COMMUNITY BUILDING IN MATHEMATICS PROFESSIONAL DEVELOPMENT

by

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(Under the Direction of Denise S. Mewborn)

ABSTRACT

This study considers the practicality of creating a mathematical community of practice among mathematics middle school teachers in a 14-week PD experience. The concept of creating a community of practice is a recent idea in education. Many professional developers and teachers have experiences that indicate developing some level of community in the classes they teach leads to greater student success. There is little written, however, about what demands are placed on the facilitator who attempts to create a community of practice. In addition, little is written about the possibility of a community of practice developing in a short period of time – though the time frame of this PD is consistent with many PD experiences in the United States. In this study, the design of the PD included focus on mathematics content knowledge and active engagement in high cognitive demand tasks with rational number concepts. Both are common recommendations for effective PD. This study found that a community of practice could be developed in this setting. Although no data were collected on the path of the community after the PD, this study provides an example of success of community of practice development within a PD setting with a facilitator intent on not only improving teachers' understanding of rational numbers but attempting to cultivate a community. This study found that development of a community of practice facilitated mathematics learning in this PD.

INDEX WORDS: Mathematics Education, Professional Development, Community of Practice, Teacher Learning, Rational Numbers

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DEDICATION

I dedicate this work to my grandparents, Clifford F. & Evelyn M. Pierce and Howard A. & Beatrice A. Eriksen, for modeling that hard work, perseverance, and dedication to career and family make life meaningful.

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CHAPTER 1

INTRODUCTION

As evidenced by increased attention of policy makers and state funding efforts, professional development (PD) has become an important aspect of the initiative to improve the quality of our nation's teachers. Research and practical experiences with PD have resulted in lists of guidelines for the structure of these experiences. Guskey (2003) compared 13 of these lists to determine the consistency of their creation and content. One of the common characteristics was regarding "promotion of collegiality and collaborative exchange" (Guskey, p. 12). Wilson and Berne (1999) analyzed literature on PD and noted a common trend, across disciplines, of building community within various PD experiences. Despite the prevalence of this finding, there is minimal research on how to build a community in PD. In addition, teacher educators must consider how an explicit goal of building community interfaces with other goals of PD such as developing teachers' content knowledge.

The National Council of Teachers of Mathematics (NCTM) has been promoting a vision of the ideal mathematics classroom. Teachers should, according to NCTM (2000), "establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create" (p. 18). This environment parallels many of the guidelines for what should occur during PD. NCTM's goal for students is to learn mathematics with understanding. This learning "can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others" (p. 21). Within the context of the mathematics classroom, the connection between learning and community appears to be strong. Goos (2004) provided a case study of a secondary teacher who successfully supported a community of mathematical inquiry in his classroom. Thus, it is not surprising that when the goal of PD is to help teachers learn content with understanding that some attention should be paid to developing community. This connection in PD, however, has not been well researched.

For teachers to change their classrooms and their teaching practices to be aligned with the NCTM (2000) standards requires "that teachers and students develop new ways to engage together around mathematical ideas—specifically, it requires new forms of mathematical discourse" (Franke, Kazemi, & Battey, 2007, p. 249). Mathematical discourse includes norms and building relationships in teachers' classrooms. Thus, teachers, however, are being encouraged to form communities with their students. Few teachers have experienced this type of learning environment themselves. Sowder (2007) wrote, "Education reformers realize that instructional improvement and increased student achievement depend on the professional development of teachers" (p. 159). PD must, then, provide experiences for teachers similar to those they are expected to be cultivating in their classrooms. Borko and Putnam (1995) noted successful PD provides "opportunities for teachers to construct knowledge of subject matter and pedagogy in an environment that supports and encourages risk taking and reflection" (p. 59).

In response to the many recommended characteristics of PD, including building community, Wilson and Berne (1999) stated, "These principles and beliefs seem reasonable. Yet, we know as little about what teachers learn in these kinds of forums as we do about what teachers learn in traditional staff development and in-service" (p. 176). After reviewing the literature on PD, Wilson and Berne concluded that despite progress with research on teacher learning and PD, few studies have examined what content teachers learn in a PD community. Wayne, Yoon, Zhu, Cronen, and Garet (2008) stated "The literature also reveals an informal consensus about the features PD programs should have in order to make them effective. But the evidence base for this consensus is weak" (p. 477). Thus, research needs to be conducted on what teachers learn in the context of PD designed with some of the recommended guidelines, including community, as the focus.

The shortage of studies of this nature may be due to the methods used to understand what teachers learn. Commonly in research on PD, teachers are asked to fill out surveys or are interviewed about their experiences. These data are then used to explain what teachers learned through the experience. For example, Garet, Porter, Desimone, Birman, and Yoon (2001) examined characteristics of PD and self-reported change in teachers' knowledge and practice. Cohen and Hill (2000) studied how teachers' practices changed compared with their opportunities to learn (PD) and collected their data through survey items. Both of these studies are valuable to the field because they reported data from a large number of teachers. These results are limited, however, because the data were all self reported. There is a need for examining teacher learning through the written materials teachers produce in PD and in the interactions of these teachers with the teacher educator who is facilitating the PD experience.

Personal Experience

My interest in community and PD resulted from my experiences as a teacher and as a teacher educator. As I interacted with teachers, as a peer or as the professional developer, some groups tended to work better together than others. In my experience, if the instructor openly discussed norms, expectations, and acceptable behavior, on some level, the group seemed to work together better, and the groups that worked together tended to learn more mathematics. Thus, I chose to study the development of community in the context of a PD program designed to enhance teachers' content knowledge.

The need to examine PD in this way became clear to me as I became a professional developer through my assistantship work. As a professional developer, I had experiences with groups of teachers (sometimes from the same district or at the same grade level) who were very different. The influence of the school district on teachers' engagement in and perceived usefulness of the workshop appeared to be extremely critical to their learning. Additionally, how the group worked together and communicated was very influential on my perceptions of the learning of the teachers in that group. These experiences and observations led me to this research focus. My hypothesis was that the importance of community is intimately connected to the learning of the participants. Thus, by designing a study to examine these questions, I tested not only the theoretical descriptions about communities of practice and learning but also my hypothesis based on my practical experiences.

In this study, the PD setting was an InterMath rational number (IM) course designed for a National Science Foundation (NSF)-funded project called *Does It Work?: Building Methods for Understanding Effects of Professional Development* (DiW). This

14-week course was offered to middle school mathematics teachers in an urban southern school district in the United States. The goal of IM was to increase teachers' mathematical understanding through engagement with open-ended tasks. Various technologies were used as ways to represent and communicate mathematical ideas. In the rational number course, teachers worked on concepts relating to operations with decimals and fractions, in addition to direct and inverse proportion.

In this sociocultural study, I examined video, transcripts, and written work from an IM course I taught. Using the frameworks of communities of practice (Wenger, 1998) and transformation of participation (Rogoff, 1997), I analyzed the teachers as a whole group and individually to investigate the culture, norms, discourse, and learning of the group and the changes that took place during the IM course. The study was guided by the following questions:

a. What are the IM instructor's intentions and actions related to developing a community of practice within the group?
b. What characteristics of a community of practice, as defined by Wenger (1998), does the group exhibit? How do these community characteristics change during the course?
c. What learning occurred?

CHAPTER 2

LITERATURE AND THEORECTICAL FRAMEWORK

Literature Review

In this section, I review literature on professional development (PD), teachers' knowledge of rational numbers, tasks, and community. As previously stated, there are many published guidelines on PD. I review literature relating to three common recommendations for PD: having a content focus, engaging teachers in active learning, and maximizing contact hours. My review of the literature on teachers' knowledge of rational numbers includes studies of both inservice and preservice teachers. Research on tasks is reported, because in this study, teachers engaged in the mathematics via tasks on rational numbers. Finally, my review of literature on the development of community helps place this study in the field.

There is a sizeable body of research on PD, and Borko (2004) provided a framework for organizing this research into three phases. Phase 1 research focuses on a single site and its goal is "to provide evidence that a professional development program can have a positive impact on teacher learning" (p. 5). The other two phases broaden the research to include multiple sites, facilitators, and PD designs. The majority of research on PD falls under Phase 1 with a few studies in Phases 2 and 3. None of the phases has been extensively explored. The study reported here was a Phase 1 research study with the goal of exploring the challenges faced by the facilitator and teachers in building a community of practice while investigating mathematical concepts. Investigating teacher learning in this climate was also a goal.

Borko (2004) discussed the concept of community in her description of Phase 1 research. She noted “strong professional learning communities can foster teacher learning and instructional improvement” (p. 6). She went on to describe two research programs– the Community of Teacher Learners project (Grossman, Wineburg, & Woolworth, 2001) and Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) (Stein, Smith, Henningsen, Silver, 2000). The Community of Teacher Learners project focused on a group of high school English and history teachers, and the purpose of the study was to attempt to create a community of learners and document the progress. From this experience Grossman et al. (2001) developed a model of community development including three phases–beginning, evolving, and mature. The QUASAR project focused on improving instruction in middle school mathematics classrooms through emphasizing the cognitive demand of tasks (Stein et al., 2000). Through the QUASAR project a task analysis guide for classifying the cognitive demand of tasks was developed. Stein, Silver, and Smith (1998) found “opportunities were available to QUASAR teachers to participate in collaborative working arrangements-forms of teacher support that represented departures from the conventional forms of teacher education” (p. 21). The results of the study indicated teachers began to value community because “something important was happening between and among the teachers” (Stein, Grover, & Henningsen, 1996, p. 28). These two programs not only supported the value of learning communities but also showed “the development of teacher communities is difficult and time-consuming work” (Borko, 2004, p. 7). By making community development one of my instructional goals, I documented the challenges faced by a group of teachers attempting to achieve this goal in a semester-long course.

There has been an extensive accumulation of research and anecdotal evidence on the characteristics of effective PD. Thus far, the research has shown the importance of PD programs having a content focus, engaging the teachers in active learning, and having a high number of contact hours.

Content Focus

Many studies of and guidelines for PD include the importance of focusing on specific content. For example, Sowder (2007) listed one goal of PD as developing content knowledge and pedagogical content knowledge. Similarly, Garet et al. (2001) reported that PD focusing on content knowledge, in this case mathematics, “is more likely to produce enhanced knowledge and skills” (p. 935). Guskey (2003) analyzed a variety of published recommendations for successful PD and found “the most frequently mentioned characteristic of effective professional development is enhancement of teachers’ content and pedagogic knowledge” (p. 9). Hill, Rowan and Ball (2005) shared results of an assessment of content knowledge and student gains and noted “teachers’ content knowledge for teaching positively predicted student gains in mathematics achievement” (p. 399). Thus, this research supported the notion that focusing on improving teachers’ content knowledge in PD has the potential to positively impact student learning. Cohen and Hill (2000) used a survey instrument to learn about PD opportunities for elementary teachers relating to state policy changes and if teachers reported practice related to the type of PD opportunities they participated in. They found empirical evidence to support the importance of content knowledge as the focus of PD in changing teachers’ practices, noting "It seems to help to change mathematics teaching practices if teachers have even more concrete, topic-specific learning opportunities" (Cohen and Hill, p. 312). Kennedy (1999) conducted an analysis of various forms of PD with a variety of structures. She found "that programs that focus on subject matter knowledge and on student learning of particular subject matter are likely to have larger positive benefits for student learning than programs that focus mainly on teaching behaviors" (p. 4).

When PD has a more generic focus on teaching (as opposed to a more specialized, specific focus on mathematics content) not only do teachers' practices remain the same, but also student learning is not enhanced. Gearhart et al. (1999) explored three forms of PD and their impact on teachers' practices and opportunities for students to learn. The teacher participants were separated into three groups called Integrating Mathematics Assessment (IMA), Support, and Traditional. Teachers in the IMA and Support groups participated in PD that addressed the content of two replacement units designed to support students' development of conceptual understanding and encourage problem solving. The IMA group experienced PD designed to enhance teacher knowledge about content, students' understanding of the same content, and assessment. The Support teachers participated in a community meeting and met with a facilitator to talk about the two replacement units, including the challenges and successes of implementation. The Support teachers set their own agendas for meetings. The Traditional group received no PD and used traditional skill-based textbooks to teach fractions. The findings showed the IMA and Support teachers "were providing students greater opportunities to engage with conceptual issues" (p. 306) than the Traditional teachers. In addition, there was a difference between the IMA and Support group in that the Support group "had relatively lower ratings for providing students opportunities to learn how numeric representations are linked to fractions concepts" (p. 306). Thus, the PD that focused on increasing teacher knowledge showed the greatest results in classrooms because students were provided with more opportunities to learn. The PD that focused on teachers discussing their experiences teaching a unit (i.e., focusing more on pedagogy without much provided structure) was not as successful in changing classroom practice.

Cohen and Hill (2000) found similar results in that teachers reported less change in their teaching practice (i.e., becoming more reform-oriented) when their PD included more generalized teaching strategies as a focus. More importantly, the workshops they examined that were not mathematics-focused did "not directly and independently affect student performance" (Cohen & Hill, 2000, p. 326).

Kennedy's (1999) analysis of various PD experiences supported the importance of content knowledge not only for teachers but for their students as well. Students of teachers who attended workshops focusing on specific teaching practices underperformed on assessments on reasoning and problem solving while their performance on basic skills assessment was comparable to teachers who attended PD focused on specific mathematical content. Thus, the difference between focusing on mathematical knowledge and pedagogical practices was evident in student performance on problem-solving assessments as well as in classroom practices.

Active Learning

A common theme in Wilson and Berne's (1999) review of PD literature was the need for PD to help activate knowledge in the participants, not to deliver knowledge. Professional developers "engage them [teachers] as learners in the area that their students will learn in but at a level that is more suitable to their own learning" (Wilson & Berne, 1999, p. 194). Garet et al. (2001) examined the opportunities for teachers to engage in active learning in PD through survey data and found "active learning also is related to enhanced knowledge and skills" (p. 933) of teachers; however, the effect was not very strong. Guskey (2003) noted a common characteristic in the guidelines he analyzed as "the promotion of collegiality and collaborative exchange" (p. 12). This characteristic included working together, sharing ideas and strategies, and becoming reflective practitioners–all things that would be classified as active learning. Thus, active learning is seen as an essential characteristic in effective PD.

For example, in a PD experience studied by Kazemi and Franke (2004), a group of teachers met monthly to discuss their students' responses to a problem provided by the facilitator, using students' written work as a springboard for discussion. The facilitator's role was to press teachers to focus on student strategies and propose strategies the teachers did not suggest. The research used a similar framework to this study (described later in Chapter 2) and was concerned with exploring the learning of the teachers as a group. Learning was defined as shifts in participation (Rogoff, 1997). Two major shifts were apparent through the analysis of the transcripts of the meetings, student work, written teacher reflections, and teacher interviews. One shift pertained to attending to the details of children's thinking, and the other related to identifying possible instructional paths to support the mathematics. For example, "teachers began to be more interested in how to create opportunities for students to generate their own efficient strategies" (p. 226). The use of student work was imperative to the shifts the researchers found; however, using student work alone was not sufficient. Kazemi and Franke argued that teachers made their classroom practices explicit in the meetings and the student work was generated in the teacher's classrooms. Another observation was the importance of the role of the facilitator. The facilitator was able to help guide the direction of the meetings and assist the group in maintaining a focus on students' mathematical work. Using the whole group as the unit of analysis, the researchers analyzed how the teachers supported each other's mathematical and pedagogical development, how the teachers examined student work from colleagues' classrooms, and how and when teachers asked each other for help. The researchers concluded these teachers were beginning to develop norms about what it means to teach and learn from each other based on their examination of student work. This development of norms was related to the community building of the group and their active engagement in examining students' mathematical work. Active learning lends itself to creating a community of practice as teachers are given opportunities to explain, compare, and contrast mathematical strategies for solving tasks as a group. Without active learning, it would be meaningless to explore whether a community of practice formed in PD.

Contact Hours

Many studies have shown the need to meet with teachers multiple times to have the greatest impact on teacher learning and change in teaching practice. Garet et al. (2001) found time span and contact hours were both important features in PD because both of these measures had a positive influence on opportunities for active learning and focus on content knowledge. Garet et al. found "professional development is likely to be of higher quality if it is both sustained over time and involves a substantial number of hours" (p. 933). In Guskey's analysis (2003), he found 10 of the 13 sets of guidelines for effective PD included the need for sufficient time. Some lists, however, did not support the idea that the influence of time was significant. Guskey concluded "although effective professional development clearly requires time, it also seems clear that such time must be well organized, carefully structured, and purposefully directed" (p. 12). Boyle, While, and Boyle (2004) conducted an extensive study based on survey data in England. They noted

> traditional approaches to professional development, such as short workshops or conference attendance, do foster teachers' awareness or interest in deepening their knowledge and skills. However, these approaches to PD appear insufficient to foster learning which fundamentally alters what teachers teach or how they teach. (p. 47)

Additionally, Banilower, Heck, and Weiss (2007) found the effects of PD to be the greatest when contact hours were high. There results suggested that if contact time was between 32 and 80 hours teachers would gain the most from the PD. Thus, time seems to be an integral piece of PD design.

On the other hand, time spent in PD cannot be divorced from the quality and content of that PD. Hill (2004) examined 13 different PD sessions and described a conference session that "emphasized students' ability to develop and use sensible strategies for multi-digit operations" (p. 226) as a counterexample to the common assumption that PD must occur over a sustained period of time in order to be effective. The session was two hours long and a one-time meeting. Hill commented "those 2 hours contained some of the most substantial learning opportunities I observed" (p. 236). By watching video of students using novel strategies to solve problems and asking teachers to discuss these strategies with other participants, the session leader encouraged teachers to reflect on the mathematics. Hill noted that although teachers talked about student strategies in small groups, "the entire group did not discuss it at length, nor did the provider address the generalizability of the solution method" (p. 226). Despite this comment, Hill thought this was a counterexample to the notion that time is an important piece of PD design. One could argue, however, if the session leader had more time with the teachers both of these key activities would have taken place. But the important message is she did not have the time, and thus the teachers were not able to completely see (or be exposed to) the full picture of what the students were doing. Hill did not conduct any follow-up research, so it is unclear which of the PD experiences she reviewed was the most effective in terms of the PD meeting its goals and had the greatest impact on teachers who participated. Thus, although this study provided a counterexample to the idea that time is essential, it was not a strong counterexample.

Another counterexample was provided by Kennedy's (1999) analysis. She found total contact hours were unrelated to student outcomes. The results were mixed as to whether distributed time was related to student outcomes. Her conclusion was "structural features alone provide no guarantee of improved teacher learning or of eventual benefit to students" (p. 6). According to Kennedy, the importance should be placed first on the content and then on the structure of PD.

The design of IM, the PD in this study, was consistent with the recommendations from the literature as it was focused on increasing teachers' mathematical knowledge for teaching rational numbers. The format of IM was to actively engage teachers with the mathematics by proposing tasks for them to work on over a sustained period of time.

Teacher Knowledge of Rational Numbers

Because the focus of IM was on rational number concepts, I looked at the literature on teachers' knowledge of these concepts. Surprisingly, there were few studies on teachers' understanding of rational number concepts. In this section, I review studies on inservice and preservice teachers' mathematical understanding related to rational numbers.

Inservice teachers. The studies of mathematical knowledge for teaching rational number concepts all reported that teachers, at least in the United States, did not have the knowledge needed to handle student questions, student-suggested algorithms, or student-generated representations. Izsák (2008) compared two middle school teachers' use of levels of units when teaching about fraction multiplication and addition. Ms. Archer, one of the teachers, demonstrated an understanding of two levels of units. In her instruction, there were opportunities for her to discuss three levels of units; however, she struggled and failed to do so. Ms. Reese, the other teacher in the study, used three levels of units in her instruction on addition and subtraction. Despite demonstrating an understanding of levels of units, she was challenged by students' ideas involving nested units. Thus, teacher knowledge did not directly translate into classroom practice. Additionally, the two teachers used drawings differently in their classrooms, which was connected to their unit structures. Izsák concluded, "the two case studies suggest that teachers' capacity to produce three levels of units is necessary but not sufficient for interpreting and evaluating the variety of ways that students might begin to evidence multi-level structures when using drawings" (p. 139).

In another study from the same project, Izsák, Tillema, and Tunc-Pekkan (2008) showed data from Ms. Reese's lessons on addition and subtraction of fractions on number lines and a student's interpretation of these lessons. These researchers suggested teachers need to focus on particular details such as language and partitioning when teaching fraction addition. Ms. Reese tended to refer to fractions as "amounts," and this added to a misunderstanding of referent unit for at least one of her students. In addition, her decision to most often partition number lines from left to right, rather than recursively, did not support proportional reasoning among her students. Proportional reasoning has the potential to lead to three levels of units and is important for teacher and student understanding of rational numbers. Thus, seemingly minor decisions related to her teaching of fraction addition impacted her students' understanding of the concept.

Armstrong and Bezuk (1995) documented challenges middle school teachers faced as they explored multiplication and division of fractions in a PD setting. They noted

> …it is a challenge at best and impossible at worst to expand one's thinking about a mathematical operation beyond rotely learned rules if those rules have become automatized. The gravity of this situation is especially critical for teachers as they try to override their own rule-based educations in order to relearn those topics that they want to teach conceptually to their students. (p. 87)

The teachers struggled with their beliefs about mathematics, their understanding of mathematics, and their understanding of students' thinking about mathematics. The aspects they found particularly difficult in writing a word problem for a fraction division sentence were understanding partitive and measurement division, recognizing whether a word problem is multiplication or division, needing practice in writing word problems, and solving word problems involving fractions generally. Another challenge teachers faced with understanding fraction operations was identifying the referent unit for each number in a fraction number sentence.

Post, Harel, Behr, and Lesh (1988) conducted research with inservice teachers who were assessed on “what we believe to be the conceptual underpinnings of rational number topics for grades 4, 5, and 6” (p. 203). Unfortunately, teachers did not perform well on the assessment with the overall means scores below 70%. The teacher assessment was created based on the researchers’ work with elementary children, which made the results more disheartening. Teachers were also asked to solve a word problem and discuss how they would explain it to a student. Again, the results showed these teachers did not have the mathematical knowledge for teaching rational numbers necessary to be supportive of students.

Ma (1999) described how U.S. and Chinese teachers solved and wrote a story problem for $1\frac{3}{4} \div \frac{1}{2}$. While 43% of the U.S. teachers could calculate the answer to division problem correctly, none of the U.S. teachers was able to explain the reasoning behind the procedure for dividing fractions. In contrast, the Chinese teachers all correctly computed the answer and were able to explain multiple ways of thinking about the problem. Whereas U.S. teachers expressed anxiety about fractions, the Chinese teachers appeared to enjoy discussing the problem. Another difference between the two groups was that the Chinese teachers were aware of the need for understanding multiplicative reasoning when reasoning about fraction division and the U.S. teachers were not. Ma concluded the education of the Chinese teachers encouraged making connections among mathematical concepts and thinking about how to develop understanding.

Preservice teachers. Ball (1990) described preservice elementary and secondary teachers' struggles with the concept of division of fractions. Only 5 out of 19 participants were able to provide an appropriate representation for the problem $1\frac{3}{4} \div \frac{1}{2}$ (the same task Ma used later in her research). Interestingly, the preservice teachers focused on the fact that the problem involved fractions and did not address the concept of division. Ball suggested this struggle came from a dependence on partitive division. Ball's study supported the idea that teachers depend on procedural knowledge and rules rather than the "underlying meanings" (p. 141). This was a recurring theme in the literature on rational number knowledge for teaching, suggesting teachers struggle with conceptual understandings of rational number concepts.

Borko et al. (1992) described one teaching episode of a student teacher reviewing the common algorithm for division of fractions in order to prepare students for a standardized test. A student in the class asked a question about why the algorithm worked. Ms. Daniels, the student teacher, recognized this as a conceptual understanding question and began to draw a representation of the situation. However, instead of drawing a representation for a division problem, she, instead, drew a representation for a multiplication problem. When her representation gave her an answer contradicting the result from the algorithm, she was puzzled and abandoned her representation. Clearly, Ms. Daniel's undergraduate coursework and her own knowledge had failed her. In addition, her confidence in her own knowledge limited her. Ms. Daniels was reluctant to ask for help or look at references for support. Borko et al. concluded that change must be made in university programs to strengthen prospective teachers' subject matter and pedagogical content knowledge. A similar argument could be made for changes to be made in PD for inservice teachers in order to strengthen content knowledge.

Studies by Graeber, Tirosh, and Glover (1989) and Tirosh and Graeber (1989) supported the notion that preservice teachers have difficulty with multiplying and dividing rational numbers. These studies examined the common misconceptions that multiplication always produces a larger product and that division always makes a number smaller. Graeber et al. (1989) found their preservice teachers had the same misconception as the students in Fischbein, Deri, Nello, and Marino's (1985) study. Tirosh and Graeber (1989) found similar results although they also collected data on explicit beliefs of preservice teachers related to common misconceptions, specifically the quotient must be smaller than the dividend. They found many American preservice teachers appeared to be influenced by primitive behavioral models even if presented with evidence to the contrary. For example, 8 of the 21 participants argued that the dividend was always greater even after being prompted to notice a contradiction by correctly solving $4 \div 0.5$. This result suggests teachers' understanding of division is grounded in the domain of whole numbers. Additionally, the preservice teachers did not have a working definition of partitive division that could be extended to fractions. These misconceptions potentially translate into student misunderstanding of these same concepts. They also raise questions about whether teachers are truly prepared to teach in the manner suggested by NCTM (2000).

The literature on inservice and preservice teachers' mathematical knowledge of rational numbers demonstrates the need for PD focused on rational number concepts. In IM, teachers engaged with the rational number concepts through tasks that emphasized proportionality and units. These tasks encouraged teachers to explore various representations and to think proportionally about rational number concepts. One theme of IM was to focus on the concept of referent unit in a variety of situations. A goal of IM was to have teachers communicate about such pedagogical and mathematical topics verbally and through their write-ups of tasks. Thus, this study adds to the field by reporting what teachers learned about rational number concepts in the IM setting. Clearly, mathematical knowledge for teaching rational numbers is complex.

Tasks

The literature on mathematical tasks and their implementation was relevant to my actions as the facilitator of the PD in this study. In this section, I discuss the importance of using tasks and the challenges often faced by teachers as they implement high cognitive demand tasks. Finally, I describe the use of technology with tasks.

Stein et al. (1996) defined a mathematical task as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (p. 460). Task features included “the existence of multiple-solution strategies, the extent to which the task lends itself to multiple representations, and the extent to which the task demands explanations and/or justifications” (Stein et al., p. 461). For this study, the tasks were designed to have multiple entry points leading to a variety of solution paths and had the potential to generate rich discussion about the mathematical concepts involved. Additionally, teachers were prompted to explain decisions on the approach used, the mathematical concepts involved, reasons for choosing specific representations, and any connections to their mathematical practices. Thus, the tasks in this study align with Stein et al.’s definition.

Both the *Principles and Standards* (NCTM, 2000) and the *Georgia Performance Standards* documents suggest problem solving and using tasks as important elements of teaching mathematics. NCTM recommended teachers make problem solving an integral part of their classes "by choosing interesting problems that incorporate important mathematical ideas from the curriculum" (p. 258). Teachers use tasks "both to stimulate students' investigation and to develop facility with mathematical reasoning and argumentation" (p. 265). For all middle grades, the *Georgia Performance Standards* includes process standards including solving problems, reasoning mathematically, and communicating mathematically. Thus, teaching with tasks is a national and local recommendation. In order to support this recommendation, teachers need opportunities to experience learning this way in a supportive environment.

Teaching with rich tasks is a challenging enterprise. Bennett and Desforges (1988) conducted research on elementary teachers' intentions behind tasks and students' perceptions of those tasks and reported mismatches between the two. For example, teachers tended to overestimate low attainers by asking these students to work on tasks that were too difficult. This led to students working very slowly and not progressing through the whole task. A similar problem was seen with the mismatching of tasks for high attaining students. These students often spent their time working on "the production features of tasks" (Bennett & Desforges, p. 227), such as drawing a picture of the manipulative used to solve the task. Not only is the job of choosing appropriate tasks for students difficult, but maintaining the appropriateness of the task is as well. Bennett and Desforges also reported teachers making compromises on the chosen tasks in order to ease emotional and curriculum tension. Teachers were concerned with students' feelings as well as making sure they covered the concepts in a timely manner. Thus, the nature of the classroom environment and the norms of participation influence how teachers support students' engagement with tasks.

Stein et al. (2000) provided a framework for identifying the cognitive level of instructional tasks using four categories on a continuum of low to high demand tasks. The tasks progress from memorization tasks to procedures without connections tasks to procedures with connections tasks and finally to doing mathematics tasks. This framework was extremely useful to QUASAR teachers as they planned lessons involving challenging instructional tasks. However, maintaining the high cognitive demand of the task during implementation was challenging for teachers (Stein et al., 1996). Factors found to be associated with the maintenance of a high cognitive demand include appropriate task selection, scaffolding, modeling a high level of performance, insisting on explanations, and providing adequate time for exploration (Stein et al., 1996). In the studies related to the maintenance of the cognitive demand of tasks (Stein et al., 1996; Boston & Smith, 2009) the researchers found teachers often lowered the cognitive demand of tasks, sometimes intentionally and sometimes unintentionally. Thus, choosing demanding tasks was only the first step in the tasks being implemented in a way that gave students an opportunity to grapple with challenging mathematical ideas.

Arbaugh and Brown (2004) reported their experiences with preservice and inservice high school mathematics teachers using the same framework (Stein et al., 2000) to sort a variety of tasks. Important conversations such as the difference between a difficult task and a task with a high cognitive demand were a result of the activity. In addition, the inservice teachers discussed the challenges in choosing a task to use in their classrooms. The teachers thought many surface characteristics of problems were misleading, making it difficult to classify the cognitive demand of tasks. This study did not follow teachers into their classrooms to see the impact of the task sorting activity on teaching practices. However, teachers now had a common language and way to categorize tasks to assist them in lesson planning and choosing texts. This common language is essential to community development, because teachers are able to communicate their ideas clearly. As a group struggles with defining a concept, such as a high cognitive demand task, they begin to make collective sense of the concept.

Zbiek, Heid, and Blume (2007) discussed the relationship between the cognitive demand of tasks and the use of technology (like those in IM). If the demand of the task is low, then the technology may not advance the thinking of the user about the concept and may, for example, simply aid with a computation. If the task's demand is high, then the use of technology as a tool might aid in generalizing a situation. Zbiek et al. provided two terms for thinking about tasks and technology: exploratory activity and expressive activity. They defined these two terms by stating, "When students are given a procedure to carry out, they are engaging in exploratory activity; however, when students decide which procedures to use they are engaging in expressive activity" (p. 1181). Tasks in this study were designed to engage teachers in both types of activities in order to learn how to use the technology and also to use the technology as an aid in developing understanding of the mathematics. The knowledge of how technology interfaces with tasks is helpful to a PD facilitator as she designs and plans activities to support teacher learning.

I considered the challenges implicit in using high cognitive demand tasks in teaching the class. Knowing the common challenges teachers face in choosing and implementing appropriate tasks, I was especially aware of this added challenge as the facilitator. Clearly, using tasks for learning has tremendous potential and is recommended by NCTM (2000). My intent was not only to model for the participants how to teach with tasks but also to record how teaching with tasks affects the instructor.

Community

Research on community building has been conducted in mathematics education in both PD settings and school classrooms with similar results in each. In Kazemi and Franke's (2004) study (described previously) the mathematics PD focused teachers on analyzing student work from their classrooms. Sherin and Han (2004) described the learning of a group of four middle grades mathematics teachers as they examined video clips of their own classrooms. Finally, the Community of Teacher Learners project (Grossman et al., 2001) provided data on a non-mathematical community of teachers at the secondary level. This study adds to the literature described here because the intent of the project was to examine the formation of teacher learner communities through PD, which has rarely been done in the mathematics education community. All three of these studies reflect the importance of active engagement of teachers in communities. Additionally, the development of norms relating to how the group was communicating and what the group was focusing their attention on was common to all of these studies. Finally, the role of the facilitator and its relationship with the community development was described.

As previously described, Kazemi and Franke (2004) examined PD with a group of elementary teachers. These teachers became actively engaged in examining their student's mathematical work on problems chosen by the PD facilitator. The facilitator's role was important in supporting the group as they developed norms about what it means to teach, learn, and be a part of a group of teachers examining student work. These teachers began to support each other's professional growth and learned the value of examining student work for enhancing their teaching practice. The group was cohesive and became a community of practice. Active engagement, regularly meeting for a year, and the role of the facilitator were connected to community development in this study.

Another mathematics PD study of a video club (Sherin & Han, 2004) reported changes in how the group of teachers discussed the video and the elements of the video on which they focused their discussion. These changes came about in an environment that was respectful and supportive of teacher growth and with a facilitator who had set goals. Evidence about the environment was seen in transcripts of the teachers' interactions with each other and in the decision to allow teachers to not be videotaped if they were uncomfortable. Two of the four teachers asked to not be videotaped; however they contributed meaningfully to the discussions of videos. The teachers met monthly 10 times to review video clips. At the first video club meeting, the teachers focused on pedagogical issues such as whether the teacher should have allowed the mathematical discussion to continue for as long as he did. These conversations focused on actions of the teacher and possible changes to the teacher's decisions. In the seventh video club meeting, the teachers initiated more conversations about student comments and the mathematical concepts from the clip. The facilitator participated in the video club meetings in two important ways. Her intention was to encourage teachers to talk about what struck them about the video clip and to focus the attention of the group on "student conceptions" (Sherin & Han, p. 167). To support these intentions, the facilitator encouraged the teachers to clarify their statements and to expand their thoughts when appropriate. She also posed questions to direct the teachers to look at students' mathematics in the video clips. As the group developed norms for interacting and agreed upon the purpose of their meetings, the facilitator did not have to encourage clarification or direct the conversation as much. The teachers began to more thoroughly explain themselves and were able to all contribute to a collective understanding of a student's mathematical idea, for example. The teachers also focused on student understanding more often. This study demonstrates the importance of active engagement, sustained PD meetings, and the role of the facilitator in community development.

The PD group in Grossman et al.'s (2001) study consisted of teachers from English and history departments in the same high school, and their purpose for meeting was to create an interdisciplinary humanities curriculum. The design of the PD included reading various texts and meeting in the form of a book club. The group met monthly for a whole day to read and discuss texts, had after school meetings, and participated in two five-day long summer institutes over two years. The facilitators declared the group a community and because of this were reluctant to set an agenda for the meetings. Grossman et al. explained "We straddled a rather ambiguous line: as researchers, documenting the progress of the project and as co-participants, reading books, discussing curriculum, and sharing our own ideas with the group of learners" (p. 950). Being a facilitator with the intention of forming a community was a challenging enterprise.

Grossman et al. (2001) developed a model of the formation of a professional teacher community. This model provides a rubric for classifying communities as beginning, evolving, or mature along four dimensions. The four dimensions are formation of group identity and norms of interaction, navigating fault lines, negotiating the essential tension, and communal responsibility for individual growth. Each dimension includes indicators of community growth along that dimension.

Similar dimensions involving norms and communal responsibility are seen in K-12 classroom studies. For example, Yackel and Cobb (1996) reported a study with a group of second- and third-grade teachers as they changed their teaching practice to become more inquiry-based. By analyzing classroom episodes they revealed the sociomathematical norms (classroom norms that are specific mathematical practices) created by the class. Examples of such norms included mathematical difference, mathematical explanation, and mathematical argumentation. Mathematical difference was defined as the characteristics that made a solution different from another. Mathematical explanation included "describ[ing] actions on mathematical objects" (p. 470). Acknowledging challenges and "making judgments about how other children might make sense of" (p. 471) an explanation were characterized as mathematical argumentation. The norms were created and defined by the class as a whole–teacher and students–and reflected the culture of the classroom, in turn influencing the learning of all involved. Few research studies exist on examining mathematics PD similarly to the elementary classrooms described here.

Lampert (1990) reported a self-study while she was teaching fifth grade students. Her goal was to create a classroom in which students considered their beliefs about mathematics while building mathematical knowledge. In order to better define this goal, she referred to ideas of Lakatos and Polya about engaging in mathematics as following "a 'zig-zag' path" (p. 30) and "assum[ing] 'the inductive attitude'" (p. 31). Lampert believed the social virtues of honesty and courage were important to support in the classroom environment. She was successful in meeting her goal by establishing norms with her students and providing them with appropriate language to engage in mathematical discussions. Her students analyzed assumptions thoughtfully by considering observations and generalizations about mathematical ideas. Lampert connected this success with the culture of the classroom and her role as the teacher. She found when classroom culture is considered, teaching is not only about content but also about how to participate in a lesson. Lampert discussed three different roles of the teacher: telling students which activities are appropriate or inappropriate, modeling the roles each of them would take, and doing mathematics with them. Little is written about the role of the facilitator in PD; instead guidelines, such as those examined by Guskey (2003), have been provided to facilitators. Lampert's work also supports the idea that examining culture (or community) is important in learning about classrooms.

In a study conducted by Cobb, Boufi, McClain, and Whitenack (1997) the focus was "to suggest possible relationships between classroom discourse and the mathematical development of the students who participate in, and contribute to it" (p. 258). Two types of discourse, reflective discourse and collective reflection, were defined and analyzed. Reflective discourse involved objectifying repeated mathematical activity. For example, students generated combinations of 5 for a given context. Collective reflection "refers to the joint or communal activity of making what was previously done in action an object of reflection" (p. 258). One student explained she thought all the combinations of 5 were found. Thus, she reflected on the actions she and others in the class were taking to solve the problem. Cobb et al. (1997) used discourse to explore the learning of the students and the classroom community.

Another study describing elements of community was a comparison of four elementary teachers' implementations of the same lesson on addition of fractions (Kazemi & Stipek, 2001). The four teachers all had experience with reform-oriented instruction, and all four classrooms were determined to have a positive social environment prior to the study. The difference between the four teachers' practices was described as their "press for learning" (p. 61). This variable "reflected the degree to which teachers engaged students in mathematical thinking" (p. 61). The intent of the study was to use the construct of social norms and sociomathematical norms from Cobb and his colleagues to attempt to distinguish between the two types of press for learning-high press and low press. The four teachers were intentionally chosen so two were labeled as high press and the other two as a lower press. (The low press teachers were intentionally chosen not to be the lowest from the data set.) The researchers found in the high press classrooms problem-solving strategies were linked to mathematical explanations and justifications; students justified their actions by triangulating verbal, graphical, and numerical strategies. In addition, comparing strategies, including adequate and inadequate strategies, was common. These sociomathematical norms were observed regularly in the lessons of the high press teachers. The other teachers demonstrated the social norms commonly associated with reform-oriented teaching, such as explaining one's thinking, discussing different strategies, and working in small groups. These activities were not necessarily mathematically rich and focused more on behaviors of students and not mathematical understanding. One of the low press teachers avoided discussing incorrect or inadequate student solutions in her classroom. Kazemi and Stipek (2001) claimed "The more superficial changes mentioned above [social norms] are necessary, but they are not sufficient for helping students build sophisticated understandings of mathematics" (p. 60). Thus, distinguishing between high and low press teachers correlated with sociomathematical norms and social norms, respectively. They also commented that using sociomathematical norms as a framework were useful for thinking about teacher actions that support student mathematical development. Kazemi and Stipek concluded, "It is also important to investigate, with longitudinal data, how sociomathematical norms are created and sustained, and how they influence students' mathematical understanding" (p. 79).

The literature on community demonstrates the importance of examining the types of norms developed by the group in order to determine if a community develops. Additionally, the importance of active engagement with mathematics and the role of the facilitator were evident. I used Grossman et al.'s (2001) model to determine the extent to which the IM group could be classified as a community. Additionally, the literature shows the challenges faced by the group and the instructor as they engage in mathematics together. Some of these studies examined groups that had been working together previously. In this study, the data were collected as the group began working together on mathematics so it adds to the understanding of the formation of teacher communities.

Theoretical Framework

In this section, I will explain the theoretical framework of the current study. Two important dimensions, community of practice and learning, are described. These two elements form the basis of my analysis.

Community of Practice

Wenger (1998) described a way to view learning socially by observing how groups of people interact. In this study, I examined whether the IM group formed a community of practice. Wenger described a community of practice as exhibiting "practices that reflect both the pursuit of our enterprises and the attendant social relations. These practices are thus the property of a kind of community created over time by the sustained pursuit of a shared enterprise" (p. 45). A group of teachers and an instructor of a PD experience would not necessarily qualify as a community of practice. Wenger described three necessary characteristics of a community of practice: mutual engagement, a shared repertoire, and a joint enterprise. These characteristics are the elements I looked for in the data from the IM course.

Mutual engagement. Mutual engagement referred to the aspects of individuals working together for a common purpose. In the case of IM, I began class believing the purpose was to expand our knowledge of rational numbers and be reflective about our teaching practices. This was what I brought to the community. The other participants came with different intentions. Together, if we formed a community of practice, we determined our collective purpose. This should not be viewed as something easy to do or something trivial. In order to work together, we negotiated norms related to mathematical discussions, write-ups, and working together to understand mathematical ideas. Wenger (1998) wrote, "Mutual engagement involves not only our competence, but also the competence of others" (p. 76). My biggest challenge was to encourage all members to be on an equal playing field. As the instructor, I could be viewed as having more knowledge than the teachers, or the teachers might spend time trying to figure out my agenda. In order to build a community of practice we had to work through these issues and others that arose. Another potential challenge was if the teachers did not want to focus on their mathematical understanding but instead focus on pedagogical issues related to rational numbers. Pedagogical decisions were a topic we discussed and wrote about as a group; however, the design of IM was to address teachers' own personal mathematical knowledge. Thus, the confines of the design of the course influenced us, which leads to the next characteristic of a community of practice.

Joint enterprise. This characteristic referred to the greater contexts that influence a community of practice and the way in which the community responds to these external conditions. "Because members produce a practice to deal with what they understand to be their enterprise, their practice as it unfolds belongs to their community in a fundamental sense" (Wenger, 1998, p. 80). Negotiating a joint enterprise included defining accountability for the group. For example, in IM, arriving to class on time became something important to the group. Determining whether to share your mathematical confusion and ask for help was a negotiated accountability. "An enterprise is a resource of coordination, of sense-making, of mutual engagement; it is like rhythm to music" (p. 82). For IM, this included the requirements of the course and research as well as any requirements of the school district. An example of requirements of the course was the ten required write-ups. An example from the research included the weekly phone interviews made to the IM participants. Participants were required to sign in every class meeting; the district imposed this external condition on the community. Additionally, joint enterprise did not mean the community members were similar: "their responses to their conditions – similar or dissimilar – are interconnected because they are engaged together" (p. 79).

Therefore, some participants were dissatisfied and even annoyed by the design of IM. A few participants were completely gratified by the design. We all responded to the external conditions placed upon us together; however, certain aspects of the shared enterprise may not have been pleasant for everyone.

Shared repertoire. The final characteristic of a community of practice as defined by Wenger (1998) included the resources, routines, and references we use together and become part of our collective vocabulary. For IM these included problems we worked as a group that we referred back to when we communicated. Additionally, the technological tools we used to communicate would be classified under shared repertoire. Another example of shared repertoire was a hypothesis a participant had suggested but that the group did not determine to be true or false immediately. Something as simple as the structure of our class meetings also fit in this category. Shared repertoire cannot be found right away in a group, as it is dependent on some shared history together. This characteristic also referred to the necessity of ambiguity in practice. These common resources were all a shared experience but could be interpreted differently by every member of the community. This allowed for negotiation of meaning.

Learning

For the purposes of this study, learning in the IM course was viewed as a *transformation of participation* (Rogoff, 1997). Transformation of participation referred to changes in roles, engagement, and involvement of each individual. For example, a participant could begin IM by not saying much and working quietly on problems. As the course progressed, this person could become more apt to propose a solution to the class.

This change in behavior would be evidence of learning. The change was toward the ideal participation in the community.

This ideal participation included engaging in mathematical discussion and argumentation, being open to explore errors and misconceptions, and striving to understand the connections between various strategies. This participation was also characterized by the teacher's participation in class discussions in a respectful manner: the focus was on mathematical understanding and this needed to be perpetuated as the goal. If a participant "over-participated" (by dominating) or "under-participated" (by not engaging) this was not classified as learning. The participants needed to begin to take over my role as facilitator by asking others to provide justification for claims, questioning incomplete or incorrect mathematical ideas, providing counterexamples, and pushing for generalizations. I modeled learning and this model was what I expected participants to change toward. The ideal learner was open to exploring mathematical ideas generated by peers and by me. Another important aspect of learning was to work to connect mathematical ideas to further conceptual understanding. An ideal learner communicated his mathematical ideas and understandings and listened to and respects others. This learner was also deeply engaged in the mathematical task at hand. Not all changes in participation were evidence of learning. If changes in participation moved away from this ideal learner, they were considered evidence of the contrary.

Furthermore, behavior was not the only source of evidence of learning. Participants produced a total of 10 write-ups and wrote a reflection response after each class meeting. These written documents also provided evidence of learning. In these documents, learning was evidenced by changes in language and depth. For example, an increase in mathematical argumentation and consideration of other strategies could be a change in participation.

This view of learning was reflected in the design of the IM course as well. Teachers were not told rules or procedures for dealing with rational numbers. Instead, teachers engaged in tasks where they used prior knowledge to create new understanding about rational number concepts. Teachers were required to turn in their own work; however, engagement with other participants, the instructor, and technology was strongly encouraged.

Rogoff (1997) described the transformation view of learning as an improvement of the transmission and acquisition views. The transmission and acquisition views of learning were “one-sided model[s] of development” (p. 266). The transmission view placed the importance on the outside world giving knowledge, and the acquisition view emphasized the learner obtaining knowledge. With this alternative concept, the world and the individual were both active in development. The unit of analysis was neither the individual nor the environment but instead was the sociocultural activity. Rogoff suggested three lenses to view the world: personal, community, and interpersonal. The personal focused on contributions by individuals. The interpersonal focused on “how people communicate and coordinate efforts,” and the community view focused on “people participating with others in culturally organized activity” (p. 269). All three lenses were present but the researcher chose to focus primarily using one (knowing the other two were influencing the data). In researching my first two questions, I used the interpersonal lens. For my last question, I used the personal lens.

In the IM course, my focus on learning was to the attention given to the referent unit, drawn representations, and proportionality. These were three themes of the course and narrowed my search for evidence of learning from rational number concepts to these three aspects. My role as facilitator was multidimensional. I needed to balance the requirements placed on me to complete the activities outlined in the course syllabus with my desire to help create a community of learning in the class. Because the design of IM was content focused, the challenge was if the community wanted to focus on pedagogy, I needed to step in and re-focus the community on mathematical content. However, if the group was struggling with understanding equivalent fractions, then I had to determine how to meet the goals of IM while supporting the teachers who I was teaching. If the IM syllabus changed based on my reaction to participants' needs, I planned to readjust my focus on learning to include the concept we spent more time on than anticipated.

CHAPTER 3

METHODOLOGY

This research study was a special case of a qualitative case study (Ball, 2000), because I was a member of the case as the instructor. In order to explore my three research questions (listed below), a qualitative case study was most appropriate because of my desire to examine community and PD. Merriam (1988) noted that using a case study design is important when the focus in on a particular phenomenon and the end product is a rich, illuminating description of the phenomenon. The intent is not to find truth or falsity in the situation, and the desired objectives are to focus on humanistic outcomes and not behavioral outcomes. In this study, not only was I a member of the case, but in order to investigate my three research questions, I needed to gather intimate data about the group and its developing characteristics. A community is characterized by three aspects as described in Chapter 2: mutual engagement, joint enterprise, and shared repertoire (Wenger, 1998). Wenger's (1998) definition of a community of practice provided a framework in order to analyze and describe the group's interactions; however it did not provide specific examples of how the group should behave in order to be classified as a community. Hence, there are no right or wrong findings for my research questions. The purpose of the study "interweaves the empirical with the conceptual" (Ball, 2000, p. 374) by examining the concept of community of practice in a setting where I gathered data.

Research Questions

a. What are the IM instructor's intentions and actions related to developing a community of practice within the group?
b. What characteristics of a community of practice, as defined by Wenger (1998), does the group exhibit? How do these community characteristics change during the course?
c. What learning occurred?

Because these research questions were grounded in my position as the instructor, this was a first-person inquiry in teaching as described by Ball (2000). Ball described this type of inquiry by stating, "Practitioners examine their practice thoughtfully with the goal of learning about teaching and learning and ultimately, improving it" (p. 367). The intention of this study was to explore the much-recommended concept of community building in a PD setting in which I was intent on trying to build a community of practice. By studying this concept from within, I was able to examine the challenges professional developers face in trying to achieve this recommended practice and the challenges the group faced together in its attempt to become a community. Thus, using a qualitative case study design was most appropriate.

More specifically, this study was an instrumental case study (Stake, 2005) where the purpose was to provide insight into the concept of community and PD. Characteristics of an instrumental case study include examining a case in order to impart insight into an issue, to facilitate our understanding, and to pursue an external interest: in this case community building (Stake, 2005). Understanding this particular group of teachers was not the sole purpose of the inquiry but a way to explore the concept of community in PD.

This was in contrast to other types of case studies where the purpose may be to describe a phenomenon or to evaluate an educational issue. Merriam (1998) also described the purpose of this type of case study, referring to it as an interpretive case study, to collect "as much information about the problem as possible with the intent of interpreting or theorizing about the phenomenon" (p. 28). The purpose of the research questions was to examine the case of my IM course and to explore community building from my perspective as instructor. In the rest of this chapter, I discuss the context, setting, participants, data collection, and data analysis.

Context

This study was embedded in a larger research project called *Does it Work?* funded by the National Science Foundation for three years. The purpose of the DiW research was to explore what teachers learned in PD and how teachers' learning impacted their teaching practice and students' learning. DiW was funded for three years in order to offer IM four times to middle school teachers working in urban school settings. Thus, larger numbers of middle school teachers from the same district were required for the statistical analysis. DiW funded numerous graduate assistantships and had been part of my assistantship assignment for two years. The DiW research team consisted of three professors and numerous graduate students from instructional technology, mathematics education, and educational measurement. The team created assessments designed to measure mathematical knowledge for teaching rational numbers as well as designed the syllabus for IM. Through this project, two IM courses were offered simultaneously during the fall of 2008 at two different school districts. In this study, only one IM course was examined.

IM was a series of six mathematics content courses for middle school teachers designed for conducting professional development. IM was developed with an NSF grant awarded to a collaborative team including members at University of Georgia and Georgia Institute of Technology (InterMath, 2007). The premise of the project was that many teachers needed to improve their mathematical knowledge for teaching in order to serve their students well. The assumption was that an increase in teachers' mathematical knowledge for teaching would translate into a positive impact on their teaching practices and their students' understanding of mathematics. A version of this particular course, the rational number course, had been offered the previous fall (2007). As a member of the DiW research team, I helped to design the syllabus, including choosing and developing tasks and determining sequencing. For this offering, I made alterations to the syllabus, with the team's help and using feedback from the previous instructors, before the course began. The team decided on three themes for IM: referent unit, drawn representations, and proportionality. Referent unit referred to the whole for a given quantity. Drawn representations included array models, area models, single number lines, double number lines, tables, and graphs. These representations were intended to be used to reason about a given problem, not just as a picture of the solution. Proportionality referred to multiplicative reasoning in fraction and decimal operations as well as situations involving direct and inverse proportions. The content of the course was directly related to the state standards for fifth, sixth, and seventh grade mathematics.

Each class meeting focused on a specific topic of rational numbers, for example decimal division. Some topics were the focus for two class meetings; however many were discussed for just one class meeting. See Appendix A for a copy of the syllabus. A typical class consisted of me greeting the class and either posing a warm-up problem or a task for the class to consider as a whole. A warm-up was either a portion of a task, an open-ended mathematical problem, or a mathematical question that I decided would provide a good start to our class. After some time for the teachers to consider the problem or task individually or with their partner, we had a mathematical discussion as a whole group. There were generally two whole group tasks that followed this format. Then teachers were asked to work on an individual task they would then write up and post to a website. (Write-ups are described in more detail later in this chapter). The write-ups were started in class, but most teachers finished them outside of class. Sometimes teachers chose the task to write up from a collection of tasks on the IM web site; other times I assigned every teacher the same task. A total of 10 write-ups were assigned in addition to two lesson plans for the course. The two lesson plans were completed outside of our class meetings. I posted general comments about the write-ups after every class meeting. The participants received four professional learning units (PLUs) toward renewing their teaching certificates and a stipend for participating in the course and the DiW research.

Setting

The class met a total of 14 times for three hours each course meeting from August to December. The first class meeting was comprised of introductions, research paperwork, and a pre-assessment. During the following 12 meetings, the class examined open-ended tasks pertaining to issues on multiplication and division with fractions and decimals, direct and inverse proportions, and worked with various technologies including Fraction Bars (Orrill, 2003) and our class web page. The tasks were chosen from a collection of mathematical tasks on the IM site. Other resources on the IM site included a mathematical dictionary, templates for the write-up and lesson plan, as well as a sample write-up. The final class meeting consisted of a discussion where we reflected on the course and time for teachers to provide written feedback on lesson plans followed by paperwork related to the DiW research, including the first post-assessment of the teachers' learning. We met in a computer lab in the district's central offices. The lab contained 20 computers organized in five rows of two pairs in each row.

The district, as a whole, did not make Adequate Yearly Progress in 2007 and was classified as a Needs Improvement School System. The district served over 30,000 students, and the student population was approximately 73% Black and 21% White. Sixty-nine percent of the student population was eligible for free or reduced price meals. Mathematics teachers at all nine middle schools in the district were invited to participate in IM. The district was selected because of the DiW research team members' ongoing relationship with the district in mathematics PD. I had assisted with or led various workshops with elementary teachers, mathematics coaches, and middle school mathematics teachers in this district during the previous two years. I did not, however, know any of the IM participants prior to the start of the study. The district mathematics coordinator was able to help us work with the district to set up the IM course and helped us handle any "housekeeping" issues that arose.

Participants

The group consisted of 13 fifth, sixth, seventh, and eighth grade teachers (see Table 1) and one central office staff member (not included in Table 1). Two of the teachers reported teaching more than one grade level.

Table 1: Grade Taught by Participants

Grade Level	# of participants
Fifth	1
Sixth	7
Seventh	6
Eighth	2

Table 2: Teaching Experience of Participants

Teaching Experience	# of participants
Less than five years	4
Between five and ten years	3
Over ten years	6

Two of the participants were special education teachers at the middle school level. The central office staff member worked in the technology division and was certified to teach secondary mathematics but had no teaching experience. Table 2 presents the mathematics teaching experience of all the participants except for the central office staff member who had no teaching experience. Table 3 shows the highest degree earned by the participants. Two participants were working toward another degree related to education.

Table 3: Highest Degree Earned by Participants

Description	# of participants
Bachelor	3
Masters	10
Education Specialist	1

Teachers from seven of the nine middle schools in the district participated in IM. Two of the participating middle schools had their two sixth and seventh grade teachers participating in IM together. Four of the 14 teachers worked at the same middle school.

Data Collection

Because of my dual role as researcher and instructor, I organized the data into two categories: researcher data and instructor data (Lampert, 2001). Table 4 depicts the data I collected in order to address my three research questions. For the first question relating to my actions, I was the unit of analysis. In the second and third questions, the group was the unit of analysis. In the paragraphs that follow I will describe the researcher data followed by the instructor data.

Table 4: Data Collected for this Research Study

Researcher Data	**Instructor data**
Video & Lesson graphs	Write-ups & Feedback
Weekly phone interviews	Email correspondence
Journal & Goals	Reflections & End of course survey
Assessments	

Video

Two researchers with two separate cameras videotaped all of my class meetings with the exception of the first meeting and most of the second meeting when participants were taking the assessment and filling out paperwork. One camera was set up at the back of the classroom to focus on my teaching actions and the group involvement as a whole. The other camera focused on inscriptions – any work on the board, chart paper, projector, and computer screens of teachers. Two microphones were used to capture sound. One was placed near me, and the camerawoman who focused on the written work moved the other microphone to capture participant comments as they happened. These two videos were then mixed to create a *restored view* (Hall, 2000) of the instructional class meetings.

Lesson Graphs

After the restored view was created, I lesson graphed (as described in Izsák, 2008) the class meeting highlighting the events of the class, including anything related to the description of community of practice and the three themes of the course. A lesson graph consisted of a table with three columns: time, description, and comments (see Appendix B for a portion of a lesson graph). The time stamp was taken from the video; the description was my retelling of a particular episode based on the video; and the comments allowed me to record ideas related to teaching and the research. A lesson graph was created for every instructional class meeting and was then reviewed (and at times, edited) by a DiW co-principal investigator (co-PI) who was not present when the class was taught but who watched each video in its entirety each week.

Weekly Phone Interviews

Another member of the DiW research team interviewed every participant, except for the central office staff member, during the week after each instructional class meeting. The central office staff member was interviewed twice at the beginning of the course. The other 13 participants were interviewed a total of 12 times for an average of 10 minutes per interview. Each interview was audio recorded and then later transcribed by DiW team members. Many of the questions pertained to the overarching DiW research; however some of the questions were specific to this study, such as “At school, what do you do if you have a question about mathematics?” See Appendix C for a sample interview protocol. Sometimes a participant had to do two weeks’ worth of interviews in one phone call because of scheduling issues. However, this was avoided where possible. Each participant was asked all of the questions in the interview protocols.

Journal and Goals

Before the course began, I created a list of goals I had for developing the community of the group (see Appendix D). These goals related to the sociomathematical norms described by Yackel and Cobb (1996) and Kazemi and Stipek (2001) as stated in Chapter 2. I included anticipated actions on my part and potential reactions of the participants. I also included different variations of the responses I could give based on these potential reactions. This allowed me to record my intentions going into the class and see how the community influenced and enacted these.

After every class meeting, before I began lesson graphing, I wrote in a journal about the class. My journal entries had no structure or preset questions. Instead, I wrote about how I felt about the class. This not only included my research interests but my concerns about the pace of the course, the mathematical knowledge of my participants, and the sequencing of topics. I wrote about events I perceived as positive and negative. I also began some preliminary data analysis as I thought through whether or not we were becoming a community of practice. The journal entries reflected my roles as both researcher and facilitator.

Assessments

All of the participants completed a written pre-assessment and a post-assessment on mathematical knowledge for teaching rational numbers. The purpose of the assessments was to measure each teacher's mathematical ability. The assessments were generated using items from the University of Michigan's *Learning Mathematics for*

Teaching assessment (SII/LMT, Study of Instructional Improvement/Learning Mathematics for Teaching, 2004) as well as items created by the DiW team (For further information on the assessment, see Izsák, Orrill, Cohen, Brown, 2009).

Write-ups and Feedback

Each participant electronically submitted write-ups on 10 tasks. If a participant missed a class meeting, then she was asked to create a write-up for one of the tasks we discussed as a group as well as one of the individual tasks. This happened once for three different participants. The participants were provided guidelines for their write-ups that included the following: title, problem statement, problem setup, plans to solve/investigate the problem, investigation/exploration of the problem, extensions of the problem, and author.

After each class meeting I read the write-ups and posted general comments on the mathematics, the quality of the write-ups, and thoughts about the discussions we had in class. For two of the 10 write-ups, I gave individual feedback privately to each participant. Feedback was given on the write-ups for our third and seventh class meetings. The feedback consisted of questions about the mathematics, suggestions to clarify the writing, and encouragement for changes to future write-ups. The teachers were not required or expected to respond to this feedback with an updated write-up. However, many teachers emailed me with some responses, and one person updated her write-up and emailed it to me.

Reflections

At the end of every class meeting, excluding the last meeting, the participants were given a reflection question to complete before leaving class. The reflection questions were focused on the mathematical topic of the day and served as an informal assessment. Generally, two questions were asked: one content-oriented and the other pedagogically oriented. For example, the reflection questions for the first instructional class meeting asked teachers to identify the referent unit for each number in two given expressions and to write about why referent units are important for students to understand. Each week these questions were provided on a sheet of paper with plenty of space to work and write and then collected. I read all participants' reflections at least one time before planning the next class meeting.

End of Course Survey

During the last class meeting, the participants were asked to complete a two-page end of course survey (see Appendix E). The survey was created to collect feedback from the participants on the IM course. Part one of the survey consisted of five short answer questions; part two contained 17 Likert scale responses. The participants had completed a beginning of course survey during the first day of instruction; however that survey was not used in this study.

Data Analysis

Qualitative case studies involve ongoing analysis during the data collection as well as intensive analysis when collection is complete (Merriam, 1988). In order to be successful in analysis at either stage, the data must be organized. I organized my data (with the help of the DiW research team) by lesson graphing all of the class meetings, transcribing the weekly phone interviews, and recording all written documents on the website. My goal table, journal, and written comments to participants via email all documented my intentions and actions as an instructor hoping to create a community of practice. Finally, the write-ups and reflection questions provided data on the learning of the group in this context. As I organized the data and conducted ongoing analysis, I recorded comments I had about the data and wrote memos to myself about what I was learning through the process (Merriam, 1988). My preliminary analysis included creating the lesson graphs, reflecting on the community development and participant understanding of the mathematics in my journal entries, and reading the reflection responses each week.

In order to conduct more intensive analysis, I coded the data using Wenger's (1998) three-prong definition of community of practice and the three themes of IM. It is common to start with preconceived codes in an instrumental case study (Stake, 2005). I began by analyzing the lesson graphs for each of my three questions. For community, I used the codes mutual engagement, shared repertoire, and joint enterprise. For my intentions and actions, I developed codes based on the data such as "work together" and "clarifying question." For learning, I used referent unit, drawn representation, and proportionality. I returned to the data and my coding numerous times in order to be sure my analysis was identifying patterns that were supported by multiple data sources.

My goal through analysis was to look for confirming and disconfirming evidence of the group forming a community of practice, my goals being realized by the group, and learning about referent unit, drawn representation, and proportionality. After analyzing the lesson graphs, I then turned to the transcriptions of the weekly phone interviews, the participants' write-ups, and my journal and searched for confirming and disconfirming evidence. When I found disconfirming evidence, I re-evaluated my analysis and made necessary changes in codes and descriptions of data. The written materials were analyzed similarly to the interview data and lesson graphs of the video (Merriam, 1988).

Triangulation

In order to support any conclusions or hypotheses I drew about the case, I gathered multiple data types on each of the three questions. Ball (2000) wrote about the researcher's role in a first-person inquiry as "to gain alternative perspectives and interpretations of his own and others' actions and thoughts in the session, while also seeking to use the intimate and the personal as resources" (p. 393). This makes triangulation especially important in this study as I was the instructor and the researcher. Triangulation of data includes "redundancy of data gathering" and ensuring different ways of seeing the case from different perceptions (Stake, 2005, p. 454). Combining dissimilar data sources, in this case lesson graphs, interviews, journal entries, and assessments, in order to examine the same unit of study is triangulation (Merriam, 1988). In this study, I used the lesson graphs (and video to verify any vagueness in the lesson graph) of each class meeting to identify my repeated actions as the facilitator who was attempting to develop a community of practice with her group. I then analyzed my journal entries and lesson plans in addition to the participants' phone interview transcripts. To address my second and third research question, I again analyzed the lesson graphs to look for evidence for and against any of the group's development related to mutual engagement, joint enterprise, and shared repertoire. The phone interview transcripts provided evidence to either support or contradict my analysis of the lesson graphs. The participants' write-ups were analyzed to look for confirming and disconfirming evidence for the final two research questions. Finally, the pre and post assessment of teacher learning provided another way to describe the learning of the participants in order to compare and contrast it to Rogoff's (1997) definition.

Another member of the DiW research team conducted the phone interviews. Ball (2000) discussed the importance of using another researcher to collect some data when conducting a first-person inquiry in teaching as a way to complement the data set "without complicating the data collection with his [the researcher's] complex role as both researcher and teacher" (p. 391). The interviewer was a camerawoman in the IM course and attended every class meeting. This potentially provided me with more objective data because the participants could be more honest and share ideas they might not be comfortable telling me.

As I analyzed data, I consulted with members of the DiW team, especially the two members who videotaped our class meetings. This aided my analysis by offering additional insights and different perspectives. The consultations mediated my biases as the facilitator attempting to create a community. I asked the DiW PI who videotaped most of our class meetings to read a draft of my analysis to ensure I was representing the data fairly. Her feedback supported me in reflecting on my analysis in a more objective manner.

Finally, I consulted my major advisor regularly either in person or via email about observations I made and things I noticed during data collection. I continued to discuss my analysis with her after the IM course finished when I was conducting more intensive analysis. These discussions supported me in thinking about my data as the teacher and as a researcher. My major advisor often asked questions that supported me in revisiting my analysis from alternative perspectives.

Limitations

The fact I was the instructor and a member of the group being researched may be seen as a limitation. My view of what took place in the classroom was limited because I was not be able to take field notes during class and had to rely on stimulated recall of my thoughts based on the review of the class video. However, as the instructor of the course, I was able to explore the challenges instructors face as they attempt to create a community of practice. I also had an "inside view" of the data set (Ball, 2000, p. 392), which is important to the design of this research study.

Other possible limitations related to the two research agendas at play in this data collection. The DiW project had broader research questions than my research study, which limited the number of questions in the phone interviews relating specifically to community building. In addition, the video focused on my actions as that focus aligned with that of DiW. If the focus had been on a select number of participant groups, the data would have shown how teachers were interacting with and without my presence. This would have also provided more data on learning by recording teachers' exploration of tasks. I often provoked perturbation while teachers were working in small groups on a task. Finally, because this study was part of a larger project, another member of the DiW team who did not know my research purpose well conducted the phone interviews. Thus, she may not have understood the purpose of a question or was limited in her prompting or follow up questioning.

CHAPTER 4

DATA ANALYSIS

My Actions

In this section I present data pertaining to my first research question: What are the IM instructor's intentions and actions related to developing a community of practice within the group? Before IM began, I stated three instructional goals for myself: to create a safe environment where all mathematical ideas are welcome, to stress that learning is collective, and to emphasize that errors and sharing tentative, ill-formed ideas are part of the learning process. By analyzing lesson graphs of the 12 instructional class sessions I identified behaviors I repeatedly employed to meet these goals. I clustered these behaviors into three categories related to managing the discussion, questioning the mathematics, and stressing the importance of others. I also provide evidence that the group began to mimic some of those behaviors and comment on them during phone interviews.

Managing the Discussion

Because the emphasis in the IM course was on engaging with mathematical tasks and debriefing about them, my ability to facilitate a discussion was important to the learning of the participants. In addition, my interest in creating a community of practice made facilitation a top priority. My desire was to emphasize how to learn as a group in a safe environment, including being respectful of other's ideas, listening, and being patient. Five of my actions fit into this category: giving the participant the floor, checking for understanding, using positive reinforcement, giving wait time, and determining the direction of the conversation as a group.

Giving the participant the floor. As the leader of the group when IM began, my intention was to model good communication and dialogue. I enacted this intention by making sure that a variety of participants contributed to the conversation about mathematical tasks and that everyone who wanted the attention of the group could get it. I accomplished this by simply giving the floor to a participant who asked to speak, looked like she was going to speak, or mumbled something I overheard. Rarely did participants raise their hands in class to indicate they wanted to speak; they were able to get the floor without this behavior. I used the tactic of giving a participant the floor during every class period; however the frequency reached a plateau after Week 6 with the exception of Week 10. The participants began to give each other the floor starting at Week 2. This behavior was observed only once or not at all each class meeting until Week 10 when it was observed happening three times. Interestingly, we began discussing proportions during Week 10 and our group task was simplistic and straightforward (more on this in Tensions section). By giving each other the floor, the participants were taking more of a lead in the discussion and recognized the value of other participant's thoughts and comments.

In addition, every participant, in response to phone interview questions relating to describing IM to a colleague, my role as facilitator, or what she was learning from IM, reported the importance of sharing ideas and hearing other perspectives. One participant stated "I like the way she [Rachael] brings people into the lesson. She's very observant. She monitors everybody, she monitors body language, she monitors comments, and she brings everybody into the discussion" (Sharlene[1], phone interview, Week 3). Another participant noted "In this class, I've been trying to make an effort to make sure that I'm communicating with several teachers around me and that we're sharing ideas" (Keith, phone interview, Week 4). A third participant described my role in this way: "She is going to open up the discussion, but then she is going to step back and let us have reign to freely talk and discuss problems" (King[2], phone interview, Week 3).

Checking for understanding. In all but two weeks, I asked for questions or comments after a participant explained his or her model, shared a suggestion, or noted how the mathematics connected to their classroom during whole group discussion. Another common strategy I used was to stop the conversation and check in with participants to see if they understood what was being discussed. I did this while a participant was explaining a model or idea. I generally implemented this behavior if a participant was looking confused or made a funny face or if a participant mumbled something under her breath. One participant described my teaching approach by noting "She's very good at reading our facial features – our facial expressions and knowing when we don't quite understand something that's been said" (Carrie, phone interview, Week 3). Checking for understanding is a common teaching practice; however my intent was not only to understand how participants were making sense of the mathematics but also to send the message that learning is a collective experience. I also wanted to emphasize that every person's ideas were valued and should be considered seriously by the group. This behavior of checking for understanding was imitated three times by

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[1] All names are pseudonyms.
[2] King was the only teacher who went by his last name.

participants. In two of the instances, a participant was explaining an idea to the whole group and paused to ask if the explanation made sense or if the group had questions. The third instance of a participant checking for understanding occurred during Week 5 when two participants, Walt and Will, were discussing a problem related to fraction division in a small group setting. Will was trying to make sense of Walt's drawings. In order to help Will, Walt explained where he saw the numbers involved in the problem in his drawing. Walt then asked Will if he could see it. The discussion continued as the two worked together to make sense of Walt's drawings and the problem.

Checking for understanding was useful in IM to convey the importance of all of us learning from each other and respecting all of our ideas. Errors, mistakes, and tentative ideas were treated the same as well thought out ideas.

Using positive reinforcement or encouragement. I encouraged the participants' community-building behaviors by praising individual participants for sharing models and mathematical thoughts and commending the whole group for great conversation and debate. For example, at the conclusion of a discussion in our eighth class meeting (October 28, 63700) I said, "We had a great conversation about this task!" When teachers expressed frustration with or confusion about a task, I would give positive encouragement by sympathizing with the participants and encouraging them to continue to engage with the task.

The participants began to give each other positive feedback during Week 4 and consistently did this for the remaining class meetings with the exception of Week 7. The typical positive reinforcement consisted of praising a fellow participant for his idea, model, or thoughts. An extraordinary example of this type of praise occurred during

Week 10 when a participant shared his quadruple number line model using Fraction Bars, the software we used regularly, to model a ratio problem involving mixing paint. Once his model was posted and the group had considered it, the group spontaneously began clapping for his work.

Interestingly, in Week 7, I gave everyone assigned seats, and this was the only class after Week 4 where no positive reinforcement was noted during the class meetings. My intention in assigning seats was threefold. I was curious to see if the behaviors the group was exhibiting would persist with a new arrangement. I also noticed a few pairs were not pushing each other mathematically, so I was concerned some participants were already too comfortable with each other and were not willing to correct each other's mistakes. Lastly, I wanted to insure all participants had a partner, because some participants were choosing to work alone. Some of the participants were upset with my decision to change their partner and vocalized their discontent in class and in phone interviews. Others did not seem to care at all. The participants, and myself, appeared to resort back to some of our earlier behaviors like not using positive reinforcement.

Wait time. Another strategy I employed to create a safe environment where learning was viewed as collective was to provide participants plenty of time to think. The tasks used in IM had a high cognitive demand and were explored for a substantial length of time so I provided time to be thoughtful and engage in each task. When I was ready to move to whole group discussion, after a participant shared an idea, or we had a discussion about a task but was met by silence from the participants, I asked them whether their silence indicated a need for more think time, confusion, or something else.

My intention was to get them to take more control of the direction of the conversation. For example, during Week 2, I told a participant in front of the whole group, “Hey, this is your class, man. You tell us to zip it if you need some time” (September 16, 5:22:55). A few times throughout the course I reminded them this was their conversation. For three class meetings (Weeks 1, 6, and 7) there were no instances of wait time. Week 1 was our first instructional class meeting, and we spend a lot of time getting comfortable with Fraction Bars. Many of the discussions pertained to technology and not mathematics. The participants reviewing each other’s lesson plans largely consumed the Week 6 class. Week 7 was when I assigned seats, and the participants were getting to know their new partners.

There were four instances where participants asked for wait time while thinking about a task. The first two occurred in Week 4, the first class on division. Two participants were working together on modeling 1/3 divided by 3 and I was talking with them about their thoughts. One of the participants was explaining her area model and asked her partner and I to “hold on” twice while she collected her thoughts and reminded herself how the area model worked (September 30, 6:13). Later in the same class during a whole group debriefing, a participant was sharing an area model for 2 divided by ¾. Another participant asked her if she would instead share a model for 2/3 divided by ¾ so she could compare it to a number line model that had just been shared with the group. The participant responded that she had not done that one and she needed to think about it. After about a minute, the participant was at the front computer creating the requested model. During Week 8 a participant asked another participant a probing question during her explanation of a model. The participant asked for time to think. Finally, during Week

11 as I was trying to get the whole group back together to debrief a task, a pair of participants asked for a few more minutes to finish talking before the whole group began discussing.

In the phone interviews some participants spoke about having plenty of time to think. Nine of the fourteen participants commented about having plenty of time to think in class. One said, “She really gives you an opportunity to think about why you’ve chosen what you’ve chosen” (Salihah, Week 8). Another said, “She’s almost like a psychology major. She’s, like, she puts it out there and then she waits for us to help ourselves” (Joyce, Week 3).

Determining the direction of the conversation as a group. Sixteen times over the course of the 14-week class, I asked the group to vote on which aspects of a task to discuss or asked them how they would like to proceed. My intention was to give them authority and to lessen my role as leader of the group. The participants routinely asked questions and engaged in the discussion; however there were four instances when a participant changed the direction of the conversation as opposed to supporting the conversation in its current direction. All of these instances occurred in the first 6 weeks of class.

Two of the four instances occurred during pedagogical conversations, and the other two occurred during mathematical discussion. An example of a pedagogical lead change occurred during Week 4 while we were debriefing about a double number line model for a problem. I noticed two participants mumbling to each other and teased them about it openly. One of them began to share his experiences with using a more discovery-based approach when he first starting teaching. The participant completely changed the conversation from whether the double number line model really demonstrated division to how teaching in a discovery manner was a challenge.

An example of a lead change related to mathematics occurred in Week 5 while the group was debriefing about a task in which the participants examined models for 3 ÷ ½. The participants were whether the models showed 3 things cut *in* half or 3 things cut *into* halves. Another participant addressed the group and asked, “Are we sometime gonna just stop and discuss what is division?” (5:40:40). He continued by saying he believed division was more than just sharing. In response, I suggested we look at the next part of the task where it seemed the sharing model did not work.

Questioning the Mathematics

In order to convey the notion that errors and tentative ideas were just as welcome as well thought out responses to tasks, I used clarifying questions and probing questions. My purpose was to consider everyone’s ideas equally by putting them all through the same rigorous questioning in order to show that everyone’s mathematical ideas were worth considering and to press the importance of explanation.

Clarifying questions. I asked clarifying questions to understand what a participant was saying. I might not have quite understood the mathematics, the model, or what was being said. At times I was concerned that a thought could be interpreted multiple ways and asked a question to illuminate the participant’s intended meaning. Throughout the 12 instructional class meetings of IM, I asked 207 clarifying questions. I was surprised to see that I asked no clarifying questions on the first night of class. However, upon reflection, I noted that the primary focus during Weeks 1 and 2 was on learning the expectations of the group, the IM website, the write-up formats, and the operation of the Fraction Bars software, so I had few opportunities to ask clarifying questions because we were not doing a lot of intensive mathematical discussion. We began intently working on tasks related to fraction multiplication during Week 3, and I asked 33 clarifying questions that week. After Week 3, I averaged 17 clarifying questions per class meeting, and these questions were asked both during whole group and small group conversations.

The teachers began asking clarifying questions the same night I began asking them myself, which was Week 2. Someone other than me asked a clarifying question at least once each week, and there were an average of seven participant-generated clarifying questions asked each week. The first instance of a participant asking a clarifying question occurred during a whole group discussion. After a lengthy discussion of the beaker problem (see Figure 1), which was the first task participants were expected to work on individually, I asked the group if there were any questions. Carrie struggled with making sense of this task and in her write-up had concluded "Sam was the one with the correct explanation". This comment demonstrated that she was not flexible in identifying the referent unit for different parts of the task. In class, her body language indicated to me that she was confused by the explanations offered by two other participants. As I encouraged her to take direction of the conversation, she explained she needed to go back to an idea that was shared earlier by a participant. I began to suggest that the participant re-explain her idea when Carrie asked for herself. Although this was not a specific clarifying question, such as asking about a particular part of her explanation, Carrie needed clarification on the participant's mathematical reasoning. Thus, this instance was the beginning of participants asking each other clarifying questions.

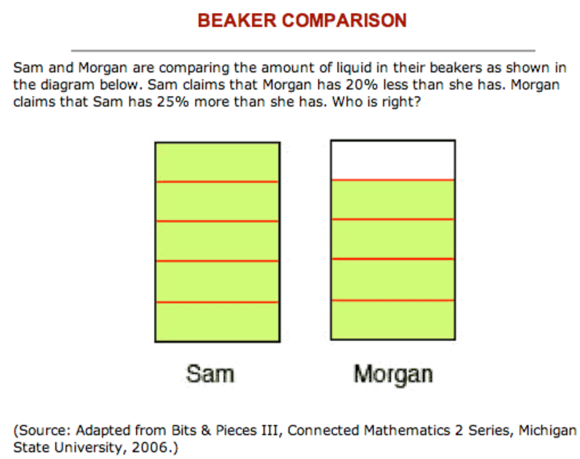

Figure 1: Beaker Comparison Task

Over an hour and half later in the same class when another participant was sharing her model for a new problem, Carrie, without any encouragement from me, asked her to repeat her explanation, indicating to me that Carrie did not understand the explanation. Others began to ask questions during this class meeting as well. In fact, toward the end of class, Carrie was explaining her solution to a problem. She was one of two who vocalized a differing opinion than the rest of the class. As she explained to the group why she disagreed, another participant asked her, “So how do you understand it?” Carrie sarcastically thanked him but proceeded to explain her thinking. The intent of the question was to ask Carrie to explain her understanding of the problem further. Although the question was sarcastic and provoked some laughter, Carrie proceeded to explain her mathematical thinking in more detail.

Probing questions. I asked probing questions when I was attempting to push our understanding of the mathematics at hand. The first probing question I asked the group occurred an hour into the Week 1 class meeting when the participants were engaged in answering a question using Fraction Bars. The group was discussing the copy feature and

I asked, “Why do we care about copying the bar? Why not just make two bars?” (September 9, 6:00:53). I knew that the result of the pre-assessment showed that the idea of a referent unit was challenging for some members of the group, so this seemed like a perfect opportunity to push the teachers to reflect on the link between the copy function of the software and the mathematical idea of referent unit, which was one of the themes of the course. Similarly, in Week 7, I used a probing question as a warm-up for the class on decimal division to again push the participants’ understanding of referent unit. I knew that the importance of referent unit had not been fully explored and understood by the group during our discussions of fraction division in Weeks 4 and 5, so to start Week 7 I asked, “What is the referent unit for each number in 2/3 divided by ¼ = 8/3?” I asked an average of 14 probing questions per class meeting with the greatest number of probing questions (n = 24 and 29) being asked during weeks that we discussed fraction and decimal division, which were two of the more challenging concepts we tackled during the course.

The participants began asking probing questions of each other and about the mathematics during the Week 3 class meeting and continued to ask probing questions every class meeting after Week 3, averaging 7 probing questions per class period. One of the first probing questions from a participant occurred in a small group discussion in Week 3 when I was trying to support a participant’s understanding of a task. I had asked a question of the participant, and her partner was patiently listening to our discussion. I was not having success so the partner stepped in and asked, “What would you multiply 1/3 by to get 1?” (September 23, 7:36:50, Diane). I coded this a probing question because the participant was struggling with the task and this question gave her a specific example with numbers to explore without immediately giving her the answer. Thus, the question pushed her mathematical understanding.

Participants not only asked probing questions during small group discussion but also during whole group discussions. For example, during the final class meeting, Week 13, a participant asked the group, “Did we ever decide why the division algorithm works?” (December 16, 5:40:00). This was the ultimate probing question for our group, because we had not (and never were able to) resolved why the division algorithm worked despite all of the time spent on exploring division. The group dismissed this question by suggesting we did a task or a reflection about this question and everyone seemed satisfied. We had only 17 minutes until our break before taking the post-assessment, so I did not pursue this question (although I wished I had the time to).

Participants' views of questioning. Ten of the fourteen participants talked explicitly about questioning during phone interviews. For example, one participant said, “She asks what-if questions to cause us to think, but she never gives a direct answer” (Patton, Phone Interview, Week 3). Two participants described my teaching as similar to the Socratic method. One said, “[She is] more of a guide, a person who poses the question and guides us through the discussion. I don’t think she really tells us right or wrong or tells us where to talk so it’s more of a I guess kind of Socratic method” (Diane, Week 8). Finally, one participant noted that my questioning and patience were things he learned from IM that would bring back to his classroom (Keith, Week 12).

Stressing the Importance of Others

The final category of my behaviors that were intended to support the development of a community of practice in IM pertained to the value of others. Two of my goals as the instructor were to convey that learning is a collaborative activity and that all ideas and thoughts were valued, whether right or wrong. To achieve these goals, I routinely encouraged collaboration, restated ideas, connected ideas, and avoided providing answers.

Encouraging collaboration. One of the simplest strategies I employed to convey that learning is a collaborative activity was to remind the participants to work together on tasks. If things became quiet during small group discussion time, I would comment on the quietness. Throughout the course I suggested they work together or commented on the silence 33 times, and 20 of those times were in the first 6 weeks of the class. The participants generally responded by working together on a task. Two participants often sat by themselves in class at the beginning of IM, which is what eventually led me to assign seats during Week 7. Each week I quietly encouraged these participants to move to sit next to other people to work. During the whole group task portion of class in Week 2, these participants moved so everyone was paired with at least one other participant. However, they returned to working alone during the individual task work later in the class period. After assigning seats in Week 7, I only encouraged collaboration eight times, suggesting that my strategy of assigning seats was effective.

In phone interviews, the benefits of hearing other viewpoints and knowing and understanding how others were thinking was referred to by many. One participant, in explaining what she had learned from others in IM that she would not have learned alone said, “You know, it’s every time we talk. Every week, I hear a point of view or hear math concepts explained and thought about in ways that were a little different from mine.” (Donna, Week 12). Another spoke of my teaching approaches when he said, “She has the approach that you know the answers, there are no right or wrong answers. But everyone should be able to learn from each other. And she tries to make sure that she is not the center focal point, is the center focal point, that it is on us the students." (King, Week 8).

Connecting ideas. I frequently connected our mathematical ideas and conversations together by referring to the similarities and differences between participant's models or ideas or referencing previous classes and previous conversations we had as a group. My goal was to make mathematical connections and emphasize that by connecting our ideas together we were able to learn more than if we had worked individually. I offered connections 49 times and did so each week except Weeks 1 and 3. In Week 1, we were beginning to work together and no shared mathematical history. During Week 3, I was transitioning the class from focusing on referent unit and the task from Week 1 and 2 to fraction multiplication. The participants' second write-up was due this class.

For example, participants often referred to each other's models and solutions to tasks we had solved. A participant referred to another participant's idea or a past conversation in small group discussion or whole group discussion during every class meeting with the exception of Week 1. Often in whole group conversations references were made to mathematical ideas that had been discussed previously. The most common references were to two tasks having to do with referent unit. In particular, participants repeatedly returned to the ideas illustrated by these tasks: 1. The referent unit does not have to be one whole, and 2. A person's perspective on the question being asked can change the referent unit. The participants commonly referred to the double number line model, which was a new model to almost everyone in the group. The task used to introduce this model was often referred to when participants were considering making another double number line or talking about types of situations where a double number line was a reasonable model.

Refraining from providing answers. In order to de-emphasize my role as an authority figure, I rarely provided direct answers to questions about correctness or the answer to a task. In the first few weeks of class, many participants asked if their ideas were correct or if the group was right after a debriefing. My response was either “I don’t know” or “What did the group decide?” I continued to use these responses every time a participant asked for external validation of an idea. In my journal entry for Week 1, I described a debate between two participants during a whole group discussion. I wrote, “I decided to let the conversation stop there because I felt like we were at a standstill and only two people were deeply participating. Although others were definitely following along. I did not say what I thought the answer was which caused a minor stir.” (Journal entry, September 10, 2008). By establishing the routine that I did not give the final answer after our discussions, I hoped to convey that decisions about correctness would come from our individual and collective mathematical reasoning and not from me, which pointed out the value of our group discussions.

Participants noticed my behavior and commented on it occasionally in class. During Week 6, one participant commented that I had never said he was right in the past 7 weeks of class (October 14, 5:47:30). Many spoke about my avoidance of answers in the phone interviews. In response to a question about the strengths and weakness of IM, a participant said, “I like the fact that she doesn’t really give us the answer. We actually have to come to some type of consensus after we investigate a problem. We have to come to a consensus on the answer and then make sense of it" (Sharlene, Week 12).

Summary

Many of my behaviors were found in the high press classrooms in Kazemi and Stipek's (2001) research study. In these high press classrooms students justified their reasoning, an importance was placed on explanation, and comparing strategies was valued. Similarly, some of my behaviors have been associated with maintaining the high cognitive demand of tasks (Stein et al., 2000). Providing sufficient time for task exploration, asking probing questions, pushing for explanation, connecting mathematical ideas, and expecting participants to justify the correctness of their responses are all characteristics of teachers who keep the cognitive demand of a task high rather than succumbing to participants' attempts to reduce the cognitive demand of a task. The fact that participants began to mimic my behaviors in both whole group and small group discussions suggests that they were taking ownership of the class and beginning to form a community.

Tensions

I struggled with the tensions between my nested roles as the IM instructor, a researcher working on her dissertation, and a member of the DiW research team. As the instructor I felt a constant tension between my intentions of increasing participant's mathematical content knowledge (the goal of IM) and building a mathematical community of practice (the goal of my dissertation research). As the instructor and a member of the DiW research team I felt tremendous responsibility to keep the participants and the members of the research team satisfied with the experience. At times the needs of the participants and the research team were incompatible with one another and at other times their needs were incompatible with my dissertation research goals. For example, the DiW team expected me to be true to the design of IM with respect to content coverage and pacing, but at times my participants needed to spend more time on a topic or were only able to get to a certain level of understanding in the time allotted. For my dissertation research, building a community was a priority, but balancing community building and pushing the participants' mathematical knowledge was a challenge. In addition, between IM class meetings, members of the research team were giving me feedback on how the class was progressing and suggestions for how to proceed. I welcomed this input, but it added to the tension. Finally, one of the DiW PIs was on my doctoral committee, which made me all the more self-conscious about my multiple and competing roles and hers as well. All of these competing demands influenced how I led the class meetings and worked toward building a community of practice. With all of these pieces shaping my actions, I reached a point in Week 9 where I could no longer manage all of the competing goals harmoniously and had to pick one over the others. In this section, I will describe the various tensions I felt in more detail and explain the choices I made in Week 9 and the results of those choices.

My Role as the IM Instructor

The original InterMath project developed syllabi for several mathematics content courses for middle school teachers, including one on rational numbers. In my role as one of the IM instructors for the DiW project, I took a leading role in tailoring the original IM syllabus to needs of the DiW project and the participating teachers. In refining the syllabus, I considered feedback from participants and instructors in a previous version of the IM rational numbers course that spanned over 50 hours of instruction. Based on the feedback from the participants and instructors from the previous offering, the DiW research team decided to cut the class back to just over 40 hours. The design of the modified syllabus was especially important to me as I planned my dissertation research because the pacing and tasks allowed me time and opportunity to build a community. For example, there was time at the beginning of the course for me to lead a discussion about the expectations participants had about IM, myself as the facilitator, and themselves as participants. I felt passionate about all the tasks and the sequencing of the concepts, so my goal was to be as true to the modified syllabus as possible. The pressure to examine all of the content topics we had determined before the course began got stronger as the weeks passed. Knowing the importance of flexibility in teaching, the research team included two class meetings with no set agenda on the syllabus so the instructor could make decisions based on the students' progress. As I taught the class, various situations arose and some modifications to tasks and class meetings were made. All of the modifications I made were the result of conversations with at least one other member of the DiW research team involved in data collection. I wanted the course to be successful not only for the participants but also for my research and the DiW research project.

Mathematics Versus Community

As the instructor of a PD course with a primary goal of increasing the content knowledge of the participating teachers, I struggled with balancing the participants' mathematical development and my interest in building a community. One of the principal ways I tried to build community was by fostering productive conversations as we debriefed our mathematical work on tasks in whole group setting, but I rarely had enough time for all the tasks and conversations I had planned. Thus, I was faced with making decisions about doing more tasks to meet the aim of the course versus doing fewer tasks and having longer conversations about them to meet my goal of building community. On October 8th I wrote, "I was nervous about being able to "cover" everything and being able to talk about everything." The tension between building the teachers' mathematical knowledge and building community was overwhelming at times and made me feel like I was unable to be successful on either front. After the Week 3 class, I wrote, "There is so much that we needed to talk about and do but we didn't have time. I'm concerned because we move on to division of fractions next. I feel like I'm leaving many participants with holes in their knowledge" (Journal, September 24, 2008). Because of the time crunch and the tug I felt between debriefing the mathematics and debriefing our conversations, I never explicitly addressed my community building efforts with the participants. Although I praised the group at the conclusion of a good conversation, we never discussed what made those conversations special in terms of either building a community or developing mathematical knowledge.

Pressure of Research Team

I placed a significant amount of pressure on myself because I knew the success of the DiW research project was partially dependent on the quality of data that were collected during the IM course. I knew people's careers were invested in this project and another researcher was basing her dissertation on what happened in the course. Although there was another research site implementing this IM course and generating data, I felt especially responsible for ensuring that my course produced good data because my group of participants was closer to the ideal profile identified in the grant and because one of the PIs was attending every week of my class and the other was watching the video every week.

This perceived pressure caused me to feel defensive about my teaching and to feel a tension between being a member of the research team and being the object of discussion as the instructor. In a journal entry after the Week 3 class, I wrote about a comment one of the PIs made about me missing a teachable moment. I was disappointed when I heard the comment and also defensive because I wondered what more could be expected of me. A few weeks later I wrote, “Last week when [PI 1], [PI 2], and [graduate student 1] were talking about my teaching and what they wanted me to do, I was completely defensive” (Journal, October 8, 2008). They had suggested a task for my participants to do, and I felt like I was witnessing a conversation about me but no one bothered to ask me what I wanted or thought. At other times I felt like a genuine member of the research team who was responsible for making shared decisions and did not feel singled out for discussion as the instructor. I had numerous previous experiences as part of a team that both organized and led PD efforts, and many of these experiences were with the very same people who were on the DiW team. Given these prior experiences, I was surprised by my defensiveness in this situation and attribute it to being the sole facilitator of the PD.

Another factor that led me to feel pressure was a sense of competition between my IM class and another IM class that was being offered in different school district. Originally the other instructor and I were going to meet weekly and plan the classes together. Due to scheduling difficulties, our weekly meetings ended after just a few weeks. Because the research team was videotaping both classes, I often heard my colleagues talking about how different the classes were from each other. On October 1st I wrote, “[Other instructor] and I have very different styles of teaching – that’s what I hear the rest of the research team saying.” I never felt the need to find out the specifics of these differences, and I did not perceive that the research team was making a value judgment about my class versus the other one. I did feel uncomfortable with the comparisons though. In addition, in my role as a DiW researcher I was conducting phone interviews with half of the participants in the other course during the same semester I was teaching IM. Listening to those participant responses made me nervous about what my participants were saying and caused me some anxiety as I wondered how others, particularly the DiW PIs, were comparing our classes.

My Breaking Point

The preplanned syllabus for the IM course called for a switch in focus from multiplication and division of fractions and decimals to a focus on ratios and proportions in Week 9. As we began to get closer to Week 9, I began to question the wisdom of this change in topic. I thought the teachers were really making some progress in their understanding of fraction and decimal operations, but I also knew they had some holes in their understanding. I was confident that with a few more hours of work on carefully chosen tasks, we could close at least some of those holes. I was also concerned that these holes in the participants’ knowledge about fractions and decimals would cause problems in their ability to explore ratios and proportions deeply. Further, knowing how long it had taken us to thoroughly address fraction and decimal operations and knowing that the teachers had less prior experience with ratios and proportions, I was concerned that I was going to expose more gaps in their knowledge that would leave everyone frustrated because we would not have time to thoroughly explore these topics. Finally, I was concerned about my own ability to handle complicated mathematical conversations about ratio and proportion, given my relative inexperience with leading PD on these topics and given the fragility of the teachers' knowledge. Thus, I thought it was in the best interest of the teachers to continue to focus on fraction and decimal operations, and I made my concerns known to the rest of the research team.

After discussion and deliberation, the team decided to forge ahead with ratio and proportion despite my feelings (and, I thought, sound rationale) that it was not a good idea. Addressing ratio and proportion was important to the research team because these topics aligned with the state curriculum, and there was potential for collecting data on teachers' instruction on these topics in the following semester. Thus, the goals of the research project trumped my goals as a teacher and, to some extent, my goals as a researcher because I was concerned our community might erode as we tackled the difficult topics of ratio and proportion without a sound footing in fraction operations.

My reaction to the team's decision surprised me: I became extremely direct in my teaching. I gave a direct answer or shared a personal idea about a problem at an increasingly high rate from Week 9 until the end of the course. I observed this behavior 74 times in the course with 89% of those times occurring after Week 8. Thirty-five of those times were in Weeks 11 and 12. Clearly, my teaching had become much more direct and very different than the first eight weeks of IM. Because there was little time left to address these significant and complicated mathematical ideas, I wanted to carefully lead the group through the IM tasks. There were many big ideas left unfinished from the first 8 weeks of class, and I was afraid to add more to the list.

The participants never mentioned the change in my teaching, and no one on the research team commented on the change either although I was vocal about it to them. In my December 3rd journal entry I wrote,

> I've been getting progressively more directive in my leading of class. I would say this has been happening the last two classes. I know time is limited and I've felt like it was a "mistake" to push teachers into the ratio and proportion content. They were really challenged by it and we had so many loose ends from earlier in the course. Since we are forging ahead and the number of classes is dwindling down, I have felt like I need to be more directive. There are certain issues that we HAVE to talk about and things that MUST be said. At least that's how I'm feeling.

After reflecting on my decision to become more direct, I am embarrassed by my behavior because it did not support any of my goals as the instructor of IM (to create a safe environment, to stress that learning is collective, and to emphasize that errors are part of the learning process), and it subverted my research interest in community building. Fortunately, the community did not erode; however this drastic change in my teaching style became a test for our group, which I describe in detail in the Mutual Engagement section of this chapter.

Community

In this section, I present data relating to my second research question about the characteristics of a community of practice exhibited by the group and how our community developed. I used Wenger's (1988) definition that a community must exhibit mutual engagement, joint enterprise, and a shared repertoire to organize this section.

The participants had very different reasons for enrolling in the IM class but similar goals, which they articulated in the first phone interview. Seven of the participants signed up for the IM class because they were told to or felt some administrative pressure to participate. Five participants wanted to learn new ways of teaching rational numbers or to become better teachers, and one participant enrolled to earn credit to renew his teaching license. Most of the participants identified goals related to the teaching and learning of rational numbers. Ten of the participants wanted to learn new ways to teach or become better teachers while two participants wanted to learn the "why" behind the mathematics. One participant wanted to learn new visual aids for teaching rational numbers, and another stated her desire to be more comfortable with the material to be able to share it with her colleagues. The participants were also asked about their history of working with other group members. While some of the participants knew each other casually, most of them had not worked together professionally in the past.

Mutual Engagement

Wenger (1988) described mutual engagement as having a collective purpose, which involves negotiation. Wenger (1998) noted that practice "exists because people are engaged in actions whose meanings they negotiate with one another" (p. 73). In this section, I provide evidence of the mutual engagement of the IM participants by describing the characteristics of our whole group discussions, the importance of explanation, a discontinuity we faced involving the nature of tasks, and the role of pushing our mathematical limits.

Characteristics of whole group discussions. Our whole group discussions around mathematical tasks provide evidence of the participants' mutual engagement. I illustrate mutual engagement with an excerpt from a group discussion in Week 8 that spanned 41 minutes. As the excerpt begins, Walt and Salihah had noted a possible connection between the algorithm for multiplying fractions and array models (Walt & Salihah, October 28, 5:33:50). They had discovered that the array model for a fraction multiplication situation such as $\frac{1}{3} \bullet \frac{2}{5}$ generated two pieces of the standard algorithm. In particular, they noted that the numerator was represented with a 1 by 2 array, and counting all of the boxes appeared to give the new denominator. In order to explain the possible connection they saw between the array model and the algorithm, Walt and Salihah showed the group an example using $\frac{2}{3} \cdot 5$ and drew an array model (see Figure 2).

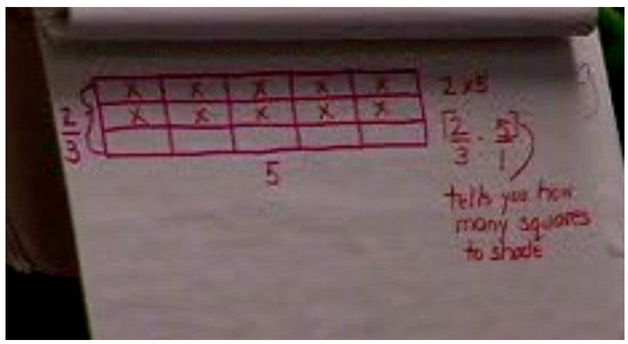

Figure 2: Walt and Salihah's Array Model for $\frac{2}{3} \cdot 5$

As the group considered this model and the explanation given, one participant raised a possible source of confusion for students with regard to the model. During this discussion I was standing at the back of the room, and the chart paper with the model was at the front of the room.

[05:39:40.02]

Joyce: (looking toward front) How do-how do we keep kids from being confused that it's not 10 fifteenths?
Walt: Yeah, that it's not. It is 10 fifteenths!
Will: It is!
Walt: That is what it is.
Rose: It's just two-thirds 5 times.
Rachael: What-so what are you wanting your kids to say, I guess?
Joyce: I mean ho-how do our kids not look at that after some of the stuff we've been doing and go, "Oh, the answer is 10 fifteenths"? Instead of-

Walt: Oh, oh, oh.
Joyce: Which is a very different answer.
Rose: Oh, 10 fifteenths. I see what you're saying.

A few participants talk at the same time.

Claire: Those are 5, not fifths.
Rachael: Right. Those aren't fifths. Those are 5 wholes.
Claire: They're 5 wholes.
Joyce: But-but traditionally we've been doing it. I mean like if you-we could look at every picture I drew where all the blocks make one unit. And so, if our whole becomes the unit and we teach our kids to think that way, then they look at this and they go, "Oh, 10 fifteenths." And it's wrong.
Diane: I tell you what. I kept doing that and I was like "I'm getting the wrong answer." But then I measured it on Fraction Bars and it turned out right. So, I-
Carrie: (quietly) I don't see 10 fifteenths.
Rose: How many-how many total blocks are there?
Will: There are 15 total blocks. And 10 of them colored.
Carrie: Gotcha. Ok.

Sharlene, Carrie's partner, is also talking to Carrie about the 10 and the 15 in the model as a sidebar.

Walt: I'm not sure.
Joyce: Wait, I get it.
Walt: Ah-ha, yeah.
Joyce: But-but I'm not sure the students I teach can make that leap.

There was a one-minute segment where I inserted myself into the conversation by asking questions to direct their attention to referent unit issues. Specifically, I was attempting to help the group untangle the issue of the difference between the bar representing 1 or 5 and the implications of each of those answers. Following this segment, the group returned to the issue of the model and its utility in teaching.

[05:42:18.28]
Walt: Then it's not really a very good teaching tool?
Will: (looking at Walt) Well, yeah, I think that it can be modified simply by-simply by putting a space between each one of those units. And that simplifies the whole thing. Instead of doing these (and he puts his hands together)
Walt: Close, but not touching.
Will: closer but not-but.

I clarified what Will was saying, and there were a lot of comments on his idea.

Walt:	(to Joyce) would that work?
Joyce:	Yeah, that would work.

I interjected for about a minute to push the group to try to make sense of the algorithm and the picture.

[05:43:46.10]

Rachael:	Can we make sense of where we see the denominators being multiplied here? Cuz that's the trick, right?
Walt:	Yeah. Yeah.
Claire:	That's the 3 times the 5.
King:	But is the 5 the denominator?

A few say no.

Claire:	1 is the denominator.

Rachael and Walt agree with 1 being denominator of 5.

Claire:	So we'd have to separate them out.
Walt:	Yeah, that's where the separating comes out a little bit.
Donna:	(looking at Walt) Well, with the space between them - just a little space!

A few participants laugh.

Walt:	And still-still make it a legitimate array.
Donna:	It still could be clear that it's 10 thirds.
Walt:	Ok.
Donna:	Yeah. And there's your denominators.
Walt:	Instead of 10, 10 fifths.
Donna:	10 thirds.
Walt:	Yeah.

[05:44:32.28]

Joyce sparked this discussion by questioning whether the area model would be misleading to students, something that many in the group, myself included, had not recognized. In fact, the model was confusing to participants in the group, as evidenced by Will and Walt's initial claim that the answer was 10 fifteenths. As Joyce continued explaining the dilemma she saw based on her previous work with similar models, other participants commented on their understanding of her question and the pedagogical and mathematical implications of the model. The group gave Joyce the time to explain herself, and no one tried to interject or change the direction of the conversation, suggesting that Joyce's thoughts were considered important enough by the group to be heard and understood before moving forward.

The group not only considered Joyce's question, but participants returned to the question after I interjected to focus the group on referent unit issues that were inherent in the model. Will and Walt suggested a modification to the model (putting space between each unit of 1) to address the question posed by Joyce. When they reached a conclusion, Walt asked Joyce if the modification they settled on would work, and she agreed. This portion of the discussion shows that participants were looking to each other for guidance on the mathematics and for validation on their ideas. The group never brought me directly into the conversation, and no one asked me if they were right.

Further evidence of mutual engagement comes from the fact that the participants were willing to help one another and used questioning to help each other think through the situation. For example, when Carrie quietly articulated her confusion, a participant a few desks away asked Carrie a question to help her through her confusion. A different participant responded to the question, and Carrie's partner was also quietly responding directly to Carrie. Later in the discussion King asked Claire a question to help her see that she had made a mathematical mistake.

In this discussion, no one raised her hand or asked to speak, but the conversation flowed well. The participants looked at each other and the array model as they conversed. I was standing in the back of the room and was never explicitly invited to participate in the conversation as the facilitator, so I did not direct or shape the group's discussion. Rather, the discussion was based on a question posed by Joyce and unfolded as other participants offered observations and ideas.

The example above occurred roughly 2/3 of the way through the course (and before the change in my teaching style as described in the Tensions section) and illustrates the level of mutual engagement we eventually achieved. However, at the beginning of the course I was more directly involved in monitoring the conversation, and the participants did not always talk and listen to one another. In the episode presented below from Week 2 (September 16), the group was considering the Beaker Comparison task (see Figure 1), and participants were sharing their solutions with the whole group. The debriefing of this task lasted for 27 minutes, and this excerpt is seven minutes into that discussion.

[05:26:54.06]

Rachael: Um, Carrie I think you were about to say something too and they-they interrupted.

Carrie: (looking at me) Well, I think if I wouldn't have seen it if the top portion of Morgan's wouldn't have been there I would have been able to look at it differently. And I would have seen that Morgan thinks that part that you're sharing right there is her whole.

A few participants say yeah.

Carrie: And Sam thinks that part right there is her whole. But because of how it looks visually to me, my-I was interpreting it as something else.

Rachael: So, I guess one question that's making me wonder is why do we need this empty space on Morgan's?

Carrie: Right.

Linda: (looking at me) That makes it looks like the beakers are the same.

Carrie and King both say "Right."

Linda: And the way that we're looking at it now is that one beaker is smaller than the other-not as tall as the other beaker? So-

Rachael: So would the problem be the same without this one white piece?

Diane: No.
Rachael: I mean do we need that piece? What is that piece doing for us?
Will: With or without it's the same.
Rachael: With or without it's the same is what I hear some people saying.
Will: But by being blank means it-it's not filled to that capacity.
King: Well by having it there the illustration throws you off. Because you naturally assume that the first one is the whole. The whole is 5 fifths. (looks at Carrie, his partner)
Carrie: Right.
King: And that the second one is incomplete. (glances at Carrie)
Carrie: Right.
King: (looking at me) But if you had taken that space away, you'd be looking-you'd say "Alright well, we're looking at two different wholes." And when you're looking at two different wholes then it's easier to interpret "OK, one is 20% less, one is 25% more." You're able to kinda, you know, deal with that on a different perspective. But when you look at, you know we naturally associate this one is complete, so it's the whole. So that-that illustration throws you off.

Mike raises his hand.

Will: I disagree with that because the illustration says that each part is equal. And that's the only way that that would work. That-I mean in that ah-
Rachael: So show me what you mean by each part. Can you go up there and point?
[05:29:08.06]

This excerpt demonstrates the difference in my role as the group moved from less mutual engagement to more mutual engagement. Early in the instructional class meetings I was very involved in facilitating the conversation. In this example I served as a mediator for the group because I posed a question for the group to discuss, posed more follow-up questions, and restated a participant's response. I determined who got the floor to speak and restated participants' comments as questions. In contrast, in the Week 8 example, I asked a clarifying question of Joyce only because I, like many of the other participants, was not seeing the correct solution using the model. I did not mediate the conversation; rather, I was able to participate in the conversation as the other participants did and was not viewed as the sole mathematical authority in the group.

The degree to which the group took up an idea offered by a participant was evidence of our progress toward mutual engagement. In the Week 8 example, Joyce posed a question without prompting from anyone, and the group considered her idea seriously and extensively. A significant number of people participated in the free flowing discussion with the intention of understanding what she was saying and the implications of her concern. In contrast, in the Week 2 discussion, no one, including me, asked Carrie to say more about how she interpreted the model for the Beaker Comparison problem. Instead, after Carrie partially explained her idea I posed a question to articulate my perception of the issue Carrie was having with the drawings of the two beakers. My decision to articulate Carrie's confusion for her was based on my knowledge of common misconceptions associated with this problem and based on my reading of the participants' write-up for this problem. I had additional knowledge that was not shared by the participants, and I used this knowledge to direct the conversation toward what I though was important mathematically. Although I presented the dilemma of the meaning of the model for the group to consider, only Linda and Will participated verbally in the conversation, and the participants' comments and visual attention were directed largely to me rather than to one another.

Additional evidence of our evolution toward mutual engagement comes from the participants' imitation of my teaching actions, which slowly occurred as we became more of a community. As noted in the My Actions section, the participants began to ask clarifying and probing questions of each other starting in Week 2 and began to give each other the floor starting in Week 3. As the weeks progressed, they increasingly restated each other's ideas and gave each other positive reinforcement for their ideas and models.

Participants recognized the importance of mutual engagement as it related to their learning. When asked to describe his role as a learner in the IM class during phone interviews, one participant said, "When we hear the thinking of others, then that kind of expands our views on the subject of teaching fractions" (Will, Week 3). Another participant said, "It's not like you can sit and just receive information. If you don't understand something, you're going to have to dig deep. You're going to have to go inside yourself; you're going to have to be able to collaborate with other people and talk to other people" (Sharlene, Week 9). These comments suggest that the teachers valued the mutual engagement that came from thinking carefully about other participants' thoughts and ideas.

Explanation. Another characteristic of our group that reflected mutual engagement was the expectation to explain when presenting a drawn representation or a mathematical model and the norm that others would question those representations and models. For example, during Week 5 I checked in with Rose and Claire as they worked on one of the group tasks. Rose told me they were done and then said, "But I'm not ready to argue though," suggesting that she understood that the had an obligation to be prepared to explain her thinking (Sept 30, 5:24:08) In another instance (Sept 23, 5:57:32), Keith said that his group disagreed with Linda's group, and she turned, looked at him and said, "Convince me! Convince me." This short exchange shows that participants were not reluctant to challenge one another and that they expected each other to back up their claims with explanations.

A more extensive example of the participants' expectations for explanation comes from a whole group discussion in Week 4 (September 30) where Keith explained his group's number line model for $\frac{2}{3} \div \frac{3}{4}$ (see Figure 3).

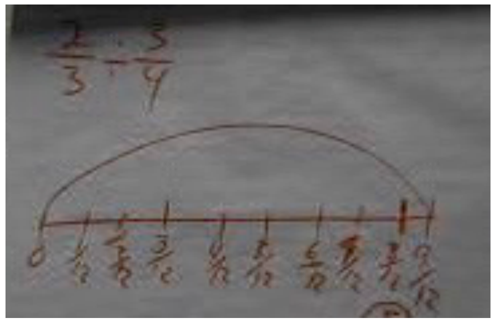

Figure 3: Keith's Number Line Model

[06:28:55.02]
Keith: Alright, well, this is 2/3 [darkens mark for 8/12]. This is the one that we were like "Well, we don't really understand what's going on here." But this is 3/4 (draws arc). And you can't quite get 3/4 into 2/3. Right?

A few participants say "Huh?" and I repeat what Keith said.

Keith: You can't get 3/4 into 2/3.
Rachael: So how can you-
Keith: But I was-we were dividing. Our question, if it was a fraction, we were asking ourselves how many 3/4 can we put into 2/3?

I repeat Keith's question for division. A few participants say "yes" or "right."

Keith: Well you can't put 3/4 into 2/3. Not if it's-
Rachael: Because it's bigger than-
Keith: Right, 'cuz it's bigger than 2/3. BUT you can get 1, 2, 3, 4, 5, 6, 7, 8 (counting marks on number line up to 8/12) ninths of 3/4 into 2/3.

Seven second silence.

Carrie: So lost, it's <u>unreal</u>.
King: OK.
Keith: See I've taken my 3/4 and I've divided it up into ninths. I can't get the entire 9/12 into my 2/3. I can only get 8/12. So I can get 8 ninths. Right? 'Cuz I've divided my 3/4 into ninths.
Carrie: I don't see that at all.

Donna: I understand it better with instead of into, out of. How many times can I subtract 3/4 from 2/3? Not quite one time. 8/9 of one time.
Claire: Same thing.

A few participants start to talk at once in agreement.

Mike: That-that's exactly right.
Claire: Same thing.
Mike: It's-that-to me, both things. One is how many 3/4 inside of 2/3? And-and it's not a whole one because it's bigger. The other one is-the other way to look at it is how many times can you subtract 3/4 from 2/3? Both of these to me is part of the definition of division. And you do division. And so also there's not quite 3/4 in 2/3. So when you subtract, it will be less than that. So you have to have-the unit has got to be the 3/4 here. That's the unit that you're working with to divide into 2/3. So that's-that's how he came up with the 9/12 and all that. 'Cuz you end up with 3/4 has to be the unit.
[06:31:29.04]

This example was typical in that when a participant shared a model, solution, or idea, it was standard for an explanation to follow. The purpose of explanations was not only to present an argument supporting a solution but also to assist other participants in understanding the task from this perspective. When Donna described how she thought about division and Mike concurred, they were relating their understanding of division to Keith's explanation. When Mike commented that 3/4 did not fit a whole time into 2/3, making it complicated, he revoiced the challenge that Carrie had indicated and that several other some participants were encountering with the model.

As this example demonstrates the participants posed probing questions when they were unclear about concepts or to support another participant's mathematical understanding. We also used questioning to improve our explanations by pushing one another to explain ideas and thoughts more clearly. As we worked together our explanations and questions became more regular (as described in My Actions section).

Together we established the norm that to engage in our community you must be prepared to explain your reasoning to the group.

The norm for explanation was also evident in the write-ups from the participants. By Week 5, some subtle changes in the quality of write-ups were evident. In the Week 5 write-ups, only five participants talked explicitly about referent units (see Shared Repertoire for further analysis of this vocabulary term). All five write-ups included models of the division expression $\frac{3}{4} \div \frac{2}{5}$ and used a quotative model to make sense of the expression. For example, Claire wrote about asking herself, “How many 2/5 are contained within ¾?” in order to reason using her area model. Eight of the 14 participants used models to explain Jacob’s Idea. Seven participants wrote thorough accounts of their reasoning by using a model, the algorithm, and another example to provide multiple ways to verify their reasoning. The additional examples used included a second area model partitioned differently than the first, using the context of money, using an algebraic argument, or relating the mathematics to a previously explored IM task. These norms reflected the collective agreement of the need for explanation, part of our mutual engagement.

Characteristics of tasks. Most of the problems in IM could be classified as mathematical tasks using Stein et al.’s (1996) definition and had the characteristics of higher-level demand tasks identified in the task analysis guide developed through the QUASAR project (Stein et al., 2000). The IM tasks were designed to have multiple entry points and high cognitive demand, which fostered mutual engagement by leading to rich mathematical discussions. The Beaker Comparison Task (see Figure 1) provides an example of a typical IM task where it was not clear how to start or approach the problem.

Clearly, the given model could provide a place to start; however, many participants used their number sense to reach a conclusion. Many IM tasks provided drawn representations or required the solver to produce a drawn representation, which also stimulated mutual engagement as we tried to compare, contrast, and reconcile differing representations. For example, in the Exploring Division with Fractions problem (see Figure 4), teachers created and analyzed multiple representations and word problems for each of the numerical expressions. When we discussed this task, I asked participants to share two different area models and a number line model for one of the expressions. After the group made sense of the models, we compared and contrasted the models. The conversation about the three models lasted 34 minutes, and we spent another 24 minutes exploring the patterns participants noticed.

EXPLORING DIVISION WITH FRACTIONS

Without using an algorithm, consider the quotients for the following problems:

$2 \div 3$	$\frac{1}{3} \div 3$
$2 \div \frac{1}{4}$	$\frac{2}{3} \div 3$
$2 \div \frac{3}{4}$	$\frac{2}{3} \div \frac{1}{4}$
$\frac{2}{3} \div \frac{3}{4}$	$\frac{2}{3} \div 4$

What do you notice about the similarities and differences in the different division problems?

What relationships do you see in these problems?

What real-world problem could you use for each of these problems?

Figure 4: Exploring Division with Fractions Task

Some tasks provided a solution to a word problem and called for an assessment of the correctness of the mathematical thinking in the solution. These tasks were especially helpful in fostering mutual engagement early in the course because the discussion did not depend on the participants producing solutions. Frequently, an IM task required teachers to find more than one solution path to an answer, which allowed for many different mathematical conversations during our discussions.

The importance of the nature of the tasks used in IM was highlighted during Week 10 when I created a task, Purple Paint (see Figure 5) to address my concerns about moving forward with the ratios and proportions portion of the syllabus. (See the Tension section for a thorough discussion of this dilemma and the resulting change in my teaching style.) Purple Paint was an extremely straightforward and directive task because my goal was to assess what participants understood about ratios and models in the least amount of time possible. The task had a lower cognitive demand because directions on how to begin and what models to create were embedded in the problem (Stein et al., 2000). The participants noticed the change in the nature of the task because they expressed concern that the task felt too easy, and they were sure they were missing something.

PURPLE PAINT

One batch of a certain shade of purple paint is made by mixing 3 pails of blue paint with 2 pails of red paint.

A. Complete the following ratio table:

# of batches	1	2	3	4	5	6	7	8	9	10
# of pails blue paint										
# of pails red paint										
# of pails purple paint produced										

B. What patterns do you notice in the rows and columns? How would the entries in the *n*th column of the table be related to the entries in the 1st column of the table?

C. Create a diagram of this ratio with Fraction Bars. How can you use your diagram to find how many pails of red paint you will need if you have 24 pails of blue paint and want to make the same shade of purple described above?

D. Use a double number line to show equivalent paint ratios for the shade of purple in this problem. What are some advantages and limitations of double number lines?

Adapted from: *Mathematics for Elementary Teachers*, Sybilla Beckmann, 2nd ed., 2008.

Figure 5: Purple Paint Task

The discussion surrounding this task was very different from the examples I gave above, and the group exhibited very little mutual engagement. Many worked on the task alone without interacting with their partners and were ready to discuss the solution with the whole group before they had consulted their partners. This behavior was a change from previous sessions where participants typically worked together to generate a plan for attacking the problem or discussed their solutions and solution paths before we held a whole group conversation about the task.

During the whole group discussion of the Purple Paint task participants explained their solutions but did not collectively work together to make sense of ideas, questions, or models for the task put forth to the group as described in the Week 8 example. As a result, in order to encourage explanation and consideration of patterns in the table, I was extremely involved in the conversation and resorted to the mediator role described in the Week 2 excerpt. I asked multiple clarifying questions and added my opinion to the discussion.

The lower cognitive demand of the task appeared to impact the engagement of the group. For example, when Walt posed a question about the relevant referent unit for a portion of the table (Walt, Nov 18, 6:01:47), the participants and I shared opinions but did not thoughtfully explore any one suggestion. The discussion consisted of many people sharing ideas with no conclusion reached (similar to our beginning discussions). After roughly five minutes of discussion, Donna asked Walt if he had an answer to his question yet, and Walt responded with “No” (November 18, 6:06:28). Other changes occurred during this class meeting that could have influenced the changes in our behavior. My teaching became more directed due to the tensions I felt (as described in

Tensions section). The reaction of the group to the Purple Paint task confirmed my biases about the decision to proceed with ratios and proportions. In addition, this was the one class meeting when we met in the afternoon as opposed to the evening.

The changes caused our community to be tested, and we (myself included) resorted back to how we interacted in the beginning weeks. Interestingly, the following class meeting, Week 11, our conversations and interactions returned to the levels described in the Week 8 and Week 4 transcripts. My teaching style, unfortunately, remained more direct and pushy. However, the tasks we examined returned to the typical level of cognitive demand of IM tasks. Thus, it appeared the cognitive demand of the task had more influence on the mutual engagement of the group than my teaching style.

Mathematical limits. Rational numbers is an important topic in middle grades mathematics in the United States, and research has shown teachers' understandings of these concepts are generally not strong (cf. Post et al., 1998; Ma, 1999). IM engaged teachers in exploring high cognitive demand tasks related to rational numbers, and at times these tasks pushed the participants to their mathematical limits, which was often uncomfortable for them and for me. This discomfort was not surprising as research has shown that exploring rational number concepts is often an uncomfortable enterprise (cf. Armstrong & Bezuk, 1995; Ma, 1999). The discomfort had the potential to inhibit our mutual engagement because participants might have been reluctant to expose the gaps in their knowledge to their peers, particularly because those peers taught in the same district. In the following example, however, I offer evidence that pushing the participants' mathematical limits actually enhanced our mutual engagement and that our sense of community allowed us to feel comfortable enough to push our mathematical limits even further.

In the Week 8 discussion example when Joyce suggested a possible student confusion, many members of the group were being pushed in their mathematical understanding about Walt and Salihah's observation about seeing the fraction multiplication algorithm in the area model. Most of the group had not seen any referent unit issues with the model provided until Joyce suggested a problem. The idea that the model represented 5 and not 1 was surprising to many (see more on referent unit in the Shared Repertoire section). Instead of ignoring Joyce's suggestion, the group discussed the correct referent unit and how our construction of area models up to this point was potentially misleading. Our mathematical limits were being pushed by each other's comments and the discussions that followed. Without an agreement to thoughtfully consider each other's ideas and to willingly expose mathematical confusion, the mutual engagement of the group could have been limited.

At 5:42:18:28 in the Week 8 discussion, Walt suggested the area model was not a good teaching tool. He was expressing doubt about the validity of his idea that the area model would assist students in seeing why the algorithm works. This demonstrates the willingness of a participant to share his doubts and concerns to the group. The group's response was to persevere and continue to work on finding a way to make sense of his idea and the example they were examining. The excerpt ended with the group discussing the proposed solution (separating the columns in the area model to emphasize 5) that Joyce agreed would be acceptable.

With the support of the community, Walt was successful in making sense of his original idea, which was tested by Joyce's question. Joyce's mathematical concerns about the proposed area model were considered and the group agreed upon a solution. Additionally, the group was open enough to trust another participant's observation even when they were unclear on the mathematical issue. The act of grappling with difficult mathematical concepts strengthened our community because it fostered mutual engagement.

During weekly phone interviews 7 of the 13 participants mentioned feeling confused or in a state of disequilibrium from the class. One participant, when asked what was challenging about the course stated, "That's a challenge for me is trying to get outside of the box, you know, seeing things the same way for so long and the way she's [Rachael's] delivering-how the class is going-it makes you get outside the box and think about other ways so that part is a challenge" (Sharlene, Week 3). Three participants recognized I was deliberately provoking them and pushing their mathematical limits. Two stated I "stir things up" in class (King & Joyce, Week 8), while another said I was sometimes an "aggravator" (Donna, Week 8).

Participants also mentioned their discomfort with me pushing their mathematical limits directly to me during class. For example, in Week 11, Walt said to me, "You guys are about to approach my level of proficiency." I responded playfully with "That's good. You hung in for 11 weeks! I've been trying to push everyone out of their comfort zone!" and laughed (Dec 2, 5:22:24). During Week 12 (December 9, 7:54:43), Linda, Sharlene, Joyce, and I were discussing their thoughts on a problem, and I had to ask many questions to understand what their thinking. Joyce commented my questioning made her doubt herself, and Sharlene noted her students got worried when she asked them questions about their thinking. Clearly, for some participants, being questioned created a tension.

Wenger (1998) wrote, “The kind of coherence that transforms mutual engagement into a community of practice requires work” (p. 74), and this was certainly true for the IM group. Discomfort and tension were inevitable because of the mathematical challenges and the high cognitive demand of IM tasks. Our sense of community and mutual engagement allowed us to maintain a high level of engagement despite the discomfort and tension.

Joint Enterprise

Joint enterprise is the second characteristic Wenger (1988) identified for a group of people to be considered a community of practice. The joint enterprise is the result of a collective process of negotiation, is constantly evolving, and creates mutual accountability. Three aspects of our work together seemed to help create our joint enterprise: shared expectations, participant engagement, and consistency with district norms.

Shared expectations. In this case, the group did not collectively decide on our joint enterprise because the DiW research project drove much of the activity, and the IM syllabus was predetermined. However, within these constraints, I engaged the group in creating our norms so as to establish a sense of ownership over our joint enterprise and to establish a sense of mutual accountability. I thought it was particularly important to foster a feeling of joint enterprise because half of the participants were forced or strongly encouraged to enroll in IM by their administration. Two of the participants came to the first class meeting and told me they had no idea what the class was about but were told they had to come (Walt and Will). In addition, I was quite familiar with the busy schedules of teachers and knew they were likely to face competing demands on their time, so I wanted us to establish a clear set of expectations from the outset so that the teachers could make a decision about how they prioritized participating in the IM class.

On the first night of class we spent approximately 25 minutes discussing expectations we had for the course, participants, and me (September 9, 2008). I gave time for participants to think about each element (course, themselves, me) and then time to talk with another person. The groups reported their expectations to the whole group, and when we agreed on an expectation, it was written on chart paper. These expectations were posted on our website as well. Although neither the participants nor I referred to these expectations after the first few class meetings, the group consistently adhered to the expectations they set.

One of the expectations was to be on time to all class meetings because the participants valued our time together. They demonstrated their commitment to our joint enterprise by coming to class on time, trying to avoid absences, and completing extra work for missed time. Only three participants missed a class meeting, and everyone generally arrived to class within 5 minutes of the scheduled start time. One participant had to be significantly late to class one night because of a mandatory meeting scheduled by her principal. She told me she would not have missed IM class time if the meeting had not been mandatory, suggesting that she recognized the value of our joint enterprise and took seriously her obligation to be part of the community.

Another mutually-agreed upon expectation was that each participant would complete all assignments for the course and research project. The assignment for the course mostly consisted of completing 10 write-ups of individual tasks and posting them to our class web site for others to review. Working on improving write-ups was a standard set by the group. For the most part, participants were conscientious about their write-ups; they wanted them to look nice, make sense, and convey their mathematical understanding. They could have just as easily dismissed the write-ups because they were not graded. Two aspects of accountability seemed to affect the teachers' commitment to completing the write-ups satisfactorily. First, the write-ups were used as evidence of participation in order to award the participants credit toward renewing their teaching certificates. Second, and more important, the requirement to post write-ups on the web for others in the group to see created a sense of mutual accountability.

Participant engagement. Three additional expectations the participants generated serve as a way to judge the nature of our joint enterprise. The expectations were to contribute positively to the class, to better understand rational numbers for themselves so as to support their students. Although all participants completed requirements and attended IM class meetings, there was a range of perceived seriousness with which participants engaged in the mathematics. Wenger (1998) wrote, "Mutual accountability plays a central role in defining the circumstances under which, as a community and as individuals, members feel concerned or unconcerned by what they are doing" (p. 81). Some participants were reflective about how IM connected to their teaching practice, were disturbed by their lack of understanding middle school mathematics, and worked to improve their communication about mathematical ideas. For example, Carrie emailed me after receiving feedback on a write-up and said, “I understand that I need to be pushed and that is part of this course. I look at it from the aspect that I am better than I was when I walked into the course and will be even better when I leave” (Carrie, November 3, email). Participants at the other extreme only occasionally participated verbally in the discussions or engaged with their partners to make sense enough of a task to write about it. These participants were not as engaged in reflecting on their teaching practice and personal mathematical understanding. During the Week 11 phone interview three participants identified too much time debating or exploring as a weakness of IM. The majority of the participants fell in between these two extremes.

As a group, the participants often talked about their students and teaching in relation to a task or concept we were discussing. Often times those individuals who were reflecting about this experience initiated these conversations. The group continued to have lengthy discussions about the tasks and teaching experiences they were reflecting on despite the occasional negative comment by one or two participants, which shows they valued our joint enterprise enough to get beyond the negative comments.

Consistency with district norms. Because of my previous PD work in this school district and my relationship with the mathematics coordinator, I knew the design of IM and my teaching style were compatible with the characteristics of mathematics PD in the district. For example, the participants in my IM class had previously participated in PD where expectations were clearly stated and where negotiating ways to communicate was valued. I also knew they had participated in PD in a safe environment where ideas, questions, and thoughts could be shared regardless of correctness. The willingness of IM participants to vocalize ideas, thoughts, and questions in the first couple of class sessions was a testament to the already established norms of mathematics PD in the district. Whether consciously or not, many participants recognized this similarity by engaging in IM class meetings and valuing our time together as was typical for mathematics PD in the district.

Shared Repertoire

The final characteristic of a community of practice according to Wenger (1998) is a shared repertoire, which consists of "routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice" (p. 83). In this section, I provide data on the routine of IM, models and software used in IM, and important tasks and vocabulary.

Routine. A group's reaction to the routine placed upon them is an element of Wenger's (1998) definition of community of practice. For IM, I set the routine of the course as the instructor, and none of the participants ever suggested changes. Once I had posed a group task for us to consider, the group typically worked in small groups on the task and then we would share as a whole group. After working on the group tasks, I shared the individual tasks with the group by reading each of them to the group. Participants chose which task they wanted to write-up and spent time talking with other participants about the task. Occasionally, I required every participant to work on the same individual task to write-up. There was only one class period where a portion of the routine changed, and this occurred during Week 9, our first week on ratios, when I posed the whole group task and suggested they work together in small groups. The group began to work aloud as a whole group to figure out the solution rather than working in small groups and then sharing in a whole group setting. The class was structured in this way in order to maximize the amount of time the teachers were discussing mathematics together. By limiting our engagement and discussions on two to three tasks a class meeting, we were able to focus on the mathematics and to have rich discussions that permitted the development of norms. I felt these activities were necessary in order to foster a community.

Models and software. Many drawn representations for multiplying and dividing fractions and decimals, including area models, arrays, and single and double number lines, were introduced by me or via the tasks. The majority of our whole group discussions consisted of making sense of models participants had created in response to a task. The participants were familiar with area models and arrays at the start of IM, but double number lines were new to most of them. Across the class meetings, participants began to use double number lines more frequently, especially as we engaged with proportions, suggesting they had become part of our shared repertoire. For example, participants used double number line models in 8 of the 10 individual task write-ups. When asked about what they learned in IM, four participants mentioned the double number line model (Week 12 phone interview).

Various technologies were introduced to the participants in order to help them explore the mathematics and communicate with each other. The software used for exploring area models was Fraction Bars, and the participants consistently used this software throughout the course. Many began to use Excel in order to create double number line models for their write-ups, and a few participants used the shading of cells in a spreadsheet to create area models similar to those in Fraction Bars. I had not anticipated this use of Excel; however some participants preferred to use it in order to easily label units and parts of their area models. All of the participants seemed to improve their ability to use Fraction Bars and Excel as they used the software to create write-ups and communicate with the whole group. Because the representations the participants produced with the software were used to communicate with one another, I consider them part of our shared repertoire.

The research team developed a web site for the class and envisioned it being used in multiple ways for communication and interaction between class sessions. The website, however, was used mainly for participants to post write-ups and for me to post general comments about class, the write-ups, and the mathematics. I posted only nine times, and participants responded online to my postings only three times throughout the course. Thus, the website became more of a communal place to save work as opposed to a place for discourse and was not part of our shared repertoire.

Tasks. A few of the tasks became especially meaningful to the group as we continued to explore rational number concepts. The "Candy Bar problem," as the group called it, was a simple mathematical situation that generated discussion in two consecutive class meetings and was revisited near the end of the course. The problem was "Share two candy bars equally among five people. How much of one candy bar does one person get?" Two solutions were debated: one-fifth and two-fifths. Questions about the congruence of the candy bars and which unit the solution referred to (one candy bar or both) generated discourse, and we, as a group, had to reach a consensus on the issue. The concept of referent unit became important to the group through this task. For all of these reasons, the group referred to this task at least six times in later class periods. For example, in Week 4, Carrie told the group the solution of $2\frac{2}{5}$ for the task we were debriefing reminded her of the Candy Bar problem (September 30, 5:33). Out of curiosity and because the group mentioned the problem during Week 11, I revisited this problem with the group as a warm-up for our last instructional class meeting. The participants greeted the problem with laughter. The conversation was much the same, although many participants said they could understand both arguments now–the argument for one-fifth and the argument for two-fifths. Participants referred back to this task twice later in the class session after we had moved on to other tasks.

The Beaker Problem (Figure 1) was also significant for participants because it again highlighted the issue of referent unit, although in a different way than the Candy Bar Problem. The idea the referent unit can be different for different people emerged from this problem and was an important idea for the group. Two of the references to the Beaker Problem occurred in our second class, which involved a continuation of exploring referent unit. The other occurrence was during Week 7 during a whole group discussion clarifying our definition of referent unit. A participant reminded the group about the Beaker Problem because both Sam and Morgan thought they had the whole. (See the Vocabulary section for further data on this group discussion.)

Referencing previous tasks allowed us to communicate with each other differently because we could reflect on the issues, ideas, and questions explored earlier and relate them to the present conversation. They were an example of connecting mathematical ideas, a behavior we all employed regularly (previously described in the My Actions section). These references would not have made sense to anyone outside of our IM group because the reference was not just to a task but also to a discussion about the task.

Reference to the task brought forth a concept the group had previously made sense of but possibly needed to reconsider (for example, Carrie's comment on the Beaker Problem). Some times, the reference was just to classify a current discussion or task as similar to a previous experience the group had. Only members of the group could have understood both types of references; therefore these tasks were part of our shared repertoire.

Vocabulary. Four terms became especially important for the IM group as we explored rational number concepts, particularly those related to division, which was the focus of 5 class meetings. *Referent unit* was a concept regularly discussed in all of the class meetings. I introduced the terms *partitive division* and *quotative division* during Week 5, and these terms were new to most participants so we struggled a bit to understand each and differentiate them. As we explored ratios and proportions, there were many discussions about what is meant by the phrase *multiplicative relationship*.

Of these terms, referent unit was a concept we struggled with more often than the others and was the one vocabulary term that was also an underlying theme of the course. Our first whole group conversation where we worked to define referent unit occurred at the end of Week 2 at my initiation (September 16, 2008, 6:58:59). The previous week I had suggested the group think of referent unit as "Where is the one?" After reflecting on the class, I realized the potential confusion I was creating by saying "one." So during Week 2 I posed the question, "Is there a problem with saying the referent unit is 1?" in order to have the group reflect on my vocabulary from the week before. Participants had a variety of thoughts on this question, and we did not reach consensus on a definition this week. Claire noted that using a definition of referent unit as one was incorrect for the Beaker problem because both Morgan and Sam thought they had found one. Will responded that "Whole means one." Donna connected this discussion to her teaching practice by noting that her students struggled when the whole referred to a set of numbers. I summarized the discussion by reluctantly saying, "Maybe one is OK."

At this point I redirected everyone's attention to the Candy Bar problem and asked what the referent unit was for that situation. Again there was discussion as to whether the question was referring to one or two candy bars. Mike then posed a question about a definition of referent unit he was working to develop.

[07:29:05.07]

Mike: When-when you use the term referent unit-

Rachael: Yeah

Mike: then anything that's a referent unit by definition is exactly the same. Is that correct?

Rachael: I need you to talk me-

Mike: In this example here if we said a-a candy bar is the referent unit.

Rachael: Right.

Mike: In this problem. Then any candy bar is equal to any other candy bar if-

Rachael: (over Mike) Right, I'm saying. Right. We're pretending they're all the same.

Mike: a candy bar is the referent unit. So that's part of the definition of a referent unit. Is-is that correct? I'm-that's the question.

Rachael: So you're saying, if I say one candy bar is the referent unit than I'm going to take that same size candy bar and always measure my stuff to it.

Mike: I'm saying-I'm saying if another candy bar-if a candy bar is the referent unit, than the candy bars all have to be the same. Is that-is that correct? As part of the definition of referent unit.

Rachael: Yes. Just like if I say one inch is the referent unit, than I imagine one inch always being the same.

Mike: (over Rachael) Every inch has to be exactly the same to each other.

Rachael: Yes. Yes.

Mike: That-that would make this problem a different light than if the two candy bars were-together were the referent unit.

[07:30:13.03]

Mike continued by connecting this idea to other participant's comments about the solution being 1/5 or 2/5. The discussion about the candy bar problem and the correct solution continued for another 15 minutes. I then suggested we let the problem be and move on to something else.

The participants were working on definitions for referent unit for their own sake and also for the group's sake because in order to reach a conclusion on the solution of the candy bar problem and many other tasks we had to agree on a working definition for referent unit. Mike was explicitly stating a part of the definition for referent unit that he felt was important to the conversation. Although he and I were the only two talking, the other participants were paying close attention to what was said. Mike's question summarized the previous discussion by concluding the referent unit must be consistent for us to measure with it. His view could be limiting because it did not allow for flexibility with the referent unit. However, he was contributing to a public definition of referent unit, and no one commented on any weaknesses his suggestion posed.

At the end of the Week 2 class, I asked the participants to provide a definition of referent unit as the reflection question. Nine participants referred to the referent unit as the whole or base that was being used for comparison. Three participants talked about the referent unit being the same as one. Thus, although we had a working definition for the group, individuals were interpreting the definition differently. This is to be expected in a shared repertoire. Wenger (1998) explained, "When combined with history, ambiguity is not an absence or a lack of meaning. Rather, it is a condition of negotiability" (p. 83). Therefore, even though various participants interpreted our definition of referent unit differently, it provided a concept for us to negotiate as a group. Again, if someone were to join our group after the fact, this definition would not make sense to the new member because she missed all of our discussion and debate about the tasks and ideas that led us to this definition, so it was part of our shared repertoire.

In the next four class meetings we explored multiplication and division of fractions and decimals, and our working definition of referent unit was put to the test. I thought the group would benefit from exploring the differences between partitive and quotative division by exploring the differences in the referent units for each number in a division number sentence. I began the Week 7 class (October 21, 2008) by posing the following warm-up question: “What is the referent unit for each number in 2/3 divided by ¼ = 8/3?” I had suggested using a model and was expecting a discussion about the differences in the referent unit for each number depending on which type of division a person was thinking about, not a discussion on referent unit. Based on our previous discussions and the participants’ write-ups and reflection questions, I had thought the group had a solid understanding of referent unit. Instead, Donna and Walt asked for a working definition of referent unit, and for the next 19 minutes the group grappled with what referent unit meant in the context of this division sentence. Donna and Mike had suggested the definition of one and King responded with "Well, then we need to decide what we mean when we say one" (05:12:50). I summarized the three ideas offered: less than one, the unit you are talking about, and one and wrote the ideas on chart paper. Then I asked King to repeat his comment about the ambiguity in saying one. I suggested asking Donna or Mike (the two who suggested one as the definition of referent unit) to clarify what they meant by one.

[05:14:37.09]
Donna: To me it says 2/3 of 1 divided by 1/4 of 1 equals 8/3 of 1.
Rachael: So when-
Donna: One, one, one.

Rachael: when you say the number one-I mean when you say one you mean the number 1?
Donna: The number-yes. All of whatever it is.
Rachael: Wait, so all of
Donna: All of-all of one of whatever we're talking about-cups of flour.
Rachael: OK.
Mike: Yeah. So one-one can represent anything or just the number one or one pizza.
Donna: Right.
Mike: Or one anything.
Rachael: So could 70 cups of flour be your one?

Donna and Mike nod.

Carrie: If you're making one recipe of something.
Rachael: That's a mighty big recipe. I think that's what you're asking though, Mr. King, right?

King agreed. Other participants chimed in. Will talked about the specific context and the idea of referent unit. Walt raised his hand and commented that we had not yet decided what the referent unit is.

Walt: I'm just gonna hang on the fact that it's one. Then would the reference [sic] unit for 2/3 be 3 over 3 and the reference unit for 1/4 be four over four and consequently the a-for the answer there still be 3 over 3? The reference unit.
Carrie: But you all need to think back, remember the Beaker problem. There was five in one and four in another. But each one thought that theirs was filled to the top and that was their one - their whole. And on this one were only dealing with 2 out of the 3.

Quiet for 10 seconds.

Mike: You still have to say that it's 2/3 of something divided by 1/4 of something. If those two somethings aren't the same, it doesn't mean anything.
Rachael: If the 2 thirds aren't the same?
Mike: 2/3 of something divided by 1/4 of something. If those two some things aren't the same, that problem has no meaning.

A few participants were mumbling.

Rachael: I think what I hear Mike saying is that 2/3 and 1/4 have to have the same referent unit. Which we're right now working with

> one as the referent unit. As the definition, I mean, for referent unit.

Wenger noted that a shared repertoire reflects a history of mutual engagement, which was clearly the case for the IM group. Our struggle to understand the concept of referent unit was a part of shared repertoire and the persistence and probing questions that helped us clarify our understanding were elements of mutual engagement. The discussion above also clearly illustrates Wenger's claim that ambiguity is important in order for the repertoire "to become a resource for the negotiation of meaning" (p. 83). It was the ambiguity of the term referent unit and the necessity of developing a clearly stated definition that led us to engage in protracted discussions to negotiate the meaning of this term.

Learning

In this section I present data relating to my third research question about the learning that occurred during the IM course. To answer this research question I provide three different types of evidence of the participants' learning. First, consistent with Rogoff's (1997) definition of learning as changes in participation, I provide evidence of three levels of change in participation toward the ideal learner among the participants. Second, I present data showing changes in participants' mathematical understandings of referent unit, drawn representations, and proportion–the three themes of IM. Finally, I provide data from the statistical measure of knowledge from the pre and post assessments used in the larger research project and discuss how these views of learning are connected.

Changes in Participation

My definition of an ideal learner in IM was someone who engaged in mathematical discussion and argumentation. He or she was open to explore mathematical errors and misconceptions and strived to understand connections between various strategies. Finally, the ideal learner began to take over my role as facilitator by asking for justifications, questioning incomplete ideas, and pushing for generalizations. This connection between learning and community was integrated in my view of the ideal learner. Three levels of change in participation were evident in the participants: a substantial change in participation, a moderate amount of change in participation, and a minor change in participation. None of the participants achieved the level of ideal learner.

Substantial change in participation. Three of the participants demonstrated substantial change in participation toward becoming the ideal learner over the course of IM. All three participants easily shared their mathematical ideas, models, or thoughts on a task we were considering in small or whole group discussions at the beginning of IM. However, the participation of these three individuals advanced beyond just sharing their mathematical ideas to asking questions about ideas others shared, pushing to connect strategies and mathematical ideas, and trying to generalize ideas. These changes were not evident in the rest of the participants. One participant was very hesitant in the beginning about asking for help or admitting confusion. She often said things under her breath, which I would then ask her to repeat to the whole group. As the course went on, she ended up encouraging others to share and ask questions. These participants also mimicked many of my behaviors as described in My Actions section. For example, Carrie was the first participant to ask a clarifying question and continued to consistently ask questions throughout the course. Walt and Donna pushed for the group to state a working definition of referent unit (described in the Shared Repertoire section).

Moderate change in participation. Half of participants exhibited moderate change in their participation. At the beginning of IM, these participants were not always willing to participate in whole group discussions and were not always open to new mathematical ideas. Some participated regularly but focused on sharing their mathematical idea with the group and rarely pushed for generalizations. Others participated irregularly in the discussions. Many of these participants seemed to prefer working by themselves on the tasks and did not always value the conversation with peers. As the class progressed, these same participants were more willing to participate in mathematical discussions. In addition to sharing mathematical ideas, many began to pose questions to the group about confusions they were experiencing or to direct the conversation to a mathematical challenge.

Two of these participants began IM by consistently sitting alone during the class meetings. After the week where I assigned seats, these participants were actively engaged in making sense of the task and helping their partners understand, as well as trying to understand their partners' ideas about the task. One of these participants took on a leadership role in her small group for the second half of IM. She worked to get her partner participating and was concerned with her partner's understanding as well as her own. She was also forthcoming with her mathematical confusion and her frustration with this new way of thinking about mathematics.

Many of the participants who exhibited moderate changes in participation were focused on completing their write-ups, and helping other participants improve their mathematical understanding through questioning was not a priority. All of these participants engaged in discussions and were respectful to the group. Interestingly, two of the eight participants became less engaged as the course progressed. One participant contemplated withdrawing from the course but was encouraged to complete it. The other was interested in pedagogical issues and struggled with understanding the intentions of IM.

Minor change in participation. Four of the participants made minor changes in participation. These participants were often quiet in class during whole group and small group discussions. They completed all assignments, but their write-ups contained minimal information. When asked if they learned anything from other people in IM that they would not have learned by themselves, one of the three participants in this category responded with “nothing.” Another participant stated it would have taken longer to learn the mathematics by herself, implying that the only value she saw in working with others was efficiency. These were the only two participants who did not appear to value other people’s ideas and thoughts in terms of their own personal mathematical learning. These participants arrived to class on time and were respectful of the group; however, their participation in IM hardly changed.

Three Mathematical Themes in IM

The three themes of IM were referent unit, drawn representations, and proportion. Using these three themes as lenses for viewing learning, the participants grew in their understanding of referent unit and drawn representations. Less was learned related to proportion, but little time was spent on this topic (2 out of 14 weeks). Data related to referent unit were described in the Shared Repertoire section. The group struggled to make sense of the definition of referent unit and began to see the importance of referent unit in making sense of the tasks. The tasks that became part of our shared repertoire and were referred to by participants pertained to issues with referent unit. In addition, referent unit became central to our discussions and debate. Our working definition focused on referent unit representing something other than one. Participants began to understand that the quantities in the same problem could have different referent units and began to view referent unit flexibly. The group recognized that the concept of referent unit was integral to understanding rational number concepts.

Many of our conversations revolved around making sense of drawn representations, and the participants struggled to make sense of the various models that could be applied to the same mathematical situation, such as the Week 8 discussion in the Mutual Engagement section. Another example was when Keith shared his single number line representation for a fraction division problem (in the Mutual Engagement section). Some of the participants struggled to make sense of how his representation demonstrated a division problem. The concepts of referent unit, fraction division, and representations came to a head in this example, and the group pushed Keith to explain his representation in order to make sense of it themselves. Many participants began IM preferring to use number manipulation to make sense of a task. The participants exhibited evidence of growth in their understanding of all of the models we used, but this was most noticeable with the double number line. The participants often attempted to create a drawn representation for a mathematical situation without prompting. Participants also began to use models that they thought were most appropriate for the given mathematical problem. For example, participants regularly used the double number line when thinking about proportional situations. None of the participants had seen or worked with double number line models prior to IM, and when this model was first introduced, many participants did not like the model and saw no value in it. As evidenced in data provided in previous sections, participants ultimately made sense of and began to use double number line models regularly as they engaged in tasks.

In addition, many of our mathematical discussions focused on comparing and contrasting various models for the same task. For example, during Week 3 I asked two participants to share models for a fraction division problem (September 30, 2008). Keith shared his number line model, and when Claire began creating her area model for another fraction division problem, Carrie interrupted her and asked if she would create an area model for the same problem that Keith's number line modeled. This shows that the participants thought it was important to understand multiple models for the same situation. Participants also began to consider the value in using the different models for different problems, as evidenced by the double number line discussion above. In the end of course survey, the participants were asked to circle three words that best described the most important aspects of IM. There were 14 words given relating to community, content, and the themes of IM. Half of the participants selected Representations, suggesting they saw some value in the various models they had learned about and used during the course.

We did not devote much time to the study of proportions, and, as noted earlier, the participants' tenuous understanding of key ideas of multiplication and division of fractions and decimals precluded us from engaging deeply with these ideas. Not surprisingly, the write-ups on the proportion tasks were not strong, despite being completed toward the end of IM. At least one participant confused situations involving direct and inverse proportions. The participants were not successful in identifying characteristics that differentiated direct proportions or inverse proportions from other mathematical situations. There was no evidence that the participants saw the connections between proportions and partitive division. One participant suggested a connection to the whole group, but the group did not pursue the idea (Carrie, September 30, 2008). They might have shied away from this connection due to their discomfort with the partitive model of division or the fact that we spent very little time exploring proportions. I was working hard to push the group to consider the connection between proportional reasoning and partitive division deeply and make sense out of it but was unsuccessful. This was evident in the final class meeting where the tasks could be considered as proportions or division, and every participant viewed the questions as proportions.

Statistical Measure of Learning

All of the participants completed a pre-assessment and a post-assessment of their mathematical knowledge for teaching rational numbers. The ability scores were presented as *z*-scores with 0 representing the average score of the 201 teachers who took the assessment. The average ability score of the group before IM was 0.54. After IM, the average score was 0.91, which was a statistically significant increase. More specifically, nine participants had a significant increase in their scores while two had a significant decrease. Three participants had no significant change in their ability score.

Interestingly, the participants with substantial changes in participation were not always the ones whose ability scores changed significantly and positively. One participant who changed his participation drastically had no significant change in his ability score from the pre to post assessment. Six participants had a negative ability score on the pre-assessment, and one of these participants had a negative ability score on the post-assessment. All of these participants had significant increases in their scores. Two of these participants were classified as having moderate changes in participation. Three participants classified as exhibiting minor change in participation began with a negative ability score and had statistically significant increases in their ability scores.

Using the definition of change of participation for learning, the participants in IM did learn because the majority displayed at least moderate changes in participation. Using a statistical measure of learning, the participants demonstrated significant increases in mathematical ability. Surprisingly, these two views of learning did not correspond because participants with substantial changes in participation did not all increase their mathematical knowledge significantly. Learning as change in participation was not focused on the mathematical ability or understanding. Instead, this learning was about being open to mathematically different ideas and models as well as making mathematical connections between tasks and ideas. Although these two views of learning are expected to coordinate, two participants with substantial changes in participation began the course with high ability levels. Additionally, the ability score alone may not have accurately measured the participants' conceptual understanding.

Synthesis of Data

To assess the extent to which our group became a community I used the model of the formation of a teacher community created by Grossman et al. (2001) based on research conducted with high school English and history teachers. They identified four markers or dimensions of community and created a model (see Figure 6). In the sections that follow I describe the status of our community using these four dimensions and the relevant indicators.

Beginning	Evolving	Mature
1. Formation of Group Identity and Norms of Interaction		
Identification with subgroups	Pseudocommunity (false sense of unity: suppression of conflict)	Identification with whole group
Individuals are interchangeable and expendable	Recognition of unique contributions of individual members	Recognition that group is enriched by multiple perspectives (sense of loss when member leaves)
Undercurrent of incivility	Open discussion of interactional norms	Developing new interactional norms
Sense of individualism overrides responsibility to group	Recognition of need to regulate group behavior	Communal responsibility for and regulation of group behavior
2. Navigating Fault Lines		
Denial of difference	Appropriation of divergent views by dominant position	Understanding and productive use of difference
Conflict goes backstage, hidden from view	Conflict erupts onto main stage and is feared	Conflict is expected feature of group life dealt with openly and honestly
3. Negotiating the Essential Tension		
Lack of agreement over purposes of professional community; different positions viewed as irreconcilable	Begrudging willingness to let different people pursue different activities	Recognition that teacher learning and student learning are fundamentally intertwined
4. Communal Responsibility for Individual Growth		
Belief that teachers' responsibility is to students not colleagues; intellectual growth is the responsibility of the individual	Recognition that colleagues can be resources for one's learning	Commitment to colleagues' growth
Contributions to group are acts of individual volition	Recognition that participation is expected from all members	Acceptance of rights and obligations of community membership (e.g., "intellectual midwifery," "press for clarification")

Figure 6: Grossman et al.'s (2001) Model of the Formation of Teacher Professional Community (p. 988)

Formation of Group Identity and Norms of Interaction

For the dimension of formation of group identity and norms of interaction, the indicators include the development of interactional norms, the sense of group responsibility for regulating group behavior, an appreciation for the contributions of individual members, and the extent to which the group functioned as a whole. Because the first two indicators are similar, I discuss them together. Because I was intently focused on building a community with the IM participants, I very consciously stated specific goals for the group and identified actions I might take as the instructor in order to aid the creation of a community of practice within the group prior to the start of the class.

The first week of class I led a whole group discussion about expectations for the course and how we would interact with one another. The norms on which we agreed were recorded on chart paper and on our web site. This activity provided us with a starting ground for understanding each other's aspirations for the course and for interacting respectfully with one another. There were never any undercurrents of incivility among the participants or toward me. I intentionally used teaching moves such as giving a participant the floor, providing positive reinforcement, asking clarifying and probing questions, restating a participant's mathematical idea or model, and making mathematical connections between models and ideas in order to model for the participants how one engages in intense mathematical discussions with others. A few weeks into the course, the participants began to use these behaviors themselves in their interactions with others. Although I maintained a certain degree of authority as the instructor, I was able to fade into the background during most of our discussion because the participants took leading roles in discussions about tasks and aided others in their explanations by asking questions. These behaviors suggest the group was responsible for how we interacted together and they recognized the importance of each member of the group by encouraging every participant to contribute to the discussion. Participants rarely raised hands in class, indicating the group had a method of regulating whole group discussions that was not formal. It would have been challenging for a new participant to jump right into the conversation without spending some time observing how we interacted with each other. Based on our development of norms and the group regulation of behavior, I believe the group was at the evolving stage with respect to these indicators.

The above-mentioned behaviors of the participants and myself demonstrated respect and appreciation for the contributions of individual members of the group. We encouraged each other to share mathematical ideas and praised each other for hard work and sharing a new perspective. The group dialogue about the definition of referent unit (in Shared Enterprise section) showed the group appreciated the value of collective deliberation on a thorny issue. Although the discussion was prompted by the confusion of one person, it quickly became apparent that most of us were unsure of the definition of referent unit, and the group collectively analyzed the pros and cons of various definitions in an attempt to reach consensus on a workable definition. In the phone interviews, the participants said they appreciated the multiple perspectives shared in our discussions of tasks because it helped them grow as individuals. Thus, members of the group were able to see the value of both individual and collective understanding and were willing to share their ideas to help others reach clarity on a topic.

Another indicator of this marker of community is the extent to which the individuals identify with the whole group rather than with subgroups. Although participants consistently worked together in the same small groups, there was never any sense of competition among the subgroups or a sense of hostility between the whole group and a subgroup. Because our whole group discussions flowed freely and most members participated in these discussions, I infer that the individuals identified with the group and that we had a genuine, rather than false, sense of community. Considering the four indicators of formation of group identity and norms of interaction together, the group was an evolving community.

Navigating a Fault Line

The two indicators in the navigating a fault line dimension of a teacher professional community are how the group handles difference and conflict (Grossman et al., 2001). This dimension was challenging to apply to my study because of some fundamental differences between my PD context and the context of Grossman et al.'s study. First, Grossman et al.'s model was based on research with humanities teachers, and the study of humanities is a more emotional endeavor than the study of mathematics. Thus, in this sense, we did not have to deal with emotionally-laden differences tied to personal identity. In addition, the role of the facilitator was quite different in Grossman et al.'s study and my study. In their study, the facilitators did not take a leading role in shaping the PD. In contrast, my role was central in the design and implementation of IM. IM had a predetermined syllabus that we followed, so there were not opportunities for conflict to erupt over the direction the group was taking. Thus, our group did not experience fault lines in the sense that Grossman et al.'s group experienced them. In the next two paragraphs, however, I reinterpret the notions of difference and conflict in the context of this study and argue that the community was mature with respect to this dimension of community formation.

The differences in our mathematics teacher community related to the diversity in educational background, teaching experience, and mathematical development. We did not specifically address these differences; the group was thoughtful and respectful of each other regardless of a teacher's grade level or years of experience. Regardless of background, if a participant shared a mathematical idea or model, then the participant was expected to provide a solid explanation, and the group was expected to give serious consideration to the shared idea and to push the participant through questioning and restating to strengthen his/her explanation.

The only conflict in our group was the cognitive conflict created by working on challenging tasks in an environment where all ideas were open for scrutiny by others. Because this type of conflict was the norm rather than the exception in our class, the group quickly learned to expect this conflict. We engaged in rich discussions about the mathematics that made many teachers uneasy. However when the cognitive demand of the task was lowered, the participants were concerned about misunderstanding the task. The phone interview data and the group discussion data show that the group handled difference and conflict in productive ways, suggesting that the community was at the mature state with respect to navigating fault lines.

Negotiating the Essential Tension

"Recognition that teacher learning and student learning are fundamentally intertwined" is the indicator of a mature teacher professional community under the negotiating the essential tension dimension of the model (Grossman et al., 2001, p. 988). Some researchers (Scribner, 1999; Stout, 1996) have found teachers resist PD requiring them to learn new mathematics content for themselves because they do not see a direct connection to their classrooms. In contrast, Grossman et al. (2001) wrote about models of teacher community in elementary mathematics where the essential tension was addressed by having teachers relearn the mathematics they teach. They noted "at the core of this kind of teacher community is the assumption that teachers cannot teach concepts they themselves have not mastered" (p. 962). This statement captures the premise behind IM and the rationale for the design of the course. At the start of IM, I was upfront about the requirements and purposes of the course. Thus, despite their varied reasons for enrolling in IM, the participants learned about their expected roles in the course. As noted earlier, the IM class was consistent with the structure of other PD in the district in which the teachers had participated, which gave our community a head start on developing toward a mature level of formation.

The tasks used in IM were designed for teachers and often included a student's mathematical work, idea, or model for the teacher to consider. The task frequently asked if a student's idea was correct, would always work, and was generalizable. The participants often related these tasks to incidences in their classrooms or issues they had faced in their teaching. The majority of the participants seriously engaged in the mathematics in IM by reflecting on themselves as learners and as teachers. In addition, many participants reflected on my teaching and expressed a desire to make some changes in their own practices. Finally, the majority of the participants stated a goal at the beginning of IM related to improving their teaching practice. Thus, the IM group can be rated as a mature community with respect to negotiating the essential tension.

Communal Responsibility for Individual Growth

The final dimension of the model of community formation is communal responsibility for individual growth. Grossman et al. noted that a community is maturing when "Members begin to accept the obligations of community membership, which include the obligation to press for clarification of ideas and to help colleagues articulate developing understandings" (Grossman et al., 2001, p. 990). The two indicators suggesting a community is mature are that the participants are committed to their colleagues' growth and that the participants accept their rights and obligations to help others in the community learn and grow. The norms developed by the IM group included an expectation that participants would explain their ideas and that other participants would press them for clarification as needed. I encouraged these norms by asking clarifying and probing questions about the mathematics, restating ideas, and checking for understanding. The participants also used these behaviors to support each other in producing strong explanations. The participants and I implemented positive encouragement to support each other as we treaded on uncomfortable ground or shared tentative ideas. We also valued a unique idea or model. In addition, I always encouraged sharing ideas with another person to emphasize my belief that learning is collaborative.

The majority of the participants exhibited substantial or moderate changes in participation during the IM course. These participants changed toward the ideal learner who was committed to the community and to learning mathematics. Although I presented learning as a collective activity, some participants focused more on their own learning than others' learning. This internal focus could have been due to not understanding the material well enough to support another participant, a lack of confidence in supporting another learner, or a lack of a understanding of what it means to be a community member. Part of my motivation for assigning seats was to try and encourage this focus on growth in others. Unfortunately, this change did not cause the group to pay close attention to each other's understanding and learning. Thus, not all participants consistently worked to improve their colleagues' growth.

I was surprised at the inconsistent commitment to the growth of colleagues, because 13 of the 14 participants were practicing teachers. However, Grossman et al. (2001) noted "To assume that teachers, just because they have experience in creating social organizations among children, can spontaneously organize themselves into congenial social units reflects a romanticism" (p. 991). Community development was much more complex than asking teachers to use their acquired teaching skills in a new setting with adult learners who were their colleagues. The IM group was an evolving community in terms of this dimension of community.

Conclusion. Based on Grossman et al.'s (2001) model of community formation, the IM group advanced from a beginning level to become a community of practice in 14 weeks. Table 5 shows the status of the group at the start and end of the IM course.

Table 5. Status of Group Using Grossman et al.'s (2001) Indicators of Community

	Start	End
Group Identity, Norms	Beginning	Evolving
Fault Lines	Evolving	Mature
Essential Tension	Beginning	Mature
Communal Responsibility	Beginning	Mature

The design of IM, including the use of challenging tasks, and the district culture related to mathematics PD supported the growth and development of the community.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Summary

The purpose of this study was to investigate the formation of a professional community among a group of middle school mathematics teachers participating in a content-specific professional development (PD) experience. PD is an important component of the initiatives to improve education in the United States, and many guidelines have been published on effective PD, several of which highlight the importance of community formation. However, little is known about how challenging it is for the facilitator to maintain the focus of the PD while also encouraging community development.

In this study, the PD was a course for middle school mathematics teachers focusing on rational number concepts called InterMath (IM). This course was designed and offered through a National Science Foundation-funded project called *Does it Work?*. In this sociocultural study, I investigated the following questions:

a. What are the IM instructor's intentions and actions related to developing a community of practice within the group?
b. What characteristics of a community of practice, as defined by Wenger (1998), does the group exhibit? How do these community characteristics change during the course?
c. What learning occurred?

The research literature shows rational number concepts are complex and often teachers do not have conceptual understanding of the operations (Ma, 1999). Instead, for example, many teachers focus on their procedural understanding of rational concepts and are unable to make sense of drawn representations. The professional development experience that was the setting for this study focused exclusively on rational number concepts and emphasized the use and interpretation of drawn representations.

Research has demonstrated the importance of the choice of tasks and their implementation when trying to teach for conceptual understanding. Stein et al. (2000) provided a framework for classifying tasks based on their cognitive demand and explained the changes in demand often occurring through implementation of the task. These studies focused on changing teachers' practices and using tasks to support this change. In this study, the teachers were engaged in solving tasks with high cognitive demand, and the nature of the tasks was shown to be important to community formation.

While there have been studies of K-12 classrooms where teachers attempted to build community and a culture of inquiry (Yackel & Cobb, 1996; Lampert, 1990; Cobb et al., 1997), few studies of this nature have been conducted in professional development settings. The existing literature points to the importance of establishing norms and fostering discourse, both of which appeared to be important in the formation of the IM community.

In order to explore the three research questions stated in this study, I used Wenger's (1998) definition of community of practice and Rogoff's (1997) view of learning as a transformation of participation. Wenger's definition included three elements: mutual engagement, joint enterprise, and shared repertoire. Mutual engagement refers to the group working together for a common purpose. Joint enterprise includes the group's response to the greater contexts influencing the group. Shared repertoire includes resources and routines the group used as they worked together over time. None of these elements is trivial, and a group of individuals meeting regularly does not imply a community.

Rogoff (1997) described learning as a change in participation. For this study I defined learning as changes in the participants' behavior, discussions, and writing. I modeled the behavior of an ideal learner by asking questions, encouraging discussion and collaboration, and supporting teachers as they explained their mathematical ideas and strategies so as to encourage the teachers to engage in these behaviors.

This research study was a special case of a qualitative case study (Ball, 2000) because of my role as the instructor. In the fall of 2008 I was the instructor of an IM course that met 14 times for 42 hours in a large, urban, Southern school district. The data collected for this study included videotapes and lesson graphs of the 12 instructional meetings, my weekly journal entries, my goals for the course, my lesson plans, transcripts of weekly phone interviews with each participant, and 10 write-ups of mathematical tasks from each participant. My analysis involved ongoing analysis during data collection as well as intensive analysis when collection was complete (Merriam, 1988).

In order to address the first research question, my actions, recorded through video and lesson graphs, were analyzed. Three major categories arose from the data–managing the discussion, questioning the mathematics, and stressing the importance of others. These behaviors occurred often throughout the 12 instructional class meetings, and the participants began to mimic many of these actions. My belief that learning is a shared experience led me to be intentional in my actions to support the formation of a community. My strong mathematical knowledge of rational numbers and self-confidence in my understanding enabled me to let the participants direct the conversation without knowing where it would lead, which supported our growth as a community.

For the second research question, I analyzed video, phone interview transcripts, and write-ups to describe the extent to which we exhibited the three components of a community of practice–mutual engagement, joint enterprise, and shared repertoire (Wenger, 1998). Evidence of our mutual engagement came from our whole group discussions, the importance of explanation, the importance of tasks, the role of cognitive demand, and norms for write-ups. Our joint enterprise consisted of shared expectations, consistency with district norms, and responding to external pressures. Our shared repertoire was characterized by the routine of IM, models and software used in IM, salient tasks, and vocabulary. The group became a community of practice as we engaged together throughout IM. Norms relating to explanation, willingness to be open to others' mathematical ideas, and the expectation that everyone shares their ideas were developed. There was an emerging consensus about mathematical concepts and ways of interacting.

Finally, the third research question was explored by analyzing video, lesson graphs, write-ups, and weekly phone interviews. I characterized the three levels of change in participation among the participants as a substantial change in participation, a moderate amount of change in participation, and a minor change in participation. Using the three themes of IM as lenses for viewing learning, the participants grew in their understanding of referent unit and drawn representations; however less was learned related to proportional reasoning. Lastly, the changes in participation were compared to statistical measures of learning from the pre- and post-assessments. Interestingly, the changes in participation did not always align with ability scores changes; however the participants grew in their mathematical understanding of concepts and in becoming an integral part of a community.

Conclusions

Community formation is challenging yet possible. The IM group was able to develop as a community of practice during the 14 class meetings. The group became an evolving community in one of the dimensions of Grossman et al.'s (2001) model and a mature community in three of the dimensions. Part of the community's development can be credited to school district's clear guidelines for the purpose of mathematics PD. Because the IM class was consistent with district guidelines and teachers' prior experiences, the teachers were willing to engage in challenging work to grow as professionals. Evidence of the durability of this community comes from one of the participants who emailed after IM ended to inform the DiW team that "Those of us who attended the Tuesday night class have tried to stay in touch and keep our lines of communication open so we can become better teachers--today at professional development several of us were able to talk and discuss how we have used some of the "stuff" we learned!!! I think it is so important that we are all still in contact and still using all that Rachael taught us!" (email, Carrie, March 25, 2009).

The difficulty of creating a community of practice is real. I experienced the strain to maintain a high quality mathematical experience while balancing the needs of the group as it developed into a community. I experienced tension between the expectations of the DiW team, the design of IM, and my own goals of developing a community. It was difficult for me to justify spending time on community-building activities when I was concerned about the mathematical understanding of the teachers.

This study demonstrated that tasks provide not only a good pedagogical tool for improving teachers' mathematical understanding of rational numbers, but they can be used to stimulate community development. Because the tasks called for multiple solution paths and multiple representations, the participants had to engage in discourse to make sense of one another's solutions and reconcile different solutions. The high cognitive demand of the tasks allowed us to engage in mathematics that pushed our understanding, and the multiple solution paths provided us with a rich context for discussing the mathematical concepts involved in solving the tasks. Many participants reported this experience was uncomfortable. Engaging in this type of activity, however, was important to our community building.

Two important aspects of the design of IM supported our growth as a community of practice: the active learning of the participants as they engaged with the mathematical tasks and the number of contact hours we worked together as a group. If the design of IM included multiple lectures, the participants would not have interacted together to develop their mathematical understanding. The focus of their interactions could have been diminished to talking about requirements of the course, for example. The tasks allowed each of us to engage in the mathematics at an appropriate level and learn together. If the IM class had only met for a few class meetings, our development of a community would not have been as mature. In addition, the perceived pressure I felt to cover the content would have only increased. Thus, these two characteristics of PD are important in community development.

Implications

If building a community is a goal of a PD experience, then community-building opportunities should be incorporated explicitly into the design of the PD from the outset. PD should incorporate activities that are engaging for the participants, provide time for participants to engage and discuss the activities, and have a clear focus on content. These characteristics do not guarantee the development of a community of practice because of all the outside influences that are uncontrollable by a facilitator. However, without the time to engage together in content, to develop norms, and to agree on a collective purpose, community formation would be nearly impossible.

Tasks with high cognitive demand tasks, multiple entry points, and multiple solution paths can foster community development. Because teachers find these tasks challenging, it is important to establish norms and expectations for interaction. Such norms might include the expectation that participants always provide explanations for their ideas, that other participants question anything they do not understand, and that all contributions, whether correct or incorrect, be treated with respect and interrogated thoroughly.

The preparation of PD facilitators should include explicit attention to community-building actions the facilitator can use. Most PD facilitators have little formal preparation for their roles, and their expertise is often in facilitating learning of the content. But if a focus of the PD is to be on community building in order to facilitate teacher learning, then the leader of the PD needs to be prepared to promote community development.

Future Research

Future research on community development in mathematics education should have a goal of developing a mathematics-specific model of community formation. The models provided by Wenger (1998) and Grossman et al. (2001) are a starting point for exploring community development, but their models were developed in other disciplines (learning sciences and anthropology for Wenger and history and English for Grossman et al.) and therefore do not apply seamlessly to mathematics education.

Future research could compare the findings of this study with the same PD conducted in a different district, with a different group of teachers, or with a different facilitator to try to tease out the variables that affect community development. Research conducted in another school district could provide insight into ways that district cultures encourage or hinder community development. A study in which I taught the same IM course to a different group of teachers could highlight participant characteristics that are important to group formation. However, if I were to teach the course again, my actions would be different based on my reflections on this study. If a new facilitator taught the course, it would interesting to compare his or her experience to mine in balancing building a community and supporting teachers knowledge growth.

Changing the design of the IM syllabus to include time devoted to community building activities, such as debriefing the discussion, would also be an interesting modification to explore. Conducting a study with the same research questions and methods in a different PD context could lead to an understanding of the benefits and drawbacks of IM for promoting a community.

Finally, little is known about the learning and growth of PD facilitators. PD is viewed as a learning experience for teachers; however this experience was a tremendous learning experience for me. Future research should explore the growth and learning of facilitators.

The ultimate goal of effective PD is to have a lasting, positive impact on teacher's knowledge and practice. This, in turn, will support the learning of their students. Developing a community of practice has the potential to further the impact of a PD experience by creating a supportive group of teachers who continue to push each other to grow professionally.

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APPENDICES

APPENDIX A: SYLLABUS OF IM

Week 0 (August 26^{th}): Pre-assessment

Week 1 (September 9^{th}): Parts

Week 2 (September 16^{th}): More Parts

Week 3 (September 23^{rd}): Parts of Parts (Fraction Multiplication)

Week 4 (September 30^{th}): Pesky Parts I (Fraction Division)

Week 5 (October 7^{th}): Pesky Parts II

Week 6 (October 14^{th}): Decimal Multiplication

Week 7 (October 21^{st}): Decimal Division

Week 8 (October 28^{th}): Recap

Week 9 (November 4^{th}): Ratios

Week 10 (November 18^{th}): Proportions

Week 11 (December 2^{nd}): Inverse Proportions

Week 12 (December 9^{th}): Mathematical Connections

Week 13 (December 16^{th}): Post-assessment

APPENDIX B: PORTION OF A LESSON GRAPH FROM IM

Teacher: RB
Date: 09/09/08
Lesson Graphed by: Rachael Brown

Time	Description	Comments
Pt 1		
5:11:25	Introductions by all participants. RB asked that they say their name, the name of their favorite math teacher, and why that person was their favorite. A minute or so was given to let people think. First two people stood and said name, where they work, favorite teacher, and why. The rest sat and said the same information.	Interesting that only first two stood. And that they all said where they worked. They sat by school, so two people sitting alone were only teacher from their school. I said mine too in an attempt to show that I am "one of them".
5:18:34	Transition into a description of IM. RB explains typical class of 2 group investigations, organized by a theme related to rational number, choose one of up to 4 individual investigations – encouraged collaboration, and do a write-up by end of night or end of week.	Tried to speak about need for working together
5:20:04	Goal for night: Fraction Bars – new software and then we'll do the same individual investigation. Three themes of IM are referent unit or reference whole, proportionality, and drawn representations. Most teachers wrote down the three themes as RB talked about them. Tonight the focus will be on referent unit: where is one?	I made a comment to the effect there are no secrets here. I talk about What is 1 or whole? Here. I've got to be careful. And we should talk about using the word 1 versus whole.
5:22:30	Expectation activity. Three questions: What do you expect from class? What do you expect from facilitator? What do you expect from yourself? Think-pair-share (~1 min each step) What we write down represents an agreement and RB described why she thinks we should do this.	I will want to spend some more thorough time analyzing this next semester.
5:25:30	Sharing out of expectations for course. RB goes back and forth and checks for agreement. RB explains why she is pushing this. (5:30:14 – discussion on words manipulative, drawing, visual. RB asked where software fits in.)	When I was facilitating this discussion/conversation I felt like I was using interviewing skills/techniques. What is

		the difference between researcher and teacher?

APPENDIX C: SAMPLE TELEPHONE INTERVIEW PROTOCOL

1. Now that you have gone to a few InterMath classes, how would you describe it to another teacher?
    a. What are the goals?
    b. What is a typical class like?
    c. What are you learning?
2. How would you describe Rachael's role as the facilitator?
    a. How would you describe her teaching approach?
    b. What kinds of things does she do to promote mathematical thinking?
    c. How does this compare to other professional development you've participated in?
    d. How does this compare to the approach you use in your classroom?
3. So far, have you worked mostly alone or with other people on the investigations (both whole group and write-up)?
    a. How do you choose when to work alone and when to work with someone else?
    b. Do you generally work alone or with others on issues related to teaching mathematics?
4. So far, how difficult has the course been for you?
    a. What about the course is challenging?
    b. What about the course is easy?

APPENDIX D: MY INSTRUCTIONAL GOALS, MY POSSIBLE ACTIONS AND THE GROUP'S POSSIBLE REACTIONS

Key to my goals column:

1. Safe environment where all mathematical ideas are welcome
2. Learning is collective – interaction and discussion are necessary to learn, we all have something worthwhile to contribute
3. Errors, mistakes, or putting forward tentative, ill-formed ideas are a part of the learning process

Goal	My actions	Possible scenarios	My possible reactions
1	I like to start the first class with a conversation on expectations so that we are all on the same page as to the purpose of the course and the roles of the participants and instructors. My expectations include that we will not know how to do all problems in course immediately or quickly and that part of the learning process is frustration and confusion. Everyone learns differently so comparison to others is not the purpose.	Participants will either respond positively, negatively, or ambiguously	If it is negative, I won't spend much time on it. If its positive or ambiguous I will refer to this conversation throughout the course of the class both to reinforce positive interactions and to extinguish negative ones
1	I will encourage people to share ideas and thoughts by being good about wait time, who I choose to speak, my body and facial reactions, checking other participants' negative reactions, expressing enthusiasm and excitement about the math shared. Additionally, I will either revoice or ask a teacher to revoice another person's idea. I will ask for teachers to agree or disagree with a proposed idea and I will also ask a teacher to finish another teacher's solution or idea or have them explain how this same teacher would do a similar problem.	Participants not respectful of each other. Participants not participating. One person hogging the floor.	If participants are not respectful to each other, my plan is to talk about it openly with the class. If it is just one or two people, again I will publicly ask them to stop and privately talk to them. I hope that the class will be able to regulate its own conversation without much "work" on my part.
1, 2	I will model what I think learning is and be open about my doubts, insecurities, etc about the mathematics in their ideas or when I first worked on a problem myself	Participants won't "get" it. Participants will want my validation of their ideas. They will want	Try to talk openly about choices and decisions I am making. Highlight multiple solution strategies whenever possible. Get them to talk about what is

		to be told how to do the problem. They might assume there is only one correct way to do it and I am withholding information.	frustrating about not knowing how to get started or what to do when you get stuck. Also have them talk about having the assumption that there is right way to do things.
1,2	I will resist saying that's right or wrong when I am trying to encourage a collective discussion and will encourage participants to comment on each others' ideas.	Participants get frustrated by my lack of authority. One person may feel like they know everything and jump in and try to play teacher.	Publicly talk about this frustration and connect it to how they view learning and teaching. Think-pair-share. Address the know-it-all privately about what is going on.
2	When we are working on tasks in class, I will encourage people to work together and communicate what they are learning. This is reinforced by posting our write-ups on the website.	Participants will work independently or I will have cliques form.	I will assign people to work together if this becomes an issue. I may also talk about learning and working styles. I may also encourage them to view small group time as an opportunity to practice teaching skills – don't give away answer, give hints, ask questions, find solutions on own then compare.
2	I'm going to try not to be a primary voice in the class by asking participants to lead discussions on group tasks	Participants could hesitate on doing this. They could be lousy at it and tell everyone the answer.	Encourage and be fair by asking everyone to participate. Monitor and moderate when others are leading; ask questions, pose extensions, etc.
1,2	Share the role of "police" – to make sure everyone has the opportunity to share and to prevent domination of talk time and opinion sharing by setting norms as a group	Participants reluctant to do this because of previous interactions with each other or intimidation	Discussion about norms for participating, getting "talk time", asking for quiet time, write-ups Think-pair-share If need be, I can use the 3 chip rule – everyone has three chips when you talk it costs 1 chip, when you are out of chips you are done talking.

2	When I do guided learning experiences I will try to minimize the view that I am the bearer of knowledge and the participants are there to listen and "receive" by being explicit about my actions and why I am choosing to do a this type of activity, de-emphasize fact that this is knowledge unique to me and encourage input by group so that we all can gain knowledge through our common experience	Participants could view this as just my dog and pony show	Be as sincere as possible. Encourage collective discussion of the concept. I am choosing guided learning experiences for topics that I find really challenging and the research team has as well.
2	Refer to previous tasks and people's reasoning that we have done together – build a shared repertoire, model this by doing and ask questions of participants such as does this remind you of any other tasks, etc.	Participants may feel like this is shallow or unnatural	Explicitly talk about the themes of InterMath and these provide a thread for which all these tasks are connected.
1	During the first class with both IM and control teachers, I will explain my role as instructor and how I fit in to all the research pieces of the project	Possibly my role will still be confused – there may be explicit questions or behavior that indicates confusion	I will try to openly address any issues, concerns, and confusion
1	Every day when there are new people from the team videotaping, I will introduce them to the class to prevent any insecurities and confusion	Presence of new people may disrupt the building of the community	Depending on how I see this is as positive or negative I will try to address it. For example, if the videotaping is making people uncomfortable I will probably tell a joke or share a story about videotaping.
1, 2	I plan to explicitly talk about how we, as a class, should build a community and have norms for interacting. I will avoid using research terms.	In the past, participants react to this positively or neutrally.	If participants react negatively to this, I'm not sure how I will handle it. I generally like to talk about "the elephant in the room" to try and get past it.

APPENDIX E: END OF COURSE SURVEY

Part 1

1. If you were to give the InterMath – Rational Numbers course a letter grade, what grade would you give it and why?

2. To you, what are the 3-4 most important things you learned in the InterMath course?

3. Was there anything you wanted to learn about Rational Numbers that was not adequately addressed in the activities you participated in?

4. Why did you take this course?

5. What did you think about using investigations to explore math concepts?

    a. What was your favorite aspect of using them?

    b. What are your concerns about using investigations to explore math concepts?

Part 2
Please rate how much you agree with agree with the following statements using the following scale:
1 – Strongly Disagree; 2 – Disagree; 3 – Neutral; 4 – Agree; 5 – Strongly Agree

1. The InterMath – Rational Numbers course helped me further develop my own understanding of mathematics.	1	2	3	4	5
2. The InterMath – Rational Number course has shown me some news to teach math to my students.	1	2	3	4	5
3. I think InterMath – Rational Number met my needs as a mathematics teacher well.	1	2	3	4	5
4. InterMath-Rational Numbers has taught me a lot about using investigations to explore math.	1	2	3	4	5
5. InterMath-Rational Numbers has shown me some new ways to teach math to my students.	1	2	3	4	5
6. I feel like I learned a lot about rational numbers in this course.	1	2	3	4	5
7. I feel confident in my ability to use technology tools to construct new and personally meaningful ideas about mathematics.	1	2	3	4	5
8. I feel confident in my ability to use pictures to *explain* mathematical ideas.	1	2	3	4	5
9. I feel confident in my ability to *explore* mathematical ideas or solve	1	2	3	4	5

problems.	
10. I feel confident to explain my own mathematical thinking in writing or out loud.	1 2 3 4 5
11. I feel confident in making conjectures about solutions before I solve a math problem.	1 2 3 4 5
12. I feel confident in using more than one model to demonstrate a mathematical concept (e.g., number line, algorithm, area model, etc.)	1 2 3 4 5
13. I believe that students are more capable of understanding mathematics concepts if they can model them using software.	1 2 3 4 5
14. It is easier for students to develop mathematical understanding if they are allowed to engage in open-ended exploration.	1 2 3 4 5
15. As a teacher, I design learning environments that foster my students' higher order thinking.	1 2 3 4 5
16. A lot of mathematical concepts must just be accepted as true and sound explanations cannot be given for them.	1 2 3 4 5
17. Students become more proficient at math by practicing the same skills and processes repeatedly.	1 2 3 4 5