A STUDY OF ADULT RE-ENTRY STUDENTS’ AFFECTIVE ORIENTATION TOWARD MATHEMATICS AND THEIR MATHEMATICAL THINKING IN THE CONTEXT OF PROBLEM-SOLVING

by

JACQUELYN TERNAN COHEN

(Under the direction of John Olive, PhD)

ABSTRACT

This modified teaching experiment explored the perspectives of adult re-entry students enrolled in the study of mathematics at the most basic level in higher education. Specifically, it investigated characteristics of the students' affective orientation toward learning, the nature of their mathematical thinking in the context of mathematical problem-solving in contrast to procedural learning, and the possible relationship between their affective orientation toward the learning of mathematics and their mathematical thinking.

Eighty-nine students enrolled in remedial level mathematics classes at a large state university completed initial questionnaires. Of the 19 re-entry students in this initial group, three participated in the entire study, which included initial and exit interviews, teaching sessions and classroom observations. Case studies of those 3 students are presented in this dissertation along with a case study of the interaction between two of them.
This research yielded a rich array of results concerning re-entry students’ affective orientation toward the study of mathematics at the remedial level, and the interplay between that affective orientation and their ways and means of operating in a problem-solving setting. These results would have been difficult to derive with methodology other than an adaptation of the teaching experiment. The initial questionnaires administered to all 89 students laid the foundation, but it was within the interviews and, more significantly, the teaching sessions that the three participants’ ways of knowing were revealed and knowledge voices were heard. Indeed, the teaching sessions not only added to the results but corrected misconceptions about the participants’ ways of operating had the conclusions been based only on questionnaires and interviews. Those participants who completed the project provided strong contrasts and an abundance of findings. One of them demonstrated a particularly vivid example of differences between stated and revealed beliefs, thus alerting us to the possibilities of such differences in other individuals. The findings suggest modifications to some of the models and research methods employed in order to better investigate re-entry students’ study of mathematics in college. Moreover, they suggest modifications in the remedial mathematics courses offered to students reentering higher education that would take advantage of the experiences and world-views of such students.

INDEX WORDS: Remedial mathematics in college, Re-entry students, Non-traditional students, Mathematical thinking, Affective orientation toward mathematics, Teaching experiment
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by

JACQUELYN TERMAN COHEN
B.S., Emory University, 1969
M.S., The University of Houston, 1971

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

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JACQUELYN TERMAN COHEN

Approved:

Major Professor: John Olive

Committee: Ronald M. Cervero
Nicholas Oppong
Theodore Shifrin
Patricia S. Wilson

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
December 2002
DEDICATION

This work is dedicated

- to all who defer their goals, particularly educational ones, and later are sufficiently blessed, lucky or just plain stubborn enough to return to them and succeed.

- to re-entry students, many of whom populate the classrooms in which I teach and three of whom worked closely with me to make this work possible.

- to me, and to all those close to me, as a token of a promise I made to myself and finally managed to keep.
ACKNOWLEDGMENTS

I wish to acknowledge Augusta State University (ASU) as an institution as well as many individuals there who recognize the importance of faculty development in the form of higher education. Many colleagues, including Dr. William Dodd, chair of the Department of Learning Support, supplied much needed words of wisdom, encouragement and characteristic insight throughout the entire process. ASU supported me in numerous ways including granting me leaves of absence for two years, enabling me to attend the University of Georgia full time and complete my coursework.

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The UGA faculty resurrected and nourished my love of learning and assisted me in developing important skills for research and intellectual inquiry, in addition to rekindling my love of the study of mathematics as well as of the exploration of the art and science of teaching. Returning to the university classroom as a student challenged and invigorated me. In addition, I appreciate the cooperation of those UGA faculty
members who permitted me to observe their classes and make contact with their students in order to recruit participants for my research.

Needless to say, the support of my Committee – John Olive, Pat Wilson and Nicholas Oppong (Mathematics Education); Ron Cervero (Adult Education) and Ted Shifrin (Mathematics) – was essential to the development of my dissertation. I must particularly thank John Olive, who I am honored to call my professor, for his expert guidance and for the confidence he had in me. Beyond all else, he was truly a role model for me. From the way he interacted with me, I learned so very much about more constructive ways to interact with my students as well as others with whom I work.

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CHAPTER 1
INTRODUCTION
Overview of the Study

This research examined characteristics of adult learners in the context of the study of mathematics at a remedial or basic level in higher education. The participants were involved in mathematical problem-solving sessions in an effort to engage them in mathematical thinking that might be different from their previous or present experiences in mathematics classroom settings. I was interested in how the characteristics of adult learners look in the realm of the study of mathematics, both in the usual classroom setting and in a mathematical problem-solving context that might be unusual for the students.

The study was situated in a multifaceted model of adult student participation in higher education that included the students’ past and present experiences and psychosocial orientations. I attempted to listen to the students’ knowledge voices (Kasworm, 1997, March) as they told me their beliefs and showed me how their belief structures operated in the academic world and in the study of mathematics. I also attempted to examine their mathematical learning and thinking by focusing on what Skemp (1979; 1987) refers to as the frontier zone. I employed the methodology of a teaching experiment to investigate both the students’ beliefs in action and their ways and means of thinking in the context of problem-solving.

The concepts that I have mentioned are explored further in the background information and literature review that follow. At this point, it may be helpful to provide
some working definitions for the phrases adult re-entry students and affective orientation toward learning before stating the research questions.

- Adults who are attending post-secondary educational institutions for the purpose of earning 2-year or 4-year degrees are commonly referred to as re-entry students or adult re-entry students. More precisely, this designation is usually reserved for students who are returning to formal education after a substantial time lapse, and are perceived as significantly older than the traditional aged students who are typically 18 years old at entry. The definition that is most common and most useful for my purposes will be students who were at least 25 years of age when they first enrolled in higher education or enrolled in higher education after not having been enrolled in a formal educational program for at least 5 years. Thus, the population would include students who dropped out of high school at age 16 or 17; were involved in family, job or other activity until age 20 or 21; took a review course before receiving a Graduate Equivalency Diploma (GED) at age 21 or 22; and then enrolled in higher education. Such a student would have been away from formal education for at least 5 years; the review course for the GED would not be considered formal education. In addition, I will focus on those adults who enter their mathematics studies in higher education at a remedial or the most basic entry level.

- Students' affective orientation toward learning includes their beliefs and attitudes about that which is to be studied and their inclination to study it. It includes beliefs about the role of the student and of the instructor and the nature of knowledge. It is a result of experience, environment or setting, level of maturity, personal goals, and factors working both in favor of and against their participation in learning. It might be described as a student's emotional predisposition, motivation-to-learn or reasons for participation in
learning. Thus, a student's affective orientation toward learning is comprised of a collection of essentially non-cognitive factors that impact significantly on their perspectives about learning.

Research Questions

The major question addressed in this study was: With what perspective do adult re-entry students engage in the study of mathematics at the most basic level in higher education? Specifically, I investigated characteristics of the students' affective orientation toward learning, the nature of their mathematical thinking in the context of mathematical problem-solving in contrast to procedural learning, and finally, the possible relationship between their affective orientation toward the learning of mathematics and their mathematical thinking.

1. The investigation of the students’ affective orientation toward the study of mathematics included their stated and demonstrated beliefs about themselves as learners and as mathematics learners and about the nature of knowledge and the nature of mathematical knowledge as well as their stated and demonstrated beliefs about the role of the student and the role of the instructor in general in higher education and in the study of mathematics. In the case of the study of mathematics, I investigated the contextual nature of their beliefs; that is, did the students perceive these roles differently in the setting of the mathematics classroom than in the setting of the teaching experiment? I also investigated their reasons for participating in higher education and for studying mathematics and the factors that have positively or negatively influenced that participation.
2. With regard to the students' mathematical thinking, I focused on their ways and means of approaching mathematical tasks that went beyond procedural skills, when engaged in mathematical problem-solving. In particular, I was interested in the connections students might make between their problem-solving abilities, perhaps developed in out-of-school contexts, and their work on mathematical tasks. For example, to what extent did the students call on their prior experiences and current life-world environment when engaged in a mathematical problem-solving setting? Did assisting the students in perceiving connections between the academic world and their prior and current environments enhance their level of engagement in mathematical problem-solving tasks?

3. Finally, I investigated the relationships that might exist between the above characteristics of the students' affective orientation toward mathematics and their mathematical thinking in a problem-solving setting.

Rationale for the Study

In our culture, mathematics holds a position of fear and respect in the hearts of most individuals, including many students in higher education. Its study is often considered both exceedingly difficult and increasingly necessary. Classes in the entry level and remedial courses in mathematics at college are frequently populated by a significant number of re-entry students, adults who have decided to return to formal education for a variety of reasons. In my pilot study, I began an examination of the interactions of these students with mathematics; it might be described as a mathematical baggage check. That is, I explored and attempted to describe the variety of in-school and out-of-school knowledge and experiences; beliefs about mathematics and themselves as
mathematics learners; as well as ways of knowing, in general, and ways of approaching mathematics, in particular that these adult students bring with them.

Using the pilot study as a backdrop, this research took a closer look at these students’ interactions with mathematics in the context of problem-solving. This provided a venue for examining the students’ mathematical thinking and level of cognitive development with respect to mathematics. In particular, this research attempted to make explicit for the students and for the researcher the mathematical abilities and knowledge the students brought with them that could assist them in being more proficient mathematical problem solvers.

Literature in adult learning and development and in mathematics education emphasizes the interplay between the affective and cognitive domains. Hence, I was interested in the motivation of these students to learn, both generally and in mathematics, and in the influences between their motivation and their ways of thinking about mathematics and about themselves as mathematics learners.

Much of the literature in adult education suggests that these students tend to prefer a deeper approach to learning, while many of the courses they are offered are focused on procedural learning. This research served to demonstrate that, to some extent, the adult education literature applies not only in general to higher education but also to mathematics education for re-entry adults. Based on that observation, the profession may want to take another look at the curriculum and format choices made for these students.
Theoretical Background

A Model of Adult Student Participation

In their AERA presentation Adult Undergraduates' Participation and Involvement: Future Directions for Theory and Research, Donaldson, Graham, Kasworm and Dirx (1999, April) observed that although "adult students are dramatically changing the nature of higher education today . . . , most of the insights about the undergraduate experience are drawn from the past two decades of research on young adults and their development" (p. 3). They explained that previous research predominantly had employed either the lens of the "cultural community frame . . . [or] the talent development frame" (p. 9). The cultural community frame assumes that the student is to be integrated into the culture via complete involvement. The talent development frame assumes that students need to acquire certain knowledge, skills and behaviors to have the talents necessary to be an educated person.

Both of these frameworks focus upon the young adult passage into a higher education experience from high school . . . [and] suggest that the student should be fully immersed in the collegiate culture . . . . When using these frameworks and related assumptions to examine the adult, the adult student presents characteristics and actions reflecting deficiency, marginalization, and incongruence with potential future success and positive impact from an undergraduate higher education. (p. 11)

In response, Donaldson et al. (1999, April) suggested these three potential alternative frameworks: 1) learner participation through adult life roles; 2) learner participation as lifelong learning; and 3) learner participation based on a post-modern society.
1) Learner involvement through adult life role. The adult learner is multicultural. "At the heart of this reframing is the shift of the unit of analysis from the student within an institutional context to an adult, who is also a student, within a life context of many other significant adult roles" (p. 14).

2) Learner participation as lifelong learning. The assumption has been that all collegiate experience should be measured against the ideal full-time residential experience following secondary school. The unit of analysis will "look at significant adult learning experiences that occur through multiple higher education involvements, through intermittent participation, and through an increasing diversity of delivery systems and learning communities" (p. 17).

3) Learner participation based on a post-modern society. This framework suggests that all undergraduate students, both young and old, are making sense of their lives and their learning through the experiences of a "post-modern context, which suggests that individuals who reflect otherness are marginalized and experience incongruence with organizational structures, power relations, and traditional student cultural context of the collegiate world" (p. 18). This framework seems to build on each of the other two as it places "the unit of analysis as the critical examination of current constructs of collegiate involvement, relationships of learning to development and the notion of salient cultural learning communities" (p. 19).

They also presented the "proposed Model of College Outcomes for Adults (see Figure 1.1), developed by Graham and Donaldson, [which represents] a multifaceted explanatory model based in current research of adults in higher education" (Donaldson et al., 1999, April, p. 12). They reviewed research on adult learners and higher education in
light of this model, which they described as a comprehensive model to "reconceptualize
the nature of participation and involvement for adult learners" (p. 12). The review was
organized under the following headings: understanding adults' experiences in higher
education, social and psychological concerns, adult cognition, the adult world
surroundings, and adults' success in college. (Where the headings are different from the
labels on the model, I have indicated each heading's relationship to the model in
parentheses.)

Figure 1.1: Model of College Outcomes for Adults developed by Graham and Donaldson
(Donaldson et al., 1999, April)

Understanding adults' experiences in higher education (prior experience and general
overview)

Often adults' involvement on campus is limited to the academic classroom, but
neither their grades nor their satisfaction with their experiences seem to suffer for this.
"Despite lower levels of campus involvement, rusty academic skills, and busy lifestyles, adults report significant progress from their academic endeavors" (p. 4). Donaldson et al. (1999, April) noted that research attributed this to several reasons. Adults make meaning by reflecting on personal experiences and previous knowledge and wisdom and integrating new learning with existing knowledge schema. Adults achieve a more 'authentic involvement' by applying the new knowledge immediately in real-life contexts. Adults use the classroom as a stage to intensify their learning and enhance their interactions with peers and instructors. Finally, adults are purposeful and intent in their learning and take the experience and instructors' advice seriously (p. 4-5).

Social and psychological concerns (psycho-social & value orientation)

Many adults are concerned with the impact of their age and with the possibility that they may lack academic skills. They rely on support of those around them. With adequate study skills and a clear purpose, they usually remain in college. Many authors report that adults expend more effort and take the process more seriously than younger students (p. 5).

The role of the classroom

Many researchers report that the classroom and the relationships built there are of utmost importance to adult students. They value highly the community of learning developed there and the presence of a respectful and caring instructor. Researchers generally acknowledged the primary role of the classroom for adult learning in higher education (p. 6).
Adult cognition

Researchers highlighted the rich learning and cognitive schema adults bring to the classroom, the role of prior experiences, the use of complex metacognitive strategies in their approaches to studying and their approach to learning as major factors in adults' cognition (p. 6).

The adult world surroundings (life-world environment)

The research reviewed pointed out that for adults the multiple life roles and duties outside of campus life provide both a context for applying school-learning and alternative opportunities for the kinds of experiences younger students may gain from extracurricular activities, often seen as the definition of involvement in traditional settings. People with whom adults interact in their other non-student roles at times provide either support or discouragement for their academic pursuit (p. 7).

Adults' success in college (outcomes)

Adults generally do as well or better than traditional-age students despite different “patterns of enrollment and involvement in higher education . . . regardless of age, students profited when they felt they were supported by the educational ethos [or climate for learning] of college" (p. 7).

Although there is much overlap between the categories listed above, these findings all suggested the strong impact of adults' prior experiences and current lifestyles on their educational experiences. The Model of College Outcomes for Adults provides a "more complex, interactive, and dynamic view of the adult learner" (p. 21). Examining the characteristics within each of the boxes of the model and the interaction between those boxes with the frameworks in mind "helps to illuminate issues critical to our
For the purposes of this research, the model situated the adult students' learning. It assisted me in seeing the way that adults construct their knowledge (make meaning) within the context of their prior and their life-world experiences, and the way they make connections between their life-world and the academic world. It provided a framework in which to observe the forces at work in the adult students' experience in higher education. The questions I asked guided my exploration into how the dynamic described in general for these students plays out in their study of mathematics, particularly with regard to what they brought with them into the college classroom.

**Zones for Learning**

Now that I have situated the study in the context of adult participation in higher education, I would like to explore the region in which a student's learning actually takes place. Several researchers/educators have described such a region, frequently referring to it as a zone. After exploring the notions presented by Vygotsky and Steffe, I focus on Skemp's frontier zone. This discussion serves to strengthen my theoretical basis for selecting mathematical problem-solving as a context for the research, for examining the students' affective orientation toward learning and their cognition concurrently, and for employing a teaching experiment in which to do so.

Vygotsky identified the region in which a person can learn with some external assistance as that person's zone of proximal development. That is, the person has the potential to be successful given the benefit of reasonable assistance. One might visualize the learning task as being just barely outside of one's grasp, so that with minimal help one
can accomplish the task. The help might take the form of a teacher, a peer or some form of technology (e.g., Berliner & Calfee, 1996).

Steffe's idea of a zone of potential construction is slightly different from Vygotsky's zone of proximal development. Steffe's notion focuses on the person's own conceptual development, and ways and means of thinking (potential) that may permit learning to take place (construction of knowledge), with or without apparent external assistance. Steffe's zone of potential construction is more compatible with Skemp's model (presented next) than it is with Vygotsky's zone of proximal development (Olive & Steffe, 2002).

To understand the Skemp's (1979, 1989) frontier zone, I need to explore several terms and notions that Skemp employed. The model that Skemp provided (see Figure 1.2) guides this discussion. The established domain is that region in which we are competent; that is, we are able to reach our goals and avoid our antigoals with ease. Outside of that domain, we are not competent and are unable to reach our goals and avoid our antigoals. The boundary between these two regions is usually not clear and precise. Skemp claims that within this ill-defined boundary "There is a frontier zone in which we can achieve our goals, and avoid our antigoals, sometimes not reliably. It is in this area that learning takes place; and learning is thus a process of changing frontier zone to established domain" (Skemp, 1989, p. 195).
The processes involved in attaining cognitive goals, and avoiding antigoals, are explored in the section on mathematical thinking. It becomes even clearer that most of the time the context of mathematical problem-solving is in the frontier zone for most adult re-entry students, particularly those who enter at the most basic levels of mathematical coursework in higher education. Thus, this model (Figure 1.2) and Skemp's notion of a frontier zone provides a theoretical basis for the choice of the context of problem-solving for this research.

The model also suggests that within the frontier zone one is likely to experience mixed emotions; this aspect is explored further in the section on affect and mathematical thinking. However, it is important to note here that Skemp's work also provides a theoretical basis for exploring both emotions (affective orientation) and mathematical thinking in order to understand students' learning of mathematics. Both with the broader lens in the previous model of adult student participation and with this finer lens focused on the frontier zone, affective orientation and cognition are significantly intertwined.
The teaching experiment is defined and explored fully in the methodology section. Suffice it to say here that the only way to examine students’ ways and means of learning and to observe their affective orientation toward learning is to go to the place where that learning takes place. Skemp's work directs me to the student's frontier zone, and a teaching experiment is the appropriate methodology to use in order to be situated properly in that place.
CHAPTER 2

LITERATURE

Perceptions About the Study of Mathematics

Societal Perceptions

Numerous books and articles (Ginsburg, 1998; Nesbit, 1996; Paulos, 1988; Steen, 1990; Steen, 1997b) have highlighted this country's growing problem with mathematics. These authors and others have established that adults' abilities and knowledge have declined in recent years, while numeracy remains an important determinant for job and career choices. Some students see the learning of mathematics as an opportunity to overcome obstacles and attain goals previously thought to be out of their reach; others see the taking of required mathematics courses as a barrier in their paths toward getting degrees. Some view mathematics mainly as arithmetic with a few tricky word problems thrown in for a good challenge; others see it as a highly esoteric and mysterious arena, accessible only to the very few. Their views of mathematics and its study are to a great extent a reflection of the views held in school, at home and in the community at large. Mathematics educators do battle with what has been observed as a cultural predisposition about mathematics that is an amalgamation of fear and respect (Jacobsen, 1991; Steen, 1990; Steen, 1997b).

Perceptions Within the Discipline

Adding to the confusion is an ambivalence in the profession about whether mathematics is primarily the queen or the servant of other disciplines (American
Association for the Advancement of Science, 1989; Shulman, 1986a; Shulman, 1986b; Skemp, 1987; Steen, 1997a; Steen, 1997b). There remains what may be described as the dual nature of mathematics best characterized by the distinction between pure and applied mathematics and, perhaps, best linked by the activity of mathematical modeling of real-world situations (e.g., Corte, Greer, & Verschaffel, 1996). So the questions of what mathematics to study and why students should study it are not clearly answered, making curricular decisions all the more complex in higher education.

Nevertheless, society will need more workers who are able to do more than perform straightforward calculations and apply given formulae. They must know which procedures to use and when to apply them; they must possess mathematical understanding. Meanwhile, although schools (K-12) and institutions of higher education have claimed, for example in the Standards (National Council of Teachers of Mathematics, 1989) and in the reform movement (e.g., Asiala et al., 1997), to have the development of such understanding as one of their principle goals, programs at all levels seem to be very much unchanged. As Nesbit (1996) declared, adults have learned through years of schooling that mathematics is an "abstract and hierarchical series of objective and decontextualized facts, rules and answers. . . . Knowledge is portrayed as largely separate from learners' thought processes and mathematics education is experienced as a static, rather than dynamic process" (p. 71). There is apparently little in these adults' experiences in entry-level mathematics to change these perceptions. The practice of mathematics education does not seem to heed the "widely-held assumptions about and practices within adult education that are built on ideas and theories about how adults learn and should be taught" (p. 71); that is, the same assumptions and practices that will
be highlighted in the following section. All of these observations serve to justify looking for ways to understand adults' interactions with mathematics at the entry level and to seek avenues for improving their situation.

Re-entry Adults’ Participation in Higher Education

There is no question that the populations on the campuses of institutions of higher education have become more diverse and there is no reason to expect that trend to abate. One aspect of this diversity is the age of the students. Higher education in the United States is no longer dominated by 18 – 22 year-olds who have just completed their secondary education. Somewhat in response to this trend, over the past 20 years an extensive literature has been developed around issues related to adults attending post-secondary educational institutions for the purpose of earning 2-year or 4-year degrees. As was indicated in the definitions presented earlier, this population is commonly referred to as re-entry students or adult re-entry students.

In preparation for this research, I reviewed general theoretical perspectives for adult learning and adult development theories with an emphasis on their application in education. I focused on what the research suggests that we know about adults’ motivation to learn and their cognitive development. Finally, I considered the implications of these observations for the study of re-entry students in mathematics courses.

A review of adult learning theory perspectives directed me to look at these students' learning in mathematics as transformational learning and situated cognition. A review of adult developmental theory encouraged me to consider structural developmental theory, as opposed to life-cycle theory, in order to accommodate better the diversity within this adult student population.
Several researchers in the area of adult education have employed these perspectives, developed them further and constructed models useful in further research. Two of the frameworks presented by Donaldson et al. (1999, April), learner participation through adult life roles and learner participation as lifelong learning and the Model of College Outcomes for Adults (see Figure 1.1, presented in the section on theoretical background, p. 8) seem to incorporate the ideas, particularly of Kasworm (1993, April; 1990), and to have the most potential in the study of re-entry students in mathematics.

My review of studies on motivation revealed that although adults' motives for enrolling in higher education are varied, they are characteristically linked to their life-world experiences. Some of the key concepts that were of interest in studying re-entry students were the notion of decisions based on triggers and barriers, adults’ emphasis on practical concerns rather than social interactions, their focus on classroom interactions, and the observation that adults are more likely to be internally or intrinsically motivated. Although some studies framed these issues as a comparison between younger and older students, not all of them did; such a comparison is not only potentially flawed but also actually unrelated to my intention to look only at the adults.

I noted two different approaches in studies that examined adult cognition: 1) those that focused on study skills and intellectual ability of students and 2) those that focused on meaning-making as the cognitive activity of adults. In research addressing study skills and intellectual ability, again, in some cases there are comparisons between traditional aged students and older students, while other studies attempt to describe adult students more completely and, in some studies, compare observations with previous studies of younger students or students in general. Of particular interest were the researchers’
claims that adults are more interested in developing deep conceptual understanding rather than surface procedural skills. This was a recurring theme that was crucial in my study of re-entry students in mathematics. Many studies noted that although adults' scores on mathematics entrance assessments are sometimes lower than younger students' scores, there is some confusion about whether this reflects a decline in intellectual ability or less exposure via previous coursework.

With regard to meaning-making as the cognitive activity of adults, the observation was made that development of more complex cognitive structures will better equip adults to function more satisfactorily. Higher education has a role to play in facilitating the process of making connections between meaning-making, learning and development. The "knowledge voices" described by Kasworm (1997, March), added a much needed dimension to Baxter Magolda's (1992) epistemological reflection model in examining the meaning-making processes of adult re-entry students. Both Kasworm’s and Baxter Magolda’s work are discussed more fully in the section on affective orientation.

One major question for me was whether these concepts and observations could be specifically applied to adult re-entry students when examining their learning experiences in mathematics. And, once applied, what would be the impact on practice and research? For example, the point has been made that adult students are a diverse group with diverse backgrounds and social settings. I questioned whether the characteristics described are uniform even for a single student. That is, it seemed that a student's mathematical background, prior knowledge, and current setting may have a very different flavor than that same student's general educational background, prior knowledge and current setting.
To use Kasworm's (1997, March) terms, students may speak with different voices for different disciplines.

Of particular concern to me was the issue of mathematical cognition. It became necessary to ask the questions: Where are these adult students really and where do they need to go? Much of the above research supports the notion that adults exhibit a conceptual orientation toward learning, and that developmentally they are ready for deep consideration of alternative perspectives (transformational learning) and problem solving. Meanwhile, in entry tests, adults often display weaker backgrounds in mathematics than in verbal skills and are then enrolled in courses to bolster their procedural skills, the same skills that are usually evaluated on the entrance tests. Questions arise out of these observations as to whether we are trying to repair perceived deficiencies in the adult students as well as whether there are alternative structures that could be put into place to improve the situation for the adult students as well as students in general. It is also interesting to note that the adult students demonstrate an orientation toward learning that many institutions are trying to foster in all of their students.

Looking at cognition again, the Model of College Outcomes for Adults (Donaldson et al., 1999, April) is quite helpful in conceptualizing the issue of mathematical cognitive development that I introduced in my pilot study. In this research, I was interested also in examining the mathematical schema that adults bring with them from their prior experience. In addition, I argue that the students' psycho-social and value orientation (I assume the area where affect and motivation reside) is not only influenced by cognition but also interacts with cognition - perhaps this occurs in the connecting classroom. That is, a student who is motivated to learn mathematics externally (for
example, to pass a perceived 'gate' into further education) may not invest the energy necessary to learn deeply and develop conceptual knowledge. The bridge between the affective and cognitive domains is discussed below in the section on affect and mathematical thinking.

Affective Orientation to Learning: Beliefs and Voices

In their examinations of how students learn and their ways of knowing, both Baxter Magolda (1992) and Kasworm (1997, March) incorporated the students' beliefs in their epistemological reflection model and belief structures (knowledge voices), respectively. I will first review Baxter Magolda's model, then explore Kasworm's voices, and finally explain their applications in my research.

Beliefs

Baxter Magolda (1992) followed traditional aged students at a large residential university through four years of college work and one year beyond. She refrained from classifying students, preferring to describe what she called ‘prevailing winds,’ based on her observations as the students journeyed through their college years. She described students as demonstrating absolute, transitional, independent and contextual learning by examining their learning beliefs and behaviors against a matrix that included five domains: role of learner, role of peers, role of instructor, evaluation and nature of knowledge. The epistemological reflection model that resulted from her work is reproduced as Table 2.1 below.
Table 2.1: Baxter Magolda’s Epistemological Reflection Model (1992)

<table>
<thead>
<tr>
<th>Domains</th>
<th>Absolute Knowing</th>
<th>Transitional Knowing</th>
<th>Independent Knowing</th>
<th>Contextual Knowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Role of learner</td>
<td>▪ Obtains knowledge from instructor</td>
<td>▪ Understands knowledge</td>
<td>▪ Thinks for self</td>
<td>▪ Exchanges and compares perspectives</td>
</tr>
<tr>
<td></td>
<td>▪ Obtains knowledge from instructor</td>
<td>▪ Understands knowledge</td>
<td>▪ Shares views with others</td>
<td>▪ Thinks through problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>▪ Creates own perspective</td>
<td>▪ Integrates and applies knowledge</td>
</tr>
<tr>
<td>Role of peers</td>
<td>▪ Share materials</td>
<td>▪ Provide active exchanges</td>
<td>▪ Share views</td>
<td>▪ Enhance learning via quality contributions</td>
</tr>
<tr>
<td></td>
<td>▪ Explain what they have learned to each other</td>
<td></td>
<td>▪ Serve as a source of knowledge</td>
<td></td>
</tr>
<tr>
<td>Role of instructor</td>
<td>▪ Communicates knowledge appropriately</td>
<td>▪ Uses methods aimed at understanding</td>
<td>▪ Promotes independent thinking</td>
<td>▪ Promotes application of knowledge in context</td>
</tr>
<tr>
<td></td>
<td>▪ Communicates knowledge appropriately</td>
<td>▪ Uses methods aimed at understanding</td>
<td>▪ Promotes independent thinking</td>
<td>▪ Promotes evaluative discussion of perspectives</td>
</tr>
<tr>
<td></td>
<td>▪ Ensures that students understand knowledge</td>
<td>▪ Employs methods that help apply knowledge</td>
<td>▪ Promotes exchange of opinions</td>
<td>▪ Student &amp; teacher critique each other</td>
</tr>
<tr>
<td>Evaluation</td>
<td>▪ Provides vehicle to show instructor what was learned</td>
<td>▪ Measures students’ understanding of the material</td>
<td>▪ Rewards independent thinking</td>
<td>▪ Accurately measures competence</td>
</tr>
<tr>
<td>Nature of knowledge</td>
<td>▪ Is certain or absolute</td>
<td>▪ Is partially certain &amp; partially uncertain</td>
<td>▪ Is uncertain – everyone has own beliefs</td>
<td>▪ Student &amp; teacher work toward goal &amp; measure progress</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Belief Structures

I explored two different approaches to examining adult cognition in this literature. One approach, illustrated by the work of Sewall (1984), Richardson (1994; 1995), and Richardson and King (1998), concentrated on the study skills and intellectual ability of students. In some cases there were comparisons between traditional aged students and older students, while other studies attempted to describe adult students more completely and, in some studies, compare observations with previous studies of younger students or students in general. The second approach looked at meaning-making as the cognitive activity of adults. The two recent papers that best illustrated this approach to connecting beliefs and learning were *Meaning-Making, Adult Learning and Development: A Model with Implications for Practice* by Merriam and Heuer (1996), and *Adult Meaning-Making in the Undergraduate Classroom* by Kasworm (1997, March). Kasworm's work was most helpful in my research.

Kasworm (1997, March) conducted a study of adult undergraduates to examine the influence of past and current life experiences on their learning, particularly with in the formal classroom environment. In exploring adult students' engagement with undergraduate learning, employing either a deep approach or a surface approach, Kasworm discerned five belief structures articulated by these students. She described these belief structures as *knowledge voices*: the *entry voice*, the *outside voice*, the *critical voice*, the *straddling voice* and the *inclusion voice*. Table 2.2 below organizes the voices according to the expressed *perspective* on learning, understanding of the relationship between *academic knowledge* and *real-world knowledge*, and perception of the *power structure* in a learning environment.
Table 2.2: Kasworm’s five belief structures or knowledge voices (1997, March)

<table>
<thead>
<tr>
<th>Knowledge Voice</th>
<th>Perspective</th>
<th>Academic &amp; Real-World Knowledge</th>
<th>Power structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Could not judge or make initial personal sense of classroom knowledge</td>
<td>Different worlds; uncertain in new and confusing academic world</td>
<td>Attempt to learn the ways of the culture. All-knowing instructor.</td>
</tr>
<tr>
<td>Outside</td>
<td>Anchored in real-world of work, family &amp; life</td>
<td>Perceived discrepancies. Frustrated in attempts to directly apply college experiences. Usually the knowledge learned beyond short term: to reinforce current knowledge, to further illuminate past personal knowledge, to validate knowledge expertise.</td>
<td>Academic game; faculty neutral or suspect because they are typically anchored in academic world.</td>
</tr>
<tr>
<td>Critical</td>
<td>Cynical involvement. Seeking credential</td>
<td>Perceived silliness in academics &amp; real knowledge in real-world. Dilemma between perceptions of personal competence and judgment by professors of their lesser abilities in the academic world.</td>
<td>Faculty-student game. External compliance without intrinsic engagement.</td>
</tr>
<tr>
<td>Straddling</td>
<td>Intersecting and connecting both academic and adult world knowledge meaning structures</td>
<td>Equally valuing two worlds of knowing and doing. Believed each world informed the other. Able to articulate value in the broad curriculum.</td>
<td>Saw faculty in quasi-peer relationship. Viewed themselves as on a learning journey. Appreciated opportunity for discussion, small-group projects, and flexibility of paper topics.</td>
</tr>
<tr>
<td>Inclusion</td>
<td>Self-directed lifelong learners; world of learning transformative &amp; generative</td>
<td>Actively sought immersion into the academic world &amp; knowledge. Spoke to building connections between the academic and other world settings</td>
<td>Spoke to building meaning bridges and to creating and generating new knowledge from connections of meaning and application. Spoke of their own meta-cognitive actions and cognitive activity.</td>
</tr>
</tbody>
</table>
Kasworm (1997, March) described the students with whom she worked in terms of these voices in the following way.

- The **Entry Voice** belief structure reflected adult students who could not make initial personal sense of classroom knowledge. For them, the collegiate classroom learning transaction was a new and confusing culture. They entered into courses with content areas which were unknown and emotionally charged. To these students, the two knowledge categories of academic knowledge and real world knowledge represented two different worlds with fundamentally different ways of knowing and understanding. Collegiate expert knowledge structures, language and inquiry were not part of their current life worlds. They perceived faculty as all knowing. These students viewed their own deficiencies of knowledge and skills as hurdles to overcome (p. 11 - 14).

- Students who represented the belief structure of the **Outside Voice** brought a strong set of beliefs and actions, which anchored them within their real world of work, family, and life. Often they viewed college as a necessary involvement for their future, and characterized it as a culturally unique place with only fragmentary connection to the world of adult life and work. They continually faced meaning discrepancies between their knowledge of the real world and what was presented in the academic knowledge world, and selectively made meaning of classroom knowledge. Because faculty are typically anchored in the academic world, several of these students viewed faculty as neutral or suspect. However, they were also mindful of the public form of respect for the faculty member and of the valuing of academic knowledge (p. 15 – 19).
Adult learners in the **Critical Voice** suggested that they entered the collegiate environment from a private cynical involvement. These learners attended college to get a credential as a ‘societal ticket’ to gain access to preferred jobs, as necessary validation of expertise for job promotion or job security, or to resolve social pressures in either work or family settings for a college degree. Academic learning was judged to be a faculty-student game. They believed valuable knowledge came from “real world experiences and action” (p. 19).

Adult learners with a **Straddling Voice** belief structure placed their beliefs and actions as intersecting and connecting both academic and adult world knowledge meaning structures. They viewed themselves as working across two knowledge structures and equally valuing those two worlds of knowing and doing. They actively attempted to make applications and connections between academic learning and adult world knowing. They spoke of bringing new understandings or actions of classroom learning into their family work and community lives. Some also utilized their knowledge and insights from their life roles within the classroom for elaboration and illumination of academic knowledge within the classroom (p. 20 – 21).

Adult learners in the **Inclusion Voice** were a unique breed of individuals who actively sought immersion into the academic world and academic knowledge. They suggested that through their academic learning they had begun to see a new world-view perspective of their adult life knowledge. They actively spoke to building meaning bridges and creating an integration of thought and action between their life-world outside the academy and their academic world of knowledge and understandings. In
essence, they acted upon their past and current knowledge and created new understandings and applications (p. 22 – 23).

Incorporating the Beliefs with the Voices

Like those in Baxter Magolda’s (1992) study, the students in Kasworm’s (1997, March) study certainly addressed the nature of knowledge and the roles of themselves as learners and the roles of instructors. However, more than Baxter Magolda, Kasworm focused on what she heard these students say about the “socio-political power between faculty, student, and knowledge in the learning transaction, and of socially negotiated understandings/actions upon knowledge between the world of undergraduate classroom and their adult life worlds” (p. 11).

A major difference in the two studies can be found in the settings. Kasworm’s (1997, March) work involved students who were currently enrolled at three different institutions of higher education. The students were at least 30 years of age; attempts were made to include a balanced representation according to gender, ethnicity/race, varied hours of enrollment (part-time to full-time), varied levels of academic status (freshman through senior) and varied academic majors, in addition to including students at varying commuting distance to the institution they attended. The population in Baxter Magolda’s work was a much more homogeneous group of traditional aged students at a large residential university. She followed these students through four years of college work and one year beyond and based her observations on the students’ journey through their college years. She claimed that they tended to move to varying degrees through the perspectives from primarily absolute, to transitional, to independent (to some extent), and (to a very limited degree) to contextual. Baxter Magolda first observed what she called
contextual knowing (the most advanced level) in the junior and senior years of college. The number of observations of contextual knowledge increased to 12% of the students in the fifth year. Although Kasworm followed the students in her study for two years, she reported neither movement from one voice to another, nor changes in what proportion of the students fell into each perspective. She focused on how the students engaged in learning and to what extent each student’s “voice” was an expression of the interplay between students’ perceptions of their academic and real worlds, and the relationship between those worlds.

Although there is considerable overlap between the issues addressed and the observations made by Kasworm (1997, March) and Baxter Magolda (1992), it seems to me that the differences are both dramatic and revealing. The students in Kasworm’s work were at a different place, and their “voices” spoke to that. In working with adult students in the classroom and in my pilot study, I have heard those voices. I have heard the entry voice of students eager to find the way and appreciative of any help in doing so, and the outside voices of students looking for validation of the years of experience and hard work and effort already expended in years outside the classroom. I have definitely heard the critical voices of students who are paying dues to a system that requires a degree in order to gain better pay or position. I have heard the straddling voices of students who are trying to make sense of the connections between the two worlds of their existence, and, occasionally, I have heard the inclusion voices of the students for whom education is a true transformative experience from the outset. In my pilot study, I struggled with the application of Baxter Magolda’s epistemological reflection model in my work with adult students in a very different setting from hers. Kasworm’s “voices,” which incorporate the
students’ experiences as a crucial dimension, focus more precisely on the adult students’ ways of making meaning in the college environment. I do not suggest that Kasworm’s approach of listening for the students’ voices is unique to her or uniquely applicable to adult students. However, I do suggest that her approach can make Baxter Magolda’s model speak more clearly for adult students.

For the purposes of my research, I followed the lead of Baxter Magolda's (1992) model by examining the students' beliefs about the roles of the student and the instructor and the nature of knowledge. I added to it the dimensions suggested by Kasworm's (1997, March) model by examining their perspectives on learning, their understanding of the relationship between academic knowledge and real-world knowledge, and their perception of the power structure in a learning environment. Moreover, I followed Kasworm's lead in listening for the knowledge voices that the participants in my study express, which may be different from those she heard, especially within the context of the study of mathematics. I was also interested in whether their belief structures or knowledge voices are different for mathematics than for learning in general in higher education.

Mathematical Thinking and Problem-Solving

From the previous section, it should be clear that adult students' willingness and ability to engage in higher level thinking needed to be the focus of this research. Hence the area of mathematical problem solving was the venue of this study.

Mathematical Thinking

I found Skemp's work quite helpful in framing my research and turn to it now to help me in describing the mathematical thinking of the students in my study. Skemp
(1979; 1987) developed a model for intelligence (see Figure 2.1) in *Intelligence, Learning and Action: A Foundation for Theory and Practice in Education* (1979) and explored his model in the realm of mathematical thinking in *The Psychology of Learning Mathematics: Expanded American Edition* (1987).

![Figure 2.1: Skemp's Model of Intelligence (1987, p. 165)](image)

The following definitions are helpful in discussing the model:

**Concept.** A mental entity which embodies certain regularities of experience, and may be changed in the light of further experience. (This may be experience of actuality, or mental experience.)

**Conceptual structure.** A set of connected concepts.

**Schema.** A conceptual structure.

**Delta-one.** A teachable director system whose operand is the physical environment.

**Delta-two.** A director system which has a delta-one as its operand.

**Director system.** That which directs the ways in which the energies available to the operators are applied so as to change the state of the operand from its present state to a goal state.
Teachable (of a director system). Which can be changed towards a state in which it functions better.

Learning. The construction and/or improvement of director systems within the lifetime of an individual.

From “Glossary” in *Intelligence, Learning and Action* (Skemp, 1979)

The roles of intelligence (i.e., thinking) include the construction of schemas . . . for all the different kinds of jobs that delta-1 does . . . [and] deriving from these schemas particular plans appropriate to different initial states and goal states. These plans can then form the basis of goal-directed action.

These are two of the goals of delta-two. The first is a learning goal. The second goal is a planning goal: that of finding an appropriate plan for the achievement of a particular purpose. If such a plan is hard to find, even given the requisite knowledge, we call this activity problem solving. (Skemp, 1987, p. 109)

Skemp (1987) emphasized that knowledge structures of this type cannot be communicated directly. "They can only be constructed by activity of a learner's own delta-2 operating on his own delta-1" (p. 109), an admittedly constructivist viewpoint. He outlined the way that schemas are constructed (built and tested) in Table 2.3. This outline provides guidance to educators interested in students' schema construction and to researchers interested in learning about students' schemas. The roles of experience, discourse and reflection in the learning process certainly are emphasized.
Table 2.3: **Schema Construction** (Skemp, 1987, p. 110)

<table>
<thead>
<tr>
<th>Schema Building</th>
<th>Mode</th>
<th>Schema Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>From our own encounters with the physical world: experience</td>
<td>Mode 1</td>
<td>Against expectations of physical event: experiment</td>
</tr>
<tr>
<td>From the schemas of others: communication</td>
<td>Mode 2</td>
<td>Comparison with the schemas of others: discussion</td>
</tr>
<tr>
<td>From within, by formation of higher order concepts; by extrapolation, imagination, intuition: creativity</td>
<td>Mode 3</td>
<td>Comparison with one's own existing knowledge and beliefs; internal consistency: reflection</td>
</tr>
</tbody>
</table>

The idea of a cognitive map “generalizes very nicely into the concept of a knowledge structure, for which another name is schema” (Skemp, 1987, p. 109). Thus, it should be clear that, although an educator/researcher may wish to know what schemas a student possesses, these cognitive structures cannot be observed directly. The educator/researcher may observe only the results of actions or operations, which are in turn the results of plans that the student has made. In the section on teaching sessions in the chapter on Methodology, I present in detail the sorts of observations I planned to make while these students were engaged with mathematical tasks. From those observations, I hoped to understand better the students' ways and means of thinking, their cognitive structures, with respect to mathematics.

Skemp (1979) also points out that teaching must be a collaborative act, in which the teacher attempts to cooperate with the student's delta-2 in constructing new schemas or improving the functioning of existing schemas in the student's delta-1 (pp. 249-252). In order to cooperate effectively the teacher must construct viable models of the student's
schemas, both delta-1 and delta-2 schemas. The teaching experiment methodology provides the appropriate situation for this construction and collaboration to take place.

**Problem-Solving**

The topic of problem-solving and how to teach it is of supreme importance in education, and particularly in mathematics education. The mere number of pertinent texts (e.g., Brown & Walter, 1990; Charles & Silver, 1988; McLeod & Adams, 1989; Schoenfeld, 1994; Silver, 1985) and sections in current handbooks (Mayer & Wittrock, 1996; Schoenfeld, 1992) testify to this.

According to the chapter on problem solving transfer in the *Handbook of Educational Psychology* (1996), problem solving is a goal-directed cognitive process. Problem solving is a personal process because "the individual knowledge and skills of the problem solver help determine the difficulty or ease with which obstacles to solutions can be overcome" (Mayer & Wittrock, 1996, p. 47). Problems may be classified as well-defined (i.e., the given state, goal state and allowable operators are made clear in the problem) or ill-defined; they may be classified as routine (i.e., the problem solver has a ready-made procedure for solving the problem) or non-routine. Academic problems tend to be more well-defined and routine, while difficult real-world problems are usually ill-defined and non-routine.

Problem solving can be analyzed in sub-processes, including representing, planning and executing. Representing occurs when a problem solver converts an externally presented problem . . . into an internal mental representation . . . Planning involves devising and monitoring a method for solving a problem, such as breaking a problem into parts. Executing occurs when a problem solver actually carries out planned
Although executing is emphasized in classroom instruction, the major difficulty for most problem solvers involves representing and planning. (Mayer & Wittrock, 1996, p. 48)

I have chosen to introduce problem-solving as defined in this general educational source before proceeding to consider problem-solving in a mathematical context in order to emphasize the general interest in this area and the overarching cognitive processes involved. Basic to my research is my claim that through adults' experiences they learn and apply problem solving skills in a variety of settings and that the recognition by both faculty and students of the applicability of those skills in mathematics settings and, in fact, in mathematics classrooms would enhance those adults' ability to solve mathematics problems.

In Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Schoenfeld, 1992), a chapter in the Handbook of Research on Mathematics Teaching and Learning (Grouws, 1992), Schoenfeld attempted to clarify terms like problem and problem solving. There has been an historical development of interpretations, and there is currently variance in perspectives within mathematics education regarding the meanings of these terms and their rightful role in mathematical thinking.

Lester (1983) referred to the “elusive and intractable nature of mathematical problem-solving” (p. 230). After acknowledging that problem and problem-solving are the most often defined terms in literature on cognition and thinking, he chose the following definition as the most appropriate when discussing mathematical problems.
A problem is a task for which:

1. the individual or group confronting it wants or needs to find a solution;
2. there is not a readily accessible procedure that guarantees or completely determines the solution; and
3. the individual or group must make an attempt to find a solution. (p. 231-232)

This definition includes the elements of interest or engagement on the part of the student; possible struggle, failure in attempts or perturbation; and effort different from an automatic response. The instructor must know a great deal about a student in order to introduce tasks that will be problems, in this sense, for a student. The definition is also reminiscent of Skemp’s (1987) observation that real learning (and problem-solving) takes place in the frontier zone where productive effort is both necessary and possible. Skemp himself referred to the essential feature of a problem situation as one in which one needs to “find a way to achieve a desired goal when one has no existing plan which would enable one to do so” (p. 170). He pointed out that although we may know we have found a way (i.e. solved the problem), we usually do not know how we have done so. That is, “we have no conceptual model of the problem-solving process itself; and . . . there is little in the literature to guide us” (p. 170).

The work of Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier and Wearne (1996) underscored the importance of investigating students’ approaches to problems and the effects of focusing their attention on the problem-solving process. Their article, Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics, made a particularly strong statement about the role of problems in the reform of mathematics curriculum and instruction. They propose that
students should be allowed to make the subject problematic. We argue that this single principle captures what is essential for instructional practice . . . [but] does not specify the curriculum nor prescribe instruction . . . . Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solution, and to resolve incongruities . . . . [S]tudents should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills. (p. 12)

Contrasting the strategy that they advocated with the traditional view that particular procedures can be used for solving particular problems, Hiebert et al. suggested that students learn “general approaches or ways of thought that are needed to construct the procedures” (p.17). They suggested that two kinds of “strategic residue” (p.17) result. One is that the students learn specific procedures for specific tasks. Second, and perhaps more important, “students learn how to construct strategies and how to adjust strategies to solve new kinds of problems” (p.17). This second “meta-strategic” residue calls to mind Skemp’s (1987) description of mathematical thinking on what he refers to as delta-one and delta-two levels. Simply put, at the delta-one level procedural thinking occurs as one uses one’s existing schemas to work on input from the environment. At the delta-two level, the schemas of delta-one are the input and one rearranges, combines existing schemas or invents new schemas to handle the problem at hand. The authors of Calculus: The Dynamics of Change (1996) referred to this as developing the heuristics of problem solving. Hiebert et al. took this interaction a step further by noting that when students develop methods for constructing new procedures they are integrating their conceptual knowledge with their procedural skill . . . . Students who treat the
development of procedures as problematic must rely on their conceptual understandings to drive their procedural advances. The two necessarily are linked. (p. 17)

Researchers interested in this strategy would ask whether allowing students to problematize their mathematical learning environment results in better mathematical understanding, that is conceptual understanding. Are the students prepared to approach unfamiliar problems and grapple with selecting appropriate methods or creating new ones in order to solve them? Does the procedural knowledge come along as a residue with this mode of instruction; are these students able, as claimed by Hiebert et al. (1996), to “perform just as well on routine tasks as their more traditionally taught peers” (p. 17)?

Affect and Mathematical Thinking

As Schoenfeld (1992) articulated it: "Once upon a time there was a sharply delineated distinction between the cognitive and affective domains. . . . As our vision gets clearer, however, the boundaries between those two domains become increasingly blurred" (p. 358). The topic certainly has gained the interest of the mathematics education field (e.g., Lester, Garofalo, & Kroll, 1989; McLeod, 1992; McLeod & Adams, 1989; Owens, Perry, Conroy, Geoghegan, & Howe, 1998).

The work of Kloosterman, some with his colleague Stage (Kloosterman, 1988; Kloosterman, 1995; Kloosterman & Stage, 1992; Stage & Kloosterman, 1995), also addressed the intertwining of affect and achievement in mathematics, sometimes specifically achievement in problem-solving. In fact, both the initial questionnaire that I used in my pilot study and the one I used in this research were based in part on some of this work (Kloosterman, 1995; Kloosterman & Stage, 1992).
In their examinations of how students learn and their ways of knowing, both Baxter Magolda (1992) and Kasworm (1997, March) incorporated the students' beliefs in their epistemological reflection model and descriptions of knowledge voices, respectively. In fact, the combined models of Baxter Magolda and Kasworm assisted me in making the connection between the affective and cognitive domains of my participants. I presented Baxter Magolda’s epistemological reflection model and Kasworm’s knowledge voices in the earlier section on affective orientation.

Another important bridge between the affective and cognitive domains is made in the work of Skemp (1979; 1987). Skemp (1979) declared that emotions “play an essential part in human experience, which experimental psychology has had little success in bringing into a satisfactory theoretical framework” (p. 11). In fact he stated that he believed that "the separation of cognitive from affective processes is an artificial one" (p. 189). He explored thoroughly the relation of emotions to action and to learning.

The first four categories of emotion (pleasure, unpleasure, fear, and relief) relate to situations that have actually arisen, and call our attention to the need to do something. The next four (confidence, frustration, security, and anxiety) relate to whether or not we are in fact able to do whatever is necessary to bring about the goal state, and to prevent the anti-goal state. They signal our own competence relative to the situation. (Skemp, 1987, p. 193, emphasis added) The emotions that act as signals indicating that we are moving toward or away from a desired goal state are depicted in Figure 2.2 below, and emotions that signal knowledge of ability or inability to change toward or away from the desired goal state are depicted in Figure 2.3 below.
As Skemp observes, "it is in the nature of learning that learning takes place in regions where we are not yet competent." If we define our **domain** as a region in which we can achieve our goals and avoid our anti-goals, this would be our region of competence in which we feel confident and secure. Outside of this region, we do not feel competent; rather we are apt to get signals of frustration and anxiety. The boundary between the inside and outside of this domain is not likely to be clear and sharp. In this fuzzy boundary area we are apt to experience mixed emotions resulting from varying
degrees of ability to achieve our desired goal states and inability to avoid our anti-goal states. This frontier zone (Skemp, 1987) was discussed in the section on learning zones in the theoretical background. (See Figure 2.4)

![Learning - Frontier Zone](skemp_1987_p.195)

The focus of this study on re-entry students’ learning in the context of mathematical problem solving necessitates a probing of the students’ frontier zones. Such probing requires the researcher to be intimately involved with the students’ problem solving activity, thus a research methodology must be chosen or designed to permit such involvement.
CHAPTER 3
METHODOLOGY

Choice of Methodology

It goes without saying that the research method should be selected carefully in light of the research questions and the theoretical perspective of the study. Skemp expressed this particularly well considering the context of my research in mathematical problem solving.

By method I mean what a researcher does, his plan of action; by methodology I mean the more general body of knowledge or beliefs from which he derives a particular method and by which he can justify it. A person who uses a method unrelated to a methodology is thus in somewhat the same position as a pupil who uses an algorithm in mathematics without having the underlying mathematical conceptual structures from which the algorithm is derived, and by which it can be understood as a correct procedure. (Skemp, 1981, p. 3-4)

In much of the literature that I reviewed, particularly on adult learning and development, there was considerable attention invested in the choice of methodology. Although large quantitative studies have, in some cases, highlighted areas of interest and provided provocative questions to be asked about the phenomenon of adults in higher education, qualitative methods seem to provide preferable ways to explore those areas and seek possible answers to the questions.
Much that I wanted to know about the students in my research could be asked in the form of a survey. In fact, I sought initial information by administering a questionnaire. However, my experience with my pilot study reinforced the observation that I made in the last paragraph. That is, the information from such a survey can at best provide trends and starting places. Although the surveys included open-ended items to which the participants could respond freely in their own words, as I had observed in my pilot study, students often do not invest the time and effort required to answer in written form as fully as they do orally in the interview setting. The data gathered from the questionnaires seemed to help refine the questions and direction of conversation more than they provided definitive answers. Therefore, I followed-up the questionnaires with diagnostic interviews.

In order to investigate the students' approaches to mathematical problem-solving, I also engaged them in mathematical tasks. I was not interested in how the students performed before and after a uniform treatment. I was interested in teasing out the problem-solving skills they bring with them from their experiences and assisting them in recognizing what those might be. In addition to hearing what they were thinking and doing as they approached these problems, I wanted to know what assistance they required if they are having difficulties. If they stumbled, I wanted to try to identify where in the problem-solving process there were barriers for them. I wanted to see how their beliefs about mathematics and themselves as mathematics learners played out in their approach to these problems. As Skemp (1981) observed, "behaviorist models . . . have been remarkable [sic] unsuccessful in explaining, predicting, or controlling the higher forms of learning, in which man most differs from the laboratory rat and pigeon, and of which
mathematics is a particularly clear example" (p. 6). Hence, I chose to employ a modified teaching experiment to better fit my explorations.

I would like to recount two incidents that are illustrative of my affinity for employing a constructivist teaching experiment in this research. The first time that I formally encountered constructivism (in the person of a professor in my doctoral program), I was asked what I needed to know in order to teach my students mathematics. With no knowledge about this perspective, I responded that I needed to know what they knew. This began a dialog about how I might characterize that knowledge and how I might go about learning it. I had often mused about how little I knew about what my students knew based on entrance tests, and how little I knew about how to find out not only what they knew (i.e. content) but also how they knew it (i.e. thought processes). As I began this doctoral research to investigate just those sorts of questions, it became clear that a constructivist teaching experiment was the appropriate path to follow in this quest.

The second incident was more recent and applies directly to this research. My role as a researcher versus my usual role as a teacher became an ethical dilemma for me in my pilot study. That study was based on diagnostic interviews that included talk-alouds during which I had been directed not to respond to the students’ comments and questions. While the students were working on the mathematical tasks, they frequently looked to me for guidance. When I considered the participants' commitment of time to the project, it seemed inappropriate or, in fact, unethical to leave them with unchallenged misconceptions. Indeed, after the students had taken things as far as they could and we considered the session over, the ensuing interchange between us often provided additional insight into their ways of knowing. This further reinforced my observation that
appropriate discourse and the necessity for the students to explain their thinking and to justify their actions resulted in improved performances, a powerful implication for teaching. Such discourse is quite appropriate in the setting of a teaching experiment.

Skemp (1981) addressed this issue in his report to the National Institute for Education, *Theories and Methodologies*. He indicated that "one of the features of the diagnostic interview is the care taken by the experimenter not to teach" (1981, p. 17). He also noted that in the process of only asking questions, one could be teaching anyway because "even when teaching is not intended, questioning can have the effect of initiating lines of thinking which might not have happened if the questions had not been put" (1981, p. 17). This observation confounded further my hesitation about relying on the diagnostic interview alone for my research. I was not comfortable with removing all teaching from the sessions as required for a diagnostic interview, and I would be concerned that in the asking of questions in those interviews I would be inadvertently teaching anyway.

Then I was introduced to the methodology of the teaching experiment (Steffe & Thompson, 2000). The opportunity to investigate the students' ways of knowing and thinking using appropriate discourse appealed to me, and seemed to address my dilemma. My reading of Skemp (1981) reinforced this line of thinking. In his comment about a study conducted by Resnick in 1979, Skemp wrote:

It included a successful piece of remedial teaching. I would here like personally to endorse the professional ethic expressed by the experimenter, that when children who are helping us by taking part in our experiments themselves need help which it is
appropriate and practically possible for us to give, then we own [sic] it to them to take
the time to give it. (Skemp, 1981, p. 22)

The teaching experiment is explained more fully below. I have attempted here to clarify
my own predisposition to employ this methodology in this research.

The Teaching Experiment

The teaching experiment as a research methodology has its origins in the work of
Vygotsky in the 1920’s (Rachlin, Matsumoto, & Wada, 1987) and other researchers in
the then Union of the Soviet Socialist Republics (Steffe & Thompson, 2000). The
methodology makes “visible processes that are ordinarily hidden beneath the surface of
habitual behavior. . . . [It] was designed to ‘catch’ processes in their
development” (Rachlin et al., 1987, p. 21). “A primary purpose for using teaching
experiment methodology is for the researchers to experience, firsthand, the students’
mathematical learning and reasoning” (Steffe & Thompson, 2000, p. 267). The teaching
experiment emphasizes process and asks these central questions: “What are the [students]
doing? How are they trying to satisfy task demands?” (Rachlin et al., 1987, p. 22).

“The label ‘teaching experiment’ is actually a generic term for a variety of
research forms that attempt to reach [their] goal through the subjective analysis of
qualitative data on a regular basis” (Rachlin et al., 1987, p. 22). Researchers have
employed the teaching experiment in a variety of settings with a variety of adaptations.
For example, Rachlin, Matsumoto and Wada (1987) employed a teaching experiment to
explored how 2 sixteen-year-olds were enabled to capitalize on errors to stimulate and
support mathematical inquiry. Simon (1995) utilized a whole-class teaching experiment
with prospective elementary teachers to develop a model of teacher decision-making with respect to mathematical tasks. Tzur (1999) studied fourth graders construction of improper fractions and the teacher’s role in promoting that learning.

All of these researchers supported the methodology’s compatibility with a constructivist perspective toward the teaching and learning of mathematics. Steffe and Thompson claimed that “the teaching experiment was designed for the purpose of eliminating the separation between the practice of research and the practice of teaching” (Steffe & Thompson, 2000, p. 305), and it was evident in each of the studies that this purpose was, to a significant extent, fulfilled. Moreover, all demonstrated agreement with the above general characteristics and included the following elements in their research designs.

A teaching experiment is an experiment in the sense that hypotheses are generated and tested in a recursive cycle (Steffe & Thompson, 2000). These hypotheses are based on the students’ actions and intend to model the students’ schemas. They are tested in subsequent interactions either within the same teaching session in which they are generated, or, perhaps more often, within subsequent teaching sessions. Plans for teaching sessions are based on observations made in prior sessions. The design is more flexible than it is fixed. The researcher is engaged in model building; that is, building and testing models of the students’ mathematics.

A teaching experiment involves teaching on several levels (Steffe & Thompson, 2000). Certainly, there is something the students are to learn, but also, and perhaps more importantly, the researcher/teacher is also to learn. He or she is to learn from the students their ways and means of operating. In the present research, the students may have learned
to approach the mathematical tasks more effectively. They may have learned to identify the mathematical problem-solving abilities they possess. As the researcher, I hoped to construct schemas about the way they operate in the situations that are presented (Skemp, 1979; Skemp, 1987).

Steffe and Thompson (2000) recommended including two other ingredients in a teaching experiment: the engagement of the teacher-researcher in exploratory teaching prior to conducting a teaching experiment and the involvement of an observer in addition to the teacher-researcher. Prior teaching enhances the possibility of establishing communication with the students. An observer can assist the teacher-researcher in reflecting on interactions when she finds it difficult to “step out of the interaction, reflect on what is happening, and then take action on that basis. This is very difficult because the teacher-researcher would ‘be’ in two places in a very short time” (p. 19). Video taping each of the sessions and viewing them as soon after the interactions as possible actually assisted not only in analyzing the data but also in allowing the teacher-researcher to be her own observer.

Rachlin, Matsumoto and Wada (1987) listed the following general characteristics of a teaching experiment; most of the above ingredients are included in these characteristics.

- Subjective analysis of qualitative data obtained in a clinical-interaction setting by recording students’ verbal and written statements over an extended period of time.
- Planning instruction in light of observations made during the previous session.
- Use of small samples with probing interviews and exchanges with individual students. (p. 22)
Borasi (1994) remarked that she was motivated to focus on a teaching experiment for two major reasons:

The greater freedom from curriculum constraints . . . enabled [her] to create a learning situation fully reflecting an inquiry approach . . . and the small number of students involved made it possible to carefully monitor the experience and its learning outcomes. (p. 173)

Rachlin, Matsumoto and Wada (1987) stated that the goal of a teaching experiment is to “provide a dynamic research base for the modification of instruction and/or curriculum” (p. 22). They went on to observe that secondary mathematics curriculum and instruction have tended to be designed by knowledgeable and successful adults under the assumption that all students think the same way. Further, these adults tend to assume that the common way that students think parallels their own way. (p. 28)

This observation applies as well to college curriculum even for students who have not experienced success in the past. The teaching experiment in this study was designed to begin to uncover the ways and means of thinking of these students as a first step toward designing curriculum and instruction more appropriate for them.

Design of this Study

Each of the parts of my teaching experiment - the selection of the participants, the questionnaires, the diagnostic interviews, classroom observations and the teaching sessions - are discussed in more detail below.
Selection of Participants

Although there are a variety of definitions for this population, as stated in the Overview of the Study, the most common and useful one for my purposes was students who were at least 25 years of age when they first enrolled in higher education or enrolled in higher education after not having been enrolled in a formal educational program for at least 5 years.

I engaged the cooperation of the faculty and administration of the Department of Academic Assistance in order to involve as many adult re-entry students as possible initially. I gained entry to seven classes, two of each of three of the instructors’ classes and one of another. A total of seventy traditional students and nineteen re-entry students completed the Initial Questionnaires, which are described fully below as are the Initial and Exit Interviews, Teaching Sessions and Classroom Observations. Six of the re-entry students agreed to schedule an initial interview, before which Participant Consent Forms needed to be filed. One of these students terminated her participation before the initial interview, and one completed the initial interview, but chose to terminate her participation at that point. A third student completed the initial interview and one teaching session before dropping out of the study. In all three cases, the students stated that committing time to a study that did not have an obvious link to their current course work was too much of a burden, given the many demands on their schedules. The remaining three students each completed the initial interview, five individual teaching sessions and an exit interview. In addition, two of the students completed two Joint teaching sessions. I made two classroom observations in each of the two classes that the three students attended.
Thus, three participants – Jennifer, Hillary and Peggy - were involved in the entire study. In addition to completing an initial questionnaire, each of them participated in an initial interview, five individual teaching sessions and an exit interview. In addition, Hillary and Peggy participated together in two joint teaching sessions. I observed in each of their classes twice during the semester. Each of the elements of the study – the questionnaire, interviews, teaching sessions and classroom visitations are explained in the following sections.

**Initial Questionnaires**

All eighty-nine students in the seven classes I visited completed an initial questionnaire (see Appendix) similar to the questionnaire used in my pilot study. This questionnaire gathered demographic data that I used to determine prospective participants who were adult re-entry student as I have defined above. It also began the process of investigating the mathematical background of the students, their in-school and out-of-school experiences, and their motivations for returning to a formal educational setting and for learning mathematics, as well as their stated beliefs about many aspects of the study of mathematics. The initial questionnaire is included in the Appendix.

The form and function of the questionnaires were similar to those described for the questionnaires in my pilot study. The initial questionnaires included requests for demographic information, the 5 beliefs scales used by Kloosterman and Stage (Kloosterman & Stage, 1992; Stage & Kloosterman, 1995), the 6-item Fennema-Sherman (1976) usefulness scale, the six items gleaned from Cooney’s (1992) study of inservice teachers, additional Likert-type items and open-ended items, all of which explore beliefs, experiences and motivation. I was interested in comparing the responses of the adult
participants in my research with the responses reported by Kloosterman and Stage (1992; 1995) in their study of traditional age college students. The responses to all the items, both Likert-type and open-ended questions provided a springboard for discussion in the initial interview. In that sense, the responses assisted me in beginning to answer my research questions regarding the students’ motivation both to reenter education and to learn mathematics, and their willingness to engage in mathematical problem-solving. The students began to establish their knowledge voices (Kasworm, 1997, March) and their epistemological perspectives (Baxter Magolda, 1992). This information was helpful in viewing their actions within the teaching sessions against the backdrop of their stated beliefs.

Interviews

Initial Interviews. These semi-structured interviews were designed to follow-up on the initial questionnaires. They also provided an opportunity to explore some of the students’ beliefs with regard to the nature of mathematics, the role of teachers and students in learning mathematics, their perspectives with regard to the relationship between the academic and the real-world, and the factors at work in favor of their re-entry into formal education. Of interest were the similarities and differences observed in their beliefs and perspectives with regard to learning in general as compared to their beliefs and perspectives with regard to learning of mathematics. These issues cannot be fully discussed in one interview, so the conversations continued as part of the teaching sessions to follow.

In my pilot study, the interviews were audio taped. In this research, I videotaped all sessions with each student. In addition to providing a richer record of behavior and
affect, this permitted other observers to have a clearer picture of what occurred in each
session. I was fortunate to be able to involve two graduate students who each acted as a
second observer for each participant. When the graduate students’ schedules did not
permit them to attend sessions with the students, the videotapes were a useful way for
them to make their observations. Most importantly, the videotapes provided an excellent
way for me to review the students’ affect along with the verbal interchange available on
the audiotapes. They provided an incredibly rich record of what occurred during the
interviews and teaching sessions.

**Exit Interviews.** An exit interview was conducted with each participant who
completed this study. The items from the initial questionnaire were modified slightly for
this purpose. For example, the items requesting demographic information were removed
and a question asking for the students’ reactions to participating in the research was
inserted. These follow-up interviews also included items intended to encourage the
participants to reflect on their experiences in the research and to compare them to their
usual classroom experiences. It was important to note whether the students perceived any
difference in the mathematical thinking that they are called upon to employ or revealed
any difference in attitude toward the mathematics in the different settings.

**Teaching Sessions**

Each teaching session was approximately 60 minutes long. The individual
teaching sessions were held in a small office. The students sat behind a large, flat top
desk supplied with paper, pens, pencils, colored pencils and a calculator. I sat on the
opposite side of the desk. Both an audiotape recorder and a video camera were also setup
in the room. Although some noise could be heard at times from the main hall, the office was off a side hall in which there was very little traffic.

The joint teaching sessions were also approximately 60 minutes long. They were held in a departmental conference room in which the participants and I sat opposite a large table. They were seated adjacent to each other to encourage collaboration. Again, the students were provided with various supplies and both audio- and videotape recorders were set up in the room. The door was closed, the room was reserved for this purpose, and there was minimal hall noise, which did not present any apparent distraction.

The students were presented with tasks and encouraged to approach them in any manner they chose and to talk about what they were thinking and doing. The tasks for the first session were selected with the students’ experiences in mind and in an effort to begin to establish where each of their frontier zones (Skemp, 1987) might be; that is, where they would be most likely to learn. Tasks were selected for each of the following sessions in order to continue that process and also in response to behaviors that were demonstrated. A guide for investigation by using tasks and a description of the actions and operations that students might use during problem-solving are presented below. Specific descriptions of segments of the teaching sessions are included in the analyses that follow in later chapters.

Relying significantly on the work of Skemp (1987), I constructed Figure 3.1 as a guide for investigation during the presentation of problems to students.
Figure 3.1: *Investigation Initiated by Task Presentation and Guided by Students' Plans and Actions*

**Routine problem.** A task was considered routine for the student if, when it was presented, the student recognized it as familiar, assimilated it into an existing conceptual structure (or schema), which was appropriate (in his/her view) for it, performed an action, and completed the task. For this student, the task was apparently a routine problem. If the schema into which the student assimilated the problem was indeed appropriate, whether it was based on typical school-taught mathematics concepts or not, the student’s work still may have resulted in an incorrect response. That is, the student may have experienced procedural difficulty (difficulty within the chosen plans or actions). We could term this sort of difficulty as occurring within the execution phase of problem-solving. It was of
interest to observe whether the student recognized his/her error and whether this error provided a perturbation for further work on the task.

A different situation transpired if the student inappropriately conceptualized the problem, that is, if she assimilated the task into a schema that was inappropriate. Then, whether or not the student executed the resulting actions (procedures) correctly, this situation indicated difficulty with representation and planning, and it definitely was pertinent to my research. That is, the student was having difficulty in selecting or, perhaps, constructing an appropriate schema. It was of interest to observe whether the student discovered this error and how she handled it.

**Non-routine problem.** When a task was presented, if the student did not recognize the task as familiar; that is, she was uncertain of an appropriate schema in which to assimilate the problem, then, for this student, this problem was non-routine. It is important to note that the determination of whether the problem is routine or non-routine is based not only on the problem itself. For example, a “work” problem may be routine for one student and non-routine for another. The student’s prior experience and knowledge base are factors that are as important as the level of thinking in which the student is willing and able to be engaged.

In the case in which the problem was non-routine, the student sometimes ceased working on the problem (i.e., gave up). The student’s reasons for giving up were of great interest. I attempted to discern whether the reasons were based in the student’s beliefs about mathematics and about herself as a mathematics learner. In some cases, the student seemed to be unwilling or unable to examine her schema to determine if modifying a schema, combining schemas, or constructing a new schema would permit the problem to
be addressed (i.e., engage in delta-2 level thinking) (Skemp, 1979). At times the student did not at first seem to know how to approach a task (i.e., could not conceptualize it in such a way that it was assimilated into an existing schema for the student). In that case, I attempted to find out whether that student could operate in delta-2 thinking in such a way as to figure out how to approach the task (i.e., find an appropriate schema or modify one, or construct a new one perhaps by combining existing schemas). That is, I attempted to discern whether the student brought with her ways of thinking that could be brought to bear on the situation.

The resulting action or operations employed by the student were of interest. Attempts were made to tease out the origin of those actions and operations in order to assist the student in recognizing those that may be based in out-of-school experiences. Once the student conceptualized the task (selected or constructed a schema) and then performed an operation or took an action, the student often found that further work needed be done and the process began again (indicated by the dashed arrow in Figure 3.1). The investigation proceeded along the same path as a new task. Since a task could often be broken into parts or might have lead a student through several steps, this process proceeded through the above figure on a variety of paths.

**Actions and operations.** Along similar lines to work by Schoenfeld (1992), I proposed a list of actions and operations which students might employ as they sought appropriate ways to address their problems (see Table 3.1). I anticipated that my observations of the students at work might reveal different or additional actions and operations.
Table 3.1: **Students’ actions and operations during problem solving**

<table>
<thead>
<tr>
<th>Conceptualizing the problem; mental and physical representation, and planning</th>
</tr>
</thead>
<tbody>
<tr>
<td>• the big picture; what the problem is asking; what kind of problem it is</td>
</tr>
<tr>
<td>• necessary information and/or data; determining what’s there and how it can be used;</td>
</tr>
<tr>
<td>• what may be extraneous and what may be missing; figuring out how to find what may be missing</td>
</tr>
<tr>
<td>• order of events; whether some depend on others; considering alternative paths</td>
</tr>
<tr>
<td>• relationship between pieces of information and/or data</td>
</tr>
<tr>
<td>• putting it all together; sketching;</td>
</tr>
<tr>
<td>• writing equations or planning other heuristic approaches</td>
</tr>
<tr>
<td><strong>Solving the problem: executing</strong></td>
</tr>
<tr>
<td>• the small picture; the details; the procedures</td>
</tr>
<tr>
<td>• recognizing possible errors; attempting to correct them</td>
</tr>
<tr>
<td>• checking the result with respect to the problem posed; confirming whether the question was answered or the problem was solved</td>
</tr>
</tbody>
</table>

Classroom instruction tends to focus on the second to the last line – the small picture; the details; the procedures. This teaching experiment focused on the rest of the list and the investigations indicated above. As the students encountered tasks, their ways of operating were noted and plans for further sessions based on those observations.

I also invited the students to pose problems themselves (Brown & Walter, 1990). Not only do we all learn more when we ourselves are seeking the information, but also posing appropriate questions inevitably requires an even deeper understanding of the topic than answering questions about it. The questions posed by the students could have been revealing and assisted me in constructing schemas of their learning. However, only one participant brought in a question of her own construction.
Classroom Observations

Much that I reviewed in the literature on adult re-entry students claimed that the classroom setting is crucial to the students’ learning in higher education. In particular, the Model of College Outcomes for Adults presented by Donaldson et al. (1999, April) gave the connecting classroom a central location. This model provides a valuable tool for viewing adults’ experiences in higher education. Although this research primarily is focused on the individual student, observations of the students in their classroom setting added a necessary dimension to the data. I obtained permission from the instructors and the participants in order to make classroom observations. During the observations, I recorded written notes, which I annotated following the observation to make them useful artifacts.

Two of the students attended the same class and the third attended a different class; the same instructor taught both classes. I observed in each class twice, remaining in the classroom for the entire period. The first visit was near the beginning of the semester and the second was shortly after Midterm. In addition, the students would frequently mention to me what topics they were addressing in the classroom or were to be on a test they were preparing to take.

Strengths and Limitations of the Methodology

Throughout the previous sections, I have indicated several reasons for my choice of methodology. For example, I included questionnaires, diagnostic interviews, and classroom observations, as well as teaching sessions in order to have a variety of sources of data and enhance the possibility of gathering both confirming and disconfirming evidence for my claims. I mentioned the advantage of following up the initial
questionnaires with interviews in order to enrich the written responses of the participants.

I explained the development of a teaching experiment after my experience with my pilot study, which I referred to as an ethical dilemma between my role as a researcher and my usual role as an instructor. I also indicated that the students' actions while engaged in mathematical problem-solving should illuminate their belief structures beyond statements they may make in response to questionnaires or interview questions. Most importantly, I emphasized that, in order to learn about the students' ways and means of approaching the study of mathematics, it is necessary to engage them in mathematical activity that is "at the boundaries of the students' knowledge" (Steffe & Thompson, 2000, p. 300) in what Skemp (1979; 1987) referred to as their frontier zones. The teaching experiment methodology was designed for these purposes. In the following sections, I will discuss some other issues that may be perceived as sources of weakness in teaching experiment methodology, such as my experience as an educator, logistics, and research issues such as viability and generalizability.

Effect of My Teaching Experience

If the experience of an educator/researcher is employed correctly, it will enhance the effectiveness of a teaching experiment conducted by that educator/researcher. In their manuscript detailing the Teaching Experiment Methodology: Underlying Principles and Essential Elements, Steffe and Thompson (2000) strongly recommend that any researcher first engage in exploratory teaching in order to become “thoroughly acquainted, at an experiential level, with students' ways and means of operating” (p. 275), before conducting a teaching experiment. They also insisted that the researcher/teacher must
"put his or her own concepts and operations 'on the side,' as it were, and not insist that the students learn what he or she knows" (p. 275).

The purpose of a teaching experiment is to learn about the students' conceptual structures. Turning back to Skemp's (1979, 1987) description of schema building and testing (see Table 2.3, p. 32) we see that it is in the purview of the educator/researcher to provide experiences, discourse and opportunities for reflection in which the students can construct schemas, and her experience will assist her in performing those tasks. For example, Steffe and Thompson (2000) claimed that the teacher/researcher gains confidence that communication is established when she recognizes mathematical language and actions of current students in an interaction as similar to those she has heard before. Of course, they also cautioned to be alert to unusual language and actions, to be ready for the inevitable surprise that can be so revealing. Recognizing teaching as a human endeavor, they also asserted, "In their attempts to learn students' mathematics, the researchers create situations and ways of interacting with students that encourage the students to modify their current thinking" (pp. 287-8). The experienced teacher is more apt to be familiar with the status quo of the students in these areas. The experience of the researcher/educator also assists her in recognizing the frontier zone (Skemp, 1979; 1987) for each of the students, and it is in the frontier zone that the students' learning takes place.

Much has been written (e.g., Shulman, 1986) about the wider range of both pedagogical knowledge and pedagogical content knowledge that accompanies experience for a teacher. During a teaching experiment, it is necessary both to recognize alternative ways of thinking about the tasks at hand and to present alternative questions or tasks to
assist the student when he or she is experiencing difficulty. The experienced educator should have a wider repertoire from which to draw in the setting of a teaching experiment.

Thus, I claim that my experience as an educator with adult students at the entry level of mathematics strengthened my ability to conduct a teaching experiment. I used my experience to determine the students’ frontier zones and, by working within these zones, helped to bring forth in them their “knowledge voices” and their ways of approaching mathematics. My role as a teacher in the experiment determined how the students' mathematical thinking was brought forth but could not determine the nature of that thinking. Thus, my effectiveness as a teacher in the teaching experiment influenced how well I addressed my research questions but did not determine the answers to those questions. The involvement of observers, as discussed in the sections on teaching sessions and on logistics, reinforces this claim.

**Logistics**

The planning and implementation of the teaching sessions is the greatest challenge of the teaching experiment. Certainly, technical support is a central ingredient. Each session must be video taped so that the data – the students’ and the researchers’ activity and discourse – can be reviewed. This imposes significant logistical difficulties on the process.

Someone else besides the researcher should observe each session. I was most fortunate to be able to arrange for two of my fellow doctoral students to serve as observers. The observers each attended and/or reviewed the tapes of teaching sessions and reviewed other artifacts (e.g., responses to questionnaires, transcripts of interviews,
transcripts of classroom observations, written work during teaching sessions) for specific participants. That is, each participant was assigned to an observer so that the observer could participate fully in the analysis of the data concerning that participant.

As Skemp (1981) pointed out, "A salient characteristic of this methodology is that it takes up a great deal of the experimenter's time, and the data thus derived come from a relatively small number of children" (p. 20). A major challenge in conducting a teaching experiment is the time that must be devoted to it not only by the researcher but also by the participants. Although I could devote more time to teaching sessions, my experience with the students in conducting my pilot study lead me to believe that their schedules would be a constraint; experience confirmed that belief. This constraint was one of the reasons I incorporated the questionnaires, interviews and classroom observations into the methodology.

**Viability, Generalizability and Objectivity (or Subjectivity)**

The business of a teaching experiment is the constructing of a model of the students' ways and means of thinking. As has been noted, there is no way to actually observe the workings of the students' minds. Researchers rely on other manifestations, hypothesize about what a model might be and then interact with the students, thereby collecting data with which to examine the model. The researcher then adjusts the model and makes more hypotheses and continues in a recursive cycle. Steffe and Thompson (2000) explained that "Thus, a model is viable as long as it remains adequate to explain students' independent contributions. But no amount of fit can turn a model into a description of what may be going on. It remains an interpretation that seems viable from a particular perspective" (p. 302).
Steffe and Thompson (2000, p. 304) explained that generalizability is to be considered differently in a teaching experiment. The models or conceptual schemas that a researcher develops are explanatory and dynamic concepts to be used and possibly modified in further interactions with students. The principle aspects of generalizability included:

1) The results are useful in "interpreting the mathematical activity of students other than those in the original teaching experiment . . . [and] in organizing and guiding our experience of students doing mathematics" (p. 304).

2) We can "reorganize our previous ways of thinking in a new teaching experiment, that is, if we can learn, aspects of the old model become involved in new relations in the new model and, thus, become generalized conceptually" (p. 304).

3) We can "communicate with other researchers doing teaching experiments independently of us" (p. 304).

4) "The element of generalization that is involved is strengthened if that other researcher launched his or her teaching experiment for the purpose of constructing a superseding model of our current model of students' mathematics." (p. 304).

5) In some cases, sampling procedures and interviewing of groups of students of the same age as those in the teaching experiment may help locate "disconfirmations of the aspects of the model with an eye toward building a superseding model in a future teaching experiment" (p. 305).

If the results of my research inform my conceptual models of other students as I return to the classroom, if I endeavor to engage in further teaching experiments based on what I have learned in this one, if I communicate my conclusions to my colleagues through such
avenues as publications and presentations, and if I continue to interview other students to look for confirming and disconfirming evidence relative to my findings, then I will have complied with a majority of the principles put forth by Steffe and Thompson. I can only hope to encourage others to engage in similar activity to comply with them totally.

In the world of qualitative research methodology (e.g., Denzin & Lincoln, 1994) the emphasis is on making clear the subjectivity of the researcher in lieu of making claims of objectivity. Indeed, the claim is that there is inevitable bias and subjectivity even within the selection of the research questions to be explored in quantitative research. Here, I make clear my personal bias in this teaching experiment. My experience as an instructor and as a doctoral student told me that there was something amiss in the typical mathematics classroom experiences of adult re-entry students. The literature and theory that I reviewed in these three chapters guided me to ask the questions in this research and to pursue the answers in the methodology I presented. My bias was that there was something to be learned and that this was the best way to learn it.

In the following chapters I will present and analyze the results from this study. In Chapter 4, I will focus primarily on the results gleaned from the initial questionnaires regarding all the students who completed them. I will also present findings from further exploration of the topics presented in the questionnaires in light of the responses of the three full participants during the exit interviews. In Chapter 5, the analyses will include the previous results plus the findings from the teaching sessions held with the participants.
CHAPTER 4

RESULTS FROM QUESTIONNAIRES AND INTERVIEWS

Several types of results were gleaned from this research. The initial questionnaires (see Appendix A) were administered to all the students present in seven classes of 4 instructors in the department charged with providing remedial work for potentially at risk students at a major state university. The responses to the Likert-scale items were examined in a variety of ways discussed below, including a comparison to results from previous research upon which five of the ten items were modeled. The responses to the open-ended items on the initial questionnaires assisted in characterizing the views of the re-entry students who were potential participants in the teaching experiment conducted for this research. The initial and exit interviews (see Protocols in Appendix B) enriched the results from these items for those students who participated beyond the initial questionnaire. These analyses are also discussed below.

The results from the initial questionnaire and the initial and exit interviews were further enriched by the findings from the teaching sessions. These results are explored at length in Chapter 5 in the case studies of the three participants, Jennifer, Hillary and Peggy as well as the case study of the collaboration between Hillary and Peggy. The Research Questions are specifically addressed in Chapter 6 through a synthesis of all of these results.
Demographic Information

The students who completed the initial questionnaire were asked to provide demographic information about themselves. Eighty-nine students in 7 different classes taught by four different instructors completed the questionnaires. Seventy of the students were traditional students, based on the information they reported about their ages and years between high school and entrance into higher education.

Of the nineteen re-entry students completing the initial questionnaires, fourteen were female and five were male. Two females and one male did not indicate their race. Ten females and four males identified themselves as white, while two females identified themselves as African-American or black.

All nineteen re-entry students reported their ages on the questionnaires. Their ages ranged from 25 to 52 years, with the mode being 37 (three students); the median age was also 37, while the mean age of the 19 students was 36.8 years.

One 28-year-old black female completed a consent form but decided her schedule would not permit her to participate in the study. One 36 year-old black female completed a consent form and participated in an initial interview, but withdrew from classes and could not continue her participation. One 46-year-old white female completed a consent form and participated in an initial interview and one teaching session before deciding that her schedule would not permit her to continue participating in the research.

The three re-entry students who participated fully in the research were all white females. Using their pseudonyms to identify them: Peggy was 35-years old, Jennifer was 36 and Hillary was 37.
Responses to Likert-Scale Items

The First 5 Scales Compared to Responses in Study by Kloosterman & Stage (1992)

The first five belief scales in the initial questionnaires were the same as those used in the study that Kloosterman and Stage conducted in 1992 involving 517 traditional-aged students enrolled in a large state university. In the tables below, the means of those scales of the students in their study are compared to the means of the traditional students and then of the re-entry students who completed the initial questionnaires in my study. On each of these 5 scales there were 6 items to which the students could respond that they agreed strongly (2 points), agreed (1 point), had no opinion (0), disagreed (-1 point) or disagreed strongly (-2 points). Within each scale, agreement with three of the items indicated support of the belief and disagreement with three of the items indicated support of the belief. The scores of the three later items were reversed before summing. Thus, the point total for each scale could vary from –12 to 12.

An examination of the results presented in Table 4.1 below revealed that only in the case of the first belief scale, in which the student indicated whether he or she could solve time-consuming mathematics problems, was there no significant difference between the means for the students in the prior study by Kloosterman and stage (1992) and the traditional age students in this study. On the other four belief scales, a significant difference between means was found (p<.005); one must therefore reject the conclusion that the population means are the same. However, in the second, third and fifth belief scales, the means for each group are both of the same sign (both positive or both negative). In the second and fifth scales, the means for the group being currently studied were more extreme than the means for the Kloosterman and Stage population. For Belief
4 concerning the importance of word problems, the means for the two groups are of opposite sign but within the interval between negative and positive one. This reflects a general sense of ambivalence about this item that was also reflected in discussion with students during this research and was reported by Kloosterman and Stage in their study.

In general, there is not a statistical reason to conclude that, based on these five belief scales, the traditional students in the Kloosterman and Stage’s work and the traditional students in the classes I surveyed have the same beliefs with regard to these items, but there are important indications of a strong similarity of perspective, with differences in degree.

Table 4.1: Students from Kloosterman & Stage, 1992 (n=517) and Traditional Students in this Study (n=70)

<table>
<thead>
<tr>
<th>Belief</th>
<th>Mean: Kloosterman &amp; Stage 1992</th>
<th>Standard deviation: Kloosterman &amp; Stage 1992</th>
<th>Mean: Traditional</th>
<th>Standard deviation: Traditional</th>
<th>t-statistic with (70+517-2=585) degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: I can solve time-consuming mathematics problems.</td>
<td>2.5</td>
<td>3.7</td>
<td>2.03</td>
<td>3.88</td>
<td>-0.99556</td>
</tr>
<tr>
<td>2: There are word problems that cannot be solved with simple, step-by-step procedures.</td>
<td>-1.5</td>
<td>3.4</td>
<td>-3.13</td>
<td>2.82</td>
<td>-3.83583</td>
</tr>
<tr>
<td>3: Understanding concepts is important in mathematics.</td>
<td>7.3</td>
<td>2.8</td>
<td>3.84</td>
<td>4.41</td>
<td>-8.95328</td>
</tr>
<tr>
<td>4: Word-problems are important in mathematics.</td>
<td>0.8</td>
<td>3</td>
<td>-0.29</td>
<td>2.62</td>
<td>-2.88431</td>
</tr>
<tr>
<td>5: Effort can increase mathematical ability.</td>
<td>4.4</td>
<td>3.8</td>
<td>6.07</td>
<td>3.72</td>
<td>3.465163</td>
</tr>
</tbody>
</table>

reject equal population means at p<.005
In Table 4.2, the means for the students in the Kloosterman and Stage (1992) study are compared to the means for the re-entry students in this study. Except for the fourth belief concerning the importance of word-problems, in each case the mean for the students in my study is in the same direction but more extreme than the mean for the students in Kloosterman and Stage’s study. Again, for Belief 4, both means are between positive and negative one, reflecting the same ambivalent viewpoint observed in the previous case.

Table 4.2: Students from Kloosterman & Stage, 1992 (n=517) and Re-entry Students in this Study (n=19)

<table>
<thead>
<tr>
<th>Belief</th>
<th>Kloosterman &amp; Stage 1992 Mean: 2.5</th>
<th>Standard deviation: 3.7</th>
<th>Standard deviation: 2.95</th>
<th>Mean: re-entry 5.18</th>
<th>t-statistic 0.490962</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief 1: I can solve time-consuming mathematics problems.</td>
<td>2.5</td>
<td>3.7</td>
<td>2.95</td>
<td>5.18</td>
<td>0.490962</td>
</tr>
<tr>
<td>Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures.</td>
<td>-1.5</td>
<td>3.4</td>
<td>-5.47</td>
<td>3.03</td>
<td>-5.07034</td>
</tr>
<tr>
<td>Belief 3: Understanding concepts is important in mathematics.</td>
<td>7.3</td>
<td>2.8</td>
<td>8.00</td>
<td>3.13</td>
<td>1.055846</td>
</tr>
<tr>
<td>Belief 4: Word-problems are important in mathematics.</td>
<td>0.8</td>
<td>3.0</td>
<td>-0.11</td>
<td>3.80</td>
<td>-1.24912</td>
</tr>
<tr>
<td>Belief 5: Effort can increase mathematical ability.</td>
<td>4.4</td>
<td>3.8</td>
<td>7.05</td>
<td>3.95</td>
<td>2.976726</td>
</tr>
</tbody>
</table>

In this comparison, it is interesting that for three of the belief scales (one, three and four) one need not reject the conclusion that the population means are equal and that the students have essentially the same views in those areas. Also, for the other two belief scales (two and five), the means for the two groups are of the same sign (in the same direction) and, in each case, the mean of the re-entry students is more extreme than the
mean for the traditional students. That is, one can conclude that the two groups hold essentially the same views on three scales, and that although their views may differ on the other two scales, they are in the same direction, with the beliefs being more intense or more strongly held in the case of the re-entry students.

All Likert-Scale Items on Initial Questionnaire

Table 4.3 below includes a summary of the results from all the Likert-Scale items that appeared on the initial questionnaires in this study and a statistical analysis of those results. Belief scales 1 – 5 are the same as those discussed above, belief 6 is based on the Fennema-Sherman (1976) usefulness scale, and belief scale 7 is gleaned from Cooney’s (1992) study of in-service teachers. Like the first five belief scales, scales 6 and 7 consist of six items each, scored in the manner described previously; hence the scores on these scales also could vary between –12 and 12. The last four additional Likert-type items were added to elicit beliefs about the individual’s general mathematical ability as well as his/her beliefs about the study of mathematics in comparison to other areas of study. For each of these individual items, the scores can vary between –2 and 2.
Table 4.3: Traditional and Re-entry Students’ Responses

<table>
<thead>
<tr>
<th>Belief</th>
<th>Mean Traditional Students (n=70)</th>
<th>Mean Re-entry Students (n=19)</th>
<th>t-statistic with (70+19-2 = 87) degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief 1: I can solve time-consuming mathematics problems.</td>
<td>2.03</td>
<td>2.95</td>
<td>-0.857</td>
</tr>
<tr>
<td>Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures.</td>
<td>-3.13</td>
<td>-5.47</td>
<td>3.188 reject equal population means at p&lt;.005</td>
</tr>
<tr>
<td>Belief 3: Understanding concepts is important in mathematics.</td>
<td>3.84</td>
<td>8.00</td>
<td>-3.862 reject equal population means at p&lt;.005</td>
</tr>
<tr>
<td>Belief 4: Word-problems are important in mathematics.</td>
<td>-0.29</td>
<td>-0.11</td>
<td>-0.242</td>
</tr>
<tr>
<td>Belief 5: Effort can increase mathematical ability.</td>
<td>6.07</td>
<td>7.05</td>
<td>-1.013</td>
</tr>
<tr>
<td>Belief 6: Mathematics is useful in daily life.</td>
<td>3.21</td>
<td>6.00</td>
<td>-2.100 reject equal population means at p&lt;.025</td>
</tr>
<tr>
<td>Belief 7: In mathematics, processes are more important than rules and calculations</td>
<td>0.86</td>
<td>0.95</td>
<td>-0.160</td>
</tr>
<tr>
<td>General I have never been able to do mathematics very well.</td>
<td>0.41</td>
<td>0.58</td>
<td>-0.536</td>
</tr>
<tr>
<td>General I think that people either have a talent for doing mathematics or they don't.</td>
<td>0.59</td>
<td>0.42</td>
<td>0.606</td>
</tr>
<tr>
<td>General With hard work, I can usually succeed at mathematics.</td>
<td>0.86</td>
<td>1.16</td>
<td>-1.360</td>
</tr>
<tr>
<td>General With hard work, I can usually succeed at non-mathematical subjects, like English or history.</td>
<td>1.48</td>
<td>1.37</td>
<td>0.708</td>
</tr>
</tbody>
</table>

For all except the second, third and sixth belief scales, the t-statistic indicates that the population means of the two groups are not significantly different, thus, one may conclude that the students hold essentially the same views on belief scales 1, 4, 5 and 7,
and the four general items. For each of the belief scales 2 and 3, the means are statistically different for the two groups (p<.005); for belief scale 6, they are also statistically different at p<.025. Furthermore, the means in each case are of the same sign (in the same direction), while the mean of the re-entry group is more extreme. It is reasonable to conclude that, based on these observations, the students in the two different groups hold similar viewpoints with respect to these beliefs, and the view of the re-entry group could be described as being of a stronger or more intense nature.

Observations

The Likert-scale items on the initial questionnaires did not predict strong differences between the views of Traditional students and Re-entry students. In fact, in many ways indicated above, similarities were indicated with the possibility that in some cases Re-entry students may hold more extreme views or hold their beliefs more intensely.

The responses to the Likert-scale items on the initial questionnaires indicated that most students believe that they can solve time-consuming mathematics problems. However, the means of all the groups studied were rather low, between 2 and 3, indicating that, in general, this was not a very intensely held belief.

Most of the students also agreed with the beliefs that understanding concepts is important in mathematics and that mathematics is useful in daily life. These two beliefs were more intensely held by the re-entry students than by the traditional students in my study.

The responses indicated that most of the students in my study agreed with the belief that effort can increase mathematical ability; that, in mathematics, processes are
more important than rules and calculations; that they have never been able to do mathematics very well and that people either have a talent for doing mathematics or they don’t. Most also agree that, with hard work, they can usually succeed in both mathematics and non-mathematical subjects, but their belief regarding non-mathematical subjects apparently was held more strongly.

Most students disagreed with the belief that there are word problems that cannot be solved by simple step-by-step procedures. That is, they seemed to believe that all word problems could be solved by simple step-by-step procedures. This belief was held more strongly by re-entry students than by traditional students in my study.

The responses indicated that most of the students also disagreed with the belief that word-problems are important in mathematics. However, the means for this item for both groups were very close to zero (-0.29 for traditional and -0.11 for re-entry), which would indicate that this was not a strongly held belief or that the students were somewhat ambivalent about this belief. In conversation, one can quickly surmise that students are aware that knowing how to solve word problems is important for their success, but they often do not perceive the importance of the problems themselves. The participants in my study voiced this sort of confusion in their beliefs about the importance of word problems. This belief proved to be particularly difficult to assess using only a questionnaire, an observation also made by Kloosterman and Stage (1992, 1995).

Above all, the usefulness of these items remained in two areas: as a springboard for discussion during the interviews and teaching sessions and as a basis of comparison between stated beliefs and behaviors. In addition, it could be argued that, given similar
belief structures, findings resulting from research with re-entry students might be applicable to traditional students as well.

Responses to Open-Ended Items for Re-entry Students

**Open-Ended Items on Initial Questionnaires**

Six open-ended items were included on the initial questionnaires and were further discussed during the initial interviews. These items addressed: 1) what the students did during the years between the completion of high school and enrolling in higher education; 2) activities besides school in which they are involved, or would like to be involved, such as work, hobbies or interests; 3) the students’ main reasons for attending college; 4) the students’ primary purposes for studying mathematics in college; 5) to what they attributed their own good performance in mathematics; and 6) to what they attributed their own poor performance in mathematics. The last four items were specifically revisited during the exit interview.

These items were included in order to elicit the students’ explicit input. I hoped to begin to formulate their views about knowledge in general and mathematics in particular – their knowledge voices as described by Kasworm (1997, March) - and to examine their meaning making processes in light of Baxter Magolda’s (1992) epistemological reflection model. I also hoped to begin to situate these views relative to their experiences, as modeled by Donaldson et al. (1999, April). In addition, the past experiences of those who continued further in the research guided the selection of tasks to be used in the teaching sessions.

Table 4.4 below displays the types of responses that were given on the initial questionnaires by the nineteen re-entry students who completed them. The numbers
indicate the number of students who provided that type of response. Each student was free to give as many responses as she or he wished, so the total of the responses for each item is more than nineteen. The responses were then coded according to the numbers of students giving each type of response. That is, the response given by the most students was coded A, the one given by the next highest number of students was coded B, and so forth. If the same number of students gave two responses, those types of responses were coded with numbers as well. That is, the same number of students gave two different types of responses, one would be coded 1 and the other 2, with no significance to the number that was assigned. This coding system simply permitted to me to see how the potential participants’ responses compared to the group of 19 re-entry students’ responses.

Table 4.4: Summary of open-ended responses for all 19 re-entry students

<table>
<thead>
<tr>
<th>number responded category</th>
<th>Activities between H.S. and college</th>
<th>Outside activities</th>
<th>Reasons to return to college</th>
<th>Reasons for study of math in college</th>
<th>I do well in math because…</th>
<th>I do poorly in math because…</th>
</tr>
</thead>
<tbody>
<tr>
<td>code</td>
<td>18</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Worked</td>
<td>Hobbies &amp; interests</td>
<td>Work or career related</td>
<td>Requirement, perhaps for a degree or major</td>
<td>Student’s efforts</td>
<td>Student’s efforts</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>number responded category</td>
<td>8</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>code</td>
<td>Family</td>
<td>Work related</td>
<td>Degree related</td>
<td>To apply in life or other areas</td>
<td>Teacher’s efforts</td>
<td>Not getting help when needed</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>
Although 18 of the 19 students indicated that they had worked between high school and college, only 5 of them listed work related activities as activities besides school that they were involved in or wished to be involved in. This may have been due to the fact that they had just listed work as an activity in the prior item and felt the need to list something else as an outside activity, or they may have considered work as a requirement and listed activities other than work as outside activities. Those whom I interviewed certainly drew from both work and other activities for examples of experiences outside of school. Eight students indicated that family activities had occupied them during the time between high school and college. Only two listed other activities: getting a degree from a technical school and military service.
The predominant stated purpose for returning to academia – the one given by 12 of the 19 students - was a very practical one and usually job related; that is, it was either to open new opportunities or to progress within their present setting. Eight students indicated that they were returning to get a degree; three linked that degree to a specific academic major or career goal. During the interviews, this connection between getting a degree and changing or improving career situations was made by all the participants. Five students stated such self-esteem related purposes as bettering themselves and proving to themselves that they could do it. Five indicated that they were interested in pursuing knowledge or getting an education. Three students stated reasons related to their families such as having a similar schedule or setting an example for their children.

Eleven of the nineteen students, a little over half of them, stated that their purpose for taking mathematics was that it was required. Ten students indicated that they felt they would be able to apply it in their lives; eight of them mentioned work or family arenas such as helping their children while two specified that they thought they would need it in other coursework. Three of them stated that they wished to prove their ability to be successful in mathematics.

The last two open-ended items addressed the area of attribution of success and failure in the study of mathematics in school. In completing the sentence beginning: When I do well in mathematics, it is usually because . . ., sixteen of the nineteen re-entry students credited their own efforts. Five of them mentioned the skills of their teachers, but only one credited only the teacher for her own successes. Two students mentioned other forces such as luck or divine guidance, while one credited work experience as well as simply liking mathematics.
In completing the sentence beginning: When I do poorly in mathematics, it is usually because . . ., seventeen of the students blamed their own efforts and ability by indicating such things as that they had not studied sufficiently or appropriately, that they were not motivated or that they were confused or frustrated. Only two of the students indicated that the teacher/instructor was responsible, and both of these students shared the blame by indicating that it was a combination of the two: the teacher not being skillful and the student preparing or performing inadequately. Three students indicated that they had not gotten help when it was needed and three listed such interferences as illness or other commitments. The issues of not taking advantage of available assistance and time commitments outside of class arose repeatedly during the teaching sessions, particularly with one of the participants. Only one student indicated that he avoided math by not attending class. He wrote, “I have not shown up in class. What can I say? I am not a ‘math person.’” This particular student did not participate further in the study, so I was unable to clarify whether being a math person would mean that a person has an innate ability or that a person likes mathematics. Although the participants in my study were quite responsible about attending class, each of them wondered on occasion about her innate ability as well as bemoaning a poor background from earlier school experiences.

Below is a table listing the responses to the open-ended items of the three re-entry students who participated in the entire study. The responses are coded to indicate how their responses fit in with the group of 19 re-entry students. Their demographic information is also indicated. In the chapters that follow, these responses along with those given in the initial interview and throughout the teaching sessions will be explored. For
example, comparison between these stated beliefs along with those gleaned from the Likert-scale items and their demonstrated behaviors will be of interest.

Table 4.5: **Open-ended responses for participants, with demographic information and codes indicated**

<table>
<thead>
<tr>
<th>Code</th>
<th>Activities between H.S. and college</th>
<th>Outside activities</th>
<th>Reasons to return to college</th>
<th>Purpose for study of math in college</th>
<th>I do well in math because . . .</th>
<th>I do poorly in math because . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hillary White female</td>
<td>Married, worked in various university depts, raised a family</td>
<td>Singing in choir, reading, cross stitch</td>
<td>Working on a BS Ed in middle school education</td>
<td>Part of the BS Ed program</td>
<td>I have worked hard, studied and stayed focus on succeeding.</td>
<td>I have not prepared properly or I do not understand the material.</td>
</tr>
<tr>
<td>Age 37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peggy White female</td>
<td>Worked (both in university setting and elsewhere)</td>
<td>Reading, crocheting; Admin. Secretary (for campus department)</td>
<td>To increase knowledge. My degree will give me the ability to work in a field I have always loved</td>
<td>Required</td>
<td>My teacher is incredible.</td>
<td>My teacher is not as good explaining and I do not make any extra effort.</td>
</tr>
<tr>
<td>Age 35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jennifer White female</td>
<td>Worked (at the university)</td>
<td>Hobbies are collecting first editions of favorite authors; interests are mainly in my children’s activities, family and community</td>
<td>To be promoted further at job.</td>
<td>Need to obtain degree</td>
<td>I have studied extremely hard and have a teacher good at explaining</td>
<td>Have not studied due to other commitments</td>
</tr>
<tr>
<td>Age 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Code: A, B, C, 1
Open-Ended Items on Initial Questionnaire and in Exit Interview

The four open ended items concerning the students’ reasons for returning to college, their purposes for studying mathematics in college, to what they attribute doing well in mathematics and to what they attribute doing poorly in mathematics appeared on the initial questionnaires, were further discussed during the initial interviews and were again presented in the exit interviews. The responses to the four items for the three participants who completed the entire project are indicated in the tables below; my observations are also included. Next to Initial are the written entries from the students’ initial questionnaires. During discussion in the initial interviews, these responses were expanded upon, but not substantially altered. Next to Exit are synopses of the comments made by the students during the exit interviews. These exit interviews were conducted individually on May 11, 2000. The actual responses made by the students are discussed further in other sections. However, the shifts that did or did not take place between initial responses and those upon exit are worthy of comment and are noted in the Observations column.

Jennifer’s responses are summarized in Table 4.6 below. Throughout the teaching experiment, she was the least effusive and the most stable in her demeanor. These responses are consistent with Jennifer’s involvement within the entire project.
### Jennifer’s responses to open ended items initially and upon exit

<table>
<thead>
<tr>
<th></th>
<th>Jennifer</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reasons to return to</strong></td>
<td>Initial: To be promoted further at job.</td>
<td>Same</td>
</tr>
<tr>
<td></td>
<td>Exit: Maxed out in terms of promotions without a degree.</td>
<td></td>
</tr>
<tr>
<td><strong>Purpose for study of math in college</strong></td>
<td>Initial: Need to obtain degree</td>
<td>Same</td>
</tr>
<tr>
<td></td>
<td>Exit: Got to have it. Taking it because I have to. If I go into business, I will have to take a lot of math. May not pursue that goal because of the amount of time it takes to take math classes.</td>
<td></td>
</tr>
<tr>
<td><strong>When I do well in math it is usually because . . .</strong></td>
<td>Initial: I have studied extremely hard and have a teacher good at explaining</td>
<td>Essentially the same. Understanding involved her own effort coupled with the teacher’s explanations.</td>
</tr>
<tr>
<td></td>
<td>Exit: I understood what the teacher was saying and understood the method she was using. I do better when I understand</td>
<td></td>
</tr>
<tr>
<td><strong>When I do poorly in math it is usually because . . .</strong></td>
<td>Initial: Have not studied due to other commitments</td>
<td>Essentially the same: Jennifer’s comments during the initial interview emphasized the pressure of other commitments and her inability (due to time constraints) or her hesitancy (due to shyness and reluctance) to seek assistance that was available.</td>
</tr>
<tr>
<td></td>
<td>Exit: Not having enough time to study or think about it, to see instructor, to use help. Did not get good notes in class or understand in class and then could not do it at home or on her own. (Most of the time she could tell whether she was lost.) Not the teacher’s fault – The main issues were that she either did not have the material before or forgot it – Usually did not ask for help in class, hated to take up time in class because the rest of the class seemed to remember from high school (Watched a tape of a class she missed – only one time – wished that she could take the tapes home.)</td>
<td></td>
</tr>
</tbody>
</table>

Hillary’s involvement in the teaching experiment was certainly an active one, but she repeatedly exhibited resistance to different ways of thinking about things. In her case, each of the differences in her responses revealed slight shifts of emphasis. The environment may have nudged her into expressing her thoughts about her encounter with
education and with mathematics in a slightly different way. Hillary’s responses are summarized in Table 4.7 below.

Table 4.7: Hillary’s responses to open ended items initially and upon exit

<table>
<thead>
<tr>
<th>Reasons to return to college</th>
<th>Initial</th>
<th>Exit</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working on a BSEd in middle school education</td>
<td>To be a middle school teacher</td>
<td>Shift from focus on degree to focus on application of skills</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Purpose for study of math in college</th>
<th>Initial</th>
<th>Exit</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part of the BSEd program</td>
<td>1- required 2- may have to teach it 3- help children in their school work</td>
<td>Some shifts from fulfilling a requirement to application of skills in other settings – work and home.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>When I do well in math it is usually because . . .</th>
<th>Initial</th>
<th>Exit</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have worked hard, studied and stayed focus on succeeding. Also, when I have an instructor who goes above and beyond to help</td>
<td>I’ve worked hard. Teacher is good. Between those two. Right now: Probably more effort on my part.</td>
<td>Slight shift to more emphasis on own effort</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>When I do poorly in math it is usually because . . .</th>
<th>Initial</th>
<th>Exit</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have not prepared properly or I do not understand the material.</td>
<td>Probably effort on my part as well. Or I don’t understand and either the instructor hasn’t explained it where I can understand it or the textbook doesn’t help me even though the instructor has explained it, there’s something where I haven’t made the connection.</td>
<td>Slight shift to defining what happens when she does not understand with some assignment of responsibility outside of self.</td>
<td></td>
</tr>
</tbody>
</table>

Peggy not only totally immersed herself in the teaching experiment but also revealed herself to be quite open to alternative ways of thinking about things. She may have been pondering the topics presented initially throughout the semester. She may also have responded to the atmosphere provided within the sessions and accepted the tacit
challenge to explore the ideas more deeply. Peggy’s responses are summarized in Table 4.8 below.

Table 4.8: Peggy’s responses to open ended items initially and upon exit

<table>
<thead>
<tr>
<th>Reasons to return to college</th>
<th>Initial</th>
<th>Exit</th>
</tr>
</thead>
</table>
| Peggy                        | 1 to increase knowledge  
                             | 2 my degree will give me the ability to work in a field I have always loved | Because I am tired of working for other people. (She was very hesitant to say so, but finally stated that she knew that she was as smart as or maybe smarter than some people in higher positions, and she wanted to have those positions instead of doing what she does. Even if she never used her degree, she indicated that she believed that she would be a better person for having gotten it.) |

<table>
<thead>
<tr>
<th>Purpose for study of math in college</th>
<th>Initial</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peggy</td>
<td>Required</td>
<td>This is my philosophy on the whole college experience: Regardless of whether your major is in a specific area or not, I think it increases all of us as human beings to be familiar with all of it. Sounds really hokey, but I really do feel that way. Makes you a more well rounded human being. Knowledge never hurts.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>When I do well in math it is usually because . . .</th>
<th>Initial</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peggy</td>
<td>My teacher is incredible.</td>
<td>I have a good instructor and I have applied myself . . . [that is,] I have put as much in it as the instructor has. One thing that I did learn . . . is that for so long I had disliked it [math] so much that I didn’t put as much in it as I should have . . . So I found out that it was not something that I could not do, it was just a matter of whether I put the effort into it to learn how to do it. It’s not as big to me. I can do it, if I just take the time to learn it.</td>
</tr>
</tbody>
</table>

Observations |
| Major shift to issues of self-esteem and self-worth. Work related, but more personal. More emphasis on the value of education to make her a better person. |


Major shift away from assigning total responsibility to instructor. More emphasis on personal attitude and willingness to try. |
Initial | My teacher is not as good explaining and I do not make any extra effort.
---|---
Exit | I just didn’t try. I have to admit that when I did not do well on the test, I did not spend as much time as the times I did really well. (In the fall, she took one class as an independent study course; that was the first remedial math course and she made a B. This semester, she took two classes and felt she was constantly going from one to the other; she felt that was a major reason that she made a C in math this semester.)

More intensive exploration of these three students’ responses together with their comments during the interviews and their behaviors during the teaching sessions will be presented as case studies in the following chapter. The first three sections of Chapter 5 focus on Jennifer, Hillary and Peggy, respectively, and the last section addresses the collaboration between Hillary and Peggy.
CHAPTER 5

CASE STUDIES

As has already been indicated, much more and, more importantly, much richer results were gleaned from the interviews and the teaching sessions than from the questionnaires alone. The methodology of the teaching experiment permitted me to examine both the participants’ affective orientation and their mathematical thinking in the context of problem solving. Their levels of involvement and the very quality of that involvement varied greatly. Each of the three participants made very different contributions to the sessions, so I learned different things from each of them. These differences influenced not only the substance but also the form of the case studies in this chapter. Although the brevity of Jennifer’s case study reflects the lack of complexity revealed in both her perspective and her approach to problem solving, her pragmatic and somewhat detached involvement in this study and in the study of mathematics in general generated some important themes.

Hillary’s participation in the teaching sessions proved a challenge to her and to me as the researcher; her eagerness to tell me about her perspective was not well matched by a willingness to explore her perspective and thinking in the atmosphere of a teaching experiment. Yet, it was actually her critical voice and the degree to which she separated the mathematics of the classroom from the mathematics of her own experience that provided a generous amount of data on which to base her more lengthy case study.
Peggy revealed a fascinating and perplexing picture, which is presented in the longest of the case studies. She was highly involved in the sessions and displayed both significant problem areas in certain aspects of her mathematical thinking and intensely desirable qualities in her perspective and in other areas of mathematical thinking.

The final case study, that of the collaboration between Peggy and Hillary, not only highlights the interplay between the two participants but also underscores many of the observations made about them individually. Within the case studies, the tables of the timelines and tasks for the sessions with the participants involved are presented; each case study ends with a summary.

Jennifer

Introduction

When she began participating in this teaching experiment, Jennifer was a 36 year-old white female, married with 3 young children. She had been working at the university in various capacities since she graduated from high school 18 years previously. She had been involved in personnel matters, budgeting, analyzing and reporting statistical data, and troubleshooting and implementing research grants. Her supervisor had advised her that pursuing a degree would enhance her opportunities for advancement. In fact, during the time we were engaged in this study, Jennifer received a promotion, which she believed was based not only on her experience, but also on the fact that she had shown that she was making the effort to get a degree. She had considered going back to school previously, but time, pressures of a young family and the cost precluded her from doing so. The encouragement of her supervisor seemed to tip the balance, and a reimbursement
program for employees at the university made it financially possible for her to enroll at this time.

Jennifer took only general mathematics courses when she was in high school and was not surprised to be required to enroll in the most basic level of mathematics in Academic Assistance. At the time of this study, she was enrolled in the second semester of that two-semester sequence. Although no pressure had been exerted on Jennifer to major in any particular area, she believed that a business degree would relate better to her employment. On the other hand, she preferred to avoid the mathematics courses that were required. Her real interests were in the area of English or literature; in fact, she had enjoyed a hobby of collecting first editions of her favorite authors.

**Jennifer’s Purposes and Goals**

Jennifer’s reasons for enrolling in higher education at this time and her feelings toward the study of mathematics seemed to cause considerable conflict in her thinking about her academic goals. Mathematics is a subject in which Jennifer felt that she had to put a great deal of effort in order to pass; to some extent she ascribed this to the fact that she did not take mathematics for college-bound students when she was in high school. If she had the option to avoid taking mathematics in college, she would do so. She did not seem to be trying to avoid hard work, as much as she was more interested in other things. She made good grades and ranked well in high school even though she had enrolled in some challenging English courses and also had studied a foreign language. When discussing her thoughts about what she might select as a major field of study, she explained as follows.
Jennifer: I’m still undecided. . . . Probably something in business would probably be the ideal choice, if you’re going for . . . job related, but I know there are a lot of math classes associated with business, and I am not that good at math. I . . . only took general math in high school and, you know, I struggled in [college instructor’s name]’s class. I mean, she’s a very good teacher, but because I’ve never had any of that – It’s just some of the things, I should know – that I should already know – that she assumes, and I should know based on the other kids who are in the class. I really have to struggle just to keep up with that. And I don’t know if any of the other math classes – I’m sure they’re not going to be any easier – When you get to calculus and some of the things you have to have to have a business degree – so that’s something I’ve got to think about, if I really want to put the effort – I mean, maybe not the word effort. But, and then if I do something for fun or something I’m interested in – English or something in that area would be more in line with what I like to do. But as far as my job – I don’t know – if that really relates to my job at all. (Speech #26, emphasis of the speaker)

**Jennifer and her Involvement with her Mathematics Course**

The amount of time that Jennifer had to devote to educational activity coupled with her personality seemed to limit the extent to which she capitalized on opportunities for assistance both in and outside of the classroom. It also appeared that those factors – time and personality – affected her views of the roles of instructors, students in general and herself in particular.

Jennifer was not very out-going and was therefore a bit hesitant about asking questions or openly seeking help. During the initial interview, she reflected on the fact
that she tended not to ask questions in class. She compared the situation in the two semesters of mathematics that she had experienced so far at the university; in the first there were more students who often asked questions.

Jennifer: [I]t's helpful when questions are asked. I have a tendency - my personality - not to - I might be thinking of a question. I think that's what I have missed about some of the students that were probably in that first class. We had some that would just ask a question about everything. And I would be thinking, "Good, because I don't know that either." So, it's helpful to have other questions asked, and I should make more of an effort to ask them myself. It's just - my personality - I'm not as open about asking as many others. (Speech #50)

Listening to the questions other students ask was the only peer interaction to which Jennifer referred within the context of the study of mathematics. During a summer orientation class, “it was fun to hear others' point of view in that. With math - you know, I don't know. I'm probably not a good one to ask, because I'm more stick-to-myself in working” (Speech #92). Outside of class, Jennifer did not seek out other students to study together. She ascribed this mainly to her full schedule, but also acknowledged the role that her personality plays.

Jennifer: It's mainly the time - I just don't have the time to - for, you know, socializing with other students. After class I have to go back to work, and I have to pick my kids up after work. And that is probably the drawback of non-traditional students, because you're not interacting socially with other students. And a lot of it too is probably my personality. I really don't miss it. (Speech #96)
On her questionnaire, Jennifer indicated that, when she did well in mathematics, it was because “I have studied extremely hard and have a teacher good at explaining.”

When asked about the important ingredients in the mathematics classroom, Jennifer first mentioned how helpful it was that her instructor usually wrote down all the steps to problems and how carefully she recorded all of those steps in her notes. Outside of class, she reviewed carefully the material that had already been presented in class. “And then I go over and over what she's given us - She'll give us practice tests, and I'll just keep practicing the problems, going over the steps, until I feel comfortable” (Speech #76).

Although she read the material in the textbook, Jennifer avoided reading ahead or reading anything that conflicted with what the instructor had presented.

Jennifer: I never read ahead. And there are times she - she shows us a way - you know, a certain way to do it - I guess she feels like the chapter's confusing - You know, there's more than one way to do it, and she'll show us her preferred way, and I don't read those sections [that show other ways to do the work], because I don't want to get confused. I don't worry about it. She says, “There's several ways to do this, but I'm showing you this way.” I don't even look at it [in the book], because I don't want to get confused. (Speech #78)

Jennifer observed that, although she had no other courses in college with which to compare, it was possible that she might consider reading ahead in other subjects given the time and the interest in the material.

During the initial interview, Jennifer described a situation that occurred when a different instructor substituted for her regular instructor.
Jennifer: She even had us go through the book and made us work problems for our homework and that was helpful, too. Because we normally don't do that. [The regular instructor] gives us examples out of the book, and she assigns our homework, and we go home and do it. But [the other instructor] - you know, she walked around and made sure you were doing that problem correctly. That was helpful. (Speech #40)

Although the actual instruction delivered by the instructor and the clarifying questions of students were clearly valued elements of the mathematics classroom for Jennifer, this practice of having the students work problems during class and receive feedback from the instructor became a major segment in her representation of the important ingredients in the mathematics classroom. (See pie charts in Figures 5.1 and 5.2 below.)

![Figure 5.1: Jennifer’s Ranking of Classroom Ingredients by Importance](image1)

![Figure 5.2: Jennifer’s Ranking of Classroom Ingredients by Time to be Allotted to Them](image2)
Jennifer mentioned the time pressures of her job and her family as reasons that she did not do well in mathematics or take advantage of the help available outside of the classroom as much as she should. On her questionnaire, she indicated that when she did poorly in mathematics it was usually because she had “not studied due to other commitments.” Jennifer was attending the class of which videotapes were being produced, and those videotapes were available for any student enrolled in any section of the course to view in the Learning Center on campus. During the initial interview, she specifically mentioned that she would like to take advantage of this form of assistance.

Jennifer: I would love to be able to check those tapes out and take them home, but you know, right now, you have to go over to the Learning Center. If I had the time, it would be wonderful. The tapes she did of that class that day - to take them home at night - I don't know if it's even feasible. (Speech #42)

She continued, “you can go over to the Learning Center and use them, but for me it's just very - The time - After class, I've got to get back to work. I have to get back immediately” (Speech #48).

Jennifer acknowledged that she could benefit from other services that were available outside of class, but that she did not take advantage of them. For example, during the initial interview, she made this comment.

Jennifer: I know there are tutors over at the Learning Center, and those places, but just because of my time and my family, it's just difficult. . . . I know we do have that, it's just a matter of - I don't take advantage of it - That would be helpful. (Speech #74)
Jennifer reported that she was very meticulous in her notetaking, and she appreciated it when the instructor carefully wrote all the steps while demonstrating the way to do problems.

Jennifer: Everything that she writes down, I write down. . . . I have that at night to remember . . . how she worked that problem . . . She gives us an example, she shows us how to work it, she'll list steps - You know, you do this first, do this second, do this third . . . If you looked at her tape [of each class session] and my notebook is the exact same thing. That's been helpful for me. (Speech #40)

Thus, Jennifer’s personality and the time pressures of her life-situation had a tremendous impact on her views of the ideal classroom for her mathematics in college. Her shyness precluded her from feeling comfortable about asking questions in class, but she benefited from hearing the questions other students ask and the explanations the instructor gave them in response. It appeared that the fact that she found it inconvenient to take advantage of assistance available outside of class, such as the videotapes of the class sessions and tutoring, made it even more attractive to her to have time taken within the class for students to work problems and get feedback from the instructor. Most importantly, she valued the instructor’s complete, step-by-step presentation of the material, which she recorded carefully in her notes.

Jennifer’s Meticulous Practices

Jennifer carried though with this notion of meticulously recording steps when she was engaged in the tasks during our sessions together. During those sessions, she appeared to be committed to the concept that problem-solving situations can always be handled with step-by-step procedures. At the same time, we will see that it was to her
advantage that she recorded her own steps when addressing the tasks presented to her. At times, this practice assisted her in recognizing patterns and generalizing procedures.

The timeline for the initial interview and teaching sessions, together with the sequences of tasks in which Jennifer engaged are presented in Table 5.1.

**Table 5.1: Timeline and Tasks for Jennifer**

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of session</th>
<th>Tasks and activities (All tasks are listed in Appendix C and are described as they are discussed in the text that follows.)</th>
</tr>
</thead>
</table>
| February 2, 2000 | Initial Interview   | Follow-up discussion from Initial Interview  
Task #1 (Picture Hanging)  
Task #2 (Retirement Party)  
Task #17 (Paint and Carpet) |
| February 16, 2000| Teaching Session #1 | Task #6 (Handshake)  
Task #8 (Fieldtrip)  
Task #10 (Jobs/salaries)  
Task #11 (Gas Budget)  
Task #22 (Cows and Chickens) |
| March 15, 2000   | Teaching Session #2 | Task #21 (1-6 in a Triangle)  
Task #9 (Map)  
Task #24 (Change in Bills) and symbolizing  
Task #28 (Irregular Shapes on a Grid) |
| March 17, 2000   | Teaching Session #3 | Discussion of researcher’s visit to the classroom on the day before the test  
Task #20 (How many Triangles?)  
Task #4 (Fence Painting)  
Task #30 (Array of 7 Squares)  
Task #32 (Areas on a Grid, including a Lake) |
| March 29, 2000   | Teaching Session #4 | Task #33 (How many Triangles? #2)  
Task #3 (Merchant and 3 Fairs)  
Task #34 (Couples)  
Task #35 (Rectangles – Area and Dimensions)  
Task #36 (Balancing) |
| April 6, 2000    | Teaching Session #5 | |
| May 11, 2000     | Exit Interview      | |

During our first teaching session, the first task Jennifer was asked to address was the Picture Hanging Task. (Given various placements on a wall, the student is asked to
determine how far to the left should the picture be moved so that it is centered on the wall. See Task #1 in Appendix C.) Jennifer displayed both her low self-confidence, referring to her appropriate conceptualizing of the problem as guessing, and her habit of recording her work as she addressed each part of the task. It seemed that the combination of her initial conceptualization of the task and her ability to literally look back at her work in each part eased her path for writing a general procedure to follow. Another excellent example of the extent to which her record keeping while addressing tasks seemed to guide her thinking came in Teaching Session #2 when she addressed the Cows-and-Chickens Task. (The student is asked to determine how many chickens and cows are in a barnyard if there are 35 heads and 100 legs. See Task #22 in Appendix C.). Jennifer began by looking at the extremes and noting that because $100 \div 4 = 25$, there would be 25 animals (which she equated immediately to the number of heads) if they were all cows. Similarly, she observed that because $100 \div 2 = 50$, there would be 50 animals if they were all chickens. Neither of the extremes resulted in the required 35 heads. She then went back to the scenario with all cows, writing $25 \times 4 = 100$, and started “backing down” (Teaching Session #2, 11:40:00 – 11:46:10 a.m.) on the number of cows and putting in enough chickens to result in 100 legs. In her first iteration she wrote,

$$23 \text{ cows} \times 4 = 92 \text{ legs}.$$  

She mentally subtracted 92 from 100 and commented that she needs 8 legs. She then mentally divided 8 by 2 wrote below the first line,

$$4 \text{ chickens} \times 2 = 8 \text{ legs}.$$
She added the 92 and the 8, and wrote below the column of legs:

100 legs

She then wrote

27 heads

under the cows and chickens and concluded that this was not the solution, because there were not enough heads. She continued this process, recording with labels each time, beginning with 20 cows, then, 17 cows, then 16 cows and, finally with 15 cows. She drew a box around the final iteration because it gave her the required 100 legs and 35 heads. After she completed the problem, I commented on the care she had taken to record her own steps and asked her for feedback about that habit. (I refer to myself as Researcher in all such presentations from the teaching sessions.)

Researcher: I noticed that you kept track of what you were doing. . . In the work that you do, do you have to keep track what you are doing and demonstrate how you have done it?

Jennifer: Oh yes, because we are audited. And so you have to show the steps as to why you made that change. . . . So, I’m used to doing that at work. . . .

Researcher: What about in your math course work?

Jennifer: Do I keep track of what I do?

[Researcher ascertains that Jennifer does keep track of her own work, and continues on that topic.]

Researcher: . . . Do you find that is something that is usually encouraged or is required on a test or . . .
Jennifer: I don’t know if she requires it, but she wants to see how you work the problem. . . . If you can just sit there and look at it and do it in your head then . . . As long as you have the right answer, I don’t guess she would care. (11:46:47 - 11:48:25 a.m.)

Jennifer commented that the instructor administered tests that were a combination of multiple choice questions and problems to be worked out. The course included some computer testing for which work was done on scratch paper that was not submitted for any evaluation; it was placed in the recycle box. It would be difficult to conclude whether Jennifer had learned from her employment setting to meticulously record her work in a problem-solving setting, but this practice was certainly more linked in her mind to the requirements of her job than to her experiences in mathematics coursework. In any case, it seemed apparent that she brought this habit with her and applied it to her learning in the classroom.

One More Look at the Issue of Time for Jennifer

Jennifer’s concern about time surfaced when she was talking about solving problems as well as when she was actually involved in a problem-solving setting. On her questionnaire, Jennifer’s responses to the questions related to Belief 1: I can solve time-consuming mathematics problems combined to give her a minimum score on that scale relative to all the students who answered the questionnaire. That is, that score indicated that, in comparison to all who answered the questionnaire, she most strongly felt that she could not solve time-consuming mathematics problems. When I asked her to comment on this issue during the initial interview, she made this statement.
Jennifer: I am so booked for time - You're sitting there at night and you're having to read through those word problems and really think and the more time it takes - It's almost like I just - It's almost like your mind shuts down or your brain shuts down. And you think: Oh, I'll just go on to the next one. I can't do that one. So, I don't know. I don't like word problems. (Speech #34)

While engaged in working on the tasks during our teaching sessions together, Jennifer occasionally commented that solving a problem would take a long time or that there must be a shorter way to do it. However, this hesitation or the frustration referred to in the above quote did not preclude her from proceeding with the task, usually quite successfully. For example, while working on the Cows-and-Chickens Task discussed in the previous section, she made several comments. After the first iteration when she realized that having 100 legs was not sufficient; it was necessary to have 35 heads also. She said, “It takes a while to keep backing in” (Teaching Session #2, 11:42:06 a.m.). As she went through a few iterations, she commented, “There’s probably a faster way to do this” (11:43:47 a.m.). After she made a slight error and noticed it, she hesitated before moving on. She mused that it might never work out, but that she just needed to keep doing the same process. She commented, “It would take time for me to do it!” (11:46:10 a.m.). As she completed the task, she acknowledged, “I guess that’s why I don’t like them [i.e., word problems], because they just take so much time” (11:46:41 – 11:46:44 a.m.). At least in the context of the teaching experiment, these concerns about time did not impede Jennifer’s progress. She actually made the comments as asides as she kept working. She did not require prodding from me to continue; she seemed a bit impatient, but not frustrated as she proceeded.
Summary

Jennifer presented what I choose to refer to as a pragmatic voice. (In chapter 7, I comment further on the addition of this voice to Kasworm’s matrix of knowledge voices.) She was very practical in her decision-making about her major of study in college and in her choices for her use of time. Although she felt she had an inadequate background from previous school experiences and lacked confidence, she recognized that she had the ability to learn what was needed to be successful in mathematics and demonstrated that potential while addressing the tasks during the teaching sessions. She also recognized the need to devote more time to her study of mathematics and felt that she could not reconcile that demand with those of her real world situation. Although she claimed that she could not solve time-consuming problems, she demonstrated this ability within the sessions and indicated that she appreciated the opportunity to explore alternative ways to solve problems and to examine the meaning of answers to problems. Her meticulous record keeping practices, which may have emanated from, or at least been encouraged by, her workplace experiences, served her well when exploring mathematical tasks. It seemed that the atmosphere of the teaching sessions was supportive to Jennifer’s exploration of mathematics. Furthermore, it seemed that, if her evaluation in coursework were at least in part based on similar activities, she could be convinced to invest her time and efforts in them and benefit from that investment. In a setting such as this research in which the teaching sessions could be considered extracurricular activities, Jennifer’s minimal emotional involvement prevented her from being fully engaged in learning. It seemed that from Jennifer’s pragmatic perspective there was insufficient reason to encounter the mixed emotions present in her frontier zone (Skemp, 1987).
Hillary

Introduction

Hillary’s Background

Hillary, a 37 year-old white female, was enrolled in the university eighteen years after graduating from high school. She had previously enrolled in higher education 7 years prior to this enrollment, but the responsibility of a household with two young children caused “too much stress. And so I ended up withdrawing . . . [I]t’s been back there on the back burner” (Initial Interview, Speech #14). This time around things seemed to be going more smoothly for her; her children were older, so she felt more comfortable about turning her attention to higher education. This was her second year as a part-time university student, and she seemed to be highly motivated to be successful. Mathematics was the only area in which she was required to take an Academic Assistance course, and she had to begin at the most basic level offered in that program.

During the years since her high school graduation, Hillary had been employed for 5 years at a credit union and for 15 years in various departments on campus. Her duties had run the gambit from general typing to preparing materials for legal action or for educational publication as well as budget-related responsibilities. She had a variety of experiences that she reported as being related to teaching through activities with her children and in her church; her intention was to earn a BS in middle school education and to teach at the middle school level.
Hillary’s Views about the Roles of the Student, the Instructor and Her Peers in Higher Education

Hillary made a strong connection between her instructors’ behavior and both her long-standing disdain for mathematics and her current success. During her initial interview, she related several incidents that illuminated this connection. In 8th grade algebra, she felt that she had received an inappropriately reduced grade from a teacher; “she put on there [i.e. the report card] that she failed me because I talked too much” (Initial Interview, Speech #20). Hillary repeated the course in 9th grade and passed with a B, a situation about which she made no further comment. She struggled with 10th grade geometry, and was told by her teacher that if she worked every extra credit problem in the book, he would pass her. Although she complied with this requirement, she still failed the course because so many problems were not done correctly. It was notable that although Hillary went on to indicate some of the behaviors of instructors that assisted her in her learning, her criticism of teachers seemed primarily based on a sense of justice and fair play. Actions that she viewed as inappropriate such as the two indicated above continued to have an overwhelming impact on her comfort not only with the individuals involved, but also with her study of the mathematics that they taught. In addition, her employment for the last few years within the college of education had enhanced her sense of what she saw as the way things ought to be in the college classroom; this pattern of judgment and evaluation based on the propriety of others’ actions surfaced on occasion throughout our sessions together.

When asked to indicate the ingredients in the mathematics classroom, Hillary described at length five areas of importance to her: (1) the instructor’s attitude about
students’ success, (2) instructor’s explanation for student understanding, (3) students’ questions, (4) student focus, and (5) the presence of other re-entry students in the classroom. When asked to allocate importance in a pie chart to each of these ingredients, she assigned two large sections to the first two and much smaller sections to the last three. (See pie chart in figure 5.3 below.) There was no doubt from her descriptions or from her pie chart that the instructor’s attitude and behavior were the most crucial elements for Hillary in the mathematics classroom.

Figure 5.3: Hillary’s Ranking of Classroom Ingredients in the Mathematics Classroom

Hillary stated that the pie chart would be the same for other subjects, but she also indicated that student participation would have a greater role for other subjects and placed a bit more emphasis on the instructor’s attitude for the study of mathematics. For example, she reported that she emailed her mathematics instructor before the semester during which she was to take the first part of the two-course sequence, and she gave her instructor her mathematics background and indicated how uneasy she was to be taking a mathematics class. Hillary appreciated the fact that she received an encouraging response from the instructor. (Speech #46) She did not report any interchanges of this sort in other classes. It would seem that she was not as concerned with the instructor’s attitude in other
coursework in which she has more personal confidence. Thus, her image of the distribution of importance for the five items on her list seemed somewhat different for subjects other than mathematics.

It was important to Hillary for students to have an opportunity to ask questions, and it was the responsibility of the instructor not only to allow time for questions, but also to use those questions to guide his or her instruction. She praised her present instructor by indicating, “there's plenty of time for us to ask questions. I mean she allows us to talk to her enough for understanding” (Initial Interview, Speech #62). During the course of this study, Hillary mentioned times that a test was delayed or a topic was revisited because of students’ questions and confusion. When asked to prioritize the ingredients in the classroom in terms of importance and of time, Hillary commented, “I think it's important that students have time to ask questions, too. Um, but I think for a student to ask questions, they've got to have some kind of an explanation as to how to understand it” (Initial Interview, Speech #62). In mathematics, in her view, the students’ questions emanated principally from the instruction and work on previously assigned homework.

When asked about the possibility of using the book as a source of information, much like Jennifer and Peggy, Hillary described the mathematics textbook as confusing and unapproachable. For other subjects she found it appropriate to read ahead in order to prepare assignments or participate in classroom discussions. However, she found that the mathematics textbook approached the material differently from the way that the instructor presented it. “They take things and make a lot more steps out of it than [the instructor] does when she's teaching. . . . I look at it just to get my problems to do my
homework” (Speech #78). Thus the instructor’s explanation was the primary source, perhaps the only source, of mathematical knowledge for Hillary.

Hillary believed that she was capable of learning mathematics. While discussing the item on the questionnaire that asked about whether people either have a talent for mathematics or they don’t, she asserted that “I think that a majority of people can learn whatever they want to learn” (Speech #38) including herself. She explained that even when she had difficulty comprehending something in mathematics, “if someone could just explain to me why I need it, then it makes a whole lot of sense and I can grasp it” (Speech #30). Again, the role of the teacher was paramount in the process for Hillary.

Hillary based her evaluation of the academic setting on her life experience and what she learned via her employment. While discussing the ingredients in a classroom, she emphatically made the following statement.

Hillary: [T]o me as a student growing up, and for me as a student now, and for me as a person that works at this institution and hears students' feedback [speaker’s emphasis], the instructor's attitude about whether they care if that student learns or not makes a huge difference. And them being able to explain it well where somebody else can understand it is a huge factor. (Initial Interview, Speech #70)

She later expounded at length on the level of dedication required of instructors and on the level of understanding that instructors and students, particularly re-entry students, should have about each other’s situations (Speech #98). She went on to bemoan the situation when an instructor is inappropriately placed in a classroom.

Hillary: [L]et's just take a grad student, for instance, that comes in here as a first year student and is thrown into a classroom situation, teaching something they've never
taught. You basically have from the time you're hired - which that could be up to 2 weeks before classes start - to prepare for it - to teach something you have never taught in your life. There's no way - humanly way - that person can get prepared to teach that effectively that first time. (Speech #102)

Hillary’s comments about student focus and the presence of re-entry students in the classroom (items 4 and 5 on her list of ingredients) were closely intertwined. The mere presence of other re-entry students in the classroom had been helpful to Hillary. She felt strongly that the focus of the older students was more constructive and that their focus affected both their own learning and the atmosphere in the classroom.

Similar to the comments that Hillary made about some of the reasons instructors may not be as effective as they should be, those she made about her interaction as an older student with recent high school graduates also reflected her sense of knowing how the academic world ought to function. While discussing the kinds of interactions there might be between students in different classes, she indicated that in an English class, although she “felt extremely out of place as the only non-traditional student in that classroom. . . . The interaction was good, because I was able to give them a perspective they didn't have [my emphasis] - being 17 - 18 year-old kids” (Initial Interview, Speech #54). She reported a similar situation in an orientation class.

When discussing the characteristics of a good student, she emphasized the goal-orientation of most re-entry students as different from that of traditional aged students.

Hillary: There's a reason that you start this as a non-traditional student. You don't just do it just for the fun of doing it. I guess there may be people that do, but um. You
need to have some kind of goal in mind. What are you wanting to do with this when you're finished? (Initial Interview, Speech #104)

She observed, “people that are 17 or 18 years old may not have clear idea of what they want to do yet” (Speech #104).

In addition to the apparent advantages to having the perspective of an older student, Hillary alluded to some of the disadvantages as well. In response to my follow-up question about her statement on the questionnaire that she sometimes did poorly in mathematics when she did not fully understand the material, she explained

Hillary: I would say probably at this point, it's the fact that I'm not just a student. I'm a full time employee. I'm a mother, wife; I'm extremely involved in my church. I think it's a matter of not having enough time to really focus - And I think it's probably the fact that I go right from class back to work with no time to sit and let that sink in. So by the time that I actually get to sit down with that work at night, it could be 10 o'clock, which has been 12 hours or so since I actually did it. And even though I take what I think are extremely good notes, sometimes . . . I don't understand that [i.e. something that has been presented in class]. . . . The fact that sometimes, by the time it gets to be 10 o'clock at night, you're tired and you're not at your probably best to sit down and figure out something that you haven't done for 20 years! I think that has a lot to do with it - at least for me it does. (Speech #42)

Hillary did not list interaction between students as an important ingredient in the classroom. She also indicated several times that students of varying ages have limited interactions both in and outside of class in any subject. Although there may be some reason for interaction between students in other classes, she did not see a need for it in a
mathematics class. Outside of class, for the other classes she had taken, she explained the
difference as follows.

Hillary: I did not have a study-buddy. Those weren't classes that you had tests, per se.
. . . [In the other subject I had to] write 5 papers. You had to do them yourself; you
couldn't have a partner. And for me, that comes a whole lot easier than math does, so
I didn't have any trouble with those. So, no, I don't know that it would be the thing for
any other subject. (Speech #84)

The structure of the course and her relative ease with the material precluded her from
working with other students outside of class.

The most constructive interaction she reported was that with her study-buddy, a
woman in her mathematics class who was also a re-entry student. They sat together in
class and went over notes and studied outside of class. Coincidently, both Hillary and
Peggy, her study-buddy, were participants in this study. They each mentioned their study
relationship, and two joint sessions in which they participated together were included in
this teaching experiment, permitting me to explore their interaction further. Hillary
described their relationship this way.

Hillary: Because there are things that I don't understand sometimes that she's got, and
vice-versa, and that's been real helpful. We studied together for every single test that
we had last semester, and when - ah - I guess just like siblings and just like in any
class setting, when you get close to somebody, there's been some - a little competition
on the side. O.K. let's see who can get the highest score this time. But, it's been a
motivation for both of us. And things that she doesn't feel real confident in - we've
really tried to boost each other's ego. I'll say, yes, you can do this. Don't say you can't,
because, yes, you can. We - There aren't too many weeks go by that we don't call each other on Tuesday night or Thursday night to homework-help either. (Speech #50)

The purposes of these phone calls and study sessions were to reinforce that which had been presented in the classroom by the instructor and to motivate each other. The two students did not see each other as independent sources of knowledge.

There were several indications that both Hillary and Peggy saw this relationship as mutually beneficial, but felt that Hillary was the stronger student of the pair. Their participation in this study both individually and as a pair in two sessions permitted me to form my own opinion about this comparison; that topic is explored later in a section addressing the collaboration between Hillary and Peggy.

Hillary’s Views about Her Place in Academia

On her questionnaire, Hillary indicated that her only purpose for taking mathematics in college was because it was part of her degree program. Hillary related in her initial interview that although her admittance score was not very far from the required score for a regular admittance, the slower pace of the two-course sequence in mathematics in Academic Assistance had benefited her. She had done very well and had enjoyed the feelings of success. Even so, she reiterated that she would opt not to take mathematics in college if it were not required. She questioned the value of taking “that kind of math. I am not going to use that in what I do. My math is adding and subtracting, basically” (Initial Interview, Speech #30). Furthermore, during the initial interview, she told of a paper that she was assigned in her college freshman class about her best or worst
subject in school. She wrote about her experiences with mathematics. She explained as follows:

Hillary: Those feelings, whether they were self-inflicted or I felt like I had been told that I could not do math, stayed with me that whole time period [from high school until now] . . . – After I made an A in my math class last semester – This is the first time in my life that I feel like I can do math ever. Do it well; I’ll put it that way. But, yeah, if I could get through without taking a math course, I probably would. I’m going to take the math and the science courses, and get them out of the way. Then the rest will be fun for me. (Speech #22)

When I inquired whether her current success could change her mind, she replied that faculty members with whom she worked in her on-campus job had encouraged her to move in that direction and take more mathematics courses. She had considered taking another course from the instructor she currently had if she could fit it into her program, because “I just really like her teaching style” (Speech #22). Here her study of mathematics was not her focus; rather it was the behavior of the instructor that was attracting her. The fact that she displayed considerable self-confidence and had expectations of success with the tasks she was asked to perform in this study lead me to conclude at first that the positive experience perhaps had desensitized her to some extent with regard to the study of mathematics. However, she was reporting a positive reaction to the teaching style of the instructor, not to the study of mathematics.

On her questionnaire, Hillary stated that her reason for returning to higher education was to obtain a degree in order to teach at the middle school level. She reiterated this goal during her initial interview. She mentioned the influence of her
experiences with her children on her choice of grade level to teach. She frequently remarked rather passionately about the impact that teachers make on their students.

She then added that, although she did not pursue her education at that young age, she had these ideas about her goals.

Hillary: I knew at 18 what I wanted to do, even though I'm 38 and I'm still not doing what I really feel like I wanted to be doing. I've done it in different circles, just not teaching in the classroom per se. But I've done it in other circles - teaching Sunday school, being the office manager. I'm teaching other people how to do what I do. So, I've done this teacher thing, just not in the context that I want to do it in. (Speech #104)

Hillary seemed to feel that she had some experience and considerable expertise and that she simply needed the validation of completing the degree.

Hillary’s Views about the Pursuit of Mathematical Knowledge

As was noted above, Hillary differentiated between mathematics and other subjects in terms of her ability to access and, indeed, the appropriateness of accessing the material on her own. Although she would read ahead in other subjects, she would not do so for the mathematics class. Although she stated that discussions in classes in which students contribute their personal views was appropriate in other subjects, such participation by students in mathematics classes, she believed, would not add to the value of the class. For a mathematics class, the questions that the students ask and the study sessions that she has outside of class were focused on reviewing and clarifying that which had been presented by the instructor. For her, mathematical knowledge was simply not as accessible as other subject knowledge unless an effective instructor was guiding the way.
The following table outlines the sequence of tasks in which Hilary engaged during the initial interview and each teaching episode.

Table 5.2: Timeline and Tasks for Hillary

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of session</th>
<th>Tasks and activities (All tasks are listed in Appendix C and are described as they are discussed in the text that follows.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 2, 2000</td>
<td>Initial Interview</td>
<td>Follow-up discussion from Initial Interview Task #1 (Picture Hanging) Task #2 (Retirement Party) Task #6 (Handshake) Discussion about formulas Task #11 (Gas Budget)</td>
</tr>
<tr>
<td>February 9, 2000</td>
<td>Teaching Session #1</td>
<td>Task #21 (1-6 in a Triangle) Discussion of student’s example about reimbursements for travel Task #1 (Picture Hanging), again Task #8 (Fieldtrip) Task #19 (Shop Stock) Task #10 (Jobs/salaries) Task #22 (Cows and Chickens)</td>
</tr>
<tr>
<td>February 16, 2000</td>
<td>Teaching Session #2</td>
<td>Task #21 (1-6 in a Triangle) Discussion of student’s example about reimbursements for travel Task #1 (Picture Hanging), again Task #8 (Fieldtrip) Task #19 (Shop Stock) Task #10 (Jobs/salaries) Task #22 (Cows and Chickens)</td>
</tr>
<tr>
<td>March 17, 2000</td>
<td>Joint Session #1 with Peggy</td>
<td>Task #12 (Spaghetti Supper) Task #4 (Fence Painting) Task #9 (Map)</td>
</tr>
<tr>
<td>March 20, 2000</td>
<td>Teaching Session #3</td>
<td>Task #17 (Paint and Carpet) Task #21 (1-6 in a Triangle), again Task #24 (Change in Bills) and symbolizing Task #22 (Cows and Chickens), again, symbolizing</td>
</tr>
<tr>
<td>March 29, 2000</td>
<td>Teaching Session #4</td>
<td>Discussion of recent test in course Discussion of researcher’s visit to the classroom on the day before the test Return to Task #4 (Fence Painting), which was explored in Joint Session #1 with Peggy Return to Task #9 (Map), which was explored in Joint Session #1 with Peggy Return to Task #22 (Cows and Chickens) to attempt to symbolize Task #20 (How many Triangles?) Task #29 (Farmer’s Field) Task #27 (Gameboard)</td>
</tr>
</tbody>
</table>
During the initial interview, Hillary stated that there have been times that she did not understand the material. “I just don’t get it! But, [the instructor] will go back, and she’ll show us a different way of working it, and I’ll say, ‘Oh, I see’” (Initial Interview, Speech #38). At the beginning of the First Teaching Session, in the process of clarifying some of the comments that she had made previously, I asked her to elaborate on this statement.

Hillary: I don’t know that she actually does it a different way [speaker’s emphasis].

She may take it slower in explaining it, and maybe go more step-by-step . . . She might say, ‘O.K. Do you have this step?’ Where she can figure what step we don’t actually have. I don’t think she explains it a different way.

Researcher: If somebody does that – presents a problem and then shows you another way to go about doing it, how do you feel about that?

Hillary: Well, I would think that probably at the time it would make me feel extremely frustrated. But if the second way made me get it, probably the end result would be o.k. (Teaching Session #1, 1:00:00 to 1:02:07 p.m.)
I then asked her about whether this sort of thing ever occurred when she was working with other students. She responded that when she was working with her study-buddy, one might have needed to actually do more of the steps than the other. However, their notes were basically alike, so the way they did the problems was the same.

There were several occasions during teaching sessions when Hillary displayed discomfort with the notion that there could be several ways to approach a task or became concerned that she might not be doing them in what she would consider an acceptable way. During Teaching Session #1, an interchange that highlighted well this concern occurred between us. About 20 minutes into the session, Hillary began working on the Handshake Task. (If there are 10 people in a room, how many individual handshakes will there be if each person shakes hands with every other person? See Task #6 in Appendix C) Frustration arose, and I restated the problem, at which time Hillary indicated that she did not like this type of task. I asked her what it was about this task that she disliked. She replied that it was hard for her to visualize it. After I asked what would help her to visualize it, she reluctantly began sketching the ten people, but again expressed her frustration.

Hillary: . . . I guess if I just had the straight numbers. Rather than just saying basically – And I want to say that this is because I don’t think mathematically, and I’m not going to say that because that’s not necessarily true. For me, I would really have to sit, almost, and draw 10 people out.

[Researcher encourages student to do what she needs to do.]

Hillary: Now, I’m going to be a realist here for a minute. If I was taking a test, I would not have time probably to draw out 10 people and try to figure that out.
Researcher: What we want to do is not to think about what you would do when you are taking a test, but rather what it takes for you to understand what that problem is saying. (1:21:47 - 1:22:37 p.m., speaker’s emphasis)

Hillary began drawing out the task and talking it out. She required lots of support and encouragement to take the time to sketch as much as needed. She began to see the pattern of one less handshake for each person, but remained uncomfortable.

Hillary: I’m sure that I should be able to just look at it now that I know that and do it. For me, this is when it gets frustrating. It becomes really frustrating to me personally [pointing to herself] that I can’t automatically figure that out. I would have to sit here and do that [i.e., draw a sketch] for every one of those ten people or nine people or however many people that are here.

Researcher: You just said – Did you hear what you just said? – You said that it [the number of handshakes] would go down one each time. So, would you need to draw them all out? (1:25:01 – 1:25:36 p.m.)

Hillary completed the task by listing the numbers of handshakes for each person and adding them up. Her discomfort prompted me to attempt to encourage her to think differently about what had transpired.

Researcher: The fact that you cannot think of an automatic way to do that does not mean -

Hillary: [Interrupting] It means that I don’t know how to do it! I don’t know how to do it efficiently. That’s the way I feel. (1:28:53-1:29:05 p.m.)

This type of interchange occurred frequently during the teaching sessions with Hillary. She rather clearly equated doing well in mathematics with those skills for which she
would expect to be rewarded in her usual classroom setting: recognizing a problem as
being like those that have been done in class or as homework, recalling the efficient way
to approach that type of problem and working quickly to get the answer. Not only did she
become frustrated when she could not accomplish these things on the tasks presented in
the sessions, but, in addition, when she perceived that I was not necessarily looking for
the skills that she chose to use. She required considerable support to approach non-
routine problem-solving tasks. She ascribed her inability to accomplish these tasks in a
routine way to either her inadequacy or to not having yet learned the method or formula. I
surmised that she considered mathematic knowledge as a well-defined package of
information; she seemed to think that when one was well informed, one could not only
get the right answer, but could do so in the most efficient way.

Hillary and Symbolizing Mathematical Situations

Several interesting interchanges occurred with Hillary regarding the symbolizing
of mathematical situations and the use of formulas. Her understanding of how formulas
come into being seemed to affect her ability to use them effectively.

Teaching Session #1

As I report elsewhere, Hillary became quite frustrated when she did not know the
way to work out a non-routine task directly. She frequently mentioned that she was sure
that there must be a formula that would make her work more efficient. Her first reference
to formulas came during the first teaching session after she had struggled with the
Handshake Task (See Task #6 in Appendix C) and expressed a desire for a formula that
would assist her.
Researcher: Where do those formulas come from, that people come up with?

Hillary: I don’t know. Certainly not from here [gesturing to her own head, laughing in a self-deprecating manner]. Well, that’s why we need formulas. How about that [speaker’s emphasis]? Because it does make things easier. Even if sometimes the formulas are hard to memorize. (Teaching Session #1, 1:29:05 - 1:29:29 p.m.)

Hillary indicated that one way to learn difficult formulas was to understand the way that they work, but, when pressed to explain, she discussed the need to use a formula correctly rather than to have a clear understanding of the underpinnings or derivation of the formula. She mentioned, as an example, the formula for finding the vertex of a parabola and emphasized the care that needs to be taken in substituting the negative of the coefficient b into that formula. When Hillary alluded to a time when she had to find a formula, I assumed that she was referring to deriving a formula, and encouraged her to talk about that.

Hillary: There was a problem on our practice test - Find the area of a triangle. I did not have a clue that, number 1 there was a formula to find that out, or what it was. But I studied it on my practice test and hopefully on my bonus question on the test, I got it right. Because I spent some time with that one working it out on the practice test in case it was on the test.

Researcher: So, you needed to find the formula for the area of a triangle?

Hillary: Uh, huh.

Researcher: What did you use to find that?

Hillary: The front of our book. A bunch of formulas are in the front of the book. So, I had to find it. But I know that one now. Now, I don’t know if I’ll remember it a
year from now when I’m not doing this, but I would like to think it would come back. (1:30:36 - 1:31:33 p.m.)

In Hillary’s way of thinking, formulas are derived elsewhere and come to her from external sources such as the instructor or the text. She considered it her responsibility with the assistance of the instructor to learn to utilize formulas correctly and to memorize them. This lack of a clear understanding of formulas in general as well as the formulas with which she deals was an apparent stumbling block in her learning. The episodes in the following pages from Teaching Sessions #2, #3 and #4 trace my attempts to explore this assumption further and assist Hillary in a clearer understanding of formulas. It was clear that Hillary considered formulas to be external to her own thinking. I must observe that I cannot imagine learning these things about Hillary without the kind of personal interchange that occurred within the teaching experiment.

Teaching Session #2

During teaching session #2, Hillary brought in and explained a procedure with which she was familiar in her work environment. The procedure was the one to be used when a faculty member has returned from travel and wants to be reimbursed for travel expenses. Following Hillary’s presentation of her own example procedure, I asked her to keep in mind the observations she had made about balancing expenses, and look back at the Picture Hanging Task (See Task #1 in Appendix C), which she first faced in Teaching Session #1. At that time, she had recoiled from the task indicating that she hated math and that she did not do measuring. She claimed that she would just visualize where the center of the wall was, but that her “husband would get out the measuring tape and put it dead center” (Teaching Session #1, 1:03:43 p.m.).
In Teaching Session #2, after her discussion of the example procedure she had brought in from work concerning reimbursement of expenses, the following interchange ensued.

Researcher: Keep in mind what you just said, and let’s go back to this [The Picture Task] . . . You said you would just eyeball it. I believe that’s what you said.

Hillary: That’s what I said. (Teaching Session #2, 1:20:07 – 1:20:14 p.m.)

I asked her to consider the possible results if she were to eyeball the situation and hang the picture. Hillary first mentioned needing the measurement of the whole wall and then said, “Once I had moved it, I would probably take a measuring tape and measure from here to the wall [one side of picture] and here to the wall [other side of picture] to see if it was centered” (1:20:37 - 1:20:43 p.m.). She acknowledged that if the picture were centered, the two measurements would be the same. I then encouraged her to consider what she would need to do if her estimate had been incorrect.

Researcher: So let’s say you eyeballed it, and you moved it over, and you discovered that this distance [pointing to the left side of the picture] was still 2” longer than that distance [pointing to the right side of the picture]. What would you then do?

Hillary: [immediately] I would move it over an inch [pointing to the left side of the picture].

Researcher: Why an inch?

Hillary: Because, if it’s two inches [too much] still this way, an inch would balance that out. (1:20:58 – 1:21:21 p.m.)

Hillary easily applied the same line of thinking to the task and repeated the process, at my suggestion, for an 8” difference. Then I asked what would need to happen
if the measurements were the ones given in the first part of the task. With little difficulty, she found the difference between the two measurements and divided by 2 to get the proper adjustment. At this point she laughed and agreed with me that she did indeed know what to do. (Teaching Session #2, 1:21:22 - 1:22:38 p.m.) She proceeded with the rest of the task, stumbling with the fractions, but each time finding the difference between the two measurements and dividing by 2 to get the proper adjustment to the right or to the left.

Finally, when I asked Hillary for a procedure for me to follow in general, she reverted to the idea of measuring the entire wall, finding the center and hanging the picture at that spot, and then laughingly added that she would either do that or “call my husband” to do it. (Teaching Session #2, 1:25:15 p.m.) Although she had demonstrated that she had mentally formulated a procedure and applied it repeatedly to the various steps of the task, she was resistant to formally stating that procedure in a general way. She seemed to avoid thinking of a formula as a way to express a rule or a procedure to follow or of the cases she had considered as examples of the use of a procedure. At least, her statements did not indicate that she observed the pattern and relationship between the examples and a general procedure.

Teaching Session #3

During Teaching Session #3, Hillary reached an untenable level of frustration with her inability to solve the Triangle Task (See Task #21 in Appendix C and Figure 5.4 below.) This task asks the student to place the numbers 1, 2, 3, 4, 5, and 6 in the spaces provided at the vertices and midpoints of the sides of an equilateral triangle so that the sum of the numbers on each side of the triangle is 10.
In an effort to reduce some of the tension that might be counterproductive and to continue in my exploration with Hillary in a more constructive way, I presented the Change-in-Bills Task (See Task #24 in Appendix C). In this task a customer pays for a $44 purchase with a $100-bill. The student is asked several questions regarding the way change could be made. The last question is: Using only $10-bills and $1-bills, how would the change be given using exactly 11 bills? Hillary easily worked through this task; then I asked her to symbolize the situation. She was unsure of what I wanted her to do until I suggested that this might be referred to as writing equations. With some guidance from me, Hillary had relative ease in writing a pair of equations that symbolized this situation. She knowingly chuckled when I remarked, “We sometimes think that formulas and equations are somehow magic, and are always given to use by somebody else. So, you have created your own” (Teaching Session #3, 12:58:15 - 12:58:22 p.m.).

During the 2nd teaching session, Hillary had grappled with the Cows-and-Chickens Task in which the student is to determine how many cows and chickens there are in a barnyard in which one sees 100 legs and 35 heads (see Task #22 in Appendix C). She was successful at finding a solution, but at that time she expressed displeasure with that task. Acknowledging that I understood that she was uncomfortable with that task, during Teaching Session #3, I asked her if she would mind trying to symbolize the
situation in a similar way to what she had just done with the Change-in-Bills task. Hillary willingly made an attempt to write equations to symbolize that task. Using $X$ for chicken legs and $Y$ for cow legs, she wrote $X + Y = 100$. Using $A$ for chicken heads and $B$ for cow heads, she wrote $A + B = 35$. So that she could write both of the equations in terms of the animals, she began to think about the relationship between the heads and the legs; that is, the relationship between $A$ and $X$ and between $B$ and $Y$. However, we ran out of time before she could complete her work. We agreed that, given adequate time, she would have been able to refine the equations. I was encouraged by her attempts, sometimes successful, to translate the relationships that she had encountered into equations. It seemed to me that the task of symbolizing a situation was not necessarily a stumbling block for Hillary. I was still interested in her beliefs about formulas and an incident in Teaching Session #4 was very revealing.

**Teaching Session #4**

During Teaching Session #4, I asked Hillary to re-visit the Map Task (see Task #9 in Appendix C). She and Peggy had worked on this task during a previous joint session. In this task, the students are given a map and asked to find the shortest distance, in terms of time and in terms of distance, between two locations. On one path, the speed limit is 45 mph on the entire path. On the other path, for one section that is common to both paths, the speed limit is 45 mph. For some of the rest of that path, the speed limit is 60 mph, and for the remainder, it is 70 mph. The two students had agreed on the approximate length in inches of each of the sections of the paths. Given the scale of 1 inch representing approximately 50 miles, they had developed a table of values, reproduced below as Figure 5.5. They agreed to head the columns of the table D for
distance, R for rate and T for time, commenting that this was similar to something they were doing in their course work.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>400</td>
<td>45 mph</td>
</tr>
<tr>
<td>B</td>
<td>100 + 50</td>
<td>60 mph</td>
</tr>
<tr>
<td>C</td>
<td>300</td>
<td>70 mph</td>
</tr>
</tbody>
</table>

Figure 5.5: Hillary and Peggy’s Table for Task #9, the Map Task

They had then struggled with remembering the formula they needed to use to compute the time needed to travel each path. In their discussion, they proposed that the formula relating D, R and T ought to be one of the following: D = T/R or D = R/T, but could not come up with a way to use either of those to arrive at a response.

Then, during the 4th teaching session, I asked Hillary to consider the task again.

Researcher: Now, tell me about the D-R-T [on the table]. What is the relationship between these [pointing to D, R, and T on the table]?

Hillary: Distance, Rate and Time is what those represent.

Researcher: What is the relationship between those?

Hillary: Because, you can use the distance and the rate to find the time.

Researcher: How would you go about doing that?

Hillary: [laughing] . . . I do think it’s distance equals rate over time [pointing to D = R/T, which is written on the sheet from the previous session], and then you would solve for T which would give you your time. (Teaching session #4, 1:12:19 – 1:12:45 p.m.)
I was interested in relating this information with something she had experienced, in order to assist her in thinking through the situation and understanding the formula properly.

Researcher: If I travel 70 miles per hour for 2 hours, how far will I have gone?

Hillary: [confirms the information in the question and then pauses to think for several seconds, then softly] I don’t know.

Researcher: What does 70 miles per hour mean?

Hillary: That’s your rate. That’s how fast you’re going. And I have my time, but I don’t know how to take those two numbers and get the distance. And I know there’s a way and I know there’s a way using that formula right there [pointing to \( D = R/T \)] just working it a different way. But, umm, I mean [pause] -

Researcher: Well, if I go 70 miles per hour for one hour, how far will I go?

Hillary: [pause, softly] I guess you could – [pause, more confident voice] I mean, I guess you could take and multiply those two numbers to come up with your distance how far you drove. You know, I’m trying to think in my mind. When I go somewhere for 2 hours and if I’m going at that speed, am I really covering 140 miles. That may be the case, but – Because I’m sitting here thinking about how long it takes me to get from here to [another city in the state] which I know is a little over –

Researcher: It’s about 90 miles.

Hillary: Yeah, and it takes me close to 2 hours to get from here to [the other city].

Researcher: But you don’t travel 70 miles an hour the whole time. (Teaching session #4, 1:12:47 – 1:14:48 p.m.)
Hillary agreed, and more confidently asserted that one should multiply 70 times 2 to get a distance of 140 miles.

We worked for a few more minutes with some rate situations, and Hillary seemed comfortable with the notion that rate multiplied times time should yield distance. During this discussion she recorded $R \cdot T = D$ on the paper, and I encouraged her to recognize that formula as the relationship that holds the table together.

Returning to the original task, Hillary noted that the path identified on the table as A was 400 miles long and that the speed limit on the entire path A is 45 miles per hour. I asked her to use what she had observed to calculate the approximate time it would take to travel that path.

Researcher: So now that you have this [pointing to the formula $R \cdot T = D$], how long will it take to go that distance?

Hillary: [long pause]

Researcher: What’s the formula that you decided kept this [pointing to the table] together?

Hillary: [long pause] I’m just playing, O.K.? [Writing and calculating on calculator, trying different approaches] I know that is not right!

Researcher: O.K., what is the stumbling block that you’ve run into?

Hillary: I’m trying to think, O.K., is it distance equals time over rate or rate over time, and there’s something in my head that says it’s rate over time. But if that is the case then we have 400, which is the distance, equal to the rate over the time, which is our unknown. Which means, I’ll have to solve for that unknown. And to
do that, because it’s a fraction, I have to multiply both sides times that [pointing to the T].

Researcher: Tell me again what you decided was the relationship between these [pointing to the D, R, and T on the table], the formula that made sense for distance, rate and time.

Hillary: Well, I mean, this [R•T=D] is what I decided. Which is – You would – [long pause]

Researcher: I’m wondering where –

Hillary: Hold on; hold on just a sec, O.K.? Let me play another minute. [Long pause, working, mumbling to self] Well, if I use this same formula and I do the rate which was 45 miles per hour, times the time which is unknown and put it equal to the distance, I can still solve for that variable and I guess the time is a little over 8 hours. [Not sounding convinced]

Researcher: Why were you troubled with this [the interim calculation using a different version of the formula]? [Pause from Hillary.] You calculated using this other formula, and you said, “let me play for a while.” What kept you feeling as though something you were doing was not correct?

Hillary: Well, because I was trying to use whichever one of these things [pointing to the two incorrect versions] works because I know you can use that formula that way, and when I did it this way [using D = t/r] I came up with 18,000. I tried to take that and divide it by 60 minutes. Umm, which didn’t come up. You know that came up to [looking on paper] 300. I knew it did not take 300 hours to get there. And so that’s when I thought, O.K., fine, which was using it this way [the
correct version] and that’s how I came up with that [8.8 hours] Where if I had
done it this way [using D = R/T, she writes 400 = 45/X] then I would have had to
multiply times X, which would have cancelled it out here and 45 would have
equaled 400X . . . [Some stumbling here with the computation, but she quickly
corrects herself] I should have divided both sides by 400, because I’m trying to
get the X alone. I don’t know. Obviously, none of that works, but I can do it that
way [using D = R • T] and that works. O.K.? Which tells me that even though the
formula in the book might have been set up this way [D = T/R], this stuff is
interchangeable. I mean, even though, it’s a division thing here, you can still work
it out using the multiplication formula.

Researcher: So, you’re comfortable with the fact that the formula for distance could
be rate times time or time over rate?

Hillary: I don’t know if it could be time over rate or rate over time. I’m not sure
which one, but I do know that these two things [one of the so-called division
formulas and the multiplication formula] will give you distance. I don’t know it
[that is, which one of the two division statements] works here I just can’t
remember.

Researcher: But you feel like one of those –

Hillary: It’s either distance equal time over rate or distance equal rate over time, and I
can’t remember. My gut says it’s this one right here [D = T/R], but I am just
going on trying to look at my little sheet of paper that that’s on, and I can’t
visualize it.
Researcher: O.K. Well, I just wanted to go back and see how you had built this
[gesturing to the table] based on what you know [gesturing to the formula, \( D = R \times T \)] –

Hillary: Well, this is really good to know, because I really hadn’t thought about it this way. If, when I get to my test, because I’m sure there will be one of those on there, if I can’t remember which way it is [gesturing between the two division formulas \( D = T/R \) or \( D = R/T \)], I can do it that way [gesturing to the formula \( D = R \times T \)], so, I mean, that helps me. (Teaching Session #4, 1:16:13 – 1:23:53 p.m.)

Hillary was strongly committed to what she believed that she recalled from the text. The sections in that text to which she was referring address the issue of solving the formula \( D = R \times T \) for the variable \( R \) in order to find rate or for the variable \( T \) to find \( T \). Thus, she was apparently misremembering the rewritten formulas \( R = D/T \) and \( T = D/R \), referring to them as the division formulas. Although her experience told her that distance, rate and time have a fixed relationship, Hillary apparently had a strong enough faith in what had been presented by the official sources of knowledge (i.e., the textbook or the instructor) to be able to accommodate having other possible, even contradictory, relationships hold simultaneously.

I personally have observed in students in my classroom this tenacious devotion to formulas and techniques that have been misremembered from previous classes. The students can demonstrate total competence in working in the correct way, only to revert to the misremembered information under pressure. Much like small children often remember those behaviors or vocabulary we wish they had never seen or heard, students seem to find it difficult to unlearn something that was miss-learned in a prior situation.
This behavior also seemed indicative of a willingness to separate what she knew to be true from her experience from what she was expected to know and do within a school setting. This willingness to separate out-of-school from in-school experience reduces the possibility for one to enrich or inform the other and permits such logically inconsistent notions to coexist as were reported above.

\textbf{Hillary and the Case of Being Too Close for Comfort}

During the first teaching session with each student, I made a special effort to select tasks that were related to the participants’ out-of-school experiences. Hillary’s description of her various work settings and other activities suggested to me that she might have shared the expenses for events with her co-workers, family or friends. She agreed that she had, so I presented to her, as the second task of the session, the Retirement Party Task. (See Task #2 in Appendix C.) In this task, the student is told that she and 4 other people have decided to put on a retirement party for someone in their office and plan to split the expenses evenly among all of them. Assuming that one person offers to use his home for the party and each person keeps receipts for expenses for stated amounts, the student is asked to describe how the expenses would be settled.

Hillary appeared quite confident as she began work on this task. She totaled the various amounts that were spent and divided by the number of people involved to get the amount for which each should be responsible. Feeling that she had completed the task, she moved right on to part (b) of the task in which the scenario is unchanged except for the amounts that each person spent. (I provided a calculator for the students to use, and I was interested in when they employed it. My assumption here was that part (a) might be completed at least partially mentally, while part (b) might require either paper-and-pencil
or calculator computation. In fact, Hillary chose to use the calculator for most of her calculations.

Referring to part (b), Hillary stated, “The same rule would apply, because . . . you are still going to split the expenses evenly” (Teaching Session #1, 1:05:17 – 1:05:29 p.m.). It seemed to me that Hillary felt that her work on part (a) was complete when she determined what each person’s share was. In order to redirect her to complete that part, I asked her to go back to part (a) and tell me what should happen next, once she had determined that each person’s share is $60. She remarked that

Hillary: I would probably just let them work it out amongst themselves [chuckle].

Had I been doing this to begin with, I would have known each person probably would not put the same amount in or I would have given them a specific amount. “Do not go over this amount” because everyone’s going to get the same amount of reimbursement. (1:06:07 – 1:06:23 p.m.)

Thus, she first suggested, with some humor, a way to avoid the task by leaving it up to those involved, and then gave me the first indication that she saw the task in a different context. I then understood that she did not see this as a situation in which those who spent more needed to be repaid by those who spent less in order to equalize their expenditures. Recalling her discussion of mathematics teachers from her high school experience and the role fairness played in her thinking, I tried to call her attention to that issue by pretending to be one of those who spent over the average and wondering how I could recoup the overage. However, she continued to insist that everyone should be reimbursed the same amount, the result being that those who spent less would get an extra amount back in their reimbursement, and those who spent more would not be reimbursed fully.
To apply the unfairness more directly to her, I point out that the task was worded in such a way that the person who was working on it (that is, Hillary herself) was listed as spending $80 while the fair share is $60; thus, her approach also short changed herself. Hillary then seemed to become irritated and affirmed that “Someone should have decided at the beginning how much money they were talking about” (1:08:17 p.m.). I pressed her to see if she could suggest a way to resolve the situation as it was written, and she finally became exasperated and declared that there really was no way to reimburse the two people who spent more than the fair share “other than these other people [the ones who spent less] pulling money out of their pockets, to make us all come up to sixty!” (1:08:56 – 1:09:03 p.m.). Although I was concerned about her level of frustration, I was encouraged by the thinking that was revealed in her remark and took this opportunity to ask her, “How would that look? . . . How much should they each pull out?” (1:09:06 – 1:09:12 p.m.). Hillary’s mood shifted dramatically; she laughed and waved her hands at me and reminded me that she did not like word problems. She agreed that her main stumbling block with the task was that she had assumed that the party was budgeted out of the department and the intent of the task was for the individuals to share the expenses. She finally proceeded to think about it this different way.

For Hillary, some of this computation came easily, but she also met some stumbling blocks. When she did, her frustration rose to the surface. At one point she said that she “doesn’t want to play anymore” and then vacillated between wanting someone to explain to her what she was doing wrong and immediately trying some other tactic. Even as she seemed to be very close to completing the task successfully, she expressed
exasperation by commenting, “O.K. Did anybody ever decide how much they were going to spend?” (1:13:57 – 1:14:06 p.m.).

Eventually, Hillary did complete part (a) of the task, bemoaning the amount of time that she had to take to do it. I asked her for only a brief comment about part (b), and directed her attention to part (c), which asked her to design a procedure to follow for situations like this. Hillary sighed and said, “Yeah, my situation would be to decide up front how much everybody’s going to spend, and don’t spend over that. That would be my procedure” (1:19:18 - 1:19:28 p.m.).

It seemed clear that Hillary never accepted the premise of the task, that the co-workers should equalize their expenses. Once she was convinced that she needed to do so, she had only minimal trouble conceptualizing what needed to be done. Although she used the calculator liberally, she had no difficulty performing the mathematical operations that were necessary. I surmised that she had had more experience with budgeting in advance than with sharing expenses in the way it was done in the task.

I had asked each of the participants to try to bring in samples of mathematics that they used outside the classroom, and Hillary inadvertently supported my observation with the sample procedure that she brought to our next teaching session. The procedure was the one to be used when a faculty member had returned from travel and wished to be reimbursed for travel expenses. Funds were encumbered prior to the actual travel and the “amount reimbursed . . . cannot exceed the amount of the original travel authority” (Document submitted by participant, February 16, 2000). The task that I had asked her to complete was also about reimbursement, but the context was different. The two incidents (The Retirement Party Task and Hillary’s submitted procedure) shed light on what
appeared to be Hillary’s difficulty with addressing a task that seemed so similar to her experience, but differed in such a crucial way.

**Summary**

Hillary spoke clearly with the critical voice that Kasworm (1997, March) described in her work. She was enrolled in higher education to gain credentials in order to pursue a career for which she felt that in many ways she was already qualified. She presented herself as fully savvy about the workings of the academic world and seemed confident about what it took to be successful in her coursework. She expressed some initial phobias about the study of mathematics, but she knew well how to operate in a mathematics classroom focused on rules and procedures. Hillary depended on formulas without regard to their meaning or how they might be derived. This dependence coupled with her separation between the mathematics of the classroom and the mathematics of her experience permitted her to either blindly apply formulas without thinking more deeply about the tasks or to apply incorrectly remembered formulas without troubling over the inconsistencies that resulted. The atmosphere of the teaching experiment, which asked that she think differently about these issues, was very uncomfortable for Hillary. Using Skemp’s image of the frontier zone (1987), Hillary’s frontier zone seemed to be very narrow. At times, it seemed to be a wall. She either stayed behind it, remaining in her established zone, or resisted participation, indicating the task was in her outside domain. Faced with a non-routine task, she usually attempted to make it routine by making it fit some established pattern from classroom experience (a formula or procedure that she had learned) or resisted exploring it. This behavior also affected her ability to link her real world mathematics experience with the classroom or to the tasks presented to her in the
teaching sessions, unless the tasks were virtually identical to those with which she dealt in other settings.

Peggy

Introduction

Peggy’s Background

Peggy, a 35 year-old white female, graduated from high school 18 years ago. She enrolled in college briefly 12 years ago, but family situation and finances caused her to drop out. Having been employed on campus, she was able to take advantage of a special accommodation for employees, work one instead of two jobs and “concentrate on school” (Initial Interview, Speech #20). She still had the responsibility of caring for her mother, but the financial support of the university had permitted her to return to higher education. She was required to take Academic Assistance courses in English at the second of two levels and in mathematics at the most basic level. She also was advised to begin with the mathematics course, and was enrolled in the second semester of the two-semester sequence while involved in this study.

During the years since her high school graduation, Peggy held a number of secretarial positions each of which she described as including additional responsibilities in a variety of areas. For example, working for the real estate agent involved personnel work. At the newspaper, she would sometimes be involved with the paste-up of articles; when she moved to the accounting department, the job included more bookkeeping. On campus, the logistics of working with both students and faculty add to the dimensions of her secretarial duties. Peggy stated on her questionnaire that she returned to higher education to “increase knowledge” and to get a “degree that will give me the ability to
work in a field I have always loved.” She explained in her initial interview that she planned to get her degree in Art History because of her love for art and that her dream was to work at a renowned museum in antiquities.

Peggy’s Views about the Roles of the Learners, the Instructor and the Nature of Knowledge

Peggy’s views about her own role as a student, the role of her peers, the role of the instructor, and the nature of knowledge were closely intertwined. The fact that she seemed to see the nature of mathematical knowledge as different from knowledge in other areas impacted on differences she saw in the roles of the participants in the process. Her feelings of inadequacy relative to the study of mathematics were also at work here. I begin with a discussion of her views about these roles and explore some of the impact of her views about the nature of knowledge and her feelings of inadequacy in mathematics. This discussion assists me in situating Peggy in the academic setting. These same views and feelings – her belief structure – are further illuminated as I continue in my exploration of her mathematical thinking.

Situating Peggy in the Academic Setting

Peggy stated on her questionnaire that when she did well in mathematics it was because “My teacher is incredible,” and that when she did poorly, it was because “My teacher is not as good at explaining and I do not make any extra effort.” She expanded on this partnership in the initial interview when she made this statement.

Peggy: But [the instructor in which she is presently enrolled] is really good at explaining it so I can grasp it. And even if I can’t grasp it, I can work at it long
enough to where I can understand it without getting frustrated and giving up. And in my past experience, I was intimidated. (Initial Interview, Speech #24)

While discussing the possible differences in her views about different subjects, Peggy indicated that although she did not recall that her high school teachers in other subjects were any better than in mathematics, the other subjects “came to me easily . . . I grasped it easier. Whereas with the math, I needed that extra that wasn’t there and because it wasn’t there, I just gave up on it” (Initial Interview, Speech #30). She later continued this thought.

Peggy: I’m not as afraid of it as I used to be. I actually think now that whatever it is, I can learn it given the proper instruction and a certain amount of time. I probably could learn it. Whereas before, I thought I just could not learn it . . . I just did not have that part of my brain going. I don’t feel that way anymore. (Initial Interview, Speech #38, emphasis added)

Toward the end of the interview, when asked about what she brought to the classroom from her experiences since high school, she became a bit emotional about the fact that when she was in high school, “if I couldn’t get something right away, then it had whipped me. But I think that experience has taught me that if you will stick it out and try hard enough, there’s not much of anything that you can’t get” (Initial Interview, Speech #110). Although Peggy may not have been describing a deep understanding of knowledge, she was clearly acknowledging the definitive roll of the learner (herself) in the learning process, whereas she indicated that during her high school career the responsibility for learning in areas where she struggled was primarily in the hands of the teacher.
As was already explained, Peggy and Hillary had become study partners prior to their involvement in this study. Their study sessions had included clarifying notes from class and explaining to one another what they had learned in the classroom. It had not been unusual for them at least to telephone each other after each class meeting to confirm that they had the same impressions about the way the instructor had presented material and the way they should complete the assignments. As Hillary had referred to her, Peggy also referred to Hillary as her study-buddy and mentioned the interaction between them frequently during our sessions, usually indicating that Hillary was a “quicker study” (Initial Interview, Speech #60) or more apt to be the helper while Peggy was the one being helped. Peggy’s perception of herself as a learner became a key element throughout this study; I explore it further in another section. Although Peggy reported that these exchanges were active, she indicated that they had been solely based on the information and the method the instructor had presented in class. That is, Peggy saw her peers as possible reinforcers of that which the instructor had presented but not as sources of information themselves. She had also gotten together with another friend who was not in this class and was planning to choose mathematics as a minor. This friend had functioned as a tutor, providing Peggy with some “one-on-one [assistance] that is hard for the teacher to do” (Initial Interview, Speech #76). Thus, within the study of mathematics, the instruction delivered by the instructor was of paramount importance to Peggy. Her view was that the primary role of the student was to receive that instruction, and peers could be helpful only in assisting one another in that task.

In addition, there was another rather significant dimension to Peggy’s views with regard to the role of peers within the classroom. (This dimension was also reflected in
Jennifer’s remarks, as we saw in her case study in the previous section of this chapter.

Peggy valued the interaction between students and the instructor, as well as the interaction among the students themselves, but to a different extent. When asked during the initial interview what the important ingredients were in the classroom setting, she listed (a) the atmosphere in the classroom permitting student interaction with the instructor by asking questions, (b) the interaction between the students themselves, (c) the instruction including good explanation, and (d) the instructor’s attitude. (See pie charts in Figures 5.6 and 5.7 below.)

![Figure 5.6: Peggy’s Ranking of Classroom Ingredients by Importance](image)

Figure 5.6: Peggy’s Ranking of Classroom Ingredients by Importance

![Figure 5.7: Peggy’s Ranking of Classroom Ingredients by Time](image)

Figure 5.7: Peggy’s Ranking of Classroom Ingredients by Time

She then was asked to indicate on a pie chart how she would indicate the relative importance of these ingredients and, on a second pie chart, to indicate how the time in the
classroom should be allocated to these ingredients. She assigned large sections of about
the same size in each of these charts to (a) atmosphere in the classroom (students’
questioning of the instructor) and (c) instruction, and a smaller section to (b) student
interaction between themselves. In the importance pie chart, she assigned a medium sized
section to (d) the instructor’s attitude. Peggy indicated that student interaction was very
important outside the classroom, but not as important to her learning within the classroom
setting. For her, interaction between the students and the instructor in the mathematics
classroom consisted almost exclusively of students asking questions and seeking
clarification based either on the instructor’s presentation or on difficulty with previously
assigned work.

Peggy: A lot of times someone else will ask a question about something that you
haven’t got and it will clarify it for you where you didn’t really understand it . . .
[T]he students feel comfortable enough to say: I don’t understand. Could you go over
that again? Could you go over that again? [Repetition in the student’s speech.] I think
that makes a huge difference. And I think that’s the most important thing to me.
(Initial Interview, Speech #48)

She continued,

Peggy: Sometimes I think that, well, this is a certain way . . . and then someone will
ask a question about it and it will clarify it a little bit further, even though I thought I
understood it. I’ll understand it a little bit more than I did just simply because
someone else asked it, but I hadn’t thought it. (Initial Interview, Speech #50)

Thus, the interaction between other students and the instructor as her peers ask questions
enriched the learning experience for Peggy in the mathematics classroom. I would like to
make two observations about this way of thinking. First, as I delved further in my exploration of Peggy’s perspective about the nature of knowledge, she revealed that she saw the kind of interaction between students and instructor as having a different focus in other subjects than it had in mathematics. In the mathematics classroom, interaction between the students and the instructor was limited for the most part to students indicating when they were unsure of the material previously explored, and seeking and receiving clarification. Secondly, Peggy did not see peers as a source of different points of view or of knowledge about the subject. On the other hand, she did see their role as supportive to her learning as their freedom and willingness to air their points of confusion helped her to uncover inadequacies in her own understanding. Most importantly, she remained open to the conversation in the classroom about something that she previously had believed that she had full understanding and was able to listen and accommodate to what was being discussed by others. Using Skemp’s model of the frontier zone (1987), her behavior might be described this way: Although Peggy believed that certain knowledge could be within her established zone, she seemed to let it remain in her frontier zone, open to further adjustments. The alternative behavior that one might observe in the classroom would be for a student to ignore questions by peers when the student him/herself feels confident of the material; at times such students seem resentful of having class time taken in such activity. It is interesting to note that, during the initial interview, when Peggy was asked about the ingredients in the role of a student that are important to being a good student, she replied,

Peggy: You’ve got to be open-minded . . . I had a closed mind when it came to math. . . I had to tell myself that I could not be that way or I would defeat myself before I
ever started. . . . If it’s any subject you don’t like, you’ve got to be open-minded when you go into it, and have a positive attitude . . . You know, I can learn it. (Initial Interview, Speech #104)

Peggy was referring to open-mindedness with regard to learning the subject in general in spite of fears or feelings of inadequacy, but her comments about the way she learned from others’ questions lead me to conclude that her open-mindedness extended to more specific learning situations.

Although, as reported above, Peggy had expanded her view on the responsibility of the student to make an effort in learning, she still described the role of the instructor as being dominant in the classroom. She described a good instructor as one who both establishes a supportive atmosphere in the classroom and communicates the knowledge to be learned appropriately. She defined good instruction as “the professor’s ability to explain things in a way that everyone in there can understand, without making anyone feel stupid” (Initial Interview, Speech #62). Peggy saw the role of the instructor differently for different subjects. When faced with learning basic information in areas such as history,

Peggy: those are things that I feel comfortable enough to grasp whether my instructor’s any good or not. But now, in my math class, if I don’t have a good instructor, I’m in deep . . . I’d want a good instructor, but it would not be important to me as it is in my math class. (Initial Interview, Speech #86)

Above, it can be seen that Peggy attributed this enhanced need for good instruction in mathematics to her estimation of her inability to grasp the information. She observed that for her this need resided in the study of mathematics whereas for another
student who is weak in another subject, that student would need stronger instruction in that subject.

She also differentiated between the roles of instructors and of students in the different subject areas based on the nature of the knowledge itself. In comparing a literature class with her experience in mathematics, she indicated,

Peggy: There’s more interaction between the students in the [literature] class. It’s more because it’s all about discussing what you read and your opinion about it. Whereas in math it’s more: Now we’re going to study factoring, and this is how you do it. So, there’s a lot of instruction and watching and learning. Now, the literature class is more talking . . . There’s more classroom interaction. It’s just a different subject matter. I mean, you don’t sit around and talk about why does x = a. It doesn’t matter; because that’s just the way you do it. I mean, I think math has more rules to it, so it’s done a certain way. (Initial Interview, Speech #90)

Another example of the interplay between Peggy’s perception of her own ability to learn mathematics, her views about the nature of knowledge in mathematics and her views about good instruction arose in her comments about being shown applications and alternative methods. During the initial interview, Peggy reported that

Peggy: [S]ometimes [the instructor] will start talking about something we’re not doing and I’m like, “La, la, la, la [with ears covered and eyes closed].” I don’t want to know! And I also can’t deal with multiple ways of doing things in math. I need to know one way to do it. Don’t give me options because it just all gets confused. (Initial Interview, Speech #84)

She reported the same behavior when working Hillary during their study sessions. In fact,
this behavior occurred during one of their joint sessions in this study. However, I must add that I would associate Peggy’s rejection of listening to different approaches and applications only to her own self-reporting of her actions in the classroom and in connection with school mathematics. While dealing with Peggy during the teaching experiment, in spite of her claims to the contrary, her willingness to modify her thinking and consider alternative approaches was quite evident as is reflected in the last section of her case study.

To summarize some of the subject-related differences in Peggy’s perspectives, I stated the following:

Researcher: What I am hearing is that within the class itself the teacher is foremost in the math class and, in other subjects, there’s more interaction between the students and the teacher. Outside of class, for a math class, you really do value being able to interact with other students or someone else, whereas in other subjects you don’t.

(Initial Interview, Speech #93)

Peggy agreed completely with my summary. It seems to me that the driving forces in these differences were Peggy’s perception of herself as a mathematics learner (resulting in added dependence on the instructor for support in the classroom) and her views about the nature of mathematical knowledge as different from other subjects.

As has been previously discussed, Peggy regarded the mathematics instructor as the primary, perhaps the sole, source of knowledge. She avoided using the textbook as a source (a topic to which I return later) and considered the most constructive interaction with peers as that which further clarified what the instructor had presented. Thus, for her, the power structure in the mathematics classroom was based on an all-knowing
instructor, but she did not portray herself, the student, as powerless. She did not see herself as a novice in the academic world, nor did she behave as one. Indeed there was an element of joy in Peggy’s attitude toward being a student at the university.

Peggy stated on her questionnaire her reasons for returning to college as (1) “to increase knowledge” and (2) “my degree will give me the ability to work in a field I have always loved” and indicated during the initial interview that she wanted to get her degree in Art History in order to pursue a dream of working in an art museum (Speech #12). Although she was working toward a degree in order to pursue a specific occupational goal, she did not present a cynical view about this requirement. She valued both the worlds of knowing and doing and viewed herself on a learning journey. She seemed to acknowledge the connection between the academic world and real-world knowledge, at least with regard to knowledge in general.

Peggy stated on her questionnaire that her only purpose for study of mathematics in college was that it was required and further explained, during the initial interview, that if a mathematics course were not required she would not take one (Speech #22). She talked about memories of not liking mathematics and not doing well, of teachers that were either not knowledgeable or not helpful, and of a lack of encouragement from others. Hence, at the outset, it was only for the purpose of getting her degree that she studied mathematics. However, even with regard to mathematics, she seemed to be attempting to articulate the value of its study. When asked in the initial interview to comment on the usefulness of mathematics, Peggy shared these thoughts:

Peggy: I never really considered it past basic, general math, until I started coming to this class. . . . I complained before I started last semester. I’ve got to start with this
algebra class. Now, when am I ever going to use this again – You know, the typical thing. And [the director that I work for] said you know your whole life is spent on finding x. In every problem you’ve got to solve, that’s what you’re doing. And when he said that, I thought, gee, I never really thought about that before. So, I kind of looked at it a different way. And then when – This past weekend, I work for the school of music, so we went to a conference in Atlanta, and we had this big, collapsible – I don’t know how to explain it – but anyway – you pop it out and when you collapse it, this huge thing that would fill up this room comes to about this wide [indicating a few inches with her fingers]. And it fascinates me. It just totally fascinates me. And we popped it down, and my friend says, “That’s just mathematics – That’s all that is; just mathematics.” And I was like, yeah, it is! So I think I appreciate its contribution more now, than I did before. I know really how important it is, where before I thought it was just the bane of my existence. (Initial Interview, Speech #106)

Peggy protested that she was not interested in learning about applications in the classroom, but responded positively to the opportunity to see this connection within the setting of the teaching experiment. It is possible that these changes in her perspective about the nature of mathematical knowledge could portend shifts in her views concerning the participants in the classroom environment, especially considering how intertwined all of these beliefs are for her.

To understand further Peggy’s perspective about knowledge in general and mathematical knowledge in particular, I explored her views about the accessibility of that knowledge to her. During the initial interview, while Peggy was talking about the
activities in which she engaged outside of class, she mentioned time spent alone studying. When I asked her what kinds of things she did when studying, she replied that she worked problems, more than those that are assigned in areas where she was having difficulty. To clarify, I asked if the problems were always from material that already had been presented in the classroom, and she said that they were. I then asked whether she ever read ahead in the textbook, and she replied,

Peggy: No, that confuses me and then I get confused with what is coming up and what I’m doing now. So, I don’t do that. Now, I do that in other things, but I don’t do that in math. I concentrate on whatever I have to know right now. (Speech 84)

She later continued to explain that she needed more guidance in the study of mathematics.

Peggy: When it comes to facts like in a history class where, you know, we signed the declaration of Independence in 17 hundred and things like that, you need a good instructor, but I can usually learn those dates and I can read regardless of how good my instructor is. I mean, those are things that I feel comfortable enough to grasp whether my instructor’s any good or not. But now, in my math class, if I don’t have a good instructor, I’m in deep – And I guess I would – I’d want a good instructor, but it would not be important to me as it is in my math class. (Speech 86)

As was discussed earlier, for Peggy, student interaction outside the mathematics classroom was focused on what had already been introduced by the instructor, and in the mathematics classroom students’ participation was involved in clarifying that instruction. In other subjects, students’ opinions and interpretations might be appropriate. Thus, from every angle, Peggy expressed the perspective that neither she nor students in general can
access mathematical knowledge on their own. (This issue of the accessibility of mathematical knowledge also arose during discussions with Jennifer and Hillary, and I explored it in their case studies in the two prior sections of this chapter as well.) The following sections explore Peggy’s mathematical knowledge, skills and thinking, as well as her approach to problem solving as they emerged during the teaching sessions. Table 5.3 outlines the sequence of tasks in which Peggy engaged during each of the teaching sessions.

Table 5.3: Timeline and Tasks for Peggy

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of session</th>
<th>Tasks and activities (All tasks are listed in Appendix C and are described as they are discussed in the text that follows.)</th>
</tr>
</thead>
</table>
| February 2, 2000 | Initial Interview      | Follow-up discussion from Initial Interview  
Task #1 (Picture Hanging)  
Task #2 (Retirement Party)  
Task #6 (Handshakes)  
Task #8 (Fieldtrip)  
Task #11 (Gas Budget) |
| February 9, 2000 | Teaching Session #1    | Task #8 (Fieldtrip – modified)  
Task #17 (Paint and Carpet)  
Task #10 (Jobs/salaries)  
Task #21 (1-6 in a Triangle)  
Task #22 (Cows and Chickens) |
| February 16, 2000 | Teaching Session #2    | Discussion of Task #21 (1-6 in a Triangle)  
Task #26 (Arrays)  
Task #27 (Gameboard)  
Task #24 (Change in Bills)  
Task #25 (Change in Coins)  
Symbolizing Tasks #24 & #25  
Return to Task #22 (Cows and Chickens) |
| March 15, 2000   | Teaching Session #3    | Task #12 (Spaghetti Supper)  
Task #4 (Fence Painting)  
Task #9 (Map) |
| March 17, 2000   | Joint Session #1 with Hillary |  

March 22, 2000  Teaching Session #4  Continue with Task #27 (Gameboard)  Task #29 (Farmer’s Field)  Return to Task #17 (Paint and Carpet)  Return to Task #9 (Map), which was explored in Joint Session #1 with Hillary

April 5, 2000  Teaching Session #5  Task #33 (How many Triangles? #2)  Task #34 (Couples)  Task #36 (Balancing)  Task #35 (Rectangles – Area and Dimensions)

April 13, 2000  Joint Session #2 with Hillary  Task #3 (Merchant and 3 Fairs)  Task #28 (Area of a Lake)  Task #38 (Fencing)  Task #37 (Space Station)

May 11, 2000  Exit Interview

**Peggy – Problems with Area, Perimeter and Multiplicative Reasoning**

Peggy’s computational skills were very weak, but she was rather adept in her use of the calculator. It became evident rather quickly that the weakness in her multiplicative thinking could be a much larger stumbling block for her in dealing with mathematics. She recognized what she described as her phobia with regard to fractions and admitted her confusion concerning area and perimeter. Multiplicative reasoning is central to so much of the mathematics which Peggy needed to master that it seemed important to try to plumb the depth of her deficit in this area and to attempt to help her strengthen this weakness.

It was during the second teaching session that Peggy’s confusion about the concepts of area and perimeter surfaced while she was addressing the Carpet and Paint Task (see Task #17 in Appendix C). Although it seemed at first that her confusion between the two concepts could be addressed rather easily, it became evident that Peggy had considerable difficulty with the concept of area. Also, her work on the Cows and Chickens Task (see task #22 in Appendix C) highlighted her tendency to think in terms
of repeated addition rather than multiplication. I concluded that these difficulties were both related to her multiplicative reasoning. I therefore decided to select for Peggy tasks that would help her focus primarily on the concept of area and her apparent confusion between area and perimeter. In addition to addressing that particular concept, my intent was to assist Peggy in her development of multiplicative reasoning skills. The context of the teaching experiment permitted this exploration into an arena of specific concern to her. Even in our limited time together, noticeable progress was made. In this section, I present some of the actions that were taken, both by Peggy and by me, as well as some of the results.

During the first teaching session, Peggy demonstrated considerable ease with the quantities and concepts involved in the five tasks as long as she could avoid fractions. She generally accomplished this by either ignoring them or converting them to decimals. When she converted between fractional notation and decimal notation, she did so only with familiar quantities (such as the equivalence of 0.5 and one-half or of 0.25 and one-fourth). This remained the case throughout the teaching experiment. Due to this underlying difficulty with fractional expressions, I often took the opportunity to include examples that involved fractions within the explorations of multiplicative reasoning and, more specifically, area.

Our First Look at the Carpet and Paint Task

During Teaching Session #2, as soon as I presented the Carpet and Paint Task to Peggy, she claimed that she did not know how to calculate area and perimeter. Thus began our ongoing exploration of the area and perimeter. She immediately recalled the formula for the area of a rectangle as \( A = H \times W \) and described the perimeter as the
distance around the figure, but had great difficulty applying either the formula or the
description. It seemed crucial to me that she stated the formula of the area of a rectangle
and the definition of perimeter, but still knew that she lacked understanding of the
concepts. In the case of the area, this appeared to be a case of Peggy having memorized a
formula but not understanding the concept. As I learned throughout this teaching
experiment, even if she recalled a formula or technique to employ, Peggy was reticent to
employ it if she did not comprehend what she was doing. In the case of the perimeter, I
was uncertain about the barrier to Peggy using a concept she had clearly described.

Using my suggestion that she imagine that she was tiling the floor of a 10 foot by
15 foot room with 1-foot by 1-foot tile squares, Peggy was able to calculate that one
would cover the floor with 10 x 15 or 150 tiles, but she was still troubled. “I feel like I’m
leaving out part of the room when I don’t do all the sides” (Teaching Session #2, 5:59:24
– 5:59:29 p.m.). This was the first of several times that Peggy expressed concern about
not using all the sides when calculating the area of a rectangular region. Although I had
called upon an image of filling the floor with squares of tile, Peggy did not see the
multiplication of the two dimensions as a way of counting those tiles. She was
multiplying because the formula told her to multiply and was left with the feeling that she
had not taken the entire room into account by leaving out the other two sides’ lengths.

Among other tasks, during Teaching Session #3, I presented Peggy with several tasks
involving arrays in order to address this possible barrier in her thinking.

In the continuation of the carpeting the room problem in session #2, I asked
Peggy how much baseboard she would need to go completely around the room.
Peggy: 150 feet? No. . . . You’re only going to put it this way [tracing around the outside of the rectangle, then pausing] I don’t know.


Peggy: . . . I’m not covering the whole floor. I’m just doing the outside [as she continues tracing the shape]. You need 10 feet here and 10 feet here and 15 feet here and 15 feet here. [Soft voice and small smile of satisfaction] O.K. That’s the perimeter.

Researcher: Do you see the difference [between the area and the perimeter]?

Peggy: Yeah. One is actually filling the floor. The other is going around. [Peggy uses her pencil to demonstrate the two.] (Teaching Session #2, 5:59:56 – 6:00:58 p.m.)

Three interesting events occurred here. First, although Peggy was asked the amount of baseboard needed for the room, not to find the perimeter of the room, she quickly made the association for herself. Second, although she never stated a formula for the perimeter as such, she revealed the possibility of some knowledge of it when she indicated what the calculation would be. That is, although she traced around the room sequentially (length, then width, then length, then width), she referred to adding the two widths and the two lengths to get the total needed. Finally, as she became more comfortable with her understanding of the difference between the two concepts of area and perimeter, she used a description of covering for area (instead of the formula) along with her description of going around the figure for perimeter. The gameboard tasks along with the arrays in Teaching Session #3 were selected to reinforce this way of thinking and to strengthen Peggy’s understanding of the concepts.
In the continuation of Teaching Session #2 we returned to the original task of carpeting and painting a room. Peggy was able to handle rectangular rooms but not non-rectangular rooms such as L-shaped ones. Thus, her actions had provided me with suggestions as to how to proceed, but did not indicate a well-developed understanding. It should be noted that although she was not able to complete the task and seemed intimidated by the non-rectangular rooms, when Peggy did experience some success, she displayed pleasure and satisfaction and showed appreciation to the researcher for, as she saw it, having shown her the difference between area and perimeter. Peggy’s general behavior with this task and the mixed emotions that she displayed fit well into Skemp’s description of the frontier zone (Skemp, 1987, p. 196), where, given appropriate tasks, she would be most apt to learn. This reinforced my decision to focus on area and perimeter during part of my sessions with Peggy.

**The Chickens and Cows Task – A Challenge for Peggy’s Multiplicative Thinking**

Later in Teaching Session #2 Peggy worked on Task #22, which asked her to determine how many cows and chickens were in a field given that there were 35 heads and 100 legs. It was during her work with this task that Peggy’s difficulty with multiplicative reasoning seemed to prevent her from capitalizing on her intuition.

After she had examined a series of proposed scenarios, she reached the stage in which she had 25 chickens (providing 50 legs) and 10 cows (providing 40 legs) and sat pondering it. I asked her to review what she had and what she needed.

**Researcher:** What are you looking for? What is your goal? What do you have?

**Peggy:** Ninety [legs]

**Researcher:** What is your goal?
Peggy: 100 [legs]

Researcher: What did you just tell me [in the previous stage]?

Peggy: Every time I go up [1 cow] and down [1 chicken], I only gain 2 [legs]. I mean, 2 will go into the 10 five times. (Teaching Session #2, 6:36:59 – 6:37:22 p.m.)

That is, she had observed that each time she decreased the number of chickens by one and increased the number of cows by one, the result was to increase the total number of legs by 2, while keeping the number of animals (and therefore animals’ heads) at the required 35. She further acknowledged that ten, the number of legs she needed to gain in order to get the required 100 legs, divided by 2 was five. However, Peggy did not realize the implication of this observation (and thus reduce the number of chickens and increase the number cows by five each). Instead, she first lost this train of thought completely and interpreted the need for 10 legs as a need to add 1 chicken and 2 cows. This action increased the number of animals from the required 35 to 38. Then, after I redirected her to her previous observation, she was able to make the necessary adjustment by changing the number of animals one at a time, but not by changing the number of animals by 5 each. This use of repeated addition rather than multiplication supported and expanded on my interpretation of Peggy’s behavior earlier in the Carpeting Task. In the prior case, she did not understand that by following the formula and multiplying she was summing the tiles in the rows. In this case, she did not see multiplication as a way to add repeatedly. That is, for Peggy, it did not seem that repeated addition and multiplication were related.

Thus, it appeared that the difficulties that Peggy had previously demonstrated with the calculation of area and the above apparent weakness in her multiplicative
thinking were related. Again, the use of tasks involving arrays appeared to be an appropriate choice for the following session.

During our conversation as Teaching Session #2 ended, Peggy concurred that she had not been listening to her own observations and that she had been intimidated by the problem. Although, as I had indicated earlier, I had chosen to focus to a certain extent on area and perimeter with Peggy, I presented her with other tasks not necessarily related to this realm of mathematical thinking for several reasons. First, I thought it was important for the purposes of my research to observe how she operated in other mathematical settings. Second, other issues such as whether Peggy could be encouraged to listen to her own observations were worthy of pursuing. Finally, the frustration and anxiety that Peggy exhibited during certain tasks and described here by her as intimidation seemed to need to be relieved in order for her to be able to explore further (Skemp, 1987, p. 192). Because this section of this document concentrates on Peggy’s problems with area and perimeter, I will not report on the other tasks selected except to the extent that her actions reflect on those concepts and her multiplicative thinking. More extensive discussion related to other aspects of Peggy’s mathematical thinking demonstrated during her work in the sessions is presented in the following section of this paper titled “Peggy as a Mathematician.”

Our First Look at Arrays of Items

During Teaching Session #3, after re-visiting the Triangle Task (see Task #21 in Appendix C), we began considering the arrangement of arrays of items by introducing Task #26 (see Appendix C). It should be noted that the context of Task #26 was selected with Peggy’s recent work experience in mind. When asked in the first part of the task to give the possible arrangements in a room of 36 desks, Peggy immediately suggested 6
rows of 6. She then used a calculator and concluded that one could have 2 rows of 18, 3 rows of 12, and 4 rows of 9. She mumbled that she would skip 5 and then used a calculator to confirm that 7 would not divide into 36 evenly. (When division of 36 by 7 resulted in a decimal, she laughed about it being part of a row.) When asked to explain what she was doing, she responded as follows:

Peggy: I just knew 6 times 6 is 36. And then I knew that 2 would go into 36. And I was pretty sure that 3 would go into 36 and then I just started going up. And I knew that 5 wouldn’t [divide into 36] because it’s not a multiple of 5. And then I just went from there. (Teaching Session #3, 5:57:55 – 5:58:10 p.m.)

After finding that she could use 9 rows of 4 by dividing 36 by 9, Peggy remarked that that case was the same as the previously listed 4 rows of 9. She still kept dividing by candidates but stopped at 16; that is, at dividing 36 by 16. Discussion lead Peggy to acknowledge that having found 4 rows of 9, one could automatically list 9 rows of 4 as a possibility, and, similarly, 12 rows of 3 and 18 rows of 2 would follow from the cases of 3 rows of 12 and 2 rows of 18, respectively. It is of interest that Peggy could acknowledge the commutativity of the multiplication in discussion, but did not use it to generate other possibilities until pressed to do so. This apparent inconsistency was reminiscent of Peggy’s initial stating of the formula for area and the description of perimeter followed by her inability to employ either of them. In each case, Peggy seemed to be repeating something she might describe as knowing but not understanding; these were facts she knew but not concepts that she understood.

In this part of the task, Peggy was asked to consider only cases in which there was the same number of desks in each row. She immediately understood that she was looking
for pairs of factors whose products were thirty-six, but omitted the pair, thirty-six and one, that perhaps seemed unreasonable in this setting. Thus, she used division to find a partner for each suggested first member of a pair and multiplication, not repeated addition, to confirm that 36 desks were in each of her proposed arrays. It is possible that the stipulation that there was the same number of desks in each row influenced her thinking as well as her actions.

We then moved to the second part of the Arrays Task (Task #26). Peggy was at first baffled when asked what would happen if 62 students were marching in rows of 8 students each. She divided 62 by 8 and interpreted the result of 7.75 to mean that she would have 7 rows with “point 75 students left” (Teaching Session #3, 5:59:59 p.m.). I directed her back to the example with the rows of desks, and asked her what the room would look like if as many rows of 8 desks as possible were used. She thought about this, made several calculations on the calculator and then automatically returned to the marching-band example. She concluded that she could have 7 rows of 8 with 6 students left over. She arrived at this result by checking that the product of 7 times 8 is 56, and 62 minus 56 is 6.

Peggy: I did 62 divided by 8 to see if it would go in there an even number of times, and of course it didn’t. That’s what led me to the 8 times 7 with the 6 left over.

(6:04:34 – 6:04:50 p.m.)

This was an interesting application of the division algorithm to find a remainder while using the calculator. The result of 7.75 in the initial division gave Peggy the candidate of 7 for the number of full rows of 8. The decimal part of the result had no
meaning for her. She chose to ignore it and apply her knowledge of the sense of the situation to arrive at the result of 6 students remaining.

Use of Rectangular Gameboards to Explore the Concept of Area

Continuing my exploration with Peggy and the concept of area and her multiplicative thinking, I asked her to take a look at the Gameboard Task. (See Task #27 in Appendix C.) This task presented arrays from a different perspective by indicating the dimensions of the array and asking for the total number in the array, and by focusing on other features in an effort to assist the student in exploring the components of the array more deeply.

In the first part of this task, a gameboard was to be constructed with white and black squares according to the usual alternating pattern with a black square in the corner spot. The task stated that there were 15 squares across the top and 9 squares down the side and included a sketch of a few squares in the corner of the board to indicate the pattern. The student was asked how many squares there would be on the board, and how many white squares and how many black squares there would be. Peggy immediately multiplied 15 times 9 and concluded that there would be 135 squares on the board.

She pondered the question about how many white squares and how many black squares and then made the following observations. “It’s not going to be even. [I asked her what she meant.] Well, there’s got to be one more of one than there is of the other one” (Teaching Session #3, 6:06:14 - 6:06:22 p.m.). She indicated that she did not know how to proceed and pondered this for several minutes, but again indicated that she did not know what to do. Twice she suggested adding the two measurements together and taking half of the result, but did not know how to proceed from there.
Peggy: Well, I was thinking that you have to account for what’s here and what’s here [gesturing to the two dimensions]. If you have 15 across here and 9 down here, then to account for both, you’d have to add them together. (6:07:40 - 6:07:52 p.m.)

It seemed reminiscent of her confusion between area and perimeter that Peggy confidently multiplied the two dimensions to find the number of squares in the board, but was tempted to add the measurements together in order take them all into consideration. It was a pleasant surprise that she noticed that there had to be one more square of one color than the other; it is not clear that there was anything except good intuition behind this potentially important observation.

At this point, Peggy was simply stuck, so I suggested that she either extend the given drawing or sketch a smaller example. She chose to draw another example with 7 squares across and 3 squares down. She acknowledged later that she had chosen the odd numbers 7 and 3 because the dimensions of the original gameboard were to be the odd numbers 15 and 9. She made a complete sketch, stated that there were 21 squares in the entire board and counted the white and black squares finding that there were 10 of one and 11 of the other. She observed that there was one more of one that the other, as she had predicted, and pondered whether the number of black ones would be more in the original example because the corners were black. She was still at a loss for finding how many of each without drawing the entire figure. Although she was offered the opportunity to do so, she opted to think more about the question in the hope of being able to figure it out without making a physical representation.

We moved on to the second part the Gameboard Task (Task #27), in which the student was asked, how many red squares would be needed to make a border using red
squares the same size as the other squares. After creating the border, the student was asked how many squares there would be in the expanded board. Finally, in the third part, assuming that each square in the boards in parts A and B was 1 inch by 1 inch, the student was asked what the area and the perimeter would be of each board, the original one and the expanded one.

In response to the question about the number of red squares that would be needed to make a border with red squares the same size as the others, Peggy replied, “So you’ve got 15 and 15 and 9 and 9. [Looks up for approval and immediately smiles at herself and declares], but that’s area and not perimeter! Right?” (Teaching Session #3, 6:12:03 – 6:12:13 p.m.) We discussed her previous calculations, and she repeated her descriptions of the area as what is inside the figure and the perimeter as going around the figure. I asked her to consider the task of putting a red border around her smaller example (7 by 3). She began to sketch those squares and made the following comments.

Peggy: So, you do the same number that you have. [Referring to the 7 squares on the top and bottom] And then you do the same here. [Referring to the 3 squares at either end] (6:13:20 – 6:13:36 p.m.) She stated that this was similar to when she was computing perimeter and then immediately went through the following self-talk.

Peggy: But it’s not the same as when you compute perimeter. [voice gets very soft, appearance very doubtful] Because when I think - To me that’s like doing area – not perimeter. I don’t know, maybe I’m confusing myself. The perimeter is these outlying things, while the area is what’s inside. All right. All right. (6:13:43-6:14:06 p.m.)
Throughout, I had tried to call attention to the fact that the perimeter was associated only with the lines or edges of the figures. At this point, I asked about the complication of putting squares around the edge. Peggy immediately realized that 4 squares must be placed in the corners to complete the border. When I asked Peggy to explain the difference between putting a border around the board and computing its perimeter, she stated, “The perimeter is just this line. Whereas, what we’re doing [putting a border on the figure] is more a little outside of what’s all the way around that line” (Teaching Session #3, 6:15:00 – 6:15:14 p.m.). Thus, once Peggy had cleared up her confusion, computing the number of red squares needed to make borders around each of the gameboards was easily accomplished except for some calculation errors. She easily corrected those errors using her calculator.

In response to the question about the area and perimeter of the 15 by 9 board, Peggy quickly computed area, but she again hesitated when considering the perimeter and seemed to be stuck at the same place as before.

Peggy: Well, the area is 15 times 9, because that includes everything inside. Then the perimeter would be the sides.

Researcher: The perimeter would be what?

Peggy: [Hesitation, then slowly and softly] Length times width. [Looking off in the distance and then literally squirming in her chair] This is where I confuse myself. I know the area would include all of this, whereas with perimeter, you just want one side and one side [pointing to a length and a width].

Researcher: What did you tell me that you would do to get the area?
Peggy: 15 times 9. So that’s the area. Because I mean that’s everything inside. But the perimeter is just going to be the outside. [Pause] So it can’t be. [Pause] I don’t know.

Researcher: Can you do it for the little one [referring to her 7 by 3 example]?

Peggy: [Glancing at her smaller example, but then going immediately back to the one she was given] The 15 and the 15 and the 9 and the 9, but that’s what we just did, which was different-

Researcher: What we just did?

Peggy: But we added. We added the sides. That’s right. (Teaching Session #3, 6:16:54-6:18:30 p.m.)

Peggy then proceeded to successfully compute the perimeter of the larger board by adding 15 plus 15 to get 30, then adding 9 and 9 to get 18 and finally adding 30 and 18 to get 48. She did not employ a formula for the calculation and completed most of the arithmetic on her calculator. She also readily observed that the difference between the calculation of perimeter and of the squares needed for the red border was the inclusion of the 4 corners for the border.

In each instance, the difficulty for Peggy seemed to be in using all four sides to compute perimeter. She appeared to stall after referring to two sides and to feel that she would be computing area if she took all 4 sides into consideration. Once she looked back at the work she had done with the concepts using the smaller example that she had constructed, Peggy was able to compute both area and perimeter of the larger board.
Peggy then addressed the final section of the task. Given that each square was 1-inch by 1-inch and asked to state the units for her previous results, she immediately identified the perimeter as inches.

Peggy: Area is – There’s a different unit of measurement, I think. . . .

Researcher: How did you get 135 [area of large board] again?

Peggy: Is it square feet. 15 times 9, which is 15 squares times 9 squares, which is 135 squares.

Researcher: 135 squares? What are they?


Researcher: If each one of these [pointing to original drawing] is 1-inch on this side and 1-inch on that side-

Peggy: Square inches. [chuckle]. (Teaching Session #3, 6:19:43-6:20:20 p.m.)

Although I was hopeful that Peggy’s reference to 15 squares times 9 squares as a way to compute the area was evidence of a shift in her thinking based on her work with arrays, using the phrase “15 squares times 9 squares” as a way to compute the area of the gameboard could also have indicated that she did not connect multiplication with the structure of an array: number of rows times number of squares per row. Rather, the phrase “15 squares times 9 squares” might have indicated an application of her formula: $A=LxW$ if she saw the length of the gameboard as being 15 squares and the width as being 9 squares. Thus the area would be “15 squares times 9 squares.” This lack of a 2-dimensional mental structure for multiplication (number of items per row times number of rows) could have accounted for Peggy’s difficulties with thinking multiplicatively in
these tasks. Indeed, this was precisely what I was attempting to assist her in constructing by employing the tasks based on arrays.

It can be seen that it took considerable thought and discussion for Peggy to be able to work through these parts of the Gameboard Task. Her work on adding the border forced her to extend the concepts of area and perimeter, and assisted me in concluding that some progress had been made, but that she still was not secure in her understanding of these concepts.

At the beginning of Teaching Session #4, we returned to the Gameboard Task and explored a variety of gameboards, some having even numbers of squares and others having odd numbers of squares. Peggy consistently could determine the number of squares given a description of the boards. She could also predict the number of each colored square as long as the total number of squares was even. That is, she was able to divide the number of squares by two in order to determine the number of squares of each color. However, she could not predict the number of each colored square when the total number of squares was odd; she observed that when she divided the odd number of squares by two, the result was a decimal ending point-five, which she automatically identified as one-half. Moreover, Peggy recalled that by counting the squares in the example that consisted of 3 squares by 7 squares she had learned that it had 10 squares of one color and 11 squares of the other. When I asked her to see what she would get if she divided 21, the number of squares in the smaller board, by 2, she used the calculator and got 10.5, which she immediately identified as 10-and-one-half (Teaching Session #4, 5:53:30 - 5:53:44 p.m.). Although this stumbling block for Peggy was both bothersome
and interesting, I chose at this point to redirect her to other tasks in an effort to return to addressing her confusion between the concepts of perimeter and area.

With that in mind, I asked Peggy to examine a gameboard that was 8 squares by 8 squares and to determine how long a string would be needed to go around it. Peggy measured one square and found that it measured 2 inches on each side. With considerable struggling she was able to calculate the perimeter, to calculate the area both by multiplying the area of one square by the number of squares and by multiplying the length times the width, and to put the appropriate labels on each result. Her gestures assisted me in seeing that she was developing greater understanding of the two concepts of perimeter and area. Finally, she was able to calculate and appropriately label both the area and the perimeter of the board that was 10 squares on each side and in which each side of the squares measured one and one-half inches (Teaching Session #4, 5:54:10 – 6:04:10 p.m.). Peggy used a calculator for virtually every calculation, expressing any fraction as its equivalent decimal representation.

Using the Farmer’s Field Task (Task #29) to Explore Area and Perimeter Further

Although Peggy seemed to be developing a greater understanding of the concepts, I presented her with some follow-up tasks to check my impression as well as to help her both to solidify her understanding and to gain confidence. We first examined the Farmer’s Field Task in which Peggy was presented with various dimensions for fields and was required to use her understanding of perimeter and area to determine how many square meters of topsoil and how many meters of fencing would be required for the fields. She was able to discuss why the perimeter remained the same for fields measuring
100 meters by 60 meters and 90 meters by 70 meters even though the areas that she had calculated for the fields were different.

What Does a Square Meter Look Like?

I became concerned about Peggy’s understanding of the language that we were using to describe area and asked her what a square meter looked like. With little hesitation, Peggy suggested a square that measures 1 meter by 1 meter. When I asked her how else it might look, she immediately made another sketch (by dividing the 1 meter by 1 meter in half horizontally and sketching a rectangle that looked like one half next to the bottom half) and identified it as measuring one-half meter by 2 meters, or one-third meter by 3 meters, and so forth. Although it was gratifying to see Peggy move easily into using fractions, I chose to remain focused primarily on the concepts of area and perimeter at this point. I returned to some work with fractions during Teaching Session #5, first in a different setting with some Balancing Tasks (See Task #36 in Appendix C) and then with area and perimeter (see below).

Returning at Last to the Paint and Carpet Task (Task #17)

Although she had expressed great hesitation about doing so previously, in the continuation of Teaching Session #4, Peggy agreed that she was ready to go back to the task involving carpeting the rooms of various shapes. She was able to compute the area and perimeter two different ways. That is, in one case she dealt with overlapping regions and subtracted off the part that was duplicated. In the other case she eliminated the overlap by selecting different regions. Her two approaches are illustrated in Figures 5.7 and 5.8 below. It is notable that she was quite comfortable with the notion that there was
more than one way to do this and that there were probably others. (Teaching Session #4, 6:15:18 – 6:24:04 p.m.)

Sketching a line separating the room into two rectangles (see Figure 5.8 below), Peggy observed that width of the smaller rectangle would be 4. She calculated that the area of the larger part was 10 times 8 or 80 and that the area of the smaller side was 4 times 7 or 28. She then added 80 and 28 to get a total area of 108 and later added the units to make the area 108 square feet.

Figure 5.8: One of Peggy’s sketches for finding the area of the room in Task #17

Figure 5.9 below illustrates Peggy’s other solution; she used colors to identify regions. This time she found the area of the two overlapping rectangles by multiplying 10 times 8 to get 80 for one and multiplying 7 times 12 to get 84 for the other. She added these together to get 164. She then observed that she needed to multiply 7 times 8 and subtract the result, 56, from the previous sum. This again yielded 108, which she later labeled as square feet.
Peggy was able to compute the perimeter of both the rectangular and the L-shaped room with little guidance. She expressed great pleasure in her level of understanding of the concepts of area and perimeter. (6:24:00 – 6:28:28 p.m.)

A Brief Exploration Including Fractional Measurements

During Teaching Session #5, after completing the Counting Triangles Task (Task #33) and the Couples Task (Task #34) and exploring several Balancing Tasks (Task #36), I asked Peggy to recall once more our work with area and perimeter. It was at the end of this, our last individual session, that she made an interesting discovery. First Peggy reiterated that to find the perimeter of a rectangle she added the sides, and to find the area she multiplied two sides. We noted that previously, during our work with the Farmer’s Field Task (Task #29), we had looked at situations in which the perimeter remained the same and the area varied. With all of that in mind, I asked her to consider a rectangle whose area remained 48 square inches and determine what the dimensions of that rectangle could be in order to have the smallest perimeter, limiting the sides to whole number measurements. She successively divided 48 by 2, 3, 4, etc. on the calculator and concluded that 6 by 8 was the correct response. I asked her to consider the same question, keeping the area to 10 square inches, again limiting the sides to whole number
measurements. She quickly concluded that the only whole number pairings would be 10 inches by 1 inch and 5 inches by 2 inches and that the later dimensions would yield the smaller perimeter.

Then, I asked her to consider fractional measurements for the sides. After a chuckled denial that she could handle the task with fractions, I attempted to assist her in recalling the groundwork she previously had done with regard to rectangles of area equal to 1 square meter with sides having fractional measurements.

Researcher: What if we don’t limit it to integers?

Peggy: [large grin and chuckle] Then you have left me out of the picture! I have no idea. I’m serious. I really don’t. (Teaching Session #5, 6:27:52 - 6:28:04 p.m.)

I reminded Peggy that she had dealt with this idea before and asked that she consider a rectangle with the area of one square meter. Peggy sank in her seat holding her head in both hands as she pondered, then looked up and made the following suggestions.

Peggy: A half and a half?

Researcher: If it’s a half and a half, then it’s area [voice rising in a question] -

Peggy: Oh, that’s right. A quarter and a quarter and a quarter and a quarter [pointing at the 4 sides of the figure she has sketched]

Researcher: That will give you [voice rising in a question] –

Peggy: One square. [Pause]

Researcher: What’s the area of that rectangle?

Peggy: One square meter.

Researcher: Calculate the area given those measurements. How do you calculate the area?

I asked Peggy to look back at her work with the previous two rectangles to remind her about how to calculate area. She quickly remarked that she understood, but remained quiet and thoughtful as she pondered the present question about a rectangle with the area of 1 square meter. I then guided her in recalling that the area of a rectangle is computed by multiplying the length times the width and that a square that measures 1 meter on each side has an area of 1 square meter. Again, I asked her to try to describe a rectangle that would have the same area.

Researcher: Now, if you have a rectangle that has the same area as that, but it’s not a square so it’s not the same on each side – What’s one example of some dimensions it could have?

Peggy: What’s throwing me off is that you have to get that area . . . It could be a half times a half -

Researcher: What happens if it’s a half times a half? What happens when you multiply a half times a half?

[Peggy uses calculator to multiply the decimal equivalents 0.5 times 0.5, and appears disappointed in the result.]

Researcher: What did you get when you multiplied a half times a half?

Peggy: Point twenty-five. (Teaching Session #5, 6:31:49 - 6:32:54 p.m.)

I guided Peggy in sketching what she had just computed; she used the color pencils to help visualize what occurred. Her sketch appears as Figure 5.10 below. She appeared to be comfortable with the result that a square that measures one-half by one-half would
occupy one-fourth of the original one-by-one square. Together we enjoyed having made this discovery.

![Diagram of a one-by-one square with one-fourth shaded]

Figure 5.10: Peggy’s demonstration that \( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \)

I then tried once more to get Peggy to return to the rectangle measuring one-half unit on one side and to figure out what the other dimension would need to be in order to have an area of one square meter. However, both the time for the session and Peggy’s patience were running low, so the session ended with Peggy commenting that she would look at this question further. Unfortunately, Peggy was not able to reproduce and apply her previous observation that 1 square unit could be the area of a 1 by 1 square and a one-half by 2 rectangle, and so forth. As soon as fractions were part of the picture, it seemed that she reverted to her apparent confusion between area and perimeter. That is, even though she had previously linked multiplication of the dimensions to the notion of area sufficiently to observe that the areas of rectangles whose dimensions were \( N \)-units by \( 1/N \)-units, that construction was not robust enough to be called forth again during this session. Instead, it seemed she was thinking in terms of addition, or at least that she was not sure whether to add or multiply the dimensions because one-half plus one-half is one. This may not have been so much a sign of confusion between the concepts of area and perimeter as a lack of understanding of area and its association with multiplication as well as an inadequate construct of multiplication itself. Furthermore, in applying the
formula $A=L \times W$ to the calculation of area, Peggy may well have simply multiplied those quantities without utilizing the notion I hoped to introduce by examining arrays. That is, she did not seem to think in terms of, for example, a rectangle that measured 3 units by 7 units as 3 columns of square units by 7 rows of square units. Then, when faced with the more challenging context of fractional measurements, Peggy lost her tenuous hold on both area and multiplication.

Notably, she stumbled upon the situation in which a square is one-half meter on each side and found that the area would be one-fourth square meter. Although I was disappointed that she could not reprise her good observation from before, I was pleased at her exploration of a square measuring one-half meter by one-half meter and shared that reaction with her. It should be noted that Peggy herself again expressed great satisfaction with her improved understanding of area and perimeter.

**Peggy’s Actions During Joint Session #2**

During Joint Session #2 for Peggy and Hillary, there were two opportunities to explore the notions of area and perimeter. I was interested in how the two participants would work together within a setting about which Peggy had expressed such insecurity and then explored extensively in this teaching experiment. To some extent, I was interested in how robust her understanding of the concepts would be when engaged collaboratively with Hillary. Finally, I was interested in how they would operate within the non-routine settings (for students with their level of formal mathematical experience) of finding the area of an irregular shape and of attempting to maximize the area of a figure.
In Task #28b, the participants were presented with an irregular shape (which might be referred to as a lake) superimposed on a grid. (See Figure 5.11 below.) In this task the focus was on the approximation of area.

As Peggy and Hillary attacked Task #28, Peggy, at first, misunderstood Hillary’s indication that she would begin by finding an enclosed rectangle. That is, Peggy was troubled when she interpreted Hillary’s initial actions to mean that she was going to form a rectangle that did not have opposite sides of the same length. Peggy keyed in on the individual squares and the notion of estimating and began by simply counting squares that were predominantly included within the lake. Then, noticing Hillary’s first stab at finding the included rectangle, Peggy followed suit by identifying the full rows of squares along each dimension. She began the process of estimating by including squares of which
more than half appeared to be included in the figure. She seemed to be struggling to use this information in order to find the rectangles of interest. See Figure 5.12 below for Peggy’s first estimations.

Peggy: I was trying to figure out, to get an estimation if I could take it from what I considered its first almost full point where they were almost all full squares [gesturing along the left side width of about 10 squares] by its almost full length [gesturing along bottom row of 12 squares] I could get some kind of close estimation. [Pause] But I’m not sure because it’s not that wide all the way across. (Joint Session #2, 12:49:50-12:50:18 p.m.)

After some discussion between them, Peggy and Hillary agreed that this would be an acceptable way to estimate.
At this point in the session, Hillary had drawn a 10 by 10 rectangle (which she mislabeled as being 9 by 10) that was totally enclosed by the lake. (See Figure 5.13 below.) When asked how they could then continue to more closely approximate the area of the figure, Peggy began looking for rectangles within the parts of the figure not included in Hillary’s rectangle.

Hillary indicated that she might approach the task of making a closer approximation differently. First, she drew a second rectangle that included more of the main body of the figure, but whose vertices extended outside of the figure. Peggy immediately suggested, “Then it would be easy to just shave this off [gesturing to the pieces that fall outside the figure]” (Joint Session #2, 12:52:31 - 12:53:33 p.m.). Eventually, Hillary proposed making a rectangle that completely enclosed the figure and removing the squares that extended beyond it. See Figure 5.14 below.
Hillary and Peggy examined the two methods of approximating the area of the figure: the one suggested by Peggy best described as finding increasing under-approximations and the last one suggested by Hillary closely akin to finding an over-approximation and reducing it. When asked to explain how they would decide which pieces of squares to include, Hillary first indicated that if a square was predominately in the figure it should be included, but if it was primarily outside of it, it should be excluded. Peggy suggested including fractional parts by commenting, “You could count half of it. As long as you don’t get into quarters, I’m o.k. I can do half [laughter]” (Joint Session #2, 12:53:46-12:53:52 p.m.).

Hillary and Peggy then addressed Task #38 concerning the use of 400 meters of fencing to enclose a field (see Appendix C). Together they sketched a square 100 meters by 100 meters and a rectangle 150 meters by 50 meters. During their work, they frequently confused the terms square and rectangle, but this did not seem to affect their
endeavors. They agreed to compare each of their above candidates by calculating each area.

Although Peggy commented that there were a number of ways to arrange the fence and began by dividing the 400 meters by 4 to find the length of the side of the square, she continuously demonstrated her confusion about the relationship between area and perimeter. In particular, she was confused by the situation in which the perimeter remained the same while the area changed. This seemed much more bothersome to her than the situation in which the area remained the same while the perimeter changed.

Peggy: I mean to me, if you’ve got – Aside from the way the land might look, which could impair whether it is practical or not, aside from that – You’ve got 400 meters of fence, so there is only so much land you can - So it’s 400 meters. I mean – If it’s this way, or it’s this way, or if it’s this way, it’s 400 meters. (Joint Session #2, 12:57:22 – 12:57:49 p.m.)

When she seemed to indicate that all the figures with a perimeter of 400 meters should have the same area, Hillary reminded her of the two examples they had sketched and the different areas they had calculated. Peggy agreed but remained uneasy, and the following interchange ensued.

Peggy: But see, I don’t understand why. Because if you’ve got – I don’t know – If you’ve got 400 meters of fencing, you’ve got 400 meters of fencing. So, why does this give you more area than this? . . .

Hillary: Because the lengths of the sides are different, and to get the area, you multiply the side times the side.

Peggy: I understand – [apparently meaning she knows the formula]
Hillary: Now, I can’t explain it any further than that, that’s as far as my knowledge goes.

Peggy: I understand that, but you still have only 400 meters of fencing. (Joint Session #2, 12:58:52 - 12:59:34 p.m.)

After I suggested that they create additional examples using 400 meters of fencing, they noticed that the length of a rectangle could not be 200 meters because there would be no fencing available for the width. They described a trapezoid with 200 meters on one side, 100 meters on the opposite side and 50 meters for the other two sides. Although Hillary indicated that to find the area they would have to do something like what they had done with the previous task and Peggy agreed, they rejected the notion of pursuing it. [I felt that clarifying the situation while considering only rectangular shapes both was more comfortable for them at this time and presented a more reasonable goal for the session.] They then produced the rectangle with length of 125 meters and width of 75 meters, and calculated the area to be 9,375 square meters. They examined the pattern that had been exhibited in the three examples that they had sketched, and Peggy seemed to be more comfortable with their conclusion that the rectangles had different areas and that the square field would provide the largest area among them.

In the interest of including the notion of fractional parts in the calculation of area, I then chose to have the participants move on to part c of Task #28 in which they were asked to determine the shape of the largest area that could be fenced with 390 meters of fencing. They immediately proposed the rectangles that can be made by selecting 100 meters and then 150 meters as the length of one side. Peggy suggested that a square would probably have the largest area. Hillary agreed that the work they had done so far
would lead to that idea. Peggy divided 390 by 4 to determine that each side would be 97.5 meters. Hillary calculated the area to be 9506.25 square meters and reluctantly identified 0.25 as a quarter of a square meter. Noticing the reference to the fourth of a square meter, Peggy commented that the researcher was probably trying to get them to see something.

When attempts to have Peggy reprise her work with the area of square measuring one-half meter on each side were not successful, I chose to present an alternative task. Sensing that the representation of 1 meter as a small measurement on the paper may be interfering with their thinking, I guided them in using a sketch drawn to scale of a square that measured 1 inch by 1 inch. They were then asked to construct within that square one that measured one-half inch by one-half inch. Both Peggy and Hillary displayed some hesitancy, but persevered. They demonstrated increasing ease with the calculations of areas and specifically with the area of one-fourth square inch. In fact, there seemed to be a note of recognition on Peggy’s part, perhaps based on her prior experiences in this study.

After they examined a rectangle that was one-half inch by 1 inch, Hillary and Peggy demonstrated their understanding that rectangles measuring one-half inch by two inches and 1 inch by 1 inch both had areas of 1 square inch. Furthermore, they noted that the perimeter of the square was 4 inches and the perimeter of the rectangle was 5 inches. Here Peggy seemed comfortable with the fractional parts and with the situation in which the area remained the same while the perimeter varied.

Peggy had not reconciled totally the issues she had demonstrated regarding area and perimeter. However, the context of the teaching experiment permitted me to observe
what she sensed that she was confused about, to uncover other sources of concern and to provide tasks through which she could address those concepts in a variety of ways. It also permitted me to test my assumptions and the conclusions I was drawing as well as her level of understanding. Peggy’s work on Task #28 (the area of the “lake”) with Hillary during Joint Session #2 pointed out an important feature of Peggy’s thinking. She seemed to have a very clear understanding of the area of the lake and easily considered both under-approximations by looking for additional squares (and even halves of squares) to fill in the rest of the area and with over-approximations by taking off the parts that the squares (and halves of squares) that extended beyond the lake. As long as she was faced with this concrete example, the concept and the calculations were not difficult. However, when working with dimensions that she multiplied on her calculator, she did not seem to have a clear image of those dimensions as representing the rows and columns of units and the result of the multiplication being the number of square units within the figure. Thus, it did not seem to be her understanding of the concept of area that was the stumbling block.

Peggy’s multiplicative thinking seemed to be the primary issue. As was pointed out earlier, her work on Task #22 with the Chickens and Cows provided an example of her tendency to choose repeated addition over multiplication. Her limited application of the commutativity of multiplication in her work with the first arrays in Task #26 was another area of concern. She certainly had not tied multiplication of whole numbers with the construct of arrays. She did not seem to have a deep understanding or mental construct of multiplication and, hence, had difficulty using it as a tool in exploring other concepts such as area. Because fractions were apparently a very artificial concept to her unless they were the familiar ones that she could interpret easily as decimals, introducing
fractional measurements into calculation of area simply complicated her thinking further. Although she was able to deal with them on a limited basis briefly, her understanding was not sufficiently robust to persist.

**Peggy as a Mathematician**

Although Peggy depicted herself during the initial interview as having inadequacies when it came to the study of mathematics and even though, as the previous section illustrated, she did have serious defects in some areas of mathematics, she displayed several characteristics during our teaching sessions conducive to her being a successful mathematician. Often her behavior during the teaching experiment was quite different from the behavior that she ascribed to herself. To some extent, her perception of what it means to think mathematically was at the root of this conflict. In the following sections, I first illustrate Peggy’s revealed views about what it means to think and act mathematically. I then recount several incidents that occurred during various teaching sessions with Peggy in order to illustrate and support my claim that she possessed the following attributes of a student of mathematics: a willingness and ability to be reflective, an instinct to seek a balance in quantities, an inclination to make and then modify her assumptions as necessary, a willingness to explore tasks from more than one perspective, the use of insight to develop strategies, a tendency to notice patterns, the ability to classify trials and explore cases, an understanding of the use of simpler examples to focus on basic concepts, the ability to symbolize actions and concepts, and a willingness to accept a challenge in order to expand on a concept.
What it Means to Think and Act Mathematically

The Picture Task (see Task #1 in Appendix C) was the first task that I presented to Peggy during our first teaching session. In this task the student is given several pairs of measurements from the left and the right edge of a picture to the respective end of the wall and asked to determine how far to the left the picture should be moved so that it would be centered on the wall. Peggy pondered the task and commented

I have no idea if I am going about this the right way. . . . This is where I have a problem. I cannot- I know what needs to be done but I cannot figure how to get there mathematically. . . . I know that somehow, I’ve got to find the center, and I’ve got to do it with some formula with these two numbers. But, I don’t know how to get there.  

(Teaching Session #1, 5:39:50 – 5:41:40 p.m.)

In spite of these protestations, she then proceeded to devise a strategy with which to approach the task.

This behavior, which may have appeared initially to reflect resistance, continued throughout the teaching experiment. During the fifth teaching session I presented Peggy with the Couples Task (see Task #34 in Appendix C) in which limited information is given about eight individuals and the participant is to determine who is married to whom. It was not surprising to me that she commented, “Oh G-d, I hate word problems. [She paused to read the task completely, and continued,] I hate this” (Teaching Session #5, 5:51:12 – 15 p.m.). Neither was it surprising to me, at this point in the teaching experiment, that she began working in an organized way even as she was making these comments. She engaged in self-talk, while recording the facts and reflecting on the implications of information that was given. With very little encouragement and no
guidance, given the time to think and reflect, Peggy was able to come up with a solution and to explain it completely. The type of encouragement that Peggy usually required and her effective use of reflection is explored further later in this section to assist in demonstrating what I consider to be other aspects of Peggy’s mathematical thinking.

During our sessions, Peggy frequently claimed, as she did in these examples, to not be able to address a task in what she referred to as a mathematical or the correct way. She usually meant that she believed that there must be a formula or some standard procedure with which she was not familiar, and that, if she was not doing it in that way, then she must not be working and thinking mathematically. And, as in these examples, Peggy characteristically followed such a comment with actions that did indeed demonstrate mathematical thinking, from my point of view if not hers.

**Willingness and Ability to be Reflective**

I learned early in the teaching experiment that I could encourage Peggy by asking questions that simply repeated back to her what she said needed to happen or by directing her to look at what she had already done. I frequently asked her what she knew and what her goal was or simply to tell me how she had gotten what she already had as an interim response. She seemed to be establishing our relationship when she asked me, “You’re not going to tell me if I’m right or wrong, are you?” To which, I made the rhetorical reply, “Well, how could you tell if you were right or wrong?” In many cases, she initially would state that she did not know how to go about substantiating her response, but she would then immediately check appropriately to confirm it.

To put this attribute another way, I attempted to ask questions that directed her to examine her own thinking, and she was quite open and responsive to this approach. I
found that it was important to assist Peggy in understanding that not only self-talk, but
listening to her self-talk was potentially helpful to her. As she was struggling to write a
general procedure for the Picture Task (see Task #1 in Appendix C), this conversation
occurred:

Peggy: I know I want $x + y$ divided by $xy$. Or by the sum of $x$ and $y$. – But, I don’t
know how to write it.

Researcher: Tell me what you did in words.

Peggy: O.K. I added $x$ and $y$ and then I divided the sum by 2. [Looks chagrined as she
immediately recognizes the conflict between what she wrote at first as a symbolic
representation and what she actually did in the examples; begins erasing and
correcting her formula.]

. . .

Peggy: See you sound like [the instructor of her math class]. She would say, “If you’d
just read it.”

Researcher: Remember, you’re not reading. That’s what you’re telling me that you
did. These are your words. They are not on that paper anywhere. (Teaching
Session #1, 5:54:06 – 5:54:42 p.m.)

Later in the first session, she attacked the Handshake Task (see Task #6 in
Appendix C), which asks the student to figure out how many individual handshakes will
there be if there are 10 people in a room and if each person shakes hands with every other
person. She repeatedly verbalized appropriate observations, but her sketch reflected
something different. I urged her to listen to her own thoughts and to be willing to write
down more instead of trying to keep track of so much in her head. Eventually, she resolved the task.

**Balancing Quantities**

Continuing with the Picture Task, when I asked her what had to happen in order for the picture to be centered on the wall, she immediately responded

Peggy: It’s got to be even between both of the two of these [uses both hands to indicate the balance] . . . There has to be the same number of inches on each side. . . . So if I add this and this together and then divide it by two, then it has to be 58 inches on each side. (Teaching Session #1, 5:41:41 – 5:42:07 p.m.)

Peggy was very quick to recognize the element of balance and effectively used physical representations (such as the hand and arm motions used here) to demonstrate what needed to occur.

Peggy’s sense of balance between quantities supported her in other tasks. In fact, during Teaching Session #5, when I decided to make one more effort to encourage her to work with fractions, it seemed that Peggy’s understanding of balance was most helpful to her. I presented a series of tasks to her in which items such as bricks or bags of grain and weights balanced with each other, backing off to less complex ones when she struggled and then moving to the more complex ones as she progressed. The following interchange was illustrative of Peggy’s thinking during this session. I offered a modification of a previous question with which she was struggling by asking her, “What if I had a brick balancing with three-quarters of brick plus 7 pounds?” (Teaching Session #5, 6:12:51 – 6:13:06 p.m.) Her response was, “You gotta make up for the quarter of a brick that’s over here, but not over here, which is evidentially made up by this 7 pounds. But, I don’t know
how to figure that from here!” (6:13:58 – 6:14:05 p.m.). She said this several different times in different ways, for example observing that, “There’s 7 pounds in each quarter of the brick – but I don’t know how to – I don’t know how to go from there” (6:16:25 – 6:16:44 p.m.). Eventually, she concluded that the brick weighed 7 times 4 or 28 pounds. In each stage of this work, Peggy’s intuitive sense of balance persisted and seemed to guide her thinking.

Making and Modifying Assumptions

Toward the end of Teaching Session #1, Peggy addressed the Fieldtrip Task (see Task #8-A in the Appendix C) and then returned to a modified version of it at the beginning of Teaching Session #2. The Fieldtrip Task states that the participant has been asked to arrange for drivers to drive for a trip for a group at school; she must determine the number of cars and lunches that will be needed. The task states that two staff members and 35 students will be attending, but neither staff member will be driving a car. Peggy automatically placed this situation in a context familiar to her by assuming that those who were attending were of driving age; she also assumed that each vehicle could accommodate 5 people in seat belts, and easily concluded that she would need 8 cars and 37 lunches.

At the beginning of Teaching Session #2, I asked Peggy to consider a modified Fieldtrip Task (see Task #8-B in Appendix C). The modified version asked her to assume the context was a grade school class field trip in which all the students were younger than driving age. She understood immediately, accepted the change in assumptions that I had imposed on the task, modified the assumptions she had made about the number of
passengers and drivers, and successfully completed this task. More about the way she approached this task, including the details of these adjustments, follows.

**Exploring Tasks From More Than One Perspective**

As Peggy began to work on the modified Fieldtrip Task (see Task #8-B in Appendix C), she used her prior work on the first version as a starting point and figured that the 8 cars would mean that she would need 8 parents to drive. When she added the 8 drivers to the original 37 people, she then had 45 people. This caused her to conclude that she needed 45 divided by 5 or 9 automobiles. She proceeded to compute the number of tickets and lunches, but seemed troubled. I asked her what was bothering her, and her comments indicated that she was thinking about the problem from two different perspectives.

I could do it one of two ways. I could take 35 plus 2 and count it as a four-passenger vehicle and divide by four and that would automatically leave one spot for the driver.

... And that would tell me that I need 10 automobiles. But [working from the previous version’s results,] if I take 35 plus 2 plus 8, that tells me I need nine. ...

There’s something that I’m not computing correctly but I’m not sure what it is.

(Teaching Session #2, 5:50:21 – 5:51:07 p.m.)

I suggested that she explain again the different approaches she had taken. Peggy soon became convinced that she had done something in error on the second method (the one extending the problem from the previous version) and that the first one (assuming that each car can accommodate 4 passengers plus a driver) made more sense to her anyway.

During the fourth teaching session, Peggy’s demonstration of her ability to consider alternative approaches was particularly gratifying. As indicated in the previous
section, I had offered her several opportunities to explore area and perimeter, concepts with which she had displayed considerable discomfort. I had asked her to return to the Paint and Carpet Task (see Task #17 in Appendix C), which required that she compute the area of the combined space of two rectangular spaces that abut one another in an \( L \) shape. As described in the previous section, Peggy was able to break the non-rectangular space into rectangles and identify the dimensions in order to compute the area and perimeter two different ways. That is, in one case she dealt with overlapping regions and subtracted off the part that was duplicated, and in the other case she eliminated the overlap by selecting different regions. She was quite comfortable with the notion that there was more than one way to do this and that there were probably several. She continued by computing the perimeter of all the rooms involved and expressed great pleasure in her level of understanding of the concepts involved.

There are several things to note here. First of all, although Peggy stated several times that she was not willing to consider alternative methods in class or on homework, as was demonstrated by these examples, in her work within the teaching experiment she voluntarily explored at least two different ways to look at the task at hand. Secondly, Peggy was confident that if the alternative methods were both correct and completed correctly, they should have the same result. That is, she did not consider one method inherently right and the other one, wrong. Finally, she willingly reflected on her own work and thinking, and greatly benefited from that reflection. Here again, her willingness to be reflective surfaced.
Using Insight to Develop Strategies

Peggy’s work on the revised Fieldtrip Task also served to illustrate her ability to develop a strategy based on the information given. I commented to her at the time that she had demonstrated good insight into the task by allocating only 4 of the 5 seats in the vehicles to the students and the staff who were going on the fieldtrip and saving the fifth seat for the driver.

During Teaching Session #2, Peggy’s work on the Cows-and-Chickens Task (see Task #22 in the Appendix C) was both enlightening with regard to her insight into the task and disappointing with regard to her ability to listen to her own observations. In this task, the participant is asked to determine how many cows and chickens are in a barnyard if there are a total of 35 heads and 100 legs.

Peggy indicated immediately that it was clear that if there were 35 heads then there would be 35 animals and referred to distributing the 100 legs. She repeatedly commented that she needed to somehow separate the legs from the heads. To me, this seems a natural precursor to developing the system of two equations that would classically be used for such a task. Because I was interested in observing how she thought about the task without any pressure from me to make it fit into traditional methods, I waited until after she had worked through the task in more informal and intuitive ways before I asked her to attempt to write it symbolically.

After grappling with several ways to work with the numbers, I reminded her how helpful it was for her to visualize the situation in the Handshake Task. Eventually, she began the following exploration.
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Peggy: If I have 35 heads, then 30 of them could be this and 5 could be that. . . .
Thirty of them could be chickens and 5 of them could be cows.
Researcher: What would happen if 30 of them were chickens and 5 of them were
cows?
Peggy: The legs! (Teaching Session #2, 6:30:07 p.m.)
This was a eureka-type of exclamation accompanied by a change to a happy facial
expression and indicated that she was beginning to understand the balancing act that
needed to occur between the number of heads and the number of legs. However, she was
getting lost in thinking and visualizing the situation. I urged her to write down what she
was calculating and observing, and she recorded: “30 chickens – 60 [chicken] legs and 5
cows – 20 [cow] legs.” When she commented, “I just feel like this tells me I can make it
anything I want it to be, within reason” (6:31:30 – 6:31:36 p.m.), I tried to get her to
focus on what would be within reason. As she tried different combinations, she explained
that she had decreased the number of chickens and increased the number of cows,
because cows have more legs and she wanted to increase the number of legs.
I was both frustrated and impressed when she made the powerful observation that
“Every time I decrease this [the chicken legs] 2 and increase this [the cow legs] 4, I’m
only [emphasis added] gaining 2 [legs]” (Teaching Session #2, 6:35:42 – 6:32:50 p.m.). I
encouraged her to listen to her own observation. She continued to make the adjustments
by changing the number of animals one at a time, but she never understood fully the
implications of her observation. That is, when she had reached the stage of having 90 legs
and recalled that her goal was 100 legs, she made the additional observation that 10
divided by 2 is 5. However, as described in the previous section, she quickly lost this


train of thought and considered the possibility that the 10-leg deficit could be gained by adding 1 chicken and 2 cows. I redirected her to her own observations, and she was able make the rest of the adjustments one at a time, but did not accomplish the task by changing the number of animals by 5 each. She remarked that she had been intimidated by the problem. When I reflected on this incident, it seemed to me that Peggy’s level of multiplicative thinking was not sufficient for her to capitalize on her insight into the task. She did indeed develop a strategy for working out the task, but could not take full advantage of the observations she made to develop a more efficient strategy.

In another incident, what appeared to both of us to be a lucky guess was actually the result of powerful insight into the task and creative strategy development. During Teaching Session #2, I asked Peggy to take a look at the task involving the placement of the numerals 1 through 6 at the vertices and midpoints of an equilateral triangle so that each side would sum to 10. (See Task #21 in Appendix C.) At the time, her quick solution after only a few trials caught both of us off guard. Because we ran out of time for this session, it was not until the beginning of Teaching Session #3 that I invited Peggy to reflect on what she was thinking when she arrived at the solution. (See Figure 5.15 below.) Peggy was quite willing to reflect on what she had been thinking when she completed the task. After mulling over some of the details of what she had done in each of the trials, she observed,

I remember thinking that I needed to have a large number at some pivotal point because it had to add [to ten] on all sides. But I knew I couldn’t put a 5 here and a 6 here [pointing to two spots on the same side]. I had to have one large number at a crossroads, so to speak, and one large number where it did not add to anything else.
That was my train of thought. [Hand gestures indicate a separation of the two larger numbers.] (Teaching Session #3, 5:51:15 – 5:51:38 p.m.)

Figure 5.15: Peggy’s Solution for the Triangle Task (Task #21)

Noticing Patterns

Two clear examples of Peggy’s ability to notice patterns surfaced during Teaching Session #4. After she investigated several different gameboards (rectangular boards made up of alternating squares of two colors) of various dimensions, I asked Peggy to look at the similarities and differences among them. She observed that they all had squares of alternating colors and noted that on the gameboards that had an even number of squares, half of the squares were of each color. She also noted for the boards that each had an odd number of squares in them, she had difficulty predicting how many squares would be each color without seeing the entire board. As described in the previous section, in the case in which she was given only the information that the board would be 15 squares by 9 squares and the pattern would be the usual alternating pattern, she could not determine how many squares would be of each color without drawing out the entire board. She observed that when she divided 135, the total number of squares in the board, by two, the result was 67.5. She recalled that by counting the squares she had learned that
the board that was 3 by 7 had 10 squares of one color and 11 squares of the other. When I asked her to see what she would get if she divided 21, the number of squares in the smaller board, by 2, she used the calculator and got 10.5. Peggy remarked, “I came up with the same problem. . . . [I got] 10 ½ [automatically identifying 10.5 as 10 ½]” (5:53:30 - 5:53:44 p.m.). She recognized that in each case, division by 2 did not automatically provide her with the number of squares of one color, but, unfortunately, could not apply the similarity to what she had observed in her smaller example to resolve her confusion about the larger board. (See previous section on area and perimeter of rectangles.)

Later in Teaching Session #4, I asked Peggy to continue her exploration of area and perimeter of rectangular surfaces by computing the area and perimeter of fields measuring 100 meters by 60 meters and then a field measuring 90 feet by 70 meters. After easily finding the area and perimeter of each field, I asked her what she had noticed about those two fields. She noted that length had decreased and the width had increased. She also observed that the area had increased, while “the perimeter stayed the same because you made up the difference right here [gesturing movement from the length to the width]” (6:09:17 – 6:09:32 p.m.).

Classifying Trials and Looking at Cases

Closely related to the ability to notice patterns is the ability to consider cases. In the development of her strategy, Peggy was able to classify trials as cases, another important aspect of mathematical thought. A striking example occurred when she was working on the task involving the placement of the numerals 1 through 6 at the vertices and midpoints of an equilateral triangle so that each side would sum to 10. (See Task #21
in the Appendix C.) She realized that placing a particular number in the middle of one side of the triangle was equivalent to putting it in the middle of any side. Her hand motions and comments such “or whichever side that you put it on” (Teaching Session #3, 5:52:46 – 5:52:51 p.m.) reinforced this observation. At the same time, she realized the restrictions imposed by the conditions of that case. That is, Peggy understood that once a number was placed, the other positions on the triangle were no longer equivalent. In other words, she had defined a case by placing one large number at a vertex and was able to look at the ramifications of that placement. As she explained:

So I had to have, in my way of thinking, one large number at a pivotal point [pointing to a vertex of the triangle] and one [pointing to the middle of the opposite side] where it didn’t interact with another large number [hands overlapping]. (5:52:51 – 5:53:01 p.m.)

Understanding of the Use of Simpler Examples to Focus on Basic Concepts

Repeatedly, while we were exploring her multiplicative reasoning, I found it useful to present a simpler example to assist Peggy in focusing on a basic concept. Not only was she able to apply the concepts that were demonstrated back to the original task, but she also demonstrated in two major ways her own adoption of this strategy for focusing on the essence of a concept.

One such incident occurred during Teaching Session #3. I had asked Peggy to consider the second part of the Arrays Task (see Task #26 in the Appendix C) in which 62 students of a marching band can march no more than 8 to a row. When asked what would happen if the 62 students were marching in rows of 8 students each, Peggy was baffled. She divided 62 by 8 and interpreted the result of 7.75 to mean that she would
have 7 rows with “point 75 students left” (Teaching Session #3, 5:59:59). I directed her back to her work on Part 1 of the task concerning 36 desks, and asked her what the room would look like if as many rows of 8 desks as possible were used. She thought about this for a moment, made several calculations on the calculator and then, without responding to my question about the desks, automatically returned to the marching-band example. She went on to conclude that she could have 7 rows of 8 with 6 students left over. She arrived at this result by checking that the product of 7 times 8 is 56, and 62 minus 56 is 6. It seemed to me that she understood that I wanted her to look back at the previous example as a way to understand the concept. That is, she seemed to understand that there was an overall concept and that the specific examples were not isolated from each other.

Later in Teaching Session #3, continuing my exploration with Peggy and the concept of area and her multiplicative thinking described in the previous section of this chapter, I presented her with the Gameboard Task. (See Task #27 in Appendix.C) In this task, a gameboard is constructed with white and black squares according to the pattern indicated by displaying several squares in the upper left-hand corner of the board. The complete board would have 15 squares across the top and 9 squares down the side. The student is asked how many squares there are on the board, and how many white squares and how many black squares there are. Peggy immediately multiplied 15 times 9 and concluded that there are 135 squares on the board. She pondered the question about how many white squares and how many black squares and then observed, “It’s not going to be even. [I asked her what she meant.] Well, there’s got to be one more of one than there is of the other one” (Teaching Session #3, 6:06:14 - 6:06:22).
Peggy pondered this observation and made some unsuccessful suggestions, but indicated several times that she did not know what to do. She was simply stuck, so I suggested that she either extend the given drawing or sketch a smaller example. She chose to draw another example with 7 squares across and 3 squares down. She made a complete sketch and counted the white and black squares finding that there were 10 of one and 11 of the other. She observed that there was indeed one more of one than of the other, as she had predicted, and pondered whether the number of black ones would be more in the original example because it was on the corners. As described in the previous section on area and perimeter, she was still at a loss for finding how many of each color there would be in the original situation without drawing the entire figure. I note here her choice to look at a smaller example, choosing odd numbers of squares on each edge, in the hopes of learning something that will help her address the original situation. Although it did not yet yield success, it did reveal her understanding of selecting an appropriate simpler example in order to focus on a concept. (I also note her ability to present a hypothesis that the numbers would differ by 1 and to test that hypothesis.)

During the following session, Teaching Session #4, we returned to the Gameboard Task, and Peggy acknowledged that she had chosen a sample that was 3 squares by 7 squares in order to keep the number of squares like in the original problem. We examined other examples, some with even numbers of squares, some with odd. When I directed her back to the question of the number of black and white squares in the original board that was 15 squares by 9 squares, she still could not see how to answer the questions about how many squares were black and how many were white. She observed that dividing 135, the total number of squares, by two resulted in a fraction, and she was
not sure what to do with that. She postulated that she could assign it to the black squares because it starts in the corner, but, without drawing it out, she was not sure what would happen at the other end. She was trying to see how to answer the questions without drawing more of the board. That is, she persisted in trying to gain the insight into the task that she needed by looking at other examples.

Symbolizing Actions and Concepts

The last section of the Picture Task (see Task #1 in the Appendix) asks the student to write a procedure for accomplishing the task. Peggy wondered whether procedure meant equation, and I told her that she might begin with a verbal description, but I would like for her to try to write the procedure symbolically. She laughed good-naturedly, as she began writing and commented, “And you want me to write it the way that I did it?” (Teaching Session #1, 5:53:20 p.m.). Peggy then wrote a procedure in symbols. As she did so, she hesitated while pondering the difference between the order in which she actually performed the operations and the order in which she wrote them in her formula. That is, the order in which she told me that she entered quantities in the calculator, was (1) add x and y together, and divide by 2, then (2) subtract the result from x. However, the symbolic representation of the procedure that she wrote looked like this:

\[
\frac{x - \left(\frac{x + y}{2}\right)}{}
\]

Peggy commented that the parentheses were needed to keep the order of operations straight. It was instructive to me to think carefully about this apparent conflict between the way that we write (from left to right) and the way that we might logically follow a process (from the middle outwards or even from right to left). Perhaps this
contributes to what some students see as an artificial or magical quality of formulas. At least, this seemed to be the case for Peggy.

An incident in Teaching Session #4 further underscores Peggy’s understanding of symbolized statements. During a joint session, Peggy and Hillary explored the Map Task (see Task #9 in Appendix C) in which a map of two paths between two cities is displayed and the participant is to determine which path requires less time to complete and which path is the shorter distance. The relationship between distance, rate and time and the related formulas, as the participants had recalled them, had been problematic. In Teaching Session #4, I asked Peggy to return to the Map Task, and she indicated that she was still not certain about the concepts. I asked her how far I would travel if I drove 60 miles per hour for 2 hours. Peggy immediately multiplied to get the distance and wrote the formula \( D = R \times T \). With some discussion about how the rate is usually expressed, Peggy wrote the formula \( R = D/T \) and her comments confirmed that she realized that the two formulas expressed the same concepts and were simply written in different forms.

Acceptance of a Challenge in Order to Expand on a Concept

This final attribute that I identified in Peggy was best illustrated by our brief sojourn into the computation of the volume of a rectangular solid. Initially, she had demonstrated considerable confusion about the concept of area. Throughout the sessions, I offered a variety of tasks to assist her in its exploration. As we began working in Teaching Session #4, Peggy seemed to have arrived at a fairly good level of understanding of the concept in the physical settings of both the gameboards and the farmers’ fields that we had explored. To explore beyond area for a moment, I introduced the notion of 1 cubic meter and suggested that one way to visualize 1 cubic meter is as a
cube that is one meter on each side. With no additional guidance, Peggy was able to calculate the volume of topsoil needed for the field of given length and width for both the case of it being one meter deep and the case of it being ½ meter deep. Her demeanor seemed to indicate a certain pride in being able to explore a concept that was totally new to her. Furthermore, when we immediately returned to working with area, she took on the air of someone going back to a familiar concept. Stretching to a more complicated level seemed to enhance her confidence at a level that had previously been problematic.

Conclusion

Thus, although Peggy did not describe herself as a mathematician, I would conclude that she possessed many of the attributes that qualify her to be a worthy student of mathematics – indeed, many of the attributes that mathematics professors should be pleased to find in their students. As I have illustrated above, there are two main reasons for this difference in Peggy’s and my opinions of Peggy as a mathematician; one concerns our definitions of mathematical thinking and actions while the other involves an apparent difference between how Peggy described herself, perhaps what she believed about herself, and how she actually behaved during the teaching experiment. The examples that were described above serve both to point out these differences and to demonstrate the attributes of a mathematical thinker that I observed in Peggy. The attributes were her willingness and ability to be reflective, her instinct for balancing quantities, her inclination toward making and modifying assumptions, her willingness to explore tasks from more than one perspective, her use of insight to develop strategies for addressing tasks, her habit of noticing patterns, her ability to classify trials and look at cases, her understanding of the use of simpler examples to focus on basic concepts, her
ability to symbolize actions and concepts, and her acceptance of a challenge in order to expand on a concept.

Summary

Peggy presented a most fascinating and yet perplexing picture. It was evident that Peggy operated in her frontier zone (Skemp, 1987) in the teaching sessions, but that she did not seem to do so in the classroom setting. Peggy thrived in the atmosphere of the teaching sessions and might well function more productively in the classroom setting given the opportunities to explore learning in similar ways. She repeatedly displayed significant weakness in computation skills and a deficit in her multiplicative thinking. She was aware of this deficit, particularly with regard to the concept of area, and willingly participated in activities to address it. At the same time, Peggy brought to the enterprise of learning powerful intuition and important logic and reasoning skills as well as an open and inquisitive perspective – all of which are extremely beneficial to operating as a mathematician. Peggy spoke with the straddling voice (Kasworm, 1997) as she attempted to make sense of what she was exploring as a student and to connect it with her life experiences and her goals. She valued and was excited about learning and saw her education as a personally enhancing experience. Peggy’s case study raises many questions for mathematics educators, for example: Would her weaknesses in multiplicative thinking be better addressed within a setting different from the traditionally structured classroom and more similar to the teaching experiment? Should her computational inadequacies prevent her from exploring mathematics, given the qualities and skills that would make her a worthy student of mathematics?
Collaboration between Peggy and Hillary

Both Peggy and Hillary reported to me, during their initial interviews, that the two of them had become study partners prior to their involvement in this teaching experiment. They referred to each other as study-buddies. They sat together in class and frequently studied together outside of class. They telephoned each other regularly and got together on occasion to study, particularly prior to a test. Hillary and Peggy agreed to hold two Joint Sessions during our work together. Thus, I was able both to hear from each of them what their perceptions were of their collaboration and to observe a sampling of their style of interaction. There were several indications that both Hillary and Peggy saw this relationship as mutually beneficial, but felt that Hillary was the stronger student of the pair while Peggy was more likely to be the recipient of assistance. The behaviors I observed were not uniformly consistent with those perceptions. The sequence of tasks that I presented to Peggy and Hillary in the joint sessions is listed in Table 5.4.

Table 5.4: Timeline and Tasks for Sessions with Hillary and Peggy

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of session</th>
<th>Tasks and activities (All tasks are listed in Appendix C and are described as they are discussed in the text that follows.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 2, 2000</td>
<td>Initial Interviews with Hillary and with Peggy</td>
<td></td>
</tr>
<tr>
<td>February 9, 2000</td>
<td>Teaching Session #1 with Hillary and Teaching Session #1 with Peggy</td>
<td>(Tasks for individual sessions are listed in timelines for each of the participants..)</td>
</tr>
<tr>
<td>February 16, 2000</td>
<td>Teaching Session #2 with Hillary and Teaching Session #2 with Peggy</td>
<td></td>
</tr>
<tr>
<td>March 15, 2000</td>
<td>Teaching Session #3 with Peggy</td>
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<tr>
<td>Date</td>
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| March 17, 2000     | Joint Session #1 with Peggy and Hillary    | Task #12 (Spaghetti Supper)  
                             | Task #4 (Fence Painting)  
                             | Task #9 (Map)                                                      |
| March 20, 2000     | Teaching Session #3 with Hillary           |                                                                       |
| March 22, 2000     | Teaching Session #4 with Peggy             |                                                                       |
| March 29, 2000     | Teaching Session #4 with Hillary           |                                                                       |
| April 5, 2000      | Teaching Session #5 with Hillary and       | Task #3 (Merchant and 3 Fairs)  
                             | Teaching Session #5 with Peggy                                     | Task #28 (Area of a Lake)  
                             | Task #38 (Fencing)                                                    | Task #37 (Space Station)                                                |
| April 13, 2000     | Joint Session #2 with Peggy and Hillary    |                                                                       |

**Peggy’s Point of View**

Peggy reported that she and Hillary had become study partners. Their study sessions had included clarifying notes from class and explaining to one another what they had learned in the classroom. They usually telephoned each other after each class meeting to confirm that they had the same impressions about the way the instructor had presented material and the way they should complete the assignments. Peggy referred to this interaction between herself and Hillary frequently during our sessions, usually indicating that Hillary was a “quicker study” (Initial Interview, speech #60) or more apt to be the helper while Peggy was the one being helped. Although Peggy reported that these exchanges were active, she indicated that they had been solely based on the information and the method the instructor had presented in class.
Hillary’s Point of View

Hillary found her interaction with Peggy as a study partner to be very constructive. She reported that they sat together in class and that outside of class they went over notes and studied. Hillary described their relationship this way:

Hillary: Because there are things that I don't understand sometimes that she's got, and vice-versa, and that's been real helpful. We studied together for every single test that we had last semester, and when - ah - I guess just like siblings and just like in any class setting, when you get close to somebody, there's been some - a little competition on the side. O.K. let's see who can get the highest score this time? But, it's been a motivation for both of us. And things that she doesn't feel real confident in - we've really tried to boost each other's ego. I'll say, yes, you can do this. Don't say you can't, because, yes, you can. We - There aren't too many weeks go by that we don't call each other on Tuesday night or Thursday night to homework-help either. (Initial Interview, Speech #50)

Hillary indicated that the purposes of these phone calls and study sessions were to reinforce that which had been presented in the classroom by the instructor and to motivate each other.

Joint Reactions

At the beginning of the first Joint Session with Hillary and Peggy, they reiterated their descriptions of their study-partnership. Peggy again claimed that it was usually she who was requesting assistance from Hillary. They agreed that it was often difficult to explain mathematical information while talking on the phone. When pressed, it seemed that the issue was one of communicating images rather than explaining concepts. Peggy
commented, “It’s hard for me to visualize, because I’m used to [the instructor] writing it on the board, so I’m having to get a mental picture as opposed to an actual physical picture” (Joint Session #1, 1:26:10 – 1:26:16 p.m.).

They explained the possible effects on their learning as follows:

Peggy: In the few instances when I have been describing something to her, I think it just helps – I don’t know about her, but it helps me under- I already understand it or I wouldn’t be describing it, I think it helps cement it a little bit more. But I don’t usually end up explaining very many things to you.

Hillary: Yeah, it does help internalize it though. (1:26:45 – 1:27:10 p.m.)

They agreed that a major difficulty they had experienced in their mathematics course work had been deciding what technique to employ. Their instructor had urged them to practice, and they had complied. However, when different types of situations were presented to them, they have had trouble knowing how to begin. Hillary’s and Peggy’s comments revealed their dilemma.

Peggy: To me with math, the way I’ve learned to deal with it- it’s a repetitive function. If I do it over and over and over and over – Sooner or later it gets there.

Hillary: But in that same token when you say that . . . In class . . . I said how do you know that’s one of those you have to set to zero to solve though, and she [the instructor] said “Just Practice’ . . . I’ve got it when I’m doing just that kind of problem, but when you mix all the problems together, some of them, yeah, I know immediately, but some of them . . . I look at it and I . . . don’t know what to do with it.
Peggy: When you mix it with something else, all of it convoluted in my mind and I can’t separate it out. I can’t separate out the ones that we set equal to zero and the ones that we work like just a normal problem or we just factor or we just – [groan and hold hands over eyes] - I don’t want to think about it. (Joint Session #1, 1:27:12 - 1:28:12 p.m.)

They had learned that mathematics is a repetitive process, and the instructor had told them to practice, but students had difficulty determining how to begin. However, during their study sessions, both individually and together, the students had not addressed this major concern. Furthermore, I witnessed many incidents during this teaching experiment in which the students appeared to be searching for the appropriate technique, previously learned in the mathematics classroom, that would fit the tasks being presented – that would give them a way to get started. This behavior validated that they did indeed have difficulty in this area. It is notable that these two seemingly responsible students were perceptive enough to recognize the difficulty they were having but either were not able to or did not think it appropriate to address it.

During the teaching experiment, the students frequently were grappling with answering their very basic question of how to begin by searching their prior experience in mathematics class in hopes of finding a previously practiced pattern with which to approach the tasks. The fact that these were non-routine tasks and that I was trying to draw from them knowledge not necessarily learned in the classroom presented an unusual twist for them. Occasionally, they searched their prior experience from outside the mathematics classroom, but this was not an automatic response. It seemed that they had
learned that it was better to put the outside of the classroom experience aside in favor of the textbook or the instructor’s guidance.

The Nature of Their Collaboration

The degree to which Peggy and Hillary collaborated varied throughout the Joint Sessions. I found myself reminding them often to discuss with each other how they might approach a task, how they had chosen to address it, and what they were thinking in the process. It seemed clear that working together for them had previously meant one helping the other and, as they had stated, reinforcing the work as presented in the classroom, rather than exploring new or different ways of thinking. I comment further about the nature of their collaboration at the end of this section.

Peggy’s Inclination Toward Collaboration

Peggy was significantly more inclined toward collaboration than Hillary. Her comments were peppered with plural pronouns (e.g., What should we do now? p.m.) as she pondered how they should begin or proceed or adjust their thinking when things seemed to be going awry. She often issued invitations by stating what she had done and wondering whether Hillary had done the same thing. As she had claimed, Peggy often turned to Hillary for technical assistance, but many of her other comments were of a different, more cooperative nature. The setting and my frequent reminders that they should work together, help each other and tell each other what they were thinking may have influenced her. In any event, achieving collaboration was not automatic for the two of them, and Hillary was much less amenable than Peggy. This was one of Peggy’s strengths on which the two self-proclaimed study-buddies had not capitalized.
Effect of Familiarity with Context

The Spaghetti Supper Task (Task #12 in Appendix C) asks what information would be needed to plan a Spaghetti Supper as a fund-raising project for an organization.

It was interesting that when Peggy and Hillary were addressing this task during Joint Session #1, Peggy found this to be a very familiar situation. She actually stated that it was reminiscent of situations with which she had dealt at work. Her actions substantiated this familiarity, because when interpreting and modeling this task, she very easily presented specific, even quantitative, hypothetical scenarios. Hillary acknowledged that the situation did not appear similar to any prior experiences and her comments were more general.

Peggy: If you’ve got $500 and you’ve got 50 people, then you could probably do it. But if you’ve got $500 and 1000 people, then you probably can’t do that. I mean it’s all relative, to me, I mean, the dollars in relation to the people. Now if you can trim back people, that’s O.K., but it may be something where there’s not anybody you can leave out without someone’s being offended, so it might be better not to have it at all. I have to deal with this kind of stuff all the time!

Hillary: [mutters] I don’t.

Peggy: I mean that’s why I’m saying that. Me, I can’t go past that point, ‘til I know how much money I have. (Joint Session #1, 1:30:10-1:30:45 p.m.)

Concerning the fund-raising aspect of the event,

Peggy: Yeah, because you’ve got to find out – If it costs $10 per head to feed ‘em, can you charge 25 to raise money? To me, the fact that it’s a fund-raising project makes this [the budget] even more important. (1:30:57-1:31:14 p.m.)
Hillary was ready to summarize and move on, while Peggy was still making suggestions.

Hillary: It might be something where you have x number of dollars and you already know how many people. This place may charge something ridiculous, and this place may – you say you can do it, if we go with this place, so –

Peggy: Or you may have one of these people who can donate it – there may be a caterer who can donate. (1:31:39-1:31:55 p.m.)

Just as Hillary’s familiarity with some of the technical aspects of other tasks provided her a comfort zone in which to operate, Peggy’s experience in this particular setting permitted her to operate with greater ease.

Joint Visualization Experiences

One area in which Peggy and Hillary differed in their approaches to the tasks was in the visualization of the situations that were described. This difference created some stumbling blocks to their cooperation. Peggy was usually quite successful in this endeavor, while Hillary would sometimes offer an interpretation that missed the mark, and she then frequently remained resistant to thinking through the task with a fresh perspective. Two clear examples of this phenomenon occurred during Joint Session #1 when the two were working on the Fence Posts Task (see Task #4 in Appendix C) and during Joint Session #2 when they were addressing Task #3: The Merchant & the 3 Fairs (see Task #3 in Appendix C).

The Fence Posts Task contained a number of details that may have contributed to the variance in interpretation by the participants. It read as follows:

Two girls, Jan and Tanya, have a job painting fence posts that line both sides of a path leading to a barn. Jan arrived early and had already painted 5 posts on one side
when Tanya arrived. Before she started painting, Tanya said, "Jan, I'm left-handed and it's easier for me to paint this side of the path while you paint the other side." Jan agreed and went over to the other side of the path. Tanya painted all the posts on her side, then went across the path and painted 10 posts on Jan's side. This finished the job. If there were the same number of posts on each side of the path, who painted more posts and how many more? (Task #4 in Appendix C, based on #138 in Krulik & Rudnick, 1989)

When the Fence Posts Task was first presented during Joint Session #1, Peggy
and Hillary began working together to understand the setting. At first, they searched mentally through recent formulae with which they had worked, but they quickly abandoned those to just talk it through. They encountered differences of opinion about the wording but seemed to resolve them. They then each worked individually and discovered that they had arrived at slightly different conclusions. They were urged to explain their approaches to each other in an effort to come to a united conclusion.

Almost from the outset, Peggy had a clear image of what was physically occurring. She combined the five posts that Jan had already painted on one side before Tanya completed them with ten additional posts that Tanya did on the other side to conclude that Tanya actually painted 5 more posts than Jan. She referred to this exchange as having “plussed” herself only 5 rather than 10 because 5 were already done for her. Hilary agreed that Tanya painted more posts, but concluded that she simply painted the 10 extra posts as it was written. Their interaction was particularly troublesome because Hilary arrived at the correct numerical answer somewhat accidentally while Peggy’s more careful but incomplete analysis of the situation resulted in a numerically incorrect
response. They were both certain that Tanya had painted more posts and Hillary was willing to go along with Peggy’s claim that the difference was 5.

Toward the end of this process of summarizing what had transpired, Hillary suggested that she had seen the situation totally differently from the beginning, and that was the reason that their results were different. In this alternate interpretation, Tanya and Jan were painting opposite sides of individual posts rather than posts on opposite sides of the path. This startled both Peggy and myself because nothing in her comments to that point indicated that she had had a different mental image of the task. There had been many references by both participants to sides of the path and none to the sides of each post. Rather quickly, Hillary willingly acquiesced to Peggy’s interpretation and results, more as an observer than as a colleague in the enterprise. She was not willing to return to the task with the amended understanding.

Task #3, the Merchant and Three Fairs, also included several details that required sequencing in an appropriate way.

A merchant visited three fairs. At the first, he doubled his money and then spent $30. At the second, he tripled his money and then spent $54. At the third fair, he quadrupled his money and then spent $72. He then found that he had $48. How much money did he start with? (Task #3, based on #139 in Krulik & Rudnick, 1989)

The encounter between the participants as they addressed this task during Joint Session #2 was particularly revealing. From the beginning, both Peggy and Hillary symbolized what occurred at the first fair as 2x-30, at the second fair as 3x-54 and at the third fair as 4x-72, where x represented the amount of money the merchant brought to the first fair.
The difference in the way that the two perceived the situation was evidenced in the equation that each one constructed as well as in the manner in which each one initially checked her results. Hillary began by indicating that she had used $x$ to represent the money that the merchant started with and then contradicted herself by saying that the expression $2x - 30 + 3x - 54 + 4x - 72 + 48$ represented the amount they were looking for and wrote $2x - 30 + 3x - 54 + 4x - 72 + 48 = ?$. She proceeded to work with this statement as though it were the equation $2x - 30 + 3x - 54 + 4x - 72 + 48 = 0$, and from this she concluded that $x = 12$. Meanwhile, Peggy explained that $x$ represented the amount of money that the merchant started with and built the equation: $(2x - 30) + (3x - 54) + (4x - 72) = 48$. After careful calculation aided by a calculator, she concluded that $x = 22.66$.

Peggy and Hillary observed that both answers were reasonable but that they are different. At this point, Hillary seemed discouraged, perhaps because she typically first checked her responses by determining whether they were reasonable. Noting the two reasonable but different responses, she was resistant to pursuing it further. When asked how she might determine whether her results were correct, she suggested plugging her answer into her equation. Meanwhile, Peggy had began talking through what would need to be done in order to check the results in the original situation, but wanted to go back to check her arithmetic once more. She had, as usual, little confidence in the accuracy of her computations even though all were done on the calculator.

Hillary was persuaded that she should check her results in the original situation. When she began explaining to Peggy how her answer would check in the situation, she soon arrived at the impossible situation of the merchant having a negative amount of
money before leaving the fair. At this point Hillary asked whether the merchant is supposed to be tripling at the second fair what he had left after the first fair or starting out with the same amount of money at each fair. It was agreed that the former was the case. Hillary indicated that she had misinterpreted the situation and had thought he was starting out with the same amount at each fair. To adjust, she tried the approach of setting equal to zero the expression that represented the activity in the first fair and solved the equation $2x - 30 = 0$, concluding that the merchant left the fair with $15. Even though she observed that this would result in the merchant having a negative amount of money after the second fair, she insisted that they must solve this first step before going on; that she must find out how much the merchant took to the next fair.

Peggy indicated that she had understood the task correctly; that is, that the merchant had taken the money from each fair to the following one and had not necessarily started with the same amount of money at each fair. When Peggy attempted to check her result by following the directions in the original problem, she doubled her result of $22.66$, got $45.32$ and subtracted $30.00$ from that, concluding that the merchant left the first fair with $15.32$. She then multiplied that amount by 3 and realized that the resulting $45.96$ was not sufficient for him to be able to spend $54.00$ at the third fair. Peggy concluded that she had done something wrong. She believed that although she read the task correctly, she had not set it up correctly and began trying to restate it.

In verbalizing the situation, Peggy finally was able to move toward a more accurate representation.

Peggy: O.K. he goes to the first place, and he has whatever his money is and he doubles it and he spends thirty. So, whatever the results of that is is what he takes to
the next one that he triples. (Joint Session #2, 12:41:30-12:41:45 p.m., emphasis added)

While talking, Peggy wrote \((2x – 30 =)\). From her words and hand motions, it seemed that she was on the way to viewing the expression \(2x – 30\) as the quantity that the merchant took to the second fair, and so forth. She was interrupted in her thinking by Hillary’s insistence that they set \(2x – 30\) equal to zero and that the merchant therefore had $15 to take to the second fair. Peggy hesitantly went along with this suggestion – setting the expression equal to zero had a note of familiarity for both of them - but still looked unsure. She remarked, “O.K. Let’s see how this works out . . . . See, we’re not doing something right” (12:42:48 – 12:43:03 p.m.).

Peggy seemed to have become convinced by Hillary that they must solve this first step before going on; that they must find out how much money the merchant took to the second fair. She acknowledged that, based on the information given, one could not know this, rubbing her face and eyes in frustration, but Hillary underscored this way of thinking insisting that they must find out how much the merchant took from the first fair to the second fair. Peggy, pondered a different way to think about it, then, gesturing to some spot in the air in front of her, said, “It’s right here and we can’t find it” (Joint Session #2 12:44:27 – 12:44:29 p.m.). After talking through the situation once more, Peggy was ready to pursue it further, but Hillary again asserted she did not see it any differently. It seemed that Hillary was adamant about not trying any other approaches and that, although Peggy was still thinking of ways to approach the task, she was heavily swayed by Hillary’s viewpoints. We put this task aside.
Hillary Taking the Lead

When it came to the technical aspects of working through the solution, Peggy frequently displayed the same lack of confidence in her own abilities that she had in the individual teaching sessions, and often turned to Hillary for guidance. Hillary assumed an authoritative stance at those moments. In fact, she was much more comfortable encountering the tasks at the technical level.

During Joint Session #1, when Hillary and Peggy were working on the Map Task #9, they had found an approximate location at which the car had traveled each of the given distances, and they needed to calculate the time traveled. As they sought an appropriate formula, this interchange occurred.

Peggy: And isn’t it [the formula] rate over distance?
Hillary: [hesitantly, softly] I think it’s distance equals rate over time. I don’t know.
Peggy: I’m not sure.
Hillary: I think that’s right.
Peggy: So you would have 400 equals 45 over x, right? We don’t know what the time is yet . . .
Hillary: Well, we need to solve for x. [voice getting stronger] so – (Joint Session #1, 1:53:23 - 1:54:05 p.m.)

Hillary asserted herself more as they moved into the technical manipulations and Peggy looked to her for assistance.

Peggy: See, I don’t know how to go from here.
Hillary: Yes, you do [voice very firm]. All right, yes, you do too . . . What do you have to do with that fraction?
Peggy: Do we want to multiply it all by $x$?

Hillary: Uh, huh, that’s what she [the instructor] told us to do. And the $x$’s will cancel out on that side.

Peggy agrees and proceeds. (1:54:19 - 1:54:46 p.m.)

The result was not reasonable and, unfortunately, the session came to a close. In later independent sessions, the participants each returned to work further with this task, as was reported in Chapters 6 and 7.

**Dependence on and Comfort with Formulas**

It should be noted that Hillary was much more comfortable with encountering the tasks at an operational level; that is, she was pleased when she could apply a formula, even if it was misremembered, or employ an established routine to a task. When the participants were examining rectangles whose areas were 1 square inch, Hillary calculated the area of a rectangle that was one-half inch by 2 inches to be 1 square inch. When asked whether she believed that the area of the rectangle was one square inch, Hillary was incredulous at being asked such a question. She was confident of the formula. If the calculation turned out that way, and the result seemed reasonable, there was no reason to question or think further about what the result meant.

Meanwhile, Peggy was more willing, and actually eager, to substantiate such an outcome with physical evidence. Perhaps her lack of confidence in her recall of the formula was at play, but she also appeared to want to substantiate the outcome against the concept. Even when she was confident of the formula, she wanted the outcome to make sense to her.
This same dynamic was also at play when the two were addressing the Merchant and the Three Fairs Task during Joint Session #2. Hillary was confident of the notion that setting quantities equal to zero was a valid way to arrive at a solution and campaigned with vigor for them to apply that routine to find a solution. Peggy understood that routine and felt pressured to follow Hillary’s lead, but realized that not only was the result unreasonable but, even more crucial, it did not seem to fit the situation.

Another vivid example occurred during their work in Joint Session #2 on the Fencing Task #33. Peggy expressed her confusion with the possibility that one could have rectangles with the same area but different perimeters. It seemed that the results of the formulae were sufficient to convince Hillary, but that Peggy found those results hard to believe in the face of her lack of comprehension of the concepts.

Peggy: But see, I don’t understand why. Because if you’ve got – I don’t know – If you’ve got 400 meters of fencing, you’ve got 400 meters of fencing. So, why does this give you more area than this [gesturing to the two examples]? . . .

Hillary: Because the lengths of the sides are different, and to get the area, you multiply the side times the side.

Peggy: I understand – [meaning she knows the formula]

Hillary: Now, I can’t explain it any further than that, that’s as far as my knowledge goes

Peggy: I understand that, but you still have only 400 meters of fencing.

(Joint Session #2 12:58:52 - 12:59:34 p.m.)
Peggy Taking the Lead

On the occasion that Hillary followed Peggy’s lead, it seemed that Hillary was frequently unaware that she had been guided by Peggy. However, Peggy took pride when she sensed that her suggestions had been accepted.

During Joint Session #2, while working on the Area of the Lake Task #28, revealing interplay occurred between the two participants. Hillary took the initiative in sketching a rectangle entirely enclosed within the lake. Noting how she began, Peggy sought the maximum dimensions for an enclosed rectangle and chose to sketch one whose vertices extended only slightly beyond the rim of the lake. (See Figure 5.16 below.)

![Figure 5.16: Peggy’s First Rectangular Approximation.](image)

Peggy: I was trying to figure out to get a estimation if I could take it from what I considered its first almost full point where they were almost all full squares [gesturing along the left side width of about 10 squares] by its almost full length [gesturing...
along bottom row of 12 squares] I could get some kind of close estimation. [pause]

But I’m not sure because it’s not that wide all the way across. (Joint Session #2,
12:49:50 - 12:50:18 p.m.)

When asked to compare their approaches, this conversation occurred.

Peggy: Well, you took out a full square, but see, some of mine [trails off]

Hillary: You’re taking a rectangle.

Peggy: But I don’t have a full end over here [gesturing to the right end of the base

line she has chosen]

Hillary: Well, we don’t have to have a full end if we’re just estimating.

Peggy agrees . . . and then: Well, what are you going to do now?

Hillary: Well, as I’m looking at this now, I probably would have dropped this down

another line down here [gesturing to the bottom of her rectangle, which she

labeled 9 by 10 but actually was 10 by 10] because other than a little teeny tiny

piece here. And up here [gesturing to the top], I mean it pretty much encompasses

- So I probably would have extended it another row [gesturing to top and bottom

of her original figure] now that I’m looking at it. (Joint Session #2 12:50:25 -

12:51:59 p.m.)

Peggy laughed, seeming to enjoy the evidence that Hillary had been influenced to rethink

her solution after observing Peggy’s approach.

When asked how they could then continue to more closely approximate the area

of the figure, Peggy began looking for rectangles within the regions of the lake that were

not included in the rectangle that Hillary had sketched totally within the lake. Peggy

suggested, “Could you go – 11 by 1 or 10 by 2?” (Joint Session #2 12:51:27 p.m.) Hillary
seemed to back away from the drawing and commented, “I mean that’s fine, but I probably would not do that” (12:51:31 p.m.). Hillary insisted that she understood what Peggy was doing, but that she would not have approached the task in the same way. Only after being pressed by both the researcher and Peggy to suggest how she might approach it, Hillary drew a second rectangle that included more the main body of the figure, but whose vertices extended outside of the figure. The following comments were made.

Hillary: I probably would have gone to that as my next thing.

Peggy: Then it would be easy to just shave this off. [Gesturing to the pieces that fall outside the figure]

Hillary: [not showing any outward signs of having heard Peggy’s last comment] And I mean, I mean, I know I have pieces that are not included in that [the original figure], but if each of these little squares is a square foot, then I can count the full squares here and knock them out. (12:52:27-12:52:41 p.m.)

Hillary eventually extended her rectangle completely around the lake. (See Figure 5.17 below.)
They proceeded with a discussion about how to decide which parts of squares to include and with observations not far removed from the processes of constructing under-approximations and over-approximations. Peggy’s ideas were often expressed softly and tentatively, as was demonstrated above. She had difficulty making her voice heard, but appreciated it when it was heeded.

A Final Look at Their Collaboration.

The two outstanding examples of collaborative work for Hillary and Peggy both occurred during Joint Session #2. As was discussed earlier, both of the participants were apparently fully involved and agreeable to the two methods of approximating the area of the figure in the Area of the Lake Task #28. They repeatedly not only encouraged each other with positive feedback but also fed off of each other’s ideas to develop their own.

At the end of the second Joint Session, they attacked the Space Station Task #37. In this task, the participants were asked to place eleven astronauts at relay stations for the
Intergalactic Space Ship Line according to certain specifications. Peggy immediately understood the premise that people needed to be placed according to the number of people with whom they were friendly. Hillary suggested that they make a tabulation of the people and their friends. Peggy actually wrote the list, but they collaborated completely on the process. Together they placed the person who would associate with the most people at the hub with the most contacts and worked from there. Peggy seemed more willing to move people about as needed. For Hillary, once they were placed, she was more hesitant about making choices, but yielded to Peggy’s suggestions. Time did not permit them to complete the task, but they were well on their way to an appropriate solution. They appreciated the compliments given them about how they cooperated and how immediately they tackled the task. A discussion ensued about the value of group work and their minimal opportunities to engage in it. Clearly, the last task that the two addressed during Joint Session #2, the Space Station Task, seemed to elicit the most constructive collaboration between Hillary and Peggy.

One might conclude that the experience of participating in the teaching experiment contributed to the students’ cooperative behavior during this final session. Perhaps they had become accustomed to the environment and to my expectations, that is, to the culture of the setting. It could also be observed that for each of these tasks, the students were on relatively equal footing. Neither seemed significantly more comfortable with the tasks or more familiar with the needed tools.

Something else seemed to be at play in the final task, the Space Station Task. The students were significantly more relaxed and cooperative in their work. It seemed that because there are no apparent quantities or calculations with which to deal, one could
safely assume that Peggy and Hillary probably saw this task as one of the least mathematical of those they explored. During the conversation that ensued following this task, they reiterated that opportunities for students to explore topics with their peers were not usually offered in mathematics and did not seem as appropriate in mathematics as in other course work.

The students’ behavior viewed in this way underscores the notion that for Peggy and Hillary, mathematics was a topic that they felt neither able nor empowered to explore on their own. As they addressed tasks that were, in their view, more distant from mathematics, they were freer to creatively take control of the situations. When encountering tasks that resembled to some degree the kind of tasks and problems they confronted in mathematics courses, they felt compelled to unearth some predetermined correct way to do them. Hillary, in particular, became more comfortable with such tasks when she could get to the point that she could apply a method that she recalled from her course work. Peggy often became frustrated when she thought there was some technique that she ought to remember. Basically, they each tried to recall what an instructor had told them.

**Summary**

Hillary and Peggy’s work as a pair on these tasks underscored the observations that I had made about their individual approaches during the sessions I held with each of them alone. Once Hillary had a result, for her the activity (and her own thinking about it) was curtailed. Peggy, on the other hand, continued to question the result in light of her own intuitions about the situation. This behavior indicated a more open-minded approach to the tasks. On the whole, Hillary’s approach seemed more closed minded than Peggy’s
approach, and this difference impacted on their individual performance as well as on their collaborations.

Significant aspects of Peggy and Hillary’s collaboration were both the way that each of them evaluated it and the degree to which they each benefited from it. Both of them found their work as study-buddies to be constructive, and both perceived that Hillary was more apt to be the helper while Peggy was the one being helped. In the context of their traditional classroom, Hillary’s superior procedural or technical skills would be valued more highly; Peggy did turn to Hillary for support and assistance in those areas. However, in the context of the teaching sessions, Peggy’s intuition, observational skills and logic, were more valuable. Peggy was able to exhibit some leadership in these areas during the joint sessions, but Hillary’s general discomfort with the atmosphere seemed to prevent her from taking full advantage of Peggy’s assistance. The question remains as to whether further training and experience in operating in an environment more similar to the teaching sessions would permit Hillary and Peggy to collaborate in ways that would be more beneficial to both of them and to their learning.
CHAPTER 6

ANSWERING THE RESEARCH QUESTIONS

It is now time to return to the research questions and to address them formally. Most of what is presented in this chapter is an analysis of what has been already stated in the previous chapters. It is organized here in such a way as to directly address the research questions as stated in Chapter 1.

Research Questions - Restated

The major question addressed in this study was: With what perspective do adult re-entry students engage in the study of mathematics at the most basic level in higher education? Specifically, I investigated characteristics of the students' affective orientation toward learning, the nature of their mathematical thinking in the context of mathematical problem-solving in contrast to procedural learning, and finally, the possible relationship between their affective orientation toward the learning of mathematics and their mathematical thinking.

1. The investigation of the students’ affective orientation toward the study of mathematics included their stated and demonstrated beliefs about themselves as learners and as mathematics learners and about the nature of knowledge and the nature of mathematical knowledge and their stated and demonstrated beliefs about the role of the student and the role of the instructor in general in higher education and in the study of mathematics. In the case of the study of mathematics, I investigated the contextual nature of their beliefs; that is, did the students' perceive these roles
differently in the setting of their mathematics classroom than in the setting of the teaching experiment? I also investigated their reasons for participating in higher education and for studying mathematics and the factors that have positively or negatively influenced that participation.

2. With regard to the students' mathematical thinking, I focused on their ways and means of approaching mathematical tasks that go beyond procedural skills, when engaged in mathematical problem-solving. I was interested in the connections students might have made between their problem-solving abilities, perhaps developed in out-of-school contexts, and their work on mathematical tasks. For example, to what extent did the students call on their prior experiences and current life-world environment when engaged in a mathematical problem-solving setting? Did assisting the students in perceiving connections between the academic world and their prior and current environments enhance their level of engagement in mathematical problem-solving tasks?

3. Finally, I investigated the relationships that might have existed between the above characteristics of the students’ affective orientation toward mathematics and their mathematical thinking in a problem-solving setting.

Question 1: Students’ Beliefs and the Factors Influencing their Re-entry into Education and the Study of Mathematics

Students’ Stated Beliefs

As was reported in Chapter 4, the eighty-nine students who were present in seven classes completed the initial questionnaires. Their responses to the Likert-scale items on the initial questionnaires did not indicate strong differences between beliefs of the 70
Traditional students and the 19 Re-entry students. In fact, in many ways, similarities were indicated with the possibility that in some cases Re-entry students may hold more extreme views or hold their beliefs more intensely.

The responses to the Likert-scale items on the initial questionnaires indicated that most students believed that they could solve time-consuming mathematics problems. However, the means of all the groups studied were rather low, indicating that, in general, this was not a very intensely held belief. Most of the students also agreed with the beliefs that understanding concepts is important in mathematics and that mathematics is useful in daily life. These two beliefs were more intensely held by the re-entry students than by the traditional students in my study. The responses indicated that most of the students in my study agreed with the belief that effort can increase mathematical ability; that, in mathematics, processes are more important than rules and calculations; that they have never been able to do mathematics very well and that people either have a talent for doing mathematics or they don’t. Most also agreed that, with hard work, they could usually succeed in both mathematics and non-mathematical subjects, but their belief regarding non-mathematical subjects apparently was held more strongly.

Most students disagreed with the belief that there are word problems that cannot be solved by simple step-by-step procedures. That is, they seemed to believe that all word problems could be solved by simple step-by-step procedures. Re-entry students tended to hold this belief more strongly than did the Traditional students in my study. The responses indicated that most of the students also disagreed with the belief that word-problems are important in mathematics. However, the means for this item for both groups indicated that this was not a strongly held belief or that the students were somewhat
ambivalent about this belief. In conversation, the participants in this study revealed that the students were aware that knowing how to solve word problems was important for their success, but they often did not perceive the importance of the problems themselves. Thus, they underscored the confusion in students’ beliefs about the importance of word problems that the data seemed to suggest.

To summarize, most of the students claimed that they had never been able to do mathematics very well and that people either have a talent for doing mathematics or they don’t. In spite of that, most agreed that with effort, mathematical ability could be increased and that with hard work, they would be able to succeed in both mathematics and non-mathematical subjects. It should be noted that their belief about the effectiveness of hard work was apparently stronger with regard to non-mathematical subjects. Furthermore, re-entry students reported stronger beliefs on these scales.

The responses to the questionnaires indicate that most students stated that they believed that mathematics is useful in daily life. They also indicated that most students were either ambivalent about or took issue with, to a small extent, the importance of studying word-problems. It would seem that many of the students did not see a linkage between the word-problems they usually study in mathematics and the application of mathematics in their daily lives. Thus, although they stated that they believed that mathematics was useful in other areas of their lives, one might observe that at least with regard to this aspect of the study of mathematics, the students tended to not see a connection between the classroom study and their life-world environment. It is helpful to see the potential effect of this disconnect through the lens of the Model of College Outcomes developed by Graham and Donaldson (Donaldson et al., 1999, April). This
model was presented in Chapter 2, and I have reproduced it here to assist the reader in following this and other discussion in the current chapter.

Figure 6.1: Model of College Outcomes for Adults developed by Graham and Donaldson (Donaldson et al., 1999, April) Reprint of Figure 1.1, page 8

Most students reported that understanding concepts was more important than rote memorization and that processes were more important than rules and calculations. However, most also believed that word problems could be solved by simple step-by-step procedures. Here it would seem that there was a difference between what the students believed about mathematics in general and how they believed they needed to operate in order to be successful in the study of mathematics in their coursework. This conflict could be considered in a number of ways in terms of the Model of College Outcomes for Adults shown above. In one sense, the students’ value orientation toward mathematics was different from the tasks they were typically presented in the mathematics classroom. Their success (the outcome of their work) was measured by the degree to which they could memorize rules, perform routine calculations and solve problems using proscribed
step-by-step procedures, even though their value orientation would lead them to focus on understanding processes and concepts. This conflict could also be seen as a disconnect between the students’ patterns of cognition and the behaviors demanded in the coursework, as well as between the possible applications in their life-world environments and the sorts of applications presented in the classroom. These conflicts in combination can be seen as a potential breakdown of positive interaction between the classroom and all three of the realms with which it interacts: cognition, value orientation and environment. A negative impact on outcomes would be the inevitable result.

The conflict with regard to understanding concepts could also stem from different interpretations of the word “understanding.” When students indicate a desire to have a better understanding they could mean that they want to understand the task sufficiently to know what to do or even how to begin. This could indicate their desire simply to know which formula to use in a given situation and how to apply it. For other students, a desire to understand could take on the deeper meaning that would indicate a more mature perspective. Similar to the confusion that has been discussed regarding students’ attitudes about word problems, this confusion about the term “understanding concepts” emphasizes the need to examine more than written or verbal responses to a questionnaire in order to make conclusions about students’ perspectives in this area.

Participants’ Stated and Demonstrated Beliefs

I was interested in students’ stated and demonstrated beliefs about themselves as learners and as mathematics learners and about the nature of knowledge and the nature of mathematical knowledge and their stated and demonstrated beliefs about the role of the student and the role of the instructor in general in higher education and in the study of
mathematics. In the case of the three participants who completed the study, in addition to their statements on the initial questionnaire, during the interviews and teaching sessions, I observed the three participants’ demonstrated beliefs along with hearing other statements about what they believed. Many of these observations are presented in the sections of Chapter 5 that focus on the participants, but I will summarize them here to more specifically address this research question.

Throughout this analysis, I have found it useful to employ the lenses of Baxter Magolda as presented in her Epistemological Reflection Model (1992) and of Kasworm as described in her five belief structures or knowledge voices (1997, March). I have reproduced them in this chapter to assist the reader in following the discussion. I would like to emphasize that my use of these models was not an attempt to label the students but rather a way of organizing the information that I had learned about the students. In addition, I was interested in discovering whether these models (constructed in more general settings) were appropriate when examining students’ perspectives with regard to learning mathematics, especially re-entry students in remedial coursework.

Ways of Knowing

Table 6.1 below presents again Baxter Magolda’s Epistemological Reflection Model (1992). The following discussion refers to this table.
Table 6.1: Baxter Magolda’s Epistemological Reflection Model (1992) Reprint of Table 2.1, page 22

<table>
<thead>
<tr>
<th>Domains</th>
<th>Absolute Knowing</th>
<th>Transitional Knowing</th>
<th>Independent Knowing</th>
<th>Contextual Knowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Role of learner</td>
<td>▪ Obtains knowledge from instructor</td>
<td>▪ Understands knowledge</td>
<td>▪ Thinks for self</td>
<td>▪ Exchanges and compares perspectives</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>▪ Shares views with others</td>
<td>▪ Thinks through problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>▪ Creates own perspective</td>
<td>▪ Integrates and applies knowledge</td>
</tr>
<tr>
<td>Role of peers</td>
<td>▪ Share materials</td>
<td>▪ Provide active exchanges</td>
<td>▪ Share views</td>
<td>▪ Enhance learning via quality contributions</td>
</tr>
<tr>
<td></td>
<td>▪ Explain what they have learned to each other</td>
<td></td>
<td>▪ Serve as a source of knowledge</td>
<td></td>
</tr>
<tr>
<td>Role of instructor</td>
<td>▪ Communicates knowledge appropriately</td>
<td>▪ Uses methods aimed at understanding</td>
<td>▪ Promotes independent thinking</td>
<td>▪ Promotes application of knowledge in context</td>
</tr>
<tr>
<td></td>
<td>▪ Ensures that students understand knowledge</td>
<td>▪ Employs methods that help apply knowledge</td>
<td>▪ Promotes exchange of opinions</td>
<td>▪ Promotes evaluative discussion of perspectives</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>▪ Student &amp; teacher critique each other</td>
</tr>
<tr>
<td>Evaluation</td>
<td>▪ Provides vehicle to show instructor what was learned</td>
<td>▪ Measures students’ understanding of the material</td>
<td>▪ Rewards independent thinking</td>
<td>▪ Accurately measures competence</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>▪ Student &amp; teacher work toward goal &amp; measure progress</td>
</tr>
<tr>
<td>Nature of knowledge</td>
<td>▪ Is certain or absolute</td>
<td>▪ Is partially certain &amp; partially uncertain</td>
<td>▪ Is uncertain – everyone has own beliefs</td>
<td>▪ Is contextual; judge on basis of evidence in context</td>
</tr>
</tbody>
</table>
The nature of (the exploration of) knowledge. I would like to take this opportunity to make a comment pertinent to my use of Baxter Magolda’s model (1992). It became useful to reinterpret the description of the nature of knowledge by thinking of it in terms of the nature of the exploration of knowledge. In the context of mathematics, we might consider absolute knowing as believing that there is one correct way to proceed for any given task; one must learn that way and apply it. A transitional knower might recognize there are other ways, but feel that one way is the best; he/she might desire to learn the one best way to solve the given task. An independent knower might understand that there are multiple ways to work a problem and attempt to understand and select an appropriate way. Attempts to apply prior knowledge to develop one’s own way to work through a task might signal contextual knowing in this area.

Evaluation. I did not overtly inquire about the participants’ views in the domain of evaluation nor did I devise specific tools for exploring it. However, the students’ perspectives in this domain certainly had an impact on their involvement in the study. In fact, I claim that the students’ decisions whether to participate at all were to some degree influenced by their proclivity to evaluate their learning only in terms of the vehicles provided by instructors within courses. In all fairness, given the various demands on the students’ time, it is a matter of practicality to focus time and energy on that which is evaluated by instructors. As was discussed in the previous section, the three participants revealed through their comments and behaviors that their success was evaluated by the degree to which they could memorize rules, perform routine calculations and solve problems using proscribed step-by-step procedures, rather than on their understanding of processes and concepts. Their major tests were administered on a computer; only answers
were checked. Thus, the participants brought a perspective of absolute knowing with respect to evaluation. Jennifer was hesitant about her explorations into transitional or independent thinking and often questioned whether she was correct rather than focusing on her own understanding. She seemed surprised when I praised her for those explorations. Hillary was most frustrated by the shift in emphasis that is central to the teaching experiment. She seemed to take comfort in the absolute perspective’s apparent clarity. Peggy, on the other hand, seemed to be liberated by the opportunity to explore and appreciated the encouragement to understand the concepts and venture into independent thinking.

**Peggy.** Peggy’s responses and behaviors were too complex to identify her perspective in only one of the ways of knowing described by Baxter Magolda (1992). First of all, viewing Peggy only in light of her responses to the initial questionnaire and her self-description in the initial interview, it might have seemed that she approached the learning situation primarily as an absolute knower, especially with regard to mathematics. However, it is important to note that viewing her within the context of the teaching experiment, a very different picture emerged. This was particularly obvious when examining her beliefs about the roles of herself as a learner, herself with her peers and her instructors.

During the exit interview, Peggy summarized her views about learning and knowledge in a rather impassioned way when she claimed that learning about all subjects makes students better people, a perspective that might at least be described as transitional knowing. During the teaching experiment, Peggy demonstrated her ability to think
independently, to share views with others and to create her own perspective. In fact she
was reaching to think through problems and to integrate and apply her knowledge.
Thus she demonstrated all four of the ways of knowing described by Baxter Magolda
with regard to herself as a learner.

With regard to the role of peers, Peggy’s work in the joint sessions with Hillary
showcased her efforts to provide active interchanges and to become a resource. Both
participants reported that these were not customary activities in their study of
mathematics. Peggy found them to be beneficial ways of working; it seemed that she was
prepared to engage in transitional and independent knowing as described by Baxter
Magolda. Similarly, with regard to the role of instructors, Peggy was accustomed to
thinking of instructors of mathematics as the dispensers of knowledge. She responded
well to my role in the teaching experiment as a promoter of independent thinking. As
with the role of peers, Peggy displayed the capacity to engage in other ways of knowing
rather than the absolute way of knowing that she described in her previous experience.

Although Peggy initially described herself as an absolute knower with regard to
the nature of the exploration of mathematics – even to the point of describing her own
resistance to hearing about alternative ways to approach problems in class, Peggy
acknowledged and demonstrated her acceptance of there being more than one way to
approach a task and her interest in understanding the process. Thus, she demonstrated the
capacity for transitional and perhaps independent knowing in this area.

I claim that Peggy was able to utilize different ways of knowing within the
context of the teaching experiment – ways that she did not feel comfortable employing
and that are typically not encouraged in the usual mathematics classroom setting. For Hillary and Jennifer, the picture was quite different.

Jennifer and Hillary. Although they presented a variety of perspectives when talking about the study of non-mathematical subjects, both Hillary and Jennifer described themselves as absolute knowers as defined by Baxter Magolda (1992) in the domains of the role of the learner, the role of peers and of instructors and the nature of knowledge in the area of mathematics. Jennifer demonstrated the capacity for independent thinking and knowing and indeed acknowledged that she explored more deeply and thought more constructively about her own work in the teaching experiment, but she clearly described a preference for the perspective of the absolute knower with regard to the roles of herself as a learner, her peers and instructors in the mathematics classroom setting. She lacked confidence in her own ideas and did not see herself as a resource in mathematics. Thus, although there was a slight mismatch between Jennifer’s stated and demonstrated beliefs in these areas, she remained most comfortable maintaining the perspective that she had described for herself, that of the absolute knower, in the study of mathematics. Considering the amount of potential that she displayed, there remains some question as to whether a supportive environment would encourage her to develop her independent thinking.

Hillary’s stated and demonstrated beliefs were the most consistent. She described herself as expecting – in fact needing - to obtain knowledge from a mathematics instructor who could explain it well and completely. She repeatedly declared that there was one best way to address each task and was disappointed during the teaching experiment that I would not tell her what that way was. She resisted opportunities to think
independently and during the exit interview described my urging her to explore the tasks as prodding that was at times uncomfortable. Although she worked with another student both on their own time and during joint sessions with me, she described and demonstrated that she saw the role of peers as explaining what had been learned from the instructor. She clearly did not see herself or her peers as resources in the study of mathematics. Hillary’s stated and demonstrated beliefs with regard to the study of mathematics fit well in Baxter Magolda’s description of the absolute knower.

Knowledge Voices

To assist in the examination of the participants’ knowledge voices, Kasworm’s five knowledge voices (1997, March) are presented again in Table 6.2 below.

Table 6.2: Kasworm’s five knowledge voices (1997, March) Reprint of Table 2.2, page 24

<table>
<thead>
<tr>
<th>Knowledge Voice</th>
<th>Perspective</th>
<th>Academic &amp; Real-World Knowledge</th>
<th>Power structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Could not judge or make initial personal sense of classroom knowledge</td>
<td>Different worlds; uncertain in new and confusing academic world</td>
<td>Attempt to learn the ways of the culture. All-knowing instructor.</td>
</tr>
<tr>
<td>Outside</td>
<td>Anchored in real-world of work, family &amp; life</td>
<td>Perceived discrepancies. Frustrated in attempts to directly apply college experiences. Usually the knowledge learned beyond short term: to reinforce current knowledge, to further illuminate past personal knowledge, to validate knowledge expertise.</td>
<td>Academic game; faculty neutral or suspect because they are typically anchored in academic world.</td>
</tr>
</tbody>
</table>
Critical Cynical involvement. Seeking credential. Perceived silliness in academics & real knowledge in real-world. Dilemma between perceptions of personal competence and judgment by professors of their lesser abilities in the academic world. Faculty-student game. External compliance without intrinsic engagement.

Straddling Intersecting and connecting both academic and adult world knowledge meaning structures. Equally valuing two worlds of knowing and doing. Believed each world informed the other. Able to articulate value in the broad curriculum. Saw faculty in quasi-peer relationship. Viewed themselves as on a learning journey. Appreciated opportunity for discussion, small-group projects, and flexibility of paper topics.

Inclusion Self-directed lifelong learners; world of learning transformative & generative. Actively sought immersion into the academic world & knowledge. Spoke to building connections between the academic and other world settings. Spoke to building meaning bridges and to creating and generating new knowledge from connections of meaning and application. Spoke of their own meta-cognitive actions and cognitive activity.

Peggy – listening for the straddling voice. Viewing Peggy one more time, this time using the lens of Kasworm’s (1997, March) knowledge voices, I again identify a shift between the initial impression and the demonstrated belief structure. Peggy described herself upon entrance into higher education very much in what Kasworm describes as the “entry voice.” She was attempting to make sense of the new culture. However, as the sessions unfolded, I began to hear her “straddling voice.” She valued both what she learned in her environment outside of class and what she could gain from higher education. She viewed herself as being on a learning journey and was able to both demonstrate and articulate these beliefs. Although Peggy was seeking a degree in order to pursue other vocational goals, she also saw education as an opportunity to learn, to grow.
and to change. On the other hand, for Hillary and for Jennifer, the gaining of a degree was primarily a credentialing process.

**Hillary – echos of the cynical voice.** Hillary was somewhat cynical about her involvement in higher education. She referred to settings in which she had already demonstrated many of the skills she would need to succeed in her chosen vocation in teaching. Her work on campus had acquainted her with the culture to the extent that she almost felt demeaned by her involvement as an undergraduate student. She expressed some resentment that her skills and experiences were not valued in the academic world and was highly critical of the inadequate performance of some of the instructors she had observed. Except for some specific areas of mathematics that she might have to teach in the future, she seemed to see the taking of mathematics as simply fulfilling a requirement. It was not clear that other courses had any greater meaning for her; she was simply more at ease in approaching them. Thus, Hillary could be described as speaking with a “critical” voice as described by Kasworm.

**Jennifer - hearing a different voice.** Jennifer’s perspective was not cynical; she took a more practical and detached view of higher education. She needed a degree in order to advance in her employment situation. She saw links between her real-world environment or her interests and the academic world but seemed to brush them aside. Academic requirements, time pressures and the logistics of family life and work drove her decision making on any educational choices presented to her. Jennifer’s perspective had a quality that is not reflected in Kasworm’s knowledge voices. Although she was seeking a degree as a credential, she did not express the other conflicts and frustrations of the critical voice. Although she seemed to remain aloof from the process, she also did not
express many of the frustrations or confusions of the outside voice. Neither did she seem to be that strongly attached to her work environment; she displayed a pragmatic attitude about it that was similar to the one she presented about education. She was proud of the opportunity to advance and of the confidence that her supervisors had shown in her, but she demonstrated no personal connection to her work. Although she valued each of her environments, she did not speak of the connections between the environments inherent in the straddling voice. Perhaps Jennifer could best be described as speaking with a “pragmatic voice,” one that has a rather restrained affective quality to it. This voice is different from those presented in Kasworm’s model.

Mathematics and Other Subjects

It should also be noted that all three participants reported differences in their perspectives with regard to mathematics as compared to other subjects. They based those differentiations on both previous and current experiences. They described levels of independent knowing or even contextual knowing when talking about subjects such as literature and history to which they felt more personal access. That is, in non-mathematical arenas, they felt empowered to think for themselves, share views with others and serve as a source of knowledge. They were willing to compare perspectives with others and felt that learning could be enhanced in some other settings by quality contributions of their peers. Their common description of their utilization of the texts in the various subject areas was very revealing. In each case, the students found it appropriate to read material prior to classes in other subjects, but not in mathematics. They felt they could access and process information in the other areas, but depended on the instructor to digest it and deliver it to them in the case of mathematics. Perhaps
because they claimed to view mathematics as absolute, they were skeptical about an
instructor promoting independent thinking.

As was discussed above, only Peggy revealed in teaching sessions by her actions
and her later comments that these stated beliefs about the study of mathematics did not
tell the whole story with regard to her perspectives. In her case, the behaviors she
described may well have been accurate for her in a classroom setting. That is, there may
not have been a mismatch between her stated and demonstrated beliefs within a
classroom setting, and those beliefs were quite different from those she described with
regard to other subjects.

For Hillary and Jennifer, the perspectives they demonstrated toward mathematics
were basically consistent with their behaviors toward mathematics in the teaching
experiment. At the same time, the perspectives they both stated and demonstrated toward
mathematics were quite different from those they described toward other subjects.

Contextual Nature of Participants’ Beliefs

In the case of the study of mathematics, I investigated the contextual nature of the
beliefs of the students who participated fully in this study; that is, did the participants
perceive the roles of teachers and students and the nature of mathematics differently in
the setting of their mathematics classroom than in the setting of the teaching experiment?

During the exit interviews, I asked each of the three participants who completed
the study in what ways her experience as a participant was similar to or different from her
usual experience in a mathematics classroom and in what ways the mathematics that she
explored was similar to or different from the mathematics that she usually explored in the
mathematics classroom. Those responses together with what I had observed in both the
classrooms and during our teaching sessions provided me with insight into the contextual nature of the students’ beliefs about the roles of students and instructors and the nature of mathematics.

Roles of Instructors and Students

All three participants described the roles of instructors and students as being different in the two contexts. Peggy mentioned what she saw as benefits and detriments of the one-on-one situation in the individual teaching sessions. She noted that an instructor, even with a relatively small class, could not focus on and be concerned about just one student as a researcher can in a teaching experiment. She appreciated the fact that, as the researcher, I had the time to talk just to her and to present tasks that were on the level that she could approach. She also acknowledged that a student in an individual teaching session could not rely on other students in the class to raise questions about topics she did not understand as she often did in a classroom setting. She seemed to feel the pressure of the responsibility for her own learning more in the teaching experiment where she could not blame anyone else for not asking for assistance.

Jennifer acknowledged that she usually had a difficulty with word problems, but in the teaching sessions there were tasks that she completed with my prompting her to continue. She said it was good to have someone encouraging her to try to “think of another angle.” She described the atmosphere of the teaching experiment as being different from not only the mathematics classroom but also other one-on-one settings. Specifically, she reported that when she received help from another individual, she was often shown other ways of doing the work, some of which she found to be beyond her ability or confusing. In the teaching experiment, she felt that she was encouraged to think
more about the tasks, to consider other ways to think about them and to think about her answers when she was finished. This she found different from either the classroom or individual tutoring that she had received. She described her role in the classroom as an observer rather than a participant and asserted that in the teaching experiment the reverse was the case.

Hillary also alluded to the differences between the atmospheres of the two settings and reported considerable discomfort with that of the teaching experiment. She ascribed some of this discomfort to her anxiety about studying mathematics. She referred to my prompting as pushing her to look at things a different way, which often made her uncertain about what she was doing. She became more comfortable as time went on, but remained frustrated when explorations were left open if she could not accomplish them. She acknowledged that it was beneficial to revisit problems that she could not do at first and to realize that a problem that may have seemed like a trick was actually approachable. She claimed that what she described as prodding was extremely frustrating, especially at the beginning, because she was accustomed to depending on her own initiative. In some sense, Hillary did not perceive the roles of the instructor and the student as being different in the setting of the teaching experiment. Rather, much of her frustration seemed to be based on her impression that I was neither performing the role of the classroom teacher nor encouraging her to perform the role of the student in the usual classroom.

Nature of Mathematics

The participants had differing views about the nature of mathematics in each of the settings. In spite of Hillary’s frequent frustration with both the atmosphere and the
content of the tasks that were presented, she claimed that the tasks themselves were more real-world related and more generally useful than what she encountered in the classroom. However, she went on to indicate that the mathematics that is presented in the classroom would be more useful to her when she began teaching in middle school. Thus, she perceived a definite difference in the nature of the mathematics in each of the settings.

Jennifer observed that the teaching experiment focused on more word problems than are usually addressed in a classroom setting. Assignments typically included problems much like the examples that had been presented in class. Although during the research sessions, the emphasis was on exploration of a variety of types of tasks, the focus of classroom instruction was on reproducing that which had been shown to the students by the instructor. For Jennifer, it seemed that the nature of the subject matter itself was different in the two settings.

Peggy claimed that the mathematics and the actual types of tasks were the same in the teaching experiment as those presented in the mathematics classroom. For her, only the type of instruction differed, as previously indicated. She saw the classroom and the teaching experiment as two different approaches to the study of mathematics, the nature of which remained the same.

Factors Influencing Students’ Re-entry into Education and Study of Mathematics

Two of the open-ended items on the initial questionnaire asked the students to state their main reasons for attending college and their primary purposes for studying mathematics in college. The first of these items was addressed primarily to the re-entry students in an effort to discern their reasons for entering higher education after a hiatus from school. The predominant stated purpose for returning to academia – the one given
by 12 of the 19 students - was a very practical one and usually job related; that is, it was either to open new opportunities or to progress within their present setting. Eight students indicated that they were returning to get a degree, and three linked that degree to a specific academic major or career goal.

In response to the question about reasons for entering college, six of the nineteen re-entry students stated such self-esteem related purposes as bettering themselves and proving to themselves that they can do it. Five indicated that they were interested in pursuing knowledge or simply getting an education. Three students stated reasons related to their families such as having a similar schedule as other family members or setting an example for their children.

The Study of Mathematics in College

Eleven of the nineteen students, a little over half of them, stated that their purpose for taking mathematics was that it was required. Ten students indicated that they felt they would be able to apply it in their lives; eight of them mentioned work or family arenas such as helping their children while two specified that they thought they would need it in other coursework. Three of them stated that they wished to prove their ability to be successful in mathematics.

Responses of the Three Participants

Both on the initial questionnaires and during the interviews, the connection between getting a degree and changing or improving career situations was made by all three of the participants. Hillary was rather passionate about her goal of teaching at the middle school level. She implied in many of her remarks that she already had learned and demonstrated many of the necessary skills in previous experiences and that getting the
degree was in many ways simply fulfilling a requirement. Although she stated on the questionnaire that her only reason for studying mathematics in college was that it was part of the program for her degree, she acknowledged during interviews and sessions that she needed to learn the mathematics in order to function more effectively in her chosen career. She made no comments on her questionnaire about her interaction with her own children or her interest in proving her ability to learn mathematics, but she spoke about both of them during interviews and sessions. In terms of the model developed by Graham and Donaldson (Donaldson et al., 1999, April -- see Figure 6.1, p. 226), Hillary perceived a slight connection between the classroom and her life-world environment with regard to mathematics, but in general she saw these as separate domains. For her, getting a degree was simply fulfilling a requirement in order to do something for which she already possessed most of the necessary skills.

Jennifer displayed an even stronger disconnect in this area. Her responses were blatantly practical, and the comments she made throughout the sessions were consistent with her answers on the questionnaire. She had entered college in order to be promoted at work, and she was studying mathematics because it was necessary for attaining a degree. In fact, she was in the process of selecting a major that would be acceptable to her supervisors, but would require as little mathematics as possible. Her interests may have drawn her to a major in business, but she was avoiding that major because of the mathematics requirements.

Peggy’s reasons for entering college were more complex both on the questionnaire and in discussion. She stated that she wanted to increase her own knowledge and demonstrated an intellectual curiosity throughout the research. She
indicated that her goal was not only to get a degree, but also to get a degree that would give her the ability to work in a field that she had always loved. For her, gaining the knowledge that lead to a degree would enable her to change her life. She had come to realize that she was capable of doing the kind of work people who supervised her were doing and was eager to earn a position of that sort. For her, the classroom and her life-world environment were very much connected. With regard to the study of mathematics, Peggy stated on the questionnaire that it was simply required. However, during sessions she displayed a significantly less superficial involvement in its study, as was explored at length in her case study in Chapter 5. During the exit interview she reiterated how much she valued learning of all subjects, including mathematics, beyond fulfilling requirements. She indicated that students should be required to study all subjects in order to make them better people. Again referring to the Model of College Outcomes for Adults (Donaldson et al., 1999, April), Peggy’s value orientation with regard to learning in general interacts positively with the classroom setting to enhance possible outcomes.

Question 2: Participants’ Ways and Means of Approaching Mathematical Tasks and Application of Out-of-School Knowledge

There are two major issues addressed in this question: a) the participants’ ways and means of approaching mathematical tasks and b) their application of out-of-school knowledge. With respect to the three participants in this study, there are three different answers for each part. The relationship between those answers can be explained to some extent by the answer to the next research question, which deals with the participants’ affective orientations toward higher education in general and the study of mathematics in particular.
Ways and Means of Approaching Mathematical Tasks

Below I have reproduced the diagram I presented in Chapter 3 to guide my exploration of the students’ plans and actions. Following it, I explore some of the paths that each of the three participants typically followed during the teaching sessions.

Figure 6.2: Investigation Initiated by Task Presentation and Guided by Students’ Plans and Actions (Reprint of Figure 3.1, page 54)

Hillary’s resistance. When a task was presented to Hillary, she would typically respond one of two ways. If she perceived it as non-routine (based on her experience), she usually rejected it, claiming that she simply did not know how to do it. When urged to formulate a plan, she often expressed frustration. She interpreted my prompting as
meaning that she ought to know how to accomplish the task, and she saw her need to explore the task as incompetence on her part. Hillary’s other common response was to attempt to make the task into a routine problem. That is, she searched her memory for problems she had previously learned how to approach that were similar to it. Often the actions were inappropriate, because the task did not fit into the pattern of routine problems in her experience or because the formulas that she chose to apply were inappropriate or not accurate. Trouble-shooting such situations and exploring the formulas she had chosen or needed were also frustrating activities for Hillary. Presenting a task to Hillary that was similar to problems she faced in coursework or situations with which she might deal at work heightened rather than eased her frustration. In those cases, the tasks seemed to be too close for comfort. This connection between Hillary’s affective orientation and her ways and means of approaching the tasks is explored further in the final section of this chapter, which addresses the third research question.

Jennifer’s level of engagement. When Jennifer was presented with a task, she thought carefully about it and then usually approached it in a very methodical way. She kept careful track of any steps she took, a habit she ascribed to both her practices in her work setting and to her note-taking in her classroom setting. Even if based on her own checking she was apparently working in a reasonable way, if she perceived that a task did not fit into her usual routine, she was quite hesitant about the possibility that what she was doing was correct. She remarked that she appreciated the encouragement to think carefully about her outcomes in order to evaluate whether they were reasonable and to consider alternative ways of approaching tasks. However, she required this encouragement and when she could think of no apparent approach or a task seemed
totally unfamiliar, she was quite willing to give up and not invest herself in the exploration. She was not resistant nor was she incapable once she was prodded to begin, but she displayed a distinct lack of engagement in the process.

**Peggy’s involvement.** In spite of her protests to the contrary, Peggy met non-routine tasks with curiosity and energy. When she recognized that she had taken inappropriate actions, she responded well to suggestions that she explore background material. Although she requested to put off a task while she explored background material, she was willing to return to it and formulate a new plan. Her intuition and logic at times permitted her to work through a task with no apparent (to her) plan. She was willing to assist in investigating her own thinking in order to determine the underlying plan in her work.

**Application of Out-of-School Knowledge**

The one commonality between the participants was that there was practically no overt display of out-of-school knowledge. All three seemed to systematically separate that which they knew from outside of a school environment from that which they considered school mathematics. In fact, they displayed skepticism about the possibility that there was any application. When I called to their attention any possible such applications of their out-of-school knowledge and skills, they generally reacted with some combination of disbelief and surprise. Hillary remained skeptical so any such comments were not helpful to her. Jennifer found my observations particularly about her methodical approach and organizational skills interesting but was not encouraged by them. She was unable to identify any connections for herself. Peggy was pleased and
encouraged and began to speak of connections between her environment and mathematics in areas she had not previously observed such connections.

**Question 3: Connections Between Participants’ Affective Orientation and Ways of Operating in a Problem-Solving Context**

**Background from Skemp (1987)**

It is helpful to review here that, as Skemp observed, "it is in the nature of learning that learning takes place in regions where we are not yet competent." If we define our **domain** as a region in which we can achieve our goals and avoid our antigoals, this would be our region of competence in which we feel confident and secure. Outside of this region, we do not feel competent; rather we are apt to get signals of frustration and anxiety. The boundary between the inside and outside of this domain is not likely to be clear and sharp. In this fuzzy boundary area we are apt to experience mixed emotions resulting from varying degrees of ability to achieve our desired goal states and inability to avoid our antigoal states. As has been pointed out before, Skemp referred to this zone as the frontier zone (1987); it is presented in the figure below.

![Image](image-url)

**Figure 6.3: Learning - Frontier Zone (Skemp, 1987, p. 195) Reprint of Figure 1.2, page 13**
The emotions that act as signals indicating that we are moving toward or away from a desired goal state are depicted below in Figure 6.4. Following that figure, emotions that signal knowledge of ability or inability to change toward or away from the desired goal state are depicted in Figure 6.5.

Figure 6.4: Emotions signaling what has occurred (Skemp, 1987, p. 192) Reprint of Figure 2.2 page 39

Figure 6.5: Emotions signaling competence (Skemp, 1987, p. 192) Reprint of Figure 2.3 page 39
Examining the participants’ actions through the lens of Skemp’s work outlined above provided me with a way to characterize the connections I had observed between their affective orientation toward the study of mathematics and their ways and means of operating in a problem-solving setting.

Hillary

For Hillary, the entire context of the teaching experiment was primarily outside of her domain for learning. The roles of the instructor and the student and the nature of the exploration of knowledge were quite different from those with which she was comfortable. Attempting to select content that was within Hillary’s established domain did not seem to help us to move into her frontier zone. In fact, as was mentioned earlier, Hillary herself tried to fit the non-routine tasks into the patterns of tasks within her established zone, but her definition for what was in her established zone was so narrow that the tactic was usually ineffective. In fact, the process tended to leave her outside her frontier zone in terms of her affective orientation even if the content was on the inside of the boundary. To make matters worse, she did not tolerate well the mixed emotions that inevitably accompany working within one’s frontier zone.

Tracking her emotions clarified further Hillary’s experience. If we take her present state as being within a teaching session and having been presented with a mathematical task, then we see that her initial reaction was generally fear and anxiety, emotions that signaled her movement toward an antigoal state. That is, she felt she was unable to do the task. With some encouragement, she might gain some confidence and begin to take pleasure in her accomplishments, but her basic distaste (unpleasure) for the context and her frustration with not being able to work as quickly or as easily as she
thought she should work would take over. These emotions would rob her of feelings of competence and block her access to her goal.

Jennifer

While Hillary’s emotions got in her way, Jennifer’s lack of emotional involvement seemed to block her engagement. Although she acknowledged in word and deed her capacity for operating constructively in the problem-solving setting, she remained detached and displayed very little affect. She voiced her feelings that as a returning student, she needed to be pragmatic and realistic about her approach – that the time for her to explore and involve herself in more creative learning experiences had passed. In many of the tasks, Jennifer was able to reach a possible goal state of simply completing the task, but she took little pleasure in it. In fact, I claim that she often did not fully explore the task in order to reach the underlying goal state of the teaching experiment; knowing that, she gained little satisfaction. Thus, although using her organized and systematic approach permitted her to avoid most frustration, she gained little confidence; and although she felt little anxiety, she was never fully secure in the environment.

In effect, Jennifer’s detached and pragmatic affective orientation kept her frozen in her initial state. In terms of Skemp’s model of the frontier zone, Jennifer was not willing to risk the mixed emotions that are inevitable in moving out of one’s established zone and into the frontier zone. Although she explored the tasks and revealed abilities to think mathematically that she did not seem to think she had, her level of engagement was rarely enough to take her fully into her frontier zone where real learning could take place.
Jennifer’s affective orientation toward the study of mathematics and herself as a student of mathematics seemed to be at the heart of this lack of engagement.

**Peggy**

We were able to very quickly begin working in Peggy’s frontier zone and remained there for most of the sessions. She tolerated well the mixed emotions that she experienced there. In fact, Peggy’s awareness of her own emotional reactions permitted her to control the effects of the negative ones and gain energy from the positive ones. Although she was fearful of the risks and the possible failure, she felt relief that the context permitted her to take those risks without impacting a grade. It seemed that the teaching sessions provided a secure environment in which she could overcome the anxiety of working in unfamiliar ways. She took pleasure in any small success, and the confidence that she gained assisted her in overcoming any frustration that she felt. Peggy’s rather open affective orientation permitted her to be actively engaged in the problem-solving environment of the teaching experiment.
CHAPTER 7
IMPLICATIONS FOR TEACHING AND RESEARCH

The results of this research speak to administrators and educators involved with adult re-entry students in the study of mathematics as well as to other researchers. There is considerable overlap in these messages, because the primary lesson derived from this study was the confirmation of my belief that it is only when we are at the business of teaching and learning that we truly can explore the double headed topic of education. Furthermore, much of what made the research successful could be applied in programs and in classrooms to make a difference in instruction.

In this final chapter, I comment first on various aspects of the research methodology and the models employed as well as modifications that I would suggest in further research. I then turn to the potential disconnections that were noted between the typical classroom setting and the adult students’ cognition, value orientation and life-world environments. I suggest that repairs to these disconnections would best take place programmatically but that adjustments to individual classrooms could be therapeutic as well. Finally, I suggest the re-examination of outcomes to aid in these repairs.

Employing the Teaching Experiment Methodology

This research yielded a rich array of results concerning re-entry students’ affective orientation toward the study of mathematics at the remedial level and the interplay between that affective orientation and their ways and means of operating in a problem-solving setting. These results would have been difficult to access with any other
methodology. The initial questionnaires administered to all 89 students laid the foundation, but it was within the interviews and, more significantly, the teaching sessions that the three participants’ ways of knowing were revealed and knowledge voices were heard. Indeed, the teaching sessions not only added to the results but corrected misconceptions about the participants’ ways of operating that might have been held otherwise. Although there were only three students who persisted throughout the entire study, those participants provided strong contrasts and an abundance of findings. One of them demonstrated a particularly vivid example of differences between stated and revealed beliefs, thus alerting us to the possibilities of such differences in other individuals. As was presented in detail in Chapters 4 and 5, for each of the participants and regarding each of the research questions of this study, much more and much richer data were gathered in the setting of the teaching sessions than from the interviews and questionnaires alone, greatly enhancing the conclusions that were reached. Certainly further research employing the methodology of the teaching experiment is indicated. Moreover, it was with the atmosphere of the teaching experiment that we were able to observe the possibilities for the participants; practitioners might consider employing elements of this atmosphere in order to explore the possibilities for their students. Further discussion of this issue appears later in this chapter.

**Application of the Models**

Two models defined what I would describe as the macro-view and the micro-view of my study. Using the Model of College Outcomes for Adults developed by Graham and Donaldson (Donaldson et al., 1999, April) as a framework for the macro-view, I was able to situate the study and to focus on appropriate issues for re-entry students in higher
education. The model again became useful in assessing the results and identifying potential disconnections that could interfere with positive outcomes for students. Suggestions for some ways to repair these disconnections as well as the need to examine the intended outcomes are mentioned in this chapter.

The micro-view of this study is best seen through the lens of Skemp’s model for learning in what he referred to as the frontier zone (1987, p. 196). This powerful model is elegant in its simplicity. Because I wished to examine students’ affective orientation toward the learning of mathematics together with their mathematical thinking in a problem-solving setting, Skemp’s model identifying an individual’s frontier zone as the arena where both learning takes place and mixed emotions are prevalent provided excellent guidance for both the methodology and the analysis of the results in this study. His figures depicting emotional signals enriched that analysis in addition to aiding the investigation of the interplay between the affect and mathematical thinking.

Researchers exploring areas related to the one presented here would do well to employ the models of Graham and Donaldson (Donaldson et al., 1999, April) and of Skemp (1987). Practitioners wishing to improve the quality of their work would benefit also from their use. In addition to repairing the disconnections indicated by the use of the Graham/Donaldson model, educators need to come to know students’ frontier zones as described by Skemp in order to situate our teaching within those zones.

The models of Baxter Magolda (1992) and Kasworm (1997, March) that were used to assist in examining the beliefs and perspectives of the participants were enormously helpful, but certain adaptations and observations made them even more useful. First, it should be noted that as was anticipated, the participants’ perspectives with
regard to the study of mathematics were usually different from those with regard to the study of other subjects. Indeed, even when considering only the study of mathematics, each individual student usually could not be described as using only one way of knowing (Baxter Magolda) or speaking with only one voice (Kasworm). One must avoid the pitfalls of both assuming that a student’s perspectives toward education in general necessarily are the same as his or her perspectives toward a specific subject, particularly mathematics, and of assigning a student to a single way of knowing or knowledge voice with regard to all aspects of learning even in a single subject. In addition, as was the case for Peggy and to some extent for Jennifer, these perspectives may be contextual in nature; that is, certain perspectives may be evident within a classroom setting while others may surface in other environments such as the teaching experiment in this study. Furthermore, as I discussed earlier, Jennifer presented an additional voice not described by Kasworm (1997, March). I referred to that voice as the pragmatic voice – one that has a somewhat detached quality to it. Studies with other students could well lead researchers to discover other knowledge voices. Finally, I found it useful to interpret Baxter Magolda’s domain of the nature of knowledge as the domain of the nature of inquiry and thereby to consider the participants’ willingness to consider alternative ways of approaching tasks and the level to which they considered the process as more significant than the product or the answer. With the above modifications in place, both Baxter Magolda’s and Kasworm’s models would prove valuable both for guiding research and in order to assist educators in listening for their students’ ways of knowing and listening for their knowledge voices.
Not Good at Math

Most of the students who completed questionnaires indicated that they have never been able to do mathematics very well and that people either have a talent for doing mathematics or they don’t. On the other hand, the responses indicated that most of them agreed with the belief that effort can increase mathematical ability and that, with hard work, they could usually succeed in both mathematics and non-mathematical subjects, but their belief regarding non-mathematical subjects apparently was held more strongly. This apparent conflict between the belief that one requires some innate talent in order to be successful in mathematics and the belief that one can increase one’s mathematical ability is problematic. The three full participants demonstrated a wide range of beliefs regarding their ability to be successful at mathematics. In spite of her weaknesses in certain areas, Peggy was excited about learning. Jennifer seemed to feel that she had passed the time in her life when she could devote the time to learning although she had the capacity to do so. Hillary was proud of the successes she had had in the procedural setting of the classroom but intimidated by challenges to move into a process oriented setting. This entire area is worthy of further exploration.

Connecting to the Life-World Environment

It was noted in previous chapters that one of the potential disconnections in light of the Model of College Outcomes for Adults (Donaldson et al., 1999, April), was that between the classroom and the life-world environment. It was also observed that the students who responded to the questionnaires as well as the three full participants in the study displayed considerable confusion about the importance of word problems. The data from all 89 students and the behavior of the 3 participants also seemed to suggest
confusion about the usefulness of mathematics in their lives. Most of the students stated that they perceived a usefulness in mathematics in their lives, but saw very little linkage between the curriculum of their courses and, specifically, the mathematical tasks they might be asked to perform in their coursework and their home and work environments, at present or in the future. Close examination of what applications would have meaning for the students would address these issues. This proved to be the most challenging area in this research and is perhaps the most challenging area when dealing with adult students at the remedial level of college mathematics.

In any further research of this type, some modifications would be helpful in order to select tasks that would represent such meaningful applications. Visits to places of employment in order to observe the participants in that environment would provide the researcher with added insight into the mathematics inherent there. The researcher might also interview employers in order to learn what behaviors and characteristics have been observed in the participants and what mathematical skills are required in their jobs. This information would strengthen the researcher’s ability to select and construct tasks that call upon the participants’ knowledge of mathematics from their outside of school experiences.

Repairing this lack of connection for the classroom instructor is even more difficult. However, selecting tasks and utilizing examples related to students’ major areas of study and career goals, current events such as elections and economic events or campus issues such as tuition and enrollment might begin the process. Individual sessions may enlighten instructors about students’ workplace and other experiences but, without
outside visitations, the same weakness in this approach that was experienced in this study may be encountered.

Established Practice as Compared to Students’ Perspectives

This study revealed that the stated beliefs of the students involved in my study with regard to the roles of instructors and students in the study of mathematics were to a great extent in concert with the methods and goals of the courses in which they were enrolled. The courses they were taking as remedial courses at the college level were structured in a way that is probably fairly common. The methods in these courses frequently emphasize a teacher-centered approach, reinforcing the view that the instructor is the primary source of knowledge.

The three full participants indicated a different perspective toward these roles in the study of other subjects and described different atmospheres in the other classrooms. Most importantly, they reflected that in subjects other than mathematics they had a higher degree of access to the subject matter and their input as students in discussion in the classroom was encouraged and valued. The issues of power in the academic world and access to knowledge seemed to be at play here. They clearly did not feel that they were empowered to explore mathematics on their own; they indicated that they did not have access to mathematical knowledge.

While involved in the teaching experiment, one participant resisted changing her perspectives; the second displayed the capacity to do so, but did not fully engage herself in the process; the third took full advantage of the opportunity to adapt to the changed environment. There remains the question as to whether, given greater control over their own learning in their coursework and the sense of access to the subject matter, these and
other students would reinterpret the various roles played out in the mathematics classroom and in their study of mathematics.

It was interesting to note that most of the 89 students who completed the initial questionnaire agreed with the beliefs that understanding concepts is important in mathematics, that processes are more important than rules and calculations and that mathematics is useful in daily life. The curriculum of the courses in which these students were enrolled is usually severely limited, and time constraints often result in the courses being quite procedural and test-oriented. Deep misunderstandings are typically not addressed. Many behaviors and characteristics that we might identify as assets of serious students of mathematics are not reinforced or encouraged. Instead rote memorization is the norm. This apparent mismatch between the students’ value orientations toward the study of mathematics and the structure of the courses in which they are enrolled deserves attention.

For two out of the three participants, it seemed apparent that the atmosphere of the teaching experiment provided an opportunity to acknowledge, and moreover to display, a different perspective toward the learning process. The question remains as to whether a change in the environment of the coursework for which they are being evaluated would assist such students in adapting their perspectives and engaging in different ways of knowing in their study of mathematics. That is, could students with perspectives similar to Jennifer’s be encouraged to utilize different ways of knowing and thinking if the coursework for which she were evaluated were structured differently? Moreover, would those with perspectives more similar to Hillary’s reject learning in a modified classroom environment or would they be encouraged to develop what could be
considered more mature ways of knowing? Clearly, Peggy seemed to thrive in the atmosphere of the teaching experiment, and one might assume that similar modifications in the classroom environment would be beneficial to her learning. Skemp’s model of affective influences on learning would seem to support restructuring the environment in order to repair the disconnections discussed here. Students who were receptive to that new environment would be working more towards their stated goals and away from their anti-goals, thus improving their confidence and security, as well as having a more pleasurable experience. The improved emotional climate for those students could honor their affective orientation and thereby improve the cognitive outcome for them. Future research that included a modified classroom setting could begin to test these assumptions and answer these questions.

Altering the Classroom Environment

Several of the observations that I have made would suggest making alterations in the classroom environment for the re-entry student at the remedial level for the purposes of both interesting research and improved instruction. However, the underlying structure, goals and objectives of the courses offered for students in the remedial programs at the college level present significant barriers to conducting courses with dramatically altered environments. For the most part in many states, such classes are state mandated. At many institutions, additional topics are included in an effort to adequately prepare the students for the entry-level mathematics courses that the students will be required to take for college credit. At most institutions, a uniform curriculum is applied to all sections of the remedial courses, along with a departmental final examination in addition to the standardized objective exit examination, which must be passed within a specified number
of terms in order to exit the remedial program. Thus curricular demands combine with
significant time restraints to impact on the courses in such a way as to cause them to tend
to be teacher-centered and test-driven. In that context, exploration on the part of the
students, which by definition requires considerable energy and time on the part of both
instructors and students, is generally considered additional work or extracurricular in
nature. As was noted in this study, for most of the students in these programs, time and
effort spent in activities that will not be evaluated is often seen as time and effort wasted.
This perception is often reinforced by the administration. Although an atmosphere similar
to that in the teaching sessions of this study may elicit from students, and perhaps
encourage them to develop, what may be considered more mature ways of thinking and
knowing, the students are routinely evaluated on the basis of their application of
numerous specific skills, regardless of their understanding. For these reasons, an
instructor might well choose to avoid dramatically altering the course. However, it may
still be desirable to include elements of exploration of non-routine tasks and to focus on
the development of critical mathematical thinking.

It is particularly important for administrators and instructors to keep in mind that
just as the results of the questionnaire did not reveal the complete or, in some sense, the
accurate picture of the students in this study, scores on entrance tests do not tell as much
about our students’ mathematical thinking as we could benefit them by knowing. Equally
significant to observe was the interplay between affect and mathematical thinking for the
students. Lack of affective engagement can result in minimal cognitive involvement, as in
the case of Jennifer. Strong affective engagement can result in either a positive or a
negative effect on cognitive gains, as in the case of Peggy and Hillary, respectively.
Similarly, experiences within a problem-solving setting can have a strong impact on a student’s affective orientation toward further such engagement with mathematical thinking. This personal dynamic for adult students must not be brushed aside as separate from the classroom or labeled as immature behavior that can be ignored. More individual contact with students may be the most obvious, albeit inconvenient, avenue for both locating students’ frontier zones and acquainting ourselves with their affective orientations toward the learning of mathematics.

Using this study as a guide, I suggest concentrating on repairing some of the possible disconnections in the Model of College Outcomes for Adults (Donaldson et al., 1999, April) that were observed, in order to improve outcomes for the students. Specifically, those disconnections were between the classroom and the adult students’ cognition, value orientation and life-world environment. To accomplish the repair, the instructor could operate in the classroom in a manner that would respond to students whose ways of knowing (Baxter Magolda, 1992) would be considered transitional, independent or contextual, rather than absolute. Doing so would honor these students’ cognitive habits while assisting others to develop more mature habits. It may elicit from students the level of cognition they tend to employ in other areas of study and were to some extent seen to use in this research. That is, the instructor could employ methods aimed at understanding of concepts and application of knowledge; that promote independent thinking, an exchange of opinions from the students and evaluative discussion of the students’ perspectives; that promote application of knowledge in the students’ life-world contexts.
Although the final examination is usually of a different nature, throughout the term the instructor can employ evaluation techniques that focus on the students’ understanding, that reward independent thinking and the exploration of alternative techniques and elicit the students’ active involvement in the process of evaluation. This sort of evaluation would not only be consistent with the methods described above but also honor the adult students’ predominant value orientation with regard to mathematics; that is, that the understanding of processes is more important than the memorization of rules. Adjustments that could address the disconnection between the classroom and the life-world environment were suggested in a previous special section.

In studying the effects of these adjustments, there remains one major consideration, that is, the definition of outcomes. Within the context of the remedial programs, the good outcomes are generally measured in terms of success on exit requirements in the short term and success in the mathematics course in the core, which is usually the college algebra course, in the long term. Indeed, these core courses are often skill-based and teacher-centered, that is, structured much like the remedial courses that have been discussed here. There is some vague sense that deeper mathematical thinking would be a worthy outcome, but that is neither the outcome that is usually measured nor the outcome whose measurement is used in evaluating the success of a remedial program or indeed of the instructors themselves. However, it is deeper mathematical thinking that should be fostered by the modifications that are suggested by this study, an outcome that is much more difficult to measure. Thus, without major changes in the established remedial programs, the challenge would be to accomplish both the goals of those programs and the goals of the modifications. I claim that accepting the challenge and
attaining these goals would vastly improve instruction for re-entry students in remedial mathematics courses.
REFERENCES


V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75-103). New York: Springer-Verlag.


APPENDICES

A. INITIAL QUESTIONNAIRE
B. INTERVIEW PROTOCOLS
C. MATHEMATICAL TASKS
APPENDIX A: INITIAL QUESTIONNAIRE

COVER SHEET

Some students may be contacted and asked to participate in further research. The entire project will be explained before students decide whether to participate. Please provide the following information so you can be contacted to discuss the possibility of participating further in this research.

Please print clearly.

Name: _______________________________________________________

Phone number(s): _____________________________

Email address: ____________________________
Initial Questionnaire

Please answer the following questions as honestly and completely as you can. You may continue on the back or on a separate sheet if you need more space for any question.

1. Age: _______ Male or Female (circle one)  Race: ______________ (optional)

2. List the math courses you took in high school: _______________________________
   _______________________________________________________________________
   List any math course you have taken at any college or university: ______________
   _______________________________________________________________________
   Year you graduated from high school _______ or received your GED _______.
   How many years has it been since you last attended a formal school program, such as
   high school or college? _______ What did you do during this time?

3. Have you been required to take any Academic Assistance courses? YES or NO
   If so, which Academic Assistance courses have you been required to take? Circle
   both those you have already taken and those you still need to take:
   ACAE0098   ACAE0099   ACAM0098   ACAM0099   ACAR0098   ACAR0099
   Have you volunteered to take any courses in the Department of Academic
   Assistance?
   YES or NO  If YES, then please indicate what influenced you to make that decision.

4. Describe some activities besides school in which you are involved, or would like to
   be involved. (For example: work or hobbies or interests)
5. What are the main reasons that you are enrolled in college? If you list more than one, please try to rank them, with #1 as the most important, #2 as second in importance, and so on.

6. What purpose does taking mathematics in college have for you? If you list more than one, please try to rank them, with #1 as the most important, #2 as second in importance, and so on.

7. Please complete this statement in your own words: When I do well in mathematics, it is usually because . . ..

8. Please complete this statement in your own words: When I do poorly in mathematics, it is usually because . . ..
For each of the following statements, darken the response on the answer sheet that best indicates your position. (SD) = strongly disagree, (D) = disagree, (U) = undecided, (A) = agree, (SA) = strongly agree

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
<th>Response</th>
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<tbody>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>1. Math problems that take a long time don’t bother me.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>2. Word Problems can be solved without remembering formulas.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>3. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>4. Learning computational skills is more important than learning to solve word problems.</td>
<td></td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>5. With effort, one can improve one’s ability to do math.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>6. Studying mathematics is a waste of time.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>7. I feel I can do math problems that take a long time to complete.</td>
<td></td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>8. Memorizing steps is not that useful for learning to solve word problems.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>9. It’s not important to understand why a mathematical procedure works as long as it gives a correct answer.</td>
<td></td>
</tr>
<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>10. Math classes should not emphasize word problems.</td>
<td></td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>11. I can get smarter in math if I try hard.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>12. I study mathematics because I know how useful it is.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>13. I find I can do hard math problems if I just hang in there.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>14. Any word problem can be solved if you know the right steps to follow.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>15. Getting a right answer in math is more important than understanding why the answer works.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>16. Word problems are not a very important part of mathematics.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>17. By trying hard, one can become smarter in math.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>18. Knowing mathematics will help me earn a living.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>19. If I can’t do a math problem in a few minutes, I probably can’t do it at all.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>20. Most word problems can be solved by using the correct step-by-step procedure.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>21. It doesn’t really matter if you understand a math problem if you can get the right answer.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>22. A person who can’t solve word problems really can’t do math.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>23. Working cannot improve one’s ability in mathematics.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>24. Mathematics is a worthwhile and necessary subject.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>25. If I can’t solve a math problem quickly, I quit trying.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>26. Learning to do word problems is mostly a matter of memorizing the right steps to follow.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>27. Time used to investigate why a solution to a math problem works is time well spent.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>28. Computational skills are of little value if you can’t use them to solve word problems.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>29. Ability in math will not increase when one studies hard.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>30. Mathematics will not be important to me in my life’s work.</td>
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<td>(SD) (D) (U) (A) (SA)</td>
<td>31. I’m not very good at solving math problems that take a while to figure out.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>32. There are word problems that just can’t be solved by following a predetermined sequence of steps.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>33. A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>34. Computational skills are useless if you can’t apply them to real life situations.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>35. Hard work cannot increase one’s ability to do math.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>36. Mathematics is of no relevance to my life.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>37. Use of a calculator eliminates most of the difficulty students have in mathematics.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>38. Estimating is an important mathematical skill.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>39. Memorizing rules and formulas is the most important part of learning mathematics.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>40. Justifying the mathematical statements a person makes is an extremely important part of mathematics.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>41. Doing mathematics requires one to think according to strict rules.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>42. There are many different ways to solve most mathematics problems</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>43. I have never been able to do mathematics very well.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>44. I think that people either have a talent for doing mathematics or they don’t.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>45. With hard work, I can usually succeed at mathematics.</td>
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<tr>
<td>(SD) (D) (U) (A) (SA)</td>
<td>46. With hard work, I can usually succeed at non-mathematical subjects, like English or history.</td>
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</tbody>
</table>

Please add on the back any comments you would like to make about anything on this questionnaire.
APPENDIX B. INTERVIEW PROTOCOLS

Initial Interview Protocol

The purposes of this interview are

- to follow-up on the questionnaire by clarifying any items for the researcher or the student,
- to begin to explore the students’ beliefs about the roles of the instructor and of the student in the mathematics classroom and about the nature of mathematics, and
- to engage the student in mathematical tasks as time permits.

1) Begin by thanking the student for taking the time to complete the questionnaire.
   Invite the student to react to it by saying, “I have copied it so we can both look back over it. Is there anything that you have thought about that you would like to add or any comments you would like to make?”

2) Prior to the interview, the researcher will review the questionnaire and select one or more of items to probe further and ask questions such as:
   - Could you tell me a little more about the learning experience that you described in #4?
   - Could you give me some (more) examples of your outside activities?
   - This activity (mention one) sounds interesting to me. Would you tell me more about your responsibilities?

3) Classroom activities:
   a) Ask the student about the kinds of activity he or she thinks should occur in a mathematics classroom, such as what the instructor should be doing and what the
students should be doing. Once some kinds of activity are identified, ask the student to describe these activities in terms of relative importance to his or her learning and then in terms of how much time should be devoted to them. (Providing a ‘circle’ for the students to make a pie chart in each case may be helpful.)

b) Continue this discussion with regard to the activity of the student outside of class.

c) Ask whether the answers to a and b would be the same or different for other areas of study besides mathematics.

4) Characteristics of a good instructor:

a) What are the important characteristics of a good instructor for you, in general?

b) What are the important characteristics of a good instructor for you, in mathematics?

5) Characteristics of a good student:

a) What are the important characteristics of a good student, in general?

b) What are the important characteristics of a good student, in mathematics?

6) Usefulness of mathematics:

a) In what ways has the mathematics that you have learned in school been useful to you?

b) Is there anything about mathematics that you have learned outside of your own school experience that you think will be helpful to you in college? If so, please describe those things.
7) Engage the student in one or more of the mathematical tasks as time permits. Print the problems one to a page. Provide a calculator and plenty of scratch paper for her/his use. Save all of the student’s work.

Exit Interview Protocol

The purposes of this interview are

- to explore the experience of the participants in the research in light of their other experiences with the learning of mathematics, and
- to provide an opportunity for a participant check, that is for the participants to react to the analyses made by the researcher.

1) Review of topics from Initial Questionnaire and Initial Interview:

a) Please complete this statement in your own words: When I do well in mathematics, it is usually because . . ..

b) Please complete this statement in your own words: When I do poorly in mathematics, it is usually because . . ..

c) What purpose does taking mathematics in college have for you? If you list more than one, please try to rank them, with #1 as the most important, #2 as second in importance, and so on.

d) What are the main reasons that you are enrolled in college? If you list more than one, please try to rank them, with #1 as the most important, #2 as second in importance, and so on.

e) Describe examples of mathematical thinking that you employ in activities besides school in which you are involved. (For example: work or hobbies or interests)
f) Is there anything about mathematics that you have learned outside of your own school experience that you think will be helpful to you in college?  
______________ If so, please describe those things.

2) Reactions to participation in the research activities  
   a) In what ways was your experience as a participant similar to or different from your usual experience in a mathematics classroom?  
   b) In what ways was the mathematics that you explored similar to or different from the mathematics that you usually explore in the mathematics classroom?

3) Provide an opportunity for a “participant check”
APPENDIX C: MATHEMATICAL TASKS

Task #1: Picture Hanging Task

How far to the left should the picture be moved so that it is centered on these walls?

a) Given that X = 70 inches and Y = 46 inches
b) Given that X = 65 inches and Y = 34 inches
c) Given that X = 42 inches and Y = 74 inches
d) Given that X = 64 ¾ inches and Y = 54 inches

e) Write a procedure to follow for any values for X and Y.

(adapted from (Cooney, 1992))

Task #2: Retirement Party Task

You and 4 others decide to put on a retirement party for someone in your office.

a) You plan to split the expenses evenly among all of you. Andy offers to use his home for the party. Each person keeps receipts for expenses, including a nice gift for the retiree: Andy - $50, Barbara - $45, Carol – $100, Donald - $25 and you - $80. Describe how the expenses will be settled.
b) What if the expenses broke down like this: Andy - $58.73, Barbara - $47.09, Carol - $99.87, Donald - $24.76 and yourself - $92.55.

c) Can you design a procedure to follow for situations like this and write it as simply as possible?

d) After everything is settled in part b, Andy tells everyone that he really thinks that he should be reimbursed for the use of his home considering that he had to be sure it was ready for the party and then clean up afterwards. Even though others helped, he shouldered most of the burden. Besides utilities and wear and tear, someone spilled something on the carpet and it will have to be shampooed! He thinks $25.00 would be a reasonable amount for his troubles. How can this be handled?

e) Now Barbara adds to the confusion by indicating that because they all had not worked with the person retiring – actually for the company at all – for the same amount of time, they should not all contribute evenly. She requests that the expenses be split proportionally to the length of time they have each worked for the company. Their years of service are Andy – 15 years, Barbara - 5 years, Carol – 9 years, Don - 10 years and you – 11 years. Now what is the situation?

Task #3: The Merchant and the 3 Fairs

A merchant visited three fairs. At the first, he doubled his money and then spent $30. At the second, he tripled his money and then spent $54. At the third fair, he quadrupled his money and then spent $72. He then found that he had $48. How much money did he start with?

(Krulik & Rudnick, 1989)
Task #4: Fence Posts Task

Two girls, Jan and Tanya, have a job painting fence posts that line both sides of a path leading to a barn. Jan arrived early and had already painted 5 posts on one side when Tanya arrived. Before she started painting, Tanya said, "Jan, I'm left-handed and it's easier for me to paint this side of the path while you paint the other side." Jan agreed and went over to the other side of the path. Tanya painted all the posts on her side, then went across the path and painted 10 posts on Jan's side. This finished the job. If there were the same number of posts on each side of the path, who painted more posts and how many more?

(Krulik & Rudnick, 1989)

Task #6 Handshake Task

If there are 10 people in a room, how many individual handshakes will there be if each person shakes hands with every other person?

(Cooney, 1992; Krulik & Rudnick, 1989)

Task #8: Fieldtrip Task

a) You have been asked to arrange for drivers to drive for a trip for a group at school. Two staff members and 35 students will be attending. Neither staff member will be driving a car. Each person must wear a seatbelt. How many cars will be needed for the trip? If everyone eats lunch, how many lunches will be needed?

b) You have been asked to arrange for parents to drive for a fieldtrip for your child’s class at school. A teacher, a teacher’s aide and 35 children will be attending. Neither the teacher nor the teacher’s aide will be driving a car. Each
person must wear a seatbelt. How many cars will be needed for the trip? If everyone, including drivers, visits the museum and eats lunch, how many tickets will be needed for the museum? How many adults’ lunches? How many children’s lunches?

Task #9: Map Task

Using the map below,

a. Determine which route is the shortest in terms of distance.

b. Determine which route is the shortest in terms of time.

Task #12: Spaghetti Supper Task

You are responsible for planning the Spaghetti Supper as a fund-raising project for an organization that you support. What information do you need? Explain how you would use that information to plan the event.
Task #17: Carpet and Paint Task

Below are sketches of two rooms in my home.

a) What is the area of each room?

b) I want to paint the ceilings of each room. The paint costs about $7 per gallon. The directions say to allow one gallon of paint for about 400 square feet of ceiling, if the ceiling is smooth or 150 square feet of ceiling, if the ceiling is textured. The ceilings in both rooms are rough. How much money should I allow for the cost of the paint?

c) The carpet that I want to use is sold from rolls that are 9 feet wide. The price of the carpet I want to use is about $30 per square yard. How much money should I allow for the cost of the carpet?

![Floor Plan 1](image1.jpg)

![Floor Plan 2](image2.jpg)

Task #19: Shop Stock

A merchant who was liquidating her stock offered various items at the following prices:

- 7 items at $10 per item
- 12 items at $8 per item
- 15 items at $6 per item
- 6 items at $5 per item

If she sold one-half of this stock on the first day, what is the most money she could have received?
Task #20: How many Triangles?

How many triangles are there in this figure?

![Diagram of a triangle with smaller triangles inside it.]

Task #21: Triangle Task

Place the numbers 1, 2, 3, 4, 5, and 6 in the spaces provided in this figure, so that each side of the triangle adds up to 10. (Provide several copies of the figure.)

![Diagram of a triangle with circles at each vertex and sides.]

(Krulik & Rudnick, 1989)

Task #22: Cows-and-Chickens Task

I noticed some chickens and some cows in a barnyard. I counted 35 heads and 100 legs.

How many chickens and how many cows were there?

(Krulik & Rudnick, 1989)
Task #24: Change-in-Bills Task

A customer pays for a $44 purchase with a $100-bill.

a) How much change should the clerk give the customer?

b) If the clerk wants to give the change in the smallest number of bills, how many of what kind of bills should he use?

c) If the clerk has only $10-bills and $5-bills in his drawer, will he be able to “make change” for the customer using only those bills?

d) If the clerk has only $10-bills and $1-bills in his drawer, how many of which type are needed in order to use the MOST bills? The FEWEST bills?

e) Using only $10-bills and $1-bills, how would the change be given using exactly 11 bills?

Task #25: Change-in-Coins Task

There are 30 nickels and dimes on the table. They are worth $1.90. How many nickels and how many dimes are there?

Task #26: Arrays Task

a) I have to arrange 36 desks in a classroom so that there are the same number of desks in each row. How many different ways can I arrange the desks?

b) There are 62 students in the marching band of a high school. There is room at the parade site for the students to march no more than 8 to a row. If I put them in rows of 8, how many students will be “left-over?” Can I have them march with the same number in each row without putting them in single file?
Task #27: Gameboard Task

A gameboard is constructed with white and black squares according to the pattern indicated below. There are 15 squares across the top and 9 squares down the side.

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  _ _ _ _ _ _ _ _ _ _
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a) How many squares are there on the board? How many white squares and how many black squares are there?

b) If I want to make a border with red squares, how many red squares will I need? After I add that red border, how many squares will there be in the board?

c) (continued on a second page) Suppose that each square in the boards in parts A and B is 1 inch by 1 inch. What is the area of each board? What is the perimeter of each board?

Task #28: Areas of Shapes

(a) Each square in the grid below measures 1 foot by 1 foot. How would you estimate the area of the shape shown below?
(b) Each square in the grid below measures 1 foot by 1 foot. How would you estimate the area of the shape below?
Task #29: Farmer’s Field Task

a) A farmer has a field that measures 100 meters by 60 feet. What area must he cover with topsoil? How much fencing would he need in order to enclose the entire field?

b) What if the field is bounded on one edge by a river? The farmer plans to raise crops in the field, not restrain animals. Will either the amount of topsoil or the amount of fencing change for this situation?
c) What if the field really measures 90 meters by 70 meters? Will either of the
answers in part (a) change?

d) What if the topsoil is sold by volume (in cubic meters) and the topsoil must be
distributed to a depth of ½ meter? How much topsoil would be needed in each of
the situations above?

Task #30: Array of 7 Squares

The drawing in this figure shows an arrangement of 7 squares. If the perimeter and the
area of the figure are numerically equal, find the side of one square.

Task #32: Areas on a Grid:
Task #33: Counting Triangles Task

How many triangles are there in this figure?
Task #34: Couples Task

Four married couples went to a baseball game last week. The wives' names are Carol, Sue, Jeanette, and Arlene. The husbands' names are Dan, Bob, Gary, and Frank. Bob and Jeanette are brother and sister. Jeanette and Frank were once engaged, but broke up when Jeanette met her husband. Arlene has a brother and a sister, but her husband is an only child. Carol is married to Gary. Who is married to whom?

Task #36: Balancing Tasks

(a) If a brick balances with three-quarters of a brick plus three-quarters of a pound, then how much does the brick weigh?

(b) If a bag of grain balances with a half of a bag of grain and 5 pounds, then how much does a bag of grain weigh?

(c) If 3 bags of rice balances with 2 bags of rice and 10 pounds, then how much does one bag of rice weigh?

Task #37: Space Station Task

In the outer reaches of space, there are eleven relay stations for the Intergalactic Space Ship Line. There are space ship routes between the relay stations as shown in the map below. Eleven astronauts have been engaged as communications operators, one for each station. The people are Alex, Barbara, Cindy, Donna, Elvis, Frances, Gloria, Hal, Irene, Johnny, and Karl. The two people in stations with connecting routes will be talking to each other a great deal, to discuss space ships that fly form station to station. It would be helpful if these people were friendly with each other. Here are the pairs of people who are friends:

Alex - Barbara     Hal - Frances     Irene - Karl
Gloria - Johnny    Gloria - Irene    Donna - Elvis
Donna - Irene    Alex - Gloria    Karl - Elvis
Cindy - Hal     Alex - Donna    Johnny - Irene
Johnny - Cindy  Donna - Karl

Place the eleven people in the eleven stations so that the people on connecting stations are friends.

Task #38: Fencing a Field

A farmer has 400 meters of fencing.

a) What possible dimensions would a field enclosed with that fencing have?

b) What shape should he choose to enclose with that fencing in order to have the largest possible area to cultivate?

c) After he unrolls the fencing, he realizes that someone has used 10 meters of the fencing. How would questions (a) and (b) now be answered?