Preferential Trade Agreements and Bilateral International Relationships

by

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(Under the direction of John Turner)

Abstract

Preferential Trade Agreements are a major aspect of international trade and are the focus of many discussions of current policy and trade discussions. The common belief is that the current GATT/WTO policies of tariff manipulation and market access are wholly incapable of their desired impact due to the vast complexities in the global trade environment. I study these established methods in a property rights framework and show these approaches are not as ineffective as believed. There are two competing effects that a PTA has on relationship-specific investment, and for sufficiently productive firms, a trade agreement yields positive welfare gains.

Index words: Trade policy, Intermediate inputs, Sourcing, Tariffs,
Preferential Trade Agreements and Bilateral International Relationships

by

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Chapter 1

Introduction

The past few decades have been marked by a significant change in the nature of international trade, notably the formation of Preferential Trade Agreements (PTAs). According to the World Trade Organization’s list of reported agreements, the number of PTAs since 1948 was as high as 194 in 2007, compared with 91 in 2000 and just 25 in 1991. The sharp increase in the number of these agreements in the past 23 years is enough to warrant attention, as they are extraordinarily prevalent in the world of international trade. This phenomenon is not specific to just one region, with the WTO’s PTA Database listing 216 countries participating in recognized agreements\(^1\). The World Trade Organization’s 2011 Trade Report shows that as a percentage of global trade, intra-PTA trade grew from 18% to 35% in the period between 1990 and 2008.\(^2\).

Figure 1.1 shows the explosive growth in the number of trade agreements over the past few decades. It is not hard to see why they have garnered increased attention in trade policy and industrial organization literature, and I hope to add a new method for analyzing and examining them.

An example of explosive growth in intermediate input outsourcing due to trade liberalization is automobile seat production. According to Klier and Rubenstein (2009), U.S. imports of seat parts grew from $621 million in 1989 to $4.1 billion in 2008, with $2.8 billion being attributed to Mexico and $667 million from Canada in that year, the two other members of NAFTA (signed in 1994). The majority of the growth in this timespan came from the two member nations, especially

\(^1\)http://ptadb.wto.org/SearchByCountry.aspx
\(^2\)For a more in depth look at the growth of trade agreements and preferential trade, see Urata and Okabe (2010) and Krishna (2012)
after entering into the trade agreement. Figure 1.2 shows levels of imported parts during this timespan with dashed lines representing nations that the U.S. has entered a free trade agreement with, and solid lines representing countries without one. There is significant post-agreement growth in imports relative to non-agreement nations.

A commonly accepted endogeneity of trade agreements is that they have two competing effects from the tariff manipulation, as first outlined in *The Customs Union Issue* (Viner 1951). In it, Viner defines trade creation and trade diversion as the two (potentially) simultaneous results as follows:

- **trade creation** - Lower cost foreign member-country firms producing instead of less efficient domestic firms

- **trade diversion** - Member-country firms with a higher cost structure producing instead of nonmember firms with lower costs

Viner views trade creation as the positive effect of a PTA, while trade diversion is the economically undesired result (from a welfare standpoint). His analysis relies on the principle of market-clearing conditions determining price, as opposed to bargaining. The next question is if there are pre-
existing synergies of a bilateral relationship for firms in both countries, could a trade agreement be a mechanism by which that relationship is strengthened? And if so, what characteristics of the nature of the agreement as well as the industries involved would become important for the construction of the PTA.

I build a model that shows how a Preferential Trade Agreement can, in fact, induce welfare-increasing levels of investment by member-country firms, offering a novel rebuttal to the existing literature. By entering into a PTA, intermediate suppliers in member countries receive a level of protection (in the form of the still existing tariff) that strengthens the incentives to invest in the existing vertical relationship. Tariff manipulation as a means of altering investment incentives has already been shown to be a potential solution to certain international trade scenarios (Ornelas and Turner, 2012) and I look to extend this to a trade agreement framework.

Intermediate inputs have recently garnered increased attention because they have grown to be a significantly large portion of world trade (Antras and Staiger, 2011). The major problem associated with multiple production stages is the imperfect nature of contracts which leads to misaligned supplier incentives. When relationship specific investment can be undertaken to increase the benefit of entering into a vertical relationship, contractual inefficiencies lead to a loss of welfare compared with the social optimum. Attempts have been made at coming up with a solution for this problem, but the general consensus seems to be that as a mechanism for dealing with incomplete contracts,
trade agreements in their current form are ineffective at inducing desired levels of relationship-specific investment.

There are two organizational choices for a firm who deals with foreign suppliers, vertical integration and arm’s length trade. For a more complete analysis of these decisions, see Ornelas and Turner (2008), Antras and Helpman (2004) and Antras and Chor (2012). These papers look at how firms decide whether or not to purchase intermediate suppliers in order to better align production incentives. I eschew the structural decision and instead focus on only foreign outsourcing. Under vertical integration, the choice of investment becomes trivial, because the owner firm naturally chooses a profit maximizing level. By eliminating this option the focus is strictly on the effects of tariff manipulation on relationship-specific investment.

The results indicate that in a model of incomplete contracts, certain levels of protectionism brought by the PTA can be preferable. Hence, the Vinerian trade diversion effect is significantly more complicated than previously believed. There are two main effects that are the driving force of a PTA, relationship strengthening and sourcing diversion. The first effect causes firms to invest more in a bilateral relationship (due to the exogenous protection of the tariff), while the second effect causes inefficiencies associated with a protective tariff.

From here, I incorporate firm heterogeneity into the model to capture whether different industries should be included in these agreements. The basic structure of this differentiation is a productivity parameter over a continuum in the supplier cost function, in the same vein as Melitz (2003). This productivity parameter enters the cost function as a shifter of the marginal cost curve, and can be thought of as an industry-specific unit cost. As it increases in value, firms face a higher cost curve, and are therefore less productive. Because final good demand is irrelevant to the model, the structure needs only to ensure dual sourcing of intermediate inputs by the final good producer. The dual sourcing component implies that inputs are purchased from both member and non-member countries, which is essential for capturing the effects of a trade agreement. This is a very useful feature, and any number of functional forms of consumer utility can be used with the model.

Solving for the equilibrium values, I find that as firms become more productive, society is relatively more well off by that industry being included in the PTA. After setting a specific range of productivity, there is a specific cutoff level that separates firms that should be included in the
agreement from those that should not. Continuing with this approach, I then find that as the initial tariff level rises, the PTA becomes less effective as a means of increasing welfare. This comes as no surprise, as it has been shown that large tariffs may lead to an inoptimal amount of deadweight loss (Ornelas and Turner, 2012).

The model relates in a variety of interesting ways with existing literature in multiple fields. As an analysis of a sector of international trade, it adds a new view of Preferential Trade Agreements arguing against current thoughts. The most prevalent view is that trade agreements under traditional GATT/WTO rules are ineffective at dealing with the current needs of international trade. Baldwin (2011) lays out a case for the vastly increasing complexities in today’s supply-chains and how Vinerian economics is no longer an adequate framework for analyzing world trade. He stresses regulatory economics as opposed to tariff analysis as the necessary guideline of trade policy. The complete inability of the WTO’s current function could, in his opinion, lead to drastically undesirable developments in the world trade structure. This paper instead lays out an extension of the old methodology as a means of coping with these recent developments, as opposed to tearing it down.

Another paper arguing against the GATT/WTO concepts as able mechanisms for trade policy is Antras and Staiger (2011). In it, they posit that offshoring and integration of the global supply chain necessitate trade agreements focused on deep-integration as opposed to market access and tariff manipulation. They show that tariff concessions from trade agreements for industries with high input-customization were significantly lower than for industries with low input-customization. Using the methodology from Nunn (2007), the level of input-customization is found by looking at the share of an industry’s inputs that are not traded on an open exchange. This is motivation for them to conclude that countries have increased difficulty in negotiating trade agreements for customized industries, but these results are also encouraging for the approach in this paper. Assuming that these tariff concessions were the ones necessary to induce desired levels of relationship-specific investment, these results are malleable to my framework.

In the model, as industries become more productive, the level of specialized inputs goes up and generic ones goes down, with the total number remaining the same - translating to more productive industries have higher input-customization. The results of the model indicate higher productivity industries should be included in a PTA given a sufficiently low tariff, because too high of a tariff distorts sourcing too much and is welfare reducing. When examined in this light, Antras
and Staiger’s findings could suggest that countries negotiate trade agreements to induce desired levels of investment, but that they include industries that should not necessarily be afforded the protection (the ones with lower input-customization and higher tariff levels). While not a glowing endorsement for all PTAs, this suggests that under the right motivation, some trade agreements using traditional approaches could be beneficial.

In their 1993 paper, Grossman and Helpman also analyze the decision of countries entering a trade agreement and the inclusion/exclusion of certain industries. Their work, however, is focused on the political motives of this decision, as opposed to the purely economic framework this paper uses.

Chapter 2 introduces the general form of the model and presents the results proposed earlier. Chapter 3 is an example of the model, which is used to highlight the effects previously proposed. Finally, Chapter 4 offers a conclusion as well as possibilities for further extensions of the paper.
Chapter 2

Model

2.1 Basic Structure

There are a spectrum of final goods available for consumption in the world economy, as well as a numeraire good that enters a consumer’s utility function linearly. Consumption of the final goods increase utility with diminishing marginal returns.

Final good production is done by Buyer (B) firms located in the country Home. These firms act as aggregators, transforming intermediate inputs into marketable goods. Furthermore, each firm produces a differentiated good, so that each good in the aforementioned spectrum matches up with one and only one final producer. Intermediate goods enter the production function under decreasing returns to scale.

Under these conditions, final good producers in each industry obtain revenue $V(Q)$ from purchasing a total of $Q$ intermediate inputs. The characteristics of $V(Q)$ are $V' > 0$ and $V'' < 0$. Simply put, from purchasing more inputs (and therefore producing more final goods), the final good producers obtain more revenue but at a decreasing rate. The Buyer faces a per-unit tariff $t$ on imported intermediate goods.

When purchasing intermediate inputs, there are two options available to $B$, a generic good available in the world market, $g$, and a specialized intermediate input, $q$, produced specifically for that firm. The specialized good is produced by firms located in the country Away, and to highlight the hold-up effects of bilateral relationships, each buyer $B$ can only source inputs from a specific Supplier ($S$). Each particular supplier firm is identified by $\omega$, a heterogeneity parameter that is a
part of the supplier’s cost function. The specialized goods are not traded on an open market, and therefore have no value outside the dealings between $B$ and $S$. The generic input is produced by a competitive fringe, located in neither Home nor Away and is available for purchase by $B$ at price $p_w$. This price is inclusive of compatibility costs, so the total expenditure by $B$ on generic inputs is $G = (p_w + t) \cdot g$. To ensure production of the final good, the initial level of marginal revenue for $B$ is $V'(0) > p_w + t$.

Sourcing by $B$ is simple enough and quite intuitive - he will choose levels of $q$ and $g$ where the relative marginal cost of purchasing one is equal to the relative marginal cost of the other.

The intermediate supplier faces a cost of producing inputs dependant upon the level of production, $q$, the amount of investment it decides to undertake prior to production, $i$, and its heterogeneity parameter $\omega$. Explicitly, the cost function takes the following functional form:

$$C(q, i, \omega) = (A + \omega + bi)q + \frac{c}{2}q^2$$

As $S$ increases production, cost rises at an increasing rate. The conditions for this are $C_q > 0$ and $C_{qq} > 0$, where subscripts here denote partial derivatives. This implies a quadratic nature for the cost function, $C_{qqq} = 0$, and $C_{qq} = c$, a positive constant. To highlight the case where $B$ engages in dual sourcing with both specialized and generic inputs, $C_q(0) < p_w$. This means that $S$ initially produces inputs at a cost less than the generic market, but as the level rises, their marginal cost eventually exceeds it. Again, to ensure dual sourcing of inputs, the level of $Q$ where $V'(Q)$ intersects $p_w + t$, denoted by $Q^*$, is greater than the value of $q$ where $C_q(q) = p_w + t$. Continuing, $S$’s cost function also satisfies the following conditions: $C_{qi} = b$, a negative constant, and I normalize the effect of the productivity parameter on the marginal cost curve, $C_{q\omega} = 1$. Looking at the relationship between the parameters of the cost function, the final assumption is that $2c \geq b^2$.

With these constraints, $S$ undertakes relationship-specific investment that drives their marginal cost curve down, but does not affect the slope of said curve. Investment is bounded by $i \in (0, i^{max})$, where $i^{max}$ is the level of investment that leads to $C_q(Q^*, i^{max}) = p_w + t$ for the most productive firm ($\omega = 0$). The heterogeneity parameter has a similar effect, acting only as a shifter of marginal cost, but higher levels of $\omega$ move the curve up, so it can be thought of as a unique unit cost each

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1This ensures that the effect of investment on marginal cost is not too large relative to the elasticity of the cost function. If $b$ were too large, the supplier investment incentive would lead to unrealistically large levels of $i$
firm faces. The higher the unit cost, the less efficient the firm is, so lower levels of $\omega$ indicate a more efficient industry. The heterogeneity parameter is bounded by $\omega \in (0, \omega_{max})$, where $\omega_{max}$ is the level that leads to $C_q(0) > p_w + t$. These two assumptions complete the necessary constraints to ensure inputs are purchased from both the specialized supplier and competitive fringe.

Adding the incomplete contracting element to the model, investment is seen by both $B$ and $S$, but is not verifiable in a court of law. The Supplier decides on the level of investment, facing a cost of $I(i)$:

$$I(i) = i^2$$

Qualitatively, as firms increase their level of investment, it is more expensive at the margin.

### 2.2 No Trade Agreement

Initially, there is no trade agreement between Home and Away, and all inputs imported into Home are subject to a per-unit tariff of $t$. In addition to the non-verifiability of investment cost, $B$ and $S$ can not enter into a complete, binding contract prior to production due to inefficiencies and irregularities in their legal systems. Instead, these two firms determine the price of the specialized intermediate inputs, $p_s$, by Generalized Nash bargaining over the surplus due to them trading, where the supplier has bargaining power $\alpha \in (0, 1)$. Examining surplus from trading, $B$ will purchase a total of $Q^*$ units regardless of the sourcing decision.

With the assumptions above, sourcing decisions by $B$ will satisfy the following two conditions with the equilibrium values $q_{No}$ and $g_{No}$:

- $Q^* = q_{No} + g_{No}$, with $V'(Q^*) = p_w + t$
- $C_q(q_{No}, i_{No}, \omega) = p_w$

From a social welfare standpoint, the optimal level of investment for $S$ to undertake maximizes the difference in cost of production of inputs versus the world market, net of investment cost. This value, $\Psi$, is defined below

$$\Psi = p_w g_{No} - C(q_{No}, i_{No}, \omega) - I(i_{No})$$
Differentiating with respect to $i$ and noting that $q_{No}$ increases with investment, the first-best level of investment with no PTA is $i_{No}^{fb}$ such that

$$I'(i_{No}^{fb}) = -C_i(\cdot)$$

because in equilibrium with no trade agreement, $p_w = C_q(\cdot)$.

Under Generalized Nash bargaining, the two firms maximize the following expression with respect to $p_s$, the price of the goods being bargained over:

$$\Lambda = (U_T^B - U_N^B)^{(1-\alpha)}(U_T^S - U_N^S)^\alpha$$

where $U_T^J$ is the verifiable profit that firm $k$ (either B or S) would receive under scenario $J$. The two scenarios possible are them cooperating and trading ($T$) or not being able to reach an agreement and not trading ($N$). These values are laid out below:

- $U_T^B = V(Q^*) - (p_w + t)q_{No} - (p_s + t)q_{No}$
- $U_N^B = V(Q^*) - (p_w + t)Q^*$
- $U_T^S = p_s q_{No} - C(q_{No}, i_{No}, \omega)$
- $U_N^S = 0$

The reason the cost of investment is not included in the expression for $S$’s profit is that it is non-verifiable (from above) and therefore impossible to include in any contracting or negotiations.

Defining $\beta = (U_T^B - U_N^B) + (U_T^T - U_N^T)$ as the bargaining surplus, it can easily be shown that this process will result in a price by which the two firms split the proceeds with $S$ receiving $\alpha \beta$ and $B$ receiving $(1 - \alpha) \beta$.

With no trade agreement, $\beta_{No} = p_w q_{No} - C(q_{No}, i_{No}, \omega)$ which is the difference in cost of producing the intermediate inputs that $S$ would produce under trading. This would indicate that $S$’s investment decision is now simply to maximize their portion of the bargaining surplus net of investment cost with respect to $i$:

$$\max_i \alpha \beta_{No} - I(i_{No})$$
Table 2.1: Explicit Values - No PTA

<table>
<thead>
<tr>
<th>$q_{eq}^{No}$</th>
<th>$p_w - A - \omega - bi_{No}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{eq}^{No}$</td>
<td>$(b - \frac{2c}{a'b})^{-1}(p_w - A - \omega)$</td>
</tr>
</tbody>
</table>

Equilibrium investment $i_{eq}^{No}$ is such that $I'(i_{eq}^{No}) = -\alpha C_i(\cdot)$. Comparing this to the condition of the first-best level of investment above, it should be intuitive that $i_{eq}^{No} < i_{fb}^{No}$. Under bargaining negotiations, $S$ chooses $i$ to maximize his share of the surplus, as opposed to maximizing total surplus. The more bargaining power that $S$ has (and the greater $\alpha$ is), the closer equilibrium investment comes to the first best level. In fact, if $\alpha = 1$, $i_{eq}^{No} = i_{fb}^{No}$, if $\alpha = 0$, $i_{eq}^{No} = 0$ and if $0 < \alpha < 1$, $0 < i_{eq}^{No} < i_{fb}^{No}$.

Table 2.1 outlines explicit values of $q_{No}$ and $i_{No}$ based on the functional forms of production and investment cost:

### 2.3 Entering into a Trade Agreement

The next step is outlining what happens if Home and Away enter into a Preferential Trade Agreement and include industry $\omega$ in it. Attention is restricted to the tariff manipulation aspect of a trade agreement, so the impact of the PTA is the complete removal of the tariff for intermediate goods traded for that industry. Because its producers are located in nonmember nations, the generic good still faces the tariff, and the relative price of it for $B$ is still $p_w + t$. However, the specialized supplier’s relative price drops to $p_s$, as opposed to $p_s + t$ without the agreement.

Total inputs purchased by $B$ is still $Q^* = q_{PTA} + g_{PTA}$, but the sourcing decision changes to reflect the change in relative prices. This is summarized by the condition $C_q(q_{PTA}, i_{PTA}, \omega) = p_w + t$. The initial effect on $q$ (and subsequently $g$) is for $q_{PTA} > q_{No}$ and $g_{PTA} < g_{No}$.

Only one of the potential firm profits, $U^T_B$, has structurally changed and is now

$$U^T_B = V(Q^*) - (p_w + t)g_{PTA} - p_sq_{PTA}$$
Table 2.2: Explicit Values - PTA

<table>
<thead>
<tr>
<th>( q_{PTA}^{eq} )</th>
<th>( \frac{p + t - A - \omega - bi_{PTA}}{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_{PTA}^{eq} )</td>
<td>( \left( b - \frac{2c}{ab} \right)^{-1}(p + t - A - \omega) )</td>
</tr>
</tbody>
</table>

\( \beta_{PTA} \), the bargaining surplus under a trade agreement, is still defined in the same manner as above, and noting the change in buyer profit with trade,

\[
\beta_{PTA} = (p + t)q_{PTA} - C(q_{PTA}, i_{PTA}, \omega)
\]

Generalized Nash bargaining still leads to \( B \) and \( S \) retaining the same shares of \( \beta_{PTA} \) as they do without a trade agreement, so the investment decision is described below:

\[
\max_i \alpha \beta_{PTA} - I(i_{PTA})
\]

which will lead to

\[
I'(i_{PTA}^{eq}) = -\alpha C_i(\cdot)
\]

These results show that the first effect of a trade agreement is increasing relationship-specific investment levels. By no longer facing the tariff, \( S \) enjoys an immediate increase in the level of \( q \) it produces for \( B \), which increases the amount of surplus to be gained from bargaining. The investment decision is directly tied to this surplus, and the marginal cost of \( i \) at the non-agreement level is now lower than the marginal benefit to be had in terms of cost savings (as well as a further increase in the level of \( q \)). Investment is then increased to compensate for this, based on the amount of increased surplus the Supplier can now capture.

The relative size of the increase then depends on the Supplier’s bargaining power. If \( \alpha = 0 \), then there is no change in level of investment due to a PTA, but if \( \alpha > 0 \), a PTA will unambiguously increase investment undertaken by \( S \). Again, the explicit values for \( q_{PTA} \) and \( i_{PTA} \) are outlined in Table 2.2.

Figure 2.1 summarizes the equilibrium sourcing decisions prior to entering a PTA as well as after one.
Entering into the PTA and shifting relative prices increases the level of $q$ through multiple channels. First, the direct effect of relative pricing gives $S$ a competitive advantage, which in turn induces higher levels of investment, driving the marginal cost curve down. One additional level of $q$ is notated on the figure, $q_1$, which is the level of intermediate inputs that leads to $C(q, i_{PTA}, \omega) = p_w$. Explicitly, $q_1 = \frac{p_w - A - \omega - bi_{PTA}}{c}$.

This specific level of $q$ marks the point at which $S$ is no longer producing inputs at a marginal cost under that of the world market. For all $q \in (q_1, q_{PTA})$, their production is inefficient relative to the optimal, because the competitive fringe could produce them at a lower marginal cost than $S$. This deadweight loss is highlighted in the figure with horizontal stripes, and it is exactly what Viner was referring to when he spoke of the trade diversion effect.

What Viner did not account for is the potential cost savings associated with alleviating under-investment in bilateral relationships, marked by the shaded area in the figure. Of course, the next step is to examine whether or not the alleviating effect is large enough to offset the deadweight loss from the sourcing diversion (as well as the increased investment cost) a PTA causes.
2.4 Welfare Effects of a PTA

The effects of entering into a trade agreement can be significantly more nuanced than believed, and now the next step is to analyze these effects quantitatively. Examining the previous assertions made about entering into a trade agreement, consumer welfare from the final good remains constant in the model, because the total level of inputs purchased, $Q^*$, does not change. The only aspects that are changing due to the PTA is the cost of producing the intermediate inputs, as well as the cost of investment incurred by $S$. This implies that the expression for change in welfare from a PTA is the difference in production and investment cost, defined as $\Delta \Psi$.

For a PTA to be welfare enhancing, the total cost of production of intermediate inputs inclusive of investment cost under the agreement must be less than the total cost not under it. This translates into a positive value for $\Delta \Psi$ being desired.

From here, $\Delta \Psi$ is separated into the two effects previously highlighted, *relationship strengthening* and *sourcing diversion* to see how they change with firm productivity.

2.4.1 Relationship Strengthening Effect

The positive desired effect of a PTA is the increased investment by $S$ prior to bargaining, which lowers the marginal cost of production for intermediate inputs. Quantitatively, this effect is defined as the difference in cost of inputs produced under the marginal cost of the world market ($q \in (0, q_1)$) net of increased investment cost. Explicitly, this is

$$\Delta \Psi_R = C(q_{No}, i_{No}) + p_w(q_1 - q_{No}) - C(q_1, i_{PTA}) - (I(i_{PTA}) - I(i_{No}))$$

To determine how productivity level affects welfare from an agreement, I take the partial derivative of this expression with respect to the heterogeneity parameter, $\omega$. It is worth noting that the quadratic nature of the cost function implies that the levels of $q$ and $i$ change at a constant rate.

$$\frac{\partial \Delta \Psi_R}{\partial \omega} = [C_q(q_{No}, i_{No}) - C_q(q_1, i_{PTA})]\frac{\partial q}{\partial \omega} + [C_i(q_{No}, i_{No}) - C_i(q_1, i_{PTA})]\frac{\partial i}{\partial \omega}$$

$$+ [C_{\omega}(q_{No}, i_{No}) - C_{\omega}(q_{No}, i_{PTA})] - 2[i_{PTA} - i_{No}]{\frac{\partial i}{\partial \omega}}$$
In terms of the parameters of the model, this is

\[
\frac{\partial \Delta \Psi_R}{\partial \omega} = \left[ \frac{b}{c} \left( \frac{b - 2c}{\alpha b} \right)^{-1} + \left( 2 - \frac{b^2}{c} \right) \left( \frac{b - 2c}{\alpha b} \right)^{-2} \right] t
\]

which is strictly negative. This implies that as firms become less productive, the cost saving aspects of a PTA unambiguously shrink.

### 2.4.2 Sourcing Diversion

As previously pointed out, the negative aspect of an agreement is qualitatively thought of as the specialized inputs produced over the world market marginal cost. This is the direct result of the protection the tariff affords \( S \) by skewing the sourcing decision away from the initial equilibrium. Explicitly, this is

\[
\Delta \Psi_S = C_q(q_{PTA}, i_{PTA}) - C_q(q_1, i_{PTA}) - p_w(q_{PTA} - q_1)
\]

Differentiating this with respect to \( \omega \) and remembering the effect of the heterogeneity parameter on our values of \( q \) and \( i \), the first-order partial is:

\[
\frac{\partial \Delta \Psi_S}{\partial \omega} = \left[ C_q(q_{PTA}, i_{PTA}) - C_q(q_1, i_{PTA}) \right] \frac{\partial q}{\partial \omega} + \left[ C_i(q_{PTA}, i_{PTA}) - C_i(q_1, i_{PTA}) \right] \frac{\partial i}{\partial \omega} + C_\omega(q_{PTA}, i_{PTA}) - C_\omega(q_1, i_{PTA})
\]

Which, when put in terms of the parameters of the model, this equals 0. This result implies that the negative effect of a PTA stays constant regardless of productivity. Qualitatively, this means that the downside of an agreement stays the same, while the upside varies with the productivity of the supplier firm.

### 2.4.3 Total Change

I summarize the results regarding productivity and welfare in the following proposition, whose proof (along with the proof of the following corollary) is in the Appendix:

**Proposition 1** As \( S \) becomes less productive, the cost savings to be had from a Preferential Trade Agreement unambiguously decrease.
This result shows that the desirability of the inclusion of specific industries in the PTA is not all the same. In fact, there exists a level of the heterogeneity parameter, \(\omega^*\), that is defined as \textit{the minimally sufficient level of productivity}, where the \textit{relationship strengthening effect} no longer outweighs the \textit{sourcing diversion effect} and \(\Delta\Psi = 0\). The nature of this value is described in the following corollary.

**Corollary 1** \textit{For sufficient values of the exogenous parameters of the model, all firms} \(\omega < \omega^*\) \textit{will enjoy welfare gains from the agreement, while all firms} \(\omega > \omega^*\) \textit{will lose welfare from the PTA.}

### 2.5 The Effect of the Tariff Level

Now that there is a definitive set of statements concerning firm productivity, the tariff level becomes the focus. This begins with a qualitative discussion of the size of \(t\). Obviously, if the tariff does not exist (\(t = 0\)), then a PTA has no effect on welfare making this case irrelevant. Increasing from this level, it is clear that both specialized inputs and investment level under an agreement increase, but the question is to what extent does this affect welfare.

First examining how the tariff affects the \textit{sourcing diversion effect}, it should not be hard to see that increasing the size of \(t\) will exacerbate this distortion. The greater the tariff, the farther the spread between the marginal cost of the specialized supplier and the world market, adding more inefficiency to the market.

Now, looking at the \textit{relationship strengthening effect}, the tariff has a much more nuanced effect. Investment increases, lowering the marginal cost function, but at an increased cost. As already noted, a portion of the additional specialized inputs are produced at a lower relative marginal cost while the other portion is produced above it. Both of these sections are exacerbated by an increase in the tariff, as well as the units that would have been produced by \(S\) without a PTA. Sizing the two competing effects is key, however the current structure does not allow us to definitely sign the total change the tariff has.

Even though this yields an ambiguous result regarding change in welfare, the following proposition concerning the \textit{minimally sufficient level of productivity} can be made, with the proof in the Appendix:
**Proposition 2** As the tariff approaches the limiting case $t = 0$, there are always some firms who will realize surplus gains from a trade agreement. As the tariff level rises, the minimally sufficient level of productivity unambiguously decreases and fewer firms will exhibit net positive effects from a PTA.

Figure 2.2 shows the relationship between $\omega^*$ and the tariff level graphically, offering a concise picture of the effect $t$ has. From here, the next step is presenting a specific example of the model in order to add clarity to the effects of the heterogeneity parameter and tariff level on the change in welfare from a PTA.
Chapter 3

An Example

In order to quantify the welfare effects of entering a PTA, the example uses simplified values for certain parameters, as well as choosing specific forms of previously undefined functions. The general model yields multiple clear results concerning productivity, and this is to highlight some of the more ambiguous effects by looking closer at the tariff level.

The first step in this specification is setting a functional form of $B$’s value function, $V(Q)$, defined as:

$$V(Q) = 3Q - \frac{1}{4}Q^2$$

which implies a linear form of the marginal value function, $V’(Q)$. A world price of the generic input that will ensure a sufficient level of final good production is $p_w = 2$, implying the total level of inputs, $Q^*$, purchased is decided by the condition:

$$2 + t = 3 - \frac{1}{2}Q^*$$

Because the parameters of the Supplier cost function play such a large role in the analysis, setting specific values is paramount in quantifying the model’s effects:

- $A = 1$

- $b = -1$ - investment has the same sized effect on the marginal cost function as productivity

- $c = 1$ - elasticity of the cost function is normalized to 1
Table 3.1: Specific Model Results

<table>
<thead>
<tr>
<th>Result</th>
<th>No PTA</th>
<th>PTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>$2 - 2t$</td>
<td>$2 - 2t$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\frac{2}{1-\alpha}(1 - \omega)$</td>
<td>$\frac{2}{1-\alpha}(1 + t - \omega)$</td>
</tr>
<tr>
<td>$g$</td>
<td>$\frac{2}{1-\alpha}(1 - 2t - \alpha + \alpha t + \omega)$</td>
<td>$\frac{2}{1-\alpha}(1 - 3t - \alpha + \alpha t + \omega)$</td>
</tr>
<tr>
<td>$i_f^b$</td>
<td>$1 - \omega$</td>
<td>$1 - \omega + t$</td>
</tr>
<tr>
<td>$i_e^c$</td>
<td>$\frac{\alpha}{1-\alpha}(1 - \omega)$</td>
<td>$\frac{\alpha}{1-\alpha}(1 - \omega + t)$</td>
</tr>
</tbody>
</table>

which yields a cost function of

$$C(q, i, \omega) = (1 + \omega - i)q + \frac{1}{2}q^2$$

and marginal cost of

$$C_q(q, i, \omega) = 1 + \omega - i + q$$

Investment cost remains unaltered. The bargaining process between $S$ and $B$ is still the same, with bargaining power of $\alpha$ for $S$ and $1 - \alpha$ for $B$. The sourcing and investment decisions also remain unchanged, so the equilibrium decisions follow the same path. Table 3.1 outlines the resulting levels of total, specialized and generic inputs as well as investment levels in equilibrium:

Levels of $q$ and $i$ are decreasing in $\omega$ and increasing in $t$, with the opposite effect holding for generic inputs.

Using the same methodology as before, $\Delta \Psi$ is defined as the difference between total cost under both a PTA and without one. Again, defining $\omega^*$ as the value of the productivity parameter that separates welfare-enhancing firms under a PTA from welfare-reducing ones, the expression for the minimally sufficient level of productivity is:

$$\omega^* = \frac{2\alpha^2 - 2\alpha - 2\alpha t + 2t + \alpha^2 t}{2\alpha^2 - 2\alpha}$$

The two effects of a PTA remain the same, however they are split up into finer components to highlight tariff level effects more acutely.
As defined in Figure 3.1, there are three separable changes in welfare from production due to a PTA, as well as a fourth that encompasses change in investment cost. These changes are separated into the two previously defined effects of a PTA - relationship strengthening and sourcing diversion. First, the quantitative definitions of each area are laid out, and then broken down by the effect of changing the tariff level.

- **R1** is the cost-savings in the production of inputs produced without the PTA, and is mathematically defined as $C(q_0, i_{No}) - C(q_0, i_{PTA})$

- **R2** is the change in cost of production of additional inputs due to the PTA that are produced under the generic marginal cost of $p_w$, defined as $p_w(q_1 - q_0) - [C(q_1, i_{PTA}) - C(q_0, i_{PTA})]$

- **R3** is the additional cost of specialized inputs that are produced above a marginal cost of $p_w$, $[C(q_2, i_{PTA}) - C(q_1, i_{PTA})] - p_w(q_2 - q_1)$

- **R4** is the change in investment cost, defined as $I(i_{No}) - I(i_{PTA})$

### 3.1 Relationship Strengthening Effect

As previously discussed, the positive effect of a trade agreement is the increased level of cooperation between $B$ and $S$. By increasing the relationship-specific investment level, the supplier’s cost curve shifts down, lowering the cost of production for a portion of the inputs produced. In terms of
the areas identified, R1, R2 and R4 will be included in this effect, because these three deal with surplus gained from inputs produced at a lower marginal cost as well as the cost of this increased investment.

Taking the derivative of R1 with respect to t:

\[
\frac{\partial R1}{\partial t} = C_q^{N_o}(\cdot) \frac{\partial q_0}{\partial t} + C_i^{N_o}(\cdot) \frac{\partial i_{N_o}}{\partial t} - \left[ C_q^{PTA}(\cdot) \frac{\partial q_0}{\partial t} + C_i^{PTA}(\cdot) \frac{\partial i_{PTA}}{\partial t} \right]
\]

Plugging in the respective values gives the following expression for \( \frac{\partial R1}{\partial t} \) in terms of the exogenous parameters of the model:

\[
\frac{\partial R1}{\partial t} = \left( \frac{2}{2 - \alpha} \right) \left( \frac{\alpha}{2 - \alpha} \right) (1 - \omega)
\]

By construction, R1 is always positive, because \( C(q_0, i_{PTA}) < C(q_0, i_{N_o}) \). \( \frac{\partial R1}{\partial t} \) is unambiguously positive, which means that as the tariff level rises, the amount of surplus gained from a PTA in production of units \( q \in (0, q_0) \) increases.

Moving on, R2 is the surplus gained from S producing units above the non-PTA level but at marginal cost below that of the generic market's. Taking the first order partial with respect to t:

\[
\frac{\partial R2}{\partial t} = p_w \left( \frac{\partial q_1}{\partial t} - \frac{\partial q_0}{\partial t} \right) - \left( \frac{\partial C(q_1, i_{PTA}, \omega)}{\partial t} - \frac{\partial C(q_0, i_{PTA}, \omega)}{\partial t} \right)
\]

Thinking through this area intuitively, a higher tariff increases both \( q_1 \) and investment under a PTA, so R2 grows with the tariff. With the parameters of the model:

\[
\frac{\partial R2}{\partial t} = \frac{2 \alpha}{2 - \alpha} + \left( \frac{\alpha}{2 - \alpha} \right)^2 t
\]

which is also an unambiguously positive value.

The final component of the relationship strengthening effect is the difference in cost of investment incurred by S prior to bargaining. Because \( i_{PTA} > i_{N_o} \), R4 is defined as \( I(i_{PTA}) - I(i_{N_o}) \) so the value is positive (and therefore comparable with the other sections). Once again, differentiating with respect to t:

\[
\frac{\partial R4}{\partial t} = I'(i_{PTA}) \frac{\partial i_{PTA}}{\partial t} - I'(i_{N_o}) \frac{\partial i_{N_o}}{\partial t}
\]
but under no PTA, the tariff does not enter the equilibrium value of investment. This means that the only change in investment cost from a change in $t$ is in the PTA value. Quantitatively, this value is

$$\frac{\partial R4}{\partial t} = \left(\frac{2\alpha}{2 - \alpha}\right) \left(1 + t - \omega\right) \left(\frac{\alpha}{2 - \alpha}\right)$$

which is once again a positive value.

This result implies that as the initial tariff level increases, entering into a PTA leads to a greater increase in investment cost the supplier incurs.

### 3.2 Sourcing Diversion Effect

The second effect highlighted is the negative side of a trade agreement, *sourcing diversion*. When suppliers from *Away* are afforded protection from the tariff, the allocation of input production is no longer optimal. $S$ is now producing at a marginal cost above the world market, but this shift causes higher investment (which increases investment cost accordingly). For these reasons, the sourcing diversion effect is defined as $R3$, the extra cost of producing inputs above the marginal cost of the world market.

$R3$ is the net surplus lost from the production of specialized goods over the marginal cost of the generic market. Here is the negative effect of a Preferential Trade Agreement. Looking at how $R3$ changes with $t$:

$$\frac{\partial R3}{\partial t} = \left(\frac{\partial C(q_2, i_{PTA}, \omega)}{\partial t} - \frac{\partial C(q_1, i_{PTA}, \omega)}{\partial t}\right) - p_w \left(\frac{\partial q_2}{\partial t} - \frac{\partial q_1}{\partial t}\right)$$

Again, defining this in terms of the parameters of the model:

$$\frac{\partial R3}{\partial t} = \frac{2\alpha}{2 - \alpha} - \left(\frac{\alpha}{2 - \alpha}\right)^2 t + \left(\frac{2 - 2\alpha}{2 - \alpha}\right) t$$

Over the relative values of $t$ and $\alpha$, this value is also always positive. Intuitively, this makes sense because as the tariff increases, the gap between the maximum level of $S$’s marginal cost and the marginal cost of the generic market gets wider. In addition to this, $q_{PTA}$, or the total number of specialized inputs produced under a PTA, rises faster than $q_1$ from a change in $t$, exacerbating the effect.
3.3 Total Surplus Change

Each of the component partials is positive, meaning that an increase in the initial tariff exacerbates all aspects of the differences in welfare from a trade agreement. The next step in determining the effectiveness of a PTA is sizing the positive and negative effects.

Taking note that the total change in welfare from a PTA is

\[ \Delta \Psi = R_1 + R_2 - R_3 - R_4 \]

where positive values of \( \Delta \Psi \) are economically desired, this implies

\[ \frac{\partial \Delta \Psi}{\partial t} = \frac{\partial R_1}{\partial t} + \frac{\partial R_2}{\partial t} - \frac{\partial R_3}{\partial t} - \frac{\partial R_4}{\partial t} \]

and in the terms of the model:

\[ \frac{\partial \Delta \Psi}{\partial t} = (1 - \omega) \frac{2\alpha}{(2 - \alpha)^2} (1 - \alpha) - \left( \frac{2 - 2\alpha}{2 - \alpha} \right) t \]

This is a negatively sloped linear function of \( t \), with the vertical intercept corresponding to some positive value. With this information alone, a relatively complete picture of the tariff level effect on the welfare change from a PTA can be constructed.

At \( t = 0 \), a PTA is simply a novelty, because there is no difference between either case, so \( \Delta \Psi_{t=0} = 0 \). Referring back to \( \frac{\partial \Delta \Psi}{\partial t} \), at \( t = 0 \) this is positive, so an incremental increase in \( t \) from the initial zero level leads to an unambiguously positive (and therefore preferred) change in welfare. This result indicates that for a firm of any productivity level specified in the model, there is some initial tariff level that results in a trade agreement being welfare enhancing.

As \( t \) increases from this zero value, the marginal change in surplus from a PTA is strictly decreasing at a constant rate. This implies an inverted U shape for \( \Delta \Psi(t) \), meaning as the tariff level increases, welfare from a PTA first increases at an ever slower rate until it levels off and begins decreasing. This continues until the welfare change is no longer positive. This value of \( t \) (for a given \( \omega \)) has similar implications for welfare as \( \omega^* \), or the minimally sufficient level of productivity.
does. For the given level of productivity, this is the maximum welfare-enhancing tariff level where anything greater will yield societal welfare losses.

Again looking at $\frac{\partial \Delta \Psi}{\partial t}$, the effect of $\omega$ is very clear. It enters the expression only in the intercept, meaning it will act as a shifter of the marginal welfare change, but not affecting the slope. The cross partial of $\Delta \Psi$ with respect to $t$ and $\omega$ is:

$$\frac{\partial^2 \Delta \Psi}{\partial t \partial \omega} = -(1 - \alpha) \frac{2\alpha}{(2 - \alpha)^2}$$

which is a negative value. This result can be interpreted to mean that as firms become less productive, the rate at which their marginal welfare changes will be less than a firm with higher productivity’s, and the gains to be had from a PTA are relatively lower for each specific level of $t$.

Figure 3.2 provides graphs of both $\frac{\partial \Delta \Psi}{\partial t}$ and $\Delta \Psi$ for a high- and low-productivity firm.

Looking first at $\frac{\partial \Delta \Psi}{\partial t}$, the level of $t$ that sets this equal to 0 is identified, which is the tariff level that maximizes welfare for a given level of productivity. It should be apparent that this value of $t$ is strictly higher for higher productivity firms. This indicates that higher productivity firms enjoy positive welfare gains from an increased range of tariff levels than lower productivity firms.
Turning now to $\Delta \Psi$, the maximum levels of surplus from a PTA for both cases are indicated, and it should come as no surprise that higher productivity firms have a larger possible maximum surplus level, as well as higher levels of surplus for each level of $t$.

By looking at the figure above, a clear trend can be seen in the effect of increasing $t$. Initially, gains are had from an agreement, but they grow at a decreasing rate until they are maximized at some given level of the tariff. Continuing, the extra surplus begins to fall at an ever faster rate until all gains to be had are erased and the change in welfare is equal to 0. At this point, a trade agreement begins to result in welfare losses from the *sourcing diversion effect* outweighing the *relationship strengthening* one. These losses grow at a faster rate than the welfare gains, which would indicate that while tariff manipulation can be an effective mechanism for strengthening international bilateral relationships, it must be used with some caution because of the severity of the results of overly strong trade barriers.

The figure shows that there exists a maximum possible positive increase in welfare to be gained from a PTA for firms of all productivity levels, and it strictly increases for more productive firms. The focus of the paper is the desirability of trade agreements, but this result is promising for optimal tariff policy analysis.
Chapter 4

Conclusion

Preferential Trade Agreements are the focus of a significant amount of recent trade literature because of their rapidly increasing prevalence in the global economy. The current belief about these trade agreements is that they are antiquated and unable to effectively govern international trade. The model in this paper shows that the WTO/GATT approach of tariff manipulation and market access can be effective due to the significantly more nuanced implications they have for trade than traditional Vinerian economics suggests.

The model first shows that the previously defined Vinerian trade diversion effect of a PTA actually consists of two juxtaposed effects, relationship strengthening and sourcing diversion. The latter is the traditional view of the effect of a PTA, where inefficient member firms produce as opposed to more cost-effective foreign ones, but the former involves increased relationship-specific investment building synergies between two naturally vertically related firms.

These effects are then examined over a spectrum of heterogeneous firms, where it is shown that more productive firms have a larger amount of surplus to potentially gain from the tariff protection a trade agreement provides. This increase in surplus is dependant, however, on the size of the tariff faced by nonmember firms. The model, concurrent with other trade literature, shows how large protectionist tariffs can have severe consequences for welfare, especially for less productive firms. The analysis focuses only on the case of eliminating the tariff for the specialized supplier, but the results from the example can also be used as an argument for optimal tariff policy dependant on firm productivity.
While the model achieves a novel set of results, it is quite adaptable to a number of extensions. The exogeneity of organizational form is one of the first assumptions made, and acknowledging that choice would certainly be a useful step in broadening the results in the context of trade policy work. Antras and Chor (2012) build a model determining optimal organizational structure for heterogeneous suppliers ignoring the political economy decisions, and a combination of these approaches could certainly yield further interesting results. Another factor that is shut down in the model is the Vinerian idea of trade creation, with member country firms producing more at the expense of domestic firms. It is certainly plausible that the nuances brought along by the bargaining framework to trade diversion will also alter the way the positive effect of an agreement is viewed, and this option is open for further exploration.

Both Home and Away are small relative to the global economy, and their signing of a PTA has negligible impact on the rest of the world. While this narrows the focus to the implications of the agreement on member country firms, it would be of interest to extend the analysis to include the global impact of this deal. In their 2009 paper, Antras and Foley examine changes in the levels of U.S. foreign direct investment in member countries due to the signing of the ASEAN FTA using a model that incorporates firm-level heterogeneity. They find that U.S. multinationals became more active in member countries and that these firms exhibited greater-than-average growth rates. This result is encouraging, and further analysis of agreement-induced investment could yield further knowledge.

Continuing with this approach, incorporating the effect of the relationship-specific investment into an intertemporal DSGE model could yield results beyond the scope of the current literature. Tracking the effects that agreement-induced shocks have on certain macroeconomic indicators would be of interest from both a theoretical and empirical standpoint.

Theoretical extensions are not the only continuation that I would like to see for the model. Empirical analysis of the effects on trade patterns a PTA has is a natural next step, and would be a welcome addition to the already burgeoning literature. Attempts have already been made at determining the welfare effects of trade agreements (for examples, see Krueger (1999) and Chomo (2002)) but their approach is based upon the traditional ideas of trade creation and trade diversion which has a much higher level of complexity than previously thought.
Chapter 5

Appendix

Proof of Proposition 1. It is relatively intuitive and apparent that $\frac{\partial \Delta C}{\partial \omega} > 0$ and $\frac{\partial \Delta I}{\partial \omega} < 0$ (where the spreads are of the form $K_{PTA} - K_{No} \Rightarrow$ a negative value of $\Delta \Psi$ is desired), so in trying to prove

$$|\frac{\partial \Delta C}{\partial \omega}| > |\frac{\partial \Delta I}{\partial \omega}|$$

it is the same as showing

$$\frac{\partial \Delta C}{\partial \omega} > -\frac{\partial \Delta I}{\partial \omega}$$

because the cost savings in generic good production remain constant with respect to $\omega^1$, or in terms of the parameters of the model:

$$\frac{-tb}{c} \left(b - \frac{2c}{ab}\right)^{-1} \left(1 - b \left(b - \frac{2c}{ab}\right)^{-1}\right) > (2t) \left(b - \frac{2c}{ab}\right)^{-1}$$

Dividing by the common terms of $t$ and $(b - \frac{2c}{ab})^{-1}$:

$$\frac{-b}{c} \left(1 - b \left(b - \frac{2c}{ab}\right)^{-1}\right) > (2) \left(b - \frac{2c}{ab}\right)^{-1}$$

$$\frac{-b}{c} + \frac{b^2}{c} \left(b - \frac{2c}{ab}\right)^{-1} > (2) \left(b - \frac{2c}{ab}\right)^{-1}$$

Footnote:

The cost savings are equal to their marginal cost, $p_w$ multiplied by the difference in levels of $q$, but because the quadratic cost curve implies that the first order partial derivative of $q_{PTA}$ and $q_{No}$ with respect to $\omega$ are equal, this will equal 0.
This implies that the overall sign of $\frac{\partial \Delta \Psi}{\partial \omega}$ is positive, which means that as firms become less productive, the cost savings to be had from the decreased investment are outweighed by the extra cost of specialized input production and welfare is unambiguously decreased.

**Proof of Corollary 1.** Starting with investment cost:

$$
\Delta I = J - \left( b - \frac{2c}{\alpha b} \right)^{-2} 2t \omega
$$

where $J = \left( b - \frac{2c}{\alpha b} \right)^{-2} (t^2 + 2p_w t - 2At)$.

Moving to generic good cost:

$$
\Delta G = \frac{p_w}{c} \left( b \left( b - \frac{2c}{\alpha b} \right)^{-1} - 1 \right) t
$$

and finally specialized good cost:

$$
\Delta C = \frac{1}{2c} \left[ EF - BD - 2tb \left( b - \frac{2c}{\alpha b} \right)^{-1} H \omega \right]
$$

where

- $B = A + p_w + b \left( b - \frac{2c}{\alpha b} \right)^{-1} (p_w - A)$
- $D = p_w - A - b \left( b - \frac{2c}{\alpha b} \right)^{-1} (p_w - A)$
- $E = A + p_w + t + b \left( b - \frac{2c}{\alpha b} \right)^{-1} (p_w - A + t)$
- $F = p_w + t - A - b \left( b - \frac{2c}{\alpha b} \right)^{-1} (p_w + t - A)$
- $H = 1 - b \left( b - \frac{2c}{\alpha b} \right)^{-1}$
It can be seen with each of the component spreads that they are either a linear function of $\omega$ or do not include it at all. This is a very encouraging result, because the value of $\omega^*$ that will set $\Delta TC$ equal to 0 can be solved for relatively cleanly. This is the least productive firm for which a trade agreement is weakly welfare enhancing, where all firms with $\omega \in (0, \omega^*]$ should be included and all other firms should not.

The actual expression for the cutoff firm is

$$
\omega^* = \frac{\frac{p_w}{c} H t + \frac{1}{2c} (BD - EF) - J}{-\frac{tb}{c} \left( b - \frac{2c}{ab} \right)^{-1} H - 2t \left( b - \frac{2c}{ab} \right)^{-2}} \quad \blacksquare
$$

**Proof of Proposition 2.** First, it is shown that in the limiting case of $t = 0$, $\omega^*$ is positive, and then that it is decreasing in $t$. Writing out the expression for $\omega^*$ slightly differently than above:

$$
\omega^* = \frac{\frac{p_w}{c} H t + \frac{1}{2c} \left[ AHb \left( b - \frac{2c}{ab} \right)^{-1} t - 2p_w H (1 + b \left( b - \frac{2c}{ab} \right)^{-1}) t - H (1 + b \left( b - \frac{2c}{ab} \right)^{-1}) t^2 \right] - J}{[-\frac{tb}{c} b \left( b - \frac{2c}{ab} \right)^{-1} H - 2 \left( b - \frac{2c}{ab} \right)^{-2}] t}
$$

Everything in the numerator and denominator can be simplified down to the form $Mt + Nt^2$ and $Lt$ respectively. With this, the *minimally sufficient level* can be written as:

$$
\omega^* = \frac{M + Nt}{L}
$$

For the limiting case, $t = 0$, this simply comes down to the fraction $\frac{M}{L}$. Examining the denominator first:

$$
L = -\frac{b}{c} b \left( b - \frac{2c}{ab} \right)^{-1} H - 2 \left( b - \frac{2c}{ab} \right)^{-2}
$$

$$
= - \left( b - \frac{2c}{ab} \right)^{-1} \left( b \frac{c}{c} H - 2 \left( b - \frac{2c}{ab} \right)^{-1} \right)
$$

$$
= - \left( b - \frac{2c}{ab} \right)^{-1} \left( b - \frac{b^2}{c} \left( b - \frac{2c}{ab} \right)^{-1} - 2 \left( b - \frac{2c}{ab} \right)^{-1} \right)
$$

$$
= - \left( b - \frac{2c}{ab} \right)^{-1} \left( b - \left( \frac{b^2}{c} + 2 \right) \left( b - \frac{2c}{ab} \right)^{-1} \right)
$$
which is strictly positive. Turning now to the numerator component \(M\),

\[
M = \frac{p_w}{c} H + \frac{1}{c} \left[ A H b \left( b - \frac{2c}{\alpha b} \right)^{-1} - p_w H (1 + b \left( b - \frac{2c}{\alpha b} \right)^{-1}) - 2 \left( b - \frac{2c}{\alpha b} \right)^{-2} (p_w - A) \right]
\]

\[
= \frac{p_w}{c} H + \frac{H}{c} \left[ -(p_w - A) b \left( b - \frac{2c}{\alpha b} \right)^{-1} - p_w H (1 + b \left( b - \frac{2c}{\alpha b} \right)^{-1}) - 2 \left( b - \frac{2c}{\alpha b} \right)^{-2} (p_w - A) \right]
\]

\[
= -\frac{H}{c} b (p_w - A) \left( b - \frac{2c}{\alpha b} \right)^{-1} - 2 \left( b - \frac{2c}{\alpha b} \right)^{-2} (p_w - A)
\]

\[
= -(p_w - A) \left( b - \frac{2c}{\alpha b} \right)^{-1} \left( \frac{H}{c} b + 2 \left( b - \frac{2c}{\alpha b} \right)^{-1} \right)
\]

Noting that \(-(p_w - A) \left( b - \frac{2c}{\alpha b} \right)^{-1}\) is a negative value, the result depends on the sign of the far right value in the parantheses - if it is negative (making \(M\) positive), then the value of \(\omega^*\) at \(t = 0\) is positive because the denominator is also strictly positive. This can be determined by seeing if the following inequality holds:

\[
\frac{H}{c} b + 2 \left( b - \frac{2c}{\alpha b} \right)^{-1} < 0
\]

\[
\frac{H}{c} b < -2 \left( b - \frac{2c}{\alpha b} \right)^{-1}
\]

\[
\frac{b}{c} \left[ 1 - b \left( b - \frac{2c}{\alpha b} \right)^{-1} \right] < -2 \left( b - \frac{2c}{\alpha b} \right)^{-1}
\]

\[
\frac{b}{c} < \left( \frac{b^2}{c} - 2 \right) \left( b - \frac{2c}{\alpha b} \right)^{-1}
\]

\[
\frac{b^2}{c} - \frac{2}{\alpha} < \frac{b^2}{c} - 2
\]

\[
\frac{2}{\alpha} > 2 \Rightarrow M > 0
\]

This result shows that at \(t = 0\), the minimally sufficient productivity level is always positive.

Now examining how \(\omega^*\) changes with \(t\), the sign on \(N\), the coefficient of \(t\) becomes the focus:

\[
N = -\frac{1}{2c} H \left( 1 + b \left( b - \frac{2c}{\alpha b} \right)^{-1} \right) - \left( b - \frac{2c}{\alpha b} \right)^{-2}
\]
\[-\frac{1}{2c} \left( 1 - 2b \left( b - \frac{2c}{\alpha b} \right)^{-1} - \left( b - \frac{2c}{\alpha b} \right)^{-2} \right) - \left( b - \frac{2c}{\alpha b} \right)^{-2} \]

\[= -\frac{1}{2c} + \frac{b}{c} \left( b - \frac{2c}{\alpha b} \right)^{-1} + \left( \frac{b^2}{2c} - 1 \right) \left( b - \frac{2c}{\alpha b} \right)^{-2} \]

\[= -\frac{1}{2c} + \left( b - \frac{2c}{\alpha b} \right)^{-1} \left( b + \left( \frac{b^2}{2c} - 1 \right) \left( b - \frac{2c}{\alpha b} \right)^{-2} \right) \]

This expression can be signed as negative because \( b^2 < 2c \). This implies that \( \omega^* \) is strictly decreasing at a constant rate (of \( \frac{N}{L} \)) with respect to the tariff. \( \blacksquare \)
Chapter 6

Bibliography


