section

Response	Brief survey (n=22)	Extended survey (n=18)	Interview (n=7)
What factors do you think might cause a student to have poor participation in the online assignments?			
Busy schedule		6	2
Issues with using technology		7	2
Issues with accessing technology		5	2
Difficult questions		6	2
Student characteristics		1	3
Preference for paper-and-pencil work		4	0
Time consuming assignments		3	0
No one to help outside of class		0	1
No sense of community		0	1
If there are any online assignments that you didn't do, why did you not do them?			
Busy schedule	6	5	5
Forgot	6	4	3
Difficult questions	4	10	1
Issues with accessing technology	4	2	0
Issues with using technology	1	2	0

Attributes of the student. Of the three categories of factors influencing participation, attributes of the students appeared most often in the responses to the survey and interview questions. Many of the responses concerned situational factors. With the student's hectic school and work schedules, it was not a surprise that a lack of time appeared often in their responses. On the brief survey, SW9, who was frequently absent and did not pass the class, gave the following reason for not completing some of the assignments:

> I work 6 days a week and am also take [*sic*] 4 classes in which I have classes everyday. When I'm not at school or work I have other things and activities that I am in.

Although the students in this study decided to register for the course on their own, taking on the responsibilities involved, situational factors clearly affected their participation. During her interview, Student SW4 added another situational factor, saying that she had assistance from tutors and a family member, whereas some students might not have had those resources.

In addition to situational factors, several students also responded that dispositional factors were a potential issue. The most common reason the students gave for not attempting an assignment was that they forgot. This might have been an easy excuse rather than an honest explanation, but there were a number of reasons that a student could have "forgotten" to do an assignment. With their busy schedules, some students might have lost track of the deadlines, or they might have been forced to prioritize their obligations and decided to sacrifice an assignment. Forgetting to do an assignment could also be an indication of a lack of responsibility or motivation. A few students specifically mentioned those characteristics, saying that some students are lazy and uninterested in school work. Other responses included a strong preference for working with pencil and paper instead of with a computer and not feeling a sense of community.

Attributes of the medium. The second most common category of responses given by students was attributes of the medium. Attributes of the medium included issues related to technology and the asynchronous format. Two types of technology difficulties were reported. First, some students reported that obtaining access to a computer, the Internet, and an appropriate version of Java to run the graphing applet was a problem. Access to the needed technology should not have affected most students in the shared-work section because only four students reported not having a computer with Internet access at home, and Java can be obtained free on the Internet. In addition to accessing technology, students commented on difficulties involved in using the technology, such as having to type responses and not knowing how to use the software. None of the students in the shared-work section specifically mentioned the asynchronous format of the assignments, but the time independence did require students to manage their time effectively, which has already been discussed.

Attributes of the learning activities. I discussed the online assignments in chapter 2 and noted that I attempted to create challenging problems. According to several students, the nature of those problems did influence their participation. On the brief survey, only 4 out of 22 students said that they did not post anything for an assignment, because the problem was too difficult, but the number of students making that type of response was 10 out of 18 on the extended survey. In addition to comments on the difficulty of the problems, a few students said that the assignments were time consuming. During their interview, Students SW18 and SW25 agreed that the amount of time required to complete the problems was a major drawback.

Factors Influencing Interactions

In the shared-work section, there were three forms of interaction: viewing a classmate's work, incorporating a classmate's work in one's own work, and interacting directly through dialogue posted in a classmate's square. I asked students about each of those interactions using the surveys and interviews.

Viewing a classmate's work. Table 21 contains the frequency of student responses, from the extended survey and interviews, to the question of why students did not view a classmate's work on one or more assignments. As with participation, there were attributes of the student, the medium, and the learning activities, as well as perceived attributes of the other students. The most common response was that some of the problems could be solved independently. All but one of the interviewed students said that they did not look at a classmate's problem when certain

Table 21
Frequency of Responses to Reasons for Not Viewing a Classmate's Work by Data Source

Response	Extended survey (n=18)	Interview (n=7)
Understood how to solve the problem without help	4	6
Classmates' work might be incomplete or incorrect	5	1
Did not want to be confused by classmates' answers	4	0
Busy schedule	2	0
Classmates posted privately	2	0
Forgot	1	0
Did not know it was allowed	1	0

of their solution. I have already addressed this claim in my discussion of the nature of the interactions (see p. 70). The next most common responses contained the word *confused*. Five students said that their classmates might be confused, incorrect, or not have answers to their questions, and 4 students said that they might get confused themselves by reading other students' answers. Less common responses included busy schedules, classmates posting privately, forgetting to look, and not knowing it was allowed. Several students did post privately at the beginning of the class, but by Assignment 6, only two students consistently posted privately.

Incorporating a classmate's work. On the extended survey and during the interviews, I asked the students how they had found viewing a classmate's work helpful. The results are in Table 11 (p. 71). As with viewing work, most of the students said that they used classmates' work only when they needed help or wanted to check their work, and a few students said that they had found it helpful to view alternate solutions to problems. Four students provided reasons for preferring not to look, which included a desire to work independently, private posts, and not understanding the work.

Posting in a classmate's square. The students gave a wide variety of reasons that they did not make posts in classmates' squares (Table 22). As with other questions, some of the students mentioned a lack of time, because of situational factors. Several students indicated that they were not comfortable interacting, either not wanting to interfere with their classmates or not knowing what to say, and a few students indicated that they did not feel a need to interact, preferring to work independently or posting only when required. Three students said that they did not know that they were allowed to make posts in classmates' squares. This comment surprised me because I had spent time the first week of classes discussing the ways that students could interact through the software.

Table 22
Frequency of Responses to Reasons for Not Posting Comments or Questions in a Classmate's Tensor Square by Data Source

Reason	Extended survey (n=18)	Interview (n=7)
Not enough time	4	2
Did not want to criticize or interfere with others	4	1
Did not know it was allowed	3	1
Did when required	2	1
Preferred to work independently	2	0
Did not know what to say	1	1
Did not think classmates would respond	0	1

Small-Groups Section

Factors Influencing Participation

Table 23 contains the frequencies of the students' responses to the interview and survey questions concerning the factors that influenced participation in the online assignments. As with the shared-work section, the responses can be classified as attributes of the student, the medium, and the learning activities.

Table 23
Frequency of Responses to Questions Concerning Factors Influencing Participation in the Online Assignments by Data Source for the Small-Groups Section

Response	Brief survey (n=15)	Extended survey (n=15)	Interview (n=15)
What factors do you think might cause a student to have poor participation in the online assignments?			
Student characteristics		9	5
Busy schedule		8	1
Issues with accessing technology		7	2
Issues with using technology		4	0
Difficult questions		2	0
Not being able to meet face-to-face		0	1
If there are any online assignments that you didn't do, why did you not do them?			
Forgot	7	6	
Busy schedule	3	4	
Issues with accessing technology	2	3	

Attributes of the student. As in the shared-work section, the students in the small-groups section mentioned dispositional and situational student attributes that influenced their participation. Student characteristics, such as laziness and apathy, and busy schedules were the most commonly mentioned factors related to poor participation. A busy schedule was also given by several students as a reason for not doing an assignment, but the most common response was forgetting. During her interview, Student SG24 said that face-to-face contact was essential, and therefore that not being able to find time for a group meeting could also be a problem.

Attributes of the medium. The student responses that mentioned attributes of medium were similar to the responses from the shared-work section. A number of students responded that difficulties using or accessing technology could be a factor that limited their participation. Asked why they did not attempt an assignment, no one reported problems using the technology, but a few students reported difficultly accessing a computer or installing Java.

Attributes of the learning activities. According to the student responses, attributes of the learning activities did not have a large influence on participation in the small-groups section. Not a single student mentioned attributes of the learning activities as a reason for not attempting an assignment, and only two students listed the difficulty of the assignments as a factor that might cause low participation.

Factors Influencing Interaction

The most obvious factor influencing interaction in the small-groups section was the small-groups approach. Unless a student made the first post in a group square, he or she could not participate without viewing a classmate's post. This feature of the small-groups approach increased the number of PSR posts and required interaction, at least in the form of viewing a classmate's work. Other factors that I identified through the surveys and interviews were the impact of nonparticipants, establishing a sense of community, and the asynchronous format.

Impact of nonparticipants. References to students not participating or making small contributions, such as posting "I agree," were abundant in the survey and interview responses. Nonparticipation is always a potential problem when students are asked to collaborate, but the problem may be magnified in the online setting. In a face-to-face setting or synchronous discussion, the mere presence of a group member may reinforce the perception that a group is working together and that the students are not alone. In an asynchronous setting when students do not participate by making timely posts, other students may come to feel that they are working alone, a phenomenon that seems to have occurred in this study. Asked on the extended survey what factors could be harmful to a group's success, 10 out of 16 students gave responses referring to other group members not participating, and when asked what factors were necessary for success, 5 students mentioned participation by all members.

On the brief survey, I asked the students to explain whether their group had a successful experience on the first three assignments. Because I was the instructor, it is not surprising that most students said that they had done well and worked together. Three students, however, did mention the impact of nonparticipating students:

> SG13: No, I feel like I'm the only one who made an attempt. Maybe they don't know the answer.
>
> SG15: With assignments 1-3, I think my group was not really successful because not all of us did the work.
>
> SG18: No. I did ALL the work.

Student SG18 was the only one to participate in her group on the first three assignments. Students SG13 and SG15 were in the same group, and neither of the other two group members contributed to the first three assignments.

The effect of nonparticipants also arose on the extended survey when I asked the students how I could have helped them more on the problem that they found most challenging. Student SG4 wrote, "I probably would have had more success with this problem if my group had helped me when [I] asked for help in the group discussion section."

A sense of community. Asked on the extended survey what factors are necessary for successful online groups, five students gave responses related to building a sense of community. All five mentioned a need to work together in class or outside of the online environment. Student SG10, who consistently expressed a preference for face-to-face interactions, wrote, "The main [factor] I believe is communication outside of the online, if they are able to form a bond it could mean more." Four other students gave responses that expressed a general need to communicate and work together as a team.

The one group of students who asked to work together provided an example that a strong sense of community is not sufficient to ensure meaningful participation and interaction. These three students made it clear in class that they were friends prior to the semester and knew each other well, but all three had low participation in the online assignments, leaving Student SG4 to work alone on most of the problems.

The asynchronous format. The fact that the students did not have to work together at the same time and location definitely had an influence on their interactions. On the extended survey, two students said that not being able to work together at the same time was a major drawback to the online assignments, and as previously noted, five students said that a group needed to work together face-to-face in order to be successful. Two specific consequences of the asynchronous format arose during the interviews when I asked why there had not been more interactions and discussions in the tensor. Student SG19 said that once a student believed that he or she had posted a correct solution to a problem, he or she was not likely to return to that problem. Student SG7 addressed the inability to physically get someone's attention in the online environment:

> People would either not participate, or they would post late. And you can't force them because it's on a computer. So you can't be like "hey, what did you write?" So, it's less personal. I guess [that] is the way you would say it. … I mean it's obvious that it's a computer, but it can be a bit of a drawback. [In person] you can talk. You can tap them on the shoulder [and say] "is that what you meant?" [On] the computer, you just write it.

Asynchronous collaboration certainly has limitations created by the loss of real-time, face-to-face interaction.

Comparison Between Sections

Factors Influencing Participation

The students in the shared-work and small-groups sections provided responses indicating that participation was influenced by attributes of the student, medium, and learning activities. In both sections, students mentioned difficulties accessing and using technology as possibly leading to poor participation, and in both sections only a few students claimed that attributes of the technology kept them from attempting a problem. I expected this response because in both sections, all but a small number of students reported having a computer with Internet access at home. In both sections, a large number of responses included statements about dispositional and situational student attributes, which included 11 students in each section responding that they did not attempt to do an assignment because they forgot.

There were considerably more students who referred to a busy schedule as a reason for not attempting an assignment in the shared-work section, but the most noticeable difference was the number of references to attributes of the learning activities. In the shared-work section, 9 students included comments about the difficulty of the problems or the time required to complete the assignments as potential reasons for low participation, and 11 students responded that they did not do a problem because it was too difficult. In the small-groups section, no one provided an attribute of the learning activities as a reason for not attempting a problem, and the difficulty of the problems was mentioned by only two students as a potential factor causing poor participation. In both sections, I told the students that I had attempted to write problems that would require considerable effort and collaboration and that they were not expected to completely solve the problems on their own. Apparently the explicitly defined group structure in the small-groups section did a better job of conveying that message because the students in that section rarely made comments about the difficulty of the problems, whereas students in the shared-work section frequently did.

Factors Influencing Interaction

Because participation was necessary for interaction, every factor that influenced participation had some influence on the interactions. Thus, the factors such as busy schedules and student dispositions affected interactions in both classes. With the different approaches to collaboration, however, there were significant differences in the factors that influenced the interactions. The students in the shared-work section viewed classmates' work on only 68% of the assignments that they attempted, giving reasons such as being able to solve a problem alone, not trusting other students' answers, and not wanting to be confused by reading other students' solutions. The use of groups made it very difficult for a student in the small-groups section to participate without viewing a classmate's post. Even if a student had solved a problem independently, he or she had to access the group square and make a post, which promoted some form of interaction. The requirement of working with specified groups also created issues for the small-groups students, such as nonparticipating group members and the lack of a sense of community that might not have affected the shared-work students as significantly.

CHAPTER 5

CONCLUSIONS

In this chapter, I present a brief summary of the study, including the purpose, method, and results. I then present conclusions and interpretations of the results. I also address limitations of the study and implications for teaching and future research. I conclude with some personal reflections on the study and my experiences teaching with the tensor.

Summary

The purpose of this study was to gain a better understanding of the nature and quality of student interactions when solving online asynchronous mathematics assignments using a communication tensor (an array of hyperlinks called tensor squares that provide access to each student's work on each assignment) and to explore factors influencing those interactions. Because few researchers have addressed the use of asynchronous computer conferencing in mathematics (Kosiak, 2004; Myers, 2002; Simonsen & Banfield, 2006), and no one has attempted a formal analysis of the work produced with a communication tensor, my initial goal was to classify student behavior when using asynchronous computer conferencing. I applied coding schemes to two sections of college algebra that I taught with different approaches to the online assignments. One section operated similarly to classes taught by the communication tensor's originator, Thomas Banchoff. Students in this shared-work approach worked on assignments individually for an initial phase and then had access to one another's work for a secondary phase. The second section operated with a more traditional small-groups approach.

Students in the small-groups section worked on the same problems as their shared-work counterparts, but they worked in designated groups of 3 to 4 students.

Using the archived student work, I developed a general coding scheme for asynchronous interactions. I adapted a scheme designed by Stacey and Gooding (1998) for face-to-face collaboration. I combined some of Stacey and Gooding's categories and added some additional categories. To determine how the students used classmates' work, I also developed a multi-dimensional coding scheme to analyze the units of meaning for which a student posted a solution after viewing a classmate's work in the tensor. I applied the coding schemes to data from both sections to determine the nature and quality of their interactions and to make comparisons. To obtain additional information about student achievement on the online assignments, I also reported and examined the average student scores on each problem in each section.

I also administered a series of open-ended surveys and interviewed 13 students. I asked the students questions about their experiences with the online assignments and factors that might have influenced those experiences. Guided by my research questions, I analyzed the surveys and interview transcripts for recurring themes using a constant comparative method (Glaser, 1965).

The Nature of Interactions

In both sections, I coded the vast majority of the units of analysis as cases of students posting solutions independently or students posting solutions created after viewing a classmate's work. I posted approximately three times as many messages to students in the shared-work section as I did in the small-groups section, and that resulted in more units in the shared-work section coded as responding to the instructor. The students in the small-groups section displayed a wider variety of student-to-student interactions, with a fair amount of commenting, questioning, and responding. Most such responses in the shared-work section were between the students and me. In the shared-work section, no messages were coded as explaining proposed solutions to classmates, giving directions to classmates, or refocusing discussions, and only a small number of messages received such codes in the small-groups section. The distribution of messages by student revealed that the small-groups section was more interactive, with more students identified as leaders and active followers.

The Quality of Student Work and Interactions

The posts in which students proposed a solution after viewing a classmate's post were slightly more common in the small-groups section than in the shared-work section. The percent of students incorporating classmates' work into their own were much the same in each section, but the percents making minor and major individual contributions were slightly higher in the small-groups section. In both sections, the majority of the solutions posted after a student had viewed a classmate's work contained essentially the same information as the viewed post, but a higher percentage of students in the small-groups section made improvements in their classmates' work. The only result of the coding that favored the shared-work section was that the small-groups section had more instances of students changing their proposed solutions to less correct solutions after having viewed their classmates' work, but the number of such changes was small in both sections.

Achievement on the online assignments favored the small-groups section. Although the students were operating under different approaches to collaboration, the average score per problem on a 6-point rubric was higher for students in the small-groups section than the shared-work section on 33 out of 40 online problems, and the overall mean score was nearly 1 point higher in the small-groups section.

Student Perceptions

I used surveys and student interviews to obtain information about factors that might have influenced the nature and quality of the interactions and participation in the online assignments. Some of the questions applied to both sections, and others were specific to the shared-work or small-groups approach. All of the participants were asked a series of questions aimed at discovering factors that promoted or hindered participation and interaction. To analyze the data on participation, I adapted Bullen's (1998) classification of influencing factors. The participants saw attributes of the student, including situational and dispositional characteristics, as potentially contributing to low participation. Several students in each class saw attributes of the medium, including difficulty accessing and difficulty using technology, as potentially harmful to participation, but only a few students reported that those attributes prevented them from attempting a problem. Most of the students in the shared-work section reported that the difficulty level was a reason that they did not attempt one or more problems, but no students made such a claim in the small-groups section.

In the shared-work section, I investigated factors influencing interactions by using the interview and survey questions that concerned reasons for not viewing a classmate's work, how classmate's work was helpful, and why more students had not posted messages in classmates' tensor squares. The most common reasons reported by the students for not viewing a classmate's work were that they could solve some problems by themselves, classmates might be incorrect, or that they did not want to become confused by reading classmates' solutions. Common responses by students to how classmates' work was helpful included providing assistance when they were stuck and helping them check their own solutions. A few students said that it was helpful to see different ways to solve a problem. The most common reasons reported by the students for not posting messages in classmates' tensor squares included not wanting to criticize or interfere and not having enough time.

The main themes concerning interaction in the small-groups section were the impact of nonparticipating students, the need for a sense of community, and effects of the asynchronous format. The majority of the students expressed the view that meaningful participation from all group members was necessary for success, and numerous responses to interview and survey questions contained references to students not participating or simply posting "I agree." A few students said that group work could be successful only if the team members knew each other well, and a few students said that face-to-face contact or real-time discussions were necessary for groups to work together.

Conclusions and Interpretation

The Nature of Asynchronous Interactions

I began using computer conferencing in my teaching of undergraduate mathematics because I hoped that it would help me break free from the time constraints imposed by three 50-minute classes per week and develop an environment in which students could work on challenging, nonroutine problems while interacting with each other and with me. Researchers such as Myers (2002) have argued that asynchronous communication is not suited for collaborative learning. I believe that the results of this study suggest that although asynchronous collaboration may be difficult, especially in a class in which the majority of the students have been required to take remedial mathematics and some are repeating the course, the use of tools such as a communication tensor has potential for improving learning.

I picked the terms *shared-work* and *small-groups* going into this study, but having completed the analysis and thought about the results, I think both sections operated in a shared-work fashion, which might be a reasonable alternative to true collaboration. When I asked the students in the small-groups section how they had worked on the assignments, the overwhelming response was that most students worked alone, posted their best answers, viewed other students' solutions, and worked from there. That type of behavior is what I had in mind when I came up with the term *shared-work*. Each student works alone for awhile, and then work is shared and, hopefully, built upon by classmates.

The coding of the students' interactions indicated that both sections operated primarily in a shared-work mode, but the designation of specific groups was helpful. In both classes, the vast majority of the interactions were instances of students proposing solutions, and the second highest percentage in each class was proposing solutions after having viewed another student's work. That percentage was higher in the small-groups section, with fewer students producing more units of interaction. The students in the small-groups section were more likely to build on and improve classmates' solutions than the shared-work students.

In both sections, there were a few interactions such as questioning, responding, or commenting. In the shared-work section, the comments and questions were mainly directed at me. In the small-groups section, questions to classmates often went without answers, and responses were typically statements of agreement. Moreover, only one student explained a solution using evidence, which was mainly because explanations tended to accompany solutions rather than being made in response to requests for clarification. The students in this study had many demands on their time other than learning or doing mathematics. Most of them had a history of bad experiences in mathematics. The average student in each class was working over 20 hours a week and taking at least four classes. A few students had children, and there were many other personal duties and life situations for these students to deal with. It is understandable;

therefore, that many of these students focused on their own assignments and tried to do what was necessary to achieve a passing grade and no more.

I consider student situational and dispositional factors to be the most challenging to combat. If a student has spent more than a decade developing a belief that mathematics is too difficult, is not important, or should be taught as a series of step-by-step procedures, a teacher is going to have a hard time changing those opinions. If a student has to take several classes and work more than 20 hours a week to pay for tuition or provide for a family, a teacher cannot add more hours to the day. It is the attributes of the medium and the design of the learning activities that teachers have the most control over and should continue to study and search for ways to improve.

The Benefits of Using Groups

While interactions were limited in both sections, the students in the small-groups section outperformed the students in the shared-work section in a number of ways. Both sections had 8 nonparticipating students. Considering only the participating students, the small-groups section had fewer students but produced more messages per student, created more overall units of meaning, and displayed higher levels of interaction than the shared-work section. Moreover, the students in the small-groups section outscored the students in the shared-work section on most of the online problems, and the students in the small-groups section tended to perform better in class than the students in the shared-work section did. On the final exam, which contained problems similar to those found in the textbook (Blitzer, 2007), the participating students in the small-groups section had a class average of 74, whereas the participating students in the small-groups section had a class average of 68. I cannot conclude a causal relationship, but the use of small groups certainly seems to have been beneficial to the students.

Benefits of the Tensor

Whether an instructor uses a shared-work or a small-groups approach, the major benefits of using the tensor are that students can get timely feedback from the instructor and see the work of their classmates. For example, consider Problem 6.3 (see Appendix A). On this problem, I asked the students to determine values of B and C such that the line $2x - By = C$ has negative slope and a negative y-intercept. One student in the shared-work section incorrectly solved the equation for y, getting $y = -2x/B - C/B$. He added, "I did get a negative slope and a negative y-intercept but somehow I still think my answer is wrong." After the tensor had unlocked, this student posted a new message, which stated that he had looked at a classmate's response, caught his mistake, and created an alternate solution. He had caught his sign error and realized that B and C did not have to be positive whole numbers. This type of behavior also occurred in the small-groups section as students caught mistakes or misconceptions by looking at their group member's responses.

Limitations

I served as both the researcher and the instructor in this study, which can be viewed as a strength and a weakness. Obviously, I had a vested interest and wanted the students to succeed, but I believe whatever bias I brought to the study was more than made up for by the knowledge I gained from having worked with the students as their teacher in the classroom and online over the entire semester. I have tried to make my methods of collecting and analyzing data as transparent as possible so that readers can judge the merit of the results for themselves. I hope that more instructors will embrace the use of computer conferencing for online collaboration in mathematics, and I would like to see more studies conducted with different instructors and different researchers.

A limited amount of feedback from nonparticipating students was also a limitation of this study. In the shared-work section, there were 8 students that I classified as nonparticipants because they created less than 1% of the online posts. Because of their limited attendance or withdrawal from the course, I was not able to interview any of these nonparticipating students. I did receive survey responses from 6 of the nonparticipating students on the brief survey and from 3 of them on the extended survey. I obtained less information from the 8 nonparticipants in the small-groups section. I was not able to interview any of the nonparticipants in that section, and the survey data were limited to the brief survey from one student. It is likely that the nonparticipants could have provided valuable insight about the factors influencing participation.

Another limitation of this study, common to qualitative research, is the inability to generalize the results. The sample size was small, and I used two of my intact College Algebra classes to ensure the implementation of the shared-work and small-groups approaches. I could not randomly assign students to sections and therefore could not account for such variables as time of day and individual characteristics. The purpose of this study was not to show the superiority of one approach over another. Instead, I sought to gain some initial understanding of the way students use a communication tensor and factors that affect their behavior. There is still much room for future research.

Recommendation for Teachers

Based on the results of this study, I would recommend that teachers interested in using a communication tensor or similar software to facilitate online problem solving use an approach with an initial individual phase and a secondary group phase. An initial individual phase requires each student to participate and attempt to solve the problems, and a secondary group phase provides opportunities for a focused collaborative effort. Without the initial phase, some students might wait until other group members have solved a problem and contribute only by superficially agreeing. Placing the students into groups for a secondary phase might lessen a student's tendency to work independently, and the use of groups reduces the number of classmates' posts that each student has access to in the tensor, allowing the student to focus more on his or her group members' posts. Furthermore, it is much easier for a teacher to manage a 5 to 10 group squares as opposed to 20 to 30 individual squares.

Another recommendation that I have for teachers using asynchronous online assignments is to take an ample amount of class time throughout the semester to discuss technical issues and to model productive behavior for students. When I asked students why they did not look at classmates' work more often, one student said that he did not know that he was allowed to do that. When I asked why they did not ask questions or make comments in classmates' assignment squares, three students said that they did not know that they were allowed, and one student said that she did not know what to say. I devoted a lot of time at the beginning of the course to discussing the software and my expectations, but a couple of the students who said that they did not know that they were allowed to interact had enrolled late. Therefore, I suggest that teachers continuously remind students of productive behavior throughout the semester. Such reminders would also be helpful if students become stuck in a routine of focusing on their own work as the semester progresses.

The coding schemes used in this study can be used to help students understand productive behavior. For instance, a teacher could provide examples of students asking good questions and responses including explaining with evidence. Teachers could also describe the main ideas of the coding scheme that I designed for solutions made after having viewed classmate's work and provide examples of responses falling into the various categories, which might allow students to understand how they can use a classmate's post and make their own major contributions.

Recommendations for Future Research

This study provided some initial evidence that the approach that I have called small-groups may have some advantages over the individualized shared-work approach. Without using a more experimental research design, one cannot make causal inferences about the approaches. In a future study, I would like to see a design more closely resembling an experiment to test whether one approach leads to higher levels of achievement than the other does. I would also like to see the tensor used with a wide variety of students in different types of courses at different types of institutions to determine where it is most effective. Thomas Banchoff has used his tensor software with a variety of courses including Calculus, Differential Geometry, and Foundations of Geometry for Teachers at Brown University and other institutions, including the University of Georgia. I was a student in one of those courses at the University of Georgia, and I have used Banchoff' tensor and my own version at a community college with students in College Algebra, Precalculus, and Calculus III. I would definitely like to see the tensor become more widely used and studied. My final recommendation is to explore the effect of explicitly using the coding schemes with students. It would be useful to see if explicitly training students with the scheme to create specific types of messages would actually result in more interaction or higher achievement.

Personal Reflections

When I began teaching undergraduate mathematics, I tried to emulate the professors that I thought had taught me well as a student. Considering myself an independent learner, I preferred teachers who spent most of the class time presenting well-prepared lectures. I always felt that as long as a teacher presented the material logically, answered students' questions, and assessed the students fairly with material similar to that seen in lectures and homework, then the teacher had done an excellent job. When faced with the task of helping students learn mathematics, I soon discovered that my best prepared lectures were often ineffective, with several hardworking students failing to master the material. My professors at the University of Georgia helped me realize that mathematics is not a static body of knowledge that can be acquired from books. Instead, mathematics is a dynamic way for people to describe and analyze the world. I am convinced that each student brings a unique set of strengths and weakness, based on personal experiences, to the classroom, and there is no single method of teaching that is best for all students.

Knowing that research has shown benefits of collaborative assignments (Springer, Stanne, & Donovan, 1999), I have tried to incorporate collaborative activities into my teaching. My experiences as a student did not include many collaborative activities, and I have always felt a pressure to cover a great deal of material in what seems like a short amount of time. I was excited when I saw that Thomas Banchoff's tensor could facilitate online collaborative problem solving. I still use most of my face-to-face class time lecturing and discussing material with students, but the tensor has allowed me to include collaborative problem-solving activities that students can do outside of the classroom. I had students work together on several in-class assignments during this study, but time constraints limited the types of problems that I could assign. With the tensor, I was able to give problems that were unfamiliar to the students and required time for thinking and exploring. I simply did not have enough time for the students to work on these types of problems in class, and hectic schedules at a nonresidential college prevented most students from working together in person outside of class. The tensor allowed me to assign challenging problems and facilitated communication among students and between the students and myself.

When I began this study, I intended for one section to work on the online assignments in small groups with no individualized phase, but I realized that requiring initial posts by each student prior to group work could increase participation. This approach of combining an initial individual phase with a secondary group phase, which I have recommended to teachers, has turned out to be one of the most effective methods that I have seen for facilitating online group work. Using a two-phase approach, the students receive the benefits of group work, and requiring initial posts helps alleviate the potential problems of students not participating and students dominating discussions. If one student starts off a group discussion with a correct solution to a problem and the other group members respond in agreement, an instructor has no way of knowing if the agreeing students could have solved the problem by themselves. By requiring initial posts, instructors gain more insight into each group member's abilities. Based on my experiences during this study, I definitely intend use a two-phase group approach in the future.

Finally, I would like to address a question that I often hear when I discuss the tensor with other teachers. Inevitably, people tend to ask me about the amount of time required to monitor the students and provide feedback in the tensor. Using the tensor is time consuming for an instructor. While I have never kept detailed records, I would estimate that reading and making comments in 20 to 30 tensor squares for an individual assignment has usually taken me between one and two hours, and I have always tried to provide feedback several times between the initial release of an assignment and the final deadline. If a computer has a slow connection to the Internet, it can be frustrating waiting for the contents of a tensor square to load. The use of small groups definitely helped in this respect. When the students were required to work in group squares, I only had to access and work with the contents of 5 to 10 group squares. When I included individual squares and an initial phase for students, I had more tensor squares to manage, but I felt less need to make comments in every individual square than I did in the shared-work section, because I knew that the students were going to work together in the group squares during the second phase. In summary, using the tensor is time consuming, but I have found it rewarding. The tensor has allowed me to use more challenging problems than I have used without it, and I would argue that the time spent communicating with my students in the tensor has allowed me to get to know them better than I would have otherwise.

REFERENCES

Allen, G. D. (2001). Online calculus: The course and survey results. *Computers in the Schools, 17*(1–2), 17–30.

American College Testing Program. (2008). *COMPASS: College placement tests.* Retrieved May 10, 2008, from http://www.act.org/compass/index.html

American Mathematical Association of Two-Year Colleges. (2006). *Beyond crossroads: Implementing mathematics standards in the first two years of college.* Memphis, TN: Author.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245–274.

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction*, *9*(2), 137–175.

Banchoff, T. (2005, May). Interactive geometry and multivariable calculus on the Internet. *Proceedings of KAIST International Symposium on Enhancing University Mathematics Teaching, Daejeon, Korea*. Retrieved April 24, 2007, from http://www.mathnet.or.kr/kaist2005/article/banchoff.pdf

Blitzer, R. (2003). *College algebra* (3rd ed.). Upper Saddle River, NJ: Prentice Hall.

Blitzer, R. (2007). *College algebra: An early functions approach.* Upper Saddle River, NJ: Prentice Hall.

Bullen, M. (1998). A case study of participation and critical thinking in a university-level course delivered by computer conferencing. *Dissertation Abstracts International, 59*(1), 51A. (UMI No. AAT NQ25024)

Cengage Learning. (2008). CengageNOW [Computer software]. Available from http://west.ilrn.com/ilrn/

Chickering, A. W., & Ehrmann, S. C. (1996). Implementing the seven principles: Technology as a level. *American Association for Higher Education Bulletin*, *49*(2) 3–6. Retrieved July 15, 2006, from http://www.tltgroup.org/programs/seven.html.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement, 20*, 37–46.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Erlbaum.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, *64*, 1–35.

Cooney, T., Sanchez, W., Leatham, K., & Mewborn, D. (n.d.). *Open-ended assessment in math.* Retrieved May 26, 2008, from http://books.heinemann.com/math/

Davidson, N. A., Reynolds, B. E., & Rogers, E. C. (2001). Introduction to cooperative learning in undergraduate mathematics. In E. C. Rogers, B. E. Reynolds, N. A. Davidson, & A. D. Thomas (Eds.). *Cooperative learning in undergraduate mathematics: Issues that matter and strategies that work.* (MAA Notes 55, pp. 1–11). Washington, DC: Mathematical Association of America.

De Corte, E., Verschaffel, L., Dhert, J. L. S., & Vandeput, L. (2002, August). *Collaborative learning of mathematical problem solving and problem posing supported by* *'webknowledge forum': A design experiment.* Paper presented at the International Federation for Information Processing 17th World Computer Congress - TC3 Stream on Tele-Learning: The Challenge for the Third Millennium, Montreal.

Fullilove, R. E., & Treisman, P. U. (1990). Mathematics achievement among African American undergraduates at the University of California, Berkeley: An evaluation of the Mathematics Workshop Program. *Journal of Negro Education, 59*, 463–478.

Garofalo, L. R., & Lester, F. K. Jr. (1985). Metacognition, cognitive monitoring and mathematical performance. *Journal for Research in Mathematics Education*, *16*(3), 163-175.

Glaser, B. G. (1965). The constant comparative method of qualitative analysis. *Social Problems, 12*(4), 436-445.

Gunawardena, C. N., Lowe, C. A., & Anderson, T. (1997). Analysis of a global online debate and the development of an interaction analysis model for examining social construction of knowledge in computer conferencing. *Journal of Educational Computing Research*, *17*, 397–431.

Guzdial, M., Lodovice, P., Realf, M., Morley, T., & Carroll, K. (2002, October). *When collaboration doesn't work*. Paper presented at the 5th International Conference of the Learning Sciences, Seattle.

Harasim, L. (1986). Educational applications of computer conferencing. *Journal of Distance Education, 1*(1), 59–70.

Hodges, C. B. (2006). Self-efficacy, motivational email, and achievement in an asynchronous mathematics course. *Dissertation Abstracts International, 66*(11). (UMI No. AAT 3197970) Retrieved June 2, 2008, from http://scholar.lib.vt.edu/theses/

Hornsby, J., Lial, M., & Rockswold, G. (2007). *A graphical approach to college algebra* (4th ed.). Boston: Addison Wesley.

Huck, S. W. (2004). *Reading statistics and research* (4th ed.). Boston: Pearson Education.

Hurme, T., & Jarvela, S. (2005). Students' activity in computer supported collaborative problem solving in mathematics. *International Journal of Computers for Mathematical Learning, 10,* 49–73.

Klemm, W. R. (1998). Eight ways to get students more engaged in online conferences. *Technology in Higher Education Journal, 26*(1), 62–64.

Kosiak, J. J. (2004). Using asynchronous discussions to facilitate collaborative problem solving in college algebra. *Dissertation Abstracts International*, *65*(5), 2442B. (UMI No. AAT 3134399)

Lamb, C. E., & Klemm, W. R. (1997). Computer conferencing in mathematics classrooms: Distance education--the long and the short of it*. Annual Proceedings of the National Distance Education Conference* (pp. 89–93). (ERIC Document Reproduction Service No. ED 416 103)

Lincoln, Y., & Guba, E., (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.

McCallum, M. E. (2005). *A comparison of instructional delivery models for teaching mathematics at the college level.* Unpublished doctoral dissertation, University of Georgia, Athens.

Myers, J. (2002). A content analysis of computer-mediated collaborative mathematical problem solving. *Dissertation Abstracts International, 63*(05), 1756A. (UMI No. AAT 3054205)

Nagai, M., Okabe, Y., Nagata, J., & Akahori, K. (2000, November). *A study on the effectiveness of Web-based collaborative learning system on school mathematics: Through a practice of three junior high schools.* Paper presented at the International Conference on Computers in Education/International Conference on Computer Assisted Instruction, Taipei.

Nagai, M., Shiraki, K., Koshikawa, H., & Akahori, K. (2001, November). *Development and evaluation of a Web bulletin board enhanced with a knowledge map.* Paper presented at the ICCE/SchoolNet 2001, Seoul, Korea. Retrieved July 18, 2006, from http://www.ak.cradle.titech.ac.jp/Publication/pdf/nagai/nagai_ICCE2001.pdf

O'Daffer, P., Charles, R., Cooney, T., Dossey, J., & Schielack, J. (2005). *Mathematics for elementary school teachers* (3rd ed.). Boston: Addison-Wesley.

Patton, M. Q. (2002). *Qualitative research & evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.

Pearson Education. (2006). MyMathLab [Computer software]. Available from http://www.mymathlab.com/

Pragnell, M. V., Roselli, T., & Rossano, V. (2006). Can a hypermedia cooperative e-learning environment stimulate constructive collaboration? *Educational Technology and Society, 9*, 119–132.

Rho, K. (2000, April). *A case study on the changes of university students' function concept in a virtual environment*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED440869)

Rogers, E. C., Reynolds, B. E., Davidson, N. A., & Thomas, A. D. (Eds.). (2001). *Cooperative learning in undergraduate mathematics: Issues that matter and strategies that work.* (MAA Notes 55). Washington, DC: Mathematical Association of America.

Rouhani, B. (2005). A case study of students' knowledge of functions in an online college algebra course. *Dissertation Abstracts International*, *65*(12), 4501A. (UMI No. AAT 3159321)

Sands, P. (2002). Inside outside, upside down: Strategies for connecting online and face-to-face instruction in hybrid classes. *Teaching with Technology Today, 8*(6). Retrieved July 23, 2006, from http://www.uwsa.edu/ttt/articles/sands2.htm.

Satler, N. J. (2005). AMATYC members offer perceptions of interactions in on-line developmental mathematics courses at two-year colleges. *AMATYC Review, 27*(1), 62–81.

Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Simonsen, L., & Banfield, J. (2006). Fostering mathematical discourse in online asynchronous discussions: An analysis of instructor interventions. *Journal of Computers in Mathematics and Science Teaching*, *25*, 41–59

Smogor, L. (2002, April). *Computer conferencing in the university classroom*. Paper presented at the Information Technology and Universities in Asia (ITUA) 2002 Conference. Retrieved April 16, 2007, from http://www.asean-eulemlife.org/ITUA/Papers_for_ITUA_ Proceedings/itua-smogor.pdf

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research*, *69*, 21–51.

Stacey, K., & Gooding, A. (1998). Communication and learning in small-group discussions. In H. Steinbring, M. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 191–206). Reston, VA: National Council of Teachers of Mathematics.

Zafeirou, G., Nunes, J., & Ford, N. (2001). Using students' perceptions of participation in collaborative learning activities in designing online learning environments. *Education for Information, 19,* 83–106.

APPENDICES

APPENDIX A

The Online Assignments

Assignment 1: Is It a Function?

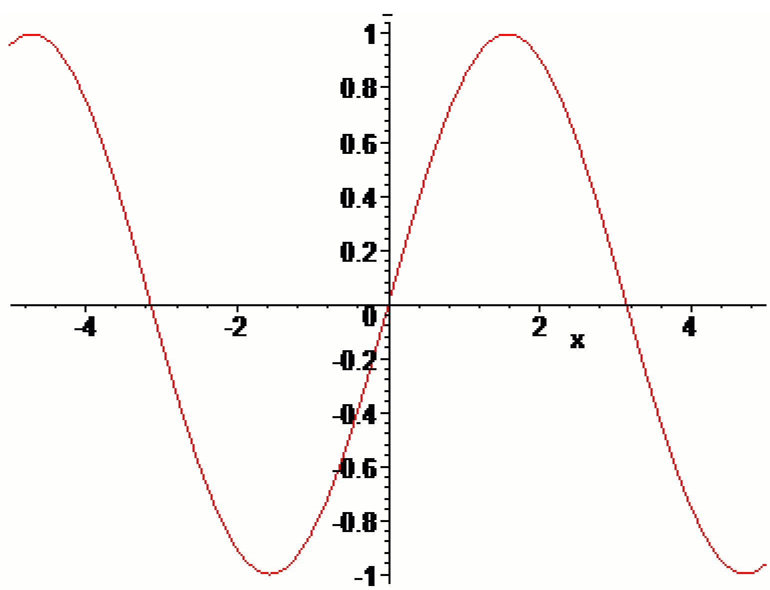
1. Determine whether or not the given relation represents y as a function of x. Explain your answer.

a) x+1 = |y|

b) x: 1 2 3 4 -1 -3

y: 5 5 5 3 2 3

c)



2. The following list contains real world relationships. Some of these represent functions, and some do not. A couple could probably be argued either way depending on your interpretation.

Pick one that you think is a function and explain how you can tell it is a function.

Then pick one that you think is not a function and explain how you can tell that it is not.

I am going to use the phrasing x is assigned to y to mean that an input x has an output y.

a) Each person in your class is assigned a grade.

b) A parent is assigned to his or her children.

c) Each person is assigned to his or her social security number.

d) Each amount of money is assigned to the object that it will buy.

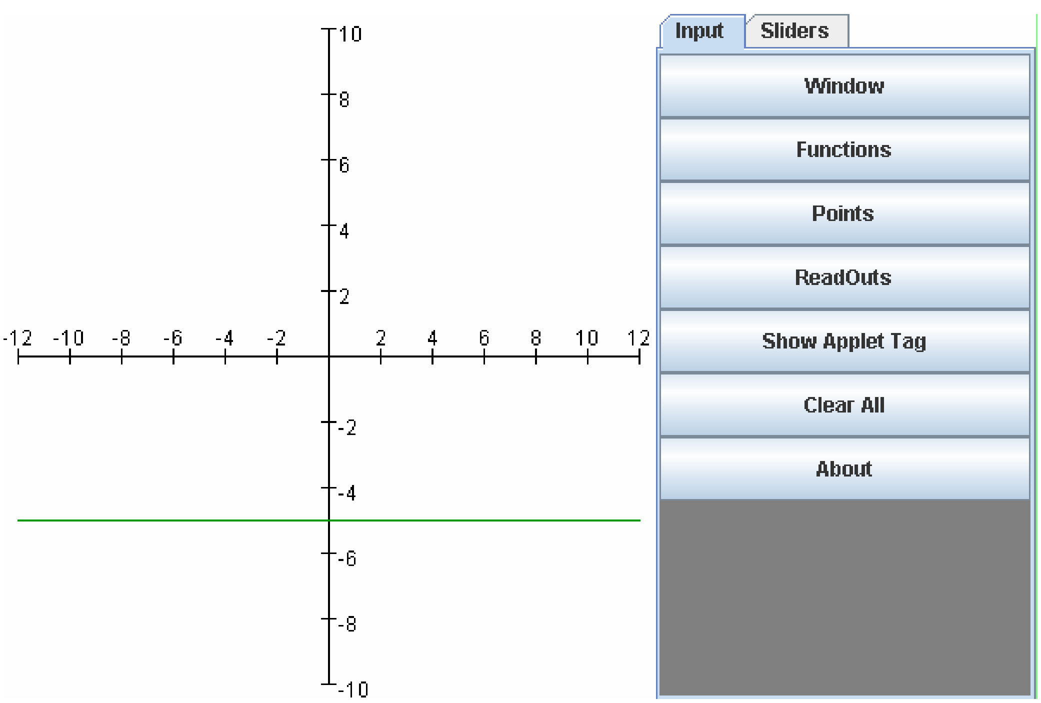
e) People are assigned to their telephone numbers.

f) A person is assigned to a person who is older.

Assignment 2: Domain & Range

This assignment will allow you to explore domain and range as well as the idea of parameters, sliders, and families of functions. I will first show an example and then present your questions.

Example 1: The demo below shows the function y = k. k is defined to be a slider that we can change it. If you move the slider for k, you will see the horizontal line move up and down.



Note: With the use of the parameter **k** defined as a slider, this **demo** shows not one but many graphs for different values of k.

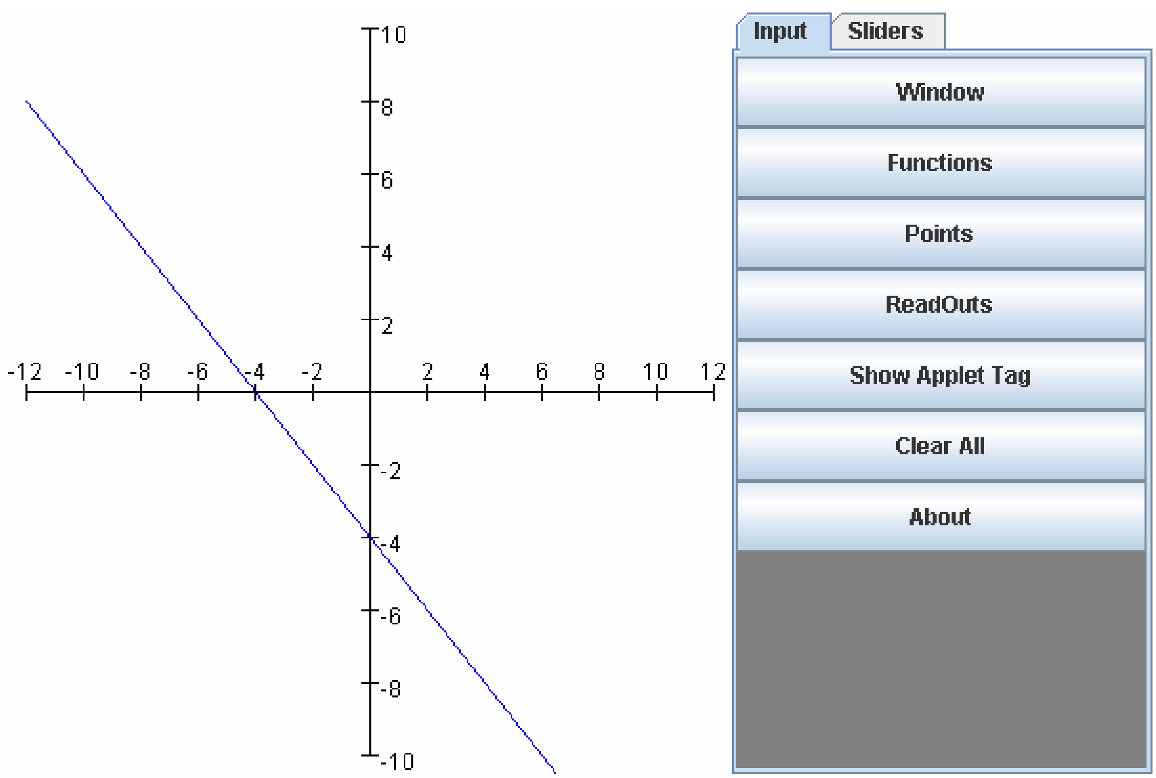
What is the domain and range of y = k?

Solution: The domain is all possible inputs (x-values). That is all real numbers or the interval $(-\infty,\infty)$. For any x that we pick, we get a real number, k as an output.

We can also see that the graph runs all the way from left to right in the demo.

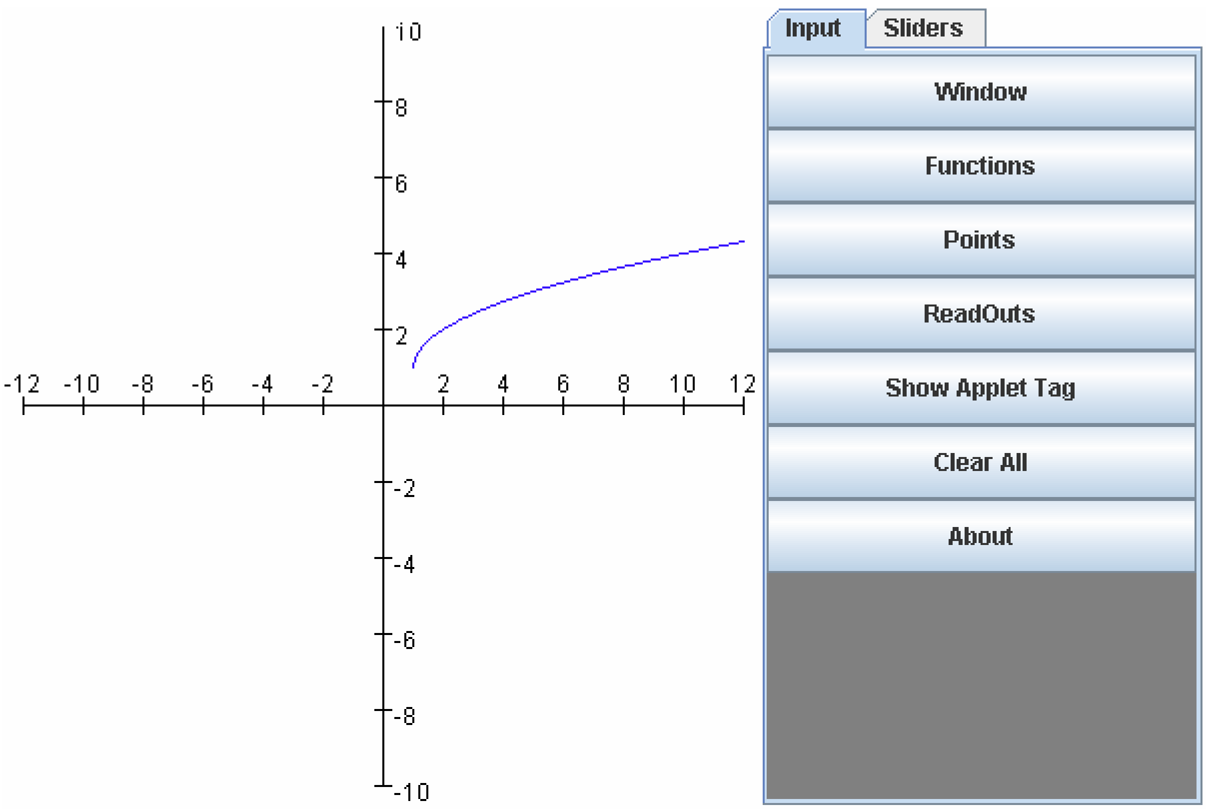
The range depends on the parameter. For this function it is always {k}. For example, y = 2 has range {2} and y = 3 has range {3}. In general, y=k has range {k}.

1. This demo shows y = m*x+b. Sliders have been defined for m and b. That means they are parameters that can change values.



What is the domain and range of y = m*x+b? Consider all possible cases of m and b.

2. This demo shows √(x-a)+b.

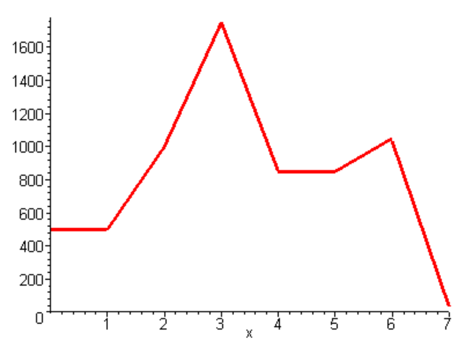


What is the domain and range of √(x-a)+b? Explain how the answers can be seen on the graph and from the formula.

Assignment 3: Increasing, Decreasing, and Constant

1. The following graph depicts the tale of a man and his trip to a casino.

The x-axis shows the number of hours since he entered the casino, and the y-axis shows the amount of money that he has in his possession.



Over what intervals is the function increasing? (That is, when is he winning?)

Over what intervals is the function decreasing? (That is, when is he losing?)

Over what intervals is the function constant? (That is, when is he breaking even?)

2. Make up your own story and accompanying function. Be creative and have at least one interval each of increasing, decreasing, and constant. Give your story and present the function as a graph or table. List the intervals where your function is increasing, decreasing, or constant.

Assignment 4: Translations

1.

A. Vertical Shifts

i) Complete the tables

x	$f(x)=x^3+2$
-2	-6
-1	
0	
1	
2	

x	$g(x)=f(x) + 1$
-2	-5
-1	
0	
1	
2	

ii) Make a graph of f(x) and g(x) to show the translation.

2.

B. Horizontal Shifts

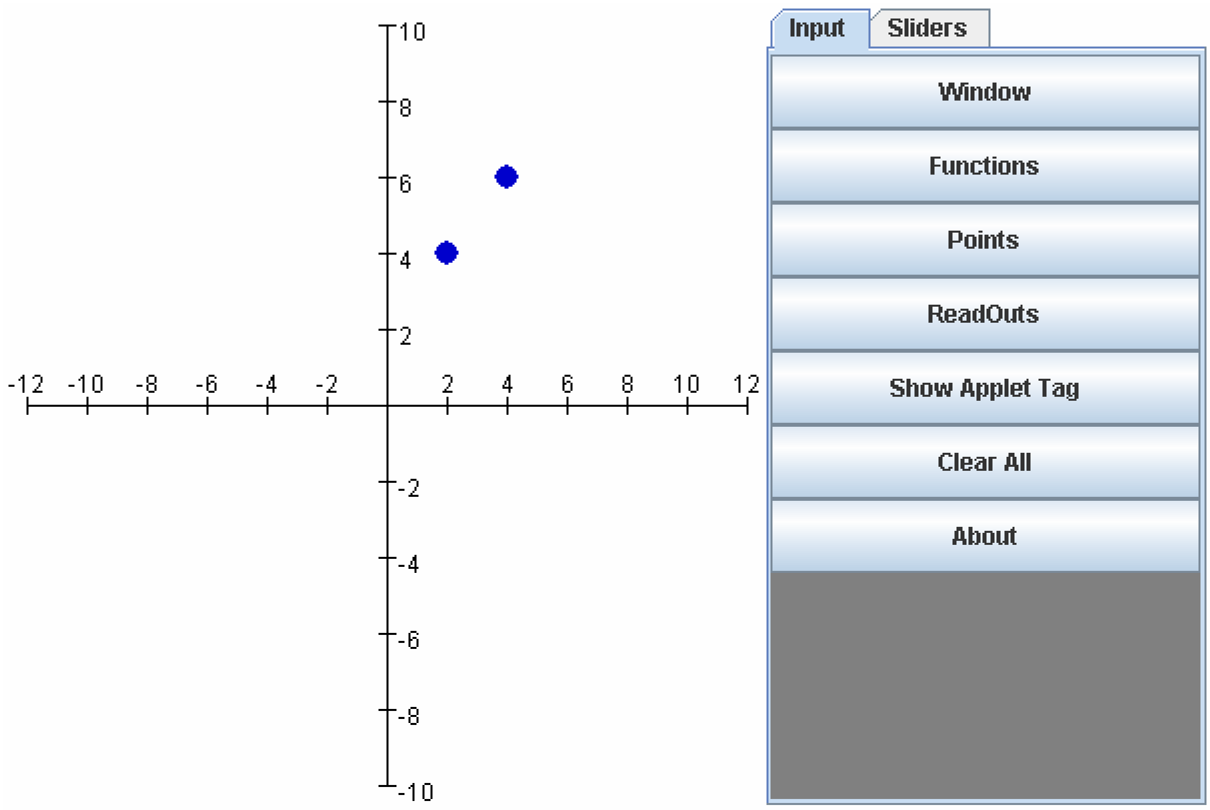
i) Complete the tables

x	$f(x)=x^2-1$
-2	3
-1	
0	
1	
2	

x	g(x) = f(x+2)
-4	
-3	
-2	
-1	
0	

ii) Make a graph of f(x) and g(x) to show the translation.

3. Find a formula for a translation of $f(x) = x^2$ that passes through the two points on the graph. Explain how you found your solution.



Assignment 5: Compositions

1. Complete the third table

x	f(x)	x	g(x)	x	$f \circ g$
1	4	1	3	1	
2	2	2	4	2	
3	-2	3	2	3	
4	1	4	1	4	

For at least one x value, explain in detail how to find $f \circ g(x)$.

2. Find formulas for f, g, and f∘g. Try your best to explain how you found them.

x	f(x)	x	g(x)	x	f∘g
1	-1	-1	1	1	7
2	2	0	2	2	14
3	7	1	3	3	23
4	14	2	4	4	34
5	23	3	5	5	47

3. My car gets approximately 28 miles per gallon. The day that I wrote this problem, gas was about $2.90 per gallon.

a) Considering gas as the only expense, come up with a function that gives my cost to drive x miles.

b) If it is 45 miles from my house to UGA, how much would it cost me to drive there and back?

c) How many miles could I drive in a given month for $300?

Assignment 6: Linear Functions

1. Write an equation of a linear function with negative slope that contains no points in the second quadrant (negative x, positive y). Explain why your function satisfies the conditions or argue why it is impossible to construct such a function. (Source: http://books.heinemann.com/math/)

Once the tensor has unlocked (even if you think your answer is correct), find at least one person's response that you agree with and explain why. If you disagree with everyone else's responses, explain why.

2. Find equations for three lines whose graphs form a triangle with area 12 square units. Explain why your equations satisfy the conditions. There are lots (infinitely many) of possible answers for this. (Source: http://books.heinemann.com/math/)

When the tensor unlocks

- if you think your response is correct, try to find a different but also correct example in the tensor and discuss how they are both correct.
- if you think your response is incorrect or incomplete, try to find a correct response in the tensor and use the ideas that you find to make a different example.

3. Determine values of B and C such that the line 2x - By = C has negative slope and a negative y-intercept. Justify your answer. (Source: http://books.heinemann.com/math/)

This one also has many different solutions.

When the tensor unlocks

- if you think your response is correct, try to find a different but also correct example in the tensor and discuss how they are both correct.
- if you think your response is incorrect or incomplete, try to find a correct response in the tensor and use the ideas that you find to make a different example.

Assignment 7: Linear Models

1. Jared has a job selling skin cream at the local mall. He gets a weekly salary of $300 plus $1.75 for each bottle of the product that he sells. We are going to answer the following question in a couple of ways: How many bottles would he have to sell in order to get a weekly paycheck of $657.00? Make sure you follow the directions in order.

a) Complete the table below:

items sold: 0 50 100 150 200 250

paycheck : 300

Between what two numbers in the table would the answer be found?

b) Make a graph (use the applet) of $f(x) = 300+1.75x$ and $g(x) = 657$. Estimate the solution as best as you can from the graph. Place a point that you believe estimates the answer.

c) Algebraically solve $300+1.75x = 657$.

2. A city recreation department plans to build a rectangular playground with a perimeter of 810 meters. The playground is to be surrounded by a fence which is 3 meters longer than twice the width.

a) Express the length of the fence in terms of its width.

b) Express the width of the fence in terms of its length.

c) Find the dimensions of the fence.

3. Write your own problem similar to problem 2 (above). Use different numbers and a different relationship between the length and width. Post the problem publicly and a solution privately. After the tensor opens, find and solve a classmate's problem.

Assignment 8: Number of Solutions

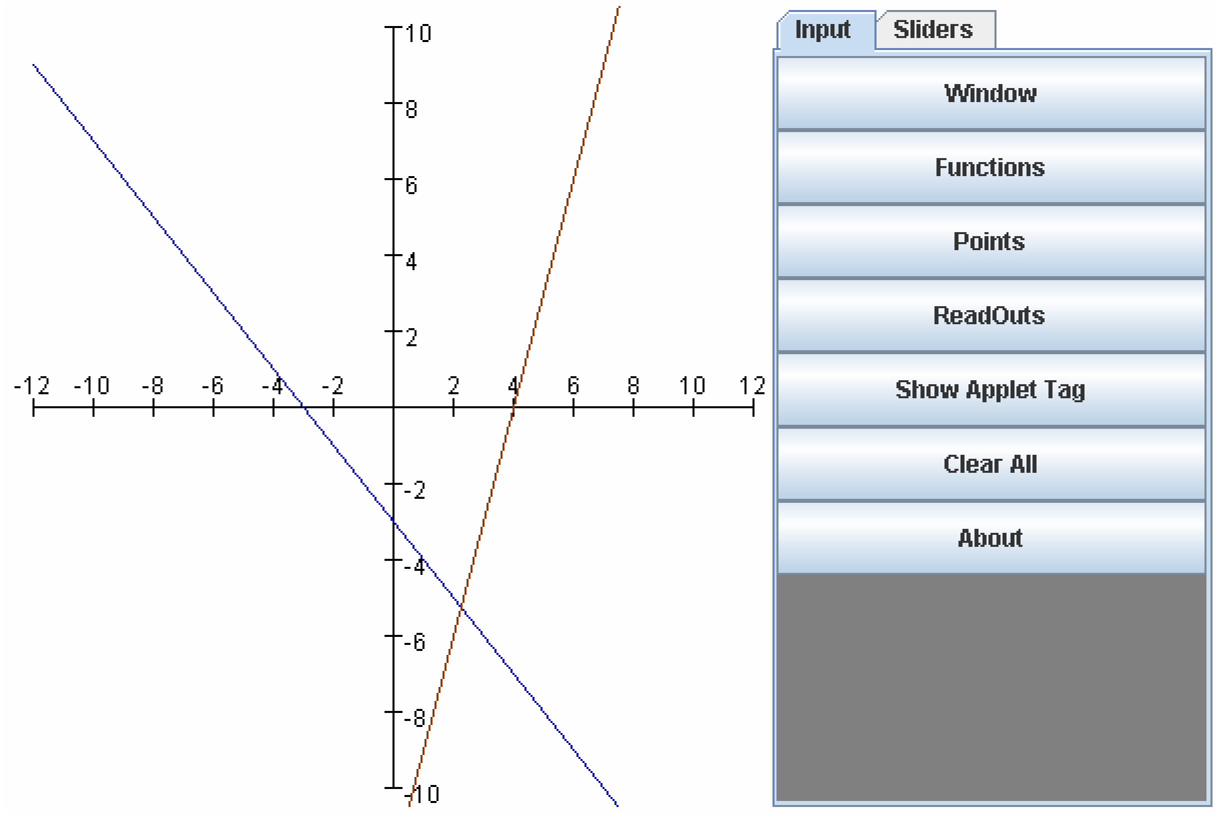
1. For each of the following, first make graphs to determine the number of solutions to the equation. Post your graphs so that I know you did it. Tell if the equation is an identity, inconsistent, or conditional. Then solve the equation algebraically, showing all steps.

a) $-x - 1 = - [(2x+1) - (4+x)]$

b) $2[(x+1)-4(x-3)] = 2x - 6$

2. This applet has two lines, $y = m*x+b$ and $y = -(c*x+2)+d-x$.

You can change m, b, c, and d with the sliders.



a) Adjust the slider values so that m*x+b = -(c*x+2)+d-x has one solution. Post the graph with those values and solve for the one solution using the particular values of m, b, c, and d that you have chosen.

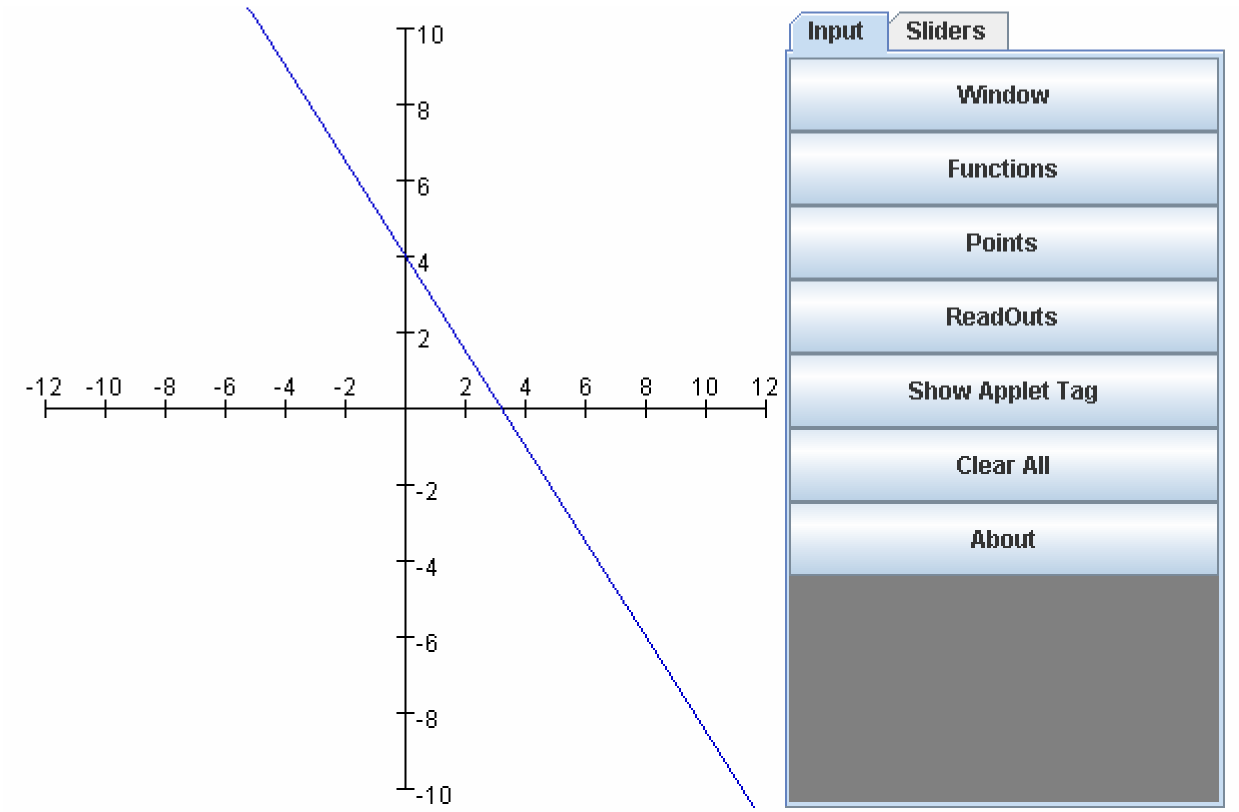
b) Adjust the slider values so that m*x+b = -(c*x+2)+d-x has no solution. Post the graph with those values and show algebraically that there is no solution with the particular values of m, b, c, and d that you have chosen.

c) Adjust the slider values so that m*x+b = -(c*x+2)+d-x has infinitely many solutions. Post the graph with those values and show algebraically that there are infinitely many solutions with the particular values of m, b, c, and d that you have chosen.

Note: If you are unsuccessful at completely solving this task during the initial phase, it should be easy to get help from the tensor without copying. There are lots of different choices for the values of the sliders. If you read someone's answer, you should be able to do the problem in a similar way with different slider values.

Assignment 9: Linear Function Graphs

1. The applet below shows y = m*x+b. The values of m and b are controlled by the sliders.



Do any three of a through e. If you do more than three clearly indicate which three you want counted. Otherwise, I'll grade the first three.

a) While keeping b constant, change values of m to see the effect that it has on the graph. Be sure to explore positive and negative values. Describe the effect that m has on the graph in as much detail as possible.

b) While keeping m constant, change values of b to see the effect that it has on the graph. Be sure to explore positive and negative values. Describe the effect that b has on the graph in as much detail as possible.

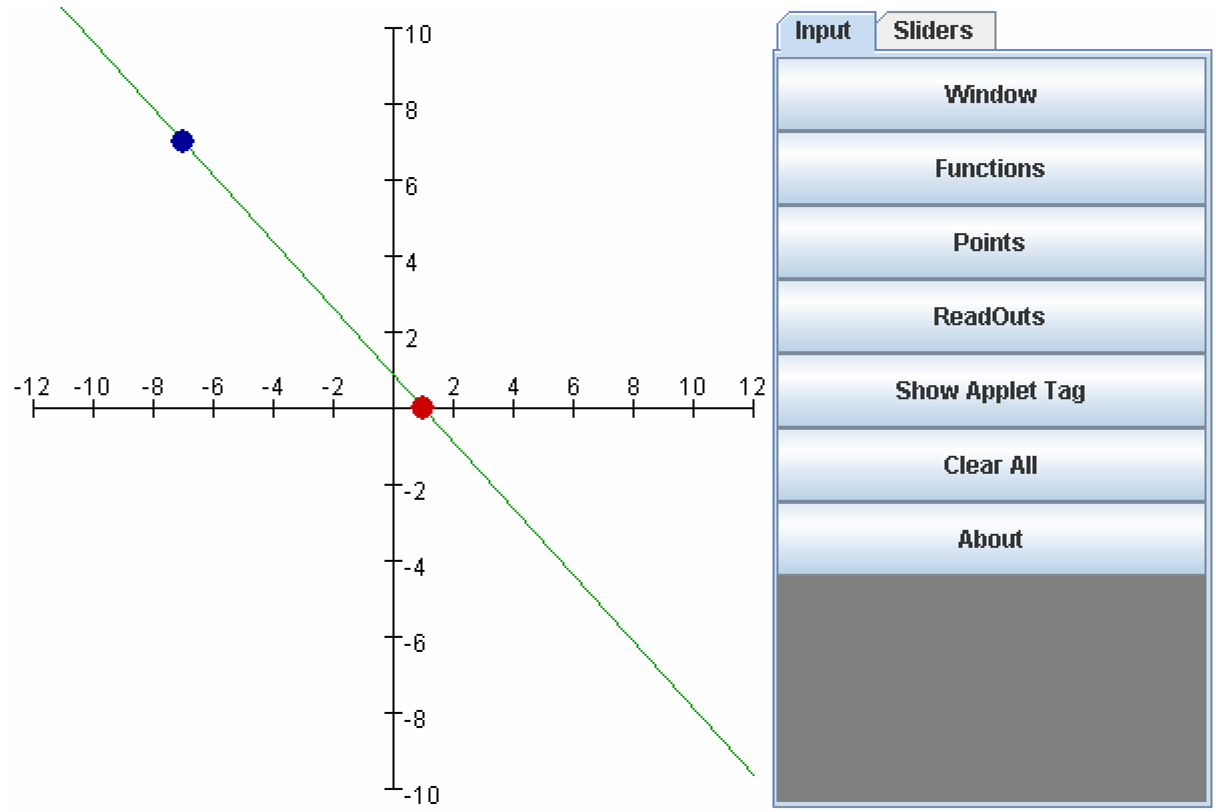
c) Add a line to the applet that is parallel to y = m*x+b and passes through the point (1,5). The line should adjust as you change the values of the sliders to always remain parallel. Post the applet in your response.

d) Add a line to the applet that is perpendicular to y = m*x+b and passes through the point (1,5). The line should adjust as you change the values of the sliders to always remain perpendicular. Post the applet in your response.

e) Add a line to the applet that has the same y-intercept as y = m*x+b, but has a slope of -2. The line should adjust when you change the sliders, but always has a slope of -2. Post the applet in your response.

If you are stumped on c through e, here is how to use someone's work from the tensor without copying: On c or d, do the problem with a point other than (1, 5). Pick any other point you like. On e, do the problem with a slope other than -2. Pick some other number for it.

2. The following demo contains the points (a,b) and (c,d). You can change the values of a, b, c, or d using the sliders. This allows you to move the points around the plane. The graph also contains the line between the two points. Its equation is y = (b-d)/(a-c)(x-a)+b. (b-d)/(a-c) is the slope. You can see numerical values by clicking on readout. If you resize the window, you can see the slope and move the sliders at the same time.



a) Can you adjust the sliders to create a horizontal line?

If yes, what must be true about the slider values? If no, why not?

b) Can you adjust the sliders to create a vertical line?

If yes, what must be true about the slider values? If no, why not?

c) Add a line to the applet that is perpendicular to the one given and passing through the point (2,5). The line should adjust with the sliders. Post your applet in your response.

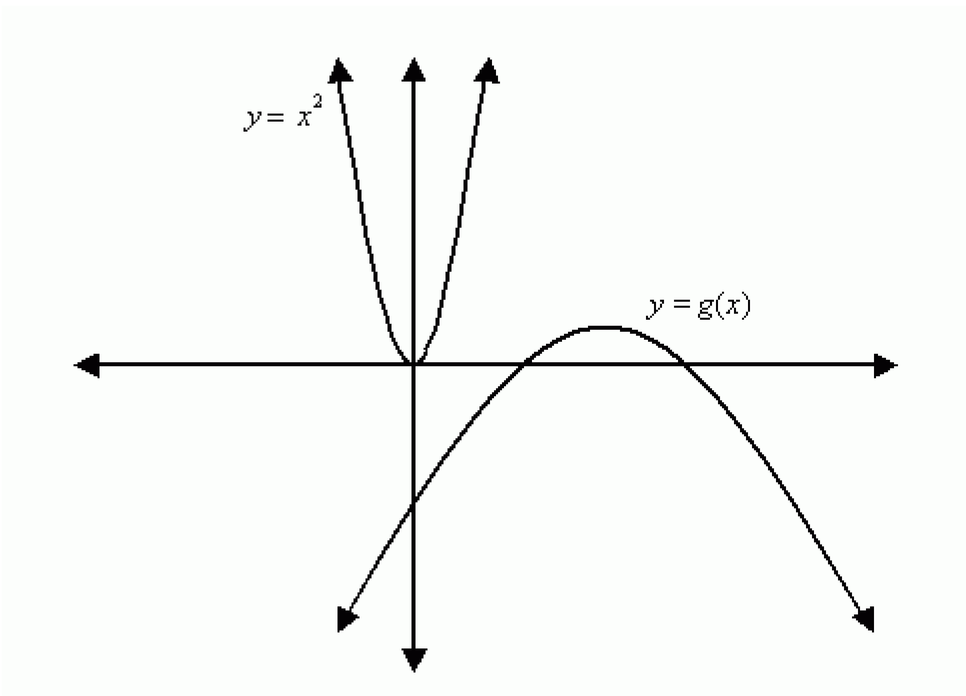
If you look in the tensor and find out how to do c from someone, please do the problem yourself with another point of your choice other than (2,5).

Assignment 10: Quadratic Functions

1. Is it possible to choose values for b and c in the following equation such that there is exactly one real solution? Explain why or why not? $-x^2+bx+c = 0$

(Source: http://books.heinemann.com/math/)

2. Give a reasonable formula for g(x) on the form $g(x) = a(x-h)^2 + k$. Explain why your formula is reasonable.



(Source: http://books.heinemann.com/math/)

3. Take the last two digits in your student id and add them together. Call this number n. Don't post your digits, just the n.

a) Write an equation for a quadratic function that has x-intercepts (n,0) and (n+1,0) and a y-intercept (0,n+2). Provide the formula and the graph.

b) Write an equation for a quadratic function that has vertex (n,-n) and y-intercept (0,3). Provide the formula and the graph.

Assignment 11: Quadratic Models

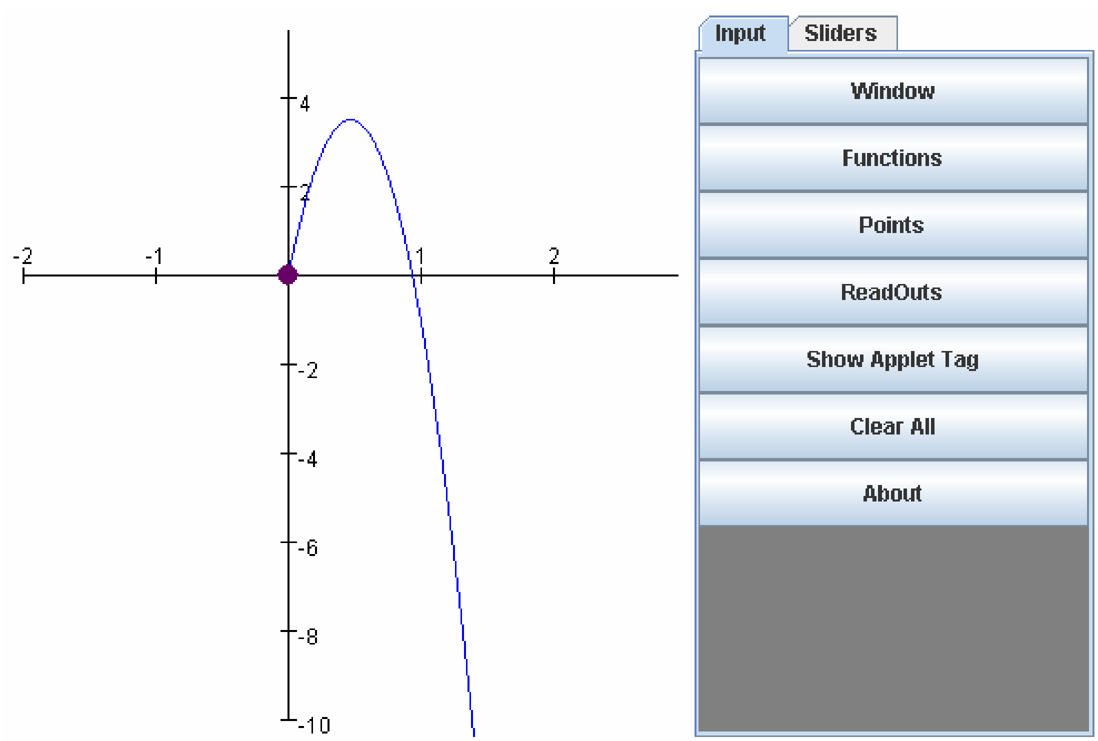
1. A) Solve this problem:

A farmer is going to build a rectangular pig pen. He wants it to be 10 feet longer than twice the width. If the enclosed area is to be 1612 feet, what are the possible dimensions of the pen?

B) Make up a similar problem by changing the numbers and the relationship between width and length.

C) Once the tensor opens, find someone's problem and solve it.

2. A common example of an application of quadratic functions is projectile motion. An object projected into the air from an initial height of s0 feet with an initial velocity of v0 ft/sec will obey the formula $h(x) = -16x^2+v0*x+s0$, where h(x) is the height of the object after x seconds. This demo allows you to see what happens as v0 and s0 vary. The slider a controls a purple point that has its coordinates in the readout panel.



Let v0 = 25 ft/sec and s0 = 5 ft. For each of the following, first estimate the answer from the graph. Then find the exact solutions algebraically. Post algebraic work and graphs.

a) What is the maximum height of the object? and when does it reach it?

b) How long is the object in the air before it hits the ground?

Assignment 12: Polynomials

1. Take the last two digits of your student id and add them together to get a number n.

Using that specific value for n, do the following problem:

Let $f(x) = (x - n)(x + n + 2)^2$.

Give the exact values of the x and y intercepts. Estimate any local maxima and minima. Make a graph of f(x). Put a red point at each x-intercept. Put a blue point at the y-intercept. Put a green point at each local max. Put a yellow point at each local min.

2. With the same value for n as in problem 1, do the following:

Construct a polynomial with x-intercepts at x = -n, x = n, and x = n+3 and y-intercept at y = 4. Make a graph of your polynomial. Put a red point at each x-intercept. Put a blue point at the y-intercept. Put a green point at each local max. Put a yellow point at each local min.

Assignment 13: Rational Functions

You will need two unique numbers for this assignment. Let n be the sum of the last two digits of your student id number. Let m be the number of the month in which you were born.

(Ex: September = 9)

1. Let f(x) = 2/(x - n) + m

What is the horizontal and vertical asymptote?

What is the domain and range of f(x)?

Find any x and y intercepts.

2. Create a function that satisfies the given properties.

Provide graphs to support your answers.

a) This function should have the same shape as 1/x, but moved right m, down n, and vertically stretched by n+1.

b) This function satisfies:

$f(x) \to -\infty$ as $x \to m^-$

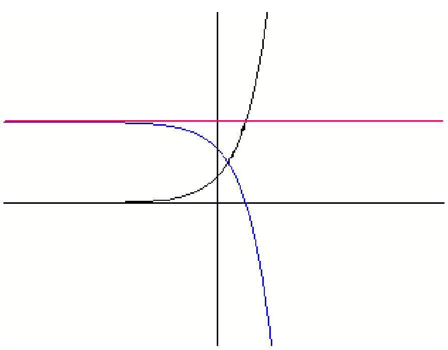
$f(x) \to -\infty$ as $x \to m^+$

$f(x) \to n$ as $x \to \infty$

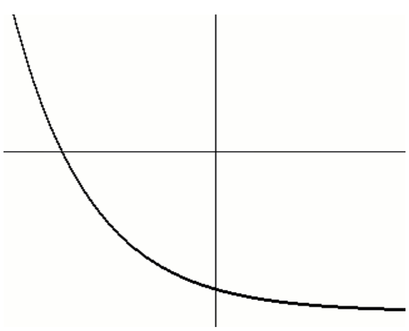
$f(x) \to n$ as $x \to -\infty$

Assignment 14: Exponential Functions

1. The blue graph is a translation of $y = b^x$ (the black graph). Give a reasonable value for b and a reasonable formula for the function shown in blue.



2. Find a reasonable formula for the exponential function shown below:



3. Let n be the sum of the last two digits in your student id and let m be the digit of the month you were born in (January $\Rightarrow$ m = 1, February $\Rightarrow$ m = 2, etc...).

Let $f(x) = 3(n+2)^{(x+m)}-m$.

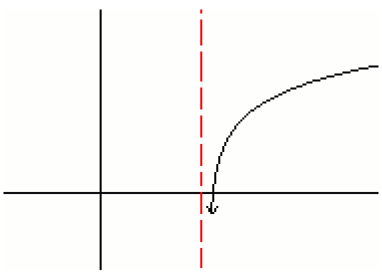
a) List the translations needed to obtain a graph of f(x) from a graph of $(n+2)^x$.

b) Give equations for any asymptotes that f(x) has.

c) List the domain and range of f(x).

Assignment 15: Logarithmic Functions

1. The function shown is a translation of a logarithmic function. Give a reasonable formula for the function shown.



2. Let n be the sum of the last two digits of your student id and let m be the number for the month you were born in. Let f(x) = -2*log(x-m)+n.

a) List the translations needed to obtain a graph of f(x) from a graph of log(x).

b) List equations for all asymptotes of f(x).

c) List the domain and range of f(x).

3. Choose values of a, b, and c that make the following equation true, and justify your answer.

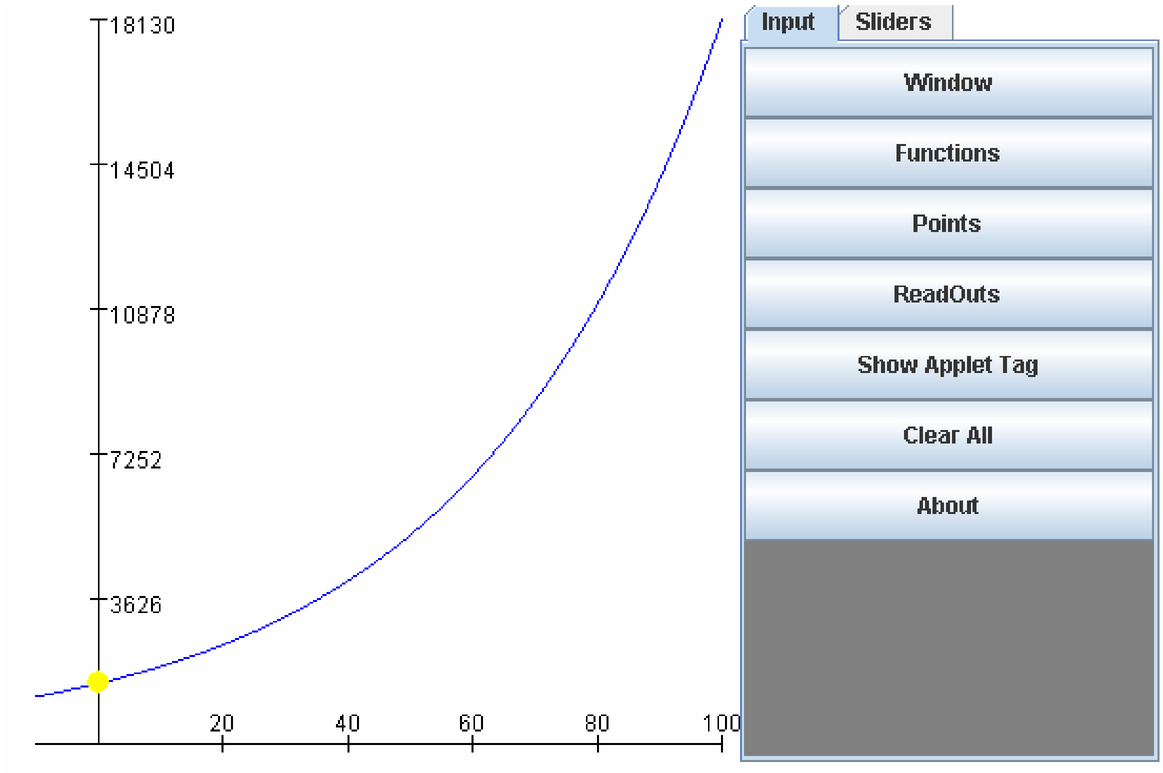
$\log_c(\log_a(\log_b(c))) = 0$

Assignment 16: Models and Equations

1. Show me the money!

The following demo shows A(x) = P(1+r/n)^(n*x). P, r, and n can be controlled by sliders.

The slider a controls a point on the graph that has a readout available.



For each of the following, first use demos to estimate answers graphically and post the demos.

Then solve algebraically, using logarithms when necessary showing your work.

a) If the initial deposit is $2000, the interest rate is 3.5%, and it is compounded quarterly, when will the account be worth $4500?

b) If the interest rate is 3.5% compounded monthly, how much money would need to be deposited in order to have the account worth $7500 after 25 years?

2. Solve log(x)+log(x-1)=1 algebraically, then provide the best graph that you can to demonstrate the solution.

3. Let n be the sum of the last 2 digits of your student id:

Consider the equation: $3(n)^{x+1} - 4 = 23$

Solve this algebraically, showing work. Also post a graph that verifies your solution.

APPENDIX B

Shared-Work Section Survey Questions

Shared-Work Brief Survey

1. Have you found it helpful so far to view the work of other students in the tensor? Why or why not?
2. If there are any online assignments that you didn't do, why did you not do them?
3. If there are any assignments that you posted late, what kept you from meeting the deadline?

Shared-Work Extended Survey

1. What do you see as major benefits to using the online assignments?
2. What do you see as major drawbacks to using the online assignments?
3. How often do you look at the work of other students? In what ways is this helpful?
4. If you rarely read other students' work, why not?
5. Do you look at the work of some students more often than others? If so, how do you decide?
6. How comfortable are you having your work available for others to read? Have your feelings changed over the course of the semester?
7. Have you posted comments or questions in other students' assignments? Why or why not?
8. How much time do you typically spend on the online assignments?
9. If there are any problems that you did not attempt, why?
10. What problems, if any, have you experienced using the tensor software? Any suggestions for improving it?

11. What factors do you think might cause a student to have poor participation in the online assignments?
12. What factors do you think might cause a student to have poor achievement on the online assignments?
13. Have you used feedback from the instructor or classmates to improve your solutions between the initial and final deadlines on any problems? If so, was it helpful? If not, why not?

Pick one or two problems that you found most challenging and answer the following (14 and 15):

14. What made the problem challenging?
15. How could I have helped you be more successful with this problem?

Shared-Work Demographic Survey

1. Do you have access to a computer at home? Yes____ No____
2. If you do have computer access at home, does it have internet access? Yes____ No____
3. How many classes are you taking this semester? ______
4. How many credit hours are you taking this semester? ______
5. If you have a job beyond being a student, on average this semester, how many hours per week did you work?

APPENDIX C

Small-Groups Section Survey Questions

Small-Groups Brief Survey

1. If there were any online assignments (1-3) for which you did not make an online post, why not?
2. Do you feel that your group had a successful experience with assignments 1-3? What do you think are the reasons for this?
3. What do you think I could do to increase participation and performance with the online assignments?

Small-Groups Extended Survey

1. What do you see as major benefits to doing the online group work?
2. What do you see as major drawbacks to doing the online group work?
3. How comfortable are you working in groups on the online assignments? Have your feelings changed over the course of the semester?
4. How much time do you typically spend working on the online assignments?
5. Aside from what is obvious in the software, how have you worked on these problems? Do you work on them alone? Do you work with group members offline? Do you work with other people who are not in your group offline?
6. What factors do you think might cause a student to have poor participation in the online group work?

7. What factors do you think are necessary for a group to be successful in the online group work?
8. What factors do you think are harmful to the success of a group working on the online assignments?
9. Do you feel that each member of your group is contributing to the group's success? Explain.
10. Has your group been interactive? Have you helped each other solve problems, or has each participant done his or her own thing?
11. If there are any assignments for which you did not participate, why?

Pick one or two problems that you found most challenging and answer the following (12-13):

12. What made the problem challenging?
13. How could I have helped you be more successful with this problem?

Small-Groups Demographic Survey

1. Do you have access to a computer at home? Yes____ No____
2. If you do have computer access at home, does it have internet access? Yes____ No____
3. How many classes are you taking this semester? ______
4. How many credit hours are you taking this semester? ______
5. If you have a job beyond being a student, on average this semester, how many hours per week did you work?

APPENDIX D

Shared-Work Section Interview Guide

Interaction

Could you describe for me how you would go about a typical assignment in the courseware?

Did you use other students' work?

If yes, then how did you use it?

If no, why not?

Did you look at the work of some students more often than others? If so, how did you decide?

How comfortable are you having your work available for others to read? Have your feelings changed over the course of the semester?

Have you posted comments or questions in other students' assignments? Why or why not?

Have other students posted comments or question in your assignments? If so, have you found them helpful?

Participation and Performance Factors

How much time did you typically spend on the online assignments?

What do you see as major benefits to using the online assignments?

What do you like most about the online assignments?

What do you see as major drawbacks to using the online assignments?

What do you like least about the online assignments?

If there are any problems that you did not attempt, why?

What problems, if any, have you experienced using the tensor software? Do you have any suggestions for improving it?

Why do you think it is hard to get some students to participate in the online assignments?

We had several different kinds of problems in the courseware. What types of problems gave you the most trouble?

Attitudes toward Mathematics and Computers

How would you describe your feelings toward mathematics in general?

Tell me about your past experiences with mathematics.

How has this course compared to your past experiences?

How would you describe your feelings toward computers in general? How comfortable are you using them? How often do you use computers outside of class?

Could you describe your overall feelings toward using computer conferencing for mathematics?

APPENDIX E

Small-Groups Section Interview Guide

Interaction

Could you describe for me how you would go about a typical assignment in the courseware?

How did your group interact to complete the assignments?

Was it helpful to work in groups? Why or why not?

Did some of your group members make posts that were more helpful than others? Can you discuss that more?

How comfortable were you working in groups? Have your feelings changed over the course of the semester?

Participation and Performance Factors

How much time did you typically spend on the online assignments?

What do you see as major benefits to doing online assignments in groups?

What do you like most about the online group work?

What do you see as major drawbacks to doing the online group work?

What do you like least about the online group work?

What problems, if any, have you experienced using the software? Any suggestions for improving it?

Why do you think it is hard to get some students to participate in the online group work?

We had several different kinds of problems in the courseware. What types of problems gave your group the most trouble?

Attitudes toward Mathematics and Computers

How would you describe your feelings toward mathematics in general?

Tell me about your past experiences with mathematics.

How has this course compared to your past experiences?

How would you describe your feelings toward computers in general? How comfortable are you using them? How often do you use computers outside of class?

Could you describe your overall feelings toward using computer conferencing for mathematics?

APPENDIX F

The Online Problem Solving Rubrics

The 4 Point Problem Solving Rubric (Adapted from Kosiak, 2004)

Score	Criteria
4	A correct solution and an appropriate strategy are shown or explained and the solution is shown with correct labels or description if necessary.
3	A complete, appropriate strategy is shown or explained but: • an incorrect solution is given due to a simple computational or other error or • no solution is given. A correct solution is given with no solution strategy or explanation shown. A correct solution and appropriate strategy is shown or explained, but not labeled correctly when necessary.
2	Some parts of an appropriate strategy are shown or explained, but some key elements are missing. Some parts of an appropriate strategy are shown or explained, along with some inappropriate parts. Appropriate strategy shown or explained, but implemented incorrectly.
1	Some work or explanation beyond re-copying data, but work would not lead to a correct solution. One or more incorrect approaches attempted or explained.
0	No work or solution shown or explained. Incorrect solution and no work shown or explained. Some data from the problem copied over, but no evidence of any strategy is shown or explained.

Special Instructions for the Shared-Work Section

Problems will be graded in two phases.

Private phase: Students work independently with access to their own assignments only.

Collaborative Phase: Students continue to work with access to others' work in the tensor.

The private phase will be graded 0-2. So you only have to get as high as a 2 on the rubric to receive full credit during this phase. The public phase will be graded with the full 4 point rubric. Therefore, each problem is worth 6 points total.

The following expectations apply to the collaborative phase:

-If you use something that you have read on someone else's work, you must cite it.

-Your responses need to be in your own words and based on your own understanding.

-When appropriate you should try to build on what you have learned from others.

(You are highly encouraged to use each other's work during the collaborative phase.)

-Even if you think you have solved a problem correctly, you are encouraged to

check yourself against your classmates. Many problems will have multiple solution paths

and/or multiple possible answers.

Special Instructions for the Small-Groups Section

A four point rubric will be used to grade assignments. In addition, each group member will be held accountable for participation in the following way:

Each student will receive an individual participation score out of 2 points:

2 - Member makes a major contribution to the group's success.

1 - Member makes a minor contribution to the group's success.

0 - Member makes no significant contribution to the group's success.

Ways to contribute to the group's success could be offering potential solutions, explaining possible solutions or concepts, asking relevant questions, or addressing questions raised by other students. Each assignment will be worth 6 points (2 for the individual + 4 for the group). However, the following applies: **Any student who does not participate at all in an assignment will receive a score of zero for that assignment.**

APPENDIX G

Asynchronous Collaboration Coding Scheme

Proposing Solutions

The unit of meaning contains a proposed solution to all or part of a problem.

- D) This is in my oppionion [*sic*] NOT a function. The amount of money would be the x value and what item that one would be buying would be the y value. Although all the items maybe different in style or device, two different devices may cost the same price causing an x value to repeat itself therefore it not being a function!
- -x-1=-[(2x+1)-(4+x)]

  -x-1=-[2x+1-4-x]

  -x-1=-[x-3]

  -x-1=-x+3

  Inconsistent solution

Proposing Solutions in Response

The unit of meaning contains a proposed solution to all or part of a problem, and the student made the unit after viewing another student's proposed solution.

- SG4: I'm not quite sure but I think as long as c=1 the equation will equal 0, and the values of a and b can be any real number.

  SG7: a & b = 10 c = 1

Asking Questions

The unit of meaning contains one or more questions concerning content, correctness of proposed solutions, or logistics directed to other students or the instructor.

- Do you'll want me to post the final response tomorrow? If you do which set of answers would you all rather I type in? Does it matter????
- WOULD YOU HAVE TO TURN INTO SLOPE INTERCEPT FORM TO FIND THE SLOPE AND THE TRY TO DETERMINE WHAT ARE GOING TO BE THE VALUES
- can someone please explain this a little to me in class on Friday or something, I still dont get it :(
- Is it right so far??

Responding

The unit of meaning contains a response to a question, an expression of agreement, or an expression of disagreement.

- I would agree with SG7 on his solution to this question.
- No, I don't think that's right. Because when you multiply the logs together, it equals x^2-x.
- SG12: Do you'll want me to post the final response tomorrow? If you do which set of answers would you all rather I type in? Does it matter????

  SG17: i dont really care which you choose. both are correct, just worded differently.

Responding to the Instructor

The unit of meaning contains a response to a question made by the instructor or a proposed solution that obviously uses the instructor's feedback.

- Instructor: What if m = 0?

  SG6: If m=0 then it has no slope, which means it is a straight horizontal line.

- Instructor: That's close:

  c)2(2w+3)+2w=810

  4w+6+2w=810

  8w+6-6=810-6 (Right here: 4w+2w=6w≠8w)

  8w/8=804/8

  w=100.5

  Once you get width, make sure you find the length too.

  SW20:

  c)2(2w+3)+2w=810

  4w+6+2w=810

  6w+6-6=810-6

  6w/6=804/6

  w=134

  l=2(134)+3

  =268+3

  =271 meters

Commenting

The content of the unit of meaning is not directly related to solving the mathematical task. This is the default category for units of meaning that do not fit into other categories.

- i honestly have no idea how to do this.
- I am sorry, I can see nothing but a gray box
- umm i can't see the graph or whatever it is..it's just a big gray box.

Reposting

The unit of meaning contains the same content as a prior post made by the same student.

- First Post: i used y=-2x+6, y=2x+6, and y=-6 for my three functions. I know the bottom part of the triangle is 12 units away but i don't really know how to tell if the rest of it is correct.

  Repost: This is what i put but i'm not sure if it's correct.

  i used y=-2x+6, y=2x+6, and y=-6 for my three functions. I know the bottom part of the triangle is 12 units away but i don't really know how to tell if the rest of it is correct.

Directing

The unit of meaning contains a request or directs classmates to do something.

- Move the sliders on the demo to get a visual.
- Somebody needs to solve this

Refocusing Discussion

The unit of meaning contains an attempt to change the focus of the discussion.

- We still need some work on the second tensor
- The previous graphs by SG18 are good according to Mr. Cooper. Now what we have left to do is solve the problems.

Explaining with Evidence

The primary purpose of the unit of meaning is to explain to classmates the content of a previous post using mathematical evidence.

- for the first set they both did it correctly and the work looks good.
  for the 2nd set SW18 is correct.

  if x=-4, g(x)= f(-4+2)
   = f(-2)
  then after you get that you use the first equation to solve for g(x), so:
  g(x)= f(-2)= (-2)^2-1
   = 4-1
   = 3
  that's how you solve for the 2nd set, so SW23 is correct. :)

APPENDIX H

Sample Problem-Solving Episode and Coding of Interactions

The Problem Statement

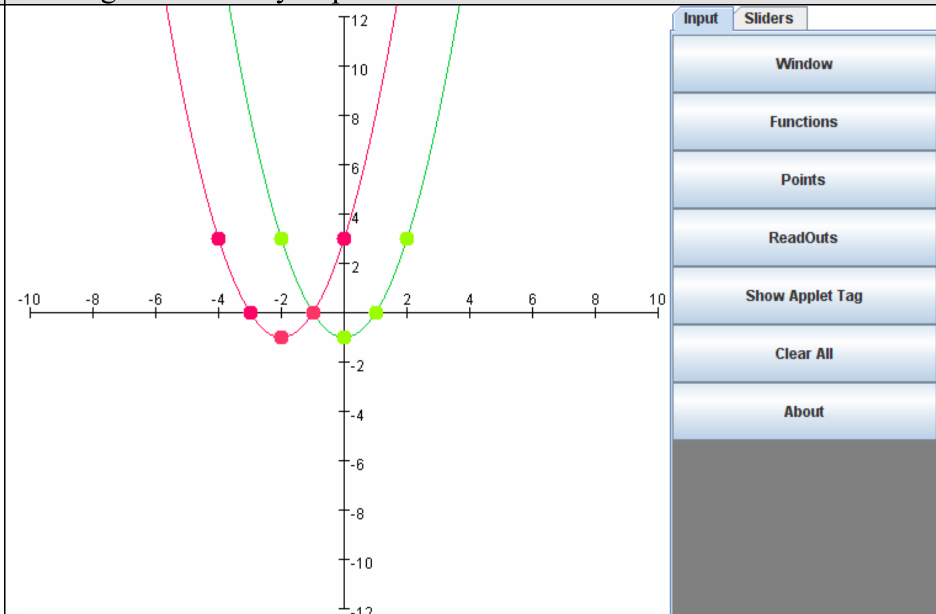
Problem 4.2:
B. Horizontal Shifts
i) Complete the tables

x	$f(x)=x^2-1$	x	g(x) = f(x+2)
-2	3	-4	
-1		-3	
0		-2	
1		-1	
2		0	

ii) Make a graph of f(x) and g(x) to show the translation.

The Student Work

Author	Message Thursday September 06th 2007 02:03:55 PM
SG23	x f(x) -2 3 -1 0 0 -1 1 0 2 3 x f(x) -4 3 -3 0 -2 -1 -1 0 0 3

Author	Message Thursday September 06th 2007 02:04:03 PM
SG23	
Author	**Message Friday September 07th 2007 01:28:06 PM**
SG12	f(x) (-1)^2-1=0 (0)^2-1=-1 (1)^2-1=0 (2)^2-1=3
Author	**Message Friday September 07th 2007 01:38:36 PM**
SG12	g(x) -4^2-1(-4+2)=34 -3^2-1(-3+2)=10 -2^2-1(-2+2)=0 -1^2-1(-1+2)=0 0^2-1(0+2)=-2 Did I do something wrong here?
Author	**Message Saturday September 08th 2007 08:21:26 AM**
TC	This is a great chance to see the benefits of a group. We have a table and graph that certainly looks like a horizontal shift, and we have a different table with work shown. What do the other two members think? Can one of you explain why SW23 says g(-2) = -1 for instance? What are SW23 and SW12 doing differently to get their numbers?
Author	**Message Monday September 10th 2007 09:03:45 PM**
SG17	Everyones answers look good. i dont really think that there is much for me to add to make it any better. i feel like a jerk always saying "oh yea, i agree".

Author	Message Tuesday September 11th 2007 06:25:21 AM
TC	SG23 and SG12 have different answers. The group needs to come to agreement.
Author	Message Tuesday September 11th 2007 07:04:25 PM
SG19	for the first set they both did it correctly and the work looks good. for the 2nd set SG23 is correct. if x=-4, g(x)= f(-4+2) = f(-2) then after you get that you use the first equation to solve for g(x), so: g(x)= f(-2)= (-2)^2-1 = 4-1 = 3 that's how you solve for the 2nd set, so SG23 is correct. :)
Author	Message Saturday September 15th 2007 08:17:52 AM
TC	Good work. Good explanation RMR.

Results of the Asynchronous Interaction Coding Scheme

Post	Code
SW23 Post 1	PS
SW23 Post 2	PS
SW12 Post 1	PSR
SW12 Post 2	PSR, Q
TC (The Instructor) Post 1	Not applicable
SW17	R, C
TC Post 2	Not applicable
SW19	RI, EE
TC Post 3	Not applicable

APPENDIX I

Coding Scheme for Proposing Solutions in Response

The Dimensions

Dimension	Description
Incorporation of Classmates' Work	This dimension assesses whether the student's post contains evidence that it incorporates the work of one or more classmates.
Individual Contribution	This dimension assess the level of original contributions that the student contributed to the response. This dimension is only applicable if the student incorporates classmates' work.
Relative Correctness to Others	This dimension assesses the correctness of a student's post in comparison to those viewed in the tensor prior to the posting.
Relative Correctness to Prior Posts	This dimension assesses the correctness of a student's post in comparison his or her own prior posts.

Dimension 1: Incorporation of Classmates' Work

Category	Criteria
Yes	There is some obvious similarity between the student's work and the work of another student viewed prior to making the post.
No	There is no apparent relationship between the student's work and any posts viewed prior to making the post.
SP (Similar Problem/ Alternate but Related Solution)	The student has solved a problem with his or her own parameters that is similar to a one viewed or has created an alternate but conceptually equivalent solution to one viewed in the tensor
Other	The student attempted to solve a classmate's proposed problem.

Dimension 2: Individual Contribution

Category	Criterion
Copied	The student has copied a classmate's post verbatim.
Reworded	The student has made a post that is essentially the same as one that has been viewed in the tensor, and only minor changes have been made in the wording or presentation.
Minor contribution	The majority of the content in a post is similar to the work viewed in the tensor, but the student has made a minor contribution such as adding an explanation, adding an extra graph or diagram, or correcting a minor mistake.
Major contribution	The post contains some similarities to those viewed in the tensor, but the student has made major contributions such as correcting major errors, completing a partial solution, finding an alternate solution, or solving a similar problem.

Dimension 3: Relative Correctness to Others

Category	Criteria
Improvement	When combined with any prior posts by that student, the response forms a more correct solution than any solution viewed in the tensor.
Regression	The proposed solution is less correct than the most accurate response viewed in the tensor when comparing the portion of a question that the response addresses.
Equivalent	The response addresses the same portions of a problem as those viewed in the tensor and has the same level of correctness on all parts.
Equivalent/ Partial	The response addresses a subset of the portions of a problem when compared to those viewed in the tensor and has the same level of correctness on all parts addressed.
Mixed	On a question containing multiple parts, a student's response is less correct on some portion of the question and more correct on other portions when compared to those viewed in the tensor.

Dimension 4: Relative Correctness to Prior Posts

Category	Criteria
Improvement	The student's response makes his or her overall solution more correct.
Regression	The student's response makes his or her overall solution less correct.
Equivalent	The response does not change the level of correctness for the students overall response.
Mixed	The students post improves some portions of a solution and makes other portions less correct.

APPENDIX J

Examples of the PSR Coding Scheme

Student SW9's Problem 1.1 Episode

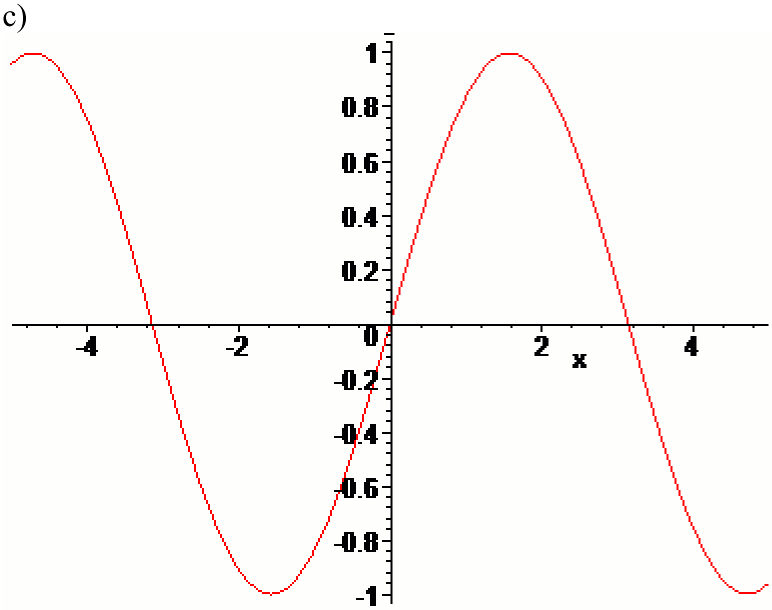
The Problem Statement

Determine whether or not the given relation represents y as a function of x. Explain your answer.

a) x+1 = |y|

b) x: 1 2 3 4 -1 -3
y: 5 5 5 3 2 3

c)



SW8's Post Viewed by SW9

Author	Message Friday August 24th 2007 07:58:34 AM
SW8	A) in the first linear equation the answer is that when graphed it does come out to be an upward sloping line that does pass the verticle line test, so yes EX:a is an f(x) B)the data represented here, yes, is indeed an f(x) because for every X imput there is exactly one Y output C)in the graph represented the verticle lilne test shows the the relationship of the data is ,yes, an f(x). All examples are functions.

SW9's Post

Author	Message Wednesday September 12th 2007 10:51:42 AM
SW9	A.) This is a function because once you graph the equation it passes the vertical line test. B.) This is also a function because for every X value there is exactly one Y value. C.)This graph is a function because it passes the vertical line test.

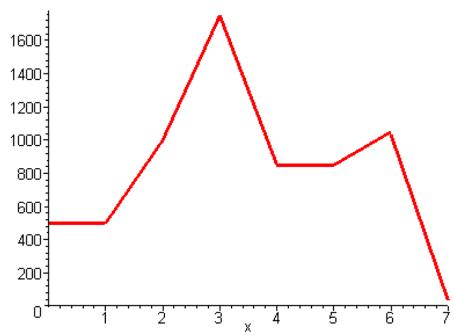
PSR Coding of SW9's Post

Dimension	Code
Incorporation of classmates' work	Y
Individual contribution	Reworded
Relative correctness to others	Equivalent
Relative correctness to prior posts	Not applicable

Student SW14's Problem 3.1 Episode

The Problem Statement

The following graph depicts the tale of a man and his trip to a casino. The x-axis shows the number of hours since he entered the casino, and the y-axis shows the amount of money that he has in his possession.



Over what intervals is the function increasing? (That is, when is he winning?)
Over what intervals is the function decreasing? (That is, when is he losing?)
Over what intervals is the function constant? (That is, when is he breaking even?)

SW25's Post Viewed by SW14

Author	Message Wednesday August 29th 2007 09:06:28 PM
SW 25	increasing-(1,3)U(5,6) decreasing-(3,4)U(6,7) Constant -(0,1)U(4,5)

SW14's Post

Author	Message Wednesday September 05th 2007 12:41:55 AM
SW14	He is winning: (1,2) U (2,3) U (5,6) you can see this by looking at the graph from left to right. The slope of the line increases from 1 to 2 from 2 to 3 and from 5 to 6; therefore giving us his winnings. He is loosing: (3,4) U (6,7) you can see this by looking at the graph decreasing from 3 to4 and from 6 to 7 therefore, giving us is loses. he is breaking even: (0,1) U (4,5) you can see this by looking at the graph from left to right. The slope of the line does not move up or down from 0 to 1 and from 4 to 5: therefore giving the man a content line.

PSR Coding of SG6's Post

Dimension	Code
Incorporation of classmates' work	Y
Individual contribution	Minor contribution
Relative correctness to others	Equivalent
Relative correctness to prior posts	Not applicable

Student SW12's Problem 5.1 Episode

The Problem Statement

Complete the third table

x	f(x)	x	g(x)	x	$f \circ g$
1	4	1	3	1	
2	2	2	4	2	
3	-2	3	2	3	
4	1	4	1	4	

For at least one x value, explain in detail how to find $f \circ g(x)$.

SW4's Post Viewed by SW12

Author	Message Wednesday August 29th 2007 06:10:34 PM
SW4	x fog 1 -2 2 1 3 2 4 1

SW12's Post

Author	Message Friday August 31st 2007 02:59:49 PM
SW12	x f(x) x g(x) x Fog 1 -3 1 4 1 12 2 0 2 5 2 21 3 6 3 6 3 4 12 4 7 4 5 22 5 8 5 x^2-4 x+3 (x+3)^2-4 First I created the tables and found their equations. I then plugged them into F(g(x)). Then I plugged in x which gave me (1+3)^2-4= 12 and that is how I got Fog.

PSR Coding of SW12's Post

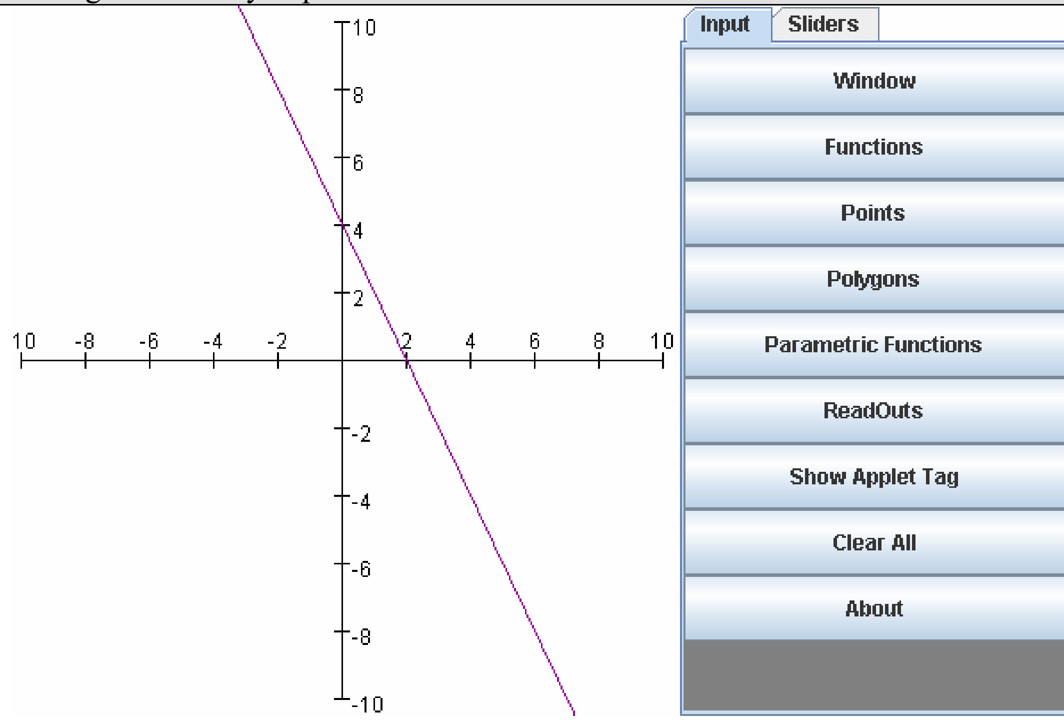
Dimension	Code
Incorporation of classmates' work	N
Individual contribution	Not applicable
Relative correctness to others	Regression
Relative correctness to prior posts	Not applicable

Student SW25's Problem 6.1 Episode

The Problem Statement

Write an equation of a linear function with negative slope that contains no points in the second quadrant (negative x, positive y). Explain why your function satisfies the conditions or argue why it is impossible to construct such a function. Once the tensor has unlocked (even if you think your answer is correct), find at least one person's response that you agree with and explain why. If you disagree with everyone else's responses, explain why.

Contents of SW25's Tensor Square at the End of the Initial Phase

<table>
<tr><th>Author</th><th>Message Monday September 17th 2007 09:34:04 AM</th></tr>
<tr><td>SW25</td><td>

first I plotted points (1,2) then I plotted point on (2,0) b/c slope is negative and the points can not be in quadrant II.
Second I found the slope then find the formula of Y=mx+b function statisfy because the slope is -2x and the line is decreasing on right.</td></tr>
<tr><th>Author</th><th>Message Thursday September 20th 2007 03:39:07 AM</th></tr>
<tr><td>TC</td><td>That line has points in quadrants I, II, and IV.</td></tr>
<tr><th>Author</th><th>Message Friday September 21st 2007 06:03:04 AM</th></tr>
<tr><td>SW25</td><td>(1,-1)U(2,-2)
Slope = -1
y=-1x+0</td></tr>
<tr><th>Author</th><th>Message Sunday September 23rd 2007 05:03:35 PM</th></tr>
<tr><td>TC</td><td>That hits quadrants II and IV.</td></tr>
<tr><th>Author</th><th>Message Monday September 24th 2007 06:16:13 AM</th></tr>
<tr><td>SW25</td><td>Y=-1/2+0</td></tr>
<tr><th>Author</th><th>Message Monday September 24th 2007 08:18:04 AM</th></tr>
<tr><td>TC</td><td>I know that you like to try to do these on your own, but go ahead and take a look around the tensor. See what some others have said.</td></tr>
</table>

SW26's Post Viewed by SW25

Author	Message Thursday September 20th 2007 05:21:24 AM
SW26	it is impossible to construct a line with negative slope and not have it go into the second quadrant because if the line is never ending and negative it eventually has to go into the second quadrant.

SW27's Post

Author	Message Wednesday September 26th 2007 06:05:11 AM
SW27	I relaized [*sic*] that it is impossible to graph a line and leave the quadrant II out. I wanted make sure so I used source(SW26)

PSR Coding of SG6's Post

Dimension	Code
Incorporation of classmates' work	Y
Individual contribution	Reworded
Relative correctness to others	Equivalent
Relative correctness to prior posts	Improvement

Portion of Student SW27's Problem 11.1 Episode

The Problem Statement

A) Solve this problem: A farmer is going to build a rectangular pig pen. He wants it to be 10 feet longer than twice the width. If the enclosed area is to be 1612 feet, what are the possible dimensions of the pen? B) Make up a similar problem by changing the numbers and the relationship between width and length. C) Once the tensor opens, find someone's problem and solve it.

SW18's Posed Problem Viewed by SW27

Author	Message Friday October 05th 2007 08:19:46 AM
SW18	A farmer is going to build a rectangular pig pen. He wants it to be 5 feet longer than QUADRIPLE the width. If the enclosed area is to be 800 feet, what are the possible dimensions of the pen?

SW27's Response

Author	Message Friday October 19th 2007 06:34:19 AM
SW27	Hi I did your math model, here's what I got: 800 = (5+4w)w 800 - 800 = (5+4w)w -800 0 = (5+4w)w - 800 0 = 4w^2 + 5w - 800 Used the QUAD program W = -14.8 W = 13.5 Width cannot be a negative number so W = 13.5 L = 5+4(13.5) L = 59 Dimensions: L = 59 W = 13.5

PSR Coding of SW27's Post

Dimension	Code
Incorporation of classmates' work	Other
Individual contribution	Not applicable
Relative correctness to others	Not applicable
Relative correctness to prior posts	Not applicable

Student SG6's Problem 15.2 Episode

The Problem Statement

Let n be the sum of the last two digits of your student id and let m be the number for the month you were born in. Let f(x) = -2*log(x-m)+n. a) List the translations needed to obtain a graph of f(x) from a graph of log(x). b) List equations for all asymptotes of f(x). c) List the domain and range of f(x).

SG12's Post Viewed by SG6

Author	Message Wednesday November 07th 2007 11:31:57 AM
SG12	n=1 m=12 f(x)= -2*log(x-12)+1 vertical stretch of 2 -= flip x-12= y=-12 up 1 domain- (-infinity, infinity) range- (-12, infinity)

SG6's Post

Author	Message Friday November 09th 2007 08:07:06 PM
SG6	n=13 m=3 f(x)=-2*log(x-3)+3 a) -2 vertically stretches and flips across x-axis -3 makes it move right 3 on y-axis +3 moves it up 3 b)VA: x=3 c)Domain: (3, infinity) Range: all real numbers

PSR Coding of SG6's Post

Dimension	Code
Incorporation of classmates' work	SP
Individual contribution	Major contribution
Relative correctness to others	Improvement
Relative correctness to prior posts	Not applicable

APPENDIX K

Frequency of Assignments Attempted, Assignments for Which Classmates' Work Was Viewed, and Total Posts by Student

	Shared-work section				Small-groups section		
Student	Assignments attempted	Viewed others	Total posts	Student	Assignments attempted	Viewed others	Total posts
SW1	11	2	20	SG1[a]	1	0	2
SW2	13	7	32	SG2[b]	0	2	0
SW3	9	9	24	SG3[b]	1	1	1
SW4	14	13	45	SG4	14	14	54
SW5	7	11	16	SG5	16	16	56
SW6	16	6	68	SG6	16	16	85
SW7	7	1	17	SG7	16	13	92
SW8[a]	7	1	15	SG8	14	15	41
SW9	7	5	13	SG9[a]	0	1	0
SW10[b]	4	3	10	SG10	9	5	23
SW11	16	8	67	SG11	14	8	38
SW12	16	13	45	SG12	16	16	110
SW13[b]	4	2	6	SG13	16	8	41
SW14[b]	3	2	11	SG14	0	3	0
SW15	11	5	42	SG15	15	15	52
SW16	5	0	6	SG16	12	7	29
SW17	15	12	38	SG17	12	10	33
SW18	3	3	58	SG18	15	11	67
SW19[b]	16	15	10	SG19	16	14	52
SW20	13	9	79	SG20[b]	2	3	3
SW21	12	9	42	SG21[a]	2	3	3
SW22	11	8	17	SG22	1	3	3
SW23	6	0	10	SG23	14	15	54
SW24	4	6	10	SG24	11	15	37
SW25	16	13	89				
SW26	16	9	71				
SW27	16	16	65				

Note. Ignoring students who withdrew or ceased attending class, the shared-work section means are 11.1, 7.5, and 37.5, and the small-groups section means are 12.6, 11.3, and 48.2 for assignments attempted, assignments for which students viewed classmates' work, and total posts per student, respectively.
[a]Stopped attending class.
[b]Withdrew from the course.

APPENDIX L

Frequency of Asynchronous Interaction by Coding Scheme Category per Problem for the Shared-Work Section

Problem	Posts	Units	PS	PSR	RP	RI	C	Q	R	D	RD	EE
1.1	29	33	25	2	2	3	1	0	0	0	0	0
1.2	24	26	22	2	0	1	0	1	0	0	0	0
2.1	32	35	13	10	0	8	3	1	0	0	0	0
2.2	26	29	13	8	0	4	3	1	0	0	0	0
3.1	28	28	21	3	2	2	0	0	0	0	0	0
3.2	26	26	14	9	1	2	0	0	0	0	0	0
4.1	31	38	19	7	0	5	6	1	0	0	0	0
4.2	30	32	19	9	0	2	2	0	0	0	0	0
4.3	23	25	12	8	0	3	1	1	0	0	0	0
5.1	22	24	11	10	0	1	2	0	0	0	0	0
5.2	16	16	6	8	0	1	1	0	0	0	0	0
5.3	14	14	5	7	0	2	0	0	0	0	0	0
6.1	35	39	23	3	2	2	2	0	7	0	0	0
6.2	29	33	17	2	0	4	3	1	6	0	0	0
6.3	24	27	14	4	0	2	3	0	4	0	0	0
7.1	32	38	22	2	1	6	5	1	1	0	0	0
7.2	26	30	16	3	0	6	3	1	1	0	0	0
7.3	33	34	20	11	1	1	0	0	1	0	0	0
8.1	31	35	18	5	1	8	3	0	0	0	0	0
8.2	33	35	16	6	0	11	1	1	0	0	0	0
9.1	25	29	16	1	0	7	4	1	0	0	0	0
9.2	24	32	13	7	0	5	4	3	0	0	0	0
10.1	27	33	14	4	0	9	5	1	0	0	0	0
10.2	22	24	12	2	0	7	1	2	0	0	0	0
10.3	28	29	12	9	0	4	2	2	0	0	0	0
11.1	25	31	10	10	2	4	4	1	0	0	0	0
11.2	18	25	12	4	0	3	3	3	0	0	0	0
12.1	18	23	12	3	0	3	3	2	0	0	0	0
12.2	20	23	9	4	3	3	4	0	0	0	0	0
13.1	17	18	12	3	0	3	0	0	0	0	0	0
13.2	14	15	12	2	0	0	1	0	0	0	0	0
14.1	18	18	12	3	0	3	0	0	0	0	0	0
14.2	14	14	11	3	0	0	0	0	0	0	0	0
14.3	22	24	13	5	0	3	2	1	0	0	0	0
15.1	16	19	11	3	0	3	1	1	0	0	0	0
15.2	15	15	11	3	0	1	0	0	0	0	0	0
15.3	21	26	9	4	0	7	4	2	0	0	0	0
16.1	14	15	12	0	0	2	1	0	0	0	0	0
16.2	9	9	6	1	0	2	0	0	0	0	0	0
16.3	13	13	11	0	0	2	0	0	0	0	0	0
Total	924	1032	556	190	15	145	78	28	20	0	0	0

APPENDIX M

Frequency of Asynchronous Interaction by Coding Scheme Category per Problem for the Small-Groups Section

Problem	Posts	Units	PS	PSR	RP	RI	C	Q	R	D	RD	EE
1.1	27	35	11	7	5	0	4	4	4	0	0	0
1.2	21	26	7	8	3	0	0	4	4	0	0	0
2.1	30	44	7	8	3	4	6	9	7	0	0	0
2.2	20	23	6	8	3	0	2	2	1	1	0	0
3.1	19	19	7	8	2	2	0	0	0	0	0	0
3.2	10	10	4	2	1	1	0	0	2	0	0	0
4.1	20	22	7	8	2	0	0	0	5	0	0	0
4.2	19	26	8	4	2	2	4	3	2	0	0	1
4.3	16	18	7	2	2	0	0	2	4	0	1	0
5.1	14	15	5	5	2	0	0	0	3	0	0	0
5.2	10	10	4	1	1	1	1	0	2	0	0	0
5.3	10	13	4	2	1	1	2	1	2	0	0	0
6.1	26	32	16	1	2	5	3	2	2	1	0	0
6.2	30	35	16	4	4	4	4	2	1	0	0	0
6.3	26	31	15	2	6	1	2	3	1	1	0	0
7.1	34	38	16	3	8	3	3	0	5	0	0	0
7.2	33	41	13	3	5	5	7	4	3	1	0	0
7.3	35	44	15	7	6	3	6	6	0	1	0	0
8.1	35	39	18	6	7	1	3	1	3	0	0	0
8.2	38	42	14	8	7	1	5	2	4	0	1	0
9.1	36	41	18	4	8	0	5	0	6	0	0	0
9.2	26	32	14	4	6	1	5	2	0	0	0	0
10.1	22	23	13	2	1	4	1	1	1	0	0	0
10.2	13	13	10	1	0	0	2	0	0	0	0	0
10.3	24	33	13	3	0	3	8	4	2	0	0	0
11.1	27	34	17	7	2	3	4	1	0	0	0	0
11.2	16	24	9	2	1	4	5	2	1	0	0	0
12.1	18	18	14	2	0	0	1	0	1	0	0	0
12.2	19	22	11	4	0	0	4	2	1	0	0	0
13.1	17	23	12	4	0	0	4	2	1	0	0	0
13.2	16	18	13	0	0	1	2	0	2	0	0	0
14.1	21	23	9	8	1	1	1	2	1	0	0	0
14.2	20	25	8	6	0	4	4	2	1	0	0	0
14.3	20	22	10	8	0	2	1	1	0	0	0	0
15.1	19	21	7	10	0	2	1	1	0	0	0	0
15.2	18	20	8	8	0	3	1	0	0	0	0	0
15.3	16	20	6	8	0	1	4	0	1	0	0	0
16.1	27	28	13	8	0	0	4	3	0	0	0	0
16.2	13	18	6	6	0	0	3	2	1	0	0	0
16.3	17	19	8	6	0	0	4	0	1	0	0	0
Total	878	1040	419	198	91	63	116	70	75	5	2	1

APPENDIX N

Frequency of Each Asynchronous Interaction Category Contributed by Student and Interaction Type in the Shared-Work Section

Student	PS	PSR	RI	C	Q	R	RP	D	RD	EE	Total units	Interaction type
SW1	18	2	0	0	0	0	0	0	0	0	20	LI
SW2	22	6	4	5	1	0	1	0	0	0	39	AI
SW3	15	5	4	0	0	0	0	0	0	0	24	LI
SW4	29	8	4	2	0	6	0	0	0	0	49	AI
SW5	3	13	0	0	0	0	0	0	0	0	16	LF
SW6	50	5	9	1	0	3	0	0	0	0	68	AI
SW7	13	0	1	3	0	0	0	0	0	0	17	LI
SW8	15	0	0	1	0	0	0	0	0	0	16	NP
SW9	5	8	0	0	0	0	0	0	0	0	13	LF
SW10	3	7	0	0	0	0	0	0	0	0	10	NP
SW11	45	6	10	14	5	2	3	0	0	0	85	AI
SW12	21	23	1	2	0	0	0	0	0	0	47	NF
SW13	11	0	0	1	1	0	0	0	0	0	13	NP
SW14	2	4	0	0	0	0	0	0	0	0	6	NP
SW15	23	8	8	6	4	1	0	0	0	0	50	AI
SW16	6	0	0	0	0	0	0	0	0	0	6	NP
SW17	22	12	6	0	0	0	0	0	0	0	40	AI
SW18	30	17	9	4	1	0	4	0	0	0	65	AI
SW19	6	1	3	2	0	0	1	0	0	0	13	NP
SW20	44	7	23	5	1	3	3	0	0	0	86	AI
SW21	28	6	7	5	4	3	0	0	0	0	53	AI
SW22	8	10	0	0	2	0	0	0	0	0	20	LF
SW23	8	0	0	2	0	0	0	0	0	0	10	NP
SW24	9	1	0	0	0	0	0	0	0	0	10	NP
SW25	50	8	30	8	0	0	1	0	0	0	97	AI
SW26	39	11	19	3	1	0	0	0	0	0	73	AI
SW27	31	22	7	14	8	2	2	0	0	0	86	AI

Note: AI = active independent, LI = low-participating independent, LF = low-participating follower, NP = nonparticipant, and NF = noninteractive follower

APPENDIX O

Frequency of Each Asynchronous Interaction Category Contributed by Student and Interaction Type in the Small-Groups Section

Student	PS	PSR	RI	C	Q	R	RP	D	RD	EE	Total units	Interaction type
SG1	2	0	0	0	0	0	0	0	0	0	2	NP
SG2	0	0	0	0	0	0	0	0	0	0	0	NP
SG3	1	0	0	0	0	0	0	0	0	0	1	NP
SG4	32	8	2	7	0	0	11	1	0	0	61	AI
SG5	16	16	9	13	11	5	0	0	0	0	70	AF
SG6	36	18	10	28	18	9	8	1	0	0	128	L
SG7	45	13	14	1	1	0	21	1	0	0	96	AI
SG8	14	23	5	6	6	3	0	0	1	0	58	AF
SG9	0	0	0	0	0	0	0	0	0	0	0	NP
SG10	16	1	0	5	1	0	3	0	0	0	26	LI
SG11	22	7	0	3	0	2	4	0	0	0	38	LI
SG12	48	22	11	14	14	9	11	1	1	0	131	L
SG13	26	11	3	1	0	0	1	0	0	0	42	LI
SG14	0	0	0	0	0	0	0	0	0	0	0	NP
SG15	15	24	0	3	1	13	4	0	0	0	60	AF
SG16	22	4	0	2	1	2	0	0	0	0	31	LI
SG17	13	5	0	11	1	10	0	1	0	0	41	LF
SG18	36	10	4	3	2	2	15	0	0	0	72	AI
SG19	26	9	5	15	8	8	3	0	0	1	75	L
SG20	0	2	0	0	0	1	0	0	0	0	3	NP
SG21	0	3	0	0	0	0	0	0	0	0	3	NP
SG22	0	0	0	0	0	3	0	0	0	0	3	NP
SG23	37	5	0	0	2	3	10	0	0	0	57	AI
SG24	12	17	0	4	4	5	0	0	0	0	42	LF

Note: AI = active independent, AF = active follower, L = leader, LI = low-participating independent, LF = low-participating follower, and NP = nonparticipant

APPENDIX P

Average Class Score per Problem by Section

Problem	Shared-work score[a]	Small-groups score[a]
1.1	4.2	4.9
1.2	4.2	4.9
2.1	3.2	4.1
2.2	2.5	3.6
3.1	4.5	4.3
3.2	2.9	2.3
4.1	4.0	4.2
4.2	3.4	3.1
4.3	3.2	2.9
5.1	3.3	3.4
5.2	2.7	1.5
5.3	1.9	1.9
6.1	4.6	5.5
6.2	3.8	4.1
6.3	3.7	4.6
7.1	4.2	5.1
7.2	3.1	4.4
7.3	2.9	3.1
8.1	3.6	3.8
8.2	3.0	4.1
9.1	2.7	4.0
9.2	2.0	3.9
10.1	3.1	4.9
10.2	3.5	3.9
10.3	2.8	3.3
11.1	2.2	4.6
11.2	1.7	2.7
12.1	2.7	3.7
12.2	2.3	3.4
13.1	1.9	2.5
13.2	1.7	1.6
14.1	3.2	4.7
14.2	3.3	4.5
14.3	3.0	5.1
15.1	2.7	5.6
15.2	2.1	4.9
15.3	1.3	2.9
16.1	1.5	3.3
16.2	1.1	2.6
16.3	1.5	3.0

Note. The average section scores were computed from the students who did not withdraw and who contributed at least 1% of the units of meaning.
[a]Each score is out of 6 possible points.