FOSTERING MATHEMATICAL DISCOURSE IN AN ONLINE COLLEGE ALGEBRA CLASS

by

KIMBERLY NICOLE RICE BENNEKIN

(Under the Direction of Patricia Wilson)

ABSTRACT

Students are attracted to the convenience and the autonomy of taking online classes, and, as a result, the demand for online classes is increasing at a much higher rate than traditional face-to-face classes. Researchers in mathematics education and online education agree that enhancing mathematical discourse and online interactions will improve knowledge construction, respectively. In this case study, I created a discussion board activity, with the intention of fostering more meaningful mathematical discourse in an online college algebra course. I completed the study in three stages. In the exploration stage, I collected data on students' affective responses to mathematics and their current online course materials. In the development stage, I used this information to create a purposefully designed discussion activity. In the experimental stage, I administered the activity then analyzed the students' discussion board postings. The week the purposefully designed activity was given, more meaningful mathematical discourse took place. This study provided some initial evidence that mathematical discourse can be fostered and enhanced in an online environment.

INDEX WORDS: mathematical discourse, online, interactions, learning, mathematics

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DEDICATION

This dissertation is dedicated to my family. To my son, Asa, who has inspired me to do my best in everything I encounter. To my mother, Gloria, who helped me hold down the home front, doing whatever she could to help, as I balanced graduate school with working full-time. Last, but not least, to my husband, Mario, who always believed in me and is my biggest fan. I love you all.

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CHAPTER 1

INTRODUCTION

Technology has changed our lives, allowing us to do things that were not imaginable even 25 years ago, personally and professionally. The introduction of technology into the classroom has had a large-scale impact on the way we teach and learn, directly and indirectly. For example, online courses for businesses, colleges and universities provided accessibility and a self-paced structure that enabled people to continue their education within the constraints of their daily lives. This chapter will showcase the growth and demand of online courses, quality concerns with creating online course components, and the need for research in the area.

Growth & Demand of Online Education

Each year there are more students taking classes online and this number continues to rapidly increase. The number of students enrolled in online classes doubled between 1995–1998 (Wadsworth, L., Husman, J., Duggan, M., & Pennington, M., 2007). During the 1997-1998 academic year, 1.6 million students were enrolled in online courses (Institute of Higher Education Policy, 2000). In the fall of 2009, there were 5.6 million students taking at least one online course (Sloan Consortium, 2010), which equates to about thirty percent of all higher education students. In addition, “The twenty-one percent growth rate for online enrollments far exceeds the less than two percent growth of the overall higher education student population” (Sloan Consortium, 2010, p. 2). Colleges also report that the decline of our country’s economy has created an increasing demand for online courses (Sloan Consortium, 2010). It is not unusual to expect people to return to school when the economy is bad for further training or to change career paths; however, many of these people are not able to quit their jobs to attend school. Online classes are more accessible and appealing to students. Students are attracted to the convenience and the autonomy of taking online classes (Tallent-Runnels et al., 2006). The appearance of this trend and the demand of accessibility have put pressure on colleges to provide more online course offerings (The Sloan Consortium, 2010).

Quality Concerns

Even with the increased demand for online courses and the support of chief academic officers, many professors are still unconvinced about the quality of online instruction. Since the creation of online classes, their validity and effectiveness have been in question. There are some that believe the quality of online classes is deficient due to the lack of face-to-face interaction, i.e. lack of socialization (Allen & Seaman, 2003). There is a perception of isolation and inability to form student-teacher and student-student relationships. Some even go as far to say that they "doubt strongly that a totally virtual institution could demonstrate that it provides an education equivalent to that offered at traditional colleges and universities," (Perley & Tanguay, 1999, p. B4). Nish Sonwalkar (2008) states that one of the reasons online classes have been ineffective is because "there is no pedagogical framework provided for instructors to be effective online," (p. 45). However, there has been ongoing research emphasis on creating standards for the design and implementation of online courses (Brown & Corkill, 2007; Matsuo, Barolli, Xhafa, Koyama, & Durresi, 2008; Trotter, 2008a, 2008b). Even with researchers starting to take a general interest in online courses, there has not been much research on the design and implementation of course components specifically designed for online mathematics classes.

Many believe it is possible to educate a student in a totally online environment and still maintain, if not enhance, the quality of the instruction. Online interaction increases anonymity and gives all students a chance to be heard, in some cases, eliminating time constraints and reducing the fear of speaking in front of others (Fountain, 2006, p. 78). Sharon Fountain (2006) discovered that students, faculty and administrators concur that an increased amount of interaction is needed in online classes. As a result, some believe that more research should be done on how interactions in online courses can impact learning outcomes. Instructional designers believe online courses created in a semi-structured format are ideal for promoting interaction and increasing learning outcomes (Woo & Reeves, 2007). Although her research supports face-to-face classroom discourse, Sfard (2001b) agrees that communication should also be the focal point of students' mathematical learning. Combining the theories of both online learning and mathematics education will better prepare researchers for creating appropriate materials for online mathematics courses.

The Institute of Higher Education Policy's (IHEP) study on the quality of online courses provides a good start to identifying needed research in this area. The IHEP is a non-profit organization dedicated to promoting success in postsecondary education world-wide. Their mission is to "increase access and success in postsecondary education around the world through unique research and innovative programs that inform key decision makers who shape public policy and support economic and social development" (IHEP, n.d.). The IHEP was commissioned by the National Education Association

(NEA), the nation's largest professional association of higher education, and Blackboard, Inc., a widely used Web-based platform provider for online education, to explore issues and examine benchmarks related to quality in online education. In 2000, the IHEP identified 24 benchmarks essential to quality assurance in online education. The benchmarks spanned 7 categories: institutional support, course development, teaching/learning, course structure, student support, faculty support and evaluation/assessment (IHEP, 2000). My area of interest falls within the teaching and learning category. The first benchmark under teaching and learning states, "Student interaction with faculty and other students is an essential characteristic and is facilitated through a variety of ways…" (IHEP, 2000, p. 2). Online interaction is an area identified to be in need of further investigation. The need to foster meaningful online student interaction is a familiar thread among online education researchers (Tallent-Runnels et al., 2006; Woo & Reeves, 2007). Gilbert and Dabbagh agree that "Despite a growing body of research on the instructional benefits of asynchronous communication, there is little research about the impact of the protocols and criteria that guide online discussions on meaningful discourse," (Gilbert & Dabbagh, 2005, p.5).

Purpose of the Study

This study investigated how to design course activities that enhance mathematical discourse in online mathematics courses. I used theories from both online education and mathematics education as a guide to design online activities that promote online mathematical discourse. There are many researchers who agree that mathematics is a social activity (Balacheff, 1990, Davydov, 1995, van Oers, 1996). In terms of online education, researchers agree that meaningful online student interaction has the ability to increase learning outcomes (Woo & Reeves, 2007). The goal of this study was to unite research in mathematics and online education by their common threads to design an online activity that will enhance meaningful online mathematical discourse. The research questions concentrated on the discourse patterns of students participating in an activity purposefully designed to enhance meaningful student interaction, i.e. online mathematical discourse. It also addressed the students' perceptions of their discussion activities and their interactions.

Research Questions

To examine the impact of a purposefully designed discussion activity on students' online mathematical discourse, the following research questions were formulated:

1. What is the nature of the mathematical discourse of participants in an online college algebra course?
    a. What is the nature of the individual students' patterns of online mathematical discourse with standard discussion activities?
    b. What is the nature of the collective group's online mathematical discourse with standard discussion activities?
    c. What is the nature of the individual students' patterns of online mathematical discourse with a purposefully designed discussion board activity?
    d. What is the nature of the collective group's online mathematical discourse with a purposefully designed discussion board activity?
2. What are the participants' perceptions of their online discussion board interactions (including mathematical discourse and other interactions)?

CHAPTER 2

LITERATURE REVIEW

The literature review begins with a summary of relevant sociocultural theories that form a pragmatic foundation for my study. Next, I review research on mathematical discourse and the nature of mathematical learning. Then, I connect this research with research on interactions in online learning. Next, I review instructional and motivational design, in relation to designing online course materials. Last, I discuss the role of designing asynchronous discussion activities for the online mathematics classroom.

A Review of Social Theories

Social theories have been present in mathematics education literature, in various forms, since people began to recognize that mathematical learning involves a social process. Social cognitive theory is one of the first social theories that emerged from the premises of symbolic interactionism (Driscoll, 2005). It is a way of looking at how individuals learn, in relation to their environment and their interactions with their environment. Driscoll argues that “what people perceive, think, and do, develops in a fundamentally social context” (2005, p.157). However, it was Lev S. Vygotsky’s sociocultural theory that strongly influenced the field of social psychology and mathematics. Vygotsky carried out his research in Russia, without much exposure to the rest of the world. He passed away at the early age of 37, but his work was continued by his students at the Vygotsky School. It was the fall of the Soviet Union in the late 1980’s, long after Vygotsky’s death, which opened the door for his ideas on human development to begin to spread throughout the world. There was a push towards the development of an individual's personality and creativity (Davydov, 1995). Vygotsky's ideas offered a revolutionary view of teaching and learning, different from Piaget's notion of child development stages. This introduced a framework that allows the consideration of social and cultural contexts on learning and development (Vygotsky, 1978). There are three principles that arose from Vygotsky's school. They are, "education must first of all provide the development of human personality; education must be carried out on the basis of the individual activity of each student and take into account the particularities of his or her interests; and education must create in each student the complete variety of general human values" (Davydov, 1995, p. 12). Vygotsky believed higher order thinking comes from social interaction during the learning process, external and internal (Driscoll, 2005, Vygotsky, 1978). He also believed there was a distinct connection between what we think and what we communicate to others (Vygotskii, 1986). Within this framework, more focus is put on the communication process of learning than on the outcomes. As a result, teachers ought to consider social learning activities that engage the learner to actively participate in an academic setting.

A social cognition theory that emerged out of Vygotsky's work was activity theory. Activity theory looks at cognition as it relates to an activity, in which learning could be altered through cultural participation with that activity (Nardi, 1996). More specifically, van Oers (2006) explains, "From an activity point of view, cognition is seen as a collection of psychological functions that are necessary for the accomplishment of mediated activities, particularly for the participation in cultural practices (socio-cultural activity settings)" (p. 120). From this view, participating in a learning activity is a way of connecting to artifacts as they are acted upon in their environment through interaction, within the cultural settings of the classroom. The activity and the actions taking place should be studied within the environment, as they are interrelated and cannot be separated (van Oers, 2006). It is important to consider this viewpoint when designing activities that promote the best environment to make learning processes as effective as possible.

An interrelated cognitive theory under the umbrella of activity theory is situated cognition (Nardi, 1996). Situated cognition is the act of learning through participation with one's environment. Under the same premises stated before, we cannot separate what one knows and the context in which one knows it from what one does. All knowledge is subjective, from the perspective of the learner's reaction to the environment. It changes the focus from looking just at the individual, to also including the individual's surroundings and how they relate to him or her while acquiring knowledge. Driscoll states that this type of "knowledge accrues through the lived practices of the people in a society" (2005, p. 158) and that "learning is conceived as increasing participation in communities of practice" (2005, p. 159).

Community of practice is defined as "a group of people who share an interest in a domain of human endeavor and engage in a process of collective learning that creates bonds between them" (Wenger, 2001, p. 2). One cannot separate what and how people learn from the environment in which they are learning, as they are all interconnected. Wenger (1998) explains that the following assumptions must be made before considering a situated framework:

1. We are social beings. Far from being trivially true, this fact is a central aspect of learning.
2. Knowledge is a matter of competence with respect to valued enterprises – such as singing in tune, discovering scientific facts, fixing machines, writing poetry, being convivial, growing up as a boy or a girl, and so forth.
3. Knowing is a matter of participating in the pursuit of such enterprises, that is, of active engagement in the world.
4. Meaning – our ability to experience the world and our engagement with it as meaningful – is ultimately what learning is to produce. (p. 4)

The students and the teacher are members of the community known as the mathematics classroom. As they begin to discuss material and exchange ideas, they are creating a community of practice, with the understanding that they are engaging with the mathematics collectively as a group. It is through this community from which one's individual knowledge evolves, by way of engaging with the group. Wenger argues that this community can be modeled within an online environment (Wenger, 2001). "New technologies such as the Internet have extended the reach of our interactions beyond the geographical limitations of traditional communities," (Wenger, 2001, p.4). Within this community of practice, social relationships are created with the goal of working together to gain shared knowledge. Mathematics educators Cobb, Yackel & Wood (1992) refer to shared knowledge as "taken-as-shared" (p. 8). It is the knowledge that has been negotiated by the group until it has become accepted by the collective. This process is allowing the students to alter and confirm their ways of thinking by the direction of those involved in that community or group (Cobb, Yackel, & Wood, 1992). I have characterized this process by Figure 1.

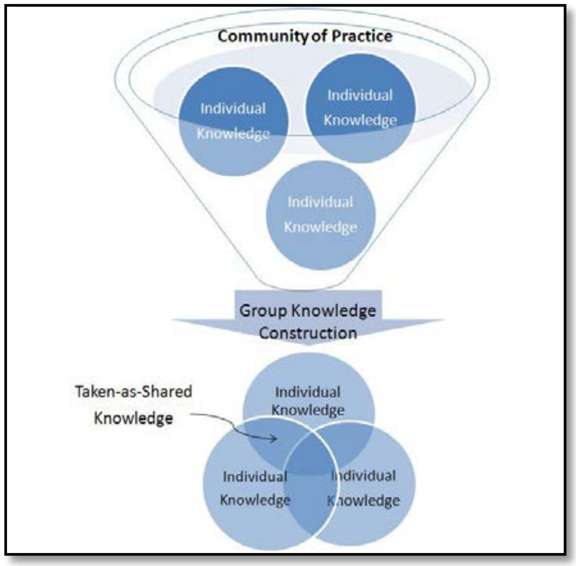

Figure 1. The social construction of knowledge is modeled in the figure above. Individuals interact in a community of practice as group knowledge is constructed. Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematical education. *Journal for Research in Mathematics Education*, *23*(1), 2-33.

By analyzing the process of learning as a community of practice, I would like to explore how a class' group communication can help describe the group construction of knowledge. It is implicit that one will be able to develop knowledge that is shared with others through interacting in that particular setting, i.e. developing a community of practice (Driscoll, 2005).

With situated cognition, the object of study is the student's interaction with a particular learning activity in that moment in time (Nardi, 1996). Since the students' online interactions are my objects of study, I plan to utilize situated cognition to analyze the mathematical discourse of the group. Understanding the basis of sociocultural theory has helped to solidify my belief about the importance of interaction and communication in the mathematics classroom and optimizing them to enhance student learning.

When considering discourse in the mathematics classroom, one should recognize the social aspect of learning mathematics and embrace contributions of individuals and that of the group. The analysis of mathematical discourse for this study contextualized the discourse for both the individual and the collective group as it relates to a specific activity in which they are engaged. This approach positioned the problem within the situated cognition framework. Fostering mathematical discourse lies in the hands of teachers and researchers. It is our job to find ways to support engagement through the use of activities that encourage students' participation in mathematical discourse. Purposefully creating activities that provide the opportunity for all students to be successful at various tasks will support student learning, regardless of students' past experiences and knowledge. Some researchers argue that the mixed ability approach to teaching promotes an equitable face-to-face classroom, by providing opportunities for success for every student, regardless of their varying abilities (Boaler, 2006, 2008; Cohen, Lotan, Scarloss & Arellano, 1999). Online courses that use whole class online discussions are the online version of mixed ability classrooms. Creating structured discussion board activities can promote the interactive environment needed for students on various levels to collaborate and debate their ideas in an online classroom. The goal was to create an activity that promoted mathematical discourse, allowed students the opportunity to explore mathematics individually, then debate and validate their ideas in a social setting.

This way of teaching and learning could be novel for both the teacher and the student. It may take time to prepare teachers and teach students how to operate in this type of student-centered classroom, but it is recognized as an effective method to support learning outcomes in both mathematics education and online education. Using a conceptual framework to study online interactions in terms of contextualizing mathematical discourse of the individual student and of the social collective will provide the best insight into online mathematics pedagogy. We cannot separate the individual from the collective group and the environment, because the environment and the people in it contribute to the nature of the discourse that is occurring (Han, 2005). Cobb (1994) agrees:

> In this view, individual students are seen as actively contributing to the development of classroom mathematical practices, and these both enable and constrain their individual mathematical activities. Consequently, it is argued that neither an individual student's mathematical activity nor the classroom microculture can be adequately accounted for without considering the other. (p. 15)

Here, Cobb argues for a pragmatic stance for combining the constructivist view of knowledge as being constructed by the individual, with the situated view of knowledge as being influenced by a community of practice. Cobb (2007) also argues that this view is somewhat pragmatic in nature, in that the phenomenon is studied in terms of both the individual and the collective group, which is seen by some as conflicting theoretical stances. He contends that each view is important and must not be ignored for the sake of the other. This analysis can be carried out a number of ways, depending on what the researcher decides to focus on as the unit of analysis. Cobb adds that "The task facing both the mathematics teacher and the instructional designer is therefore framed as that of supporting and organizing students' induction into a specific discourse practices…" (Cobb, 2006, p. 148). There are many facets involved, which is why separate discipline-specific research is very important. It is up to online mathematics practitioners to design online course materials, based on research, which will foster more meaningful mathematical discourse.

A Review of Mathematical Discourse

The view of *mathematics as a language* is emphasized in Sfard's (2008) work. This view can be found in many forms in mathematics education research (Cuoco, Goldenberg, & Mark, 1996; Franke, Kazemi, & Batey, 2007; Lampert, 1990; Lerman, 2001; van Oers, 1996). Thinking of mathematics as a language implies the importance of meaningful mathematical discourse as a tool for learning the subject. Mathematics has a unique type of discourse (Cuoco, Goldenberg, & Mark, 1996; Sfard, 2008; van Oers, 1996), and this discourse is about abstract objects that are often acted upon. Whereas, with other subjects the discourse is about concrete objects that can be touched and/or examined physically in some way. This unique distinction sets mathematical discourse apart from the discourse in other subjects, thus special attention is necessary in order to describe mathematical discourse. Sfard (2008) identified four characteristics to determine whether or not a discourse can be considered mathematical. These characteristics include:

1. Word use: The use of the words are mathematical in nature, in that they refer to shapes, quantities, etc. in the context of solving a problem.
2. Visual Mediators: The discourse involves using visual mediators, or symbolic artifacts, that are operated on in order to communicate about mathematics.
3. Narrative: The discourse includes a description of the mathematical objects and the relations between them. This discourse is subject to endorsement or rejection.
4. Routines: Mathematical routines or repetitive patterns are used in the discourse to communicate about numbers or shapes (Sfard, 2008, p.133-134).

These characteristics can be paraphrased and shaped to define online mathematical discourse as *speaking in a mathematical nature, using the language, practices, symbols, patterns, and images unique to mathematics, or relating real world problem solving to mathematical concepts* (Sfard, 2008) in an online setting. This definition was used a guide to distinguish between what was online mathematical discourse, and what was not, in my study.

Interactions in a mathematics classroom are like no other, and there are many who believe that sociocultural interaction about the material is innate to the nature of mathematics and how it is taught and learned (Sfard, 2001a & 2001b, van Oers, B., 1996). The nature of the subject of mathematics is unique from all other subjects. Mathematics has a unique vocabulary, often utilizing pictures and diagrams, and has a dependence on the use of specialized symbols, making it a very unique language. Interactions and discourse in the mathematics classroom have been researched in many ways. Some have studied mathematical discourse from a linguistic point of view, by analyzing how words and group of words are used in the classroom in terms of socially producing meaning (Herbal-Eisenmann, Wagner & Cortes, 2010). Other researchers have looked at discourse from a semiotic perspective, which includes the linguistic perspective, but also includes an analysis of one's behavior and their use of symbolism and visual aids embedded in the nature of the mathematics (Chapman, 1993; O'Halloran, 2000). Others look at an entire episode of a conversation to seek evidence of the effectiveness of the communication taking place (Sfard, 2001a & 2001b). There are several perspectives that provide a different lens for understanding and studying mathematical discourse in the face-to-face classroom. With mathematical learning, there are more facets that are involved, which is why separate discipline-specific research is very important when studying online mathematical discourse. Sfard states that "Careful analyses of diverse classroom episodes can be trusted to provide a good idea about what could be done in order to make mathematical communication, and thus mathematical learning, more effective," (Sfard, 2001b, p. 44). Thus, I first analyzed how students currently participated in mathematical discourse in an online setting before I designed new online activities. Researchers in mathematics education and online education agree that students will not participate in interactive communication unless they are already motivated to do so (Keller, 2010, Sfard, 2006), whether intrinsically or extrinsically. Including motivational factors to influence participation and communication about mathematics in an online setting is supported by both online and mathematics educators. The framework one uses to view this problem positions what one believes is important to measure during that process. For this study, I considered techniques for encouraging online mathematical discourse and techniques on how to analyze that discourse.

Conceptual foundations for social cognitive theories previously discussed in the literature have been deficient in giving researchers and practitioners clear guidance on how to carry out an investigation of classroom discourse situations, which have the potential to improve instruction (Cobb, 2006). In addition, social cognitive theories lack practical direction on how to engage students to take part in creating and participating in a community of learning and practice in the mathematics classroom. Researchers' groundwork on mathematical discourse could be helpful in creating a mathematical community of practice, and could provide more direction on how to engage students. Sfard's extensive research on mathematical discourse focuses on analyzing mathematics classroom discourse from a social cognition perspective. Sfard (2008) argues that thinking can be seen as a form of communication, whether communicating with others or internally, thus coining the term *commognition* to describe this process. Sfard also argues that mathematics itself can be viewed as a form of communication, where we are constantly negotiating about the mathematical objects that we are acting upon (Sfard, 2008). Mathematical discourse/communication is essential to the development of mathematical thinking (Cobb, 2006; NCTM, 2000; Sfard, 2008), thus we should study the communication of mathematics to find ways to enhance mathematical discourse. We should "no longer regard thinking as a self-sustained, stand-alone individual function, prior to and independent of the activity of communication in its various manifestations," (Sfard & Kieran, 2001, p. 47). Since thinking is the act of communicating, we should be able to describe a student's thinking and learning process in relation to the communication taking place. What a student can express as knowledge should not be taken out of its context since "Students' thinking is only understood in the context of demands and patterns of the overall communicative activity of which it is an inseparable part" (Sfard & Kieran, 2001, p. 47).

The nature of mathematics. Aside from defining mathematical discourse, we can go further to characterize it using methods unique to the nature of mathematics. The nature of creating and doing mathematics has long been examined as a social activity where communication is essential (Cobb, 1994; Davydov, 1995; Sfard, 2008; van Oers, 1996). Mathematical learning should allow students the chance to explore mathematics individually, but still provide the opportunity for students to validate their findings and discoveries with the teacher and classmates through socialization (Cuoco, Goldenberg & Mark, 1996, Franke, Kazemi & Batey, 2007, Lampert, 1990). Many researchers argue that mathematical knowledge is constructed socially (Balacheff, 1990, Davydov, 1995, van Oers, 1996) as a mathematics learning community (Balacheff, 1990). It is important not only to provide the opportunity for individual construction of knowledge, but also for the negotiation of that knowledge within a social setting to promote a sense of community and support learning outcomes. This socialization can take place in various ways online (e.g., email, discussion boards, chat rooms, online whiteboards).

Some believe that mathematics should be taught in the same manner in which it is created, (Cuoco, Goldenberg & Mark, 1996). One must understand the nature of the subject in order to provide a stronger foundation for learning the mathematics itself. Cuoco, Goldenberg and Mark (1996) argue that mathematics students should participate in creating mathematics like mathematicians, by applying the "habits of mind," which include experimenting, visualizing, making conjectures, and inventing new methods, to name a few. Once the student (or mathematician) thinks their problem has been solved after applying these habits, they must disclose their ideas for public review. Mathematicians do not create mathematics in isolation, but participate in constant explanation and negotiation until the new idea is accepted by a body of peers with the same intellectual background. Thus, the nature of mathematical learning should be centered on the interactions that take place as mathematics is socially debated and constructed (Franke, Kazemi, & Battey, 2007). It is common practice for mathematicians to negotiate meanings with one another until a new concept has been accepted by the mathematical community as factual. There is a constant "zig-zag between revising conclusions and revising assumptions in the process of coming to know" (Lampert, 1990, p.30). Many argue that mathematical learning activities should allow students the opportunity to explore and discover mathematics the same way, by validating findings and discoveries through the act of socialization (Cuoco, Goldenberg & Mark, 1996; Franke, Kazemi & Batey, 2007; Lampert, 1990). So, social cognitive theories as well as mathematical discourse literature could be helpful in finding ways to foster social, mathematical communications, which could enhance learning. Encouraging these different types of communication could help foster meaningful interaction. Modeled online, this discourse will be without the constraints of time and space, which allows time for reflection and deep thought, fostering more meaningful interaction. An online environment could also benefit students who experience anxiety during face-to-face communication (Fountain, 2006), adding a level of anonymity.

The National Council of Teachers of Mathematics [NCTM] also contends that communication in the mathematics classroom is essential, and they have listed communication as one of the necessary standards for teaching and learning mathematics

(NCTM, 2000). The NCTM argue that ‘Instructional programs…should enable all students to – organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000). They also recognize that this type of communication may not come naturally to the students, thus reinforcing the need for an embedded scaffolding approach to instruction through the course design, focusing on peer communication as the means for learning. This is consistent with social learning theories discussed earlier. The idea of scaffolding used for this study is different from the traditional idea of what educators think scaffolding should be. The process of scaffolding was originally characterized as an expert, such as a teacher, assisting the student in a task where needed, until eventually the student can continue with the task on their own (Davydov, 1995; Driscoll, 2005). Instead, this study will use the Sherin, Reiser, & Edelson (2004) definition within the educational technology field, which considers embedded scaffolding techniques within an activity to encourage instructional interactions. These scaffolds can take various forms. It could be a person, but it can also be a calculator or computer program, artifacts that assist in moving the student from the point of not knowing to knowing.

Based on the review of literature on mathematics education and mathematical discourse, I chose to contextualize and classify mathematical discourse using the following categories: making conjectures or suggestions (Cuoco, Goldenberg & Mark, 1996, Trouche, 2006), explicating work or giving further evidence (van Oers, 1996), debating or questioning of ideas (Franke, Kazemi & Batey, 2007, Trouche, 2006), and validating or confirmation of ideas (Cuoco, Goldenberg, & Mark, 1996, Lampert, 1990, van Oers, 1996). These categories emerged from the literature review as the types of discourse that should be encouraged in mathematics classrooms. Thus, I will focus on ways to foster these types of discourse by purposefully designing an online discussion activity encouraging online interaction. These basic categories will serve as a basis to describe the nature of the online mathematical discourse of my participants.

Interactions in Online Learning

Online interactions are essential to online learning and sustaining that interaction is essential to achieve quality in online courses (e.g., Yang, 2006). In fact, fostering classroom interactions in an online environment may provide benefits to encouraging rich communication. Online interaction increases anonymity and gives all students a chance to be heard, by eliminating time constraints and the fear of speaking in front of others (Fountain, 2006). Sharon Fountain (2006) discovered that students, faculty and administrators agree that an increased amount of interaction is needed in online classes. Others argue that more research is needed that focuses on studying the interactions in online courses and exploring how those interactions can affect learning outcomes (Woo & Reeves, 2007).

Woo & Reeves (2007) define meaningful interaction as the interaction that "has a direct influence on the learners' intellectual growth," (p. 15). They also state that many instructional designers who design online courses "lack sound theoretical foundations for determining what is good quality or meaningful interaction" although, there is belief that meaningful interaction supports higher order thinking (Woo & Reeves, 2007, p. 16). I believe it is the responsibility of the discipline to determine what should be considered meaningful interaction. According to Sfard (2001), "most of our learning is nothing else than a special kind of social interaction aimed at modification of other social interactions," which emphasizes a "communicational approach to learning" (p. 3). Knowing that interaction is important to both online learning and mathematical learning, it would make sense to incorporate the theories that overlap in both disciplines to maximize the success of designing an online mathematics course.

Instructional and motivational design. Sfard (2001) agrees that instructors can mold discourse and communication, and that we should first analyze our students to see what can be done to motivate them to participate in more meaningful mathematical discourse. Motivation should be considered in the design of instruction that aims to facilitate mathematical discourse because "Strong motivation is necessary to engage in mathematical conversation and make it work" (Sfard & Kieran, 2001, p. 70). This is where instructional design may provide guidance. After an initial evaluation of the current state of the participants' mathematical discourse, one can better explore ideas of how to design components which would facilitate students' meaningful interactions, i.e. mathematical discourse.

Instructional design is the systematic process of using learning theories and principles to make plans for instruction. This design process, and variations of it, has been widely adopted by instructional designers of online course materials. The instructional design process involves five stages: analysis, design, development, implementation and evaluation (Dick, Carey and Carey, 2009). It was previously stated that students may need to be motivated to participate in mathematical discourse online, so I opted to use a motivational design structure to create new materials. Keller (1983,

2010) has created an instructional design model, called the ARCS (Attention, Relevance, Confidence, and Satisfaction) model of motivational design, based on motivation theories and research. This motivational design model provides techniques for designing course materials to enhance learner motivation by focusing on the components and strategies that could increase the attention, relevance, confidence and satisfaction of the learner (Keller, 1983, 2010). There is no "one size fits all" and the design is tailored for the motivational needs of the learner, which makes it critical to collect the information of learners' needs. That is, the design process relies heavily on the initial needs assessment of current learners (Dick, Carey & Carey, 2009). It takes a heuristic approach to design, where each part is connected and dependent on the other. There are some who believe that the ARCS model has "inconsistent results on motivation levels and learning outcomes in different groups of learners" (Means, Jonassen, & Dwyer, 1997, p. 5). However, these researchers dissected each part as a stand-alone entity. Keller believes the model should be approached holistically, with each part interrelated and depending on the other (Keller, 1987).

Even with many agreeing that social interaction is essential for mathematical and online learning, there has not been much research investigating mathematical discourse in online mathematics classes. Ryan and Deci (2000) agree that more self-directed, extrinsic and intrinsic motivation will yield to greater interaction, better performance, less dropping out, and higher quality learning. Using the motivational design of ARCS model (Keller, 2010) could provide the format for providing that needed motivation within the online setting.

Designing Discussion Activities for the Online Mathematics Classroom

What can we learn from current classroom discourse to create activities that foster mathematical discourse in a way that will positively impact student learning? Some researchers believe that the sociocultural interaction about the material is innate to the nature of mathematics and how it is taught and learned (Cobb, 1994; Sfard, 2001a & 2001b; van Oers, 1996) and "Careful analyses of diverse classroom episodes can be trusted to provide a good idea about what could be done in order to make mathematical communication, and thus mathematical learning, more effective," (Sfard, 2001b, p. 44). It is the work of dedicated and curious researchers that will find ways to improve learning activities that foster mathematical discourse online, a practice which could be unfamiliar to many students and teachers. Due to the nature of mathematics and the characterization of mathematics as a language, it would be wise to follow the advice of those who are well versed in mathematics education research. Part of creating a good activity includes finding the right context in which to present it.

With the use of intelligent technology changing, there will ultimately be significant consequences on teachers and students roles. With time, hopefully research can support teachers' use of technology by encouraging activities that will guide students to explore and experiment with mathematics, and come back to validate their findings through socialization. This process is consistent with the way mathematics is created (Cuoco, Goldenberg, & Mark, 1996; Lampert, 1990). As described in the earlier section, mathematical concepts are debated by mathematicians until consensus is reached. This is how formal mathematical concepts are still tested and agreed upon today. Merely having appropriate activities is not enough. Students should be given the opportunity to interact with their peers during the mathematical learning process. They need opportunities to make conjectures, debate ideas and validate procedures (Trouche, 2006).

Many of the underlying principles that support teaching mathematics in face-to-face classrooms mirror several of the same desired characteristics for teaching online. Interaction and active participation are important factors in mathematics education, and they are also important in online education as well (Tallent-Runnels et al., 2006; Wallace, 2003; Woo & Reeves, 2007). Since online interaction is based within a virtual social environment, one must study the phenomenon in terms of the individual, the group and the environment in which they are interacting, because the construction of knowledge of the individual and the group are interdependent within that learning activity (Han, 2005). Han (2005) refers to this group of learners as a virtual learning environment and argues that the learning is dependent on how the individual contributes to the learning of the group and how the group can influence the learning of the individual. This viewpoint, of some researchers in online education, has focused on research based on comparable theoretical foundations positioned within sociocultural theory, activity theory, situated theory and distributed theory (Han, 2005; Tallent-Runnels et al., 2006), social theoretical foundations also used by several mathematics education researchers (Cobb, 2006; Lerman, 2006). Pena-Shaff and Nicholls (2004) argued that:

> Dialogue serves as an instrument for thinking because in the process of explaining, clarifying, elaborating, and defending our ideas and thoughts we engage in cognitive processes such as integrating, elaborating and structuring. Therefore, it is in the process of articulating, reflecting and negotiating that we engage in a meaning making or knowledge construction process. This process

> can become even more powerful when communication among peers is done in written form because writing, done without the immediate feedback of another person as in oral communication, requires a fuller elaboration in order to successfully convey meaning. (p. 244-245)

This viewpoint of creating a medium for rich, interactive, online discussion is imperative to support the learning outcomes of an online mathematics student. Some of the same characteristics, such as explaining, clarifying, elaborating, and defending ideas, were also seen as desirable types of classroom discourse in mathematics education literature.

Technology that supports creating online communities of practice can be facilitated by way of online mediums, such as chat rooms and discussion boards. Chat rooms are online "meeting spaces," where students and teachers can meet online synchronously, for live, real-time, typed discussions. Some chat rooms have the capability to add audio or video, by connecting through a phone line or computer webcam. Discussion boards are asynchronous posting boards, where the instructor and the students can post and/or respond to written posts at any time. Discussion boards allow students the flexibility to post responses twenty-four hours a day, as well as provide students the time to think and reflect on their response before they post them. Pena-Shaff and Nicholls (2004) agree that discussion boards or "asynchronous, online environments can provide students with opportunities to develop sophisticated cognitive skills such as self-reflection, elaboration, and in-depth analysis of course content, allowing the purposeful construction of knowledge" (p. 248). The nature of responding to the discussion board adds flexibility for the learner, giving them the time to reflect on their response before posting it and the choice to decide when to respond and read other posts. The discussion board provides an ideal outlet for facilitating asynchronous group communication in the online environment. Hakkarainen (2009) agrees that mediums providing students with an "environment that provides sophisticated tools for creatively externalizing students' ideas, storing them in a shared collaborative space in which the other students can comment on them, build on them, or rise above them" are ideal (p. 219). It gives students the opportunity to take part in discussions over time that would not have been feasible in a face-to-face classroom. The discussion board as a mediator to learning in this manner "allows the users to entertain more complex thought, engage in deeper inquiries, immerse in more intensive collaborative processes than would otherwise be possible at all" (Hakkarainen, 2009, p. 220). Some researchers argue that more time should be taken in designing online discussion board activities to achieve the desired outcome (McCarthy, Smith & DeLuca, 2010). Regardless, it is clear that active participation is important for the learning of mathematics. Thus, it is the job of researchers and practitioners to come up with appropriate activities that foster this interaction in an online setting.

Researchers recognize that interactions are essential to online learning and more research is needed in analyzing online interactions, to see which techniques promote active participation and support learning outcomes (Tallent-Runnels et al., 2006; Wallace, 2003; Woo & Reeves, 2007). In one study, students were not given any guidelines or timeframes for participating online, other than a minimum of one post per week. As a result they found that most students rarely posted above the minimum amount required (Pena-Shaff & Nicholls, 2004). This is not a surprise, since we have said that students need clear guidance and motivation to participate in an activity that may be new or novel for them. Students should be given more structure for discussion board activities that encourage a variety of content related conversations and interactions, presented within a student-centered environment. Even Pena-Shaff and Nicholls (2004) agree that instructors should (1) motivate students to actively participate, (2) give clear objectives and guidelines for participating in the activity and (3) embed it naturally into the course components, to increase active participation online.

Both mathematics and online educators agree that interaction, or active communication, is important with learning mathematics and online learning. Wallace reviewed several articles that found students to be more satisfied with an online course when they were socially connected to their classmates (Wallace, 2003). Without the interaction, it is not possible for the student to acquire or utilize social meaning, which is embedded within mathematical knowledge. Thus, when developing online courses, one must think beyond presenting the content. Creating an active learning environment for students to personally engage and collaborate on topics in a social setting is just as important as the content itself. It seems that participating actively online is imperative not only to learning the material, but for the student to feel connected to the classroom community. Wallace's (2003) thorough analysis of several online course studies found that, "students participate actively in online classes in which discussion is valued. They create social presence by the nature and content of their participation, and social presence seems to be an important element of both satisfaction and learning" (2003, p. 253). She also stated that more research is needed the area of online discourse.

Fostering the needed social interaction within online group activities could potentially impact the student's online learning outcomes. Woo and Reeves (2007) argue that fostering meaningful online interaction can “stimulate the learners’ intellectual curiosity, engage them in productive instructional activities, and directly influence their learning” (p. 16). Discussion boards can be used as the ideal technological medium for supporting this social interaction. The use of the discussion board in online mathematics courses gives students the chance to communicate ideas with one another in a relaxed open forum, while still supporting the construction of individual knowledge. (Woo & Reeves, 2007). It also provides a place for students to create an ongoing mathematical community of practice, supporting the need for knowledge-building communities that “transform local classroom practices toward inquiry-based ones, involving students’ participation in collaborative knowledge building” (Hakkarainen, 2009, p. 221). Moreover, discussions within online classes have the potential to be dynamic, interesting and relevant to the real world since “the interactive nature of the Web allows learners to explore a variety of resources and establish connections with other knowledge domains that are meaningful to them” (Woo & Reeves, 2007, p. 20). In this case, the other knowledge domains I want to encourage them to use would be their classmates’ points of view. The use of the discussion board in online courses presents an environment that could encourage students to explore online resources, making the mathematics more attainable and relevant to them. In addition, the asynchronous environment gives the student the flexibility to reflect on the mathematics individually in their own time, and then come back to share and debate their discoveries with classmates, by way of the discussion board. Some also believe that a discussion board activity given in debate format could foster the desired types of communication needed to enhance learning (Gunawardena, Lowe and Anderson, 1997).

A structured online discussion board activity, with embedded online scaffolding artifacts, challenging students to solve more complex problems which are relevant to them, and that encourages active participation through peer-peer interaction, seems to be the most desirable activity to support mathematical discourse. This activity could include solving real-world, open-ended problems requiring the student to calculate the time and money it would take to complete a trip, for example. The problem should be complex enough such that the problem cannot be solved immediately; however, the problem could have embedded scaffolding techniques, asking students to explore certain parts of the task on different days or in consecutive parts. This helps to relieve pressure for students to answer immediately, and gives them time to evaluate, confer, and/or dispute classmates' responses. The activity would force the student to share, question, debate, validate ideas, and take a stance. Embedding these techniques into the activity may assist in fostering the type of discourse desired.

CHAPTER 3

RESEARCH DESIGN AND METHODS

This study was designed to investigate the nature of the mathematical discourse of students in an online college algebra course in relation to their discussion board activities. The focus of this study was to determine if a purposefully designed discussion activity can enhance the online mathematical discourse among students. The literature review revealed that the factors of creating course components through motivational design (Keller, 1983, 2010) overlapped with factors that foster mathematical discourse (Sfard, 2001a & 2001b), and shows the potential for integration within the design of online mathematics activities. In addition, the categories for desired types of discourse in the mathematics classroom were characterized as making conjectures or suggestions (Cuoco, Goldenberg & Mark, 1996, Trouche, 2006), explicating work or giving further evidence (van Oers, 1996), debating or questioning of ideas (Franke, Kazemi & Batey, 2007, Trouche, 2006), and validating or confirmation of ideas (Cuoco, Goldenberg, & Mark, 1996, Lampert, 1990, van Oers, 1996). These categories guided me to find a comparable analysis model that could characterize this type of discourse, within this online format.

In this chapter, I review the methodology used to address my research questions. The two main research questions for this study are, “What is the nature of the mathematical discourse of participants in an online college algebra course, with and without a purposefully designed activity; and what are the participants’ perceptions of their online discussion board interactions?” I begin this chapter with a description of the college, the college algebra course used in the study, and the participants. Next, I discuss the instruments used for the study, including the coding scheme for analyzing the asynchronous discussion board postings. I then explain the various stages of the research study, based on the motivational design process. Next, I explain the data collection process and analysis methods used. Last, I discuss the validity and reliability of the data. The methodology for this study included techniques for analyzing qualitative and quantitative data with the intention of describing the nature of individual and group online mathematical discourse patterns.

Setting

The host school was a two-year community college in the Southeastern United States with a population of about 27,000 students at the time of the study. This particular college offers the largest number of online mathematics courses of all the colleges & universities in that state. Approximately 90% of the students transfer to a four-year college within 2-3 years. The students at this college vary in age, background and ethnicity. The host school has a 70% minority enrollment with an average student age of 26. All of the participants in this study were at least 18 years of age. General background information was available through the college's website; however, some specific participant demographics were collected on the first survey administered in the study.

At the time of the study, most of the online mathematics courses at this college followed a course template that was created by current faculty members. All part-time instructors were required to use the template for their courses, but some full-time faculty members also used them. About three years before the study, I developed the college algebra template that served as the main structure of online college algebra courses at the college during the time of this study. The template course used the textbook *College Algebra: An early functions approach*, 2nd edition (Blitzer, 2010). When the template was created, there were no specific guidelines provided for the developers, other than minimal structure and quality concern issues. The college utilized quality standards for teaching online by subscribing to Quality Matters, a nationally recognized peer review process for reviewing online courses (Legon, 2012). This helped ensure that the course template had some basic structure, similar to other online classes taught in the country.

The college algebra course taught at the institution was one of two college-level introductory mathematics courses. This course was designed for students majoring in science, technology, engineering, or mathematics, but any student is allowed to take it to fulfill their minimum mathematics requirement for other majors. About 75% of the student population received Associate of Science or Applied Science degrees between 2007 and 2012, so this course has high demand and large enrollment numbers. The course description stated that emphasis was placed on the study of functions, and their graphs, inequalities, and linear, quadratic, piece-wise defined, rational, polynomial, exponential, and logarithmic functions. The content of the course was aimed to prepare students to move on to Precalculus and other higher level mathematics courses. Students enrolled in this course showed minimum proficiency through the college's mathematics placement examination, or moved into the course after taking required learning support mathematics courses to make up for deficiencies when they entered the college.

The classroom model entailed students completing various online assignments in a weekly module format. The assignments included watching instructional videos, completing online homework, quizzes and tests, reading the electronic book, and participating on the asynchronous discussion boards by responding to weekly discussion questions, asking questions, and responding to classmates' questions. A separate discussion board was provided for students to pose questions directly to the instructor, however all content questions were directed to the weekly discussion board. The weekly discussion boards were also used for peer-peer communication about the discussion questions and related mathematical content. The norms of the online discussion included a requirement that students post to the weekly discussion board at least three days a week. Although students were given a weekly discussion question to respond to, the expected content of all other posts were not explained or elaborated on in the template syllabus. The standard template discussion questions drew from a variety of mathematical topics relating to everyday events in their lives where mathematics was used. For example, one of the discussion questions for week three in the template course states, "Sections … deal with linear functions and slope. Every linear function has a slope. Give an example where the concept of slope is used in real life. Describe the increase/decrease in the context of the example." The format of the standard template discussion questions did not necessarily encourage further exploration and additional interaction on the topic. In my experience using these questions, after students give their response they were no longer interested in the question, and communication about the question is no longer a high priority. Designing activities that encourage more meaningful interaction, specifically online mathematical discourse, was the focus of my study.

Participants

The participants of this study were drawn from one online college algebra class at the host institution. This class had 24 enrolled students. There were 21 students who gave consent to participate in the study. Two of these students dropped the course during the first week of classes. The remaining 19 students, who gave consent, completed the first survey. One of the 19 students who completed the first survey had no discussion postings through week seven, leaving discussion board postings available from 18 participants. Four more participants dropped the course before completing the final survey, which left survey results from 14 participants. The mean age of those who completed the first survey was 31, with the youngest age 20 and the oldest 54 (SD = 8.15). There were no mathematics majors in this group of participants. Their majors were listed as biology, education, social work, psychology, nursing, radiology, engineering, pre-medicine, business, art, computer science and criminal justice. Of the 18 participants that provided discussion board postings through week seven, 15 were female and three were male. There were also ten students who agreed to participate in the optional interview after the study ended on the pre-survey. There were five students who followed through with an interview.

This class was selected because of my access to the college as a current online mathematics instructor. This allowed for a point of view of the instructor, as well as the researcher. Having access to every detailed account of the class contributed to a more comprehensive case-study analysis. This particular college was also preferred because it mandated a widely-used online Vista-based course management system that is used for every online class and many on-campus classes at the college, in addition to several colleges in the state. This helped ensure students were familiar with the online learning system and reduced the need to learn new routines for navigating the system. It guaranteed uniformity of the teaching and research online environments. It also provided ease of transfer of data and reduced human error with collecting data. Lastly, it preserved and archived the raw data in a safe and password protected environment.

Instruments

There were two surveys used to collect information about students' attitudes towards mathematics and their online course components. To collect information about the participants' current attitudes about their online course components, I used an instrument embedded in the motivational design process called Course Interest Survey (Keller & Subhiyah, 1993). To collect information about participants' attitudes about mathematics, I used a modified version of the Fennema-Sherman Math Attitude Scales survey (Fennema & Sherman, 1976). To analyze the discussion board postings, I chose to use the Interaction Analysis Model (IAM) (Gunawardena, Lowe, & Anderson, 1997). The phases of discourse of the IAM are very similar to the desired types of mathematical discourse found from the literature review, and thus is an ideal fit for this study.

Surveys. This study was based on literature that unites mathematics and online education theories. To ensure that I continued this approach in every aspect of the study, I used surveys in both areas that were similar to each other, but measured attitudes in each area independently. Since I chose to use motivational design to create the discussion activity, I was guided to administer the ARCS model course interest survey (Keller & Subhiyah, 1993), to collect information about students' perceptions of online course materials. I also chose to administer a modified version of the Fennema-Sherman Math Attitude Scales survey (Fennema & Sherman, 1976), to collect information about students' perceptions of their mathematics attitudes (see Appendix A). The ARCS model course survey is an instrument that is embedded within the motivational design process

(Keller, 2010). It was created specifically for motivational design to address the areas identified as essential for the process of creating course materials based on students' needs. I chose to use a modified Fennema-Sherman survey because the categories tested on this survey were very similar to those of the ARCS model survey. This study was guided by a conceptual framework that united mathematics and online education learning theories. My choice of instruments was made to unite two areas of research. The modified Fennema-Sherman survey measured the students' mathematics attitudes in terms of: Success, Anxiety, Motivation, Usefulness, and Confidence. In particular, measures in these categories will help characterize how successful students feel about doing mathematics, what degree of anxiety they feel while doing mathematics, how motivated they feel about doing mathematics, how useful they feel mathematics can be, and how confident they feel about mathematics. The ARCS model survey measured the students' perception of their class materials in terms of: Attention, Relevance, Confidence, and Satisfaction (Keller, 2010; Keller & Subhiyah, 1993). In other words, it will measure if course materials catch and hold their attention, if they see the relevance in the materials, if they feel confident they can be successful completing the materials, and if they feel satisfied after completing the task. Both instruments test Confidence in each area. Relevance and Usefulness are similar categories, as are Satisfaction and Success. In addition, Keller argues that Attention is inversely related to Anxiety (Keller, 2010). The more attentive and stimulated a student appears to be about an activity or course component, the less likely it is that this student is experiencing anxiety about participating in that activity. Last, Motivation about learning mathematics and the motivational designed process based on the ARCS model are analogous in nature. The participants overall motivation is supported by the design of the project, as motivational design was used to create the purposefully designed course activity (Keller, 2010). My plan was to use the information from these surveys to design an online mathematics course activity that motivates the learner to participate in more meaningful types of online mathematical discourse.

Interaction Analysis Model. As discussed in chapter 2, the following categories were important when learning mathematics through communicating: making conjectures or suggestions (Cuoco, Goldenberg, & Mark, 1996; Trouche, 2006), explicating work or giving further evidence (van Oers, 1996), debating or questioning of ideas (Franke, Kazemi, & Batey, 2007; Trouche, 2006), and validating or confirmation of ideas (Cuoco, Goldenberg, & Mark, 1996; Lampert, 1990, van Oers, 1996). I chose an analysis model that was specifically created to analyze asynchronous discussion postings, and situate it within social learning theory. The phases of this analysis model closely align with the categories of discourse for mathematical learning previously found from the literature review. Using an instrument specifically designed for asynchronous postings that encompasses the same categories of ideal mathematical discourse found from the literature review, proved to be an ideal match for this study. Table 1 shows the similarities between the categories found in the literature and the phases of the Interaction Analysis Model (IAM).

Table 1

Comparison of Mathematical Discourse Categories and Interaction Analysis Model Phases

Ideal Mathematical Discourse Categories	Interaction Analysis Model Phases
(1) Explicating their work or giving evidence	Phase I: Sharing/Comparing Information
(2) making conjectures or suggestions	Phase II: Discovery & Exploration of Dissonance
(3) debating or questioning of ideas	Phase III: Negotiation of Meaning/Co-Construction of Knowledge
(4) validating ideas	Phase IV: Testing & Modification
(5) confirmation of ideas	Phase V: Agreement/Application

Note. The mathematical discourse categories were found from various researchers through the literature review. The categories from the interaction analysis model matches the categories found from the mathematics literature review. Gunawardena, C., Lowe, C., & Anderson, T. (1997). Analysis of a global online debate and the development of an interaction analysis model for examining social construction of knowledge in computer conferencing. Educational Computing Research, 17(4), 397-431.

The IAM was created for examining the social construction of knowledge within asynchronous postings through a gradual evolution in the types of interaction. This methodology acknowledges the design of an activity as contributing to the discourse and allows me to focus on analyzing of the social negotiation of knowledge in this setting (Gunawardena, Lowe, & Anderson, 1997) as a means of describing the nature of the online mathematical discourse. Finding a model that analyzes the social construction of knowledge with data in this electronic format was an ideal fit for this study. It not only incorporates the categories of mathematical discourse that I have found important, it was designed for use with the same data type for which it was designed, asynchronous discussion postings. Using my framework as a guide to finding the best methodology for this project has shown to be a beneficial process.

The Interaction Analysis Model (IAM), which characterizes the discourse of a collective group, was created by a group of professionals who debated the statement "No Interaction, No Education," (Gunawardena, Lowe, & Anderson, 1997, p.401) during a virtual pre-conference to the XVI World Conference of the International Council on Distance Education (ICDE). Through the detailed debate of this statement, Gunawardena, Lowe and Anderson (1997) analyzed their posts to create an analysis tool designed by the suggestions of a group of experts in the field of online education. The authors not only used the content of the posts in the study, but they also analyzed the participants' interaction patterns to gain evidence of how they debated this issue to come to an agreement. The categories of the IAM came about as a result of this study. The IAM categorizes group discourse in terms of the social construction of knowledge and addressed my research questions related to the nature of online mathematical discourse. The IAM uses five phases to characterize discourse, in the form of asynchronous posts, (see Appendix D) in terms of the lowest to the highest levels of group knowledge construction (Gunawardena, Lowe, & Anderson, 1997), by categorizing individual postings using the provided phases. This analysis model not only aligns with the categories for ideal mathematical discourse, it is designed for asynchronous online discussions, presenting itself as the best method of analysis for this study.

Research Stages

This research study took place in three stages: exploration stage, development stage and experimental stage, similar to the stages of the instructional design process (Dick, Carey, & Carey, 2009; Keller, 2010). During the exploration stage, I conducted a needs assessment of the current student population to determine the main areas of concern in relation to the participants' perceptions of their online course materials and their mathematics attitudes. Next, during the development stage, I created a purposefully designed discussion activity based on the information found from the exploration stage and the literature review. The experimental stage was the main part of the study where I administered pre and post surveys, along with the purposefully designed discussion activity, to an online college algebra class during week four of the course.

Exploration stage. To begin the motivational design process, I conducted a needs assessment to collect information about current student attitudes in my population of future participants. With the students' consent, I administered both the ARCS Model Survey and the modified Fennema-Sherman Survey during this stage. The surveys were administered to an online college algebra course I taught during the spring of 2011. The categories on the survey with the lowest average scores showed the areas that needed the most attention when designing new course materials. The two main topics of interest when designing activities that enhance online mathematical discourse were the current levels of motivation (Sfard & Kieran, 2001) and current attitudes about online course materials (Keller, 2010), measured by the surveys given. This pre-assessment was essential to determine which motivational factors needed to be addressed when designing new course materials. Once this information was collected, I used it to purposefully design an online activity, aimed to motivate students to participate and foster communication. This is the beginning stage of the instructional design process.

I believe those who are qualified to teach mathematics should be the ones who take the lead in designing activities for online mathematics courses. The complexity of motivating students to learn how to generate meaningful mathematical discourse should take precedence when designing materials for this learning environment. Researchers suggest that students should be properly motivated to participate in discourse in order to yield the highest levels of communication (Keller, 2010, Sfard, 2006). Instructional designer Keller (1983) agrees that some may opt to use subject experts to answer questions about students' motivation in specific disciplines. My research focus was to understand the nature of online mathematical discourse, but since motivation was an underlying factor, I used motivational design (Keller, 2010) to create a purposefully designed discussion activity. The purpose of this research was to inform the design and content of online course components that could enhance online mathematical discourse.

Results of exploration stage. The results of the surveys participants completed during the exploration stage provided information used to purposefully design a new discussion activity. I concentrated on the deficient factors identified by the survey to design a new activity tailored to motivate students to participate in more meaningful mathematical discourse, based on their needs. Motivation should be considered in the design of instruction that aims to facilitate mathematical discourse because "Strong motivation is necessary to engage in mathematical conversation and make it work" (Sfard & Kieran, 2001, p. 70). Using an instructional design process based on motivation theories and research supports this theory.

Analyzing the survey data from the exploration stage revealed that students scored lowest in the areas of Attention on the ARCS Survey and Mathematics Anxiety from the mathematics attitude survey. These are the areas that need the most consideration. The area of Attention on the ARCS Model Survey was the only mean score below 4.0, and Math Anxiety on the Fennema-Sherman Survey was the lowest mean score of all categories, and the only one falling below a mean of 3.0 (see Table 2).

Table 2

ARCS & Fennema-Sherman Survey Means and Standard Deviations (Exploration Stage)

Construct	Variables	Means (SD)
ARCS Course Interest Survey	Attention	3.57 (1.1)
	Relevance	4.38 (.73)
	Confidence	4.24 (.81)
	Satisfaction	4.28 (.64)
Fennema-Sherman Mathematics Attitudes Survey	Success	4.47 (.87)
	Math Anxiety	2.79 (1.35)
	Motivation	3.12 (1.31)
	Usefulness	4.00 (1.19)
	Confidence	3.40 (1.36)

Notes. Possible range for each variable: 1-5

The results provided guidance for designing activities that are tailored to these students. These ideas were applied during the development stage.

The discussion board data from this stage provided samples of how students are currently participating in online mathematical discourse in relation to their discussion board activities. The exploration stage analysis showed that students participated at the

Phase I-Sharing/Comparing Information level of the IAM (see Appendix D). The goal was to get students to participate at higher phases of interaction, according to the IAM. In particular, the plan was to develop discussion board activities that will impact online mathematical discourse.

Development Stage

The information acquired during the exploration stage supported a discussion board activity that encompasses every aspect of scholarly research discussed in the literature review, in addition to the perspectives of a sample of the current participant population. The exploration stage revealed Mathematics Anxiety and Attention to be the areas indicated as the most deficient on the surveys administered. Keller articulates that, "Learner attention is necessary for both motivation and learning. In motivation the issue is with how to stimulate and sustain the learner's attention. In learning, the concern is with how to direct the learner's attention…" (Keller, 2010, p. 76). We know that attention, motivation and learning are all connected. In fact, low levels of attention, i.e. arousal levels, are inversely connected to high levels of anxiety (Keller, 2010). This was confirmed by the exploration data, which showed that low attention and high anxiety as the areas of most concern, having the lowest score for students. The more comfortable someone feels with their surroundings, the more apt they are to explore and allow their curiosity to take control. To catch and hold participants' attention, the discussion activity was designed using the premise of a familiar television show that relates to many age groups in the participant population. The show uses people of different ages and backgrounds to compete through a variety of tasks. The purposefully designed activity was made with the intention of making it enjoyable, believable, and easy to understand.

Addressing the attention and arousal aspects of the activity has the ability to directly affect the motivation of the participants and could help enhance their online discourse.

The activity was designed so that there were contestants that had to participate in a variety of tasks to win a competition. The tasks and data were given in the activity and participants had to work together and use mathematics to determine who would proceed from Round One to Round Two, then students must decide who will ultimately be the winner of the competition. Figure 2 shows a screen shot of the web-based activity.

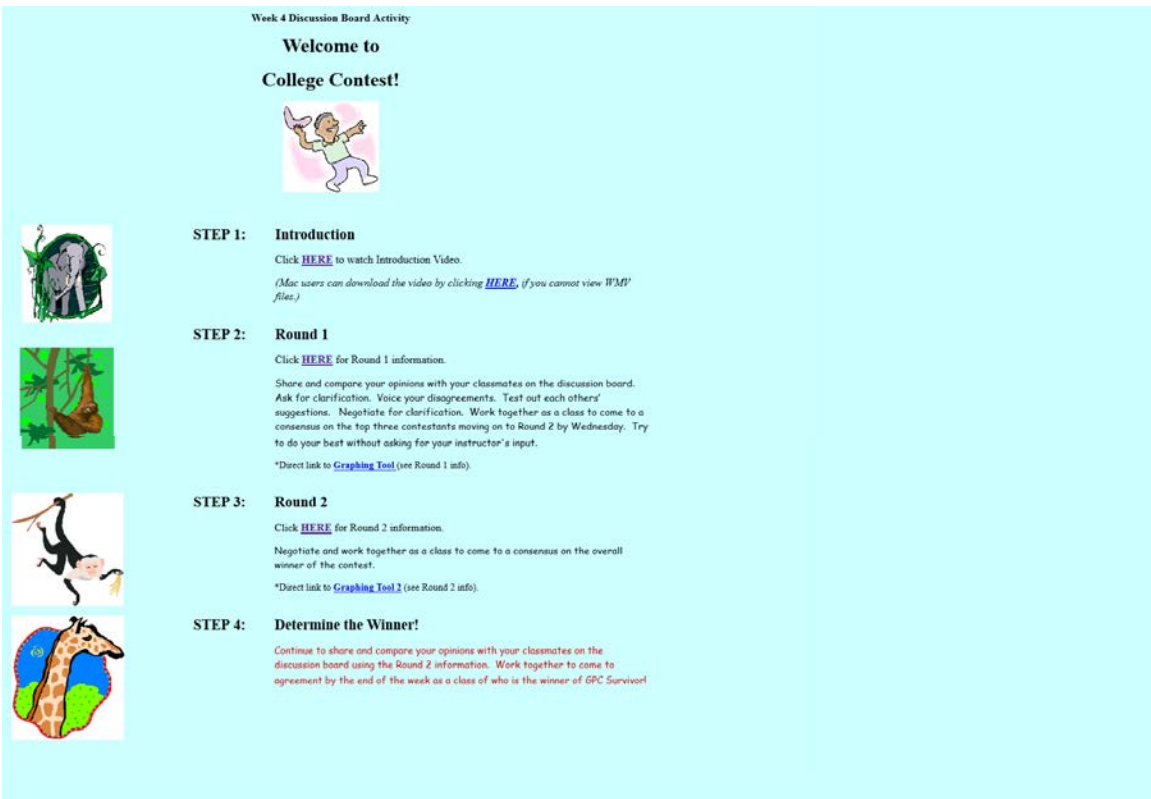

Figure 2. This is a screenshot of the purposefully activity, which was delivered online through the web. The activity is located at the URL http://facstaff.gpc.edu/~kbenneki/UGA/Week_4_Activity.htm

For the activity, students were asked to participate as a group and decide, by consensus, on the final decisions. This method of working to find a group consensus is suggested by Gunawardena, Lowe, and Anderson (1997), which encourages students to work together through a process involving asking questions, debating answers, and verifying solutions. For the first round, students were given two ordered pairs for 5 imaginary contestants that modeled the path of a dart shot in a straight line by the contestant. The participants were asked to use the data to predict which 3 contestants hit closest to the target. For round two, additional data were given for the remaining contestants, which modeled the path of launching a stone over an obstacle to hit a target. The path in the second round modeled a quadratic function. For both parts, students had to express, discuss, debate, and validate their ideas with one another to come to a consensus as a class. This peer-peer interaction encouraged all of these types of discourse through the use of the purposefully designed discussion board activity and supported the cultural aspects of peer-peer motivation (Keller, 2010). Encouraging different kinds of discourse has the potential to help support a higher level of group knowledge construction to take place (Gunawardena, Lowe, & Anderson, 1997).

The interview questions were written with the intention of investigating the students' perceptions of the online discussion board activities and their interactions (Appendix B). There were twelve interview questions, designed in semi-structured format. The questions asked about the participants' experiences with participating on the discussion boards. I was interested in knowing the participants' perceptions of the importance of their online discussions and the role they may have played in their learning. The interviews were conducted within the same online learning system in which the online course was delivered. The optional, live, online interviews took place after the study had ended in May 2012. It was important that I included the students' view of the discussion activities, because students' perceptions "can provide an in-depth understanding of the effectiveness of web-based learning" (Yang, 2006, p. 4). I value students' views and opinions, which reveal their needs, and used the student input as a guide on how to design the purposefully designed online discussion activity.

All of the instruments were piloted in the development stage, during the fall of 2011 in three sections of college algebra courses I taught that semester. The participants were expected to complete the pre and post research surveys and participate in the purposefully designed discussion activity. They were also asked to volunteer for an online interview after the study ended to pilot the discussion questions as well. Now that the activity has been designed, the main purpose of piloting the instruments was to determine if there are any logistical problems with implementing the discussion board activity or the interviews. No major modifications to the activity or the interview questions were necessary and it continued as planned during the experimental stage.

Experimental stage. The experimental stage was the main part of the research study. It took place during the spring 2012, in a 15-week course that ran from January until early May. It was a single case-study analysis of students' mathematical discourse in an online college algebra course in relation to the discussion board activities. The study focused on the three weeks right before and right after the purposefully designed discussion activity was administered. The unit of analysis was an in-tact online class of college algebra students. A case-study design was chosen since each mathematics class is a well-defined community and each class will have a distinct group mathematical discourse pattern. In addition, a "case study design is employed to gain an in-depth understanding of the situation and meaning for those involved," (Merriam, 1998, p. 19), including that of the instructor/researcher. It is also seen as one of the best methods to study a problem in its natural setting (Berg, 2007). This is imperative, since this is an online course and the learning environment is very distinct. The reason for the study is to see if an activity created with motivational design, based on social learning theories, can promote online mathematical discourse among the students.

Students were asked to participate in the study at the beginning of the course by completing an online consent form. The consent form notified participants of their responsibility to complete two surveys, and give the researcher consent to download their discussion board posts and achievement data. In exchange for participating in the study, students were given the opportunity to earn 10 extra credit points to be added to an assignment chosen by their instructor. An alternate, but equivalent, extra credit option was available for those students who did not wish to participate in the study. There were two students who chose the alternate option, but they withdrew from the course before midterm. After students showed interest in participating in the study by completing the consent form, they were given immediate access to the first survey. The first survey was the same one administered in the exploration stage, which included the ARCS model survey, the modified Fennema-Sherman survey, and personal information (Appendix A). They were given two weeks to complete the first survey, which included a question asking interest in participating in an optional online interview once the study was complete. The students proceeded in their course as they normally would, reading their online text and completing weekly homework assignments and weekly quizzes. The weekly discussion questions from the original course template were used for every week, except week four. The purposefully designed discussion activity was administered during week four of the course. The activity was given after some weeks had passed in the course, to give the students a chance to become familiar with the technology and the format of the online class. Smith (2003) argues that adult learners need to be allowed time to learn the technology. Waiting a few weeks to administer the activity gave students time to adjust, and could have helped decrease some of the anxiety they may have had about participating in an online class (Smith, 2003). Students were administered Test 1, covering chapter 1 of their text (Blitzer, 2010), which covers functions and their graphs, due at the end of week four. The motivationally designed discussion board activity required students to use a variety of mathematics skills learned from chapter 1 and was administered one week before, leading up to the due date of their chapter 1 test. During weeks five and six, students were administered the post-survey (Appendix C) after participating in the motivationally designed activity. Completing the second survey concluded each participant's responsibility to the study, since the interviews were optional. However, I did have five students follow through with an interview after the study ended. I downloaded all discussion board postings from weeks one through seven. These data provided the opportunity to apply qualitative data analysis using computer techniques, to look for similarities and differences within the discourse patterns of the weeks the purposefully designed activity was not used and the patterns when it was used. NVivo9 research software was the software used for the comparative analysis. Researchers agree that qualitative data analysis computer software can add detail to an analysis that may have been overlooked without the use of the software. In a research study by Putten and Nolan (2010) comparing manual constant comparative techniques to computer software techniques, they recognized that “The sophisticated data representation tools in the software allowed for deeper levels of analysis that were too labor intensive for the individual researcher to produce” (p. 108). In addition to the discussion board data, I collected the survey, achievement, and interview data.

Data Collection

During the experimental stage there were several types of data collected. The students participated in a pre and post survey which provided information in relation to their attitudes about mathematics, before and after the purposefully designed activity was administered, from the modified Fennema-Sherman survey. In addition, the pre-survey collected personal information along with their attitudes towards course components, from the ARCS Model survey, to verify this class is comparable with the sample population used in the exploration stage. Also, the students’ achievement data were collected, including assignment grades and final course grades. The bulk of data collected were the participants’ discussion board postings from weeks one through seven. This was collected to describe the participants’ group and individual online mathematical discourse and the perceptions of the participants’ discussion board interactions. Last, the interview data was recorded and transcribed for analysis.

Data

The students were asked to participate in their everyday classroom activities as they normally would. The consent form gave me permission to download their pre and post survey data, discussion board postings and their course grades. The consent form also gave them the opportunity to indicate if they wanted to participate in the optional interview. Part of their course requirements was to participate on the weekly discussion boards by posting messages at least three days a week. The template discussion board questions were used for each week, except for week four. During week four, the purposefully designed discussion question was administered. The purposefully designed discussion activity was administered in two parts, where students had to work together to come to a consensus on each part within a time frame of three and a half days for each. The other activities were designed to have individual responses to a single question asked, with no guided discussion techniques embedded in the question. At the end of the study, the students' discussion board postings, survey, interview, and achievement data were collected and analyzed.

Discussion board postings. The main unit of analysis was the participants' weekly discussion board postings from weeks one through seven. All discussion board postings were downloaded and saved, along with the date, time, and author of each post. Participants' names were replaced with random three-digit numbers. There were originally 21 students who gave consent to participate in the study, but only 18 of those participants continued in the course and participated on the discussion boards during weeks one through seven. Of the 18 participants that provided discussion board postings, there were fifteen females and three males. The participants were given random 3-digit numbers to identify each student. The purposefully designed activity was administered during week four. The first seven weeks of discussion board postings were collected, three weeks before and three weeks after the activity was given, to look for any variations or similarities in group discourse patterns. The posts were first coded by an assistant using the IAM, then double-checked and/or corrected by myself, as the principal researcher. Corrections were discussed with the assistant for agreement according to the IAM. All posts that were direct responses to, or about, the weekly discussion questions, as well as, posts that were about mathematics or used mathematical language were coded as online mathematical discourse. All other posts were coded as non-mathematical discourse. The mathematical discourse discussion board postings were coded deductively using the predetermined categories from the IAM (Appendix D). The phases of the IAM are understood as levels of lower mental functions to higher ones, through group communication (Gunawardena, Lowe, & Anderson, 1997).

Interview data. Ten participants originally agreed to participate in the optional interview, once the study was over. Of those ten, a total of five participants from the class followed through and participated in the online interviews, which were recorded through the online learning system. There were twelve interview questions (Appendix B), delivered in a semi-structured format. The audio of the interviews were recorded and transcribed. The same random participant three-digit numbers were replaced for real names. Qualitative data analysis computer techniques were used to look for similarities and differences in the interview data, in terms of their perceptions of the discussion board activities. After each interview was transcribed, a text search query was run within nVivo9 on the set of interviews.

Survey data. Other than the descriptive statistics obtained from the discussion board postings, additional quantitative data included the survey data and student course grades. The modified Fennema-Sherman survey was administered to collect information from students on their mathematics attitudes. These attitude scales were used to help characterize the individual students' mathematical discourse patterns, by looking at groups of similar students' discourse, based on their motivation levels. The survey scores allowed me to group participants into categories, which allowed for various comparisons of their individual discourse patterns. This information helped to categorize individual students in order to answer research questions related to the individual participant's pattern of discourse.

Of the 21 students who gave consent, there were 14 participants that took both the pre and post surveys. The loss of the seven participants was due to their withdrawal from the course before the post-survey was given. The pre-survey from the development stage was used again here for the experimental stage (see Appendix A). The ARCS survey measured the students' attitude towards the course materials, to verify that the attitudes of the participants align with that of the sample of participants used during the exploration stage. In both instances, Attention had the lowest mean of all the categories of the ARCS model survey. Mathematics attitudes were measured by the modified Fennema-Sherman survey, and it also showed Mathematics Anxiety as the lowest mean at the start of both stages (see Table 3).

Table 3

ARCS & Fennema-Sherman Survey Means and Standard Deviations (Experimental Stage, pre-survey)

Construct	Variables	Means (SD)
ARCS Course Interest Survey	Attention	3.46 (1.03)
	Relevance	4.05 (.95)
	Confidence	3.92 (1.04)
	Satisfaction	3.82 (.95)
Fennema-Sherman Mathematics Attitudes Survey	Success	4.19 (.99)
	Math Anxiety	2.89 (1.32)
	Motivation	3.02 (1.09)
	Usefulness	3.87 (1.05)
	Confidence	3.28 (1.32)

Notes. Possible range for each variable: 1-5

Achievement data. The achievement data considered for the study were the test scores on the first test and final grades. The chapter test scores that covered the mathematical content from the purposefully designed activity of participants were reviewed. Also, motivation levels and final course grade scores were looked at to see if there are any interesting comparisons that may exist. It is important to remember that this information was used to help answer research questions related to the individual students' patterns of mathematical discourse, by grouping students by individual achievement and anxiety levels. It is not my intention to suggest that discourse patterns had an effect on achievement and/or anxiety levels, as the sample size is not large enough to suggest a causal relationship. However, I did look at detailed comparisons to get better picture of the overall phenomena, remembering the discourse patterns themselves are my main focus of study.

Validity & reliability. Validity and reliability attest to the trustworthiness of the inferences made from the data. I used several techniques in my research to support the validity and reliability of the data. One of those techniques was participant debriefing, also known as member checking. I allowed the participants the opportunity to review my interpretations of their interview responses after they have been transcribed and themes have been identified. There was one student who followed through and reviewed my interpretations, but he/she had no corrections or additions. Also, the use of qualitative data software more easily allowed for continuous testing of hypotheses, confirming and disconfirming ideas, adding to the validity of the data (Merriam, 1998). Peer examination was used when coding data, deductively and inductively, to test validity. I worked with a collaborator to do the initial coding. She went through the process of coding data using the IAM, and then I double-checked the coding. Next, we resolved any conflict through a last round of verification. The process of double-checking data increased internal validity (Merriam, 1998). We coded posts separately, and then I compared it to see if we got the same results and asked for feedback to iron out any discrepancies, in addition, I kept a research journal of all the processes during the research study. Also referred to as an audit trail, the research journal helped me keep track of all decisions made throughout the study. Keeping the research process transparent will help maintain the reliability of the research process (Merriam, 1998).

Both surveys used in the exploration and experimental stages have been around for a while, and have reliability measures published by several authors. The ARCS model course interest survey has been found to have high reliability estimates of 0.95 and 0.81 (Keller & Subhiyah, 1993; Kim & Keller, 2011). The Fennema-Sherman Attitude Scales survey was shown to have a reliability estimate of 0.87 (Fennema & Sherman, 1976). The original Fennema-Sherman survey has nine sections: attitude towards math, math as a male domain, mother's influence, father's influence, teacher's perceived attitude, confidence in math, math anxiety, motivation, and usefulness. My sample consisted of adults in an online college setting, so I modified the survey by only using the relevant categories. I used the five sections related to attitude towards math, confidence in math, math anxiety, motivation and usefulness. There are several authors who have used modified versions of this survey over the years and still found high reliability values. According to an analysis done by Borg and Gall (1996) using a modified version of the survey, they found a reliability estimate of 0.93 (compared to an estimate of 0.96 with the full survey) and indicated that using fewer domains would not significantly reduce the reliability of the instrument.

Summary

The data collected during this study included participants' weekly discussion board postings from weeks one through seven, the pre and post surveys, achievement data, and interview transcripts. This data helped characterize the individual and group patterns of discourse and the perceptions of their interactions, in particular, in relation to the discussion questions listed in the following table (Table 4).

Table 4

Connections between Data Sources & Research Questions

Research Question	Weekly Discussion Board Postings (weeks 1-7)	Post-Activity Participant Interviews	Student Achievement Data	Pre- & Post-Surveys
1. What is the nature of the mathematical discourse of participants in an online college algebra course?				
1a. What is the nature of the individual students' patterns of mathematical discourse when using the standard discussion board activity?	Downloaded and analyzed discussion board data using the IAM.		Used achievement data to look at groups of similar types of students' individual discourse patterns.	Used survey data to look at groups of similar types of students' individual discourse patterns.
1b. What is the nature of the collective group's mathematical discourse when using the standard discussion board activity?	Downloaded and analyzed discussion board data using the IAM.			
1c. What is the nature of the individual students' patterns of mathematical discourse when using a purposefully designed discussion board activity?	Downloaded and analyzed discussion board data using the IAM.		Used achievement data to look at groups of similar types of students' individual discourse patterns.	Used survey data to look at groups of similar types of students' individual discourse patterns.
1d. What is the nature of the collective group's mathematical discourse when using a purposefully designed discussion board activity?	Downloaded and analyzed discussion board data using the IAM.			
2. What are the participants' perceptions of their online discussion board interactions (including mathematical discourse and other interactions)?		Interviewed participants to collect additional information about the students' perceptions of the discussion board activities and interactions.		

The discussion board postings were coded using the Interaction Analysis Model (IAM) and the group's weekly patterns of online mathematical discourse were characterized by that data. The individual patterns of discourse were characterized using the descriptive statistics of the posts, the survey data, and achievement data, by exploring how certain groups of individual students participated in online mathematical discourse. Lastly, the interview data, along with some discussion board postings, provided information on the participants' perceptions of their online interactions.

CHAPTER 4

FINDINGS & INTERPRETATIONS

In this chapter, I discuss the findings and interpretations of the study. First I review the findings in terms of the types of data collected from the discussion board postings, interview data, survey data, and achievement data. Next, I review my interpretations in terms of the research search questions, by discussing the nature of interactions in terms of the group and individual patterns of discourse. Last, I discuss the perceptions of the participants in terms of their discussion activities and their interactions.

Findings

The experimental stage was the main part of the study and it took place in a college algebra class during the spring semester in 2012. This section of college algebra had 24 students, and 21 of them signed the consent form to participate in the research study. The participants were asked to complete a pre and post survey, as well as participate in an optional interview that was given after the study ended. Ten students agreed to participate in the optional interview on the consent form. By the end of the study, five students provided interviews, 18 participants provided discussion board postings for weeks 1 through 7, and 14 provided pre and post survey data for the study.

Discussion board postings. There were a total of 360 posts provided by the eighteen participants on the weekly discussion boards from weeks one through seven. There are 252 posts coded as mathematical discourse and 108 coded as non-mathematical discourse. All posts with information that fell into both categories were coded as mathematical discourse. There was an average of 35.1 mathematical discourse posts per week. The participants posted an average of 13.16 mathematical discourse postings and 5.68 non-mathematical posts per person during the 7-week period.

The mathematical discourse discussion board postings were coded deductively using the predetermined categories from the IAM (Appendix D). A collaborator completed the first round of coding. Then, I reviewed the coding and reconciled discrepancies with the collaborator. All mathematical discourse posts with information that fell into more than one phase of the IAM, were coded within the higher phase. The phases of the IAM are understood as levels of lower mental functions to higher ones, through group communication (Gunawardena, Lowe, & Anderson, 1997). Table 5 summarizes the number of posts coded within each category of the IAM.

Table 5

Summary of Mathematical Discourse Posts coded within the IAM phases

Phase	Description	# of math discourse posts	% of math discourse posts	% of all posts
Phase I	Sharing/Comparing	246	97.6%	68.3%
Phase I-A	Statement of observation/opinion	123	48.8%	34.2%
Phase I-B	Statement of agreement	51	20.2%	14.2%
Phase I-C	Corroborating examples	60	23.8%	16.7%
Phase I-D	Asking/answering questions	12	4.8%	3.3%
Phase II	Discovery/exploration of ideas/concepts	5	2%	1.4%
Phase II-A	Identify area of disagreement	3	1.2%	0.8%
Phase II-C	Restating position, advance argument	2	0.8%	0.6%
Phase III	Negotiation of meaning/co-construction of knowledge	1	0.4%	0.3%
Phase III-D	Proposal/negotiation of new statements/co-construction	1	0.4%	0.3%

Note. Phases with no posts coded were omitted from the table. The nVivo9 software was used to organize the data displayed.

Most of the posts coded within Phase I included direct statements, such as “Graphs are used to show how trend or data relate to each other. It shows whether there is an increase or decrease in certain data.” Phase II included the discovery or explanation of dissonance, such as “You may want to look at that again. The numbers are not different tries for throwing at the target but rather two points for which we know coordinates along the dart’s path.” This student disagrees with someone and is identifying the area of disagreement. The one post that fell into Phase III showed a negotiation of knowledge, “After further review and recalculating, I determined that my calculations were wrong and that Thomas was higher at 40. He would actually be in 4th.” This was one, out of a total of 10 posts that week for this participant. In addition, this student also had two posts coded within Phase II that same week. We can infer that this student worked on the problem carefully and looked at others’ posts before posting their answer, since solutions had already been posted by other students. Table 6 displays the number of mathematical discourse postings in each phase, per week.

Table 6

Number of Posts Coded within Phases by Week

Phase	Week						
	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7
Phase I-A	23	20	16	18	19	15	12
Phase I-B	7	7	10	8	6	8	5
Phase I-C	14	14	6	4	6	7	9
Phase I-D	3	2	0	6	0	1	0
Phase II-A				3			
Phase II-C				2			
Phase III-D				1			
TOTAL	47	43	32	42	31	31	26

Note. This shows the number of mathematical discourse postings per phase by week.

Every week had posts categorized within Phase I. The purposefully designed discussion activity was administered during Week 4. Weeks 1 and 2 were the only weeks that had over 40 mathematical discourse posts made. However, week 4 is the only week showing posts categorized in Phases II and III, suggesting higher levels of mental functions taking place through group communication during that week. There were no posts categorized in Phases IV and V during the study.

Having several weeks where discussion board posts fell into the lowest level of communication, Phase I, is an indication that the lowest level group knowledge construction was taking place during that time. As a conversation transforms into higher phases of communication, there is more indication of the creation of higher levels of group knowledge construction (Gunawardena, Lowe, & Anderson, 1997). Week 4 includes posts coded within Phase I, II and III. The majority of the posts fell within Phase 1 with 36 posts. There were 5 posts that fell within Phase II and 1 post within Phase 3. This is not a significant number of posts in Phases II and III; however, the existence of posts coded within these categories is an indication of more group knowledge construction than in other weeks. This information answers research questions related to the group's mathematical discourse (Research Questions #1b and 1d). Week 4, in which the purposefully designed activity was administered, shows an indication of higher a level of group knowledge construction than in weeks which used the standard template discussion board questions, according to the IAM. Since these categories were used deductively in this approach, the analysis more closely aligns with classic content analysis where "the codes are produced deductively and then can be either included as descriptive information about the data, can be analyzed using quantitative procedures, or both" (Leech & Onwuegbuzie, 2007, p.569). Week 4 had the most number of non-mathematical discourse postings, compared to the other weeks, and was the only week to show an increase in postings of all discourse from the previous week. The time frames before and after week 4, (i.e., weeks 1-3 and weeks 5-7) showed a decline in the number of posts from week to week (see Table 7).

Table 7

Classification of participants' weekly online discourse postings, weeks 1-7

Week Number	Total Number of Posts by Participants	Total Number of Mathematical Discourse Posts	Total Number of Non-Mathematical Discourse Posts
Week 1	57	47	10
Week 2	59	43	16
Week 3	59	32	27
Week 4	72	42	30
Week 5	36	31	5
Week 6	45	31	14
Week 7	32	26	6

There was also an investigation of the themes that appeared in the 108 non-mathematical discourse postings. The information in these posts had no mathematical content. They were labeled in terms of the content within categories such as, praise, question, thanks, etc. After the non-mathematical discourse postings were gathered, a word frequency analysis was run in nVivo9 on these data. The analysis showed the highest frequency of the non-mathematical posts coded were those that were either direct statements of observation (38) or statements of praise (33). Other categories included statements of organization (18), thanks (13), or personal comments (12). These categories were discovered by word frequency, thus some of these posts may have been coded into more than one category. One example of a non-mathematical post of praise was, “Great example. Amazing how many different viewpoints posted so far.” During Week 4, the majority of the non-mathematical posts were for organization purposes, to keep everyone’s responses categorized. One student, participant #901, posted several posts for the purpose of organizing responses. This participant was also the student who had the most number of non-mathematical discourse postings, with 20 posts total over the 7-week period. However, of those 20 posts, 17 of those were for organization of the posts by the class during week 4, such as “Use this space ONLY TO VOTE for the candidate listed to advance. Explain why she should advance…” This participant took the lead to make sure posts related to the purposefully designed activity were kept organized. Most of the other participants’ non-mathematical posts during week 4 included some of their feelings about the purposefully designed activity. These statements included comments such as “I enjoyed it. This was a good way for the class to work together,” and “Very enjoyable project.” They also posted, “The activity went well and it gave us something to do besides the usual posts. Pretty fun…” and, “I enjoyed the challenge. It also reminded me that I need to check and double check my solutions before responding.” There were no negative comments posted on the discussion board about the purposefully designed activity. The existance for the large number of total posts during week four was due to the unfamilar nature of the purposefully designed discussion activity. The participants’ desire to keep their posts organized that week by posting several non-mathematical discourse postings showed their interest in keeping responses organized.

Participant #859 had the most number of online mathematical discourse postings, with 37 posts over the 7-week period. His/her posts were distributed within three categories of Phase I: Phase IA-12 posts, Phase IB-12 posts, and Phase IC-13 posts. This student had the math anxiety average score that increased the largest amount, with a +0.75 increase between the pre to post-surveys (2.5 to 3.25), which indicated a drop in their anxiety levels. There was also an increase in the mode from 3 to 4, confirming a considerable improvement in anxiety during the times between the pre and post-survey. Thus, the student who participarticipated the most in mathematical discourse on the discussion boards, had the sharpest decrease of anxiety about mathematics. This suggests that the more students "talk about math," the less anxious they feel about the subject.

On the other end of the spectrum, those with the smallest number of mathematical discourse postings showed little to no change in their mathematics anxiety scores. Particpant #669 did not participate on the discussion boards, with only 1 post, coded as non-mathematical, during the entire 7-week period. This student placed in the top one-third average score on the pre and post-surveys in math anxiety, however there was a decrease in the average score and mode, suggesting an increase in their anxiety. The average mathematical anxiety scores changed from 4.42 to 4.17, and the mode went from 5 to 4.

The particpant with the next lowest number of postings was #758, who had 3 mathematical discourse postings and 1 non-mathematical discourse post for the seven week period. The partcipant stopped posting on the boards after week 2, and did not participate at all in the purposefully designed activity. This participant also had a decrease in their mathematical anxiety average score from 2.08 to 1.58, with a mode contstant at 2, which indicated a slight increase in their mathematics anxiety. Participant #882 did not have any posts during week 4, and thus did not pariticpate in the purposefully designed discussion question. This participant also had the lowest drop in average math anxiety score, going from 3.17 to 1.25, and a mode going from 5 to 1, which indicated a huge increase in their mathematics anxiety during the study. This data show that those who partcipated the least on the discussion boards had decreases in their average mathematical anxiety scores, indicating an increase in anxiety. Other participants who completed the study had at least 8 mathematical discourse postings, suggesting they at least tried to partcipate each week. The results of those participants with anxiety scores closer to the class average had mixed results.

By looking at the pattern of those individual participants posting the most and least, we are able to give more of an individual characteriztion in the types of students that particpate on the discusson boards the most and those who do not. Participant #927 was the only particpant who posted a mathematical discourse post falling within Phase III (Phase III-D). He/she had an above average number of mathematical discourse postings with 22, falling in Phases I, II and III. All of the posts from Phases II and III were posted during week 4, during the purposefully designed activity. Participant #927 also had a slight increase in his/her average mathematical anxiety score from 2.42 to 2.58, with a mode constant at 2, suggesitng a small decrease in their anxiety. Participants #901 and #220 also provided posts that were coded into Phase II. Participant #901 had a slight decrease in their average math anxiety score, with a constant mode of 5, however he/she took the lead in the activity and helped the class get organized for finding the answer to the activity. Since the mode did not change, it confirms that the change in the score was not very significant. Participant #220 also had a post that fell into Phase II during week 4. This participant had a pre-survey average math anxiety score of 3.25 and a mode of 4, but he/she dropped the class after week 6 and did not take the final survey. Thus, there are no conclusive results for this individual participant.

After sifting through the individual data of the participants, the pattern that emerged was those who particpated the most, with a total of at least 20 mathematical discourse postings over the 7-week period, had postings that fell into at least three different sub-cagtegoreies, showing more of a variety in the type of conversation taking place. This suggests, that the more students participate in mathematical discourse on the discussion board, the more advanced their conversations can eventually become and thus, the better chance of higher levels of group construction of knowledge. The information from the individual participant data helped to answer the research questions related to nature of the individual's patterns of mathematical discourse (Research Questions #1a and 1c). By creating an activity that embeds collaboration, I was able to show that the communication can be tailored and mathematical discourse can be fostered within an online setting.

Interview data. The word query in nVivo9 showed the purposefully designed activity mentioned by four out of the five participants interviewed. Participant #759 stated that it was their favorite activity because they "worked as a group," and "It was more of a group effort, rather than just my opinion." Participant #927 also enjoyed the purposeful activity for "being able to interact with other class members, working more closely together." This participant went further to compare the purposefully designed activity to others when she/he stated, "I know we interact through normal discussion boards, but this was more back and forth … and giving more interaction." Participants #583 and #882 expressed some difficulty with the purposefully designed activity.

Participant #583 was confused about the activity at the beginning, because he/she thought it was a real situation. Once it was realized that this was a fictitious class activity, everything went fine. Participant #882 stated that it was difficult because the activity was presented in several parts. Getting earlier parts of the activity incorrect hindered their ability to continue. This student was the only participant who expressed this difficulty, although, this participant did not make any posts during the week 4, when this purposeful activity was administered. Thus, they did not participate in the activity, nor did they ask questions to get help.

After generalizing the interview responses, I was able to create a short narrative that summed up the participants' perceptions. The interviewees described their experience with the discussion boards as interesting, challenging and enjoyable. It was seen as a medium to relate to and interact with other students in the class. The discussion questions helped them think more about how math relates to the real world. Having the ability to see what their classmates were thinking was valuable for them. Responding to the discussion questions helped them make sure they understood what they were learning. Some of the discussion questions provoked their curiosity to pursue further math investigations. They did not feel any of the questions were too easy or too difficult, but that they balanced out, and the amount of required participation per student was also just about right. Two of the interviewees mentioned that having one or two questions similar to the purposefully designed activity will help make the class more interactive. The responses to the interview questions were positive and the students' perceptions will go a long way in tweaking the current activity and for creating new ones.

The interviewees were asked if they would like to review the transcriptions and the narrative of their interviews, to see if any ideas were misrepresented through transcription and the reduction of data, or to offer additional clarity. There was only one participant who asked to see their transcription and the summary. That participant did not provide any corrections or additions to make. This technique will help produce a level of validation within the process, which contributes to constructing premises that more closely models reality of the case at hand. This information provided additional guidance on the preferences of online students, which could assist in more accurately gauging the needs of the students when designing course materials.

Survey data. Mathematics anxiety had the lowest average on the survey for both the exploration and the experimental stages of the study, signifying relatively high feelings of anxiety. During the exploration stage, the mathematics anxiety score for the class averaged at 2.78922. During the experimental stage, the survey was given twice, before and after the purposefully designed activity was administered. On the pre-survey, the mathematics anxiety average score was 2.89035, with a mode of 2. During the post-survey, the mathematics anxiety average score slightly fell to 2.7, however the mode remained at 2.

Table 8

Means, standard deviations, & modes of Fennema-Sherman Survey (Experimental Stage, post-survey)

Construct	Variables	Means (SD)	Modes
Fennema-Sherman Mathematics Attitudes Pre-Survey	Success	4.19 (.99)	5
	Math Anxiety	2.89 (1.32)	2
	Motivation	3.02 (1.09)	4
	Usefulness	3.87 (1.05)	4
	Confidence	3.28 (1.32)	4
Fennema-Sherman Mathematics Attitudes Post-Survey	Success	4.14 (.91)	5
	Math Anxiety	2.70 (1.32)	2
	Motivation	3.08 (1.05)	3
	Usefulness	3.85 (.92)	4
	Confidence	3.05 (1.29)	2

Note. Possible range for each variable: 1-5

This may suggest that the participants' overall anxiety slightly rose during this time period; however, with no change in the mode the rise is not large enough to indicate any considerable change as a group. An individual analysis may provide more information about what is occurring.

As we look more closely at the individual patterns in the quantitative data, we can acquire more information about the phenomenon taking place. According to the pre-survey, participants #358, #901, #669, #712 and #882 had the highest average score in the area of mathematics anxiety, suggesting they had the lowest anxiety among the group of participants when the course began. Of this group, four out of the five students remained in the top five highest average score in this area. This suggests that those students who already had low math anxiety going into the course continued to have the lowest anxiety at the end, regardless of the course materials.

One of the participants, #882, that started in the top five highest averages in math anxiety (denoting very low feelings of anxiety), fell to the very bottom of the list on the post-survey. By looking more closely at this student's records, it shows that he/she did not participate at all in the purposefully designed activity and had no posts for that week. This participant had a total of 22 posts for all seven weeks, and 12 of those 22 were coded as non-mathematical discourse. In addition, this participant had low participation overall, by posting less than the minimum required on several weeks. Those posts that were coded as mathematical discourse, all fell into either Phase I-A or I-B. It seemed that this student was not motivated to participate in the discussion boards from the beginning, which worked against them in terms of their mathematics anxiety during the course.

This participant (#882) was also one who provided an interview. After further review of the transcripts, it seemed that the participant had issues with adjusting to the online format in general. The participant stated that he/she was, "used to being in a classroom setting where you would raise your hand, it wasn't the same." This suggests that it was not one particular activity or course component that caused difficulty, but the ability learn how to navigate through the system and regulate online assignments. Even though this participant did not fully participate on the discussion boards, he/she expressed that they participated in the discussion board only, "as a part of the assignment," suggesting that they would not have participated had it not been a required component of the course. This is consistent with research, which says students don't usually participate more than what they are required (Pena-Shaff & Nicholls, 2004). He/she also talked further about the role of the discussion boards stating, “It helped … to interact with your classmates, even though you’re not in the classroom setting.” The participant also admitted that classmates’ posts were supportive saying “I thought they were helpful. Reading their comments on some of the questions that I didn’t understand, their posts help me to understand what it is you were looking for.” This suggests that the participant is aware of the usefulness and the role of the discussion board as a mode of communication and learning, even though they did not fully participate on the boards.

The nine participants with a math anxiety average that fell below the top five during the pre-survey had slight increases or decreases in score of no more than ±0.75 of a point. Of these nine participants, five had an increase in their average mathematics anxiety scores, one stayed the same, and three slightly fell. Three of the nine participants had an increase in mode and the others stayed the same. There were no decreases in mode of the mathematics anxiety scores. The student who started with the lowest mathematics anxiety average score showed an increase from 1.75 to 2.33, with a mode staying at 2. Collectively, this information suggests that the purposefully designed activity did not hinder the students’ mathematics anxiety, and could have possibly had a positive effect on participants’ anxiety.

The main units of analysis were the discussion board postings of each participant. Considering the individual participants’ nature of their mathematical discourse in relation to their mathematics anxiety scores showed another level of individual analysis that proved to be beneficial. There were nine participants that met the minimum participation requirement, and had at least three posts per week. If we look at all the postings for those nine participants’ total discussion, three students (#859, #256, & #759) had an increase in both their mode and average mathematics anxiety score (indicating a drop in anxiety). There were four participants that had slight increases or decreases in their average scores, but kept the same mode (#583, #929, #927, & #829), not indicating any significant change. There were two students who had a decrease in their average participation score and their mode score, participants #901 and #882. Despite the decrease, participant #901 had the second highest mathematics anxiety score on both the pre and post-surveys, indicating low anxiety before and after the purposefully designed activity. Participant #901 had the most number of posts of all the participants, with 42 posts. #882 did not pass the course, making a final grade of 69, and had close to the minimum with 22 posts during the seven week period. Considering the individuals who passed the course, of the nine who posted the minimum amount on the discussion boards, seven of them passed with a grade of B or better. Of the remaining nine participants that posted on the discussion boards below the minimum requirement, seven of those students either dropped or failed the course with a grade of F. Data suggest that those who met the minimum participation requirement seemed more likely to have average mathematics anxiety scores and modes higher than those who did not meet the minimum requirement. Some of this is due to students who dropped the course before the study ended, and thus did not complete the last survey to be included in the final comparison. However, all but one those who participated more than the minimum amount and completed the course, and ended with a grade of B or better. Of all 18 participants, there were eight that received a final course grade of B or higher. Of those eight, seven of them had a mathematics anxiety mode score that either increased (showing lowered anxiety) or stayed the same. The individual mode scores gave a better indication that the overall mathematics anxiety decreased or stayed constant as a class during the study. However, a larger scaled study is needed for more conclusive quantitative comparisons.

Achievement data. Of the fourteen participants who finished the course, participants #901, 927 and 829 made A's on the first test, #882 made a B, #759, 859, and 929, made a C. Participant # 712 did not take Test 1. The remaining six participants scored below 70 on the first test. However, these participants' final course grades were not all aligned with the scores on the first test. Participant #901 was the only one who made an A in the course. This student is one of those which scored highly on the pre and post-test in the area of anxiety, showing low feelings of mathematics anxiety. There were seven participants who made a final grade of B in the course, #583, 859, 256, 358, 929, 759 and 927. There was one participant who made a C in the course, #673. The remaining participants did not pass, with #882 making a D, and #669, 758, 712, and 829 making a final grade of F. Of the four students who failed the course, three of them did not take the final exam. One of the three had a passing average at the end, but did not take the final exam. Also, participant #882 had an A average going into the final exam, but failed the final, bringing his/her final grade to D. Looking at the final grades of the 11 students who took the final exam, the average mode of anxiety was 2. The student with the highest mode of 5, made a final grade of B in the course. The two students with the lowest mode of 1, made a final grade of D and F in the course. After looking at the individual achievement scores of the participants, there does not seem to be a relationship between the achievement scores of the participants and their mathematics anxiety scores. This could be because of the small size of the sample. A larger study with more participants is needed to statistically test if there is a correlation between the anxiety scores and the achievement scores of the participants.

Comparing the final course grades of the students with how they participated shows a more appropriate comparison for this study. Participant #901, who received a final grade of A in the course, had the highest number of posts of all the participants, with 47 posts. Of these 47 posts, 27 of them were coded as mathematical discourse. Of the posts coded as mathematical discourse, two of those posts fell into Phase II (Phase II-A and Phase II-C). Participant #901 was one of three students who posted into Phase II.

Participant #927 had the second highest final grade of 89, B. This participant also had posts coded within Phase II and had one post coded within Phase III. All the postings for Phase II and III were done during week 4. There were a total of 29 posts, 22 coded as mathematical discourse, for this participant over the seven week period. Participants #901, 927, and 220 were the only participants with posting coded within Phases II and III, but Participant #220 withdrew from the course, thus there are no final achievement scores available to compare. By looking at the scores of the participants who passed the course with a grade of C or better, the average number of total posts was 29 posts and the average number of mathematical discourse posts was 20.9 posts. Of those students who did not pass, receiving a final grade of D or F, the average total number of posts was 12.6 posts and the average number of mathematical discourse posts was 8 for the seven week period. Of all the participants who completed the course, the average number of posts was 23 posts and the average number of mathematical discourse posts was 16 posts. Overall, the six students who had the most number of mathematical discourse postings during the study, with posting 20 posts or more, all received final grades of either A or B in the course. Of the eight students with less than 20 mathematical discourse postings, five failed the course with grades of D or F (see Table 9).

Table 9

Total number of posts, number mathematical discourse posts, and final course grade per participant

Participant ID Number	Total Number of Posts	Total Number of Mathematical Discourse Posts	Final Course Grade
#859	42	37	83-B
#583	37	29	81-B
#901	47	27	97-A
#927	29	22	89-B
#256	24	21	83-B
#929	31	21	86-B
#829	22	16	57-F
#759	21	12	87-B
#712	14	11	53-F
#882	22	10	69-D
#673	19	10	74-C
#358	11	9	84-B
#758	4	3	45-F
#669	1	0	10-F

Looking at data from this perspective showed that those who participated more overall on the discussion board, as well as those with more mathematical discourse postings, received higher overall course grades. The nature of the individual students' pattern of discourse for this particular study shows that students who participated more on the discussion boards received better grades. Since there were only two students to complete the study that posted within Phases II and III, there is not enough data to determine if there is a relationship between those posting into higher phases of discussion and final course grades received. In addition, after reviewing the scores from Test 1, there were no connections found between the score on the first test, and the number of mathematical discourse postings or their mathematics anxiety scores.

Interpretations

My interpretations were based on the analyses of the data and previous research. The descriptive data of the phases of the IAM were used to characterize the group's online mathematical discourse. The descriptive data of the phases of the IAM, along with the survey and achievement data were used to characterize individual students' online mathematical discourse. The descriptive data of the phases of the IAM was used to describe the nature of the discourse of the class as a whole. The interview data and discussion board postings were used to characterize the participants' perceptions of their online interactions and discussion activities. An overview of all the participants' descriptive statistics and research study activities can be found in the appendix (see Appendix E).

Nature of interactions. The nature of participants' interactions was conceptualized by first coding and analyzing data from the weekly discussion board postings using the IAM. I used the literature as my guide for finding desired discourse characteristics for online mathematical discourse. The literature base showed me what to look for in terms of desired behavior and led me to choose this analysis tool that worked best for characterizing asynchronous discussion board data with categories important to learning mathematics. The pragmatic stance allowed for the use of two different perspectives to study the same complex phenomenon of online mathematical discourse. One perspective was used to characterize the nature of the individual students' discourse in terms of the students' individual survey scores and achievement data. The other perspective allowed for me to characterize the nature of the group's discourse solely in terms of the levels of group construction of knowledge in the IAM (Appendix D). This approach gave a more detailed picture for characterizing the discourse. It also preserved the pragmatic stance used to address both the individual and the group discourse patterns, uniting the constructivist view of knowledge as being constructed by the individual, with the situated view of knowledge as being influenced by a community of practice (Cobb, 2007).

Group patterns of discourse. As a collective, I was able to describe the group's pattern of discourse using the IAM. I was interested in characterizing the groups' discourse by collectively looking at the different phases of discourse that occurred each week. As explained in my theoretical background, thinking is a form of communication (Sfard, 2001b). Thus, a groups' communication is the closest piece of tangible data I can get to characterize the group's construction of knowledge. Discourse postings coded within the first phase, Phase I, is an indication that the lowest level of group knowledge is being constructed. Posts coded within Phases II – V, are an indication that more group knowledge construction is taking place, the higher the phase. I do not presume to measure what that group knowledge represents, but I can determine to what extent group knowledge was taking place during that particular time by characterizing the group's discourse in relation to each weekly discussion activity. This supports the use of social theories which frame this problem, taking into account the individual students, the group as a whole, and the activity in which they are participating.

There is clear evidence that a higher group construction of knowledge occurred during week 4 than in any other week. This shows that the purposefully designed discussion activity fostered more meaningful online mathematical discourse, in other words, higher levels of group knowledge construction (Gunawardena, Lowe, & Anderson, 1997). The IAM phases from the lowest to highest phases indicated the different stages of co-construction of knowledge, starting at Phase I and working up to Phase V. The nature of the participants' mathematical discourse during the weeks that used the standard template discussion board questions fell into Phase I, sharing or comparing of information. This phase indicates the lowest level of group knowledge construction. The nature of the participants' mathematical discourse during week 4, when the purposefully designed activity was administered, fell into Phases I, II and III. Week 4 also had the most number of posts coded into Phase I-D, asking and answering questions to clarify details of statements. This is an indication of students reaching out to one another to seek their classmates' help or to provide assistance. This suggests that the purposefully designed activity guided the discussion to value every participant's point of view, and encouraged participants to ask questions and offer explanations. Since week 4 showed posts categorized into higher phases of the IAM, this showing higher levels of group knowledge construction took place. Regardless of the number of postings, week 4 was the only week with evidence of higher co-construction of knowledge compared to the other weeks, with posts coded within Phases II and III. This is a direct indication that the purposefully designed activity fostered higher levels of group knowledge construction, or more meaningful online mathematical discourse, compared to the standard discussion questions. Thus, the nature of the group's mathematical discourse was enhanced when the purposefully designed activity was used, than when it was not used.

Individual patterns of discourse. The natures of the individual participants' patterns of discourse were better characterized by the final course grades, than by their mathematics anxiety scores. I discovered a relationship between the number of posts of each participant and their final course grade. All except one participant who met the minimum participation requirement, with 21 or more postings for the seven week period, and completed the course, received a final grade of A or B. Thus, the nature of the overall individual students' patterns of discourse showed that students who participated more on the discussion boards received the better grades in the course.

When comparing the weeks, week 4 had the most number of posts compared to all weeks, with a total of 72 posts made by participants. Of the 72 posts, there were 36 that fell into Phase I, 5 into Phase II, and 1 into Phase III, giving a total of 42 online mathematical discourse postings. Even though week 4 had the most number of total posts, it did not have the most number of posts coded as online mathematical discourse. There were a very large number of non-mathematical discourse postings for that week as well. There were 30 posts coded as non-mathematical discourse, 18 of those were coded as organizational posts. One student took the initiative to attempt to organize students' posts ahead of time. Almost all (17) of these posts were submitted by Participant #901, so the class could organize their answers to the purposefully designed discussion activity. This student took the initiative to do this for the class, which helped participants organize their thoughts on the week four discussion board.

When looking at the number of posts per week for each individual participant, there were only two participants that stood out with a large number of posts during week 4. Participant #901 had 27 posts and Participant #927 had 10 posts for week 4. All other participants posted about the same number of posts each week. There were six participants who did not post at all during week four, Participant #882, 358,758, 669, 820, and 136. Of these six participants, two of these participants posted every week except week four, but only one of them received a passing course grade. Also, of the six participants that did not post during week four, five of them either failed the course with a grade of D or F, or they withdrew from the course. In addition, of the nine participants that posted at least once every week, eight of them passed the course. The one student who posted every week, but did not pass the class, was Participant #712, who had a passing average at the end of the course, but did not take the final exam. It is my belief that if this student had taken the final exam, he/she would have passed the course as well. This is additional evidence that participants with consistent weekly participation were more likely to receive a higher grade in the course than those who did not have regular participation.

The overall nature of the individual patterns of online mathematical discourse when using the purposefully designed discussion question was no different than when the template discussion questions were used, in terms of the number of posts per week. When looking at the types of posts every week, the data showed the number of online mathematical discourse posts slightly decreased from week one through week seven, with exception of week 4. This is common, as students withdraw from the course and the excitement of starting a new course wears off. The two participants who posted the most during week four were the two students with posts categorized within Phases II and III; in addition, they were the two students who receive the top two final course grades. So, even though the individual patterns of the participants' discourse were similar from week to week regardless of the activity, those who posted the most during week four did the best in the course. In addition, those participants who kept a regular pattern of participation from week to week and met the minimum requirement had the best final course grades.

After analyzing the mathematical discourse postings, the individual students' discourse patterns were revealed, showing how each contributed to the weekly discourse. By comparing discussion data for each student, to their individual survey scores and motivational levels, I was able to describe how particular students interacted on the discussion boards. Analyzing the discussion board postings helped characterize the types of students that are participating on the boards and those who are not, which is valuable information to have in order to improve course components that will enhance online interactions.

Even though mathematics anxiety was not the main focus of my study, I measured the students' mathematics attitudes before and after the activity for three reasons. The first reason was to ensure my participants' anxiety levels showed to be at similar levels to those tested during the exploration stage of the study. The second and main reason was to use this measure to group students by anxiety levels in order to characterize their individual mathematical discourse patterns. The last reason for collecting these data was to see if the purposefully designed activity could have had an effect on this small sample of students' mathematics anxiety levels. I do not propose to make any conjectures of the direct effects of the activity, as it would take a larger, multi-case study, to investigate that. However, the descriptive data from this study could identify areas that suggest a need for further research.

When I compared the descriptive data of the discussion board postings to the achievement data, I discovered that those who participated the most on the discussion boards made better grades. In particular, of the nine participants that met the minimum participation requirement of three posts per week, seven of them made a grade of B or better. On the other hand, of the remaining nine participants who did not meet the minimum requirement, seven of them either dropped or failed the class with a grade of D or F. This suggests that the students who participate more on the discussion boards will more likely make the better grades.

When I analyzed survey data, I discovered that four of the top five students showing the lowest anxiety during the time of the pre-survey (i.e., showing the highest survey scores), also showed the lowest anxiety during the post-survey. The one student who fell from the top ranking did not participate in the purposefully designed activity and did not meet the minimum participation requirement. Of the remaining students who took both surveys, 11 showed a decrease in their anxiety (i.e. an increase in their mode score) or it stayed constant. Thus, 15 of the 18 participants showed no negative effects on the students' anxiety score between the pre and post surveys.

Perceptions of participants. The perceptions of the participants were characterized by their online discussion board postings and the interview data. These data were used to answer the research question, "What were the participants' perceptions of their online discussion board interactions?" According to the interview narrative, in general the participants' perceptions were positive. They felt the discussion board was a valuable tool to their mathematical learning in various ways. It helped them relate the mathematics to real-world examples, helped them see what their classmates are thinking about the mathematics, and helped spark further mathematical investigation and learning. Specifically, in terms of the purposefully designed activity, the interview comments and the comments on the discussion board focused on their interaction and the ability to work together as a group. Overall, the participants seemed to enjoy the structure of the discussion board and understood why it was used in the course. Each of the participants discussed their experiences with the discussion board questions. Many were frank stating that they would not have participated had it not been required, but found some interest in particular topics discussed each week. Even though the participants were not directly asked about the purposefully designed discussion activity, four of the five interviewees mentioned the purposefully designed discussion activity. This is evidence that the activity caught and held their attention enough to discuss it in the post interview. This supports the use of the ARCS model to create the purposefully designed activity (Keller, 2010). According to the ARCS model survey, attention was the area showing the lowest score, identifying it as the area needing the most consideration when designing new materials for this student population. This was taken into consideration when the purposefully design activity was created. The fact that four of the five interviewees remembered this particular discussion activity over others helps to show that addressing the area of attention to design the activity proved to be successful.

Most of the interviewees mentioned that the discussion questions were not too difficult or too easy, however, Participant #882, mentioned that they thought the purposefully designed activity was too complicated for them to follow. The purposefully designed discussion activity was administered in two parts, where students had to work together to come to a consensus on each part. The other activities were designed to have individual responses to a single question asked, and did not encourage to group discourse. So, students were not forced to work together during the other weeks. It was not the mathematics this student had trouble with, but the nature of the activity and how it was designed. However, participant #882 did not post at all during this week, even though he/she did post during all the other weeks. This student did not even try to participate during week four, and he/she fell one point shy of passing the course. I knew from the research of Sfard (2001b) that I would have students that were unfamiliar with this type of discussion, and it make take time for every student to feel comfortable communicating in this manner. This could be addressed in a future study by embedding several discussion activities into the course, giving students more chances to practice and learn how to communicate this way.

All of the remaining four interviewees did participate in the purposefully designed activity. Participant #901 was the one who did not mention the purposefully designed discussion activity during the interview; however, this was the student who had the most number of posts during that week by taking the initiative to help organize the class' postings. Taking this initiative is the way this student expressed his/her interest in the class activity. The other three interviewees who mentioned the purposefully designed activity had no negative comments. Participant #583 mentioned that he/she thought the activity was so life-like that they believed it was a real scenario; however they also mentioned that this did not hinder their ability to participate. In fact, Participant #583 had the third highest number of posts that week of all the participants.

Overall, the comments made about the discussion board interactions were positive. The interviewees described their entire online discussion board experience as interesting, challenging and enjoyable. They stated that the discussion boards gave them the opportunity to communicate with their classmates and it was a valuable experience for them. Many of them stated that the discussion board was a way to experience how mathematics related to the real world. They enjoyed the interactions, however, they stated they were not inclined to participate more on the discussion boards, unless it was required. They agreed that the discussion boards did not hinder their learning. Most of them mentioned that the discussion boards helped their learning, because they were able to get ideas and communicate with others about the mathematics and see how it was relevant to the real world. Participants were asked to post comments about the purposefully designed discussion activity on the week four discussion board, the week after the activity was complete. These comments were all positive; stating that they enjoyed the activity and thought it was a good way for the class to work together. One student mentioned that it was good to have an activity that was different than the others, and posted "The activity went well and it gave us something to do besides the usual posts. Pretty fun." After reviewing all the comments, it conveyed that the overall perceptions of the discussion activities were positive and participants considered the discussion board an integral part of the course. Using students' perceptions to evaluate learner's needs when creating new course components has proven to be useful, and validated the student-centered process for designing materials. If designed specifically with the students' opinions in mind, the discussion board could be a major component to bridging school mathematics to real-world mathematics through discourse in online courses.

CHAPTER 5

CONCLUSIONS

In this section, I present an overview of the study. The overview includes a brief description of the purpose, methods, and results of the study. I then present my conclusions and interpretations of the results. Last, I address implications for teaching and future research and discuss my personal reflections about the study.

Overview of the Study

The purpose of this single case-study was to gain a better understanding of how to purposefully design online course materials that could enhance peer-peer interaction, in particular, online mathematical discourse. My research began with a literature review which revealed interaction as an important factor in both mathematical learning and online learning. I connected the research based on interactions and focused on the similarities in mathematical and online learning. This directed me to focus on the important factors for increasing meaningful online mathematical discourse while designing a new activity for an online college algebra course. Using the information from the literature review allowed me I created a purposefully designed asynchronous discussion board activity, using motivational design (Keller, 2010), that combined the needed aspects from mathematics education and online learning that foster discourse. The study took place in three stages: exploration stage, development stage, and experimental stage. During the exploration stage, I administered a modified version of the Fennema-Sherman Math Attitude Scales survey (Fennema & Sherman, 1976) and the

ARCS model course interest survey (Keller & Subhiyah, 1993) to collect information on the participants' mathematics attitudes and their perceptions of online course materials, respectively (see Appendix A). The Fennema-Sherman survey showed mathematics anxiety as the lowest scoring area, indicating feelings of high mathematics anxiety. Attention was the lowest scoring area on the ARCS model survey, indicating that the online course materials were not grasping and/or keeping their attention. The survey results, along with my literature review of research, provided a basis for designing course materials that addressed these two areas, as well as fostered online mathematical discourse to promote higher levels of group knowledge construction. Next was the development stage, where the purposefully designed activity was created and piloted. After information from the exploration stage was collected and reviewed, the purposefully designed discussion activity was created for an asynchronous weekly discussion board setting. The activity was pilot tested during the fall of 2011. There were no major concerns with administering the activity. As a result, no adjustments were made to the discussion activity. The last phase was the experimental stage, which took place in the spring of 2012. During this stage, the purposefully designed activity was administered to one section of an online college algebra course. The content of the activity included material that the participants learned within chapter 1 of their textbook (Blitzer, 2009). The activity was given during week 4, which coincided with the time the course completed chapter 1 materials. Standard template discussion questions were used for the other weeks of the course. Pre and post surveys were administered before and after the purposefully designed activity to see how participants' individual discourse patterns related to their mathematics anxiety scores. After the study ended, the

Interaction Analysis Model (Appendix D) was used to code the discussion board postings for weeks one through seven. The literature review uncovered comparable categories for ideal discourse for learning mathematics and for online learning. All of these categories were represented within various levels of the IAM. This model was chosen because of its close alignment with my literature review on the types of communication ideal for the deep learning of mathematics. I also interviewed five participants to get their perception of the discussion board activities. After the data were collected, the qualitative data were analyzed using nVivo9, a qualitative software program. Descriptive statistics were analyzed from the quantitative data. These data were used to characterize the participants' group and individual discourse patterns, as well as the perceptions of their interactions. The nature of the group discourse patterns showed to have more meaningful mathematical discourse during week 4, when the purposefully designed discussion activity was administered. The nature of the individual patterns of discourse showed that students who participated the most on the discussion boards, received the better grades. In addition, all students who posted at least three mathematical discourse posts per week for the seven week period passed the course. Of the eight students who did not have at least three mathematical posts per week, five of them did not pass the course. This suggests that those who make an effort to participate in online mathematical discourse are more likely to pass the course. There were some interesting comparisons looked at between the mathematics anxiety scores and the natures of the individual students' discussion, but no convincing results were found from that detailed comparison. Those students who provided interviews provided positive feedback about their online discussions. They agreed that the discussion board was an integral part of the course, allowing them the ability to communicate with each other about the subject matter.

Conclusions

This study allowed me the opportunity to use motivational design to create a discussion board activity that helped to enhance the mathematical discourse taking place on the asynchronous discussion boards of an online college algebra course. The sociocultural interaction about the material is innate to the nature of mathematics and how it is taught and learned (Cobb, 1994; Sfard, 2001b; van Oers, 1996). With my activity, I was able to navigate the discourse and direct learners how to communicate with one another. I was also able to encourage higher levels of group knowledge construction using the purposefully designed discussion question during week 4. Taking the view that mathematics is a language and focusing on improving online discussions allowed me to foster online mathematical discourse in terms of levels of group knowledge construction. Researchers agree that mathematical discourse is essential to the development of mathematical thinking (Cobb, 2006; NCTM, 2000; Sfard, 2008). By focusing on the discourse, I was able to look at mathematical learning from a social cognition perspective and focus on students' communications. According to Sfard & Kieran (2001), thinking is an act of communicating, and I was able to describe the group's levels of knowledge construction in relation to the communication taking place on the discussion boards. This proved to be a valuable approach and opened the doors for research in the area of online discourse and creating online discussion activities.

Implications for Teaching and Future Research

The motivational design process that took place for this study can be tailored to various student populations (Keller, 2010). I encourage online mathematics instructors to take the lead in designing their own course materials, in particular, discussion board activities. “In the early days of computer-based training (CBT) and through the early transition to Web-based delivery of asynchronous, self-paced learning, instructional development teams were most often organized with clear delineation between those who *designed* the instruction and those who programmed or *authored* course material…Over time, however, the lines between these roles continued to blur, with increased pressure on instructional designers to assume a more active role” (Chapman, 2008, p. 673). For so many years, instructional designers have had the task of designing online course materials for various subjects, without necessarily having been trained as a teacher or having content knowledge in that subject area. As technology quickly grows and changes, it can be difficult to keep current with new technologies within new teaching environments. The more mathematics teachers teach online, the more aware they become of the tools that will be most beneficial to delivering their course content to their student population. It is my belief that we should use online mathematics teachers as our primary resource when designing course materials. No one knows what students need to be successful and the pitfalls for mathematics students better than mathematics instructors. So, they should be used as the primary source when designing new materials. Instructional designers can be used in a supportive role to connect instructors’ ideas with technologies that could be helpful.

This study provided some initial evidence that mathematical discourse can be fostered and enhanced in an online, asynchronous environment, to create higher levels of group construction of knowledge. This study also contributed to a body of knowledge supporting pedagogical techniques for online mathematics instruction and added to both online education and mathematics education research. In addition, the methodology created for this study contributed to the research in both research areas, by creating a methodology that united the theories in both fields. Teacher-lead research is one of our best assets for future investigations in this area. Colleges and universities that currently offer large numbers of online mathematics courses have the best resources available for wide-scale research in this area, their instructors. A wide-scale study that focuses on student participation would have large amounts of qualitative data to be analyzed, and could provide the numbers needed to run certain statistical analyses to support a larger study and provide generalization to a larger population. This study provided me the opportunity look deeper into a phenomenon occurring in my own classroom and to design a plan to improve the online mathematical discourse. It also encouraged me to continue conducting research in this area to improve course materials for online students that foster online mathematical discourse.

Personal Reflections

For this study, I served as both the instructor of the course and the researcher for this study. Because of this, I did my best to make my methods as transparent as possible. I have been a full-time college mathematics instructor since 1995, immediately after receiving a Masters in Mathematics. I began teaching online classes in 2002, two years after receiving a Specialist in Mathematics Education. When I first began teaching online, I was given very little guidance for designing online course materials. We had the resources and excellent technical support that assisted us with all technology related issues; however, we had no knowledge of existing pedagogy for online instruction. For so long, I used trial-and-error to find the best techniques for designing online components for the course. Back then, it was assumed that if you transferred the material from your face-to-face class to an electronic format, then that was enough to facilitate an online class. I soon found out, through my experience teaching online, this was inadequate instruction and was a disservice to our students. They deserved better and I wanted to be able to provide it for them. Eventually, I realized that I wanted to conduct my own research in this area to help improve how online mathematics course components are created. I returned to school to become a trained researcher and my studies led me to focus on online communications and discourse from a social cognition perspective. It was my responsibility to provide the best possible instruction with the available resources. One of those possible resources is reading literature and conducting research in this area to find out more information on the best ways to teach mathematics online and design online course materials. Through this process, I found that looking at the discourse of online math students was a good place to start my research journey.

The purpose of this study was to see how one could create discussion board activities to enhance online mathematical discourse. The descriptive statistics of the quantitative data were used to help characterize the individual participants' patterns of discourse. There were no statistical tests run on the quantitative data, due to the small sample size. I kept the focus of my study on the interaction of the students, in particular, their online mathematical discourse patterns. It was important to make these biases known to keep the study as transparent as possible. Research to see the correlation of how students' mathematics anxiety and achievement scores are affected by using purposefully designed discussion activities would need a much larger sample size and some level of random assignment. There is a lot of room for future research, using various research perspectives. For example, if several class sections are available, a more mixed methods approach could be used with stronger statistical analysis. In addition, administering more purposefully designed discussion activities could positively enhance online mathematical discourse for additional weeks. My future plans are to extend this study by creating three or four additional discussion board activities for this course, so students have more opportunities to become comfortable with learning how to communicate in this manner. Sfard (2008) agrees that students have to be taught to communicate this way, so having more opportunities for students to interact in this format will help students become comfortable with communicating with each other about mathematics. Once the new activities are created and piloted, I plan to conduct a multi-case study to look at this problem again, on a larger scale. Since this was a single case study with an intact class and the majority of this study was based on qualitative data, it makes this study incapable of generalizing to a larger population. A multi-case study will provide the numbers needed to help generalize the study. In summary, conducting this study showed me the many opportunities going forward for continuing my work to improve online mathematics course components that foster online mathematical discourse.

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APPENDICES

APPENDIX A

Student Survey #1

(ARCS Model Survey & modified Fennema-Sherman Survey)

Instructions: Please read the following questions and choose the answer that best tells how you really feel. There is no “right” answer. The answers that you give will not be accessible to your instructor and will not affect your grade. Please select one choice between A-Strongly Agree, B- Somewhat Agree, C-Neither Agree, Nor Disagree, D-Somewhat Disagree, E-Strongly Disagree.

ARCS survey	Rating Scale for A, B, C, D, E
The things I learn in this course will be useful to me.	A=5, B=4, C=3, D=2, E=1
I feel confident that I will do well in this course.	A=5, B=4, C=3, D=2, E=1
This class will have many things in it that will capture my attention.	A=5, B=4, C=3, D=2, E=1
I expect that the amount of work I will have to do will be appropriate for this type of course.	A=5, B=4, C=3, D=2, E=1
I expect that the instructor will make the subject matter of this course seem important.	A=5, B=4, C=3, D=2, E=1
I expect that the instructor will use an interesting variety of teaching techniques.	A=5, B=4, C=3, D=2, E=1
I expect that a person has to be lucky to get good grades in this course.	A=1, B=2, C=3, D=4, E=5
I do NOT see how the content of this course will relate to anything I already know.	A=1, B=2, C=3, D=4, E=5
Whether or not I will succeed in this course is up to me.	A=5, B=4, C=3, D=2, E=1
I feel that the grades or other recognition I will receive will be fair compared to other students.	A=5, B=4, C=3, D=2, E=1
I expect that the instructor will do unusual or	A=5, B=4, C=3, D=2, E=1

surprising things that are interesting.	
To accomplish my goals, it is important that I do well in this course.	A=5, B=4, C=3, D=2, E=1
I expect to feel satisfied with what I will get from this course.	A=5, B=4, C=3, D=2, E=1
I expect that my curiosity will be stimulated by the questions asked or the assignments given on the subject matter in this class.	A=5, B=4, C=3, D=2, E=1
I expect to find the challenge level in this course to be about right: neither too easy nor too hard.	A=5, B=4, C=3, D=2, E=1
I feel that I will get enough recognition of my work in this course by means of grades, comments, or other feedback.	A=5, B=4, C=3, D=2, E=1

Attitude Towards Math Success	
I like math.	A=5, B=4, C=3, D=2, E=1
I'd be proud to be the outstanding math student.	A=5, B=4, C=3, D=2, E=1
I am happy to get good grades in math.	A=5, B=4, C=3, D=2, E=1
It would be great to win a prize in math.	A=5, B=4, C=3, D=2, E=1
Being first in a math competition would make me happy.	A=5, B=4, C=3, D=2, E=1
Being thought of as smart in math would be a great thing.	A=5, B=4, C=3, D=2, E=1
Winning a prize in math would make me feel embarrassed.	A=1, B=2, C=3, D=4, E=5
Other kids will think I'm weird if I get good grades in math.	A=1, B=2, C=3, D=4, E=5
If I get good grades in math, I would try to hide it.	A=1, B=2, C=3, D=4, E=5

If I got the highest grade in math, I'd prefer no one knew.	A=1, B=2, C=3, D=4, E=5
It would make kids like me less, if I were a really good math student.	A=1, B=2, C=3, D=4, E=5
I don't like people to think I'm smart in math.	A=1, B=2, C=3, D=4, E=5

Math Anxiety	
Math does not scare me at all.	A=5, B=4, C=3, D=2, E=1
It wouldn't bother me at all to take more math courses.	A=5, B=4, C=3, D=2, E=1
I don't usually worry about being able to solve math problems.	A=5, B=4, C=3, D=2, E=1
I almost never get nervous during a math test.	A=5, B=4, C=3, D=2, E=1
I am usually calm during math tests.	A=5, B=4, C=3, D=2, E=1
I am usually calm in math class.	A=5, B=4, C=3, D=2, E=1
Math usually makes me feel uncomfortable and nervous.	A=1, B=2, C=3, D=4, E=5
Math makes me feel uncomfortable, restless, irritable and impatient.	A=1, B=2, C=3, D=4, E=5
I get a sick feeling when I think of trying to do math problems.	A=1, B=2, C=3, D=4, E=5
My mind goes blank and I am unable to think clearly when working math problems.	A=1, B=2, C=3, D=4, E=5
A math test scares me.	A=1, B=2, C=3, D=4, E=5
Math makes me feel uneasy, confused, and nervous.	A=1, B=2, C=3, D=4, E=5

Motivation	

I like math puzzles.	A=5, B=4, C=3, D=2, E=1
Math is enjoyable to me.	A=5, B=4, C=3, D=2, E=1
When a math problem comes up that I cannot solve right away, I stick with it until I find the solution.	A=5, B=4, C=3, D=2, E=1
Once I start working on a math puzzle, it is hard to stop.	A=5, B=4, C=3, D=2, E=1
When I have a question that doesn’t get answered in math class, I keep thinking about it.	A=5, B=4, C=3, D=2, E=1
I am challenged by math problems I cannot understand right away.	A=1, B=2, C=3, D=4, E=5
Figuring out math problems is not something I like to do.	A=1, B=2, C=3, D=4, E=5
The challenge of math problems does not appeal to me.	A=1, B=2, C=3, D=4, E=5
Math puzzles are boring. I do not understand how some people can spend so much time on math and seem to like it.	A=1, B=2, C=3, D=4, E=5
I would rather have someone else figure out a tough math problem than to have to work it out myself.	A=1, B=2, C=3, D=4, E=5
I do as little work in math as possible.	A=1, B=2, C=3, D=4, E=5

Usefulness	
I’ll need math for my career.	A=5, B=4, C=3, D=2, E=1
I study math because I know how useful it is.	A=5, B=4, C=3, D=2, E=1
Knowing math will help me earn a living.	A=5, B=4, C=3, D=2, E=1
Math is an important and useful subject.	A=5, B=4, C=3, D=2, E=1

I need to master math for my future work.	A=5, B=4, C=3, D=2, E=1
I will use math in many ways.	A=5, B=4, C=3, D=2, E=1
Math is not important in my life.	A=1, B=2, C=3, D=4, E=5
Math will not be important in my life's work.	A=1, B=2, C=3, D=4, E=5
I see math as a subject that I won't use very much in daily life.	A=1, B=2, C=3, D=4, E=5
Taking math is a waste of time.	A=1, B=2, C=3, D=4, E=5
It's not important for me to do well in math.	A=1, B=2, C=3, D=4, E=5
I expect to have little use for math when I get out of school.	A=1, B=2, C=3, D=4, E=5

Confidence in learning mathematics	
I feel confident in trying math.	A=5, B=4, C=3, D=2, E=1
I am sure that I could do advanced work in math.	A=5, B=4, C=3, D=2, E=1
I am sure that I can learn math.	A=5, B=4, C=3, D=2, E=1
I think I could handle more difficult math.	A=5, B=4, C=3, D=2, E=1
I can get good grades in math.	A=5, B=4, C=3, D=2, E=1
I have a lot of self-confidence when it comes to math.	A=5, B=4, C=3, D=2, E=1
I am no good at math.	A=1, B=2, C=3, D=4, E=5
I do not think I could do advanced math.	A=1, B=2, C=3, D=4, E=5
I am not the type to do well in math.	A=1, B=2, C=3, D=4, E=5
For some reason, even though I study, math is really hard for me.	A=1, B=2, C=3, D=4, E=5
I do fine in most subjects, but when it comes to	A=1, B=2, C=3, D=4, E=5

math I really mess up.	
Math is my worst subject.	A=1, B=2, C=3, D=4, E=5

Personal Information

Gender, Choose MALE or FEMALE.	A=male, B=female
What is your age?	
Top of Form What is your major?	
Would you like to volunteer for an online interview? It is optional and is not a required for you to receive the incentives. If you would like to volunteer for an interview click YES or NO, then enter your email address on the last item. If not, type DECLINE in the last item.	A=yes, B=no
Type your preferred email address for the researcher to contact you for an interview after classes have ended. (This survey has been reviewed and approved by the GPC Office of Institutional Research and Planning in accordance with 45 CFR 46 (Protection of Human Subjects).	

APPENDIX B

Interview Protocol (Semi-structured)

1. How would you describe your experience with participating on the discussion boards this semester?
2. What role do you think the discussion boards played in your online math course?
3. What was your favorite discussion question this semester? Why?
4. What was your least favorite discussion question this semester? Why?
5. Overall, did discussion board questions provoke your curiosity in terms of the mathematics, and, if so, can you give me an example of a question that was interesting to you that did spark your curiosity?
6. Overall, were discussion board questions difficult or easy, did you think the questions were doable or too challenging and can you give an example of a question that you thought was too difficult or easy?
7. How would you perceive your classmates participation? Were they helpful to you? If so, in what ways, and if not, in what ways do you wish it would have been more helpful?
8. In what ways did interacting on the discussion boards contribute to your learning this semester?
9. In what ways do you think interacting on the discussion boards hindered your learning this semester?
10. Do you wish the discussion boards were more interactive? If so, elaborate and tell me why.

11. Do you think the discussion questions were more relevant to your studies, personal life, career goal or anything else? If so, elaborate and tell me why you think so.
12. What do you think would have made you participate more on the discussion board?

APPENDIX C

Modified Fennema-Sherman Survey

Student Survey #2

Instructions: Please read the following questions and choose the answer that best tells how you really feel. There is no "right" answer. The answers that you give will not be accessible to your instructor and will not affect your grade. Please select one choice between A-Strongly Agree, B- Somewhat Agree, C-Neither Agree, Nor Disagree, D-Somewhat Disagree, E-Strongly Disagree.

Attitude Towards Math Success	
1. I like math.	A=5, B=4, C=3, D=2, E=1
2. I'd be proud to be the outstanding math student.	A=5, B=4, C=3, D=2, E=1
3. I am happy to get good grades in math.	A=5, B=4, C=3, D=2, E=1
4. It would be great to win a prize in math.	A=5, B=4, C=3, D=2, E=1
5. Being first in a math competition would make me happy.	A=5, B=4, C=3, D=2, E=1
6. Being thought of as smart in math would be a great thing.	A=5, B=4, C=3, D=2, E=1
7. Winning a prize in math would make me feel embarrassed.	**A=1, B=2, C=3, D=4, E=5**
8. Other kids will think I'm weird if I get good grades in math.	**A=1, B=2, C=3, D=4, E=5**
9. If I get good grades in math, I would try to hide it.	**A=1, B=2, C=3, D=4, E=5**
10. If I got the highest grade in math, I'd prefer no one knew.	**A=1, B=2, C=3, D=4, E=5**
11. It would make kids like me less, if I were a really good math student.	**A=1, B=2, C=3, D=4, E=5**

12. I don't like people to think I'm smart in math.	**A=1, B=2, C=3, D=4, E=5**

Math Anxiety	
13. Math does not scare me at all.	A=5, B=4, C=3, D=2, E=1
14. It wouldn't bother me at all to take more math courses.	A=5, B=4, C=3, D=2, E=1
15. I don't usually worry about being able to solve math problems.	A=5, B=4, C=3, D=2, E=1
16. I almost never get nervous during a math test.	A=5, B=4, C=3, D=2, E=1
17. I am usually calm during math tests.	A=5, B=4, C=3, D=2, E=1
18. I am usually calm in math class.	A=5, B=4, C=3, D=2, E=1
19. Math usually makes me feel uncomfortable and nervous.	**A=1, B=2, C=3, D=4, E=5**
20. Math makes me feel uncomfortable, restless, irritable and impatient.	**A=1, B=2, C=3, D=4, E=5**
21. I get a sick feeling when I think of trying to do math problems.	**A=1, B=2, C=3, D=4, E=5**
22. My mind goes blank and I am unable to think clearly when working math problems.	**A=1, B=2, C=3, D=4, E=5**
23. A math test scares me.	**A=1, B=2, C=3, D=4, E=5**
24. Math makes me feel uneasy, confused, and nervous.	**A=1, B=2, C=3, D=4, E=5**

Motivation	
25. I like math puzzles.	A=5, B=4, C=3, D=2, E=1
26. Math is enjoyable to me.	A=5, B=4, C=3, D=2, E=1
27. When a math problem comes up that I cannot solve right away, I stick with it until I find the solution.	A=5, B=4, C=3, D=2, E=1
28. Once I start working on a math puzzle, it is hard to stop.	A=5, B=4, C=3, D=2, E=1
29. When I have a question that doesn't get answered in math class, I keep thinking about it.	A=5, B=4, C=3, D=2, E=1
30. I am challenged by math problems I cannot understand right away.	**A=1, B=2, C=3, D=4, E=5**
31. Figuring out math problems is not something I like to do.	**A=1, B=2, C=3, D=4, E=5**
32. The challenge of math problems does not appeal to me.	**A=1, B=2, C=3, D=4, E=5**

33. Math puzzles are boring. I do not understand how some people can spend so much time on math and seem to like it.	**A=1, B=2, C=3, D=4, E=5**
34. I would rather have someone else figure out a tough math problem than to have to work it out myself.	**A=1, B=2, C=3, D=4, E=5**
35. I do as little work in math as possible.	**A=1, B=2, C=3, D=4, E=5**

Usefulness	
36. I'll need math for my career.	A=5, B=4, C=3, D=2, E=1
37. I study math because I know how useful it is.	A=5, B=4, C=3, D=2, E=1
38. Knowing math will help me earn a living.	A=5, B=4, C=3, D=2, E=1
39. Math is an important and useful subject.	A=5, B=4, C=3, D=2, E=1
40. I need to master math for my future work.	A=5, B=4, C=3, D=2, E=1
41. I will use math in many ways.	A=5, B=4, C=3, D=2, E=1
42. Math is not important in my life.	**A=1, B=2, C=3, D=4, E=5**
43. Math will not be important in my life's work.	**A=1, B=2, C=3, D=4, E=5**
44. I see math as a subject that I won't use very much in daily life.	**A=1, B=2, C=3, D=4, E=5**
45. Taking math is a waste of time.	**A=1, B=2, C=3, D=4, E=5**
46. It's not important for me to do well in math.	**A=1, B=2, C=3, D=4, E=5**
47. I expect to have little use for math when I get out of school.	**A=1, B=2, C=3, D=4, E=5**

Confidence in learning mathematics	
48. I feel confident in trying math.	A=5, B=4, C=3, D=2, E=1
49. I am sure that I could do advanced work in math.	A=5, B=4, C=3, D=2, E=1
50. I am sure that I can learn math.	A=5, B=4, C=3, D=2, E=1
51. I think I could handle more difficult math.	A=5, B=4, C=3, D=2, E=1
52. I can get good grades in math.	A=5, B=4, C=3, D=2, E=1
53. I have a lot of self-confidence when it comes to math.	A=5, B=4, C=3, D=2, E=1
54. I am no good at math.	**A=1, B=2, C=3, D=4, E=5**
55. I do not think I could do advanced math.	**A=1, B=2, C=3, D=4, E=5**

56. I am not the type to do well in math.	**A=1, B=2, C=3, D=4, E=5**
57. For some reason, even though I study, math is really hard for me.	**A=1, B=2, C=3, D=4, E=5**
58. I do fine in most subjects, but when it comes to math I really mess up.	**A=1, B=2, C=3, D=4, E=5**
59. Math is my worst subject.	**A=1, B=2, C=3, D=4, E=5**

APPENDIX D

Interaction Analysis Model

PHASE I: SHARING/COMPARING OF INFORMATON.	
A. A statement of observation or opinion	[PhI/A]
B. A statement of agreement from one or more other participants	[PhI/B]
C. Corroborating examples provided by one or more participants	[PhI/C]
D. Asking and answering questions to clarify details of statements	[PhI/D]
E. Definition, description, or identification of a problem	[PhI/E]
PHASE II: THE DISCOVERY AND EXPLORATION OF DISSONANCE OR INCONSISTENCY AMONG IDEAS, CONCEPTS OR STAEMENTS.	
A. Identifying and stating areas of disagreement	[PhII/A]
B. Asking and answering questions to clarify the source and extent of disagreement	[PhII/B]
C. Restating the participants position, and possibly advancing arguments or considerations in its support by references to the participant's experience, literature, formal data collected, or proposal of relevant metaphor or analogy to illustrate point of view	[PhII/C]
PHASE III: NEGOTIATION OF MEANINIG/CO-CONSTRUCTION OF KNOWLEDGE	
A. Negotiation of clarification of the meaning of terms	[PhIII/A]
B. Negotiation of the relative weight to be assigned to types of argument	[PhIII/B]
C. Identification of areas of agreement or overlap among conflicting concepts	[PhIII/C]
D. Proposal and negotiation of new statements embodying compromise, co-construction	[PhIII/D]
E. Proposal of integrating or accommodating metaphors or analogies	[PhIII/E]
PHASE IV: TESTING AND MODIFICATION OF PROPOSED SYNTHESIS OR CO-CONSTRUCTION	
A. Testing the proposed synthesis against "received fact" as shared by the participants and/or their culture.	[PhIV/A]
B. Testing against existing cognitive schema	[PhIV/B]
C. Testing against personal experience	[PhIV/C]
D. Testing against formal data collected	[PhIV/D]
E. Testing against contradictory testimony in the literature	[PhIV/E]
PHASE V: AGREEMENT STATEMENT(S)/APPLICATIONS OF NEWLY-CONSTRUCTED MEANING	
A. Summarization of agreement(x)	[PhV/A]
B. Applications of new knowledge	[PhV/B]
C. Metacognitive statements by the participants illustrating their understanding that their knowledge or ways of thinking (cognitive schema) have changed as a result of the conference interaction	[PhV/C]

(Gunawardena, Lowe, & Anderson, 1997, p. 414)

APPENDIX E

Overview of participants' research study activities.

					Participation Weeks 1-7			Participation Weeks 8-14			
Student #	Consent Form	Survey 1	Age	Major	Total # of posts per week	Participated in Discussion Activity	Survey 2	Total # of posts per week	Final Exam	Final Grade	Interview
829	X	X	26	Biology	6-7-0-6-5-4-6	X	X	3-3-5-3-3-3-3		F	
673	X	X	31	Social Work	6-5-5-3-6-5-5	X	X	2-3-3-2-0-3-3	X	C	
669	X	X	20	Finance	0-0-3-0-0-0-0		X	0-0-0-0-0-0-0		F	
572	X	X	40	Social Work	6-4-6-4-0-0-0	X		0-0-0-0-0-0-0		W	
583	X	X	40	Psychology	7-6-7-7-7-6-6	X	X	6-5-3-5-3-4-5	X	B	X
136	X	X	25	Nursing	5-4-0-0-0-0-0			0-0-0-0-0-0-0		WF	
712	X	X	25	Radiology Tech	4-7-5-4-4-6-4	X	X	4-4-1-3-1-3-4	X	F	
820	X	X	30	Engineering	4-4-0-0-0-0-0			0-0-0-0-0-0-0		F	
358	X	X	29	Pre-medicine	6-4-5-4-4-5-0	X	X	1-4-3-1-1-3-4	X	B	
759	X	X	32	Business Admin	7-6-3-5-5-5-5	X	X	3-3-2-3-3-3-3	X	B	X
859	X	X	24	Art	5-7-6-6-5-7-4	X	X	1-7-5-2-4-5-7	X	B	
927	X	X	54	Business Admin	5-6-6-11-4-6-5	X	X	5-5-3-5-4-5-3	X	B	X
929	X	X	30	Nursing	6-7-5-7-6-7-6	X	X	5-7-6-7-7-6-7	X	B	
220	X	X	35	Nursing	0-7-5-4-0-6-0	X		5-1-0-0-0-0-0		F	
256	X	X	35	Elementary Ed	5-4-4-6-4-5-5	X	X	3-4-5-4-6-5-3	X	B	
901	X	X	27	Computer Sci	6-5-5-8-5-6-5	X	X	3-3-3-3-5-3-3	X	A	X
882	X	X	39	Criminal Justice	5-5-6-0-5-5-6		X	2-3-4-0-0-1-0	X	D	X
105	X		-	-	0-0-0-0-0-0-0			0-0-0-0-0-0-0		-	
438	X	X	20	Radiology Tech	0-0-0-0-0-0-0			1-1-1-1-1-1-1	X	C	
100	X		-	-	0-0-0-0-0-0-0			0-0-0-0-0-0-0		-	
758	X	X	31	Business Admin	4-6-0-0-0-0-0		X	0-0-1-1-3-3-4		F	

Note. The X indicates that the student participated in that event or took that particular survey.