ABSTRACT

While researchers have claimed that using high cognitive demand tasks can foster students’ learning of mathematics, there is little research on teachers’ perspectives on implementing high cognitive demand tasks. In this research study, I elicited teachers’ perspectives on implementing high cognitive demand tasks and compared them to my perspective. I worked with three middle school mathematics teachers on planning and implementing high cognitive demand tasks and observed the implementation of tasks. Using the task implementation framework developed by Stein, Grover, and Henningsen (1996), I outlined factors that affected the implementation of tasks from both my perspective and the teachers’ perspectives. Factors that supported the use of high cognitive demand tasks from the teachers’ perspectives included task conditions, teachers’ instructional dispositions, and students’ learning dispositions. Factors that the teachers claimed were barriers included time, district imposed requirements and curriculum, and teachers’ instructional dispositions. From my perspective, I found time and teachers’ instructional dispositions to both positively and negatively affect the implementation of tasks.
INDEX WORDS: mathematics teacher education, high cognitive demand tasks, professional development,
MIDDLE SCHOOL MATHEMATICS TEACHERS’ PERSPECTIVES ON
IMPLEMENTING HIGH COGNITIVE DEMAND TASKS

by

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MIDDLE SCHOOL TEACHERS’ PERSPECTIVES ON IMPLEMENTING HIGH COGNITIVE DEMAND TASKS

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DEDICATION

I dedicate this dissertation to my mom and dad, Ellen and Jim and my sister and brother, Alexis and Adam, and all of my family and friends who have been there and supported me throughout this journey.
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As I have gone through the process of writing my dissertation, and on my educational journey, many people I need to thank have helped along the way. First, I would like to thank my advisor, Denise Spangler, who guided me through this journey and was the best mentor, supporter, and editor I could have asked for. I would also like to thank my committee members, AnnaMarie Conner and Jori Hall who have helped me grow as a student and a researcher. I would like to thank my professors at the University of Georgia, Appalachian State University, and St. Bonaventure, and all of my teachers who have taught me and helped me develop as an educator. I would also like to thank every one of my students, in North Carolina, New York, and Georgia who have influenced me in ways I cannot express in words. I would like to thank all of my friends and colleagues, especially Jen and Zandra, who have supported me and have been my sounding boards. Lastly, I would like to thank my participants, without whom I would not have been able to complete this dissertation.
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CHAPTER 1
INTRODUCTION

In 1991, the National Council of Teachers of Mathematics (NCTM) produced a document outlining professional standards for mathematics teachers. The first strand of the document urged teachers to pose worthwhile mathematical tasks, saying:

Teachers should choose and develop tasks that are likely to promote the development of students’ understandings of concepts and procedures in a way that also fosters their ability to solve problems and to reason and communicate mathematically. Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills that capture students’ curiosity and that invite them to speculate and to pursue their hunches. Many such tasks can be approached in more than one interesting and legitimate way; some have more than one reasonable solution (NCTM, 1991, Para 1).

Since 1991, reform-based standards documents containing an emphasis on not only mathematics content but also the processes by which children should engage in mathematics have been released. In 2001 two documents were released that provided guidelines to help students be successful in mathematics. The National Council of Teachers of Mathematics (NCTM) released the Principles and Standards of School Mathematics that included both content standards as well as process standards. The NCTM (2001) stated “the mathematical Content and Process Standards … are
inextricably linked. One cannot solve problems without understanding and using mathematical content” (Para 15). The NCTM process standards include problem solving, reasoning and proof, communication, connections, and representation. Students engaged in problem solving should build new mathematical knowledge from the problem, apply and adapt necessary strategies to solve the problem, and monitor and reflect their actions while problem solving. When engaged in reasoning and proof students should come up with and explore mathematical conjectures, develop and evaluate arguments and proofs related to those conjectures and utilize different types of reasoning and proof strategies. The connection standard includes students making connections between mathematical concepts, understanding that mathematics is connected and recognizing where mathematics is applicable in other subject areas. Students should communicate mathematical ideas clearly, analyze and evaluate others’ mathematical thinking, and express mathematical ideas using the language of mathematics. The last standard, representation, includes students’ abilities to create and use different representations to organize, record, communicate, and model mathematical ideas as well as recognizing and moving flexibly between the different mathematical representations. The purpose of the process standards was to outline a way for students to acquire the necessary content standards and build their conceptual understanding of topics.

In 2001 The National Research Council (NRC) released Adding It Up, which included a chapter on mathematical proficiency describing what the authors believed was necessary for all students to have success learning mathematics. Kilpatrick, Swafford and Findell stated there were five strands of mathematical proficiency needed to foster mathematical growth among students. These strands included conceptual understanding,
procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. Conceptual fluency is students’ understanding of mathematical concepts, operations, and relations. Procedural fluency is the ability to perform mathematical procedures. Strategic competence involves formulating, representing and solving mathematical problems. Adaptive reasoning is a students’ ability to reflect, justify, and explain their mathematical ideas. Productive disposition is when a student sees mathematics as useful and worthwhile and believes he/she has the ability to learn mathematics with enough effort. Kilpatrick et al. stress, “The five strands are interwoven and interdependent in the development of proficiency in mathematics” (p. 116). This implies that in order to have mathematical proficiency, students should be given problems or task that foster the development of all five aspects described above.

In 2010 the Common Core State Standards for Mathematics (CCSSM) combined the process standards and strands for mathematical proficiency and created the standards for mathematical practice stating that the standards for mathematical practice “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important ‘processes and proficiencies’ with longstanding importance in mathematics education” (Council of Chief State School Officers & National Governors’ Association, 2010, p. 6). The standards for mathematical practice include making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, attending to precision, looking for and making use of structure, and looking for and expressing
regularity in repeated reasoning. By engaging students in the mathematical practices, teachers can help develop students’ conceptual understanding in mathematics.

One way to address the many processes that these documents describe (which I will refer to as the mathematical processes) is through the implementation of high cognitive demand tasks. Martin (2007) outlined seven standards that represent the core dimensions of teaching and learning mathematics. One of those standards is worthwhile mathematical tasks. Martin explained that teachers should pose tasks that help students develop mathematical understanding, help students make mathematical connections, require problem formulation and problem solving, and help students communicate about mathematics. Therefore, by choosing worthwhile tasks, teachers engage their students in the mathematical processes as well as the mathematical content of the task.

Doyle (1988) explained the difference in cognitive processes of tasks and related it to the information students need to access tasks. He said most tasks in classrooms involve memorization, and students only need to remember what procedure to use to solve the problem correctly. He said tasks with a higher cognitive load require students to engage in complex thinking:

Higher cognitive processes involve decisions about how to use knowledge and skills in particular circumstances to interpret problems and generate answers. A task demanding higher cognitive processes might require students to recognize transformed versions of information or of a formula they have already learned … The focus for tasks involving higher cognitive processes, then, is on comprehension, interpretation, flexible application of knowledge and skills, and
assembly of information from several different sources to accomplish work (p. 170).

By engaging students in higher-order thinking tasks, they apply many of the mathematics processes such as problem solving and communication. Implementing high cognitive demand tasks can provide students opportunities to engage in many of the process standards and not just use procedural knowledge to solve problems.

Researchers elaborated on this idea, suggesting that teachers pay attention to the cognitive demand of tasks (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein, et al., 1996). Cognitive demand refers to the amount of effort a student needs to expend to think about a problem, and mathematical tasks are categorized in two levels, low and high cognitive demand. Smith and Stein (2011) outlined and characterized four different demands of tasks: memorization tasks, procedures without connections tasks, procedures with connections tasks, and doing mathematics tasks. Memorization and procedures without connections are low cognitive demand tasks while procedures with connections and doing mathematics are high cognitive demand tasks. Memorization tasks involve recalling facts or definitions and do not require computation. An example of a memorization task is stating the Pythagorean Theorem. Tasks labeled as procedures without connections involve using a procedure to solve a problem but not connecting it to any other mathematical ideas, such as solving equations for missing variables. Procedures with connections tasks involve using a procedure but connecting it to other mathematical ideas. One such problem is solving a quadratic function, interpreting what the solution means, and relating these values to the graph of the function. A task labeled doing mathematics does not give an explicit way to solve the problem and may include multiple
solution methods, such as figuring out a pattern and coming up with generalized formula.

Procedures with connections and doing mathematics are considered high cognitive demand tasks. See Figure 1 for descriptions of the four levels of demand. Many of the descriptors of tasks with a high level of demand align with the mathematical processes.

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<th>Low-Level Cognitive Demands</th>
<th>High-Level Cognitive Demands</th>
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<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures With Connections Tasks</strong></td>
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<td>- Involve either producing previously learned facts, rules, formulae, or definitions or committing facts, rules, formulae, or definitions to memory.</td>
<td>- Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.</td>
</tr>
<tr>
<td>- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</td>
<td>- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</td>
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<td>- Are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.</td>
<td>- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.</td>
</tr>
<tr>
<td>- Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
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<th>Procedures Without Connections Tasks</th>
<th>Doing Mathematics Tasks</th>
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<td>- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or place-learning of the task.</td>
<td>- Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</td>
</tr>
<tr>
<td>- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>- Require students to explore and to understand the nature of mathematical concepts, processes, or relationships.</td>
</tr>
<tr>
<td>- Have no connection to the concepts or meaning that underlie the procedure being used.</td>
<td>- Demand self-monitoring or self-regulation of one’s own cognitive processes.</td>
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<td>- Are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>- Require students to access relevant knowledge in working through the task.</td>
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<tr>
<td>- Require no explanations or explanations that focus solely on describing the procedure that was used.</td>
<td>- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.</td>
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For example, when students engage in a procedures with connections task, they use both procedural fluency and conceptual understanding from the strands of mathematical proficiency, representation and problem solving from the NCTM process standards, and
use appropriate tools strategically from the CCSM standards of mathematical practice, as well as other mathematical processes. Therefore, when students are provided with high cognitive demand tasks, they not only engage with mathematical content but also engage with mathematical processes to solve the tasks. While the implementation of all four types of tasks should occur in mathematics classroom, it has been my experience that the categorization of the majority of problems required low cognitive demand. Henningsen and Stein (1997) argue teachers should be able to select tasks appropriately and implement tasks at a high level of cognitive demand to support students’ mathematical thinking.

Research also shows that successful mathematics teachers give students mathematical tasks that focus on a conceptual understanding of mathematics. In doing this, teachers engaged their students in conversations about mathematics and did not focus solely on basic facts and processes (e.g., Edmonds, 1979; Gutierrez, 2000; Kitchen, Row, Lee, & Secada, 2009; Staples & Truxaw, 2010). As early as 1979, Edmonds reviewed research on urban schools in New York City, California, and Michigan. In each of the three studies, teachers in higher achieving schools had a task-oriented approach to mathematics. Kitchen et al. found “consistent evidence that teachers at highly effective schools expressed more elaborated concepts of mathematics education than teachers at typical schools” (p. 70). Staples and Truxaw found that teachers in the Mathematics Learning Discourse project focused on higher order thinking and less on routine skills, allowing students to think critically about mathematics, develop better mathematical vocabulary, and provide justification for their answers. Gutierrez found that teachers in a school she deemed as highly effective used a reform-oriented instructional curriculum
that aligned with recommendations made by NCTM (1989). This curriculum had more of an emphasis on cooperative learning, knowledge construction, and real world problems and less emphasis on drilling and rote memorization.

Realizing teachers may not have the time or necessary resources to find high cognitive demand tasks, I wanted to support teachers in finding and planning tasks characterized as having high cognitive demand, specifically doing mathematics. I also wanted to obtain the perspective of the teachers throughout the process to gain an idea of what supports and barriers affected the implementation of high cognitive demand tasks. In this particular case, I worked with three middle school mathematics teachers at Yellow Brick Middle School during the planning stages of implementing high cognitive demand tasks in their seventh grade classrooms, observed the implementation of tasks, and gained the teachers’ perspectives of the task implementation.

**Rationale**

While studies exist on factors that influence the implementation of high cognitive demand tasks, they are solely from the perspective of the researchers. I have not found a study that includes the voice of the teachers about their experiences as they implemented high cognitive demand tasks. NCTM (1991) called for students to engage in problems that require thinking that is more complex and require the use of mathematical processes and not just procedures. Stein, et al. (1996) claimed the mathematical tasks provided by the teacher can address this issue. Stein et al., Henningsen and Stein (1997) and Boston and Smith (2009) conducted research on teachers who chose and enacted high cognitive
demand tasks, but they did not include the teachers’ perspectives on the implementation process.

Thus, my goal was to give voice to middle school mathematics teachers who were trying to implement high cognitive demand tasks in order to gain their perspectives on factors affecting cognitive demand during the task implementation process. Henningsen and Stein (1997) identified factors contributing to the change of cognitive demand of enacted tasks, but they looked at archival data so were unable to ask teachers about the reasons for change in demand. My intention is to inform those working with teachers to implement high cognitive demand mathematical tasks in classrooms. By offering the teachers’ perspectives, those working with teachers can anticipate possible roadblocks and successes teachers may have when implementing tasks in the classrooms. I hope to add to the body of research on working with in-service teachers and help those working with teachers realize what factors affect teachers trying to implement high cognitive demand tasks.

**Research questions**

The research questions that guided my study were:

1. What are teachers’ perspectives of their classroom practices as they implement high cognitive demand tasks, and how do teachers’ perspectives compare to the researcher’s perspective?

2. What factors do teachers identify as affecting the implementation of high cognitive demand tasks during all phases of the task implementation process?
CHAPTER 2
LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Literature Review

The goal of my research study was to gain teachers’ perspectives on implementing high cognitive demand tasks and compare their perspectives to my perspective of their implementation. As part of the study I engaged teachers in professional development on implementing high cognitive demand tasks and maintaining the demand. In this literature review I summarized research in four areas: high cognitive demand tasks, the implementation of such tasks, professional development for teachers using such tasks, and studies that portray teachers’ perspectives.

The QUASAR Project. The QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project was a national project that introduced reform curricula in economically disadvantaged middle schools (Silver & Stein, 1996). The purpose of the project was to observe the effects of instruction comprised of high cognitive demand tasks to study the premise that the low performance of students in urban schools was due to the lack of high quality mathematics rather than the often reported lack of student ability or potential to do well in mathematics. The researchers chose participating schools based on collaborative team applications, which included administrators and mathematics teachers along with mathematics educators from nearby universities. The collaborative teams had support from QUASAR representatives but
were in charge of the design and implementation of all instructional activities. The demographics of the student population in the QUASAR schools was 50% African-American, 33% Latino, and 12% Caucasian, 75% living in poverty (Silver & Stein, 1996). The characteristics of instruction in QUASAR schools included “emphasizing meaningful learning, using tasks that involve multiple connected representations and allow multiple solutions strategies, and supporting learning and understanding through collaboration and communication” (Silver & Stein, 1996, p. 486). This type of instruction encouraged students to develop a conceptual rather than procedural understanding of the mathematical topics. Three results the researchers found from the study included: (a) students developed increased problem solving, communication, and reasoning strategies, (b) the use of instructional tasks that allowed for multiple strategies and explanations had a positive effect on student gains, (c) and all students, including those from diverse populations, have the capacity to improve their mathematical understanding when using instructional programs that stress the implementation of tasks like those found in the QUASAR project. These findings were the impetus for me to study task-based instruction because I wanted to situate my study in a diverse middle school and help teachers provide instruction that fosters students’ mathematical learning. The QUASAR project looked at multiple schools in multiple districts, but I wanted to focus my attention on one cohort of teachers in one school and understand the factors that influenced the implementation of high cognitive demand tasks and, ultimately, students’ mathematical learning. While my study focused solely on the teachers, it was my hope that they would see that high cognitive demand tasks positively affected their students’ mathematical knowledge.
High cognitive demand tasks. Research on high cognitive demand tasks to date has focused on whether teachers could identify high cognitive demand tasks and if teachers maintained the level of demand during implementation (Henningsen & Stein, 1997; Stein et al., 1996). Both Henningsen and Stein and Stein and colleagues studied the effects of mathematical tasks on student performance using data from the QUASAR project. The researchers chose three teachers to follow during each year of the QUASAR project based on criteria that included grouping students heterogeneously and being a part of the targeted grade level for the professional development at each school. Data collection took place through classroom observations over a 3-day period three times each year from the fall of 1990 to the spring of 1993. Trained observers took field notes and classroom videos and completed a classroom observation instrument (COI) that included descriptions of the physical setting and chronology of the tasks (Stein et al., 1996). One part of the COI involved paying particular attention to the two tasks occupying the most time in the lesson and recording in detail the nature of the tasks, the mathematical content involved, learning goals of the teacher, and behaviors of students in the classroom. Overall, instances of tasks were collected and coded based on four categories: (a) task description, (b) task set up, (c) task implementation, (d) and factors associated with the decline or maintenance of high-level tasks (Stein et al., 1996). From these instances, the researchers created task narratives describing the implementation. Two follow-up studies utilized data from the instances and focused on the implementation of the mathematical tasks (Henningsen & Stein, 1997; Stein, et al., 1996).
In one study, Stein, et al. (1996) wanted to observe and portray the nature of the mathematical task as implemented by the teachers in the QUASAR classrooms. The researchers had three questions guiding their study. The first entailed observing the set up of task to look for features the teachers used that developed students’ capacity to think and reason mathematically. The second question was to determine if teachers maintained the demand of the task from the set up phase, and the third question looked specifically at tasks set up at a high level of demand and what factors affected the maintenance or decline of demand. The researchers used stratified random sampling from the 620 task narratives to narrow data analysis to 144 tasks and created narrative summaries of the 3-day classroom observations collected during the initial research study. The researchers used the four categories as stated above to code the tasks: (a) task description, (b) task set up, (c) task implementation, (d) and factors associated with the decline or maintenance of high-level tasks. The task description codes described how much time the teacher spent on the task, resources used during task implementation, and the type of mathematical topic the task covered. The task set up category included codes for the features of the task and the cognitive demand of the task as the teacher set up and introduced the task. Features of the task included possible solution methods and possible representations. The codes used to categorize the cognitive demand of the task included memorization, procedures without connections, procedures with connections, and doing mathematics. The implementation category was coded according to how students engaged with the task. This category also used codes from task features and cognitive demand but looked at how the students appeared to engage with the task. The fourth category included factors affecting implementation of the task and was only used to code tasks deemed to
be set up and implemented at a high level of cognitive demand (procedures with connections or doing mathematics) or tasks that were set up at a high level of cognitive demand, but demand declined after set up. When coding tasks, the researchers chose from a list of factors they attributed to either the decline or maintenance of the task. If a factor was not on a list, the researcher chose other and gave a description of the factor affecting demand.

The researchers found that the teachers involved in the QUASAR project had success selecting and setting up mathematical tasks that encouraged students to use multiple solutions and representations, engage in group work, justify answers, and engage in complex mathematical thinking and reasoning. While teachers had success selecting and implementing some aspects of the task, they were not as successful in maintaining the intended level of cognitive demand throughout the lesson. The researchers found the higher the initial cognitive demand, the less likely it was that teachers maintained the demand at that level. Stein et al. characterized factors related to the maintenance or decline of cognitive demand during task implementation based on a database of classroom observations, but these observations were entirely from the perspective of the researchers. The researchers wanted to know if teachers could select high cognitive demand tasks, and that differs from my research because I took that element out of the study. I wanted to see if teachers could take a task deemed as high cognitive demand and implement that task at a high level. Stein and colleagues watched videos of the lessons to determine what factors affected implementation. There was no conversation with teachers to gain their perspectives on whether they realized they lowered demand and if so, their
reasons for lowering demand. I sought to understand from a teacher’s perspective the factors that affected implementation.

In a follow-up study using the data collected by Stein et al. (1996), Henningsen and Stein (1997) looked more closely at the classroom factors associated with the implementation of high cognitive demand tasks, specifically 58 tasks classified as doing mathematics. The researchers looked at how tasks at that level affected student engagement. Twenty-two of the 58 instances of tasks labeled doing mathematics had students actively engaged in doing mathematics. The researchers further analyzed the 36 instances of tasks where teachers did not maintain the cognitive demand and categorized the student engagement as unsystematic exploration, no mathematical focus, procedures without connections, and other cognitive engagement (Henningsen & Stein, 1997). The researchers created more detailed narratives of instances in which the teachers maintained the level of cognitive demand and three different types of instances where the level of demand was not maintained, excluding “other cognitive engagement” from analysis and noted the factors associated with the maintenance or decline of cognitive demand. The researchers found many factors associated with both the maintenance and decline of demand. Three patterns of decline included decline from high cognitive demand, either procedures with connections or doing mathematics, into procedures without connections, unsystematic exploration, or non-mathematical activity. Henningsen and Stein gave qualitative portraits of teachers who fit into each of the categories of the maintenance or decline of the cognitive demand of mathematical tasks. Although the researchers described and characterized factors associated with the implementation of high cognitive
demand tasks, they used archival data and therefore were unable to include the teachers’ perspectives on what happened during task implementation.

Boston and Smith (2009) conducted two years of professional development with 18 teachers, including a portion on identifying and implementing high cognitive demand tasks. The purpose of the study was to determine the impact of professional development on the instructional practice of teachers in regard to choosing and implementing high cognitive demand tasks. Specifically, the researchers sought to document whether teachers’ use and implementation of high cognitive demand tasks changed, and whether the type of curriculum (standards-based versus conventional) used by teachers in their classroom affected their use of high cognitive demand tasks. Nineteen teachers participated in professional development over two years consisting of three elements. The first element was to prepare teachers to mentor beginning teachers and involved six full days of professional development during the first year. The second element was to prepare teachers to mentor pre-service teachers and occurred for one week at the end of the first school year. The last element was to help both the mentor teachers and preservice teachers develop a shared vision for teaching and occurred during five half-days throughout the second year. The professional development included elements similar to the QUASAR project as well as a text that provided narrative cases from the QUASAR project and opportunities to solve and classify different levels of tasks.

Boston and Smith (2009) visited each teacher three times during the school year following the 2 years of professional development to collect instructional tasks and student work used over five consecutive days of instruction as well as to conduct one classroom observation during those five days. The researchers collected data from a
control group to use as a baseline when comparing the number of high cognitive demand tasks implemented and the ability to maintain intended demand. The data collected from the control group consisted of an assessment of knowledge of cognitive demand of tasks, one classroom observation, and a copy of the instructional activities used in the observed lesson. The results showed the teachers who participated in the professional development increased both the number of high cognitive demand tasks used and their ability to maintain the level of demand throughout the lesson. Thus, participation in focused professional development on task selection and implementation can increase teachers’ use of high cognitive demand tasks as well as their ability to implement these tasks with fidelity.

In a later study, Boston and Smith (2011) followed up with seven of the teachers from the 2009 study to determine whether the professional development continued to affect their teaching after its conclusion. The researchers found that the subset of teachers continued to improve their selection and implementation of high cognitive demand tasks over time, even a year after the project had ended. This suggests that teachers involved in targeted professional development on the selection and use of high cognitive demand task, can continue to improve their selection and implementation of tasks, even after professional development has ended.

Boston and Smith (2009, 2011) carried out professional development over time to improve teachers’ selection and implementation of high cognitive demand tasks and found that teachers did select and implement more high cognitive demand tasks. My study is similar to those conducted by Boston and Smith in that I focused on teachers’ implementation of tasks, but I eliminated the necessity for teachers to find tasks and
instead gave the teachers high cognitive demand tasks to implement. Boston and Smith (2009, 2011) used the Instructional Quality Assessment (IQA) rubrics to measure students’ work, the level of the task, and the implementation of the task. I only used a portion of the IQA, specifically the aspect surrounding task implementation. While Boston and Smith (2009, 2011) focused on whether the teachers could maintain the demand of the task and whether teachers could choose high cognitive demand tasks, I chose to study teachers’ implementation of tasks and their ability to maintain the demand, but I added teachers’ voices to get a holistic view of implementation from both my perspective and that of the teachers.

**Teachers’ perspectives.** While the research on teachers’ perspectives may differ with respect to topic, it is similar in that it all seeks to give voice to teachers. Some research studies on gaining teachers’ perspectives include topics that have been documented as being beneficial to student learning (e.g., technology in the classroom, teaching thematic units) but are lacking teachers’ perspectives of the reality of implementing such structures in the classroom (Handal & Bobis, 2004; Wachira & Keengwe, 2010). Some research has been done to compare different perspectives of participants, such as teacher and student or administrator and teacher (Jansen & Bartell, 2013; Stevenson, Dantley & Holcomb, 1999). Regardless of the reasons, these studies aim to fill a gap and give voice to those often not heard in research. While research may argue that integrating certain instructional practices in the classroom can improve student learning, studies that illuminate factors teachers identify as preventing them from implementing such practices can help professional developers provide targeted support to assist teachers in overcoming perceived barriers.
Both Wachira and Keengwe (2010) and Handal and Bobis (2004) wanted to identify barriers that kept teachers from implementing different aspects of instructional practices that research has shown to positively influence students’ learning of mathematics. Wachira and Keengwe wanted to gain urban teachers’ perspectives on integrating technology in their mathematics classrooms. Wachira and Keengwe wanted to see if urban teachers’ perspectives aligned with research on the benefits of using technology in the mathematics classroom, giving their purpose as follows:

While the use of technology has been found to be an effective means to produce growth in students’ understanding of mathematics content, research findings indicate that few teachers integrate technology into their teaching to enhance student learning. This study sought to explore urban teachers’ perspectives on barriers that hinder technology integration in their mathematics classroom. (p. 18)

The researchers explained that while there are many studies on the integration of technology in mathematics classroom, there was a lack of research on urban teachers’ perspectives and on barriers they identify as hindering the integration of technology in their classrooms.

Handal and Bobis (2004) wanted to capture teachers’ perspectives on implementing thematic units, or teaching mathematics around a central theme instead of content. Handal and Bobis wanted to understand what barriers were preventing teachers from teaching thematic units, even though a thematic approach can motivate students and deepen their conceptual understanding of mathematical topics and it was mandatory in the teachers’ districts. The researchers realized there were obstacles preventing teachers from implementing thematic units and wanted to understand better from the teachers’
perspective what those obstacles were. The reason for gaining teachers’ perspectives in both these studies was similar to my reason: I wanted to understand what barriers teachers face when implement certain instructional aspects to supplement research saying it will benefit mathematical learning.

While studying teachers’ perspectives on what was preventing the use of thematic units and technology in the classroom, the researchers found different barriers. Wachira and Keengwe (2010) found both external barriers (e.g., no access to technology, unreliability of technology, lack of technology support) and internal barriers (e.g., lack of time, lack of knowledge, and lack of confidence) as reasons the teachers gave for not integrating technology into their mathematics classrooms. Handal and Bobis (2004) found that the main reason teachers did not implement thematic units was the lack of curricular coherence. The units the researchers provided for the teachers did not match their course performance standards. Another reason for hesitation was that the end of year test did not consider content from thematic units.

Wachira and Keengwe (2010) proposed actions to help teachers overcome barriers and recommended strengthening teacher support and building technology learning communities. The aspect of strengthening teacher support is relevant to my study because it implies that teachers need more support when implementing a teaching practice that is new to them. Providing support with the implementation of tasks may lessen barriers that impede the implementation of high cognitive demand tasks. Handal and Bobis (2004) found that teachers thought teaching thematic units was appealing, but there were too many barriers that impeded their ability to teach thematic units. Handal and Bobis concluded that “although teachers’ responses showed a general appreciation
for the humanistic goals of teaching thematically, they disagreed … because of pressures originating from a range of instructional, curricula and organisational factors” (p. 13). Research on perspectives is used to compare teachers’ perspectives to others in the field of education, whether teacher to student, teacher to researcher, or teacher to administrator. Jansen and Bartell (2013) wanted to give voice to teachers and students and their perspectives on caring mathematics instruction while also comparing the perspectives between the two. Stevenson and colleagues (1999) wanted to compare why administrators thought teachers left the field to the reasons teachers gave. Jansen and Bartell noted that while there was research on caring instruction, there was a lack of research specific to how students viewed caring instruction in mathematics classrooms. The researchers wanted to capture “both teachers’ and students’ voices to describe caring mathematics instruction as well as [analyze] similarities and differences between teachers’ and student’ conceptions” (p. 34). Stevenson et al. wanted to learn whether administrators in a National Science Foundation funded urban schools program meant to retain mathematics and science teachers had the same ideas about teacher retention as teachers as reported in various research articles. The researchers provided surveys to administrators using factors previous researchers claimed were reasons teachers left the field as well as what retention strategies worked best. This is similar to my study of comparing the teachers’ perspectives of instruction to my perspective of their instruction, but the researchers did not gain perspectives of the teachers in the program, instead using factors from previous researchers, and only gained the perspectives of the administrators. Research on comparing perspectives yields results in which both parties agree and disagree. Jansen and Bartell (2013) found that teachers and students agreed on some
aspects of caring instruction, but students also had unique conceptions of caring instruction. Both students and teachers agreed that caring instruction includes reaching every student, but students’ perspectives went further and included explaining mathematics thoroughly, understanding how students thought, and taking time with students to explain mathematical ideas as aspects of caring instruction. Similarly, Stevenson et al. (1999) found that while teachers and administrators had similar conception of teacher retention, they did not agree about the priority of the factors. Both parties ranked dissatisfaction with teacher salary as the main reason for leaving, but teachers in one study then ranked desire to pursue another career second and dissatisfaction with teaching as a career as third whereas administrators cited inadequate support from administration and lack of teacher competency as the next two reasons for leaving. Administrators claimed higher salaries, recognitions and support from administrators, and reduced workloads were the most effective strategies for increasing teacher retention. This finding was partially consistent with one study that showed that teachers thought offering higher salaries and benefits, implementing more effective student discipline, reducing class size, and giving teachers more authority in the classroom were the top incentives to encourage teachers to remain in the field. While the Stevenson et al. did not talk directly with the teachers in the administrators’ schools, there is enough discrepancy in the findings to warrant further studies comparing administrators’ and teachers’ perspectives of why teachers leave and what retention strategies work best.

One result of the study done by Jansen and Bartell (2013) aligns with work on implementing high cognitive demand tasks. Students claimed they knew that
enacted care when they selected tasks that aligned with students’ interests, supported sense making, and honored their development as students. While I was not studying caring mathematics instruction, students’ reasoning that high cognitive demand tasks were a manifestation of caring instruction contributes to the body of knowledge that implementing high cognitive demand tasks foster student learning.

**Professional development.** I engaged the teachers in my study in professional development to support their implementation of high cognitive demand tasks. While I conceptualized a comprehensive professional development plan, circumstances led to a half-day session before the beginning of the school year and sporadic interventions throughout the semester. McGee, Wang, and Polly (2013) listed effective professional development practices as synthesized from previous research. These practices include promoting collaboration between teachers, addressing both content and pedagogy knowledge, making connections to teachers’ classroom practices, and ongoing support. I wanted to include these elements in the professional development for the participants in my study as well as including professional development specific to implementing high cognitive demand tasks. In looking at studies of professional development focused on helping teachers implement high cognitive demand tasks, I found many common elements. Researchers have studied the nature of professional development around implementing high cognitive demand tasks as well as whether teachers perceived the professional development provided to increase their use of high cognitive demand tasks (Boston, 2013; Foley, Khoshaim, Alsaeed, & Er, 2012; McGee et al., 2013; Polly, Neale, & Pugalee, 2014).
One common theme of the research was that the professional development followed a similar format across studies with the teachers engaging in an extended summer institute and follow-up sessions throughout the year. While the overarching goal of the professional development may be different (e.g. different levels of teachers, differing content), many professional development programs have used the idea of implementing high cognitive demand tasks as a central component of the program. McGee et al. (2013) evaluated a professional development project with elementary school teachers preparing to teach a standards-based curriculum. A central aspect of the curriculum was effectively implementing tasks, providing scaffolding to students, and facilitating classroom discussions. Foley et al. (2012) conducted a similar study in a high school context with eight statistics teachers who signed up for a year-long professional development program to increase their knowledge of using technology in their classrooms to implement high cognitive demand tasks. Boston (2013) sought to understand how professional development changed teachers’ knowledge of high cognitive demand tasks and whether there was a relationship between change in knowledge of high cognitive demand tasks and their learning experiences in the professional development. The studies similarly wanted to find out teachers’ perspectives of the professional development, what elements teachers implemented or planned to implement from the programs, and what supports and barriers impeded their implementation of high cognitive demand tasks.

All three professional development programs engaged teachers in analyzing the level of demand of tasks and reflecting on how to utilize those tasks in their classroom instruction. Boston and Smith (2009) additionally engaged teachers in solving high
cognitive demand tasks. Two of the professional development programs (Boston, 2013; Foley et al., 2012) used the *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein, Smith, Henningsen, & Silver, 2000) as a resource. One aspect of the casebook that both programs used was having teachers use the task analysis guide to sort and identify different levels of high cognitive demand tasks to familiarize teachers with the different levels of cognitive demand. I used articles from *Mathematics Teachers in the Middle School* journal with similar content to the casebook text. I engaged the teachers in similar activities such as doing a task sort to determine the cognitive demand of tasks and introduced teachers to the four different levels of cognitive demand (memorization, procedures without connections, procedures with connections, and doing mathematics).

Teachers in all three studies reported being more comfortable teaching high cognitive demand tasks at the end of the year, and the researchers’ observations confirmed this claim. This finding has implications for my study because I wanted teachers to integrate tasks into their classroom practices, but they were unfamiliar and uncomfortable with high cognitive demand tasks. Providing focused professional development and sustained support throughout the year can help teachers become more comfortable implementing instructional practices and curricula that are new to them.

Boston (2013) engaged teachers in solving tasks from their curriculum in order to provide them the opportunity to be learners and remember what it can be like to struggle with a task, which gave the teachers an experience their students might have when engaging with a task. Similar to Boston, when I presented the teachers in my study with high cognitive demand tasks, I first had them solve the task and come up with multiple
solution methods before we talked about implementing the task. Boston found that at the end of the professional development teachers significantly increased their knowledge of high cognitive demand tasks.

Foley et al. (2012) found that two major obstacles for teachers that prevented the implementation of high cognitive demand tasks were class duration and curriculum. Four of the eight participants cited lack of class time as a reason for not implementing high cognitive demand tasks. Some teachers said an impediment to implementing high cognitive demand tasks was the need to cover specific content during the year and the lack of fit between tasks from the professional development with the curriculum. McGee et al. (2013) found similar curricular obstacles that prevented the implementation of high cognitive demand tasks. While the teachers initially implemented aspects of the standards-based curriculum, teachers in the 3rd, 4th, and 5th grades reverted to more traditional test preparation the second half of the year to get students ready for the state test. The researchers found that teachers were concerned with the open-ended format of the standards-based curriculum and how that differed from the multiple-choice format of the state tests. The teachers thought that in order to prepare students for tests, they needed to supplement the curriculum with multiple-choice questions to prepare students for those types of test questions. One way I dealt with time and curriculum in my study was to provide the teachers with tasks that fit into their curriculum and class time schedule.

**Theoretical Framework**

The theoretical framework that I used in this study was one developed by Stein et al. (1996) to look at the implementation of tasks, called the task implementation
framework. I chose to situate my study within this framework because it highlights factors that affect the implementation of high cognitive demand tasks. From the task implementation framework, Boston and Wolf (2004) developed an instrument, the Instructional Quality Assessment (IQA), for the purpose of rating the implementation level of tasks. The IQA is a set of rubrics used to assess the level of cognitive demand of tasks, the implementation of the task, and the classroom discussion around the task. I specifically focused on the implementation of tasks and therefore used rubrics that pertained to the implementation in the classroom. When thinking about how to help my participants plan and implement high cognitive demand tasks, I drew upon the work of Smith and Stein (2011) that lays out five practices for orchestrating discussions around high cognitive demand tasks. The five practices are anticipating, monitoring, selecting, sequencing, and connecting, and provide structure when planning and implementing high cognitive demand tasks. By utilizing the task implementation framework, the IQA and the five practices, I was able position my study using research on the implementation of high cognitive demand tasks.

**Task implementation framework.** I used the task implementation framework (Stein et al., 1996) as shown in Figure 2 as the framework guiding my analysis. The task implementation framework “proposes a set of differentiated task-related variables as leading toward student learning and proposes sets of factors that may influence how the task variables relate to one another” (Stein et al, 1996, p. 458). The framework comes out of the work of Doyle (1983) and his view that curricula can be defined by a collection of tasks, and what students learn is a product of the tasks with which they engage. From this work, Stein et al. proposed the task implementation framework to account for the fact that
tasks can be altered during different stages of implementation, and ultimately students may not engage with the task at a high cognitive level due to multiple factors.

Figure 2: Task Implementation Framework (Stein et al, 1996, p. 459)

The purpose of the task implementation framework is to identify factors that change the task as it is implemented in a mathematics classroom. While a task may start out at one level, during implementation the level may change due to the factors listed in the framework. The level of the task can change between successive phases of implementation: between the task as represented and the task as set up by the teacher or between the task as set up by the teacher and the task as implemented by students. The definition of a mathematical task is “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (Stein et al., 1996, p. 460).
Factors that influence set up include teachers’ goals, teachers’ knowledge of subject matter, and teachers’ knowledge of students. While much research has focused on the latter part of the task implementation framework, there is little on factors that influence set up of the task, including the definitions of these factors. Therefore, I defined the factors as follows. Teacher goals are the goals and objectives the teacher has for implementing the task. Goals may include curricular goals, such as learning a certain mathematical concept, or environmental goals, such as setting classroom norms.

Teachers’ knowledge of subject matter is their comprehension and understanding of the content in the task. A teacher’s knowledge of students is any information the teacher has of students and uses when planning lessons. This includes students’ classroom behaviors, content knowledge, background, or teachers’ perceptions of students’ capabilities and abilities.

The task set up is the task as introduced by the teacher. The task set up can be elaborate, with the teacher giving a lengthy introduction, or as simple as telling students to get started on the problem in front of them. The task as implemented by students refers to what happens when students engage and attempt to find solutions for the task. Factors that affect how students engage with the task, whether as intended or not, include task features, cognitive demand, classroom norms, task conditions, teachers’ instructional dispositions, and students’ learning dispositions. These factors are the codes I used when analyzing my data. The definition of each of the codes is shown in Table 1. Previously, researchers used the task implementation framework to look at many different aspects of the implementation of tasks, including choosing tasks and implementing tasks (Henningsen & Stein, 1997; Stein et al., 1996). Stein and colleagues found that more than
half of tasks implemented at a high level of demand maintained that demand throughout the lesson. The higher the level of the task at the outset, the more susceptible that task was to decline in demand. Stein et al. found that on average, 2.5 factors affected the decline of the task. Factors that led to decline, in order from most to least influential, were tasks becoming non-problems, the inappropriateness of the task, focus of the task shifting to the correct answer, too much or too little time, lack of accountability, classroom management problems, and other factors.

Table 1
Factors Influencing Implementation as Adapted from Stein et al., 1996

<table>
<thead>
<tr>
<th>Factor</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task features</td>
<td>The features of the task that develop mathematical understanding, reasoning, and sense making, including multiple solution methods, multiple representations, and communicating mathematically.</td>
</tr>
<tr>
<td>Cognitive demand</td>
<td>Thinking processes in which students engage when solving a task. This includes memorization, procedures with connections, procedures without connections, and doing mathematics.</td>
</tr>
<tr>
<td>Classroom Norms</td>
<td>The established expectations regarding how academic work gets done, by whom, and with what degree of quality and accountability.</td>
</tr>
<tr>
<td>Task conditions</td>
<td>Attributes of tasks as related to a particular set of students. Examples include the extent to which tasks build on students’ prior knowledge and the appropriateness of the amount of time provided for students to complete tasks.</td>
</tr>
<tr>
<td>Teachers’ instructional dispositions</td>
<td>The amount of struggle a teacher is willing to let students have when engaging with a difficult mathematics problem. This includes the kinds of assistance that teachers typically provide students who are having difficulties.</td>
</tr>
<tr>
<td>Students’ learning dispositions</td>
<td>The amount of perseverance a student engages in when solving a difficult problem.</td>
</tr>
</tbody>
</table>

Many factors helped maintain the demand of a task. Factors that influenced maintaining demand, from most to least common occurrence, included tasks that built on students’ prior knowledge, an appropriate amount of time, teachers modeling high-level performances, sustained press for explanations and meanings, scaffolding, students’ self-
monitoring, teachers drawing connections, and other factors. Stein and colleagues found that, on average, four factors influenced the maintenance of demand during each task. The data showing that the average number of factors that affected the decline of a task was 2.5, while 4 factors were needed to maintain demand, suggest that it does not take much to decrease the level of demand but it takes a great effort to maintain demand.

In the follow up study by Henningsen and Stein (1997), in which they looked only at tasks labeled as doing mathematics, the researchers found that factors related to maintaining the demand of a task included the appropriateness of the task and teachers’ actions that supported students’ thinking, such as scaffolding or helping students make connections. Henningsen and Stein found “students’ engagement with the tasks that declined to lower levels of cognitive demand activity happened in different ways and for different reasons” (p. 546). They explained that there were three decline patterns: decline into unsystematic exploration, decline into procedures without connections, or decline into no mathematical activity, and there was variation with respect to what factors caused the decline. The one common factor, although it affected each pattern differently, was the amount of time, either too much or too little. Other factors that affected the decline included inappropriateness of the task and the challenging parts of the tasks being removed by the teacher, such as giving information away to students. I used the task implementation framework to identify factors affecting implementation when the participants in my study enacted high cognitive demand tasks.

**Instructional quality assessment.** The Instructional Quality Assessment (IQA) is a rubric-based tool used to measure the academic rigor of mathematics classrooms (Boston & Wolf, 2004; Junker et al, 2006). The IQA is based on the fact that in order for
students to engage in rigorous mathematics, they must have access to the type of problems that will allow such engagement (Boston & Wolf, 2004). The rubrics of the IQA are based on the Stein et al. (1996) task implementation framework. Boston and Wolf stated that the rubrics “were created to consist of four dimensions critical to assessing students’ opportunities to learn mathematics with understanding: potential of the task, implementing of the task, student discussion or students’ written responses; and teachers’ expectations” (p. 10). When using the rubrics, a researcher observes a lesson and rates the initial cognitive demand of the task presented as well as the implementation level of the task.

Studies in which the IQA rubrics have been used to assess the rigor in the classroom have mainly been conducted from the perspective of the researcher (e.g., Boston & Smith, 2009; Boston & Smith, 2011; Cheng, Feldmen, & Chapin, 2012; Jackson, Garrison, Wilson, Gibbons, Shahan, 2011; Kotsopoulos, Lee, & Heide, 2011). Boston and Smith (2009, 2011) used the IQA to evaluate the effectiveness of a professional development program. The researchers used the IQA during the same year as the professional development program and then used it two years later in a follow-up study. The IQA was used to measure the potential of tasks as well as the implementation level of the tasks. The researchers found that teachers who participated in the professional development program were initially able to identify high cognitive demand tasks and increased their ability to maintain the level of demand throughout the lesson. The follow up study showed that over time the teachers were still able to identify high cognitive demand tasks and continued to implement the tasks at a high level.
Cheng et al. used the IQA to measure the demand of a task during preservice teachers’ small group discussions in an elementary geometry class. The researchers analyzed the discussions of two small groups as they engaged in a high cognitive demand geometry task using the IQA-AR rubrics to determine the potential of the task, to assess the implementation of the task, and to identify any changes in levels. They found that the group of preservice teachers that maintained the demand of the task while solving it focused on the conceptual aspects of the mathematics and did not focus on just finding the correct answer. Cheng et al. hypothesized that the group that did not maintain demand did not have a common understanding of the mathematics of the task. They suggested that in order for groups to have success maintaining the demand, the discussion needs to focus on the mathematical concepts and not just on finding the solution.

Jackson et al. (2011) used the IQA with 132 middle school teachers to assess students’ opportunities to learn in a mathematics classroom. The researchers used the IQA to assess the implementation of tasks because the classroom teachers were using a curriculum that included cognitively challenging tasks. Using the IQA to measure the set up and class discussion after the task, the researchers found the quality of set up affected the quality of discussion after the task. Kotsopoulos et al. (2011) used the potential of task rubric to compare the demand of 66 mathematics tasks, some given in class and others given as homework, in one teacher’s classroom. They found that 66% of the time the demand of the homework differed from the class work, but it was both higher and lower at different times. I used the IQA to rate the level of implementation of high cognitive demand tasks based on my observation field notes. I used the academic rigor rubric on task implementation and the checklist that accompanied the rubric (see
Appendices A and B) to assess the level at which the teachers engaged students with each task.

**Five practices.** Smith, Stein, and their colleagues identified five instructional practices in which teachers should engage when implementing high cognitive demand tasks in the classroom (Smith & Stein, 2011; Stein, Engle, Smith, & Hughes, 2008). The title of Smith and Stein’s book is *5 Practices for Orchestrating Productive Mathematics Discussions* and that is how this work has come to be known in the field of mathematics education. The five practices are anticipating, monitoring, selecting, sequencing, and connecting, and all pertain to student solutions. When teachers utilize the five practices they can orchestrate mathematical discussions during the implementation of high cognitive demand tasks and can increase student engagement with tasks. When developing the five practices, Stein et al. (2008) had novice teachers in mind. The participants in my study all had less than five years experience teaching, thus falling into the novice category.

The first practice, anticipating, involves teachers predicting possible student solutions for the task. The solutions should include both correct and incorrect approaches. Along with anticipating student solutions, teachers should prepare possible questions to aid students if they are stuck or are on an incorrect solution path. Teachers should also anticipate questions that can extend students’ thinking or provide scaffolding. The second practice, monitoring, involves observing students’ mathematical thinking as they engage with the task. Teachers should circulate around the classroom and make note of how students are approaching the problem. Teachers should identify different representations of the problem and start to think about which solution methods to share with the class.
Based on what the teacher observed while monitoring, the teacher selects students to present their solutions. There is no correct way to select, but selecting is based on the teacher’s goals for the lesson. Teachers may call on specific students or combine asking for volunteers and asking students who have a specific method the teacher wants shared. While selecting student solutions to share, teachers may choose to sequence the presentation of solution methods. There is no correct way to sequence solutions, but it is again based on the goals of the teacher. A teacher may want to start with the least complex solution and move toward the most complex. A teacher may want to start with incorrect solutions and initially address misconceptions. After students have presented the solutions, teachers should connect between student solutions. Teachers should draw connections between different representations presented and help students connect solutions to other mathematical ideas. When working with teachers on planning task implementation I used the five practices to help the teachers think about implementation and then looked for evidence of the five practices during my observations of instruction.
CHAPTER 3

METHODOLOGY

According to Morrow (2007), qualitative studies are useful when trying to understand deeply experiences people have and what to make of these experiences as well as studying a phenomenon in depth. These roles for qualitative research align with my desire to study the experiences of mathematics teachers implementing high cognitive demand tasks, with the phenomenon of focus being the implementation process. The use of qualitative methods allowed an in-depth look at three teachers’ experiences as they navigated implementing high cognitive demand mathematical tasks in the classroom, and my ability to answer my research questions centered on participants’ perspectives as well as my perspective on factors affecting implementation.

In order to collect the perspectives of the teachers participating in the research project, I chose to do a multiple case study (Yin, 2003). Yin (2003) argued that qualitative research happens out of a researcher’s need to understand complex social phenomena. While there are many types of qualitative research strategies, Yin states that case studies are used most specifically to answer “how” and “why” questions, defining a case study in two parts. The first part of the definition explains the scope of the study stating, “A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident” (p.13). This implies that the use of case studies is appropriate to study conditions that a researcher believes to be pertinent to the
phenomenon. A researcher may not always be able to separate the context and phenomenon so Yin (2003) gives a second part to the definition saying,

The case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence with data needing to converge in a triangulating fashion and as another result benefit from the prior development of theoretical propositions to guide data collection and analysis. (pp. 13–14)

Taking these two parts of the definition together, I situated my research to answer questions about teachers’ perspectives with respect to the phenomenon of implementing high cognitive demand tasks. I collected multiple data points around the case of each teacher including interviews, classroom observations, and data from team planning meetings.

Yin (2003) splits case study research into two categories based on the number of cases the researcher is planning to study—single case studies and multiple case studies. A single case study looks at a singular case, and Yin cautions that it should only be used in specific situations such as when the unit of analysis represents an extreme or unique case or when using a case study to represent a typical case to capture information or experiences of a person or institution that is involved in that phenomenon. Yin warns that if at all possible, except in the above situations, a case study should involve at least two cases. I chose to do a multiple case study with three cases, each of which was a teacher. I looked at teach teacher individually and then looked across all three teachers to find similar themes. In particular, I was interested in teachers’ perceptions of their challenges, successes, and supports during implementation of high cognitive demand tasks.
Setting and Participants

During the 2012-2013 school year, I attended grade level team meetings at Yellow Brick Middle School to provide support to and collaborate with teachers when planning their mathematics lessons. I worked with the 6th, 7th, and 8th grade teams, attending their weekly Thursday planning meetings. I gave mathematical support by occasionally providing professional development on mathematical topics the teachers claimed to be unfamiliar with or topics for which the teachers wanted help coming up with activities or tasks. I collaboratively planned with the teachers by brainstorming and planning upcoming lessons and helped teachers realize what misconceptions students could have with the lesson and how to provide extensions if needed.

Yellow Brick Middle School was a Title 1 public school in the southeastern United States. The school was comprised of grades 6 through 8 and had approximately 730 students. As reported by the participants, roughly 60% of the students were African-American, 25% were White, 7% were Latino/a, 4% were Asian, were 4% are multi-racial. About 75% of students were eligible to receive free or reduced price lunches, and about 4% of students were English language learners. Students were placed heterogeneously in mathematics classes, with the exception of the accelerated mathematics class. In order to be enrolled in the accelerated class, students must have scored in the top 5% on the end of year state mathematics test for three years in a row, made all A’s in advanced mathematics, and scored at least 90% on a national mathematics skills test. Although the school did not “track students,” some mathematics classes had clusters of advanced students or clusters of students identified as needing Individualized Education Plans (IEPs). Students were considered advanced if they met
three of the following requirements: have a teacher recommendation, excellent mathematics grades, score at least an 85% on the Performance Series Test by Scantron, met a creativity requirement, or scored in the top 10% on the end of year state mathematics test. If a class had a cluster of students classified as needing an IEP, a collaborative special education teacher joined the classroom mathematics teacher to provide support for the students. Students were classified as on level or below level depending on how they scored on a national skills test. If students scored at a 7th grade level, they were considered on level while if they score below that, they were considered below level.

While working at the school I developed relationships with the teachers, the building principal, and the instructional coaches. As a result of these interactions, I wanted to continue to work with teachers at this school on implementing high cognitive demand tasks in the classroom in 2013-2014. I selected the participants using a convenience sample (Patton, 2002) because I had experience working with the mathematics teachers in Yellow Brick Road Middle School. I wanted to work with a team of teachers teaching the same grade level, both to make planning easier and so the teachers could have the benefit of working together. The building principal proposed working with the 7th grade team, which consisted of three teachers, Mr. Cone, Mrs. O’Neill, and Mr. Fielder.

Mr. Cone was a mathematics and social studies teacher in his fifth year of teaching, all at Yellow Brick Middle School. Mr. Cone received both his bachelor’s and master’s degrees from the same university, which was a large public university in the southeastern United States. For his bachelor’s degree he was a social studies education
and history double major. After student teaching in a high school, he decided he was not ready to teach and applied to graduate school. Mr. Cone was accepted into the middle grades masters program and took courses to add mathematics as a certification area, along with social studies. Mr. Cone explained that he added mathematics because he enjoyed mathematics and found many students did not and wanted the challenge of teaching the subject. Because Mr. Cone added a concentration in mathematics in graduate school, he did not take as many mathematics education courses as undergraduates in the same program had to take, only taking two–number systems and algebra. He also did not take any college-level mathematics classes during his undergraduate degree, although he took calculus in high school. His first year at Yellow Brick he taught 6th grade mathematics, his second and third year he taught 8th grade mathematics, and his fourth and fifth year he taught 7th grade mathematics and social studies. At the time of my study Mr. Cone taught two mathematics classes during first and second period. His first period mathematics class was an advanced class with 13 on level students and a cluster of 8 advanced students. His second period mathematics class included a collaborative teacher with 18 on level students and 9 students identified as needing IEPs. I observed his first period advanced level mathematics class for purposes of this study.

Mrs. O’Neill was a mathematics teacher in her third year of teaching. Mrs. O’Neill said that she never thought she would be a teacher because her parents pushed her in every direction except teaching. Her bachelor’s degree was in biology with a minor in mathematics. Mrs. O’Neill got married right out of college and worked as a substitute teacher during her first couple of years of marriage, but after having children decided to stay home. After 25 years of being a stay-at-home mom and working as a substitute
teacher again the last couple of years, she decided she liked being in the middle school and high school and pursued a teaching degree. In order to receive her teaching certification she took the state licensing test in both biology and mathematics for middle school and went through the state Teacher Academy for Pedagogy and Preparation (TAPP) program to earn her initial teaching certificate. TAPP is a state program that gives teaching certificates in 1 or 1.5 years to those who pass the state certification exam and have been hired by a school district. Mrs. O’Neill was hired by Yellow Brick to teach mathematics and received her certificate in one year through the program. The TAPP program included teachers at all levels (elementary, middle, secondary) and content areas and required them to attend a 2-week course in the summer and monthly 4-hour meetings. The monthly meetings addressed general pedagogy and included topics such as differentiation and classroom management. People from the program, including a mentor and supervisor, along with school administrators observed Mrs. O’Neill 12 times during the year for evaluative purposes. The program ended with a capstone electronic portfolio, including videos of teaching, unit plans, book studies, and assignments from throughout the year that was used for evaluation purposes. Upon successful completion of the program elements, Mrs. O’Neill received her teaching certification. At the time of the study Mrs. O’Neill taught four mathematics classes. Her first period class of 16 students was an accelerated mathematics class, which covered the last half of the 7th grade curriculum and the first half of the 8th grade curriculum. Her second period class had 24 students, 8 of whom were on-level and 16 of whom were below-level in mathematics. In her fifth period class she had 26 students, 6 of whom were below level, 14 were on-level and 6 were classified as advanced. Her seventh period class was taught with a
collaborative co-teacher and had 6 students with IEPS, 1 on level student, and 15 below level students. I observed her second period class. Mrs. O’Neill also had administrative responsibilities that included being 7th grade team leader and data team leader, she was a mentor to a first year teacher, and had a preservice teacher intern that was in her classroom all day on Tuesdays and Thursdays.

Mr. Fielder was a mathematics teacher in his first year of teaching. He graduated from a large public university with a middle school degree, concentrating in mathematics and social studies, and did his student teaching the previous year in Mr. Cone’s classroom. During his student teaching Mr. Fielder taught and planned the mathematics lessons, and Mr. Cone taught and planned the social studies lessons. At the time of this study Mr. Fielder’s course load consisted of four mathematics classes. Mr. Fielder had a collaborative teacher in two of his classes, each with 20 students, because there were clusters of students with IEPs who require additional support in mathematics. His other two classes were on level, each with an advanced cluster of students. His fifth period class had 22 students, 6 of whom were labeled as advanced, and his seventh period class had 28 students, 6 of whom were labeled as advanced. I observed his fifth period class.

For purposes of this study, I started working with the teachers during their pre-planning days in July 2013 and continued through December 2013. To help the teachers with the implementation of high cognitive demand tasks, I provided a four-hour professional development session during one day of the pre-planning week before the start of the year. I used three articles from Mathematics Teaching in the Middle School to guide my professional development. I had previously read each article, and I engaged the teachers in activities from the articles. I provided teachers with a copy of each article but
did not require them to read the articles prior to the professional development. I started with an article by Smith and Stein (1998) titled *Selecting and Creating Mathematics Tasks: From Research to Practice* to introduce the definition of high cognitive demand tasks. Based on an activity described in the article, I engaged teachers in a task sort. The purpose of the activity is to have teachers classify tasks as high or low level based on their definitions and discuss how they determined the label for each task. I had the teachers classify the tasks and discuss their reasoning for each label. I then introduced the teachers to the four levels of demand provided in the article and had a brief discussion about each of the levels and examples of tasks that were classified at each level. I had the teachers go back through each task, and label the tasks again using the four levels of demand as defined by Smith and Stein—memorization, procedures without connections, procedures with connections, and doing mathematics. We then had a short discussion about how they re-sorted the tasks and how using the four levels of cognitive demand changed their conceptions of tasks. After discussing the attributes of different levels of tasks, I introduced the five practices—anticipating, monitoring, selecting, sequencing, and connecting—utilizing the *Orchestrating Discussions* article by Smith, Hughes, Engle, and Stein (2001). I outlined each of the five practices, and we discussed how each of the five practices would look during the planning of tasks and in the classroom.

I concluded the professional development session with the article *Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms* by Smith, Hillen, and Catania (2007). I talked with the teachers about using a patterning task to create norms in the classroom that would allow students to become comfortable engaging in high cognitive demand tasks. The teachers chose a task from the article to implement
during the first week of school, and we went through each of the five practices to plan the implementation of the task. The teachers chose the Figure S task and worked through the task while also anticipating possible student solutions based on their answers. They made a recording sheet to monitor students during task implementation based on the anticipated student solutions.

I used the articles and the conversations we had during this time to develop a vocabulary and shared understanding of terms that I could use during future interventions with the teachers. During planning meetings throughout the semester I referenced the cognitive demand of the task, utilized the five practices to frame lessons, and talked about creating norms in the classroom that would allow students to engage in the high cognitive demand tasks.

Data Collection

Yin (2003) suggested that a case study should use as many different sources as possible, and he broke down the different types of sources into six categories: documents, archival records, interviews, direct observations, participant observations, and physical artifacts. Documents are letters, agendas, minutes from meetings, etc., that should be used to corroborate evidence found in the case study. Archival records include service and organizational records, personal records, surveys and maps. Physical artifacts include physical evidence produced from the study such as student work used during the study. Yin saw interviews as one of the most important forms of data collection and as a way to follow the line of inquiry and by asking participants in a conversational way about their involvement and perceptions of the phenomena. Direct observation occurs when a
researcher is outside of the phenomenon and observes what is happening in different situations. Participant observation occurs when a researcher actually participates in events that surround the phenomenon and records what is happening. Yin suggests that the researcher use as many sources of evidence as possible so as to provide for triangulation of data. Triangulating data can increase the validity of the case study.

The main data collection methods I used were classroom observations, which would be considered participant observations because I was a participant in the events, and teacher interviews, which align with Yin’s (2003) suggested data collection strategies for a case study. During the classroom observations I collected physical evidence by taking pictures of the board work by the teacher and the students. To answer my first research question about how the teachers’ perspective compared to my perspective during the implementation of tasks, I needed observational data (to aid in forming my perspective) and interview data (to gain the teachers’ perspectives). I relied heavily on the individual interviews and on data collected during team meetings to answer the second research question about teachers’ perspectives on factors that affected the implementation of high cognitive demand tasks.

I observed each teacher daily during one class period for the first 2.5 weeks of the school year to gain an understanding of how each set up norms in the classroom related to high cognitive demand tasks. In particular, I observed each teacher implementing the Figure S task during the first week of school. I conducted additional observations of each teacher during the aforementioned class period over one to two days, depending on the length of the task. For example the Border Problem was a 2-day problem for Mr. Cone and Mr. Fielder while Mrs. O’Neill only presented the task in one day. I observed Mr.
Cone in September, October, November, and December, Mrs. O’Neill in September, October, and November, and Mr. Fielder in September, October, and December. Each teacher implemented the Border Problem in September, and I was present for the duration of this task. Unfortunately, due to the lack of consistent implementation of high cognitive demand tasks and not having a complete implementation cycle of planning, implementing, and interviewing afterward, I only have complete data on the Figure S task and Border Problem for each teacher. Thus, although I have additional observation data, I focused my analysis for both research questions on the data from those two problems. During classroom observation, I took copious field notes about what occurred throughout the lesson, paying particular attention to what the teacher said to the entire class, whole class discussions, and interactions the teacher had with students if I could hear them.

I conducted three semi-structured interviews (Patton 2002), lasting between 40 and 60 minutes, with each teacher in order to gain his or her perspective on implementing tasks, to understand his/her thought processes when setting up the tasks with students, and to learn whether or not the lesson went according to the implementation plan (see Appendix D). I conducted the first interview with each teacher after the initial observation period ended in August, the second interview within one week of the implementation of the Border Problem in October, and the third interview in December. I conducted semi-structured interviews (Patton, 2002), including questions about the planning and implementation process of high cognitive demand tasks. During the initial interview, I asked each teacher about general conceptions of high cognitive demand tasks, if/how those conceptions had changed since working with me, and how they saw tasks fitting into classroom instruction. I asked specifically about the implementation of
the Figure S task, goals associated with implementing that task, perceptions of the
implementation of the task, including maintaining the demand of the task, and specific
examples of when he or she lowered or maintained the demand of the task. During the
second interview, I asked each teacher about his or her general conception of high
cognitive demand tasks and if that changed since working with me, where high cognitive
demand tasks fit into each teacher’s classroom instruction, and how often each teacher
implemented high cognitive demand tasks. I also asked specific questions related to the
Border Problem, similar to the questions asked about the Figure S task. During the final
interview, I asked similar questions to the first and second interview and included
questions about ideal conditions to implement high cognitive demand tasks. I asked about
ideal conditions to implement tasks because I was trying to gain each teacher’s
perspective about what factors supported his or her implementation of high cognitive
demand tasks in the classroom. I audio recorded and transcribed each interview.

I also attended team planning meetings and audio recorded each session. My
purpose in attending team meetings was to help the teachers plan for implementing high
cognitive demand tasks and to discuss the implementation process with them. During the
first two weeks, I attended team planning meetings when the teachers were scheduled to
discuss any aspect of teaching mathematics. After the initial two weeks, I started going to
their planning meetings on Thursdays during third period because that was their
scheduled mathematics team planning meetings. In September, scheduling conflicts
caused several meetings to be canceled due to the school scheduling other meetings such
as data team meetings or school initiated professional development meetings.
Furthermore, the agendas for many of these meetings no longer focused on planning for
implementing high cognitive demand tasks as the demands of the day-to-day activities of teaching (such as implementing and scoring district-mandated assessments) became the focus of meetings. Thus, during the October 3rd planning meeting, we agreed to start meeting Friday mornings before school in order to preserve their team planning time for activities mandated by the school administration while still affording us time to focus on implementing high cognitive demand tasks. We met on Friday mornings for 60 minutes during October, November, and December (a total of 6 times). Due to the limits of my IRB approval, I only recorded meetings where no one but the three participants and myself were present. My goal in recording the meetings was to have a record of the planning process for implementing high cognitive demand tasks in the classroom as to document they ways in which I provided professional development for the teachers. I went through each recording and transcribed the parts of the meeting pertinent to my research questions. These included times the teachers talked about planning and implementing high cognitive demand tasks, how the implementation of tasks went in the classroom, barriers that impeded the implementation of high cognitive demand tasks, and discussions around finding tasks for specific curricular units.

**Data Analysis**

To analyze the data I utilized the task implementation framework (Stein et al., 1996), the instructional quality assessment (Boston, 2012), and the five practices (Smith & Stein, 2011) as the basis for coding interview transcripts and classroom observations. The goal for data analysis was to gain teachers’ perspectives of implementing high cognitive demand tasks in order to answer both research questions. I chose to use the data
from the Figure S task and the Border Problem because I had data for a complete implementation cycle for all three teachers. An implementation cycle included planning meetings, the implementation of the task, and an interview reflecting on the task implementation. The summer before data collection, I participated in IQA training with another graduate student. We received the training materials and the two of us went through the training modules that included watching videos and rating different aspects of the implementation of tasks using the corresponding rubrics.

I began by analyzing the data for the second research question about the teachers’ perspectives on what factors affected their implementation of high cognitive demand tasks. The data I used to answer the second research question included transcripts from team meetings and individual interviews. I went through each teacher’s interview line-by-line and found instances where the teacher made a comment about a factor that affected implementation of the Border Problem, the Figure S task, or high cognitive demand tasks in general. In a separate document, I created narratives of these instances, separating the data into instances where the teacher cited factors that led to the decline of demand, instances where the teacher cited factors that maintained the demand of the task, and instances where the teacher cited factors that affected his or her decision about whether or not to implement high cognitive demand tasks. I then looked at the transcripts from the team meetings and similarly looked for instances where a teacher cited a factor affecting implementation and included it in that teacher’s narrative. I did this for each of the teachers in separate documents. Once I had each teacher’s narrative, I went through and coded using the factors from the task implementation framework. When I identified a factor that was not in the framework, I made a note of it and gave that factor a name. I
realized that the factors the teachers mentioned that were not included in the task implementation framework specifically related to events that happened before planning tasks and affected whether or not they chose to implement a high cognitive demand task. These factors included time, teachers’ knowledge of students, district requirements and curriculum, and parental dispositions, and I identified these factors as influencing the teachers’ implementation of the task. I modified the framework to include the factors in a separate circle before the mathematical task square in the task implementation framework (as seen in Figure 3). I used the new factors, along with those in the task implementation framework to code the interviews accordingly. I coded each teacher’s narrative individually first and then looked for common themes across all cases, making note of where the teachers had common barriers and supports, and then identified factors each teacher individually pointed to as being a barrier or a support. I wrote up the analysis first explaining the commonalities between teachers and then included factors unique to each teacher.

**Figure 3:** Proposed new Task Implementation Framework (adapted from Stein et al., 1996)
In order to answer the first research question about teachers’ perspectives on their classroom practice as they implemented high cognitive demand tasks compared to my perspective, I first turned my field notes into rich, thick narrative descriptions to give an in depth picture of the Figure S task lesson and the Border Problem lesson. To create my perspective, I rated the lesson using the Academic Rigor 2 (AR2) rubric from the IQA (Boston, 2012). The IQA protocol involves using a checklist to identify features of the teacher’s and students’ interactions during task implementation. The checklist then maps onto categories of the rubric. I used the AR2 rubric to rate the Figure S task and the Border Problem for each teacher. I was unable to apply the IQA to Mrs. O’Neill’s Figure S task because I had incomplete data on that lesson. For the rest of the lessons I used the checklist to check off instances where the teacher provided opportunities for students to engage in high-level thinking, instances where the teacher did not provide opportunities for students to engage in high-level thinking, and instances where the class discussion provided opportunities to engage in high-level thinking. Using AR2 protocol (see Appendix C) I applied the checklist to the rubric and rated each teaching episode using one of the four levels, the highest being a level 4 and the lowest being level 0.

After rating the lesson as a whole, I went through the field notes line-by-line and looked for specific instances, from my perspective, where the teacher lowered or maintained demand. I coded the observations using the factors from the task implementation framework. As I assigned factors, I wrote memos to provide an explanation for why I chose that specific factor and whether the factor contributed to maintaining or lowering demand. The same factor could relate to either lowering or maintaining the demand, depending on the classroom interaction. For example, if a
student asked a question and the teacher immediately gave the answer without questioning the student or allowing the student to figure it out, I classified that as teacher’s instructional disposition, and noted the instance lowered the demand of the task.

If students presented a solution method of the task and the teacher pressed for information asking how the students arrived at the solution, I classified it as teacher’s instructional disposition and noted how the instance maintained the demand of the task. I also went through the field notes and located instances where it was evident the teacher was using one of the five practices (Smith & Stein, 2011). I made note if the teacher was anticipating, monitoring, selecting, sequencing, or connecting and wrote a memo about that instance including specifically what practice was evident and why. I used the analysis from the IQA as well as identifying the factors that affected demand to write up my perspective on each of the teaching episodes for each teacher.

To analyze the tasks from the teachers’ perspectives, I went through the interviews where the teachers referenced implementing the Figure S task and the Border problem. I went through line-by-line, looked for specific instances where the teacher claimed to lower or maintain demand, and wrote narratives of those instances. I first separated the instances into two categories, factors the teachers claimed help maintain demand of the task and factors that led to the decline of demand of the task. Once I wrote up each lesson for each teacher, I went back through and coded the narratives using the factors from the task implementation framework. I documented instances of each factor and wrote a memo to justify that classification. I used this analysis to write up the teacher’s perspective using quotes from the interviews to back up my claims, making sure
to provide thick, rich descriptions of each task for each teacher. I concluded my analysis of each teaching episode by comparing my perspective to the teacher’s perspective.

In order to maintain the validity and credibility of my qualitative data I triangulated data, used member checks, and provided evidence to back up my claims to more accurately portray the teachers’ experiences. Freeman, deMarrais, Preissle, Roulston and St. Pierre (2007) claim validity is the trustworthiness of the data and therefore the research and claim the quality of data and evidence need to be evident throughout the study. Additionally, they recommend that researchers ground claims with evidence and the theory behind the study. In order to create trustworthy data, I included evidence from the data, including excerpts from the observations, interviews, and team meetings, to back up claims of the factors I identified as affecting the implementation of tasks. Triangulation, or cross checking data, involves using multiple data sources to verify themes found in data (Lincoln & Guba, 1986; Maxwell, 2005; Patton, 2002; Stake, 1995). I had multiple data sources from interviews, classroom observations, and team meetings to help provide credible claims. Patton (2002) noted that studies with only one source are more vulnerable to errors such as loaded interview questions or bias from the researcher. Maxwell (2005) concurred, saying triangulation “reduces the risk that your conclusions will reflect only the systematic biases or limitations of a specific collection method (p. 93). By using multiple methods of data collection, I increased the validity of my analysis.

Member checking involves checking back with participants to determine whether the analysis represents their experiences. Lincoln and Guba (1986) describe member checks as:
the process of continuous, informal testing of information by solidifying reactions of respondents to the investigator’s reconstruction of what he or she has been told or otherwise found out and to the constructions offered by other respondents or sources, and a terminal, formal testing of the final care report with a representative sample of stakeholders. (p. 77)

Stake (1995) related member checking to that of an actor going over a script. The actor should have a say in the character he or she is portraying. In the same way, research subjects should have a say in how the researcher is portraying them and voice concerns or alternatives. I provided each teacher with a copy of my analysis of his/her implementation of tasks in response to the first research question and my analysis of all the teachers in response to the second research question to get their input. Specifically, I wanted to find out if they agree with the factors I identified as influencing the implementation of high cognitive demand tasks in their classrooms or whether I had omitted any important factors. Mr. Fielder emailed me back fairly quickly saying that he read over the documents and was “fine” with my analysis; he provided no further comments. Mrs. O’Neill expressed in her reply email that she was not thrilled with how I portrayed the teachers in the analysis of the second research question. She thought my writing sounded very negative and that was not how she had intended for her part to sound. I replied to her email and asked if she would like to meet and discuss her comments further, to which she agreed. When I met with Mrs. O’Neill in person, she explained:

I know that we expressed to you often our lack of time that was just ours alone for planning and that kind of thing. So that filtered out into we didn’t have time to
research, to find tasks and all that, and having you do that was awesome. But, I felt like when you wrote it out, it came across as extremely negative. It didn’t come out in a very flattering way for the county, for the school, or for us.

She did not identify any specific parts of the text that were negative; rather she said she felt the overall tone was negative. She did agree with my statement that due to the teachers’ many responsibilities, there was not time in the day for quality planning, and she elaborated on her many roles within the school, which included team leader, data team leader, and having a prospective teacher intern. Mrs. O’Neill said that I accurately portrayed her students as unable to implement high cognitive demand tasks because of their lack of mathematical abilities. I believe she misinterpreted what I wrote as I did not mean that I saw the lack of students’ abilities was a barrier; rather, I said that the teachers viewed the students’ lack of abilities as a barrier to implementing tasks. Mrs. O’Neill commented that she was just putting in her “two-cents worth” and wanted to make sure I realized the negative tone of the analysis, but ultimately she said it was up to me if I wanted to change anything.

In response to Mrs. O’Neill’s comments, I looked over the second research question and did fix some repetitiveness issue, but overall I felt the tone of the analysis was accurate. While I understood Mrs. O’Neill’s view on the tone, it was my intention to showcase the lack of space within the school day for the teachers to have quality planning time. I believe that my depiction of the teachers was not overly negative, because I included both positive and negative factors that affected the implementation of tasks. I did make a claim that Mrs. O’Neill has a narrow view of mathematics, and while I could see that as being negative, she did not comment on that section. I invited Mrs. O’Neill to
make specific comments in the documents and told her I would take those comments into consideration, but I never received any additional information from her. Based on our conversation, I added her additional school roles to her teacher profile to show the complexity of her teaching responsibilities.

When I met with Mr. Cone in person, he said he did not have issues with the analysis. He said it was interesting to read my perspective of him in the classroom and of how the teachers planned their lessons together. He said that being able to read someone else’s perspective on his teaching was “good and bad at the same time, and very, very honest.” He said he found my perspective to be valuable, and he was going to use the analysis from the second research question to help him in the future:

Part of the repetitive nature that you had to include was interesting because it highlighted the things that we did over and over again. When I read about myself saying I altered things I was like “Oh, I did do that, lowering the cognitive demand or having lowering expectations for [the students].” It was something that I say I am going to reread in August before the first day of school, and maybe every month, to remind myself that I shouldn’t be doing those things.

Overall, Mr. Cone said that what I wrote was an accurate portrayal, and therefore I did not change any part of my analysis after talking with him.

**Researcher bias**

While I brought bias to my research study, I also had ways of dealing with bias through data collection and analysis. As a former classroom teacher and current teacher educator, I have ideas about how a teacher’s mathematics classroom should operate when
implementing high cognitive demand tasks. I needed to find ways to balance my views of how classrooms should operate while respecting the constraints each teacher faced and being mindful that the teachers might not share my ideals. Because I was a participant researcher in this study, the first step I took to deal with bias was to keep a researcher journal. I used the journal as a place to write down thoughts entirely from my perspective and to keep track of these thoughts. I wrote in the journal after interactions with the teachers and during times when I was struggling working with the teachers. I used the journal as a space to write down my frustrations and keep that in check during the study and during analysis. During the study, I provided interventions for the teachers as needed. For instance, I provided a mini professional development on using tape diagrams and number lines for solving proportions when the teachers told me they did not know how to use those strategies when solving proportion problems. I kept both written and audio notes after instances of interventions to make note of times when I intervened. I made note of when the intervention happened, why as well as how the teachers reacted to the intervention, and if they employed that strategy in their classrooms. My field notes consisted of a running record of classroom interactions. I tried not to make judgmental statements in my field notes and only recorded factual statements about what was happening in the classroom. On the occasions when I felt the need to make a note about my perceptions of the lesson, I did so in the margins. I did not include these in the classroom write-ups.

When answering the first research question that includes my perspective, I made sure to be transparent about my perspective versus that of the teacher. I kept the sections separate and did not include my perspective while writing up the teacher’s perspective. I
had a section for my perspective on the teaching episode and a section that compared my perspective to the teacher’s perspective. This way it is clear to the reader which is my perspective and which is the teacher’s perspective. Both Morrow (2005, 2007) and van Heugten (2004) suggest having others to talk to about the research and data analysis. I met weekly with my major professor to discuss my finding and data analysis. We discussed sections of the analysis to make sure I was not allowing my bias to skew the results. I also had her read sections of my analysis to make sure my voice did not interfere with the teachers’ perspectives, especially related to the first research question. Talking with someone about how I interpreted what happened allowed me to interpret my data with more validity. Having a person to challenge my assumptions allowed me to be confident of my analysis.
CHAPTER 4

FINDINGS

Teachers’ Classroom Practices When Implementing High Cognitive Demand Tasks

Two research questions guided my study: (1) What are teachers’ perspectives of their classroom practices as they implement high cognitive demand tasks, and how do teachers’ perspectives compare to the researcher’s perspective? (2) What factors do teachers identify as affecting the implementation of high cognitive demand tasks during all phases of the task implementation process? In this chapter, I detail the findings from both research questions. I first present my analysis of research question one. This section includes a separate analysis for each of the three teachers for both the Figure S task and the Border Problem. For each teacher I provide a narrative of each task implementation followed by my perspective on the implementation, the teacher’s perspective of the implementation, and a comparison of both perspectives. For the second research question, I begin by presenting the additions I made to the task implementation framework (Stein et al., 1996). I then describe the factors all three teachers said affected implementation and then share the factors that were unique to each teacher.

Mr. Cone

Implementation of the Figure S task. On the first day of class for the new school year, Mr. Cone presented the Figure S task (See Figure 4) to his students, saying
there were many different types of activities, the hardest one was “doing mathematics”, and today they were going to “do mathematics.” Mr. Cone instructed students to look at the Figure S pattern and answer the questions by working quietly with nearby classmates. The students engaged with the task, but bearing in mind this was the first day of class, he prompted them often to talk with neighbors. Mr. Cone circulated the room with a sheet on which he had anticipated the possible student solutions and prompted students to talk with one another when he saw partners who had similar or differing solutions.

Figure 4: Figure S task, (adapted from Smith, Hillen, & Catania, 2007, p. 41).

A student asked how many blocks would be in a figure with a base of 10 cubes, and Mr. Cone pointed out the first figure had a base of two, the second figure had a base of three, and the third figure had a base of four and told the student to imagine a figure
with a base of 10. As class wound down, Mr. Cone told students he wanted them to answer one question: How many squares were in any step? In other words, how many were in step 1, step 2, step 3, step 100, step 1000 and so on? He asked them to write down a method for finding the answer to his question. When a student shared his method, Mr. Cone asked if that worked on every other step, and the student realized it would not. The class ended and Mr. Cone told students to make sure they had a method written down to share with the class when they met again before lunch. This was the first day of school so the teachers saw their homeroom classes before lunch because the extended learning time schedule was not complete.

When the class met for the second time that day, the students presented their work. One pair explained that they multiplied the height of the figure and the width and added the two extra blocks to find the number of squares in any sized figure. Another student said, “To find how many squares there are, add one to the step it is and that equals the base, then the height equals the base plus 1, and then times the base times the height and add 2.” Mr. Cone asked a student to draw a figure of base six using what the previous student had said. The student came to the board and drew out the figure while the other student verbally explained his method to show it worked in finding the number of squares in a figure with any base.

**My perspective.** Using the Instructional Quality Assessment (IQA) rubric (Boston, 2012), shown in Figure 5, I rated Mr. Cone’s implementation of the Figure S at a level 3. Evidence from the rubric I observed included students engaging in the task and communicating mathematically with peers. Additionally, the students had appropriate knowledge to access the task because it was a patterning task and had multiple entry
The students had access to resources that supported their thinking because they had squares they could work with as well as their peers. Mr. Cone gave the students sufficient time to work on the problem. During the discussion time in class, the students identified patterns, made conjectures, and used evidence to test conjectures. During student presentations, Mr. Cone questioned students about how they arrived at their solution methods. The reason the implementation of the task cannot be rated a level 4 is that although students used multiple strategies, no connection was made between strategies or to the mathematics in the problem. While I observed students engage in the problem using complex and non-algorithmic thinking to solve the task, identify patterns, and create a generalization from the pattern, connections were not made between solutions (Boston, 2012). At the end of class, students provided explanations for finding the $n$th figure in the pattern but due to time constraints, there was little discussion around the generalization and class just ended.

Figure 5: Task implementation rubric for a level 3 (Boston, 2012, p. 20).

I noticed several factors that affected the implementation of the Figure S task as described by Henningsen and Stein (1997) in the task implementation framework. During
the set up of the Figure S task, both task features and the cognitive demand of the problem contributed to Mr. Cone successfully maintaining the demand of the task. The nature of the Figure S task, labeled as doing mathematics (Smith & Stein, 2011), meant that it had multiple entry points and allowed students to engage in finding multiple solutions.

During the implementation of the Figure S task, there was evidence of factors influencing students’ engagement with the task. As this was the first day of school, Mr. Cone attempted to set up classroom norms, including students working together and not relying on him for the answers, or only talking to him when they had questions. When he saw students with similar or different solutions, he told them to talk with their partners about the solutions. Mr. Cone’s instructional disposition helped maintain the demand of the task because he allowed students to struggle with the problem. This occurred when students asked questions and Mr. Cone did not answer but referred them to their partner, showing his disposition was not to answer students’ questions immediately. When a student told him his solution for finding the number of squares in any size figure, Mr. Cone asked if it would always work, attempting to get the student to check his work and figure it out for himself. There was an instance where Mr. Cone was helping students find how many squares would be in a figure with a base of ten. He told the students to pay attention to the number in the base and imagine what that figures looked like, but he did not tell the student explicitly how to find the answer. In another instance, a student told Mr. Cone his answer, and Mr. Cone pressed and asked how he found it. Mr. Cone’s instructional disposition throughout the Figure S task was one that allowed the students to struggle and therefore helped maintain the demand of the task.
A hindrance to maintaining the demand of the task was the task conditions. There was a short amount of time for the students to engage in the task due to it being the first day of school because the periods were shorter that day, and there were administrative details to attend to. The first day of school was a Wednesday, and Mr. Cone knew he would be absent the following two days, so he needed the students to complete the task in one class period. Mr. Cone did not return to the task the following week. Mr. Cone was able to see his one of his classes (the one I observed) for two periods that day because they returned to him for the period before lunch, but he only saw his other classes once. This did not allow the students to engage in some of the more complicated aspects of the task, including discussing solutions for generalizing for a figure with any size base and connecting between solutions.

The students learning dispositions positively affected the implementation of the task. The students engaged in the task and found different solution methods. Initially, the students mostly worked by themselves, but I attribute this to being the first day of school and not knowing other students in the class. As the lesson progressed, students started to work in pairs and groups and continued to engage with the task. The problem was a patterning task that had multiple solution methods, so the students did not need much prior knowledge to access the first part of the task. During the task, the students had success with the first three questions, and some students found a solution for any size figure. The students presented information about the task at multiple points throughout the class and briefly talked about finding the general pattern.

There was evidence throughout the lesson of Mr. Cone using the five practices (Smith & Stein, 2011) to frame his instruction. Prior to the lesson, Mr. Cone anticipated
possible student solutions during a task planning session with the seventh grade team. He used these possible solutions to monitor students at the beginning of the lesson. Mr. Cone printed out the possible solutions and walked around the room, marking down when he saw instances of each solution method. He did select students to present their work, but it did not appear sequenced in a certain way. Perhaps due to the lack of time, there was no evidence of connecting between student solutions and connecting solutions to the mathematics of the task.

**Mr. Cone’s perspective.** Mr. Cone identified several factors that affected the implementation of the task. While he did not use terms from the task implementation framework, he identified instances of his instructional disposition, the task conditions, and instances of students’ learning dispositions as factors that affected implementation.

Mr. Cone thought that overall the lesson was a success, but due to not having enough time to engage in the task, he felt the students missed key aspects of the task. Because it was the first day of school there were logistical factors that shortened the amount of class time the students had to engage in the task, and he was disappointed because he thought the students were having productive discussions around the task. He said, “The kids were really talking and thinking, and I would have liked to have more time because we didn’t get to share all of their answers. [A student] had some algebra that people did not really get to see.” Mr. Cone thought the students engaged successfully with the task, mostly because of the nature of the task:

I know some of them were miffed to do math on the first day of school, but overall I don’t think they were too upset. I mean the pattern itself, drawing out the pattern for the most part was a fairly easy task for [the students], but doing the
explaining was hard. I feel like they did part of it, and then kind of struggled with the rest of it and I am ok with that struggle, but I think that they seemed to be ok. Mr. Cone thought that although his students struggled with certain aspects of the task, overall they found some success with the task. He explained that his disposition was to allow the students to struggle with the task and not readily give away information. Mr. Cone thought he maintained the demand of the task during implementation:

Not giving [the students] information. After [the seventh grade team] had talked about [the task] and talked through the solutions, I was very aware of not giving away little pieces of information consciously. I remember making a very clear effort to ask them question that would lead them in the right place or give them options. ‘What do you think about this or this? How does the length and the width of the figure change as it increases?’ If they had the width wrong for example, instead of saying ‘how does the width change’, give them maybe both options, or instead of pointing out the width or telling them to go back and count, say ‘how does the width change?’ and asking them so they have to do the work.

This statement shows that Mr. Cone was consciously trying not to give students pieces of information as he walked around from group to group. His instructional disposition was to make an effort to question students about their solutions and to make broad statements that would not immediately point to what the student had done incorrectly.

**Comparing perspectives.** Overall, Mr. Cone and I had similar assessments of his implementation of the Figure S task. While he did not make connections between different solutions, he also did not point to ways of solving the problem and did not give
information that significantly lowered the problem. The students engaged in the activity and generalized the pattern.

**Implementation of the Border Problem.** The Border Problem (Figure 6) took place over a two-day period. In this analysis, I am looking at the two days of the Border Problem as one implementation.

![Border Problem](image)

*Figure 6: Border Problem (adapted from Boaler & Humphreys, 2005).*

**Day 1.** Mr. Cone introduced the Border Problem by showing his students the picture, saying it was a 10 by 10 square, and asked the students to find the border of the pool without counting the squares one by one. A student asked if the border was “how much is around,” and Mr. Cone said, “Yes, it is the perimeter.” Students started working by themselves, and Mr. Cone walked around and talked with students. When he realized many were getting an answer of 40, which was the perimeter of the figure, he pulled the students back together and asked how many got 40, 38, and 36. When one student raised his hand for 36, Mr. Cone asked him to come to the board and explain his reasoning. From this student’s explanation, the rest of the class understood what the problem was
asking. Mr. Cone gave students more time to figure out other solution methods. Mr. Cone brought the class back together, and students explained how they found the answer of 36 while Mr. Cone wrote their ways on the board and assigned each the name of the student who gave it to him (see Figure 7). The students came up with \((9 + 9 + 9 + 9)\), \((10 + 10 + 8 + 8)\), \((10^2 - 8^2)\) and \((10 \times 4 - 4)\) while Mr. Cone gave the students \((10 + 9 + 9 + 8)\) and \((9 \times 4)\), saying these solutions came from two students who had done the problem last year.

*Figure 7: Mr H’s student solutions for the Border Problem.*
Mr. Cone instructed students to find the perimeter of a 6 by 6 square using any method on the board. After the students had time to work, he asked for a student to explain his/her answer for each method. He called on volunteers, and these were not necessarily the students who the methods were named after. After going over all the solutions for the 6 by 6, Mr. Cone instructed students to work with their partners to find the border of any size pool. I was trying to assist Mr. Cone in getting students to talk with each other, so I intervened when a student wanted to tell Mr. Cone his solution method. I told Mr. Cone to instruct the student to first talk with his partner before telling Mr. Cone his solution method. The next time a student tried to tell Mr. Cone his method, Mr. Cone asked the student what his partner thought and when the student said he had not talked to his partner, Mr. Cone said he would not listen until the student explained it to the partner and they both understood and walked away. Mr. Cone ended class by having students write down whether they used the same method to solve the 10 by 10 and 6 by 6 squares and explain why or why not.

Day 2. During our interview, when I asked Mr. Cone what he struggled with he told me when implementing this task in his second class the co-teacher in the room asked him about his use of the word perimeter, and he realized it was not the correct word to use. Thus, in his first period class on the second day he explained that they were looking for the number of squares in the border of the pool. Mr. Cone opened the lesson with a warm-up asking students to write the steps to find the border of a 7 by 7 pool. Mr. Cone asked someone to share his or her steps, and a student said he used $(7 \times 7) - (5 \times 5)$. Mr. Cone wrote $(7^2) - (5^2)$ and asked if the answer was the same or different when solving the two expressions. A student responded that they were different and Mr. Cone told the
students to work with group members to find if the expressions were the same or
different. After the student determined the answers would be the same Mr. Cone said they
were equivalent expressions, which means they looked different but gave the same
answer.

Mr. Cone moved on to finding the border of any size square and talked about the
verbal steps a student used to find the border and how to write that algebraically (see
Figure 8). He took different expressions from the previous day and wrote them on the
board using variables. He wrote two algebraic solutions on the board \((4s - 4 = \text{border})\)
and \((2s + s - 2 + s - 2 = \text{border})\) and asked students to talk about what was the same and
different between the first and second one.

![Figure 8: Converting numerical expressions to algebraic expressions.](image-url)
He walked around as the students talked about the question and tried to make sure the
groups were holding each other accountable by instructing students to talk with one
another about possible solution methods. Then the class discussed what was the same and
different about each expression. Mr. Cone tried to connect the expressions by using
algebra tiles and showed a picture where there were four rectangular algebra tiles and
four little square algebra tiles in the shape of a square (see Figure 9).

Figure 9: Border Problem represented with algebra tiles.

He wanted students to understand that $4s + 4$ represented 4 times the side length plus the
four corners and represented $s$ with a rectangular algebra tile. The students had trouble
simplifying the expressions, especially $(s + s + s + s = 4s)$ so Mr. Cone did a mini review
on why multiplication is repeated addition, giving the example, $(5 + 5 + 5 + 5 = 20 = 4 \times 5)$. Mr. Cone then wrote $(4s - 4)$ and $(2s + s - 2 + 2 - 2)$ on the board, and I interjected
and asked what was the same and different between the two expressions. A student said
they both have the same answer, and Mr. Cone asked him to elaborate. The student explained that the two expressions had the same amount of rectangles and squares (talking about the algebra tile representation). Mr. Cone asked for more similarities and differences, and a student said they were different because one expression had a four. The students did not seem to understand what he was asking, so Mr. Cone gave them the definition of equivalent expressions: expressions that look different but give the same answer or picture. Mr. Cone tried to give another problem but ran out of time.

**My perspective.** Using the IQA I rated this lesson a level 4 (see Figure 10). The students engaged in using complex and non-algorithmic thinking and solved a genuine problem while showing evidence of reasoning throughout the task (Boston, 2012). Mr. Cone asked students for their answers, giving them mathematical authority, and then gave them freedom to use any solution method. He had students explain the method used and why that method worked. He made connections between solutions and representations of

![Figure 10: Task implementation rubric for a level 4 (Boston, 2012, p. 20).](image)

the Border Problem the second day. Students conjectured about how to find any size figure and supported their conclusions (Boston, 2012). Overall the students engaged in a doing mathematics problem, and Mr. Cone maintained the demand.
During my observation of the Border Problem, there was evidence of many factors influencing the students’ implementation of the task. The task conditions allowed all students to engage with the task. The task is a high cognitive demand task, specifically *doing mathematics* (Smith & Stein, 1998), and had multiple entry points for students to access to the task. A task condition that helped maintain demand was time. While there was not enough class time in the first day to connect finding a border in a concrete case to any size square, Mr. Cone showed that the solutions gave equivalent expressions the following day.

Mr. Cone initially set up the border task without much explanation. When a student asked a question about the border, Mr. Cone mistakenly said “perimeter,” which did not end up affecting the demand but initially affected how students engaged in the task. Most students initially found 40 as their answer, thinking they were looking for perimeter, but Mr. Cone corrected the misconceptions without lowering the demand. Going into the second part of the task, Mr. Cone allowed students to use any method to find the number of squares in the border of a 6 by 6 square. He gave just enough information to allow access to the problem and maintain the demand. After going over the solutions to the 6 by 6 square, Mr. Cone simply asked students to find the border of any size square. Mr. Cone set up each aspect of the task and maintained the demand. During the second day, Mr. Cone set up the task without much explanation and allowed students to engage without much interference.

Mr. Cone’s instructional disposition was a factor that helped maintain the demand of the task. When Mr. Cone realized students were getting the wrong answer because they were finding perimeter, he pulled the class back together to talk about what they had
so far. When he realized that one student thought about it the right way, he had that student explain his answer and made sure the rest of the class understood what to do. He then had the students try to find other ways to solve the problem. Mr. Cone did not give the students a specific procedure to follow, thus maintaining the demand. There were instances where students asked Mr. Cone a question and he did not give a specific answer but referred them to their partner. There was one instance where Mr. Cone started to answer a student, but I interjected and told Mr. Cone to have the student talk with his partner first before talking with Mr. Cone., trying to get Mr. Cone to realize he did not have to answer every time a student asked a question but could refer the student to his partner and get the students talking to one another. The next time a student tried to tell Mr. Cone his answer, Mr. Cone told him to talk with his partner first. During the second day when a student asked if the two expressions would give the same answer, Mr. Cone put it back on the students to decide. This maintained the demand because he put the mathematical authority in the hands of the students. This is an example of Mr. Cone not specifically answering a question but putting it back on the student to answer.

Classroom norms both helped and hindered the cognitive demand of the task. There were instances where it seemed as though the norm was for the students to tell Mr. Cone their answers and not discuss any aspects of the task with their peers. Mr. Cone did try to get students to talk with each other and at times was successful. The students did seem comfortable presenting their ideas in front of their peers, suggesting that presenting occurred often in the classroom.

The students’ learning dispositions affected the implementation of the task. The students had high engagement in each section of the task and successfully found
expressions for borders of the 10 by 10 and 6 by 6 squares. They briefly engaged with any size border but ran out of time. There were instances when the students wanted to check their answers with Mr. Cone or asked him questions before first talking with group members. The students mostly engaged in the task and found multiple solution methods when finding the number of squares in the border of the 10 by 10 square.

There was evidence throughout the task that Mr. Cone used the five practices (Smith & Stein, 2011) to frame his lesson. One evident practice was anticipation. When the students did not come up with two solution methods, Mr. Cone gave them to the students and explained they were solutions found by students in another class. He also anticipated incorrect solutions when he asked if students got 40, 38, or 36. The seventh grade team worked through the Border Problem and together anticipated possible student solutions. Mr. Cone showed evidence of monitoring as he walked around the classroom and observed which methods the students used. Mr. Cone selected students to share their answers and had students present their solutions. During the second day, Mr. Cone attempted to connect the algebraic model to a picture model with algebra tiles. He tried to connect the expression $4s - 4$ to the algebra tiles by representing the sides as rectangular times and the corners with the small square algebra tiles. I did not see evidence of sequencing the solutions in any intentional way.

**Mr. Cone’s perspective.** Mr. Cone described factors that influenced both the set up and the implementation of the task. Mr. Cone’s plan was to give the students the image and have them find the number of tiles in the border of the pool. He wanted students to show ways they found the border on the board and he planned to show any additional methods. Mr. Cone ideally wanted students to look at the algebra that went
along with the lesson so the students would see the expressions were equivalent, but he ran out of time. Mr. Cone explained his goals for instruction, saying:

I wanted them [the students] to see that there was a bunch of different ways to solve the problem, because I still think that as 7th graders, and 7th graders in this classroom, they don’t understand enough that there are multiple ways to solve many problems, and those ways all get correct answers. Especially as we are talking about expressions and equations, and equivalent expressions, there’s a bunch of different ways to write that algebraically, but the end result is always the same and that’s what really defines the equivalent expressions. I wanted to get at that idea; different ways, same answer.

Mr. Cone said he accomplished that goal and attributed it to the students’ learning dispositions and task conditions, saying:

Because the students were able to, by the end of the lesson, and in the next day also as we continued the discussion, the students were referring to the name of their method. So I used the B method that I made up from last year, or I used the M method or the M junior method, and would still understand they were coming up the same number of tiles as someone else who used the different method.

Mr. Cone said his students used different methods and saw that different methods led to the same answer. Mr. Cone acknowledged that his use of the word perimeter unintentionally misled the students and explained in the second class his co-teacher came up to him and asked if perimeter was the right word to use when describing finding the border. Mr. Cone realized he was incorrect using the word perimeter in the earlier class.
because the students were looking for the number of tiles that bordered the pool. Overall, he thought he set up the task well and maintained the demand.

Mr. Cone identified several factors affecting implementation, including his students’ learning dispositions and the task conditions saying, “[the students] were engaged. The task had a low floor for entry because you could kind of count and you could divide it in a bunch of different ways, but then some students had a chance to think creatively.” Mr. Cone said the task was successful because all the levels of his students engaged in the task. Mr. Cone liked that the task was one all his students could access but still provided a challenge for his more advanced students. Mr. Cone thought the work period went well and students’ learning dispositions contributed to the success because “the students were able to come up with the borders for the 6 by 6 pool, name their method, pick a method they liked using and understand it enough to continue using it.” Mr. Cone based the success of his implementation on how well the students found an expression for the border of the original pool and used that or a different method to find the border of a smaller pool.

Mr. Cone identified times that he lowered the demand, noting that when students got 100 he pointed to their answers and told them they were looking for the perimeter, not the area. Mr. Cone realized by telling students 100 was the wrong answer and why it was incorrect he was lowering the demand of the task; he attributed this to his instructional disposition, saying:

I would point to their answer and say, “We are looking for the perimeter of the pool not the area of the pool” instead of prompting them through “What’s the
difference between perimeter and area?” and leave it at that, and let them go back to work.

Conversely, he attributed his instructional disposition to helping to maintain the demand saying, “Throughout class I was making a conscious effort to not answer their questions, to require them to ask the people at their table before they asked me.” Mr. Cone realized his instructional disposition contributed to maintaining and lowering the demand of the task, depending on whether or not he gave students answers.

Mr. Cone identified task conditions as a factor that influenced the implementation of task saying there was not enough class time to get through the entire problem, saying, “We didn’t do the algebra of the problem very much, and we just kind of dipped our toes in it.” Mr. Cone thought “student engagement, student thinking, student access, and students being able to see different ways to answer the same problem correctly” were successful aspects of the lesson. These factors all describe students’ learning dispositions. According to Mr. Cone, he was successful with providing his students with a problem they could find success in, and they saw more than one way to solve the problem. This aligned with his goals for the lesson, although he did not achieve his goal of showing how the different solutions are equivalent expressions.

Comparing perspectives. When comparing my perspective to Mr. Cone’s perspective, we agree on the definition of success with the lesson. Students explained what methods they used and explained a different method when asked but did not make the connection that the solutions were equivalent expressions. I agree with Mr. Cone that using the word perimeter was problematic but thought he was able to correct the students’ thinking that resulted from his misuse of the word and still maintain the demand. Mr.
Cone had a student explain how he arrived at an answer of 36 tiles, giving the student mathematical authority in the classroom, and then once students were able to see how to find the border, Mr. Cone asked them to find other ways to find the border without counting. He never told them a set way to solve the problem or focused on getting the correct answer. Mr. Cone said that by not answering students’ questions, he was maintaining the demand of the task. Our perspectives differ on this aspect because while Mr. Cone did not answer their questions, which helps to maintain the demand, he did not help them to make connections. On the second day, Mr. Cone did a better job of putting the mathematics back into the hands of the students by asking what they thought about the different expressions. He also tried to connect different models of representations so students could better understand equivalent expressions.

**Portrait of Mr. Cone.** While working with Mr. Cone, I found him to be a reflective teacher who was eager for feedback and actively sought ways to improve his teaching. Even when the feedback on his practice was critical, he would think about it, and I could see changes in his practice. One example of this occurred when we discussed his tendency to give students the answer right away. After talking with him about strategies to question students more, he attempted to question his students more the following day, but was not always carried through in subsequent lessons. When we talked about the implementation of the Figure S task, he said one successful aspect of the task was that he did not give information to the students. He then explained he had not carried that strategy through with the same bravado with other tasks. While he did sometimes
give students the answers, throughout the semester, I saw improvement in his ability to question students and not give information away too quickly.

Mr. Cone often said that one of the supports he most liked was having another pair of eyes in the classroom. While he realized it was a critical eye, it helped him to change his practice. He said that having a second person in the classroom to watch his actions and provide feedback helped him improve his teaching. During the member check, Mr. Cone said that while it was difficult to read about his teaching, he found it helpful to read his actions and think about his teaching. He also said that he realized that he modified tasks for lower level students and should not continue to do that. He said he would read his analysis again before the beginning of the school year and periodically throughout the year in order to remind himself that he should be giving all levels of students high cognitive demand tasks.

Overall, I felt that Mr. Cone gained the most from being in my research study. He said he was going to implement the Figure S task and the Border Problem the following year, and he wanted to try to implement more high cognitive demand tasks. I think if I could have continued to work with Mr. Cone and been more explicit using the five practices to frame instruction, he would improve his ability to implement high cognitive demand tasks at a higher level and maintain the demand throughout.

Mrs. O’Neill

**Implementation of the Figure S Task.** Mrs. O’Neill introduced the Figure S task on the third day of school, which was a Friday. She started the class period by reviewing patterns and asked students about how to find patterns. For example, she displayed X 0 #,
asked what came next, and the students responded with the correct pattern. She introduced the Figure S task and asked students if there was only one correct way to do something, and they said no. She told the students that they may come up with a different way to solve the problem than their partner and that it was fun to see the different ways. She read the question and told students to start thinking individually what the answers could be and in a few minutes, they could work with their partner. She told students to draw or use blocks and if they had a question to ask their partner first and if neither could figure it out then ask either her or me. She handed out the paper, and we both walked around to help the students.

Mrs. O’Neill often saw me as a person to help her implement tasks, perhaps suggesting that I did not define my role as a researcher to her. I often would get up in the middle of documenting lessons to help because the students struggled with the mathematics or were off task. On this particular day in class, I watched her set up the lesson, and then I walked around while the students worked. Thus, I was not able to take field notes on the interactions Mrs. O’Neill had with her students.

My perspective. Because I did not take field notes during the time the students worked on the task, I was unable to apply the IQA rubric to the lesson. I was able to identify factors related to task set up and task as implemented by students. Mrs. O’Neill maintained the cognitive demand of the task when presenting the problem because she did not give any information away. Mrs. O’Neill explained there would be multiple solution methods and told students that if someone else had a different way of thinking about the task, they should talk about it with their partners. Even though Mrs. O’Neill reviewed patterns at the beginning of the lesson, she did not talk about how to find the
pattern or focus on procedural understanding related to the task. Factors influencing implementation included Mrs. O’Neill trying to set up norms. She had students work in pairs, encouraged multiple solution methods in her introduction, and suggested using blocks or drawing a picture to try to solve the task.

**Mrs. O’Neill’s perspective.** Mrs. O’Neill said she thought the patterning task went well because it was a “puzzle” and the students enjoyed puzzles, which relates to the task features. She thought the lesson was successful because the students engaged with the task and the class had a discussion around the task, which relates to students’ learning dispositions. She commented on her teaching disposition, saying she helped the students too much, thus lowering the demand. But she justified lowering demand saying, “In instances where [the students] were getting frustrated and I could see they were going to shut down, so without giving them, ‘ok why don’t you try this instead’, they wouldn’t have done it.”

**Comparing perspectives.** Overall, I would say our perspectives are the same on student engagement of the task and disagree on Mrs. O’Neill’s teaching disposition. Mrs. O’Neill thought the students needed her to tell them the answers in order to keep going whereas I think there were ways she could have helped the students by questioning and providing scaffolding that would prevent them from shutting down but without giving away solution strategies.

**Implementation of the Border Problem.** Mrs. O’Neill started the lesson with PowerPoint slide that said “key vocabulary today” with the words variable, equivalent expressions, and perimeter. She asked what a variable was, to which a student replied it was a symbol used to represent a number. She asked what it meant for something to be
equivalent, and a student said equal. Mrs. O’Neill said they would be talking about equivalent expressions, or expressions that were equal during the lesson, explaining that they may get many different answers, but they would all be the same. Mrs. O’Neill asked for the definition of perimeter, and the students said you add up all the sides. She reiterated there were three words they needed to understand: variable, equivalent expressions and perimeter. Mrs. O’Neill. put the Border Problem on the board (see Figure 11) and said, “Without counting one by one, find how many squares are in the border.” The students worked on the task, and Mrs. O’Neill walked around the classroom monitoring the students and correcting when they tried to count the border tiles individually. The students worked mostly by themselves with a few groups of students working together. Mrs. O’Neill circulated and talked with students about their solution methods. While most students engaged in solving the Border Problem, a few students were off task, and Mrs. O’Neill had to position herself by those students so the rest of class could remain on task.

Figure 11: Border Problem from Mrs. O’Neill’s class.
After about 10 minutes of work time, Mrs. O’Neill brought the class back together and asked a student how many tiles were in the border, and the student replied, “100.” Mrs. O’Neill asked if 100 was the number of squares in the orange part (See Figure 11), and the student said the orange part was 10 by 4. Mrs. O’Neill asked what 100 represented, and the student said area. Mrs. O’Neill asked if the student was thinking $10 + 10 + 10 + 10$ to get 40 and circled each row and column, making it obvious that the corners would overlap if you counted each row of 10 once (see Figure 12). Mrs. O’Neill asked another student for her answer, and the student said 36 and explained that she used the equation $10 + 10 + 8 + 8 = 36$, counting two rows of 10 and 8 blocks in the other two columns. Mrs. O’Neill circled the top two rows of 10 and then the middle rows of 8 and wrote 36 in the columns. Mrs. O’Neill then asked the class what they thought of the two solutions, emphasizing that they goal was to find the number of tiles around the “edges” of the picture.

![Figure 12: First and second student solutions for Border Problem in Mrs. O’Neill’s class.](image)
A student agreed with the answer of 40 “because you were counting the edges.” Mrs. O’Neill then explained that the first student counted the corners more than once and the second student counted the corners only once, clearly suggesting that 40 was not the correct answer. Mrs. O’Neill then asked another student how she came up with 36, and the student said there were four corners for 4 and then four sides of 8 so $4 \times 8 = 32 + 4 = 36$ (see Figure 13).

![Figure 13: Third student solution for Border Problem in Mrs. O’Neill’s class.](image)

Mrs. O’Neill then noted that both the second and third solutions counted the corners only once and asked the class why she would want to count corners twice if she wanted to find exactly how many bricks to put around the pool. Mrs. O’Neill then said the answer was 36 and told the students to find a way to count the bricks that gave them the answer of 36.

After a few minutes of the students working individually or in pairs, Mrs. O’Neill had more students explain their methods. She called on a student to explain her method, and the student explained that she circled 10, then 9, then 9, then 8 tiles to get 36 (see
Figure 14). Mrs. O’Neill asked the class how the student got 9 but did not wait for the class to answer and told students to subtract 10 – 1. Mrs. O’Neill asked the class how the student got 8, but again did not wait for students to answer and said to subtract two from 10. Mrs. O’Neill explained that the expression 10 + 9 + 9 + 8 was the same as 10 + (10 – 1) + (10 – 1) + (10 – 2). Another student explained her solution in which she counted three tiles in each corner, the corner and the two tiles on either side of it, for each of the 4 corners to get 12 tiles, leaving 6 tiles per side or 24 more tiles. Thus, because 12 + 24 = 36, there were 36 tiles in the border. (See Figure 14 for Mrs. O’Neill’s depiction of the student’s solution). Mrs. O’Neill then redirected the class’ attention to the original solution of 100 and told the class that she wanted them to use the area of the figure to find the number of tiles in the border. A student suggested finding the area of the middle section, 8 x 8 = 64, and subtracting the inner area from the total area for 100 – 64 = 36 (see Figure 14). Mrs. O’Neill then fixed the solution that said 10 x 4 = 40 and changed it to (10 x 4) – 4 = 36, crossed off where the corners were double counted, and explained this was to subtract off the corners (see Figure 14).

Mrs. O’Neill put a picture up of a 6 by 6 square and told the students to choose one solution method from the board and use it to find the expression for the number of tiles in the border of the 6 x 6 square. She told them to work first on their own and then discuss with their partners. She told the students to think about which solution strategies they liked best and then to think about what the expression would be for an n-by-n square.
The students worked for five minutes and Mrs. O’Neill started to wrap up the lesson and asked a student for the answer to the 6 x 6 problem. The student said “20,” and Mrs. O’Neill asked if the class agreed, which they did. Mrs. O’Neill asked someone to explain how he or she found 20, and a student said, “6 + 5 + 4 + 4”. Another student said “6 + 6 + 4 + 4,” and the student who used the corner strategy earlier said she got (3 x 4) + (2 x 4). Class was ending, so Mrs. O’Neill told students to write down the homework.

**My perspective.** In looking at the IQA for this task, I rated Mrs. O’Neill’s lesson at a level 3 (see Figure 5) because the task had the potential to engage students in complex thinking, but factors existed that did not allow students to fully engage with the task (Boston, 2012). The students engaged in complex thinking and identified patterns for the 10 by 10 and 6 by 6 borders, but they did not have an opportunity to make generalizations, mostly due to lack of time. The students found multiple solution methods for the Border Problem but did not make connections between the different
representations (Boston, 2012). When Mrs. O’Neill showed how to write $10 + 9 + 9 + 8$ in terms of the side length 10, she attempted to have students recognize the pattern in relation to side length, but due to time, the students did not generalize the pattern for any size square.

Many of the factors affecting implementation described by Henningsen and Stein (1997) in the task implementation framework existed in this lesson. Prior to introducing the task, Mrs. O’Neill reviewed key vocabulary words including perimeter, which initially mislead students to thinking the answer was 40. Mrs. O’Neill also told students they would be finding equivalent expressions, instead of letting students see that for themselves, thus lowering the demand. This unnecessary review also consumed class time that could have been used to help students reach a generalization. These factors influencing set up were related to Mrs. O’Neill’s knowledge of students because she said she gave the review because the students needed the review in order to access the task. After the review, Mrs. O’Neill maintained the demand of the Border Problem during set up. When she introduced the Border Problem, she gave minimal directions, which allowed students to access the task at a high level. She set up trying to find the border of a 6 by 6 in the same way, maintaining the demand.

There were many factors influencing students’ implementation that existed during the lesson. Mrs. O’Neill’s instructional disposition affected the implementation of the task, sometimes helping to maintain the demand and sometimes contributing to lowering the demand. Mrs. O’Neill gave students answers without allowing them to struggle, thus lowering the demand for those students. There was an instance when Mrs. O’Neill changed an incorrect answer to a correct one and explained to the students why but did
not ask them why it was incorrect. In contrast, there was also an instance when Mrs. O’Neill went back to a student’s misconception about finding the area and asked the class how they could use that to find the border, maintaining the demand and allowing the student to take his misconception and turn it into a solution method, which maintained the cognitive demand. Mrs. O’Neill set norms by telling students to first work individually and then work with a partner. The task conditions and students’ learning dispositions allowed students to engage in finding multiple representations and solution methods, and students had the opportunity to present solutions.

Throughout the lesson, there was evidence of Mrs. O’Neill using the five practices (Smith & Stein, 2011). Mrs. O’Neill anticipated possible student solutions prior to implementation when planning with fellow teachers on the seventh grade team, and she used this anticipation when she had the student present the solution of using the areas to determine the border when no other student found that solution. Mrs. O’Neill monitored the lesson by walking around and looking at student work. There was evidence of her selecting students to present solution methods but not evidence of her purposefully sequencing those solutions. At one point, she connected a student’s solution to representing the expression in terms of side length when she rewrote 10 + 9 + 9 + 8 as 10 + (10 – 1) + (10 – 1) + (10 – 2), but she did not explicitly say she was rewriting the expression in terms of side length.

**Mrs. O’Neill’s perspective.** Mrs. O’Neill thought the implementation of the boarder problem was successful and attributed the success to many of the factors found in Henningsen and Stein’s (1997) task implementation framework. She said:
I thought it went super well. Most of the classes after a fairly decent introduction, and then you would have those kids that the light bulb went off a little bit quicker and it kind of lead the class in the right direction, they got it. As you saw in one of my classes, I mean the girl that did the three corners, was just a really creative concept.

She explained that she set up the task by giving a review and justified the review saying it helped the students find an expression for the border of the square more quickly. She described her goals for instruction as:

The goals were to be able to introduce it and give them time to think about it on their own and then maybe collaborate with those around them...I mean they had some pretty good conversations I thought. Then with me kind of pulling it back together and saying, “Ok let’s hear the examples. What do you have and why does it work? Why does this one work? Do you agree? Does it work? Does it not? If not, then why?” I think we had some pretty good conversations, and the kids liked sharing.

Mrs. O’Neill reasoned that the lesson went well because the students engaged with the task and had good conversations about the border and relating the equivalent expressions together, which relates to students’ learning dispositions.

Mrs. O’Neill said she maintained the cognitive demand of the task because the students were successful with the task. Evidence of their success came from the fact that when she later reviewed finding expressions for perimeter the students recalled how to find the expression for perimeter from the Border Problem. Mrs. O’Neill also measured the success of the lesson in terms of future lessons because she used the problem later in
the unit saying, “I was even able to use the Border Problem when I was introducing equations because I could represent the sides of the borders as my algebra tiles and so they liked that.” Mrs. O’Neill thought because the students were successful, she maintained the demand of the lesson, which relates to students’ learning dispositions. She thought the students engaged with the task, found different solutions, and made connections between the Border Problem and other concepts later on in the unit.

Mrs. O’Neill acknowledged that she lowered the demand when she gave away too much of the answer too soon, which relates to her instructional disposition with respect to incorrect answers. She offered a specific example of a time she lowered the demand:

At first, it was hard for them to understand they couldn’t count all the tiles going around. They all wanted to start with 40 tiles, and so I felt like I kind of, in some ways I had to push them or lead them to “Maybe I can do 10 on this side and 10 on this side and then 8 and 8 or 10, 9, 9, or 8.”

She elaborated that when a student incorrectly said the answer was 40, she led too much by talking about whether or not the stones could overlap, and instead of helping the girl realize her misconception, she led her to realize the answer was 36. Mrs. O’Neill said that she made this decision because “I had to probably lead a little too much with that just so we could get through what I wanted to accomplish in that first day especially.” Mrs. O’Neill did not realize in the moment that she was leading but did recognize it upon reflection and made changes with subsequent classes. Specifically, with future class periods she said, “I would show one side and say ‘ok now how would you figure out how many are the next side without counting’ … instead of showing them that they overlap.”
She explained that she changed her line of questioning throughout the day to maintain the demand.

**Comparing perspectives.** My perspective and Mrs. O’Neill’s perspective mostly align. While Mrs. O’Neill and I have similar perspectives on students’ success in the lesson, we have differing perspectives on how to define success. Mrs. O’Neill said she questioned the students to help them make connections. While I did not see examples of these questions in the class I observed, she did ask some questions, asked for examples, and had students explain why it worked. One reason for the difference in interpretation may be that I observed the lesson the first time she taught it whereas because she taught the lesson four times, she may have been reflecting on all four lessons during the interviews. We agree the Border Problem was successful to the extent the students engaged in a mathematical problem and had found the number of tiles in the border of a 10 by 10 and a 6 by 6 square in multiple ways. We differ because I think the lesson stopped short of helping students make connections between solution strategies by realizing the expressions were equivalent, which I see as critical for a successful lesson. When Mrs. O’Neill talked about implementing the Border Problem, she referred to implementing the task in all of her classes, which may lead to our differing perspectives. My perspective is from the class I observed her teach the Border Problem, whereas her perspective includes all lessons where she implemented the task.

**Portrait of Mrs. O’Neill.** Mrs. O’Neill initially appeared to be eager to implement high cognitive demand tasks. She said that she was alternatively certified and no one had taught her how to plan or implement tasks so she was looking forward to learning how to implement high cognitive demand tasks. Her attitude toward trying to
implement high cognitive demand tasks seemed to change quickly, however, once the study began. While she implemented the two tasks that we planned together as a group, she often would turn other tasks into low-level tasks or implement skill and practice lessons instead of tasks. She explained that her students’ lack of knowledge and her inability to manage classroom behavior inhibited her from implementing tasks. Mrs. O’Neill did not implement a high cognitive demand task around proportional reasoning that the other two teachers implemented because she thought her students needed to practice more skills associated with solving proportions. I collected the least amount of observational data on Mrs. O’Neill because she would either not implement the task, I would teach the task entirely, or I would step in and help her implement the task; in the latter two situations I was not able to collect observational data.

One challenge I had with Mrs. O’Neill was her lack of mathematical knowledge. She often only found one solution to tasks we solved as a group and relied heavily on Mr. Cone and Mr. Fielder for solution methods. I think this lack of mathematical knowledge led her to be uncomfortable implementing high cognitive demand tasks. She explained that a barrier for her was the unpredictable nature of high cognitive demand tasks, and she liked to have her lessons planned out fully. She did not like situations occurring when students came up with answers she had not thought of or asking questions she could not answer. I think one reason for these barriers was her lack of content knowledge and her inability to find multiple solution methods for problems.

Overall, working with Mrs. M was a challenge for me. Although she told me she wanted to change her practice, her actions did not reflect this. I spent time trying to work with her, but I did not have enough time to help her change her practice. I am not sure
that continuing to work with her throughout the year would have changed because of the barriers she perceived as preventing the implementation of tasks. She did say many times that if a task was given to her, she could implement it. So if I were to work with her in the future, I would make sure I found tasks for her to implement and then work with her on implementing the tasks in the classroom.

**Mr. Fielder**

**Implementation of the Figure S Task.** Mr. Fielder started the lesson on Monday during the second week of school, with the following instructions on the board 1) discover independently 2) speak and share mathematics 3) group differences and similarities 4) present. He handed out the Figure S task and projected the task on the board while telling students there were different levels of mathematics with the highest being “doing mathematics,” meaning they would think rigorously about the problem from top to bottom on their own. Mr. Fielder explained that patterns were at the highest level and he would give tips but could not teach the students how to see the pattern; therefore, the students needed to think and solve the problem on their own. During the work period, Mr. Fielder walked around the classroom looking at students’ work. After three minute of individual work, he told the students to hold on to their thoughts and put students in groups of four or five to talk about their solutions. He told the students to communicate, argue, and prove they had the rationale to defend their answers. When groups excluded a member, Mr. Fielder made an effort to get all the group members to talk.

When Mr. Fielder started the group discussion, he asked students to come to the front and present solution methods. Mr. Fielder told the class there were many ways to
find solutions so they should pay attention to those at the front. A student came to the board and gave his rule to find step 3, \((3 \times 5 = 15, 15 + 2 = 17)\). Mr. Fielder explained how the student’s solution corresponded with the figure of base 3 saying the three was the base, the five was two more than the base, and the two represented the two extra pieces. Mr. Fielder asked the students if there were different methods to find the number of blocks/tiles/something in any size figure and gave the students five minutes to come up with their own method. After the students had time to figure out their methods, Mr. Fielder called a student to the board to explain. The student explained how he saw the middle as a square and gave his numerical solution, and Mr. Fielder wrote the corresponding algebraic expression on the board (See Table 2).

Table 2  
*Students’ numerical solutions with Mr. Fielder’s corresponding algebraic solution for the Figure S task.*

<table>
<thead>
<tr>
<th>Numerical Solution</th>
<th>Algebraic Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle as a square</td>
<td>(3 \times 3 + 2(3 + 1))</td>
</tr>
<tr>
<td>Middle as a rectangle</td>
<td>(3 \times (3 + 2) + 2)</td>
</tr>
</tbody>
</table>

Mr. Fielder told the class he wanted them to write expressions with a variable and to convert the figure number to a form with “\(n\)” in it because they used the figure number to find the length and width. Mr. Fielder explained to students the middle section could be seen as a rectangle and went through how to find both the numerical and algebraic solution methods (see Table 2). He explained that the algebraic solutions help with finding how many blocks are in any size figure. He ended class by telling students that if the base was 10, he followed the order of operations and got \(10 \times 12 = 120\), and then added the two blocks for a total of 122 blocks in a figure with a base 10.
My perspective. According to the IQA Mr. Fielder implemented the Figure S task at a level 3 (see Figure 5) because the students engaged in complex thinking, identified patterns, and used multiple strategies to find the pattern. The students had appropriate prior knowledge to access the task and had opportunities to serve as mathematical authorities when they explained their solution strategies to their peers. The students used multiple strategies to solve the problem but did not make connections between different strategies and representations (Boston, 2012). The students identified patterns, but Mr. Fielder started to focus students’ attention on procedures during the discussion on finding the expression for any size base.

While observing Mr. Fielder I noticed many factors that affected the implementation of the Figure S task. During the set up of the task, Mr. Fielder implemented the task at a high level by giving few instructions and allowing the students to engage in the task immediately. The task features allowed for this because the task is a high cognitive demand task, more specifically doing mathematics (Smith & Stein, 1998). The task had multiple entry points for students to gain access as well as having multiple solution methods.

During implementation, many of the factors Henningsen and Stein (1997) attributed to influencing students’ engagement with the task existed. When Mr. Fielder introduced the task he attempted to set norms by explaining to students that they needed to struggle with the task so he was not going to help them right away. He told students they were going to have to do their own thinking, putting the impetus on the student to engage in the problem and not rely on Mr. Fielder for answers. There was an instance when he talked with a student about her answer and asked how she got her total number
of boxes. She explained how to get the number in each row but could not figure out how many rows there would be. Mr. Fielder told her that was what she needed to figure out and walked away. As Mr. Fielder walked around, he told students to talk with each other in groups and said students needed to have a rationale for their solution methods, which was also an attempt to set classroom norms. If a group did not include a member, Mr. Fielder interjected and included that student. When a student hesitated giving an explanation at the board, Mr. Fielder told her he would help her along, setting a norm that the students should first try to explain and he would help if they got stuck.

Mr. Fielder’s instructional disposition affected students’ implementation of the task. There were times when he lowered the demand of the task by telling students answers when they were stuck. There was an instance when a student explained how he found the number of blocks in a figure with base 3 using language, and Mr. Fielder wrote the corresponding expression. I see this as lowering the demand because Mr. Fielder wrote the algebraic expression for the student’s numeric expression. When Mr. Fielder started to go over how to find a figure with any size base, his instructional disposition was to stand at the board and ask students what to do next, but he did all of the writing. He modeled how he found any size figure for the class, but from my perspective, the mathematics was in his hands and not the students’. Mr. Fielder’s instructional disposition helped to maintain the demand of the task when he walked around and questioned students and did not give the information away. A couple of times when students found themselves stuck and asked Mr. Fielder for help he tried to help them with minimal interference. The students’ instructional dispositions affected the implementation of the task because students engaged with the task and presented multiple
solutions for finding figures with different sized bases. It was the second week of school, but after some prompting the students worked together in groups to find solution methods. When the students presented solutions, they could explain their thinking.

During the lesson, there was evidence of Mr. Fielder using the five practices (Smith & Stein, 2011). I am aware that Mr. Fielder anticipated student solutions because I was there while he planned for this task with the other members of the seventh grade team. He did think about different possible student solutions as well as where students might have trouble. During the lesson, Mr. Fielder was monitoring, walking around, and looking at different student solutions. He did select students to present their work, but it is unclear if he purposefully selected those students or sequenced their solution. Mr. Fielder did connect the students’ work back to the figure at times by showing students how the algebraic expression related to the Figure S. While he did connect solutions to the figure, he did not connect between solutions.

**Mr. Fielder’s perspective.** Mr. Fielder identified both factors that affected the success of the lesson and obstacles to maintaining the demand. One of the reasons he gave for the success of the implementation of the task was task conditions because the students did not need prior knowledge to access the task. Mr. Fielder attributed the success to the task being a patterning task so most of his students enjoyed the challenge of finding the pattern. He said:

*It was good to see their brains buzz. For the most part, everyone answered all the problems and answered the problems in the way that the problem asked which is very rare. The questions themselves were scaffolded in such a way that it was*
easy to answer the next one because it was just one-step further, and they could see that far.

Mr. Fielder also attributed the success of the task to the conditions that allowed students to easily scaffold between questions. He thought because students just had to find the next step, they were able to apply the previous step and be successful finding the next figure in the pattern.

Mr. Fielder thought he maintained the cognitive demand of the task due to his instructional disposition. He went to every table and student to ask why each got a particular answer and asked students to show examples and prove why they were correct. Mr. Fielder said that because he questioned the students about solutions, he maintained the demand, saying:

Ninety-nine times out of 100 today I said, “Show me your examples that prove this. Tell me why.” There was not one kid that I was satisfied with an answer. I said, “Your answer is good, but it is not great.” I wanted them to have a great answer, more so just for their thinking.

When I asked about the one time out of 100, he said that was the only person he asked who had great reasoning and he was satisfied with her explanation. Mr. Fielder described his instructional disposition as being a factor that contributed to lowering the demand. He said he liked students to know where the answer came from, but sometimes he had to give them the information or be satisfied with the students knowing basic facts. Mr. Fielder explained his reasoning saying:

There are a few students that I try to get them to that next level, but I will accept it. I will not be ok with it, but I will accept the fact that sometimes, some students
will just need to know the mechanical thing. [They] will just need to know that
the commutative property does not work for subtraction, but it does work for
addition. I would like them to know why. I would like them to peer into it and
play with that idea, but there are some students, sometimes, it is a little bit out of
their reach for what they have in their cognitive toolbox. And so just being able to
say yes and no, right now, for them is huge. I have to be able to gauge each
individual student, but for the most part the bar is set high.

Mr. Fielder said that he was not sure if he maintained the cognitive demand of the task
because he claimed that many students did not have success with the problem, attributing
it to students’ learning dispositions. He said while students could see that the pattern
grew, they were unable to explain how the pattern grew. When he saw a student doing the
problem incorrectly, he tried to help students realize they were wrong without pointing
out they were wrong. He said, “I tried to re-direct them to the original pattern itself, and
say, “Well how does it grow? Does it grow just one way? Does it grow two ways?”

Comparing perspectives. My perspective of the lesson does not always align
with Mr. Fielder’s perspective of the lesson. I agree with Mr. Fielder that the task
conditions helped maintain demand. The task conditions allowed students to engage and
find multiple solution methods. I agree that his instructional disposition helped maintain
demand when he asked good questions of his students and tried to hold them accountable
for their explanations. I disagree with Mr. Fielder on his instructional disposition
maintaining demand at times. When the students did not give the explanation Mr. Fielder
was looking for, I think there was a decline in demand because he gave them the solution
or a procedure to follow, which diminished the cognitive load for the student. I disagree
with Mr. Fielder that the students did not have success with the problem and think it is due to having different definitions of success of the lesson. I think because the students engaged with the task, saw the pattern, and could explain how to find the number of blocks in a figure with a particular base, the students had success with some aspects of the task. Mr. Fielder thought because the students were unable to give the algebraic solution, they were unsuccessful with the task.

**Implementation of the Border Problem.** The Border Problem occurred over a two-day period, but for this analysis, I treated the entire implementation of the problem as a single session.

**Day one.** The class started with the following on the board: Happy Monday: How can we use equations to solve problems for any scenario? A student read the question aloud for the class, Mr. Fielder asked what an equation was, and the student gave the example $5 + 4$. Mr. Fielder said “No” and called on another student who said an equation had an equal sign. Another student read the definition of equation from her notebook: “Mathematical sentence formed by setting two expressions equal to each other.” Mr. Fielder told students that on the next slide there would be a figure with green border squares and their challenge was to figure out how many squares were in the border of a 10 by 10 square without counting and write down how they got their solution.

Mr. Fielder walked around looking at work and asking students how they solved the problem while telling them to write down how they solved it on the paper. Mr. Fielder told the students if they found one way they needed to think of multiple ways to solve the problem. He then addressed the entire class and said that $10 \times 4$ was not the correct answer. He put a slide on the interactive white board with 10 by 10 squares and asked
students to come to the board and share solutions (see Figure 15). Mr. Fielder told the students to show how they thought about the solutions. He called on a student to come to the board, and she explained that she knew there were 100 squares in the whole figure and she needed to find the inside squares so she did \((100 \times 100) - (8 \times 8)\). Another student explained that she added the top row of 10, the bottom row of 10 and the two sides of 8 to get \(10 + 10 = 20, 8 + 8 = 16,\) and \(20 + 16 = 36\). Mr. Fielder said he was going to clean up what she wrote and re-wrote her solution method as the expression \(10 + 10 + 8 + 8\).

![Figure 15: Student solutions for the Border Problem.](image)

Mr. Fielder called on another student to come to the board. The student came up and started to illustrate a solution, but the student became confused, returned to his desk
to get his notebook, returned to the board, and changed his expression. Mr. Fielder asked how he got the numbers in his expression and the student gave the pen back to Mr. Fielder and sat down without answering. Mr. Fielder told the class if anyone was going to come to the board they needed to think first about what they would say. A student explained his solution of $10 + 9 + 9 + 8$ and Mr. Fielder discussed with the class how the student could have found each number without counting each square. The students reasoned if the top row was 10 then subtract 1 to get the two sides and subtract two to find what was left on the bottom. Mr. Fielder asked if anyone else had a method, and the student who tried unsuccessfully before raised his hand again. Mr. Fielder asked him if he could explain his solution in one try, and the student said “yes” and said “four 9’s.” Mr. Fielder discussed with the class how to get four 9’s without counting each square individually. He showed how by counting one corner with each row, there were 9 in each row and drew the corresponding lines on the board (see Figure 15). Another student came to the board and drew a line through each side, overlapping on the corners and wrote -1 on each corner (see Figure 15). Mr. Fielder wrote under the diagram $10 \times 4 - 4$ and said he was surprised more students did not use that method. He asked the student to explain her method, and she said while everyone else took blocks off, she used blocks twice and then took one off for each of the corners.

After the students went through their methods, Mr. Fielder told the class to use two different methods to find the number of tiles in the border of a 6 by 6 square. The students started working, and Mr. Fielder reminded them to write something down using a method on the board. When going over the solution methods for a 6 by 6 square, Mr. Fielder told the class the answer was 20 and asked if anyone used the first method on the
board. When no students responded, Mr. Fielder showed how to convert the expression for a 10 by 10 to one for a 6 by 6 relating how the students knew where the 10 and 8 came from for finding the same solution method for the border of a 6 by 6 square (see Table 3).

Table 3

<table>
<thead>
<tr>
<th>Numerical solution methods for the border of a 10 by 10 square and a 6 by 6 square in Mr. Fielder’s class.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution Methods for 10 by 10 Border</strong></td>
</tr>
<tr>
<td>10 x 10 – 8 x 8</td>
</tr>
<tr>
<td>9 x 4</td>
</tr>
<tr>
<td>10 + 10 + 8 + 8</td>
</tr>
<tr>
<td>10 + 9 + 9 + 8</td>
</tr>
<tr>
<td>10 x 4 – 4</td>
</tr>
</tbody>
</table>

Mr. Fielder told the students the next challenge was to generalize for an \( n \)-by-\( n \) square and asked students to find the number of squares in the border of an \( n \)-by-\( n \) square. He noted that \( n \) is the side length so they should represent their expressions in terms of \( n \) or in terms of \( s \) for side length. The students worked for a while, and Mr. Fielder pulled them back together and asked who had an expression for an \( n \)-by-\( n \) square. A student replied \( n \times n – n \), and Mr. Fielder explained that an easy way to test to see if the generalization worked was to plug in 10 for \( n \) to see if the expression gave the answer of 36. He showed that plugging in 10 gave 90, so the expression was not correct. A student said \( n + n + n + n \), and Mr. Fielder asked "4\( n \)?" He asked students if that made sense, and students said it did not. Another student said, \( n – 4 + 3n \) and explained that she took the side length minus the four corners and then added the other three sides. Mr. Fielder said he would clean it up, because you cannot subtract four corners from one side length and wrote \( 4n – 4 \). He then told students this method related to the student’s method
for subtracting the four corners. He asked if anyone else had an expression to share, and a
student said he “subtracted 2 and multiplied by 4 and then subtracted 4” for which Mr.
Fielder wrote \( (n - 2) \times 4 + 4 \). At that point, the bell rang and class ended.

\textit{Day 2.} Mr. Fielder started class and asked a student to read aloud the essential
question, which was “How can we tell if expressions are equivalent?” Mr. Fielder.
reminded the class that expressions do not have equal signs. Mr. Fielder put the solution
methods from the previous day on the board (see Table 4).

Table 4
\textit{Numerical solutions for a 10 by 10 square and the algebraic solution for a square with
any size border in Mr. Fielder’s class.}

<table>
<thead>
<tr>
<th>Students’ Numerical Solution</th>
<th>Algebraic Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 + 10 + 8 + 8</td>
<td>( s + s + (s - 2) + (s - 2) )</td>
</tr>
<tr>
<td>10 x 4 – 4</td>
<td>( s \times 4 - 4 )</td>
</tr>
<tr>
<td>10 x 10 – 8 x 8</td>
<td>( (s \times s) - [(s - 2) \times (s - 2)] )</td>
</tr>
<tr>
<td>9 x 4</td>
<td>( (s - 1) \times 4 )</td>
</tr>
<tr>
<td>8 x 4 + 4</td>
<td>( [(s - 2) \times 4] + 4 )</td>
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He said that he wanted to talk about these expressions in terms of side length. The class
discussed the side lengths of each of the squares from the previous day with side lengths
10, 6, and \( n \). Mr. Fielder then discussed how to change each numerical expression into an
algebraic expression in terms of side length \( s \). He converted the first expression,
modeling for the students how to change the expression, and wrote the algebraic
expression next to the corresponding numerical expression. He then asked the students
how to re-write the next few expressions in terms of \( s \) and asked students to explain
where the numbers came from. When he discussed \( 4s - 4 \), he spent time discussing the
difference between the \( 4s \) term and the 4 term. He wanted students to realize the first
term represented the four sides, which varied depending on the side length, and the second 4 represented the four corners, which always remained constant. He told them to do the last one on their own and called on different students for each part of the expression. The students could not decide if all the algebraic expressions were the same or different so Mr. Fielder said to pick an expression and test with 6 because they knew how many squares were in the border of a 6 by 6 square from yesterday. After giving the students a few minutes to work, he went through each algebraic expression showing each expression equaled 20. Mr. Fielder ended the discussion by pointing out if you could plug 6 into each expression and get the same answer, the expression were equivalent.

**My perspective.** When applying the IQA (Boston, 2012) to this problem, I split the class into two parts, finding the borders of a 10 by 10 and 6 by 6 squares, and finding the border of an \( n \)-by-\( n \) square because Mr. Fielder implemented the two tasks at different levels. Mr. Fielder implemented finding the border of a 10 by 10 and 6 by 6 squares at a level 4 (see Figure 10). The students engaged in exploring different ways for finding the border and flexibly used different solution methods to find the border of a square with given dimensions. The students engaged in complex non-algorithmic thinking and Mr. Fielder pressed for explanations when the students presented their solution strategies. The students developed explanations for why their strategies worked and Mr. Fielder did not point at specific procedures to find solution methods. The students made connections between strategies when Mr. Fielder asked what was similar and what was different. Mr. Fielder implemented an \( n \)-by-\( n \) square at a level 2 (see Figure 16). Mr. Fielder specifically called attention to a procedure of how to change the expressions from the 6 by 6 square to an \( n \)-by-\( n \) square.
There was little ambiguity about how to change the numeric expression into an algebraic one. The focus was on finding the algebraic expression more than on finding equivalent expressions.

<table>
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<td>The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it. The task does not require students to make connections to the concepts or meaning underlying the procedure being used. Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).</td>
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<tr>
<td>OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class.</td>
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*Figure 16: Task implementation for a Level 2 (Boston, 2012, p. 20)*

Many of the factors existed from the task implementation framework (Henningsen & Stein, 1996) during the implementation of the Border Problem. Mr. Fielder set up the task in a way that maintained the demand by giving students minimal instructions and allowed students to engage without too much direction. The task conditions helped maintain demand because the categorization of the Border Problem is *doing mathematics* (Smith & Stein, 1998). The task has multiple entry points and multiple solution methods requiring non-algorithmic approaches to the solutions.

During the implementation of the Border Problem, Mr. Fielder continually tried to implement norms, helping to maintain demand. Mr. Fielder set norms in relation to how the students engaged with the problem, explaining the problem had multiple solution methods and students should find as many solutions as possible. Mr. Fielder encouraged students to write the solution method down and not just keep it in their head. During student presentations, Mr. Fielder set norms on how students should present. When a student could not remember his solution method, Mr. Fielder told students they should know what they were going to say before coming to the board to minimize wasting class
time. The next time that student volunteered he shared his method and showed he had thought out how he was going to explain his solution.

Mr. Fielder’s instructional disposition affected students’ implementation of the Border Problem. Mr. Fielder often did not wait long before giving students answers to questions he asked or when a student replied with a wrong answer, Mr. Fielder said it was incorrect and explained why, taking the mathematics out of the hands of the students. When going over the problems, Mr. Fielder gave his take on student answers, saying whether they were correct or incorrect, and wrote the problem his way on the board. Mr. Fielder modeled how he wanted the expressions to look for each of the dimensions of the square (10 by 10, 6 by 6), but often focused on the procedure of turning a numeric expression to an algebraic expression, thus lowering the cognitive demand. When students worked on finding the number of tiles in the border of any size square, Mr. Fielder did not allow students to struggle for long before intervening and explained how to find the different expressions. He quickly showed how to find the border of any size square and showed explicitly how to rewrite the numerical expressions to algebraic expressions.

Students’ learning dispositions affected the implementation of the task. The students engaged in the task and found multiple solutions for each part of the Border Problem. The students presented and explained solutions at the board, showing their engagement with the problem. When some students were stuck, they asked Mr. Fielder for help before talking with their neighbor, but there were students who solved the problem with little to no help.
Throughout the task, I saw evidence of the five practices (Smith & Stein, 2011). During implementation, it is hard to see evidence of anticipation, but I was with Mr. Fielder when he planned the task along with the seventh grade team. During that meeting, the teachers solved the task in multiple ways and anticipated student solutions. During implementation, there is evidence of monitoring as Mr. Fielder walked around the classroom and looked at students’ work. Mr. Fielder selected students to come up and present solutions, but there is no evidence that he sequenced the presentations. He mostly called for student volunteers and asked students to come up who had differing solution methods. Mr. Fielder did connect students’ work back to the Border Problem when he related the student solutions to the side lengths of the square, asking how the students could find the side lengths without counting and relating each of the numbers in the numerical expressions back to the side lengths of the square. He also connected the algebraic solutions to the numeric solutions and made the connection of equivalent expressions between the student solutions and in the expression $4s - 4$, and made sure the students realized $4s$ represented the sides and varied depending on side length, whereas the 4 represented the corners and was constant.

Mr. Fielder’s perspective. When talking to Mr. Fielder about the Border Problem, he attributed several factors to affecting implementation. The factors he mentions fit into the task implementation framework (Henningsen & Stein, 1997) and include teacher’s goals, teacher’s knowledge of students, teacher’s instructional disposition, and students’ instructional disposition. Mr. Fielder said his plan when implementing the Border Problem was for students to participate and forget they were doing mathematics while doing mathematics. Mr.
Fielder said he did not plan much because it was a straightforward problem and explained he did not bias the students saying, “I didn’t want to lead them into ‘set up the equation that expresses this.’ They did not know what that was at that point, so it was just ‘hey tell me how many are in the border without counting’.” Mr. Fielder said based on anticipating with the seventh grade team, he realized the only prompting the students needed was instructions to find border without counting. He said his plan was to give them very little prompting in the beginning but then guide the students to see they were actually doing more advanced mathematics. Another goal for the lesson was for the students to see that there were many different strategies to come up with finding any size square. Mr. Fielder said he said mostly accomplished his goals, but was unsure if all students agreed with him. Mr. Fielder was not sure if students understood they had found a universal method to solve any size square by the end of class, but may have understood as they continued with the unit saying, “I see there are more students that seem to be comfortable understanding the universal-ness of having the expression where you can plug in the variable”.

Mr. Fielder thought he maintained the cognitive demand when he set up the problem relating it to the task features, saying the problem was innately high cognitive demand. He said students figured out how to find the border of the 10 by 10 but had difficulty coming up with the algebraic expressions for the general case. He thought he maintained the cognitive demand when he asked the students to transition from finding the 10 by 10 to the 6 by 6 using two methods saying that the students successfully implemented someone else’s method and saw it worked the same as their original
method. He thought the cognitive demand declined when students transitioned from a specific number square to any size square saying:

I think the part where you had the split between maintaining the level of demand and diminishing the level of demand was going into the variable $x$ depending on the classroom or depending on the class grouping. Some students you could say “alright now let’s think in all cases of the border. Let us say the border could be a million, the border could be 5. So could we use $s$ for side length?” Some of the classes were ready and able to approach it that way. Some of the other classes you had to explain more what that meant. Somehow jumping from tangible, or the constant numbers into it could be any possibility, there is nothing concrete about any possibility for the other classes. So that required more detailed explanation. That kind of gave it away. I am not even sure if that went well, because I really do not know how I could explain better than it is for any case.

Mr. Fielder realized that in some of his classes he lowered the demand when it came to finding the expression for any size square, which he based on students’ content knowledge. He attributed the students’ inability to handle jumping from a concrete case to using a variable as the reason why he lowered the demand and explained how to find the expression for a square of size $N$. Mr. Fielder recognized that by explaining how to find that expression, he was lowering the demand saying:

I really had to say “alright, how could we represent it?” I really had to draw out the word variable and then emphasize side length. That’s what we are changing. Let us talk about this in terms of side length and having to really emphasize side length. Then we could use a variable because that is the one that changes. Talking
about the \(+4/–4\) thing that never changed when we made a variable version because those had to do with the square itself, not the side length…I also had that discussion with a class that maintained the cognitive demand and they appreciated and grasped that concept.

This relates to Mr. Fielder’s instructional disposition and his willingness to give the information to some classes. He said he maintained the demand in a class during the discussion for any size square, but I did not observe that class to see if we had the same perspective on what it means to maintain the demand during a discussion. Mr. Fielder said he struggled with some aspects of the lesson:

The only struggles I had were in those classes where I had to give it away. It was frustrating because I know theoretically that we are supposed to allow the students to develop these ideas themselves, just kind of point them in the right direction. To have to tell someone about this idea and them not get it that is frustrating because they cannot get it on their own. Then you try to tell them about it and then they get more confused so then it is almost like you have no idea, and that is the frustrating part.

Mr. Fielder was frustrated with his inability to maintain the demand because he wanted the students to solve the problem entirely on their own, which relates to his teaching disposition.

I asked Mr. Fielder what he successes he had with the lesson and he said he felt he had success with maintaining the demand with his on-level students later on in the day. He said, “Being able to see students as the day progressed do it more on their own, made me feel better because I did not have to explicitly say anything.” I was unable to see Mr.
Fielder teach a class he considered to be on-level, but it seems as though he let go later in the day and let the students do more of the mathematics. I asked how he felt the students did with the lesson and he thought the students were successful saying the task was in the students’ zone of proximal development and he had full participation from all of his classes, which relates to students’ learning dispositions. He thought even for students who struggled with the problem, it was never fully out of reach and every student had some success with the problem.

**Comparing perspectives.** Overall, my perspective on the lesson aligns with Mr. Fielder’s perspective on the lesson. We agree that he maintained the demand of the task with most aspects of the lesson. He did implement the lesson at a high level and was able to maintain the demand of the lesson for the parts of the lesson having to do with finding a 10 by 10 or 6 by 6 borders. He was unable to maintain the demand when going over how to find the border of an \( n \)-by-\( n \) square. He ended up giving the students the procedure for turning an expression of a 10 by 10 square into an algebraic expression. I agree with Mr. Fielder that the demand of the task declined when the students needed to find an expression for the border of any size square, but our perspectives differ on why the demand declined. One place my perspective differs is the students’ abilities to handle making the generalization of the pattern. I maintain he gave up too quickly and did not scaffold in a way that could have allowed his students to make those connections. Even though students were struggling. Mr. Fielder could have scaffolded and questioned in a way that would have allowed students to figure out how to find the algebraic expression. Mr. Fielder appeared to think the students needed to solve the problem on their own, and when they were unable to, he gave them a procedure for how to do it. Mr. Fielder seemed
to understand the purpose of implementing high cognitive demand tasks, but had difficulty scaffolding to maintain the demand. Being a first year teacher, he will continue to use tasks and figure out ways to scaffold and not just give the students the information.

**Portrait of Mr. Fielder.** Working with Mr. Fielder was like being on a roller coaster. There were both high and low experiences while working with him, but during our last interview, he claimed he finally saw the value in implementing more high cognitive demand tasks with his students. While I am not convinced he will include more high cognitive demand tasks, there is evidence to suggest he might.

Mr. Fielder was a first year teacher with a lot on his plate. He was a participant in my research study, he was getting his gifted certification, which required night classes, he was a participant in a federally funded mentoring program, which included after school meetings, and he had a grade level mentor outside of the mentor program, which required meeting time outside of school hours. This was all on top of his regular teaching and school duties. Initially I was hesitant about including Mr. Fielder in the study because of his many responsibilities, but the principal was confident he could handle it all, and he agreed to be in the study. Initially, he implemented high cognitive demand tasks at a high level, and I was excited that we seemed to have the same ideas about tasks. As the year progressed, his implementation of tasks declined. He did not implement as many high cognitive demand tasks as Mr. Cone and when he did implement them, he focused on procedural aspects of the task. He explained in his last interview that he needed to realize on his own that direct instruction and focusing on the skills was not working. After coming to this realization, he said he wanted to start implementing more high cognitive demand tasks. While I would like to believe this was true, in a meeting at the end of the
year to prepare a professional development activity for teachers around the Figure S task and the Border Problem, Mr. Fielder claimed he was going to start the next year off with skill and practice worksheets to show students they needed to work hard right out of the gate.

I think if I had continued to work with Mr. Fielder after he realized the value of high cognitive demand tasks, I could have helped him implement tasks more consistently on a high level. He did see the value in implementing high cognitive demand tasks, but his actions of giving many skill and practice worksheets leads me to think he would need more professional development on implementing high cognitive demand tasks and fostering students’ conceptual understanding of mathematics.

Factors Affecting Implementation

Adding to the Task Implementation Framework

Stein et al. (1996) completed classroom observations and looked at factors that influenced the implementation of a high cognitive demand task at a classroom level. This included factors that affected how a teacher set up the task in the classroom through the entire implementation of the task, along with students’ interactions with the task. During the course of my study, I looked at a more holistic view of the task implementation process and gained teachers’ perspectives on the entire process from planning, to implementing or not implementing the task, and then into the classroom for the task implementation. In doing this, I found factors that affected the teachers’ implementation and occurred before the teachers chose to implement a task that did not fit in with the task
implementation framework. Therefore, added an additional circle to the diagram that includes factors that influence the teachers’ implementation of the task (see Figure 4). These factors include time, teachers’ knowledge of students, district requirements and curriculum, and parental dispositions. These are factors identified by the teachers as affecting whether or not they chose to implement a high cognitive demand task. Once a teacher chose to implement a task, factors that affected implementation mostly aligned with the task implementation framework. When looking at factors influencing set up, I found that teachers’ goals and teachers’ knowledge of students affected implementation. Factors influencing students’ implementation of the task included task conditions, teachers’ instructional dispositions, students’ learning dispositions, and task conditions.

**Factors Influencing Teachers’ Implementation of Tasks**

**Knowledge of students.** A factor that influenced each teacher’s implementation of the task was his or her knowledge of students. This knowledge of students influenced whether or not a teacher planned to implement a task or only to use parts of the task for a group of students. A teacher’s knowledge of students affected the teacher’s implementation of a task when he or she chose to bypass the task altogether in favor of spending the lesson as a review or skills period, deliberately lowered the demand of the task to focus on the procedural aspects, or conducted a review before the task. Mr. Cone claimed that when he taught his on-level class, he altered the tasks to focus more on the skill part than the conceptual part. During a task on integers, Mr. Cone modified the task for his on-level class saying:
I circumvented some of the investigations. When we got to subtraction, I said, “Let’s see how we can rewrite subtraction as addition” and worked on that instead of saying “What are the possible algorithms for subtraction?” so they would be less overwhelmed.

Mr. Cone often only gave small portions of the task to his on-level students and used the remainder of the time for review. One time Mr. Cone did a small portion of a task, explaining that during a task on decimals, percents and fractions, he only did part of the task with one class and then did work to “shore up basic skills.” During the implementation of the Border Problem, Mrs. O’Neill included a review of expressions before the implementation of the task. She thought the implementation of the task was successful, saying, “I thought it went super well. And most of the classes, after a fairly decent introduction, they got it.” She continued to explain how she thought students would not have been able to access the task without a review before the problem. Another time, Mrs. O’Neill bypassed a task altogether in order to spend the time on the procedural aspects of scale factor.

Sometime teachers did not implement tasks based on classroom behaviors. Examples included a teacher’s belief that the students could not remain on task and stay focused and overall classroom management issues. Mrs. O’Neill claimed that behavior issues prevented her from implementing tasks, saying:

Sometimes [the students] just haven’t had a good grasp of the skills that they need to use so they can’t even get started on it. Sometimes we may just have an incident in the classroom that has gotten everybody off task and consequently I
get frustrated and flustered or we just can’t do it. We just can’t manage to get it done, and that’s something that we’ll have to do another time, or on a better day.

Mr. Cone was less likely to implement a task if he thought the students would not remain focused throughout the task saying, “Some students don’t have practice being focused in class. They are unable to focus and stay on task. The learning that takes place during a high cognitive demand task requires focus, and that focus takes practice they don’t have.” During the final interview, Mr. Fielder explained that he had trouble setting up tasks because of wasted time during the first 30 minutes of class. He claimed this was time spent for either review or the students being off task and not talking about mathematics. He explained that his struggle was with classroom discipline and said students were so disruptive he felt could not implement tasks.

**Time.** Time was a factor that affected each teacher’s implementation of tasks. The teachers claimed they did not have time to find and plan tasks. Mrs. O’Neill said what kept her from implementing tasks was “the time to be able to find good tasks, the planning of them, there again time” and Mr. Cone said “not having the time or the knowledge of which tasks exist.” Mrs. O’Neill claimed she did not have enough time to search out tasks that were not already associated with the curriculum. All the teachers talked about not having the time to meet as a group and plan tasks as an obstacle for task implementation. In the last interview, Mrs. O’Neill said she wanted her planning times to be used for planning. Mrs. O’Neill said district or school required meetings such as data team meetings or professional development sessions, meeting with parents about students, and individualized education program (IEP) meetings concerning students took precedence over planning and interfered with time to plan tasks. During one team
meeting on a Thursday, Mrs. O’Neill commented that she had not had individual planning time that entire week and the schedule had been changed that day to eliminate the common planning, which she and the other teachers found frustrating. The teachers wanted more time to plan and discuss tasks together as a grade level team, but it was difficult to find that time within the space of their school week. Mr. Cone explained that the seventh grade team often did not have the time to plan tasks and anticipate possible student solutions. He said they were often pressed for time and if there was a little time, they talk generally about how to implement tasks. He said more often than not, there was no time for common planning, and that lead to each teacher implementing the task in his or her own way and they were unable to get ideas from each. Mr. Cone said ideal conditions to implement tasks included protected planning each week where the team could plan tasks.

When the team did have time to plan and discuss a task, the teachers felt the implementation of those tasks was extremely successful. Mr. Cone said the work he did with the seventh grade team and me helped him to anticipate students’ responses and contributed to the success of implementing the Border Problem. He said planning how to implement the lesson, deciding on questions to give students, and talking about how to introduce the lesson was helpful because the teachers talked about it in advance and decided on what to do and say. He thought implementation was more successful when the team had time to talk and plan the tasks ahead of time because they could all look at the mathematics involved and be more prepared to help students. Mrs. O’Neill said that being able to discuss the task with the seventh grade team was extremely helpful when implementing tasks. An example of this was working out the Border Problem with the
seventh grade team. When I asked if the team could replicate the planning without me, she said they could except for the fact that they did not have planning periods where they could devote the entire time to planning a task, unlike our time working together. Mrs. O’Neill said that she loved common planning times because the team could throw around ideas and get good things going, but they often did not have that chance. She also commented that she needed her own individual time to think through the lesson and prepare mentally when giving a task. Mrs. O’Neill said she was frustrated with time and the lack of quality uninterrupted time. She wanted both time to plan with the seventh grade team as well as individual time to plan alone.

**District requirements and curriculum.** District requirements, including the district imposed curriculum, were a factor that affected the teachers’ implementation of tasks because they felt the curriculum was so expansive and with district required testing, there was not the space to implement tasks. The teachers were in a unique situation because while they had a reform-oriented curriculum, the *Connected Mathematics Project* (CMP), it did not align with the newly instated state Common Core Performance Standards. This led to the teachers often picking and choosing other resources to fill the gaps, which were often not high cognitive demand tasks. They often used workbooks or worksheets that emphasized skills practice. Each teacher reported instances of district requirements of testing and curriculum as influencing his/her decision not to implement tasks. Mrs. O’Neill explained that the curriculum needed to be covered, and testing often disrupted the schedule. Between benchmark testing and the Performance Series Scantron testing that replaced teaching days, the teachers had to push to make sure to cover the curriculum, including leaving time for review before the end of the year state tests. Mr.
Fielder said his struggles included being “bogged down by what teachers have to do,” which included paperwork, meetings, and detentions. This sentiment summarized all three teachers’ feelings on what was required of them on a weekly basis that kept them from their planning. Mr. Cone identified the pressures of implementing the curriculum as a factor when thinking about implementing tasks. He claimed that he typically implemented tasks when there was an extra day and he did not have a specific curricular goal in mind. Mrs. O’Neill said she struggled to find tasks that seamlessly fit into the curriculum.

One factor that positively influenced the teachers’ implementation of tasks was when I brought in tasks and went through the planning process with the team. This relates to time and curriculum because I found tasks that aligned with the curriculum, taking that burden from the teachers. When asked what successes they had with implementing tasks, the teachers pointed to having a task brought in for them to use and having a knowledgeable facilitator assist them with preparing the task. Mr. Cone said, “One of the things that we’ve enjoyed this year is you having the time to say ‘Here is a task that fits’ and then bring it to us.” Mrs. O’Neill expressed a similar sentiment, saying:

Certainly with this last one [the Border Problem] and the patterning one, your guidance and thoughts because you had worked through them. That was very helpful, having somebody that knows the ins and outs of the whole thing before we started it.

Mrs. O’Neill noted the importance of receiving task aligned to the curriculum. She claimed the Border Problem and Figure S task went well because she did not have to find the tasks and there was a discussion around how to implement the tasks. She said that if
someone could help her find a task and plan it out, she could pretty much implement any task. She said it might not be beautiful, but that she could do it.

Factors Influencing Students’ Implementation

**Teachers’ instructional dispositions.** Each teacher’s instructional disposition was a factor influencing implementation. All of the teachers said they often led students to the right answer. They gave different reasons as to why they led students, but each had the tendency not to allow students to productively struggle with the task. Mr. Cone was not apt to let students struggle with the problem but was working on being able to question students instead of just giving them the answer. Mr. Fielder lowered the demand during the Border Problem when the students struggled with finding the border of any size square. He had not intended to lead, but when students could not come up with an expression for any size square, Mr. Fielder showed them how to take each of the numerical expressions and turn it into an algebraic expression rather than asking questions or offering hints to help them find the expression themselves. Mr. Fielder recognized this behavior:

> The only struggles I had were in those classes where I had to give it away. It was frustrating because I know that theoretically we are supposed to allow students to develop these ideas themselves, just kind of point them in the right direction. To have to tell someone about this idea and them not get it, that is frustrating because they can’t get it on their own and then you try to tell them about it and then they get more confused so then it’s almost like you have no idea, and that’s the frustrating part.
Mrs. O’Neill also acknowledged that she struggled with leading students too much. During the implementation of the Figure S task, she said she explicitly helped students who were stuck because she was afraid the students would shut down without her help. She said she led students too much during the Border Problem when they were getting the answer 40 (an incorrect answer obtained by double counting the corner squares). She initially told a group of students they were getting 40 because they were double counting the corners, but when she realized that she was being too directive, she adjusted her questioning to help the other groups of students arrive at that conclusion without her doing it for them. She said she initially was leading her students through tasks because she wanted to make sure she had enough time to accomplish her goals. Thus, instead of letting students struggle, she gave them the information needed to solve the problem.

Teachers’ instructional dispositions positively influenced students’ implementation when the teachers held back and allowed students to productively struggle with the task. Examples of teachers holding back included using questioning techniques or referring students to work with each other instead of relying on the teacher for ideas. Both Mr. Cone and Mr. Fielder pointed to questioning students and not giving away answers as reason for maintaining the demand of tasks. The teachers recognized both instances of maintaining demand due to questioning but also lowering demand due to giving away the information too soon. Mr. Fielder said he had success with the Border Problem because he maintained demand with his on-level students by pulling back and not leading students to the answer. He said he realized the students were getting it on their own, and he could lead less and watch the students come up with the ideas of generalizing any size square on their own. Mr. Fielder said he was becoming okay with
allowing students to struggle with the mathematics and if they could not finish the task in one day, he would allow them to take two days to struggle with the task. By engaging with the task for two days, they may flail but not give up entirely. During the first interview, Mr. Cone said that he was getting better at using questions, and his goal was to focus students’ attention on the mathematical concepts with questions rather than statements. He said he had to make a conscious effort not to give away pieces of information during implementation. During the second interview, Mr. Cone said he was getting better at letting students “carry more of the cognitive burden” because he tried to answer the students’ questions less frequently and encouraged the students to engage with their peers more.

**Task conditions.** Task conditions, which include the time allotted to work on a task in class, were a factor influencing students’ implementation of the task because often there was not enough class time for students to grapple with the task. Mr. Cone attributed class time as a reason for not maintaining the demand in both the Figure S task and the Border Problem. During both tasks, he did not think the students discussed how to generalize for any size figure in the Figure S task and for any size border in the Border Problem. Mr. Cone described the implementation of the Figure S task by saying, “I am leaning towards failure because we didn’t have a lot of time in class to do it. Students worked on it, but they weren’t able to share their thinking. I didn’t have a chance to continue that discussion.” Mr. Cone also claimed he altered tasks because he did not have the time for students to grapple with the mathematics so he would often make the connections for his students instead of letting them try to figure it out. During work on integers Mr. Cone explained why he altered the task saying:
Two factors. In the time that I have, I don’t think that I can do justice to the subtraction algorithm and then have enough time for them not to be confused by it. So the time constraint, we focused a lot on addition and why that works with number lines. If I can make the connection between subtraction and addition then it might reinforce the addition without having to do the work for subtraction. It’s a time crunch and an intellectual capacity issue within that time that I have.

Mrs. O’Neill identified lack of class time as a reason for lowering demand, saying she often gave the answer away because that would save her time in class to be able to get to each part of the lesson.

For the Figure S task and Border Problem, the task condition was a factor that positively affected students’ implementation of the task because students were able to build on prior knowledge. The two tasks were high cognitive demand tasks and had multiple entry points for students that allowed students access to the task. The classification of both the Figure S task and the Border Problem is doing mathematics (Smith & Stein, 2011). The teachers commented on the task conditions as a reason for the success of the lessons. Mr. Fielder said the Figure S task was successful because the students did not need a lot of prior knowledge to engage with that task. He had the same sentiment with the Border Problem and explained that the task was both easy to set up and implement because the students could engage with the task. He noted that the Figure S task built on itself in a way that students did not need much support from the teacher. He also said the Border Problem had an easy set up in that all he needed to do was give simple instructions, and the students were able to access the task. Mr. Fielder posed the tasks as challenges or problems for the students to solve because he said lessons were
successful when he found tasks that he could set up as challenges and have students engage in without realizing they were doing mathematics. Mr. Cone attributed the success of the Border Problem to it having a low entry floor while also allowing students the option of being creative. Mrs. O’Neill spoke of the nature of the Border Problem and noted that all levels of students could access the task; saying, “The kids that were more advanced, they were looking for different, more complex strategies for solving it, so it was a puzzle for your weakest learner as opposed to the one that is more advanced.” As an extension, she had her accelerated group create their own patterns to give other students to come up with the generalization for the pattern.

**Students’ learning dispositions.** A factor that affected students’ implementation of the task was students’ learning dispositions throughout the tasks. The teachers explained that high student engagement in the class as well as the ability to have a discussion around the task contributed to maintaining the cognitive demand during successful implementations. Mrs. O’Neill said she was successful with task implementation when she was able to have a discussion around the mathematics with the students sharing multiple solution methods because she enjoyed hearing multiple ways the students had solved the task. During the final interview, Mrs. O’Neill said that she was getting better at allowing discussion in her classroom. She noted that when she introduced tape diagrams and double number lines for solving proportions, the students discussed why they used certain methods and the class was able to have a discussion around finding answers, which was different from the beginning of the year when the students just wanted to know how to get the answer. Mr. Cone said success with tasks implemented in his classroom was due to high student engagement. Mr. Cone said his
students engaged in the mathematics during the Figure S task and the Border Problem, even in classes where he had a hard time engaging students. He also said that through the implementation of tasks he had become more reflective about his practice of interacting with students, thinking about how he can clarify, extend, or prompt students’ thinking based on what the student says, which he had not thought of before. Conversely, Mr. Cone did comment that students’ learning dispositions were a barrier when he received pushback from students who did not want to engage in the complex nature of the task.

**Individual Teacher Factors**

**Mr. Cone.** While the teachers identified many of the same factors that affected task implementation, each had his or her own individual factors. A factor that existed for Mr. Cone that influenced students’ implementation with tasks was students’ learning dispositions, and a factor that influenced his implementation of tasks was parents’ dispositions towards tasks. An obstacle for Mr. Cone when he implemented high cognitive demand tasks was pushback from students and parents who were frustrated and then in turn frustrated him. He said that students sometimes thought engaging in high cognitive demand tasks was hard and often just wanted to learn the rules and get the answer. He explained during a task on scale factor, he was not able to get through the entire task due to push back from students, which led to behavioral problems and therefore he was not able to accomplish the mathematics he had hoped. He claimed parents gave him pushback because the *CMP* curriculum was not similar to the way they learned mathematics. The parents argued they had a hard time understanding the
mathematics their child was doing in class and therefore could not always help their child.

Another individual factor that affected task set up for Mr. Cone was teacher goals. He often talked about the difference between a curricular goal and a non-curricular goal. He said he was more apt to implement tasks when he did not have a curricular goal in mind. He claimed that he typically implemented tasks when there was an extra day in the schedule and did not have a specific curricular goal in mind. Mr. Cone claimed his goals with the Figure S lesson pertained to the atmosphere and environment, but he did not have a specific content goal, which made for a successful implementation. He said it was enjoyable to implement a task without a specific content goal but when giving a task that had a curricular goal, he felt rushed to make sure the students were making mathematical connections.

I feel as though I have less pressure if I know that we can explore and examine whatever the kids come up with, instead of having to try and figure out, either how to get them to the goal that I have for that day, or me having some sort of agenda that I want them to accomplish. If nothing happens that day, if [the students] don’t learn a specific skill but they are just doing math, then it is ok on that day I think. I don’t think it’s ideal, but it’s not a bad thing.

Mr. Cone said using a high cognitive demand task was more difficult when he had a curricular goal because there was not enough time for the students to engage in the task and understand the mathematical connections in one day:

But when I am teaching lessons where I want them to learn a specific skill or concept, and they don’t get that, then I struggle. Not like I’m in trouble, but like
“Oh that was the goal, I wanted them to learn this,” and if they haven’t learned it I’m like “AHHH…”we didn’t accomplish what we wanted to accomplish today, or what I wanted them to accomplish.

He said due to time constraints and the amount of curriculum he had to cover, one day might be all he had to go over a topic. He appeared to think students could not learn content from engaging with tasks.

**Mr. Fielder.** Mr. Fielder identified factors that hindered the implementation of tasks as his prior experience of how he thought mathematics instruction should look. During the final interview, Mr. Fielder seemed to realize the importance of implementing high cognitive demand tasks and said he had a revelation about wanting to include more tasks in his instruction. He explained what had been keeping him from implementing tasks, saying:

Me. The way that I thought, the way I learned in school, the way my mom taught me, the way my teachers taught me, the way that the system has been, the way mathematics is. Most math teachers are math teachers because they were good at math, but not all of your students are good at math, so there is a big separation. You are not only teaching people who are good at math. I had to get over that. I had to realize that not everyone learns the same way.

Mr. Fielder explained that he worried he would run out of ideas in the future and be unable to find problems that were difficult and demanding enough for his students.

**Mrs. O’Neill.** A factor influencing students’ implementation of the task reported solely by Mrs. O’Neill was students’ learning dispositions. Mrs. O’Neill thought students relied on her or other adults in the room for the answer and would not productively
struggle with the problem. She wanted the students to work by themselves or ask a neighbor if they were unable to figure it out, but often they wanted her to give them the answer. She said that it just took time for the students to get to know each other to feel comfortable talking with their neighbor.

Mrs. O’Neill struggled with the task conditions and the unpredictable nature of high cognitive demand tasks. When planning a lesson she wanted everything completely planned and did not like to fly by the seat of her pants. Mrs. O’Neill said she struggled with implementing tasks because she could not predict every student question or solution method that might arise during implementation. She said that when implementing tasks, questions arose that she had not thought of or something would pop up that she had not planned for, and that was difficult for her. This may have been because Mrs. O’Neill entered the classroom as a second career and had not had much training in the way of planning and implementing lessons. She often said no one taught her how to plan so she had figured it out on her own, and her method of planning was to plan every detail before teaching the lesson. Mrs. O’Neill also had a narrow view of the mathematics involved in the seventh grade curriculum. During the planning of the two tasks, she usually only found one way to solve the task and relied on others to provide other solutions. When solving the Border Problem, both Mr. Cone and Mr. Fielder found four different ways to solve, and Mrs. O’Neill wanted them to tell her what they were. During another team meeting, Mrs. O’Neill commented that she liked when someone showed her how to do things, such as someone finding and explaining how to implement the task. In general, it seems as though her lack of confidence in her students’ willingness to struggle and her inability to anticipate student solutions contributed to her need to have lessons explicitly
laid out for her to be comfortable during instruction. She did say that as she went through the day, the tasks were easier to implement because she knew how to address some of the situations that arose in prior classes.

Overall, there were many factors identified by Stein et al. (1996) that the teachers identified as contributing factors to the implementation of a high cognitive demand tasks. Factors that the teachers identified as obstacles fall into the categories of students’ learning dispositions, teachers’ instructional dispositions, and teachers’ knowledge of students. Factors teachers identified as contributing to the success of the task fall into the categories of task conditions, teachers’ instructional dispositions, and students’ learning dispositions. There were several other factors teachers identified affecting task implementation, including time, teachers’ knowledge of students, curricular and district requirements, and parental push back. These different factors illustrate the complexity that goes into planning and implanting high cognitive demand tasks in the mathematics classroom.
CHAPTER 5
DISCUSSION

Summary

Many organizations such as the NCTM, the NRC, and the National Governors Association have called for students to engage in problems that require thinking that is more complex and requires the use of the mathematical processes and not just procedures. Researchers have since expanded on this, suggesting that teachers pay attention to the cognitive demand of tasks (Henningsen & Stein, 1997; Smith & Stein, 2011; Stein et al., 1996). Mathematical tasks can be categorized in two levels, low and high cognitive demand. Cognitive demand refers to the amount of effort a student needs to expend to think about a problem. Henningsen and Stein argued that teachers should be able to select tasks appropriately and be able to implement tasks at a high level of cognitive demand in order to support students’ mathematical thinking. According to Smith and Stein, one way to frame the implementation of high cognitive demand tasks is utilizing what they described as the five practices for orchestrating productive discussions. The five practices include anticipating, monitoring, selecting, sequencing, and connecting student solutions.

Stein et al. (1996) found that the teachers involved in the QUASAR project were successful in selecting and implementing some aspects of high cognitive demand tasks but were not as successful in maintaining the intended level of cognitive demand.
Further, they found that the higher the initial cognitive demand, the less likely the teacher was to maintain that level of demand. Henningsen and Stein (1997) looked more closely at the classroom factors associated with the implementation of high cognitive demand tasks and found factors contributing to the decline or maintenance of demand. Boston and Smith (2009) provided professional development to teachers on the selection and implementation of high cognitive demand tasks. When they followed the participants in their classrooms, they found the teachers were able select high cognitive demand tasks and had success maintaining the demand of the task during lesson. The aforementioned researchers conducted their studies solely from the researchers’ perspectives. I was unable to locate any that offered the teachers’ perspectives on the selection and implementation of high cognitive demand tasks. Therefore, the purpose of my study was to observe teachers using high cognitive demand tasks, gain their perspectives on factors affecting the implementation, and compare the teachers’ perspective to my perspective.

I conducted a qualitative multiple case study of 3 seventh grade teachers at a public middle school in the Southeastern United States. Prior to the start of the school year, I provided a 4-hour professional development for the participants focused the implementation of high cognitive demand tasks using the five practices. During the first half of the school year I helped teachers plan for the implementation of high cognitive demand tasks, observed the teachers implement the tasks we planned together, and conducted interviews with each teacher after each lesson to gain his/her perspective on the implementation of tasks.

The theoretical framework guiding this study was the task implementation framework proposed by Stein et al. (1996). Stein et al. explained that various factors may
affect the cognitive demand of a task between successive phases of implementation (the task as represented, the task as set up by the teacher, and the task as implemented by students). Factors include teachers’ instructional dispositions, students’ learning dispositions, and task conditions.

Data collection included field notes from classroom observations, teacher interviews, and audio from team planning meetings. Data analysis included serial coding interview transcripts and observation field notes using the factors as described by Stein et al. (1996), coding instances of the teachers’ use of the five practices (Smith & Stein, 2011), and using Instructional Quality Assessment (Boston, 2012) to rate the implementation level of the tasks. While the task implementation framework addresses factors inside the classroom/during the lesson that affect the implementation of tasks, the teachers named several external factors, such as time to plan tasks and district requirements that affected their decisions to use or not use high cognitive demand tasks in their classrooms. Therefore, I proposed adding additional factors to the task implementation framework.

The teachers recognized factors that affected the demand of tasks during implementation. Each of the teachers attributed giving away information to lowering the demand and not giving information away to maintaining the demand. According to the task implementation framework this relates to the teachers’ instructional dispositions and their inclination as to when to allow students to struggle with the task. The teachers often did not allow students to grapple with the problem. Instead, they intervened to help students find the answer. The teachers realized they were lowering the demand and provided justification for leading students, claiming that if they did not give the
information or help the student, the student would have disengaged with the problem out of frustration. From the perspective of the teachers, lowering the demand was acceptable if it kept a student engaged.

The teachers attributed task conditions, students’ learning dispositions, and teachers’ instructional dispositions as relating to maintaining the demand of tasks. The teachers explained that the tasks having multiple solution methods and multiple entry points allowed students to engage successfully with the task. The teachers noted that students at all levels engaged with the high cognitive demand tasks, which was not always common in their classes. The teachers also attributed their instructional dispositions to helping maintain the demand of the task when they intentionally did not provide students with answers or lead them to solution methods.

Another factor that supported the teachers’ use of tasks was planning together as a team using the five practices to guide their planning. During their interviews, the teachers explained that planning with their colleagues and anticipating possible student solutions and misconceptions contributed to what they thought to be a successful implementation of the task. They said they were more inclined to implement high cognitive demand tasks when they were given time to collaborate and plan. When the teachers and I planned together, the teachers found multiple solution methods to a task and anticipated possible student solutions and misconceptions. The teachers then used these solutions to monitor and track students’ use of solution methods during instruction. The teachers selected students to present solution methods. While the teachers successfully integrated anticipating, monitoring, and selecting into their instruction, they did not appear to sequence student solutions purposefully and rarely made connections between student
solutions. I provided explicit instruction on anticipating, monitoring, and selecting but was unable to discuss sequencing and connecting due to time constraints and limited access to the teachers. Overall, I found many factors that affected the implementation of tasks, from both my perspective and the teachers’ perspectives.

Conclusions

In doing this research study I gained valuable information about working in a professional development capacity with inservice teachers. I realized that researchers and professional developers need to listen to teachers’ perspectives to ascertain what interventions will benefit teachers and will fit within the confines of teachers’ many responsibilities. Listening to teachers can also help researchers individualize professional development plans and provide more targeted support for each teacher. While I initially implemented one professional development session with all three teachers, when analyzing the data from interviews and observations, it became apparent that each teacher needed different interventions and support to help him/her implement high cognitive demand tasks. The teachers were all at different places with respect to mathematical content knowledge, classroom management, self-confidence, and classroom circumstances (i.e., size and composition of classes), so they needed more individualized support than what I could provide in a common professional development session. This finding speaks to the value of mathematics coaches and/or instructional coaches who can work with teachers on an individual basis over time in their classroom settings to personalize teacher learning.
From my perspective the most productive interventions were planning meetings where we focused on the implementation of specific tasks and talked explicitly about using the five practices (Smith & Stein, 2011) during implementation. My findings showed that teachers were able to implement tasks at a fairly high level when we planned the implementation together. When we did not plan task implementation together, it did not happen in the classroom.

During our planning sessions I was explicit about the first three of the practices of anticipating, monitoring, and selecting, and while observing the implementation of the tasks we planned together I saw evidence of the teachers anticipating, monitoring, and selecting. What was not evident was sequencing and connecting, but I attribute that to the fact that I did not provide explicit instruction on how to sequence and connect student solutions. What kept the teachers from implementing the tasks at the highest level of the Instructional Quality Assessment was a lack of sequencing and connecting student solutions.

The teachers said the support they found most helpful was spending time planning the implementation of specific high cognitive demand tasks and being provided with high cognitive demand tasks that fit seamlessly into their curriculum. However, it was difficult to find time during their planning periods to focus on task implementation as the teachers had many other competing demands on their time.

The teachers also had difficulty finding time during instruction to implement the tasks that we planned because they felt pressure to cover the curriculum. This finding is consistent with previous research on implementing high cognitive demand tasks (Foley et al., 2012; McGee et al., 2013). Teachers in previous studies often cited needing to cover
their curriculum as a barrier to implementing tasks. Foley and colleagues found that the teachers in their study did not implement high cognitive demand tasks because they felt the tasks did not fit in with their curriculum, and they were unable to fit the tasks in due to needing to cover the required curriculum. McGee and colleagues found the teachers in their study did not implement tasks due to curricular and district requirements, such as testing throughout the year and the expansive curriculum they needed to cover. The teachers in my study expressed similar sentiments and explained that it was difficult to find time to fit in high cognitive demand tasks on top of what was required of them by the district.

While previous research on implementing high cognitive demand tasks has documented factors inside the classroom that contributed to the decline in demand (Henningsen & Stein, 1997; Stein et al., 1996), I found factors outside the classroom that affected teachers’ implementation of tasks. This finding aligns with the research done by Handal and Bobis (2004) who found that while teachers may see the benefit to implementing instructional practices such as thematic units, outside factors can affect their decisions about whether to use those practices. The most common factors the teachers in my study identified that influenced whether or not they implemented a task were factors that occurred outside of the classroom and included time, teachers’ knowledge of students, and curricular and district requirements. Therefore, I think when using the task implementation framework by Stein et al., researchers need to pay attention to outside factors that may affect the use of high cognitive demand tasks (See Figure 3).
Implications

While I only worked with the teachers a short time, they made strides in their implementation of tasks. This suggests a need for more sustained professional development throughout the year to assist teachers in implementing high cognitive demand tasks. I worked with the teachers on implementing two high cognitive demand lessons and wanted to continue working with them to improve areas of implementing high cognitive demands tasks, particularly during the middle and closing of the lesson. The teachers maintained the demand during the set up but had difficulty maintaining the demand throughout the lesson. If I had continued working with the teachers, I think I could have helped improve the overall implementation of tasks from the set up through closing the task. One area of improvement for which I saw need was support for how to scaffold during a lesson without lowering the demand. The teachers would often answer students directly or not answer their questions and walk away, but did not appear to scaffold in ways that supported students thinking and maintained the demand of the task. The teachers also needed support with selecting, sequencing, and connecting from the five practices. If I had worked with the teachers longer, I could have targeted support during the latter part of the lesson.

Teachers need access to high cognitive demand tasks, especially doing mathematics tasks that align with their curriculum. It was difficult to find high cognitive demand tasks aligned with the curriculum, and the teachers did not have the time to look for resources containing high cognitive demand tasks. The teachers were willing to implement the high cognitive demand tasks I found but were unable to search for tasks on their own due to time constraints and their self-proclaimed lack of knowledge about
quality resources. One reason for this is not all content areas are conducive to high cognitive demand tasks. Integers are a mathematical concept the teachers taught in the beginning of their curriculum, and high cognitive demand tasks did not appear to exist dealing with the operations with integers. The teachers investigated patterns with integers, had students develop rules for integer operations, and taught the subject in a conceptual way, but I was unable to find tasks with multiple solution methods and access points for students to engage with in regard to integers. This occurred with other content areas including solving inequalities and properties of rational numbers such as the commutative and associate properties.

Another reason the teachers need access to high cognitive demand tasks is because of the nature of high stakes testing. The teachers felt pressure to cover their curriculum in order to prepare students for the end of the year tests. They did not feel as though high cognitive demand tasks fit in with their curriculum and did not want to lose a day with a task. One of the teachers explained that the students needed to know the tested skills and focused time on skill practice instead of implementing high cognitive demand tasks. The other teachers felt similar pressure to cover the curriculum for the end of year tests. The teachers were unwilling to use a task that did not align with their standards, and needed tasks supporting their curriculum. The teachers’ resource materials, although based in conceptual understanding, did not contain many high cognitive demand tasks, and the teachers did not have time to look for tasks with all other responsibilities in their day. There existed a misalignment between their curriculum resources and the newly adopted Common Core standards. To supplement, the teachers used resources in their classrooms containing a procedural approach to the curriculum.
An implication for inservice teachers is the need for them to find ways to help one another find productive ways to plan together. The teachers in my study identified common planning as critical to the success of implementing high cognitive demand tasks, yet they did not have much common planning time to devote to planning high cognitive demand tasks. The teachers had two common planning times a week; one was devoted to data team meetings in which the teachers talked about using assessment to drive instruction. The second meeting was a general common planning time, but there was not always time to talk about tasks because the teachers had other topics they needed to discuss, more urgent to their daily teaching lives. This implies the teachers need the time and the structure to work productively on high cognitive demand tasks. There needs to be protected space in the week teachers can devote to planning high cognitive demand tasks, either by themselves or with other mathematics teachers, depending on the structure of the school.

**Limitations**

While completing this study, I faced limitations related to the study’s design and during data collection. A limitation I faced with the study design was the small sample size and how I chose my participants. I intended to survey all middle school teachers in the district and find participants who wanted to work on implementing high cognitive demand tasks, but due to the logistics involved in working in four different middle schools, I chose to use participants at one school where I had already established relationships with the administration and teachers to allow for more access to the teachers. This decision limited the participant pool, and the pool was further limited
because the principal directed me toward this particular team of teachers due to personnel changes on the other teams. So, although the teachers on the seventh grade team volunteered for my study, they did so under some degree of pressure, and in the end, I am not convinced that these teachers fully bought into the idea of implementing high cognitive demand tasks on a regular basis. Perhaps because they were not fully invested in the study, their other school responsibilities often took precedence over my study. The daily life of a teacher includes many demands with school duties outside of teaching, and one participant noted that they often did what was urgent, not important. While they may have wanted to meet with me, they could not ignore meetings called at the last minute, and they often cancelled scheduled meetings with me due to needing to make up tests or lesson plans for that day or week. This led to significant limitations in my data collection as I was only able to complete two observations cycles of planning, implementing, and interviewing afterward with high cognitive demand tasks with all three teachers instead of one cycle each month, which I had originally planned.

The largest limitation I faced was access to the teachers, which could relate to them not having complete buy in to implementing high cognitive demand tasks. I did not anticipate my teachers having so many outside responsibilities when I asked them to participate in my study. The outside activities included having preservice teacher interns in their classrooms twice a week, participating in other professional development programs, being team leaders, data team leaders, and being a part of two different content teams. I had originally planned to have a longer professional development session with the teachers during the first week of school but was limited to one four-hour session. This caused me to have to scale back what I wanted to cover with the teachers, and it caused
me to shorten the professional development activities with the teachers. I also had
difficulty scheduling planning meetings with the teachers. I planned to meet with them
during their Thursday morning common planning, but I would often get only part of the
meeting to talk with the teachers about implementing tasks, or they would have to cancel
the meeting with me altogether, even after I had arrived at the school. The teachers
canceled meetings because other meetings were scheduled at the last minute, including
meetings with parents, administrators, or academic coaches. We eventually started
meeting Friday mornings before school, but this did not take effect until October and
only lasted until the beginning of December. This meeting time worked because meeting
Friday morning meant nothing had interrupted their day yet, but 7 am on a Friday is not
an ideal time to meet when doing professional development with teachers. While I was
able to get some good quality planning time with the teachers toward the end, it was
overall not as much as I would have liked. I was limited with access to observations
because I only had certain days and times I could observe that did not conflict with my
responsibilities as a student, and if the teachers could not implement the lesson during
those days and times, I was unable to observe them teach. There were two lessons we
planned together that I was unable to observe and only talked with them about the
implementation of those lessons.

Another limitation of my data analysis was only observing each teacher during
one class period, but the teachers talked about the implementation of the tasks in all of
their class periods. When interviewing the teachers on the implementation of the tasks,
they often cited factors that affected the demand of the task that I did not observe. For
example, one teacher claimed later in the day to be better at not leading students, but
because I did not see this occur I had a different perspective on the success of the lesson.

A smaller limitation to the study was not having video recordings of the teachers
implementing high cognitive demand tasks. I had originally planned to record the
teachers and do video stimulated recall interviews to ascertain if the teachers could point
out instructional moves where they lowered or maintained the demand. While I had
included video recording in my institutional review board (IRB) application, I was unable
to collect consent from all the students in the class. The teachers were hesitant to have
video taken in their classroom without all students having parental consent to be video
recorded. After analyzing my data using field notes, I realized that it would have been
better to have the video to refer to at times. The video also could have helped when I
assisted one of the teachers with her classroom instruction because I was unable to take
field notes when I assisted with implementation.

Having video would have helped with the reliability of using the IQA observation
instrument. For the IQA to be reliable, it needs two trained raters with a video or direct
observation for each teaching episode (Boston & Wolf, 2004). I used the IQA as a means
to classify each teaching episode, but because I was the only one rating the episode from
field notes, the analysis is not as reliable.

Future Research

This study informs my thinking about future research with both inservice and
preservice teachers. With inservice teachers, I want to conduct a similar study in a more
conducive context. I did not have ideal access to the teachers or participants who were
devoted to implementing high cognitive demand tasks. Participants in a future study would include teachers who are committed to implementing more high cognitive demand tasks at any grade level, and I would work with them for a longer time, providing interventions and helping the teachers find high cognitive demand tasks aligned with their curriculum. Sustained professional development would allow for interventions over time and ensure the teachers had time devoted to planning, with me or other participating teachers at the school. Video-recording would be an integral part of data collection and a means to interview the teachers. I would make sure to record teachers implementing high cognitive demand tasks and apply the IQA with another rater to each recording during data analysis. The video-stimulated recalls would be included as part of the intervention. I would use the VSR during interviews if a teacher was not aware of changes in cognitive demand of tasks or was not able to articulate actions affecting the cognitive demand of tasks. I would ask teachers about their reasons for particular instructional moves that changed the cognitive demand to see if teachers’ perspectives align with mine on factors affecting the demand.

In future work with inservice teachers, I would use the Instructional Quality Assessment (IQA) (Boston, 2012) as both a professional development tool and an observation tool. I used the IQA as an observation protocol in this study, but in looking back, I think the teachers should have seen the instrument I was using to rate their lessons. Using the IQA as a professional development tool could assist teachers in conceptualizing what high cognitive demand tasks should look like from beginning to end. I would use the implementation lesson checklist (Boston, 2012) to help teachers realize what is involved in maintaining the demand of a task. The task implementation
framework focuses on what students should be doing throughout the lesson to engage with the task at a high level, and by sharing this with teachers it could help them focus more on what students should be doing. I would provide the teachers with one of the IQA rubrics that includes different type of questions to elicit students’ thinking and discuss at what points in the lesson teachers could utilize the different questioning techniques. Question types include probing questions, generating discussion questions, and exploring mathematical meanings and relationships questions (Boston, 2012). By providing teachers with questions for specific instances, they could plan questions during the anticipation stage of the lesson to help the flow of the lesson. I would have the teachers work through the IQA training so they could perform observations, which will give them a window into what I am looking for when observing the teachers. I would eventually like to use the IQA with administrators in the same capacity to help with their observations in the mathematics classroom.

Boston and Smith (2009) found teachers who participated in professional development targeted at implementing high cognitive demand tasks were more successful selecting and implementing those tasks in their mathematics classrooms. From working with the teachers in this study, there were times I individualized support on different aspects of implementing tasks. For example, I intervened in one classroom when the teacher answered students’ questions too quickly. I also co-taught with one of the teachers or taught her class to demonstrate implementing high cognitive demand tasks and walked around the room to help the students. I would like to study teachers participating in professional development aimed at using the five practices (Smith & Stein, 2011) to frame the implementation of high cognitive demand tasks. The purpose of
the research would be to study what supports are needed to help teachers implement high cognitive demand tasks using the five practices and study whether supports need to be individualized for different teachers and what those individual supports look like. I would like to continue working with inservice teachers and conduct more in-depth studies into the possibility of “differentiating” professional development for teachers. One way to do this is in the context of using the five practices to implement high cognitive demand tasks because the teachers in this study appeared to benefit from using the five practices of anticipating, monitoring, selecting, sequence, and connecting to frame instruction when implementing high cognitive demand tasks. I would like to continue using the five practices to frame the implementation of high cognitive demand tasks and help teachers utilize all of the practices in their classroom instruction. With beginning teachers, I would talk about all five of the practices and support their use of all five when planning to implement high cognitive demand tasks. I would make sure to work on anticipating and monitoring first, and then move on to selecting, sequencing, and connecting. For those who are able to anticipate and monitor, I would work more closely on selecting, sequencing, and connecting student solutions. The teachers I worked with did not sequence and connect student solutions with regularity, so I would want to emphasize how to sequence and connect when planning a lesson and reflect after the lesson if the teachers made connections throughout the lesson and how they could have if did not appear. By providing teachers with the professional development needed to implement high cognitive demand tasks and integrate the five practices into their instruction, I hope to add to the body of knowledge of professional development with inservice teachers and
the possibility of individualizing professional development for teachers based on their needs.

Conducting future research with preservice teachers with the intent to help bridge research to practice is another path I want to follow. I gained the perspective of inservice teachers who were relatively unfamiliar with high cognitive demand tasks and the five practices. By doing a similar study with preservice teachers, I can gain their perspective on implementing high cognitive demand tasks. I would like to engage pre-service teachers in activities promoting the use of high cognitive demand tasks during their methods courses. These activities would be similar to the interventions I used with the inservice teachers. I want to identify pre-service teachers’ perceptions of how high cognitive demand tasks will play a role in their future classrooms and if they plan to engage students with high cognitive demand tasks. I want to identify what factors they think will most influence their use of high cognitive demand tasks and attempt to address how to overcome any perceived barriers once in the classroom. I would then follow the pre-service teachers into the field to identify what supports are needed to implement high cognitive demand tasks when teachers transition from a preservice program into their first few years of teaching. I would then attempt to provide this support in their first few years to help the teachers continue to use high cognitive demand tasks throughout their career. By identifying whether and how teachers’ perspectives change from their teacher preparation program into the field, as well as what supports are needed once they becoming practicing teachers, I can contribute to the body of work on what assistance beginning teachers need to maintain the demand of tasks during implementation.
In Closing

What I have learned from this process is that both teaching and conducting research on teaching are complex processes with many variables and moving parts. While the process of working with teachers may have been challenging, it is ultimately rewarding to capture the teachers’ voices about what factors affect the implementation of tasks. I have found with support, teachers can implement high cognitive demand tasks and ultimately would like to find how to support teachers so they can find and implement high cognitive demand tasks with fidelity on their own.
References


### RUBRIC 2: Implementation of the Task

At what level did the teacher guide students to engage with the task in implementation?

<table>
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<th>Score</th>
<th>Description</th>
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| 4     | Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
- Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR  
- Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
There is explicit evidence of students’ reasoning and understanding. For example, students may have:  
- solved a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
- developed an explanation for why formulas or procedures work;  
- identified patterns, formed and justified generalizations based on these patterns;  
- made conjectures and supported conclusions with mathematical evidence;  
- made explicit connections between representations, strategies, or mathematical concepts and procedures.  
- followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a “4” because:  
- there is no explicit evidence of students’ reasoning and understanding.  
- students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy or too hard to sustain engagement with high-level cognitive demands);  
- students identified patterns but did not form or justify generalizations;  
- students used multiple strategies or representations but connections between different strategies/representations were not explicitly evident;  
- students made conjectures but did not provide mathematical evidence or explanations to support conclusions. |
| 2     | Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task. There was little ambiguity about what needed to be done and how to do it. Students did not connect to the concepts or meaning underlying the procedure being used. Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).  
OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1     | Students engage in memorizing or reproducing facts, rules, formulae, or definitions. Students do not make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |
| 0     | Students did not engage in mathematical activity. |
| N/A   | The students did not engage with a mathematical task. |
Appendix B: Academic Rigor 2: Implementation Checklist (Boston, 2012, p. 21)

<table>
<thead>
<tr>
<th>Mathematics Lesson Checklist</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check each box that applies:</strong></td>
</tr>
<tr>
<td><strong>A</strong> The Lesson provided opportunities for students to engage with high-level cognitive demands:</td>
</tr>
<tr>
<td>Students</td>
</tr>
<tr>
<td>o engaged with the task in a way that addressed the teacher’s goals for high-level thinking and reasoning.</td>
</tr>
<tr>
<td>o communicated mathematically with peers.</td>
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<tr>
<td>o had appropriate prior knowledge to engage with the task.</td>
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<tr>
<td>o had opportunities to serve as mathematical authority in classroom</td>
</tr>
<tr>
<td>o had access to resources that supported their engagement with the task.</td>
</tr>
<tr>
<td>Teacher</td>
</tr>
<tr>
<td>o supported students to engage with the high-level demands of the task while maintaining the challenge of the task</td>
</tr>
<tr>
<td>o provided sufficient time to grapple with the demanding aspects of the task and for expanded thinking and reasoning.</td>
</tr>
<tr>
<td>o held students accountable for high-level product; and processes.</td>
</tr>
<tr>
<td>o provided consistent presses for explanation and meaning.</td>
</tr>
<tr>
<td>o provided students with sufficient modeling of high-level performance on the task.</td>
</tr>
<tr>
<td>o provided encouragement for students to make conceptual connections.</td>
</tr>
</tbody>
</table>

C The Discussion provides opportunities for students to engage with the high-level demands of the task. Students:

- Use multiple strategies and make explicit connections or comparisons between these strategies, or explain why they choose one strategy over another.
- Use or discuss multiple representations and make connections between different representations or between the representation and their strategy, underlying mathematical ideas, and/or the context of the problem.
- Identify patterns or make conjectures, predictions, or estimates that are well grounded in underlying mathematical concepts or evidence.
- Generate evidence to test their conjectures. Students use this evidence to generalize mathematical relationships, properties, formulas, or procedures.
- (rather than the teacher) determine the validity of answers, strategies or ideas.
Appendix C: Rating the Academic Rigor Rubric: Rubric and Protocol Issues (Boston, 2012, pp 23–24)

Rubric Issues:
This rubric rates the implementation of a task, or how the task "plays out" during a lesson. A task that begins with a high score for Potential (i.e., a 3 or 4) is not always enacted during instruction in ways provide students with opportunities to engage in the rigorous thinking that the task affords.

How do I know what to look at to score Implementation of the Task?

- The score for this dimension is holistic, reflecting the level at which the task was implemented during the majority of time in the lesson, including both student work time and any whole or small group discussion.
- Implementation begins when students begin to work on the task and ends at the close of the lesson.
- During live observations:
  - circulate (without interacting with students) to get a sense of how students are working on the task and the type of support and direction being given by the teacher.
  - take field notes about the nature of students' work, interactions between the teacher and students (script if possible), and teachers' questions and students' contributions during the whole-group discussion (script if possible).
  - check items on the Lesson Checklist that apply to the lesson.
  - Rate AR2 as soon after observing the lesson as possible.
- When viewing videotaped lesson, score based on what you are able to observe the teacher and students doing in the video.
- Do not rate the lesson based on what you do not have the opportunity to see, (i.e., in a class of 20, we can see 3 students working on the task at a high level but cannot tell what the other students are doing -- use as evidence only what you are able to see).

Source of evidence to consider:

- nature of students' work
- interactions between students and between the teacher and students,
- teachers' questions and students' contributions during the whole-group discussion

Mathematics Lesson Checklist: Use the Mathematics Lesson Checklist to GUIDE scoring the Implementation rubric. (You will have checked the relevant items during the lesson observation, while the rubric can be scored following the observation.)

There is no rigid formula for moving from the Mathematics Lesson Checklist to scoring the rubric. However, in general:

- a score of "4" will have checklist items that provide opportunities to engage in high-level thinking AND have strong evidence of at least one of the items in the Discussion section of the checklist; OR students provided explanations of their mathematical work and thinking (i.e., items in Section C of the Checklist) as they were working on the task rather than during a whole-group discussion.

- a score of "3" will have checklist items that provide opportunities to engage with in high-level thinking AND no items or weak evidence of items in the Discussion section of the checklist, during a whole-group discussion or during students' work on the task.

- a score of "2" will have checklist items that did not provide opportunities to engage with in high-level thinking AND no items in the Discussion section of the checklist.

Protocol Issues:

Why include the discussion as part of Implementation when there is a separate Discussion rubric (AR2)?
- The implementation score is intended to be an overall rating of students' opportunities to engage in high-level thinking and reasoning throughout the entire lesson. The AR2 score indicates whether the level of cognitive demand of the instructional task (AR1) was maintained, increased or decreased throughout the lesson.
- The presence and level of the whole-group discussion is an important subset of implementation, and thus receives its own rating, as well.
Appendix D: Interview Protocols

The following are the interview protocols used with each interview. The interviews were semi-structured so while these questions were the guidelines.

First Interview Protocol

Questions about general planning and instruction
1. How do you approach your lesson planning?
2. What do you do when you plan?
3. Is there a general pattern to your class every day?
4. Is your plan different with different classes? Why or why not?
5. How do you approach your instruction?
6. How do you typically structure your mathematics lesson?
7. Do you structure your lessons according to CMP2? Why or why not?

Questions about High Cognitive Demand (HCD) tasks
8. What do you think a HCD mathematical task entails?
9. Is this the same or different as before working with me, here?
10. Where do HCD tasks fit in with your teaching?
11. Do you use HCD tasks mainly for review, preview, everyday teaching, etc?
12. During the enactment of HCD tasks, what is your role in the classroom?
13. What are your expectations for students?
14. How often do you give HCD tasks?
15. What kinds of HCD tasks have you implemented this year?

Questions about the implementation of the Figure S task
16. How do you think the implementation of the Figure S task went?
17. How did the students react? How do you think the students reacted?
18. Do you think that you maintained the demand of the task as you implemented it? Why or why not?
19. What support has been helpful when planning to implement HCD Tasks?
20. What support has been helpful when implementing HCD tasks when you are in the classroom?
21. What support would you like to receive going forward when planning to implement HCD tasks?
22. What support would you like to receive going forward when implementing tasks?

Questions about goals for students
23. Last year, what were your mathematical goals for your students?
24. Do you believe they achieved your goals? Why or why not?
25. What are your mathematical goals for students this year?
26. Are your goals different for different class periods of students? Why or why not?
27. Are they the same as last year? Why or why not?
Second Interview Protocol

1. What is your conception of a mathematical task and has it changed at all since last time we talked?

Questions about the Implementation of the Border Problem
2. What was your plan when implementing the Border Problem?
3. How do you think it went?
4. What were your goals for instruction during the Border Problem?
5. Do you feel as though you accomplished those goals? Why or why not?
6. Do you feel like you maintained the cognitive demand the whole way through?
7. Do you feel like you maintained the demand during the set up of the task?
8. How did you set up the task in the classroom?
9. How do you think the work period went? Do you think you maintained the demand while students worked on the problem?
10. How do you think the discussion went?
11. Did you ever feel like you lowered the cognitive demand in any of the class periods? Can you explain that time?
12. Do you remember a time when you realized you were maintaining the demand? Can you explain that time?
13. How do you feel as though your students handled the lesson?
14. What struggles did you have with the lesson?
15. What do you think went well, or what did you have success with when implementing the lesson?
16. One of my RQs is what obstacles/successes do teachers’ identify when attempting to implement HCD tasks. This was a HCD task, what happened before/during/after implementation that you felt was a success when implementing this task?
17. Were there any obstacles you felt you were facing when trying to implement the task?

General Questions about Implementing HCD tasks
18. Have you done any other tasks, HCD tasks, on your own, since the border problem in this unit. If yes, what types. If no, why not?
19. It is now October, do you feel as though it is easier or harder to implement more HCD tasks in the classroom and explain why.
Interview 3 Protocol

Background Question:
1. Could you talk a little bit about your experience teaching, who you are, your background, why mathematics?

Questions about HCD tasks
2. What do you think a HCD task entails?
3. How do you see HCD tasks fitting into your curriculum?
4. If everything was ideal, how would you integrate HCD tasks in your classroom, and what are the ideal conditions?
5. In a perfect world, what needs to happen for you to be able to implement more HCD tasks?
6. What keeps you from implementing HCD tasks?
7. Do you do things differently in different math classes, in relation to HCD tasks Does who you are teaching influence what you t