# Price Discrimination with Congestion Effects

by

Stephanie Chapman

(Under the direction of John Turner)

## Abstract

I examine a model of price discrimination with congestion and find that when service providers are allowed to sell priority access to networks, consumers are made better off in most cases. In particular, profit is greatest when priority access is sold to a low value consumer, though high value consumers have a higher willingness to pay for priority when both consumers are served. Selling a priority right makes it profitable to serve all consumers in all sections of the parameter space. This result is robust to both single price monopoly pricing and third degree price discrimination. When no priority is offered, greater flexibility in pricing leads to greater profit for the firm, with the highest profit being achieved under fully nonlinear pricing. This analysis has implications for the net neutrality debate, particularly that consumer welfare may be made improved if net neutrality is relaxed.

Index words: price discrimination, net neutrality, congestion

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# Chapter 1

# Introduction

Communications networks have become increasingly important in consumer life over the past several decades. In developed countries it is difficult to find someone who does *not* own a cell phone or use the Internet daily, when those items were virtually nonexistent only twenty years ago. Despite the pervasiveness of these networks, little has been done to investigate the specific structure of consumer/service provider interaction in markets for network connectivity. This market is unique in that the product service providers sell (access) is simply a means of obtaining the desired good (internet content), rather than the desired good in and of itself. It is also unique in that consumers may suffer a non-monetary cost of consuming the access good, namely the congestion cost that occurs when many users are demanding content.

Recent literature has focused mainly on the interaction between service provider firms and content provider firms because of news coverage of the net neutrality debate. Net neutrality is the idea that all content should be treated equally in Internet networks, passing through on a first-come, first-serve basis. While net neutrality has been maintained for many years, service providers are now challenging the paradigm (Higginbotham 2010, McQuillen and Shields 2010, Whitt 2010). They argue that in order to better manage congestion in their networks they must be allowed to manage how content flows through the network, rather than just acting as a "dumb pipe." They go on to claim that without the abolition of net neutrality, they will have no incentive to expand network capacity, a situation that would clearly impact consumer welfare if current trends in internet usage continue (Clarke 2009).

Moreover, providers of mobile internet service have begun to adopt nonlinear pricing schemes as a tool to manage network congestion in spite of harsh criticisms from consumers that tiering pricing according to data usage "potentially stifles future mobile application usage and innovation" (Reardon 2010). In June 2010, AT&T introduced a tiered pricing plan for data usage on mobile internet networks, citing a need to manage exploding network congestion. T-Mobile and Verizon Wireless followed suit in May 2011 and July 2011, respectively (Goldman 2011). These recent developments make an examination of how consumers respond to nonlinear pricing schemes, and what those schemes may look like, particularly relevant.

In order to address these questions, I create a model that examines the specific relationship between the service provider and the consumer, allowing for congestion costs to be included in the calculation of consumer utility. In order to address the net neutrality issue, a consumer may purchase a priority right of access to the network, allowing that consumer to suffer only from the congestion he creates rather than the total congestion in the network. Within the framework of net neutrality, I then examine how consumers react to nonlinear pricing bundles, as well as the form that those bundles might take.

In support of service provider claims in the net neutrality debate, I find that when service providers are allowed to sell priority access to their network, consumers are made better off individually and in aggregate in most cases, while profit is simultaneously increased. Preceding analyses have found ambiguous results with regards to consumer welfare, but most of them have addressed consumers only as a side note in a discussion that centers more closely on profit motives of service providers and content providers (Krämer and Wiewiorra 2010, Choi and Kim 2010). Several different pricing schemes are examined here, including single price monopoly pricing and nonlinear pricing, to determine what effect selling a priority right of access to a high or low value consumer has on profit, welfare and total congestion.

In each case, profit under a priority regime is greater than profit achieved in a non priority regime, and in each case profit is maximized in the case where the monopolist sells the priority right to a low demand consumer, though it is not always the case that the low demand consumer is willing to purchase the priority right in an auction. In particular, the high value consumer is always more willing to purchase the priority right in an auction when both consumers are being served. When the high value consumer having the priority right leads to the monopolist pricing such that the low value consumer does not demand content at all, awarding a *de facto* priority right to the high value consumer, the high value consumer has a negative willingness to pay for the priority right in that parameter space, as it is a right he has already been awarded by default. In that case, the low value consumer has a higher willingness to pay for the priority and will win the right in an auction. Therefore, auctioning the priority right will lead to consumers in all of the parameter space being served when a priority right is offered, even though not all consumers would be served in the analogous, no priority case.

In order to outline these results, this paper examines several different pricing structures a monopolist might adopt depending on the legal environment and information available, including single price monopoly pricing, third degree price discrimination, fixed fee pricing and fully nonlinear pricing. For the single price monopoly and third degree price discrimination sections three separate cases are examined, one without a priority right and one with a priority right offered for purchase to each consumer in the model. The result that profit is greatest when the low demand consumer is awarded the priority right is robust to each of the pricing schemes. This result may seem counter-intuitive, but consider that when no priority is awarded, the high value consumer will demand more content than the low value consumer, so the high value consumer's demand composes a larger share of the congestion cost. Thus, the low value consumer has more to gain in savings on congestion than does the high value consumer. In the final chapter, three different types of nonlinear pricing are examined. Although none are explicitly worked out with the monopolist offering a priority right to either consumer, greater flexibility of pricing leads to greater profit for the monopolist.

# Chapter 2

# Related Literature

Only a handful of papers have been written on the topic of net neutrality, all of which have contained a theoretical model representing different features of how the authors perceive the structure of the market. Each focuses primarily on the interactions between content providers and service providers, neglecting the pricing and content decisions that occur between the consumer and the service provider which are the focus of this paper. This paper applies the policy of net neutrality to consumers rather than content providers, providing a stronger basis from which to assess the consumer welfare effects of net neutrality.

Choi and Kim (2010) propose a model to examine total social welfare effects of relaxing the net neutrality policy that introduces congestion effects in the form of a waiting cost implied by M/M/1 queuing theory. A unit mass of consumers distributed between two content providers incurs a fixed fee from the ISP for accessing the Internet, a transport cost of choosing between the two content providers and a waiting cost imposed by the average amount of time it takes for a request to be serviced in the system. The waiting cost suffered by each consumer is based on the average waiting cost of all consumers, obscuring the decision a consumer may make to request more or less content in the face of congestion. Service providers gain revenue only from consumers under net neutrality in the form of a flat fee for accessing the Internet, and under the discriminatory regime they gain additional revenue from a fee charged to the single content provider that wins a priority contract. Under this structure, the effect of net neutrality on investment incentives is ultimately ambiguous, though they do conclude that if the two content providers are sufficiently symmetric, short run social welfare is higher under net neutrality.

Krämer and Wiewiorra (2010) is similar to Choi and Kim in that M/M/1 queuing theory is used to represent congestion in the network, though they weave measures of network congestion into the objective functions of all three players in the system: the consumers, content providers and the service provider. With the adjustments to the model, they find that in the short run innovation is unaffected by the network discrimination policy, and that though welfare is unambiguously higher in the case of network discrimination, all content providers are worse off because the additional welfare is completely captured by the ISP in the fee charged to content providers. Though this paper improves upon the extent to which congestion is included in the market, the authors do not address the source of congestion, consumers. Additionally, they use aggregate welfare, the sum of profits and consumer surplus, as the relevant metric of comparison between regimes with and without net neutrality. While this may be a useful measure to assess value being created in the system, it is more policy relevant to consider the effect a policy change may have on consumers and service providers separately.

Economides and Tåg (2009) also examine a model of net neutrality that differs somewhat from those previously examined. The primary difference between this paper and Choi and Kim (2010) discussed above is that content providers engage in perfect competition, as in Krämer and Wiewiorra, with firms freely entering until profits fall to zero. Consumers are differentiated in their preferences for internet access along a continuum, $x \in [0, \infty]$, so that those closer to x = 0 have more of a preference for internet, or pay less of a cost to access the internet. Consumers gain marginal utility from each additional content provider in the market, and both consumers and content providers enter until their utility/profit reaches zero.

Economides and Tåg find that under a service provider monopoly, the monopolist would like to set a positive fee to content providers, while a social planner would set a negative fee, subsidizing content. They also find that the effect of net neutrality on total welfare is ambiguous and dependant on specific parameter values. Intuitively, although the removal of net neutrality would lead to fewer content providers under their model, it would also lead to lower access fees for consumers, and the lower access fee may be sufficient to compensate the consumer for fewer content providers.

Each of the papers discussed here obscures some part of the market that may have a substantial impact on how profit and welfare are affected by the removal of net neutrality, in particular the content decisions of consumers and the related pricing decisions of service providers. For instance, Economides and Tåg address the question of how the number of content providers in the market is affected by the removal of net neutrality, but do not consider congestion effects in their model. In Choi and Kim, the waiting cost paid by each consumer is constant, which does not allow the researcher to examine how consumers may change their consumption of content to affect that waiting cost in reaction to a policy change.

This paper focuses exclusively on the interaction between service provider and consumers in order to address this failing. It introduces the ability for consumers to choose the quantity of content they demand while varying pricing and structural policies. In contrast to previous studies, this paper applies the policy of net neutrality (or lack thereof) to consumers rather than content providers, abstracting away from content providers entirely and assuming that consumers select the amount of content they consume from a potentially infinite, competitive market for content.

This provides a framework that more closely approximates the existing market for network connectivity service, allowing a monopolistic ISP to sell "faster" or "slower" service to one consumer or another and examining the changes in profit, content demanded and welfare that result. Independently of the policy debate of net neutrality, this paper examines more complex pricing structures that may be adopted by the service provider, examining how classical price discrimination results may change when consumers are subject to a congestion externality.

# Chapter 3

# The Model

This model consists of two types of consumers facing a monopolistic service provider who costlessly provides content under some pricing structure to all consumers. The consumers suffer from a congestion cost generated by their usage of the network. The monopolist is only sensitive to this congestion indirectly through the reduction in demand caused by an increase in congestion. In many other models of Internet network access, a two sided model is utilized with interactions between content providers and the service provider forming the second side of the market. This paper examines only the interaction between consumers and the service provider, though content is generated implicitly by content providers, not the service provider. This simplification is motivated by the observation that consumers demand different types of content from multiple different content outlets with widely varying network usage requirements. For instance, reading email for an hour consumes vastly less bandwidth than streaming a movie from a website such as Netflix, but the consumer may obtain the same utility from each.

I abstract away from the need to distinguish between types of content and types of content providers by allowing the consumer to choose simply a quantity of content demanded from a potentially infinite, perfectly competitive market for internet content. Content quantity here could be interpreted as the bandwidth consumed by a consumer, the amount of time spent on the network or the amount of data downloaded. This assumption is advantageous in that it allows the focus to rest squarely on consumers, resulting in cleaner predictions of welfare and congestion changes in response to net neutrality or the removal thereof. Additionally, consumers may be highly differentiated in their types of internet content usage while not begin differentiated in their volume of content demanded, the main focus of this paper. Abstracting away from types of content allows greater attention to be paid to the volume of congestion created in the network, without worrying over what particular form that congestion might take.

Making this assumption limits the ability of this paper to address certain concerns of supporters of net neutrality. One major concern of proponents of net neutrality is that if the policy were to be abandoned, incentives of content providers to "innovate at the edge" would be diminished. By abstracting away from content providers, this analysis can no longer address how differentiated content providers may be adversely affected by the removal of net neutrality. The gain in richness of specification of the consumers' problem, however, provides new and unique insights into how consumers will be affected by a policy change, outweighing the loss of insight into the content providers' reaction to the same policy adjustment.

## 3.1 Consumers

In each of the following pricing scenarios there are two different consumers, one with a high value of content and one with a low value. Each has a valuation of content quadratic in the amount of quantity he consumes, in particular $V = q_H - q_H^2$ for the high value consumer. The low demand consumer values content at a lower rate than his high demand counterpart by a scale factor of $\alpha$, namely $V = \alpha q_L - q_L^2$, where $\alpha$ is bounded below by $\frac{1}{2}$ and above by one.[1] The size of $\alpha$ represents the degree of differentiation between consumers. The lower $\alpha$, the greater the degree of differentiation between the two consumers. Each consumer will pay a fee ($T = A + pq$) to the monopolist for access to the internet. In order to compare different pricing regimes, $A$ and $p$ may in different sections be set to zero, may be common to both consumers or may be differentiated between the two consumers.

## 3.2 Congestion Cost

Each consumer suffers from a congestion cost incurred from his and his counterpart's use of the network. In previous literature (Choi and Kim 2010, Krämer and Wiewiorra 2010), this congestion cost has been derived from M/M/1 queuing theory. While the foundations of this form are attractive and deeply rooted in statistical theories of networks, it is analytically intractable in a problem in which consumers may choose the amount of content they consume.

To determine what type of waiting cost to adopt, consider the qualitative properties of the waiting cost implied by M/M/1 queuing theory. Letting $w$ denote the waiting cost implied for all consumers when there is no priority available,[2] it is the case that $\frac{\partial w}{\partial \lambda} > 0$ and $\frac{\partial^2 w}{\partial \lambda^2} > 0$, where $\lambda$ is the amount of content generated by all consumers. That is, the waiting cost is increasing at an increasing rate in content demanded. This result is true of the waiting costs implied when one consumer group has priority.[3] Additionally, waiting cost is decreasing in total network capacity, $\mu$.

The functional form proposed for congestion retains these properties while providing a

[1] As we will see, this restriction is necessary to ensure that demand is positive for at least some non-negative values of $p$. This can be interpreted as simply a restriction that the low value consumer values the good sufficiently to be willing to purchase, but does not value the good more than the high value consumer (without loss of generality)

[2] $w = \frac{1}{\mu - \lambda}$ where $\mu$ is the network capacity and $\lambda$ is the total amount of content generated by all consumers.

[3] When one consumer group has priority, the waiting cost imposed on that group ($w_P$) has the same form as the general waiting cost, though it depends only the content generated by that group. $w_C$ denotes the waiting cost of the low priority group, which does change form. In particular, $w_P = \frac{1}{\mu - \lambda_1}$ and $w_C = \left(\frac{\mu}{\mu - \lambda}\right)\left(\frac{1}{\mu - \lambda_1}\right)$ where $\lambda_1$ is the amount of content demanded by the high priority group only.

form that allows for explicitly solving and examining the implications of model. Consider the function $C_H(q_H, q_L) = tq_H(q_H + q_L)$, which will be symmetrically defined for the low demand consumer. Intuitively, the consumer suffers from the total congestion in the network, $(q_H + q_L)$, scaled by the amount of content he himself generates. If a consumer uses the internet only a little, his sensitivity to total congestion is lessened by the lack of exposure, whereas if a consumer accesses the internet a great deal, his sensitivity to congestion is increased. Note that when the consumer demands no content, his congestion cost is zero. The parameter $t$ is a scale factor on total congestion in the network, common to both consumers. This may be interpreted as the quality of the underlying network. For example, a high bandwidth network would be represented by a low value of $t$, whereas a low bandwidth network would be represented by a high quality of $t$. While total network capacity does not enter into this congestion function explicitly, $t$ functions much the same as the network parameter $\mu$ in that it dictates the magnitude of congestion for all consumers.

Aggregating the aspects of the consumer side of the model outlined above, consumers strategically interact to solve the following problem

$$\begin{aligned} \max_{q_H} U_H &= q_H - q_H^2 - T_H - tq_H(q_H + q_L) \\ \max_{q_L} U_L &= \alpha q_L - q_L^2 - T_L - tq_L(q_H + q_L) \end{aligned} \tag{3.1}$$

General demand functions for this problem are characterized by a Nash equilibrium, though they are functions in terms of the marginal prices charged to consumers

$$\begin{aligned} q_H^* &= \frac{2(1+t)(1-p_H) - t(\alpha - p_L)}{(2+3t)(2+t)} \\ q_L^* &= \frac{2(1+t)(\alpha - p_L) - t(1-p_H)}{(2+3t)(2+t)} \end{aligned} \tag{3.2}$$

To examine the case of one consumer having a priority right of access to the network over another, in some cases one consumer will purchase the right to be insensitive to the congestion created by the other consumer. That is, the consumer will purchase a new congestion function of $C = q_i^2$, where $i \in \{H, L\}$. This will represent the service provider serving all content requests from one consumer before serving any requests from the other consumer, differentiating the consumers on congestion cost as well as content value. The consumer with the priority right will have access to the network as if it were his network alone, and as such will have "faster" service. This will allow an examination of the welfare effects of consumers being asymmetric in their congestion cost.

The assignment of priority access is assumed to be costless to the firm at the margin, just as it is assumed that the marginal cost to the firm of providing service to consumers is costless. Given the specific nature of how service is provided in this industry, it is a reasonable assumption that assigning priority to one group or another would entail a large fixed cost (to write the programming necessary to make prioritization of content automatic) but negligible marginal costs. This leads to the conclusion that including a cost of prioritization would needlessly complicate the analysis. Additionally, a purpose of this paper is to assess the validity of service provider claims that everyone would be better off were service providers afforded the ability to prioritize content. Thus, the costless prioritization assumption gives a "best-case-scenario" under which this assertion may be examined. Consumers and the service provider would only be made worse off if there were a cost to prioritization of content, especially if that cost were significant enough for the service provider to elect not to offer prioritization for sale. In either case, examining the situation where prioritization is costless is sufficient to determine the validity of service provider claims.

## 3.3 The Firm

Consumers face prices set by a single service provider that costlessly transports content to the consumer. The assumption of a single firm is motivated by the reality that many consumers of internet or cellular service face. In many areas there are only one or two internet and/or cellular service providers from which to choose. This is a convention that has also been adopted in other papers that address net neutrality and the nature of the Internet (Economides 2009, Choi and Kim 2010). The monopolist sets fees by maximizing his profit, given demand functions generated by the two consumers. The monopolist may price many different ways that will be addressed throughout the paper, but in general, the monopolist will solve the following profit maximization problem.

$$\max_{\{A_H, A_L, p_H, p_L\}} \Pi = A_H + p_H q_H + A_L + p_L q_L \tag{3.3}$$

The particular structure of the problem will be different in each section based on the allowed pricing scheme. For instance, in the initial case of pure monopoly pricing the monopolist may only select a single per-unit price $p = p_H = p_L$, while $A_H = A_L = 0$. Depending on the pricing scheme, this maximization problem will also be subject to constraints ensuring that at any given price bundle, consumers will elect to consume a nonnegative quantity and that they will choose their intended pricing bundle rather than the bundle intended for the other consumer. Additionally, the monopolist will always be subject to the constraint that his profit gained from pricing such that both consumers purchase service is greater than his profit gained from pricing such that only the high value consumer wishes to purchase, i.e. pricing such that the high value consumer receives a *de facto* priority right. This will be a particularly interesting constraint in the cases when the monopolist may introduce fixed fees into his pricing bundles.

## 3.4 Timing

Before consumers or the firm make any decisions, it is common knowledge that there are exactly two consumers of different types, thus each consumer knows his type and the type of the other consumer. The firm first decides whether to offer a priority right of access to the network or not, which becomes common knowledge when he makes the decision. If he decides not to offer a priority right, consumers strategically interact to reach a Nash equilibrium that describes their demand functions in prices, which the firm then uses to select the profit-maximizing price(s).

In chapter 6, consumers must choose between two different pricing bundles, and there is the possibility that one or the other may deviate and select the bundle intended for the other party. In determining the quantities demanded in the case of a deviation, I will assume that the two consumers strategically interact based on the packages they do choose, resulting in different quantities demanded for both consumers when one deviates and selects a different package from the one intended.

If the monopolist decides to offer a priority right, he may sell it to either the high or low value consumer. In order to determine to which consumer the priority will be sold, it will be auctioned off to the consumer with the higher willingness to pay before consumers strategically interact to determine their demand functions and before the monopolist makes pricing decisions. For simplicity, consider the mechanism by which the priority is auctioned to be a sealed bid, second price auction which induces consumers to report their willingness to pay truthfully. The mechanism then awards the priority right to the consumer with the higher willingness to pay at a cost of the greater of the other consumer's willingness to pay or zero. In order to form their bids, each consumer looks ahead and calculates his utility for each possible case, the case where he has the priority right and the other consumer does not, or visa versa. The willingness to pay of each consumer is then the difference between a consumer's utility given that he has the priority right and the same consumer's utility given that the other consumer has the priority right and he does not. Within the context of this paper, both situations will be calculated under the assumption that one consumer or the other has the priority right, then a conclusion will be made about which consumer would purchase the priority in an appropriate auction.

# Chapter 4

# Single Price Monopoly Pricing

Consider the case where the monopolist may only select a single per unit price. In this case, he may not distinguish between the two types of consumer, and must set a single price based on the total demand in the market, composed of the demand from both types of consumer. This gives a basis for understanding how consumers might react to a single price that is distorted away from their individual marginal cost of content. In particular, this case serves as a point of comparison for understanding how adding a priority right auction in a nonlinear price setting might affect whether both consumers are served in equilibrium.

Consumer demand depends on the single optimal price that is chosen:

$$
\begin{aligned}
q_H &= \begin{cases} \frac{1-p}{2(1+t)} & \text{when } p > \frac{2\alpha(1+t)-t}{2+t} \\ \frac{2(1+t)(1-p)-t(\alpha-p)}{(2+3t)(2+t)} & \text{when } p < \frac{2\alpha(1+t)-t}{2+t} \end{cases} \\
q_L &= \begin{cases} 0 & \text{when } p > \frac{2\alpha(1+t)-t}{2+t} \\ \frac{2(1+t)(\alpha-p)-t(1-p)}{(2+3t)(2+t)} & \text{when } p < \frac{2\alpha(1+t)-t}{2+t} \end{cases}
\end{aligned}
\tag{4.1}
$$

Because the monopolist cannot distinguish between the two consumer types, he must price to total demand only, thus the demand function faced by the monopolist is

$$Q = \begin{cases} \frac{1-p}{2(1+t)} & \text{when } p > \frac{2\alpha(1+t)-t}{2+t} \\ \frac{1+\alpha-2p}{(2+3t)} & \text{when } p < \frac{2\alpha(1+t)-t}{2+t} \end{cases} \tag{4.2}$$

When either consumer group has priority, the form of his congestion cost changes as discussed in Chapter 3, and the demand function faced by the monopolist changes accordingly. In this case, the monopolist may not charge differentiated prices when he offers the priority right to one consumer group, so gains from the priority right stem entirely from changes in demand.

## 4.1 Monopoly Pricing - No Priority

When the monopolist faces two consumers that have symmetric congestion costs, the marginal price he sets depends on the value of alpha only when the low value consumer chooses to purchase service. Recall that $\alpha$ is bounded below by $\frac{1}{2}$. When $\alpha$ is low and t is high, the monopolist does not profit from pricing such that the low demand group buys. Maximizing profit ($\Pi = pQ$) yields that when the monopolist prices to both types (i.e., when $\alpha$ is high), he sets a price of $p = \frac{1+\alpha}{4}$. When he is pricing such that only the high value consumer buys, he will set a price of $p = \frac{1}{2}$. Upon examining how these prices compare to the bounds set on $p$, it becomes clear that there is a region of the parameter space where both prices satisfy the given constraints, namely where $\alpha \in [\frac{2+5t}{6+7t}, \frac{2+3t}{4(1+t)}]$. When t is zero, this range is $\alpha \in [\frac{1}{3}, \frac{1}{2}]$ while when t goes to infinity, it approaches $\alpha \in [\frac{5}{7}, \frac{3}{4}]$ Within this space the monopolist will choose the more profitable arrangement, which leads to the result that he will price to both consumers when the expression $\frac{1+2t}{1+t} < \alpha(2+\alpha)$ is satisfied and to the high type consumer only when the complement is satisfied. Thus quantities demanded are

$$q_H = \begin{cases} \frac{1}{4(1+t)} & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{2(3-\alpha)+t(7-5\alpha)}{4(2+t)(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases}$$
$$q_L = \begin{cases} 0 & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{2(3\alpha-1)+t(7\alpha-5)}{4(2+t)(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases} \quad (4.3)$$

Total congestion is the sum of low and high value quantities demanded, denoted by $Q$. Profit is then the product of total congestion and the price charged at the particular value of $\alpha$.

$$Q = \begin{cases} \frac{1}{4(1+t)} & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{1+\alpha}{2(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases} \qquad \Pi = \begin{cases} \frac{1}{8(1+t)} & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{(1+\alpha)^2}{8(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases} \quad (4.4)$$

## 4.2 Monopoly Pricing with a Priority Right

Now consider the monopolist facing the total demand of two consumers, one of whom is assigned the priority right by an auction wherein the one with the higher willingness to pay is awarded the right. Both consumers still pay the same per unit price for content. This may be the case where one consumer has a substantially better computer and/or network connection than the other, so much so that additional usage of the network by other consumers becomes negligible. In this framework, the auction cost could be interpreted as the equipment cost to the consumer.

### 4.2.1 High demand consumer has priority

When the high demand consumer has priority, his utility function is altered such that he does not detect the congestion generated by the low value consumer. In particular the utility function $U_H = q_H - q_H^2 - pq_H - tq_H^2$ for the high value consumer only leads to new Nash equilibrium demand functions

$$q_H^* = \frac{1-p}{2(1+t)} \qquad q_L^* = \begin{cases} 0 & \text{when } p > \frac{2\alpha(1+t)-t}{2+t} \\ \frac{2(1+t)(\alpha-p)-t(1-p)}{4(1+t)^2} & \text{when } p < \frac{2\alpha(1+t)-t}{2+t} \end{cases} \tag{4.5}$$

Using the new specification of utility for the high demand consumer and the procedure outlined in the previous section gives a pricing schedule of

$$p = \begin{cases} \frac{1}{2} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{2+t+2\alpha(1+t)}{2(4+3t)} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \tag{4.6}$$

Equilibrium quantities, total congestion and profit are then

$$\begin{gathered} q_H = \begin{cases} \frac{1}{4(1+t)} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{2(3-\alpha)+t(5-2\alpha)}{4(1+t)(4+3t)} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \\ q_L = \begin{cases} 0 & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{(10\alpha-7)t^2+2(11\alpha-6)t+4(3\alpha-1)}{8(1+t)(4+3t)} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \\ Q = \begin{cases} \frac{1}{4(1+t)} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{2+t+2\alpha(1+t)}{8(1+t)^2} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \\ \Pi = \begin{cases} \frac{1}{8(1+t)} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{[1+(1+t)(1+2\alpha)]^2}{16(4+3t)(1+t)^2} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \end{gathered} \tag{4.7}$$

### 4.2.2 Low demand consumer has priority

Following the same procedure as the previous section, we find that the monopolist will charge the same price for all allowable values of $\alpha$, because when the low value consumer has priority, his consumption of content is sufficiently subsidized for him to consume even when $\alpha$ is low.

Price for all values of $\alpha$ is then

$$p = \frac{t + (2+t)(1+\alpha)}{2(4+3t)} \tag{4.8}$$

resulting in quantities demanded of

$$q_H = \frac{(10\alpha - 7)t^2 + 2(11 - 6\alpha)t + 4(3-\alpha)^2}{8(4+3t)(1+t)} \qquad q_L = \frac{2(3\alpha - 1) + (5\alpha - 2)t}{4(1+t)(4+3t)} \tag{4.9}$$

and profit and total congestion of

$$\Pi = \frac{[2(1+\alpha) + t(2+\alpha)]^2}{16(4+3t)(1+t)^2} \qquad Q = \frac{2(1+\alpha) + t(2+\alpha)}{8(1+t)^2} \tag{4.10}$$

Note again that in this case, there is no parameter space where the low value consumer is not willing to consume. When the low value consumer is given access to the priority right, the absence of congestion generated by the high value consumer lowers his cost of content sufficiently for him to demand a positive amount of content at all values of $\alpha$.

**Lemma 1.** *Under single price monopoly pricing, profit and congestion are maximized when the low value consumer is awarded the priority right.*

A cursory examination of the pricing schemes and resulting quantities demanded for this section reveal an intuitive explanation for this assertion. The low value consumer gains more from having priority because the content demanded by the high value consumer constitutes a greater proportion of total content than the content demanded by the low value consumer, thus the reduction in congestion when the low value consumer has priority is greater than the reduction in congestion for the high value consumer when he has priority, which generates more surplus that the monopolist is able to capture as profit. This result stems from the two consumers having symmetric reactions to congestion, even though they are differentiated in their valuation of content. A potential extension to consider in order to check the robustness of this result may be one in which consumers are differentiated in their assessment of congestion as well as in their valuation of content.

### 4.2.3 Priority right auction

To determine which consumer would receive the priority right in an auction setting, compare the willingness to pay of each consumer across the states in which one consumer has priority over the other. The willingness to pay of the high value consumer is $W_H = U^H_{priority} - U^H_{no\ priority}$ while the willingness to pay of the low value consumer is $W_L = U^L_{priority} - U^L_{nopriority}$. Because prices and quantities are piecewise defined in the section in which the high value consumer has priority, the willingness to pay function for each consumer will also be piecewise defined according to the same bounds.

**Lemma 2.** *The high value consumer has a higher willingness to pay for priority when both consumers purchase service.*

This result can be seen in figure 4.1, where each branch represents the high and low value consumers' willingness to pay for different values of $\alpha$. The discontinuity in the in the high value consumer's willingness to pay occurs at the parameter combination where the monopolist decides to sell only to the high value consumer. Thus the high value consumer is less willing to pay for priority than the low value consumer when he is awarded priority by default through the profit maximization problem of the monopolist.

In the auction to determine which consumer will receive the priority right, the high value consumer always wins the priority right if both consumers are served under that regime. The low value consumer wins the priority right when he would be priced out of the market if he did not have priority. When the low value consumer purchases priority, he pays nothing for it, since the high value consumer's willingness to pay is zero. When the high value consumer purchases priority, however, profit may still be less than it would be if the low value consumer

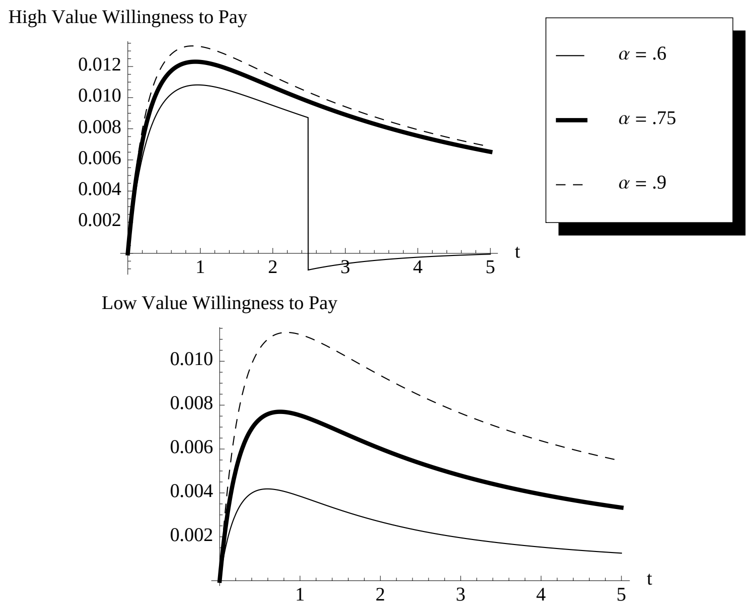


Figure 4.1: Willingness to pay for priority

were assigned priority.

The end result of the priority right being available for auction is that there is no area of the parameter space in which the low value consumer is not served. The area in which the low value consumer would not be served under a high priority regime is exactly the parameter space in which he is willing to purchase the priority right and thus service. Profit is greater than it would be if the priority right were not available for purchase, though whether total profit is greater than profit in the case where the low value consumer has the priority right for all of the parameter space is ambiguous because of the nature of the low value consumer's willingness to pay for priority. The amount of the low value consumer's willingness to pay is the fee that the high value consumer pays to purchase the priority right in the second price auction. Because that willingness to pay decreases with $\alpha$, the revenue generated from auctioning the priority right is less when $\alpha$ is low, generating a region of the parameter space

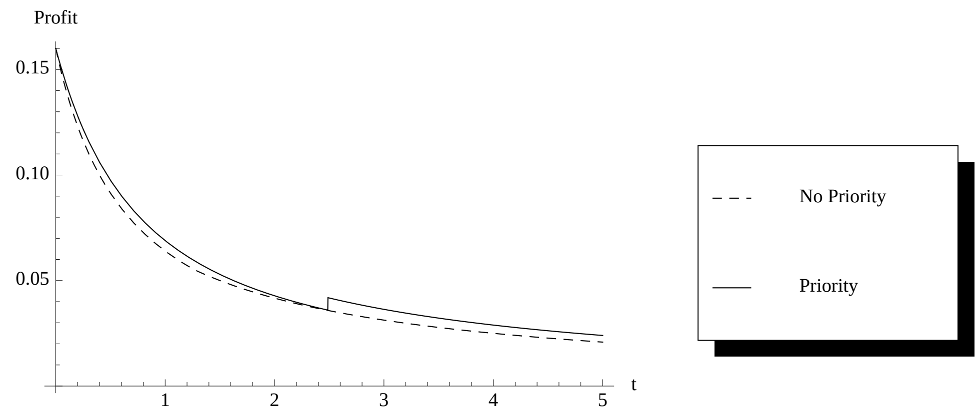


Figure 4.2: Profit under each regime; $\alpha = .6$

in which it is still more profitable for the low value consumer to have the priority right, even though he is not more willing to pay for the right than the high value consumer. When $\alpha$ is high, however, the auction of the priority right produces the most profitable arrangement for the monopolist. Total profit generated with the auction compared to profit under the no priority regime can be seen in figure 4.2.

## 4.3 Welfare Analysis

Note in figure 4.3 that the effect of assigning a priority right to either consumer is positive for all ranges of the parameters $\alpha$ and $t$. The utility of both consumers is improved when a priority right is offered, and total utility is therefore improved. For the low value consumer, this improvement stems entirely from the ability to "purchase" the priority right at no cost when the alternative is zero utility. For the high value consumer, the key observation is that utility drops when he is awarded the *de facto* priority right under the regime when no priority right is available for purchase. His utility is higher when both consumers are being served because the monopolist cannot charge a specific price to the consumer with priority,

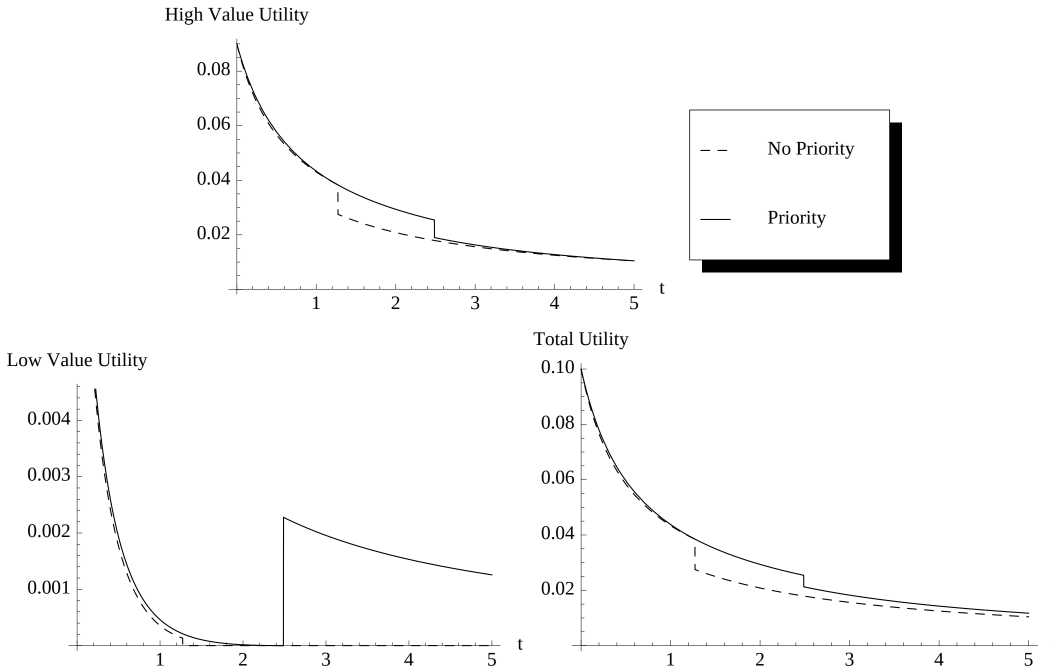


Figure 4.3: Utility under each priority regime; $\alpha = .6$

and the price the monopolist sets is lower than the price set when the high value consumer is the only participant in the market. When the low value consumer is awarded priority, the high value consumer's utility is no higher than when he has he *de facto* priority right under a no priority regime. When a priority right is awarded, each consumer is at least as well off when he is able to purchase the priority right as he was when no priority right was available, so total utility is improved when a priority right is available.

# Chapter 5

# Third Degree Price Discrimination

The monopolist most effectively price discriminates if he can distinguish between the high and low demand consumers and price to each group exactly. In this case, consumers will have Nash equilibrium demand functions of

$$\begin{aligned} q_H^* &= \frac{2(1+t)(1-p_H) - t(\alpha - p_L)}{(2+t)(2+3t)} \\ q_L^* &= \frac{2(1+t)(\alpha - p_L) - t(1-p_H)}{(2+t)(2+3t)} \end{aligned} \tag{5.1}$$

where $p_H$ is the price charged to the high demand consumer and $p_L$ is the price charged to the low demand consumer. It is a reasonable assumption that the monopolist would be able to crudely distinguish between high and low demand consumer groups based on demographic factors. Consider specifically the senior-oriented cell phone service "Jitterbug" or specially designed phones for children as examples of how the monopolist may distinguish between consumer groups with differing values of content and price accordingly. Because the firm can distinguish between the two consumers, it may select which consumer receives priority based on which arrangement would be most profitable. In order to assess that decision, we calculate profit under each regime, then determine which would be most profitable.

## 5.1 No Priority

When neither consumer has a priority right, the monopolist will price at $p_H = 1/2$ and $p_L = \alpha/2$, which is consistent with the monopolist facing two consumers with demand functions as outlined above separately. Because the firm can now distinguish between the two consumers and price to each exactly, the problem does not arise in which the firm prices such that the low value group is not willing to purchase service. Quantities and total congestion are simply

$$q_H = \frac{2 + 2t - t\alpha}{2(2+t)(2+3t)} \qquad q_L = \frac{2\alpha + t(2\alpha - 1)}{2(2+t)(2+3t)} \qquad Q = \frac{1+\alpha}{2(2+3t)} \tag{5.2}$$

while profit under this regime is

$$\Pi = \frac{(1+t)(1+\alpha^2) - t\alpha}{2(2+t)(2+3t)} \tag{5.3}$$

Congestion is unchanged versus the single price monopoly situation, though profit is increased even over the low value $\alpha$ cases when low value consumers do not purchase service. This result mirrors the well-known result obtained for linear demand absent of congestion that total quantity is unchanged between single price monopoly pricing and third degree price discrimination. In this case, the decrease in quantity demanded by the high value consumer resulting from the increase in price he faces is exactly offset by the increase in quantity demanded by the low value consumer. There is no distortion created by the congestion cost.

## 5.2 Third Degree Price Discrimination with a Priority

Consider now the case where the priority right is awarded to one consumer group or the other. Because the monopolist may distinguish between the two consumer groups, this section maps out the monopolists decision between offering the priority right to the high or low demand groups by comparing the profitability of each arrangement.

### 5.2.1 Priority right to high value consumer

Carrying out this maximization by taking first order conditions with respect to $p_H$ and $p_L$ we have the following prices

$$p_H = \frac{(7+2\alpha)t^2 + 2(8+\alpha)t + 8}{(4+3t)(4+5t)} \qquad p_L = \frac{2(1+t)[4\alpha + t(4\alpha - 1)]}{(4+3t)(4+5t)} \tag{5.4}$$

Which result quantities and total congestion of

$$q_H = \frac{4(1+t) - t\alpha}{(4+3t)(4+5t)} \qquad q_L = \frac{4\alpha + t(4\alpha - 1)}{(4+3t)(4+5t)} \qquad Q = \frac{1+\alpha}{4+5t} \tag{5.5}$$

The resulting profit is then

$$\Pi = \frac{2(1+t)(1+\alpha^2) - t\alpha}{(4+3t)(4+5t)} \tag{5.6}$$

### 5.2.2 Priority right to the low value consumer

Now consider the monopolist offering the priority right to the low demand consumer. Profit maximization leads to prices of

$$p_H = \frac{2(1+t)[4 + t(4-\alpha)]}{(4+3t)(4+5t)} \qquad p_L = \frac{(7\alpha+2)t^2 + 2(8\alpha+1)t + 8\alpha}{(4+3t)(4+5t)} \tag{5.7}$$

Total congestion resulting from these prices is the same, though it is generated by the two consumers adjusting their consumption to exactly as much as each demanded in the high value priority situation. That is,

$$q_H = \frac{4(1+t) - t\alpha}{(4+3t)(4+5t)} \qquad q_L = \frac{4\alpha(1+t) - t}{(4+3t)(4+5t)} \qquad Q = \frac{1+\alpha}{4+5t} \tag{5.8}$$

This indicates that selling priority to the low value consumer has a greater impact on his consumption, since the increase in quantity demanded over the no priority case is greater for the low value consumer than it is for the high value consumer. This is the case because in all cases, the high value consumer demands more content than the low value consumer. Thus, the content generated by the high value consumer is a larger share of the total congestion created. Eliminating that congestion for the low value consumer then has a greater impact than eliminating the congestion created by the low value consumer from the high value consumer's congestion function. Since the monopolist is able to price to each group individually, he is able to maximize his profit by exactly offsetting the varying congestion effects, resulting in the same quantities and profit in each priority setting.

Because prices and corresponding quantities are the same as in the previous section, profit is also unchanged.

$$\Pi = \frac{2(1+t)(1+\alpha^2) - t\alpha}{(4+3t)(4+5t)} \tag{5.9}$$

Examining total quantity demanded and total profits across each of the preceding sections leads to the following lemma.

**Lemma 3.** *Under third degree price discrimination, profit and congestion are maximized when either consumer purchases a priority right.*

This echoes the result obtained in the previous chapter, that profit and congestion are maximized when the priority right is offered to the low value consumer, with the interesting twist that quantities and profit are exactly the same regardless of which consumer is offered the priority. Because the monopolist can see the demand functions of the two consumers separately, and because he is affected by the additional congestion generated by the consumer with priority, he internalizes that negative externality. In maximizing profit, he then sets prices such that each consumer demands exactly the same amount of content, leading to the same profit in either case. This result is not generated in the previous, single price monopoly case because the monopolist cannot distinguish between the two consumers to manage this externality, he simply sees total demand. If each consumer was equally affected by having priority in the single price monopoly case, this result would arise, but because consumers have different valuations of content, they benefit asymmetrically from having the priority right, resulting in different quantities and profit in the single price monopoly case.

### 5.2.3 Priority right auction

The willingness of each consumer to pay for the priority right is exactly zero, because the monopolist exactly internalizes the congestion externality and sets prices such that each consumer yields the same utility regardless of which has priority. Intuitively, this is why quantities are unchanged from one priority setting to another. Thus, there is no difference between offering the priority right to either consumer, though it is more profitable for one of them to have priority access than it is for neither to have priority access. It is interesting to note that most of the increases in congestion moving from the no-priority state to the priority state are generated by the low value consumer, since there is an unambiguous increase in the content demanded by the low value consumer but an ambiguous change in the content demanded by the high value consumer.

## 5.3 Welfare Analysis

Although consumers are indifferent between having and not having the priority themselves, there is an improvement in welfare realized for most parameter values when one or the other

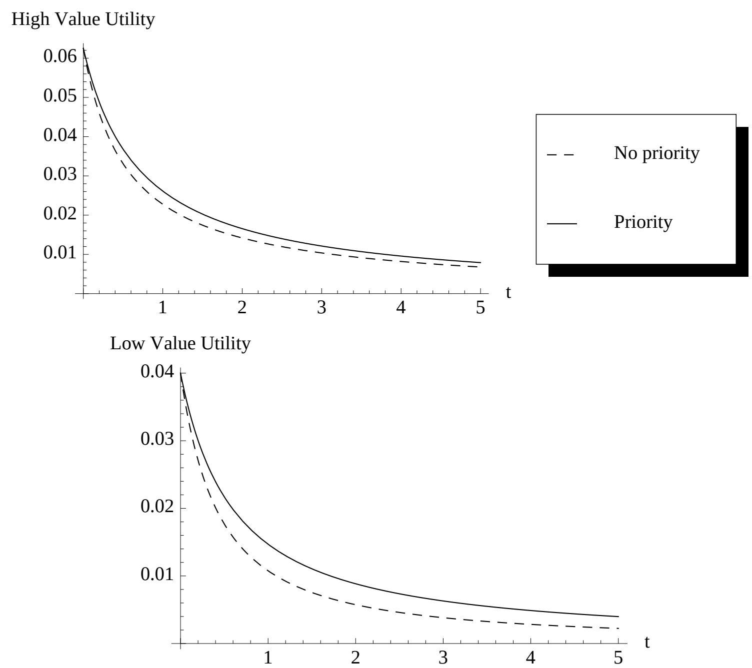


Figure 5.1: Utility improvements; $\alpha = .8$

consumer has the priority right over the case when neither has priority. The improvements in utility are pictured in figure 5.1. This result holds for the majority of the parameter space, though there is a small region when $\alpha$ is quite low and $t$ is very large that total utility is not improved over the no priority case. This ambiguous result originates entirely with the high value consumer, as the low value consumer's utility is strictly improved in the priority case regardless of parameter values. The region of the parameter space in which the priority is welfare diminishing for the high value consumer is pictured as the shaded region in figure 5.2, while the darker region is the parameter space in which the negative gains of the high value consumer overwhelm the positive gains of the low value consumer, resulting in a loss in total utility.

The high value consumer's utility is diminished in the case pictured in figure 5.2 because in that area of the parameter space, the amount of content demanded by the high value

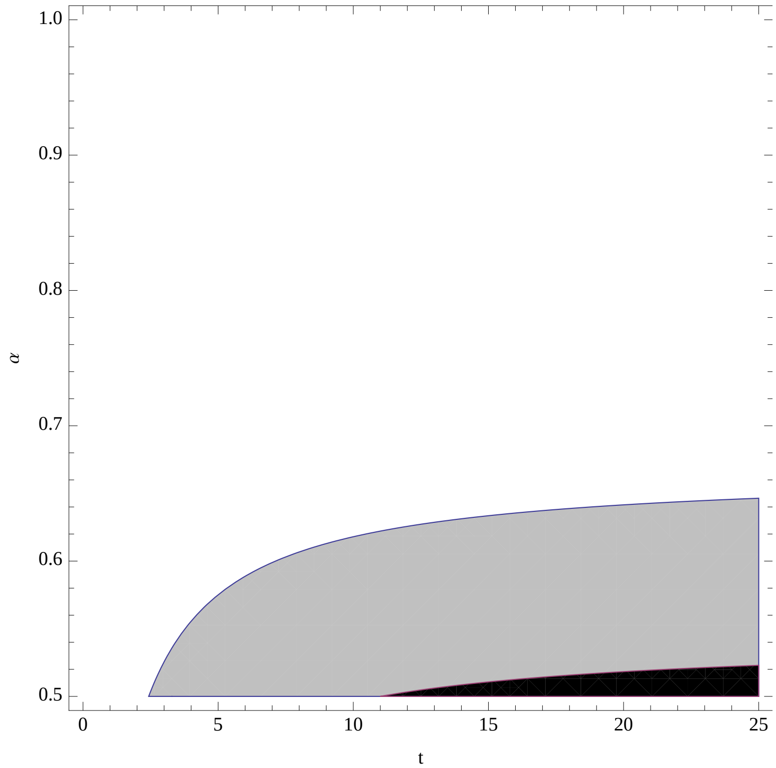


Figure 5.2: Parameter space in which high value utility decreases with priority

consumer under the no priority regime is greater than the amount of content he demands under the priority regimes. This is a result of the diminished congestion externality created by the low value consumer when $\alpha$ is relatively low. Because the low value consumer is demanding very little content under the no priority regime, the high value consumer demands more content, resulting in higher utility.

# Chapter 6

# Nonlinear Pricing

This chapter examines the effect of adopting several different unorthodox pricing strategies, though none with the priority convention assigned. The first section examines the effect of the monopolist simply charging a single fixed fee to consumers, the second the effect of charging the high value consumer a fixed fee and the low value consumer a marginal price, and the third allows for a fully nonlinear pricing scheme, with a different fixed fee and marginal pricing bundle offered to each consumer. In each pricing scheme, there is an area of the parameter space, when $\alpha$ is sufficiently high and $t$ sufficiently low, that the monopolist is more profitable selling service to both consumers. When $\alpha$ is low and $t$ is high, however, the monopolist will set a single pricing bundle that captures all of the surplus of the high value consumer, ensuring that the low value consumer does not purchase service. In this way, the monopolist maximizes profit by offering a *de facto* priority right to the high value consumer for some areas of the parameter space. This mirrors the result obtained in the single price monopoly case, that it is not always profitable for the monopolist to price such that both consumers purchase service.

## 6.1 Fixed Fee only

Consider the case where the monopolist may set only a single fixed fee for unlimited content. This case closely approximates many pricing schemes observed in reality that charge a flat monthly fee for unlimited network access.

When both consumers are induced to purchase service with no priority right given, the Nash equilibrium quantities reached will be constant, as there is no marginal cost of content to the consumer. The resulting quantities are

$$q_H^* = \frac{2(1+t) - t\alpha}{(2+t)(2+3t)} \qquad q_L^* = \frac{2\alpha(1+t) - t}{(2+t)(2+3t)} \tag{6.1}$$

whereas the quantity demanded by the high demand consumer when the low demand consumer is not induced to buy will be $q_H^* = \frac{1}{2(1+t)}$.

If both consumers are to purchase service, the monopolist will set the fee to be equal to the surplus of the consumer with the lesser amount of surplus, given demand at the per-unit price of zero. This occurs when

$$U_H = (1+t)[q_H^*]^2 - A > (1+t)[q_L^*]^2 - A = U_L \tag{6.2}$$

Thus, the monopolist will set $A$ to be equal to the right hand side of this expression. At this price, monopolist profit is

$$\Pi = \frac{2(1+t)[2\alpha(1+t) - t]^2}{(2+t)^2(2+3t)^2} \tag{6.3}$$

However, if the monopolist prices such that the low value consumer does not purchase service, the high value consumer will then choose a quantity $q_H = \frac{1}{2(1+t)}$, resulting in utility of $U_H = \frac{1}{4(1+t)}$, which is the maximum fixed fee the monopolist may charge. In this case, profit is simply $\Pi = \frac{1}{4(1+t)}$ because the monopolist will only yield the fee from one of the two consumers. The profit generated from awarding a *de facto* priority right to the high value

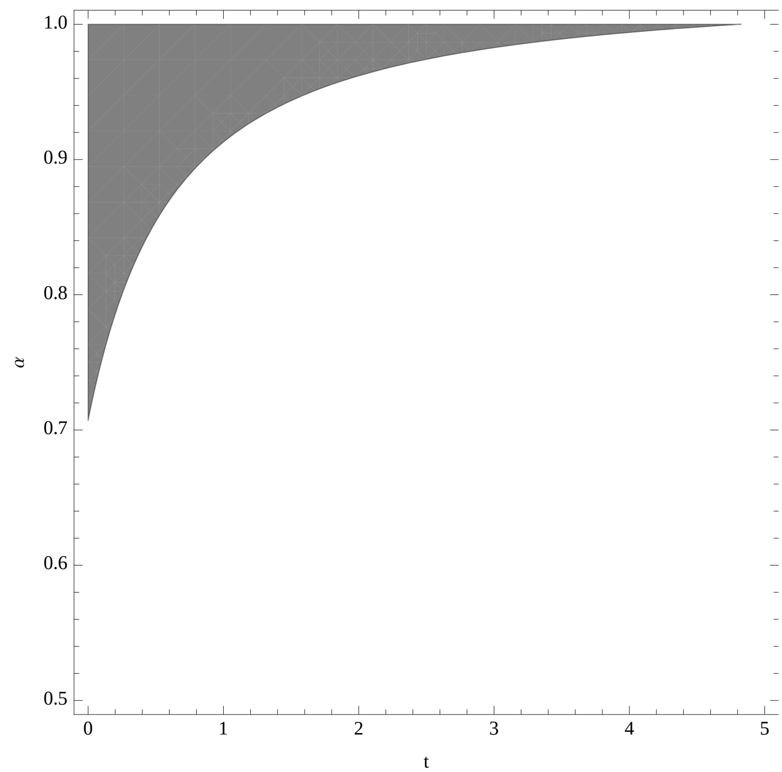


Figure 6.1: Parameter space in which providing service to both consumers is profitable

consumers is greater than a single fixed fee set such that both consumers purchase service in the area of the parameter space pictured in figure 6.1. When $t = 0$, the cutoff value of $\alpha = \frac{\sqrt{2}}{2}$. Because the profit achieved by selling to both consumers is dependent on $\alpha$ but the profit achieved by selling to only the high value consumer is not, and because profit is increasing in $\alpha$, there is a point at which the low value consumer does not value content sufficiently to for the monopolist to be willing to sell him service. Profit is also decreasing in $t$ for both cases, since the consumers' marginal value of content diminishes as congestion increases, an effect that is stronger when both consumers are induced to purchase service since both are using the network. Thus, as $t$ increases, the value of $\alpha$ at which the monopolist wishes to sell to both consumers increases as well.

## 6.2 Restricted Nonlinear Pricing

Consider next a restricted version of the nonlinear pricing situation, in which the monopolist offers consumers a choice between a fixed fee bundled with unlimited network access or a marginal price of access. Observation and intuition support that high value consumers will select the fixed fee and low value consumers will select a marginal price, so the following section will assume that to be the case, adopting a pricing structure that imposes the constraints $A_L = 0$ and $p_H = 0$.

This restricted case allows an examination of how consumers might react when offered access to two different pricing structures. It is particularly interesting because it closely mirrors the existing situation observed in cell phone plans. One class of plans offers unlimited service for a fixed fee each month (the pricing bundle intended for the high value consumer) while pay-as-you-go phones allow the user to pay a per-minute cost of "content" (the pricing bundle intended for the low value consumer). Under this scheme the monopolist intends for the high value consumer to choose the fixed fee pricing bundle and the low value consumer to choose the marginal price, which is represented by the following utility expressions.

$$\begin{aligned} U_H &= q_H - q_H^2 - A - tq_H(q_H + q_L) \\ U_L &= \alpha q_L - q_L^2 - pq_L - tq_L(q_H + q_L) \end{aligned} \tag{6.4}$$

Maximizing utility by a strategic interaction between consumers yields a Nash equilibrium system of quantities demanded

$$q_H^* = \frac{2(1+t) - t(\alpha - p)}{(2+t)(2+3t)} \qquad q_L^* = \frac{2(1+t)(\alpha - p) - t}{(2+t)(2+3t)} \tag{6.5}$$

Note that $q_H$ is in fact increasing in $p$ in this case. This occurs because when the high value consumer chooses to purchase the fixed fee pricing bundle, he is only affected by the price of content through the effect of price on the low value consumer's choice of quantity demanded, i.e. the amount of congestion generated by the low value consumer. Thus, the higher the marginal price, the lower the amount of content the low value consumer will demand and the lower the congestion cost to the high value consumer. Because the high value consumer pays only a fixed fee in order to access unlimited content, the increase in price generates a decrease in congestion without generating an increase in the cost of content to the high value consumer, leading to an increase in the demand for content from the high value consumer.

Using these quantities yields the first set of constraints imposed on the monopolist's profit maximization problem, namely the constraints that ensure that consumers will gain some non-negative utility from purchasing the bundle intended for their consumption.

$$U_H = (1+t)[q_H^*]^2 - A \geq 0 \qquad U_L = (1+t)[q_L^*]^2 \geq 0 \tag{6.6}$$

In order to ensure that consumers choose the bundle intended for them instead of the other bundle, the monopolist will maximize profit subject to the constraints that consumers derive more utility from behaving as the monopolist intends for them to than they would from deviating and selecting the other pricing bundle. If one consumer deviates, the two consumers reach a new Nash equilibrium based on their new incentives. Thus, if the high value consumer deviates, both consumers are then consuming at the same marginal price, generating the demand functions from the single price monopoly case. If the low value consumer deviates, both consumers are consuming at the same fixed fee bundle, generating the demand functions observed in the previous section.

This leads to the following incentive compatibility constraints

$$
\begin{aligned}
(1+t)\left(\frac{2(1+t)-t(\alpha-p)}{(2+t)(2+3t)}\right)^2 - A &\geq \frac{(1+t)[2(1+t)(1-p)-t(\alpha-p)]^2}{(2+t)^2(2+3t)^2} \\
(1+t)\left(\frac{2(1+t)(\alpha-p)-t}{(2+t)(2+3t)}\right)^2 &\geq \frac{(1+t)[2\alpha(1+t)-t]^2}{(2+t)^2(2+3t)^2} - A
\end{aligned}
\tag{6.7}
$$

Simplifying and combining these constraints places bounds on the possible values of $A$ when the monopolist is restricted to setting the pricing scheme such that both consumers purchase service, and purchase the service plan intended for them

$$
\frac{4p(1+t)[(1+t)(2-p)-t(\alpha-p)]}{(2+t)^2(2+3t)^2} \geq A \geq \frac{4p(1+t)[(1+t)(2\alpha-p)-t]}{(2+t)^2(2+3t)^2} \tag{6.8}
$$

Again, there is a limited region of the parameter space in which the monopolist attains the highest profit by selling to both consumer groups, pictured in figure 6.2. Otherwise, the monopolist is best off setting $A = \frac{1}{4(1+t)}$ and setting $p$ sufficiently high (consider $p = \infty$) such that neither consumer is willing to purchase service under marginal pricing. This awards a *de facto* priority right to the high demand consumer and results in profit of $\Pi = \frac{1}{4(1+t)}$ and total congestion of $Q = \frac{1}{2(1+t)}$. It is interesting to note that the parameter space in which the monopolist gains by selling service to both consumers is more restricted under this pricing scheme than it is in the previous, single fixed fee case. This occurs because of the particular pricing structure adopted in this section. When the monopolist is bound to charge a marginal price to the low value consumer, charging a higher price simultaneously and directly reduces the amount of surplus the monopolist may capture from the low value consumer, whereas the same effect does not take place when only a fixed fee is charged. Thus, the surplus the monopolist may extract from the low value consumer by use of a marginal price is more limited, restricting the parameter space in which it is profitable more severely than in the case where a fixed fee is assessed. This suggests that there are reasons other than

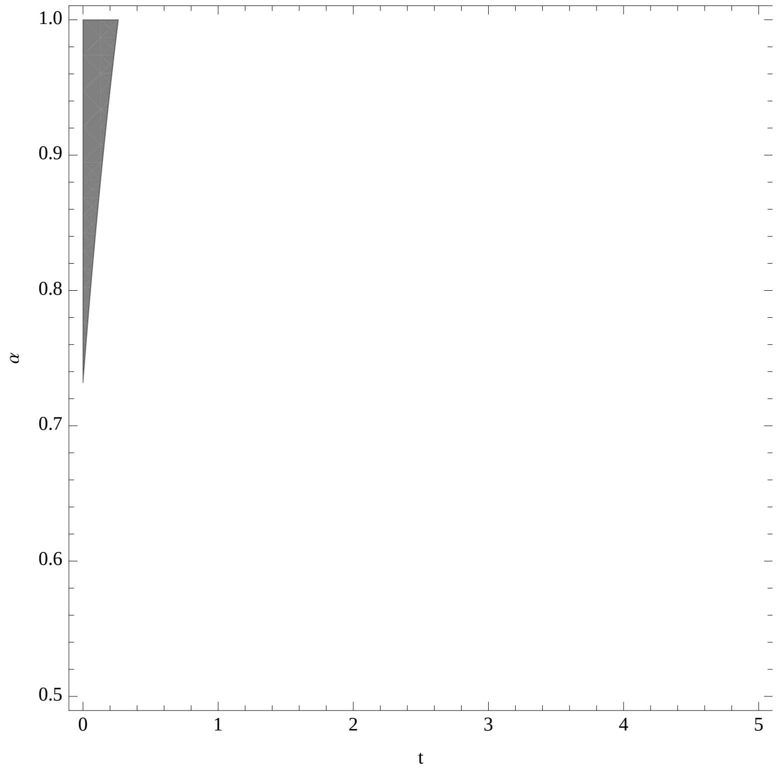


Figure 6.2: Parameter space in which providing service to both consumers is profitable

profit maximization for the existence of different types of cellular service plans as discussed in the beginning of the section (i.e. regulatory constraints).

## 6.3 Fully Nonlinear Pricing

Consider finally the case of fully nonlinear pricing, where the monopolist selects two different price bundles, $(p_H, A_H)$ and $(p_L, A_L)$, between which the consumers must decide. The Nash equilibrium implied when the two consumers strategically interact as intended to determine demand curves is as described in Chapter 3.

$$q_H^* = \frac{2(1+t)(1-p_H) - t(\alpha - p_L)}{(2+3t)(2+t)}$$
$$q_L^* = \frac{2(1+t)(\alpha - p_L) - t(1-p_H)}{(2+3t)(2+t)} \tag{6.9}$$

The problem is bound by several constraints that are by now familiar. There are two individual rationality constraints that bind the monopolist to price such that the two consumers are willing to purchase at their intended pricing bundles.

$$U_H = (1+t)[q_H^*]^2 - A_H \geq 0 \qquad U_L = (1+t)[q_L^*]^2 - A_L \geq 0 \tag{6.10}$$

There are also two incentive compatibility constraints binding each consumer to choose the bundle that is intended for him

$$(1+t)[q_H^*]^2 - A_H \geq \frac{(1+t)[(2-\alpha)t - (2+t)p_L + 2]^2}{(2+t)(2+3t)} - A_L$$
$$(1+t)[q_L^*]^2 - A_L \geq \frac{(1+t)[t + (2+t)p_H - 2\alpha(1+t)]^2}{(2+t)(2+3t)} - A_H \tag{6.11}$$

Additionally, there are two final constraints that bind the monopolist to price such that quantities consumers demand is positive. Given these constraints, four separate sections of the parameter space are implied in each of which a different combination of constraints is binding. The sectioning of the parameter space is pictured in figure 6.3. The shaded regions represent regions in which it is more profitable for the monopolist to sell service to both consumers than it is for the monopolist to sell service to only the high value consumer. In the darkest region, classic second degree price discrimination result is replicated, as the low value individual rationality constraint and the high value incentive compatibility constraint are binding. In the middle gray region, the two incentive compatibility constraints as well as the low value individual rationality constraint are binding, leading to the monopolist offering

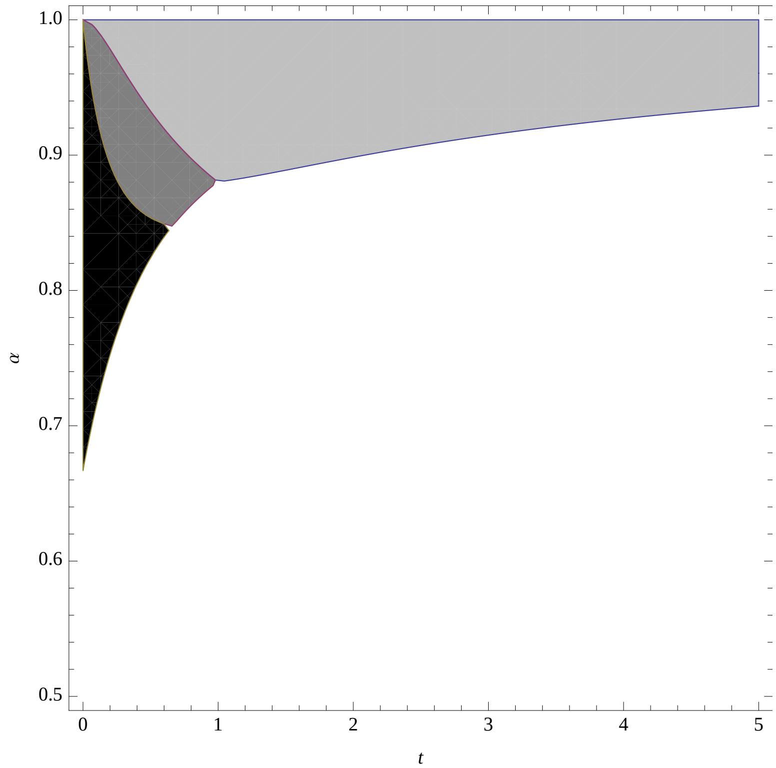


Figure 6.3: Maximum profit parameter space sections

only one pricing bundle to both consumers. In the light gray region, the two individual rationality constraints and the high value incentive compatibility constraint are binding, generating the intriguing result that the monopolist captures all surplus of both consumers with that particular pricing structure.

Generally, when $\alpha$ is high and congestion ($t$) is low, it is profitable to price such that both consumers purchase service. As $\alpha$ decreases, the quantity demanded by the low value consumer decreases, and the profit added by the low value consumer fails to exceed the profit that can be achieved by capturing the surplus generated when the high value consumer does not suffer from congestion. As $t$ increases, each unit of content demanded generates a stronger negative effect on the utility of the high value consumer, resulting in a higher value of $\alpha$ being necessary to sustain the result. The result that the monopolist will price to only the high value consumer in much of the parameter space is the result of two forces: the low value consumer not valuing content enough for it to be profitable to serve him, and the low value consumer hindering the ability of the monopolist to extract surplus from the high value consumer.

For a benchmark case, and to further explore the first of the two effects creating the boundary pictured in figure 6.3, consider the case when $t = 0$, when neither consumer is affected by congestion. In this case the consumers will have demand curves of

$$q_H^* = \frac{1 - p_H}{2} \qquad q_L^* = \frac{\alpha - p_L}{2} \tag{6.12}$$

The case of $t = 0$ falls into the region of the parameter space in which the low value individual rationality constraint and the high value incentive compatibility constraint are binding, which leads to the conclusion that the monopolist will set pricing bundles of

$$(p_H, A_H) = \left(0, \frac{3\alpha^2 - 4\alpha + 2}{4}\right) \qquad (p_L, A_L) = \left(1 - \alpha, \frac{(2\alpha - 1)^2}{4}\right) \tag{6.13}$$

The resulting profit and total congestion when $t = 0$ are

$$\Pi = \frac{3\alpha^2 - 2\alpha + 1}{4} \qquad Q = \alpha \tag{6.14}$$

It is clear that profit here is greater than the benchmark profit of $\Pi = \frac{1}{4}$ when $\alpha > \frac{2}{3}$, which gives confidence to the parameter regions outlined in figure 6.3. Note that in particular, $p_H = 0$, which mirrors the result from traditional second degree price discrimination that there is no distortion for the higher value consumer; all of the revenue generated from that consumer is generated through a fixed fee (Tirole 1989). This result does not persist when the monopolist offers service to both consumers. In fact, for some regions of the parameter space, the marginal price charged to the high value consumer is very high, generating a

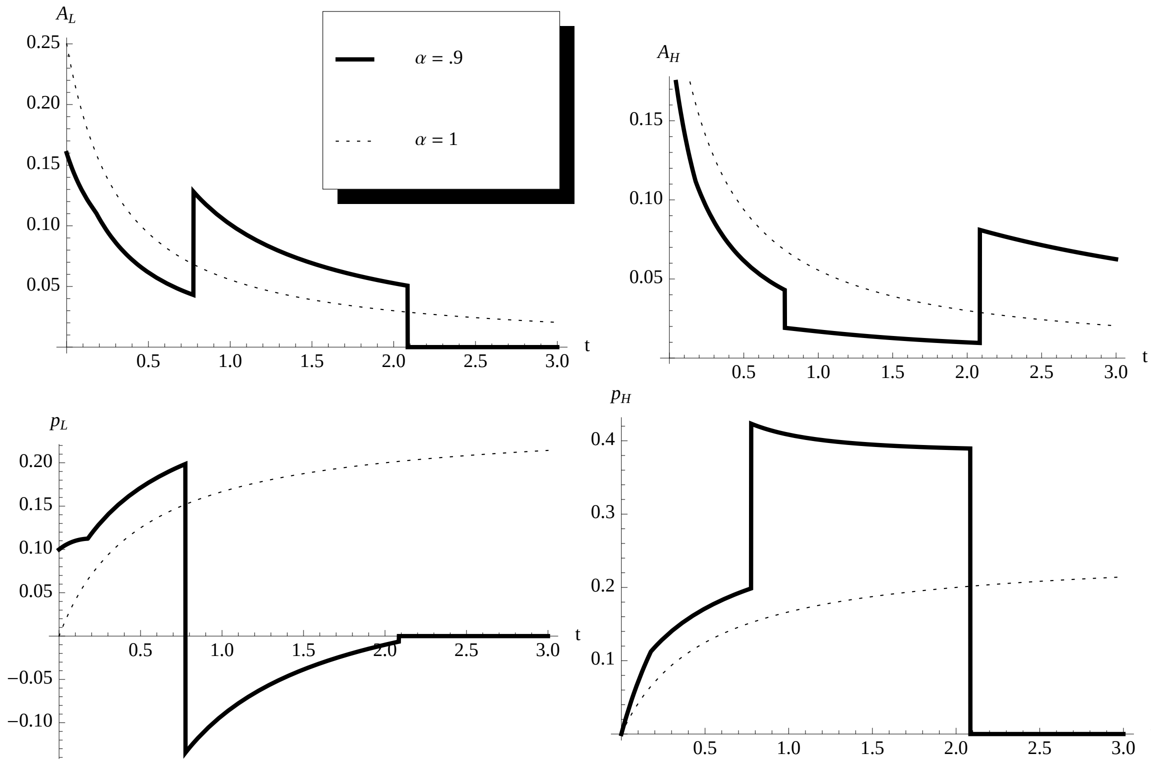


Figure 6.4: Prices and fixed fees as $t$ changes

substantial distortion.

When $t > 0$, several interesting results emerge when prices and fixed fees are examined across each of the four sections of the parameter space. Setting $\alpha = .9$ allows for a comparison of all four parameter regions because $\alpha = .9$ crosses all four regions of the parameter space. This can be seen in figure 6.3. Each discontinuity in prices in figure 6.4 represents a shift from one area of the parameter space to another as $t$ changes. The dotted line represents the fixed fees and prices when $\alpha = 1$, or the fixed fees and prices charged to two identical consumers subject to congestion.

It is interesting to note that in the second section of the parameter space, the middle gray region in figure 6.3 and the space approximately where $t \in [.2, .8]$ in figure 6.4, the two pricing bundles are the same, that is, one bundle with a fixed fee and marginal cost is offered

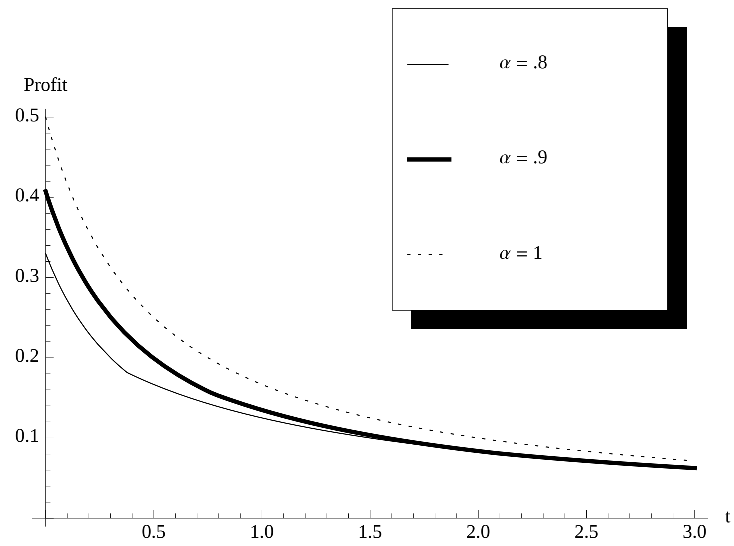


Figure 6.5: Fully nonlinear profit as $t$ changes

to both consumers. Although the two consumers are offered the same bundle, that bundle is different from the one that would be offered if both consumers were the same. Marginal prices are higher and the total fixed fee charged is lower than in the identical consumer case. This occurs because the presence of the $\alpha$ parameter in the utility of the low value consumer drives down the total fixed fee the monopolist may collect for a given price and quantity combination. Thus, to compensate the monopolist charges a slightly higher marginal price, trading the additional distortion that is created for a greater total profit.

For the $\alpha = .9$ case pictured in figure 6.4, at approximately $t = .8$ the market transitions into the third section of the parameter space, the light gray region in figure 6.3. In this case, the low value marginal price is in fact negative for some values of $t$ and $\alpha$, indicating that the monopolist subsidizes the consumption of the low value consumer in order to generate and capture a greater amount of surplus. When this is the case, the fixed fee charged to the low value consumer is relatively high. In fact, the fixed fee charged to the low value consumer exceeds the amount paid to the low consumer by the monopolist in per unit

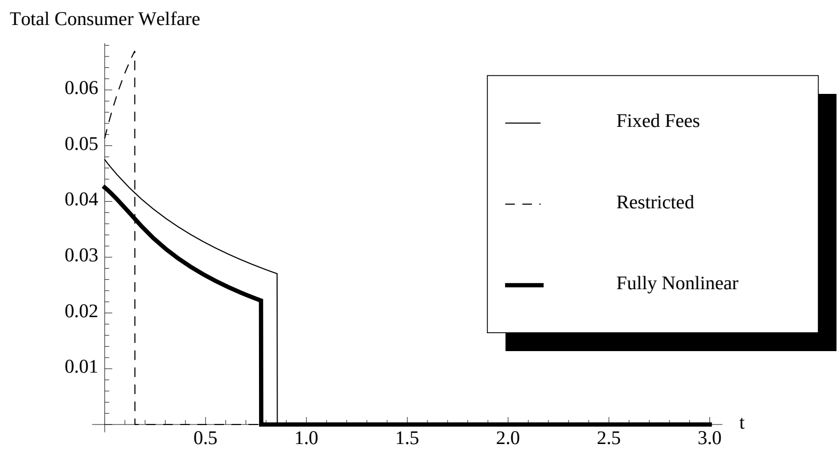


Figure 6.6: Total consumer welfare as $t$ changes; $\alpha = .9$

rebates, generating a net gain for the monopolist despite paying the low value consumer.

From figure 6.5 it is clear that profit is decreasing in $t$ and increasing in $\alpha$ for all pricing specifications. That is, the greater congestion, the less profit the monopolist is able to extract, and the greater the degree of differentiation between consumers, the less surplus is available to extract as profit. It is surprising that profit is relatively smooth, despite being composed of several different pricing bundles across different values of $\alpha$ and $t$. This gives some confidence to the great discontinuities observed in pricing bundles across different sections of the parameter space.

## 6.4 Welfare Analysis

Total consumer welfare pictured in figure 6.6 gives some insight into why consumers may be incensed over a movement towards nonlinear pricing in the market for mobile internet service. Consumer welfare is almost completely captured by the monopolist under some parameter regions of the fully nonlinear pricing scheme, and is completely captured in other parameter regions. Noting the scale of figure 6.5 relative to figure 6.6 provides the observation that monopolist profit is almost an order of magnitude greater than consumer welfare under the fully nonlinear pricing scheme. In most of the parameter space (the white and lightest gray regions in figure 6.3) consumer welfare is completely captured by the monopolist.

# Chapter 7

# Discussion

After examining several different pricing schemes, it is clear that several statements may be made about the general behavior of consumers and the service provider firm under congestion externalities.

**Proposition 1.** *Profit and congestion are maximized under single price monopoly pricing and third degree price discrimination pricing when the low value consumer is offered the priority right for purchase.*

This result may be counter-intuitive, but consider the marginal benefit to each consumer when priority is purchased. Because the high value consumer has a greater demand for content, the benefit to the low value consumer of being freed from suffering from the congestion created by the high value consumer is greater than the symmetric benefit to the high value consumer when he has priority access. Thus, the low value consumer has a greater increase in content consumed, leading to greater profit in any pricing scheme, either through increased quantity of service purchased or through increased utility which the monopolist may extract through a fixed fee.

Congestion is also maximized when the low value consumer purchases priority access, which works through the same mechanism that increases profit, particularly when the pricing scheme involves a marginal price. The monopolist benefits from increased total quantity demanded, and the consumer that generates the increase does not internalize the full negative externality of increasing his demand. Because the monopolist benefits from increased demand, not necessarily from the increased congestion, it seems likely that providing a priority right would provide an incentive to the monopolist to increase network capacity. While it is not within the scope of this model to examine how an endogenous change in network capacity would affect consumer demand, it seems reasonable to assume that increased capacity would increase consumer demand for content, since the negative effects of congestion would be reduced.

The structural parameter $t$ provides the closest approximation in this model to a measure of network capacity, as it represents a scale factor on congestion common to all consumers. To consider how the monopolist might consider a long term decisions to increase network capacity, consider how profit changes with respect to changes in $t$. This leads to the following proposition.

**Proposition 2.** *Profit under any scheme is decreasing in the congestion parameter $t$.*

This includes the profit that is calculated in the cases where the monopolist prices in the face of one consumer or the other having a priority right. Consumers will demand an increasingly large amount of content as $t$ goes to zero. Within the context of internet networks, we can see this sort of demand effect at work as the capacity of networks has been expanded over that past decade. Consider an increase in capacity as a downward shock to the value of $t$. As network capacity has increased in recent times, consumers have demanded more content in terms of the number of websites visited and in terms of the bandwidth requirements of content downloaded (Clarke 2009). The analysis here indicates that under any of the many possible pricing schemes the service provider would have incentive to expand the capacity of networks as much as possible, subject to the costs that that capacity expansion entails. While capacity does not enter explicitly into this model, the lower the amount of congestion the consumer feels, the more he demands, which translates into greater profit for the monopolist. A useful extension of this model might be one in which the congestion parameter $t$ is a function of network capacity.

**Proposition 3.** *Profit is maximized under fully nonlinear pricing.*

This result is clear from the discussion in previous sections because profit is greater when fixed fees are allowed than when they are not, and the fully nonlinear case is the most general and most profitable case in which fixed fees are permitted. This result is explained by the observation that the more flexible the monopolist is, the more effectively he may extract surplus from consumers and the more profitable he will be. In particular, the set of pricing strategies in which the monopolist may charge a fixed fee are substantially more profitable than the pricing strategies in which he may only charge a marginal cost to congestion, because a fixed fee does not distort the amount of content the consumer will demand, resulting in higher quantities demanded and higher total surplus that may be captured. This suggests a reason for the market observation that many service plans for internet and cellular service are based on a flat fee per month, with either unlimited or constrained consumption. This analysis suggests that service providers are able to extract more profit by applying fixed fees than they would be if they charged marginal prices for their content. In particular, in the benchmark case of $t = 0$ under fully nonlinear pricing, the result arises that the optimal marginal price to the high value consumer is in fact 0, further suggesting that it is optimal for the firm to charge a low marginal cost of content, allowing consumer created congestion to regulate the amount of content requested in the network.

When $\alpha$ is relatively low or $t$ is relatively high, however, it is more profitable for the monopolist to price such that the low value consumer does not purchase service at all, awarding a *de facto* priority right to the high value consumer. In this case, the marginal congestion cost to the high value consumer created by the low value consumer is greater than the marginal value generated by the high value consumer demanding more content. This result is also generated in several other pricing schemes, particularly when the monopolist cannot distinguish between the two consumers, when there is no priority available or when the priority is sold to the high value consumer.

Although the nonlinear pricing bundles were calculated without the convention of priority imposed upon them, the analysis in section 4.3 suggests that were a priority to be auctioned off to consumers, the priority auction may be won by the low value consumer in the regions of the parameter space where he would not be served in the no priority or high value priority situations. In this way, offering a priority right for auction may induce the monopolist to offer service to both consumers in a greater region of the parameter space than would be possible without the option to offer priority.

# Chapter 8

# Conclusion

When a service provider is allowed to offer priority access to networks, consumer is improved even as congestion is increased. Profit is greatest when the monopolist can sell priority to a low demand consumer specifically, or when he can flexibly set prices to include a fixed fee for access, either exclusive of or in addition to a marginal price. The analysis of willingness to pay for priority in the single price monopoly suggests that introducing priority along with flexible pricing schemes such as fully nonlinear pricing may induce the monopolist to serve a greater span of the parameter space, which may be welfare improving. In light of these results, relaxing net neutrality may in fact lead to welfare and service improvements for all consumers of network connectivity services by ensuring that it is profitable for the monopolist to serve all consumers, rather than just those with a high value of content.

# Appendix A

# Proofs of Propositions

For each of the following proofs, let a subscript N denote profit or congestion under a no priority scheme, H denote profit or congestion under a scheme where the high value consumer has priority and L denote profit or congestion under a scheme where the low value consumer has priority.

## A.1 Lemma 1

*Under single price monopoly pricing, profit and congestion are maximized when the low value consumer is awarded the priority right.*

*Proof.* Recall that profit under each pricing scheme is

$$\Pi_N = \begin{cases} \frac{1}{8(1+t)} & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{(1+\alpha)^2}{8(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases}$$

$$\Pi_H = \begin{cases} \frac{1}{8(1+t)} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{[1+(1+t)(1+2\alpha)]^2}{16(4+3t)(1+t)^2} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \qquad \Pi_L = \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} \tag{A.1}$$

Let $\Pi^*$ denote the low value of a piecewise defined function, then

$$\begin{aligned}\Pi_L - \Pi_H^* =& \Pi_L - \Pi_N^* \\ =& \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} - \frac{1}{8(1+t)} \\ =& \left(\frac{1}{8(1+t)}\right)\left(\frac{[2(1+\alpha)+t(2+\alpha)]^2}{2(4+3t)(1+t)} - 1\right) \\ =& \left(\frac{2+t}{8(1+t)}\right)\left(\frac{2\alpha^2+2(2\alpha-1)+t\alpha^2+2t(2\alpha-1)}{2(4+3t)(1+t)}\right) \\ =& \left(\frac{2+t}{8(1+t)}\right)\left(\frac{\alpha^2(2+t)+2(1+t)(2\alpha-1)}{2(4+3t)(1+t)}\right)\end{aligned} \tag{A.2}$$

Because $\alpha > \frac{1}{2}$ and $t > 0$, all terms in the numerator of the fraction are positive, thus $\Pi_L - \Pi_H^* = \Pi_L - \Pi_N^* > 0$. Now let $\Pi^{**}$ denote the high value of a piecewise defined function

$$\begin{aligned}\Pi_L - \Pi_N^{**} =& \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} - \frac{(1+\alpha)^2}{8(2+3t)} \\ =& \frac{4(1+t)(1+\alpha)+(2+6t+3t^2)(1-\alpha^2)+t(2+3t)}{16(2+3t)(4+3t)(1+t)^2} > 0\end{aligned} \tag{A.3}$$

Thus $\Pi_L > \Pi_N$. Finally consider

$$\begin{aligned}\Pi_L - \Pi_H^{**} =& \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} - \frac{[1+(1+t)(1+2\alpha)]^2}{16(4+3t)(1+t)^2} \\ =& \frac{[t(1-\alpha)][4+4\alpha+3t+3t\alpha]}{16(4+3t)(1+t)^2} \\ =& \frac{t(1-\alpha)(4+3t)(1+\alpha)}{16(4+3t)(1+t)^2} \\ =& \frac{t(1-\alpha^2)}{16(1+t)^2} > 0\end{aligned} \tag{A.4}$$

when $\alpha > \frac{1}{2}$ and $t > 0$, so $\Pi_L > \Pi_H$.

Now recall that total congestion under each pricing scheme is

$$Q_N = \begin{cases} \frac{1}{4(1+t)} & \text{when } \frac{1+2t}{1+t} > \alpha(2+\alpha) \\ \frac{1+\alpha}{2(2+3t)} & \text{when } \frac{1+2t}{1+t} \leq \alpha(2+\alpha) \end{cases}$$
$$Q_H = \begin{cases} \frac{1}{4(1+t)} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{2+t+2\alpha(1+t)}{8(1+t)^2} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \qquad Q_L = \frac{2(1+\alpha)+t(2+\alpha)}{8(1+t)^2} \tag{A.5}$$

Following the same procedure as above, we have

$$\begin{aligned} Q_L - Q_N^* =& Q_L - Q_H^* \\ =& \frac{2(1+t)+2\alpha+t\alpha}{8(1+t)^2} - \frac{1}{4(1+t)} \\ =& \frac{\alpha(2+t)}{8(1+t)^2} > 0 \end{aligned} \tag{A.6}$$

for $t > 0$. Then

$$\begin{aligned} Q_L - Q_N^{**} =& \frac{2(1+t)+2\alpha+t\alpha}{8(1+t)^2} - \frac{1+\alpha}{2(2+3t)} \\ =& \frac{2(1+t)(2+3t)+\alpha(2+t)(2+3t)-4(1+\alpha)(1+t)^2}{8(2+3t)(1+t)^2} \\ =& \frac{2t+t^2(2-\alpha)}{8(2+3t)(1+t)^2} > 0 \end{aligned} \tag{A.7}$$

and

$$\begin{aligned} Q_L - Q_H^{**} =& \frac{2(1+t)+2\alpha+t\alpha-2-t-2\alpha(1+t)}{8(1+t)^2} \\ =& \frac{2(1-\alpha)(1+t)-(2+t)(1-\alpha)}{8(1+t)^2} \\ =& \frac{t(1-\alpha)}{8(1+t)^2} > 0 \end{aligned} \tag{A.8}$$

Thus, $Q_L > Q_H$ and $Q_L > Q_N$ as required. □

## A.2 Lemma 2

*The high value consumer has higher willingness to pay for priority when both consumers are served.*

*Proof.* The willingness to pay of the high value consumer is

$$W_H = \begin{cases} \frac{\left(t^2(7\alpha-4)+4t(3\alpha-2)-4(1-\alpha)\right)\left(t^2(16-7\alpha)+12t(3-\alpha)+4(5-\alpha)\right)}{64(1+t)^3(4+3t)^2} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{t\alpha\left(t^2(20-11\alpha)+4t(11-5\alpha)+8(3-\alpha)\right)}{64(1+t)^3(4+3t)} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \quad \text{(A.9)}$$

while the willingness to pay of the low value consumer is

$$W_L = \begin{cases} \frac{[2(3\alpha-1)+t(5\alpha-2)]^2}{16(1+t)(4+3t)^2} & \text{when } \alpha < \frac{4+12t+7t^2}{2(6+11t+5t^2)} \\ \frac{t\left(t^2(20\alpha-11)+4t(11\alpha-5)+8(3\alpha-1)\right)}{64(1+t)^3(4+3t)} & \text{when } \alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)} \end{cases} \quad \text{(A.10)}$$

Both consumers are served when $\alpha \geq \frac{4+12t+7t^2}{2(6+11t+5t^2)}$, so it is necessary only to check that the high value willingness to pay is greater than the low value willingness to pay in that case only. Thus,

$$\begin{aligned} W_H - W_L =& \frac{t\left(t^2(11-11\alpha^2)+4t(5-5\alpha^2)+8(1-\alpha^2)\right)}{64(1+t)^3(4+3t)} \\ =& \frac{t(1-\alpha^2)(11t^2+20t+8)}{64(1+t)^3(4+3t)} \geq 0 \end{aligned} \quad \text{(A.11)}$$

since $t > 0$ and $\alpha < 1$, as required. □

## A.3 Lemma 3

*Under third degree price discrimination, profit and congestion are maximized when either consumer purchases a priority right.*

*Proof.* Recall that

$$\Pi_N = \frac{(1+t)(1+\alpha^2) - t\alpha}{2(2+t)(2+3t)} \qquad \Pi_H = \Pi_L = \frac{2(1+t)(1+\alpha^2) - t\alpha}{(4+3t)(4+5t)} \tag{A.12}$$

Then

$$\begin{aligned} \Pi_L - \Pi_N =& \Pi_H - \Pi_N \\ =& \frac{2(1+t)(1+\alpha^2) - t\alpha}{(4+3t)(4+5t)} - \frac{(1+t)(1+\alpha^2) - t\alpha}{2(2+t)(2+3t)} \\ =& \frac{-3t^2(1+t)(1+\alpha^2) + t\alpha(9t^2+16t+8)}{2(2+t)(2+3t)(4+3t)(4+5t)} \\ =& \frac{3t(1+t)(\alpha(3-\alpha)-1) + \alpha(7t+8)}{2(2+t)(2+3t)(4+3t)(4+5t)} > 0 \end{aligned} \tag{A.13}$$

because $\alpha(3-\alpha) > 1$ for all $\alpha \in (\frac{1}{2}, 1)$. Thus, $\Pi_L = \Pi_H > \Pi_N$ as required. Next recall that

$$Q_N = \frac{1+\alpha}{2(2+3t)} \qquad Q_L = Q_H = \frac{1+\alpha}{4+5t} \tag{A.14}$$

Then

$$\begin{aligned} Q_H - Q_N =& Q_L - Q_N \\ =& \frac{1+\alpha}{4+5t} - \frac{1+\alpha}{2(2+3t)} \\ =& \frac{t(1+\alpha)}{2(2+3t)(4+5t)} > 0 \end{aligned} \tag{A.15}$$

Thus $Q_H = Q_L > Q_N$ as required. □

## A.4 Proposition 1

*Profit and congestion are maximized under single price monopoly pricing and third degree price discrimination pricing when the low value consumer is offered the priority right for purchase.*

*Proof.* By Lemmas 1 and 3. □

## A.5 Proposition 2

*Profit under any scheme is decreasing in the congestion parameter t.*

*Proof.* Profit from all cases in which the monopolist sells only to the high value consumer, as well as the single price monopoly case when neither consumer has priority, is of the form $\Pi = \frac{C}{(1+t)}$ where $C$ is a positive constant with respect to $t$. Then the first derivative with respect to $t$ is

$$\frac{\partial \Pi}{\partial t} = \frac{-C}{(1+t)^2} < 0 \tag{A.16}$$

thus profit in all cases in which only the high value consumer purchases service is decreasing in $t$. Turning attention to remaining cases, first consider the profit in each case of single price monopoly. Profit when both consumers purchase service and the high value consumer has priority is

$$\begin{aligned}\frac{\partial \Pi}{\partial t} =& \frac{\partial}{\partial t}\left(\frac{[1+(1+t)(1+2\alpha)]^2}{16(4+3t)(1+t)^2}\right) \\ =&\Pi\left(\frac{-3t^2(1+2\alpha)-2(7+3\alpha)-3t(5+4\alpha)}{(1+(1+t)(1+2\alpha))(4+3t)(1+t)}\right) \leq 0\end{aligned} \tag{A.17}$$

When the low value consumer has priority, the equivalent calculation is

$$\begin{aligned}\frac{\partial \Pi}{\partial t} =& \frac{\partial}{\partial t}\left(\frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2}\right)\\ =&\Pi\left(\frac{-3t^2(2+\alpha)-3t(4+5\alpha)-2(3+7\alpha)}{16(4+3t)(1+t)^2}\right)\leq 0\end{aligned} \tag{A.18}$$

as required. Next, the profit achieved under third degree price discrimination when no priority is awarded is as follows

$$\begin{aligned}\frac{\partial \Pi}{\partial t} =& \frac{\partial}{\partial t}\left(\frac{(1+t)(1+\alpha^2)-t\alpha}{2(2+t)(2+3t)}\right)\\ =&\Pi\left(\frac{-6t(1+\alpha^2)-3t^2(1-\alpha+\alpha^2)-4(1+\alpha+\alpha^2)}{(2+t)(2+3t)(1+\alpha^2+t(1-\alpha+\alpha^2))}\right)\leq 0\end{aligned} \tag{A.19}$$

while profit achieved in either case of priority being awarded under third degree price discrimination is

$$\begin{aligned}\frac{\partial \Pi}{\partial t} =& \frac{\partial}{\partial t}\left(\frac{2(1+t)(1+\alpha^2)-t\alpha}{(4+3t)(4+5t)}\right)\\ =&\Pi\left(\frac{-t^2(6-3\alpha+6\alpha^2)-12t(1+\alpha^2)-2(3+2\alpha+3\alpha^2)}{(1+t)(4+3t)(2(1+\alpha^2)+t(2-\alpha+2\alpha^2))}\right)\leq 0\end{aligned} \tag{A.20}$$

as required. In the case of profit achieved under nonlinear pricing as described in Chapter 6, refer to figure 6.5, which clearly shows that profit under nonlinear pricing schemes is decreasing in $t$ under each of the sections of the parameter space in which there is a different pricing bundle, as required. □

## A.6 Proposition 3

*Profit is maximized under fully nonlinear pricing*

*Proof.* Let $\Pi_{NL}$ be the profit achieved under fully nonlinear pricing. $\Pi_{NL}$ is greater than or equal to any other profit in Chapter 6, because they are each special cases of fully nonlinear pricing and could be achieved if they were optimal. It is also the case that $\Pi_{NL} \geq \frac{1}{4(1+t)}$. The highest profit in the single price monopoly case is

$$\Pi_{SP} = \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} \tag{A.21}$$

Then

$$\begin{aligned}
\Pi_{NL} - \Pi_{SP} \geq & \frac{1}{4(1+t)} - \Pi_{SP} \\
= & \frac{1}{4(1+t)} - \frac{[2(1+\alpha)+t(2+\alpha)]^2}{16(4+3t)(1+t)^2} \\
= & \frac{4(3-2\alpha-\alpha^2)+4t(5-3\alpha-\alpha^2)+t^2(8-4\alpha-\alpha^2)}{16(1+t)^2(4+3t)} \geq 0
\end{aligned} \tag{A.22}$$

since $3 > 2\alpha+\alpha^2$, $5 > 3\alpha+\alpha^2$ and $8 > 4\alpha+\alpha^2$ for all $\alpha \in [\frac{1}{2}, 1]$, so it is the case that $\Pi_{NL} \geq \frac{1}{4(1+t)} \geq \Pi_{SP}$. Next examine the profit achieved under third degree price discrimination, $\Pi_{TD}$.

$$\begin{aligned}
\Pi_{NL} - \Pi_{TD} \geq & \frac{1}{4(1+t)} - \Pi_{TD} \\
= & \frac{1}{4(1+t)} - \frac{2(1+t)(1+\alpha^2)-t\alpha}{(4+3t)(4+5t)} \\
= & \frac{8(1-\alpha^2)+16t(1-\alpha^2)+4t\alpha+7t^2+4\alpha t^2(1-2\alpha)}{4(1+t)(4+3t)(4+5t)} \\
= & \frac{8(1+2t)(1-\alpha^2)+7t^2+4\alpha t^2(1-2\alpha)}{4(1+t)(4+3t)(4+5t)} \geq 0
\end{aligned} \tag{A.23}$$

since all terms are greater than zero. Thus $\Pi_{NL} \geq \frac{1}{4(1+t)} \geq \Pi_{TD}$, as required. □

# Appendix B

# Pricing Bundles from Chapter 6

## B.1 Bundle from Section 6.2

In the section where both consumers are served, the fixed fee set for the high value consumer is

$$\begin{aligned} A =& ((3t^3(2\alpha - 1) + 18t^2\alpha + 4t(3 + 5\alpha) + 8(1 + \alpha)) \\ & (12t^4(2 - \alpha) + 5t^3(23 - 10\alpha) + t^2(200 - 74\alpha) \\ & + 4t(37 - 11\alpha) + 8(5 - \alpha))/(4(1 + t)(2 + t)^2(2 + 3t)^2(6 + 8t + 3t^2)^2) \end{aligned} \tag{B.1}$$

while the marginal price set for the low value consumer is

$$p = \frac{3t^3(2\alpha - 1) + 18t^2\alpha + 4t(3 + 5\alpha) + 8(1 + \alpha)}{4(1 + t)(6 + 8t + 3t^2)} \tag{B.2}$$

## B.2 Bundles from Section 6.3

Recall the sections of the parameter space in which there are different pricing bundles, pictured in figure B.1. In the light gray region, the pricing bundle intended for the high value consumer is

$$\begin{aligned} p_H &= \frac{(4+14t+11t^2)(1-\alpha)+2t^2}{4t(1+2t)} \\ A_H &= \frac{(1+t)[4(1-\alpha)+2t(5-7\alpha)+t^2(7-9\alpha)]^2}{16t^2(2+t)^2(1+2t)^2} \end{aligned} \tag{B.3}$$

and the pricing bundle intended for the low value consumer is

$$\begin{aligned} p_L &= \frac{-(4+14t+11t^2)(1-\alpha)+2\alpha t^2}{4t(1+2t)} \\ A_L &= \frac{(1+t)[4(1-\alpha)+2t(7-5\alpha)+t^2(9-7\alpha)]^2}{16t^2(2+t)^2(1+2t)^2} \end{aligned} \tag{B.4}$$

In the medium gray area, the pricing bundle intended for the high value consumer and the

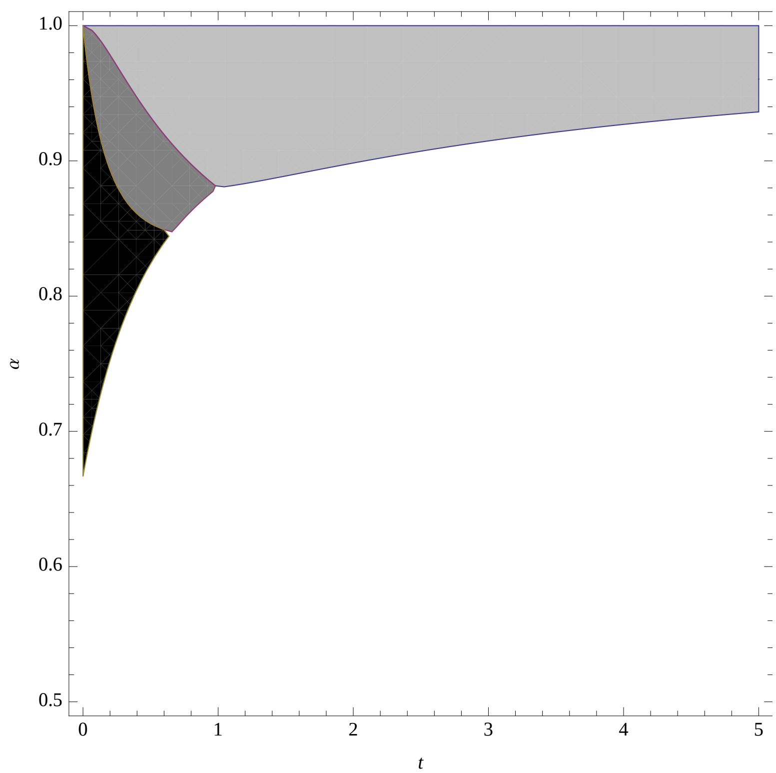


Figure B.1: Maximum profit parameter space sections pricing bundle intended for the low value consumer are the same

$$\begin{aligned} p_H = p_L &= \frac{4(1-\alpha)+4t(3-2\alpha)+t^2(7-5\alpha)}{4(1+2t)(2+t)} \\ A_H = A_L &= \frac{(1+t)[2(3\alpha-1)+t(7\alpha-5)]^2}{16(2+t)^2(1+2t)^2} \end{aligned} \tag{B.5}$$

In the darkest area, the pricing bundle intended for the high value consumer is

$$\begin{aligned} p_H =& \frac{t\left(8-24\alpha+3t^4(9-8\alpha)+2t^2(55-72\alpha)+t^3(94-96\alpha)+t(52-96\alpha)\right)}{2(2+t)\left(-4-16t-16t^2+4t^3+9t^4\right)} \\ A_H =& ((1+t)(64\left(2-4\alpha+3\alpha^2\right)+128t\left(10-21\alpha+13\alpha^2\right) \\ &+32t^2\left(170-365\alpha+204\alpha^2\right)+t^8\left(45-336\alpha+332\alpha^2\right) \\ &+4t^7\left(351-1044\alpha+751\alpha^2\right)+16t^3\left(795-1730\alpha+931\alpha^2\right) \\ &+16t^5\left(916-2099\alpha+1195\alpha^2\right)+4t^6\left(1691-4176\alpha+2592\alpha^2\right) \\ &+4t^4\left(4421-9788\alpha+5316\alpha^2\right)))/\left(4(2+t)^2\left(4+16t+16t^2-4t^3-9t^4\right)^2\right) \end{aligned} \tag{B.6}$$

while the pricing bundle intended for the low value consumer is

$$\begin{aligned} p_L =& (16(1-\alpha)+96t(1-\alpha)+9t^5(2-3\alpha)+12t^2(18-19\alpha) \\ &+2t^4(55-71\alpha)+t^3(228-262\alpha))/\left(2(2+t)\left(4+16t+16t^2-4t^3-9t^4\right)\right) \\ A_L =& ((1+t)(8-16\alpha+t^4(15-14\alpha)+2t^2(41-52\alpha) \\ &+t^3(62-66\alpha)+t(44-68\alpha))^2)/\left(-4(2+t)^2\left(4+16t+16t^2-4t^3-9t^4\right)^2\right) \end{aligned} \tag{B.7}$$

Finally, in the white area, the monopolist prices such that only the high value consumer demands content. In this case all of the monopolist's profit is captured in a single fixed fee. The pricing bundle intended for the low value consumer is set such that the low value consumer is not induced to purchase service at all. Thus, it is the case that the pricing bundles are $p_H = 0,\ p_L = \infty$ while $A_H = A_L = \frac{1}{4(1+t)}$.

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