Three Essays on

Financing Public Spending in a Small Open Economy

by

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(Under the direction of William Lastrapes)

Abstract

While foreign aid transfers, in *theory*, serve to relieve the national savings constraint as government's finance their development agenda, the empirical evidence of its effectiveness is mixed. In fact, from the perspective of the donor community, the performance of foreign aid has been overwhelmingly unsatisfactory. But with aid transfers expected to increase, donors intensify their efforts towards improving the delivery and monitoring of aid transfers. The three essays examine how foreign aid and the government's response to those aid transfers may influence output, total consumption, debt accumulation, public and private capital accumulation, and the real exchange rate.

In the first chapter, I use a neoclassical growth model to examine how the allocation of aid among public investment expenditure, public consumption expenditure and a pure transfer can generate sometimes opposing long-run effects on key outcomes such as output, consumption, private and public capital accumulation. This study is not the first to do this. However, the paper uses a neoclassical growth model that allows technology and capital accumulation to be endogenous, but where policy variables alone influence the long-run growth rate through their influence on population growth and the technological parameters. This model does not generate the scale effects that were a source of concern and limit the usefulness of predictions from endogenous growth models. One finding is that the allocation of foreign aid to different public expenditure categories matters for the responses. Also, the results suggest that there is no scenario in which the long-run levels of public and private capital are enhanced when governments alter their public spending commitments.

Building on the first chapter, the second paper presents another neoclassical growth model, but one to examine how the real exchange rate responds to foreign aid transfers. The link between the real exchange rate and foreign aid has received attention in the recent literature as researchers try to find possible channels through which foreign aid mitigates its own effectiveness. The result highlight foreign aid scenarios in which the real exchange rate may either appreciate or depreciate in the long-run. The depreciation occurs: (i) under low adjustment costs to either the accumulation of public or private capital and when the foreign aid finances public investment expenditure; and (ii) when the government can alter its spending commitments in response to the allocation of aid to any type of expenditure.

The third paper uses panel and cross-sectional data methods to analyze data from sixty-six aid recipient countries for any effect of foreign aid transfers on the real exchange rate. The results suggest economic significance, and highlight the depreciation effect of foreign aid inflows.

Overall the results suggest that to the extent that donors and recipients can engage in macroeconomic management of the aid resources, countries can benefit from the growth potential while at the same time avert some of the adverse effects of foreign aid flows.

INDEX WORDS: non-scale growth, level effect of foreign aid, foreign aid and real exchange rate, endogenous labor, endogenous growth, neo-classical growth

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by

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Dedication

To my parents, Cecil and Kimlan, who instilled the desire to ask questions and the courage to search for the answers.

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Chapter 1

Foreign Aid and Macroeconomic Performance

1.1 Motivation and Background

Foreign aid may be in need of aid. In 2005, total official development assistance rose by 32% from USD 80.9 billion in 2004, and tens of billions of dollars will continue to be allocated each year with the goal of fostering economic growth and development in recipient countries. But while donor agencies are ready to provide assistance, there remains a lingering concern for whether aid transfers can create lasting development results. In fact, concern for the impact of aid has intensified as the donor community works towards achieving the Millennium Development Goals. With the donor community worried about the practice of foreign aid, the research community has contributed to the debate with both theoretical and empirical studies on how aid transfers may facilitate or hinder growth. At best, the recent empirical evidence on the effectiveness of aid has been mixed, and the theoretical predictions limited[1]. Some researchers have gone so far as to say that recipients suffer from the aid 'curse' (Hartford & Klein (2005) and Djankov, Montalvo & Reynal-Querol (2005)). One policy implication to emerge from both theoretical and empirical literature has been that effectiveness can be improved if the aid is directed to specific projects in the recipient's country development agenda[2].

But, even when programs and projects are carefully planned, monitored, and implemented, the growth and development impacts can be limited if the aid is fungible. Foreign

[1]Some of the more influential aid effectiveness literature include Boone (1996), Feyzioglu, Swaroop & Zhu (1998), Burnside & Dollar (2000), Easterly (2003), Burnside & Dollar (2004), Easterly, Levine & Roodman (2004), Dalgaard, Hansen & Tarp (2004), Collier & Dollar (2004), and Cordella & Dell' Ariccia (2007).

[2]See Dollar & Levin (2006), Wane (2004), and Easterly & Pfutze (2008).

aid is *fungible* when either the recipient government diverts resources from public expenditure programs that are earmarked to receive foreign aid transfers (Pack & Pack (1993)), or when it crowds out spending that the recipient government would otherwise take if the resources were not available (Cordella & Dell' Ariccia (2007)). Some argue that fungibility is an empirical question because it is hard in practice to monitor how government's allocate the aid[3] (Feyzioglu, Swaroop & Zhu (1998)). Others argue that it is a theoretical construct (Pack & Pack (1993)). For Hartford & Klein (2005), aid creates a morale hazard problem as recipients spent without a binding budget constraint[4]. Whether empirical or not, understanding theoretically how government behavior may affect the accumulation of both private and public capital is useful for refining the practice of foreign aid.

This paper adds to the aid effectiveness literature by using a neoclassical growth model to examine how the allocation of foreign aid among public investment expenditure (referred as *tied aid*), public consumption expenditure, and a pure transfer (referred as *untied aid*) may influence the long-run levels of key outcomes such as output, consumption, and private and public capital accumulation[5]. While there have been recent

[3]Two studies show empirically that growth may not be an outcome where institutions are weak and aid is not tied to specific expenditures (Feyzioglu, Swaroop, and Zhu 1998; and Dollar and Levin 2006). But even with this view of designing aid packages that meet the objectives of both donors and recipients, (Burnside & Dollar (2000), Hansen & Tarp (2001) and Wane (2004), the empirical evidence has not been very encouraging but did highlight that: (i) aid is as effective as the policy environment in which it is introduced; and (ii) initial country characteristics are important - results that have influenced widely, for better or worse, the practice of aid.

[4]To varying degrees, researchers have discussed the ways in which aid can weaken and remove the incentives by governments to develop sustainable fiscal institutions (Corden & Neary (1982), Burnside and Dollar (2000), Boone (1996), Svensson (2000), Svensson (2003), Easterly (2003), Dalgaard, Hansen & Tarp (2004) and Collier & Dollar (2004)). Other studies agree and further warn that aid encourages rent seeking, which can weaken political and economic institutions (Tornell & Lane (1999), Acemoglu & Robinson (2000), Sachs & Warner (2001) and Sala-i-Martin & Subramanian (2003)).

[5]Chatterjee and Turnovsky (2007) use Bhagwati (1967) and recent World Bank studies to classify tied and untied aid. For these authors, tied aid refers to transfers that can be used to enhance public investment projects or augment the provision of public consumption goods and may include procurement from specific donor countries. In contrast, untied aid are those transfers that are 'pure' in the sense that they are not restricted to be used for procurement of services from the donor country and facilitate a tax reduction.

theoretical studies of foreign aid, the limitations of some of the underlying assumptions may affect the results that are generated. The main thesis of the paper is that foreign aid does not permanently alter the long-run growth rate but can have long-run level effects on macroeconomics variables. This argument is consistent with the growth and convergence literature which presents economies as characterized by neo-classical growth. The assumption of neo-classical growth is a critical point of departure from some of the more recent aid literature that adopt the endogenous growth approach, and may be important in explaining why the empirical evidence of aid effectiveness has been mixed.

To the extent that the empirical models were examining for the effect of aid on long-run growth rates, foreign aid may be deemed ineffective if in fact the growth rate of the economy is not affected in the long-run. In terms of the fungibility question, to my knowledge there has not been an attempt to model aid fungibility in a neoclassical growth framework. The paper argues that the positive outcomes from aid can be mitigated when government alters its own spending commitments or when government spending is crowded out. From a policy perspective, the results are significant as the donor community is currently reassessing the benefits of aid in light of donor fatigue and falling donor commitments (See Wane (2004) and Easterly & Pfutze (2008).).

Jones (1995*a*, 1995*b*, 1999) are among the first in the literature to suggest how a neoclassical growth model can address at least two of the shortcomings of the endogenous growth theory. First, and the most severe criticism of the endogenous growth approach has been the lack of evidence in empirical studies on growth and convergence of the on-going long-run equilibrium growth rate (Jones (1995*a*), Jones (1995*b*), Segerstrom (1998), Young (1998), and Sala-i-Martin & Subramanian (2003)). Second, models of endogenous growth are characterized as exhibiting scale effects. Scale effects, as discussed by Jones (1995*a*), imply that an increase in population must necessarily lead to faster growth rates - a hypothesis that is also not supported by empirical evidence. Some studies find that after controlling for technological changes, countries converge to a long-run growth rate

(Eicher & Turnovsky (1999*a*), citing Mankiw, Romer, and Weil (1992); and Barro and Sali-i-Martin (1995)). In respect of aid specific studies, Dalgaard, Hansen & Tarp (2004) find their model of endogenous growth does not show a robust link between aid transfers and growth even after attempting to account for offsetting measures such as corruption and political rent seeking.

The approach here uses a neoclassical model in which there are no scale effects (referred as a *non-scale growth* model)[6]. First, Eicher & Turnovsky (1999*c*), Eicher & Turnovsky (1999*a*), Eicher & Turnovsky (2001) and Turnovsky (2004) reason that the predictions from the non-scale approach are more robust and reliable than those generated from a model in which the accumulating factors are subject to the constant returns to scale restriction. The constant return to scale restriction is a necessary assumption of the endogenous growth model that imposes a knife-edge condition for the stability of the long-run equilibrium growth rate[7]. Second, unlike in endogenous growth models, several studies show policy changes in non-scale models as having level effects and affecting the growth rate only during transition (Eicher and Turnovsky (1999*b*), Eicher & Turnovsky (2001) and Turnovsky (2004)). Eicher & Turnovsky (1999*b*) and Eicher & Turnovsky (2001) find longer transitional dynamics and slower convergence rates using this variant of a neoclassical model. They also argue that the accumulation effects observed during transition help identify the level effects of aid shocks. As such, application of this theoretical approach to the aid debate can therefore extend the understanding of aid shocks and its effectiveness: (i) through the generation of results that are more robust and reliable; and (ii) by allowing for longer transitional dynamics.

[6]Non-scale growth refers to the case where variations in the levels of variables do not permanently affect their long-run equilibrium growth rates. That is, a country's growth rate is independent of the scale of the economy (Jones (1995*b*), Segerstrom (1998), and Young (1998)).

[7]Eicher & Turnovsky (1999*d*) emphasize that models of endogenous growth require the restriction of constant returns to scale on the accumulating factors which imposes a knife-edge condition that is necessary for stability. The production restriction is unrealistic and empirical studies have not supported conclusively this strict assumption.

The paper also benefits from recent research on aid effectiveness[8] (Chatterjee, Sakoulis & Turnovsky (2003), Chatterjee & Turnovsky (2005) and Chatterjee & Turnovsky (2007)). These three papers, which are part of a comprehensive research program, examine the effect of the composition of aid that is allocated across public expenditure. The model used in their papers finds the effects of aid shocks on long-run growth to depend on the characteristics (initial conditions) in the recipient country[9]. Chatterjee & Turnovsky (2007) find aid allocations across public investment and consumption expenditure may have opposing growth and welfare effects. Aid to public investment expenditure is growth-enhancing, and generates changes in steady-state ratios and growth rates that are consistent with increases in productive capacity. However, an allocation to either a public consumption expenditure or to a pure transfer does not have the same impact on the economy's productive capacity. The results from their work provided important insight on the impact of foreign aid on public expenditure. However, it is unclear from their predictions the magnitudes of the aid shock on the individual *levels* key variables of interest. Moreover, the shortcomings of the endogenous growth theory limit the reliability of the hypothesis that were generated.

The model set-up uses the framework of Chatterjee and Turnovsky (2007) as a starting point. As in their model, labor is endogenous so that aid transfers may affect the labor-leisure decision as well as private capital accumulation. The model also allows for capital market imperfections through the increasing supply curve for debt and for variations in: (i) the intertemporal adjustments costs associated with public and private capital accumulation; and (ii) the substitutability between private and public capital in the production technology. The model is highly non-linear and the analysis is done using numerical methods as it would be difficult to obtain closed form analytical solutions. The paper focuses on

[8]This research program benefitted from earlier theoretical research on the impact of public capital on macroeconomic performance, see Chatterjee and Turnovsky (2007).

[9]None of their models incorporate political support groups, rent-seeking activities, and corruption directly. That these models do not account for the political economy of aid, a factor aid agencies admit affects aid effectiveness, is one of the significant criticisms that has been leveled at their research agenda.

the dynamics responses and the long-run level responses of: leisure, consumption, output, debt, and public and private capital. The paper also examines the dynamic responses to allocations among the expenditure types and to aid fungibility.

The results suggest the change in the marginal productivity of the private factors in response to the portion of aid that is allocated to public investment expenditure as the main trigger for the dynamic responses. Aid to public investment expenditure generates transitional dynamics and long-run responses that are in sharp contrast to aid that either finances a pure transfer or is allocated to public consumption expenditure. Aid that is allocated fully to public consumption expenditure or between consumption expenditure and a pure transfer have only welfare effects in the short-run and do not affect the levels of either private or public capital accumulation. A pure transfer facilitates lump-sum tax reduction and is welfare enhancing in the long-run. Any allocation to public expenditure (either to investment or consumption) generates proportionally higher level effects on output, and public and private capital than a pure transfer. Overall, any amount of aid that is allocated to productive uses have higher long-run level effects, but not without the long-run tradeoffs to welfare. By varying the composition of aid and the possible responses by government, the results show that both welfare effects and level responses are reduced when governments can alter their spending commitments.

1.2 A Neoclassical Growth Model of Foreign Aid without Scale Effects

1.2.1 The Model Setup

The Private Sector

Consider a small, open economy that comprises N identical infinitely lived agents. The population grows exponentially by $\dot{N} = nN$. Each agent has an infinite planning horizon and possesses perfect foresight. Individuals produce domestic output, X_i. Agents consume some amount of the domestic output and a public consumption good, G_C. Each individual allocates one unit of labor between leisure, l_i and labor, $(1 - l_i)$. The agent can accumulate wealth in the form of physical capital, K_i and government issued bonds, B_i. Private capital accumulation is subject to adjustment and depreciation costs. Each household produces domestic output described by the Cobb-Douglas production function:

$$X_i = \alpha(1 - l_i)^{1-\sigma} K_i^{\sigma} K_G^{\eta}, \tag{1.1a}$$

where K_i is the private agent's stock of capital and K_G is the economy's aggregate stock of infrastructure or a pure public good that is not subject to congestion. The public capital stock generates a positive externality to production. σ measures the output elasticity of labor in the production function, and η is the output elasticity with respect to public capital or the marginal product of public capital. Equation (1.1a) is well-behaved in that each private factor has positive but diminishing marginal physical productivity. To ensure the existence of an equilibrium, the production function exhibits constant returns to scale in the private factors, labor and private capital. Domestic output can be allocated to either consumption or private capital.

Aggregate output is:

$$\begin{aligned} X &= NX_i \\ &= \alpha((1-l)N)^{1-\sigma} K^{\sigma} K_G^{\eta}, \qquad (1.1b) \end{aligned}$$

$$0 < \alpha < 1, \sigma > 0, \eta \gtrless 0, 1 > \sigma + \eta.$$

Following Eicher and Turnovsky (1999*b*) and Chatterjee and Turnovsky (2002), there are no restrictions on the accumulated factors, K_G and K. The agent's welfare is given by the intertemporal utility function:

$$U = \int_0^{\infty} \frac{1}{\gamma}(C_i l_i^{\theta} G_C^{\phi})^{\gamma} e^{-\beta t} dt, \theta < 0, \phi < 0, -\infty < \gamma < 1, \tag{1.2a}$$

where C_i is the individual agent's consumption; G_C denotes the consumption services of the public consumption good; and l_i is the fraction of labor to be allocated to leisure. The parameters, θ and ϕ, measure the impact of leisure and the public consumption good on the agent's welfare respectively; β is the rate of time preference; and γ measures the rate of intertemporal substitution. The agent accumulates capital according to:

$$\dot{K}_i = I_i - (n + \delta_K)K_i. \tag{1.2b}$$

In Equation (1.2b), individual capital accumulation is adjusted for population growth by the factor nK_i and depreciates by the scale factor of $\delta_K K_i$. Private capital in Equation (1.2b) is subject to non-linear adjustment costs such that total private investment is:

$$\Phi(I_i, K_i) = I_i + \frac{h_1}{2}\frac{I_i^2}{K_i}. \tag{1.2c}$$

The agent also accumulates bonds to finance private consumption expenditure. Following the specification of Eicher & Turnovsky (1999c), the credit-worthiness of the economy influences the domestic interest rate on bonds, r(.)[10], that agents face when making bond accumulation decisions. In his optimization problem, the agent assumes that the borrowing rate on bonds is exogenously determined. The agent accumulates bonds as by:

$$\dot{B}_i = C_i(1 + \tau_c) + ((1 + \tau_b)r(.) - n)B_i + \Phi(I_i, K_i) - (1 - \tau_X)X_i + T_i, \tag{1.2d}$$

where the accumulation is adjusted for population growth by the factor, nB_i.

The agent maximizes his intertemporal utility in Equation(1.2a), subject to the flow budget constraint equations, Equation(1.2b) and Equation(1.2d), such that the agent's decision problem is:

$$H = \frac{1}{\gamma}(U(C_i))^{\gamma} e^{-\beta t} - q'(\dot{K}_i)e^{-\beta t} + \nu(\dot{B}_i)e^{-\beta t}.$$

The optimality conditions with respect to the individual's choices for C_i, l_i, I_i and the rates of accumulation of capital and bonds, K_i and B_i respectively, are given by:

$$u_c(C)\frac{\partial C(C_i, l_i, G_c)}{\partial C_i} = \nu(1 + \tau_c) \tag{1.3a}$$

[10] I derive the domestic borrowing rate from the upward sloping supply curve of debt in the following section.

$$u_c(C)\frac{\partial C(C_i, l_i, G_c)}{\partial l_i} + \nu(1-\tau_X)\frac{\delta X_i}{\delta l_i} = 0 \tag{1.3b}$$

$$\Phi_I(I) = \frac{q'}{\nu} \tag{1.3c}$$

$$\frac{q'}{\nu}\beta - \frac{\dot{q}'}{\nu} = -\frac{q'}{\nu}(n+\delta_K) + \frac{h_1}{2}\left(\frac{I_i}{K_i}\right)^2 + (1-\tau_X)X_K(K, KG) \tag{1.3d}$$

$$\dot{\nu} = \nu(\beta - (1-\tau_b)r(.) - n). \tag{1.3e}$$

The transversality conditions which enforce that the agent cannot accumulate private capital and bonds indefinitely are:

$$\lim_{t\to\infty} q' K_i e^{-\beta t} = \lim_{t\to\infty} \nu_i B_i e^{-\beta t} = 0.$$

In Equation (1.3a) - Equation (1.3e), ν is the shadow price on wealth in the form of internationally traded bonds. $q^{'}$ is the shadow price on the agent's private capital stock or the market value of private capital, such that $q = \frac{q^{'}}{\nu}$ is the shadow price of private capital stock in terms of bonds. Equation (1.3a) equates the marginal utility of consumption to the tax-adjusted shadow price of wealth. Equation (1.3b) equates the marginal utility of leisure to the after-tax marginal product of labor. Equation (1.3c) equates the marginal cost of an additional unit of investment to the market value of capital. Equation (1.3d) gives the relative price of bonds in terms of private capital. Equation (1.3e) equates the rate of return on consumption to the after-tax return on bonds.

The Public Sector

Government revenue comprises taxes on consumption, bonds, income, lump-sum taxes, and the foreign aid transfer. Government revenue finances public investment, K_G, and the public consumption good, G_C. The government is passive and takes the individual agent's optimization as given in determining the amount of the public good to provide.

Following Turnovsky (2004), government revenue and foreign aid receipts finance the public consumption good by:

$$G_C = g_C^d X + \lambda_C F = (g_C^d + \lambda_C f)X, \tag{1.4a}$$

and public investment by:

$$G_I = g_I^d X + \lambda_I F = (g_I^d + \lambda_I f)X, \tag{1.4b}$$

where g_C^d and g_I^d represent the government's domestic expenditure rates on public consumption and investment (public capital) respectively. λ_C and λ_I are the proportions of the foreign aid to be allocate to the public consumption and investment goods respectively. Foreign aid proportions may be determined by either the donor alone or in consultation with recipient governments. The choices of λ_C and λ_I allow for the possibility that incentives are aligned between donors and recipients, for instance when donors base total aid transfers on past public expenditure or, perhaps, when recipients indicate specific programs that need aid funding. The total amount of the foreign aid transfer is proportional to the level of economic activity in the recipient country and is given by:

$$F = fX; 0 \leq f \leq 1, \tag{1.5}$$

where, f, is the ratio of the foreign aid transfer to GDP.
The government accumulates public capital in accordance with:

$$\begin{aligned} \dot{K}_G &= G_I - \delta_G K_G \\ &= (g_I^d + \lambda_I f)X - \delta_G K_G, \end{aligned}$$

where G_I is gross public investment expenditure, and δ_G is the rate of depreciation of the stock of public capital. The stock of public capital is also subject to adjustment costs such that total investment in public capital is:

$$\Omega(G_I, K_G) = G_I + \left(\frac{h_2}{2}\right)\left(\frac{G_I^2}{K_G}\right). \tag{1.6}$$

The government can also borrow from international capital markets to finance its expenditure. The government's budget constraint in terms of bonds issued by the domestic government, A, is given by:

$$\dot{A} = \Omega(G_I, K_G) + G_C + r(.)A - \tau_b r(.)B - \tau_X X - T - F, \tag{1.7}$$

The government must satisfy the intertemporal budget constraint:

$$\lim_{t\to\infty} Ae^{-r(.)t} = 0. \tag{1.8}$$

Combining the government budget constraint, Equation (1.7), and the budget constraints across all private agents, Equation (1.2d), gives the aggregate resource constraint[11]:

$$\begin{aligned} \dot{Z} &= \dot{A} + N\dot{B}_i \\ &= r(.)Z + \Omega(G_I, K_G) + \Phi(I_i, K_i) + G_C + C - X - F. \end{aligned} \tag{1.9}$$

The interest rate, $r(.)$, in Equation (1.9), is determined in international capital markets. International capital markets consider the country's debt-to-capital ratio as an indicator of the country default risk in determining the interest rate on borrowing. Following the Turnovsky (1997) and Eicher & Turnovsky (1999*c*) specification[12], the bond interest rate is proportional to the debt-servicing capacity of the aggregate economy:

$$r\left(\frac{Z}{K}\right) = r^* + \omega\left(\frac{Z}{K}\right), \omega' > 0, \tag{1.10}$$

where $r(.)$, is expressed as a function of the ratio of national debt, Z, to the aggregate stock of private capital, K, and r^* is the exogenous world interest rate.

[11]The aggregate resource constraint is also considered in the literature to be a measure of national debt and the current account.

[12]Their specification for the bond interest rate as a function of the debt servicing capacity benefits from Obstfeld (1982), Cooper & Sachs (1985) and van der Ploeg (1996). The original specification is attributed to Bardham (1967).

1.2.2 Macroeconomic Equilibrium

Macroeconomic Dynamics

A feature of the non-scale models is that the economy converges to a unique long-run growth rate. That is, in the long-run, aggregate output, private capital, private consumption as well as public capital are assumed to grow at the same constant rate, so that the output-to-capital ratio, the public capital-to-private capital ratio and the fraction of time devoted to leisure remain constant[13]. Therefore, $\dot{Z} = \dot{q} = \dot{l} = \dot{K} = \dot{K_G} = 0$. Taking percentage changes of the aggregate production function, Equation (1.1b), the long-run equilibrium growth rate of output, private capital, and public capital, ψ, is given by:

$$\psi = \left(\frac{1-\sigma}{1-\sigma-\eta}\right) n. \tag{1.11}$$

The growth rate of the economy, ψ, is independent of policy parameters (and the scale of the economy) and is a function of the growth in population and the productive elasticities of the two types of capital. Jones (1999) shows that the long-run per capita growth, $(\psi - n)$, is positive as long as private and public capital are productive. More specifically, assuming that $\sigma < 1 - \eta$, the long-run equilibrium growth rate is non-negative, $\psi > 0$[14].

To analyze the transitional dynamics, the system is expressed in terms of stationary variables: the fraction of time allocated to leisure, l, and the scale-adjusted per capita

[13] In contrast, in endogenous growth models it is the rate of change of these ratios that is constant (zero) in the long run.

[14] Turnovsky and Chatterjee (2002) show that the stability of the steady state rests on $\sigma + \eta < 1$ in Equation (1.11), in which case $\psi > 0$.

quantities:

$$
\begin{aligned}
k &= \frac{K}{N^{\frac{1-\sigma}{1-\sigma-\eta}}} \\
k_G &= \frac{k_G}{N^{\frac{(1-\sigma)}{(1-\sigma-\eta)}}} \\
x &= \frac{X}{N^{\frac{(1-\sigma)}{(1-\sigma-\eta)}}} \\
c &= \frac{C}{N^{\frac{(1-\sigma)}{(1-\sigma-\eta)}}} \\
z &= \frac{Z}{N^{\frac{(1-\sigma)}{(1-\sigma-\eta)}}}.
\end{aligned} \tag{1.12}
$$

Appendix A presents the macroeconomic dynamics for l, q and the scale-adjusted stationary variables z, k, and k_G. Note that under constant returns to scale, Equation (1.12) reduces to per capita quantities. Noting Equation (1.11) and that individual consumption can be expressed in terms of aggregate consumption, $C_i = \frac{C}{N}$, the macroeconomic dynamics for the economy is summarized as follows:

$$\frac{\dot{k}}{k} = \frac{\tilde{q}-1}{h_1} - \delta_K - \left(\frac{1-\sigma}{1-\sigma-\eta}\right) n \tag{1.13a}$$

$$\frac{\dot{k_G}}{k_G} = (g_I^d + \lambda_I f)\frac{\tilde{x}}{\tilde{k}_G} - \delta_G - \left(\frac{1-\sigma}{1-\sigma-\eta}\right) n \tag{1.13b}$$

$$
\begin{aligned}
\frac{\dot{z}}{z} \quad = \quad & r(.) + \frac{\tilde{c}}{\tilde{z}} + \frac{\tilde{k}}{\tilde{z}}\left(\frac{\tilde{q}^2-1}{2h_1}\right) + \frac{\tilde{x}}{\tilde{z}}\left(g_d^C + \lambda_C f + g_I^d + \lambda_I f - 1 - f\right) \\
& + \frac{h_2}{2}(g_I^d + \lambda_I f)^2 \frac{\tilde{x}^2}{\tilde{z}\tilde{k}_g} - \left(\frac{1-\sigma}{1-\sigma-\eta}\right) n
\end{aligned} \tag{1.13c}
$$

$$\dot{q} = (1+\tau_b)r(.) - \sigma\left(\frac{1-\tau_X}{q}\right)\frac{\tilde{x}}{\tilde{k}} - \frac{(\tilde{q}-1)^2}{2h_1\tilde{q}} + \delta_K \tag{1.13d}$$

$$\dot{l} = F(.)G(.) \tag{1.13e}$$

where

$$c = \left(\frac{1-\tau_X}{1-\tau_C}\right)\left(\frac{1-\sigma}{\theta}\right)\left(\frac{\tilde{l}}{1-\tilde{l}}\right) x \tag{1.13f}$$

and the aggregate scale adjusted production function is:

$$x = \alpha(1-l)^{1-\sigma}\tilde{k}^{\sigma}\tilde{k_G}^{\eta}. \tag{1.13g}$$

Finally, F(.) and G(.) in Equation (1.13e) are given by:

$$F(\tilde{l}) = \frac{\tilde{l}(1-\tilde{l})}{\theta\gamma(1-l) - (1-\gamma) - (1-\sigma)(\gamma(1+\phi)-1)\tilde{l}}$$

and

$$\begin{aligned} G(\tilde{z}, \tilde{k}, \tilde{k}_G, \tilde{l}, \tilde{q}) &= \beta + n\gamma - (1+\tau_b)r(.) - (\gamma(1+\phi)-1)(1-\sigma)n \\ &\quad -\sigma(\gamma(1+\phi)-1)[\frac{\tilde{q}-1}{h_1} - \delta_K] - (\gamma(1+\phi)-1)\eta\delta_G \\ &\quad -(\gamma(1+\phi)-1)\eta(g_I^d + \lambda_I f)\alpha(1-\tilde{l})^{1-\sigma}\tilde{k}^{\sigma}\tilde{k}_g^{\eta-1}. \end{aligned}$$

The system in Equation (1.13a) - Equation(1.13g) along with Equation (1.11) can be solved for $\tilde{q}$, $\tilde{l}$, $\tilde{\psi}$, scale-adjusted steady state values $\tilde{k}$, $\tilde{k}_G$, $\tilde{z}$, $\tilde{c}$, and the ratio $\frac{\tilde{z}}{\tilde{k}}$. First Equation (1.11) solves for the long-run growth rate, $\tilde{\psi}$. Equation (1.13a) solves for the steady-state rate of return on private capital in terms of bonds, $\tilde{q}$. Equation (1.13e) solves for the ratio of steady-state debt-to-private capital ratio, $\frac{\tilde{z}}{\tilde{k}}$. Once the debt-to-private capital ratio is known, then the domestic borrowing rate can be determined from Equation (1.10). Equation (1.13f) gives an expression for the steady-state, scale-adjusted consumption, $\tilde{c}$, in terms of $\tilde{l}$, $\tilde{k}$, and $\tilde{k}_G$. Equation (1.13b), Equation (1.13c), and Equation (1.13d) can be used to solve for steady state $\tilde{l}$, $\tilde{k}$, and $\tilde{k}_G$. Steady-state debt, $\tilde{z}$, can be deduced using $\tilde{k}$ and $\frac{\tilde{z}}{\tilde{k}}$. $\tilde{l}$, $\tilde{k}$, and $\tilde{k}_G$, and Equation (1.13g) can be used to obtain scale-adjusted output, $\tilde{x}$.

The dynamic system involves three state variables, k, k_G and z, and two jump variables, q and l. The system in Equation (1.13) is highly non-linear and would be difficult to solve analytically. Instead, a linear approximation is obtained, and the steady state is evaluated by numerical simulation. The numerical simulations reveal a saddle-point equilibrium with three stable (negative) eigenvalues for the state variables and two unstable (positive) eigenvalues for the jump variables. The model is parameterized to examine the dynamic response of the economy to a foreign aid shock that is allocated across public expenditure. The parameter values are chosen to: (i) characterize a small open economy; and (ii) yield the required pattern of eigenvalues - three negative stable roots for the state variable and two positive eigenvalues for the jump variables - that would be consistent with a steady-state equilibrium.

Parameterization of The Model

Table 1.1 presents the parameter values that lead to the initial steady-state values of the benchmark economy[15].

Table 1.1: Base Economy Parameter Values

Preference parameters	$\gamma = -1.5, \beta = 0.04, \theta = 1.75, \phi = 0.3$
Production parameters	$\alpha = 1, \sigma = 0.35, \eta = 0.2, h_1 = 15, h_2 = 15$
Depreciation rates	$\delta_K = 0.05, \delta_{KG} = 0.05$
World interest rate	$r* = 0.06$
Premium on borrowing	$a = 0.10$
Policy parameters	$\tau_b = 0, \tau_Y = .028, \tau_c = 0, g_C^d = 0.14, g_I^d = 0.05$
Transfers	$\lambda_I = 0, \lambda_C = 0$

The preference parameters, β and γ, the depreciation rates, δ_K and δ_G, and the world interest rates, r^*, are standard and are based on the original work of Ogaki & Reinhart (1998). The intertemporal elasticity of substitution is plausible and within the range of 0.1 to 1 as supported by the literature (Hall (1988) and Beaudry & van Wincoop (1995). The choice of the production parameters, α, σ, and η, are discussed in Gramlich (1994) and are chosen to lie within realistic ranges. The borrowing premium, α, is chosen to ensure a plausible debt-to-capital ratio; the elasticity on leisure, θ, is chosen to ensure a leisure allocation of 0.76; and the impact of public consumption on utility, ϕ, is set at a level which is consistent with empirical evidence. The adjustment costs to both capital stocks are set at $h_1 = h_2 = 15$ to achieve a plausible benchmark equilibrium. The tax parameters on bonds, wage income and consumption are included for completeness, although their values are not changed during the analysis. Government expenditure parameters g_I^d and g_C^d are U.S. averages and are calibrated to result in a plausible benchmark parameterization. Since the model is solved numerically, I perform sensitivity tests for changes in work effort

[15]The parameter values used to calibrate the model benefit from previous literature (Eicher and Turnovsky (1999*b*), Turnovsky (2004) and Chatterjee and Turnovsky (2007)).

and adjustment costs to both types of capital to ensure the stability of the steady state equilibrium and the robustness of the model.

1.3 Results

The succeeding sections present both the short- and long-run responses in a total of eighteen different foreign aid scenarios. The analysis focuses on the level responses in leisure, output, welfare, consumption, and public and private capital accumulation. Sensitivity analyses to changes in work effort and the adjustment costs to both public and private capital as well as a discussion of how fungibility may undermine the effectiveness of foreign aid are presented in this section. Table 1.2 presents the benchmark equilibrium for the economy as characterized by the parameters in Table 1.1.

Table 1.2: Benchmark Equilibrium

ψ	l	c	K_G	k	q	z	x
0.022	0.709	0.099	0.106	0.163	2.075	0.035	0.152

Throughout the analysis, foreign aid increases from its benchmark level of no aid ($f = 0.00$) to 5.0% of GDP, ($f = 0.05$). $\lambda_I = 1$ implies that the aid transfer is allocated fully to public investment expenditure. When $\lambda_C = 1$, the aid is allocated fully to public consumption expenditure. $\lambda_I = \lambda_C = 0$ implies that the aid is not provided to either public goods and is a pure transfer to the agent. All responses are expressed relative to their respective initial benchmark steady state levels in Table 1.2.

Table 1.3 presents the steady-state level responses of the aid transfer that is allocated fully to public investment expenditure or as a pure transfer. The long-run responses are presented in Table 1.3a, while Table 1.3b summarizes the short-run responses. The last column in Table 1.3 is the equivalent variation calculation of welfare and represents the percentage change required in the initial income level to keep welfare unchanged at the initial level. Table 1.4 presents the results when aid is allocated among public investment, public consumption expenditure, and a pure transfer. Table 1.5 and Table 1.6 present the responses to changes in work effort and the intertemporal adjustment costs of both stocks of capital. Finally, Table 1.7 presents the responses when the government can alter its spending commitment in response to the aid allocation.

1.3.1 The Long-run Macroeconomic Responses to a Foreign Aid Shock

Table 1.3 presents both the long- and short-run level effects of an aid transfer that is equivalent to 5% of GDP. the aid transfer is allocated to either public investment expenditure (tied aid) or a pure transfer (untied aid).

Foreign Aid Transfers That Finance Public Investment Expenditure (Tied Aid) - $f = 0.05, \lambda I = 1, \lambda C = 0$

Table 1.3 presents the long-run level effects of an increase in foreign aid that is equivalent to 5% of GDP that is allocated fully to public investment expenditure (Row 2 of Table 1.3a). Tied aid increases the long-run stock of public capital by almost three times the initial steady state level. The larger stock of public capital makes the existing private capital and labor more productive. Agents respond in two ways. First, agents reduce leisure. Second, increases in private capital productivity boosts its accumulation. The steady-state stock of private capital increases by 42.0% from its initial level(Table 1.3a Row 2). Aggregate output also increases as labor, and the stocks of private and public capital increase. Consumption also increases. However, with a more than proportionate increase in output, the ratio of consumption to output declines in the new steady state (not shown). Initially, as output increases (relative to the stock of debt), the debt-to-output ratio decreases[16]. Over time agents increase their borrowing to finance consumption and capital accumulation. The long-run debt level rises by 42.0% from its initial level to accommodate the higher capital stocks and consumption. The lower marginal utility of consumption and leisure as well as the marginal increases in the stock of debt act to reduce long-run welfare by 4.0%

[16]Recall that the debt-to-output ratio is required to be constant in the long-run so as to ensure a stable steady state equilibrium.

from the initial steady state. The welfare loss, associated with the increases in output, private, and public capital, confirms the tradeoff between welfare and the level effects under tied aid that have been discussed in the previous literature. The welfare results found here differ from those under the endogenous growth where tied aid improves intertemporal welfare[17]. A possible explanation for the observed negative welfare effect is that, here, the model is not characterized by on-going, long-run growth in which agents continue to acquire more capital in response to changes in the return to capital[18]. In this model, all accumulation is constrained by the characteristics of the economy - population growth and technological considerations[19]. Here, in the cost of borrowing outpaces the the rate of return to capital, and so the incentive to continue acquiring private capital decreases as the debt-servicing capacity decreases. The results suggest that tied aid can have positive long-run effects on economic activity - in terms of the levels of output and capital accumulation, but the higher long-run levels of debt and increased work effort serve to decrease long-run welfare effects.

Foreign Aid Transfers as Pure Transfers - $f = 0.05, \lambda I = \lambda C = 0$

Table 1.3 presents the long-run effects of a pure transfer that is equivalent to 5.0% of GDP (Row 3 of Table 1.3a). Both private and public capital decrease to 92.6% of their initial steady state levels. Agents experience a wealth effect as their debt stock decrease and respond by reducing labor in the long-run. Output falls in the long-run with the reduction in work effort as well as private and public capital. The slight fall-off in consumption may be the result from the substitution in the labor-leisure decision. Income falls as agents increase leisure to 102.1% of its initial steady state level. However, since agents increase their leisure and reduce their debt stock, welfare increases by 6.8% from its initial level. Compared with the results in Row 1 of Table 1.3a, a pure transfer generates welfare effects

[17] See Chatterjee and Turnovsky (2007).
[18] Recall the assumption of constant returns to scale in the accumulating factors.
[19] Recall Equation (1.11).

as it reduces the tax burden to the agents, but at the cost of lower long-run levels of work effort, private capital accumulation, and output.

1.3.2 Transitional Dynamics

The assumption of balanced growth that is made in the model set-up implies that the growth rates of the variables can vary during transition, but must all converge to the unique growth rate in the long-run [20]. These long-run results were presented in the previous section. But while the long-run level effects are essential for assessing the level of the responses to the aid shock, the immediate effects and transitional dynamics can also be insightful. Figures (1.1) and (1.2) present the immediate and dynamic responses of scale adjusted consumption, output, debt, capital accumulation to the tied and untied aid shocks respectively. Figures (1.1.1)-(1.1.6) and Figures (1.2.1)-(1.2.6) give the transition paths for leisure and the level effects of the aid shock on consumption, private capital, public capital, debt, and output relative to their initial steady-state levels. Figures (1.1.7) and (1.2.7) illustrate that the growth rates for output, and public and private capital may vary during transition but converge to the long-run growth rate of the economy.

Foreign Aid Transfers That Finance Public Investment Expenditure (Tied Aid) - $f = 0.05, \lambda I = 1, \lambda C = 0$

Figure 1.1 shows the dynamic responses to an aid shock of 5% of GDP that is allocated fully to public investment expenditure. Table 1.3b Row 2 presents the instantaneous responses.

Agents decrease leisure and consumption immediately in response to the aid transfer that increases the stock of public capital (Figures (1.1.1) and (1.1.4)). Output increases slowly as the marginal productivity of public capital and the private factors increase (Figure (1.1.6)). Debt decreases initially but rises to its new steady state level as agents

[20] Addressing how realistic is the balanced growth argument is not within the scope of this paper. Eicher & Turnovsky (1999*a*) support the notion of some rationale for a long-run balanced growth rate by discussing country experiences that have been presented in the empirical growth and convergence literature.

borrow to finance both consumption and private capital. Figure (1.1.7) shows that the growth rates of the accumulating factors and output converge to the long-run growth rate in the long-run as the rate of return to private capital falls to equates rising the cost of borrowing.

FOREIGN AID TRANSFERS AS PURE TRANSFERS - $f = 0.05, \lambda I = \lambda C = 0$

Figure 1.2 provides the transitional dynamics of a pure transfer that is equivalent to 5% of GDP. Table 1.3b Row 3 presents the instantaneous responses. The welfare effects are immediate as agents increase leisure and consumption (Figures (1.2.1) and (1.2.4). As discussed earlier, there are no accumulation effects under a pure transfer and the stock of capital depreciate to their new lower steady state levels (Figures (1.2.2) and (1.2.5)). While output decreases in the long-run, the initial decline caused the marginal product of labor to rise, which served to stimulate individuals to increase work effort (Figure 1.2.1). But as the marginal productivity of the remaining capital and labor rise, the equilibrium between the return to private capital and the cost of borrowing is restored. The results in this scenario suggest strongly that a pure transfer facilitates consumption by reducing lump-sum taxes, leisure, and debt reduction during transition. These positive changes generate the instantaneous welfare increases (Table 1.3b Row 3).

1.3.3 Allocating Foreign Aid Across Public Expenditure and a Pure Transfer

This section discusses the macroeconomic responses when foreign aid is allocated among public investment and consumption expenditure, and a pure transfer. The four scenarios that are examined are as follows:

- Scenario 1: The aid is allocated equally between public investment and consumption expenditure, ($\lambda_I = \lambda_C = 0.5$).

- Scenario 2: The aid is allocated equally between a pure transfer and public consumption expenditure, ($\lambda_C = 0.5, \lambda_I = 0$).

- Scenario 3: The aid is allocated fully to public consumption expenditure, ($\lambda_C = 1, \lambda_I = 0$).

- Scenario 4: The aid is allocated equally between a pure transfer and public investment expenditure, ($\lambda_I = 0.5, \lambda_C = 0$).

A scenario in which the aid is allocated equally among the two types of public expenditure and a pure transfer ($\lambda_I = \lambda_C = \frac{1}{3}$) was also considered. However the results did not differ qualitatively from those in Scenarios 1 and 4 above. Table 1.4 presents the level responses for each scenario relative to the initial benchmark level of no aid allocation.

Aid that is allocated equally between a pure transfer and public consumption expenditure (Scenario 2) generates the highest long-run welfare gains of all the scenarios considered. This allocation scenario facilitates debt reduction and increases leisure and consumption in the long-run. Comparing these results with those generated when aid is allocated fully as a pure transfer (Table 1.3 Row 3a) suggest that an allocation to public expenditure can generate higher level increases to the accumulating factors than a pure transfer.

The short-run welfare gains when aid is allocated fully to public consumption expenditure (Scenario 3) is the highest for all the allocation scenarios examined in in this section (Table 1.4b Row 3) but is lower than when aid is a pure transfer. The long-run welfare gains that are generated are the lowest off all the aid scenarios. The intuition is that public consumption expenditure, g_C^d, does not factor into the determination of steady-state values relevant to the private agents[21]. The exogeneity would account for both the absence of both transitional dynamics and long-run welfare effects. Agents are only better off during transition as instantaneous consumption and leisure increase.

Scenarios 1 and 4 generate qualitatively similar level effects owing to the portion of aid allocation that finances public investment expenditure. As discussed earlier, the externalities from public capital encourage private capital accumulation and increase work effort.

[21]In terms of the core dynamics, g_C^d is exogenous to determining $\tilde{k}$, $\tilde{k}_G$, $\tilde{q}$, and $\tilde{l}$. In determining private borrowing, $\tilde{z}$, the agent takes the borrowing rate, r(.), which is a function of aggregated debt, to be exogenous.

A comparison of the long-run results from Scenario 1 with those from Scenario 4 suggest that allocations to public expenditure increase both capital accumulation and welfare gains more than a pure transfer. In comparing Scenario 4 against the case where aid is allocated equally among the two types of expenditure and a pure transfer, ($\lambda_I = \lambda_C = \frac{1}{3}$, not displayed), there is a positive correlation between the extent of allocation to public investment expenditure and the accumulation of capital and output. The results suggest that capital accumulation, consumption, and output are highest when the allocation to public capital exceed the amounts allocated to either public consumption expenditure or a pure transfer.

In sum, any allocation to public expenditure (either investment or consumption) generates proportionately higher level effects relative to untied aid. Tied aid generates level effects that are higher than those generated under either a pure transfer or an allocation to public consumption expenditure. Aid that is allocated fully to public consumption expenditure generates welfare effects in the short-run and has neither transitional dynamics nor long-run level effects. Untied aid (a pure transfer) facilitates tax reduction and is welfare-superior in the long-run, whereas an allocation between tax reduction and public consumption expenditure are welfare-superior in the short-run. The results suggest that any amount of foreign aid that is allocated to productive uses have higher long-run level effects, but not without intertemporal welfare tradeoffs. A possible explanation for the observed long-run welfare losses may be, as discussed by Turnovsky & Chatterjee (2002), that the impact of public investment and consumption expenditure on the economy are small and the output elasticity of public capital may be low.

1.4 Sensitivity Analysis

The values of the parameters in Table 1.1 that are used to solve the non-linear system generate plausible benchmark estimates for a small, open economy. Their values impact directly the steady-state equilibrium for the model. This section considers how robust the dynamic results are to changes in specific parameter values under the polar cases - aid that is allocated completely to public investment or aid that is fully a pure transfer. Table 1.5 presents the level effects as well as the instantaneous and intertemporal welfare effects to changes in the importance of leisure. Robustness checks to changes in the adjustment costs to both private and public capital are in Table 1.6.

1.4.1 Sensitivity to Leisure

Table 1.5 presents the long-run level effects as leisure becomes more important in Equation (1.2a). That is, as θ increases, leisure becomes more important to the agent.

Table 1.5a shows that the positive level effects decrease and welfare effects increase as agents decrease work effort under a tied aid allocation. Chatterjee and Turnovsky (2007) finds similar results in conducting robustness checks to the importance of leisure when aid is tied. When aid is a pure transfer and agents decrease work effort, capital accumulation and the instantaneous and long-run welfare effects are lower than in the initial benchmark pure transfer case in Table 1.5b. A quick calculation of the effect of the rate of change of leisure on welfare shows a diminishing effect. That is, diminishing marginal utility of leisure affects the labor-leisure decision. When the importance of leisure increases, agents increase leisure at a diminishing rate, which lessens the extent of the welfare losses when aid is a pure transfer.

1.4.2 Sensitivity to the Adjustment Costs of Capital

Table 1.6 presents the long-run response in welfare to variations in the adjustment costs to both private and public capital. h_1 and h_2 refer to the intertemporal adjustment costs of private and public capital respectively. The higher the installation costs, for instance if $h_1 = 50$, the less effective is private capital in response to the aid shock.

Table 1.6a shows that for a given level of public adjustment costs ($h_1 = 15$), decreasing private capital adjustment costs, h_1, are welfare-deteriorating under a pure transfer but welfare improving when aid is allocated to public investment expenditure. In Table 1.6b, for a given level of private costs, falling public capital adjustment costs are welfare improving in the long-run when aid is tied but decrease the welfare gains when aid is a pure transfer. When private adjustment costs are low and agents can acquire and relinquish capital freely, high public capital adjustment costs ($h_1 \simeq 50$) is welfare deteriorating. Chatterjee and Turnovsky (2003) find a similar result and reason that by allocating aid to public infrastructure expenditure, donors are requiring recipients to commit larger portions of resources to the costly task of installation, thereby lowering the intertemporal welfare effects that accrue to private agents.

1.5 The Fungibility of Foreign Aid

Table 1.7 presents the results to four scenarios of fungibility in which the government can alter its domestic expenditure commitments in response to the foreign aid transfer[22]. The four scenarios are as follows:

- Scenario 6: Government receives 5% of GDP in foreign aid, allocates the aid fully to the public investment expenditure, and simultaneously reduces its own public investment expenditure commitment by 5% of GDP. ($\lambda_I = 1, \lambda_C = 0.5, g_I^d = 0$).

- Scenario 7: Government receives 5% of GDP in foreign aid, allocates half to the public investment expenditure, and simultaneously reduces its own public investment expenditure commitment by half, ($\lambda_I = 0.5, \lambda_C = 0, g_I^d = 0.025$).

- Scenario 8: Government receives 5% of GDP in foreign aid, allocates all as a pure transfer, and simultaneously reduces its own public consumption expenditure commitment by 5% of GDP, ($\lambda_I = \lambda_C = 0, g_C^d = 0.05$).

- Scenario 9: Government receives 5% of GDP in foreign aid, allocates the aid equally among a pure transfer and both types of public expenditure, and reduces each of

[22]White (1992) and Feyzioglu, Swaroop & Zhu (1998).

its own domestic public expenditure commitments by half, ($\lambda_I = \lambda_C = \frac{1}{3}, g_C^d = 0.115, g_I^d = 0.025$).

The examination of the behavior of the economy to varying degrees of fungibility leads to three conclusions. First, when funds are fungible, foreign aid does not have the same long-run effects as when government commits itself to financing either type of public expenditure. Second, tied aid enhances the level outcomes (output, consumption, private and public capital accumulation) when there is no response by the government to reduce either type of its own spending allocations. Third, any reduction in domestic expenditure that offsets aid that is allocated to either type of public expenditure acts as a pure transfer and is welfare enhancing but at the cost of accumulating capital and output. When aid is fungible, any reduction in government spending acts as a transfer that has a pure wealth effect and distorts the incentives for private investment and work. The rate of investment in public capital does not generate enough of an incentive for private capital accumulation and so the wealth effect dominates. Agents respond, as they do with any transfer from government, by reducing both debt and work effort. The results suggest that foreign aid complements public expenditure, but cannot replace the government's own commitment to financing public goods.

The results show that long-run welfare and levels of the accumulating factors fall where there is opportunity for the recipient government to divert public spending. Consequently, the lack of consistent positive aid outcomes observed in the literature may be due to fungibility from, for instance, poor fiscal structures, rent seeking, and corruption. The analysis suggests that policies that reduce the opportunity for fungibility may improve the effectiveness of foreign aid. As Easterly (2008) discusses, aligning donor goals with those of the aid recipients in packages that have observable measures and outcomes may be a step in improving aid's effectiveness.

1.6 Conclusion

The results provide a possible theoretical explanation for why aid may not show consistent positive outcomes. The results focus on the scenarios in which aid is allocated towards public investment expenditure, public consumption expenditure or a pure transfer. The paper contributes to the literature by identifying the direction and magnitudes of the level effects, particularly in the case where governments can alter their spending commitments in response to the aid transfer. As such the results can, at minimum, be a robustness check to the predictions of the current theoretical literature on the effects of foreign aid.

The results suggest that the allocation of foreign aid across expenditure categories matters for the response of key variables. The marginal allocation to public investment expenditure generate positive externalities that trigger changes in the marginal productivities of private factors. Agents respond by adjusting their behavior towards labor, leisure, and private capital accumulation. The change in marginal productivity that leads to private capital accumulation occur when aid is allocated to public investment; but do not occur when the aid is allocated to public consumption expenditure or is a pure transfer. In fact, aid that is directed to public consumption expenditure acts as a lump-sum transfer and agents increase their long-run levels of consumption, leisure, and debt reduction. Agents experience significant instantaneous and long-run welfare gains, but output, and public and private capital decline in the long-run.

The fungibility analysis suggests no scenario in which the long-run level effects to public and private capital are enhanced when governments can alter its spending commitments. With the exception of the scenario in which the government disproportionately reduces its spending on investment and consumption expenditure (in which case, agents experience intertemporal welfare gains but long-run welfare losses), agents always experience both intertemporal and long-run welfare gains. One important policy implication is that aid can be more effective, as the corruption literature suggests, in the presence of strong institutions, or when donor and recipient incentives are transparent through for instance, directing aid to specific projects, monitoring projects through benchmarking, and implementing conditional allocation structures. This active monitoring and goal oriented agenda would, as the more recent aid policy literature suggests, minimize the high costs of asymmetric information and closer align donor and recipient preferences.

Table 1.3: Long-run Level Effects of a Foreign Aid Shock

The table presents the level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$. λ_I is the proportion of foreign aid allocated to public investment expenditure while λ_C is the proportion of foreign aid allocated to public consumption expenditure. q^1 is the shadow price of private capital in terms of bonds. The last column is the equivalent variation calculation of welfare and is the percentage change in the initial income level that would be required to keep welfare unchanged at the pre-shock aid initial level. The responses are expressed *relative* to the initial benchmark steady state levels.

a) Long-run Level Effects

	Debt	Private Capital	Public Capital	Leisure	Output	Cosumption	$\Delta(W)\%$
Benchmark equilibrium: $f = 0, \lambda_I = 0, \lambda_C = 0.$	0.035	0.163	0.106	0.709	0.152	0.099	
Fully tied to public investment: $f = 0.05, \lambda_I = 1, \lambda_C = 0.$	1.420	1.420	2.840	0.987	1.420	1.362	-4.006
Pure transfer: $f = 0.05, \lambda_I = 0, \lambda_C = 0.$	0.926	0.926	0.926	1.021	0.926	0.997	6.786

b) Short-run Level Effects

	Debt	Private Capital	Public Capital	Leisure	Output	Cosumption	q^1	$\Delta(W(0))\%$
Benchmark equilibrium: $f = 0, \lambda_I = 0, \lambda_C = 0.$	0.346	0.163	0.106	0.709	0.152	0.099	2.075	
Fully tied to public investment: $f = 0.05, \lambda_I = 1, \lambda_C = 0.$	1.000	1.000	1.000	0.756	1.354	0.642	0.982	-56.876
Pure transfer: $f = 0.05, \lambda_I = 0, \lambda_C = 0.$	1.000	1.000	1.000	1.049	0.922	1.056	0.973	16.152

Table 1.4: Long-run Level Effects to Allocations Across Expenditure Type

The table presents the level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$. Here the aid is allocated among public investment and consumption expenditure and a pure transfer. The table presents four allocation scenarios. λ_I is the proportion of foreign aid allocated to public investment expenditure while λ_C is the proportion of foreign aid allocated to public consumption expenditure. q^1 is the shadow price of private capital in terms of bonds. The last column is the equivalent variation calculation of welfare and is the percentage change in the initial income level that would be required to keep welfare unchanged at the pre-shock aid initial level. The responses are expressed *relative* to the initial benchmark steady state levels.

The scenarios are as follows:
Scenario 1: The aid is equally allocated to public investment and to the public consumption good, that is $\lambda_I = \lambda_C = 0.5$.
Scenario 2: The aid is allocated as a 50% pure transfer with the remainder allotted to public consumption expenditure, $(\lambda_C = 0.5, \lambda_I = 0)$.
Scenario 3: The aid is allocated fully to public consumption expenditure, $(\lambda_C = 1, \lambda_I = 0)$.
Scenario 4: The aid is allocated as a 50% pure transfer with the remainder allotted to public investment expenditure, $(\lambda_I = 0.5, \lambda_C = 0)$.

a) Long-run Level Effects

	Debt	Private Capital	Public Capital	Leisure	Output	Cosumption	$\Delta(W)\%$
Scenario 1: $\lambda_I = \lambda_C = 0.5$	1.200	1.227	1.830	0.993	1.224	1.192	3.807
Scenario 2: $\lambda_I = 0, \lambda_C = 0.5$	0.943	0.963	0.962	1.011	0.961	1.000	8.176
Scenario 3: $\lambda_I = 0, \lambda_C = 1$	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Scenario 4: $\lambda_I = 0.5, \lambda_C = 0$	1.171	1.178	1.164	1.004	1.178	1.192	3.199

b) Short-run Level Effects

	Debt	Private Capital	Public Capital	Leisure	Output	Cosumption	q^1	$\Delta(W(0)))\%$
Scenario 1: $\lambda_I = \lambda_C = 0.5$	1.000	1.000	1.000	0.804	1.289	0.697	1.175	-45.633
Scenario 2: $\lambda_I = 0, \lambda_C = 0.5$	1.000	1.000	1.000	1.038	0.934	1.071	0.967	-18.521
Scenario 3: $\lambda_I = 0, \lambda_C = 1$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	9.594
Scenario 4: $\lambda_I = 0.5, \lambda_C = 0$	1.000	1.000	1.000	0.841	1.237	0.747	1.146	-40.859

Table 1.5: Long-run Level Effects to Changes in the Importance of Leisure

This table presents the long-run effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$, to changes in the contribution of leisure in the agent's utility function, Equation (1.2a). Labor becomes more important in the utility function as the elasticity, θ, increases. The last two columns are equivalent variation calculations of welfare and represents the long-run and instantaneous percentage changes in the initial income levels that would be required to keep welfare unchanged at the pre-shock aid initial level.

All of the responses in the table are expressed *relative* to the initial benchmark steady state levels in Table 1.3a.

	Public Capital	Private Capital	Debt	Leisure	Consumption	$\Delta(W)\%$	$\Delta(W(0))\%$
a) Effects under Allocation to Public Investment							
θ =0.75	2.801	1.403	1.403	0.979	1.345	-29.141	-70.684
θ =2.75	2.856	1.428	1.428	0.991	1.369	1.793	-52.527
θ =5.00	2.870	1.435	1.435	0.995	1.376	6.416	-48.883
	Public Capital	Private Capital	Debt	Leisure	Consumption	$\Delta(W)\%$	$\Delta(W(0))\%$
b) Effects under Allocation as Pure Transfer							
θ =0.75	0.946	0.946	0.946	1.036	1.019	7.987	16.497
θ =2.75	0.918	0.918	0.918	1.015	0.989	6.263	16.058
θ =5.00	0.910	0.910	0.910	1.009	0.980	5.742	15.992

Table 1.6: Long-run Level Effects to Changes in Adjustment Costs

The table presents the long-run level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$, to variations in the adjustment costs to both forms of capital. h_1 and h_2 are the intertemporal adjustment costs of private and public capital respectively. The higher the installation costs, for instance if $h_1 = 50$, the less effective will be private capital in response to the aid shock. The table presents only the intertemporal welfare effects.

The responses in the table are expressed *relative* to the initial benchmark steady state levels in Table 1.3a.

a) Adjustment costs for private capital

	Aid Allocation	
	Public Investment	**Pure Transfer**
$h_1 \simeq 50$, $h_2 = 15$	-77.588	19.732
$h_1 = 15$, $h_2 = 15$	-4.006	6.786
$h_1 \simeq 1$, $h_2 = 15$	-31.240	7.290

b) Adjustment costs for public capital

	Aid Allocation	
	Public Investment	**Pure Transfer**
$h_1 = 15$, $h_2 \simeq 50$	-52.533	8.677
$h_1 = 15$, $h_2 = 15$	-4.006	6.786
$h_1 = 15$, $h_2 \simeq 1$	7.458	6.207

Table 1.7: Level Effects When Aid is Fungible

The table presents the level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$. λ_I is the proportion of foreign aid allocated to public investment expenditure, while λ_C is the proportion of foreign aid allocated to public consumption expenditure. q^1 is the shadow price of private capital in terms of bonds. The last column is the equivalent variation calculation of welfare and is the percentage change in the initial income level that would be required to keep welfare unchanged at the pre-shock aid initial level.

g_C^d and g_I^d refer to the government's allocation from revenues to public consumption and investment expenditure respectively.
The scenarios are as follows:
Scenario 6: Government receives 5% of GDP in foreign aid, allocates the aid fully to the public investment good, and simultaneously reduces its domestic public infrastructure expenditure by 5% of GDP. ($\lambda_I = 1, \lambda_C = 0.5, g_I^d = 0$).
Scenario 7: Government receives 5% of GDP in foreign aid, allocates half to the public investment good, and simultaneously reduces its domestic public infrastructure expenditure by half, ($\lambda_I = 0.5, \lambda_C = 0, g_I^d = 0.025$).
Scenario 8: Government receives 5% of GDP in foreign aid, all the aid is untied and government reduces domestic public consumption expenditure by 5% of GDP, ($\lambda_I = \lambda_C = 0, g_C^d = 0.05$).
Scenario 9: Government receives 5% of GDP in foreign aid, allocates the aid equally among a pure transfer and aid tied to the two types of expenditure and reduces each of its domestic expenditure commitments by half, ($\lambda_I = \lambda_C = \frac{1}{3}, g_C^d = 0.115, g_I^d = 0.025$).

The responses in the table are expressed *relative* to the initial benchmark steady state levels.

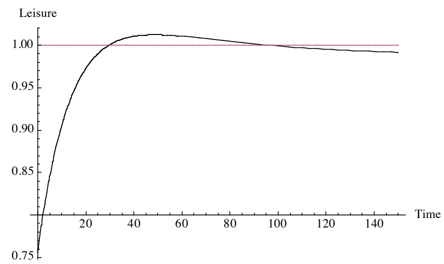
a) Long-run Effects

	Private Capital	Public Capital	Leisure	Output	Debt	Consumption	$\Delta(W)\%$
Scenario 6: $\lambda_I = 1, \lambda_C = 0.5, g_I^d = 0$	0.926	0.926	1.021	0.926	0.926	0.977	6.786
Scenario 7: $\lambda_I = 0.5, \lambda_C = 0, g_I^d = 0.025$	0.926	0.926	1.021	0.926	0.926	0.977	6.786
Scenario 8: $\lambda_I = \lambda_C = 0, g_C^d = 0.05$	0.861	0.861	1.040	0.861	0.861	0.994	-1.231
Scenario 9: $\lambda_I = \lambda_C = \frac{1}{3}, g_C^d = 0.115, g_I^d = 0.025$	0.828	0.828	1.030	0.828	0.828	0.918	-8.779

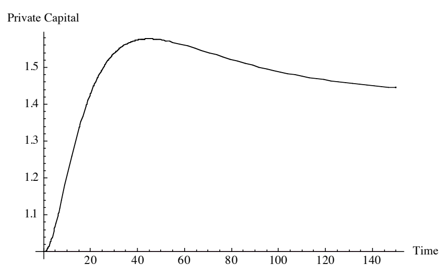
b) Short-run Effects

	Private Capital	Public Capital	Leisure	Output	Consumption	q^1	$\Delta(W(0))\%$
Scenario 6: $\lambda_I =, \lambda_C = 0.5, g_I^d = 0$	1.000	1.000	1.049	0.922	1.096	0.973	16.152
Scenario 7: $\lambda_I = 0.5, \lambda_C = 0, g_I^d = 0.025$	1.000	1.000	1.049	0.922	1.096	0.973	16.152
Scenario 8: $\lambda_I = \lambda_C = 0, g_C^d = 0.05$	1.000	1.000	1.094	0.845	1.197	0.944	16.584
Scenario 9: $\lambda_I = \lambda_C = \frac{1}{3}, g_C^d = 0.115, g_I^d = 0.025$	1.000	1.000	1.141	1.000	1.322	0.968	50.713

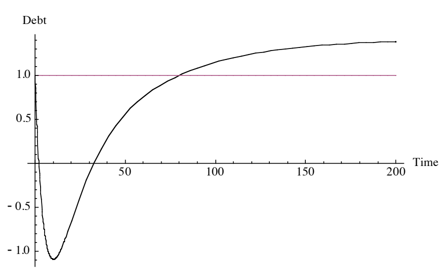
Figure 1.1: Dynamic Responses To A Foreign Aid Shock under the Assumption of Non-scale Growth



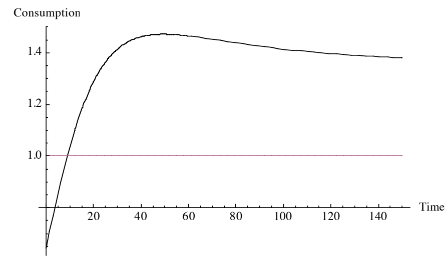
1.1.1 – Leisure relative to initial steady state



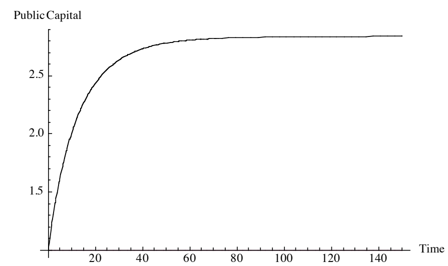
1.1.2 – Private capital relative to initial steady state



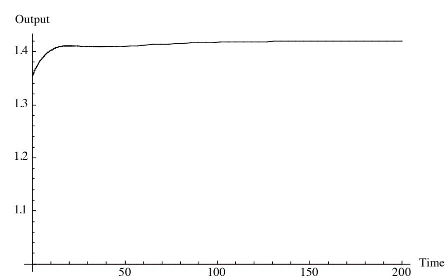
1.1.3 – Debt relative to initial steady state



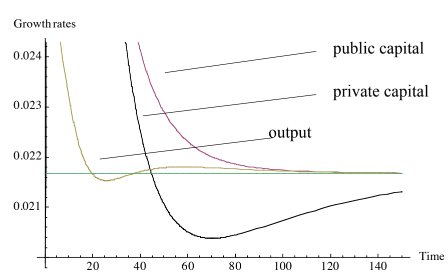
1.1.4 – Consumption relative to initial steady state



1.1.5 – Public capital relative to initial steady state

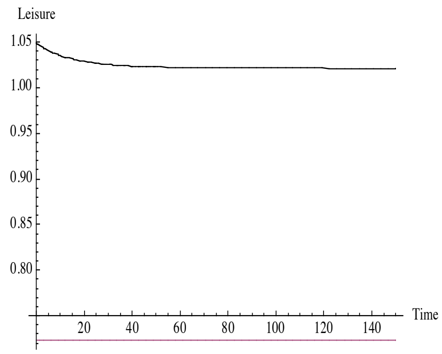


1.1.6 – Output relative to initial steady state

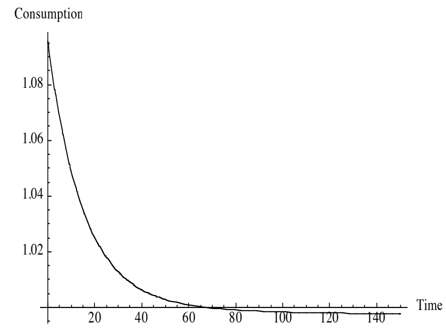


1.1.7 – Time paths for growth rates of private capital, public capital and output

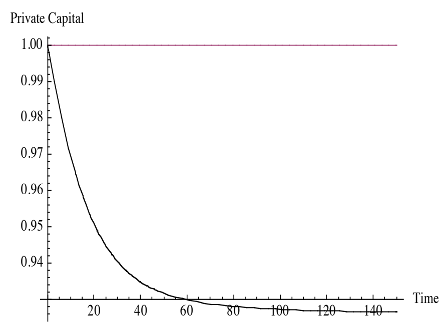
Figure 1.2: Dynamic Responses To An Untied Aid Shock under the Assumption of Non-scale Growth



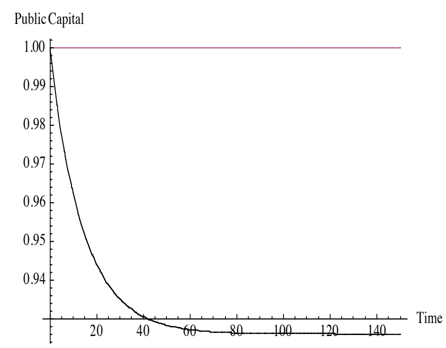
1.2.1 – Leisure relative to initial steady state



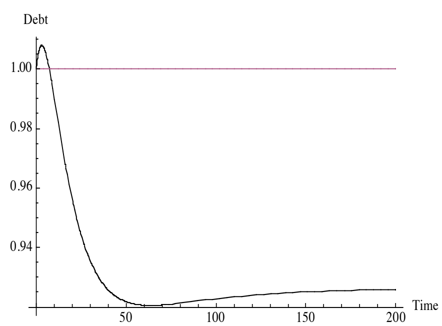
1.2.4 – Consumption relative to initial steady state



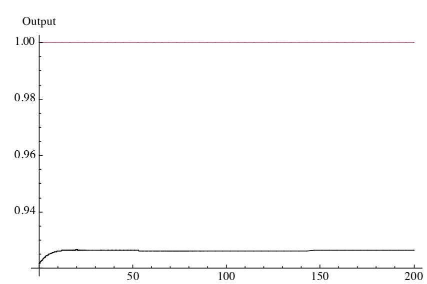
1.2.2 – Private capital relative to initial steady state



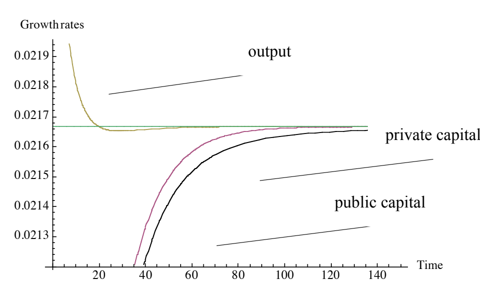
1.2.5 – Public capital relative to initial steady state



1.2.3 – Debt relative to initial steady state



1.2.6 – Output relative to initial steady state



1.2.7 – Time paths for growth rates of private capital, public capital and output

Chapter 2

Foreign Aid, Real Exchange Rate Dynamics and Macroeconomic Performance

2.1 Motivation and Background

With the Millennium Development Goals as set out by the donor community in mind, researchers continue to explore the ways in which foreign aid may mitigate economic growth and development. One such channel has been the real exchange rate channel (Rajan & Subramanian (2005*a*), Rajan & Subramanian (2005*b*), Dollar & Levin (2006), Djankov, Montalvo & Reynal-Querol (2005), Rajan & Subramanian (2007), Elbadawi, Kaltani & Schmidt-Hebbel (2008), and Cerra, Tekin & Turnovsky (2008)). The common idea to emerge from these literature - both empirical and theoretical - is that aid used to finance expenditure may adversely affect the real exchange rate through its effect on the relative prices of domestic and foreign goods[1]. The literature also discusses how the fungibility of aid may affect the the real exchange rate. Aid may be fungible if the government can redirect its expenditure commitment toward non-productive activities (White

[1]The real exchange rate is defined as the relative price of the foreign good in terms of the domestic good. This is the definition of Edwards (1989) in which the long-run real effective exchange rate is defined as:

$$p = \frac{EP_F}{P_D},$$

where E is the nominal exchange rate, P_F is the price of the foreign good, and P_D is the price of the domestic good. An increase in the ratio leads to a depreciation in the real exchange rate. The real exchange rate is misaligned when it diverges from its equilibrium level and is overvalued (undervalued) when it is above (below) its equilibrium. Underlying fundamentals of the economy determine the equilibrium. In a series of empirical papers, Rajan and Subramanian (2005a, 2005b, and 2007) highlight that aid affects the exchange rate differently depending on the sector to which it is directed. In the case where aid is spent on non-tradables, Rajan and Subramanian (2005a) state that if aid does not lead to an increase in the supply of non-tradables, then increases in the price of non-tradables can lead to an appreciation in the real exchange rate. Other studies report similar findings (Hartford & Klein (2005) and Djankov et al (2005)).

(1992)). To my knowledge, the most recent theoretical models are Chatterjee, Giuliano & Kaya (2009) and Byron (2008) in which fungibility matters for the responses in macroeconomic variables and for the effectiveness of foreign aid. However, neither of these studies set out a theoretical model to establish how the real exchange rate responds to various foreign aid allocation scenarios. This paper builds a simple neoclassical growth model: (i) to explore how aid, and its fungibility, may affect the dynamics of the real exchange rate; and (ii) in which robust and causal relationships can be established and tested. The model allows public expenditure to be financed by some combination of foreign aid transfers, income and lump-sum taxes, and tariff revenue, however the focus is on foreign aid as the main source for additional financing. *Tied aid* finances public investment expenditure, while *untied aid* refers to pure transfers to private agents.

A number of recent studies examine the relationship among sectoral productivities, real exchange rate adjustments, government expenditure financing, aid transfers, and aid fungibility[2]. However, three studies are most relevant to this work. First, in an endogenous growth model, Chatterjee, Sakoulis & Turnovsky (2003) consider the consequences of permanent and temporary foreign aid transfers on capital accumulation and economic growth. Three of their main findings are relevant to this study. First, permanent transfers that finance public consumption expenditure give pure wealth effects, generate no transitional dynamics, and do not affect long-run growth rates. Second, permanent transfers that are allocated to investment expenditure affect the ratios of key variables as well as

[2]The benefits of public investment for economic development have been examined extensively from both theoretical and empirical studies. For instance, the classic work by Romer (1990) which examines the effect of government spending in a growth model and the more recent theoretical work by Rajan & Subramanian (2005*b*) as noted above that looks at aid flows, the real exchange rate and the effect on a country's competitiveness. In respect of the theoretical work on expenditure financing, the general consensus is that public capital generates externalities to private capital which are the driving force for growth in an economy. For instance, Turnovsky & Chatterjee (2002) find that while the growth paths for different financing options may vary greatly, government spending on either a consumption good or investment good enhances the productivity of the economy. That is, there is not one financing strategy that may be 'optimal' over another, but rather that there are opportunity costs to both growth and welfare with any choice for financing public investment.

the long-run growth rate of key macroeconomic variables. Third, with the exception of the responses under the aid allocation to public consumption expenditure, the growth and welfare effects of permanent aid transfers are similar to those under temporary aid transfers. The model in this paper considers only permanent aid transfers. By including the foreign consumption good however, the paper considers how changes in relative prices may interact with public and private capital, consumption and output.

In the second relevant work, Cerra, Tekin & Turnovsky (2008) establish a two-sector, two-good endogenous growth model to look at the effect of foreign aid on the real exchange rate. A key feature is that labor and capital adjust as sectoral productivities change in response to the aid transfer. Their results suggest that long-run real exchange rate adjustments depend on the sector (traded or non-traded), and whether the aid is allocated to either tax reduction or public investment expenditure. The long-run effects of tied aid are due to its impact on sectoral capital adjustments and relative price adjustments, as well as on the market-clearing conditions for the traded and non-traded goods. The dynamics are permanent as tied aid can have accumulation effects on both capital stocks. Additionally, when the aid finances public investment expenditure in the non-traded sector, non-traded prices increase and the real exchange rate depreciates, while the real exchange rate appreciates when the transfers are directed towards increasing the productivity of the traded sector[3]. Untied aid has no long-run effects in the non-traded sector, as the level of private capital is determined independently from aid flows, and the real exchange rate equals the ratio of marginal product of capital in both sectors (assuming perfect capital mobility), which is also independent of such aid transfers.

While Cerra et al. (2008) provide an understanding of the complex sector adjustments, their study lacks tractability as the hypotheses generated cannot be tested with the level of aggregated data currently available. It would be useful to see if their predictions hold in a simpler model in which robust, causal relationships between foreign aid and the real

[3]This is the Balassa-Samuelson effect.

exchange rate can be established and tested empirically. First, labor is exogenous and so, unlike the Cerra et at. (2008) model in which labor is endogenous, the effect of the aid transfers on the dynamics of the real exchange rate can be isolated. Second, agents can either consume the domestic good or trade some for the foreign good. The change in the relative price of the domestic good to the foreign good is the real exchange rate. These assumptions generate results that can be tested empirically with the existing data; making robust some of the more complex theoretical results in Cerra et al (2008). For instance, Cerra et al (2008) find that irrespective of factor intensities across sectors, aid allocated to the non-traded sector leads to a substantial depreciation in the real exchange rate. Productivity enhancements drove an expansion in the non-traded sector and led to a decrease in its price relative to those in the traded sector. So the inclusion of a tradable sector in a theoretical model clarifies how factors adjust across sectors; but at the cost of not producing clear empirically testable hypotheses of the effect of aid on the real exchange rate. In my simpler model, when aid is allocated to public investment expenditure, and private factors experience high adjustment costs, the real exchange rate can appreciate in the long-run; but depreciates in the long-run as adjustment costs to both forms of capital are lowered. In this sense, the lowering of adjustment costs captures the effects of factors being able to move more easily across sectors as in the more complex Cerra et al (2008) model.

The third relevant work focuses on the fungibility of aid. In a neo-classical growth model with endogenous labor supply, Byron (2008) finds that any domestic expenditure reductions that offset any portion of aid that had been allocated to public investment expenditure acts as a pure transfer. Also, when governments are able to alter expenditure commitments in response to aid inflows, the absence of changes in marginal productivities of private factors (to which agents respond by altering labor supply and capital and bond accumulation) may result in short-run welfare gains but generally contribute to lower long-run levels to both welfare and key macroeconomic variables when compared to the case where aid is not fungible. In sum, fungibility may mitigate aid effectiveness. The current paper extends Byron (2008) by considering the response by the real exchange rate to aid that is fungible.

In the model, output can be consumed, traded for foreign goods, or accumulated as private capital. Agents chooses a level of consumption of the domestic and foreign good, as well as the levels and rates of bond and private capital accumulation. In borrowing, agents face an imperfect capital market where the interest rate is a function of the level of indebtedness of the recipient country. The dynamics for the real exchange rate increases the complexity of the model, so to keep the model tractable, private capital is assumed to include both labor and physical capital[4]. To capture how rigidities to private capital affect the macroeconomic dynamics, I perform robustness checks where I alter the adjustment costs to private capital[5]. Lowering the adjustment cost to private capital may capture of the some of dynamics of a model in which labor was endogenous. Of course, this is a simplification as I am not able to consider the full extent of the labor-leisure decision.

In general, the results confirm some in the existing literature. First, the results suggest the Dutch Disease effect of aid is possible (a real net appreciation accompanied by productivity increases) as when the aid is tied, not fungible but when the adjustment costs to private public capital are high. Second, it is possible for both tied and untied aid to exacerbate the current account as when the transfers finance consumption of foreign goods and therefore 'substitute' for production. Third, adjustment costs to both forms of capital matter for the effect of tied aid on the real exchange rate. Lower adjustment costs (to both forms of capital) are more favorable under tied aid for increasing the long-run levels of capital, output, total consumption, and decreasing debt accumulation. The long-run real exchange rate depreciates to reflect the overall the increased productivity.

[4]Some of the theoretical literature show how the labor-leisure decision adds to the richness of the dynamic effects of a foreign aid transfer, for instance Chatterjee & Turnovsky (2007) .

[5]Private and public capital are subject to intertemporal adjustment costs. Adjustment costs slow the speed of accumulation of both private and public capital. See Morshed & Turnovsky (2004).

With the fungibility analysis, I find that altering government expenditure in response to anticipated tied and untied aid flows may generate intertemporal welfare gains by sustaining consumption levels in light of the lower long-run output levels. But I found, as in Byron (2008), that the opportunity cost of these consumption levels is the lower long-run levels of accumulation of private capital. In all cases of fungible aid, the current account worsens and the real exchange rate appreciates in the long-run.

2.2 A Neoclassical Growth Model of Foreign Aid

2.2.1 The Model Setup

Since the functional forms in the model are complex, they are presented them in Table 2.1.

The Private Sector

Consider a small, open economy that comprises identical, and infinitely lived individual agents. Each agent has an infinite planning horizon and possesses perfect foresight. Agents produce a domestic good which can either be consumed, C_H, or traded for a foreign good, C_F. Both goods are normal goods. Total domestic consumption is a composite index given by the Cobb-Douglas function:

$$C = C(C_H, C_F),$$

where the elasticity of substitution, μ, between the two consumption goods satisfies the restriction that $0 < \mu < 1$. The agent's objective is to maximize his intertemporal utility function as given by:

$$U = \int_0^\infty \frac{1}{\gamma} u(C) e^{-\beta t} dt,$$

where γ measures the intertemporal elasticity of substitution[6], and β measures the rate of time preference. The agent assumes the rate of time preference is exogenously given in the small, open economy[7].

Agents can also accumulate bonds, N, to finance private consumption expenditure according to their budget constraint:

$$\dot{N} = \frac{1}{p}\left[C_H + pC_F(1+\tau_f) + pr(.)N + \Phi(I) - (1-\tau_y)Y + T\right], \tag{2.1}$$

where τ_f is the tariff rate that agents face in consuming the foreign good, τ_y is the income tax rate, and T is the amount of his lump-sum taxes denominated in terms of the domestic output. The agent's bond accumulation is expressed in terms of the relative price of the foreign good in terms of the domestic good (the real exchange rate), p. Even though determined in equilibrium by the market-clearing condition, agents take p as given in determining bond acquisition during his optimization. The agent also takes the domestic interest rate, $r(.)$, as exogenously given in his optimization problem.

Each household produces domestic output described by the Cobb-Douglas production function:

$$Y = Y(K, K_G), Y_K > 0, Y_{K_G} > 0, Y_{KK_G} < 0.$$

K is the private agent's stock of capital in the production function[8]. K_G is the economy's stock of infrastructure or a pure public good that is not subject to congestion and generates a positive externality in the production function. Domestic production is well-behaved in that both forms of capital have positive but diminishing marginal physical product. Restrictions are imposed on the factors in the aggregate production function so that total

[6]The intertemporal elasticity of substitution is defined as $\frac{1}{1-\gamma}$.

[7]As Turnovsky (1997) argues, this is a very strict small open economy assumption. However it is not unreasonable here as the model considers only those countries that exert little influence through either their own real exchange rate or their price level on the rate of return in world capital markets and therefore take the world interest rate and the price of foreign goods as exogenously given.

[8]In this model, private capital includes both labor and physical capital.

returns to scale are decreasing in the accumulating factors. Total output in the economy is the sum of domestic output from all households.

Any domestic output that is produced but neither consumed nor traded is accumulated as private capital. The agent accumulates private capital through investment flows according to:

$$\dot{K} = I - \delta_K K,$$

where the stock of private capital depreciates by the factor, $\delta_K K$. Private capital is also subject to adjustment costs, such that the investment function is given by:

$$\Phi = \Phi(I), \Phi' > 0, \Phi'' < 0.$$

The agent decides on levels of C_H, C_F, I, and the levels and rates of asset accumulation of private capital and debt, K and N, respectively. To do this, the agent maximizes intertemporal utility subject to the flow budget constraint equation, and the capital accumulation equation. Thus, the agent's hamiltonian function is given by:

$$\begin{aligned} H &= U(C)e^{-\beta t} + \frac{q'}{p}(I - \delta_K K - \dot{K})e^{-\beta t} \\ &- \nu(\frac{1}{p}\left[C_H + pC_F(1+\tau_f) + pr(.)N + \Phi(I) - (1-\tau_Y)Y + T\right] - \dot{N})e^{-\beta t}. \end{aligned}$$

The optimality conditions with respect to the individual's choices are given by:

$$u_c(C)\frac{\partial C(C_H, C_F)}{\partial C_H} = \frac{\nu}{p} \tag{2.2a}$$

$$u_c(C)\frac{\partial C(C_H, C_F)}{\partial C_F} = (1+\tau_f)\nu \tag{2.2b}$$

$$\Phi_I(I) = \frac{q'}{\nu} \tag{2.2c}$$

$$\frac{\dot{q}}{q} + \frac{(1-\tau_y)Y_K(K,G)}{q} - (\frac{\dot{p}}{p} + \delta_K) = r(.) - \frac{\dot{\nu}}{\nu} \tag{2.2d}$$

$$\dot{\nu} = \nu(\beta - r(.)). \tag{2.2e}$$

The transversality conditions are:

$$\lim_{t\to\infty} \frac{q'}{p} K e^{-\beta t} = \lim_{t\to\infty} \nu N e^{-\beta t} = 0.$$

In Equation (2.2a) - Equation (2.2d), $q^{'}$ is the shadow price of wealth or the market value of capital. ν is the shadow price of wealth in the form of internationally traded bonds denominated in the foreign currency, such that $q = \frac{q^{'}}{\nu}$ is the shadow price of the agent's private capital stock in terms of bonds which are denominated in the foreign currency. Equation (2.2a) equates the marginal utility of consumption of the home good to the shadow price of wealth. Equation (2.2b) equates the marginal utility of consumption of the foreign good to the shadow price of wealth in terms of the relevant price faced by the agent. Equation (2.2c) equates the marginal cost of an additional unit of investment to the market value of private capital. Equation (2.2d) gives the rate of return on capital to the borrowing cost of capital. Equation (2.2e) equates the rate of return on consumption to the after-tax return on bonds.

Equation (2.2a) and Equation (2.2b) can be solved for the domestic and foreign good demand functions:

$$C_H = C_H(\nu, p) \tag{2.3a}$$

and

$$C_F = C_F(\nu, p). \tag{2.3b}$$

Equation (2.2c) gives the investment function:

$$I = I(q). \tag{2.3c}$$

The Public Sector

For simplicity, the model abstracts from the provision of a public consumption good. The government provides public capital, K_G, which accumulates as a stock over time as:

$$\dot{K}_G = G_G - \delta_G K_G,$$

where G_G represents total government expenditure on public investment. Additionally, public capital accumulation is subject to depreciation by the factor $\delta_G K_G$, and the public capital investment function is given by:

$$\Omega(G_G) = G_G + \left(\frac{h_2}{2}\right) G_G^2.$$

Public investment expenditure is financed by government revenue which comprises tariffs on the consumption of foreign goods, lump-sum taxes, and the foreign aid transfers which are denominated in units of the foreign good. Following Turnovsky (2004) and Cerra et al (2008), the government is passive and adjusts taxes continuously to maintain a balanced budget at all times giving the government's budget constraint as:

$$\Omega(G_G) = \tau_y Y + p\tau_f C_f + \bar{T}. \tag{2.4}$$

The direct effect of a pure transfer (untied aid) is to reduce the tax burden, T, and it follows that the net tax burden, $\bar{T}$, is given by:

$$\bar{T} = T - (1-\lambda)pF, \tag{2.5}$$

where λ indicates the foreign aid allocation to public investment expenditure. The foreign aid, F, which is denominated in units of the foreign good and its allocation, λ, are exogenous[9]. Foreign aid allocation, λ, is general enough to be determined either by the donor or in consultation with recipients. Alternatively, when aid finances public investment expenditure (tied aid), the total level of expenditure is given by:

$$G_G = G_G^d + \lambda pF. \tag{2.6}$$

That is, when an exogenous foreign aid transfer is made that is equivalent to 5% GDP, expressed in terms of the foreign good, is allocated fully to public infrastructure ($F = 0.05$ and $\lambda = 1$), total public expenditure is augmented in Equation (2.6) by the amount of the

[9]The treatment of foreign aid in this paper is different from Chapter 1. In Chapter 1, the amount of foreign aid was tied to the scale of the recipient economy.

aid transfer. Untied aid reduces the lump-sum tax burden ($F = 0.05$ and $\lambda = 0$) in Equation (2.5). Also note that since domestic government investment expenditure, G_G^d, foreign aid, F, and the allocation, λ, are exogenous, $G_G = G_G(p)$ and the public capital investment function is also a function of the real exchange rate, that is, $\Omega(G_G) = \Omega(p)$.

The Current Account and Domestic Market-Clearing Condition

To close the model, the market for domestic goods must clear. So output not consumed domestically must be accumulated as private capital, used in the production of public capital, or exported to allow agents to consume the foreign good, C_F. The domestic goods market-clearing condition is given by:

$$Y = C_H + Z(p) + \Phi(I) + \Omega(G_G), \tag{2.7}$$

where $Z(p)$ is the amount of the domestic good traded for C_F. This export function satisfies $Z' > 0$.

Combining the private budget constraint and the government resource constraint (Equation 2.4) give the aggregate resource constraint; and together with Equation (2.5), and Equation (2.7), gives an expression for the current account for the economy:

$$\dot{N} = C_F + r(.)N + (1 - \lambda)F - \frac{Z(p)}{p}. \tag{2.8}$$

As raised earlier, the domestic interest rate, $r(.)$, in Equation (2.8) is determined in international capital markets. International capital markets consider the country's indebtedness, as an indicator of the default risk, in determining the appropriate premium on the domestic interest rate fir borrowers. Drawing from the Turnovsky (1997) and Eicher & Turnovsky (1999*c*) specification[10], the domestic interest rate is a function of the absolute level of indebtedness of the aggregate economy and is given by:

$$r(.) = r^* + \omega(Np), \omega^{'} > 0, \tag{2.9}$$

[10]Their specification for the bond interest rate as a function of the debt servicing capacity benefits from Obstfeld (1982), Cooper & Sachs (1985) and van der Ploeg (1996). The original specification, attributable to Bardham (1967), uses the absolute level of national debt.

where the premium, ω, is expressed as a function of the absolute level of national debt expressed in domestic goods, Np, and r^* is the exogenous world interest rate.

The Real Exchange Rate Dynamics

Using the aggregate production function, along with Equation (2.3a), Equation (2.3b) and Equation (2.6) in the market clearing condition, Equation (2.7), gives:

$$Y(K, K_G) = C_H(\nu, p) + Z(p) + \Phi(q) + \Omega(G_G). \tag{2.10a}$$

Solving Equation (2.10a) numerically for the real exchange rate:

$$p = p(\nu, q, K, K_G). \tag{2.10b}$$

Further, the dynamic equation for the real exchange rate are obtained by taking the time derivative of Equation (2.10b) to give:

$$\dot{p} = p_q(.)\dot{q} + p_K(.)\dot{K} + p_{K_G}(.)\dot{K_G} + p_\nu(.)\dot{\nu}. \tag{2.10c}$$

Therefore, once the macro-dynamic equilibrium is determined, the evolution of the real exchange rate can also be determined.

2.2.2 The Macroeconomic Equilibrium

The macroeconomic dynamics for this small, open economy can be expressed in terms of K, K_G, N, q, and ν, from which the dynamics for the other variables of interest can be derived. Appendix B gives more details on how the macroeconomic dynamics are obtained. The core dynamics is given by:

$$\dot{K} = I(q) - \delta_K K \tag{2.11a}$$

$$\dot{K_G} = G_G^d + \lambda p F - \delta_G K_G \tag{2.11b}$$

$$\dot{N} = C_F + r(.)N + (1-\lambda)F - \frac{Z(p)}{p} \tag{2.11c}$$

$$\dot{\nu} = \beta\nu - r(.)\nu \tag{2.11d}$$

and

$$\dot{q} = \left(r(.) + \delta_K + \frac{\dot{p}}{p} - \frac{\dot{\nu}}{\nu} \right) q - (1 - \tau_y) F_K(K, K_G). \tag{2.11e}$$

The steady-state equilibrium occurs when $\dot{K} = \dot{K_G} = \dot{N} = \dot{\nu} = \dot{q} = 0$ and consequently $\dot{p} = 0$. The corresponding conditions are given by:

$$\frac{\tilde{q} - 1}{h_1} = \delta_K \tilde{K} \tag{2.12a}$$

$$G_G^d + \lambda p F = \delta_G \tilde{K_G} \tag{2.12b}$$

$$C_F(\tilde{v}, p) + r(\tilde{N} p)\tilde{N} - (1 - \lambda)F = \frac{Z(p)}{p} \tag{2.12c}$$

$$\beta = r(.) \tag{2.12d}$$

and

$$(\beta + \delta_K)\, \tilde{q} = (1 - \tau_y) F_K(\tilde{K}, \tilde{K_G}). \tag{2.12e}$$

In addition, the production function and the domestic market clearing condition give:

$$Y = \alpha \tilde{K}^{\eta} \tilde{K}_G^{\sigma(1-\eta)} \tag{2.12f}$$

$$F(\tilde{K}, \tilde{K_G}) = C_H(\tilde{\nu}, p) + Z(p) + \Phi(\tilde{q}) + \Omega(p). \tag{2.12g}$$

The above system in Equation (2.12a) - Equation (2.12e) along with the aggregate resource constraint, the aggregate production function, the market clearing condition, the interest rate equation, and the demand functions, Equations (2.3a and 2.3b), can be used to solve for steady-state $\tilde{K}$, $\tilde{K_G}$, $\tilde{N}$, $\tilde{q}$, and $\tilde{\nu}$, from which $\tilde{p}$, $\tilde{C_H}$, $\tilde{Y}$, $\tilde{C}$ and $\tilde{C_F}$ can also be derived.

The system above is non-linear and would be difficult to solve analytically. The dynamic system involves three state variables, K, K_G, and N, and two jump variables, q, and ν. The system is solved using a linear approximation around the steady-state equilibrium and then by evaluating the dynamics by numerical simulation. The numerical simulations reveal three stable (negative) eigenvalues and two unstable (positive) eigenvalues. The parameter values in Table 2.2 characterize a small, open economy and yield a pattern of eigenvalues - three negative stable roots for the stable variables and two positive roots for the jump variables - that would be consistent with a steady-state equilibrium. The results below present the responses of key macroeconomic variables to a series of foreign aid allocation scenarios.

2.3 Results

Using the model detailed in the previous section, I will now explore both the long-run responses and transitional dynamics of the allocation of foreign aid that either augments productive government spending (tied aid) or reduces lump-sum taxes (untied aid). Table 2.3 presents the long-run effects of the new steady state values for public and private capital, the current account balance, the real exchange rate, output and domestic and foreign consumption relative to their initial benchmark levels. The steady-state values for the calibrated benchmark economy are in Row 1 of Table 2.3.

All of the aid scenarios consider an aid transfer equivalent to 5% of GDP (expressed in terms of the traded good), that is, aid increases from its zero benchmark level ($f = 0.00$) to ($f = 0.05$) in the new steady-state. The case of $\lambda = 1$ indicates tied aid to finance public investment expenditure. $\lambda = 0$ refers to untied aid which is allocated fully as a pure transfer. Table 2.3 summarizes the long-run responses and represents the new steady-state values relative to their initial base levels. The last column in Table 2.3 presents the equivalent variation calculation of welfare and represents the percentage change in the initial welfare level that would be required to keep welfare unchanged at the pre-shock initial level[11].

[11]Positive values indicate that the agent's welfare is enhanced in the new steady-state.

2.3.1 Foreign Aid Transfers That Finance Public Investment Expenditure (Tied Aid) - $F = 0.05$, $\lambda = 1$

Row 2 of Table 2.3 presents the effect of an increase in aid, from its benchmark level (no aid) to an amount equivalent to 5% of GDP, that is allocated fully to public investment expenditure. Figures (2.1.1)-(2.1.7) give the transition paths for the level effects of the aid shock on the real exchange rate, private capital, debt (as measured by the current account), total consumption, public capital, output, and foreign consumption. Aid that finances public investment expenditure leads to an increase of 38.0% in the stock of public capital from its initial benchmark level (Row 2 of Table 2.3 and Figure 2.1.5). The increase in public capital makes private capital (physical and human capital) more productive. Agents respond in three ways to this increase in the marginal productivity. First, an immediate fall in the shadow price of private capital encourages its accumulation. And so the private capital stock increases by 6.0% from its initial level (Row 2 of Table 2.3 and Figure 2.1.2). Second, there is an obvious and immediate reduction in total consumption. Byron (2008), in which labor is endogenous, finds agents reduce consumption and increase work effort when the marginal productivity of private capital and labor increases in response to tied aid. In that paper, as reflected in these results as well, as output increases, agents increase their consumption levels over time. Since agents can choose between consumption of a domestic and foreign good, the increases remain less than proportionate to the initial steady-state levels (Row 2 of Table 2.3). Third, as the initial debt to output ratio decreases and as the marginal productivity of private capital continues to increase, agents borrow to finance their increased levels of private capital accumulation and consumption, as reflected in the worsening of the current account. With immediate increases in foreign consumption, equilibrium is attained through the market clearing condition by an immediate appreciation in the real exchange rate[12]. The consumption of the foreign consumption good along with the increased debt burden serve to worsen the current account in the long-run (Row

[12] Recall the definition of the exchange rate used in the model is the price of foreign goods relative to domestic goods.

2 of Table 2.3 and Figures 2.1.1 and 2.1.3). Note that as the domestic output constraint is relaxed (as it takes time for agents to respond to the change in marginal productivity of private capital), the exchange rate depreciated slightly in the medium-term, however there is a net appreciation in the long-run (Figure 2.1.1).

In sum, tied aid can have mixed long-run effects, in terms of the responses in levels of output, capital accumulation and real exchange rate; with the lower consumption and higher level of debt serving to decrease long-run welfare. Interestingly, these results are reflected in the more complex model of Cerra et al (2008) in which aid transfers are allocated to the traded sector and in which labor and capital are mobile across the traded and non-traded sectors. They find the long-run real exchange rate will appreciate to equate the sectoral rates of return on private capital (in the traded and non-traded sectors), however consumption of the foreign good increases more than proportionately to the consumption of the domestic non-traded good, and welfare gains are also observed. Because of the complexity of their model, they are also able to observe productivity gains that boost the country's exports of the traded good and affords a higher level of consumption of the traded good relative to the non-traded good. In a subsequent section, I examine whether there are any changes to consumption, the real exchange rate and welfare in long-run when private capital can respond more quickly to the tied aid.

2.3.2 Foreign Aid Transfers that Finance A Pure Transfer (Untied Aid) - $F = 0.05$, $\lambda = 0$

In this section, I examine the responses when the government receives the equivalent of 5% of GDP in foreign aid and allocates the aid fully as a pure transfer while not reducing its expenditure commitment. Most notably, Row 4 of Table 2.3 and Figure 2.2.5 show that there is neither a dynamic response nor a long-run level effect on public capital. As such there are no long-run level effects to private capital and output (Row 4 of Table 2.3), but there are transitional effects (Figures 2.2.2 and 2.2.6) as private capital responds to changes in the shadow price of capital (which adjusts to encourage agents to alter their allocation of income between consumption and capital accumulation). The decline in the rate of return to private capital (not shown) provides the incentive for the required reduction in private capital. Output also declines and returns to its initial steady state level (Figure 2.2.6).

In this scenario, aid is a pure transfer and represents a tax reduction that facilitates consumption and debt reduction (Figure 2.2.3 and Row 4 of Table 2.3). Agents experience a wealth effect and slight increases in consumption in the short-run but the consumption levels decrease overall in the long-run (Row 4 of Table 2.3 and Figures 2.2.4 and 2.2.7). By Figure 2.2.1, prices on domestic goods price rise relative to the price of foreign goods, resulting in an immediate long-run depreciation of the real exchange rate for two reasons. First, an improving current account, through debt reduction, requires an adjustment in price of the domestic goods relative to the price of foreign goods. And second, given the decline in output levels during transition, the real exchange rate must adjust if the market clearing condition is to be satisfied[13]. So while agents experience short-run welfare gains afforded them by the tax reduction and increases in consumption in the short-run, agents are made worse off as their total consumption levels and welfare decline in the long-run.

Cerra et al (2008) also finds that untied aid has wealth effects that decline over time[14]. These welfare gains are through the immediate reduction in the level of debt and long-run increases in consumption of traded and non-traded goods. They also find that there is a slight short-run appreciation because there exists excess capital and labor that can respond to sectoral productivities which serve to increasing output and consumption levels in the long-run[15].

[13]Recall that both forms of capital are also subject to depreciation costs. See Equation (2.12c) when $\lambda = 0$.

[14]They also find that agents also receive welfare gains through increases in leisure. The model in this paper does not capture these effects.

[15]More specifically, Cerra et al (2008) find that whether the traded or the non-traded sector is capital intensive, the real exchange rate remains close to its initial steady state level but with a slight appreciation in the short-run. The appreciation is the result of an increase in the demand for

In sum, in spite of the debt reduction, the effects of the untied aid transfer are a real exchange rate depreciation as well as lower long-run levels of capital accumulation, output, consumption. There are short-run gains but long-run welfare losses.

2.4 Sensitivity Analysis

The parameters in Table 2.1 and the underlying assumptions are used to solve the non-linear system and to generate plausible benchmark estimates for a small, open economy. Their values impact directly the steady state obtained in the model. One simplifying assumption, that led to specific parameter values, was that private capital embodies both physical and human capital[16]. This section considers how the economy responds when both forms of capital are subject to varying degrees of adjustment costs for two reasons. First, it would be useful to compare the dynamic responses when both public and private capital adjust quickly. And second, the model assumes that labor can be interpreted as either exogenous or as having the same adjustment speed as physical capital. While in reality, there may be rigidities that justify the assumption of an exogenous labor supply[17], the results may vary if physical capital can respond more slowly relative to labor. For instance, in the case of private capital, lower adjustment costs can arise where acquisition costs to both physical and labor are low, or there are fewer rigidities to labor, thereby making private capital in the model more responsive to the aid shock.

Table 2.5 presents the macroeconomic responses to a tied aid shock under various adjustment costs of private and public capital. The dynamic responses are in Figure 2.4 and Figure 2.5 respectively. h_1 and h_2 refer to the intertemporal adjustment costs of private and public capital respectively. When adjustment costs are low, $h_1 = 1, h_2 = 1$, and high

traded consumption good, however the real exchange rate returns to its initial steady state as both capital and labor re-allocate to provide the appropriate amount of the non-traded output.

[16]This assumption was made to allow for the calibration of the system, but it does not allow me to distinguish the sluggish adjustments of private capital from the responsiveness in the labor-leisure decision. See Chatterjee & Turnovsky (2007).

[17]As labor rigidities exist in many low-income countries.

adjustment costs occur when $h_1 = 50, h_2 = 50$. The higher the installation costs, $h_1 = 50$, the less effective is private capital in response to the aid shock. The transitional dynamics are interesting as they show a degree of expenditure reduction and expenditure switching between the consumption goods in response to changes in the price of the domestic good relative to the foreign good.

Table 2.5a shows that for a given level of public capital adjustment costs, decreasing private capital costs from the benchmark level, $h_1 = 15$, to $h_1 = 1$, is welfare improving in the long-run under tied aid (Column 1 of Table 2.5a). Under the assumption of low private adjustment costs, $h_1 = 1$, the shadow price of private capital falls immediately to which agents respond in two ways. First agents decrease the rate of capital accumulation in the short-run (Figure 2.4.2). And second, agents switch between consumption goods as there is an initial substitution of the foreign good for the domestic good. In respect of relative prices, the real exchange rate depreciates to encourage exports and to satisfy the market clearing condition, (Equation (2.7)) (Figures 2.4.1 and 2.4.7). Over time as the stock of public capital increases and with lower adjustment costs (relative to the benchmark case), the marginal productivity of private factors rise and agents accumulate private capital (both physical and human capital) at long-run levels that are higher than their initial steady states (Row 3 of Table 2.5a and Figure 2.4.2). Not only is there an 100% increase in public capital accumulation, but the increases in the marginal productivity of private capital results in an increase in output of over 26% from its initial steady state level. This scenario presents the only case in which there is a long-run depreciation of the real exchange rate, increases in domestic and foreign consumption, and output above their initial steady state levels accompanied by intertemporal welfare gains. In fact, the increase in the long-run level of output is higher than in the initial benchmark case of tied aid (Row 2 of Table 2.3), suggesting that the increase in marginal productivity of private capital (when private adjustment costs are low) helps to reduce output supply constraints such that domestic and foreign consumption needs are met at levels above their initial steady state levels. In respect of the current account, the initial substitution away from foreign consumption and the depreciation act to improve the current account (Figure 2.4.3). However, as agents increase their consumption of the foreign good and debt (for consumption and for capital acquisition), the current account worsens. In the short-run, the slight real exchange rate appreciation slightly discourages consumption, but debt accumulation can be accommodated in the long-run without a net appreciation because of the significant productivity gains from the higher long-run output levels. This result seems to support Cerra et al (2008) in that factor mobility and changes in the productivity of private capital affect the response of the real exchange rate to the aid transfer.

Row 1 of Table 2.5a presents the response when private capital adjustment costs are high, $h_1 = 50$. With the exception of the current account dynamics, the transitional paths resemble those of the case of the fully tied as shown in Figure 2.1[18]. The intuition for this difference is as follows. There is a more severe decrease in the shadow price of wealth when compared to the benchmark and the case of low private adjustment costs (Rows 2 and 3 of Table 2.5a respectively). Since there is no incentive to accumulate capital, agents draw down on wealth and immediately lower their consumption levels. Until the shadow price increases sufficiently, agents dispose of capital and accumulate debt to finance their consumption requirements. This results in the decline in the current account observed under $h_1 = 50$. Otherwise, when compared to the benchmark case, the dynamic responses are similar but there is attenuation in long-run level effects of the real exchange rate, current account, output and consumption. Long-run welfare losses are also incurred. These results are broadly consistent with those of Byron (2008) where labor was endogenous, suggesting that increasing adjustment costs in this simpler model could embody labor rigidities that act to lower the potential increases in long-run output.

Column 1 of Table 2.5b shows that for a given level of private installation costs, falling public capital installation costs are also welfare improving under tied aid. For complete-

[18]And therefore I do not include the dynamic responses to a tied aid shock with high private adjustment costs in the paper.

ness, the dynamic responses to a tied aid shock under low public capital adjustment costs are presented in Figure 2.5. The dynamics under low public adjustment costs are similar to those of low private installation costs, with the exception of the effect on the initial domestic consumption levels. When public adjustment costs are low, agents increase their consumption of the domestic good and foreign consumption falls more than this increase. The overall immediate result is one of expenditure reduction as there is an immediate decline in total consumption (Figure 2.5.4). However, as pubic capital accumulates, as discussed above, agents benefit in terms of output and they increase their consumption levels more than proportionately in the long-run. As discussed in the case of low private adjustment costs, over time the real exchange rate depreciates, consumption levels of both goods more than proportionately increase and the level of output rises in the long-run relative to their initial steady state levels. There are welfare losses under this scenario, but these welfare losses are not to the extent exhibited in the benchmark tied aid case as presented in Row 2 of Table 2.5b.

In sum, lowering adjustment costs to capital, in particular private capital, in the presence of tied aid transfers can lead to positive effects for the real exchange rate (depreciation) as well as capital accumulation, output, consumption and net positive welfare gains in the long-run.

2.5 The Fungibility of Foreign Aid

In the previous section, I examined the effect on the real exchange dynamics of aid transfers that were allocated fully to their intended purposes. In this section, I consider three scenarios in which the government alters its spending commitments in response to the aid transfer. These scenarios allow me to consider the effects on the real exchange rate when aid is fungible. The scenarios examined are as follows:

- Scenario 1: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid fully to the public investment expenditure, and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 1, g_G^d = 0.028$).

- Scenario 2: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid equally among a pure transfer and aid tied to public investment expenditure and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 0.5, g_G^d = 0.028$).

- Scenario 3: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid fully to the reduction of lump-sum taxes, and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 0, g_G^d = 0.028$).

Table 2.4 and Figure 2.3 present the macroeconomic responses of a tied aid transfer when the government is able to alter its spending obligations[19]. The responses are expressed relative to the initial benchmark case in Row 1 of Table 2.4.

Rows 2-4 of Table 2.4 correspond to Scenarios 1, 2, and 3 respectively. Table 2.4 shows how public and private capital, and output long-run decline relative to their initial steady state levels when aid is fungible. The transitional dynamics show immediate increases in the consumption levels, and an immediate appreciation that restores the market clearing condition to equilibrium (Figures 2.3.4 and 2.3.7). With output falling, the released government resources act as a pure wealth transfer and serve to reduce debt accumulation and boost long-run welfare under all aid fungibility scenarios. In fact, untied aid, when coupled with the released resources, results in the lowest level of public and private capital formation and the largest long-run welfare gains (Row 4 of Table 2.4). The decline in consumption over time - domestic consumption falling more proportionately than foreign consumption - requires a real exchange rate depreciation for equilibrium in the market

[19]The level effects differ in all three scenarios as in Table 2.4. Figure 2.3 presents the transitional dynamics for a tied aid shock under Scenario 1. The transitional dynamics for Scenarios 2 and 3 are similar and are therefore not presented in the paper.

clearing condition (Equation 2.7). However the depreciation is not sufficient to offset the foregone productivity increases and the initial appreciation.

In sum, when aid is fungible, fully tied, untied, or combined, the real exchange rate can appreciate and agents can obtain long-run welfare gains. Note however that while a long-run appreciation is also observed when the aid is tied but not fungible, the long-run responses for private and public capital, and output are the exact opposite for those obtained when the aid is tied and fungible.

2.6 Conclusion

This paper examined the effects of foreign aid transfers with a view to highlighting how the dynamics of the real exchange rate interacts with other key macroeconomic variables. The main conclusion is that tied and untied aid allocations can have both positive and adverse effects on the long-run level of the real exchange rate depending on: (i) the ease of responsiveness of private and public capital (that is, their respective adjustment costs); and (ii) whether governments can alter their spending commitments in response to the aid transfers. The paper presents several scenarios in which aid directed to productive investment expenditure can increase productivity, generate higher long-run levels of accumulating assets but at the same time leads to either an appreciation or depreciation. The simulations offer some theoretical possibilities as to how aid may mitigate its own effectiveness through its effects on the real exchange rate dynamics. The theoretical exercise highlights the complex interactions among the macroeconomic variables when aid transfers are introduced.

A useful extension to the paper could be to allow for an endogenous labor supply in a model that is not subject to scale effects. At present, labor's responses are subsumed in the response of private capital. In reality labor and physical capital have different speeds of adjustment. Allowing the labor-leisure decision to be endogenous could also clarify the welfare response as agents can then choose among leisure, and consumption of the domestic and foreign good. Finally, the benchmark estimates depend on the chosen parameter values. As my simulation exercise shows, the real exchange rate dynamics is affected by parameterization. As such it may be useful to perform robustness checks under alternative aid and government structures to understand truly the complexity of the effects of aid transfers.

Table 2.1: Functional Forms

The table below provides the functional forms that are used to characterize the economy as a one-good neoclassical model in which foreign aid can either finance public expenditure or a pure transfer.

Utility Function	u=$\frac{1}{\gamma}\left(C\right)^{\gamma}$
Consumption Function	$C = C_H^{\mu} C_F^{1-\mu}$
Production Function	$Y = \alpha K^{\eta} K_G^{\sigma(1-\eta)}$
Private Investment Cost Function	$\Phi(I) = I + \frac{h_1}{2} I^2$
Public Investment Cost Function	$\Omega(G_G) = G_G + \frac{h_2}{2} G_G^2$
Export Function	$Z(p) = \theta p^2; \theta > 0$

Table 2.2: Base Economy Parameter Values

This table presents the benchmark parameters that characterize a small, open economy.

Preference parameters	$\gamma = -1.5, \beta = 0.04, \mu = 0.5, \phi = 0.3$
Production parameters	$\alpha = 0.4, \sigma = 0.75, \eta = 0.8$
Cost of borrowing premium	$a = 0.1$
Export parameter	$\theta = 0.6$
Private capital adjustment	$h_1 = 15$
Public capital adjustment	$h_2 = 15$
Private capital depreciation	$\delta_K = 0.05$
Public capital depreciation	$\delta_G = 0.05$
World interest rate	$r* = 0.06$
Policy parameters	$\tau_f = 0, \tau_y = 0.25, G_G^d = 0.078$
Transfers	F, λ

Table 2.3: Long-run Level Effects of a Foreign Aid Shock

The table presents the level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$. λ is the proportion of foreign aid allocated to public investment expenditure. The last column is the equivalent variation calculation of welfare and is the percentage change in the initial income level that would be required to keep welfare unchanged at the pre-shock aid initial level. The responses are expressed *relative* to the initial benchmark steady state levels.

	Private Capital	Public Capital	Output	RER	Current Account	Consumption	C_F	C_H	ΔW
Benchmark Equilibrium: $F = 0, \lambda = 0$	1.982	1.560	0.739	0.602	-0.336	0.291	0.375	0.225	
Fully Tied Aid: $F = 0.05, \lambda = 1$	1.060	1.380	1.101	0.986	1.014	0.981	0.987	0.973	-6.415
Partially Tied Aid: $F = 0.05, \lambda = 0.5$	1.020	1.198	1.055	1.014	0.985	0.953	0.946	0.960	-7.247
Fully Untied Aid:$F = 0.05, \lambda = 0$	1.000	1.000	1.000	1.035	0.966	0.914	0.899	0.931	-8.422

Table 2.4: Long-run evel Effects When Aid is Fungible

The table presents the level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$. λ is the proportion of foreign aid allocated to public investment expenditure. The last column is the equivalent variation calculation of welfare and is the percentage change in the initial income level that would be required to keep welfare unchanged at the pre-shock aid initial level. g_G^d refer to the government's allocation from revenues to public investment expenditure. The responses are expressed *relative* to the initial benchmark steady state levels. The scenarios are as follows:

Scenario 1: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid fully to the public investment expenditure, and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 1, g_G^d = 0.028$).
Scenario 2: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid equally among a pure transfer and aid tied to public investment expenditure and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 0.5, g_G^d = 0.028$).
Scenario 3: Government receives the equivalent of 5% of GDP in foreign aid, allocates the aid fully to the reduction of lump-sum taxes, and simultaneously reduces its domestic public investment expenditure by 5% of GDP. ($\lambda = 0, g_G^d = 0.028$).

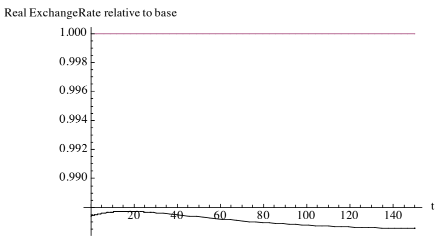
	Private Capital	Public Capital	Output	RER	Current Account	Consumption	C_F	C_H	ΔW
Benchmark Equilibrium: $F = 0, \lambda = 0$	1.982	1.560	0.739	0.602	-0.336	0.291	0.375	0.225	
Scenario 1: Fully tied aid,$F = 0.05, \lambda = 1$	0.944	0.740	0.913	0.988	1.011	0.984	0.989	0.978	3.514
Scenario 2: Partially tied aid,$F = 0.05, \lambda = 0.5$	0.891	0.548	0.833	0.982	1.018	0.909	0.917	0.901	3.117
Scenario 3: Fully untied aid,$F = 0.05, \lambda = 0$	0.820	0.359	0.732	0.955	1.047	0.807	0.825	0.788	11.773

Table 2.5: Long-run Effects of Changes in Adjustment Costs Under The Tied Aid Assumption

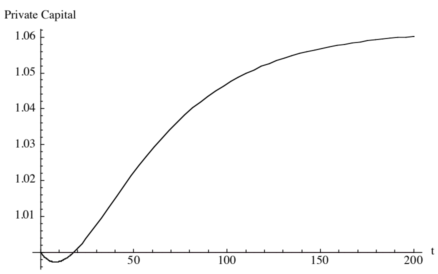
The table presents the long-run level effects of an increase in foreign aid that is equivalent to 5% of GDP, $f = 0.05$, and to variations in the adjustment costs to both forms of capital. h_1 and h_2 are the intertemporal adjustment costs of private and public capital respectively. The higher the installation costs, for instance if $h_1 = 50$, the less effective will be private capital in response to the aid shock. The table presents only the intertemporal welfare effects. The responses in the table are expressed *relative* to the initial benchmark steady state levels.

a) Varying adjustment Costs To Private Capital	ΔW	RER	Output	Consumption	Private Capital	Public Capital	Current Account
High Private Adjustment Costs: h_1 =50, h_2 =15	-13.253	0.925	1.101	0.904	1.033	1.206	1.082
Benchmark Case: h_1=15, h_2 =15	-6.415	0.986	1.055	0.953	1.020	1.198	1.014
Low Private Adjustment Costs: $h_1$1, h_2 =15	5.402	1.098	1.267	1.149	1.180	2.000	0.911
b) Varying adjustment Costs To Public Capital	ΔW	RER	Output	Consumption	Private Capital	Public Capital	Current Account
High Public Adjustment Costs: h_1=15, h_2=50	-21.863	0.873	1.078	0.828	1.049	1.293	1.146
Benchmark Case: h_1=15, h_2 =15	-6.415	0.986	1.055	0.953	1.020	1.198	1.014
Low public adjustment costs: h_1=15, h_2=1	-0.188	1.030	1.110	1.043	1.067	1.416	0.971

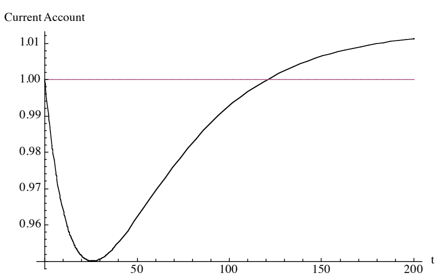
Figure 2.1: Dynamic Responses To A Tied Aid Shock under the Assumption of Neo-Classical Growth



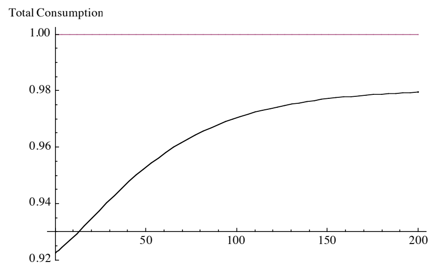
2.1.1 – Real Exchange Rate relative to initial steady state



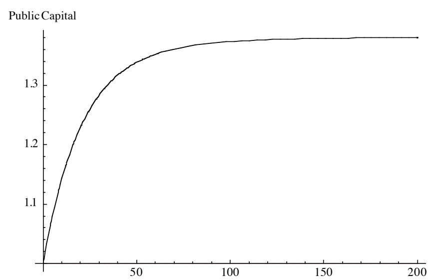
2.1.2 – Private capital relative to initial steady state



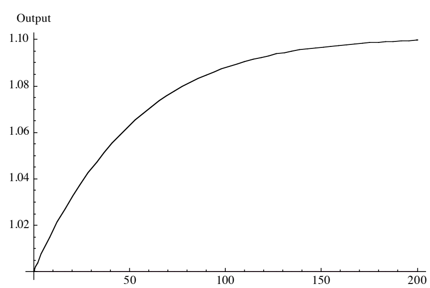
2.1.3 – Current Account relative to initial steady state



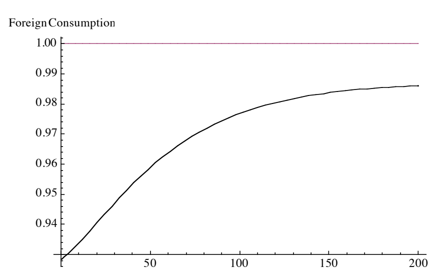
2.1.4 – Consumption relative to initial steady state



2.1.5 – Public capital relative to initial steady state



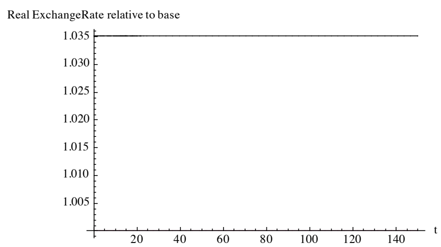
1.6 – Output relative to initial steady state



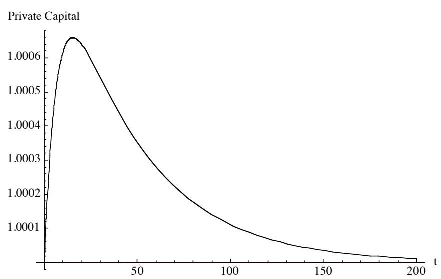
2.1.7 – Foreign Consumption relative to initial steady state

Dynamics from an aid shock equivalent to 5% of GDP.

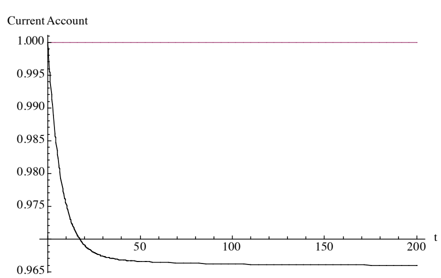
Figure 2.2: Dynamic Responses To An Untied Aid Shock under the Assumption of Neo-Classical Growth



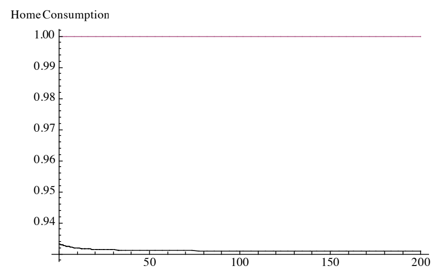
2.2.1 – Real Exchange Rate relative to initial steady state



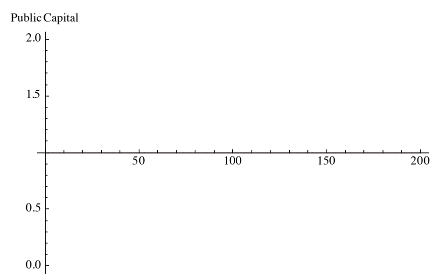
2.2.2 – Private capital relative to initial steady state



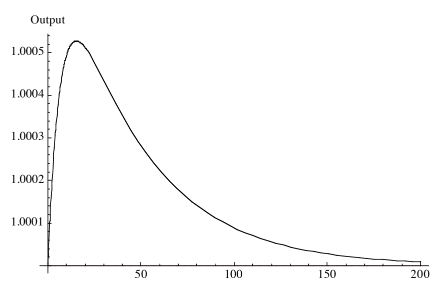
2.2.3 – Current Account relative to initial steady state



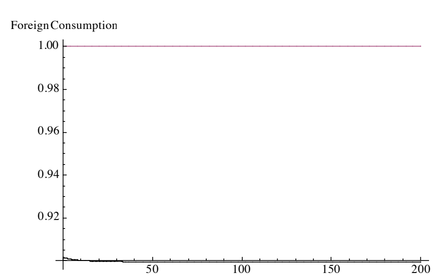
2.2.4 – Consumption relative to initial steady state



2.2.5 – Public capital relative to initial steady state



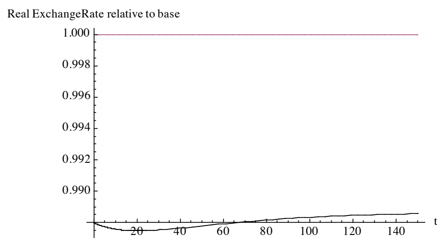
2.2.6 – Output relative to initial steady state



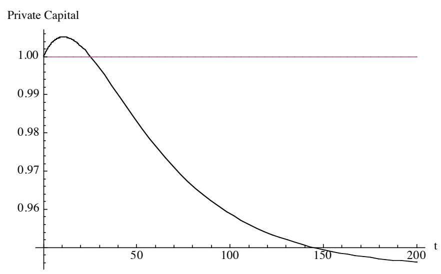
2.2.7 – Foreign Consumption relative to initial steady state

Dynamics from an aid shock equivalent to 5% of GDP.

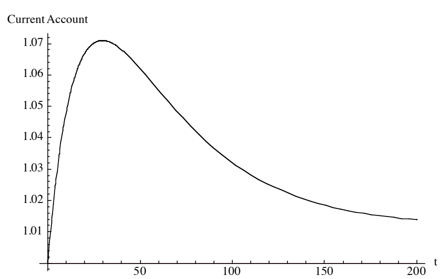
Figure 2.3: Dynamic Responses To Tied Aid Shock Under Fungible Aid



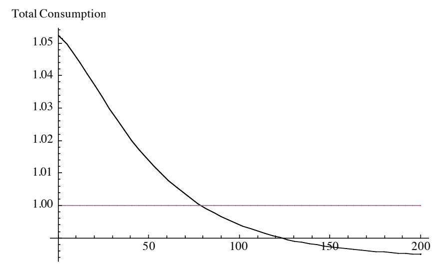
2.3.1 – Real Exchange Rate relative to initial steady state



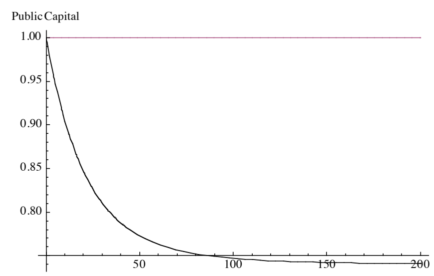
2.3.2 – Private capital relative to initial steady state



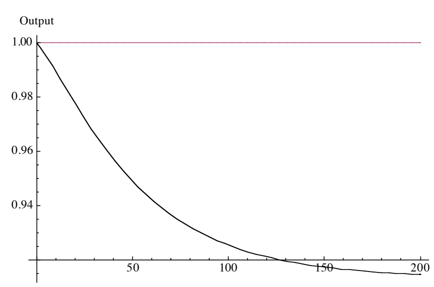
2.3.3 – Current Account relative to initial steady state



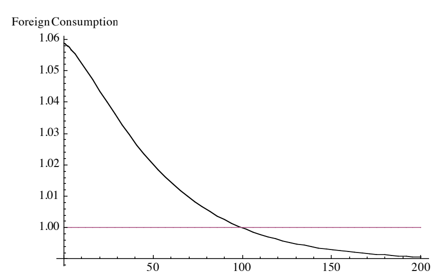
2.3.4 – Consumption relative to initial steady state



2.3.5 – Public capital relative to initial steady state



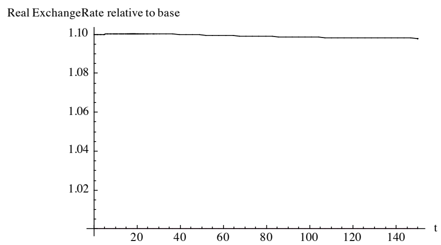
2.3.6 – Output relative to initial steady state



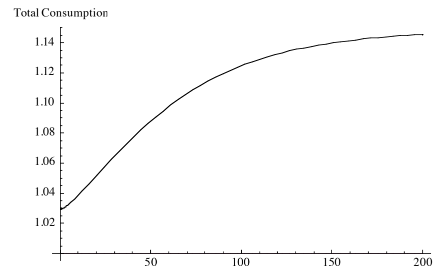
2.3.7 – Foreign Consumption relative to initial steady state

Dynamics from an aid shock equivalent to 5% of GDP.

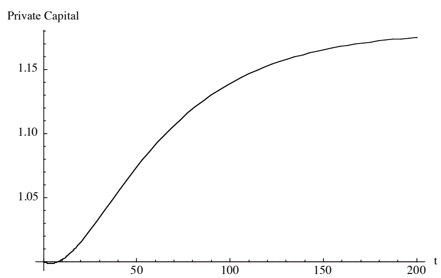
Figure 2.4: Dynamic Responses to a Tied Aid Shock With Low Private Adjustment Costs



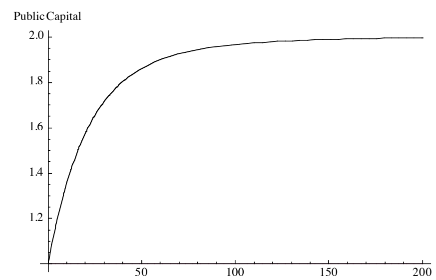
2.4.1 – Real Exchange Rate relative to initial steady state



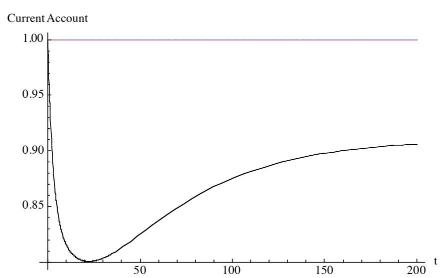
2.4.4 – Consumption relative to initial steady state



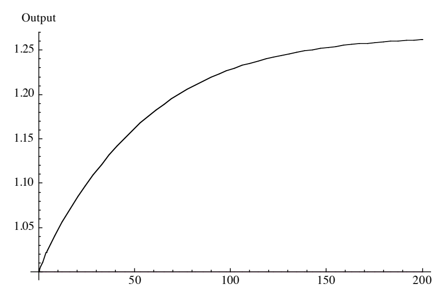
2.4.2 – Private capital relative to initial steady state



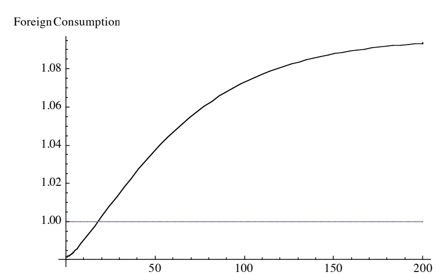
2.4.5 – Public capital relative to initial steady state



2.4.3 – Current Account relative to initial steady state



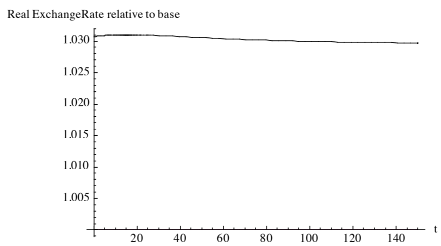
2.4.6 – Output relative to initial steady state



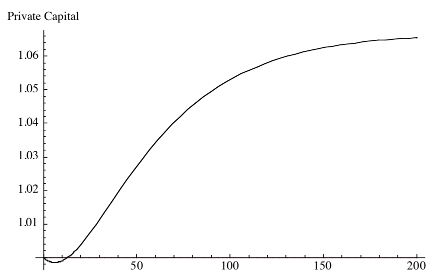
2.4.7 – Foreign Consumption relative to initial steady state

Dynamics from an aid shock equivalent to 5% of GDP.

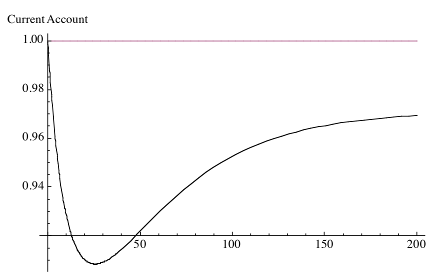
Figure 2.5: Dynamic Responses to a Tied Aid Shock With Low Public Adjustment Costs



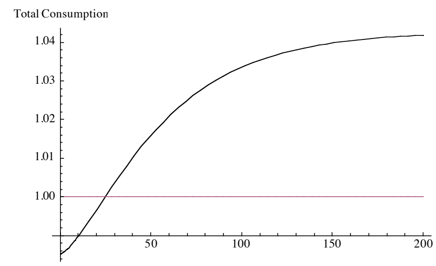
2.5.1 – Real Exchange Rate relative to initial steady state



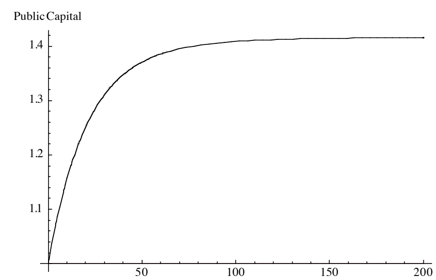
2.5.2 – Private capital relative to initial steady state



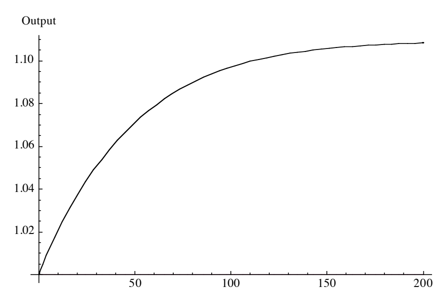
2.5.3 – Current Account relative to initial steady state



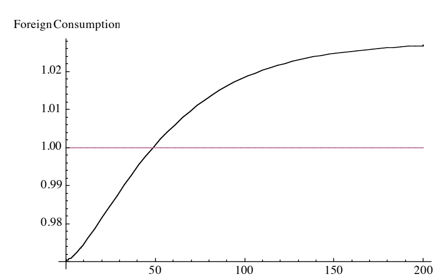
2.5.4 – Consumption relative to initial steady state



2.5.5 – Public capital relative to initial steady state



2.5.6 – Output relative to initial steady state



2.5.7 – Foreign Consumption relative to initial steady state

Dynamics from an aid shock equivalent to 5% of GDP.

CHAPTER 3

THE EFFECT OF FOREIGN AID ON THE LONG-RUN REAL EXCHANGE RATE. DOES FOREIGN AID MATTER?

3.1 MOTIVATION AND BACKGROUND

Understanding the effect of foreign aid on the real exchange rate has taken on an urgency in recent years and as a starting point, researchers draw on the literature that highlight the macroeconomic effects of large foreign capital inflows[1]. These literature on capital flows suggest that the size of the adjustment in domestic absorption has to match the size of the capital inflow. To the extent that an increase in aggregate demand is satisfied by an increase in the consumption of goods in the non-traded sector, the ratio of the prices of non-traded goods to the prices of traded goods will rise and make exports less attractive. The phenomenon is referred as the Dutch Disease effect of capital flows.

Foreign aid may affect economic activity through its impact on relative prices and on a country's export competitiveness[2]. Aid inflows may lead to resource movements away from the traded sector and towards the non-traded sector. In many small developing countries it is conceivable for aid flows spent on goods in the domestic economy to increase the demand for both traded and non-traded goods (Li and Rowe (2007)). However with

[1]Foremost in the literature is the well cited work of Edwards (1989). Calvo, Deiderman & Reihart (1993) examine the relationship between capital inflows and the real exchange rate for Latin America. Lartey et al (2008) examine the effect of remittances on the real exchange rate. Using panel data Saborowski (2009) looks at related policy implications by examining how financial sector development can contain the destabilizing macroeconomic consequences of large foreign direct investment inflows.

[2]Rajan & Subramanian (2005*a*), Rajan & Subramanian (2005*b*), Dollar & Levin (2006), Djankov, Montalvo & Reynal-Querol (2005), Rajan & Subramanian (2007), Elbadawi, Kaltani & Schmidt-Hebbel (2008), Cerra, Tekin & Turnovsky (2008), and Byron (2009) detail this phenomenon in the context of foreign aid flows.

the price of traded goods determined in international markets, for many of these countries, the price of non-tradables adjusts to ease the pressure of the expanded liquidity and the increase in aggregate demand. Given the restricted supply of non-traded goods (especially in the short-run), prices in the non-traded sector must rise, and the real exchange rate appreciates in equilibrium. On the other hand, if the aid transfers assist in removing production bottlenecks, by for example, facilitating tax reduction and trade liberalization, then it is possible for the real exchange rate to depreciate in the long-run. Another possibility may be for the effect to be muted in equilibrium if: (i) the aid is spent on imports and does not affect either the domestic money supply or the demand for non-traded goods; or (ii) resources are idle so that the increase in domestic demand is met by an immediate increase in production, therefore requiring no adjustment by the price of non-tradables.

This paper uses annual data from 66 recipient countries for the period 1972-2000 to examine whether foreign aid transfers influence the real exchange rate in the long-run. This paper contributes to the foreign aid literature in two ways. First, the paper tests empirically one of the hypotheses in Byron (2009) in which an aid allocation to productive government spending may influence the long-run level of the real exchange rate. Whether the response is a net appreciation or depreciation depends on the characteristics of the economy, and the allocation of the transfer - whether it finances a pure transfer or public consumption or investment expenditure, and extent of the adjustment costs to public and private capital. The paper also extends the recent empirical results of Chatterjee, Giuliano & Kaya (2009) by including a real exchange rate measure to consider the effect of foreign aid inflows on the real exchange rate.

The results are consistent with the theoretical results in Byron (2009) in which foreign aid can have an economically significant effect on the real exchange rate. These results however highlight the inverse relationship between foreign aid and the real exchange rate. The results highlight the role for macroeconomic and exchange rate management, such that the recipients may benefit from the growth potential of foreign aid but at the same time avert some of the adverse effects of such inflows.

A well cited study that discusses how macroeconomic variables influence the real exchange rate is Edwards (1989). For Edwards, the long-run equilibrium of the real exchange rate requires both an internal and external equilibrium. He discusses that an internal equilibrium is obtained when the relative price of traded goods to non-traded goods adjust to clear the market for non-traded goods. An external equilibrium occurs when present and expected future current accounts satisfy the government's intertemporal budget constraint and are consistent with long-run sustainable capital flows. Edwards also reasoned that the dynamics of the real exchange rate involves 'justified' and 'unjustified' changes in the macroeconomic fundamentals that drive both the internal and external equilibrium. Justified changes to the macroeconomic variables affect the structure of the 'real' economy and hence the long-run equilibrium, for instance, technological improvements and external terms of trade changes. On the other hand, the over- or under-valuation of the real exchange rate can be the result of unjustified deviations of actual from the long-run equilibrium. For example, aid inflows can affect the internal equilibrium in an unjustified way by reducing the country's international competitiveness and generating a real appreciation. Without a simultaneous response in the underlying fundamentals, the exchange rate becomes overvalued as there would now be a misalignment among fiscal, monetary and exchange rate policies. As Edwards sets out, there must be consistency across all policy choices and underlying fundamentals for both internal and external equilibrium to exist.

Li and Rowe (2007) also acknowledge the role of the underlying fundamentals in explaining the effects of foreign aid on the real exchange rate. In their view, an equilibrium real exchange rate is an unobvservable variable that " ... prevails from real macroeconomic factors acting upon the foreign exchange market without interruption ...". As such, they consider macroeconomic variables that influence both the short- and long-run deviations of the real exchange rate in their econometric specification.

In addition to the observations made in literature similar to Edwards (1989) and Li and Rowe (2007), the $\grave{a}$ priori relationship between foreign aid and the real exchange rate is informed by some recent theoretical work. Cerra, Tekin & Turnovsky (2008) build a two-sector, two-good model in which labor and private capital are mobile. Foreign aid is allocated between the two sectors (traded and non-traded) and can affect the productivity of both sectors. They find that the real exchange rate can either appreciate or depreciate in response to productive aid spending. The appreciation (depreciation) occurs when the aid is allocated to enhancing the productivity of the non-traded (traded) sector. However, the level of disaggregation of the currently available data does not allow for the testing of their hypotheses.

In Byron (2009), I derive some testable hypotheses of the impact of foreign aid on the long-run real exchange rate. In that paper, I build a one-sector, two-good neoclassical growth model of a small open economy that receives foreign aid and in which the real exchange rate plays a role in the macroeconomic dynamics. I find that the allocation of aid either to productive spending (public investment expenditure) or as a pure transfer as having implications for the long-run level of the real exchange rate. In particular, the long-run real exchange rate appreciates when foreign aid is allocated to productive spending and where the adjustment costs to both private and public capital are high. I also find that the real exchange rate depreciates as adjustment costs fall. In this paper, I focus specifically on testing empirically whether foreign aid that is allocated to productive spending has an influence on the equilibrium of the real exchange rate.

In terms of an econometric specification, many studies of capital inflows, foreign aid transfers and the real exchange rate account for the fundamentals that are necessary for internal and external equilibrium as highlighted in Edwards (1989). Some relevant work are Harry (2001), Li & Rowe (2007), Elbadawi, Kaltani & Schmidt-Hebbel (2008), Ouattara & Strobl (2004), and Saborowski (2009). These authors all acknowledge that the dynamics of the real exchange rate embody both short-run and long-run determinants, and included some measure of the following in their model specification: trade openness, terms of trade, aid transfers, government consumption expenditure, money supply, commodity prices, GDP per capita, a productivity index, and the principal and secondary market official exchange rates.

These empirical research find that the association between foreign aid and the real exchange rate to be mixed. For instance, Sackey (2001) finds, as did White (1992), that the real exchange rate can appreciate if aid transfers are spent in the non-traded sector. Using annual data for Ghana during the period 1962-1996, Sackey reasons that a low responsiveness of supply in the non-traded sector (therefore creating domestic supply bottlenecks) places greater pressure on the real exchange rate to appreciate[3]. van Wijnbergen (1986) also found that temporary aid will lead to declines in both production and exports in the traded sector, and a temporary appreciation. Li and Rowe (2007) highlight two important characteristics of aid recipient countries to influence their empirical model specification: (i) the highly export oriented production base in these economies; and (ii) that significant aid flows may require frequent macroeconomic adjustments. By including variables to control for these important characteristics of aid recipient countries and using annual 1970-2005 data for Tanzania, they find the long-run behavior of the real exchange rate to be influenced by three factors in particular: terms of trade (increases of which lead to an appreciation), trade liberalization (improvements of which lead to a depreciation), and foreign aid flows (increases of which lead to a depreciation). While arguing that the depreciation in response to aid flows is ‘non-standard’ in highly aid-dependent countries, they acknowledge that the effect is reasonable in instances where aid flows may improve the productive capacity in the economy. In fact they conclude that it is possible to create

[3] Sackey (2001) also finds that aid can have confounding effects on the real exchange rate if there is simultaneity of purpose in its allocation, for instance, for stabilization and structural adjustment programs. van Wjinbergen (1986) and White (1992) earlier suggested that directing aid across different objectives may undermine its overall objectives.

an environment in which the real exchange rate depreciates as aid flows increase if the government: (i) allocates the foreign aid to productive public expenditure; (ii) takes steps to reduce domestic supply constraints; and (iii) smoothes the flow of foreign aid.

The discussion above provides an insight into key variables used in the models to estimate the association between foreign aid and the real exchange rate using time-series data. In terms of panel data analysis, Ouattara & Strobl (2004) is one study that uses data on 12 African aid-recipient countries over the period 1980-2000 to examine the foreign aid - real exchange rate relationship. The countries in their sample have similar features in terms of monetary regimes - all highly aid dependent and with similar underlying macroeconomic fundamentals[4]. Using these data, Ouattara & Strobl (2004) find that aid inflows are associated with a real depreciation in the long-run real exchange rate. They find that trade openness is also associated with a depreciation. On the other hand, increases in the terms of trade, expansionary macroeconomic policies, and government consumption expenditure are associated with a real appreciation.

3.2 Empirical Method

3.2.1 Data Considerations

The sample for the study includes annual data on 66 aid recipient countries for the period 1972-2000. Appendix C contains a list of the countries. These data come from two main sources. The foreign aid and government expenditure data come from Chatterjee, Giuliano & Kaya (2009) (CGK) while the remaining variables are obtained from the International Monetary Fund International Financial Statistics (IFS) database website[5].

Defining the Dependent Variable - The Real Effective Exchange Rate

Following empirical studies in the literature, the dependent variable is the real effective exchange rate (REER). I use the REER series available in the IFS database, which, to my

[4]All the countries are highly aid dependent and linked to France as their major foreign aid donor.
[5]I wish to thank Ilker Kaya for graciously allowing me access to his database for this study.

knowledge, is the most consistent source for such an index. The IFS database defines the real exchange rate as the relative price of the domestic (non-traded) good in terms of the foreign (traded) good. By this definition, a decrease (increase) in the price of goods in the non-traded sector leads to a depreciation (appreciation)[6]. For countries with missing observations, I construct an index for the real effective exchange rate from the IFS database by using the principal end-of-period official market rate along with a measure of prices from the respective country and the US consumer price index[7]. I combine the IFS series with the constructed index in the analysis[8].

Defining the Explanatory Variables

The main explanatory variable is total foreign aid. I obtain this series from Chatterjee, Giuliano & Kaya (2009) (CGK). The CGK data are a compilation of annual data on foreign aid transfers to 66 countries over the period 1972-2000. The original data source, the Organization for Economic Co-operation and Development Credit Reporting System (CRS) data, provide detailed sectoral allocations that allow these authors to disaggregate total aid into three categories: (i) investment aid - referring to aid allocated to economic infrastructure and to the production sector; (ii) social infrastructure aid - referring to aid

[6]This definition is equivalent to that of Edwards (1989) in which the long-run real effective exchange rate is defined as:

$$p = \frac{E \times P_D}{P_F},$$

where E is the nominal exchange rate, P_D is the price of the domestic good, and P_F is the price of the foreign good. Given the assumption that these countries are international price takers, any change in the real exchange rate comes from adjustments to P_D. A decrease in P_D leads to a decrease in the ratio, that is, the real exchange rate depreciates. The real exchange rate is misaligned when it diverges from its equilibrium level and is overvalued (undervalued) when it is above its equilibrium. Underlying macroeconomic fundamentals determine the equilibrium.

[7]See Footnote 5. Ouattara & Strobl (2004) provide a simple alternative defintion. They construct an index as the ratio of the domestic consumer price index to a weighted average of consumer price indices of its major trading partners.

[8]Another limitation of the constructed index comes from the use of the principal end-of-period official market rate. Li and Rowe (2007) argue that the official rate in many low-income countries may be influenced by the secondary rate in a parallel market for foreign exchange. And so the official exchange rate may at times be misaligned in both the size and direction of the deviation relative to the secondary market rate.

allocated to social infrastructure and services; and (iii) non-investment aid - referring to all remaining aid components in the CRS aid series. I use total aid and these three aid measures (expressed as ratios to GDP) in the estimation model.

The CGK dataset also provides total public expenditure as compiled from the International Monetary Fund's Government Financial Statistics (GFS) data set. Using the GFS expenditure classifications, CGK define three expenditure categories: (i) public investment expenditure as the economics affairs and services component of the GFS data set; (ii) social infrastructure expenditure as the sum of general public services, education, health, social security, housing and recreation and culture components of the GFS data set; and (iii) non-investment expenditure - referring to all remaining expenditure components in the GS expenditure series. As with the foreign aid measure, I use total expenditure and the three government expenditure categories (expressed as ratios to GDP) in the empirical analysis.

For completeness, I reproduce the CRS and GFS data classifications in Appendix C. CGK also reproduced annual data on the dependency ratio, the literacy rate, and the GDP growth rate. These data were obtained originally from the World Bank National Accounts Data and the Organization for Economic Co-operation and Development National Accounts respectively.

The controls in my model are real GDP per capita, population, the consumer price index, the principal official and secondary market rates, exports, imports, direct and portfolio investment, claims on central government, and the domestic credit measures (reserve money, money supply (M1), and reserves minus gold). I obtained these controls from the IFS website.

3.2.2 Model Specification

In this section, I develop a regression equation suitable for testing the hypothesis that aid allocated to productive spending has an effect on the real exchange rate. I use the variables

highlighted in Edwards (1989) and draw on the model specifications discussed above. I consider two estimation approaches. First, I utilize the panel dimension of the data to obtain estimates of the effect of foreign aid on the real exchange rate. Second, I use an IV estimation procedure to: (i) account for the inherent simultaneity in macroeconomic data; and (ii) to minimize the measurement error due to missing observations.
Equation (1) below provides the general form of the regression equation:

$$\begin{aligned} REER_{it} &= \alpha + \beta_1 AID_{it} \\ &+ \beta_2 (AID * EXP)_{it} + \beta_3 Z_{it} + \beta_4 REER_{it-1} + \epsilon_{it}, \\ \epsilon_{it} &= \eta_i + \nu_{it}, \end{aligned} \tag{3.1}$$

where REER is the real effective exchange rate measure, AID measures the log of the ratio of total foreign aid expenditure to GDP, and Z_t is a vector of controls which include: total public expenditure, the interaction between total foreign aid and total public expenditure, trade balance, the terms of trade, money supply and price level. All variables in Equation (3.1) are expressed in logarithmic form. Here the disturbance term, ϵ_{it}, has two orthogonal components: the country fixed effect, η_i, and the idiosyncratic shock, ν_{it}. Before discussing the implications of Equation (3.1) for estimation, I consider the $\grave{a}$ priori signs on the explanatory variables.

I consider first the main explanatory variable in the model - total foreign aid expenditure. As Byron (2009) finds, it is possible for the real exchange rate to appreciate even when aid facilitates productivity gains[9]. In many of the countries in this sample it is conceivable that aid flows spent in the domestic economy can increase the demand for both traded and non-traded goods (Li and Rowe (2007)). But with the price of traded goods determined in international markets[10], it is the price of non-tradables that must adjust to

[9]Productivity gains as interpreted by the accumulation of private and public capital stocks in the long-run, which should $\grave{a}$ priori increase domestic supply, forcing the price of non-traded goods to decline (the real exchange rate should depreciate in equilibrium).
[10]Many aid-recipient countries are price takers.

ease the pressure of the expanded liquidity and the increase in aggregate demand. Given the restricted supply of domestic goods (especially in the short-run), prices in the non-traded sector must rise, and the real exchange rate appreciates in equilibrium. Second, the real exchange rate can depreciate if the aid transfers assist in removing production bottlenecks, by, for example, facilitating tax reduction, and trade liberalization. Third, the equilibrium effect can be muted if in practice: (i) the aid is spent on imports and does not affect either the domestic money supply or the demand for non-traded goods; or (ii) resources are idle so that the increase in domestic demand is met by an immediate increase in production, therefore requiring no adjustment by the price of non-tradables.

As a practical point, Li and Rowe (2007) contend that it is difficult to distinguish clearly in the data between foreign aid and government expenditure, especially when aid finances a large portion of domestic government expenditure. They argue that the disaggregation problem is most evident in the heavily aid-dependent countries. To account for this characteristic of the available data, I include interaction terms between the foreign aid and domestic government expenditure to capture their combined effect on the real exchange rate[11]. I expect that the sign on the interaction term will be in the same direction as on the aid term.

The terms of trade variable is defined as the ratio of the price of exports to the price of imports. The sign of the coefficient on the variable depends on whether the income effect or the substitution effect dominates. The income effect occurs when an increase in the price of exports increases incomes in the export sector and leads to an increase in the demand for non-traded goods (income effect). The real exchange rate will appreciate unless there is a substitution in consumption from domestic non-traded goods towards traded goods (substitution effect) - in which case the real exchange rate can depreciate.

[11] I draw on recent panel data specifications that look at the impact of capital flows on real exchange rate dynamics. In his examination of the effect of foreign direct investment (FDI) on the real excange rate, Saborowski (2009) uses similar interaction terms between FDI inflows and economic development to account for the effect that capital flows have on overall economic development, which then in turn influences the equilibrium real exchange rate.

The trade liberalization variable captures the extent of openness of the economy such that the lowering of tariff and non-tariff barriers and changes in policy toward import substitution can decrease the domestic price of the traded goods. The measure used is the sum of exports and imports as a ratio of GDP. As income from higher export volumes rises, agents respond by increasing their demand for traded goods and substitute away from non-traded goods. Demand for the non-traded goods decline; and as the prices of non-traded goods fall, the real exchange rate would depreciate in equilibrium. I expect a negative association between the trade liberalization and the real exchange rate variables.

The productivity index attempts to account for the Balassa-Samuelson effect[12]. According to this hypothesis, an increase in the productivity of the traded sector (from productive aid spending) requires that resources shift away from the non-traded sector with the effect of increasing wages in both the traded and non-traded sectors. Prices in the non-traded sector increase and the real exchange rate appreciates. Low productivity growth can lead to declines in the real effective exchange rate. I expect a positive sign for the productivity variable in the regression model.

With respect to the expenditure variables, aid allocated to any type of public spending could lead to either a depreciation or appreciation. For instance, the prices of traded goods will increase if aid finances government spending on the traded goods. The trade deficit would then require the real exchange rate to depreciate to maintain equilibrium[13]. Alternatively, it is possible for the real exchange rate to appreciate if the foreign aid is spent in the non-traded sector. I expect that the sign will be the same as on the aid term.

[12]The Balassa-Samuelson effect rests on the assumption that productivity gains in the traded goods sector are greater than productivity gains in the non-traded goods sector. The effect holds for sectors within a country as well as across countries. Li and Rowe (2007) point out that within or across country productivity differentials have the same effect on the direction of the change in the real exchange rate.

[13]This is an example of the external balance equilibrium as defined by Edwards (1989). Li and Rowe (2007) assume that an increase in government spending on traded goods captures increases in government investment expenditure. Under this scenario, the real exchange rate depreciates in equilibrium. For these authors, spending on non-traded goods is categorized as consumption expenditure. Demand and prices of the goods in the non-traded sector will rise, and the real effective exchange rate appreciates in equilibrium.

Following Saborowski (2009), I introduce controls into the base specification to allow for the monetary impact of the foreign aid flow. The controls, as expressed as a ratio to GDP, are total reserves minus gold, reserve money, and money supply (M1).

Model Estimation - Panel Data Analysis

The approach to estimating Equation (3.1) addresses at least two sources of endogenity: (i) simultaneity; (ii) lagged dependent variables; and (iii) the unobserved heterogeneity (country fixed effects)[14]. In both cases the assumption of strict exogeneity is violated. I now present an overview of the sequence of regressions that I estimate. Table 3.1 however presents only the estimates for the most parsimonious of these regression equations.

First, I include time dummies, λ_t, and estimate Equation (3.1) by pooled OLS (POLS). According to Nickell (1982), not accounting for the country effects, η_i, creates one source of dynamic panel bias. Both Nickell (1982) and Roodman (2006) establish that the POLS estimates are not consistent and the coefficients are biased because of the correlation between the lagged dependent variable and the country fixed effect component of the error term (Roodman (2006)). Column 1 of Table 3.1 presents the POLS estimates as they present an upper bound on the coefficients of interest[15].

Second, I again include time dummies and run a fixed effects model which assumes that the unobserved country effects are correlated with the independent variables. Bond (2002) shows these FE estimates are biased and inconsistent. Under the FE assumption, the additional assumption of strict exogeneity that is required for consistency is violated because of the presence of lagged dependent variables. Nonetheless, the FE estimates provide a lower bound on the coefficients of interest, and the results for the most parsimonious regression are in Column 2 of Table 3.1. Alternatively, using an IV approach to account for the lagged

[14]I consider only time invariant country effects. I do not consider the case where each country has its own unobservable time trend (that is, the unobserved heterogeneity is the form of a random trend; see Wooldridge (2002).

[15]Roodman (2006) establishes that these estimates under the POLS assumption are an upper bound on the coefficients of interest.

dependent variable alone does not solve the dynamic bias problem as I still would not have accounted for the country fixed effect[16]. Roodman (2006) argues that asymptotics in the time-series dimension (T) would make the bias problem less significant, but the current sample is characterized by a large number of cross section units (N) and a small number of time series units (T)[17].

One solution is to eliminate the country-specific effect and find instruments to deal with the lagged dependent variable (Roodman (2006)). In a third sequence of regressions, I perform Two-Staged Least Squares (2SLS estimation on first differenced data. The instruments included total aid expenditure, total government expenditure, real GDP per capita, money supply, terms of trade, trade balance, and the CPI. The transformed Equation (3.1) can be rewritten as:

$$\begin{aligned} \Delta(REER_{it}) &= \gamma + \gamma_1 \Delta(REER_{it-1}) + \gamma_2 \Delta(AID_{it}) \\ &+ \gamma_3 \Delta(AID * EXP_{it}) + \gamma_4 \Delta(Z_{it}) + \Delta\epsilon_{it}. \end{aligned} \tag{3.2}$$

Column 3 of Table 3.1 presents the results for the most parsimonious 2SLS estimates on the first differenced data. While this approach produces consistent estimators, gaps in the panel may cause problems for the estimation[18]. First, I lose observations from taking first differences and from creating the lags to account for simultaneity as discussed above. Second, taking first differences exacerbates the gaps in the panel. Third, the transformed

[16]Nickell (1981) and Wooldridge (2002) point out that now the auto-regressive lag length is chosen so that the idiosyncratic errors are serially uncorrelated. But by having performed the first difference transformation, the errors from FE estimation will be serially correlated by definition, thus creating the second source of dynamic panel bias.

[17]Roodman (2006) points out that Kiviet (1995) deals with the dynamic panel bias problem by performing Least Squares Dummy Variable (LSDV) estimation but at the same time raises two limitations. First, the approach works best for balanced panels. And second, the bias due to the presence of the lagged dependent variable still exists even after the data is transformed, but becomes less significant with large T. In fact, under the assumption of strict exogeneity, FD or FE estimation eliminates the problem of unobserved heterogeneity but magnifies measurement error bias (Roodman (2006)). So this approach does not necessarily generate parameter estimates that are consistent and asymptoticly efficient when the panel is unbalanced and T is small relative to N.

[18]These gaps in the panel existed before the data were transformed. Post estimation tests may help to either highlight or dispel whether the accuracy of the estimates is affected by poor data.

data set may not provide necessarily 'strong' instruments[19]. In respect of instruments, Bond (2002) argues that lagged levels of the dependent variable are appropriate instruments for $\Delta(REER)_{it-1}$ in Equation (3.2) above if the lags are independent of the idiosyncratic error term. There can be correlation between the levels of the explanatory variables and country effects as this correlation is eliminated by the first difference transformation. These assumptions, when they hold, identify the autoregressive parameter and generate consistent estimates provided that N is large[20]. While consistent, the estimates produced under this FD-2SLS procedure are not asymptotically efficient.

In a fourth sequence of regressions, I follow the Holtz-Eakin, Newey and Rosen approach as outlined in Roodman (2006) to attempt to improve efficiency. Here I generate instruments from lags of the explanatory variables for the lagged dependent variable and use them in a 2SLS-GMM regression in first differences. This approach transforms the instruments to make them exogenous to the country fixed effects. One major assumption of this estimator is that the instruments are orthogonal to the idiosyncratic errors. The approach substitutes zeros for the missing observations to save on sample size, but even so, the creation of the lags introduces a concern for the tradeoff between longer lags and a smaller sample. Under this estimation approach, the instrumenting matrix is 'collapsed' to save on degrees of freedom[21]. Column 4 of Table 3.1 presents these results.

Doing better than these differenced FD-IV approaches in terms of asymptotic efficiency involves: (i) using a GMM procedure, developed by Hansen (1982) and the first differenced GMM estimator as developed by Holtz-Eakin, Newey and Rosen (1988) and extended by Arellano and Bond (1991) and further developed in a series of papers (Arellano and Bond (1995) and Blundell and Bond (1998); and (ii) obtaining 'strong' instruments. As Bond (2002), Saborowski (2009) and Roodman (2006) point out, the Arellano-Bover/ Blundell-Bond procedure (ABBB), a dynamic panel estimation procedure, allows

[19]A possible extension could be to augment the existing data set.

[20]In fact, when both N and T are large, the first differencing-IV approach provides consistent parameter estimates.

[21]See Roodman 2006.

for both sequential and contemporaneous exogeneity[22], which is where the explanatory variable (foreign aid) can be correlated with current and past realizations of the dependent variable (real exchange rate), but not with future values of the dependent variables. For Equation (3.2), this assumption allows that current realizations of the real exchange rate can be influenced by past realizations but not by future values.

For Wooldridge (2002), using this ABBB approach to find good instruments is feasible if there are some variables for which strict exogeneity holds along with the lagged dependent variables. I instrument for the lagged dependent variable and assume all remaining regressors are strictly exogenous[23]. I generate lagged levels of Z_{it} as instruments for $\Delta(REER)_{it-1}$ and obtain the system GMM parameter estimates. Columns 5 of Table 3.1 presents the estimates where the instruments are lagged levels and then used in the ABBB GMM procedure. I compute standard errors that are robust to heteroskedasticity and autocorrelation.

Column 6 of Table 3.1 are the parameter estimates when differences of Z_{it} act as instruments for $\Delta(REER)_{it-1}$. Here the standard errors are made robust to both heteroskedasticity and serial correlation of the idiosyncratic errors in that I compute the Windmeijer-corrected finite sample standard errors. Since the panel is unbalanced, for both the system and differenced GMM estimators (Column 5 and 6 respectively), I compute orthogonal instruments and collapse the instrumenting matrix to minimize the trade-off between lag length and sample size (Bond (2002) and Roodman (2006)).

[22]Contemporaneous exogeneity in this model implies that foreign aid affects current and past realizations of the real exchange rate.

[23]Initially in this sequence of regressions, I allowed GDP per capita to be endogenous, in the sense that GDP per capita and the amount of aid a country receives may be correlated with the country's fixed effects. In my view, the exogeneity of foreign aid and real GDP per capital is an unrealistic assumption as most variables in macroeconomic general equilibrium models are endogenous. While the coefficient estimates were better than the ones presented, the Hansen test for identifying restriction showed the instrument matrix to be weak. As such I chose to present the results of the case where both foreign aid and GDP are assumed to be strictly exogenous.

After many iterations, I omitted the financial and monetary controls, and the interaction controls because of missing observations[24]. I also omitted the lags of the explanatory variables because the sample size became smaller as I increased the length of lags used as regressors[25].

MODEL ESTIMATION - CROSS-SECTION ANALYSIS

Although I think that a panel approach uses more appropriately and efficiently all the information in the data, I am concerned about the inherent simultaneity among macroeconomic variables such as between aid and the real exchange rate and the gaps in the panels. I now consider an alternative IV estimation procedure on country time-averaged data to account for simultaneity and reduce the data concerns.

As discussed in the panel data section, total foreign aid is endogenous in the model. I use total government expenditure, the literacy rate, the ratio of agriculture to the GDP, and population as instruments in the first stage equation. The exogenous regressors - real GDP per capita, money supply, trade balance, the terms of trade and the inflation rate - in the structural equation act as instruments for themselves in the first stage regression equation. I drop the lagged dependent estimate and rewrite the regression equation as follows:

$$REER_i = \theta_0 + \theta_1 AID_i + \theta_2 Z_i + \xi_i \tag{3.3}$$

All variables appear in logarithmic form in the regression equation. The columns of Table 3.3 present the coefficients using 2SLS, IV-LIML (limited information maximum likelihood), and GMM estimation procedures respectively. I include the LIML estimator, for as Cameron & Trivedi (2009) point out, the LIML estimator performs better in finite sample and produces smaller biases than either the 2SLS or the GMM estimation models, especially when the instruments are weak. The GMM estimator is a two step feasible GMM

[24] A possible extension would be to use appropriate techniques to impute values for the missing observations.

[25] This became more important as I began estimation on the differenced data and in constructing the lags for the dynamic panel estimation.

estimation approach that generates efficient estimates of the coefficients as well as consistent estimates of the standard errors. When not the default computation, I obtain robust-heteroskedastic-consistent errors.

Since the model is over-identified, I am able to test for the validity of the instruments and the extent to which they make strong instruments.

3.3 Results and Discussion

3.3.1 Panel Data Analysis

The results for the panel analysis are in Table 3.1. Across all specifications, the results suggest that foreign aid inflows are associated with a depreciation in the real exchange rate. Column 1 and Column 2 provide the OLS and the FE parameter estimates respectively. Both parameter estimates exhibit dynamic panel bias but they set a range within which the true parameter estimate may exist. As discussed above, the OLS estimates are biased upwards while the FE parameter estimates are biased downwards.

Column 3 reports the simple 2SLS estimates for the equation in first differences. I generate instruments for $\Delta(REER)_{it-1}$ and use them in a 2SLS regression in differences[26]. These parameter estimates are the first consistent estimates of Equation (3.2). The effect of foreign aid on the real exchange rate is negative, indicating that the long-run real exchange rate depreciates in response to increases in foreign aid transfers. The point estimate for the effect of foreign aid, -0.334, is inside the range of the OLS and Within Group point estimates (-0.490,-0.260). To improve efficiency, I perform the Bond (2002) - Holtz-Eakin, Newey, and Rosen (1988) GMM procedure to compute instruments, which generates instruments for $\Delta(REER)_{it-1}$ from the explanatory variable set. These instruments are orthogonal to the transformed idiosyncratic errors and I use them in a 2SLS regression on first differences. The point estimate on the foreign aid measure in Column 4 is now

[26]The instruments for $\Delta(REER)_{it-1}$ include lagged first differences of the explanatory variables as well as differenced time dummies.

-0.257, outside the range of the OLS and FE estimates. The standard errors here are incorrect under the assumption that the errors in the level equation are homoskedastic. That is, if the length of lags of the dependent variable in the structural equation were correctly specified, then the first differences should exhibit some form of serial correlation and/ or heteroskedasticity. As such, I proceed now to obtain efficient estimates in the presence of heteroskedastic and serially correlated errors.

Column 5 and Column 6 in Table 3.1 present the ABBB GMM estimates. The estimates in Column 5 and 6 are both consistent and asymptotically efficient. For both estimators, the Blundell and Bond (1998) stability condition criteria, is satisfied for both columns[27]. The Sargan and Hansen tests for validity of the instruments show that I cannot reject the null hypothesis that the over-identifying restrictions are valid for both the level (Column 5) and differenced (Column 6) instruments. The Arellano and Bond test for AR(1) and AR(2) under the null of zero order-l serial correlation are also satisfied. The results suggest that foreign aid is associated inversely with the real exchange rate as both models report statistical significance on the foreign aid regressor of at least the 0.1% significance level (Column 5). The point estimate for the foreign aid measure in the one step system GMM estimator is not within the range of the POLS - FE point estimate range. On the other hand, the one step differenced GMM point estimate for the foreign aid variable is within the range and is statistically significant (Column 6). Even with this favorable result, I am concerned for the effect of the gaps in the panel and whether the model was appropriately specified[28]. I now present the cross-sectional IV estimates as an alternative approach to estimating the effect of foreign aid on the real exchange rate.

[27]The criteria is that the coefficient on the lagged dependent variable is less than one in absolute value.

[28]This may be due to my assumption of strict exogeneity on the GDP per capita measure.

3.3.2 Cross- Sectional Regression

Table 3.2 presents the pairwise correlation between the instruments and the foreign aid variable. The correlation coefficients show that the overall (linear) correlations are strong. The pairwise associations are in the expected direction and do not provide any early sign of weak instruments.

Table 3.3 presents the 2SLS, LIML and GMM estimates for Equation (3.3). The first column is the 2SLS estimates, the second column presents the LIML estimates while Column 3 shows the optimal GMM estimates. I compute robust standard errors for all the estimators. Across the three estimators, there is little variation in the point estimates and the standard errors for the foreign aid. The results suggest that for the sample of country time-averaged data, foreign aid is negatively associated with the real exchange rate. However the negative association is not statistically significant - at least not at the 5% significance level. In fact the 2SLS and the GMM point estimates are not significant at even the 20 % significance level. The most promising point estimate for the association between foreign aid and the real exchange rate is the LIML estimate, which is significant at the 13.5 significance level. While the results are not statistically significant at the 95% significance level, these results suggest that there is an economically significant effect of foreign aid inflows on the determination of the real exchange rate.

The lower portion of Table 3.3 presents the post-estimation results for the first and second stage regressions of the IV estimation using the country time-averaged data. In respect of the first stage regression, the Shea Partial R^2 and the F statistics under all estimation approaches are encouraging and do not suggest weak instruments[29]. The informal diagnostic for weak instruments, the F-test statistic for joint significance of the instruments in the first stage, is in excess of 10. Under the null of under-identification, the Anderson canonical correlation likelihood ratio test statistic suggests that the models are identified and that the instruments are relevant and are not weak[30]. In fact, the Shea partial R^2,

[29]Low R^2 and F-test statistic indicate weak instruments.

[30]Weak instruments can still be present even when this test statistic suggest to reject the null.

which is robust to weak instruments, suggests that the instruments are at least 57% relevant. The Anderson-Rubin statistic at the first stage rejects the null that the endogenous regressor in the structural equation is zero. In total these first stage post-estimation results suggest that the instruments are relevant and valid.

The Hansen-Sargan J test statistic at the second stage regression for over-identifiying restrictions suggest that the joint null hypothesis that the (included and excluded) instruments are valid cannot be rejected. That is, the instruments are uncorrelated with the error term and the excluded instruments are correctly excluded from the structural estimation equation. The second stage Anderson-Rubin statistic computed for the LIML estimator rejects the null that the instruments are not valid.

While the small sample size is likely to be a reason for the lack of significance, the results demonstrate that there is not enough evidence to dismiss the inverse association between foreign aid and the real exchange rate.

3.4 Conclusion

Overall the panel data and cross-section results suggest that foreign aid is negatively associated with the real exchange rate. This broad result supports one of the hypotheses in my earlier study in which I found that foreign aid, when allocated to productive spending, may lead to a depreciation of the real exchange rate in the long-run. Overall the result is not inconsistent with some the existing empirical and theoretical work that show that show a real exchange rate depreciation in the presence of increasing aid inflows. As my previous work suggested, a depreciation effect is possible when aid flows allow for a supply-side response that more than offsets the increases in aggregate demand.

The panel data results should be interpreted with caution even though the point estimates appear significant. While the use of GMM estimators did help to minimize the problem with the missing data, taking first difference worsened the gaps in the original data set. This transformation compromised the data and could have reduced the precision with which the relationship between foreign aid and the real exchange rate could be examined in the panel dimension. With respect to the IV-cross sectional analysis, even though the results are not statistically significant at the 95% significance level, the results did suggest that there is an economically significant effect of foreign aid inflows on the determination of the real exchange rate. I address the issue of simultaneity by the cross-sectional analysis, and while the models suggest that the instruments are not weak and it may be the case that there may be ‘better’ instruments. An extension to this work could therefore be to fill in the gaps in the panels and re-estimate the sequence of regressions to see if the negative association maintains.

Table 3.1: Panel Data Estimation Results

Variable (in logs)	$POLS^1$	FE^1	$2SLS^{2,3}$	$2SLS^{5,6}$	One Step System $GMM^{3,6}$	One Step Differenced $GMM^{3,6}$
Column	(1)	(2)	(3)	(4)	(5)	(6)
Real Effective Exchange Rate$_{t-1}$	0.249**	-0.0130	0.152	-0.392	0.002	0.024
	(2.93)	(-0.36)	(1.16)	(0.00)	(0.919)	(1.43)
Total Aid	-0.490***	-0.260***	-0.334***	-0.257***	-0.537***	-0.318***
	(-4.62)	(-3.88)	(-4.99)	(-5.55)	(-5.10)	(-5.29)
GDP per capita	-0.717***	-1.236***	-1.315***	-0.976***	-0.820***	-1.312***
	(-4.05)	(-4.25)	(-16.86)	(-17.62)	(-4.35)	(-4.68)
Terms of Trade	0.293***	0.354	-0.233	0.030	0.372***	0.257
	(4.78)	(1.25)	(-1.13)	(0.230)	(5.71)	(0.84)
Trade Balance	0.0650	0.147	-0.559	0.153	0.0861	0.046
	(0.68)	(0.55)	(-0.38)	(1.50)	(0.73)	(-0.24)
N	1075	1075	850	904	1020	904
No. of groups	51	51			47	47
R^2	0.420	0.269	0.391			
F stat	23.98		35.68	18.91	12.79	41.82
F (K-1,N-K)	(32,50)	(6,50)	(34,815)		(6,46)	(6,47)
No. of Instruments			11[4]		34	32
Arellano and Bond AR(1)					0.042	0.024
Arellano and Bond AR(2)					0.250	0.140
Sargan test					0.00	0.00
Hansen test					0.643	0.430
Difference in Hansen						
Hansen test excluding group (Level)					0.707	
Difference (null H= exogenous) (Level)					0.147	
Hansen test excluding group (Differenced)					0.714	0.471
Difference (null H= exogenous) (Differenced)					0.316	0.326

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
t statistics in parentheses
Notes:
1 - Dependent variable - Real Effective Exchange Rate (in logs)
2 - Regression includes time dummies and a constant term
3 - This is the Anderson-Hsiao levels 2SLS estimator. The equation is estimated in first differences
4 - Excludes time dummies as instruments
5 - This is the Hotlz-Newy and Rosen GMM 2SLS estimator. The equation is estimated in first differences
6 - Regression includes time dummies but no constant term

In this model, the negative sign on the foreign aid component indicates a depreciation effect of foreign aid inflows.

Table 3.2: Pairwise Correlations with Foreign Aid Variable

Variable	Correlation Coefficient
Total expenditure	0.852
Literacy rate	-0.437
Dependency rate	-0.249
Ratio Agriculture spending to GDP per capita	0.591
Population	0.176

Table 3.3: Instrumental Variable Estimation Results

Variable (in logs)	2SLS	LIML	GMM
Column	(1)	(2)	(3)
Total Aid	-0.245	-0.318	-0.286
	(-1.06)	(-1.50)	(-1.14)
Real GDP per capita	-0.452*	-0.509***	-0.478*
	(-2.50)	(-3.36)	(-2.49)
Money Supply	-0.317**	-0.343***	-0.325**
	(-3.09)	(-3.38)	(-3.04)
Trade Balance	0.205	0.203	0.218
	(1.59)	(1.71)	(1.63)
Terms of Trade	-0.00481	-0.0124	-0.00596
	(-0.05)	(-0.15)	(-0.07)
CPI	0.0219	0.0131	0.0203
	(1.00)	(0.62)	(0.91)
No. of observations	49	49	49
No. of regressors	7	7	7
No. of instruments	12	12	12
No. of excluded instruments	6	6	6
Shea Partial R^2	0.569	0.569	0.569
$F_{(11,37)}$ (First Stage)	74.42	74.42	74.42
Anderson likelihood statistic (p value) (First Stage Regression)	0.000	0.000	0.000
Anderson-Rubin statistic (p-value) (First Stage Regression)	0.000	0.000	0.000
$F_{(6,42)}$ (Second Stage)	10.13	9.86	10.87
Anderson likelihood statistic (p-value) (Second Stage Regression)	0.000	0.000	0.000
Hansen J statistic (p-value) (Second Stage Regression)	0.160		0.160
Anderson-Rubin statistic (p-value) (Second Stage Regression)		0.350	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
t statistics in parentheses
Dependent variable - Real Effective Exchange Rate (in logs)
Regressions includes a constant term
In this model, the negative sign on the foreign aid component indicates a depreciation effect of foreign aid inflows.

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Appendix A

Appendices to Chapter 1

A.1 Private Sector

Each household produced domestic output given by the production function:

$$X_i = \alpha(1-l_i)^{1-\sigma} K_i^{\sigma} K_g^{\eta},$$

such that aggregate production is given by:

$$\begin{aligned} X &= NX_i \\ &= \alpha(1-l_i)^{1-\sigma} K_i^{\sigma} K_g^{\eta}. \end{aligned}$$

The agent faces the intertemporal utility function given by:

$$U = \int_0^{\infty} \frac{1}{\gamma}(C_i l_i^{\theta} G_c^{\phi})^{\gamma} e^{-\beta t}.$$

Private capital adjustment costs are given by:

$$\Phi(I_i, K_i) = I_i + \frac{h_1}{2}\frac{I_i^2}{K_i},$$

and private capital accumulates as by:

$$\dot{K}_i = I_i - (n+\delta_K)K_i.$$

Agents take the domestic interest rate as given in their optimization but the rate is determined in international capital markets as given by the upward sloping supply curve of debt:

$$r\left(\frac{Z}{K}\right) = r^* + \omega\left(\frac{Z}{K}\right), \omega' > 0.$$

The budget constraint for the private agent, his private borrowing is given by:

$$\dot{B}_i = C_i(1+\tau_c) + [(1-\tau_b)r(.) - n]B_i + \Phi(I_i, K_i) - (1-\tau_X)X_i + T_i$$

Then the private agent's hamiltonian function is given by:

$$\begin{aligned} H &= \frac{1}{\gamma}(C_i l_i^{\theta} G_c^{\phi})^{\gamma} e^{-\beta t} + q'[\dot{K}_i]e^{-\beta t} + \nu[\dot{B}_i]e^{-\beta t} \\ &= \frac{1}{\gamma}(C_i l_i^{\theta} G_c^{\phi})^{\gamma} e^{-\beta t} + q'[I_i - (n+\delta_K)K_i - \dot{K}_i]e^{-\beta t} \\ &\quad -\nu[C_i(1+\tau_c) + [(1-\tau_b)r(.) - n]B_i + \Phi(I_i, K_i) - (1-\tau_X)X_i + T_i - \dot{B}_i]e^{-\beta t} \end{aligned} \tag{A.1}$$

The optimality conditions with respect to the individual's choices for C_i, l_i, I_i, rates of accumulation K and B are given by:

$$\frac{\delta H}{\delta C_i} = C_i^{(\gamma-1)} l_i^{\theta\gamma} G_c^{\phi\gamma} = \nu(1+\tau_c) \tag{A.2a}$$

$$\nu = \frac{C_i^{(\gamma-1)} l_i^{\theta\gamma} G_c^{\phi\gamma}}{(1+\tau_c)}$$

$$\frac{\delta H}{\delta l_i} = \theta C_i^{\gamma} l_i^{\theta\gamma-1} G_c^{\phi\gamma} + \nu(1-\tau_X)\frac{\delta X_i}{\delta l_i} = 0 \tag{A.2b}$$

$$\frac{\delta H}{\delta I_i} = q' - \nu(1 + h_1\frac{I_i}{K_i}) = 0 \tag{A.2c}$$

Now for $\frac{\delta H}{\delta K_i} = \frac{\delta(\frac{\delta H}{\delta \dot{K}_i})}{\delta t}$, where

$$\frac{\delta H}{\delta K_i} = -q' e^{-\beta t}(n+\delta_k) + \nu e^{-\beta t} + \nu e^{-\beta t}(1-\tau_X)\frac{\delta X_i}{\delta K_i}$$

and

$$\frac{\delta(\frac{\delta H}{\delta \dot{K}_i})}{\delta t} = -q'(-\beta)e^{-\beta t} - \dot{q}' e^{-\beta t}$$

Equating I have,

$$q'\beta e^{-\beta t} - \dot{q}' e^{-\beta t} = -q'\beta e^{-\beta t}(n+\delta_K) + \frac{h_1}{2}(\frac{I_i}{K_i})^2 \nu e^{-\beta t} + (1-\tau_X)\frac{\delta X_i}{\delta K_i}\nu e^{-\beta t}$$

Dividing by $\nu e^{-\beta t}$ and defining $q = \frac{q'}{\nu}$, I obtain that:

$$\frac{q'}{\nu}\beta - \frac{\dot{q}'}{\nu} = -\frac{q'}{\nu}(n+\delta_K) + \frac{h_1}{2}(\frac{I_i}{K_i})^2 + (1-\tau_X)\frac{\delta X_i}{\delta K_i} \tag{A.2d}$$

Now for $\frac{\delta H}{\delta B_i} = \frac{\delta(\frac{\delta H}{\delta \dot{B}_i})}{\delta t}$, where

$$\frac{\delta H}{\delta B_i} = -\nu e^{-\beta t}[(1-\tau_b)r(.) - n]$$

and

$$\frac{\delta(\frac{\delta H}{\delta \dot{B}_i})}{\delta t} = -\beta\nu e^{-\beta t} + \dot{\nu} e^{-\beta t}$$

Equating, I have

$$-\beta\nu e^{-\beta t} + \dot{\nu} e^{-\beta t} = -\nu e^{-\beta t}[(1-\tau_b)r(.) - n]$$

Dividing by $-\nu e^{-\beta t}$ gives:

$$\beta - \frac{\dot{\nu}}{\nu} = [(1-\tau_b)r(.) - n] \tag{A.2e}$$

A.2 Public Sector

Foreign aid is tied to the economy's activity and is given by:

$$F = fX; 0 \leq f \leq 1.$$

Under the allocation of foreign aid, the expenditure on the public consumption good is given by:

$$G_C = g_c^d X + \lambda_c F = (g_c^d + \lambda_c f)X,$$

and that for public investment as given by:

$$G_I = g_I^d X + \lambda_I F = (g_I^d + \lambda_I f)X.$$

The government accumulates public capital, K_G, which is subject to depreciation as given by:

$$\begin{aligned} \dot{K_G} &= G_I - \delta_G K_g \\ &= (g_I^d + \lambda_I f)X - \delta_G K_g. \end{aligned}$$

Public capital is also subject to adjustment costs as given by:

$$\Omega(K_G, G_I) = G_I + \frac{h_2}{2}\frac{G_I^2}{K_G}.$$

The overall government budget constraint is given by:

$$\dot{A} = \Omega(K_G, G_I) + G_C + r(.)A - \tau_b r(.)B = \tau_X X - T - F,$$

such that the aggregate resource constraint, national debt (also called the current account in the literature) is:

$$\dot{Z} = \dot{A} + \dot{B}. \tag{A.2f}$$

A.3 Macroeconomic Equilibrium and Linearization

The core dynamic equations which are linearized around K, K_G, Z, q and, l, in that order, are obtained as follows:

A.3.1 Obtaining the dynamics for private capital, K

Now from eq.(A.2c),

$$\begin{aligned} q' &= \nu(1 + h_1\frac{I_i}{K_i}) \\ \Rightarrow q &= 1 + h_1\frac{I_i}{K_i} \end{aligned}$$

$$\Rightarrow \frac{I_i}{K_i} = \frac{q-1}{h_1}. \tag{A.2g}$$

Now $K = NK_i$, which implies that the rate of accumulation of the aggregate capital stock is given by:

$$\frac{\dot{K}}{K} = \frac{\dot{N}}{N} + \frac{\dot{K}_i}{K_i},$$

then using the private capital accumulation equation, I obtain that:

$$\frac{\dot{K}}{K} = \frac{\dot{K}_i}{K_i} - (n + \delta_K),$$

and by A.2g above, the rate of accumulation of private capital is:

$$\frac{\dot{K}}{K} = n + [\frac{q-1}{h_1}] - (n + \delta_K) \tag{A.2h}$$

A.3.2 Obtaining the dynamics for public capital, K_G

I can define the rate of accumulation of public capital as:

$$\frac{\dot{K}_G}{K_G} = (g_I^d + \lambda_I f)\frac{X}{K_G} - \delta_G. \tag{A.3}$$

A.3.3 Obtaining the dynamics for national debt, Z

Now for the national budget constraint. The private accumulation of bonds is given by:

$$\begin{aligned} \dot{B}_i &= C_i(1+\tau_c) + [(1-\tau_b)r(.) - n]B_i + \Phi(I_i, K_i) - (1-\tau_X)X_i + T_i \\ \dot{B} &= N\dot{B}_i = C(1+\tau_c) + [(1-\tau_b)r(.) - n]B - nNB_i + \Phi(I, K) - (1-\tau_X)X + T \end{aligned} \tag{A.4}$$

and national debt is given by eq.(**??** so that

$$\begin{aligned} \dot{Z} &= \dot{A} + \dot{B} \\ &= C(1+\tau_c) + [(1-\tau_b)r(.) - n]B - nNB_i + \Phi(I, K) - (1-\tau_X)X \\ &= +T + \Omega(K_G, G_I) + G_C + r(.)A - \tau_b r(.)B + \tau_X X - T - F \\ &= C + r(.)Z + \Phi(I, K) - X + \Omega(K_G, G_I) + G_C - F \\ &= C + r(.)Z + K[\frac{I}{K} + \frac{h_1}{2}(\frac{I}{K})^2] + [g_d^C + \lambda_C f + g_I^d \\ &\quad +\lambda_I f - 1 - f]X + \frac{h_2}{2}(g_I^d + \lambda_I f)^2\frac{X^2}{K_g} \end{aligned}$$

Following Eicher and Turnovsky (1999), when I assume that $Z \geq 0$ and that the individual faces the upward sloping demand curve for debt, τ_b is now a subsidy and is therefore no longer a source of revenue for the government but rather it increases the government's expenditure flow. Given this assumption, I can express the rate of change of national debt as:

$$\begin{aligned}\frac{\dot{Z}}{Z} &= r(.) + \frac{C}{Z} + \frac{K}{Z}\left[\frac{q^2-1}{2h_1}\right] + \frac{X}{Z}[g_C^d + \lambda_C f + g_I^d + \lambda_I f - 1 - f] \\ & \frac{h_2}{2}(g_I^d + \lambda_I f)^2 \frac{X^2}{ZK_g}\end{aligned}$$

A.3.4 Obtaining the dynamics for the shadow price of capital in terms of bonds, q

Consider 1e and 1f, and that $\frac{\dot{\nu}}{\nu}\frac{q'}{\nu} - \frac{\dot{q}'}{\nu} = -\dot{q}$. then I have from 1f that

$$\beta = \frac{\dot{\nu}}{\nu} + [(1-\tau_b)r(.) - n] \tag{A.5}$$

Substitute 14 into 1e , I get

$$\begin{aligned}\frac{q'}{\nu}\left[\frac{\dot{\nu}}{\nu} + [(1-\tau_b)r(.) - n]\right] - \frac{\dot{q}'}{\nu} &= -q(n+\delta_K) + \frac{h_1}{2}\left(\frac{I_i}{K_i}\right)^2 + (1-\tau_X)\frac{\delta X_i}{\delta K_i} \\ \Rightarrow -\dot{q} = -q(n+\delta_K) + \frac{h_1}{2}\left(\frac{I_i}{K_i}\right)^2 &+ (1-\tau_X)\frac{\delta X_i}{\delta K_i} - q[(1-\tau_b)r(.) - n] \\ \Rightarrow \dot{q} = q(n+\delta_K) - \frac{h_1}{2}\left(\frac{I_i}{K_i}\right)^2 &- (1-\tau_X)\frac{\delta X_i}{\delta K_i} + q[(1-\tau_b)r(.) - n] \\ \Rightarrow \frac{\dot{q}}{q} = (1-\tau_b)r(.) - \delta_K &- \frac{(q-1)^2}{2h_1 q} - \frac{(1-\tau_X)}{q}\sigma\frac{X}{K}\end{aligned} \tag{A.6}$$

A.3.5 Obtaining the dynamics for the labor supply, l

Equating the first order conditions 1.a and 1.b, I get

$$\begin{aligned}\theta C_i^{\gamma} l_i^{\theta\gamma-1} G_c^{\phi\gamma} &= \frac{C_i^{(\gamma-1)} l_i^{\theta\gamma} G_c^{\phi\gamma}}{(1+\tau_C)}(-1)(1-\tau_X)\frac{\delta X_i}{\delta l_i} \\ \frac{\delta X_i}{\delta l_i} &= -(1-\sigma)\frac{X_i}{(1-l_i} \\ \frac{C_i}{X_i} &= [\frac{1-\tau_X}{1+\tau_C}][\frac{1-\sigma}{\theta}][\frac{l}{1-l}] \\ &= \frac{\frac{C}{N}}{\frac{X}{N}} = \frac{C}{X}.\end{aligned} \tag{A.7}$$

Log-linearizing and differentiating A.7 with respect to time, I get:

$$\begin{aligned}\frac{\dot{C}}{C} - \frac{\dot{X}}{X} &= \frac{\dot{l}}{l} - (-1)[\frac{\dot{l}}{1-l}][\frac{1}{1+\tau_C}] \\ &= \frac{\dot{l}(1-l) + \dot{l}l}{l(1-l)} = \frac{\dot{l}}{l(1-l)}.\end{aligned} \tag{A.8}$$

Now log-linearize and differentiate 1.a with respect to time but recalling that $G_C = (g_c^d + \lambda_c f)X$ and $C_i = \frac{C}{N}$, I get that:

$$\nu = \frac{C}{N}^{\gamma-1} l^{\theta\gamma}((g_c^d + \lambda_c f)X)^{\theta\gamma}$$

and by differentiating with respect to time, the evolution of the shadow price of wealth in the form of internationally traded bonds is given by:

$$\frac{\dot{\nu}}{\nu} = (\gamma - 1)\frac{\dot{C}}{C} - (\gamma - 1)\frac{\dot{N}}{N} + \theta\gamma\frac{\dot{l}}{l} + \theta\gamma\frac{\dot{X}}{X}. \tag{A.9}$$

$$\tag{A.10}$$

Now recall that the aggregate production function is given by (1) and I can log-linearize and differentiate (1) with respect to time to obtain that:

$$\begin{aligned} \frac{\dot{X}}{X} &= (1-\sigma)(-1)\frac{\dot{l}}{(1-l)} + (1-\sigma)\frac{\dot{N}}{N} + \sigma\frac{\dot{K}}{K} + \eta\frac{\dot{K}_g}{K_g} \\ &= (1-\sigma)n - (1-\sigma)\frac{\dot{l}}{(1-l)} + \sigma\frac{\dot{K}}{K} + \eta\frac{\dot{K}_g}{K_g}. \end{aligned} \tag{A.11}$$

Now substitute 5, 3, 1g, 1i and 1f into 4, I obtain the dynamics for the labor supply, where:

$$\theta\gamma\frac{\dot{l}}{l} + \theta\gamma\frac{\dot{X}}{X} + (\gamma - 1)\frac{\dot{C}}{C} = \frac{\dot{\nu}}{\nu} + (\gamma - 1)n. \tag{A.12}$$

Substituting 3 into 4, I obtain that:

$$\begin{aligned} \theta\gamma\frac{\dot{l}}{l} + \theta\gamma\frac{\dot{X}}{X} + (\gamma-1)\frac{\dot{l}}{l(1-l)} + (\gamma-1)\frac{\dot{X}}{X} &= \frac{\dot{\nu}}{\nu} + (\gamma-1)n \\ \theta\gamma\frac{\dot{l}}{l} + [\phi\gamma + (\gamma-1)]\frac{\dot{X}}{X} + (\gamma-1)\frac{\dot{l}}{l(1-l)} &= \frac{\dot{\nu}}{\nu} + (\gamma-1)n. \end{aligned} \tag{A.13}$$

And now substituting 5 into 7, I get that:

$$\begin{aligned} &\theta\gamma\frac{\dot{l}}{l} - [\phi\gamma + (\gamma-1)](1-\sigma)\frac{\dot{l}}{(1-l)} + (\gamma-1)\frac{\dot{l}}{l(1-l)} \\ &= \frac{\dot{\nu}}{\nu} + (\gamma-1)n - [\phi\gamma + (\gamma-1)]\left[(1-\sigma)n + \sigma\frac{\dot{K}}{K} + \eta\frac{\dot{K}_g}{K_g}\right]. \end{aligned} \tag{A.14}$$

Gathering terms, I obtain that:

$$\begin{aligned} \dot{l}\left[\frac{\theta\gamma(1-l) - (1-\gamma) - (1-\sigma)(\gamma(1+\phi)-1)l}{l(1-l)}\right] &= \beta + n\gamma - (1-\tau_b)r(.) \\ &-(\gamma(1+\phi)-1)(1-\sigma)n - \sigma(\gamma(1+\phi)-1)\left[\frac{q-1}{h_1} - \delta_K\right] \\ -(\gamma(1+\phi)-1)\eta\delta_G &- (\gamma(1+\phi)-1)\eta(g_I^d + \lambda_I f)\alpha(1-l)^{1-\sigma}k^\sigma k_g^{\eta-1}. \end{aligned} \tag{A.15}$$

A.4 Obtaining the Scale-adjusted per-capita Stationary Variables

While noting that the long-run equilibrium growth rate is given by:

$$\psi = \left[\frac{1-\theta}{1-\theta-\eta}\right] n, \tag{A.16}$$

I transform the dynamic system into stationary scale-adjusted per capita variables, defined as:

$$\begin{aligned}
k &= \frac{K}{N^{\frac{(1-\theta)}{(1-\theta-\eta)}}}, \quad k_g = \frac{K_g}{N^{\frac{(1-\theta)}{(1-\theta-\eta)}}}, \\
x &= \frac{X}{N^{\frac{(1-\theta)}{(1-\theta-\eta)}}}, \quad c = \frac{C}{N^{\frac{(1-\theta)}{(1-\theta-\eta)}}}, and \\
z &= \frac{Z}{N^{\frac{(1-\theta)}{(1-\theta-\eta)}}}.
\end{aligned}$$

The (scale-adjusted) equilibrium system can be expressed as follows:

$$\frac{\dot{k}}{k} = \frac{q-1}{h_1} - \delta_K - \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.17a}$$

$$\frac{\dot{k_g}}{k_g} = (g_I^d + \lambda_I f)\frac{x}{k_g} - \delta_G - \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.17b}$$

$$\begin{aligned}
\frac{\dot{z}}{z} &= r(.) + \frac{c}{z} + \frac{k}{z}[\frac{q^2-1}{2h_1}] + \frac{x}{z}[g_d^C + \lambda_C f + g_I^d + \lambda_I f - 1 - f] \\
&+ \frac{h_2}{2}(g_I^d + \lambda_I f)^2 \frac{x^2}{zk_g} - \left[\frac{1-\theta}{1-\theta-\eta}\right] n
\end{aligned} \tag{A.17c}$$

$$\dot{q} = (1-\tau_b)r(.) - (\sigma\frac{(1-\tau_X)}{q})\frac{x}{k} - \frac{(q-1)^2}{2h_1 q} + \delta_K \tag{A.17d}$$

Now I obtain scale adjusted consumption per capita as:

$$c = [\frac{1-\tau_X}{1-\tau_C}][\frac{1-\sigma}{\theta}][\frac{l}{1-l}]x$$

And so I can obtain the rate of change of consumption as:

$$\frac{\dot{c}}{c} = \frac{\beta - (1-\tau_b)r(.) - \gamma n - \phi\gamma(1-\sigma)n - \phi\gamma\psi(\theta+\eta)}{\gamma - 1} - \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.17e}$$

$$\dot{l} = F(.)G(.) \tag{A.17f}$$

Where I define $F(.)$ and $G(.)$ as follows:

$$F(l) = \frac{l(1-l)}{\theta\gamma(1-l) - (1-\gamma) - (1-\sigma)(\gamma(1+\phi)-1)l}$$

$$\begin{aligned} G(z,k,k_g,l,q) &= \beta + n\gamma - (1-\tau_b)r(.) - (\gamma(1+\phi)-1)(1-\sigma)n \\ &\quad -\sigma(\gamma(1+\phi)-1)[\frac{q-1}{h_1} - \delta_K] - (\gamma(1+\phi)-1)\eta\delta_G \\ &\quad -(\gamma(1+\phi)-1)\eta(g_I^d + \lambda_I f)\alpha(1-l)^{1-\sigma}k^{\sigma}k_g^{\eta-1} \end{aligned}$$

A.5 Steady-State Equilibrium and Transitional Dynamics

The economy reaches a steady-state equilibrium when:

$$\dot{k} = \dot{k_G} = \dot{z} = \dot{q} = \dot{c} = \dot{l} = 0$$

I linearize the dynamic system around the steady state and obtain that:
$\dot{k} = 0$ implies that:

$$\frac{\tilde{q}-1}{h_1} - \delta_K = \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.18a}$$

$\dot{k_G} = 0$ implies that:

$$(g_I^d + \lambda_I f)\frac{\tilde{x}}{\tilde{k}_g} - \delta_G = \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.18b}$$

$\dot{z} = 0$ implies that:

$$\begin{aligned} (1-\tau_b)r\left(\frac{\tilde{z}}{\tilde{k}}\right) &+ \frac{\tilde{c}}{\tilde{z}} \\ &+ \frac{\tilde{k}}{\tilde{z}}\left[\frac{\tilde{q}^2-1}{2h_1}\right] + \frac{\tilde{x}}{\tilde{z}}[g_C^d \\ &+ \lambda_C f + g_I^d + \lambda_I f - 1 - f] + \frac{h_2}{2}(g_I^d + \lambda_I f)^2\frac{\tilde{x}^2}{\tilde{z}\tilde{k}_g} \\ &= \left[\frac{1-\theta}{1-\theta-\eta}\right] n \end{aligned} \tag{A.18c}$$

$\dot{q} = 0$ implies that:

$$(1-\tau_b)r\left(\frac{\tilde{z}}{\tilde{k}}\right) - (\sigma\frac{(1-\tau_X)}{\tilde{q}})\frac{\tilde{x}}{\tilde{k}} - \frac{(\tilde{q}-1)^2}{2h_1\tilde{q}} + \delta_K = 0 \tag{A.18d}$$

$\dot{l} = \dot{c} = 0$ implies that:

$$\left[\frac{1}{\gamma - 1}\right]\left[\beta - (1-\tau_b) r\left(\frac{\tilde{z}}{\tilde{k}}\right) - \gamma n - \phi\gamma(1-\sigma)n - \phi\gamma\psi(\theta+\eta)\right] = \left[\frac{1-\theta}{1-\theta-\eta}\right] n \tag{A.18e}$$

The long-run equilibrium growth rate, Eq.(A.17e), Eq.(A.16) along with:

$$\begin{aligned} r\left(\frac{z}{k}\right) &= r^* + exp\left[a\frac{z}{k}\right] - 1 \\ x &= \alpha(1-l)^{1-\theta} k^{\theta} k_g^{\eta} \end{aligned} \tag{A.18f}$$

will provide the steady state equilibrium values for $\psi, z, q, k, k_G, c, l, x,$ from which other variables can be derived.

A.6 Mathematica Code to Chapter 1

The Mathematica 6.0.2.1 code is as follows. (Print statements omitted).
Unprotect[In, Out]; Clear[In, Out]; Clear["Global`*"]

Off[General::spell]
Off[General::spell1]
Off[Less::nord];
(*NowNB. $\lambda_I = \pi 1$**and** $\lambda_C = \pi 2$***)**

(*STRUCTURAL PARAMETER VALUES*)
$\theta = 1.75; \gamma = -1.5; \beta = 0.04; \phi = 0.30; \alpha = 1; \sigma = .35; \eta = 0.2; n = 0.015;$
$\delta_K = 0.05; \delta_G = 0.05; h_1 = 15; h_2 = 15; a = 0.1; w = 200;$
(*base tax and expenditure rates*)
$\mathrm{tbb} = 0; \mathrm{twb} = 0.28; \mathrm{tcb} = 0.0; \mathrm{gib} = 0.05; \mathrm{gcb} = 0.14; \mathrm{rb} = 0.06; \mathrm{fb} = 0;$
$\pi 1\mathrm{b} = 0; \pi 2\mathrm{b} = 0;$
(*current tax and expenditure rates*)
$\mathrm{tb} = 0; \mathrm{tw} = 0.28; \mathrm{tc} = 0.0; \mathrm{gi} = 0.05; \mathrm{gc} = 0.14; r = 0.06; f = 0.05; \pi 1 = 0; \pi 2 = 0;$

(* DETERMINATION OF BASE EQUILIBRIUM*)
$\mathrm{rbbase} = \mathrm{rb} + \mathrm{Exp}\left[a * \left(\frac{\mathrm{zb}}{\mathrm{kb}}\right)\right] - 1;$
$\mathrm{eq1b} = \psi\mathrm{b} == \frac{(1-\sigma)*n}{(1-\sigma-\eta)};$
$\mathrm{eq2b} = \frac{\mathrm{qb}-1}{h_1} == \delta_K + \psi\mathrm{b};$
$\mathrm{eq3b} = \frac{1}{\gamma-1} * (\beta - (1+\mathrm{tbb}) * \mathrm{rbbase} + \gamma * n - \phi * \gamma * (1-\sigma) * n - \phi * \gamma * \psi\mathrm{b} * (\sigma+\eta)) ==$

$\psi\mathrm{b}\ ;$
$\mathrm{eq4b} = \left(\frac{1-\mathrm{twb}}{\mathrm{qb}}\right) * \sigma * \left(\alpha * (1-\mathrm{lb})^{1-\sigma} * \mathrm{kb}^{\sigma-1} * \mathrm{kgb}^{\eta}\right) + \frac{(\mathrm{qb}-1)^2}{2*h_1*\mathrm{qb}} ==$
$\delta_K + ((1+\mathrm{tbb}) * \mathrm{rbbase});$
$\mathrm{eq5b} = (\mathrm{gib} + (\pi 1\mathrm{b} * \mathrm{fb})) * \left(\alpha * (1-\mathrm{lb})^{1-\sigma} * \mathrm{kb}^{\sigma} * \mathrm{kgb}^{\eta-1}\right) == \delta_G + \psi\mathrm{b};$
$\mathrm{eq6b} = \mathrm{cb} == \left(\frac{(1-\sigma)}{\theta}\right) * \frac{\mathrm{lb}}{(1-\mathrm{lb})^{\sigma}} * \left(\frac{1-\mathrm{twb}}{1+\mathrm{tcb}}\right) * \alpha * \mathrm{kb}^{\sigma} * \mathrm{kgb}^{\eta};$
$\mathrm{eq7b} =$
$\mathrm{rbbase} + \frac{\mathrm{cb}}{\mathrm{zb}} + \left(\frac{\mathrm{kb}}{\mathrm{zb}} * \left(\frac{\mathrm{qb}^2-1}{2*h_1}\right)\right) +$
$\left((\alpha * (1-\mathrm{lb})^{1-\sigma} * \mathrm{kb}^{\sigma} * \mathrm{kgb}^{\eta}\right) * \frac{1}{\mathrm{zb}} *$
$(\mathrm{gib} + (\pi 1\mathrm{b} * \mathrm{fb}) + \mathrm{gcb} + (\pi 2\mathrm{b} * \mathrm{fb}) - (1+\mathrm{fb})))+$
$(\mathrm{gib} + (\pi 1\mathrm{b} * \mathrm{fb}))^2 * \frac{1}{\mathrm{zb}} * \alpha^2 * (1-\mathrm{lb})^{2-2*\sigma} * \mathrm{kb}^{2*\sigma} * \mathrm{kgb}^{2*\eta-1} * \left(\frac{h_2}{2}\right) == \psi\mathrm{b};$
$\mathrm{eq8b} = \mathrm{xb} == \alpha * (1-\mathrm{lb})^{1-\sigma} * \mathrm{kb}^{\sigma} * \mathrm{kgb}^{\eta};$
$\mathrm{sol1} = \mathrm{FindRoot}[\{\mathrm{eq1b}, \mathrm{eq2b}, \mathrm{eq3b}, \mathrm{eq4b}, \mathrm{eq5b}, \mathrm{eq6b}, \mathrm{eq7b}, \mathrm{eq8b}\},$
$\{\psi\mathrm{b}, 0.02\}, \{\mathrm{qb}, 2.0\}, \{\mathrm{kb}, 0.05\}, \{\mathrm{kgb}, 0.06\}, \{\mathrm{lb}, 0.5\}, \{\mathrm{cb}, 0.14\},$
$\{\mathrm{zb}, 0.02\}, \{\mathrm{xb}, 0.23\}]$
$\psi\mathrm{b} = \psi\mathrm{b}/.\mathrm{sol1}[[1]]; \mathrm{qb} = \mathrm{qb}/.\mathrm{sol1}[[2]]; \mathrm{kb} = \mathrm{kb}/.\mathrm{sol1}[[3]];$

$\text{kgb} = \text{kgb}/.\text{sol1}[[4]]; \text{lb} = \text{lb}/.\text{sol1}[[5]]; \text{cb} = \text{cb}/.\text{sol1}[[6]];$
$\text{zb} = \text{zb}/.\text{sol1}[[7]]; \text{xb} = \text{xb}/.\text{sol1}[[8]];$

(*Determination of new steady state*)
$\text{rbr} = r + \left(\text{Exp}\left[\frac{a*z}{k}\right]\right) - 1;$
$\text{eq1} = \psi == \frac{(1-\sigma)*n}{(1-\sigma-\eta)};$
$\text{eq2} = \frac{q-1}{h_1} == \delta_K + \psi;$
$\text{eq3} = \frac{1}{\gamma-1} * (\beta - (1+\text{tb}) * \text{rbr} + \gamma * n - \phi;$
$*\gamma * (1-\sigma) * n - \phi * \gamma * \psi * (\sigma + \eta)) == \psi;$
$\text{eq4} = \left(\frac{1-\text{tw}}{q}\right) * \sigma * (\alpha * (1-l)^{1-\sigma} * k^{\sigma-1} * \text{kg}^{\eta}) + \frac{(q-1)^2}{2*h_1*q} == \delta_K + ((1+\text{tb}) * \text{rbr});$
$\text{eq5} = (\text{gi} + (\pi 1 * f)) * \left(\alpha * (1-l)^{1-\sigma} * k^{\sigma} * \text{kg}^{\eta-1}\right) == \delta_G + \psi;$
$\text{eq6} = c == \left(\frac{(1-\sigma)}{\theta}\right) * \frac{l}{(1-l)^{\sigma}} * \left(\frac{1-\text{tw}}{1+\text{tc}}\right) * \alpha * k^{\sigma} * \text{kg}^{\eta};$
$\text{eq7} =$
$\text{rbr} + \frac{c}{z} + \left(\frac{k}{z} * \left(\frac{q^2-1}{2*h_1}\right)\right) +$
$\left((\alpha * (1-l)^{1-\sigma} * k^{\sigma} * \text{kg}^{\eta}) * \frac{1}{z} * (\text{gi} + (\pi 1 * f) + \text{gc} + (\pi 2 * f) - (1+f))\right) +$
$(\text{gi} + (\pi 1 * f))^2 * \frac{1}{z} * \alpha^2 * (1-l)^{2-2*\sigma} * k^{2*\sigma} * \text{kg}^{2*\eta-1} * \left(\frac{h_2}{2}\right) == \psi;$
$\text{eq8} = x == \alpha * (1-l)^{1-\sigma} * k^{\sigma} * \text{kg}^{\eta};$
$\text{sol2} = \text{FindRoot}[\{\text{eq1}, \text{eq2}, \text{eq3}, \text{eq4}, \text{eq5}, \text{eq6}, \text{eq7}, \text{eq8}\}, \{\psi, 0.02\},$
$\{q, 2.0\}, \{k, 0.05\}, \{\text{kg}, 0.06\}, \{l, 0.5\}, \{c, 0.14\}, \{z, 0.02\}, \{x, 0.23\}]$
$\psi = \psi/.\text{sol2}[[1]]; q = q/.\text{sol2}[[2]]; k = k/.\text{sol2}[[3]]; \text{kg} = \text{kg}/.\text{sol2}[[4]];$
$l = l/.\text{sol2}[[5]]; c = c/.\text{sol2}[[6]]; z = z/.\text{sol2}[[7]]; x = x/.\text{sol2}[[8]];$

(*Ihavetodefinethe5 $*$ 5coefficientmatrixforthelinearized
dynamicsystem*)
(*Theorderingis$k$, kg, z, q, l*)

(*Linearized k dot equation*)
$\text{a11} = \frac{q-1}{h_1} - \delta_K - \frac{(1-\sigma)*n}{(1-\sigma-\eta)};$
$\text{a12} = 0;$
$\text{a13} = 0;$
$\text{a14} = \frac{k}{h_1};$
$\text{a15} = 0;$

(*Linearized kg dot equation*)
$\text{a21} = (\text{gi} + (\pi 1 * f)) * \alpha * (1-l)^{1-\sigma} * \sigma * k^{\sigma-1} * \text{kg}^{\eta};$
$\text{a22} = (\text{gi} + (\pi 1 * f)) * \alpha * (1-l)^{1-\sigma} * k^{\sigma} * \eta * \text{kg}^{\eta-1} - \delta_G - \frac{(1-\sigma)*n}{(1-\sigma-\eta)};$
$\text{a23} = 0;$
$\text{a24} = 0;$
$\text{a25} = (-1) * (\text{gi} + (\pi 1 * f)) * \alpha * (1-\sigma) * (1-l)^{-\sigma} * k^{\sigma} * \text{kg}^{\eta};$

(*Linearized z dot equation*)
$\mathbf{drbrk} = (-1) * \left(\frac{a*z}{k^2}\right) * \left(\mathbf{Exp}\left[\frac{a*z}{k}\right]\right);$
$\mathbf{drbrz} = \left(\frac{a}{k}\right) * \left(\mathbf{Exp}\left[\frac{a*z}{k}\right]\right);$
$\mathbf{a31} = z * \mathbf{drbrk} + \frac{q^2-1}{2*h_1} + \alpha * (1-l)^{1-\sigma} * \sigma * k^{\sigma-1} * \mathbf{kg}^{\eta} *$
$(\mathbf{gi} + (\pi 1 * f) + \mathbf{gc} + (\pi 2 * f) - (1+f)) +$
$(\mathbf{gi} + (\pi 1 * f))^2 * \alpha^2 * (1-l)^{2-2*\sigma} * \sigma * k^{2*\sigma-1} * \mathbf{kg}^{2*\eta-1} * h_2;$
$\mathbf{a32} = (\mathbf{gi} + (\pi 1 * f) + \mathbf{gc} + (\pi 2 * f) - (1+f)) * \alpha * (1-l)^{1-\sigma} * k^{\sigma} * \eta * \mathbf{kg}^{\eta-1} +$
$(\mathbf{gi} + (\pi 1 * f))^2 * \alpha^2 * (1-l)^{2-2*\sigma} * k^{2*\sigma} * (2*\eta-1) * \mathbf{kg}^{2*\eta-2} * \left(\frac{h_2}{2}\right);$
$\mathbf{a33} = \left(r + \mathbf{Exp}\left[\frac{a*z}{k}\right] - 1\right) + z * \mathbf{drbrz} - \frac{(1-\sigma)*n}{(1-\sigma-\eta)};$
$\mathbf{a34} = \frac{k*q}{h_1};$
$\mathbf{a35} = (-1) * \alpha * (1-\sigma) * (1-l)^{-\sigma} * k^{\sigma} * \mathbf{kg}^{\eta} * (\mathbf{gi} + (\pi 1 * f) + \mathbf{gc} + (\pi 2 * f) - (1+f)) -$
$(\mathbf{gi} + (\pi 1 * f))^2 * \alpha^2 * (1-\sigma) * (1-l)^{1-2*\sigma} * k^{2*\sigma} * \mathbf{kg}^{2*\eta-1} * h_2;$

(*Linearized q dot equation*)
$\mathbf{a41} = (1+\mathbf{tb}) * q * \mathbf{drbrk} - (1-\mathbf{tw}) * \sigma * \alpha * (\sigma-1) * (1-l)^{1-\sigma} * k^{\sigma-2} * \mathbf{kg}^{\eta};$
$\mathbf{a42} = (-1) * (1-\mathbf{tw}) * \sigma * \alpha * \eta * (1-l)^{1-\sigma} * k^{\sigma-1} * \mathbf{kg}^{\eta-1};$
$\mathbf{a43} = (1+\mathbf{tb}) * \mathbf{drbrz} * q;$
$\mathbf{a44} = (1+\mathbf{tb}) * \left(r + \mathbf{Exp}\left[\frac{a*z}{k}\right] - 1\right) - \frac{(q-1)}{h_1} + \delta_K;$
$\mathbf{a45} = (1-\mathbf{tw}) * \sigma * \alpha * (1-\sigma) * (1-l)^{-\sigma} * k^{\sigma-1} * \mathbf{kg}^{\eta};$

(*Linearized l dot equation*)
$F = (l * (1-l))/((\theta * \gamma * (1-l)) - (1-\gamma) - (1-\sigma) * (\gamma * (1+\phi) - 1) * l);$
$\mathbf{dFl} = ((\theta * \gamma * (1-l)) - (1-\gamma) - (1-\sigma) * (\gamma * (1+\phi) - 1) * l) * (1-2l) -$
$(l * (1-l)) * ((-1) * \theta * \gamma - (1-\sigma) * (\gamma * (1+\phi) - 1))/$
$((\theta * \gamma * (1-l)) - (1-\gamma) - (1-\sigma) * (\gamma * (1+\phi) - 1) * l)^2;$
$G = \beta + n * \gamma - (1+\mathbf{tb}) * \left(r + \mathbf{Exp}\left[\frac{a*z}{k}\right] - 1\right) - (\gamma * (1+\phi) - 1) * (1-\sigma) * n -$
$(\gamma * (1+\phi) - 1) * \sigma * \left(\left(\frac{q-1}{h_1}\right) - \delta_K\right) -$
$(\gamma * (1+\phi) - 1) * \eta * (\mathbf{gi} + (\pi 1 * f)) * \alpha * (1-l)^{1-\sigma} * k^{\sigma} * \mathbf{kg}^{\eta-1} -$
$(\gamma * (1+\phi) - 1) * \eta * \delta_G;$
$\mathbf{dGl} = (\gamma * (1+\phi) - 1) * \eta * (\mathbf{gi} + (\pi 1 * f)) * \alpha * (1-\sigma) * (1-l)^{-\sigma} * k^{\sigma} * \mathbf{kg}^{\eta-1};$
$\mathbf{a51} = (-1) * (1+\mathbf{tb}) * \mathbf{drbrk} * F -$
$(\gamma * (1+\phi) - 1) * \eta * (\mathbf{gi} + (\pi 1 * f)) * \alpha * (1-l)^{1-\sigma} * \sigma * k^{\sigma-1} * \mathbf{kg}^{\eta-1} * F;$
$\mathbf{a52} = (-1) * (\gamma * (1+\phi) - 1) * \eta * (\mathbf{gi} + (\pi 1 * f)) * \alpha * (1-l);$
${}^{1-\sigma} * k^{\sigma} * (\eta-1) * \mathbf{kg}^{\eta-2} * F;$
$\mathbf{a53} = (-1) * F * (1+\mathbf{tb}) * \mathbf{drbrz};$
$\mathbf{a54} = (-1) * (\gamma * (1+\phi) - 1) * \frac{\sigma}{h_1} * F;$
$\mathbf{a55} = F * \mathbf{dGl} + G * \mathbf{dFl};$

```
M = {{a11, a12, a13, a14, a15}, {a21, a22, a23, a24, a25},
{a31, a32, a33, a34, a35}, {a41, a42, a43, a44, a45},
{a51, a52, a53, a54, a55}};

MatrixForm[M];
Eigenvalues[M];
Mdeter = Det[M − λ * IdentityMatrix[5]];

Solve[Mdeter == 0, λ];

{EE, VV} = Eigensystem[M];
u1 = EE[[1]]; u2 = EE[[2]]; u3 = EE[[3]]; u4 = EE[[4]]; u5 = EE[[5]];

sort1 = Sort[{u1, u2, u3, u4, u5}];

s6 = Select[{u1, u2, u3, u4, u5}, # < 0&];
(*firststableeigenvalue − smallest*)
u11 = s6[[3]]
(*second stable eigenvalue *)
u22 = s6[[2]]
(*thirdstableeigenvalue − largest*)
u33 = s6[[1]]

(*ObtainingA1, A2, A3toexpressthestablesolutionsinterms
ofthenormalizedsmallesteigenvalue*)

e1 = (u11 − a11) * A1 − a12 * B1 − a13 * C1 − a14 * D1 − a15 * E1 == 0;
e2 = (u22 − a11) * A2 − a12 * B2 − a13 * C2 − a14 * D2 − a15 * E2 == 0;
e3 = (u33 − a11) * A3 − a12 * B3 − a13 * C3 − a14 * D3 − a15 * E3 == 0;
e4 = (u11 − a22) * B1 − a21 * A1 − a23 * C1 − a24 * D1 − a25 * E1 == 0;
e5 = (u22 − a22) * B2 − a21 * A2 − a23 * C2 − a24 * D2 − a25 * E2 == 0;
e6 = (u33 − a22) * B3 − a21 * A3 − a23 * C3 − a24 * D3 − a25 * E3 == 0;
e7 = (u11 − a33) * C1 − a31 * A1 − a32 * B1 − a34 * D1 − a35 * E1 == 0;
e8 = (u22 − a33) * C2 − a31 * A2 − a32 * B2 − a34 * D2 − a35 * E2 == 0;
e9 = (u33 − a33) * C3 − a31 * A3 − a32 * B3 − a34 * D3 − a35 * E3 == 0;
e10 = (u11 − a44) * D1 − a41 * A1 − a42 * B1 − a43 * C1 − a45 * E1 == 0;
e11 = (u22 − a44) * D2 − a42 * A2 − a42 * B2 − a43 * C2 − a45 * E2 == 0;
e12 = (u33 − a44) * D3 − a43 * A3 − a42 * B3 − a43 * C3 − a45 * E3 == 0;
e13 = A1 + A2 + A3 == (kb − k);
e14 = B1 + B2 + B3 == (kgb − kg);
e15 = C1 + C2 + C3 == (zb − z);


s3 = Solve[{e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12,
```

$$e13, e14, e15\}, \{A1, A2, A3, B1, B2, B3, C1, C2, C3, D1, D2, D3, E1, E2, E3\}];$$

$$A1 = A1/.s3[[1]]; A2 = A2/.s3[[1]]; A3 = A3/.s3[[1]];$$
$$B1 = B1/.s3[[1]]; B2 = B2/.s3[[1]]; B3 = B3/.s3[[1]];$$
$$C1 = C1/.s3[[1]]; C2 = C2/.s3[[1]]; C3 = C3/.s3[[1]];$$
$$D1 = D1/.s3[[1]]; D2 = D2/.s3[[1]]; D3 = D3/.s3[[1]];$$
$$E1 = E1/.s3[[1]]; E2 = E2/.s3[[1]]; E3 = E3/.s3[[1]];$$

(*Optimal Time Paths*)

$$\mathrm{kt}[\mathrm{t\_}] := A1 * \mathrm{Exp}[u11 * t] + A2 * \mathrm{Exp}[u22 * t] + A3 * \mathrm{Exp}[u33 * t] + k;$$
$$\mathrm{kgt}[\mathrm{t\_}] := B1 * \mathrm{Exp}[u11 * t] + B2 * \mathrm{Exp}[u22 * t] + B3 * \mathrm{Exp}[u33 * t] + \mathrm{kg};$$
$$\mathrm{zt}[\mathrm{t\_}] := C1 * \mathrm{Exp}[u11 * t] + C2 * \mathrm{Exp}[u22 * t] + C3 * \mathrm{Exp}[u33 * t] + z;$$
$$\mathrm{qt}[\mathrm{t\_}] := D1 * \mathrm{Exp}[u11 * t] + D2 * \mathrm{Exp}[u22 * t] + D3 * \mathrm{Exp}[u33 * t] + q;$$
$$\mathrm{lt}[\mathrm{t\_}] := E1 * \mathrm{Exp}[u11 * t] + E2 * \mathrm{Exp}[u22 * t] + E3 * \mathrm{Exp}[u33 * t] + l;$$

$$\mathrm{rt}[\mathrm{t\_}] := r + \mathrm{Exp}\left[a * \left(\frac{\mathrm{zt}[t]}{\mathrm{kt}[t]}\right)\right] - 1;$$
$$\mathrm{xt}[\mathrm{t\_}] := \alpha * (1 - \mathrm{lt}[t])^{1-\sigma} * (\mathrm{kt}[t])^{\sigma} * (\mathrm{kgt}[t])^{\eta};$$

$$\mathrm{ct}[\mathrm{t\_}] := \left(\frac{1-\sigma}{\theta}\right) * \frac{\mathrm{lt}[t]}{(1-\mathrm{lt}[t])^{\sigma}} * \left(\frac{1-\mathrm{tw}}{1+\mathrm{tc}}\right) * \alpha * (\mathrm{kt}[t])^{\sigma} * (\mathrm{kgt}[t])^{\eta};$$

(*Optimal time paths ratios*)

$$\mathrm{kgkt}[\mathrm{t\_}] = \frac{\mathrm{kgt}[t]}{\mathrm{kt}[t]};$$
$$\mathrm{kgk} = \frac{\mathrm{kg}}{k};$$
$$\mathrm{kgkb} = \frac{\mathrm{kgb}}{\mathrm{kb}};$$
$$\mathrm{ckt}[\mathrm{t\_}] = \frac{\mathrm{ct}[t]}{\mathrm{kt}[t]};$$
$$\mathrm{ck} = \frac{c}{k};$$
$$\mathrm{ckb} = \frac{\mathrm{cb}}{\mathrm{kb}};$$
$$\mathrm{cxt}[\mathrm{t\_}] = \frac{\mathrm{ct}[t]}{\mathrm{xt}[t]};$$
$$\mathrm{cx} = \frac{c}{x};$$
$$\mathrm{cxb} = \frac{\mathrm{cb}}{\mathrm{xb}};$$
$$\mathrm{kxt}[\mathrm{t\_}] = \frac{\mathrm{kt}[t]}{\mathrm{xt}[t]};$$
$$\mathrm{kx} = \frac{k}{x};$$
$$\mathrm{kxb} = \frac{\mathrm{kb}}{\mathrm{xb}};$$
$$\mathrm{zkt}[\mathrm{t\_}] = \frac{\mathrm{zt}[t]}{\mathrm{kt}[t]};$$
$$\mathrm{zk} = \frac{z}{k};$$
$$\mathrm{zkb} = \frac{\mathrm{zb}}{\mathrm{kb}};$$
$$\mathrm{zxt}[\mathrm{t\_}] = \frac{\mathrm{zt}[t]}{\mathrm{xt}[t]};$$

$\mathbf{zx} = \frac{z}{x}$;
$\mathbf{zxb} = \frac{\mathbf{zb}}{\mathbf{xb}}$;

(*Welfare Calculations*)

(*Intertemporal Welfare Paths*)

$W[\mathrm{t\_}] = \left((1/\gamma) * (\mathbf{ct}[t])^{\gamma} * (\mathbf{lt}[t])^{\theta*\gamma} * (\mathbf{gc} + (\pi 2 * f))^{\phi*\gamma} * (\mathbf{xt}[t])^{\phi*\gamma}\right) *$
$\mathbf{Exp}[-\beta * t]$;

$\mathbf{Wb}[\mathrm{t\_}] = \left((1/\gamma) * (\mathbf{cb})^{\gamma} * (\mathbf{lb})^{\theta*\gamma} * (\mathbf{gcb} + (\pi\mathbf{2b} * \mathbf{fb}))^{\phi*\gamma} * (\mathbf{xb})^{\phi*\gamma}\right) * \mathbf{Exp}[-\beta * t]$;
$\mathbf{A11} = \mathbf{NIntegrate}[W[t], \{t, 0, 100\}]$;
$\mathbf{A01} = \mathbf{NIntegrate}[\mathbf{Wb}[t], \{t, 0, 100\}]$;

$\mathbf{IW}[\mathrm{t\_}] = \left((1/\gamma) * (\mathbf{ct}[t])^{\gamma} * (\mathbf{lt}[t])^{\theta*\gamma} * (\mathbf{gc} + (\pi 2 * f))^{\phi*\gamma} * (\mathbf{xt}[t])^{\phi*\gamma}\right)$;
$\mathbf{IWb}[\mathrm{t\_}] = \left((1/\gamma) * (\mathbf{cb})^{\gamma} * (\mathbf{lb})^{\theta*\gamma} * (\mathbf{gcb} + (\pi\mathbf{2b} * \mathbf{fb}))^{\phi*\gamma} * (\mathbf{xb})^{\phi*\gamma}\right)$;

(*HereItrytomaketheproductionfunctionlinearby$a$Taylor
seriesexpansion*)

$\mathbf{appxt}[\mathrm{t\_}] =$
$x+$
$\alpha * ((\sigma - 1) * (1 - l)^{-\sigma} * k^{\sigma} * \mathbf{kg}^{\eta} * (\mathbf{lt}[t] - l) + \sigma * (1 - l)^{1-\sigma} * k^{1} * \mathbf{kg}^{\eta} * (\mathbf{kt}[t] - k) +$
$\eta * (1 - l)^{1-\sigma} * k^{\sigma} * \mathbf{kg}^{\eta-1} * (\mathbf{kgt}[t] - \mathbf{kg}))$;

$\mathbf{W2}[\mathrm{t\_}] = \left((1/\gamma) * (\mathbf{ct}[t])^{\gamma} * (\mathbf{lt}[t])^{\theta*\gamma} * (\mathbf{gc} + (\pi 2 * f))^{\phi*\gamma} * (\mathbf{appxt}[t])^{\phi*\gamma}\right) *$
$\mathbf{Exp}[-\beta * t]$;
$\mathbf{A22} = \mathbf{NIntegrate}[\mathbf{W2}[t], \{t, 0, 100\}]$;

Appendix B

Appendices to Chapter 2

B.1 Private Sector

The representative agent's hamiltonian function is given by:

$$\begin{aligned} H &= \frac{1}{\gamma}(C_H^{\mu}C_F^{1-\mu})^{\gamma}e^{-\beta t} + \frac{q'}{p}(I - \delta_K K - \dot{K})e^{-\beta t} \\ &- \nu e^{-\beta t}\left(\frac{1}{p}\left[C_H + p(1+\tau_f)C_F + pr(.)N + \Phi(I) - (1-\tau_Y)Y + T\right] - \dot{B}\right) \end{aligned}$$

The optimality conditions with respect to the individual's choices for C_H, C_F, I, rates of accumulation K and N are given by:

$$\frac{\delta H}{\delta C_H} = \mu C_H^{(-1)}(C)^{\gamma} = \frac{\nu}{p} \tag{B.1a}$$

$$\frac{\delta H}{\delta C_F} = (1-\mu)C_F^{-1}(C)^{\gamma} = \nu(1+\tau_f) \tag{B.1b}$$

$$\frac{\delta H}{\delta I} = q' - \nu\left(1 + h_1 I\right) = 0 \tag{B.1c}$$

$$\beta - \frac{\dot{\nu}}{\nu} - \frac{\dot{p}}{p} = \frac{\dot{q}}{q} - \frac{(1-\tau_Y)}{q}\frac{\delta Y}{\delta K} - \delta_K \tag{B.1d}$$

$$\beta - \frac{\dot{\nu}}{\nu} = r(.). \tag{B.1e}$$

Eq.(B.1a) and Eq.(B.1b) provide the domestic and foreign good demand functions:

$$C_H = \frac{\nu}{p}p^{\gamma(1-\mu)-1} \tag{B.2a}$$

and

$$C_F = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\tau_f}\right)\frac{C_H}{p} \tag{B.2b}$$

Eq.(B.1c) gives the investment function:

$$I = \frac{q-1}{h_1}. \tag{B.2c}$$

B.2 Public Sector

Public capital accumulation is given by:

$$\dot{K_G} = G_G^d + \lambda pF - \delta_G K_G.$$

B.3 Domestic Market Clearing, Current Account and Exchange Rate Dynamics

The domestic goods market must clear by the condition that:

$$Y = C_H + Z(p) + \Phi(I) + \Omega(G_G), \tag{B.3}$$

where $Z(p) = \theta p^2$ and satisfies that $Z' > 0$.
I obtain the aggregate resource condition (a measure of the current account) by combining the private budget constraint to get:

$$\dot{N} = \frac{1}{p}\left[C_H + pC_F(1+\tau_f) + pr(.)N + \Phi(I) - (1-\tau_Y)Y + \bar{T} + \Omega(G_G)\right]$$

and the government budget constraint:

$$\Omega(G_G) = \tau_y Y + p\tau_f C_F + \lambda pF,$$

while recalling Eq.(B.3) and that:

$$\bar{T} = T - (1-\lambda)pF.$$

Then the aggregate resource constraint for the economy is given by:

$$\dot{N} = C_F + r(.)N + (1-\lambda)F - \frac{Z(p)}{p}.$$

I use the market clearing condition, Eq.(B.3), to obtain an expression for the real exchange rate, p, such that:

$$p = p(\nu, q, K, K_G).$$

I linearize Eq. (B.4) around the steady state to obtain:

$$p = \tilde{p} + \frac{\delta p}{\delta K}(.)(K - \tilde{K}) + \frac{\delta p}{\delta K_G}(.)(K_G - \tilde{K}_G) + \frac{\delta p}{\delta q}(.)(q - \tilde{q}) + \frac{\delta p}{\delta \nu}(.)(\nu - \tilde{\nu}). \tag{B.4}$$

I obtain the partials $\frac{\delta p}{\delta K}$, $\frac{\delta p}{\delta K_G}$, $\frac{\delta p}{\delta q}$, and $\frac{\delta p}{\delta \nu}$ by taking the total derivative of the market clearing condition and *evaluate these partials at the steady state*. The market clearing condition is given by:

$$\begin{aligned} Y(K, K_G) \quad &= \quad C_H(\nu, p(K, K_G, q, \nu)) + \theta(p(K, K_G, q, \nu))^2 + \frac{q^2 - 1}{2h_1} + \left(G_G^d + p(K, K_G, q, \nu)\lambda F\right) \\ &+ \quad \frac{h_2}{2}\left(G_G^d + p(K, K_G, q, \nu)\lambda F\right)^2 \end{aligned}$$

from which I obtain the partials:

$$\frac{\delta p}{\delta K} = \eta \frac{Y}{K} \left(\frac{1}{\frac{\delta C_H}{\delta p} + 2\theta p + \lambda F + h_2(G_G^d + \lambda p F)\lambda F)} \right)$$

$$\frac{\delta p}{\delta K_G} = \sigma(1-\eta) \frac{Y}{K_G} \left(\frac{1}{\frac{\delta C_H}{\delta p} + 2\theta p + \lambda F + h_2(G_G^d + \lambda p F)\lambda F)} \right)$$

$$\frac{\delta p}{\delta q} = -1 \frac{q}{h_1} \left(\frac{1}{\frac{\delta C_H}{\delta p} + 2\theta p + \lambda F + h_2(G_G^d + \lambda p F)\lambda F)} \right)$$

$$\frac{\delta p}{\delta \nu} = -1 \frac{\delta C_H}{\delta \nu} \left(\frac{1}{\frac{\delta C_H}{\delta p} + 2\theta p + \lambda F + h_2(G_G^d + \lambda p F)\lambda F)} \right)$$

I then differentiate Eq. (B.4) with respect to time to obtain the dynamics for the real exchange rate as follows:

$$\dot{p} = p_q \dot{q} + p_K \dot{K} + p_{K_G} \dot{K}_G + p_\nu \dot{\nu}$$

where $p_q = \frac{\delta p}{\delta q}$ evaluated at the steady state.

B.4 Macroeconomic Equilibrium and Linearization

The core dynamic equations which are linearized around K, K_G, N, q and, ν, in that order, as follows:

$$\dot{K} = \frac{q-1}{h_1} - \delta_K K$$

$$\dot{K_G} = G_G^d + \lambda p F - \delta_G K_G$$

$$\dot{N} = C_F + r(.)N + (1-\lambda)F - \frac{Z(p)}{p}$$

$$\dot{q} = \frac{q}{p}\dot{p} + q(r(.) + \delta_K) - \frac{(1-\tau_Y)}{q}\frac{\delta Y}{\delta K}$$

while recalling Eq.(B.6) above, and

$$\dot{\nu} = \nu(\beta - (1-\tau_b)r(.)).$$

The domestic interest rate is given by:

$$r(.) = r^* * e^{apN-1}.$$

B.5 Mathematica Code to Chapter 2

The Mathematica 6.0.2.1 code is as follows. (Print statements omitted).

Unprotect[In, Out]; Clear[In, Out]; Clear["Global`*"]
Off[General::spell]
Off[General::spell1]

$\gamma = -1.5; \mu = 0.5; \beta = 0.04; \phi = 0.3; a = 0.1;$
$\alpha = 0.4; \sigma = 0.75; \eta = 0.8; \theta = 0.6;$
$\delta_K = 0.05; \delta_G = 0.05; h_1 = 15; h_2 = 15; w = 200; \mathrm{rba} = 0.06;$
$\mathrm{tbb} = 0; \mathrm{twb} = 0.25; \mathrm{tfb} = 0.0; \mathrm{gdgb} = 0.078; \mathrm{Fb} = 0.0; \lambda\mathrm{gb} = 0;$
$\mathrm{tb} = 0; \mathrm{tw} = 0.25; \mathrm{tf} = 0.0; \mathrm{gdg} = 0.078; F = 0.05; \lambda\mathrm{g} = 1;$

(*Here I include all 15 equations*)
(*Derivative of ch and cf*)
$\mathrm{dchdp} = \left(\frac{\gamma(1-\mu)-1}{\gamma-1}\right) * \frac{\mathrm{ch}}{p};$
$\mathrm{dchdv} = \left(\frac{1}{\gamma-1}\right) * \left(\frac{\mathrm{ch}}{v}\right);$
$\mathrm{dcfdp} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\mathrm{tf}}\right) * \left(\frac{\gamma(1-\mu)-1}{\gamma-1} - 1\right) * \frac{\mathrm{ch}}{p^2};$
$\mathrm{dcfdv} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\mathrm{tf}}\right) * \left(\frac{1}{\gamma-1}\right) * \left(\frac{\mathrm{ch}}{p*v}\right);$

(*PartialDerivativesof$p$, astakingthetotalderivativeof
$y(k, \mathrm{kg}) = \mathrm{ch}(v, p(k, \mathrm{kg}, q, v)) + \theta * p^2(k, \mathrm{kg}, q, v) + \frac{q^2-1}{2*h_1} + (\mathrm{gdg} + p(k, \mathrm{kg}, q, v) * \lambda\mathrm{g} * F)+$
$\frac{h_2}{2} * (\mathrm{gdg} + p(k, \mathrm{kg}, q, v) * \lambda\mathrm{g} * F)^2$
andthensolvingforthespecificpartialderivativesdpdk,
dpdkg, dpdn, dpdq, dpdv*)

$\mathrm{denom} = (\mathrm{dchdp} + 2 * \theta * p + \lambda\mathrm{g} * F + h_2 * (\mathrm{gdg} + \lambda\mathrm{g} * p * F) * (\lambda\mathrm{g} * F));$
$\mathrm{dpdk} = \eta * \frac{y}{k} * \frac{1}{\mathrm{denom}};$
$\mathrm{dpdkg} = \sigma(1-\eta) * \frac{y}{\mathrm{kg}} * \frac{1}{\mathrm{denom}};$
$\mathrm{dpdn} = 0(*\mathrm{since}p = p(k, \mathrm{kg}, q, v)*);$
$\mathrm{dpdq} = (-1) * \frac{q}{h_1} * \frac{1}{\mathrm{denom}};$
$\mathrm{dpdv} = (-1) * \mathrm{dchdv} * \frac{1}{\mathrm{denom}};$

(*Theorderingofthelinearizationis$k$, kg, n, q, v*)
(*LinearizationofKdot $= \frac{q-1}{h_1} - \delta_K * k$***)**
$\mathrm{a11} = -\delta_K;$
$\mathrm{a12} = 0;$
$\mathrm{a13} = 0;$
$\mathrm{a14} = \frac{1}{h_1};$
$\mathrm{a15} = 0;$

(*LinearizationofKgdot $= \text{gdg} + \lambda\text{g} * p * F - \delta_G * \text{kg}$*)
a21 $= \lambda\text{g} * F * \text{dpdk}$;
a22 $= \lambda\text{g} * F * \text{dpdkg} - \delta_G$;
a23 $= 0$;
a24 $= \lambda\text{g} * F * \text{dpdq}$;
a25 $= \lambda\text{g} * F * \text{dpdv}$;

(*LinearizationofNdot $= \text{cf} + r * n + (1 - \lambda\text{g}) * F - \lambda\text{g} * F - \theta * p$*)
(*$r = \text{rb} + \text{Exp}[a * (n * p)] - 1$; *)
drdn $= a * p * \text{Exp}[a * (n * p)]$;
drdp $= a * n * \text{Exp}[a * (n * p)]$;

a31 $= \text{dcfdp} * \text{dpdk} - \theta * \text{dpdk} + \text{drdp} * \text{dpdk} * n$;
a32 $= \text{dcfdp} * \text{dpdkg} - \theta * \text{dpdkg} + \text{drdp} * \text{dpdkg} * n$;
a33 $= r + n * \text{drdn}$;
a34 $= \text{dcfdp} * \text{dpdq} - \theta * \text{dpdq} + \text{drdp} * \text{dpdq} * n$;
a35 $= \text{dcfdv} + \text{dcfdp} * \text{dpdv} - \theta * \text{dpdv} + \text{drdp} * \text{dpdv} * n$;

(*Linerizationofvdot $= \beta * v - (\text{rba} + \text{Exp}[a * (n * p)] - 1) * v$*)
a51 $= -v * \text{drdp} * \text{dpdk}$;
a52 $= -v * \text{drdp} * \text{dpdkg}$;
a53 $= -v * \text{drdn}$;
a54 $= -v * \text{drdp} * \text{dpdq}$;
a55 $= \beta - r - v * \text{drdp} * \text{dpdv}$;

(*Linearizationofqdot $= \frac{\Delta 2 + \Delta 3}{\Delta 1}$*)
(*Elements for the linearization of qdot*)
kdot $= \frac{q-1}{h_1} - \delta_K * k$;
kgdot $= \text{gdg} + p * \lambda\text{g} * F - \delta_G * \text{kg}$;
vdot $= \beta * v - r * v$;
$\Delta 1 = 1 - \frac{q}{p} * \text{dpdq}$;
$\text{d}\Delta 1\text{dk} = \frac{(-1)*q}{p^2} * (-1) * \text{dpdk} * \text{dpdq}$;
$\text{d}\Delta 1\text{dkg} = \frac{(-1)*q}{p^2} * (-1) * \text{dpdkg} * \text{dpdq}$;
$\text{d}\Delta 1\text{dn} = 0$;
$\text{d}\Delta 1\text{dv} = \frac{(-1)*q}{p^2} * (-1) * \text{dpdv} * \text{dpdq}$;
$\text{d}\Delta 1\text{dq} = \frac{(-1)}{p} * \text{dpdq} + \frac{(-1)*q}{p^2} * (-1) * \text{dpdq} * \text{dpdq}$;
(***)
$\Delta 2 = \frac{q}{p} * (\text{dpdk} * \text{kdot} + \text{dpdkg} * \text{kgdot} + \text{dpdv} * \text{vdot})$;
$\text{d}\Delta 2\text{dk} = (-1) * \frac{q}{p^2} * \text{dpdk} * (\text{dpdk} * \text{kdot} + \text{dpdkg} * \text{kgdot} + \text{dpdv} * \text{vdot}) +$
$\frac{q}{p} * (\text{dpdk} * (-\delta_K) + \text{dpdkg} * (\lambda\text{g} * F * \text{dpdk}) + \text{dpdv} * (-v * \text{drdp} * \text{dpdk}))$;

$$\text{d}\Delta\text{2dkg} = (-1)*\frac{q}{p^2}*\text{dpdkg}*(\text{dpdk}*\text{kdot}+\text{dpdkg}*\text{kgdot}+\text{dpdv}*\text{vdot})+$$
$$\frac{q}{p}*(\text{dpdk}*(0)+\text{dpdkg}*(\lambda\text{g}*F*\text{dpdkg}-\delta_G)+\text{dpdv}*(-v*\text{drdp}*\text{dpdkg}));$$
$$\text{d}\Delta\text{2dn} = \frac{q}{p}*(\text{dpdk}*(0)+\text{dpdkg}*(0)+\text{dpdv}*(-v*\text{drdn}));$$
$$\text{d}\Delta\text{2dq} = \frac{1}{p}*(\text{dpdk}*\text{kdot}+\text{dpdkg}*\text{kgdot}+\text{dpdv}*\text{vdot})+$$
$$(-1)*\frac{q}{p^2}*\text{dpdq}*(\text{dpdk}*\text{kdot}+\text{dpdkg}*\text{kgdot}+\text{dpdv}*\text{vdot})+$$
$$\frac{q}{p}*\left(\text{dpdk}*\left(\frac{1}{h_1}\right)+\text{dpdkg}*(\lambda\text{g}*F*\text{dpdq})+\text{dpdv}*(-v*\text{drdp}*\text{dpdq})\right);$$
$$\text{d}\Delta\text{2dv} = (-1)*\frac{q}{p^2}*\text{dpdv}*(\text{dpdk}*\text{kdot}+\text{dpdkg}*\text{kgdot}+\text{dpdv}*\text{vdot})+$$
$$\frac{q}{p}*(\text{dpdk}*(0)+\text{dpdkg}*(\lambda\text{g}*F*\text{dpdv})+\text{dpdv}*(\beta-r-v*\text{drdp}*\text{dpdv}));$$

(***)

$$\Delta 3 = r*q+\delta_K*q-(1-\text{tw})*\eta*\frac{y}{k};$$
$$\text{d}\Delta\text{3dk} = \text{drdp}*\text{dpdk}*q-(1-\text{tw})*\eta*(\eta-1)*\frac{y}{k^2};$$
$$\text{d}\Delta\text{3dkg} = \text{drdp}*\text{dpdkg}*q-(1-\text{tw})*\eta*\sigma(1-\eta)*\frac{y}{k*\text{kg}};$$
$$\text{d}\Delta\text{3dn} = q*\text{drdn};$$
$$\text{d}\Delta\text{3dq} = q*\text{drdp}*\text{dpdq}+r+\delta_K;$$
$$\text{d}\Delta\text{3dv} = q*\text{drdp}*\text{dpdv};$$

$$\text{a41} = \frac{1}{\Delta 1^2}(\Delta 1*(\text{d}\Delta\text{3dk}+\text{d}\Delta\text{2dk})-\text{d}\Delta\text{1dk}*(\Delta 3+\Delta 2));$$
$$\text{a42} = \frac{1}{\Delta 1^2}(\Delta 1*(\text{d}\Delta\text{3dkg}+\text{d}\Delta\text{2dkg})-\text{d}\Delta\text{1dkg}*(\Delta 3+\Delta 2));$$
$$\text{a43} = \frac{1}{\Delta 1^2}(\Delta 1*(\text{d}\Delta\text{3dn}+\text{d}\Delta\text{2dn})-\text{d}\Delta\text{1dn}*(\Delta 3+\Delta 2));$$
$$\text{a44} = \frac{1}{\Delta 1^2}(\Delta 1*(\text{d}\Delta\text{3dq}+\text{d}\Delta\text{2dq})-\text{d}\Delta\text{1dq}*(\Delta 3+\Delta 2));$$
$$\text{a45} = \frac{1}{\Delta 1^2}(\Delta 1*(\text{d}\Delta\text{3dv}+\text{d}\Delta\text{2dv})-\text{d}\Delta\text{1dv}*(\Delta 3+\Delta 2));$$

(*Linearizing the RER dynamics*)

$$\text{qdot} = \frac{\Delta 2+\Delta 3}{\Delta 1};$$
$$\text{pdot} = \text{dpdk}*\text{kdot}+\text{dpdkg}*\text{kgdot}+\text{dpdq}*\text{qdot}+\text{dpdv}*\text{vdot};$$

$$\text{dpdotdk} = \text{dpdk}*\text{a11}+\text{dpdkg}*\text{a21}+\text{dpdq}*\text{a41}+\text{dpdv}*\text{a51};$$
$$\text{dpdotdkg} = \text{dpdk}*\text{a12}+\text{dpdkg}*\text{a22}+\text{dpdq}*\text{a42}+\text{dpdv}*\text{a52};$$
$$\text{dpdotdn} = \text{dpdk}*\text{a13}+\text{dpdkg}*\text{a23}+\text{dpdq}*\text{a43}+\text{dpdv}*\text{a53};$$
$$\text{dpdotdq} = \text{dpdk}*\text{a14}+\text{dpdkg}*\text{a24}+\text{dpdq}*\text{a44}+\text{dpdv}*\text{a54};$$
$$\text{dpdotdv} = \text{dpdk}*\text{a15}+\text{dpdkg}*\text{a25}+\text{dpdq}*\text{a45}+\text{dpdv}*\text{a55};$$

(*Initial Steady State *)

$$\text{rbbase} = \text{rba}+\text{Exp}[a*\text{pb}*\text{nb}]-1;$$
$$\text{eq1b} = \frac{\text{qb}-1}{h_1} == \delta_K*\text{kb};$$
$$\text{eq2b} = \text{gdgb}+\text{pb}*\lambda\text{gb}*\text{Fb} == \delta_G*\text{kgb};$$
$$\text{eq3b} = \beta*\text{qb}+\delta_K*\text{qb}-(1-\text{twb})*\eta*\alpha*\text{kb}^{\eta-1}*\text{kgb}^{\sigma(1-\eta)}==0;$$
$$\alpha\text{bb} = \mu^{1-\gamma(1-\mu)}*((1-\mu)/(1+\text{tfb}))^{\gamma(1-\mu)};$$
$$\text{chb} = \left(\left(\text{vb}*\text{pb}^{\gamma(1-\mu)-1}\right)\Big/\alpha\text{bb}\right)^{\frac{1}{\gamma-1}};$$

$\text{cfb} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\text{tfb}}\right) * \text{pb}^{-1} * \text{chb};$
$\text{yb} = \alpha * \text{kb}^{\eta} * \text{kgb}^{\sigma(1-\eta)};$
$\text{eq4b} = \text{chb} + \theta * \text{pb}^2 + \frac{\text{qb}^2-1}{2*h_1} + (\text{gdgb} + \text{pb} * \lambda\text{gb} * \text{Fb}) + \frac{h_2}{2}$
$*(\text{gdgb} + \text{pb} * \lambda\text{gb} * \text{Fb})^2 == \text{yb};$
$\text{eq5b} = \text{cfb} + \text{rbbase} * \text{nb} + (1 - \lambda\text{gb}) * \text{Fb} == \theta * \text{pb};$
$\text{eq6b} = \text{rbbase}==\beta;$
$\text{sol1} = \text{FindRoot}[\{\text{eq1b}, \text{eq2b}, \text{eq3b}, \text{eq4b}, \text{eq5b}, \text{eq6b}\}, \{\text{qb}, 1\},$
$\{\text{kb}, 0.1\}, \{\text{kgb}, 2.3\}, \{\text{vb}, 0.6\}, \{\text{pb}, 0.6\}, \{\text{nb}, 0.6\}]$
$\text{qb} = \text{qb}/.\text{sol1}[[1]];$
$\text{kb} = \text{kb}/.\text{sol1}[[2]];$
$\text{kgb} = \text{kgb}/.\text{sol1}[[3]];$
$\text{vb} = \text{vb}/.\text{sol1}[[4]];$
$\text{pb} = \text{pb}/.\text{sol1}[[5]];$
$\text{nb} = \text{nb}/.\text{sol1}[[6]];$
$\text{cb} = \left(\frac{1-\mu}{\mu}\right)^{1-\mu}(1/(1+\text{tfb}))^{1-\mu} * \text{pb}^{\mu-1} * \text{chb};$

(*After Shock Steady State *)
$r = \text{rba} + \text{Exp}[a * p * n] - 1;$
$\text{eq1} = \frac{q-1}{h_1} == \delta_K * k;$
$\text{eq2} = \text{gdg} + p * \lambda\text{g} * F == \delta_G * \text{kg};$
$\text{eq3} = \beta * q + \delta_K * q - (1 - \text{tw}) * \eta * \alpha * k^{\eta-1} * \text{kg}^{\sigma(1-\eta)}==0;$
$\alpha\text{b} = \mu^{1-\gamma(1-\mu)} * ((1-\mu)/(1+\text{tf}))^{\gamma(1-\mu)};$
$\text{ch} = \left(\left(v * p^{\gamma(1-\mu)-1}\right)/\alpha\text{b}\right)^{\frac{1}{\gamma-1}};$
$\text{cf} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\text{tf}}\right) * p^{-1} * \text{ch};$
$y = \alpha * k^{\eta} * \text{kg}^{\sigma(1-\eta)};$
$\text{eq4} = \text{ch} + \theta * p^2 + \frac{q^2-1}{2*h_1} + (\text{gdg} + p * \lambda\text{g} * F) + \frac{h_2}{2} * (\text{gdg} + p * \lambda\text{g} * F)^2 == y;$
$\text{eq5} = \text{cf} + r * n + (1 - \lambda\text{g}) * F == \theta * p;$
$\text{eq6} = r==\beta;$

$\text{sol2} = \text{FindRoot}[\{\text{eq1}, \text{eq2}, \text{eq3}, \text{eq4}, \text{eq5}, \text{eq6}\}, \{q, 1\}, \{k, 0.1\},$
$\{\text{kg}, 2.3\}, \{v, 0.6\}, \{p, 0.6\}, \{n, 0.6\}]$
$q = q/.\text{sol2}[[1]];$
$k = k/.\text{sol2}[[2]];$
$\text{kg} = \text{kg}/.\text{sol2}[[3]];$
$v = v/.\text{sol2}[[4]];$
$p = p/.\text{sol2}[[5]];$
$n = n/.\text{sol2}[[6]];$
$c = \left(\frac{1-\mu}{\mu}\right)^{1-\mu}\left(\frac{1}{1+\text{tf}}\right)^{1-\mu} * p^{\mu-1} * \text{ch};$

MatrixForm[
$M = \{\{\text{a11}, \text{a12}, \text{a13}, \text{a14}, \text{a15}\}, \{\text{a21}, \text{a22}, \text{a23}, \text{a24}, \text{a25}\},$
$\{\text{a31}, \text{a32}, \text{a33}, \text{a34}, \text{a35}\}, \{\text{a41}, \text{a42}, \text{a43}, \text{a44}, \text{a45}\},$

```
{a51, a52, a53, a54, a55}}]

{EE, VV} = Eigensystem[N[M]];
Det[M − μ * IdentityMatrix[5]];
u1 = EE[[1]]; u2 = EE[[2]]; u3 = EE[[3]]; u4 = EE[[4]]; u5 = EE[[5]];
sort1 = Sort[{u1, u2, u3, u4, u5}]
s6 = Select[{u1, u2, u3, u4, u5}, # < 0&];

(*firststableeigenvalue − smallest*)
u11 = s6[[3]]
(*second stable eigenvalue *)
u22 = s6[[2]]
(*thirdstableeigenvalue − largest*)
u33 = s6[[1]]

(*ObtainingA1, A2, A3toexpressthestablesolutionsinterms
ofthenormalizedsmallesteigenvalue*)

(*IMPORTANT : Whatididtogetittoworkwastoswapoutthe
rowinthematrixthatwasgoingtoZEROS!!!!*)
e1 = (u11 − a11) * A1 − a12 * B1 − a13 * C1 − a14 * D1 − a15 * E1 == 0;
e2 = (u22 − a11) * A2 − a12 * B2 − a13 * C2 − a14 * D2 − a15 * E2 == 0;
e3 = (u33 − a11) * A3 − a12 * B3 − a13 * C3 − a14 * D3 − a15 * E3 == 0;

e16 = (u11 − a22) * B1 − a21 * A1 − a23 * C1 − a24 * D1 − a25 * E1 == 0;
e17 = (u22 − a22) * B2 − a21 * A2 − a23 * C2 − a24 * D2 − a25 * E2 == 0;
e18 = (u33 − a22) * B3 − a21 * A3 − a23 * C3 − a24 * D3 − a25 * E3 == 0;

e4 = (u11 − a33) * C1 − a31 * A1 − a32 * B1 − a34 * D1 − a35 * E1 == 0;
e5 = (u22 − a33) * C2 − a31 * A2 − a32 * B2 − a34 * D2 − a35 * E2 == 0;
e6 = (u33 − a33) * C3 − a31 * A3 − a32 * B3 − a34 * D3 − a35 * E3 == 0;
e7 = (u11 − a44) * D1 − a41 * A1 − a42 * B1 − a43 * C1 − a45 * E1 == 0;
e8 = (u22 − a44) * D2 − a41 * A2 − a42 * B2 − a43 * C2 − a45 * E2 == 0;
e9 = (u33 − a44) * D3 − a41 * A3 − a42 * B3 − a43 * C3 − a45 * E3 == 0;
e10 = (u11 − a55) * E1 − a51 * A1 − a52 * B1 − a53 * C1 − a54 * D1 == 0;
e11 = (u22 − a55) * E2 − a51 * A2 − a52 * B2 − a53 * C2 − a54 * D2 == 0;
e12 = (u33 − a55) * E3 − a51 * A3 − a52 * B3 − a53 * C3 − a54 * D3 == 0;
e13 = A1 + A2 + A3 == (kb − k);
e14 = B1 + B2 + B3 == (kgb − kg);
e15 = C1 + C2 + C3 == (nb − n);

s3 = Solve[{e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12,
e13, e14, e15}, {A1, A2, A3, B1, B2, B3, C1, C2, C3, D1,
D2, D3, E1, E2, E3}];
```

$\mathrm{A1} = \mathrm{A1}/.\mathrm{s3}[[1]]; \mathrm{A2} = \mathrm{A2}/.\mathrm{s3}[[1]]; \mathrm{A3} = \mathrm{A3}/.\mathrm{s3}[[1]];$
$\mathrm{B1} = \mathrm{B1}/.\mathrm{s3}[[1]]; \mathrm{B2} = \mathrm{B2}/.\mathrm{s3}[[1]]; \mathrm{B3} = \mathrm{B3}/.\mathrm{s3}[[1]];$
$\mathrm{C1} = \mathrm{C1}/.\mathrm{s3}[[1]]; \mathrm{C2} = \mathrm{C2}/.\mathrm{s3}[[1]]; \mathrm{C3} = \mathrm{C3}/.\mathrm{s3}[[1]];$
$\mathrm{D1} = \mathrm{D1}/.\mathrm{s3}[[1]]; \mathrm{D2} = \mathrm{D2}/.\mathrm{s3}[[1]]; \mathrm{D3} = \mathrm{D3}/.\mathrm{s3}[[1]];$
$\mathrm{E1} = \mathrm{E1}/.\mathrm{s3}[[1]]; \mathrm{E2} = \mathrm{E2}/.\mathrm{s3}[[1]]; \mathrm{E3} = \mathrm{E3}/.\mathrm{s3}[[1]];$

Print["A1=", A1]; Print["A2=", A2]; Print["A3=", A3];
Print["B1=", B1]; Print["B2=", B2]; Print["B3=", B3];
Print["C1=", C1]; Print["C2=", C2]; Print["C3=", C3];
Print["D1=", D1]; Print["D2=", D2]; Print["D3=", D3];
Print["E1=", E1]; Print["E2=", E2]; Print["E3=", E3];

$\mathrm{kt}[\mathrm{t\_}] := \mathrm{A1} * \mathrm{Exp}[\mathrm{u11} * t] + \mathrm{A2} * \mathrm{Exp}[\mathrm{u22} * t] + \mathrm{A3} * \mathrm{Exp}[\mathrm{u33} * t] + k;$
$\mathrm{kgt}[\mathrm{t\_}] := \mathrm{B1} * \mathrm{Exp}[\mathrm{u11} * t] + \mathrm{B2} * \mathrm{Exp}[\mathrm{u22} * t] + \mathrm{B3} * \mathrm{Exp}[\mathrm{u33} * t] + \mathrm{kg};$
$\mathrm{nt}[\mathrm{t\_}] := \mathrm{C1} * \mathrm{Exp}[\mathrm{u11} * t] + \mathrm{C2} * \mathrm{Exp}[\mathrm{u22} * t] + \mathrm{C3} * \mathrm{Exp}[\mathrm{u33} * t] + n;$
$\mathrm{qt}[\mathrm{t\_}] := \mathrm{D1} * \mathrm{Exp}[\mathrm{u11} * t] + \mathrm{D2} * \mathrm{Exp}[\mathrm{u22} * t] + \mathrm{D3} * \mathrm{Exp}[\mathrm{u33} * t] + q;$
$\mathrm{vt}[\mathrm{t\_}] := \mathrm{E1} * \mathrm{Exp}[\mathrm{u11} * t] + \mathrm{E2} * \mathrm{Exp}[\mathrm{u22} * t] + \mathrm{E3} * \mathrm{Exp}[\mathrm{u33} * t] + v;$

$\mathrm{pt}[\mathrm{t\_}] := p + \mathrm{dpdotdk} * (\mathrm{kt}[t] - k) + \mathrm{dpdotdkg} * (\mathrm{kgt}[t] - \mathrm{kg}) +$
$\mathrm{dpdotdv} * (\mathrm{vt}[t] - v) + \mathrm{dpdotdq} * (\mathrm{qt}[t] - q);$
$\mathrm{cht}[\mathrm{t\_}] := \left(\frac{\mathrm{vt}[t]}{\alpha \mathrm{b}}\right)^{\frac{1}{\gamma-1}} * \mathrm{pt}[t]^{\frac{\gamma(1-\mu)-1}{\gamma-1}};$
$\mathrm{cft}[\mathrm{t\_}] := \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1+\mathrm{tf}}\right) * \mathrm{pt}[t]^{-1} * \mathrm{cht}[t];$
$\mathrm{ct}[\mathrm{t\_}] := \left(\frac{1-\mu}{\mu}\right)^{1-\mu}\left(\frac{1}{1+\mathrm{tf}}\right)^{1-\mu} * \mathrm{pt}[t]^{\mu-1} * \mathrm{cht}[t];$
$\mathrm{yt}[\mathrm{t\_}] := \alpha * \mathrm{kt}[t]^{\eta} * \mathrm{kgt}[t]^{\sigma(1-\eta)};$

(*Welfare Calculations*)
$W[\mathrm{t\_}] := \frac{\mathrm{ct}[t]^{\gamma} * e^{-\beta t}}{\gamma};$
$\mathrm{Wb}[\mathrm{t\_}] := \frac{\mathrm{cb}^{\gamma} e^{-\beta t}}{\gamma};$
$\mathrm{IW}[\mathrm{t\_}] = \frac{\mathrm{ct}[t]^{\gamma}}{\gamma};$
$\mathrm{IWb}[\mathrm{t\_}] = \frac{\mathrm{cb}^{\gamma}}{\gamma};$
$\mathrm{A11} = \mathrm{NIntegrate}[W[t], \{t, 0, 100\}];$
$\mathrm{A01} = \mathrm{NIntegrate}[\mathrm{Wb}[t], \{t, 0, 100\}];$

Appendix C

Appendices to Chapter 3

C.1 Aid Recipient Countries

Argentina	Mexico	Nicaragua
Bahrain	Mongolia	Niger
Barbados	El Salvador	Pakistan
Belarus	Estonia	Panama
Belize	Guatemala	Paraguay
Bolivia	Honduras	Peru
Brazil	Hungary	Romania
Bulgaria	India	Russia
Burkino Faso	Indonesia	Rwanda
Burundi	Iran	Senegal
Cameroon	Jamaica	Singapore
Central African Republic	Kazakhstan	Slovenia
Chad	Kuwait	Sri Lanka
Chile	Latvia	Syria
Colombia	Lesotho	Tajikistan
Congo- Republic of	Malaysia	Thailand
Costa Rica	Mali	Togo
Cote d'Ivoire	Malta	Trinidad and Tobago
Croatia	Mauritania	Tunisia
Cyprus	Mauritius	Turkey
Dpminican Republic	Morocco	Uruguay
Egypt	Nepal	Venezuela

C.2 Variable Descriptions as obtained from CKG Study

- REER (Real Effective Exchange Rate) Index: Ratio of an index of a currency's period average exchange rare to a weighted geometric average of exchange rates for the currencies of selected countries and the euro area, and adjusted for relative movements in national price or cost indicators of the home country, selected countries, and the euro area. Base year = 2000. Source: International Financial Statistics INS (IMF).

- Official rate (ae): End of period national currency units per U.S. dollar. Source: International Financial Statistics (IMF).
- Terms of Trade: Index of a country's exports of goods and services with respect to imports. Source: International Financial Statistics (IMF).
- Trade Balance: The sum of exports plus imports as a % of GDP. Source: International Financial Statistics (IMF).
- GDP per capita: Author's calculation. National GDP deflated by the GDP deflator divided by population. Base year = 2000. Source: International Financial Statistics (IMF).
- Money (M1): Currency in circulation plus demand deposits or checking account balances as a % of GDP. Source: International Financial Statistics (IMF).
- Total domestic government expenditure: Total government expenditure as a % of GDP. Source: Chatterjee, Giuliano & Kaya (2009), Government Financial Statistics (IMF).
- Public Investment Expenditure: Economic Affairs and Services components of Expenditure Classification as a % of GDP. Chatterjee, Giuliano & Kaya (2009) calculation. Source: Expenditure Classification, Government Financial Statistics (IMF).
- Social Infrastructure Expenditure: General Public Services, Education, Health, Social Security, Housing and Recreation, and Culture components of Expenditure Classification as a % of GDP. Chatterjee, Giuliano & Kaya (2009) calculation. Source: Expenditure Classification, Government Financial Statistics (IMF).
- Public Non-investment Expenditure. Total Domestic Government Expenditure minus Social Infrastructure Expenditure and Public Investment Expenditure. Chatterjee, Giuliano & Kaya (2009) calculation. Source: Expenditure Classification, Government Financial Statistics (IMF).
- Total Aid: Total Aid as a % of GDP. Source: Chatterjee, Giuliano & Kaya (2009), Credit Reporting System, International Development Statistics database, Organization for Economic Co-operation and Development.
- Public Investment Aid: Economic Infrastructure and Production Components of Credit Reporting System (CRS) Aid Activities as a % of GDP. Source: Chatterjee, Giuliano & Kaya (2009), Credit Reporting System, International Development Statistics database, Organization for Economic Co-operation and Development.
- Social Infrastructure Aid: Social Infrastructure Component of CRS Aid Activities as a % of GDP. Source: Chatterjee, Giuliano & Kaya (2009), Credit Reporting System, International Development Statistics database, Organization for Economic Co-operation and Development.

- Public Non-investment Aid: Total Aid minus Social Infrastructure Aid and Public Investment Aid as a % of GDP. Source: Chatterjee, Giuliano & Kaya (2009), Credit Reporting System, International Development Statistics database, Organization for Economic Co-operation and Development.

- Consumer Price Index: Base year = 2000. Source: International Financial Statistics (IMF).

- Literacy Rate: Source: Chatterjee, Giuliano & Kaya (2009), World Bank National Accounts Data and OECD National Accounts.

- Dependency Ratio: Source: Chatterjee, Giuliano & Kaya (2009), World Bank National Accounts Data and OECD National Accounts.

- Infant Mortality Rate: Source: Chatterjee, Giuliano & Kaya (2009) U.S. Census Bureau's International Database.