CHARACTERIZATION AND CALIBRATION

OF AN

ARBITRARILY SHAPED PERMITTIVITY MEASUREMENT PROBE

by

PHILIP G. BARTLEY, JR.

(Under the Direction of Stuart O. Nelson)

ABSTRACT

Permittivity is a fundamental property of material. It is one of the properties that determine how a material interacts with an electromagnetic field. A material has a particular permittivity because of its molecular structure. Researchers have demonstrated that material properties such as moisture content, fruit ripeness, bacterial content, mechanical stress, biomass, tissue health and other seemingly unrelated parameters can be related to the permittivity. Many key parameters of colloids such as structure, consistency and concentration are directly related to permittivity. This is because a change in molecular structure results in a corresponding change in permittivity. Since it is possible to measure permittivity in real-time, the potential exists for measuring other material properties in real-time. There are difficulties in realizing this potential. The permittivity of many materials of interest to agricultural and biological engineers and scientists cannot be measured easily by classical permittivity measurement techniques. Most of these techniques require precise dimensioning of the material sample. This is not practical for many materials of interest. This research addresses the challenge of designing a permittivity measurement probe that is both sensitive to changes in permittivity and has a form factor suitable to its intended application.
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DEDICATION

This work is dedicated to my loving wife Evelyn. Without her continuous encouragement, I would have never started. Without her unwavering support, I would have never finished.
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CHAPTER 1
INTRODUCTION AND OVERVIEW

The permittivity (dielectric properties) of a material is one of the factors that determine how the material interacts with an electromagnetic field. It is a complex quantity consisting of a real part (dielectric constant) and an imaginary part (loss factor). Its dimensions are farad per meter (capacitance per distance). The knowledge of the dielectric properties of materials and their frequency and temperature dependence is of great importance in various areas of science and engineering in both basic and applied research. It has always been an important quantity to electrical engineers and physicists involved in the design and application of circuit components. Over the past several decades the knowledge of permittivity has become an important property to scientists and engineers involved in the design of stealth vehicles. These applications are most often associated with the defense industry. Over the last half century, this knowledge has become increasingly important to agricultural engineers, biological engineers and food scientists. The most obvious application of this knowledge is in microwave and RF heating of food products. Here the knowledge of the dielectric properties is important in determining how long a food item needs to be exposed to the RF or microwave energy for proper cooking. For prepackaged food items, the knowledge of the dielectric properties of the packaging materials is also important. The interaction with the packaging material also determines the cooking time. Besides these obvious applications there are also numerous not-so-obvious applications. Dielectric properties can often be related to a physical parameter of interest [1]. This is because a material has a particular permittivity because of its molecular structure. A change in the molecular structure or composition of a material results in a change in its permittivity.
It has been demonstrated that material properties such as moisture content [2], fruit ripeness [3], bacterial content [4], mechanical stress [5], tissue health [6] and other seemingly unrelated parameters [7] are related to the dielectric properties or permittivity of the material. Many key parameters of colloids such as structure, consistency and concentration are directly related to the dielectric properties [8]. Yeast concentration in a fermentation process [9], bacterial count in milk [10], and the detection and monitoring of microorganisms [11] are a few examples on which research has been performed. Accurate measurements of these properties can provide scientists and engineers with valuable information that allows them to properly use the material in its intended application or to monitor a process for improved quality control [1].

The permittivity is a fundamental property of the material and is independent of the measurement technique. This is important because once a relationship between the physical parameter of interest and permittivity is established, the result is independent of the measurement method used. Measurement techniques typically involve placing the material in an appropriate sample holder and determining the permittivity from measurements made on the sample holder [12][13]. The sample holder can be a parallel plate or coaxial capacitor, a resonant cavity or a transmission line. These structures are used because the relationship between the permittivity and measurements are fundamental and well understood. One disadvantage of these types of sample holders is that many materials cannot be easily placed in them. Sample preparation is almost always required. This limits their use in real-time monitoring of processes. Another disadvantage is that several of these sample holders are usable only over a narrow frequency range. Extracting physical properties from permittivity measurements often requires measurements made over a wide frequency range [14].

Techniques for which this relationship, between permittivity and measurements, is not as straightforward have also been employed. One of these techniques is the open-ended coaxial-
line probe [15]. This technique has attracted much attention because of its applicability to nondestructive testing over a relatively broad frequency range. It can be used to measure a wide variety of materials including liquids, solids and semisolids. These attributes make it a very attractive technique for measuring biological, agriculture and food materials. In its simplest form, it consists of a coaxial cable without a connector attached to one end. This end is inserted into the material being measured. Figure 1.1 illustrates such a probe.

![Open-ended coaxial-line probe](image)

**Figure 1.1. Open-ended coaxial-line probe.**

Current techniques for using this type of probe to make permittivity measurements require that the interactions between the probe and the terminating dielectric material are fully understood. This is a complex relationship that has consumed many man-years of effort without yielding an exact closed-form solution [15]-[40]. Even with this complexity, the open-ended coaxial probe is studied because its simple geometry makes it easier to analyze than other possibly more useful configurations. These other probes are not considered because of the added analytical complexity due to a more complex shape. Eliminating this restriction was one of the goals of this research. One example is a coaxial-line probe that is cut at an angle. This shape would be useful in inserting the probe in soft materials. Another is a probe without a flange at the terminating end. A ground-plane flange is often added to the end of the open-ended coaxial-
line probe to simplify the analysis of the problem. This addition allows the assumption of an infinite ground plane, which greatly simplifies the analytical problem. Even with these simplifying assumptions, this is a complex relationship. The flangeless design has several advantages when measuring permittivity of liquids and semisolids. The absence of the flange eliminates the accumulation of gas bubbles on the probe surface. This is often a problem. A probe without a flange can also be inserted into a semisolid. This research deals with a new and innovative way of modeling the open-ended coaxial-line probe and extending this technique to other often more useful probe structures. The goal is to provide a method of characterizing a broadband, nondestructive, permittivity measurement probe of arbitrary geometry. This would allow probes to be developed that are more sensitive to changes in permittivity and better suited to a given application.

The next chapter, Literature Review, summarizes the most important permittivity measurement techniques in use. Their applicability to agricultural and biological materials is considered along with their strengths and weaknesses. One conclusion of this review is that the open-ended coaxial-line probe has the best potential for providing broadband measurement capability. The challenge of designing an effective permittivity measurement probe is threefold. The probe must be sensitive to changes in permittivity. Its size and form factor must be such that is can be used in a given application. Most importantly, the relationship between the measurement made on the probe and the permittivity of the material in contact with the probe must be known.

Chapter 3, Measurement Fundamentals, gives an overview of the instrumentation required to make the measurement. This includes a discussion of measurement calibration. Measurement calibration, as it turns out, becomes a major obstacle in modeling a prospective permittivity measurement probe. If accurate measurements can be made on a probe, when the
probe is in contact with the materials, the relationship can be established by fitting the data to an appropriate function. The primary obstacle is that the relationship between the probe and terminating material must be known before a measurement calibration can be performed. Because of these difficulties several artificial intelligence techniques were investigated.

Chapters 4, 5, 6 and 7 outline the research areas considered. These chapters are papers published or submitted for publication. These papers contain the bulk of the results of the research. Chapter 4 investigates use of an artificial neural network to establish a relationship between measurements made on a probe and material permittivity. Chapter 5 reports the use of a fuzzy logic inference system to establish the relationship. While both of these techniques were highly effective in modeling the probe, both had disadvantages. These are discussed in Chapter 10. Chapter 6 documents the results of performing a dimensional analysis of a generalized permittivity measurement probe. The Buckingham Π-theorem was used to analyze the result. The analysis greatly simplified the problem of modeling a probe. The Buckingham Π-theorem was originally proposed in the early part of the twentieth century and has long been used to analyze and model structural/mechanical/fluid problems. Chapter 7 outlines the use of dimensional analysis and a genetic algorithm to search for an appropriate model for a given probe. Several improvements to the technique established in Chapter 7 are suggested in Chapter 8. Chapter 9 gives an example of implementing the techniques of Chapters 7 and 8. It contains a step by step procedure for modeling an arbitrarily shaped permittivity measurement probe. The summary and accomplishments of the research are outlined in Chapter 10. Appendix A contains citations of the author’s published papers that were associated with this research. Appendix B defines the nomenclature used.
CHAPTER 2
LITERATURE REVIEW

This Chapter reviews the techniques used to measure the permittivity of materials. Bussey [12] and Afsar et al. [13] published review papers on the measurement techniques. These papers along with a literature search of the IEEE Transactions on Microwave Theory and Techniques and the IEEE Transactions on Instrumentation and Measurement over the past several decades formed the technical basis for this research. The permittivity techniques described in these references fall into four general categories. They are capacitance techniques, transmission-line techniques, resonance techniques and open-ended transmission-line techniques. All of these techniques have been used with varying success in the measurement of agricultural and biological materials. The capacitance technique, transmission-line technique and resonant-structure method involve placing the material to be measured in an appropriate sample holder and determining the permittivity from measurements made on the sample holder. The sample holder can be a parallel-plate or coaxial capacitor, a resonant cavity or a transmission line. These structures are used because the relationship between the permittivity and the measurements are well understood. Many of these relationships are based on solving Maxwell’s equations. All textbooks on electromagnetics provide an explanation of Maxwell’s equations [41]. The solution to Maxwell’s equations determines how electromagnetic energy propagates through and interacts with a material. Maxwell’s equations can be expressed in many equivalent forms. The most general vector form is listed as follows.

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]  

(2.1)
\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \]  
(2.2)

\[ \nabla \cdot \mathbf{D} = \rho \]  
(2.3)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(2.4)

where \( \mathbf{J} \) is the current density, \( \mathbf{E} \) is the electric field intensity, \( \mathbf{D} \) is the electric flux density, \( \mathbf{H} \) is the magnetic field intensity, \( \mathbf{B} \) is the magnetic flux density, and \( \rho \) is the charge density. These equations are based on Faraday’s law and Ampere’s law. These equations must always be satisfied. Other factors that make the solution unique to a particular situation are the boundary conditions and the satisfaction of the following constitutive relations:

\[ \mathbf{D} = \varepsilon \mathbf{E} \]  
(2.5)

\[ \mathbf{B} = \mu \mathbf{H} \]  
(2.6)

\[ \mathbf{J} = \sigma \mathbf{E} \]  
(2.7)

where \( \mu \), \( \varepsilon \), and \( \sigma \) are, respectively, the permeability, permittivity and conductivity of the media. These properties, taken collectively, are commonly referred to as the electromagnetic properties of the material. If all three parameters are unknown, three independent measurements must be made to solve for the unknowns. Most agricultural and biological materials are nonmagnetic. The consequence of this is that \( \mu \) is known. At sufficiently high frequencies, the ionic contribution to \( \sigma \) can be ignored [41]. Dipolar losses contributing to \( \sigma \) vary inversely with frequency. At very high frequencies, say above a few GHz, the contribution of ionic conduction is essentially zero. This is true for all but highly conductive materials such as metals. At lower frequencies, the losses due to ionic conduction and dipole relaxation are indistinguishable. For this reason, losses from both mechanisms are combined in the loss factor.

The disadvantage of most types of sample holders is that many materials cannot be easily placed in them. In particular, they are not well suited for many biological and agricultural
materials. Because of this they are not considered as a starting point for the development of a broadband, nondestructive permittivity measurement probe. They are listed here for completeness.

There are two types of open-ended line-techniques. The open-ended waveguide probe and the open-ended coaxial-line probe. Both have attracted attention because of their relative ease of use and applicability to nondestructive testing. Of the two, the open-ended coaxial-line probe is the only one capable of making permittivity measurements over a relatively broad frequency range. Unlike the other techniques, the interaction between the probe and the terminating dielectric material is not as well understood. Analyzing this structure with Maxwell’s equations is tedious, and no useful closed form solution exists. Often assumptions are made to simply the problem. These assumptions affect the accuracy of the relationship. Because this technique comes closest to meeting the desired characteristics of being both broadband and nondestructive, it is considered as a starting point for the development of a model for an arbitrarily shaped permittivity probe. The shape of this permittivity measurement probe should be determined by the application and not by the simplicity of analysis. Because the open-ended coaxial-line probe is the closest to having the desired properties, a more extensive review of the literature on this technique was conducted.

**Capacitance Techniques**

The capacitance technique is classical in practice and conceptually the easiest to understand. It is based on calculating the permittivity of a sample of material placed between the conducting plates of a capacitor. Figure 2.1 is a picture of a fixture used to make this measurement. The technique assumes that the capacitor is a lumped element. The steady-state admittance of a capacitor is given as

\[ Y = j\omega C \]  \hspace{1cm} (2.8)
where $C$ is the capacitance of the capacitor in Farads, $j$ is the square root of $-1$ and $\omega$ is the angular frequency expressed in radians per second. The capacitance can be determined as

$$C = \frac{\varepsilon A}{d}$$

(2.9)

where $A$ is the area of one of the parallel plates, $d$ is the distance between the plates and $\varepsilon = \varepsilon' - j \varepsilon''$ is the permittivity of the material between the plates. These two equations, 2.8 and 2.9, can be used to calculate $\varepsilon$, given the other parameters. If the size of the fixture is kept small, relative to the wavelength of the measurement frequency, the technique can be used to approximately 200 MHz [42]. Von Hippel [43], Bussey [12] and Afsar el al. [13] describe this measurement in more detail. Nelson et al. [44] and others [45]-[48] successfully used this technique to measure the dielectric properties of agricultural materials. All utilized coaxial
capacitance sample holders. Saha and Williams [49] used the capacitance technique to measure the properties of biological materials. The major disadvantage is that the technique often requires sample preparation and therefore is destructive. It also suffers from a limited usable frequency range.

Resonance Techniques

Resonance techniques involve measuring the shift in resonant frequency and Q factor of a resonant structure caused by placing a material sample in the structure. These structures can be a cavity, a microstrip line or ring resonator [50]. Figure 2.2 is a picture of a resonant fixture constructed of a waveguide section and two end plates with coupling irises.

![Figure 2.2. Resonant cavity measurement fixture.](image)

Bussey [12] and Afsar et al. [13] describe this technique and other resonance techniques in detail. This technique is essentially a single frequency technique and does not satisfy the requirement of being a broadband technique. It is mentioned here for completeness and the fact that it has been used successfully in measuring the dielectric properties of biological materials. Nelson et al. [51] and Kraszewski and Nelson [52] report applications of this technique. Kraszewski and Nelson also showed that this technique could be used to detect differences in volume or mass among objects of irregular shape but of uniform permittivity [53].
Transmission-Line Techniques

With transmission-line techniques, material properties are calculated from the measurement of reflections from material samples and/or transmission through material samples. These reflection and transmission coefficients are typically represented as scattering or S-parameters [54]. A network analyzer measures S-parameters. The relationships between the measured S-parameters and the material properties are given as

\[ S_{21} = S_{12} = \frac{T (1 - \Gamma)}{1 - \Gamma^2 T^2} \]  

(2.10)

\[ S_{11} = S_{22} = \frac{\Gamma (1 - T^2)}{1 - \Gamma^2 T^2} \]  

(2.11)

where the reflection coefficient is

\[ \Gamma = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r + 1}} \]  

(2.12)

and the transmission coefficient is

\[ T = e^{-j\frac{\omega d}{c}\sqrt{\mu_r \varepsilon_r}} \]  

(2.13)

where \( \omega \) is the angular frequency of the stimulus, \( \varepsilon_r \) is the relative permittivity of the measurement material, \( \mu_r \) is the relative permeability of the measurement material, \( d \) is the material thickness and \( c \) is the speed of light. The permittivity and permeability can be calculated from the S-parameters, the frequency of measurement and the thickness of the material [55], [56]. If the material is nonmagnetic, only one of these relationships is required [56].

This technique can be subdivided into three types, coaxial-transmission-line, waveguide-transmission-line and free-space. The coaxial-transmission-line technique represents the one
with the broadest frequency range. Measurements can be routinely made from less than 1 MHz to over 26 GHz. Figure 2.3 shows a material sample placed in the coaxial sample holder.

Figure 2.3. Coaxial-line sample holder.

The major disadvantage of this technique is that the material sample has to be shaped into a form of a tube. Not only is this usually destructive, many types of materials do not lend themselves to being formed into such a sample. For solid samples, even small air gaps between the sample and the conductors can cause serious errors.

Figure 2.4 shows a waveguide sample and sample holder. With waveguide, the sample is formed into the shape of a rectangular block. The waveguide has an inherent disadvantage of having a relatively narrow bandwidth. As an example, the frequency range for X-band waveguide is approximately 8.2 to 12.4 GHz.

Free-space techniques involve placing the sample between two antennas. Figure 2.5 illustrates such an arrangement. The limitation of the free-space technique is the bandwidth of
the antennas being used. The sample size also has to be larger than the beam-width of the antenna.
Transmission-line techniques have been successfully implemented by Nelson [42], King et al. [57], Kraszewski et al. [58] and others to measure the dielectric properties of biological and agricultural materials. None of these techniques appear to have the potential to make very broadband, nondestructive permittivity measurements.

Open-Ended Waveguide Technique

The open-ended waveguide technique, like the open-ended coaxial-line technique, is a special case of the transmission line technique. With each, the material sample is placed at the end of an unterminated transmission line. The appeal of the open-ended waveguide probe is that the relationship between the measurement and the permittivity of the terminating material is known [59]. One disadvantage is that the measurement is frequency-limited to the waveguide band being used. Another is that the open-ended waveguide acts as an antenna. Because it radiates energy the assumption of an infinite sample volume must be made. This is not a problem as long as the sample has a non-zero loss factor. It is also possible to assume a conductive backing to the sample, which allows finite sample thickness to be considered [59].

Open-Ended Coaxial-Line Technique

This section deals with the open-ended coaxial-line probe. This technique has promise in providing a convenient means of measuring the permittivity of biological and agricultural materials. Blackham [19] provides an excellent overview of this topic. It provides a nondestructive means of determining the permittivity of many materials over a relatively wide frequency range. Because of this, much attention has been given to understanding the interaction of the probe and the material to be measured. Several models have been proposed. Another advantage is that, if properly designed, the probe does not radiate energy, so the measurement volume is finite, unlike the open-ended waveguide. Sheen and Woodhead [23] have reported applications of the open-ended coaxial line probe in agriculture.
The models for the open-ended coaxial line probe range from simple lumped circuit models to models based on Maxwell’s equations to models based on artificial intelligence techniques. The subsections below review the work reported in each of these areas.

**Lumped-Circuit Models**

Figure 2.6 illustrates the lumped-circuit probe model proposed by Burdette et al. [24].

![Figure 2.6. Burdette’s open-ended coaxial-line probe equivalent circuit model.](image)

The model is used to explain the interaction between the end of the probe and the material in contact with the probe. This model is based on the impedance of a short monopole antenna. In the limiting case, where the length is zero, the impedance is given by Equation 2.14.

\[
Z(\omega, \varepsilon_0) = A\omega^2 + \frac{1}{j\omega C}
\]  

(2.14)

\(A\) and \(C\) are constants that depends on the physical dimensions of the probe. Burdette et al. [24] applied Deschamps [25] theorem. This allowed him to expand Equation 2.14 to include materials with permittivity other than that of free space. The result is given by Equations 2.15 and 2.16.

\[
Z(\omega, \varepsilon_0) = \sqrt{\varepsilon_0} Z(\sqrt{\varepsilon}, \varepsilon_0)
\]  

(2.15)

\[
Z(\omega, \varepsilon_0) = A\omega^2 \sqrt{\varepsilon} + \frac{1}{j\omega C \sqrt{\varepsilon}}
\]  

(2.16)
Deschamps theorem is only approximately valid because it does not account for the effects of higher-order electromagnetic modes that may exist at the probe aperture. It also does not take into account the power radiated from the zero-length monopole antennas. This power is given by Equation 2.17.

\[
P_r = \frac{4\pi V^2}{3} \frac{\varepsilon}{\mu} \left[ \frac{\pi^2 (b^2 - a^2)}{\lambda^2 \log(b/a)} \right]
\]

where \( a \) is the diameter of the inter conductor, \( b \) is the diameter of the outer conductor, \( \lambda \) is the wavelength of the stimulus, \( \varepsilon \) is the permittivity of the measurement material, \( \mu \) is the permeability of the measurement material and \( V \) is the applied voltage. Burdette et al. [24] chose to minimize this radiation by proper selection of outer and inner conductor dimensions.

Athey et al. [60] provided many improvements to the lumped parameter model. One of their improvements is shown in Figure 2.7.

**Figure 2.7. Athey’s open-ended coaxial-line probe equivalent circuit model.**

This model splits the fringing capacitance into two components. One that accounts for the portion of the fringing field that penetrates the material being measured, \( C_f \), and another that accounts for the portion of the fringing field that exists only in the dielectric material of the coaxial line. Based on this model, the total capacitance can be calculated as

\[
C(\varepsilon) = C_0\varepsilon + C_f
\]

(2.18)
This results in a reflection coefficient at the aperture/material interface given as

\[
\Gamma = \frac{I - j\omega Z_0 \left(C_0 \varepsilon + C_f\right)}{I + j\omega Z_0 \left(C_0 \varepsilon + C_f\right)}
\]  

(2.19)

where \(C_0\) and \(C_f\) are constants that can be determined by the measurement of known materials.

\(Z_0\) is the characteristic impedance in the coaxial region. Solving (2.19) for \(\varepsilon\) yields

\[
\varepsilon = \frac{I - \Gamma}{j\omega Z_0 C_0 \left(1 + \Gamma\right)} - \frac{C_f}{C_0}
\]  

(2.20)

Kraszewski and Stuchly [26] measured the total capacitance using a resonant technique. This yielded the following empirical expression:

\[
C_r = C_{r0} \left[1 + \omega \left(b - a\right) \varepsilon_0 \sqrt{\varepsilon_r}\right]
\]  

(2.21)

where \(a\) and \(b\) are the radii of the center and outer conductors respectively, \(\varepsilon_r\) is the dielectric constant of the material between the center and outer conductor and \(C_{r0}\) is the capacitance at low frequencies.

\[\text{Figure 2.8. Improved open-ended coaxial-line probe equivalent circuit model.}\]

Gajda and Stuchly [27] showed that the fringing capacitance, \(C_f\), is not independent of the material. The method of moments and finite-element calculations were used to obtain this result. These methods will be addressed in the Numerical Models section that follows this section. This
result suggested the model shown in Figure 2.8. Based on this circuit, the total capacitance at the probe/material interface can be determined as

\[
C(\varepsilon) = C_0 \varepsilon + C_f + \frac{C_s \varepsilon}{1 + \alpha C_s \varepsilon}
\]  

(2.22)

where \( \alpha \) is a constant. To extend the upper frequency range of this model, Stuchly et al. [28] added an additional term to account for radiation. This model results in the following expression for admittance at the probe/material interface:

\[
Y(\omega, \varepsilon) = j\omega C_f + j\omega C_0 \varepsilon + G_0 \varepsilon^5
\]  

(2.23)

Marsland and Evans [29] used this model for admittance to calibrate the probe/network analyzer to account for and correct systematic errors. Vector error correction of the probe/network analyzer system is described in Chapter 3. Grant et al. [30] showed that even with the addition of a radiation term, the model fails to cover the frequency range and range of permittivity the coaxial probe is capable of measuring. They used a point matching technique to prove this. Point-matching is an example of a technique that can be used to provide a numerical solution that satisfies Maxwell's equations for given geometry and boundary conditions. While this technique can be very precise, it is computationally intensive because of the large matrix size required for accurate solutions [61]. Because of this it has little utility in a measurement application.

**Numerical Models**

The point-matching technique mentioned in the previous section is a particular example of the moment method [62]. The moment method is a numerical technique often used to provide numerical solutions to RF, microwave and millimeter-wave passive structures [61]. Other numerical techniques commonly used include the finite-difference method [63], finite-element method [64]-[69], TLM method [73][74], mode-matching method [75], transverse resonance
technique [76], method of lines [77] generalized scattering matrix method [74], spectral domain method [69], equivalent waveguide method [70] and the planar circuit model [77]. Many numerical solutions for the open-ended coaxial-line probe have been published. In almost all of these, the finite-element technique was employed. In particular, the variational method was used. This form of finite-element analysis is often used when only one parameter is required. In the case of the open-ended coaxial-line probe, the required parameter is the probe impedance or admittance. A variational expression is required to use this method. Figure 2.9 illustrates the structure normally assumed for the probe.

![Figure 2.9. Geometry of open-ended coaxial-line probe with infinite ground plane.](image)

The ground plane is assumed to be infinite along with the assumption of a semi-infinite sample. The coaxial line excitation is assumed to be TEM. The discontinuity at the probe aperture can excite higher order TM modes, which also have no angular variation due to radial symmetry. Taking advantage of this symmetry, Baker-Jarvis et al. [31] derived a full-wave solution based on the Hankel transform. Their analysis yielded Equations 2.24 and 2.25 for the coaxial region (refer to Appendix B for nomenclature)

\[
H_{\phi(l)}(r, z) = \alpha \frac{e^{-jk_{l}(r)z}}{r} + \beta \frac{e^{jk_{l}(r)z}}{r} + \sum_{n=1}^{\infty} A_{n} R_{n}(r) e^{z\sqrt{k_{m}^{2}-k_{l}^{2}}} \tag{2.24}
\]

\[
E_{r(l)}(r, z) = \frac{C_{l}}{r} \left( e^{-jk_{l}(r)z} + \Gamma_{l} e^{jk_{l}(r)z} \right) + \sum_{n=1}^{\infty} \Gamma_{n} R_{n}(r) e^{z\sqrt{k_{m}^{2}-k_{l}^{2}}} \tag{2.25}
\]
where
\[ k_{(i)}^2 = \omega^2 \mu_{(i)} \varepsilon_{(i)} \] (2.26)

and
\[ R_n (r) = C_n \left[ J_1 (k_{cn}r) N_0 (k_{cn}a) J_0 - J_0 (k_{cn}a) N_1 (k_{cn}r) \right] \] (2.27)

\[ C_n = \frac{\pi k_{cn}}{\sqrt{2}} \frac{1}{\sqrt{J_0^2 (k_{cn}a) - 1}} \] (2.28)

The eigenvalues \((k_{cn})\) of the basis function are computed using
\[ J_0 (k_{nc}a) N_0 (k_{nc}b) - J_0 (k_{nc}b) N_0 (k_{nc}a) = 0 \] (2.29)

Levine and Papas [32] showed that in the open region the magnetic field is related to the tangential electric field in the aperture by Equation 2.30.
\[ H_r (r, z) = \frac{jk_{[2]}^2}{2 \pi \omega \mu_0} \int_0^b E_r (r', \theta) \int_0^{2\pi} e^{-jk_{[2]}R} r' \cos (\psi d) \psi d \theta d r' \] (2.30)

Several papers have been published based on these equations and the variational method. Mosig et al. [33] published results based on the method of moments where the magnetic field is matched at the aperture. The electric field in the aperture is obtained from Equation 2.25 and substituted into equation 2.30. Nevels et al. [34] used a point matching method-of-moments technique to solve for the fields in the aperture. Levine and Papas [32] reported a variational solution for the special case assuming free-space conditions. Hodgetts [35] presented the solution for the general case using an arbitrary permittivity. Marcuvitz [78] provided an integral expression for the complex admittance of the coaxial line radiating into a semi-infinite space. His results are given by Equations 2.31 and 2.32.
\[ \frac{G}{Y_0} = \frac{1}{\ln (b/a)} \int_{\theta}^{\pi/2} \left[ J_0 (k_0 a \sin (\theta)) - J_0 (k_0 b \sin (\theta)) \right] d \theta \] (2.31)
Wei and Sridhar [36] applied Deschamps theorem to Marcuvitz’s integral expression. Equations 2.33 and 2.34 show the result.

\[
\frac{B}{Y_o} = \frac{1}{\ln(b/a)} \int_{\theta_0}^{\pi} 2Si \left( k_o \sqrt{a^2 + b^2 - 2ab \cos(\phi)} \right) - Si \left( 2k_o \sin \left( \frac{\phi}{2} \right) \right) - Si \left( 2k_o b \sin \left( \frac{\phi}{2} \right) \right) d\phi
\]

(2.32)

2.33 and 2.34 show the result.

\[
\frac{G}{Y_o} = \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon} \ln(b/a)} \int_{0}^{\pi} \frac{J_0(\sqrt{\varepsilon} k_o \sin(\theta)) - J_0(\sqrt{\varepsilon} k_o b \sin(\theta))}{\sin(\theta)} d\theta
\]

(2.33)

Galejs [37] presented an equivalent expression of this integral by bringing the aperture integral equation into stationary form by multiplying it by the electric field and integrating over the radius. Galejs used only the principle mode in obtaining this expression. This integral expression was expanded by Misra [38] to include arbitrary permittivity. The result is given by

\[
\frac{Y_L}{Y_o} = \frac{jk^2}{\pi k \ln(b/a)} \int_{a}^{b} \int_{a}^{b} \int_{0}^{\pi} \cos(\phi) \frac{e^{-jkR}}{R} d\phi dr' dr
\]

(2.35)

where

\[
R = \sqrt{r^2 + r'^2 - 2rr' \cos(\phi)}
\]

(2.36)

While all the results referenced above are solutions, they are not suited for use in measurement systems. This is because of the computationally intensive algorithms required. This requirement is compounded by the requirement of measurement calibration discussed in Chapter 3. Misra represented Equation 2.35 as a Taylor's series. By doing this, the integration only has to be performed once for a given probe. He presented results using the first three terms.
of the expansion [38]. This expression was well suited to measurement applications, because it was computationally efficient and the inverse of the function could be easily computed. The requirement of calculating the inverse is discussed in Chapter 9. Blackham [19] extended Misra's results by computing the first 29 coefficients. He empirically adjusted these coefficients by using the probe to make measurements on materials with known permittivity. The reasoning was that this would take into account the higher-order modes that Galejs ignored. Stuchly et al. [39] and Anderson et al. [40] also reported a method that was useful in measurement applications. They described a rational function model that has the benefit of fast computation while maintaining the accuracy of the full-wave model. Smith and Nordgard [20] also reported results with a rational function as a basis for a model. Their analysis was performed on a probe shown in Figure 2.10.

![Figure 2.10. Geometry of a generalized open-ended coaxial-line probe with infinite ground plane.](image)

If $\varepsilon$ was large enough, a TEM wave existed at the interface of the two materials. Assuming this, they showed the following relationship was valid:

$$Z = \frac{1}{\sqrt{\varepsilon}} Z_N \left( \omega \sqrt{\varepsilon} \right)$$  \hspace{1cm} (2.37)

$Z$ is the impedance of the probe and $Z_N$ is some function of $\omega \sqrt{\varepsilon}$. They chose a rational
function to represent $Z_N$. The relationship of equation 2.37 is verified using a different approach in Chapter 7.

**Artificial Intelligence Models**

As part of preparing the proposal for this research, two artificial intelligence techniques were considered. They were artificial neural networks (ANN) and fuzzy logic inference systems (FIS). Both of the techniques are usually considered modelless but are included in this section on models, since both can be used to map independent variables to dependent variables. These techniques are described in Chapters 4 and 5. In the first investigation, an artificial neural network was trained to determine permittivity from reflection coefficient measurements made with the probe analyzed by Blackham [19]. The effectiveness of the training was evaluated by comparing values of the permittivity determined by the ANN with values determined by using Blackham's polynomial. The results [80] were essentially identical ($r^2=0.99$). Another investigation was performed to determine the feasibility of training a fuzzy logic inference system to determine the permittivity from measurements of reflection coefficient with Blackham's probe. These results were also impressive. The results obtained were virtually identical to the ones obtained with the ANN. The limitation of using these techniques is that the trained systems can not be used on similar probes used with different measurement instruments. This is because the trained ANN and FIS contained the effects of the systematic errors of the measurement system that are different from system to system. Chapter 3 contains additional information on systematic errors and their effects on measurements.

**Summary**

The literature review revealed that of the four measurement techniques available, the open-ended coaxial-line probe is the only one that can, in general, be considered broadband and nondestructive. The popularity of the probe is based on the ability to analyze its simple
geometry not on its applicability to a particular measurement application. All the models reported for the open-ended coaxial-line probe contained approximations. In general, it is normally not the best-suited probe for a given application. Other better-suited probes have not been considered because of the analytical difficulty in understanding the interaction of the probe and the material being measured. Being able to accurately model these potentially superior probes is a goal of the research reported herein.
CHAPTER 3
MEASUREMENT FUNDAMENTALS

Once a probe is designed and fabricated, the permittivity of a material is determined by making a measurement. The measurement can either be the impedance, admittance or reflection coefficient of the probe. Since each parameter can be calculated from the other, the exact measurement is not important. The instrument being used in the measurement system normally determines the parameter measured. Depending on the frequency range of interest, either a network analyzer or an impedance analyzer is used. At lower frequencies, an impedance analyzer is usually the instrument of choice. At higher frequencies a network analyzer is the instrument usually employed. The subsections below discuss the theory and operation of these instruments. The requirement by each instrument to make an accurate measurement on a permittivity measurement probe is considered. This discussion will show how these requirements are in conflict with what is required to model a permittivity measurement probe from the measurement of known materials.

Impedance Analyzer

Impedance analyzers are sometime called LCR meters because they can be used to determine the resistance, R, capacitance, C, or the inductance, L, of an electrical component at a given frequency. The major distinction between the two is that an LCR meter typically makes the measurement at one selectable frequency while an impedance analyzer sweeps the frequency between a start frequency and a stop frequency making measurements at a specified frequency step. These measurements are typically shown in graphical form on the instrument display. Figure 3.1 illustrates a basic block diagram of a modern impedance analyzer. An impedance
analyzer consists of a sinusoidal electrical source and the means for simultaneously measuring the amplitude and phase of the voltage across the impedance of the device under test, DUT, being measured and the current flowing through that impedance. From these measurements, the complex impedance, $Z$, or admittance, $Y$, can be calculated as

$$Z = \frac{V}{I}$$ (3.1)

and

$$Y = \frac{I}{V}$$ (3.2)

where $V$ is the voltage across the impedance being measured and $I$ is the current flowing through that impedance. Comparing these two equations, and by definition

$$Z = \frac{I}{Y}$$ (3.3)

Before an accurate measurement can be made, a measurement calibration must be performed. This calibration involves measuring several devices with known impedance or admittance. Details of this process are discussed later in this chapter.
Network Analyzer

Figure 3.2 shows a block diagram of a network analyzer.

The diagram shown is that of a two-port network analyzer. Modern network analyzers can be configured with additional measurement ports to suit the number of inputs and/or outputs of the device being measured. Because the permittivity measurement probes being considered have only one measurement port, this discussion will be limited to a one-port analyzer. The network analyzer consists of a sinusoidal voltage source, a means of separating the voltage incident on and the voltage reflected from the device being measured (DUT). The measurement of both the amplitude and phase of these signals can be used to calculate the complex reflection coefficient, $\Gamma$, as

$$\Gamma = \frac{V_r}{V_i}$$  \hspace{1cm} (3.4)

where $V_i$ is the voltage incident on the DUT and $V_r$ is the voltage reflected from the DUT. Often the reflection coefficient is expressed as a scattering parameter. A one-port measurement
has only one S-parameter, $S_{11}$. The two suffixes refer to the port at which the voltage waveform is applied and the port at which the measurement is being made. In this instance both ports are port one. Much like the impedance analyzer, a calibration is required before an accurate measurement can be made. Details of this process are discussed in the next subsection.

**Calibration**

Even though an impedance analyzer measures impedance and a network analyzer measures reflection coefficients, the procedures for calibrating them are identical. The reason for this is that the two parameters are related as

$$\Gamma = \frac{1 + \frac{Z}{Z_0}}{1 - \frac{Z}{Z_0}} = \frac{1 + Z_n}{1 - Z_n}$$

(3.5)

where $Z_0$ is the characteristic impedance of the measurement system. Most network analyzers have a $Z_0$ of 50 ohms. $Z_0$ is determined by the geometry and construction material of the electrical connector of the instrument. The parameter $Z_n$ is the normalized impedance. Solving this expression for $Z$,

$$Z = \frac{Z_0(1 - \Gamma)}{I + \Gamma}$$

(3.6)

Measurement calibration of network analyzers has been a topic of many researchers over the past several decades [19], [54]. Here, the calibration will be discussed in terms of reflection coefficient instead of impedance. This can be done because impedance can always be expressed as a reflection coefficient.

Manufacturers of network analyzers strive to design their products to be as accurate as possible. Even with this effort, a network analyzer cannot be made perfect. This causes the same device to measure differently when the measurement is made on different analyzers.
Adding or changing the cables on the same analyzer changes the measurement. Measurement calibration is an attempt to compensate for these imperfections. These imperfections fall into two error categories, random and systematic. Random errors are associated with measurement noise and system drift. Because of their random nature, they are difficult to eliminate completely. Averaging several measurements can minimize system noise. The errors associated with system drift can be minimized by recalibration of the measurement system.

Systematic errors are errors that do not change with time. They are caused by imperfections in the components used in the manufacture of the analyzer. Consider, for example, measuring a short circuit, $Z=0$ ohms. From equation 3.1, the equivalent reflection coefficient is -1. Without doing a measurement calibration, the measurement of a short will seldom yield this result. Another example is measuring a $Z_0$ impedance. In this case, the expected reflection coefficient is zero. Once again, without doing a measurement calibration, the measurement of a $Z_0$ impedance will seldom be zero.

The error in a one-port measurement is due to three measurement system imperfections. One of these imperfections is caused by imperfect separation of the incident signal and the reflected signal. Leakage of some of the signal that is incident on the DUT being measured will appear at the port where the reflected signal is being measured, without being reflected from the impedance. This leakage is referred to as directivity. Directivity is a parameter associated with the directional coupler or bridge that is used to separate the reflected signal from the incident signal. Another imperfection is the source match of the analyzer. Source match is the reflection coefficient of the analyzer port. Ideally the source match is zero. It is not zero whenever the intrinsic impedance of the analyzer is not $Z_0$. The third systematic error is the reflection frequency response of the analyzer. A perfect analyzer would have a reflection response of one.
Imperfect reflection response is caused when the response of the signal separation device varies with frequency.

These imperfections can be modeled as a two-port device placed between a perfect analyzer and the impedance being measured. This device is often referred to as an error adapter. The directivity, source match and reflection response are collectively referred to as the error coefficients or calibration set. Figure 3.3 shows a signal flow graph of the error adapter and impedance being measured.

![Error Adapter Diagram](image)

**Figure 3.3. One-port error model**

$S_{11u}$ is the measured reflection coefficient. $E_D$ is the directivity, $E_S$ is the source match and $E_{RT}$ is the reflection response of the system. $S_{11A}$ is the actual reflection coefficient of the device being measured. Mason's loop rule can be used to reduce this signal flow graph to the following equation.

$$S_{11u} = E_D + \frac{E_{RT}S_{11A}}{1 - E_S S_{11A}}$$  \hspace{1cm} (3.6)

Note, for a perfect network analyzer, that $E_D = 0$, $E_S = 0$, $E_{RT} = 1$ and $S_{11u} = S_{11A}$. The objective of measurement calibration is to determine the directivity, source match and reflection response at each measurement frequency. Once these complex values are known their effect on the measurement can be compensated. Solving equation 3.6 for $S_{11A}$ yields
If measurements are made on three devices with known reflection coefficients, a system of equations can be established. This system of equations contains three unknowns: directivity, source match and reflection tracking. Since there are three equations, they can be solved for the three unknowns. For calibrating a system with standard electrical connectors, the three devices with known reflection coefficients are usually a short circuit, \( \Gamma = -1 \), an open circuit, \( \Gamma = 1 \), and a matched load, \( \Gamma = 0 \).

For the system being considered, the device being tested is a permittivity probe whose impedance is a function of the properties of the material in contact with the probe. The relationship between the probe and the permittivity must be known before the reflection coefficient of a known material can be calculated. This requires performing an electromagnetic analysis of the structure. The goal of this research is to avoid this requirement.

\[
S_{II_d} = \frac{S_{II_m} - E_D}{E_{RT} + E_S S_{II_m} - E_S E_D}
\]  

(3.7)
CHAPTER 4

PERMITTIVITY DETERMINATION BY USING AN ARTIFICIAL NEURAL NETWORK

PERMITTIVITY DETERMINATION BY USING AN ARTIFICIAL NEURAL NETWORK

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Abstract

An Artificial Neural Network (ANN) was trained to determine the dielectric properties of materials from reflection coefficient measurements of an open-ended coaxial probe. The ANN was trained by using measurements made on eleven water/isopropyl alcohol solutions. Coefficient of determination values of 0.999 were obtained. This approach has the potential of greatly simplifying the characterization of new dielectric probe designs.

I. Introduction.

Permittivity of a material is one of the factors that determine how the material interacts with an electromagnetic field. It is a complex quantity consisting of a real part (dielectric constant) and an imaginary part (loss factor). The knowledge of the dielectric properties of materials and their frequency and temperature dependence is of great importance in various areas of science and engineering in both basic and applied research. Dielectric properties can often be related to a physical parameter of interest. It has been demonstrated that material properties such as moisture content [1], fruit ripeness [2], bacterial content [3], mechanical stress [4] and other seemingly unrelated parameters are related to the dielectric properties or permittivity of the material [5]. Accurate measurements of these properties can provide scientists and engineers with valuable information that allows them to properly incorporate the material into its intended
application or to monitor a process for improved quality control [6]. The permittivity is a fundamental property of the material and is independent of the measurement technique. Measurement techniques typically involve placing the material in an appropriate sample holder and determining the permittivity from measurements made on the sample holder. The sample holder can be a parallel plate or coaxial capacitor, a resonant cavity or a transmission line [7, 8]. These structures are used because the relationship between the permittivity and measurements are understood. The disadvantage of these types of sample holders is that many materials cannot be easily placed in them.

Other techniques where this relationship is not as straightforward have been employed. One of these techniques is the open-ended coaxial probe [8]. This technique has attracted much attention because of its applicability to nondestructive testing over a relatively broad frequency range. In its simplest form, it consists of a coaxial cable without a connector attached to one end. This end is inserted into the material being measured. Current techniques for using this type of probe to make dielectric measurements require that the interactions between the probe and the terminating dielectric material be fully understood. This is a complex relationship that has consumed many man-years of effort without yielding an exact closed-form solution [9-15]. The following section describes the technique commonly used to characterize and open-ended coaxial probe. The problems associated with this approach are reviewed. The sections that follow describe how an ANN can be used to overcome these problems.

**II. Basic Measurement Principles.**

The relationship between the electromagnetic fields at the end of an open-ended coaxial probe and the dielectric material that the cable is terminated in is a complex. This relationship or model usually takes the form of an expression of impedance, admittance or reflection coefficient as a function of frequency and permittivity. The only closed-form solutions available involve
approximations. These approximations usually involve modeling the interaction as a simple capacitor [12] or the relationship is assumed to be a truncated power series [13] or rational function [14] that is a function of the permittivity and frequency. The coefficients of the power series or rational function are often determined by computer simulation of the electromagnetic fields problem. These coefficients are then adjusted empirically to correct known measurement errors. While these computer models yield good approximations to the coefficients, there is some error associated with the assumptions made in the computer model of the electromagnetic problem. This accounts for some of the error in determining the permittivity. The measurement and calibration processes introduce other errors.

A network analyzer is used to make reflection coefficient measurements on the probe. To make an accurate reflection coefficient measurement with a network analyzer; the network analyzer/probe combination must be calibrated [16]. The calibration involves measuring three things with known reflection coefficients. This is required because there are three systematic sources of errors in this type of measurement; directivity, source match and reflection tracking. By measuring three items with known responses, these systematic errors can be calculated and their effects removed from subsequent measurements.

In the case of the open-ended coaxial probe, measurements are made on three items for which the dielectric properties or reflection coefficient are known. For items with known dielectric properties, the model for the probe (the ones determined by the technique described earlier) can calculate the reflection coefficient. This information is used to solve for the three error coefficients at each frequency associated with the network analyzer and the probe. When a reflection coefficient measurement is made on an unknown material, the error coefficients are used to correct the measured reflection coefficient. The probe model is then used to solve for the dielectric properties of the unknown material. Because the probe model is in error, the calculated
error coefficients will also be in error. This causes an error in the corrected measured reflection coefficient of the unknown material.

**III. Artificial Neural Network Fundamentals.**

An ANN is a collection of simple interconnected analog signal processors. The purpose of the ANN is to provide a mathematical structure that can be trained to map a set of inputs to a set of outputs. In this respect, it can be thought of as mimicking the human brain. Fig.1 illustrates an ANN similar to the one used in this study.

![Figure 1. Artificial Neural Network.](image)

This ANN has three layers: the input layer, one hidden layer and the output layer. Each layer consists of nodes or neurons. Each node has an activation function associated with it. Each interconnection between the nodes has a weight associated with it. The nodes in the hidden and output layers sum the weighted inputs from sending nodes and apply this net input to the activation function. The output of the network is determined by applying the inputs and computing the output from the various node activations and interconnection weights. The differences between the ANN shown in Fig.1 and the network used are the number of inputs and number of neurons in the hidden layer.

The feed-forward, backpropagation method was used to train the ANN [17]. This method consists of initializing the network with random weights and training is accomplished by adjusting the weights to minimize the error between the predicted ANN outputs and the observed values. The weights are adjusted by using a gradient-descent error minimization technique. In
this study, 561 sample measurements were available. These samples were divided into a training set of 392 measurements, a test set of 112 measurements and a production set of 57 measurements. The training set was used to train the network. The test set was used in the feedfoward mode only during training to determine the optimal point to terminate training. This was done to avoid overfitting the training set. The trained ANN was evaluated based on the production set exclusively.

IV. ANN Design and Training.

The Hewlett Packard 85070B [16] is a commercially available system that uses an open-ended coaxial probe to determine the dielectric properties of unknown materials. The system was developed by using the techniques described in the previous section. Its typical accuracy is 5%. The system is calibrated by measuring air, a gold-plated metal shorting block and distilled water. This system was used to measure the dielectric properties of eleven liquids. These liquids were mixtures of water and isopropyl alcohol. The permittivity was measured over the frequency range from 200 MHz to 6 GHz. The measurements were made at approximately 24°C. Fig. 2 illustrates the results of these measurements.

These measurements became the known outputs of the ANN. The measurement frequency and the uncorrected reflection coefficient measurements were used as inputs to the ANN. The reflection coefficient was represented as a complex number having a real and imaginary component. Separate ANNs were trained to determine the dielectric constant (\(e'\)) and the loss factor (\(e''\)). The number of neurons in the output layer for each network was one. The number of hidden layers and the number of neurons associated with each hidden layer are somewhat arbitrary. One hidden layer was selected. This selection was based on past success in applying ANN's to microwave measurements [18]. The number of neurons associated with this hidden layer was determined by training the network for varying numbers of neurons.
Figure 2. Water-isopropyl alcohol permittivity measurements used to train the ANN.
In general, the performance of the ANN improved as the number of neurons was increased. Above ten neurons, little or no improvement was detected. For this study, the number of neurons selected was ten.

V. Results.

Tables I illustrates the effectiveness of applying the trained networks to the production set of measurements.

Table 1. Statistics of production set measurements processed by trained neural network.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$e'$</th>
<th>$e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>r squared:</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>Mean squared error:</td>
<td>0.138</td>
<td>0.013</td>
</tr>
<tr>
<td>Mean absolute error:</td>
<td>0.244</td>
<td>0.093</td>
</tr>
<tr>
<td>Min. absolute error:</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Max. absolute error:</td>
<td>1.975</td>
<td>0.384</td>
</tr>
</tbody>
</table>

As illustrated, there is a strong correlation between the uncorrected reflection coefficient measurements and permittivity. Fig. 3 shows plots that further illustrates this correlation. These plots include the training, test and production sets of data.

VI. Conclusions.

The results obtained suggest that an artificial neural network can be trained to determine the dielectric properties of materials from uncorrected reflection coefficient measurements made on an open-ended coaxial probe. This fact, while interesting in its own right, has at least two important implications. The first deals with why the open-ended coaxial probe is used. It was originally investigated because its simple geometry allows for the solution of its reflection coefficient/permittivity relationship by using traditional electromagnetic theory. It is not necessarily the best probe design.
Figure 3. Calculated values of permittivity vs measured values.
Other probe designs, which are more sensitive to dielectric changes or otherwise better suited for the application, are usually not considered because their geometry makes solving for the relationship between the reflection coefficient and the material permittivity nearly impossible (even with simplifying assumptions). This investigation suggests that a neural network could be trained to determine the permittivity from reflection coefficient measurements on these probes. The only requirement is that materials with known properties be available to train the system. Many such materials exist. Another implication is based on why many researchers are interested in measuring permittivity. It is often because it can be related to other physical properties of interest. Permittivity is then just an intermediate goal. The real goal is determining the moisture content, fruit ripeness, bacterial content, mechanical stress or some other property. It should be possible to train a neural network to determine these parameters directly from measurements of reflection coefficient.

References


CHAPTER 5

DIELECTRIC DETERMINATION USING A FUZZY LOGIC INFERENCE SYSTEM¹


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ABSTRACT
A fuzzy logic inference system (FLIS) was designed to determine the dielectric properties of material. The inputs to the FLIS are the admittance measurement made on an open-ended coaxial line immersed in the material of interest and the measurement frequency. The measurement frequency ranged from 200 MHz to 6 GHz. A network analyzer was used to make the measurements. R-squared values in excess of 0.99 were obtained when the output of the FLIS was compared to known values. This approach greatly simplifies the characterization of dielectric measurement probes.

I. Introduction.
Permittivity of a material is one of the factors that determine how the material interacts with an electromagnetic field. It is a complex quantity consisting of a real part (dielectric constant) and an imaginary part (loss factor). The knowledge of the dielectric properties of
materials and their frequency and temperature dependence is of great importance in various areas of science and engineering in both basic and applied research. Dielectric properties can often be related to a physical parameter of interest. It has been demonstrated that material properties such as moisture content [1], fruit ripeness [2], bacterial content [3], mechanical stress [4] and other seemingly unrelated parameters are related to the dielectric properties or permittivity of the material [5]. Accurate measurements of these properties can provide scientists and engineers with valuable information that allows them to properly incorporate the material into its intended application or to monitor a process for improved quality control [6]. The permittivity is a fundamental property of the material and is independent of the measurement technique. Measurement techniques typically involve placing the material in an appropriate sample holder and determining the permittivity from measurements made on the sample holder. The sample holder can be a parallel plate or coaxial capacitor, a resonant cavity or a transmission line [7, 8]. These structures are used because the relationship between the permittivity and measurements are understood. The disadvantage of these types of sample holders is that many materials cannot be easily placed in them.

Other techniques where this relationship is not as straightforward have been employed. One of these techniques is the open-ended coaxial probe [8]. This technique has attracted much attention because of its applicability to nondestructive testing over a relatively broad frequency range. In its simplest form, it consists of a coaxial cable without a connector attached to one end. This end is inserted into the material being measured. Current techniques for using this type of probe to make dielectric measurements require that the interactions between the probe and the terminating dielectric material is fully understood. This is a complex relationship that has consumed many man-years of effort without yielding an exact closed-form solution [9-15]. The following section describes the technique commonly used to characterize an open-ended
coaxial probe. The problems associated with this approach are reviewed. The section that
follows describes how artificial intelligence (AI) techniques can be used to overcome these
problems.

II. Basic Measurement Principles.

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approximations. These approximations usually involve modeling the interaction as a simple
capacitor [12] or the relationship is assumed to be a truncated power series [13] or rational
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fields problem. These coefficients are then adjusted empirically to correct known measurement
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problem. This accounts for some of the error in determining the permittivity. The measurement
and calibration processes introduce other errors.

A network analyzer is used to make reflection coefficient measurements on the probe.
To make an accurate reflection coefficient measurement with a network analyzer, the network
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By measuring three items with known responses, these systematic errors can be calculated and
their effects removed from subsequent measurements. In the case of the open-ended coaxial
probe, measurements are made on three items for which the dielectric properties or reflection coefficient are known. For items with known dielectric properties, the reflection coefficient can be calculated by using the model for the probe (the one determined by using the technique described earlier). This information is used to solve for the three error coefficients at each frequency associated with the network analyzer and the probe. When a reflection coefficient measurement is made on an unknown material, the error coefficients are used to correct the measured reflection coefficient. The probe model is then used to solve for the dielectric properties of the unknown material. Because the probe model is in error, the calculated error coefficients will also be in error. This causes an error in the corrected measured reflection coefficient of the unknown material.

**III. Permittivity Measurement.**

The Hewlett-Packard 85070B [16] is a commercially available system that uses an open-ended coaxial probe to determine the dielectric properties of unknown materials. This system was developed using the techniques described in the previous section. Its typical accuracy is 5%. The system is calibrated by measuring air, a gold-plated metal shorting block, and distilled water. This system was used to measure the dielectric properties of eleven liquids. These liquids were mixtures of water and isopropyl alcohol. The permittivity was measured over the frequency range from 200 MHz to 6 GHz. The measurements were made at approximately 24°C. Fig.1 and Fig.2 illustrate the results of these measurements.

These measurements were used to train and validate the fuzzy logic inference system investigated. The measurement frequency and the measured admittance were used as inputs to the system. The admittance was calculated from the uncorrected reflection coefficient measurements. The admittance was represented as a complex number having a real (conductance) and imaginary (susceptance) component. The following two sections describe
Figure 1. Water-isopropyl alcohol dielectric constant measurements.

Figure 2. Water-isopropyl alcohol loss factor measurements.
how this training was accomplished.

IV. Fuzzy Logic Fundamentals.

An inference system based on fuzzy logic techniques provides a non-modeled approach to relating inputs to outputs. These systems are based on the concept that all inputs have some affiliation with one or more classifications. This relationship is determined by membership functions. Over the range of the input, these functions have a value between zero and one. A value close to one indicates strong membership. Values close to zero suggest weak membership. As an example, consider the problem on the open-ended coaxial probe. One of the inputs is frequency. With traditional modeling, frequency takes on a fixed value such as 4 GHz. With fuzzy logic, the input frequency has a value associated with one or more membership functions. In this study, frequency had membership into three membership functions: low, medium and high. Figure 3 serves to illustrate this concept.

![Figure 3. Membership functions for frequency.](image)

Every value of frequency has some level of membership into low, medium and high ranging from zero to one. This process is referred to as fuzification. The membership functions used in this study were triangular. Alternative membership functions such as trapezoidal, Gaussian and others are often used. Similar membership relationships for the other two inputs (the conductance and susceptance of the measured admittance) can be defined. Once the inputs
are fuzzified, they are combined using fuzzy logic to determine the outputs. Many forms of fuzzy logic inference systems are possible [18]. The goal is to simulate human reasoning. The one used in this study is described by Jang [19] and its block diagram is shown in Figure 4.

![Figure 4. Adaptive Neuro-Fuzzy Inference System.](image)

This system is an adaptive-network-based fuzzy inference system or ANFIS. The ANFIS uses every combination of fuzzy inputs in the fuzzy logic rules to determine the output. Based on three inputs with three membership functions associated with each, twenty-seven rules were used. The output is a linear combination of these rules. Training was used to determine the placement and shape of the nine input membership functions and the coefficients associated with the twenty-seven rules. For a given input, the shape and location of the membership functions and the linear coefficients were adjusted in an effort to minimize the difference between the calculated output and the known values (Section III). These measurements were divided into a training set of 392 measurements, a test set of 112 measurements and a production set of 57 measurements. The training set was used to train the network. The test set was used only during training to determine the optimal point to terminate training. This was done to avoid overfitting the training set. The trained ANFIS was evaluated based on the production set exclusively.

**IV. Results.**

Table 1 illustrates the effectiveness of applying the trained ANFIS to the production set of measurements. The value of $r$ squared exceeded 0.99.
Table 1. Statistics of production set measurements processed by the trained ANFIS

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$e'$</th>
<th>$e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean squared error:</td>
<td>0.011</td>
<td>0.008</td>
</tr>
<tr>
<td>Mean absolute error:</td>
<td>0.061</td>
<td>0.057</td>
</tr>
<tr>
<td>Min. absolute error:</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Max. absolute error:</td>
<td>0.542</td>
<td>0.887</td>
</tr>
</tbody>
</table>

Figs. 5 and 6 shows plots of the ANFIS Calculated values of permittivity vs. measured values.

Figure 5. ANFIS Calculated values of dielectric constant vs. measured values.
VIII. Conclusions.

The results obtained suggest that an ANFIS can be trained to determine the dielectric properties of materials from uncorrected admittance measurements made on an open-ended coaxial probe. This fact, while interesting in its own right, has at least two important implications. The first deals with why the open-ended coaxial probe is used. It was originally investigated because its simple geometry allows for the solution of its admittance/permittivity relationship by using traditional electromagnetic theory. It is not necessarily the best probe design. Other probe designs, which are more sensitive to dielectric changes or otherwise better suited for the application, are usually not considered because their geometry makes solving for the relationship between the reflection coefficient and the material permittivity nearly impossible (even with simplifying assumptions). This investigation suggests that a fuzzy logic inference
system can be trained to determine the permittivity from admittance measurements on these probes. The only requirement is that materials with known properties be available to train the system. Many such materials exist. Another implication is based on why researchers are interested in measuring permittivity. It is often because it can be related to other physical properties of interest. Permittivity is then just an intermediate step. The real goal is often determining the moisture content, fruit ripeness, bacterial content, mechanical stress or some other property. It should be possible to train an ANFIS to determine these parameters directly from measurements of reflection coefficient.

References


CHAPTER 6
DIMENSIONAL ANALYSIS
OF A PERMITTIVITY MEASUREMENT PROBE

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DIMENSIONAL ANALYSIS OF A PERMITTIVITY MEASUREMENT PROBE

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ABSTRACT

Open-ended coaxial-line probes provide a convenient means of determining the dielectric properties of many materials over a relatively wide frequency range. Because of this, much attention has been given to understanding the interaction of the probe and the material that it is inserted into. In this paper a dimensional analysis was performed on a generalized open-ended coaxial-line probe. Applying the Buckingham Π-theorem revealed that the admittance of the probe/dielectric interface, scaled by the frequency, is a function of a single dimensionless variable. This fact greatly simplifies the modeling of the probe. The problem is reduced from fitting a model of two variables, frequency and permittivity, to one dimensionless variable. In addition the dimensional analysis also revealed that the same results hold for any permittivity measurement probe where the admittance of the probe is a function of permittivity, frequency, and any number of linear dimensions.

Keywords: Permittivity, permittivity measurement, dielectric measurement, dielectric probe, network analyzer, dimensional analysis, Buckingham Π-theorem, open-ended coaxial-line probe, coaxial probe.
I. INTRODUCTION

The permittivity, or dielectric properties, of a material is one of the factors that determine how the material interacts with an electromagnetic field. It is a complex quantity consisting of a real part (dielectric constant) and an imaginary part (loss factor). The knowledge of the dielectric properties of materials and their frequency and temperature dependence is of great importance in various areas of science and engineering in both basic and applied research. Dielectric properties often can be related to a physical parameter of interest. It has been demonstrated that properties such as moisture content [1], fruit ripeness [2], bacterial content [3], mechanical stress [4], and other seemingly unrelated parameters are related to the dielectric properties or permittivity of the material [5]. Accurate measurements of these properties can provide scientists and engineers with valuable information that allows them to properly incorporate the material into its intended application or to monitor a process for improved quality control [6]. The permittivity is a fundamental property of the material and is independent of the measurement technique. Measurement techniques typically involve placing the material in an appropriate sample holder and determining the permittivity from measurements made on the sample holder. The sample holder can be a parallel plate or coaxial capacitor, a resonant cavity, or a transmission line [7], [8]. These structures are used because the relationship between the permittivity and the measurements is well understood. The disadvantage of these types of sample holders is that many materials cannot be placed easily in them.

Other techniques where this relationship is not as straightforward have been employed. One of these techniques is the open-ended coaxial-line probe [9]-[17]. This technique has attracted much attention because of its applicability to nondestructive testing over a relatively broad frequency range. In its simplest form, it consists of a coaxial cable without a connector attached to one end. This end is inserted into the material being measured. A network or
impedance analyzer is used to measure the impedance, admittance or reflection coefficient at the open end of the coaxial line. This technique requires that the interaction between the probe and the terminating dielectric material be fully understood. Traditionally this understanding is derived from an electromagnetic analysis of the problem using Maxwell’s equations. This analysis is tedious and no useful closed form solution exists. Often, assumptions are made to simply the problem [13]-[15]. These assumptions affect the accuracy of the relationship.

II. DIMENSIONAL ANALYSIS

The fundamental dimensions of all physical quantities are a subset of the product of mass (M), length (L), time (T) and charge (Q) each raised to a power [18], [19]. The Buckingham Π-theorem states that in a physical problem involving n quantities (A₁, A₂, A₃, ...Aₙ) in which there are m dimensions, the quantities may be arranged into n–m independent dimensionless parameters (Π₁, Π₂, Π₃...Πₙ₋ₘ) [18], [19]. A set of independent dimensionless parameters can be calculated by selecting m of the quantities as repeating variables. The n–m Π-parameters are determined by taking the product of the m repeating variables, each raised to a power, along with one of the remaining quantities. The power to which each of the repeating variables is raised is selected such that the result is dimensionless. Any of the Π-parameters may be inverted or raised to a power without affecting their dimensional status. Alternative Π parameters can be determined by taking the product of the n–m Π-parameters each raised to an arbitrary power. This product along with the n–m–1 previous Π-parameters forms a new set of independent Π-parameters. The only stipulation is that the power that the discarded parameter is raised cannot be zero. This procedure can be repeated to find all possible combinations.

Buckingham showed that there exists a function, f, such that

\[ f(Π₁, Π₂, Π₃...Πₙ₋ₘ) = 0. \]  (1)
Buckingham also showed that there exists another function of $n-m-1$ of the $\Pi$-parameters that is equal to the other $\Pi$-parameter. That is

$$\Pi_k = g(\Pi_1, \Pi_2, \Pi_3, \ldots , \Pi_{n-m}) ; k \neq 1, 2, 3, \ldots, n-m.$$  \hfill (2)

To evaluate the use of the Buckingham $\Pi$-theorem to analyze an electromagnetic problem, consider the parallel plate capacitor shown in figure 1.

![Diagram of a parallel-plate capacitor](image)

**Figure 1. Diagram of a parallel-plate capacitor.**

The impedance of the capacitor can reasonably be assumed to be a function of the area of the conducting plates, the distance between the plates, the frequency of the sinusoidal stimulus, and the dielectric constant of the material between the plates. With these assumptions, the Buckingham $\Pi$-theorem would apply as follows. Table I lists the aforementioned quantities and their associated dimensions.

### Table I.
**Parallel plate-capacitor physical quantities and associated fundamental dimensions.**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Quantity</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Area of plates</td>
<td>$A$</td>
<td>$L^2$</td>
</tr>
<tr>
<td>2. Distance between plates</td>
<td>$d$</td>
<td>$L$</td>
</tr>
<tr>
<td>3. Frequency of stimulus</td>
<td>$\omega$</td>
<td>$T^{-1}$</td>
</tr>
<tr>
<td>4. Capacitor permittivity</td>
<td>$\varepsilon$</td>
<td>$T^2Q^2M^{-1}L^{-3}$</td>
</tr>
<tr>
<td>5. Impedance of capacitor</td>
<td>$Z$</td>
<td>$ML^2T^{-1}Q^{-2}$</td>
</tr>
</tbody>
</table>
For this case there are five quantities which contain all four dimensions. Therefore there is only one \( \Pi \)-parameter. If \( A, \varepsilon, d, \) and \( \omega \) are chosen as the four repeating quantities, the \( \Pi \)-parameter would be calculated as follows:

\[
\Pi_1 = A^{x_1} \varepsilon^{x_2} d^{x_3} \omega^{x_4} Z .
\]

(3)

The \( x_i \)'s represent the powers that each quantity is raised to make the \( \Pi \)-parameter dimensionless. Solving for \( x_1, x_2, x_3 \) and \( x_4 \), the \( \Pi \)-parameter is

\[
\Pi_1 = \frac{A \varepsilon \omega Z}{d} .
\]

(4)

Since there always exists a function \( f(\Pi_1) = 0 \), it is clear that in the case of one \( \Pi \) parameter, that \( \Pi_1 \) is equal to a constant, \( K \). Therefore, the following expression can be written:

\[
\Pi_1 = K = \frac{A \varepsilon \omega Z}{d} .
\]

(5)

Solving for \( Z \), the impedance, yields the following expression.

\[
Z = \frac{Kd}{A \varepsilon \omega} = \frac{K}{\omega \varepsilon A d} .
\]

(6)

In order to determine the constant \( K \), impedance measurements could be made in the laboratory. From these measurements, the value of \( K \) could be determined.

If the same problem were analyzed using electromagnetic theory (refer to any first year physics book), the following result would have been obtained:

\[
Z = \frac{-j}{\omega \varepsilon A d} .
\]

(7)

From this expression of \( Z \) it can be seen that \( K = -j \) where \( j = \sqrt{-1} \).
For such a simple geometry the use of the Buckingham Π-theorem was not required. This is because standard application of electromagnetic theory easily produced a closed form solution to the problem. The importance of this example is to show that the Buckingham Π-theorem produced identical results as more traditional techniques.

The following example illustrates how this theorem can be used to analyze more complex geometries where closed form solutions do not exist or can not be obtained easily. This example deals with the impedance of a generalized, open-ended, coaxial-line probe shown in figure 2.

![Figure 2. Diagram of a generalized open-ended coaxial-line probe.](image)

Table II lists the physical quantities associated with this problem along with their definitions and fundamental dimensions.

**Table II.**

**Generalized open-ended coaxial-line probe physical quantities and associated fundamental dimensions.**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Quantity</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Radius of inner conductor</td>
<td>$a$</td>
<td>L</td>
</tr>
<tr>
<td>2. Radius of outer conductor</td>
<td>$b$</td>
<td>L</td>
</tr>
<tr>
<td>3. Extension of sample material into coaxial line</td>
<td>$c$</td>
<td>L</td>
</tr>
<tr>
<td>4. Extension of inter conductor past the outer conductor</td>
<td>$d$</td>
<td>L</td>
</tr>
<tr>
<td>5. Impedance of probe</td>
<td>$Z$</td>
<td>$\text{ML}^2\text{T}^{-1}\text{Q}^{-2}$</td>
</tr>
<tr>
<td>6. Frequency of stimulus</td>
<td>$\omega$</td>
<td>$\text{T}^{-1}$</td>
</tr>
<tr>
<td>7. Permittivity of the measurement material</td>
<td>$\varepsilon_1$</td>
<td>$\text{T}^2\text{Q}^2\text{M}^{-1}\text{L}^{-3}$</td>
</tr>
<tr>
<td>8. Permeability of the measurement material</td>
<td>$\mu_1$</td>
<td>$\text{ML}^{-2}$</td>
</tr>
<tr>
<td>9. Permittivity of the material between the conductors</td>
<td>$\varepsilon_2$</td>
<td>$\text{T}^2\text{Q}^2\text{M}^{-1}\text{L}^{-3}$</td>
</tr>
<tr>
<td>10. Permeability of the material between the conductors</td>
<td>$\mu_2$</td>
<td>$\text{ML}^{-2}$</td>
</tr>
</tbody>
</table>
The impedance of the probe illustrated in figure 2 is assumed to be a function of $a$, $b$, $c$, $d$, $\varepsilon_1$, $\mu_1$, $\varepsilon_2$, $\mu_2$ and $\omega$. Since there are 10 quantities and four dimensions, then there are six $\Pi$-parameters. If $b$, $c$, $\mu_1$ and $\varepsilon_1$ are chosen as the m repeating variables, one possible set of independent $\Pi$-parameters is the following:

$$\Pi_1 = \frac{ac}{b^2},$$

$$\Pi_2 = \frac{cd}{b^2},$$

$$\Pi_3 = \frac{c\mu_2}{b\mu_1},$$

$$\Pi_4 = \frac{be_1}{ce_2},$$

$$\Pi_5 = a\omega \sqrt{\mu_1 \varepsilon_1}, \text{ and}$$

$$\Pi_6 = \frac{b}{a} \sqrt{\frac{\varepsilon_1}{\mu_1} Z}.$$

The same technique was used to determined these $\Pi$-parameters that was used earlier to determine the one for the parallel plate capacitor. This is an alternative set obtained by the technique discussed earlier in this section. The purpose of using this set will become evident in the next section.

**III. RESULTS AND DISCUSSION**

Applying the Buckingham $\Pi$-theorem to the results of the previous section yields the following expression:

$$\Pi_6 = g(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5)$$

(9)

where $g$ is some function that is known to exist. If $a$, $b$, $c$, $d$, $\varepsilon_2$, $\mu_2$, and $\mu_1$ are constants, then the following is true:
\[ \Pi_6 = g(\Pi_4, \Pi_5). \]  

(10)

This is because \( \Pi_1, \Pi_2, \) and \( \Pi_3 \) are constants. These assumptions are true for a real probe inserted into a nonmagnetic material. If \( c \) is infinite, \( \Pi_4 \) is zero and the equation reduces to

\[ Z = \frac{I}{\sqrt{\varepsilon_1}} g'(\omega \sqrt{\varepsilon_1}). \]  

(11)

The function \( g' \) differs from \( g \) by absorbing the constants in \( \Pi_6 \). This is the same result that Smith and Nordgard \cite{14} arrived at when he assumed \( e \) was large enough that a TEM wave exists at the interface of \( \varepsilon_1 \) and \( \varepsilon_2 \). Also note that when \( c \) is zero \( \Pi_4 \) is also a constant, infinity, so (11) also holds. This is shown to be true without having to make the assumptions that Smith and Nordgard made to achieve their results.

Another useful result can be obtained by taking the reciprocal of (11) and multiplying each side by \( \omega \), as is shown by

\[ \omega Y = \frac{\omega}{Z} = \omega \sqrt{\varepsilon_1} g''(\omega \sqrt{\varepsilon_1}) = g'''(\omega \sqrt{\varepsilon_1}). \]  

(12)

\( Y \) is the admittance of the probe at the probe/material interface. The function \( g''' \) differs from \( g' \) by the reciprocal and \( g'''' \) differs from \( g'' \) by incorporating the multiplicative factor \( \omega \sqrt{\varepsilon_1} \). This is the same result that Misra \cite{15} obtained for a probe with \( c \) and \( d \) equal to zero and by assuming that only the principle mode exists at the probe/material interface. The results obtained using the Buckingham \( \Pi \)-theorem produced the same results as the application of electromagnetic theory but without applying any underlying assumptions.

**IV. CONCLUSIONS**

The Buckingham \( \Pi \)-theorem was originally proposed in the early part of the twentieth century and has long been used to analyze and model structural/mechanical/fluid problems.
Application of the Buckingham Π-theorem in these fields has resulted in important dimensionless parameters such as Reynolds number, Weber number, Froude number, and Mach number [18]. Applying this technique to the analysis of an open-ended coaxial-line probe has produced identical results to those of traditional electromagnetic theory without having to make any simplifying assumptions. The analysis also simplified the problem of modeling the probe. It revealed that the relationship between the admittance of the probe/dielectric interface scaled by the frequency is a function of a single dimensionless variable. This result is also true if the problem is generalized to include any probe geometry where the impedance is a function of frequency, \( \mu_1, \varepsilon_1, \varepsilon_2, \mu_2 \) and any number of linear dimensions. If \( \mu_1, \varepsilon_2, \mu_2 \) and all the linear dimensions remain constant, the form of (12) is valid. It should be noted that the function \( g''' \) would be different for each geometry chosen but the form of the relationship would stay the same. This result is important because the open-ended coaxial-line probe was chosen because its simple geometry allowed it to be analyzed using classical electromagnetic theory along with reasonable assumptions. It was not chosen because it was the most sensitive to changes in dielectric properties or the best shape for a given application. The theorem shows that the same general form can be used to describe a class of potential permittivity measurement probes where the form of the geometry is chosen because of its utility and sensitivity to changes in dielectric properties. The problem of analyzing any such probe is to determine the function \( g''' \). Making measurements on known materials and fitting the results to a chosen mathematical form can do this.

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dielectric properties of biological substances at radio and microwave frequencies-A


1985.


CHAPTER 7

A SYSTEMATIC APPROACH TO MODELING PERMITTIVITY MEASUREMENT PROBES$^1$

A SYSTEMATIC APPROACH TO MODELING PERMITTIVITY MEASUREMENT PROBES

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Abstract – A systematic approach is presented for modeling permittivity measurement probes. The approach utilizes the results of a dimensional analysis and use of a genetic algorithm to find an appropriate model for a probe. The only information required is uncorrected reflection-coefficient measurements on a variety of materials of known permittivity. The need to perform an electromagnetic analysis, based on the geometry of a given probe and its construction, is not required. This allows candidate probes that cannot easily be analyzed by traditional electromagnetic techniques to be considered for use. The technique was used to model a 20-GHz commercially available probe. The model developed was functionally equivalent to the model provided by the manufacturer, which was developed by using traditional electromagnetic techniques. The approach was also used to model a 50-GHz probe. Measurements with this probe compared favorably with published values of permittivity for three liquids.

Keywords – Permittivity, Dielectric measurement, Dielectric probe, Network analyzer, Dimensional analysis, Open-ended coaxial-line probe, Coaxial probe, Genetic algorithm, GA.
\section*{I. INTRODUCTION}

The permittivity (dielectric properties) of a material is one of the factors that determine how the material interacts with an electromagnetic field. It is a complex quantity consisting of a real part (dielectric constant) and an imaginary part (loss factor). Its dimensions are farad per meter (capacitance per distance). The knowledge of the dielectric properties of materials and their frequency and temperature dependence is of great importance in science and engineering in both basic and applied research. It has always been an important quantity to electrical engineers and physicists involved in the design and application of circuit components. Over the past several decades, the knowledge of permittivity has become an important property to scientists and engineers involved in the design of stealth vehicles. These applications are most often associated with the defense industry. Over the last half century, this knowledge has become increasingly important to agricultural engineers, biological engineers, chemists and food scientists. The most obvious application of this knowledge is in microwave and RF heating of food products. There are also numerous not-so-obvious applications. It has been demonstrated that material properties such as moisture content \cite{1}, fruit ripeness \cite{2}, bacterial content \cite{3}, mechanical stress \cite{4} and other seemingly unrelated parameters \cite{5} are related to the dielectric properties or permittivity of the material. Measurement techniques usually involve placing the material in an appropriate sample holder and determining the permittivity from measurements made on that sample holder \cite{6}, \cite{7}. The sample holder can be a parallel-plate or coaxial capacitor, a resonant cavity or a transmission line. These structures are used because the relationship between the permittivity and measurements are fundamental and well understood. One disadvantage of these types of sample holders is that many materials cannot be easily placed in them. Sample preparation is required. Techniques for which the relationship between permittivity and measurements is not as straightforward have also been employed. One of these
techniques is the open-ended coaxial-line probe [8]. This technique has attracted much attention because of its applicability to nondestructive testing over a relatively broad frequency range. It can be used to measure the permittivity of a wide variety of materials including liquids, solids and semisolids. In its simplest form, it consists of a truncated coaxial cable. This open end is inserted into the material being measured. Fig. 1 illustrates such a probe, manufactured and marketed by Agilent Technologies. A vector network analyzer or impedance analyzer can be used to make a measurement with this probe. Techniques for using this type of probe to determine dielectric properties require that the interactions between the probe and the terminating dielectric material be fully understood. A flange is often added to the end of the open-ended coaxial probe to simplify the analysis of the problem. This is true for the probe illustrated in Fig. 1.

![Open-ended coaxial-line probe](image)

**Fig. 1. Open-ended coaxial-line probe**

This addition allows the assumption of an infinite ground plane that makes analyzing the probe with Maxwell's equations simpler. This and other assumptions are usually made to simplify the analysis. Even with this simplifying assumption, this is a complex relationship that has consumed many man-years of effort without yielding an exact closed-form solution [9]. A dimensional analysis was performed on a generalized open-ended coaxial probe [10]. It showed that the relationship between the admittance of the probe/dielectric interface, scaled by the frequency, is a function of a single dimensionless variable.
For a given probe geometry the relationship is

$$\omega Y = F(\omega \sqrt{\epsilon}).$$  \hspace{1cm} (1)$$

In this expression $\omega$ is the measurement angular frequency, $Y$ is the admittance at the probe/material interface and $\epsilon$ is the material permittivity. $F$ is the functional relationship between $\omega Y$ and $\omega \sqrt{\epsilon}$. This paper reports use of this result to develop a systematic technique to model permittivity measurement probes that avoids the necessity for an electromagnetic analysis. This technique was used to model the open-ended coaxial-line dielectric probe shown in Fig. 1. It was then used to model a new 50-GHz probe design. The measurements on known materials compared favorably with established values.

II. PROBLEM DEFINITION

The first step in establishing the relationship between the measurement on a probe and the permittivity is to assume an appropriate form for the function $F$ defined in (1). A polynomial was chosen to model the relationship. This choice was based, in part, on previously reported results [9] and the fact that functions can be expressed as a power series. For most functions, the power series can be truncated and still accurately represent the function over a given range. Assuming this functional representation, determining the coefficients of the polynomial becomes the task. This process is straightforward if the independent and dependent variables are known. If the admittance of a material with known properties can be made at $n$ frequencies, the coefficients of a polynomial of order $n-1$ can be determined by techniques of traditional linear algebra. This process is complicated here, because a calibration at the reference plane of the measurement is required [11]. To perform a one-port calibration, measurements on three known devices are required. The calibration is calculated from these measurements and knowledge of the expected values of the calibration devices. Knowing the expected values requires knowing the function $F$. The traditional technique would be to obtain the expected values by performing
an electromagnetic analysis of the probe. The goal of this research was to avoid this requirement.

III. GENETIC ALGORITHM

A genetic algorithm, GA, was used to determine the coefficients of the truncated power series that represents the function $F$ (Equation 1). A GA is a search algorithm based on natural selection [12]. The idea is to mimic nature's survival of the fittest. This selection was chosen, in part, because of GA success in this field. A recent article described how a GA was used to develop circuits [13]. A GA recreated several patented circuits. Another reason for using a GA is implicit parallelism [12]. The search is essentially speeded up as a consequence. A GA with a population of size $N$, searches $N^3$ faster than a random search. The GA starts with an initial population of individuals (possible solutions). To create the next generation, individuals are selected for mating (to produce new solutions). The higher the fitness or accuracy of an individual, the better chance that individual has to be selected for mating. The offspring is created by some chosen scheme of combining the mating individuals' characteristics. Mutation plays a part in this process. Mutation involves some form of randomly altering the offspring. The probability of mutation is usually low. The desired result is that each subsequent generation has a better average fitness. The process continues until an individual in the population has the desired degree of fitness. This individual is the solution.

The Individual and its Fitness

In the case being considered, the individuals are represented by $n$ sets of independent/dependent variable pairs chosen at random. From these data pairs, the coefficients of the truncated power series can be calculated. These coefficients are those used to represent the function $F$ in (1). Initially the fitness of an individual is determined by how well it can be used
to calibrate the probe. Measurements are made on air, a short circuit and water. The permittivity of air and water are known. The short circuit is established by placing a block of metal at the end of the probe. These measurements, along with the polynomial represented by the individual, are used to calculate the three one-port error coefficients of the measurement system. These error coefficients are used to correct the reflection coefficients of measurements made on the air and water standards. The sum of the difference between the one-port error-corrected values and the values normalized to the short circuit is the fitness of an individual. This normalization is often referred to as a one-term calibration. Fig. 2 illustrates a comparison of a reflection coefficient measurement made on water.

Fig. 2a. Water measurements using a highly fit individual's one-port and one-term calibrations

The smooth curve shown in Fig. 2a is the measurement with a highly fit individual's one-port calibration. The smooth curve shown in Fig. 2b is the measurement with a poorly fit individual's
one-port calibration. The irregular, less smooth curve in each figure is the same measurement with a one-term calibration that is normalized to a short circuit.

**Fig. 2b. Water measurements using a poorly fit individual's one-port and one-term calibrations**

Normalizing a reflection coefficient measurement is a simpler form of calibration. It lumps the errors caused by the directivity, source match and frequency response into a single frequency-response term. While error exists in measurements calculated by this technique, the expected value of the measurement is the actual value of the measurement. This is why the sum of the difference between the one-port calibration and the one-term calibration is used to calculate an individual's fitness. As the average fitness of the population improves, the fitness is calculated differently. The difference between the calculated value of permittivity and the known permittivity of materials is used to calculate fitness. The known materials include various
alcohols and liquids of known conductivity. This is done to insure that the probe model accurately calculates the values of known materials.

**Selection and mating**

Selection for mating should mimic nature. A popular opinion is that opposites attract and that people tend to mate with individuals of similar attractiveness. In this study, the concept of opposites has to do with frequency range. Individuals that exhibit better fitness at the low frequencies, referred to as good looking, will be attracted to individuals that exhibit better fitness at high frequencies, referred to as rich, and vice versa. The selection of two individuals for mating consists of randomly selecting an individual, the male, from the current population. If the male is better looking than he is rich he will date other randomly selected individuals, the female, until he meets a female that is at least as rich as he is good looking. The dating is continued until he has met his opposite-equal or 10% of the population. He marries his opposite-equal or the best of the females he has dated. If the randomly selected male is richer than he is good looking, he will date until he meets a female that is at least as good looking as he is rich. Once selected, the two individuals will mate to produce two individuals for the next generation.

Mating consists of randomly selecting a positive integer, j, less than n, the number of data pairs. Using the first j data pairs from the father and the last n-j data pairs from the mother produces one of the offspring. The other offspring is produced using the first j data pairs from the mother and the last n-j data pairs from the father. Mutations consist of randomly changing some of the values of the variable pairs. Selection and mating continues until a new population is generated.
IV. RESULTS

An initial population of 5000 individuals evolved. The probability of mutation was selected to be 0.1% for each data pair of all new individuals. The evolution of the population was terminated when no significant improvement was observed for the best-fit individual. The process was repeated with increasing order of the polynomials until no significant improvement in fitness was observed. The final order of the polynomial for this experiment was 6. Fig. 3 shows measurements of the dielectric constant and dielectric loss factor made on a 70%-30% ethanol-water mixture with the Agilent Technologies 85070D probe.
Fig. 3. Permittivity measurements of 70%-30% ethanol-water mixture

The measurements were made over the frequency range from 200 MHz to 20 GHz, the frequency range of this probe. One measurement used the GA-created probe model, and the other was made with the probe model furnished by Agilent Technologies. The latter model is based on an electromagnetic analysis of the probe and assumes an infinite ground plane [9]. For the resolution shown, the measurements are superimposed. The largest difference in the magnitude of permittivity between the two models is less than 0.7%. The typical accuracy of the Agilent Technologies model is 5% of the magnitude.

The aforementioned method was then applied to the probe illustrated in Fig. 4. This probe is based on a 0.086-inch (2.2-mm) semirigid coaxial line with a 2.4-mm coaxial connector.
Fig. 4. Fifty-GHz probe based on a 0.086-inch semirigid coaxial line with a 2.4-mm coaxial connector
Fig. 5. Permittivity measurements made on water, isopropanol and 70%-30% ethanol-water mixture with a 50-GHz probe at 20°C

Because of its geometry, it has an upper frequency range of 50 GHz. The absence of a flange makes this configuration difficult to model with traditional electromagnetic theory. Fig. 5 shows measurements made on three liquids. The measurements made on 20° C water, 70%-30% ethanol-water mixture and isopropanol agree within 5% of published results [14] over the 500 MHz - 50 GHz frequency range.

V. CONCLUSIONS

This investigation validated the genetic algorithm modeling approach considered. This approach provides a systematic method to model a family of permittivity measurement probes [10]. The model produced is functionally equivalent to the model developed by using traditional
electromagnetic techniques. The technique was also applied to a 50-GHz probe without a ground-plane flange. The measurements made with this probe on several materials agreed with previously published results. The advantage of this technique is that it can be used to model probes, such as the flangless probe, that cannot be easily analyzed by traditional electromagnetic techniques.

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CHAPTER 8

IMPROVEMENTS IN PROBE CHARACTERIZATION

The content of this chapter is a summary of the procedure described in Chapter 7 and offers several improvements to the recommended procedure for characterizing and calibrating a permittivity measurement probe. It gives a step-by-step example of the recommended procedure to model a permittivity measurement probe. This model can be used to calibrate the probe and calculate permittivity from measurements made with the probe. To calibrate the probe, the model relationship \( \omega Y = F(\omega \sqrt{\varepsilon}) \) is used to calculate the expected reflection coefficient at the probe material interface. The admittance, \( Y \), can be calculated with the knowledge of \( F \), \( \omega \) and \( \varepsilon \). When measuring, Newton’s method [81] is used to determine the unknown permittivity from an error-corrected measurement. As an example, the 50-GHz probe described in Chapter 7 will be used. The recommended procedure for modeling a probe by this technique is identical to the one described in Chapter 7 with a few exceptions. These exceptions are outlined below.

Initial Population

The first exception is the process chosen to create the initial population. As described in Chapter 7, an individual in the population consists of \( n \) sets of independent/dependent variable pairs of \( \omega Y = F(\omega \sqrt{\varepsilon}) \). Originally the data pairs of each individual in the population were chosen at random. An improvement can be made by choosing the \( n \) data pairs of \( \omega Y \) and \( \omega \sqrt{\varepsilon} \) from the short-normalized measurements of water. The \( \omega \) values would be chosen logarithmically spaced in frequency. The \( \omega \sqrt{\varepsilon} \) values are calculated from the measurement frequency and the permittivity of water, calculated with the Debye coefficients for water [2].
The $\omega Y$ values are determined from the short-normalized measurements of 20° C water. Because the network analyzer measures reflection coefficients, this information was saved as n reflection coefficients and the n corresponding frequencies. The $\omega Y$ and $\omega \sqrt{e}$ are then computed as required. This individual, calculated from the short-normalized measurement of water, will be referred to as Adam. Adam will be used to create the remainder of the initial population used by the genetic algorithm. The process of creating the initial population from an original individual mimics the biblical creation of the Earth's initial population. The process used to create the additional individuals of the initial population is based on measurement error analysis as follows. This is in contrast to the random technique used in Chapter 7.

The magnitude of the error associated with measurements, corrected by a response calibration, can be calculated. If the uncorrected source match and directivity specifications of the measurement system are known, an estimate of the worst-case error in the reflection coefficient can calculated as

$$e = E_D + |\Gamma| E_s$$  \hspace{1cm} (8.1)

where $e$ is the worst-case error estimate, $E_D$ is the measurement system directivity, $E_s$ is the measurement system source match and $|\Gamma|$ is the magnitude of the reflection coefficient being measured. As you can see, the error is a function of the item being measured along with the instrument specifications. For low reflection measurements, the directivity dominates the error. For high-reflection measurements, both the directivity and source match contribute to the error. The specifications for uncorrected source match and directivity are given in the specifications for an instrument. The worst-case source match and directivity errors for the network analyzer used in this research were 20 and 28 dB, respectively. These are equivalent to 0.10 and 0.04 in linear terms. The maximum error occurs for a large reflection coefficient. The maximum reflection coefficient is one. For a reflection coefficient of one, the error would be 0.14.
If the response-corrected data are used as an estimate of the actual value of the measurement, the worst-case error for each of the n points of Adam can be calculated. For this example, an error estimate of 0.14 will be used. This means that the solution space is contained in an area defined by a circle centered at each of "Adam's" n points. Figure 8.1 illustrates the solution space shown in the reflection coefficient plane.

![Figure 8.1 Solution Space for model](image)

The radius of each circle is that of the worst-case error. The n data pairs of each of the additional individuals in the population are randomly selected from within these circles. It should be noted that any area that falls outside of the unit circle could be disregarded. The area outside this circle represents materials that supply energy. The mechanics of creating the initial population from
Adam is to add a random vector to each of his n reflection coefficient points that defines an individual. The magnitude of this vector randomly varies from zero to the maximum worst-case error. Because adding connectors and cables to the ports of a network analyzer worsens the overall source match and directivity, twice the worst-case error was used. The phase of the error varied randomly from zero to two \( \pi \) radians.

The method described above for creating the initial population selection has several advantages. The revised choice of creating an initial population reduces the solution space. Reducing the solution space gives evolution a head start, thus reducing the search time. Effectively, the same solution was found with the original random technique and the revised technique. The only difference is that the revised technique found the solution approximately one hundred times faster.

**Mating**

Another variation of the technique described in Chapter 7 is the process of mating. The selection criterion remains the same, but the production of the offspring is modified. The original process for mating consists of randomly selecting a positive integer, \( j \), less than \( n \), the number of data pairs. Using the first \( j \) data pairs from the father and the last \( n-j \) data pairs from the mother produces one of the offspring. Using the first \( j \) data pairs from the mother and the last \( n-j \) data pairs from the father produces the other offspring. In the modified process, two additional offspring are created. Taking a weighted average of each data pair creates the additional offspring. A random number \( r \), between zero and one, is selected. Adding \( r \) times each data pair of the father with \( (1-r) \) times the corresponding data pair of the mother forms the first offspring. Adding \( (1-r) \) times each data pair of the father with \( r \) times the corresponding data pair of the mother forms the second offspring. The fitness of all four offspring is computed. The two best-fit offspring are placed in the next generation. The other two are discarded.
original offspring creation provides implicit parallelism [82] in the search for the solution. This accelerates the initial search for the best individual. The additional technique of using weighted averages of the individual characteristics provides improvements, as the population becomes better fit and more uniform.

**Fitness**

Another recommended variation in probe modeling, as outlined in Chapter 7, is the fitness criterion. In the original work, the sum of the difference between the one-port calibration and the one-term calibration was used to calculate an individual's fitness. As the average fitness of the population improved, the fitness was calculated differently. The difference between the calculated value of permittivity and the known permittivity of materials was used to calculate fitness. Changing the fitness criteria as the population evolves is an adaptive technique. The abilities that made a cave man fit are certainly different from what makes modern man fit. An expansion of this concept is now suggested.

Four fitness criteria are suggested. One replaces another as the population evolves. The first fitness function is a comparison between the full one-port calibrated measurement and the short-based response calibration measurement. This is identical to the initial fitness criteria used in the original work. Once a highly fit individual evolves, the population and fitness function is changed. A new population is created from the best individual in the last generation created. This individual will be referred to as Noah. The new population is created in the same manner as the original, except that Noah is used instead of Adam. The random error vector added to each of the n points of Noah has maximum amplitude of 0.01. This is a reasonable expected error after searching with the original fitness function. The new fitness function is the difference between the error-corrected measurement of water, when a short-circuit, air and water are used to calibrate the system, and the error-corrected measurement of water when a short-circuit, air and
another material are used to calibrate the system. The permittivity of the other material must be known. If the permittivity of water, the permittivity of the other calibration material, and the model of the probe were known, the two corrected values of the water measurements would be identical. Note that this function provides a direct relationship between the individual's characteristics and the corrected water measurement. The reflection-coefficient values that define the model are a subset of the corrected water measurements. Minimizing this fitness function would provide the model if the permittivity of the two materials, water and the other calibration material, were known exactly. Since this is not true, additional fitness functions are applied to refine the selection criteria.

A new population is created as previously described. Once again, the best fit individual serves as Noah. The new fitness function is calculated as the difference in permittivity between the value computed from the water measurement and that calculated from the Debye equation for water. Once again, a material other than water is used to calibrate the system. When the evolution is terminated, the process is repeated with a fourth fitness function. Air, a short-circuit and water are used to calibrate the system. The final fitness function described in Chapter 7 is used. The fitness is now calculated as the difference between the measured and computed values, using a Debye model, of several known materials. This final step is required because the permittivity of water is not known exactly. It insures that correct results are obtained for a broad range of materials.
CHAPTER 9

PROBE MODELING EXAMPLE

In Chapter 8, several improvements were outlined for the technique suggested in Chapter 7. These improvements increased the search speed significantly. By incorporating these improvements, searches that originally took over five days took less than one day. This Chapter contains a complete example of modeling a probe based on the techniques described in Chapters 7 and 8. The probe being modeled to demonstrate the technique is the 50-GHz coaxial-line probe described in Chapter 7.

The first step in modeling a probe is to make measurements on a variety of materials. These measurements are performed without the use of error-correction. One of the measurements needs to be that of a short circuit. The technique requires that the permittivity of at least two of the materials measured be known as a function of frequency at the measurement temperature. In general, the more measurements the better and the more known materials the better. In this example all of the measurements were made at 20° C. The materials used for this example were a short-circuit, air, deionized water, methanol, ethanol, isopropanol and deionized water with table salt (NaCl) added. Figures 9.1 and 9.2 illustrate the uncorrected reflection coefficient measurements of the short-circuit and the deionized water. Each measurement consists of 201 measurement points logarithmically distributed between 200 MHz and 50 GHz. Plots of the other materials measured would look similar. Adam, the original individual, was created from six points selected from the water measurement normalized to the short-circuit. This is accomplished by dividing the water measurement by the short-circuit measurement. The
result is multiplied by the known reflection coefficient of the short-circuit, minus one. Figure 9.3 illustrates the normalization and the six points.

Figure 9.1 Uncorrected reflection coefficient measurements of the short-circuit

Figure 9.2 Uncorrected reflection coefficient measurements of the deionized water
As previously discussed, this is a one-term calibration that lumps the effects of imperfect
directivity, source match and tracking into one tracking term. Adding a random-error vector to
each of Adam's reflection coefficient values created the additional 4999 members of the initial
population. This process was discussed in Chapter 8.

The procedure outlined in Chapter 8 utilized four fitness functions. The plots in Figures
9.4-9.6 show the results after using the best-fit individual as defined by the first fitness function.
The short-circuit, air and deionized water were used to calibrate the system. These plots
compare the measured permittivity and the Debye-calculated permittivity for the materials. The
plots also indicate the average error between the measured and expected values. Plots for air and
deionized water were not included because they were used as calibration standards and would
show no error. Figures 9.7-9.9 show the results after using the best-fit individual and the second
fitness function. Note the improvements in the measured values (Figures 9.7-9.9) as compared to the results after evolving the initial population (Figures 9.4-9.6). Figures 9.10-9.12 illustrate additional improvements when the third fitness function is used. These measurements were made using a short-circuit, air and deionized water to calibrate the measurement. The fitness function used in the third evolution compares the measured and calculated values of water when the short-circuit, air and methanol were used to calibrate. This accounts for the very small average error in the methanol measurement. Figures 9.13-9.15 are based on use of the final fitness function. The total error is improved by use of this final fitness function. The errors for the isopropanol and ethanol were improved at the expense of the methanol. The errors observed are consistent with the uncertainty for the materials [83] and the uncertainty of the exact measurement temperature.

The measurement of the loss factor of the deionized water with table salt (NaCl) added is shown in Figure 9.16. Note that the loss factor is nearly linear for frequencies below 500 MHz when plotted in a log-log format. This is expected for a conducting liquid such as salt water and thus adds validity to the final model.
Figure 9.4 The dielectric constant and loss factor of isopropanol after the first evolution

Isopropanol
Average error = 15.40%

Isopropanol
Average error = 15.40%
Figure 9.5 The dielectric constant and loss factor of ethanol after the first evolution
Figure 9.6. The dielectric constant and loss factor of methanol after the first evolution
Figure 9.7. The dielectric constant and loss factor of isopropanol after the second evolution
Figure 9.8 The dielectric constant and loss factor of ethanol after the second evolution
Figure 9.9 The dielectric constant and loss factor of methanol after the second evolution
Figure 9.10 The dielectric constant and loss factor of isopropanol after the third evolution
Figure 9.11 The dielectric constant and loss factor of ethanol after the third evolution
Figure 9.12 The dielectric constant and loss factor of methanol after the third evolution
Figure 9.13 The dielectric constant and loss factor of isopropanol after the final evolution
Figure 9.14 The dielectric constant and loss factor of ethanol after the final evolution
Figure 9.15 The dielectric constant and loss factor of methanol after the final evolution
Figure 9.16 Measurement of the loss factor of deionized water with table salt added
CHAPTER 10

CONCLUSIONS

Several proposals, each based on the use of artificial intelligence techniques, have been presented that allow a permittivity measurement probe to be modeled. These include a fuzzy logic inference system (FIS), an artificial neural network (ANN), and a technique based on the results of a dimensional analysis aided by a genetic algorithm (DAGA). All of these techniques eliminate the need for performing an electromagnetic analysis, based on Maxwell's equations, to determine the relationship, or model, between a measurement made on the probe and the measurement results. This was the goal of this research. An advantage of the DAGA technique is that it models the admittance at the probe tip to the permittivity of the material being measured at a given frequency. This allows a full one-port calibration to be performed. The advantage is that this relationship is independent of the particular instrument being used to make the measurement and the method used to connect the probe to the instrument. As long as the geometry of the actual probe tip is identical to the probe modeled, the model is valid. This is in contrast to the FIS and ANA techniques considered for which the models yielded by these two techniques are valid only for the probe/instrument system used to train these solutions. If a different instrument or probe is substituted, the models will no longer be valid. There are two reasons for this. The first reason is that every instrument has its own unique error coefficients. If an instrument different from the one that was used to train the FIS or ANN is used, the model would not produce the desired results. This problem could potentially be remedied by performing a full one-port calibration at the end of the cable or connector used to connect the instrument to the probe. This would essentially remove the effect of using different instruments.
on the model. The model, however, would have to be established using error corrected measurements.

The second reason is that the model is only valid for the probe used to train the FIS or ANN. If there is any difference between the probe being used and the probe used during training, the measurement results will be in error. Manufacturing identical probes at microwave frequencies is difficult. Phase differences caused by slight variations in length or dielectric constants of the material used to manufacture the probe are possible. Even the slight difference in attaching the connector to the probe can cause enough difference in the measurement made on the probe to cause a problem.

The DAGA probe-modeling technique avoids these problems by establishing the relationship between the measurement at the tip of the probe and the material that it contacts. This makes the model independent of the differences in measurement configurations behind the probe tip. Differences in instrumentation, cables, connectors, probe length and manufacturing are compensated by the full one-port calibration. As long as the tip of the probes is identical, the model is valid. This approach could have been used in the ANN and FIS techniques. These techniques could have been used to establish the relationship between the probe tip and the material as was done in the DAGA technique. The disadvantage is that the training would have become almost impossibly complex. Instead of mapping known results to uncorrected measurements, mapping known results to one-port error-corrected measurements would be required. The corrected measurements can not be calculated without knowledge of the mapping or model. Because of this, the DAGA technique is considered superior.

The limitation of the technique is that it depends on the use of materials with known permittivities. The DAGA technique requires knowledge of the permittivity of at least two materials. Other materials are required if the model is to be verified. The model developed in
Chapter 10 for the 50-GHz coaxial-line permittivity measurement probe gives accurate results when compared to published values for the materials considered [83]. These published values consisted of Debye coefficients derived by curve fitting DC to 5 GHz measurement data. Capacitance and transmission-line techniques were employed to obtain the measurement data. The technique could be improvement if the data used in the curve fitting included measurements up to 50 GHz. Such data are not currently available [83] and represent a need in establishing verification materials at frequencies above 5 GHz. Permittivity data for microwave and millimeter wave verification materials are also required for verification of permittivity measurement probes whose models are based on an electromagnetic analysis.

The dimensional analysis of a generalized permittivity measurement probe and the application of the Buckingham PI theory, developed in Chapter 6, are a major contribution in the modeling of permittivity measurement probes and sensors. It showed that any probe or sensor for which admittance is a function of measurement frequency, any number of material properties and any number of linear dimensions can be represented by the relationship

$$\omega Y = F\left(\omega \sqrt{\varepsilon}\right)$$  \hspace{1cm} (10.1)

where \(\omega\) is the angular measurement frequency, \(Y\) is the measured admittance, \(\varepsilon\) is the permittivity of the material being measured and \(F\) is an unknown function. This is valid as long as the material being measured is nonmagnetic and all the other parameters remain constant. This is true for all probes and sensors designed to measure nonmagnetic materials. The implication of (10.1) is that it reduces the number of variables to two, \(\omega Y\) and \(\omega \sqrt{\varepsilon}\). The modeling problem is reduced to determining the function \(F\). In this work, a polynomial was used to represent \(F\), and a genetic algorithm was used to determine the polynomial coefficients. If a calibrated measurement can be made on a probe or sensor, without knowledge of \(F\),
measurements of known materials can be made, and traditional curve fitting techniques can be used to approximate this function.
CHAPTER 11

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APPENDIX A

AUTHOR'S PUBLISHED PAPERS THAT RELATE TO THIS RESEARCH

Journal Articles


Abstracts, Workshops and Conference Proceedings


### APPENDIX B

## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>E</td>
<td>electric field intensity</td>
</tr>
<tr>
<td>D</td>
<td>electric flux density</td>
</tr>
<tr>
<td>H</td>
<td>magnetic field intensity</td>
</tr>
<tr>
<td>B</td>
<td>magnetic flux density</td>
</tr>
<tr>
<td>J</td>
<td>current density</td>
</tr>
<tr>
<td>ρ</td>
<td>charge density</td>
</tr>
<tr>
<td>P</td>
<td>electric dipole moment</td>
</tr>
<tr>
<td>M</td>
<td>magnetic dipole moment</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>j</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>f</td>
<td>frequency (Hz)</td>
</tr>
<tr>
<td>ω</td>
<td>angular frequency = $2\pi f$</td>
</tr>
<tr>
<td>ε</td>
<td>permittivity $\varepsilon = \varepsilon' - j \varepsilon''$</td>
</tr>
<tr>
<td>ε₀</td>
<td>free-space permittivity = $8.854 \times 10^{-12}$ F m$^{-1}$</td>
</tr>
<tr>
<td>εᵣ</td>
<td>relative permittivity $\varepsilon_r = \varepsilon / \varepsilon_0$</td>
</tr>
<tr>
<td>ε'</td>
<td>real part of permittivity</td>
</tr>
<tr>
<td>ε''</td>
<td>imaginary part of permittivity</td>
</tr>
<tr>
<td>εᵣ'</td>
<td>dielectric constant</td>
</tr>
<tr>
<td>εᵣ''</td>
<td>loss factor</td>
</tr>
<tr>
<td>μ</td>
<td>permeability $\mu = \mu' - j \mu''$</td>
</tr>
<tr>
<td>μ₀</td>
<td>free-space permeability = $4\pi \times 10^{-7}$ H M$^{-1}$</td>
</tr>
</tbody>
</table>
relative permeability $\mu_r = \frac{\mu}{\mu_0}$

real part of permeability $\mu'$

imaginary part of permeability $\mu''$

c free-space propagation velocity $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299892458 \text{ m s}^{-1}$

propagation velocity $v = \frac{1}{\sqrt{\mu \varepsilon}}$

wave vector $|k| = \frac{\omega}{v}$

cyindrical coordinates $r, \phi, z$

wave number in region $i = |k(1)|$

cutoff wave number $k_c$

wavelength $\lambda$

free space wavelength $\lambda_0$

static permittivity $\varepsilon_0$

optical permittivity $\varepsilon_\infty$

relaxation constant $\tau$

impedance $Z$

characteristic impedance $Z_0$

normalized impedance $Z_N$

radiated power $P_r$

reflection coefficient $\Gamma$

transmission coefficient $T$

admittance $Y$

characteristic admittance $Y_0$
$S_{ij}$  scattering parameter at $i^{th}$ port due to source at $j^{th}$ port

$S_i$  second sine-integral $\int_{0}^{\infty} \frac{\sin(u)}{u} du$

$J_n(x)$  Bessel function of first kind of order $n$

$H_n^{(1)}(x)$  Hankel function of first kind of order $n$

$H_n^{(2)}(x)$  Hankel function of second kind of order $n$

$E_D$  directivity

$E_{RT}$  reflection tracking

$E_S$  source match